Kansas State University | Salina

Statistics Workbook

Certified Inspector Training Program

Kansas
Department of Transportation

Statistics Workbook

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INTRODUCTION

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Statistical Research Analyst

Bachelor of Science – Statistics KSU ‘97

Master of Public Administration – KU ‘13

Course Format

» Topics

- Basic Statistics (Section 5.2.1)
- Quality Level Analysis (Section 5.2.1)
- Statistical Comparison of Quality Control and Verification Tests (Section 5.2.6)

» Test

- Open Book; Open Note
- Multiple Choice Test
- 70% Required to Pass

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Need Help? Ask Questions Any Time

» Discussion Board

- Leave questions on the discussion board
- View other questions and responses

» Response Time

- Within 48 hours (hopefully much sooner)

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BASIC STATISTICS

KDOT Construction Manual

Section 5.2.1

Pages 3-6

Basic Statistics
Objectives

- Calculate basic statistical measures
- Perform basic statistical calculations on calculator
- Name methods to generate random numbers

Basic Statistics
Definition of Statistics

- **Statistics**
  - » science of interpreting numerical data that has been collected systematically, summarized, and tabulated
- By knowing what has happened or the way things are helps in making decisions; in making predictions; and in taking steps to make things for the better

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Basic Statistics
Measures and Procedures

Central value measures
- *Average or Mean*
- *4-Point Moving Average*

Variability measures
- *Range*
- *Standard Deviation*
- *Sample Variance*

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Basic Statistics
Common Terms

- **Data Set** - a group of data (numbers)

  » Numbers usually represented as variable
  – represents first number in data set,
  – represents second number in data set, etc…

  » Total number of variables represented as

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Basic Statistics
Random Sampling

- **Random number (5.2.2.2)**
  » A number selected entirely by chance as from a table of random numbers.
  – Note: Other methods of generating random numbers, such as with scientific calculators, may be approved by the District Materials Engineer.

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Basic Statistics
Random Sampling

- **Sample (5.2.2.2)**
  - » A small part of a lot or a sublot which represents the whole.
- **Random Sample**
  - » Use the **random number** to determine where to take your **sample**.
- Random sampling helps eliminate bias and ensures reliability of our data.

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Basic Statistics
Average or Mean ($\bar{x}$)

- The total sum of all variables (x_i) divided by the number of variables (n)

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\Sigma x_i}{n}$$

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Basic Statistics
Moving Average (x_{ma})

- Average computed based on a fixed set of continuous data points
- The 4-point moving average will be:

$$x_{ma_i} = \frac{x_i + x_{(i-1)} + x_{(i-2)} + x_{(i-3)}}{4}$$

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Basic Statistics
Range (R)

- The difference between the largest and the smallest values in a data set.
- A simple measure of variability.

$$R = x_{\max} - x_{\min}$$

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Basic Statistics
Standard Deviation

- Measure of the variation about the average of the data set
- Provides a better measure of variability than range
- 2 types of Standard Deviation depending on sample size
  - » **Sample Standard Deviation**
  - » Population Standard Deviation

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Basic Statistics
Sample Standard Deviation (s)

- The root mean square of the deviation from the mean.
- Typically used when the sample size is 30 or less.

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

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Basic Statistics
Variance (s^2)

- The square of the sample standard deviation

$$s^2$$

Basic Statistics
Example Problem

- Example #2
  - » Section 5.2.1 (pages 3 - 6)
  - » Asphalt content for six Superpave mix sublots
  - » Data Set: 5.4, 5.8, 6.2, 5.4, 5.4, and 6.0%

Basic Statistics
Calculator Usage

- Use YouTube
- Search for calculator model + “statistics”
- Example search for “TI-30xa statistics”

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Basic Statistics
Objectives

- Calculate basic statistical measures
- Perform basic statistical calculations on calculator
- Name methods to generate random numbers

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NORMAL DISTRIBUTION CURVE

KDOT Construction Manual

Section 5.2.1

Pages 7-10

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Normal Distribution Curve
Objectives

- List at least 4 characteristics of normal distribution curves
- Describe how normal distribution curves relate to process control
- Describe how normal distribution curves serve as a basis for statistical control charts

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Normal Distribution Curve
What is a Distribution?

