Malati

Mathematics learning and teaching initiative

Statistics

Probability 1

Grades 8 and 9

Teacher document

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We acknowledge the valuable comments of Heleen Verhage and Donald Katz.

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Guidelines for Module: Probability 1

We would like learners who have worked through this module to display the following:

1. An understanding of the notion of chance. Learners should be aware that the probability/chance of an event is expressed on a scale of likelihood (rather than simply suggesting that that an event is "possible" because it "could happen").
2. The ability to describe chance using descriptive words ("The Likelihood Scale") and fractions ("The Probability Scale").
3. An understanding of the notion that not all events have an equal likelihood of happening. Identifying those events that do / do not have an equal likelihood of occurring.
4. An understanding of the difference between calculating a probability theoretically and determining a success fraction experimentally and the notion that the success fraction tends towards the probability as the number of trials increases.
5. Appropriate use of terminology e.g. "probability", "success fraction", "outcome", ...

Pre-requisite Knowledge:
In the first activity ("The Likelihood Scale") learners are required to describe probability using descriptive words such as "likely" and "impossible". In the activities that follow, probability is described using fractions. It is thus important that learners have a working knowledge of fractions (including percentages). We recommend that teachers diagnose problems in this regard and provide the necessary remediation before beginning with this module. Teachers may wish to use the Rational Numbers module, specifically designed for this purpose.

The Activities:
Core: The Likelihood Scale
(Diagnostic Activity 1)
Coins and Drawing Pins
Zama Zama
Playing a Game with Coins
The Probability Scale
A Choir Competition
Party

Catering for diversity in the classroom:
In order for a learner to proceed after the activity "The Likelihood Scale", s/he needs to have an understanding of the notion of chance. It is thus suggested that the teacher perform a short diagnostic assessment (see Diagnostic Activity 1) to identify any problems in this regard. The class discussion during the completion of "The Likelihood Scale" will also provide an opportunity for the teacher to identify these problems.

<u>Remediation:</u> Consistent with our use of the "subjectivist" approach in our wider theoretical framework (see Malati probability rationale document) , we have found that discussion amongst learners can assist in addressing this problem. By sharing their ideas with one another, learners can reflect on their own ideas and possibly re-evaluate these if necessary. It is thus recommended that the teacher identify some

learners who have a good understanding of chance and have displayed the ability to justify their responses in the class discussion. These learners should be placed with those requiring remediation. The group can work through the assessment questions or sections of the "The Likelihood Scale" together. The teacher has an important role to play here, too, in providing the correct challenges and clarifying the use of the terminology for mathematical purposes. For example, in a problem in which three balls are placed in a bag, the teacher could increase the number of balls.

Those learners who have an understanding of the notion of chance should be given extension activities from another mathematical topic.

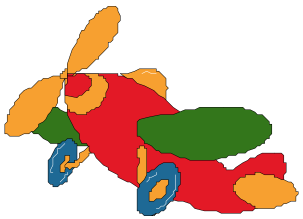
<u>Other Assessment:</u>
The outcomes numbered 2 to 5 above can be assessed at the end of Module 1: "Probability 1".

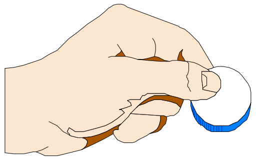
In the next module, "Probability 2", it is important that learners are able to express probabilities using fractions. In preparing for Module 2, the teacher can include an item similar to "Diagnostic Test 1", but requiring numerical answers, in the assessment to diagnose problems in this regard. Extension activities on the topic of probability or another mathematical topic could be used.

See also Guidelines for Module: Probability 2

The Likelihood Scale

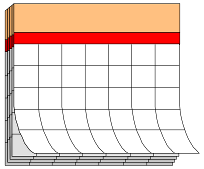
Sometimes we know that an event cannot happen, for example, we cannot fly to the sun. We say the event is **impossible**.





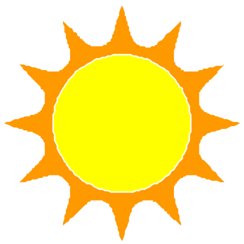
Some events have a **50% chance** of happening or not happening. For example, when we toss a coin there is an equal chance of getting ‘heads’ or ‘tails’. So we say that there is a **50% chance** that a coin will land on ‘heads’ when we toss the coin.

Sometimes we are sure that an event will happen. For example, Wednesday will come after Tuesday. We say that the event is **certain**.



Impossible	**Unlikely**	**50% chance**	**Likely**	**Certain**

In summer, it doesn’t rain much in Cape Town, so on a chosen day in December, it is unlikely that it will rain.



