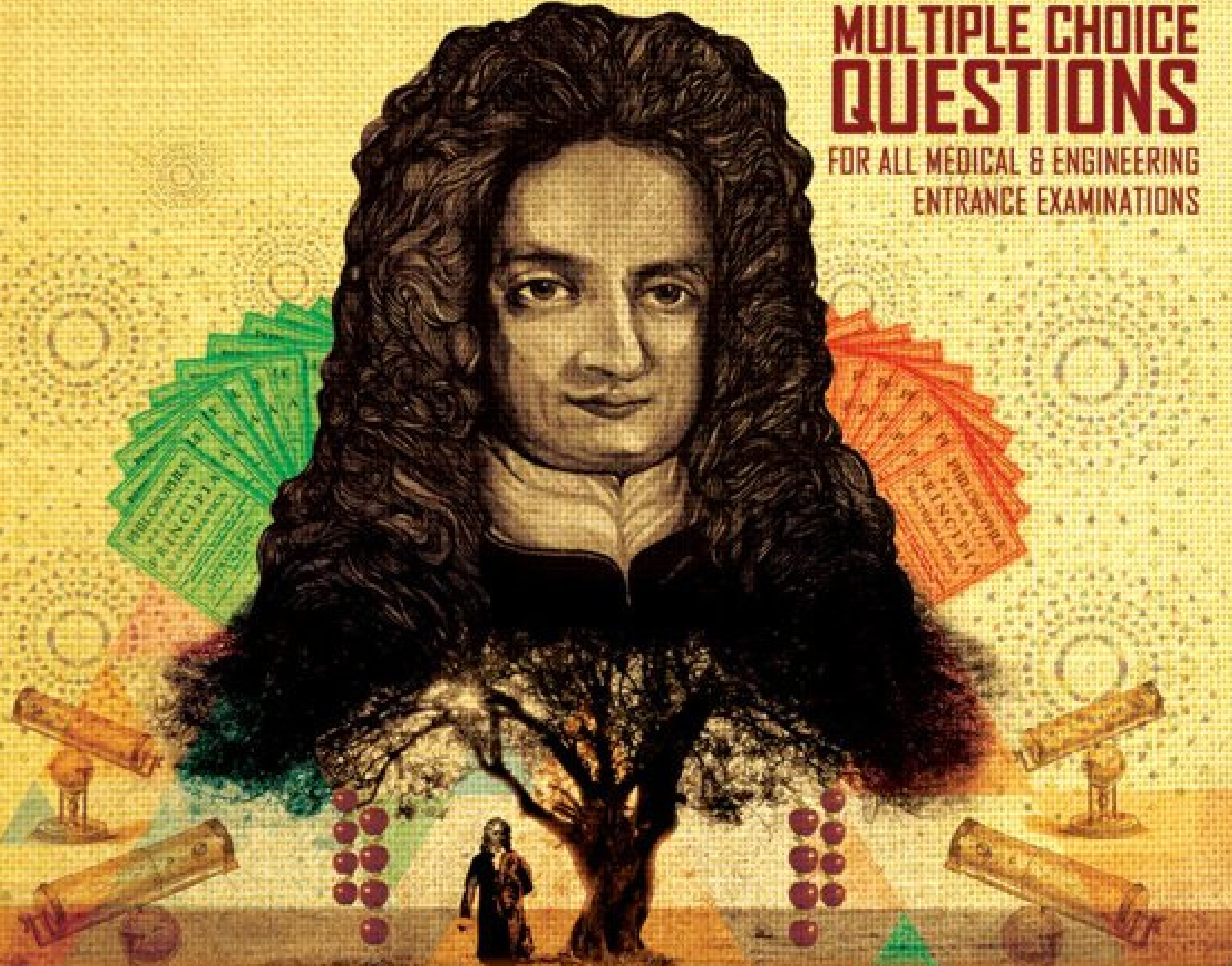
TRIUMPH PHYSICS

BASED ON STD. XII CURRICULUM (MAHARASHTRA BOARD)

MULTIPLE CHOICE QUESTIONS

FOR ALL MEDICAL & ENGINEERING ENTRANCE EXAMINATIONS



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STD. XII Sci.
Triumph Physics

Based on Maharashtra Board Syllabus

Third Edition: September 2014

Salient Features

- Exhaustive subtopic wise coverage of MCQs
- Important formulae provided in each chapter
- Hints included for relevant questions
- Various competitive exam questions updated till the latest year
- Includes solved MCQs from JEE (Main), AIPMT, CET 2014
- Evaluation test provided at the end of each chapter

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Preface

"Std. XII: Sci. Triumph Physics" is a complete and thorough guide to prepare students for a competitive level examination. The book will not only assist students with MCQs of Std. XII but will also help them to prepare for JEE, AIPMT, CET and various other competitive examinations.

The content of this book is based on the Maharashtra State Board Syllabus. **Formulae** that form a vital part of MCQ solving are provided in each chapter. **Notes** provide important information about the topic. **Shortcuts** provide easy and less tedious solving methods. **Mindbenders** have been introduced to bridge the gap between a text book topic and the student's understanding of the same. A quick reference to the notes, shortcuts and mindbenders has been provided wherever possible.

MCQs in each chapter are divided into three sections:

Classical Thinking: consists of straight forward questions including knowledge based questions.
Critical Thinking: consists of questions that require some understanding of the concept.
Competitive Thinking: consists of questions from various competitive examinations like JEE, AIPMT, CET, CPMT etc.

Hints have been provided to the MCQs which are broken down to the simplest form possible.

An **Evaluation Test** has been provided at the end of each chapter to assess the level of preparation of the student on a competitive level.

An additional feature called **"The physics of ....."** has been included in the book to foster a keen interest in the subject of physics.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on : mail@targetpublications.org

Best of luck to all the aspirants!

Yours faithfully
Authors

Contents

01 Circular Motion

Syllabus

Riding on a vertical circular arc, this roller coaster fans experience a net force and acceleration that point towards the centre of the circle

Formulae

1. Uniform Circular Motion (U.C.M.):

i. Instantaneous angular velocity,

$$\omega = \lim_{\Delta t \to 0} = \frac{\Delta\theta}{\Delta t} = \frac{v}{r} = 2\pi n = \frac{2\pi}{T}$$

ii. Average angular velocity,

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where,

θ_1 = angular position of the body at time t_1

θ_2 = angular position of the body at time t_2

iii. $\omega = \omega_{av}$ for U.C.M.

iv. If a particle makes n rotations in t second, then

$$\omega_{av} = \frac{2\pi n}{t}$$

v. Angular acceleration = $\alpha = 0$

vi. Instantaneous angular acceleration,

$$\alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

vii. Average angular acceleration,

$$\alpha_{ave.} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

where, ω_1 = instantaneous angular speed at time t_1

ω_2 = instantaneous angular speed at time t_2.

viii. Linear acceleration

$$= \text{centripetal acceleration} = \vec{\omega} \times \vec{v}$$

$$= a = v\omega = \frac{v^2}{r} = r\omega^2 = 4\pi^2 f^2 r = \frac{4\pi^2 r}{T^2}$$

ix. $$\text{Time period} = T = \frac{1}{\text{frequency}(f)} = \frac{2\pi}{\omega}$$

x. Relation between linear and angular velocity: $\vec{v} = \vec{\omega} \times \vec{r} = r\omega$ as $\theta = 90°$

2. Non-uniform circular motion:

i. Radial component of acceleration,

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

ii. Tangential component of acceleration,

$$a_t = \frac{dv}{dt}$$

iii. Net (linear) acceleration,

$$a = \sqrt{a_r^2 + a_t^2} \quad \text{....(Magnitude only)}$$

$$= \sqrt{\left(\frac{v^2}{r}\right) + \left(\frac{dv}{dt}\right)^2}$$

iv. Relation between tangential and angular acceleration,

$$\vec{a}_T = \vec{\alpha} \times \vec{r} = r\alpha$$

$\therefore$ $\theta = 0$

3. Centripetal force:

i. Centripetal force, $F_{cp} = \frac{mv^2}{r}$

$$= mr\omega^2 = mv\omega = mr(2\pi f)^2$$

$$= mr\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

ii. Magnitude of Centrifugal force,
= Magnitude of Centripetal force

i.e $F_{cf} = \frac{mv^2}{r}$ (in magnitude)

iii. When an electron moves round the nucleus of an atom along a circular path, we have

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} = m\omega^2 r$$

$$= m\,4\pi^2 n^2 r = m\frac{4\pi^2 r}{T^2}$$

where, Z = atomic number of the nucleus.

4. Motion of a vehicle on a curve road:

The maximum velocity v, with which a vehicle can take a safe turn so that there is no skidding, is $v = \sqrt{\mu rg}$

where, μ = coefficient of limiting friction between the wheels and the road.

5. Banking of roads:

The proper velocity or optimum v on a road banked by an angle θ with the horizontal is given by,

$$v = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$

where r = radius of curvature of road

g = acceleration due to gravity

μ_s = coefficient of friction between road and tyres

when $\mu_s = 0$, $v = \sqrt{rg\tan\theta}$

6. **Vertical Circular Motion:**
   i. Velocity at highest point $v_H \geq \sqrt{rg}$
   ii. Velocity at the lowest point $v_L \geq \sqrt{5rg}$
   iii. Velocity at a point along horizontal (midway position) $v_M \geq \sqrt{3rg}$
   iv. Acceleration at the highest point $a_H = g$
   v. Acceleration at the bottom point $a_L = 5g$
   vi. Acceleration along horizontal $a_M = 3g$
   vii. Tension at top most point,
   $$T_H = \frac{mv_B^2}{r} - mg \geq 0$$
   viii. Tension at the lowest point,
   $$T_L = \frac{mv_A^2}{r} + mg \geq 6\text{ mg}$$
   ix. Tension at a point where the string makes an angle θ with the lower vertical line
   $$T_\theta = \frac{mv^2}{r} + mg\cos\theta$$
   x. Tension at midway position where θ = 90° (i.e. along horizontal)
   $$T_M = \frac{mv^2}{r} \quad [\because \cos 90° = 0]$$
   xi. Total energy at different points at the top, bottom and horizontal,
   $$E_H = E_L = E_M = \frac{5}{2}mrg$$
   xii. Total energy at any point,
   $$E = \frac{1}{2}mv^2 + mgr(1 - \cos\theta)$$

7. **Conical Pendulum:**
   i. Angular velocity,
   a. $$\omega = \sqrt{\frac{g}{l\cos\theta}}$$
   b. $$\omega = \sqrt{\frac{g\tan\theta}{r}}$$
   ii. Periodic time $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$
   $$= \pi\sqrt{\frac{l\sin\theta}{g\tan\theta}}$$
   iii. Radius of horizontal circle,
   $r = l \sin\theta$

8. **Kinematical equations in circular motion in analog with linear motion:**
   i. $\omega = \omega_0 + \alpha t$
   ii. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
   iii. $\omega^2 = \omega_0^2 + 2\alpha\theta$

Notes

1. *Radian measure must be used in equations that contain linear and angular quantities.*
2. *Finite angular displacement is a scalar quantity because it does not obey the laws of vector addition.*
3. *In U.C.M., angular velocity $\left(\vec{\omega}\right)$ is only constant vector but angular acceleration $\left(\vec{\alpha}\right)$ and angular displacement $\left(\vec{\theta}\right)$ are variable vectors.*
4. *All the points on a rotating body in U.C.M. have same ω except centre as it is not rotating.*
5. *Instantaneous angular displacement is a vector quantity.*
6. *Angular speed is a scalar quantity but angular velocity is a vector quantity but both have same units i.e rad/s.*
7. *The direction of $\vec{\theta}$, $\vec{\omega}$, $\vec{\alpha}$ is given by the right hand thumb rule.*
8. *The value of ω of earth about its axis is 7×10^{-5} rad/s or 360° per day.*
9. *When a particle moves in a circle with constant speed, its velocity is variable because of changing direction.*
10. *Circular motion is a two-dimensional motion in which the linear velocity and linear acceleration vectors lie in the plane of the circle but the angular velocity and angular acceleration vectors are perpendicular to the plane of the circle.*
11. *Centrifugal force is a fictitious force and holds good in a rotating frame of reference.*

12. *An observer on the moving particle experiences only the centrifugal force, but an observer stationary with respect to the centre can experience or measure only the centripetal force.*

13. *Whenever a particle is in a U.C.M. or non U.C.M., centripetal and centrifugal forces act simultaneously. They are both equal and opposite but do not cancel each other.*

14. *Centripetal force and Centrifugal force are not action-reaction forces as action-reaction forces act on different bodies.*

15. *The direction of centripetal force is same whether the rotation of the circular path is clockwise or anticlockwise.*

16. *Centripetal force is not responsible for rotational motion of a body because only torque can produce rotational motion.*

17. *Since the centripetal force acting on a particle in circular motion acts perpendicular to its displacement (and also its velocity), the work done by it is always zero.*

18. *Centrifuge is an apparatus used to separate heavier particles from the lighter particles in a liquid.*

19. *Range of acceleration in circular motion* $90° < \theta \leq 180°$.

