Statistics & Probability in Mechanical Design

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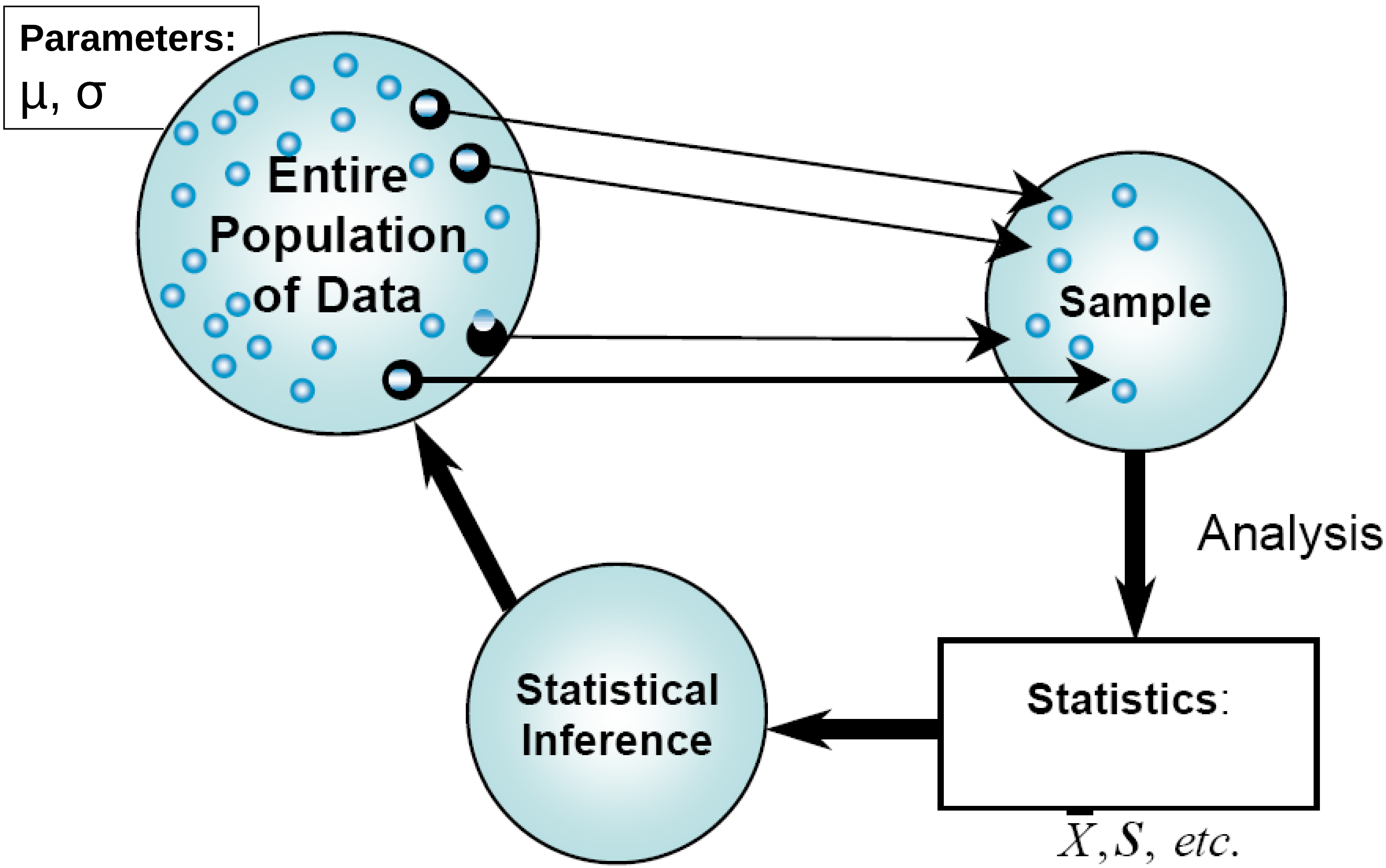
Monday, March 12th, 2012

Why Statistical Methods

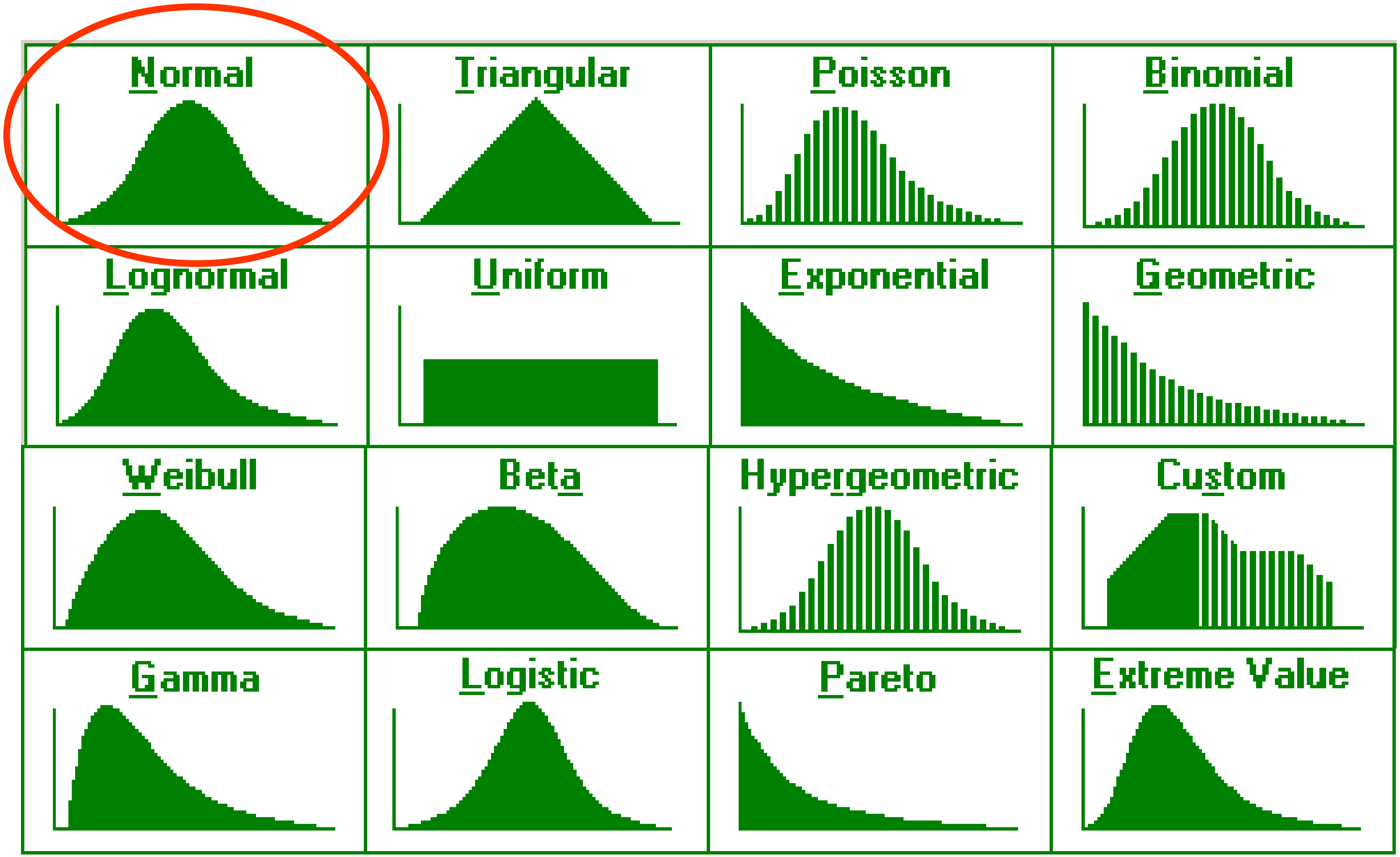
- Disciplined Approach
- Repeatable Results
- Quantifiable Decision Criteria
- Optimization