If we choose a day in June, we cannot say that it is impossible that it will rain on that day in Cape Town. We cannot say that it is certain either!

But June is in winter and it rains in Cape Town in winter so we say that it is likely that it will rain in Cape Town in June.

Impossible	Very unlikely	Unlikely	**50% chance**	Likely	Very likely	Certain

1. Choose words from the scale above to help you describe the likelihood of each of these events:

   (a) Ben has two marbles of the same size in his pocket, a green one and a red one. He puts his hand into his pocket and, without looking, takes out a **red** marble.

   (b) Cindy has three marbles of the same size in her pocket, a green, a blue and a red marble. She puts her hand into her pocket and, without looking, takes out a **red** marble.

   (c) Leroy has six red marbles of the same size in his pocket. He puts his hand into his pocket and, without looking, takes out a **blue** marble.

Impossible	Very unlikely	Unlikely	**50% chance**	Likely	Very likely	Certain

2. In each row of this table, an **event** is described. Put a tick in the column which best describes the **likelihood** of each event.

Event	**Impossible**	**Very unlikely**	**Unlikely**	**50% chance**	**Likely**	**Very likely**	**Certain**
(a) Nosipho has ten marbles of the same size in her pocket, 6 white and 4 red marbles. She takes out a white marble when she puts her hand in her pocket without looking.							
(b) A slice of bread with butter and jam spread on one side is dropped on the floor. It lands with the jam side facing up.							
(c) A coin is tossed to decide who has the kick-off in a soccer match. Bafana Bafana wins the toss at the start of their next match.							
(d) A dice is thrown and the number '3' lands on top.							
(e) A light bulb which is expected to last for 300 hours blows after 2 hours.							
(f) A drawing pin is tossed and it lands with the pin facing upwards.							
(g) You will turn 2 years old on your next birthday.							
(h) Fifteen percent of Astros are blue. You choose a blue Astro from a full pack with your eyes closed.							
(i) Two percent of the valves in new soccer balls are defective. You buy a ball with a **good** valve.							

3. In this table the likelihoods are marked by ✓s. In each case, think of an **event** which would have this likelihood. Write your answer in the column called ‘Event’.

Event	Impossible	Very unlikely	Unlikely	50% chance	Likely	Very likely	Certain
(a)	✓						
(b)				✓			
(c)						✓	
(d)			✓				
(e)							✓

Teacher Notes: Likelihood Scale

*This activity is an introduction to chance and likelihood. Learners are required to classify certain happenings (events) according to the likelihood of each event occurring. At this stage the classification is mainly **descriptive** and not numerical, and learners should become accustomed to the use of the vocabulary of chance.*

It is important the learners be encouraged to read through the introduction. In order to ensure that they have read and understood this, the teacher can ask for other examples of events that are 'impossible', 'certain', 'likely', 'unlikely' or events that have a '50% chance of happening'.

<u>It is essential that a whole-class discussion is held after the learners have completed 1(a) to 1(c) and discussed these in their groups</u>

1(a) Ben has two marbles in his pocket, one of which is red, thus there is a "50% chance" of drawing either colour (both outcomes are equally likely).

1(b) Cindy has three marbles in her pocket, one of which is red. There is thus more chance of her drawing a marble of another colour (blue or green), than of drawing a blue marble. The likelihood is thus "unlikely" (but not "very unlikely"). Another explanation is that there is LESS chance of drawing the red marble now than there was in 1(a) – because there are now 3 marbles. Thus if one looks at the scale, the likelihood must be on the 'unlikely' side of '50% chance'.

Learners may say that there is a '50% chance' or that it is 'likely' that Cindy will draw the red marble. This is probably a problem with the mathematical interpretation of the word 'likely'...it does not mean the same as 'possible'. An event may be POSSIBLE (it may or may not happen), but one needs to examine the Likelihood Scale to see whether the event is in fact LIKELY.

If learners need to be challenged, the teacher can add more marbles (e.g. 8 marbles, only one of which is red), or ask the learners whether they would bet money on the red marble.

1(c) There are only red marbles in Leroy's pocket, so he cannot remove a blue marble. The event is thus "impossible".

The Likelihood Scale activity refers to a number of contexts and introduces important ideas about probability which will be revisited in later activities, namely.

- *the notion that not all events have an equal likelihood of happening*
- *in some cases the likelihood can be determined by reasoning, that is, by considering all the possible outcomes and the chance of each occurring. For example, in question 1(a) there is an equal chance of drawing a green or a red marble so there is a "50% chance" of drawing a red marble.*
- *In other cases it is only possible to predict the likelihood based on experimentation, that is, on what **usually** happens. For example, if one tosses a drawing pin, there are two possible ways the pin can land, but the likelihood of each happening is not equal. This example also reinforces the idea that although there might be two possible results/ events, these are not always equally likely.*

These ideas should be emphasised in discussion.