20. *The radius of the curved path is the distance from the centre of curved path to the centre of gravity of the body. It is to be considered when the centre of gravity of body is at a height from the surface of road or surface of spherical body.*

21. *Whenever a car is taking a horizontal turn, the normal reaction is at the inner wheel.*

22. *While taking a turn, when car overturns, its inner wheels leave the ground first.*

23. *For a vehicle negotiating a turn along a circular path, if its speed is very high, then the vehicle starts skidding outwards. This causes the radius of the circle to increase resulting in the decrease in the centripetal force.*

[$\because F_{cp} \propto \dfrac{1}{T}$]

24. *If a body moves in a cylindrical well (well of death,) the velocity required will be minimum safest velocity and in this case the weight of the body will be balanced by component of normal reaction and the minimum safest velocity is given by the formula* $\sqrt{\mu rg}$.

25. *Cyclist leans his cycle to make an angle to avoid topling; not to provide centripetal force.*

26. *If a body is kept at rest at the highest point of convex road and pushed along the surface to perform circular motion, the body will fall after travelling a vertical distance of* $\dfrac{r}{3}$ *from the highest point where r is the radius of the circular path.*

27. *When a body moves in a circular path with constant speed, its linear momentum changes at every point, but its kinetic energy remains constant.*

28. *In horizontal uniform circular motion, kinetic energy and magnitude of linear momentum remains constant, but the direction of linear momentum keeps on changing.*

29. *Since the centripetal force is not zero for a particle in circular motion, the torque acting is zero i.e.,* $\vec{\tau} = 0$ *(as the force is central) Hence the angular momentum is constant i.e.* $\vec{L}$ *= constant.*

30. *Whenever the body moves, the force responsible for motion is the vector sum of all the forces acting at that point.*
For example, Lift going up and down with acceleration 'a'.

31. *If a particle performing circular motion comes to rest momentarily, i.e.* $\vec{v} = 0$, *then it will move along the radius towards the centre and if its radial acceleration is zero, i.e.* $a_r = 0$, *then the body will move along the tangent drawn at that point.*

32. *For non uniform circular motion*
$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

33. *When a bucket full of water is rotated in a vertical circle, water will not spill only if velocity of bucket at the highest point is* $\geq \sqrt{gr}$.

34. *If velocity imparted to body at the lowest position is equal to* $\sqrt{2rg}$, *then it will oscillate in a semicircle.*

Mindbenders

1. The maximum velocity with which a vehicle can go without toppling, is given by

   $v = \sqrt{rg\frac{d}{2h}} = \sqrt{rg\tan\theta}$

   where, $\tan\theta = \frac{d}{2h}$

   d = distance between the wheels
   h = height of centre of gravity from the road
   g = acceleration due to gravity

2. Skidding of an object placed on a rotating platform:
   The maximum angular velocity of rotation of the platform so that object will not skid on it is

   $\omega_{max} = \sqrt{(\mu g / r)}$

3. The angle made by the resultant acceleration with the radius,

   $\alpha = \tan^{-1}\left(\frac{a_t}{a_r}\right)$

Shortcuts

1. The basic formula for acceleration is $a = \omega v$.

2. In U.C.M., if central angle or angular displacement is given, then simply apply $dv = 2v\ \sin\frac{\theta}{2}$ to determine change in velocity.

3. There are two types of acceleration; a_r (radial) and a_t (tangential) acceleration.

   Formula for $a_r = \omega^2 r$ and $a_t = \frac{dv}{dt}$ or $r\alpha$

4. To calculate angular displacement, apply the formula, $\theta = \omega t + \frac{1}{2}\alpha t^2$

5. To find out number of revolutions, always apply the formula,

   Number of revolutions $= \frac{\theta}{2\pi} = \frac{\omega t}{2\pi} = \frac{2\pi nt}{2\pi} = nt$

6. Since $\vec{F_c} \perp \vec{v}$, therefore, no work is done by the centripetal force. [2π rad = 360° = 1 rev.]

7. Angle which, a cyclist should make with the vertical is the angle of banking along a curved road.

8. On frictional surface, for a body performing circular motion, the centripetal force is provided by the force of friction.

   $f_S = \mu N$ but on horizontal surface $N = mg$

9. The minimum safe velocity for not overturning is $v = \sqrt{\frac{gdr}{2h}}$

10. While rounding a curve on a level road, centripetal force required by the vehicle is provided by force of friction between the tyres and the road.

    $\frac{mv^2}{r} = F = \mu R = \mu mg$

11. To avoid dependence on friction for the supply of necessary centripetal force, curved roads are usually banked by raising outer edge of the road above the inner edge.

12. The angle of banking (θ) is given by,

    $\tan\theta = \frac{v^2}{rg} = \frac{h}{\sqrt{l^2 - h^2}}$

    where h is height of the outer edge above the inner edge and l is length of the road.

13. On the same basis, a cyclist has to bend through an angle θ from his vertical position while rounding a curve of radius r with velocity v such that $\tan\theta = \frac{v^2}{rg}$

    If θ is very very small, then

    $\tan\theta = \sin\theta = \frac{v^2}{rg}$

    $\frac{v^2}{rg} = \frac{h}{l}$

    where h is height of the outer edge from the inner edge and l is the distance between the tracks or width of the road.

14. Always remember the formulae for velocity of the body at the top, bottom and at the middle of a circle with two distinct cases:
    i. path is convex
    ii. path is concave

    Remember in both the cases, formula will be different.

    i. $\frac{mv^2}{r}$ = mg − N where N is normal reaction.
    ii. $\frac{mv^2}{r}$ = N − mg

    Remember if in the question, it is given that body falls from a certain point then at that point N = 0.

15. Effect of rotation of earth about its axis:
    The apparent loss in weight of a body on its surface = $m\omega^2 R \cos^2 \phi$ where
    m = mass of body
    ω = angular velocity of earth
    R = radius of earth
    ϕ = latitude

16. In horizontal circle, tension will be equal to centripetal force i.e. $T = \frac{mv^2}{r}$
    i. The minimum velocity of projection at the lowest point of vertical circle so that the string slacken at the highest point, is given by $v_L = \sqrt{5gr}$
    ii. velocity at the highest point is $v_H = \sqrt{gr}$

17. If T_L is the tension at the lowest point and T_H is the tension at the highest point then
    $T_L - T_H = 6$ mg

18. When
    i. $v_L = \sqrt{2gr}$, the body moves in a vertical semicircle about the lowest point L,
    ii. $v_L < \sqrt{2gr}$, then the body oscillates in a circular arc smaller than the semicircle.
    iii. For a motor cyclist to loop a vertical loop, $v_L > \sqrt{5gr}$ and $v_H > \sqrt{gr}$

19. The total energy of any body revolving in a vertical circle is (5/2) mgr.

20. The distance travelled by the particle performing uniform circular motion in t seconds is given by the formula, $d = \frac{2\pi r}{T} t$.

21. To find out any unknown quantity, if body moves in vertical circle, resolve mg and if the body moves in horizontal circle, resolve tension or normal reaction.

22. **Centripetal Force in Different Situations:**

	Situation	The centripetal force
i.	A particle tied to a string and whirled in a horizontal circle	Tension in the string
ii.	Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
iii.	A vehicle on a speed breaker	Weight of the body or a component of weight
iv.	Revolution of earth around the sun	Gravitational force exerted by the sun
v.	Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons on electrons
vi.	A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the magnetic field
vii.	Coin placed on disk	In this case frictional force gives necessary centripetal force.
viii.	Car moving on a smooth banked road	N sin θ gives necessary centripetal force.
ix.	Passenger sitting in a turning car	Necessary centripetal force is provided by seat and passenger.

Classical Thinking

1.1 Angular displacement

1. The angular displacement in circular motion is
(A) dimensional quantity.
(B) dimensionless quantity.
(C) unitless and dimensionless quantity.
(D) unitless quantity.

2. Angular displacement is measured in
(A) metre. (B) time.
(C) radian. (D) steradian.

3. A flywheel rotates at a constant speed of 3000 r.p.m. The angle described by the shaft in one second is
(A) 3π rad (B) 30π rad
(C) 100π rad (D) 3000π rad

1.2 Angular velocity and angular acceleration

4. Direction of $\vec{\alpha} \times \vec{r}$ is
(A) tangent to path.
(B) perpendicular to path.
(C) parallel to the path.
(D) along the path.

5. What is the angular speed of the seconds hand of a watch?
(A) 60 rad/s (B) π rad/s
(C) $\pi/30$ rad/s (D) 2 rad/s

6. The angular velocity of a particle rotating in a circular orbit 100 times per minute is
(A) 1.66 rad/s (B) 10.47 rad/s
(C) 10.47 deg/s (D) 60 deg/s

7. A body of mass 100 g is revolving in a horizontal circle. If its frequency of rotation is 3.5 r.p.s. and radius of circular path is 0.5 m, the angular speed of the body is
(A) 18 rad/s (B) 20 rad/s
(C) 22 rad/s (D) 24 rad/s

8. What is the angular velocity of the earth?
(A) $\frac{2\pi}{86400}$ rad/s (B) $\frac{2\pi}{3600}$ rad/s
(C) $\frac{2\pi}{24}$ rad/s (D) $\frac{2\pi}{6400}$ rad/s

9. An electric motor of 12 horse-power generates an angular velocity of 125 rad/s. What will be the frequency of rotation?
(A) 20 (B) $20/\pi$
(C) $20/2\pi$ (D) 40

10. The ratio of angular speeds of seconds hand and hour hand of a watch is
(A) 1 : 720 (B) 60 : 1
(C) 1 : 60 (D) 720 : 1

11. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration is
(A) $r\omega^2$ (B) constant
(C) zero (D) $r\omega$

12. A wheel having a diameter of 3 m starts from rest and accelerates uniformly to an angular velocity of 210 r.p.m. in 5 seconds. Angular acceleration of the wheel is
(A) 4.4 rad s^{-2} (B) 3.3 rad s^{-2}
(C) 2.2 rad s^{-2} (D) 1.1 rad s^{-2}

1.3 Relation between linear velocity and angular velocity

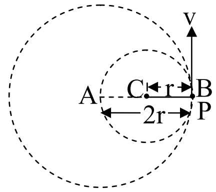
13. The vector relation between linear velocity $\vec{v}$, angular velocity $\vec{\omega}$ and radius vector $\vec{r}$ is given by
(A) $\vec{v} = \vec{\omega} \times \vec{r}$ (B) $\vec{v} = \vec{r} + \vec{\omega}$
(C) $\vec{v} = \vec{\omega} . \vec{r}$ (D) $\vec{v} = \vec{r} - \vec{\omega}$

14. A wheel has circumference C. If it makes f r.p.s., the linear speed of a point on the circumference is
(A) $2\pi fC$ (B) fC
(C) $fC/2\pi$ (D) fC/60

15. A body is whirled in a horizontal circle of radius 20 cm. It has angular velocity of 10 rad/s. What is its linear velocity at any point on circular path?
(A) 10 m/s (B) 2 m/s
(C) 20 m/s (D) $\sqrt{2}$ m/s

16. A particle moves in a circular path, 0.4 m in radius, with constant speed. If particle makes 5 revolutions in each second of its motion, the speed of the particle is
(A) 10.6 m/s (B) 11.2 m/s
(C) 12.6 m/s (D) 13.6 m/s

17. A particle P is moving in a circle of radius 'r' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B, the angular velocity of P about A and C are in the ratio
(A) 1 : 1
(B) 1 : 2
(C) 2 : 1
(D) 4 : 1



1.4 Uniform Circular Motion (U.C.M.)

18. In uniform circular motion,
(A) both velocity and acceleration are constant.
(B) velocity changes and acceleration is constant.
(C) velocity is constant and acceleration changes.
(D) both velocity and acceleration change.