Statistics Review

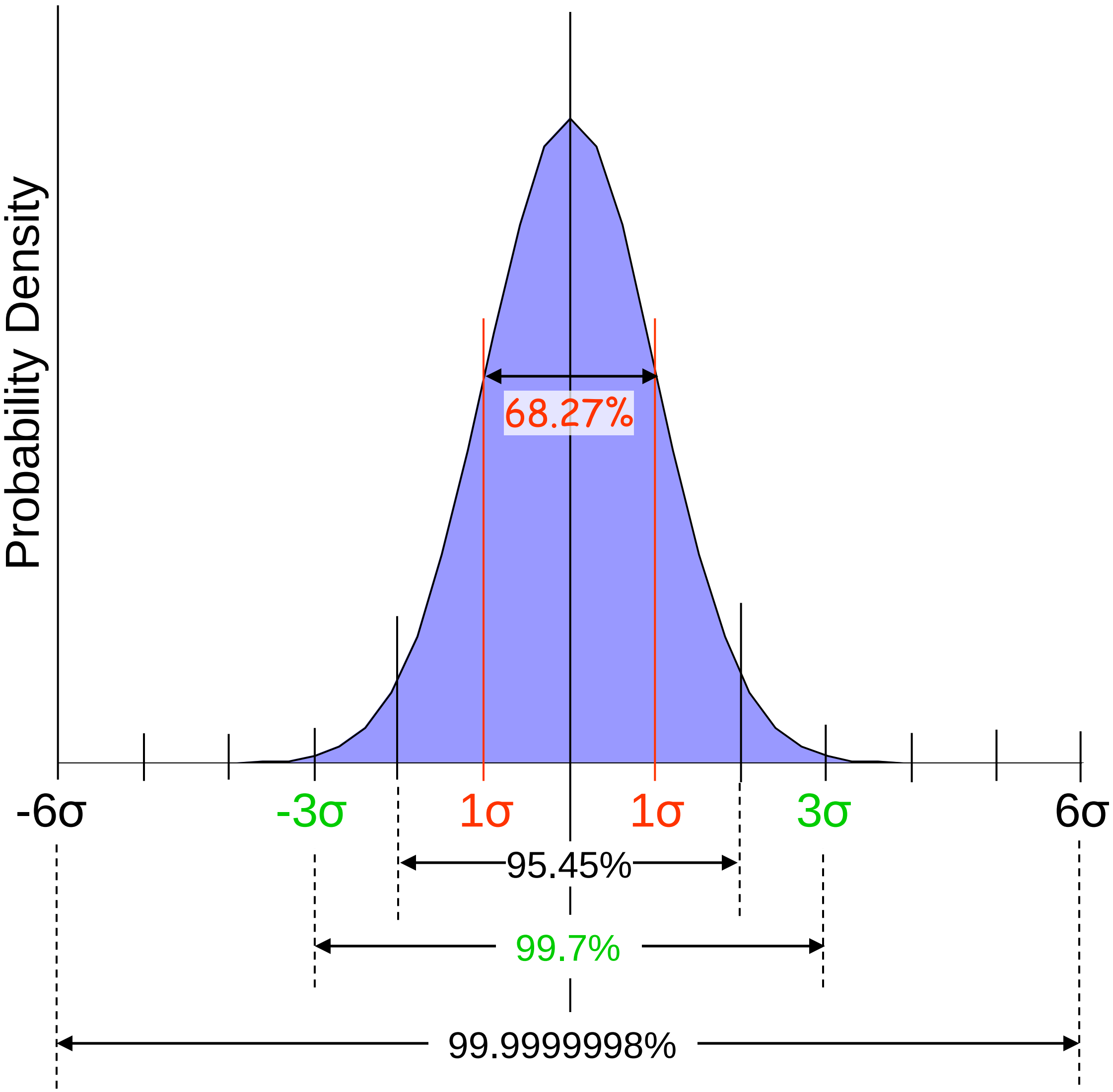
Population Sampling



Probability Distributions



Normal Distribution



$$f(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma(2\pi)^{1/2}}$$

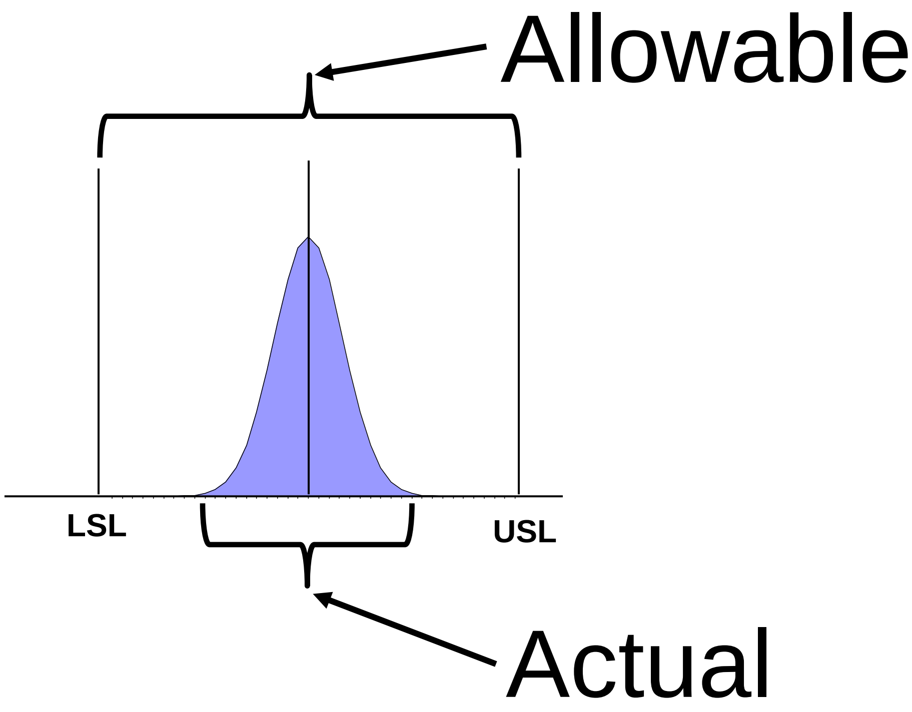
μ = mean

σ = standard deviation

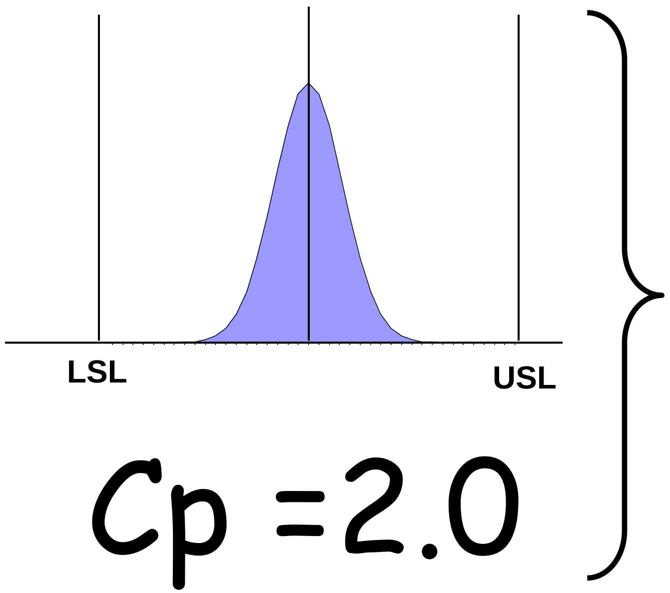
Process Capability

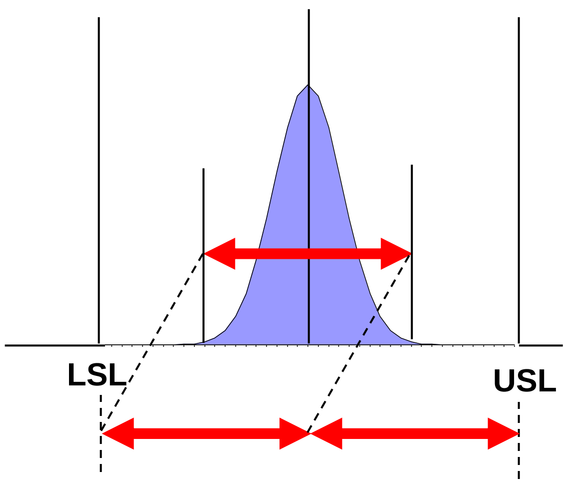
Normal Distribution

$$Cp = \frac{\text{Allowable Variation}}{\text{Actual Variation}}$$



Process Capability

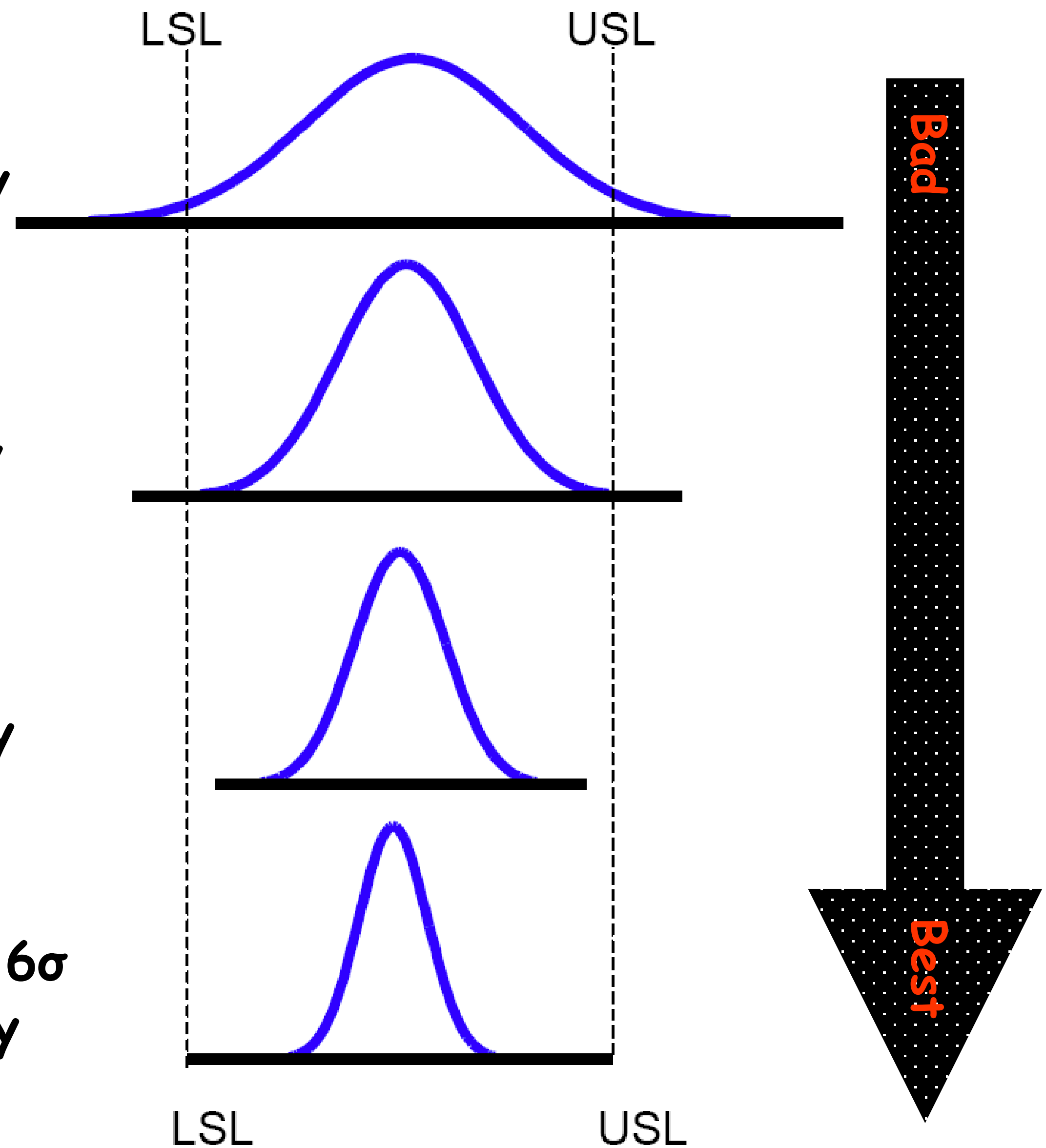




2.0 distributions can fit within the specification

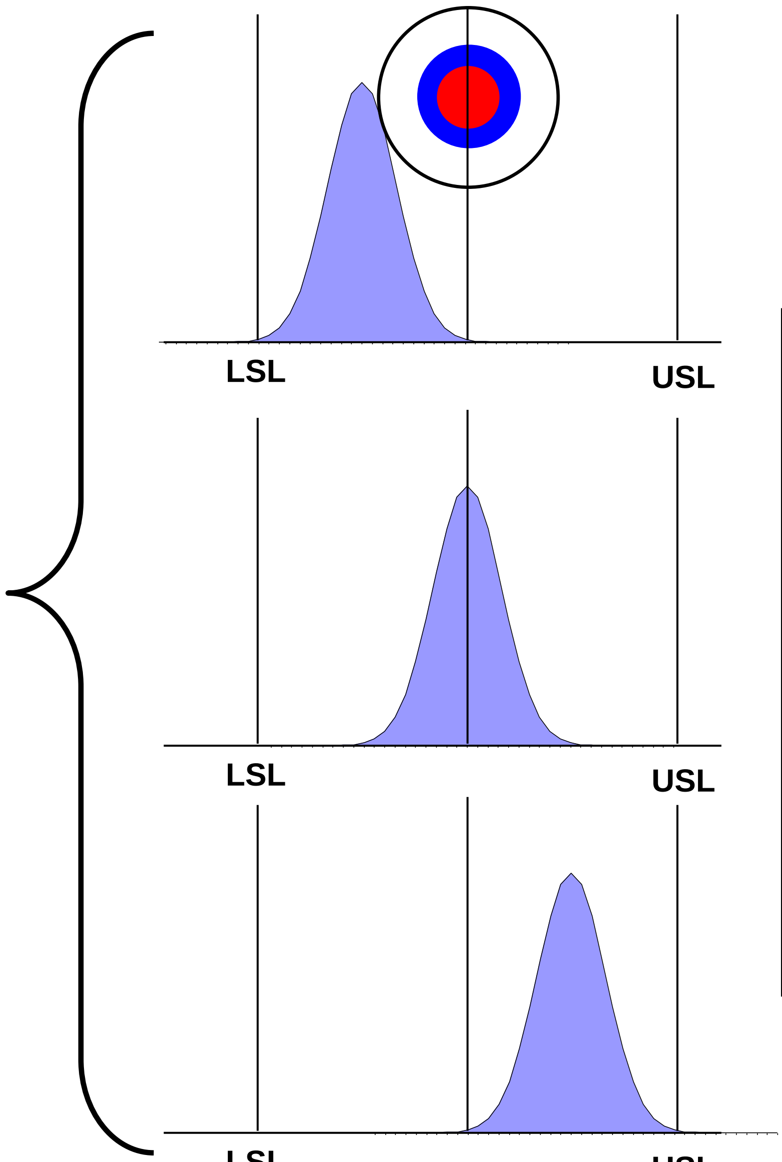
Process Capability

< 1.0	Poor Capability
1.0 - 1.5	Marginal Capability
> 1.5	Good Capability
> 2.0	Motorola 6σ Capability



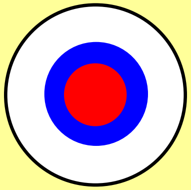
Process Capability

Cp = 2.0



Cp

Independent of the

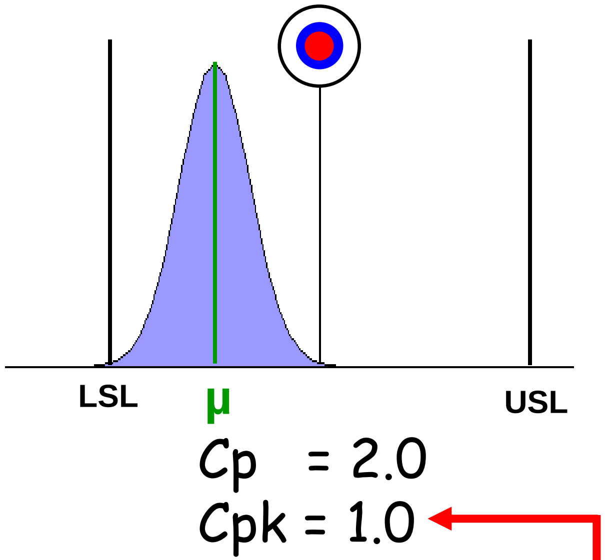


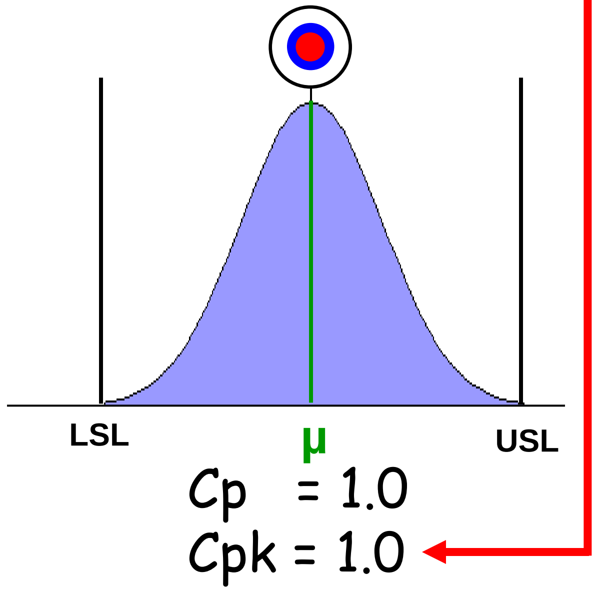
target.

Process Capability; Cp & Cpk

$$Cp = \frac{USL - LSL}{6\sigma}$$

$$Cpk = \min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right]$$



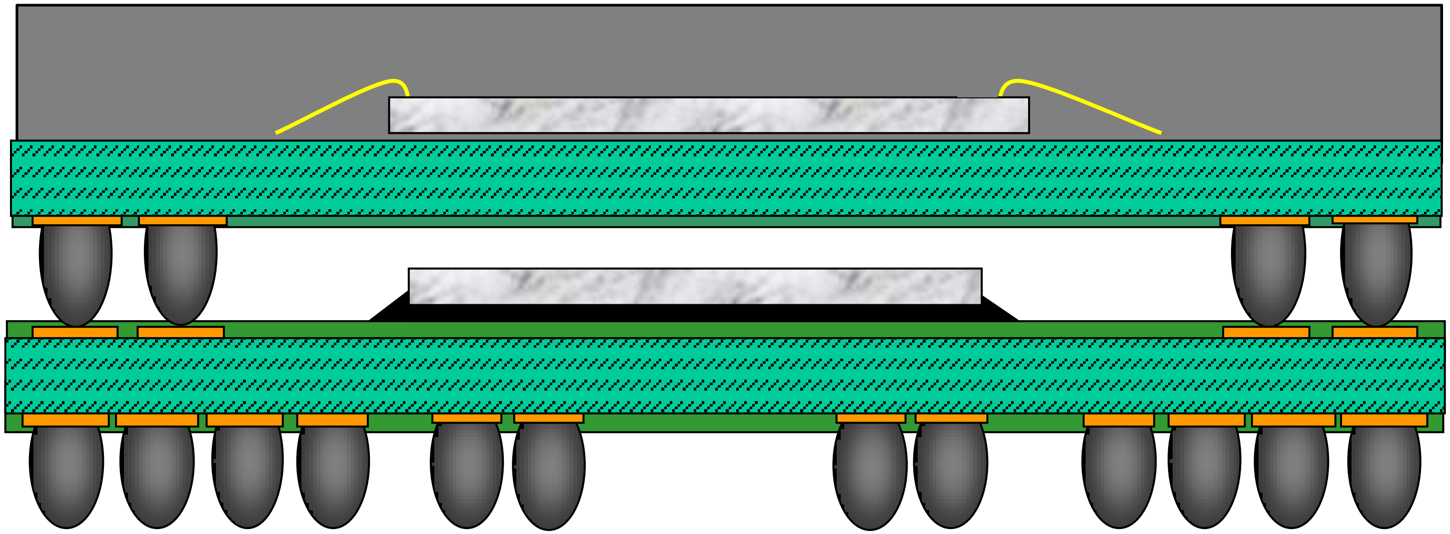


Cp, Cpk Example

Chip set Stack Up


THINNER

Chipset Stack Up

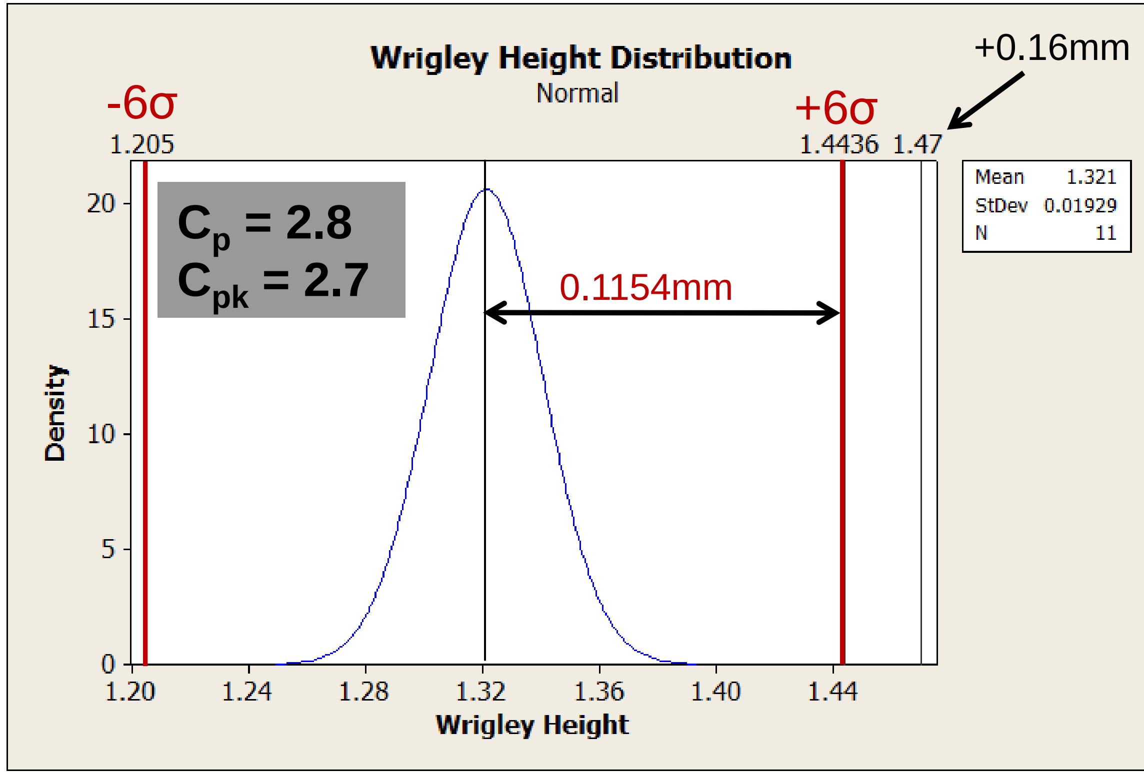


Package Height Prediction (post-reflow)	W3G (current)			
	Min.	Nom.	Max.	Tol.
Memory Mold Cap (2-die)	0.330	0.340	0.350	0.010
Memory Substrate Thk.	0.110	0.140	0.170	0.030
Memory Ball Ht. **(after reflow)**	0.140	**0.190**	0.240	0.050
OMAP Mold Cap (Ref.)	0.000	0.000	0.000	0.000
OMAP / W3G Die Thickness (um)	0.095	0.100	0.105	0.005
Die-to-Substrate Gap (um)	0.020	0.025	0.030	0.005
OMAP Top Ball Ht.	0.000	0.000	0.000	0.000
OMAP/W3G Substrate Thk. (w/o SM)	0.408	0.448	0.488	0.040
OMAP/W3G Ball Ht. **(after reflow)**	0.168	0.206	0.244	0.038
OMAP/W3G Ball Ht. **(before reflow)**	0.180	0.230	0.280	0.050
Total 2-Package POP Height (post-reflow):	1.24	1.32	**1.40**	
Max. 2-Package POP Height (post-reflow):		1.40	RSS:	0.081

Chipset Height Cp, Cpk

Current Stack Up	
Wrigley	1.31
Gap	0.16
Shield	0.13
Total	**1.60**

Proposed Stack Up	
Wrigley	1.31
Gap	0.12
Shield	0.10
Total	**1.53**



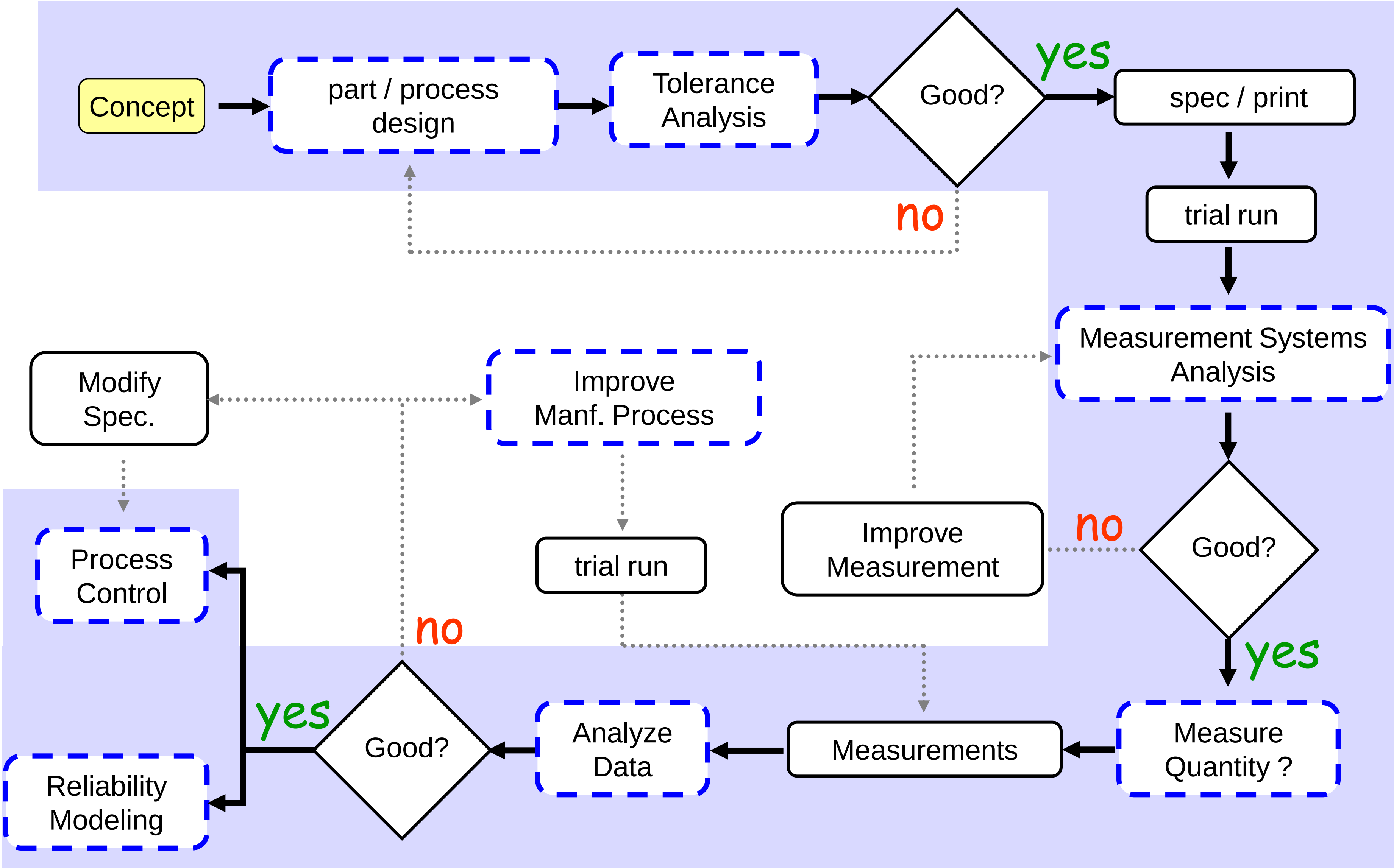
Etna

Targa

Program	Supplier	Part_Num	Point_1(mm)	Point_2(mm)	Point_3(mm)	Point_4(mm)
Etna	hynix - 031A	1	1.318	1.325	1.306	1.298
Etna	hynix - 920A	2	1.319	1.321	1.304	1.294
Etna	hynix - 920A	3	1.293	1.325	1.292	1.300
Etna	hynix - 920A	6	1.294	1.305	1.279	1.361
Etna	hynix - 046A	7	1.307	1.337	1.316	1.315
Etna	hynix - 046A	8	1.365	1.368	1.358	1.347
Targa	hynix - LV57908	A2-1	1.348	1.345	1.347	1.357
Targa	hynix - LV57908	A5-2	1.330	1.340	1.334	1.338
Targa	hynix - LV57908	B3-3	1.287	1.315	1.292	1.297
Targa	hynix - LV57908	B6-4	1.319	1.330	1.318	1.322
Targa	hynix - LV57908	C1-5	1.321	1.323	1.312	1.318