Some learners might base their decisions on personal experience, for example, in question 1(a) a learner might feel that the green marble will be drawn because green is his/her favourite colour or in question 2(d) a learner might think that a six has a greater chance of being on the top of the dice because six is his/her lucky number or because it is the biggest number. Such responses are well-documented in the literature. It is hoped that if the learners are able to discuss their responses, they might convince one another that such responses are not mathematically correct. Some of the issues are tackled in more detail in later activities.

Some learners may argue that there is a 50% chance of an event happening because it 'might or might not happen'. However, a 50% chance means that an event has an equal chance of happening or not happening.

It is important that learners can distinguish between events that are likely and very likely and similarly, between events that are unlikely and very unlikely. They could do this by considering whether an event is, in the first place, closer to "50% chance" or to "impossible", and in the second case, whether it is closer to "50% chance" or "certain".

Some learners might feel that the descriptions used here are too vague and see the need for numerical probabilities in giving an accurate description. In such a case a learner should be encouraged to suggest appropriate numbers for the likelihoods. The use of "50% chance" could provide a hint. Learners could, for example, suggest that . 'unlikely' be used for events with a chance between 25% and 50% and 'likely' for events between 50% and 75%.

It should be noted, however, that the use of rational numbers may hinder some learners and affect their grasp of probability. It is for this reason that the introduction of rational numbers as a means of describing chance is often delayed. Rather, learners could be required to mark off the approximate position of the relevant likelihood on the scale provided.

The use of numbers on a probability scale is explored in the activity "Probability Scale

<u>Answers:</u>

2(a) There are six white balls and four red balls in Nosipho's pocket, therefore it is 'likely" that a white will be drawn. The difference in the number of different coloured balls is not large enough to classify the event as "very unlikely".

2(b) Although theoretically there are only two sides on which the slice could land, the weight of the spread might affect the way it falls. It is possible that there is a greater chance of the slice landing on the jam side than on the non-jam side. The estimated difference can only be determined experimentally, but "unlikely" would be appropriate here. An answer of "50% chance" should also be accepted here. Learners might base their decisions on their personal experience, that is, the slice always lands with the jam side down!

2(c) When tossing an unbiased coin there is a "50% chance" that it will be heads and a "50% chance" that it will be tails. So there is a "50% chance" that Bafana

Bafana will win the toss in their next game. Learners might base their answers on what they know has happened in previous games, but it should be emphasised that the coin cannot remember what happened in previous games! It does not matter if Bafana Bafana have won the toss 3 times in a row: there is still a "50% chance" as each coin is tossed independent of previous tosses.

2(d) There is an equal chance of a dice landing with any of the six numbers on top. There is less chance of the '3' being on top than of any of the other five numbers being on top, so the answer is "very unlikely". Learners might be influenced by their personal experience, for example, their favourite/ lucky numbers or on the size of the numbers.

2(e) The light bulbs are expected to last for 300 hours, so it would be "very unlikely" that one would blow after just 2 hours. Learners might disagree because of personal experience. It could be suggested that other influences might affect the effectiveness of the bulb, for example, faulty fittings or irregular electricity supply. The brand being considered by the learners might be different to the hypothetical brand mentioned here. Based purely on information gathered here, however, the answer "very unlikely" should be given.

2(f) It is more likely that the pin will land with head up than with the pin facing upwards. An answer of "unlikely" is sufficient here as the estimated likelihood can only be determined experimentally.

2(g) There will not be any one-year-old children in the class, so no-one will turn 2 on his/her next birthday. Thus the event is "impossible".

2(h) For every 15 blue Astros there are 85 that are not blue. So the chances of selecting a blue Astro are "very unlikely".

*2(I) Learners should take care when reading question 2.9. Only 2% of the balls have faulty valves, which means that 98% of the balls have good valves. So the chances are "very likely" that the ball you buy will have a good valve. (These two events, "a ball with a faulty valve" and "a ball with a good valve", are **complementary events**. This concept is explored in later activities.)*

*In the second activity learners are required to think of their own events to match the given likelihoods. They should be encouraged to compare and discuss one another's suggestions. The teacher should check that the sentences used are actual 'events'. For example, "tossing a coin" is **not** an event, but "tossing a coin and getting heads" is an event. Also, "7 blue marbles and 3 red marbles" is not an event, but "drawing a red marble from a bag containing 7 blue and 3 red marbles" is acceptable.*
Learners should give sound reasons for the events that they provide.