19. A particle moves along a circular orbit with constant angular velocity. This necessarily means,
(A) its motion is confined to a single plane.
(B) its motion is not confined to a single plane.
(C) nothing can be said regarding the plane of motion.
(D) its motion is one-dimensional.

20. Select the WRONG statement.
(A) In U.C.M. linear speed is constant.
(B) In U.C.M. linear velocity is constant.
(C) In U.C.M. magnitude of angular momentum is constant.
(D) In U.C.M. angular velocity is constant.

21. If a particle moves in a circle describing equal angles in equal intervals of time, the velocity vector
(A) remains constant.
(B) changes in magnitude only.
(C) changes in direction only.
(D) changes both in magnitude and direction.

22. A particle moves along a circle with a uniform speed v. After the position vector has made an angle of 30° with the reference position, its speed will be
(A) $v\sqrt{2}$ (B) $\frac{v}{\sqrt{2}}$
(C) $\frac{v}{\sqrt{3}}$ (D) v

23. A car travels due north with a uniform velocity. As the car moves over muddy area, mud sticks to the tyre. The particles of the mud as it leaves the ground are thrown
(A) vertically upwards.
(B) vertically inwards.
(C) towards north.
(D) towards south.

1.5 Acceleration in U.C.M. (Radial acceleration)

24. A particle in U.C.M. possesses linear acceleration since
(A) its linear speed changes continuously.
(B) both magnitude and direction of linear velocity change continuously.
(C) direction of linear velocity changes continuously.
(D) its linear speed does not change continuously.

25. The acceleration of a particle in U.C.M. directed towards centre and along the radius is called
(A) centripetal acceleration.
(B) centrifugal acceleration.
(C) gravitational acceleration.
(D) tangential acceleration.

26. In an inertial frame of reference, a body performing uniform circular motion in clockwise direction has
(A) constant velocity.
(B) zero angular acceleration.
(C) centripetal acceleration.
(D) tangential acceleration.

27. An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 r.p.m., the acceleration of a point on the tip of the blade is about
(A) 1600 cm/s^2 (B) 4740 cm/s^2
(C) 2370 cm/s^2 (D) 5055 cm/s^2

28. The diameter of a flywheel is 1.2 m and it makes 900 revolutions per minute. Calculate the acceleration at a point on its rim.
(A) 540 π^2 m/s^2 (B) 270 m/s^2
(C) 360 π^2 m/s^2 (D) 540 m/s^2

29. The angular speed (in rev/min) needed for a centrifuge to produce an acceleration of 1000 g at a radius arm of 10 cm is
(Take g = 10 m/s^2)
(A) 1500 rev/min (B) 4000 rev/min
(C) 2000 rev/min (D) 3000 rev/min

30. If the angle between tangential acceleration and resultant acceleration in non U.C.M. is α, then direction of the resultant acceleration will be
(A) $\tan^{-1}\left(\frac{a_t}{a_r}\right)$ (B) $\tan^{-1}\left(\frac{a_r}{a_t}\right)$
(C) $\tan^{-1}\left(\frac{a_r}{a_\alpha}\right)$ (D) $\tan^{-1}\left(\frac{a_t}{a_\alpha}\right)$

31. A car is moving along a circular road at a speed of 20 m/s. The radius of circular road is 10 m. If the speed is increased at the rate of 30 m/s^2, what is the resultant acceleration at that moment?
(A) 10 m/s^2 (B) 50 m/s^2
(C) 250 m/s^2 (D) 80 m/s^2

1.6 Centripetal and centrifugal forces

32. The force required to keep a body in uniform circular motion is
(A) centripetal force.
(B) centrifugal force.
(C) frictional force.
(D) breaking force.

33. A vehicle moving on a horizontal road may be thrown outward due to
(A) gravitational force.
(B) normal reaction.
(C) frictional force between tyres and road.
(D) lack of proper centripetal force.

34. Select the WRONG statement.
(A) Centrifugal force has same magnitude as that of centripetal force.
(B) Centrifugal force is along the radius, away from the centre.
(C) Centrifugal force exists in inertial frame of reference.
(D) Centrifugal force is called pseudo force, as its origin cannot be explained.

35. An important consequence of centrifugal force is that the earth is,
(A) bulged at poles and flat at the equator.
(B) flat at poles and bulged at the equator.
(C) high tides and low tides.
(D) rising and setting of sun.

36. Fats can be separated from milk in a cream separator because of
(A) cohesive force.
(B) gravitational force.
(C) centrifugal force.
(D) viscous force.

37. When a car is going round a circular track, the resultant of all the forces on the car in an inertial frame is
(A) acting away from the centre.
(B) acting towards the centre.
(C) zero.
(D) acting tangential to the track.

38. Place a coin on gramophone disc near its centre and set the disc into rotation. As the speed of rotation increases, the coin will slide away from the centre of the disc. The motion of coin is due to
(A) radial force towards centre.
(B) non-conservative force.
(C) centrifugal force.
(D) centripetal force.

39. If p is the magnitude of linear momentum of a particle executing a uniform circular motion, then the ratio of centripetal force acting on the particle to its linear momentum is given by
(A) $\frac{r}{v}$ (B) $\frac{v^2}{mr}$
(C) $\frac{v}{r}$ (D) v.r

40. A racing car of mass 10^2 kg goes around a circular track (horizontal) of radius 10 m. The maximum thrust that track can withstand is 10^5 N. The maximum speed with which car can go around is
(A) 10 m/s (B) 100 m/s
(C) 50 m/s (D) 20 m/s

41. Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is
(A) $\frac{r_2}{r_1}$ (B) $\sqrt{\frac{r_2}{r_1}}$
(C) $\left(\frac{r_1}{r_2}\right)^2$ (D) $\left(\frac{r_2}{r_1}\right)^2$

42. A 10 kg object attached to a nylon cord outside a space vehicle is rotating at a speed of 5 m/s. If the force acting on the cord is 125 N, its radius of path is
(A) 2 m (B) 4 m
(C) 6 m (D) 1 m

43. The breaking tension of a string is 50 N. A body of mass 1 kg is tied to one end of a 1 m long string and whirled in a horizontal circle. The maximum speed of the body should be
(A) $5\sqrt{2}$ m/s (B) 10 m/s
(C) 7.5 m/s (D) 5 m/s

44. A proton of mass 1.6×10^{-27} kg goes round in a circular orbit of radius 0.12 m under a centripetal force of 6×10^{-14} N. Then the frequency of revolution of the proton is about
(A) 1.25×10^{6} cycles per second
(B) 2.50×10^{6} cycles per second
(C) 3.75×10^{6} cycles per second
(D) 5.00×10^{6} cycles per second

1.7 Banking of roads

45. The safety speed of a vehicle on a curve horizontal road is
(A) μrg (B) $\sqrt{\mu rg}$
(C) $\mu r^2 g$ (D) $\mu/(rg)^2$

46. The safe speed of a vehicle on a horizontal curve road is independent of
(A) mass of vehicle.
(B) coefficient of friction between road surface and tyre of vehicle.
(C) radius of curve.
(D) acceleration due to gravity.

47. The rail tracks are banked on the curves so that
(A) resultant force will be decreased.
(B) weight of train may be reduced.
(C) centrifugal force may be balanced by the horizontal component of the normal reaction of the rail.
(D) frictional force may be produced between the wheels and tracks.

48. The angle of banking of the road does not depend upon
(A) acceleration due to gravity.
(B) radius of curvature of the road.
(C) mass of the vehicle.
(D) speed of the vehicle.

49. For a banked curved road, the necessary centripetal force on any vehicle is provided by
(A) vertical component of normal reaction of the vehicle.
(B) horizontal component of the normal reaction of the vehicle.
(C) both vertical and horizontal components of the normal reaction of the vehicle.
(D) weight of the vehicle.

50. If the radius of the circular track decreases, then the angle of banking
(A) increases.
(B) decreases.
(C) first increases then decreases.
(D) does not change.

1.8 Conical Pendulum

51. When the bob of a conical pendulum is moving in a horizontal circle at constant speed, which quantity is fixed?
(A) Velocity (B) Acceleration
(C) Centripetal force (D) Kinetic energy

52. The period of a conical pendulum is
(A) equal to that of a simple pendulum of same length l.
(B) more than that of a simple pendulum of same length l.
(C) less than that of a simple pendulum of same length l.
(D) independent of length of pendulum.

53. Consider a simple pendulum of length 1 m. Its bob performs a circular motion in horizontal plane with its string making an angle 60° with the vertical. The centripetal acceleration experienced by the bob is
(A) 17.3 m/s^2 (B) 5.8 m/s^2
(C) 10 m/s^2 (D) 5 m/s^2

54. A particle of mass 1 kg is revolved in a horizontal circle of radius 1 m with the help of a string. If the maximum tension the string can withstand is $16\pi^2$ N, then the maximum frequency with which the particle can revolve is
(A) 3 Hz (B) 2 Hz
(C) 4 Hz (D) 5 Hz

1.9 Vertical Circular Motion

55. When a particle is moved in a vertical circle,
(A) it has constant radial and tangential acceleration.
(B) it has variable tangential and radial acceleration.
(C) it has only constant radial acceleration.
(D) it has only constant tangential acceleration.

56. For a particle moving in a vertical circle,
(A) kinetic energy is constant.
(B) potential energy is constant.
(C) neither K.E. nor P.E. is constant.
(D) both kinetic energy and potential energy are constant.

57. If a stone is tied to one end of the string and whirled in vertical circle, then the tension in the string at the lowest point is equal to
(A) centripetal force.
(B) the difference between centripetal force and weight of the stone.
(C) the addition of the centripetal force and weight of the stone.
(D) weight of the stone.

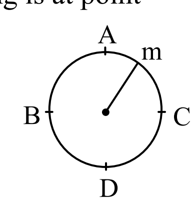
58. If a body is tied to a string and whirled in vertical circle, then the tension in the string at the highest position is
(A) maximum.
(B) minimum.
(C) between maximum and minimum values.
(D) zero.

59. A body of mass m is suspended from a string of length *l*. What is minimum horizontal velocity that should be given to the body in its lowest position so that it may complete one full revolution in the vertical plane with the point of suspension as the centre of the circle
(A) $v = \sqrt{2lg}$ (B) $v = \sqrt{3lg}$
(C) $v = \sqrt{4lg}$ (D) $v = \sqrt{5lg}$

60. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be
(A) $mg + \frac{mv^2}{r}$ (B) $mg - \frac{mv^2}{r}$
(C) $\frac{m^2v^2g}{r}$ (D) $\frac{v^2g}{r}$

61. A motor cycle is going on an over bridge of radius R. The driver maintains a constant speed. As motor cycle is descending, normal force on it
(A) increases. (B) decreases.
(C) remain the same. (D) fluctuates.

62. A particle of mass m tied with string is revolving in vertical circular motion with same speed. Maximum possibility of breaking the string is at point



(A) A (B) B
(C) C (D) D

63. A body of mass m is tied to a string of length *l* and whirled in a vertical circle. The velocity of the body at the lowest position is u. Then the tension in the string at a position when the string makes an angle θ with the vertical is
(A) $\frac{mu^2}{l}$
(B) $\frac{mu^2}{l} + mg\cos\theta$
(C) $\frac{mu^2}{l} + mg(2\cos\theta - 3)$
(D) $\frac{mu^2}{l} + mg(3\cos\theta - 2)$

64. A motorcyclist rides in a horizontal circle along the inner wall of cylindrical chamber of radius r. If the coefficient of friction between the tyres and the wall is μ, the minimum angular speed to prevent him from sliding down is
(A) $\sqrt{r\mu g}$ (B) $\frac{1}{r}\sqrt{\mu g}$
(C) $\sqrt{\frac{g}{r\mu}}$ (D) $\sqrt{\frac{r\mu}{g}}$

65. A particle is moving in a vertical circle. If v_1 is the velocity of particle at highest point and v_2 is the velocity of particle at lowest point, then the relation between v_1 and v_2 is
(A) $v_1 = v_2$
(B) $v_1 < v_2$
(C) $v_2 = \sqrt{5}\, v_1$
(D) $v_1 = \sqrt{5}\, v_2$

1.10 Kinematical equations in circular motion in analogy with linear motion

66. Calculate the angular acceleration of a centrifuge which is accelerated from rest to 350 r.p.s. in 220 s.
(A) 10 rad s^{-2} (B) 20 rad s^{-2}
(C) 25 rad s^{-2} (D) 30 rad s^{-2}

67. A wheel rotates with constant acceleration of 2.0 rad/s^2. If the wheel has an initial angular velocity of 4 rad/s, then the number of revolutions it makes in the first ten second will be approximately,
(A) 16 (B) 22
(C) 24 (D) 20

Miscellaneous

68. A car is moving at a speed of 72 km/hr. The diameter of its wheel is 0.5 m. If the wheels are stopped in 20 rotations applying brakes, then angular retardation produced by the brakes would be
    (A) -45.5 rad/s^2 (B) -33.5 rad/s^2
    (C) -25.48 rad/s^2 (D) -50.9 rad/s^2

69. A particle of mass 2 kg is rotating by means of a string in a vertical circle. The difference in the tensions at the bottom and the top would be
    (A) 12 kg wt (B) 2 kg wt
    (C) > 12 kg wt (D) < 12 kg wt

70. A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 cm. If the centripetal force F is kept constant but the angular velocity is doubled, the new radius of the path (original radius R) will be
    (A) R/4 (B) R/2
    (C) 2R (D) 4R

The physics of …..
A trapeze act in a circus….