- A listing or function showing all the possible values of the data set and how often they occur
- When organized, they're often ordered from smallest to largest, broken into reasonable sized groups and then put into graphs or charts to examine the shape, center and amount of variability in the data

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Normal Distribution Curve
Characteristics

- A typical "bell-shaped" symmetrical curve which usually describes the distribution of engineering measurements
- The **average or mean** is the curve center
- The **standard deviation** determines the curve shape

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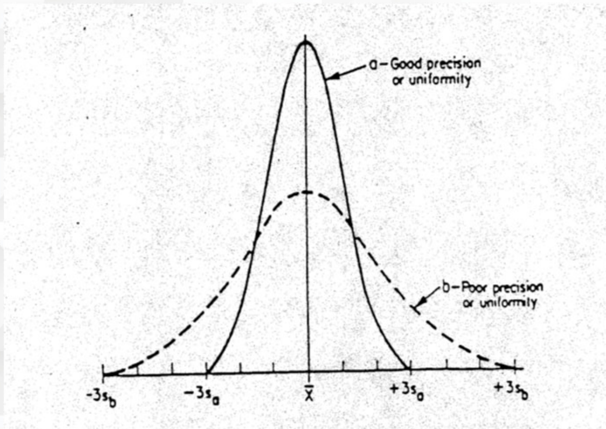
Normal Distribution Curve
Characteristics



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Normal Distribution Curve
Characteristics

- 68.26% of data falls within +/- 1 standard deviation of the average
- 95.44% of data falls within +/- 2 standard deviations of the average
- 99.74% of data falls within +/- 3 standard deviations of the average

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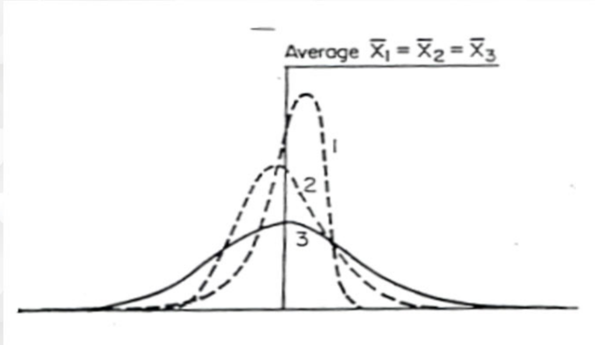
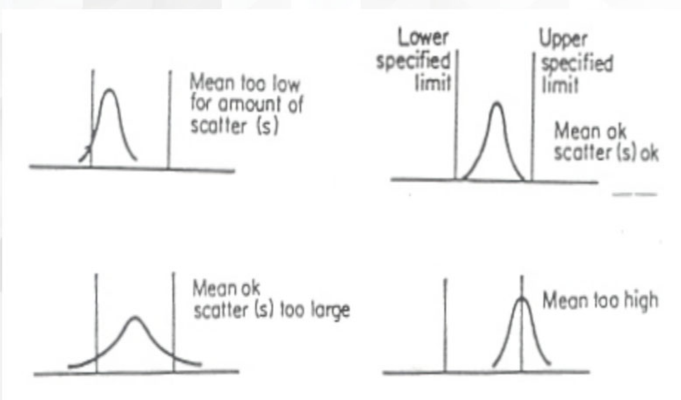
Normal Distribution Curve
Different Distributions Can Share the Same Mean



Normal Distribution Curve
Used for Process Control



Normal Distribution Curve
Used for Statistical Control Charts

- Normal distribution curves can serve as the basis for statistical control charts
- The upper specification limit (USL) and lower specification limit (LSL) can be based on multiples of standard deviation

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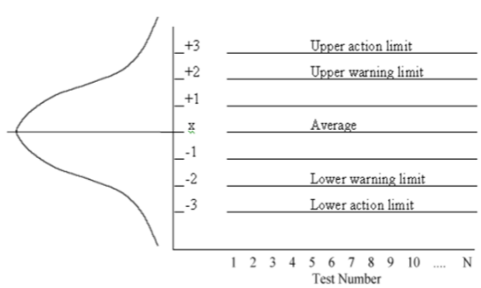
Normal Distribution Curve
Used for Statistical Control Charts