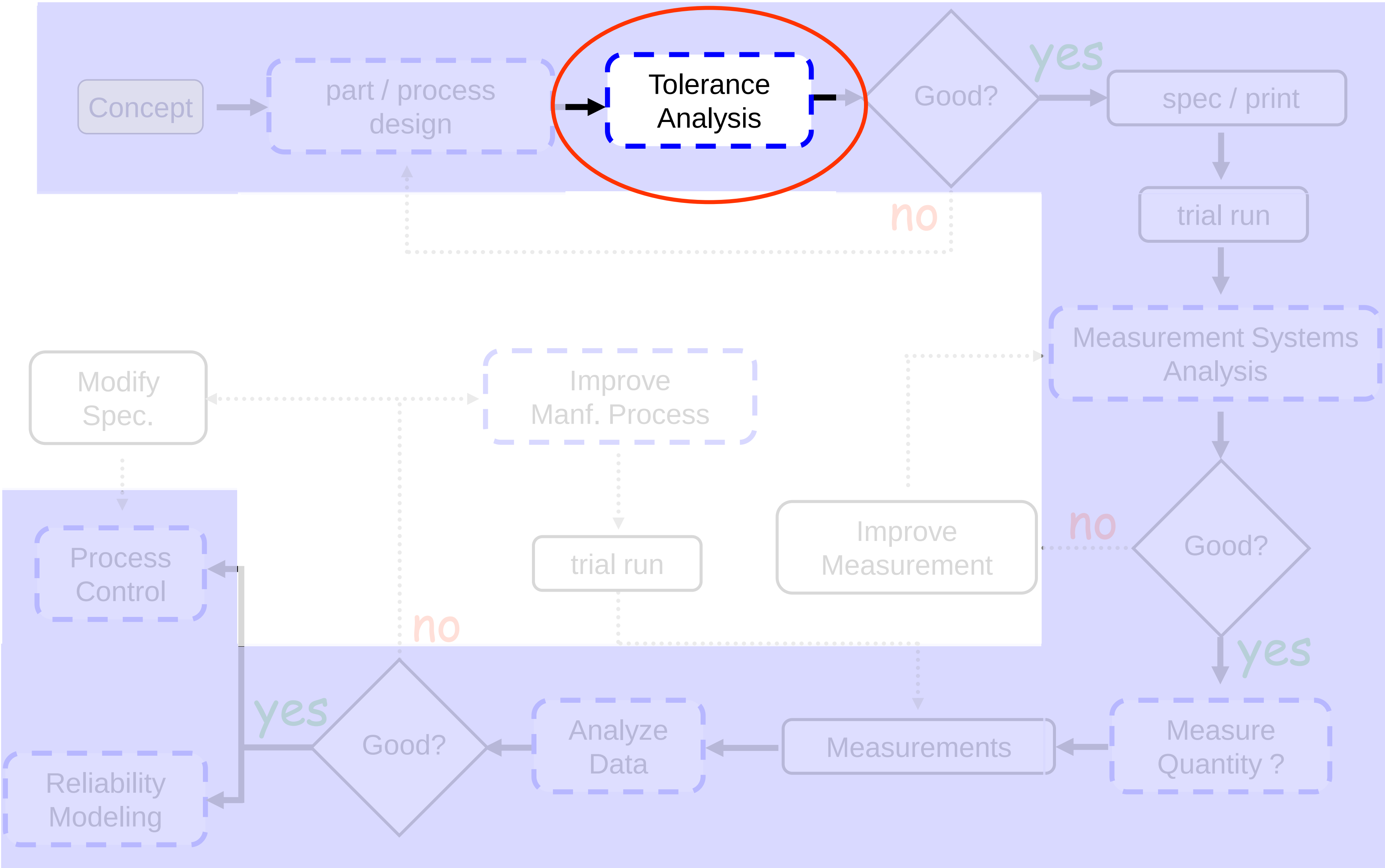
Mechanical Development Process

Development Flow (simplified)

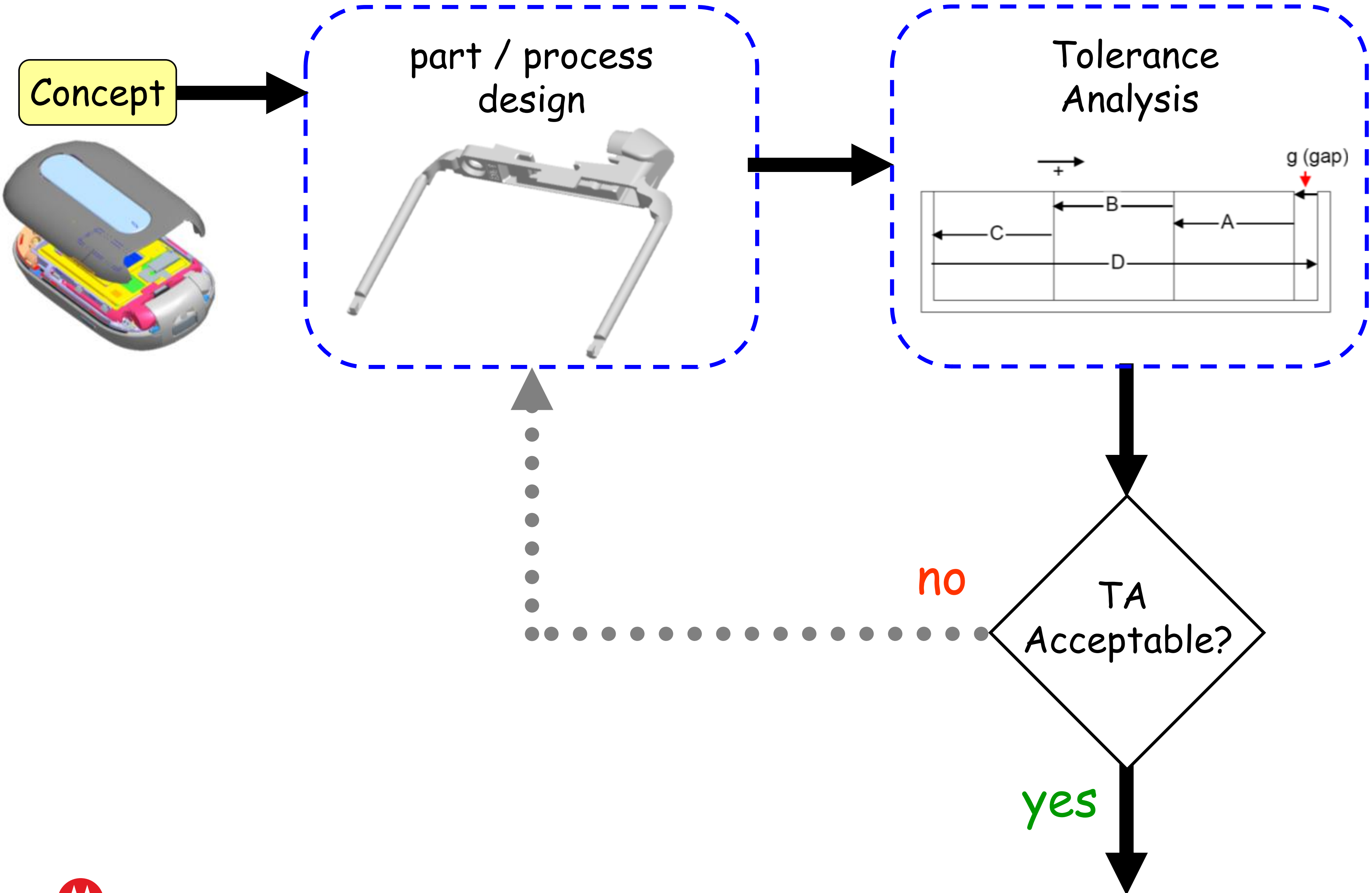


Tolerance Analysis

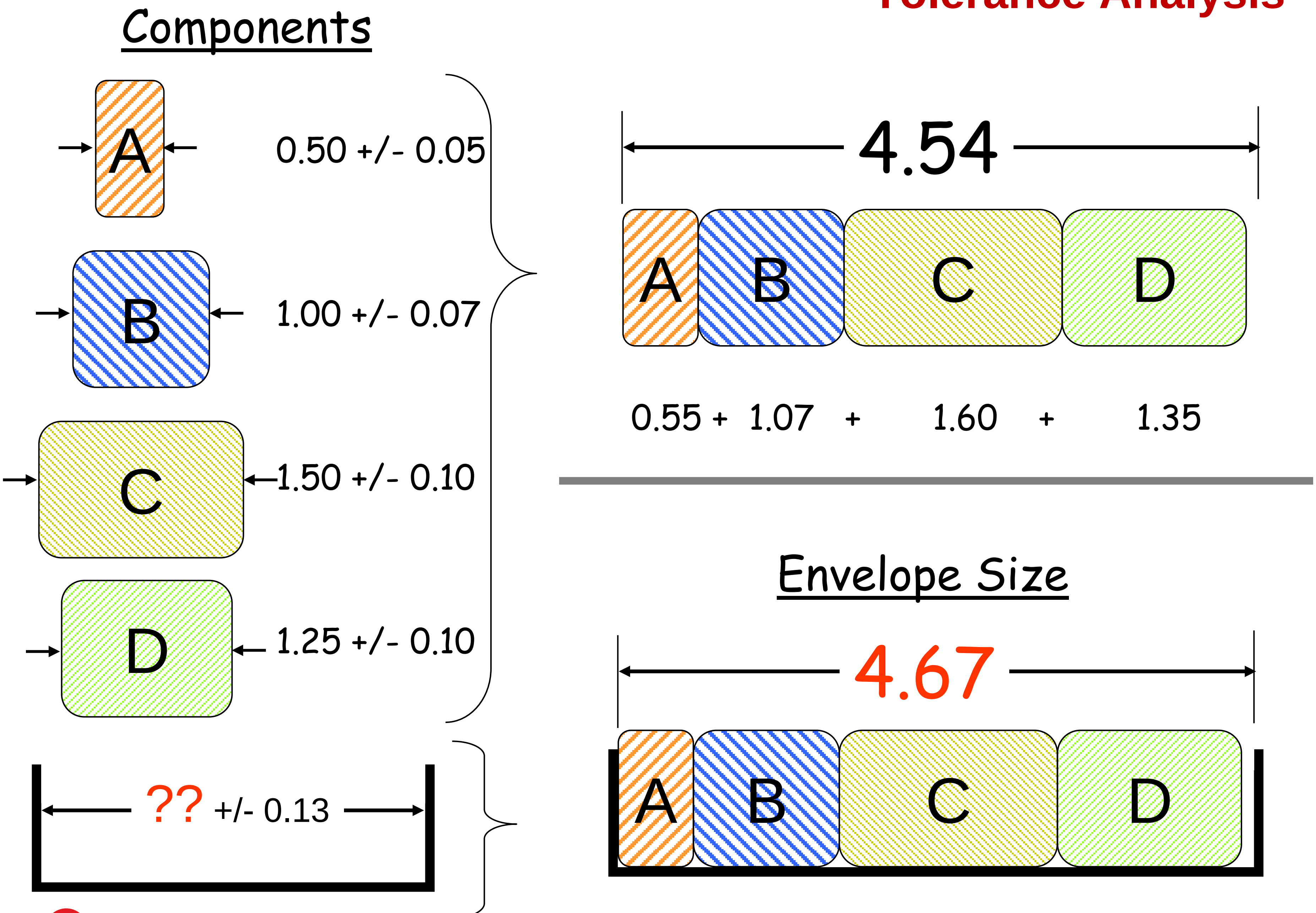
Development Flow (simplified)



Development Flow (simplified)



Tolerance Analysis



Root Sum Squared (RSS)

Variances can be added……

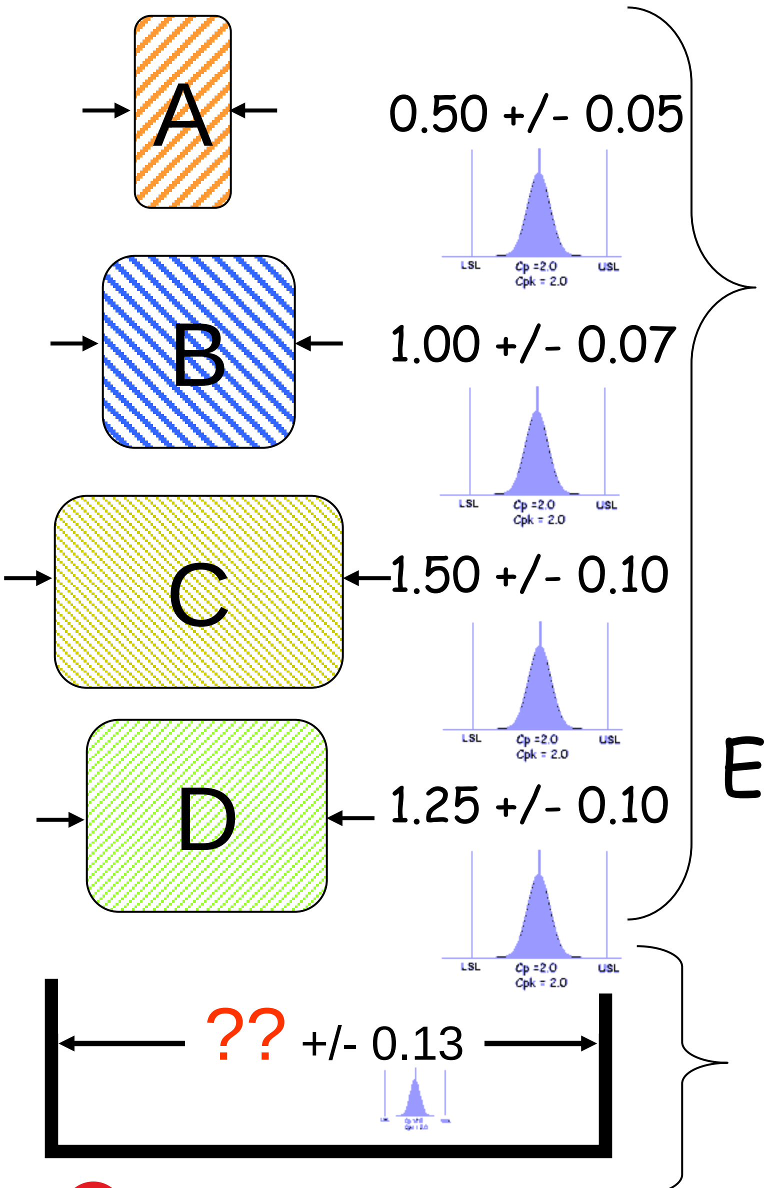
$$\sigma^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2 + \sigma_{Envelope}^2$$

$$\sigma_{gap} = \sqrt{\left(\frac{T_e}{3C_p}\right)^2 + \sum_{i=1}^{m}\left(\frac{T_{pi}}{3C_{pi}}\right)^2}$$

$$\sigma_{gap} = 0.035$$

Root Sum Squared (RSS)