Diagnostic Activity 1

Impossible	Very unlikely	Unlikely	50% chance	Likely	Very likely	Certain

Vusi has a bag containing marbles of the same size. Each time he closes his eyes and removes one marble from the bag.

Choose words from the scale above to describe the likelihood of each of these events:

1. There are two marbles in the bag, a black marble and a white marble. Vusi removes a **black** marble.

2. There are three marbles in the bag, a black one, a white one and a yellow one. Vusi removes a **black** marble.

3. There are five marbles in the bag, a black one, a white one, a red one, a blue one and a yellow one. Vusi removes a **white** marble.

4. There are ten marbles in the bag. Each marble is a different colour and there is one white marble in this bag. Vusi removes a **white** marble.

5. There are six red marbles in the bag. Vusi removes a **red** marble.

6. There are six marbles in the bag, three red marbles and three blue marbles. Vusi removes a **white** marble.

Teacher Notes: Diagnostic Activity 1

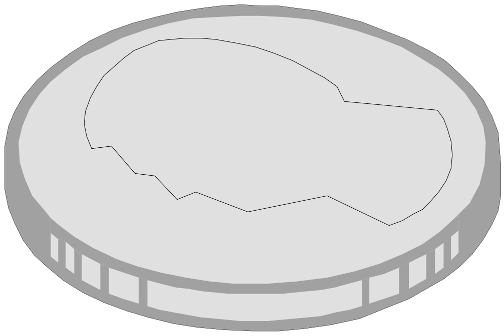
For a learner to proceed after the activity "The Likelihood Scale", s/he needs to have an understanding of the notion of chance. It is thus suggested that the teacher perform this short diagnostic assessment to identify any problems in this regard. (The class discussion during the completion of "The Likelihood Scale" will also provide an opportunity for the teacher to identify these problems.)

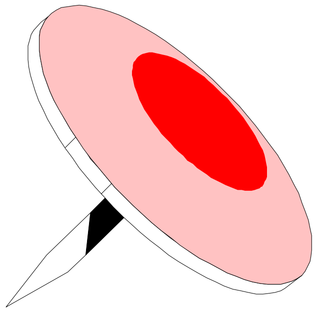
<u>Remediation:</u>

It is recommended that learners with different perspectives be given the opportunity to discuss their answers. By sharing their ideas with one another, learners can reflect on their own ideas and possibly re-evaluate these if necessary. It is thus suggested that the teacher identify some learners who have a good understanding of chance and have displayed the ability to justify their responses in the class discussion. These learners should be placed with those requiring remediation. The group can work through the assessment questions or sections of the "Likelihood Scale" together. The teacher has an important role to play here, too, in providing the correct challenges and clarifying the use of the terminology for mathematical purposes. For example, in a problem in which three balls are placed in a bag, the teacher could increase the number of balls.

Those learners who have an understanding of the notion of chance should be given extension activities from another mathematical topic.

Coins and Drawing Pins





How many sides does a coin have? How many ways can it land?

How many ways can a drawing pin land?

What are the possible *outcomes* of throwing a coin?
And of throwing a drawing pin?

If we toss a coin, what is the likelihood of it landing on each of these sides?

If we toss a drawing pin, what is the likelihood of it landing in each of these ways?

We are going to investigate both of these questions by working in groups.

1. Discuss your *predictions* for each of these experiments.

2. Toss a coin and a drawing pin 10 times each. Record your results in a logical way.

Coin

1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

RESULT:

Drawing Pin

1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

RESULT:

The result of an experiment is sometimes expressed as a *success fraction*. For example, if the coin landed on 'heads' 6 times out of 10, the success fraction for 'heads' can be expressed as $\frac{6}{10}$, where 6 is the number of *successful outcomes* and 10 is the total number of *possible outcomes*.

3. What are the success fractions for throwing 'heads' in your experiment? And for 'tails'?

   And for the different possible outcomes of throwing a drawing pin?

4. How do the results of the experiments compare with your predictions?

5. Now toss the coin and drawing pin another 20 times each. Record your success fractions for the total 30 throws here.

Coin	Drawing Pin

6. How do the results of the experiments compare with your predictions?

7. Now toss the coin and drawing pin another 20 times each. Add the success fractions of all the coin experiments and all the drawing pin experiments. Record your success fractions here:

Coin	Drawing Pin

8. Discuss how your prediction (probability) and the results (success fractions) for the '10-toss', '30-toss' and 'total' experiments were the same or different. What do you notice?