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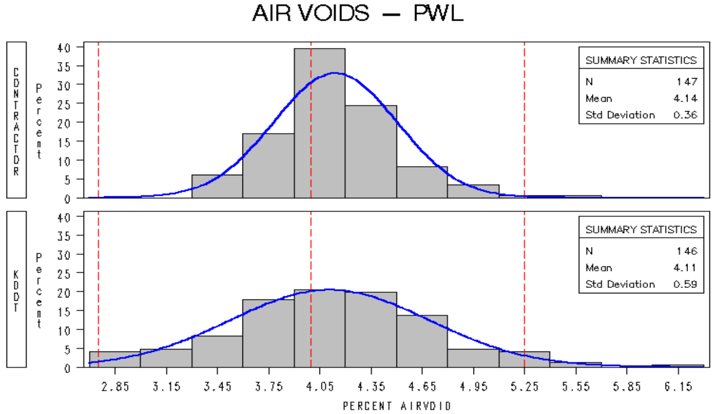
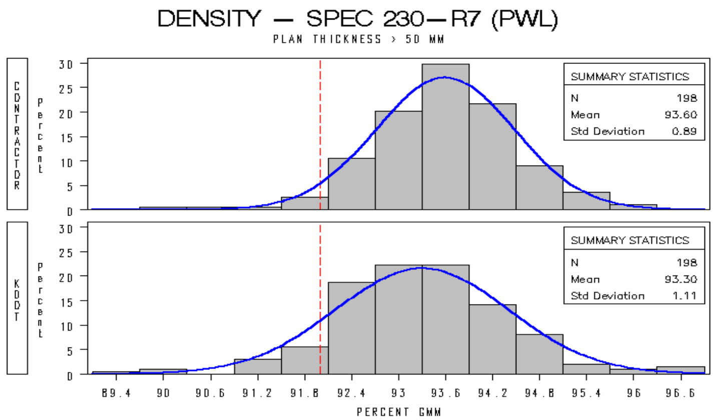
Normal Distribution Curve Examples



Normal Distribution Curve Examples



Normal Distribution Curve
Objectives

- List at least 4 characteristics of normal distribution curves
- Describe how normal distribution curves relate to process control
- Describe how normal distribution curves serve as a basis for statistical control charts

QUALITY LEVEL ANALYSIS

KDOT Construction Manual

Section 5.2.1

Pages 12-26

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Quality Level Analysis
Objectives

- Define Quality Level Analysis
- State and calculate upper and lower quality index formulas
- Use a table to find PWL
- Select and apply proper analyses

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Quality Level Analysis
Definitions

- A statistical procedure that provides a method for estimating the percentage of each lot of material, product, item of construction, or completed construction that may be expected within *specified tolerances.*

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Quality Level Analysis
Definitions

- **Percent Within Limits** (**PWL**) - amount of material or workmanship that has been determined by statistical method, to be within the pre-established characteristic boundary(ies)

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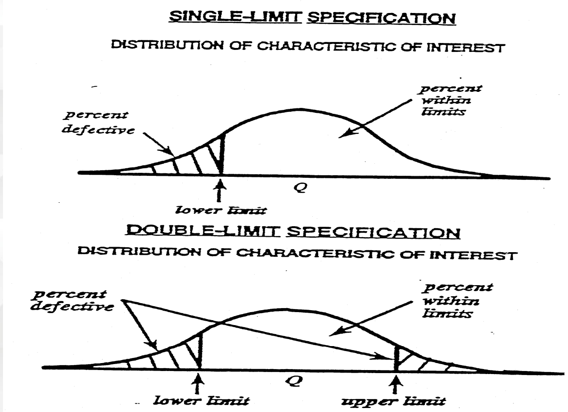
Quality Level Analysis
Concept of Percent Defective



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Quality Level Analysis
Upper Quality Index

- **Upper Quality Index** (Q_u) - subtract the average from the upper specification limit and divide by the sample standard deviation

$$Q_u = \frac{(USL - \overline{X})}{s}$$

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Quality Level Analysis
Lower Quality Index

- **Lower Quality Index** (Q_L) - subtract the lower specification limit from the average and divide by the sample standard deviation
  - » Same as Quality Index (Q) for single limit specifications with lower-limit

$$Q_L = \frac{\left(\overline{X} - LSL\right)}{s}$$

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Quality Level Analysis
Percent Within Limits

- **Percent Within Limits** (**PWL**) is determined from Table 2 after the Quality Index(es) have been computed
  - » If Quality Index is greater than 3.76, then PWL=100%
  - » If Quality Index is a negative number, the Percent Within Limits is equal to:

    100% - (PWL from Table 2)

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Quality Level Analysis
Percent Within Limits

$Q = 1.00$

$n = 5$

$PWL = 83.64\%$

Table 5.17.09-2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.91	78.89	80.33	80.93	81.22	81.39	81.49	81.56	81.61	81.73	81.77	81.81	81.83	81.85
0.92	79.34	80.67	81.23	81.51	81.67	81.77	81.84	81.89	82.00	82.04	82.08	82.10	82.11
0.93	79.81	81.00	81.54	81.81	81.96	82.05	82.12	82.16	82.27	82.31	82.34	82.36	82.37
0.94	80.27	81.33	81.84	82.10	82.24	82.33	82.39	82.44	82.54	82.57	82.60	82.62	82.63
0.95	80.75	81.67	82.14	82.39	82.52	82.61	82.67	82.71	82.80	82.84	82.86	82.88	82.89
0.96	81.25	82.00	82.45	82.67	82.80	82.88	82.94	82.97	83.06	83.10	83.12	83.13	83.14
0.97	81.75	82.33	82.75	82.96	83.08	83.15	83.21	83.24	83.32	83.35	83.37	83.39	83.39
0.98	82.26	82.67	83.04	83.24	83.35	83.43	83.47	83.51	83.58	83.61	83.63	83.64	83.64
0.99	82.79	83.00	83.34	83.52	83.63	83.69	83.74	83.77	83.84	83.86	83.88	83.88	83.89
1.00	83.33	83.33	83.64	83.80	83.90	83.96	84.00	84.03	84.09	84.11	84.12	84.13	84.13
1.01	83.89	83.67	83.93	84.08	84.17	84.22	84.26	84.28	84.34	84.36	84.37	84.37	84.38
1.02	84.47	84.00	84.22	84.36	84.44	84.49	84.52	84.54	84.59	84.60	84.61	84.62	84.62
1.03	85.07	84.33	84.52	84.63	84.70	84.75	84.77	84.79	84.83	84.85	84.85	84.85	84.85
1.04	85.69	84.67	84.81	84.91	84.97	85.00	85.03	85.04	85.08	85.09	85.09	85.09	85.09
1.05	86.34	85.00	85.09	85.18	85.23	85.26	85.28	85.29	85.32	85.33	85.33	85.32	85.32
1.06	87.02	85.33	85.38	85.45	85.49	85.51	85.53	85.54	85.56	85.56	85.56	85.55	85.55
1.07	87.73	85.67	85.67	85.71	85.74	85.76	85.78	85.78	85.80	85.80	85.79	85.78	85.78
1.08	88.49	86.00	85.95	85.98	86.00	86.01	86.02	86.03	86.03	86.03	86.02	86.01	86.00
1.09	89.29	86.33	86.24	86.24	86.25	86.26	86.27	86.27	86.26	86.26	86.25	86.23	86.23
1.10	90.16	86.67	86.52	86.50	86.51	86.51	86.51	86.50	86.49	86.48	86.47	86.46	86.45
1.11	91.11	87.00	86.80	86.76	86.75	86.75	86.74	86.74	86.72	86.71	86.69	86.68	86.66

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Quality Level Analysis
Percent Within Limits