Components



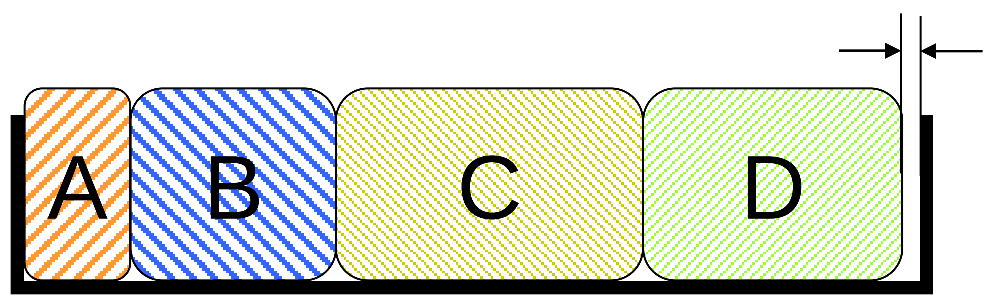
Gap Size

$$\sigma_{gap} = 0.035$$

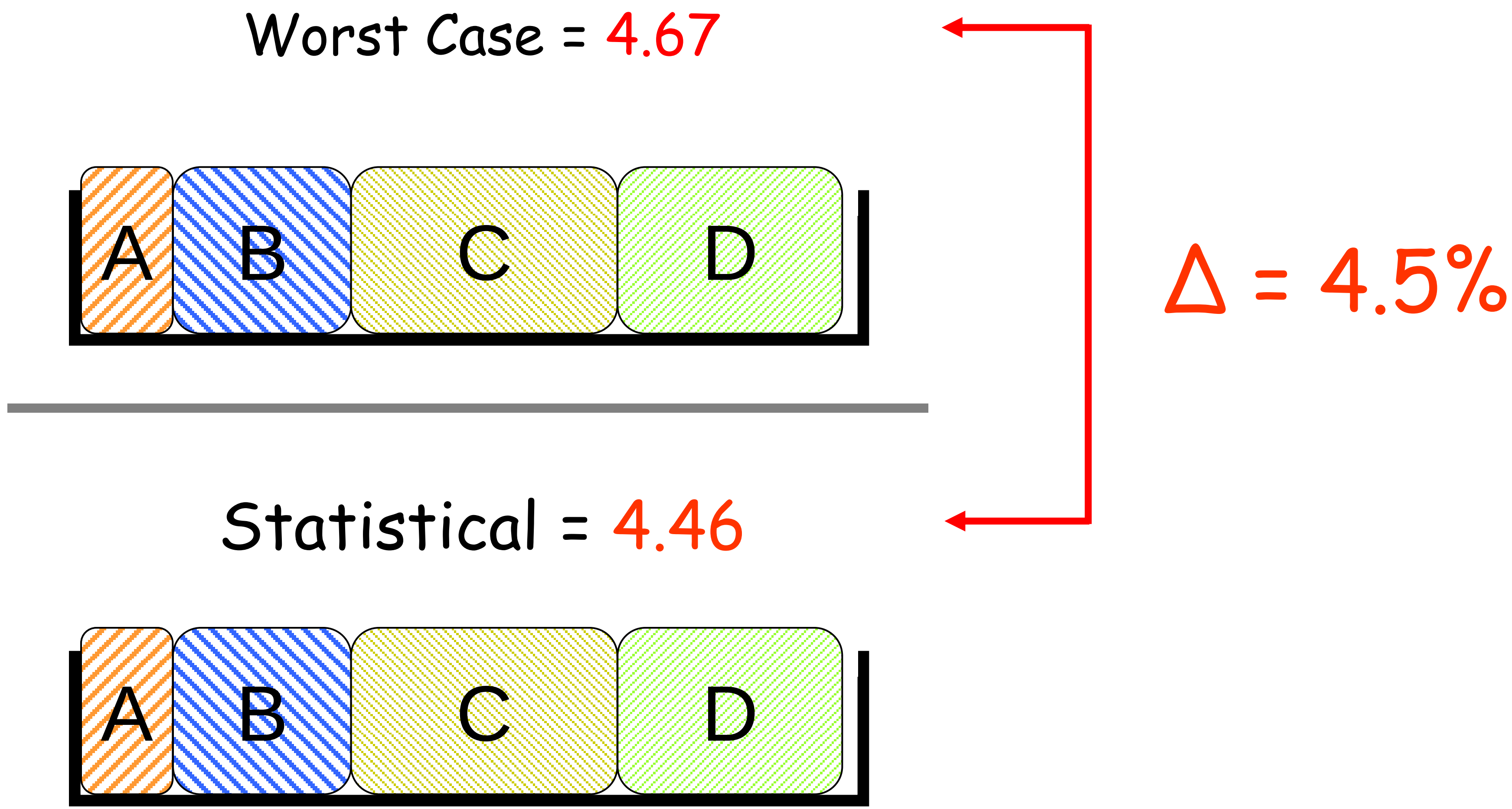
$$3\sigma_{gap} = 0.105$$

$$6\sigma_{gap} = 0.210$$

$$\text{Envelope} = A+B+C+D+6\sigma_{gap} = 4.46$$



RSS vs. Worst Case

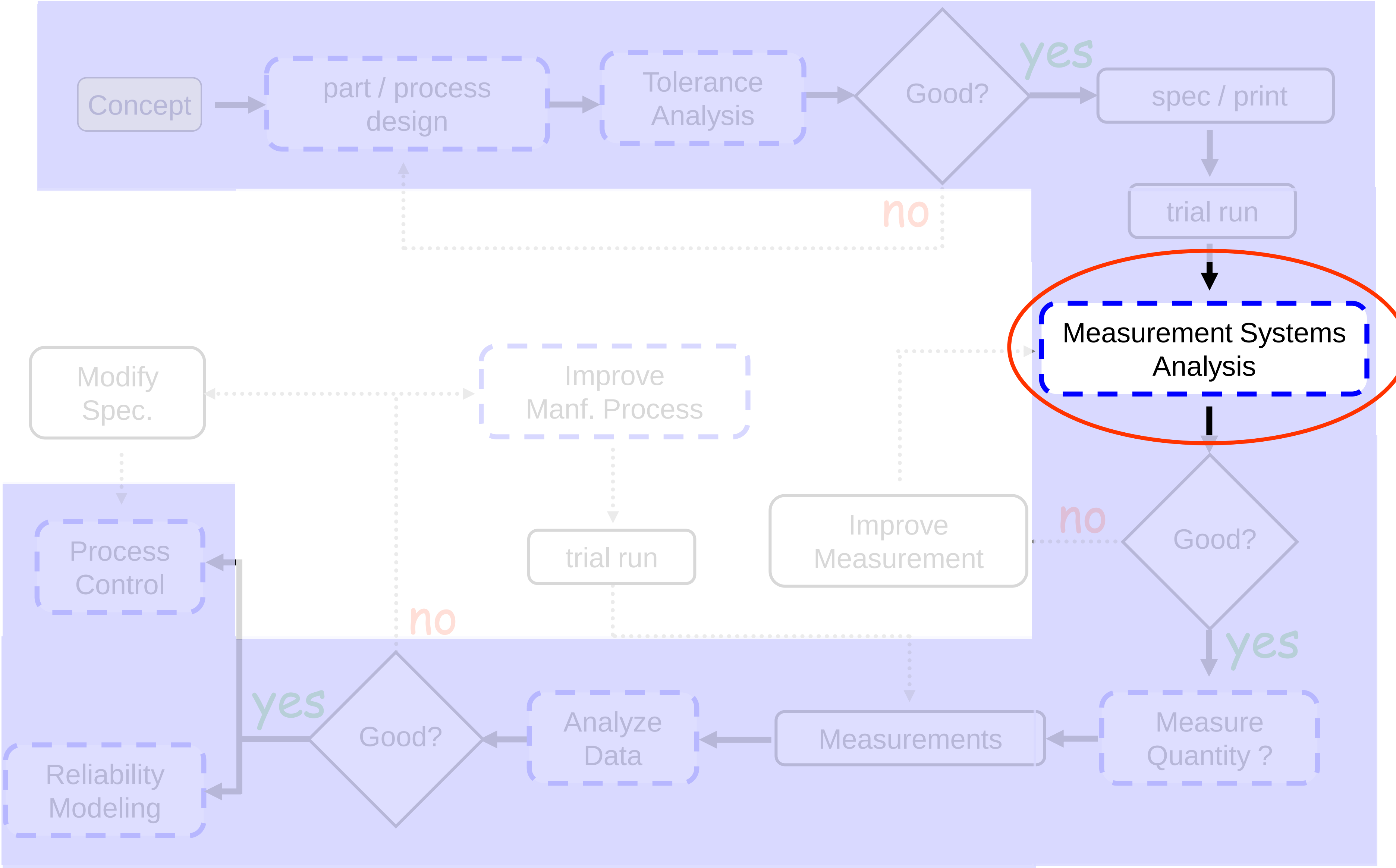


TA Example:

uUSB and HDMI Connectors

Measurement Systems Analysis (MSA)

Development Flow (simplified)

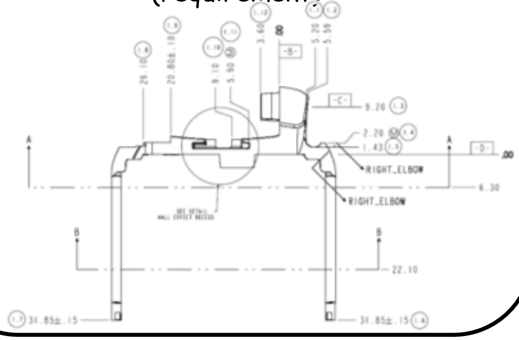


Development Flow (simplified)

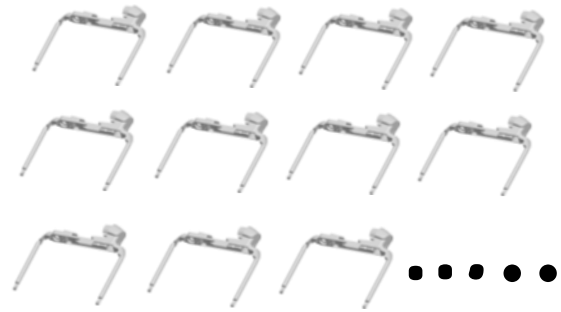
Good TA

spec / print

(requirement)



trial run



Measurement Systems Analysis

WHY?

Measurement Error → Bad Decisions

A **bad** unit might test "good"

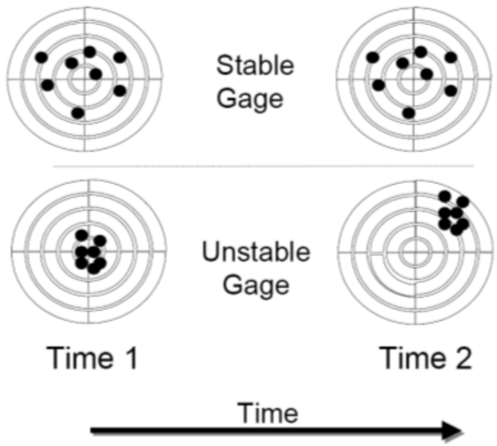
A **good** unit might test "bad"

LSL USL

Measurement Systems Analysis

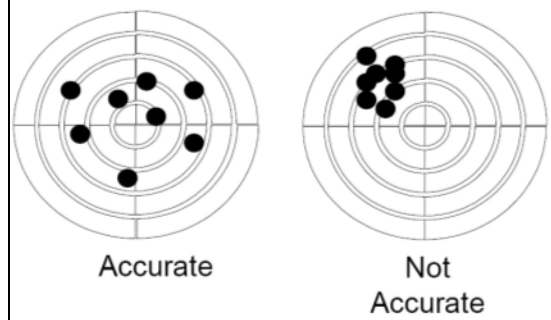
Characteristics:

Stability

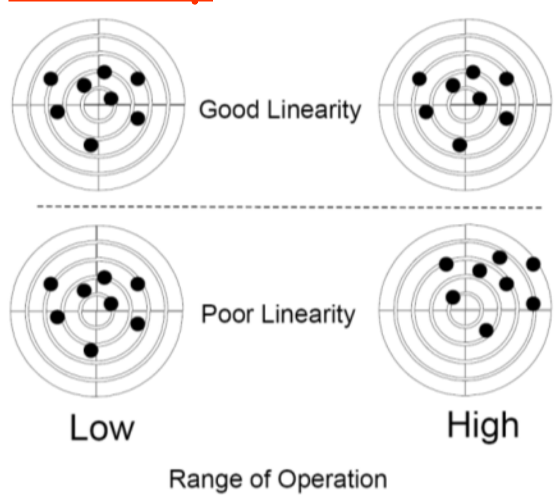


Discrimination

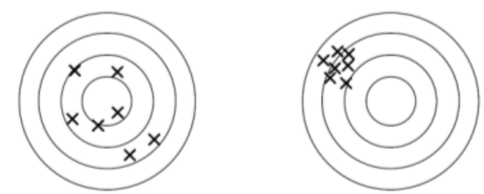
Accuracy (Bias)



Linearity

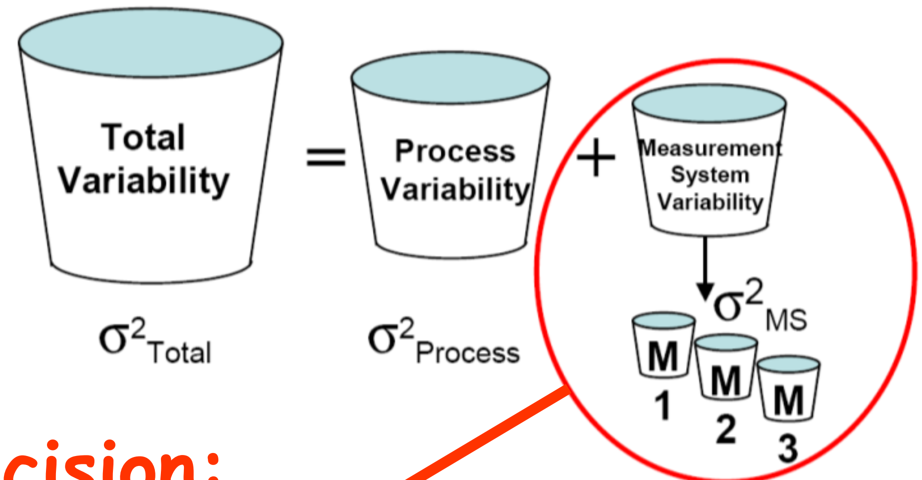


Precision

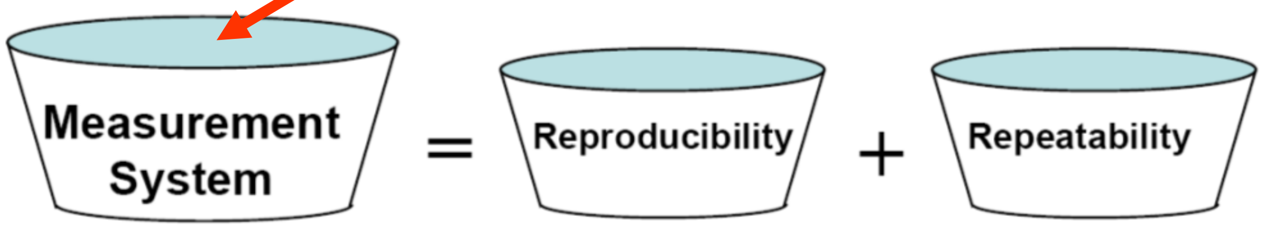


Measurement Systems Analysis

Total Variation:



Precision:



$$\sigma^2_{MS} = \sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Repeatability}}$$

$$\sigma_{MS} = \sqrt{\sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Repeatability}}}$$

MSA;   %GR & R

% GR&R	
<10%	Acceptable
10% - 30%	Ok; non-critical measurements
>30%	Unacceptable

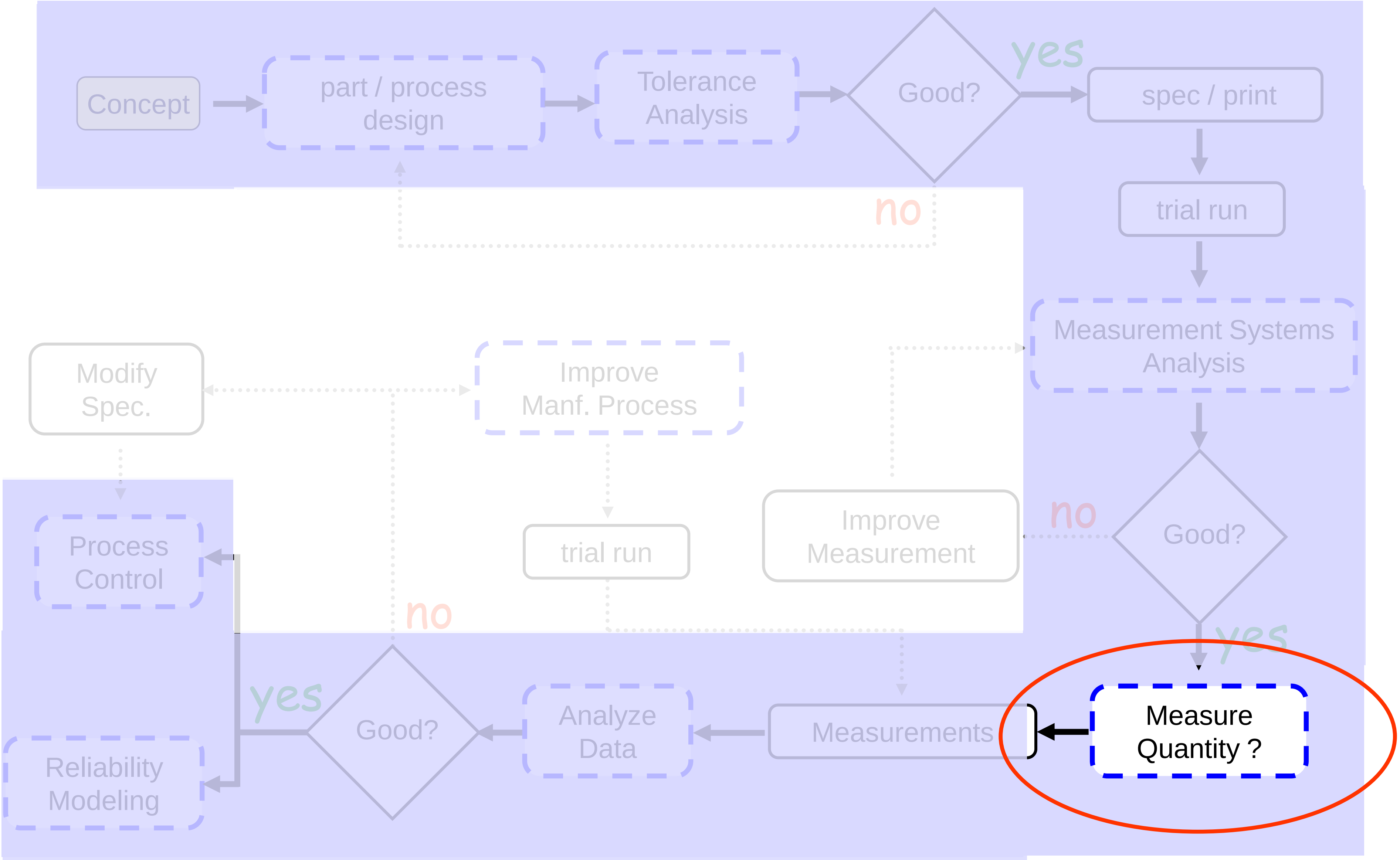
Measurement Systems Analysis

Example:

Glue Weight

Measurement Size

Development Flow (simplified)