In a circus, a man hangs upside down from a trapeze, legs bent over the bar and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight downward and is stationary or when the partner is swinging through the straight-down position?

The answer is at the end of this chapter.

Critical Thinking

1.1 Angular displacement

1. A wheel rotates with a constant angular velocity of 300 r.p.m. The angle through which the wheel rotates in one second is
   (A) π rad (B) 5π rad
   (C) 10π rad (D) 20π rad

2. For a particle in a non-uniform accelerated circular motion,
   (A) velocity is radial and acceleration is transverse only.
   (B) velocity is transverse and acceleration is radial only.
   (C) velocity is radial and acceleration has both radial and transverse components.
   (D) velocity is transverse and acceleration has both radial and transverse components.

1.2 Angular velocity and angular acceleration

3. A wheel completes 2000 revolutions to cover the 9.5 km distance. Then the diameter of the wheel is
   (A) 1.5 m (B) 1.5 cm
   (C) 7.5 cm (D) 7.5 m

4. The ratio of angular speed of second hand to that of the minute hand of a clock is
   (A) 60 : 1 (B) 1 : 60
   (C) 1 : 1 (D) 1 : 6

5. What is the angular speed of the minute hand of the clock in degrees per second?
   (A) 0.01 (B) 0.1
   (C) 1.0 (D) 0.001

6. A particle is describing the circular path of radius 20 m in every 2 s. The average angular speed of the particle during 4 s is
   (A) 20π rad s^{-1} (B) 4π rad s^{-1}
   (C) π rad s^{-1} (D) 2π rad s^{-1}

7. Calculate the angular acceleration if a flywheel gains a speed of 540 r.p.m. in 6 seconds.
   (A) 3π rad s^{-2} (B) 6π rad s^{-2}
   (C) 9π rad s^{-2} (D) 12π rad s^{-2}

8. A particle is in circular motion in a horizontal plane. It has angular velocity of 10 π rad/s at the end of 2 s and angular velocity 15 π rad/s at the end of 4 s. The angular acceleration of particle is
(A) 5 π rad/s^2 (B) 2.5 π rad/s^2
(C) 7.5 π rad/s^2 (D) 2 π rad/s^2

9. Angular displacement (θ) of a flywheel varies with time as $\theta = 2t + 3t^2$ radian. The angular acceleration at t = 2 s is given by
(A) 14 rad/s^2 (B) 18 rad/s^2
(C) 6 rad/s^2 (D) 16 rad/s^2

1.3 Relation between linear velocity and angular velocity

10. The linear velocity of a particle on the N-pole of the earth is
(A) zero. (B) 486 km/hr
(C) infinite. (D) 125 m/s

11. To enable a particle to describe a circular path, what should be the angle between its velocity and acceleration?
(A) 0° (B) 45°
(C) 90° (D) 180°

12. A body revolves n times in a circle of radius π cm in one minute. Its linear velocity is
(A) $\frac{60}{2n}$ cm/s (B) $\frac{2n}{60}$ cm/s
(C) $\frac{2\pi^2 n}{60}$ cm/s (D) $\frac{2\pi^2 n^2}{60}$ cm/s

13. Two cars C_1 and C_2 are going round concentric circles of radii R_1 and R_2. They complete the circular paths in the same time. Then $\frac{\text{Speed of } C_1}{\text{Speed of } C_2} =$
(A) 1
(B) R_1/R_2
(C) R_2/R_1
(D) can not be determined as data is insufficient

14. A wheel is 0.25 m in radius. When it makes 15 revolutions per minute, its linear speed at a point on circumference is
(A) $\frac{\pi}{2}$ m/s (B) $\frac{\pi}{8}$ m/s
(C) $\frac{\pi}{4}$ m/s (D) π m/s

15. A stone tied to the end of a string of length 50 cm is whirled in a horizontal circle with a constant speed. If the stone makes 40 revolutions in 20 s, then the speed of the stone along the circle is
(A) $\pi/2$ ms^{-1} (B) π ms^{-1}
(C) 2 π ms^{-1} (D) 4π ms^{-1}

16. The radius of the earth is 6400 km. The linear velocity of a point on the equator is nearly
(A) 1600 km/hr (B) 1675 km/hr
(C) 1500 km/hr (D) 1800 km/hr

17. What is the value of linear velocity if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?
(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (B) $-18\hat{i} - 13\hat{j} + 2\hat{k}$
(C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (D) $6\hat{i} - 2\hat{j} + 8\hat{k}$

18. If the equation for the displacement of a particle moving on a circular path is given by $\theta = 2t^3 + 0.5$, where θ is in radian and t is in seconds, then the angular velocity of the particle at t = 2 s is
(A) 8 rad/s (B) 12 rad/s
(C) 24 rad/s (D) 36 rad/s

1.4 Uniform Circular Motion (U.C.M.)

19. A particle covers equal distances around a circular path in equal intervals of time. Which of the following quantities connected with the motion of the particle remains constant with time?
(A) Displacement (B) Velocity
(C) Speed (D) Acceleration

20. A particle performing uniform circular motion has
(A) radial velocity and radial acceleration.
(B) radial velocity and transverse acceleration.
(C) transverse velocity and radial acceleration.
(D) transverse velocity and transverse acceleration.

21. **Assertion:** In circular motion, the centripetal and centrifugal forces acting in opposite direction balance each other.
Reason: Centripetal and centrifugal forces don't act at the same time.
(A) Assertion is True, Reason is True; Reason is a correct explanation for Assertion
(B) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion
(C) Assertion is True, Reason is False
(D) Assertion is False but Reason is True.

1.5 Acceleration in U.C.M. (Radial acceleration)

22. A car is travelling at a given instant 40 m/s on a circular road of radius 400 m. Its speed is increasing at the rate of 3 m/s. Its tangential acceleration is
(A) 4 m/s^2 (B) 3 m/s^2
(C) 5 m/s^2 (D) 2 m/s^2

23. For a particle in circular motion, the centripetal acceleration
(A) is less than its tangential acceleration.
(B) is equal to its tangential acceleration.
(C) is more than its tangential acceleration.
(D) may be more or less than its tangential acceleration.

24. If a body moves with constant speed along a curved path, its tangential acceleration is
(A) zero.
(B) is parallel to its velocity.
(C) perpendicular to its velocity.
(D) can make any arbitrary angle with its velocity.

25. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. Ratio of its centripetal acceleration to acceleration due to gravity is
(A) 9.2 (B) 6.25
(C) 5.0 (D) 8.25

26. A turn table which is rotating uniformly has a particle placed on it. As seen from the ground, the particle goes in a circle with speed 20 cm/s and acceleration 20 cm/s^2. The particle is now shifted to a new position where radius is half of the original value. The new values of speed and acceleration will be
(A) 10 cm/s, 10 cm/s^2
(B) 10 cm/s, 80 cm/s^2
(C) 40 cm/s, 10 cm/s^2
(D) 40 cm/s, 40 cm/s^2

27. A particle is moving on a circular path with constant speed, then its acceleration will be
(A) zero.
(B) external radial acceleration.
(C) internal radial acceleration.
(D) constant acceleration.

28. Two particles A and B are located at distances r_A and r_B respectively from the centre of a rotating disc such that $r_A > r_B$. In this case, if angular velocity ω of rotation is constant then
(A) both A and B do not have any acceleration.
(B) both A and B have same acceleration.
(C) A has greater acceleration than B.
(D) B has greater acceleration than A.

29. A particle goes round a circular path with uniform speed v. After describing half the circle, what is the change in its centripetal acceleration?
(A) $\frac{v^2}{r}$ (B) $\frac{2v^2}{r}$
(C) $\frac{2v^2}{\pi r}$ (D) $\frac{v^2}{\pi r}$

30. a_r and a_t represent radial and tangential accelerations respectively. The motion of the particle is uniformly circular only if
(A) $a_r = 0$ and $a_t = 0$
(B) $a_r = 0$ and $a_t \neq 0$
(C) $a_r \neq 0$ and $a_t = 0$
(D) $a_r \neq 0$ and $a_t \neq 0$

1.6 Centripetal and centrifugal forces

31. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same, then which of the following statements is true?
(A) The centripetal force will not suffer any change in magnitude.
(B) The centripetal force will have its direction reversed.
(C) The centripetal force will suffer change in direction.
(D) The centripetal force would be doubled.

32. A cylindrical vessel partially filled with water is rotated about its vertical central axis. Its surface will
(A) rise equally.
(B) rise from the sides.
(C) rise from the middle.
(D) lowered equally.

33. A car of mass 840 kg moves on a circular path with constant speed of 10 m/s. It is turned through 90° after travelling 660 m on the road. The centripetal force acting on the car is
(A) 324 N (B) 2640 N
(C) 284 N (D) 200 N

34. A body of mass 500 g is revolving in a horizontal circle of radius 0.49 m. The centripetal force acting on it (if its period is 11 sec) will be
(A) 0.008 N (B) 8.0 N
(C) 0.8 N (D) 0.08 N

35. The ratio of centripetal forces on two electrons which are revolving around nucleus of hydrogen atom in 2^{nd} and 3^{rd} orbits respectively is
(A) 27 : 8 (B) 81 : 16
(C) 8 : 27 (D) 16 : 81

36. A mass 2 kg describes a circle of radius 1.0 m on a smooth horizontal table at a uniform speed. It is joined to the centre of the circle by a string, which can just withstand 32 N. The greatest number of revolutions per minute the mass can make is
(A) 38 (B) 4
(C) 76 (D) 16

37. A particle performs uniform circular motion in a horizontal plane. The radius of the circle is 20 cm. The centripetal force acting on the particle is 10 N. Its kinetic energy is
(A) 0.1 J (B) 0.2 J
(C) 2.0 J (D 1.0 J

38. A coin placed on a rotating turn-table slips when it is placed at a distance of 9 cm from the centre. If the angular velocity of the turn-table is trippled, it will just slip if its distance from the centre is
(A) 27 cm (B) 9 cm
(C) 3 cm (D) 1 cm

39. A body is kept on a horizontal disc of radius 2 m at a distance of 1m from the centre. The coefficient of friction between the body and the surface of disc is 0.4. The speed of rotation of the disc at which the body starts slipping is ($g = 10\ m/s^2$)
(A) 2 rad/s (B) 4 rad/s
(C) 0.2 rad/s (D) 0.4 rad/s

40. A small coin is kept at the rim of a horizontal circular disc which is set into rotation about vertical axis passing through its centre. If radius of the disc is 5 cm and $\mu_s = 0.25$, then the angular speed at which the coin will just slip is
(A) 5 rad/s (B) 7 rad/s
(C) 10 rad/s (D) 4.9 rad/s

41. A string breaks under a load of 4 kg. A mass weighing 200 g is attached to the end of this string which is one metre long and rotated horizontally. The angular velocity of rotation when the string breaks, is nearly ($g = 10\ m/s^2$)
(A) 16 rad/s (B) 14 rad/s
(C) 12 rad/s (D) 20 rad/s

42. A bend in a level road has a radius of 100 m. The maximum speed with which a car turning this bend without skidding, if coefficient of friction between the tyres and the surface of the road is 0.8, will be ($g = 9.8\ m/s^2$)
(A) 20 m/s (B) 24 m/s
(C) 28 m/s (D) 32 m/s

43. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is 10 ms^{-1}. If the road becomes wet and $\mu' = \frac{\mu}{2}$, then what is the maximum speed permitted?
(A) 5 ms^{-1} (B) 10 ms^{-1}
(C) $10\sqrt{2}\ ms^{-1}$ (D) $5\sqrt{2}\ ms^{-1}$

44. A car moves at a speed of 36 km hr^{-1} on a level road. The coefficient of friction between the tyres and the road is 0.8. The car negotiates a curve of radius R. If $g = 10\ ms^{-2}$, then the car will skid (or slip) while negotiating the curve if the value R is
(A) 20 m (B) 12 m
(C) 14 m (D) 16 m

45. On a dry road, the maximum permissible speed of a car in a circular path is 12 ms^{-1}. If the road becomes wet, then the maximum speed is $4\sqrt{2}\ ms^{-1}$. If the coefficient of friction for dry road is μ, then that for the wet road is
(A) $\frac{2}{9}\mu$ (B) $\frac{\mu}{3}$
(C) $\frac{2\mu}{3}$ (D) $\frac{3}{4}\mu$

46. A body moves along circular path of radius 50 m and the coefficient of friction is 0.4. What should be its angular velocity in rad/s if it is not to slip from the surface? ($g = 9.8\ m/s^2$)
(A) 2.8 (B) 0.28
(C) 0.27 (D) 2.7

47. A crate of egg is located in the middle of the flat bed of a pick up truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of circle of radius 35 m. If the coefficient of friction between the crate and the flat bed of the truck is 0.6, the speed with which the truck should turn so that the crate does not slide over the bed is
(A) 14.3 m/s (B) 10.3 m/s
(C) 12.3 m/s (D) 15.3 m/s

48. The maximum frictional force between the tyres of a car and the road is 0.5 mg. The car negotiates a curve of radius 10 metre. The velocity is
(A) 10 m/s (B) 7 m/s
(C) 4.9 m/s (D) 14.2 m/s

49. A car of mass 1000 kg moves on a circular path with constant speed of 12 m/s. It turned through 90° after travelling 471 m on the road. The centripetal force acting on the car is
(A) 320 N (B) 480 N
(C) 640 N (D) 1280 N

50. A road is 10 m wide. Its radius of curvature is 50 m. The outer edge is above the lower edge by a distance of 1.5 m. This road is most suited for the velocity
(A) 2.6 m/s (B) 4.6 m/s
(C) 6.6 m/s (D) 8.6 m/s

51. A train has to negotiate a curve of radius 400 m. The speed of the train is 72 km/hour. The horizontal distance is to be raised with respect to the inner radius by h. If distance between rail is l = 1 m, the value of h will be ($g = 10\ m/s^2$)
(A) 15 cm (B) 10 cm
(C) 5 cm (D) 2.5 cm

1.7 Banking of roads

52. On a railway curve, the outside rail is laid higher than the inside one so that resultant force exerted on the wheels of the rail car by the tops of the rails will
(A) have a horizontal inward component.
(B) be vertical.
(C) equilibrate the centripetal force.
(D) be decreased.

53. A motor cyclist moves round a circular track with a certain speed and leans at an angle θ_1. If he doubles the speed, then he has to lean inward at an angle θ_2. Then
(A) $\theta_2 = 4\ \theta_1$ (B) $\theta_2 = 2\ \theta_1$
(C) $\tan\theta_1 = 4\tan\theta_2$ (D) $\tan\theta_2 = 4\tan\theta_1$

54. A railway track is banked for a speed v, by making the height of the outer rail h higher than that of the inner rail. If the distance between the rails is l and the radius of curvature of the track is r, then
(A) $\frac{h}{l} = \frac{v^2}{rg}$
(B) $\tan\left\{\sin^{-1}\left(\frac{h}{l}\right)\right\} = \frac{v^2}{rg}$
(C) $\tan^{-1}\left(\frac{h}{l}\right) = \frac{v^2}{rg}$
(D) $\frac{h}{r} = \frac{v^2}{lg}$

55. A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then
(A) $R_1 = R_2$ (B) $R_1 < R_2$
(C) $R_1 > R_2$ (D) $R_1 \geq R_2$

56. A railway line is banked with an angle of 0.01 radians. The height of the outer rail over inner rail, if the distance between the two rails of 1.5 m, will be
(A) 0.025 m (B) 0.035 m
(C) 0.015 m (D) 0.045 m

57. If angle of banking is $\sin^{-1}$ (0.2) and normal reaction is 2000 N then the weight of the car is
(A) 1959.6 N (B) 2000.8 N
(C) 21000 N (D) 22000 N

58. A bus is moving in a circular horizontal track of radius 10 m with a constant speed 10 m/s. A plumb bob is suspended from the roof of the bus by a light rigid rod of length 1.0 m. The angle made by the rod with the track is (Take $g = 10\ m/s^2$)
(A) zero (B) 30°
(C) 45° (D) 60°

59. A road is 8 m wide. Its radius of curvature is 40 m. The outer edge is above the lower edge by a distance of 1.2 m. The most suited velocity on the road is nearly
(A) $5.7\ ms^{-1}$ (B) $8\ ms^{-1}$
(C) $36.1\ ms^{-1}$ (D) $9.7\ ms^{-1}$

60. A circular road of radius 1000 m has banking angle 45°. If the coefficient of friction between tyre and road is 0.5, then the maximum safe speed of a car having mass 2000 kg will be
(A) 172 m/s (B) 124 m/s
(C) 99 m/s (D) 86 m/s

61. While driving around curve of radius 17.32 m, an engineer notes that a pendulum in his car hangs at an angle of 30° to the vertical. The speed of the car is (approximately)
(A) 10 m/s (B) 15 m/s
(C) 5 m/s (D) 6.7 m/s

62. A boy on a cycle pedals around a circle of radius 20 m at a speed of 20 m/s. The combined mass of the body and the cycle is 90 kg. The angle that the cycle makes with the vertical so that is may not fall is ($g = 9.8\ m/s^2$)
(A) 60.25° (B) 63.90°
(C) 26.12° (D) 30.00°

63. For traffic moving at 60 km/hour along a circular track of radius 0.1 km, the correct angle of banking is
(A) $\tan^{-1}\left(\frac{60^2}{0.1}\right)$
(B) $\tan^{-1}\left[\frac{(50/3)^2}{100\times 9.8}\right]$
(C) $\tan^{-1}\left[\frac{100\times 9.8}{(50/3)^2}\right]$
(D) $\tan^{-1}\sqrt{(60\times 0.1\times 9.8)}$

64. A circular racing car track has a radius of curvature of 500 m. The maximum speed of the car is 180 km/hr. The angle of banking θ is ($g = 10\ m/s^2$)
(A) $\theta = \tan^{-1}(0)$ (B) $\theta = \tan^{-1}(0.5)$
(C) $\theta = \tan^{-1}(0.3)$ (D) $\theta = \tan^{-1}(0.1)$

65. A cyclist with combined mass 80 kg goes around a curved road with a uniform speed 20 m/s. He has to bend inward by an angle $\theta = \tan^{-1}(0.50)$ with the vertical. The force of friction acting at the point of contact of tyres and road surface is
($g = 10\ m/s^2$)
(A) 300 N (B) 400 N
(C) 800 N (D) 250 N

66. The maximum safe speed for which a banked road is intended, is to be increased by 20 %. If the angle of banking is not changed, then the radius of curvature of the road should be changed from 30 m to
(A) 36.3 m (B) 21.1 m
(C) 43.2 m (D) 63.2 m

67. A cyclist going around a circular road of radius 10 m is observed to be bending inward 30° with vertical. Frictional force acting on the cyclist is (Given: $g = 10\ m/s^2$, mass of the cyclist is 90 kg)
(A) 532 N (B) 800 N
(C) 1559 N (D) 520 N

68. The maximum speed with which a vehicle can negotiate a curved road, which is banked at an angle $\theta = \tan^{-1}(0.24)$, is 54 km/hr. If the same road is flat and vehicle has to negotiate the curve with same maximum speed, the coefficient of friction between the road and tyres of the vehicle should be
(A) 0.35 (B) 0.24
(C) 0.8 (D) 0.5

1.8 Conical Pendulum

69. A mass of 10 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 r.p.m. Keeping the radius constant, the tension in the string is quadrupled. The new speed is nearly
(A) 14 r.p.m. (B) 10 r.p.m.
(C) 2.25 r.p.m. (D) 7 r.p.m.

70. Consider a simple pendulum of length 1 m. Its bob performs a circular motion in horizontal plane with its string making an angle 60° with the vertical. The period of rotation of the bob is (Take $g = 10\ m/s^2$)
(A) 2 s (B) 1.4 s
(C) 1.98 s (D) 2.4 s

71. The length of the string of a conical pendulum is 10 m and it has a bob of mass 50 g. The angle that the string makes with the vertical is 30°. If the bob covers one revolution in 3 s, then the corresponding centripetal force acting on the bob will be
(A) 10 N (B) 1 N
(C) 100 N (D) 5 N

72. In a conical pendulum, when the bob moves in a horizontal circle of radius r with uniform speed v, the string of length L describes a cone of semivertical angle θ. The tension in the string is given by

(A) $T = \frac{mgL}{\sqrt{L^2 - r^2}}$ (B) $\frac{(L^2 - r^2)^{1/2}}{mgL}$

(C) $T = \frac{mgL}{(L^2 - r^2)}$ (D) $T = \frac{mgL}{(L^2 - r^2)^2}$

1.9 Vertical Circular Motion

73. An aeroplane, flying in the sky, suddenly starts revolving in a vertical circle of radius 4 km. At the highest point of the circle, the pilot experiences weightlessness. Its velocity at the highest point will be

(A) 100 m/s (B) 200 m/s

(C) 300 m/s (D) 400 m/s

74. A hollow sphere has radius 6.4 m. Minimum velocity required by a motorcyclist at bottom to complete the circle will be

(A) 17.7 m/s (B) 10.2 m/s

(C) 12.4 m/s (D) 16.0 m/s

75. A bucket full of water is revolved in a vertical circle of radius 4 m such that water does not fall down. The time of one revolution is

(A) 10 second (B) 8 second

(C) 4 second (D) 6 second

76. A particle of mass m is rotating by means of a string in a vertical circle. The difference in tensions at the top and the bottom would be

(A) 6 mg (B) 4 mg

(C) 2 mg (D) 3 mg

77. A 2 kg stone at the end of a string 1 m long is whirled in a vertical circle at a constant speed of 4 m/s. The tension in the string will be 52 N when the stone is (Take $g = 10\ m/s^2$)

(A) at the top of the circle

(B) at the bottom of the circle

(C) halfway down

(D) at any position other than that in (A), (B) and (C)

78. A 40 kg child sits on a swing supported by two chains, each 3 m long. If the tension in each chain at lowest point is 350 N, then the child's speed at the lowest point is
[Take $g = 9.8\ m/s^2$]

(A) 4.7 m/s (B) 3 m/s

(C) 7.2 m/s (D) 9.1 m/s

79. An aeroplane flying in the sky with a uniform speed of 200 m/s moves in a vertical circle of radius 400 m. The mass of the pilot is 70 kg. The force exerted by the pilot on the seat at the highest point of the circle will be
[Take $g = 10\ m/s^2$]

(A) 3000 N (B) 6300 N

(C) 7700 N (D) 630 N

80. In the above problem, the force exerted by the pilot on the seat at the lowest point of the circle will be [Take $g = 10\ m/s^2$]

(A) 4500 N (B) 6300 N

(C) 7700 N (D) 770 N

81. A woman weighing 600 N is sitting in a car which is travelling at a constant speed on a straight road. The car suddenly goes over a hump in the road (hump may be regarded as an arc of a circle of radius 12.1 m). If the woman experiences weightlessness, calculate the speed of the car. [Take $g = 10\ m/s^2$]

(A) 11 m/s (B) 8 m/s

(C) 15 m/s (D) 5 m/s

82. A body of mass 1 kg is moving in a vertical circular path of radius 1 m. The difference between the kinetic energies at its highest and lowest positions is [Take $g = 10\ m/s^2$]

(A) 20 J (B) 10 J

(C) $4\sqrt{5}$ J (D) $10(\sqrt{5} - 1)$ J

83. The maximum and minimum tensions in the string whirling in a circle of radius 2.5 m with constant velocity are in the ratio 5 : 3. Its velocity is

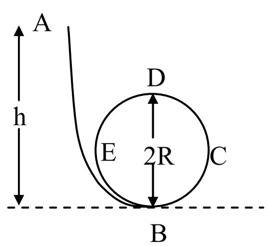
(A) $\sqrt{98}$ m/s (B) 7 m/s

(C) $\sqrt{490}$ m/s (D) $\sqrt{4.9}$ m/s

84. A student weighing 667 N rides a steadily rotating Ferris wheel (student sits upright). At the highest point, the magnitude of the normal force $\vec{N}$ on the student from the seat is 556 N. The magnitude of $\vec{N}$, if the wheel's speed is doubled, is
(A) 223 N　(B) 111 N
(C) 444 N　(D) 332 N

85. A body slides down a frictionless track which ends in a circular loop of diameter D. Then the minimum height h of the body in terms of D so that it may just complete the loop is
(A) $h = \frac{5}{2}D$　(B) $h = \frac{3}{2}D$
(C) $h = \frac{5}{4}D$　(D) $h = 2D$

86. A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is



(A) 5 cm　(B) $\frac{15}{4}$ cm
(C) $\frac{10}{3}$ cm　(D) 2 cm

87. **Assertion:** For looping a vertical loop of radius r, the minimum velocity at the lowest point should be $\sqrt{5gr}$.
Reason: Velocity at the highest point would be zero.
(A) Assertion is True, Reason is True; Reason is a correct explanation for Assertion
(B) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion
(C) Assertion is True, Reason is False
(D) Assertion is False but Reason is True.

