GRADE 8 MATHEMATICS

Statistics and Probability

Statistics and Probability (Data Analysis)—8.SP.1

Enduring Understanding:

Data are gathered and organized in different ways, which may have an impact on what the data display.

General Learning Outcome:

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
8.SP.1 Critique ways in which data are presented. [C, R, T, V]	➛ Compare the information that is provided for the same data set by a set of graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, to determine the strengths and limitations of each graph. ➛ Identify the advantages and disadvantages of different graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, in representing a specific set of data. ➛ Justify the choice of a graphical representation for a situation and its corresponding data set. ➛ Explain how a formatting choice, such as the size of the intervals, the width of bars, or the visual representation, may lead to misinterpretation of the data. ➛ Identify conclusions that are inconsistent with a data set or graph, and explain the misinterpretation.

Prior Knowledge

Students may have had experience with the following:

- Constructing and interpreting concrete graphs and pictographs to solve problems
- Constructing, labelling, and interpreting bar graphs to solve problems
- Constructing and interpreting pictographs and bar graphs involving many-to-one correspondence to draw conclusions
- Constructing and interpreting double bar graphs to draw conclusions
- Creating, labelling, and interpreting line graphs to draw conclusions
- Graphing collected data and analyzing the graph to solve problems
- Demonstrating an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings
- Determining the effect on the mean, median, and mode when an outlier is included in a data set
- Constructing, labelling, and interpreting circle graphs to solve problems

Background Information

The way data are organized and represented may have an impact on what one might interpret from the data. Graphs are a common way of organizing, representing, and communicating information. Different graphs are used, depending on what information is being represented and communicated.

Types of Graphs

Graphs are used to provide visual displays of data. It is important to know what type of data have been collected and the information that is to be communicated before deciding on what type of graph to use. Middle Years students need to have a good understanding of the advantages and disadvantages of each type of graph. This understanding will enable them to determine the appropriate graph for their data and to defend their choices.

A description of the various types of graphs is provided below, followed by a chart outlining the advantages and disadvantages of different graphs.

- **Bar Graphs**

  Bar graphs usually compare frequency of discrete data. Therefore, there are spaces between bars. Bars can be drawn either vertically or horizontally.

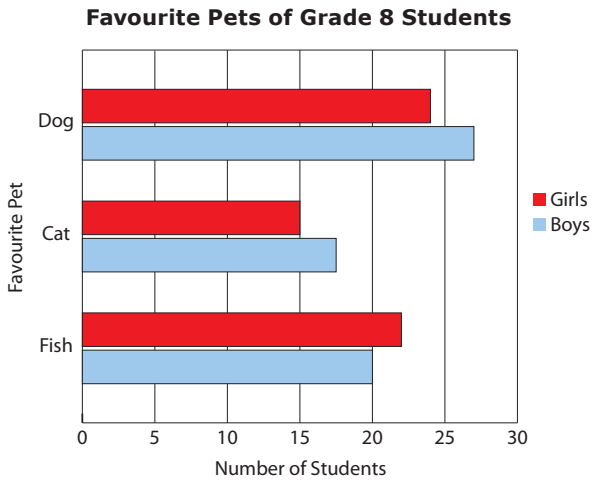
  Example:

  Note: If part of the scale is omitted, then a squiggle in the vertical axis is used. However, this tends to give a misleading visual picture.

- **Double Bar Graphs**

  These types of graphs use pairs of bars to make comparisons between and among sets of data. Bars can be vertical or horizontal.

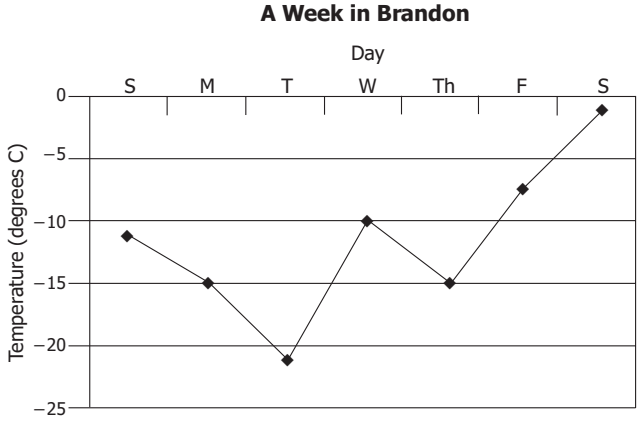
  Example:



- **Line Graphs (Broken Line Graphs)**

  These types of graphs are appropriate for indicating trends or relationships and are used primarily to show a quantity changing over time (e.g., temperature change over 24 hours, average monthly rainfall, yearly school enrolments). For example, a survey of the favourite seasons of a Grade 8 class could not be put on a line graph, as the data do not involve change over time.