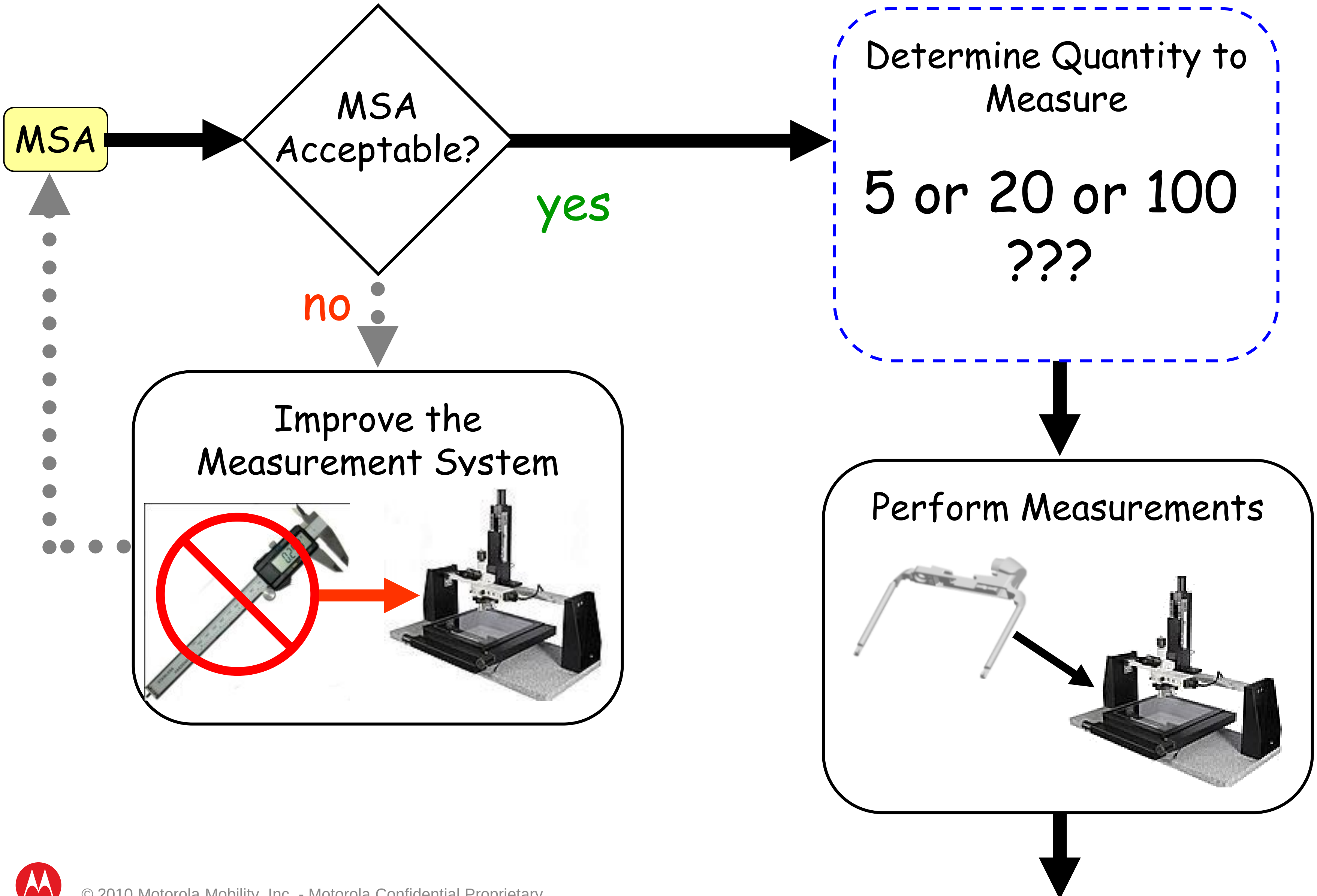
Sample Size

- Dependent on the type of analysis to be performed
- Apply applicable formula
- Example: Sample Mean to a known population

$$n = \frac{(Z_\alpha + Z_\beta) \bullet \sigma^2}{\delta^2}$$

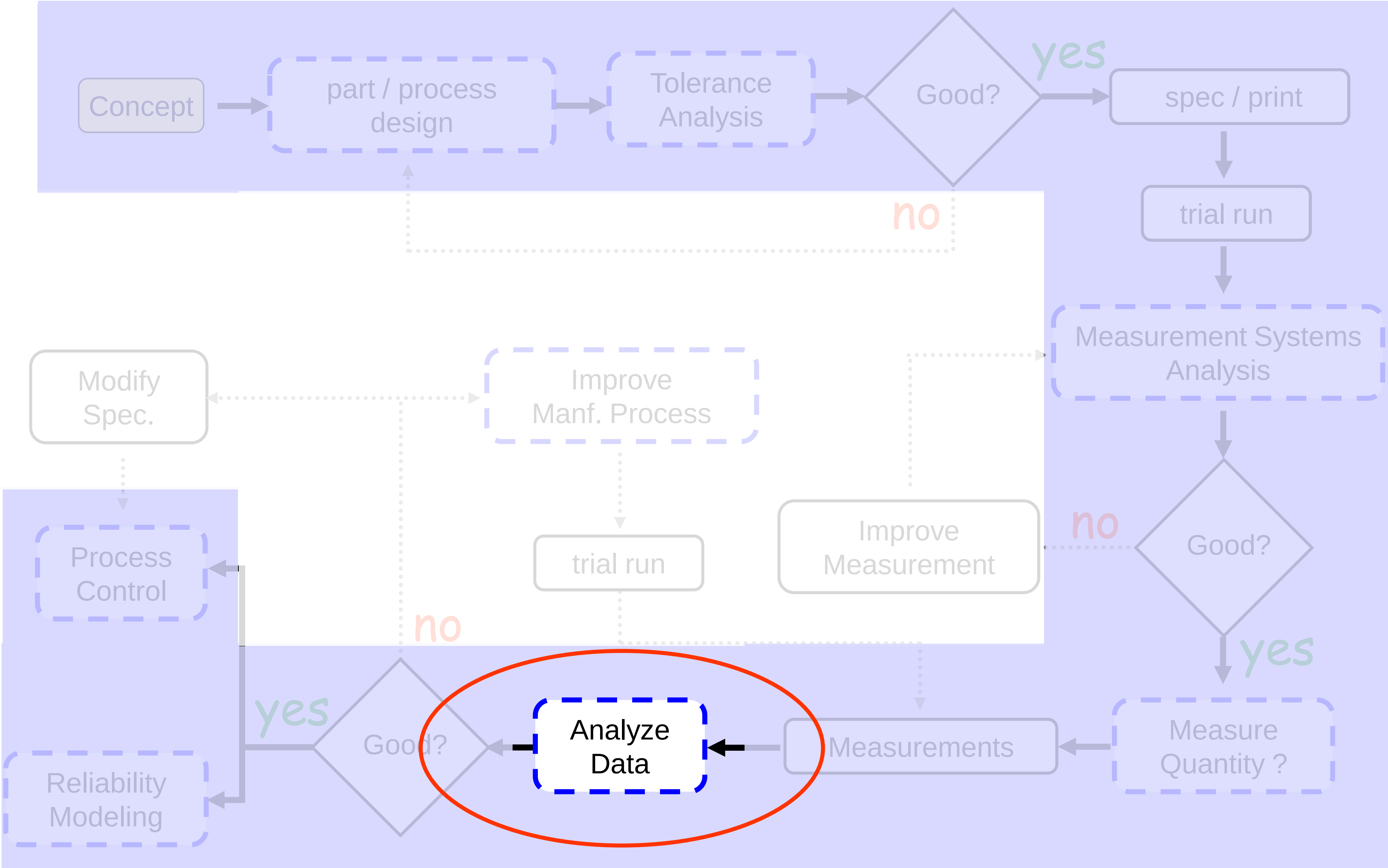
α = level of acceptability of a false positive (0.05 is typical)
β = level of acceptability of a false negative (0.10 is typical)
σ = known standard deviation
m = amount of difference that matters (practical difference)

Development Flow (simplified)

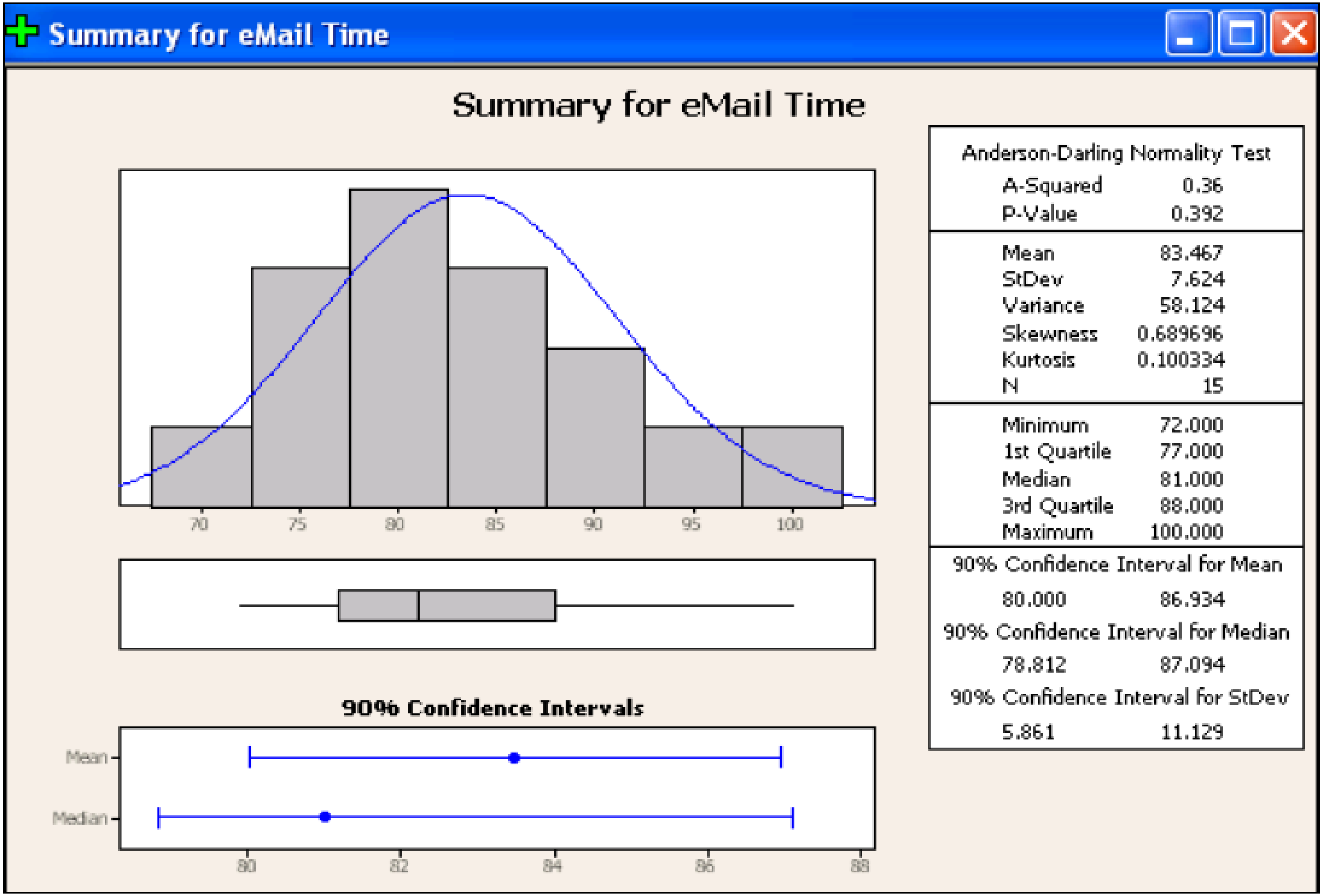


Data Analysis

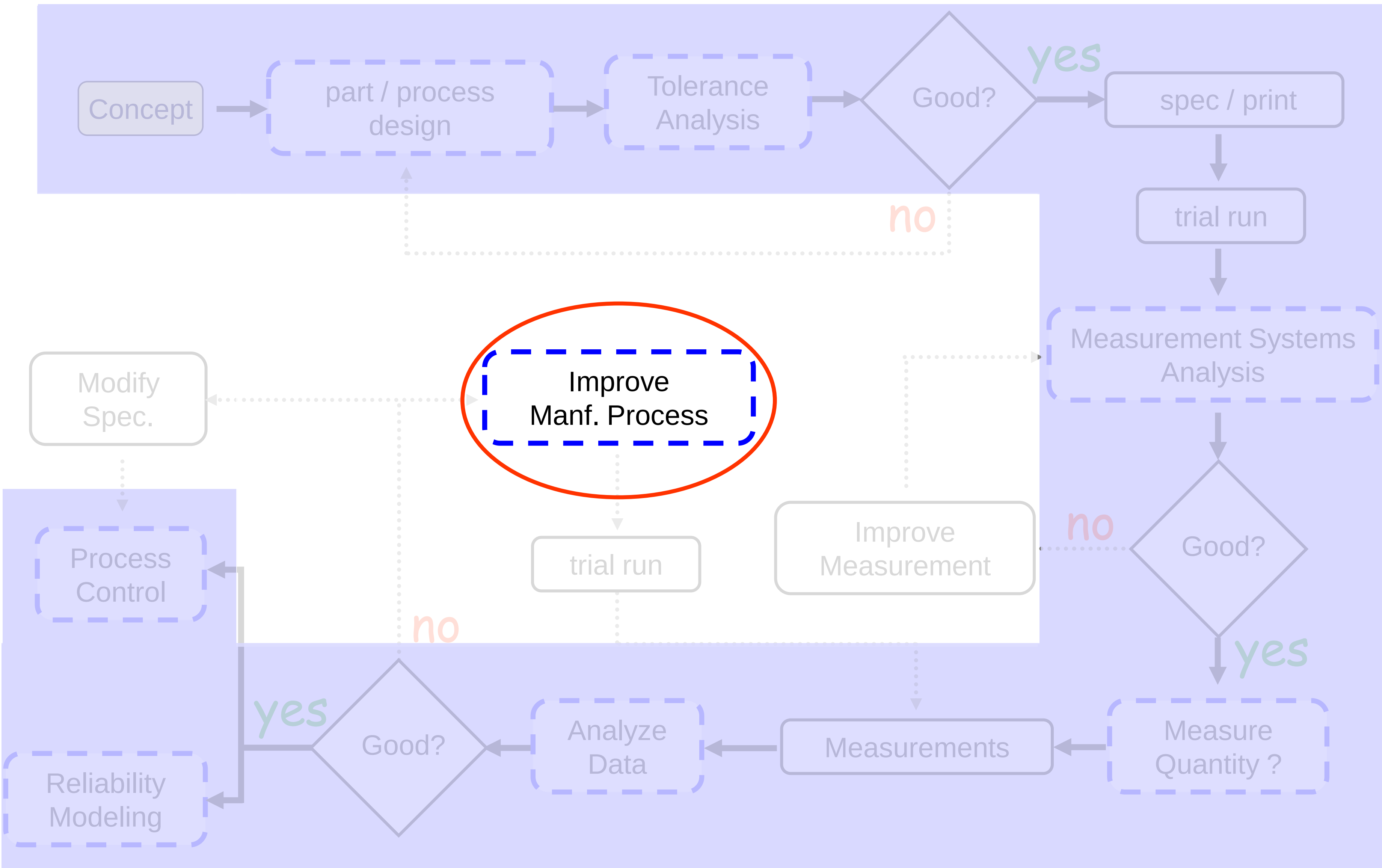
Development Flow (simplified)



Data Analysis



Development Flow (simplified)



Comparative Analysis
&
Design of Experiments
(DOE)

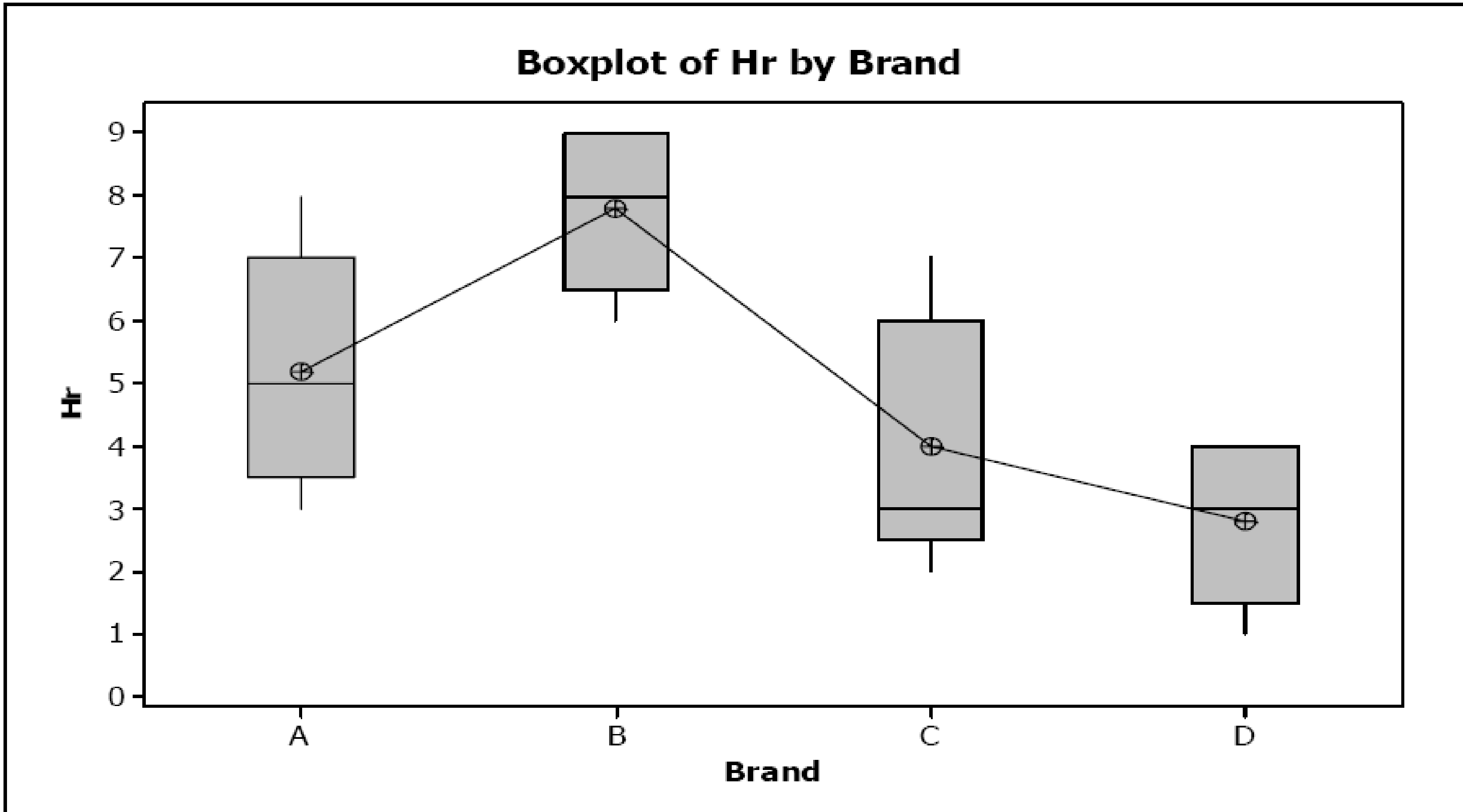
Comparative Methods

- Analytical method to evaluate changes & differences.
- **Examples:**
  - Different vendors
  - Change to a process