1.10 Kinematical equations in circular motion in analogy with linear motion

88. A flywheel at rest is reached to an angular velocity of 36 rad/s in 6 s with a constant angular acceleration. The total angle turned during this interval is
(A) 216 rad　(B) 144 rad
(C) 108 rad　(D) 72 rad

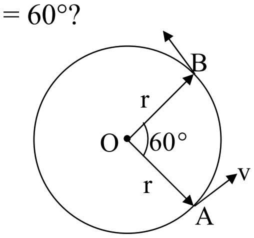
89. An engine requires 5 s to go from a speed of 600 r.p.m. to 1200 r.p.m. with constant acceleration. How many revolutions does it make in this period?
(A) 7.50　(B) 750
(C) 75　(D) 7500

90. A wheel of a vehicle is rotated to a uniform angular acceleration about its axis. Initially its angular velocity is zero. It rotates through an angle θ_1 in the first 2 s and in the next 3 s, it rotates through an additional angle θ_2. The ratio of $\frac{\theta_2}{\theta_1}$ is
(A) $\frac{4}{21}$　(B) $\frac{21}{4}$
(C) $\frac{4}{25}$　(D) $\frac{25}{4}$

91. When a ceiling fan is switched off, its angular velocity reduces to 50% while it makes 36 rotations. How many more rotations will it make before coming to rest? (Assume uniform angular retardation)
(A) 18　(B) 12
(C) 36　(D) 48

Miscellaneous

92. A particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from A to B if $\angle AOB = 60°$?



(A) 2 v sin 30°　(B) 2 v cos 30°
(C) 2 v sin 60°　(D) 2 v cos 60°

93. A particle moves along a circle of radius $20/\pi$ m with a constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the 2^{nd} revolution after motion has begun, the tangential acceleration is

(A) $40\ \pi\ m/s^2$ (B) $40\ m/s^2$

(C) $640\ \pi\ m/s^2$ (D) $160\ \pi\ m/s^2$

94. A stone of mass 1 kg tied to a light inextensible string of length (10/3) m is whirling in a circular path in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension is 4 and if g is taken to be 10 m/s^2, the speed of the stone at the highest point of the circle is

(A) $5\sqrt{2}$ m/s (B) 20 m/s

(C) 10 m/s (D) $10\sqrt{3}$ m/s

The physics of …..

Riding the bicycle in a loop the loop….

The stunt of riding the bicycle in a loop-the-loop, assuming that the loop is a circle, what is the least speed the rider could have at the top of the loop to remain in contact with it there?

The answer is at the end of this chapter.

Competitive Thinking

1.2 Angular velocity and angular acceleration

1. Which of the following statements is false for a particle moving in a circle with a constant angular speed? **[AIEEE 2004]**

(A) The velocity vector is tangent to the circle.

(B) The acceleration vector is tangent to the circle.

(C) The acceleration vector points to the centre of the circle.

(D) The velocity and acceleration vectors are perpendicular to each other.

2. If ω_E and ω_H are the angular velocities of the earth rotating about its own axis and the hour hand of the clock respectively, then **[MH CET 2009]**

(A) $\omega_E = \frac{1}{4}\ \omega_H$ (B) $\omega_E = 2\ \omega_H$

(C) $\omega_E = \omega_H$ (D) $\omega_E = \frac{1}{2}\ \omega_E$

3. A fan is making 600 revolutions per minute. If after some time it makes 1200 revolutions per minute, then increase in its angular velocity is **[BHU 1999]**

(A) 10π rad/s (B) 20π rad/s

(C) 40π rad/s (D) 60π rad/s

4. Angular velocity of hour arm of a clock, in rad/s, is **[MH CET 2005]**

(A) $\frac{\pi}{43200}$ (B) $\frac{\pi}{21600}$

(C) $\frac{\pi}{30}$ (D) $\frac{\pi}{1800}$

5. The relation between linear speed v, angular speed ω and angular acceleration α in circular motion is **[MH CET 2010]**

(A) $\alpha = \frac{a\omega}{v}$ (B) $\alpha = \frac{av}{\omega}$

(C) $\alpha = \frac{v\omega}{a}$ (D) $\alpha = \frac{\omega}{av}$

6. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then the angular acceleration of the body is **[Orissa JEE 2009]**
(A) θ_1 (B) θ_2
(C) $2\theta_1$ (D) $2\theta_2$

1.3 Relation between linear velocity and angular velocity

7. If the body is moving in a circle of radius r with a constant speed v, its angular velocity is **[CPMT 1975; R PET 1999]**
(A) v^2/r (B) vr
(C) v/r (D) r/v

8. Two particles of mass M and m are moving in a circle of radii R and r. If their time periods are same, what will be the ratio of their linear velocities? **[MH CET 2001]**
(A) MR : mr (B) M : m
(C) R : r (D) 1 : 1

9. If the length of the second's hand in a stop clock is 3 cm, the angular velocity and linear velocity of the tip is **[Kerala PET 2005]**
(A) 0.2047 rad/s, 0.0314 m/s
(B) 0.2547 rad/s, 0.314 m/s
(C) 0.1472 rad/s, 0.06314 m/s
(D) 0.1047 rad/s, 0.00314 m/s

10. A wheel of diameter 20 cm is rotating at 600 rpm. The linear velocity of particle at its rim is **[MH CET 2004]**
(A) 6.28 cm/s (B) 62.8 cm/s
(C) 0.628 cm/s (D) 628.4 cm/s

11. The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5 m, then linear velocity of the wheel is **[MH CET 2000]**
(A) 10 m/s (B) 20 m/s
(C) 35 m/s (D) 70 m/s

12. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be **[R PET 1997]**
(A) 6.28 & 0 mm/s
(B) 8.88 & 4.44 mm/s
(C) 8.88 & 6.28 mm/s
(D) 6.28 & 8.88 mm/s

13. An athlete completes one round of a circular track of radius 10 m in 40 sec. The distance covered by him in 2 min 20 sec is **[Kerala (Med.) 2002]**
(A) 70 m (B) 140 m
(C) 110 m (D) 220 m

14. The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is **[MP PMT 1987, 03]**
(A) zero (B) $\frac{\pi}{30\sqrt{2}}$ cm/s
(C) $\frac{\pi}{30}$ cm/s (D) $\frac{\pi\sqrt{2}}{30}$ cm/s

1.4 Uniform Circular Motion (U.C.M)

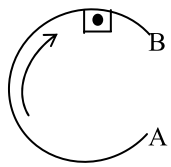
15. When a body moves with a constant speed along a circle, **[MH CET 2003]**
(A) its linear velocity remains constant.
(B) no force acts on it.
(C) no work is done on it.
(D) no acceleration is produced in it.

16. In uniform circular motion, **[MP PMT 1994]**
(A) both the angular velocity and the angular momentum vary.
(B) the angular velocity varies but the angular momentum remains constant.
(C) both the angular velocity and the angular momentum remains constant.
(D) the angular momentum varies but the angular velocity remains constant.

17. A sphere of mass m is tied to end of a string of length l and rotated through the other end along a horizontal circular path with speed v. The work done in full horizontal circle is **[C PMT 1993; JIPMER 2000]**
(A) 0 (B) $\left(\frac{mv^2}{l}\right)2\pi r$
(C) mg $(2\pi r)$ (D) $\left(\frac{mv^2}{r}\right)(l)$

18. If a particle moves with uniform speed then its tangential acceleration will be **[MH CET 2008]**
(A) $\frac{v^2}{r}$ (B) zero
(C) $r\omega^2$ (D) infinite

19. A particle moves in a circular path with decreasing speed. Choose the correct statement. **[IIT JEE 2005]**
(A) Angular momentum remains constant.
(B) Acceleration $\left(\vec{a}\right)$ is towards the centre.
(C) Particle moves in a spiral path with decreasing radius.
(D) The direction of angular momentum remains constant.

20. A body moves along a circular path with certain velocity. What will be the path of body in following figure? **[MH CET 2001]**


(A) Move radially out.
(B) Move horizontally out.
(C) Fall vertically down.
(D) Move tangentially out.

21. A particle comes round a circle of radius 1 m once. The time taken by it is 10 s. The average velocity of motion is **[JIPMER 1999]**
(A) 0.2π m/s (B) 2π m/s
(C) 2 m/s (D) zero

22. The tangential velocity of a particle making p rotations along a circle of radius π in t seconds is **[MH CET 2009]**
(A) $\dfrac{2\pi p}{t^2}$ (B) $\dfrac{2\pi p^2}{t}$
(C) $\dfrac{\pi^2 p}{2t}$ (D) $\dfrac{2\pi^2 p}{t}$

23. If K.E. of the particle of mass m performing U.C.M. in a circle of radius r is E. The acceleration of the particle is **[MH CET 2010]**
(A) $\dfrac{2E}{mr}$ (B) $\left(\dfrac{2E}{mr}\right)^2$
(C) 2Emr (D) $\dfrac{4E}{mr}$

24. **Assertion:** If a body moving in a circular path has constant speed, then there is no force acting on it.
Reason: The direction of the velocity vector of a body moving in a circular path is changing. **[EAMCET 2004]**
(A) Assertion is True, Reason is True; Reason is a correct explanation for Assertion
(B) Assertion is True, Reason is True; Reason is not a correct explanation for Assertion
(C) Assertion is True, Reason is False
(D) Assertion is False but Reason is True.

