Stats Review Chapter 9

Note:

This review is meant to highlight basic concepts from the course.   It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

The questions are displayed on one slide followed by the answers are displayed in red on the next.

This review is available in alternate formats upon request.

In a survey of 10 musicians, 2 were found to be left-handed. Is it practical to construct the 90% confidence interval for the population proportion, p?

Confidence Interval for Population Proportion

In a survey of 10 musicians, 2 were found to be left-handed. Is it practical to construct the 90% confidence interval for the population proportion, p?

- Condition 1: n(.05)≤N

  - The sample size (10) is less than 5% of the population (millions of musicians), so the condition is met.

- Condition 2: np(1-p)≥10

  - $\hat{p} = \frac{2}{10} = .2$

  - $n\hat{p}(1 - \hat{p}) = 10(.2)(1 - .2) = 1.6$. This is less than 10 so this condition is not met.

  It would not be practical to construct the confidence interval.

Confidence Interval for Population Proportion

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Is it appropriate? Yes, it satisfies both conditions.

1) point estimate

$$\hat{p} = \frac{944}{1748} = .54$$

2) Find $z_{\alpha/2}$. First take $\frac{1-.90}{2} = .05$

      The corresponding z is -1.645. We will ignore the negative and just use 1.645.

3) Find the Margin of Error

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.645\sqrt{\frac{.54(1-.54)}{1748}} = .0196$$

4) Take the point estimate and add/subtract the margin of error

      .54-.0196=.5204, .54+.0196=.5596

The confidence interval is (.5204,.5596)

Sample Size Needed of Population Proportion

A researcher wants to estimate the proportion of Americans that have sleep deprivation. How large a sample is needed in order to be 95% confident and within 3% if

    a) the researcher used a previous estimate of 60%?

    b) the researcher doesn't use a previous estimate?

Sample Size Needed of Population Proportion

A researcher wants to estimate the proportion of Americans that have sleep deprivation. How large a sample is needed in order to be 95% confident and within 3% if

    a) the researcher used a previous estimate of 60%?

$$N = \hat{p}(1 - \hat{p})\left(\frac{Z\frac{\alpha}{2}}{E}\right)^2 = .60(1 - .60)\left(\frac{1.96}{.03}\right)^2 \cong 1024.4 \approx \mathbf{1025}$$

    b) the researcher doesn't use a previous estimate?

$$N = .25\left(\frac{Z\frac{\alpha}{2}}{E}\right)^2 = .25\left(\frac{1.96}{.03}\right)^2 \cong 1067.1 \approx \mathbf{1068}$$

Remember to always round up when finding sample sizes.

Find the t-value (using the table)

a) Right tail=.1, n=6

b) Left tail=.05, n=16

c) 90% confidence, n=21

d) 95% Confidence , n=83

e) 99% Confidence, n=1200

Find the t-value (using the table)

a) Right tail=.1, n=6

   Degrees of Freedom (df)=n-1=5, **t= 1.476**

b) Left tail=.05 n=16

   df=15, **t=-1.753** (t is negative for left tail)

c) 90% confidence, n=21

   df=20, $\frac{1-.90}{2} = .05, t = \mathbf{1.725}$

d) 95% Confidence , n=83

   df= 82 (this is not in the table so choose the closest one: 80),

   $\frac{1-.95}{2} = .025, t = \mathbf{1.990}$

e) 99% Confidence, n=1200

   df=1199, since this is more than 1000 we use the z-value

   $\frac{1-.99}{2} = .005, \ \boldsymbol{t \ (or \ z) = 2.576}$

Estimating a population mean

In a sample of 81 SARS patients, the mean incubation period was 4.6 days with a standard deviation of 15.9 days. Construct a 95% confident interval and interpret.

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In a sample of 81 SARS patients, the mean incubation period was 4.6 days with a standard deviation of 15.9 days. Construct a 95% confident interval and interpret.

- We cannot use z because we do not have the population standard deviation.

- Find $t_{\alpha/2}$:

  $\frac{1-.95}{2} = .025$, degrees of freedom =n-1=81-80. The corresponding t-value is 1.99

- Find the Margin of Error

$$E = t_{\alpha/2}\frac{s}{\sqrt{n}} = 1.99\frac{15.9}{\sqrt{81}} = 3.516$$

- Add/subtract the margin of error from the sample mean

  4.6-3.516 =1.084, 4.6+3.516=8.116

We are 95% confident that the mean incubation period for SARS patients is between 1.084 and 8.116.

Sample Size for Estimating Population Mean

How large must a sample be in order to be 95% confident within 2 points given a sample standard deviation of 13.67?