9. Now add the results of all the coin experiments and all the drawing pin experiments in your class. What can you conclude about the probability of each of the possible results of tossing a coin? And a drawing pin?

10. Explain in your own words what you think the words 'probability' and 'success fraction' mean. What is the relationship between them?

Teacher notes: Coins and Drawing Pins

This activity requires work in groups (or pairs). Each group must have a coin and a drawing pin. After 10 trials, the group records their results and calculates the success fractions, and this procedure is repeated until each groups has conducted 50 trials. Then the results of all the groups in the class should be combined so that a success fraction can be obtained for a larger number of trials.

To begin with, the teacher should ensure that learners are familiar with the coin and the drawing pin and agree that there are two possible outcomes for each. They may want to negotiate 'names' for the two outcomes of a drawing pin, such as 'pin up' and 'pin down' or 'on its side' and 'on its back'.

<u>It is very important that learners make a prediction for each of the experiments before commencing</u>*, or they may simply see this as a game and not reflect on the mathematics at all. Experiments are often used to CHECK predictions, or to determine the probability in cases like the drawing pin where probability cannot be determined theoretically at this level. This should emerge in the discussion following the activity.*

Two important terms are introduced, first casually and later more formally, namely:
Estimated ***probability*** *- this is based on theoretical predictions (not always possible).*
Success fraction *- this is based on the result of an experiment.*

Probability is defined as:

$$\frac{\textit{number of possible outcomes which satisfy a specific condition}}{\textit{total number of possible outcomes}}$$

Learners should grasp this concept, without memorising the definition. They should also be able to explain how this differs from the success fraction.

Learners should be encouraged to record the results of their experiments systematically, for example on the back of the worksheets using a table or tallies or a histogram, depending on their previous experience with such tools. The need for logical recording should be clear when learners realise that they have to share their results with their peers.

This activity introduces several important concepts, for example:

- *The difference between the result of an experiment (the success fraction) and the prediction (the probability), and the fact that the success fraction tends towards the probability as the number of trials is increased (Question 10).*
- *The fact that it some cases it is not possible to theoretically predict the probability. For example, in the case of the drawing pin, one has to determine the probability by conducting an experiment (Question 11)*
- *The fact that if there are two possible outcomes, this does not necessarily imply that the probability/likelihood of each is 50%. For example, in the case of the*

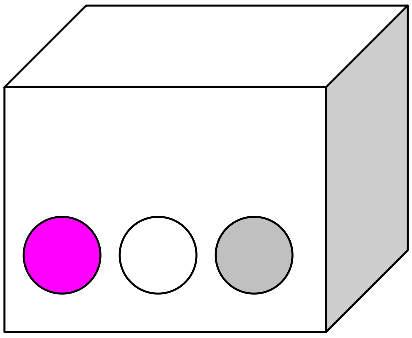
drawing pin, it has been found, after many thousands of trials, that the drawing pin is 60% likely to land on its 'side' with the pin facing downwards, and 40% likely to land 'flat on its back' with the pin facing upwards (Question 11).

- *Thus the coin and the drawing pin illustrate that even in the case where there are only two possible outcomes, these are not always equally likely, and the probability of the carious outcomes cannot always be easily determined.*

ZAMA ZAMA !

Mpande is a contestant in the Zama Zama Game Show.

She is told that a box contains three balls of the **same size**: a pink ball, a yellow ball and a blue ball. She cannot see the balls in the box.



In this game she must remove a ball from the box **and then put it back** before selecting another one.

1. What is the chance of Mpande removing the pink ball? And the blue ball?

   (a) Show your answers on the likelihood scale below.

Impossible	Very unlikely	Unlikely	**50% chance**	Likely	Very likely	Certain

   (b) Now write your answers as fractions.

Craig said that the yellow ball is most likely to be removed, because when he last watched Zama Zama, every contestant removed the yellow ball.

Craig's mathematics teacher suggested that the Grade 8's conduct an experiment to test Craig's claim. Each group was given a box containing three balls of equal size and the learners had to remove a ball without looking. They recorded the results after 10 tries and then again after another 20. The results of the experiment conducted by Craig's group are shown below:

Try	Pink	Blue	Yellow
1		✔	
2		✔	
3	✔		
4		✔	
5	✔		
6			✔
7	✔		
8	✔		
9	✔		
10		✔	
Total after 10	**5**	**4**	**1**

Try	Pink	Blue	Yellow
11			✔
12		✔	
13		✔	
14		✔	
15	✔		
16	✔		
17			✔
18		✔	
19			✔
20		✔	
21			✔
22			✔
23		✔	
24	✔		
25		✔	
26		✔	
27		✔	
28			✔
29	✔		
30		✔	
Total after 30	**9**	**14**	**7**

2. Which ball was removed most often after 10 tries? Write the result for each ball as a success fraction.

3. Which ball was removed most often after 30 tries? Write down the success fraction for each ball after 30 tries.

4. Write down anything interesting you notice about the success fractions after 10 tries and after 30 tries.

5. Do the results of this group's experiment agree with your prediction in question 1? Explain. Do the results agree with Craig's prediction?