Types of Comparisons

	Y continuous	
X discrete	Mean	Standard Dev.
1-STD	1 sample t-test	σ Confidence interval
1-1	2 sample t-test	F test
Multiple	One-way ANOVA	Graphical Bartlett Levene

Multiple Comparisons - Averages

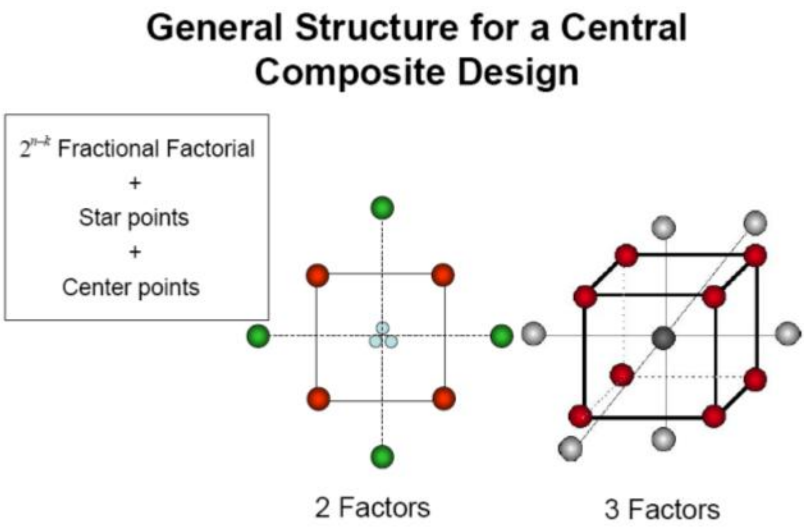


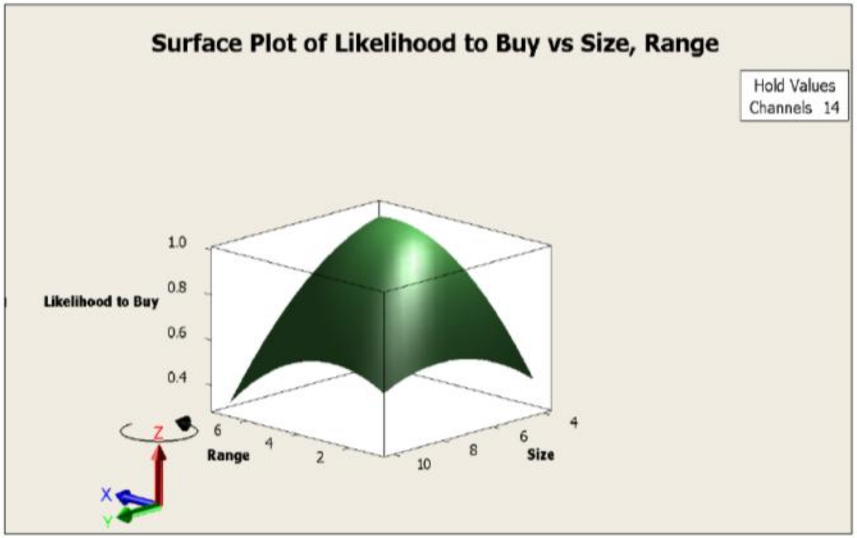
Comparative Analysis
Example:

PCB Bow

Design of Experiments

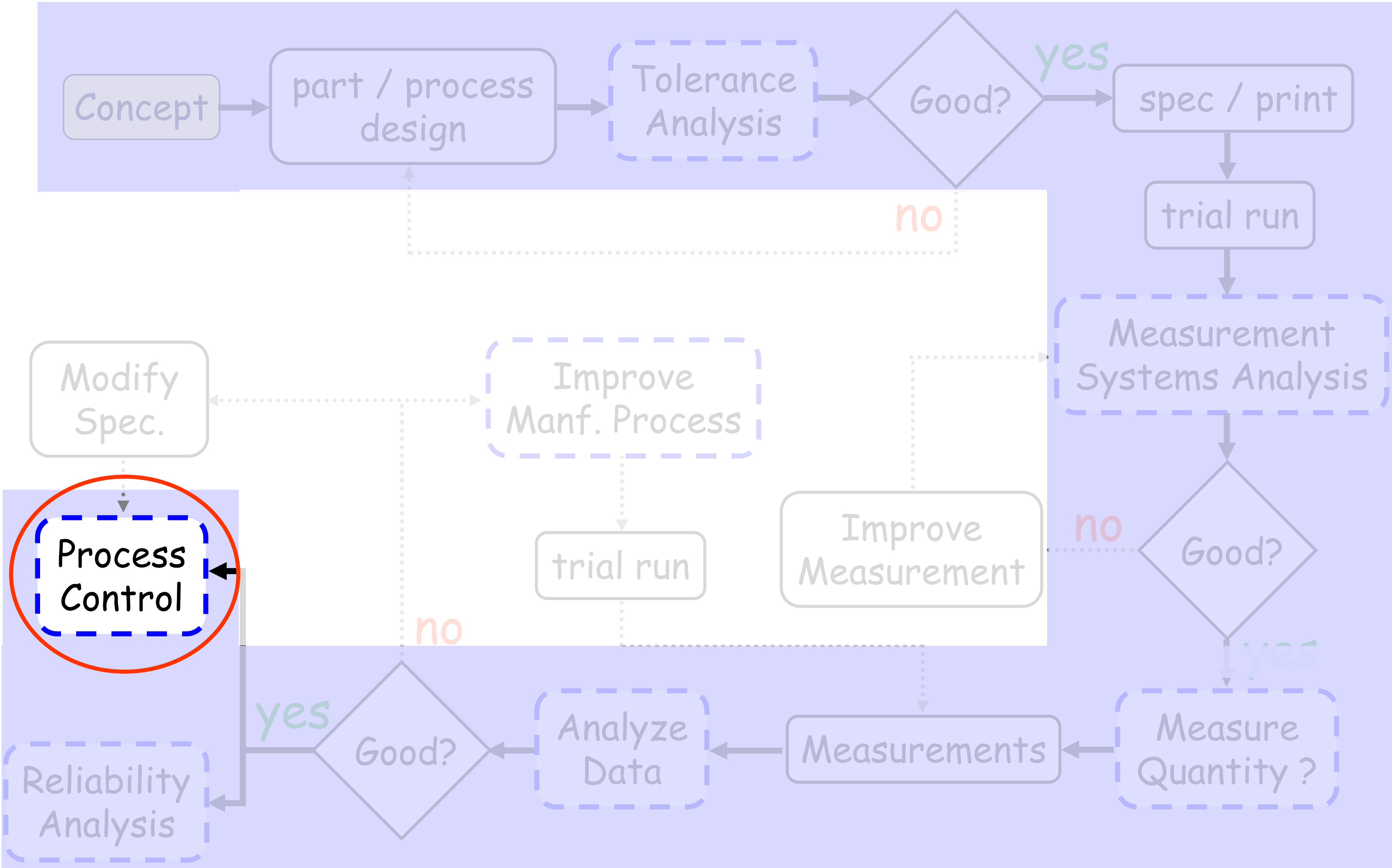
- Efficient Experimental Method
- Optimizes Processes and Designs
- Allows for the Analysis of Interactions





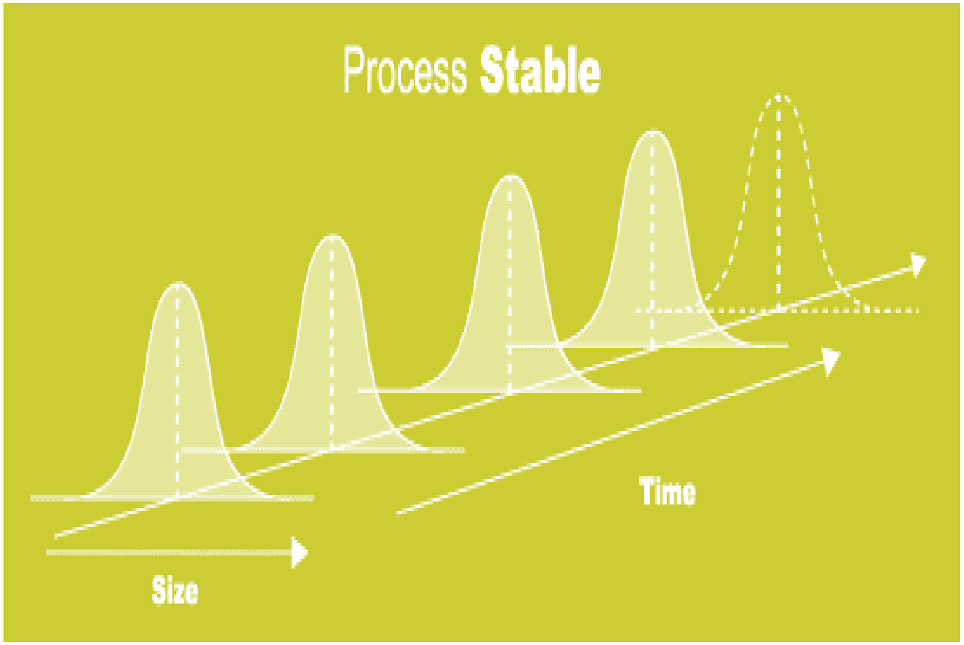
Process Control

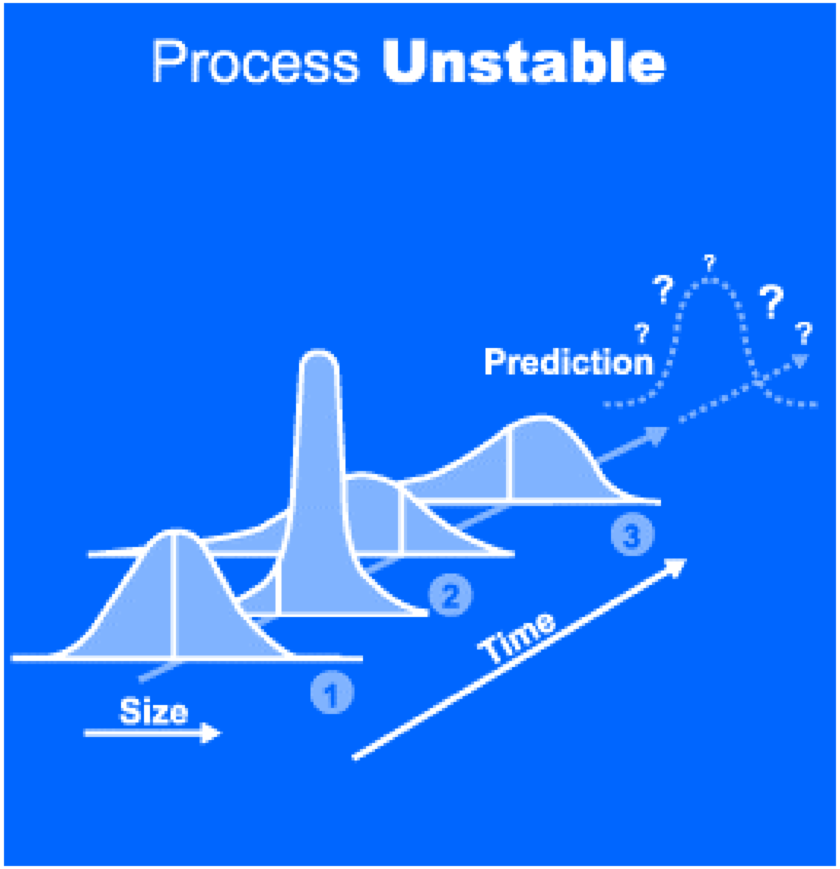
Development Flow (simplified)



Process Control

Statistical Process Control



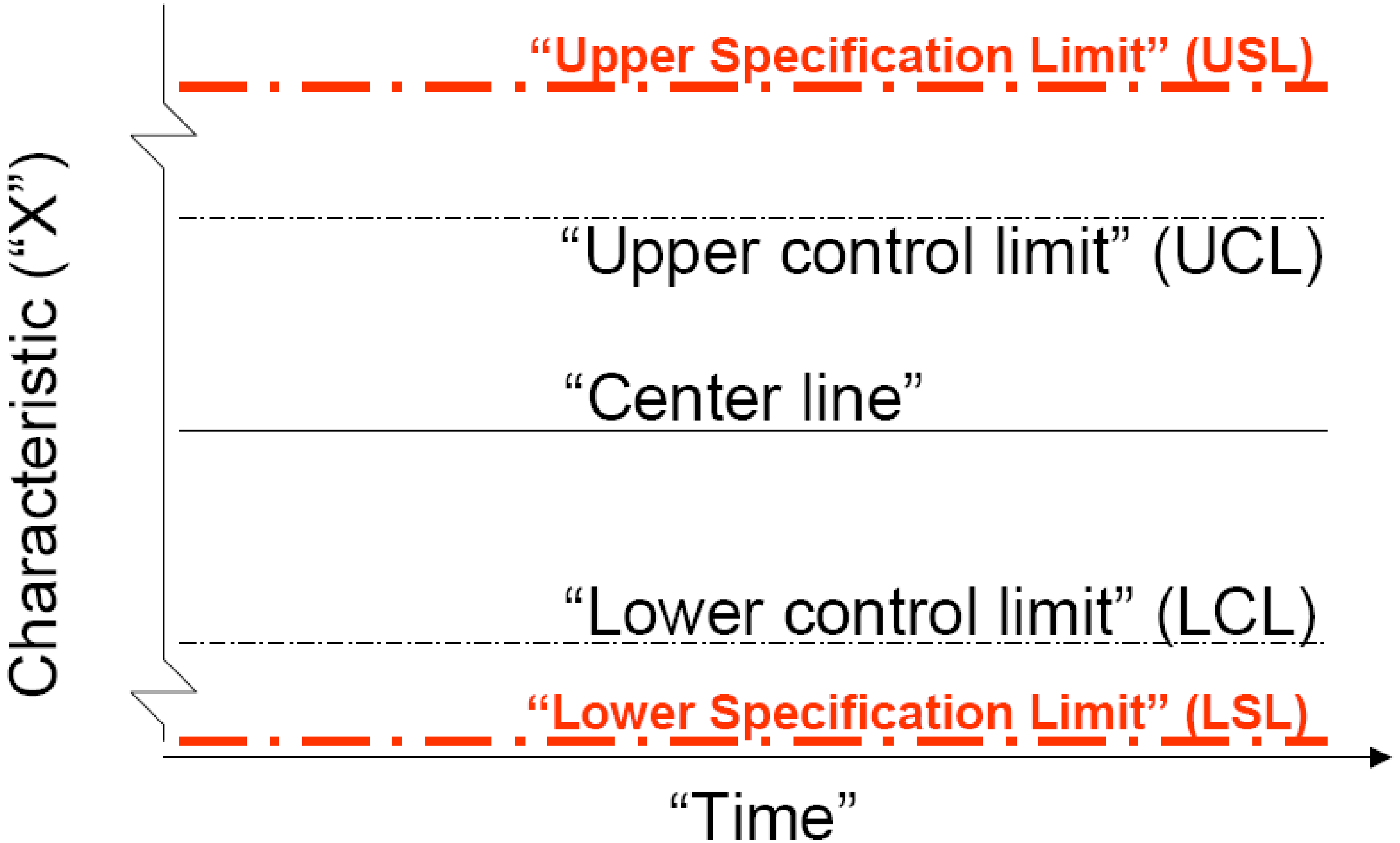


SPC Goals:

- Predicable process
  - Consistent σ (Cp)
  - Centered distribution (Cpk)