$Q_L = -.55$

$n = 4$

$PWL_L = 100 - 68.33 = 31.67$

Table 5.17.09-2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.46	63.04	65.33	66.19	66.62	66.87	67.03	67.14	67.22	67.43	67.52	67.60	67.65	67.69
0.47	63.34	65.67	66.53	66.96	67.22	67.38	67.49	67.58	67.79	67.88	67.96	68.01	68.05
0.48	63.65	66.00	66.88	67.31	67.57	67.73	67.85	67.93	68.15	68.23	68.31	68.37	68.40
0.49	63.95	66.33	67.22	67.66	67.92	68.08	68.20	68.28	68.50	68.59	68.67	68.72	68.76
0.50	64.25	66.67	67.56	68.00	68.26	68.43	68.55	68.63	68.85	68.94	69.02	69.07	69.11
0.51	64.56	67.00	67.90	68.35	68.61	68.78	68.90	68.98	69.20	69.29	69.37	69.43	69.46
0.52	64.87	67.33	68.24	68.69	68.96	69.13	69.24	69.33	69.55	69.64	69.72	69.77	69.81
0.53	65.18	67.67	68.58	69.04	69.30	69.47	69.59	69.68	69.90	69.99	70.07	70.12	70.16
0.54	65.49	68.00	68.92	69.38	69.64	69.82	69.93	70.02	70.24	70.33	70.41	70.47	70.51
0.55	65.80	68.33	69.26	69.72	69.99	70.16	70.28	70.36	70.59	70.68	70.76	70.81	70.85
0.56	66.12	68.67	69.60	70.06	70.33	70.50	70.62	70.71	70.93	71.02	71.10	71.15	71.19
0.57	66.43	69.00	69.94	70.40	70.67	70.84	70.96	71.05	71.27	71.36	71.44	71.49	71.53
0.58	66.75	69.33	70.27	70.74	71.01	71.18	71.30	71.39	71.61	71.70	71.78	71.83	71.87
0.59	67.07	69.67	70.61	71.07	71.34	71.52	71.64	71.72	71.95	72.04	72.11	72.17	72.21
0.60	67.39	70.00	70.95	71.41	71.68	71.85	71.97	72.06	72.28	72.37	72.45	72.50	72.54
0.61	67.72	70.33	71.28	71.75	72.02	72.19	72.31	72.40	72.61	72.70	72.78	72.84	72.87
0.62	68.04	70.67	71.61	72.08	72.35	72.52	72.64	72.73	72.95	73.04	73.11	73.17	73.20
0.63	68.37	71.00	71.95	72.41	72.68	72.85	72.97	73.06	73.28	73.37	73.44	73.50	73.53
0.64	68.70	71.33	72.28	72.74	73.01	73.18	73.30	73.39	73.61	73.69	73.77	73.82	73.86
0.65	69.03	71.67	72.61	73.08	73.34	73.51	73.63	73.72	73.93	74.02	74.10	74.15	74.18

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Quality Level Analysis
Two Types of Analyses

- There are two types: double-limit and single-limit specifications
- Steps in analysis for a double-limit specification on Page 13
- Steps in analysis for a single-limit specification with a lower-limit specification shown on Page 14

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QUALITY LEVEL ANALYSIS

Double Limit Specification

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Quality Level Analysis
Double Limit Specification

A contractor has run air voids tests on five lots of SM-19B. The specification limits for air voids are 4 ± 1.0 %. This sets the lower specification limit (LSL) at 3.0% air voids and the upper specification limit (USL) at 5.0% air voids. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Percent Air Voids
1	1A	4.30
	1B	3.77
	1C	4.05
	1D	4.80
2	2A	4.90
	2B	5.07
	2C	3.82
	2D	3.53

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QUALITY LEVEL ANALYSIS

Single Limit Specification

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Quality Level Analysis
Single Limit Specification

The lower specification limit for concrete pavement thickness is 10.8 in. Using the following test results, conduct a Quality Level Analysis and compute the percent within limits.

Test	Thickness (in)
1	10.9
2	10.7
3	10.9
4	11.0
5	11.0

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Quality Level Analysis
Objectives

- Define Quality Level Analysis
- State and calculate upper and lower quality index formulas
- Use a table to find PWL
- Select and apply proper analyses

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COMPARISON TESTING
F-TEST

KDOT Construction Manual

Section 5.2.6

Pages 1-4

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Comparison Testing
Objectives

- Understand the need to compare test results
- Understand level of significance (α)
- Understand test comparison procedure
- Learn and execute **F-test** method

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Comparison Testing
Why?

- Compare QC test results and KDOT verification test results.
- Determine if the material under the test came from the same population.
- Use **F-test** and **t-test** to compare test results.

Comparison Testing
Level of Significance (α)

- F-test and t-test are conducted with a selected level of significance (α)
- Level of Significance (α) used is 1%
  - » 1% chance that test results are incorrect
    1) good material/product will be rejected
    2) bad material/ product will be accepted
  - » 99% level of confidence that F-test and t-test results are correct

Comparison Testing
Procedure

1) Use F-test to compare QC and KDOT variances
2) Use F-test result to determine which t-test method to use
3) Use t-test to compare QC and KDOT means
4) Use t-test result to determine if QC and KDOT test results are statistically equal

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F-TEST

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F-test
Compare Variances

- F-test determines if QC variance (s_c^2) and KDOT variance (s_v^2) are statistically equal or not:

If $F \geq F_{CRIT}$, then variances **not** equal.
OR
If $F < F_{CRIT}$, then variances **are** equal.