Craig's teacher then took each group's results after 30 tries and combined them. The results are shown in this table:

No of Tries	Pink	Blue	Yellow
300	93	119	88

6. Compare the success fraction for each ball with those obtained in question 3. What do you notice?

7. Another Grade 8 class conducted this experiment and obtained the results for 300 tries. The two classes combined their results. Write down what you think the success fraction for each ball will be might be after 600 tries. And after 1000 tries?

Teacher Notes: Zama Zama

This activity deals with an experiment in which each of the three outcomes (removing a pink, blue or yellow ball) is equally likely, but the likelihood is not 50%.

In question 1 learners are required to predict the outcomes. This is the same situation as was discussed in Question 1(b) of the Likelihood Scale *but learners should try to write the answer (*$\frac{1}{3}$*) as a fraction as shown in the activity* Coins and Drawing Pins*.*

Learners are then required to work with the given results of an experiment. The results have been given to save time and to avoid the teacher having to rely completely on the outcomes of an experiment conducted in class, as in the activity Coins and Drawing Pins. Learners should note that the success fraction tends towards the probability (prediction) as the number of trails is increased. Learners are also given practice in determining success fractions.

Playing a Game with Coins

Imagine you are playing a game with your friend. You each have a coin, and you toss your coins together. You repeat this again and again.

The rules of the game are as follows:

- Player A scores a point if both players throw the *same* (in other words both throw heads or both throw tails).
- Player B scores a point if his coin is *different* to Player A's once they've both tossed their coins.
- The winner is the player with the most points.

1. Show all the possible *outcomes* (results).
2. Which player would you want to be if you wanted to win this game? Explain.
3. What is your chance of winning if you are Player A? And if you are Player B? Discuss.

Teacher Notes: Playing a Game with Coins

Some important concepts and terminology can be introduced using this activity. The terminology needs to be introduced by the teacher, but only <u>after</u> the meanings/concepts have been constructed and negotiated by the learners.

*Learners should be able to list all the possible outcomes (results) of throwing two coins. These results can be listed in a systematic way, by listing first all the outcomes with 'H' first (HT and HH) and then all the outcomes with T first (TH and TT). The word '**sample space**' (all the possible outcomes) should be introduced to them once this activity has been completed.*

If learners are unsure whether HT and TH are different outcomes, two learners can be asked to throw coins a few times and share the results with the rest of the class.

*The word "**chance**" is also introduced - what are the chances of scoring a point? Who has the greater chance of winning? This is a fair game because each player has an **equal chance** of winning. The teacher can talk about '**predicting**' outcomes.*
*The **likelihood or chance** of winning if you are Player A or B can be expressed using a variety of notation such as* $\frac{2}{4}$, $\frac{1}{2}$, *50% or 0,5. The teacher should encourage the use of different representations.*

The Probability Scale

The Grade 8 class are answering the following question:

*Cindy has four marbles of the same size in her pocket, a green, a blue, a yellow and a red marble. What are the chances that she takes out a **red** marble if she puts her hand in her pocket without looking?*

Mark and Kwena can't agree on an answer: Mark says this event is unlikely, but Kwena says it is "very unlikely".

Sometimes we need to be precise about how "unlikely" an event is. We can use numbers to describe the chances.

So, if one of the four marbles is red, there is a 1 in 4 chance that the marble Cindy takes out of her pocket will be red. We can also say that, provided we put our marble back after each selection, we would expect to choose the red marble in 1 out of every 4 selections, or $\frac{1}{4}$ or 0,25 of the times, or in 25 out of every 100 selections, or 25% of the selections.

- Use a ✯ to show this value on the probability scale:

0	0,1	0,2	0,3	0,4	0,5 or	0,6	0,7	0,8	0,9	1
0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

If Cindy has **only red marbles** in her pocket, we would expect her to choose a red marble. So there is 1 in 1 chance of getting a red marble, or we expect a red marble all or 1 (for whole) of the time, or in 100% of the selections.

If Cindy has **no blue marbles** in her pocket, there is a 0 in 1 chance of removing a blue marble, or we expect a blue marble for none or 0 (for none) of the time, or for 0% of the selections.