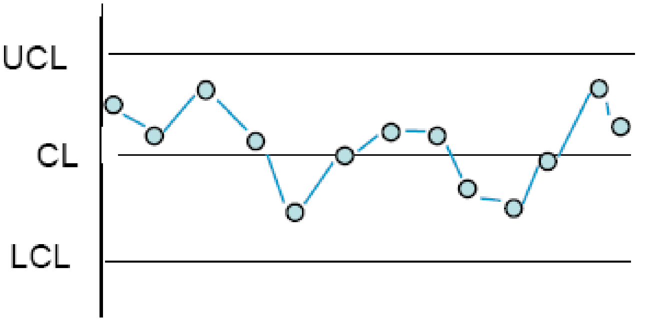
Control Charts

Control Limits vs. Specification Limits

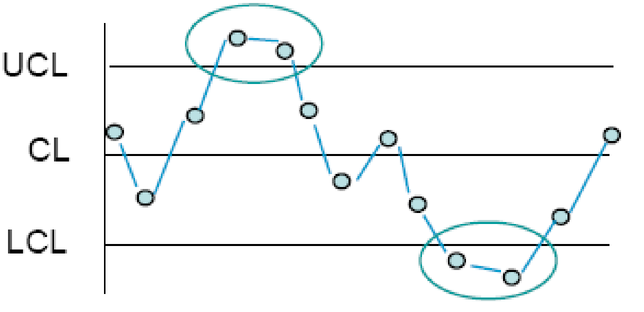


Decision Rules for Process Control

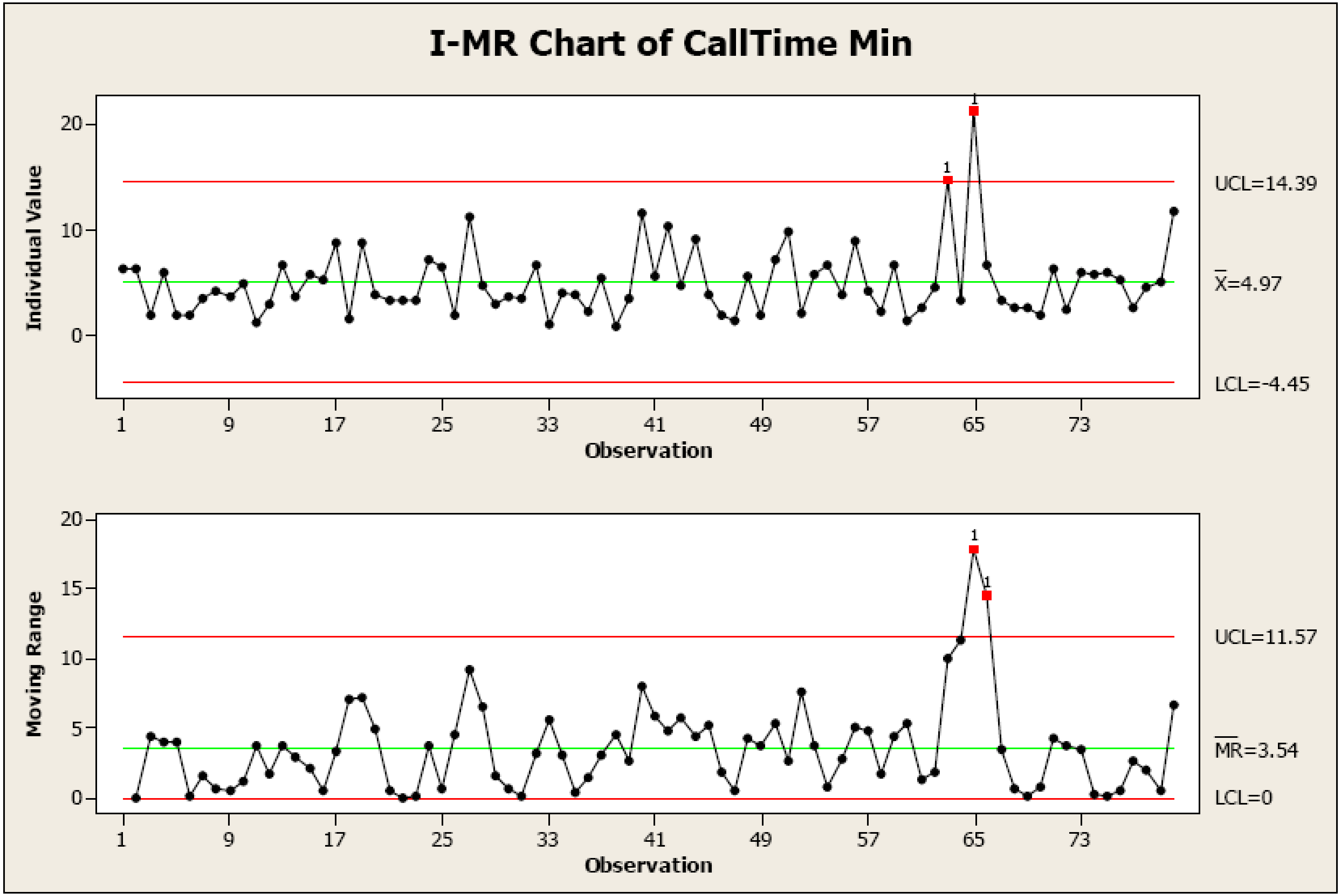
Process In Control → Chart points do not form a particular pattern AND lie within the upper and lower chart limits →



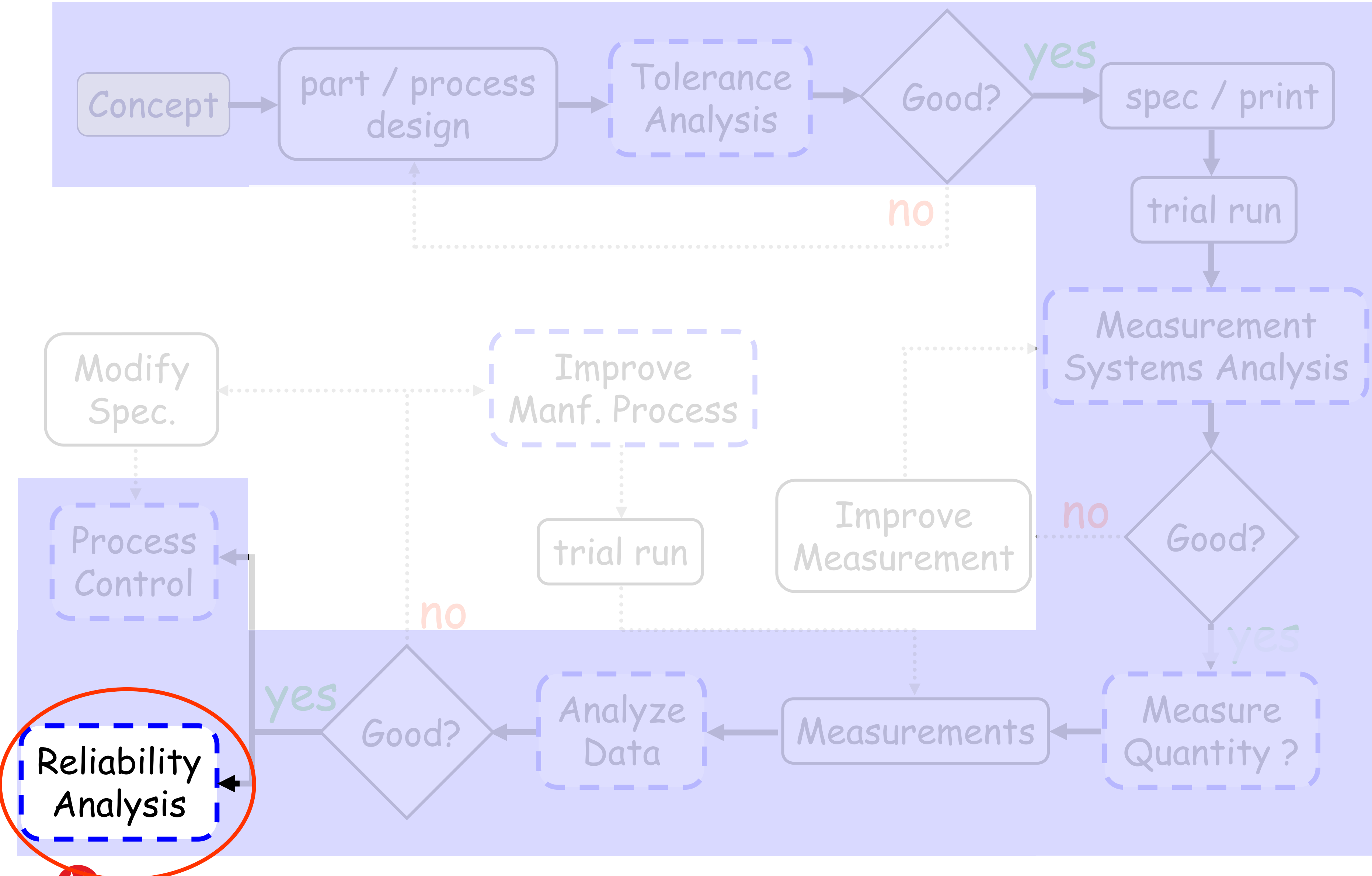
Process Out of Control → Chart points form a particular pattern OR one or more points lie beyond the upper or lower chart limits →



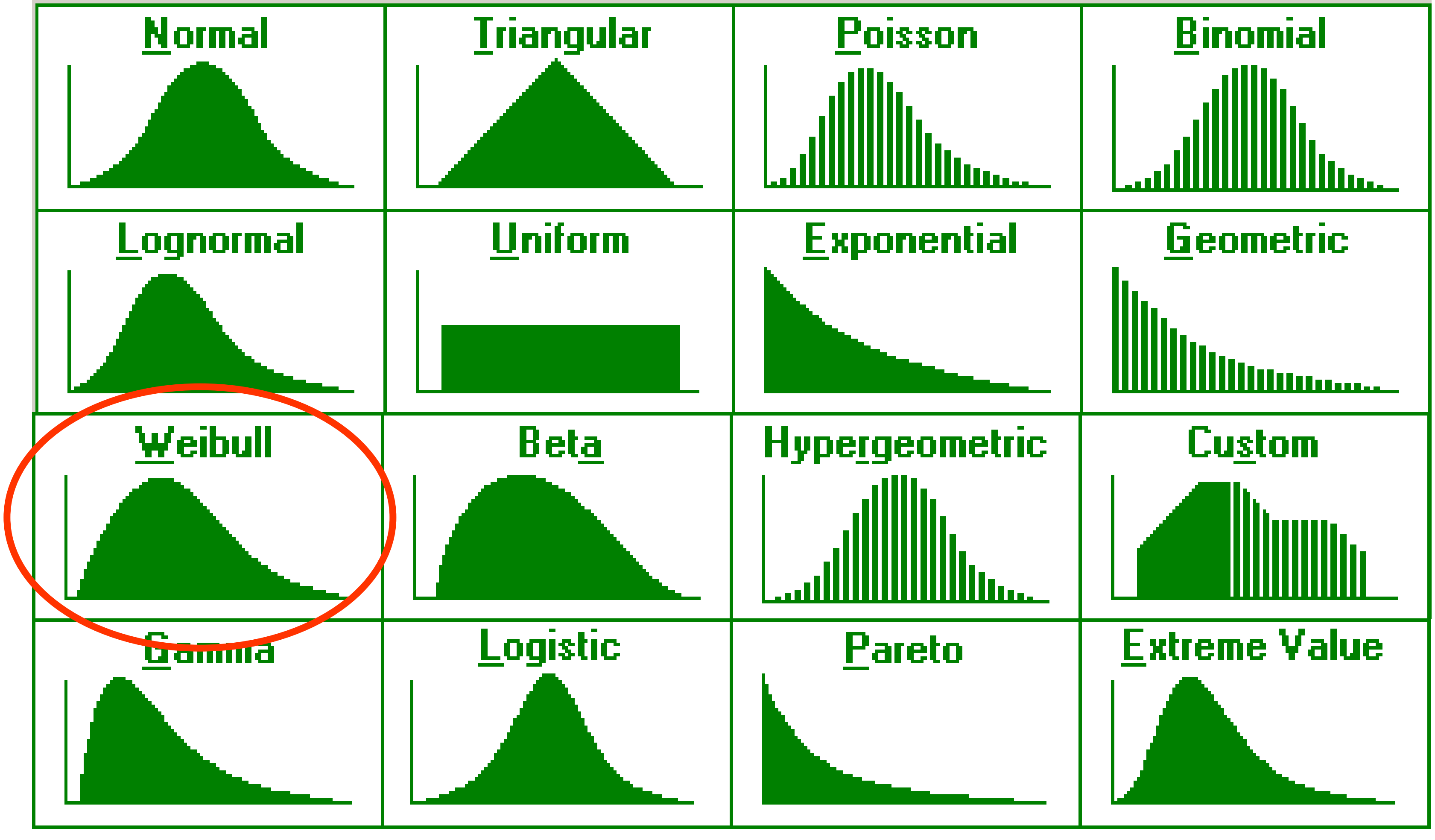
Control Charts (X-bar / R)



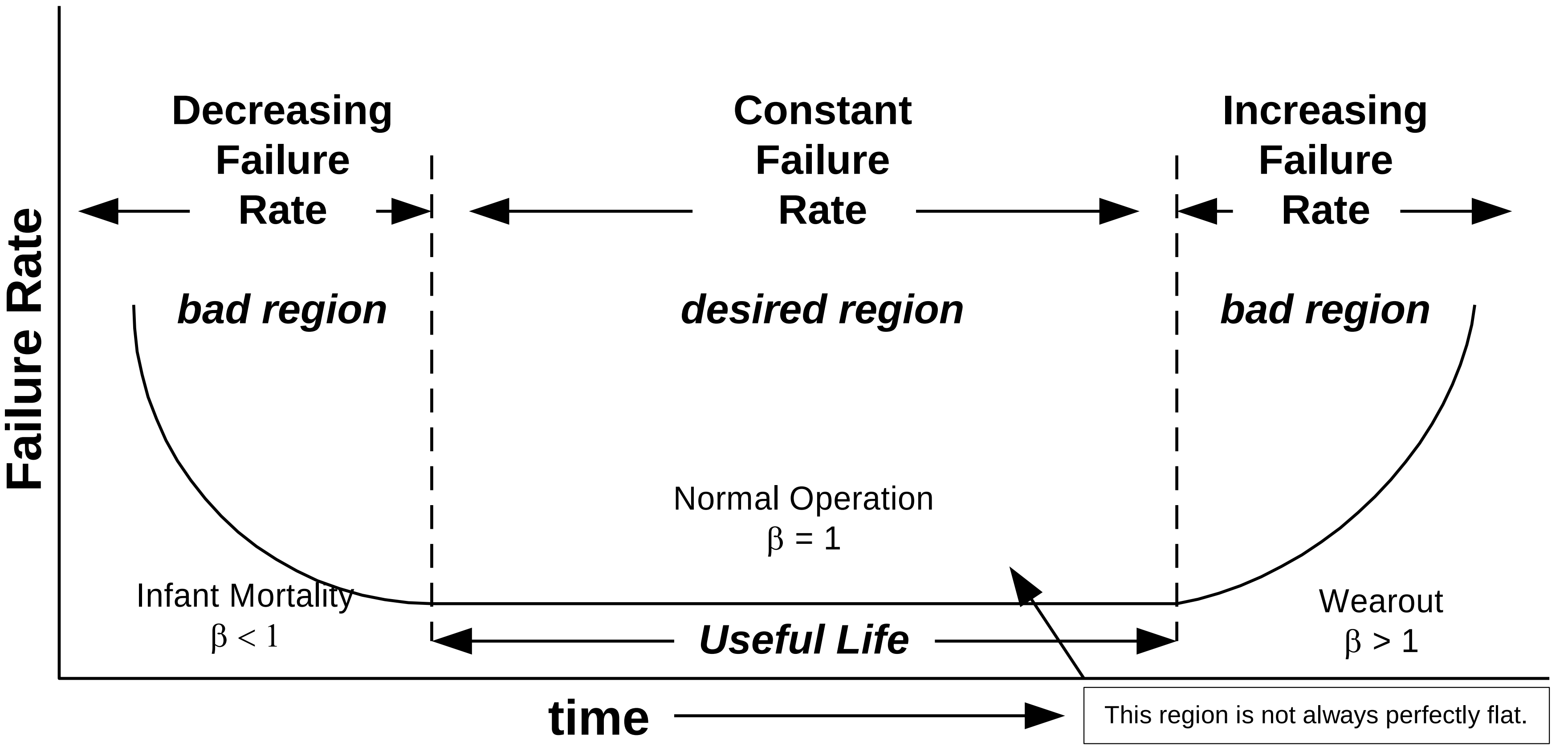
Development Flow (simplified)



Reliability Analysis



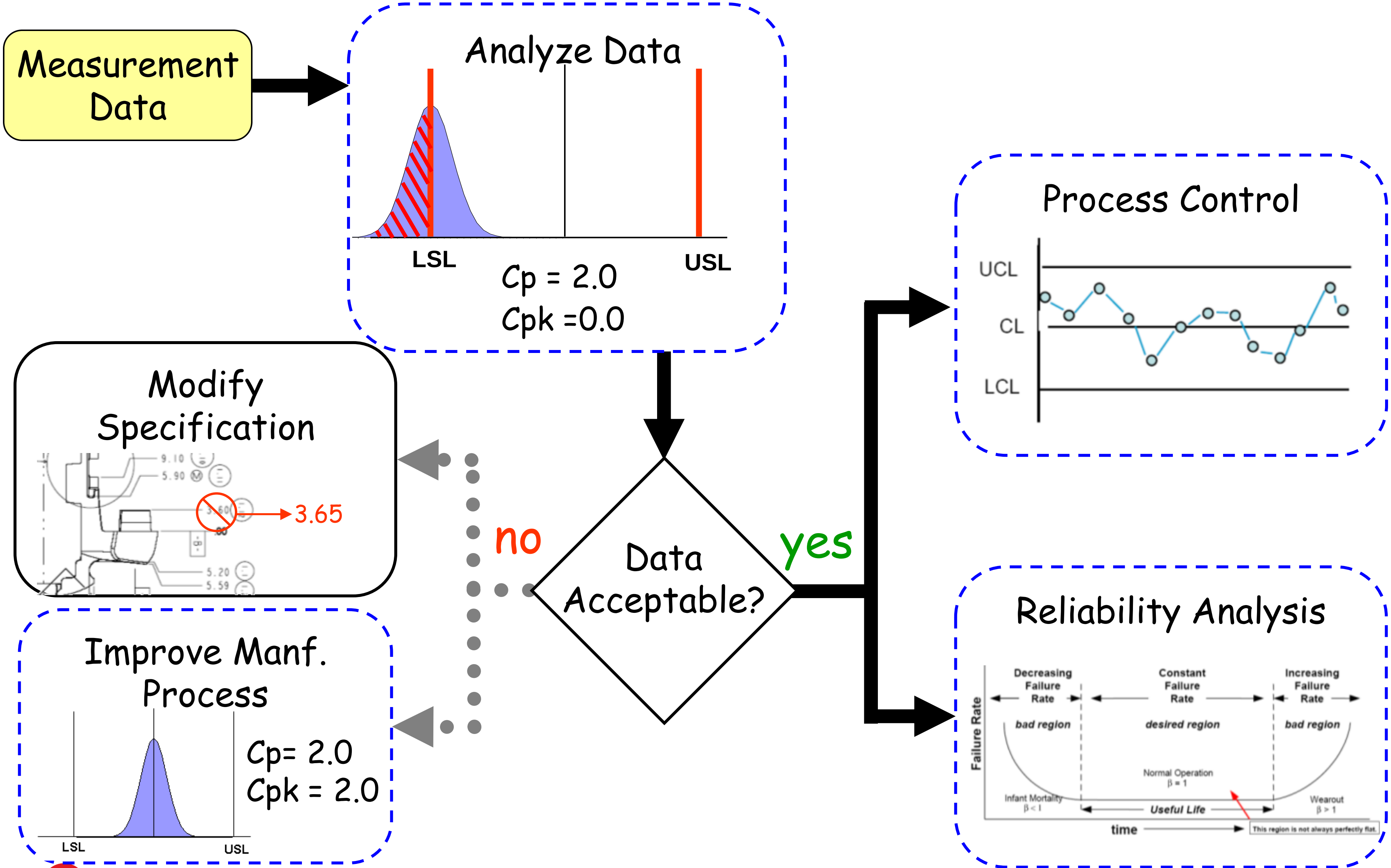
Reliability Analysis



Failure Analysis <u>Example:</u>

Display Breakage

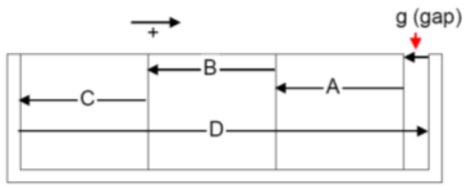
Development Flow (simplified)