- F-test determines what t-test procedure to use.

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F-test
Procedure

- Find basic statistics for both contractor and KDOT:

$$n$$

$$\bar{x}$$

$$s$$

$$s^2$$

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F-test
Procedure

- Compute F-statistic value:

$$F = \frac{s_c^2}{s_v^2} \text{ or } \frac{s_v^2}{s_c^2}$$

Note: Larger variance always goes in numerator (top)

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F-test
Procedure

- Choose $\alpha = 1\%$ level of significance
- Degrees of Freedom
  Numerator: $n_{num} - 1$
  Denominator: $n_{den} - 1$

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F-test Procedure

- Find F_{CRIT} in Table 1

Table 5.17.08-1 **Critical Values, F_{crit} for the F-test for a Level of Significance, α =1%**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

$H_o: s^2_c = s^2_v$
$H_a: s^2_c \neq s^2_v$

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F-test Procedure

Compare calculated F value and F_{CRIT}

Two conclusions:

If $F \geq F_{CRIT}$, then variances **not** equal.

OR

If $F < F_{CRIT}$, then variances **are** equal.

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EXAMPLE PROBLEM
Case 1 - Concrete

Pg. 5.2.6-8

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Example Problem – Case 1

- A contractor has run 21 QC tests for compressive strength and KDOT has run 5 verification tests over the same period of time. The results are shown below. Is it likely that the tests came from the same population?
- Contractor QC Test Results: 36.40, 36.65, 32.69, 38.05, 38.54, 37.59, 36.57, 42.48, 36.99, 38.20, 37.53, 36.00, 41.28, 40.00, 38.37, 38.72, 40.36, 30.37, 34.87, 35.62, 36.06
- KDOT Verification Test Results: 36.10, 30.00, 37.00, 32.80, 30.60

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Example Problem – Case 1

Conclusion for F-test

$F = 1.34$ $F_{CRIT} = 5.17$

- Since $F < F_{CRIT}$, assume the variances are statistically equal
- Use the ***pooled variance*** and the ***pooled degrees of freedom*** when conducting the t-test

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Takeaways

- F-test compares variances s_c^2 and s_v^2
- There is only one F-test procedure
- F-test result only tells us which t-test procedure to use
  - Variances are equal
  - Variances not equal

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Comparison Testing
Objectives

- Understand the need to compare test results
- Understand level of significance (α)
- Understand test comparison procedure
- Learn and execute **F-test** method

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COMPARISON TESTING
t-test

KDOT Construction Manual

Section 5.2.6

Pages 5-12

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Comparison Testing
Objectives

- Learn and execute the **t-test** for variances are equal
- Learn and execute the **t-test** for variances not equal

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Comparison Testing
Why?

- Compare QC test results and KDOT verification test results.
- Determine if the material under the test came from the same population.
- Use **F-test** and **t-test** to compare test results.

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Comparison Testing
Level of Significance (α)

- F-test and t-test are conducted with a selected level of significance (α)
- Level of Significance (α) used is 1%
  - » 1% chance that test results are incorrect
    1) good material/product will be rejected
    2) bad material/ product will be accepted
  - » 99% level of confidence that F-test and t-test results are correct

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Comparison Testing
Procedure

1) Use F-test to compare QC and KDOT variances
2) Use F-test result to determine which t-test method to use
3) Use t-test to compare QC and KDOT means
4) Use t-test result to determine if QC and KDOT test results are statistically equal

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t-test
Variances are Equal

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t-test
Compare Means

- t-test determines if the QC mean ($\overline{X}_c$) and KDOT mean ($\overline{X}_v$) are equal or not:

If $t \geq t_{CRIT}$, then means **not** equal.
OR
If $t < t_{CRIT}$, then means **are** equal.