1.5 Acceleration in U.C.M (Radial acceleration)

25. The centripetal acceleration is given by **[R PET 1999]**
(A) v^2/r (B) vr
(C) vr^2 (D) v/r

26. Angle between radius vector and centripetal acceleration is **[MH CET 2002]**
(A) 0^c (B) π^c
(C) $2\pi^c$ (D) none of these

27. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE 2010]**
(A) $\dfrac{v^2}{R}\hat{i}+\dfrac{v^2}{R}\hat{j}$
(B) $-\dfrac{v^2}{R}\cos\theta\,\hat{i}+\dfrac{v^2}{R}\sin\theta\,\hat{j}$
(C) $-\dfrac{v^2}{R}\sin\theta\,\hat{i}+\dfrac{v^2}{R}\cos\theta\,\hat{j}$
(D) $-\dfrac{v^2}{R}\cos\theta\,\hat{i}-\dfrac{v^2}{R}\sin\theta\,\hat{j}$

28. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is **[AIEEE 2012]**
(A) $m_1r_1 : m_2r_2$ (B) $m_1 : m_2$
(C) $r_1 : r_2$ (D) 1 : 1

29. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2 π s. The acceleration of the particle is **[CBSE PMT (Prelims) 2012]**
(A) 5 m/s^2 (B) 15 m/s^2
(C) 25 m/s^2 (D) 36 m/s^2

30. If a cycle wheel of radius 0.4 m completes one revolution in one second, then acceleration of the cycle is **[MH CET 1999]**
(A) 0.4π m/s^2 (B) 0.8π m/s^2
(C) $0.4\pi^2$ m/s^2 (D) $1.6\pi^2$ m/s^2

31. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle in m/s^2 is

[MNR 1991; D PMT 1999; UPSEAT 2000; R PET 2003; Pb PET 2004; CBSE PMT PMT (Prelims) 2011]

(A) π^2 (B) $8\pi^2$
(C) $4\pi^2$ (D) $2\pi^2$

32. Certain neutron stars are believed to be rotating at about 1 rev/s. If such a star has a radius of 20 km, the acceleration of an object on the equator of the star will be

[NCERT 1982]

(A) 20×10^8 m/s^2 (B) 8×10^5 m/s^2
(C) 120×10^5 m/s^2 (D) 4×10^8 m/s^2

33. A car is moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s^2. What is the acceleration of the car?

[Roorkee 1982; R PET 1996; MH CET 2002; MP PMT 2003]

(A) 2 m/s^2 (B) 2.7 m/s^2
(C) 1.8 m/s^2 (D) 9.8 m/s^2

1.6 Centripetal and centrifugal forces

34. Banking of roads is independent of

[MH CET 2007]

(A) radius of the path
(B) mass of the vehicle
(C) acceleration due to gravity
(D) maximum velocity of the vehicle around the curved path

35. Centripetal force in vector form can be expressed as

[MH CET 2004]

(A) $\vec{F} = -\dfrac{mv^2}{r}$ (B) $\vec{F} = -\dfrac{mv^2}{r^3}\vec{r}$
(C) $\vec{F} = -m\omega^2\vec{r}$ (D) $\vec{F} = -\dfrac{mv^2\vec{r}}{\vec{r}}$

36. A particle of mass m is executing uniform circular motion on a path of radius r. If p is the magnitude of its linear momentum, the radial force acting on the particle is **[MP PET 2010]**

(A) pmr (B) $\dfrac{rm}{p}$
(C) $\dfrac{mp^2}{r}$ (D) $\dfrac{p^2}{rm}$

37. The magnitude of the centripetal force acting on a body of mass m executing uniform motion in a circle of radius r with speed v is

[AFMC 1998; MP PET 1999]

(A) mvr (B) mv^2/r
(C) v/r^2 m (D) v/rn

38. If a tension in a string is 6.4 N. A load at the lower end of a string is 0.1 kg, the length of string is 6 m then find its angular velocity ($g = 10$ m/s^2) **[MH CET 2008]**

(A) 3 rad/s (B) 4 rad/s
(C) 2 rad/s (D) 1 rad/s

39. A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is

[C PMT 1974; AFMC 2003]

(A) doubled. (B) quadrupled.
(C) halved. (D) unchanged.

40. A body of mass m performs U.C.M. along a circular path of radius r with a velocity v If its angular momentum is L, then its centripetal force is

[MH CET 2007]

(A) L^2/mr^2 (B) L^2/mr^3
(C) L^2/mr (D) L^2/r^3

41. A string breaks if its tension exceeds 10 newton. A stone of mass 250 g tied to this string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be

[MP PMT 1999]

(A) 20 rad/s (B) 40 rad/s
(C) 100 rad/s (D) 200 rad/s

42. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is **[CBSE PMT 1999; K CET 2001; JIPMER 2001, 02]**

(A) 250 N (B) 750 N
(C) 1000 N (D) 1200 N

43. A 100 kg car is moving with a maximum velocity of 9 m/s across a circular track of radius 30 m. The maximum force of friction between the road and the car is

[Pb PMT 2000]

(A) 1000 N (B) 706 N
(C) 270 N (D) 200 N

44. A proton of mass 1.6×10^{-27} kg goes round in a circular orbit of radius 0.10 m under a centripetal force of 4×10^{-13} N. The frequency of revolution of the proton is about **[Kerala (Med.) 2007]**
(A) 0.08×10^8 cycles per s
(B) 4×10^8 cycles per s
(C) 8×10^8 cycles per s
(D) 12×10^8 cycles per s

45. If the radius of curvature of the path of two particles of same masses are in the ratio 1 : 2, then in order to have constant centripetal force, their velocity, should be in the ratio of **[Pb PET 2000]**
(A) 1 : 4 (B) 4 : 1
(C) $\sqrt{2}:1$ (D) $1:\sqrt{2}$

46. A coin is placed on a rotating turn table rotated with angular speed ω. The coin just slips if it is placed at 4 cm from the center of the table. If angular velocity is doubled, at what distance will coin starts to slip. **[MH CET 2010]**
(A) 1 cm (B) 4 cm
(C) 9 cm (D) 16 cm

47. A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be **[AFMC 2002]**
(A) double (B) half
(C) 4 times (D) $\frac{1}{4}$ times

48. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is **[IIT JEE (Screening) 2000]**
(A) $\sqrt{\frac{\mu}{a}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$
(C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

49. On the centre of a frictionless table, a small hole is made, through which a weightless string of length $2l$ is inserted. On the two ends of the string, two balls of the same mass m are attached. Arrangement is made in such a way that half of the string is on the table top and half is hanging below. The ball on the table top is made to move in a circular path with a constant speed v. What is the centripetal acceleration of the moving ball? **[DUMET 2009]**
(A) mvl (B) g
(C) Zero (D) $2mvl$

1.7 Banking of roads

50. A car sometimes overturns while taking a turn. When it overturns, it is **[AFMC 1988; MP PMT 2003]**
(A) the inner wheel which leaves the ground first.
(B) the outer wheel which leaves the ground first.
(C) both the wheels which leave the ground simultaneously.
(D) either wheel leaves the ground first.

51. A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is **[NCERT 1972; C PMT 2010]**
(A) Car is heavier than cycle.
(B) Car has four wheels while cycle has only two.
(C) Difference in the speeds of the two.
(D) Cyclist has to counteract the centrifugal force while in the case of car, only the passenger is thrown by this force.

52. A train is moving towards north. At one place, it turns towards north-east. Here, we observe that **[AIIMS 1980]**
(A) the radius of curvature of outer rail will be greater than that of the inner rail.
(B) the radius of the inner rail will be greater than that of the outer rail.
(C) the radius of curvature of one of the rails will be greater.
(D) the radius of curvature of the outer and inner rails will be the same.

53. A car is travelling on a circular banked road. The centripetal acceleration of a car is provided by **[MH CET 2006]**
(A) normal reaction.
(B) weight of a car.
(C) horizontal component of normal reaction.
(D) vertical component of normal reaction.

54. A body is moving in a circular orbit with static friction 0.2. If radius through which the body revolves is 100 m and g = 9.8 m/s^2, then maximum speed with which body revolved is **[MH CET 2004]**
(A) 14 m/s (B) 19 m/s
(C) 11 m/s (D) 13 m/s

55. The maximum speed of a car on a road–turn of radius 30 m, if the coefficient of friction between the tyres and the road is 0.4, will be **[MH CET 1999; CBSE PMT 2000]**
(A) 10.84 m/s (B) 9.84 m/s
(C) 8.84 m/s (D) 6.84 m/s

56. A cyclist goes round a circular path of circumference 34.3 m in $\sqrt{22}$ s. The angle made by him, with the vertical, is **[MH CET 2000]**
(A) 42° (B) 43°
(C) 44° (D) 45°

57. An aircraft executes a horizontal loop with a speed of 150 m/s with its wings banked at an angle of 12°. The radius of the loop is (g = 10 m/s^2) **[Pb PET 2001]**
(A) 10.6 km (B) 9.6 km
(C) 7.4 km (D) 5.8 km

58. A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is **[CBSE PMT (Prelims) 2012]**
(A) 20 ms^{-1} (B) 30 ms^{-1}
(C) 5 ms^{-1} (D) 10 ms^{-1}

59. Radius of the curved road on national highway is r. Width of the road is l. The outer edge of the road is raised by h with respect to the inner edge so that a car with velocity v can pass safely over it. The value of h is **[MP PMT 1996]**
(A) $\frac{v^2 r}{g}$ (B) $\frac{v^2 l}{r}$
(C) $\frac{v^2 l}{rg}$ (D) $\frac{v^2}{g}$

60. A person with his hands in his pockets is skating on ice at the velocity of 10 m/s and describes a circle of radius 50 m. What is his inclination with vertical? **[Pb PET 2000]**
(A) $\tan^{-1}\left(\frac{1}{10}\right)$ (B) $\tan^{-1}\left(\frac{3}{5}\right)$
(C) $\tan^{-1}(1)$ (D) $\tan^{-1}\left(\frac{1}{5}\right)$

61. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of 0.5 m/s. What is the height of the plane of circle from vertex of the funnel ? [Take g = 10 m/s^2] **[J & K CET 2005]**
(A) 0.25 cm (B) 2 cm
(C) 4 cm (D) 2.5 cm

1.9 Vertical Circular Motion

62. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break **[MP PET 1997]**
(A) when the mass is at the highest point of the circle.
(B) when the mass is at the lowest point of the circle.
(C) when the wire is horizontal.
(D) at an angle of $\cos^{-1}$ (1/3) from the upward vertical.

63. A body of mass m hangs at one end of a string of length l, the other end of which is fixed. It is given a horizontal velocity so that the string would just reach where it makes an angle of 60° with the vertical. The tension in the string at mean position is **[ISM Dhanbad 1994]**
(A) 2 mg (B) mg
(C) 3 mg (D) $\sqrt{3}$ mg

64. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical? **[Orissa JEE 2002]**
(A) 30° (B) 45°
(C) 60° (D) 90°

65. A simple pendulum of mass m and length ℓ stands in equilibrium in vertical position. The maximum horizontal velocity that should be given to the bob at the bottom so that it completes one revolution is **[MH CET 2004]**
(A) $\sqrt{\ell g}$ (B) $\sqrt{2\ell g}$
(C) $\sqrt{3\ell g}$ (D) $\sqrt{5\ell g}$

66. The relation between force acting on the electron and principle quantum number in hydrogen atom is **[MH CET 2009]**
(A) $F \propto n^4$ (B) $F \propto n^2$
(C) $F \propto \frac{1}{n^2}$ (D) $F \propto \frac{1}{n^4}$

67. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively then **[Orissa JEE 2002]**
(A) $T_1 = T_2$
(B) $T_2 > T_1$
(C) $T_1 > T_2$
(D) tension in the string always remains the same

68. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is **[Kerala (Engg.) 2001]**
(A) mg (B) $m(g + \pi nr^2)$
(C) $m(g + \pi nr)$ (D) $m\left[g + \frac{\pi^2 n^2 r}{900}\right]$

69. A bucket full of water is revolved in vertical circle of radius 2 m. What should be the maximum time-period of revolution so that the water doesn't fall-off the bucket?
[Take $g = 10\ m/s^2$] **[AFMC 2004]**
(A) 1 s (B) 2 s
(C) 3 s (D) 4 s

70. A can filled with water is revolved in a vertical circle of radius 4 m and water does not fall down. The time period of revolution will be **[MH CET 2009]**
(A) 2 s (B) 4 s
(C) 6 s (D) 8 s

71. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take $g = 10\ m/s^2$) **[AIIMS 1987]**
(A) 4 m/s
(B) 6.25 m/s
(C) 16 m/s
(D) None of the above

72. A body crosses the topmost point of a vertical circle with critical speed. Its centripetal acceleration, when the string is horizontal will be **[MH CET 2002]**
(A) 6 g (B) 3 g
(C) 2 g (D) g

73. A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If $g = 10\ ms^{-2}$, then the maximum angular velocity of the stone will be **[MP PMT 1994]**
(A) 5 rad/s (B) $\sqrt{30}$ rad/s
(C) $\sqrt{60}$ rad/s (D) 10 rad/s

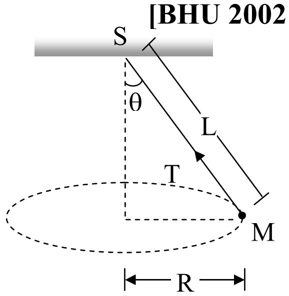
74. A weightless thread can bear tension upto 3.7 kg-wt. A stone of mass 500 g is tied to it and revolved in a circular path of radius 4 m in a vertical plane. If $g = 10\ ms^{-2}$, then the maximum angular velocity of the stone will be **[MP PMT/PET 1998]**
(A) 4 rad/s (B) 16 rad/s
(C) $\sqrt{21}$ rad/s (D) 2 rad/s