  Example:



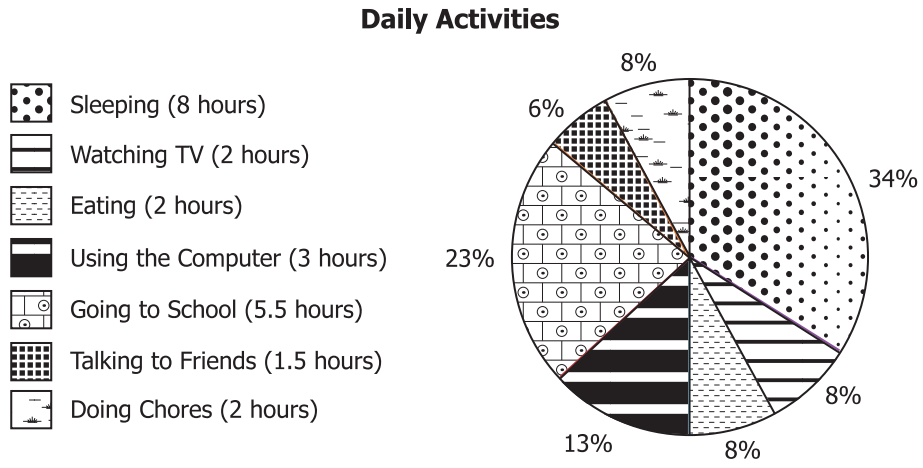
- **Circle Graphs (Pie Graphs)**

  In circle graphs, the data are represented by sectors (parts) of a circle (whole); the total of all the sectors should be 100% of the data. Each section of the circle represents a part or percentage of the whole.

  Circle graphs

  - show the ratio of each part to the whole, not quantities
  - are almost always made from data converted to percentages of the total
  - show ratios—therefore, comparisons can be made between different-sized quantities (e.g., results from the class survey can be compared to results from a whole school survey)

  Example:



Advantages and Disadvantages of Graphs			
Graphs	Purpose(s)	Advantage(s)	Disadvantage(s)
Bar graphs	■ compare frequency of data (usually discrete)	■ are easy to read and interpret ■ can be used to compare two or more related sets of data	■ can be misleading if part of the scale along one axis is compressed
Line graphs	■ show changes in a single variable over time	■ can be used to observe changes over time ■ can be used to find individual pieces of data	■ can be used only if data change over time ■ can be misleading if part of the scale along one axis is compressed
Circle graphs	■ compare groups of data to the whole set of data	■ can be used to see the ratio of each part to the whole group	■ cannot retrieve individual pieces of data because data are grouped

MATHEMATICAL LANGUAGE

bar graph
circle graph
distort
double bar graph
double line graph
interval
line graph
pictograph
trend

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials: BLM 8.SP.1.1: Data Analysis Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of data analysis over the next few lessons; however, you first need to find out what they already know about graphs.
2. Provide students with copies of BLM 8.SP.1.1: Data Analysis Pre-Assessment.
3. Have students complete the sheet individually.
4. Students will complete BLM 8.SP.1.1 at the end of the unit as a post-assessment.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize and explain why a broken line graph will not be the best way to represent which search engines students prefer.
- ❒ Understand that a circle graph must be filled completely.
- ❒ Recognize and explain why 25% of a circle graph is not 60^{o}.
- ❒ Recognize and explain that the intervals on the sample bar graph are not properly spaced (spaces are unequal, graph does not start at zero).
- ❒ Correctly interpret a circle graph.

Note: Students will be creating and reading graphs in social studies and science. Linking graphs to those subject areas would be beneficial.

Suggestions for Instruction

- **Compare the information that is provided for the same data set by a set of graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, to determine the strengths and limitations of each graph.**
- **Identify the advantages and disadvantages of different graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, in representing a specific set of data.**
- **Justify the choice of a graphical representation for a situation and its corresponding data set.**

Materials: Graphing software or white paper, graph paper, rulers, BLM 5–8.23: Understanding Words Chart, BLM 8.SP.1.2: Data, math journals, BLM 8.SP.1.1: Data Analysis Pre-Assessment

Organization: Small group/whole class/individual

Procedure:

1. Tell students that, through exploration, they will be able to determine the strengths and limitations of different graphs, identify advantages and disadvantages of graphs, and justify their choice of graphical representation for a given situation.
2. Use BLM 5–8.23: Understanding Words Chart to address the key vocabulary for this unit.
3. Divide the class into small groups, and provide each group with a copy of BLM 8.SP.1.2: Data. Ask each group to create a bar graph, a line graph, and a circle graph for each set of data presented.
4. Each group will discuss the following questions in relation to the data and present their findings to the class:
   - Based on your data, what are the strengths and limitations of each of the graphs you made?
   - What are the advantages and disadvantages of each type of graph?
   - Which graph do you feel is best for representing each set of data provided? Why?
5. As the groups present their information, record responses to each question in a chart such as the following. By the end, you will have generated a variety of strengths and limitations, advantages and disadvantages, and best choice of graph per set of data.

Strengths and Limitations	Advantages and Disadvantages	Best Choice of Graph

6. Ask students to give an example, in their math journals, of the type of data that could be collected that would best be portrayed in a bar graph, a broken line graph, and a circle graph.
7. Have students complete BLM 8.SP.1.1: Data Analysis Pre-Assessment as a post-assessment.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate an understanding of strengths and weakness of each type of graph.
- ❒ Complete BLM 8.SP.1.1 as a post-assessment.

- **Explain how a formatting choice, such as the size of the intervals, the width of bars, or the visual representation, may lead to misinterpretation of the data.**
- **Identify conclusions that are inconsistent with a data set or graph, and explain the misinterpretation.**

Materials: BLM 8.SP.1.3: Graph Samples, math journals

Organization: Small group/individual

Procedure:

1. Tell students that they will be analyzing different sets of graphs to determine which graph best represents the given data and explaining why one graph leads to a misinterpretation of the data represented.
2. Hand out graph **Sample 1**, **Sample 2**, and **Sample 3** from BLM 8.SP.1.3: Graph Samples, one at a time.
3. For **each** sample, have students, working in groups, discuss the following:
   - What can be said about these graphs?
   - What scenario do the data display?
   - Is there something that can be done to each graph to make it clearer? Explain.
   - Do the graphs display the same or different data? Explain.
   - What are the advantages and disadvantages of each graph?
   - Which graph is more accurate? Explain.
   - Can either of the graphs be misinterpreted? Explain.
4. Have students explain, in their math journals, how the format of graphs (how the graphs are made) can lead to a misinterpretation of the data. Ask them to use words and diagrams to explain their thoughts.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

❒ Explain how the format of graphs can lead to a misinterpretation of the data.

Statistics and Probability (Chance and Uncertainty)—8.SP.2

Enduring Understandings:

The principles of probability of a single event also apply to independent events.

Probability can be expressed as a fraction or decimal between 0 and 1, where 0 indicates an impossible event and 1 indicates a certain event.

Probabilities can be expressed as ratios, fractions, percents, and decimals.

General Learning Outcome:

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Learning Outcome(s):	Achievement Indicators:
8.SP.2 Solve problems involving the probability of independent events. [C, CN, PS, T]	➛ Determine the probability of two independent events and verify the probability using a different strategy. ➛ Generalize and apply a rule for determining the probability of independent events. ➛ Solve a problem that involves determining the probability of independent events.

Prior Knowledge

Students may have had experience with the following:

- Describing the likelihood of a single outcome occurring, using words such as
  - impossible
  - possible
  - certain
- Comparing the likelihood of two possible outcomes occurring, using words such as
  - less likely
  - equally likely
  - more likely

- Demonstrating an understanding of probability by
  - identifying all possible outcomes of a probability experiment
  - differentiating between experimental and theoretical probability
  - determining the theoretical probability of outcomes in a probability experiment
  - determining the experimental probability of outcomes in a probability experiment
  - comparing experimental results with the theoretical probability for an experiment
- Expressing probabilities as ratios, fractions, and percents
- Identifying the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events
- Conducting a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or another graphic organizer) and experimental probability of two independent events

Background Information

Probability

Probability refers to the chance of an event occurring. The probability of an event must be greater than or equal to 0 and less than or equal to 1.

Note: Students in Grade 8 will be working only with the probabilities of independent events.

Definitions

independent events

Events in which the theoretical probability of an event occurring does not depend on the results of another event.

Example:

Rolling a number cube and then selecting a card from a deck.

What is the probability of rolling a 6 on a number cube and then pulling a 6 from a deck of cards?

$$P(6 \text{ cube}) = \frac{1}{6} \quad P(6 \text{ card}) = \frac{1}{13} \text{ so } P(6 \text{ cube and } 6 \text{ card}) = \frac{1}{78}$$

dependent events

Events in which the theoretical probability of an event occurring does depend on the results of another event.