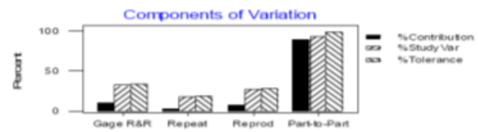
Tools Used

DFSS Tools

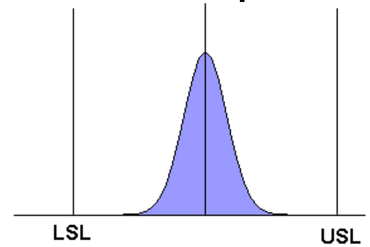
Tolerance Analysis



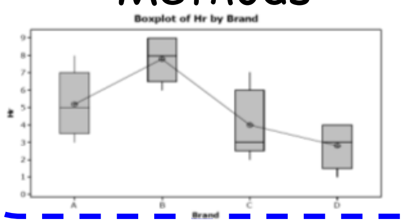
Measurement System Analysis



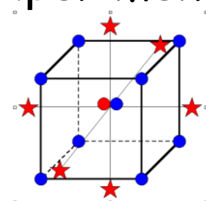
Process Capability



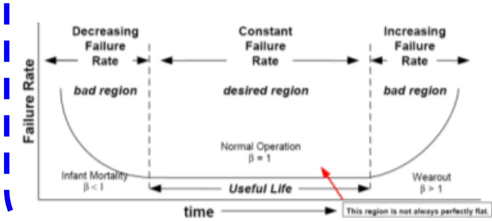
Comparitive Methods



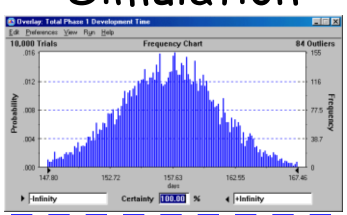
Design of Experiments



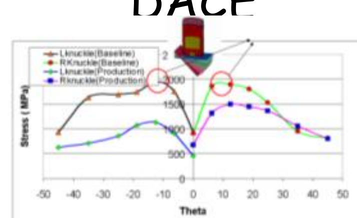
Reliability Modeling



Monte Carlo Simulation

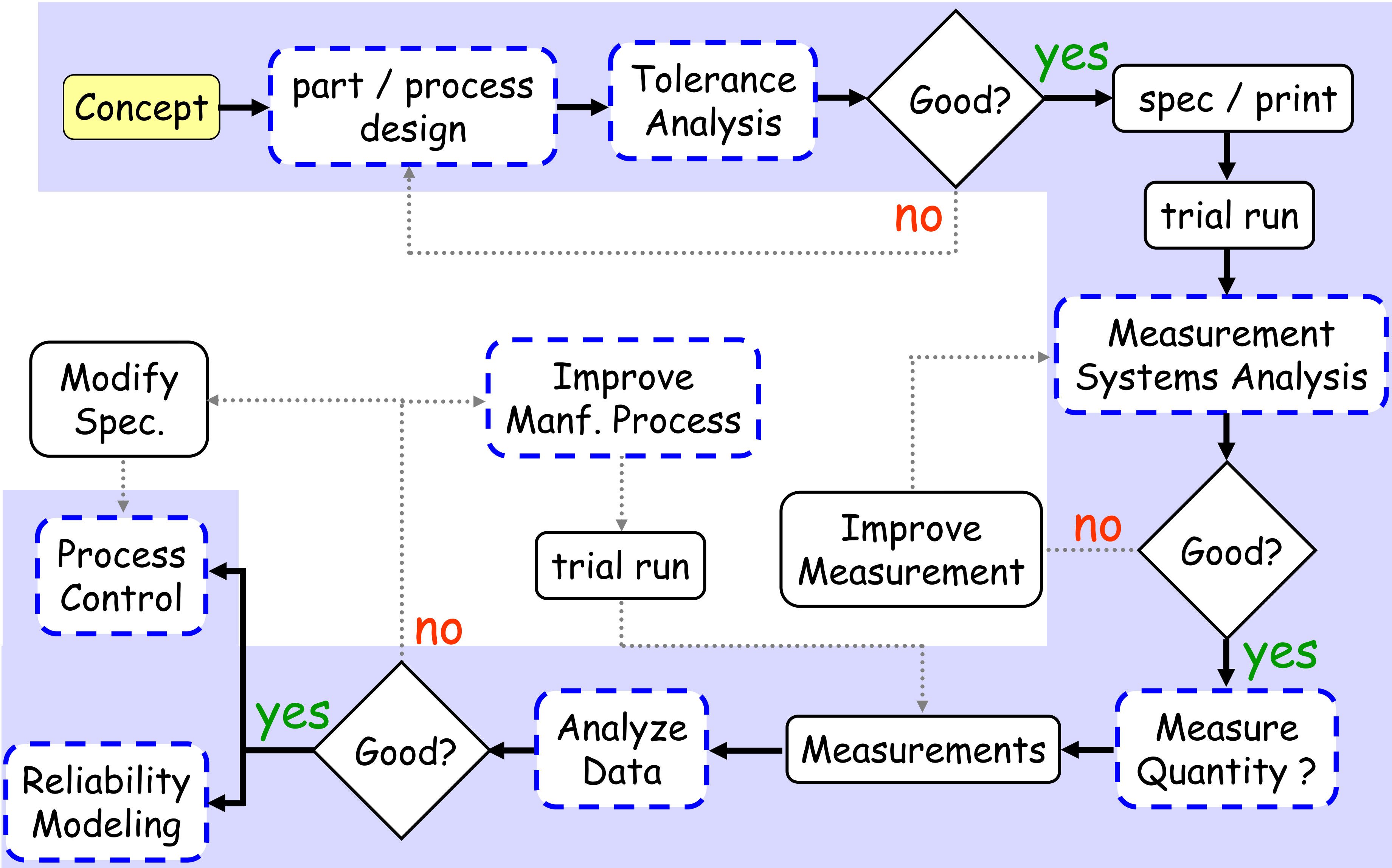


DACE



Etc....

Development Flow (simplified)



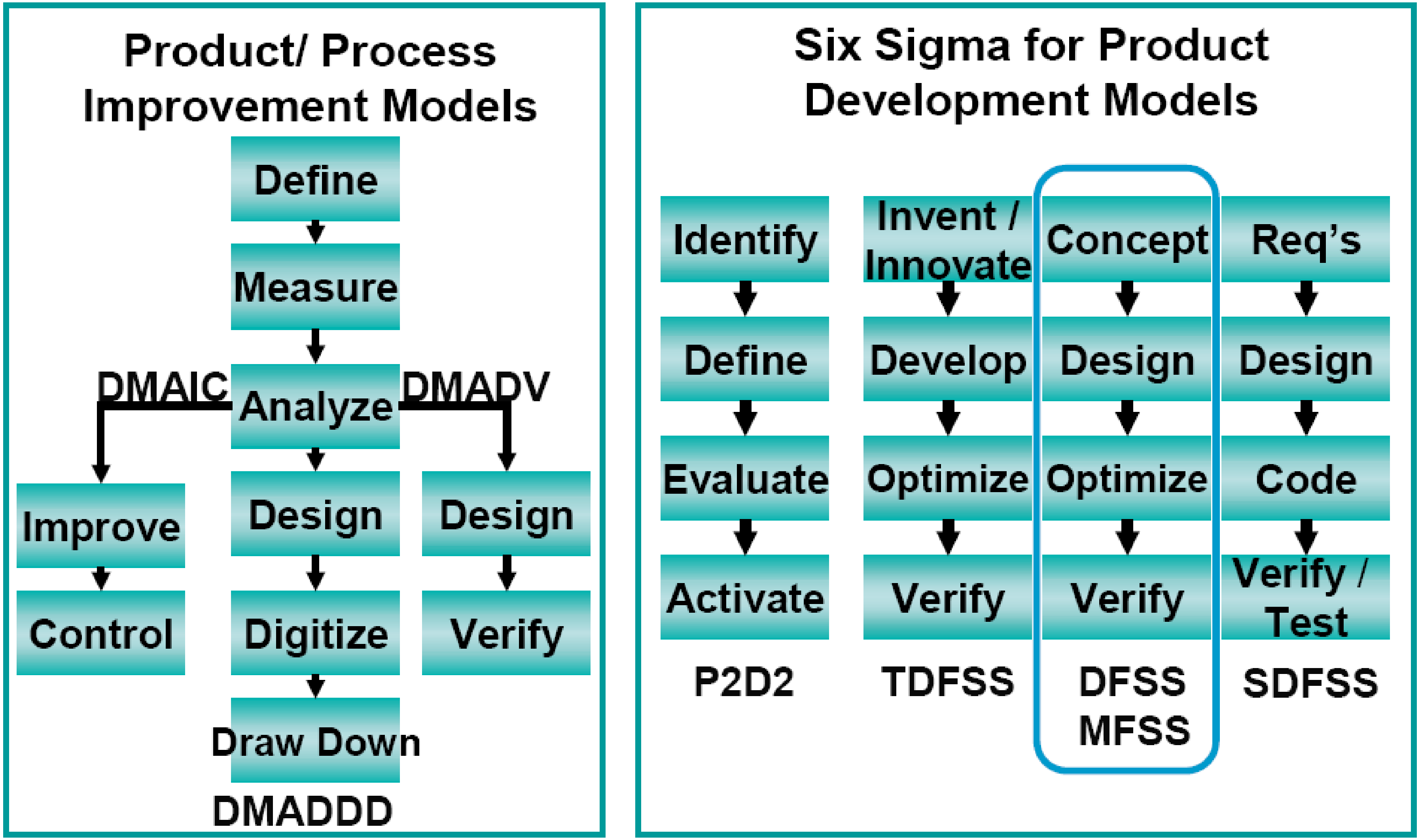
Why Statistical Methods

- Disciplined Approach
- Repeatable Results
- Quantifiable Decision Criteria
- Optimization

Q & A

Back Up Slides

6 Sigma Methodology



DFSS – CDOV Process

DFSS - Design for Six Sigma

CDOV

Concept → **Design** → **Optimize** → **Verify**

Concept
- Prioritize Customer Needs
- Select Superior Concept

Design
- Baseline Design
- Customer needs captured in Design Requirements

Optimize
- Optimize "Critical to Quality" Parameters

Verify
- Ensure long term performance

CDOV Process

Concept

Major Steps	VOC	KJ Analysis	Initiate CPM	Pugh Analysis

Design

MSA	Process Capability	Confidence Intervals	Comparative Methods	Monte Carlo	CPM	FMEA	Control Charts

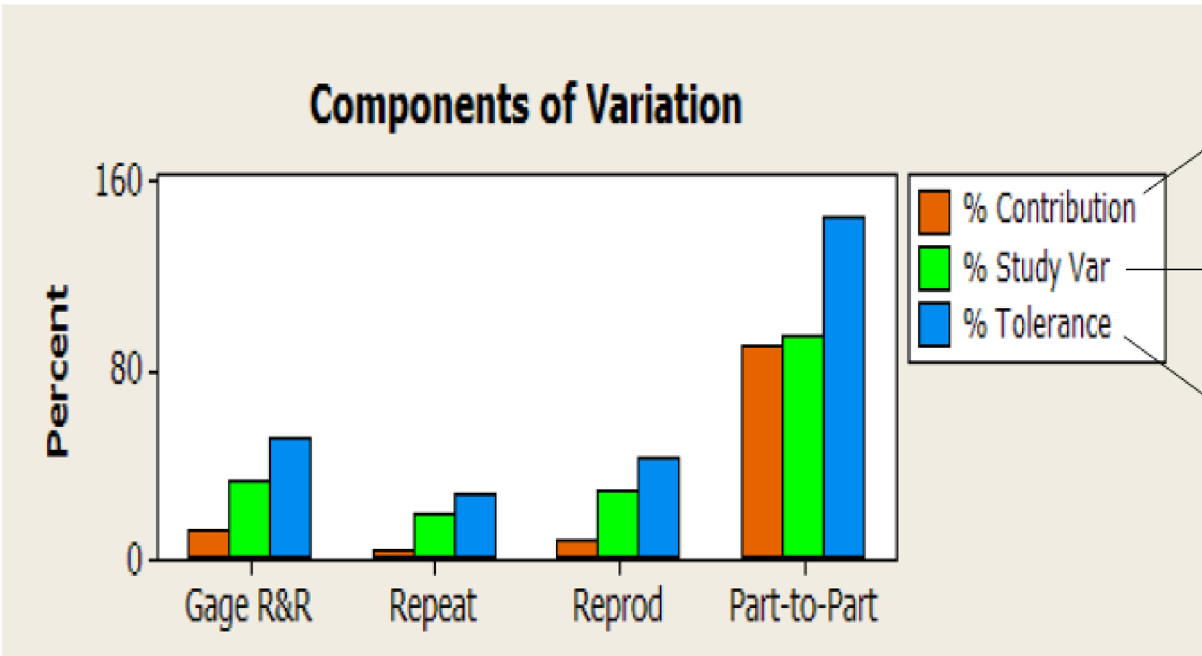
Optimize

Regression	DOE	RSM	Robust Design	Tolerance Analysis	DACE

Verify

Reliability Modeling	System Reliability	System Availability

Measurement Systems Analysis



$$\frac{\sigma_{MS}^2}{\sigma_{Total}^2} \times 100$$

$$\frac{6 \times \sigma_{MS}}{6 \times \sigma_{Total}} \times 100$$

$$\frac{6 \times \sigma_{MS}}{USL - LSL} \times 100$$

- Shows %R&R, its components and part to part variation
- We want the Gage R&R bars to be as small as possible

Design of Experiments

One Factor At a Time (OFAT) Example

	Pressure 1	Pressure 2
Temp 1	25	35
Temp 2	40	15

OFAT 1: Temperature Constant, Pressure Change

OFAT 2: Pressure Constant, Temperature Change

- **If we held Temp constant at level 1 and varied Pressure, we would conclude that Pressure at level 2 is best.**

OFAT – Disadvantages

- Easy but not efficient
- Does not allow the investigation of the combined effects of factors (Interactions)
- Does not cover a wide experimental region

Design of Experiments

Factors		Response
Speed (Mph or Kph)	Brake Force (pounds or Newtons)	Stopping Distance (yards or meters)
-1	-1	50
-1	+1	30
+1	-1	250
+1	+1	150

$$\text{Level}^{\text{Factors}} \rightarrow 2^2 = 4 \text{ Runs}$$

Design of Experiments

Analysis - Main Effects – Factor B (Brake Force)

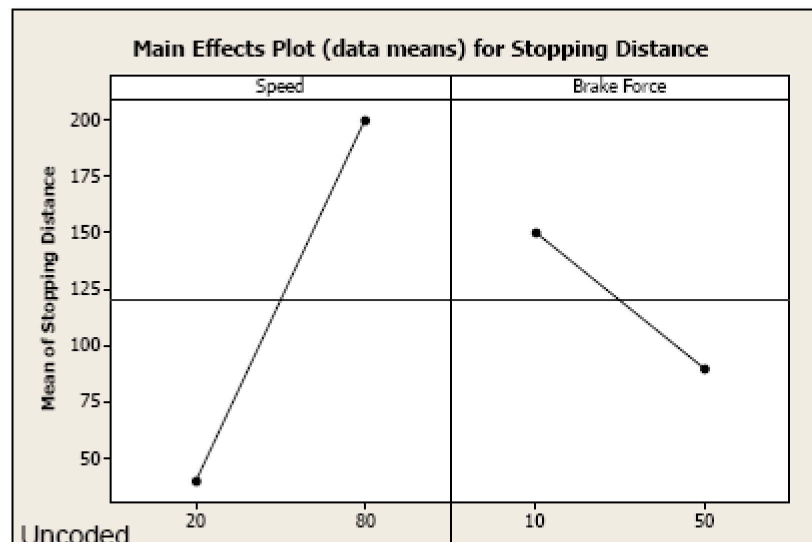
	Speed	Brake Force	Response
	-1	-1	50
	-1	+1	30
	+1	-1	250
	+1	+1	150
Average for +1	200	90	
Average for -1	40	150	
Main Effect	160	-60	

$$\text{Brake Force}_{\text{Effect}} = \frac{30+150}{2} - \frac{50+250}{2} = \frac{180}{2} - \frac{300}{2} = 90 - 150 = -60$$

The Main Effect of Brake Force is -60.

2^k -> $\text{Level}^{\text{Factors}}$

Main Effects Plots



Coded -1 +1 -1 +1

Design of Experiments

General Structure for a Central Composite Design

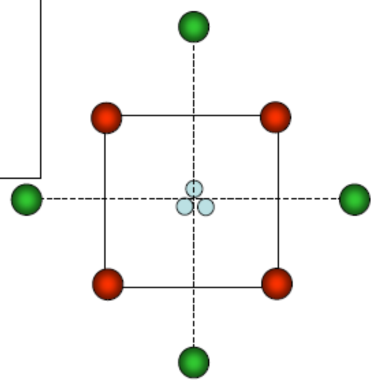
2^{n-k} Fractional Factorial

+

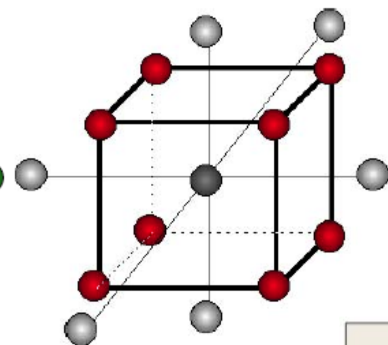
Star points

+

Center points



2 Factors



3 Factors