- **If there are no differences in the sample means, it indicates the material under test came from the same population**

t-test
Variances **are** equal ($F < F_{CRIT}$)

- Use pooled variance (s_p^2), which is the weighted average of variance from both sets of tests when calculating t factor
- Use pooled degrees of freedom ($n_c + n_v - 2$)
- When $t = 0$, the two data sets have exactly the same means

t-test

Variances **are** equal ($F < F_{CRIT}$)

- Calculate pooled variance s_p^2

$$s_p^2 = \frac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2}$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Calculate t statistic

$$t = \frac{\left|\overline{X}_c - \overline{X}_v\right|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_v}}}$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Determine $\alpha = 1\%$
- Calculate degrees of freedom

$$df = n_c + n_v - 2$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Find t_{CRIT} in Table 2

Table 5.17.08-2 Critical t values

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

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t-test

Variances **are** equal ($F < F_{CRIT}$)

Compare calculated t value and t_{CRIT}

Two conclusions:

If $t \geq t_{CRIT}$, then means **not** equal.

OR

If $t < t_{CRIT}$, then means **are** equal.

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EXAMPLE PROBLEM
Case 1 – Concrete
(continued)

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Example Problem – Case 1

Conclusion for F-test

$F = 1.34$ $F_{CRIT} = 5.17$

- Since $F < F_{CRIT}$, assume the variances are statistically equal
- Use the ***pooled variance*** and the ***pooled degrees of freedom*** when conducting the t-test

Example Problem – Case 1

Conclusion for t – test

$t = 2.865$ $t_{CRIT} = 2.797$

- Since $t > t_{CRIT}$, assume that the sample means are not equal
- Probable that the two sets of test results did not come from the same population (or lot)
  - » 1% chance two sets of test results did come from same population as we used a level of significance of 1%

t-test
Variances not Equal

t-test

Variances **not** equal ($F \geq F_{CRIT}$)

- Use individual variances from test results to calculate t factor
- Calculate effective degrees of freedom ($f^{'}$), effective degrees of freedom rounded **down** to a whole number
- When $t = 0$, the two data sets have exactly the same means

t-test

Variances not equal ($F \geq F_{CRIT}$)

- Use individual variances from test results to calculate t factor
  - No s_p^2 to calculate
- Effective degrees of freedom (f')
  - Will be provided
- When $t = 0$, the two data sets have exactly the same means

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t-test

Variances not equal ($F \geq F_{CRIT}$)

- Determine $\alpha = 1\%$
- Calculate *effective* degrees of freedom f' (given!)

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t-test
Variances **are** equal ($F < F_{CRIT}$)

- Find t_{CRIT} in Table 2

Table 5.17.08-2 Critical t values

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
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5	4.032	2.571	2.015
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8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
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15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

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t-test
Variances **not** equal ($F \geq F_{CRIT}$)

Compare calculated t value and t_{CRIT}

Two conclusions:

If $t \geq t_{CRIT}$, then means **not** equal.

OR

If $t < t_{CRIT}$, then means **are** equal.

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EXAMPLE PROBLEM
Case 2 - Asphalt

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Example Problem – Case 2

- A contractor has run 10 QC tests and KDOT has run 5 verification tests over the same period of time for the asphalt density (%Gmm). The results are shown below. Is it likely that the tests came from the same population or lot?
- Contractor QC Test Results: 93.0, 92.4, 92.9, 93.6, 92.9, 92.9, 92.4, 93.4, 92.9, 92.4
- KDOT Verification Test Results: 95.5, 93.3, 94.1, 92.5, 92.7

Example Problem – Case 2

- From the beginning
- Conduct F-test
- Choose and conduct proper t-test

Example Problem – Case 2

Conclusion for F-test

$F = 8.76$ $F_{CRIT} = 7.96$

- Since $F \geq F_{CRIT}$, assume the variances are not statistically equal
- Use t-test for variances not equal
- Use the ***individual variances*** and the ***approximate degrees of freedom (f')*** when conducting the t-test

Example Problem – Case 2

Conclusion for t-test

$t = 1.318$ $t_{CRIT} = 4.604$

- Since $t < t_{CRIT}$, assume the sample means are equal
- Even though variances were assumed not equal as result of **F-test**, the **t-test** indicates that the sets of tests results came from populations (lots) that had the same mean

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Takeaways

- Comparison testing is a two-step process
  - F-test
  - t-test
- F-test
  - Compares variances
  - Only tells you which t-test to perform
- t-test
  - Compares means
  - There are two types
  - Tells us what we want to know

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Comparison Testing
Objectives

- Learn and execute the **t-test** for variances are equal
- Learn and execute the **t-test** for variances not equal

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