Example:

Selecting a red card from a deck and then selecting the Queen of Clubs without putting the first card back.

$$P(\text{red}) = \frac{1}{2} \quad P(\text{Queen of Clubs}) = \frac{1}{51} \text{ so } P(\text{red, then Queen of Clubs}) = \frac{1}{102}$$

Organizing Outcomes/Results

There are different strategies for organizing favourable outcomes, such as tables and tree diagrams.

Example:

If Joe has six cards numbered 1 to 6 and a regular six-sided number cube, what is the probability of turning a 1 and rolling a 1 at the same time?

This scenario can be written as follows: What is P(1,1)?

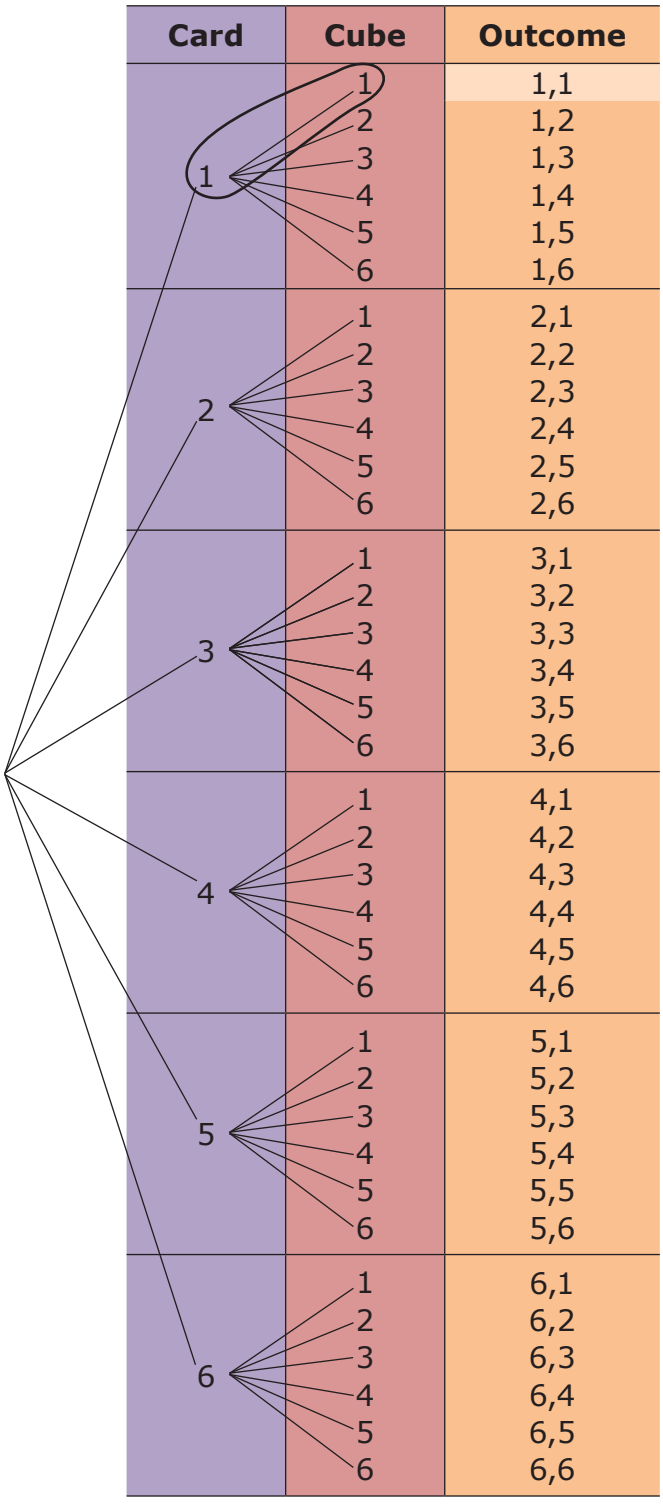
Table

		Number Cube					
		1	2	3	4	5	6
Card	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{event}) = \frac{\textit{favourable outcome}}{\textit{total number of outcomes}}$$

$$P(1,1) = \frac{1}{36}$$

Tree Diagram



Card	Cube	Outcome
1	1	1,1
	2	1,2
	3	1,3
	4	1,4
	5	1,5
	6	1,6
2	1	2,1
	2	2,2
	3	2,3
	4	2,4
	5	2,5
	6	2,6
3	1	3,1
	2	3,2
	3	3,3
	4	3,4
	5	3,5
	6	3,6
4	1	4,1
	2	4,2
	3	4,3
	4	4,4
	5	4,5
	6	4,6
5	1	5,1
	2	5,2
	3	5,3
	4	5,4
	5	5,5
	6	5,6
6	1	6,1
	2	6,2
	3	6,3
	4	6,4
	5	6,5
	6	6,6

$P(1,1) = \frac{1}{36}$ This can also be expressed as 1:36, ≈3% or ≈ 0.03.