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$$n = \left(\frac{Z_{\frac{\alpha}{2}} \cdot S}{E}\right)^2 = \left(\frac{1.96 \cdot 13.67}{2}\right)^2 \cong \mathbf{179.5} \approx \mathbf{180}$$

Width of the Confidence Interval

What Happens to the width of the Confidence Interval

a) As the sample size increases

b) As the level of confidence increase

Width of the Confidence Interval

What Happens to the width of the Confidence Interval

a) As the sample size increases
   The width decreases

b) As the level of confidence increase
   The width Increases

To find out which margin of error is smaller (smaller width) with different sample sizes or level of confidence when everything else is the same calculate
$$\frac{z_{\alpha/2}}{\sqrt{n}} \text{ (for proportion) or } \frac{t_{\alpha/2}}{\sqrt{n}} \text{ (for mean) for both cases to see which is smaller.}$$

Reasonable Interpretation of Confidence Intervals

A 90% confidence interval for the hours that college students sleep during the weekday is (6.8, 10.8). Which interpretation(s) are correct?

a)   90% of college students sleep between 6.8 and 10.8 hours

b)   We're 90% confident that the mean number of hours of sleep that college students get any day of the week is between 6.8 and 10.8 hours

c)   There is a 90% probability that the mean hours of sleep that college students get during a weekday is between 6.8 and 10.8 hours.

d)   We're 90% confident that the mean number of hours of sleep that college students during a weekday is between 6.8 and 10.8 hours.

Reasonable Interpretation of Confidence Intervals

A 90% confidence interval for the hours that college students sleep during the weekday is (6.8, 10.8). Which interpretation(s) are correct?

a)   90% of college students sleep between 6.8 and 10.8 hours

   Flawed: makes an implication about individuals rather than the mean

b)   We're 90% confident that the mean number of hours of sleep that college students get any day of the week is between 6.8 and 10.8 hours

   Flawed: should be about the weekday, not any day of the week

c)   There is a 90% probability that the mean hours of sleep that college students get during a weekday is between 6.8 and 10.8 hours.

   Flawed: implies the population mean varies rather than the interval

d)   We're 90% confident that the mean number of hours of sleep that college students during a weekday is between 6.8 and 10.8 hours.

   Correct!

Find $\chi^2_{1-\alpha/2}$ **and** $\chi^2_{\alpha/2}$

90% confidence, n=20

95% confidence, n=25

Chi-Squared Values

Find $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$

90% confidence, n=20

α=1-.90=.1; df=n-1=19

- $\chi^2_{1-\alpha/2} = \chi^2_{1-.1/2} = \chi^2_{.95} = \mathbf{10.117}$

- $\chi^2_{\alpha/2} = \chi^2_{.1/2} = \chi^2_{.05} = \mathbf{30.144}$

95% confidence, n=25

α=1-.95=.05; df=n-1=24

- $\chi^2_{1-\alpha/2} = \chi^2_{1-.05/2} = \chi^2_{.975} = \mathbf{12.401}$

- $\chi^2_{\alpha/2} = \chi^2_{..05/2} = \chi^2_{.025} = \mathbf{39.364}$

- *If the degrees of freedom is not on the table, use the closest degrees of freedom*

- *If the degrees of freedom is directly between 2 values, find the mean of the values. Ex: For 65, take the χ² values for both 60 and 70 and average their χ² values.*

Estimating a Population Standard Deviation

A student randomly selects 10 paperback books at a store. The mean price is $8.75 with a standard deviation of $1.50. Construct a 95% confidence interval for the population standard deviation, σ. Interpret the confidence interval. Assume the data are normally distributed

Estimating a Population Standard Deviation

A student randomly selects 10 paperback books at a store. The mean price is $8.75 with a standard deviation of $1.50. Construct a 95% confidence interval for the population standard deviation, σ. Interpret the confidence interval. Assume the data are normally distributed

1. Find $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ for 95% confidence level and df=9

$$\chi^2_{1-.05/2} = 2.7$$

$$\chi^2_{.05/2} = 19.023$$

2. Find the lower and upper bounds of σ²

Lower: $\dfrac{(n-1)s^2}{\chi^2_{\alpha/2}} = \dfrac{(10-1)1.5^2}{19.023} = 1.065$

Upper: $\dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2}} = \dfrac{(10-1)1.5^2}{2.7} = 7.5$

3. Since we want the confidence interval for σ and we have the lower/upper bounds of σ², we need to take the square roots of the lower and upper bound.

Lower: $\sqrt{1.065} = 1.03$

Upper: $\sqrt{7.5} = 2.74$

The confidence interval is (1.03, 2.74)

We're 95% confident that the population standard deviation of price of paperback books at this store is between $1.03 and $2.74 hours.