- Find these values on the probability scale.
- Is it possible to have a probability of more than 1 or less than 0? Explain.

1. Mark each of the following events on the above **probability scale.** Write the letter of the event in the position you have chosen.

> Choosing "randomly" is the same as choosing with your eyes closed – we don't know what we are going to get!

(a) A bag contains 3 marbles, 1 red and 2 blue marbles. What is the probability of randomly choosing a red marble?

(b) A bag contains 10 marbles, 6 white and 4 green marbles. What is the probability of randomly choosing a white marble?

(c) A dice is thrown. What is the probability of a 5 landing on top?

(d) A dice is thrown. What is the probability of a 7 landing on top?

(e) Of the estimated 14 000 000 people with Aids in Sub-Saharan Africa 7 500 000 are women.

(i) What is the probability that a person arriving with Aids at a hospital in Sub-Saharan Africa will be a woman?

(ii) What is the probability that a person arriving with Aids at a hospital in Sub-Saharan Africa will be a **man**?

2. On this probability scale there are 4 ☆s labelled A to D.

Give examples of events involving **marbles** that would have the probabilities marked A to D.
Now give exmaples of events of your own choice that would have these probabilities.

Teacher notes: Probability Scale

In the activity Likelihood Scale, *learners are required to describe chances using descriptive words, for example "likely", "certain" etc. When describing certain events, however, this is inadequate and numbers are required to describe probability more precisely. This activity requires that students consider events similar to those used in* Likelihood Scale, *but that they use fractions to describe the probabilities.*

It is important for learners to realise that a probability of any event can only range from 0 to 1. This can be clarified by explaining that something can never be less likely to happen than an impossible event or more likely to happen than a certain event. This can also be also clarified by means of the success fraction

$$\text{Probability of an event} = \frac{\text{number of successes}}{\text{number of attempts (or trials)}}$$

Clearly there can not be more successes than the number of trials so the greatest success fraction is 1. Similarly one can not have less than no successes; thus the smallest success fraction is 0.

Answers:

(a) $\frac{1}{3}$, $33\frac{1}{3}$*% or 33,33*

(b) $\frac{6}{10}$, $\frac{3}{5}$, *60% or 0,6*

(c) $\frac{1}{6}$, *16,67% or 0,167*

(d) *0*

(e)(I) $\frac{15}{28}$, *53,57% or 0,537 (or roughly* $\frac{1}{2}$*)*

(e)(ii) $\frac{13}{28}$, *46,43% or 0,464 (or roughly* $\frac{1}{2}$*).*

Question 1(e) is based on newspaper articles and is included so that learners can see the advantages of representing data as probabilities. The events in (I) and (ii) are complementary events, that is, the sum of the probabilities of the events is 1. This concept is explored in later activities where learners are required recognise complementary events and to work out the probability of the complement of an event by subtracting the probability of the event from 1.

*It is important that the teacher check the learners' examples for the given probabilities in question 2. Now that exact probabilities are given their answers should no longer be subjective, as they may have been in the '*Likelihood Scale*' worksheet. For example, learners should not give as an example of an event with a probability of*

40% "selecting a blue marble from a bag with more white than blue marbles". They should say exactly how many blue and white marbles (e.g. 6 white and 4 blue) would give the probability of selecting a blue ball to be 40%.

This is a good diagnostic activity on probability and also the learners' ability to work with fractions and percentages. If learners are having problems with fractions, there is Malati material available to help develop the necessary basic concepts.

A Choir Competition

A choir competition is being held at the Good Hope Centre in Cape Town. None of the choirs wants to sing first.

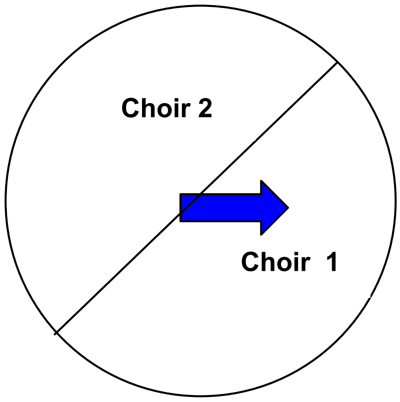
1. If there are **two** choirs in the competition, how can we decide who sings first? Remember that the method we choose must be fair to all the choirs taking part, that is, each choir must have an *equal chance* of being chosen.

2. What if there are **three** choirs in the competition?

3. What if there are **six** choirs?

4. What if there are **fourteen** choirs in the competition?

5. Marie says that she will use this ‘spinning wheel’ to make a fair choice between the two choirs in question 1:



She spins the wheel and looks where the arrow is pointing when the wheel stops moving.