Mathematical Language

certain
experimental probability
impossible
independent events
less likely
likely
more likely
outcome
probability
probable
simulation
theoretical probability

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.SP.2.1: Probability Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of probability over the next few lessons; however, you first need to find out what they already know about probability.
2. Hand out copies of BLM 8.SP.2.1: Probability Pre-Assessment.
3. Have students complete the pre-assessment individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine all possible outcomes of a specified event.
- ❒ Use an organization method to organize the outcomes.
- ❒ Determine the probability of a specified event.
- ❒ Represent probability as a ratio, fraction, decimal, and percent.

- **Determine the probability of two independent events and verify the probabilities using a different strategy.**

Materials: 1 six-sided number cube and 1 coin per group, BLM 5–8.23: Understanding Words Chart, BLM 8.SP.2.2: Tree Diagram

Organization: Individual/small group/whole class

Procedure:

1. Provide each student with a copy of BLM 5–8.23: Understanding Words Chart, and have students explore their understanding of key mathematical terms.
2. Divide the class into small groups, and present students with the following scenario:

   John rolls a six-sided number cube at the same time that Sue flips a coin. Determine all possible outcomes if they complete the task at the exact same time. (Provide each group with a number cube and a coin in case they need to manipulate the items to help them determine the outcomes.)
3. Allow groups to organize their work as they see fit. When it is time to review their work, write their outcomes on the whiteboard using a tree diagram.
4. When all possible outcomes are recorded, explain to students how a tree diagram allows favourable outcomes to be determined in an organized manner.
5. Ask students to state P(3, H), P(odd, T), P(1 or 2, H).
6. Have students complete BLM 8.SP.2.2: Tree Diagram.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Successfully use a tree diagram to determine probabilities.

Suggestions for Instruction

- **Determine the probability of two independent events and verify the probabilities using a different strategy.**

Materials: A King, Queen, and Jack of Spades, and a 1, 2, and 3 of Hearts of a deck of cards per group, BLM 8.SP.2.3: Table

Organization: Small group/whole class

Procedure:

1. Divide the class into small groups, and present students with the following scenario:

   Rita has the Jack, Queen, and King of Spades from a deck of cards, and Jessica has the 1, 2, and 3 of Hearts from the deck of cards. If they both flip a card at the same time, what are all the possible outcomes? (Provide each group with the Jack, Queen, and King of Spades and the 1, 2, and 3 of Hearts from a deck of cards in case they need to manipulate items to help them determine the outcomes.)
2. Tell students that their task is to determine all possible outcomes using a strategy for organizing the outcomes other than a tree diagram.
3. Have each group present their method to the class. If no one demonstrates how a table could be used, you will need to demonstrate it.
4. Ask students to state P(J, 1), P(face card, odd), P(Q, 1).
5. Have students complete BLM 8.SP.2.3: Table.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Successfully use a table to determine probabilities.

- **Generalize and apply a rule for determining the probability of independent events.**
- **Solve a problem that involves determining the probability of independent events.**

Materials: BLM 8.SP.2.4: Probability Problems, BLM 8.SP.2.5: Probability Problem Practice, chart paper

Organization: Small group/whole class

Procedure:

1. Divide students into small groups, and provide each group with one problem from BLM 8.SP.2.4: Probability Problems, as well as chart paper. Ask groups to solve their respective problems and be prepared to present their solutions.
2. As each group presents its problem and solution strategy to the class, allow other groups to ask questions and add to the solution.
3. Have students identify the various strategies that the groups used to solve the problems.
4. BLM 8.SP.2.5: Probability Problem Practice provides additional problems for practice. Discuss rules for finding the probability of independent events. (Students should generalize that multiplication can be used to find solutions to the probability questions.)

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Generalize a rule to determine the number of outcomes for two independent events.
- ❒ Apply a generalized rule and knowledge about probability in order to reason mathematically.
- ❒ Determine the possible outcomes of a probability experiment involving two independent events.
- ❒ Determine the probabilities of favourable outcomes.