If the arrow is pointing to Choir 1’s semicircle, then Choir 1 must sing first.

If the arrow points to Choir 2’s semicircle, Choir 2 must sing first.

(a) Is this a fair method? Explain.

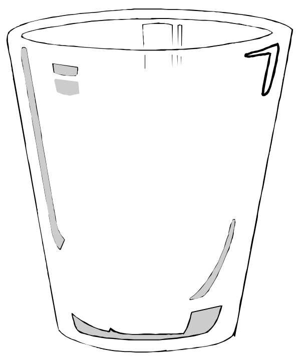
(b) Now design a spinner that could be used if there are **three** choirs.

6. Godfrey sings in Choir 1 and he does not want to sing first.

(a) Help Godfrey to design a spinning wheel that will give his choir less chance of having to sing first than Choirs 2 and 3 will have. (Remember that it must not be too obvious that Godfrey is cheating, or the spinning wheel will not be used.)

(b) Godfrey has another other idea: he says that if there are two choirs, they should toss a drawing pin to decide who sings first.
Godfrey says his choir (Choir 1) will sing first if the drawing pin lands with the pin facing up. Choir 2 must sing if the pin is facing down.
Why has Godfrey chosen this rule?

(c) Godfrey says that if there are **three** choirs he would choose to toss a plastic cup like this. Why has he chosen to use this cup?
What rules should he choose if he does not want his choir to sing first?



Teacher Notes: A Choir Competition

*This activity develops ideas introduced in the "Soccer" activity in primary Module 1). Learners are required to decide how a 'fair' choice can be made so that each choir has an **equal chance/ likelihood** of being chosen.*

Learners might suggest "taking a vote" or choosing the choir that "arrived last". They should be challenged to consider whether these methods are 'fair'. Would they be happy to accept the decision if they were singing in one of the choirs?

Possible answers:

*There are number of possibilities for questions 1 to 4, but in each case the possible outcomes must have an **equal likelihood** of occurring.*

Question 1: Tossing a coin; concealing two pieces of paper behind your back; drawing names form a hat; selecting certain numbers on a dice, eg odd and even.

*Question 2: Drawing names form a hat; selecting certain numbers on a dice, eg 1 and 2, 3 and 4, 5 and 6. Note that using a coin and the process of elimination is **not** a fair method.*

Question 3: Drawing names from a hat, tossing a dice.

Question 4: Drawing names from a hat.

*Question 5: The 'fairness' of the spinning wheel depends on the **angle** in the sector. A fair spinning wheel would have three angles of 120° at the centre.*

Question 6: The notion of 'bias' can be discussed here. In 6(b) it is more likely that the drawing pin will land with the 'head' of the pin up (refer to the activities 'Likelihood Scale' and 'Coins and Drawing Pins'). The actual likelihood cannot be calculated, but can be estimated using experiments. This differs from the use of the coin in which there is an equal chance of either outcome ("heads" and "tails"). In 6(c) there are three possible ways the cup can land, but it is most likely that the cup will land on its side.

Enrichment:

Learners can be challenged to consider the following:

- *Can a dice be used to make a decision between three choirs?*
- *Can you design a different shaped dice for making a decision between two / three etc choirs (learners could consider other polyhedra)?*
- *What if there is an even number of choirs? And an odd number of choirs?*
- *Can a triangle be used for a spinner? What other figures can be used?*

The President is holding a party for children at his house in Cape Town. He has invited sixteen learners from Mountain Senior Primary.

1. Mr Shembe has to decide which learners will go to the party. **He says that every learner should have the same chance of being chosen.**

   He takes this list of learners and says that five learners from each grade can go.

GRADE	NO. OF LEARNERS
Grade 4	48
Grade 5	52
Grade 6	43
Grade 7	40

   If you were a learner at Mountain Senior Primary, would you be happy with the method Mr Shembe has used to choose learners? Explain.

   If you do not agree, suggest a method that is fair.

2. Mr Jacobs, the Grade 5 teacher, has been told to choose 4 learners from his class. He says his method for choosing learners is fair: he places the 28 girls names in one box and the 24 boys names in another box and draws 2 names out of each box.

   Do you think this is a fair method? Explain.

   If you do not agree suggest a method which is fair.

Teacher Notes: Party

Learners are required to consider whether the decisions made have been fair, that is, whether all learners have the same chance of being selected.

In question 1 learners should note that learners in the smaller classes have a greater chance of being chosen. It would be better to place ***all*** *the names in a box and select sixteen names.*

In question 2 Mr Jacobs' method favours the boys. It could be suggested that all the names be placed in one box, or that the number drawn should be proportional to the total number in each box.