75. A mass of 5 kg is tied to a string of length 1.0 m and is rotated in vertical circle with a uniform speed of 4 m/s. The tension in the string will be 130 N when the mass is at ($g = 10\ m/s^2$) **[MH CET 2001]**
(A) highest point (B) mid way
(C) bottom (D) cannot be justified

Miscellaneous

76. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as, $a_c = k^2rt^2$. The power delivered to the particle by the forces acting on it is **[IIT 2008]**
(A) $2\pi mk^2r^2t$ (B) mk^2r^2t
(C) $\frac{mk^4r^2t^5}{3}$ (D) zero

77. A string of length L is fixed at one end and carries a mass M at the other end. The string makes $2/\pi$ revolutions per second around the vertical axis through the fixed end as shown in the figure, then tension in the string is
[BHU 2002; DPMT 2004]

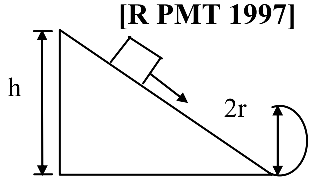


(A) ML (B) 2 ML
(C) 4 ML (D) 16 ML

78. If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force $(-k/r^2)$, the total energy is
[EAMCET (Med.) 1995; AMU (Engg.) 2001]
(A) $-\dfrac{k}{2r}$ (B) $-\dfrac{k}{r}$
(C) $-\dfrac{2k}{r}$ (D) $-\dfrac{4k}{r}$

79. A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolution/s. Assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be **[D PMT 2003]**
(A) 1.88 m/s, 35.5 m/s^2, 35.5 N
(B) 2.88 m/s, 45.5 m/s^2, 45.5 N
(C) 3.88 m/s, 55.5 m/s^2, 55.5 N
(D) None of these

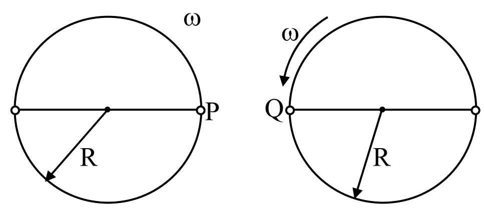
80. A block follows the path as shown in the figure from height h. If radius of circular path is r, then relation holds good to complete full circle is **[R PMT 1997]**
(A) $h < 5r/2$
(B) $h > 5r/2$
(C) $h = 5r/2$
(D) $h \geq 5r/2$

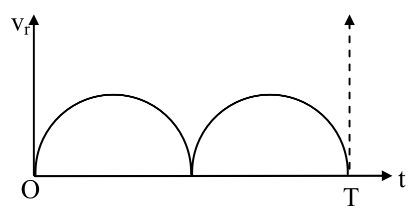


81. An electron revolves around the nucleus. The radius of the circular orbit is r. To double the kinetic energy of electron its orbit radius is
[MH CET 2008]
(A) $\dfrac{r}{\sqrt{2}}$ (B) $\sqrt{2}\ r$
(C) 2 r (D) $\dfrac{r}{2}$

82. A particle moves in a circle of radius 5 cm with constant speed and time period $0.2\ \pi$.s. the acceleration of the particle is
[AIPMT 2011]
(A) 15 m/s^2 (B) 25 m/s^2
(C) 36 m/s^2 (D) 5 m/s^2

83. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. As a function of time, it is best represented by
[IIT JEE 2012]



(A)


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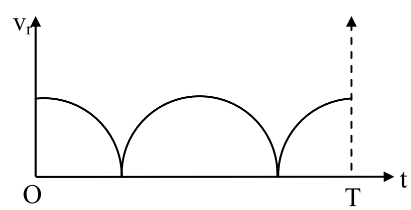
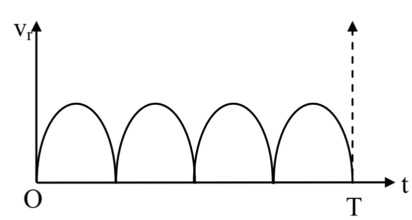
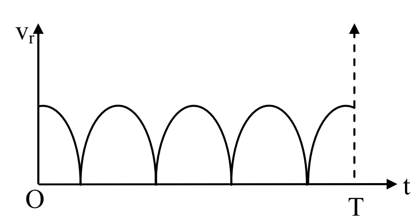


(C)


(D)


Answers Key

Classical Thinking

1. (B)	2. (C)	3. (C)	4. (A)	5. (C)	6. (B)	7. (C)	8. (A)	9. (A)	10. (D)
11. (C)	12. (A)	13. (A)	14. (B)	15. (B)	16. (C)	17. (B)	18. (D)	19. (A)	20. (B)
21. (C)	22. (D)	23. (D)	24. (C)	25. (A)	26. (C)	27. (B)	28. (A)	29. (D)	30. (B)
31. (B)	32. (A)	33. (D)	34. (C)	35. (B)	36. (C)	37. (B)	38. (C)	39. (C)	40. (B)
41. (A)	42. (A)	43. (A)	44. (D)	45. (B)	46. (A)	47. (C)	48. (C)	49. (B)	50. (A)
51. (D)	52. (C)	53. (A)	54. (B)	55. (B)	56. (C)	57. (C)	58. (C)	59. (D)	60. (A)
61. (B)	62. (B)	63. (D)	64. (C)	65. (C)	66. (A)	67. (B)	68. (C)	69. (A)	70. (A)

Critical Thinking

1. (C)	2. (D)	3. (A)	4. (A)	5. (B)	6. (C)	7. (A)	8. (B)	9. (C)	10. (A)
11. (C)	12. (C)	13. (B)	14. (B)	15. (C)	16. (B)	17. (B)	18. (C)	19. (C)	20. (C)
21. (D)	22. (B)	23. (D)	24. (A)	25. (B)	26. (A)	27. (C)	28. (C)	29. (B)	30. (C)
31. (A)	32. (B)	33. (D)	34. (D)	35. (B)	36. (A)	37. (D)	38. (D)	39. (A)	40. (B)
41. (B)	42. (C)	43. (D)	44. (B)	45. (A)	46. (B)	47. (A)	48. (B)	49. (B)	50. (D)
51. (B)	52. (A)	53. (D)	54. (B)	55. (B)	56. (C)	57. (A)	58. (C)	59. (B)	60. (A)
61. (A)	62. (B)	63. (B)	64. (B)	65. (B)	66. (C)	67. (D)	68. (B)	69. (B)	70. (B)
71. (B)	72. (A)	73. (B)	74. (A)	75. (C)	76. (A)	77. (B)	78. (A)	79. (B)	80. (C)
81. (A)	82. (A)	83. (A)	84. (A)	85. (C)	86. (D)	87. (C)	88. (C)	89. (C)	90. (B)
91. (B)	92. (A)	93. (B)	94. (C)						

Competitive Thinking

1. (B)	2. (D)	3. (B)	4. (B)	5. (A)	6. (D)	7. (C)	8. (C)	9. (D)	10. (D)
11. (C)	12. (D)	13. (D)	14. (D)	15. (C)	16. (C)	17. (A)	18. (B)	19. (D)	20. (D)
21. (D)	22. (D)	23. (A)	24. (D)	25. (A)	26. (B)	27. (D)	28. (C)	29. (A)	30. (D)
31. (C)	32. (B)	33. (B)	34. (B)	35. (B)	36. (D)	37. (B)	38. (A)	39. (B)	40. (B)
41. (A)	42. (C)	43. (C)	44. (A)	45. (D)	46. (A)	47. (C)	48. (A)	49. (B)	50. (A)
51. (D)	52. (A)	53. (C)	54. (A)	55. (A)	56. (D)	57. (A)	58. (B)	59. (C)	60. (D)
61. (D)	62. (B)	63. (A)	64. (D)	65. (D)	66. (D)	67. (C)	68. (D)	69. (C)	70. (B)
71. (A)	72. (B)	73. (A)	74. (A)	75. (C)	76. (B)	77. (D)	78. (A)	79. (A)	80. (D)
81. (D)	82. (D)	83. (A)							

Hints

Classical Thinking

3. $f = 300 \text{ r.p.m.} = \frac{3000}{60}$ r.p.s;

$$\theta = \omega.t = 2\pi \times \frac{3000}{60} \times 1 = 100\,\pi \text{ rad}$$

5. For a seconds hand of a watch, T = 60 s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{rad/s}$$

6. $n = 100 \text{ r.p.m.} = \frac{100}{60}$ r.p.s.

$$\omega = 2\pi n = \frac{2\pi\times100}{60} = 10.47 \text{ rad/s}$$

7. n = 3.5 r.p.s.

$$\omega = 2\pi n = 2 \times \pi \times 3.5 = 7\pi$$
$$= 7 \times 3.14 \approx 22 \text{ rad/s}$$

8. For earth, T = 24 hr = 24 × 3600 = 86400 s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24} \text{ rad/hr } = \frac{2\pi}{86400} \text{ rad/s}$$

9. Using, $\omega = 2\pi n$

$\therefore$ $125 = 2\pi n$

$\therefore$ $n = \dfrac{125}{2\pi}$

$\therefore$ $n \approx 20$

10. For seconds hand, $T_s = 60$ s; for hour hand, $T_H = 2 \times 3600$ s

$\therefore$ $\dfrac{\omega_S}{\omega_H} = \dfrac{T_H}{T_S} = \dfrac{12 \times 3600}{60} = 720 : 1$

....$[\because \omega \propto \dfrac{1}{T}]$

11. $\alpha = \dfrac{d\omega}{dt} = 0$ ....($\because$ ω = constant)

12. $n_1 = 0$, $n_2 = 210$ r.p.m. $= \dfrac{210}{60}$ r.p.s.

$d\omega = 2\pi(n_2 - n_1) = 2\pi\left(\dfrac{210}{60} - 0\right) = 7\ \pi$ rad/s

$\alpha = \dfrac{d\omega}{dt} = \dfrac{2\pi \times 210}{60 \times 5} = 4.4$ rad/s^2

14. $C = 2\pi r$

$\therefore$ $r = \dfrac{C}{2\pi}$

$\therefore$ $v = r(2\pi n) = \dfrac{C}{2\pi} \times 2\pi \times f = fC$ ....$[\because \omega = 2\pi n]$

15. Using, $v = r\omega$

$= 0.2 \times 10$ m/s $= 2$ m/s

16. Using, $v = r\omega$

$= r \times (2\pi n) = 0.4 \times 2\pi \times 5$

$= 0.4 \times 2 \times 3.14 \times 5 = 12.56 \approx 12.6$ m/s

17. Angular velocity of particle P about point A,

$\omega_A = \dfrac{v}{r_{AB}} = \dfrac{v}{2r}$

Angular velocity of particle P about point C,

$\omega_C = \dfrac{v}{r_{BC}} = \dfrac{v}{r}$

$\dfrac{\omega_A}{\omega_C} = \dfrac{v}{2r} \times \dfrac{r}{v}$

$\dfrac{\omega_A}{\omega_C} = \dfrac{1}{2}$

18. In U.C.M., direction of velocity and acceleration change from point to point.

22. At each point on circular path, the magnitude of velocity remains the same for any value of θ.

23. The particle performing circular motion flies-off tangentially.

27. $n = 1200$ r.p.m. $= \dfrac{1200}{60}$ r.p.s. $= 20$ r.p.s.

$a = \omega^2 r = (4\ \pi^2\ n^2)\ r = 4 \times (3.142)^2 \times (20)^2 \times 0.3$

≈ 4740 cm/s^2

28. $n = 900$ r.p.m. $= \dfrac{900}{60}$ r.p.s $= 15$ r.p.s,

$d = 1.2$ m $\Rightarrow r = \dfrac{1.2}{2} = 0.6$ m

$a = \omega^2 r = (2\ \pi n)^2 \times \dfrac{1.2}{2} = 540\ \pi^2$ m/s^2

29. $r = 10$ cm $= 0.1$ m, $a = 1000 \times 10$ m/s^2

$a = \omega^2 r$

$\therefore$ $\omega^2 = \dfrac{a}{r}$

$\therefore$ $\omega = \sqrt{\dfrac{a}{r}} = \sqrt{\dfrac{1000 \times 10}{10 \times 10^{-2}}} \approx 316$ rad/s

$n = 316/2\pi = 50.3$ r.p.s. ≈ 50 r.p.s.

$\therefore$ $n = 3000$ r.p.m.

31. Using,

$a_r = \dfrac{v^2}{r} = \dfrac{20 \times 20}{10} = 40$ m/s^2, $a_t = 30$ m/s^2

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{40^2 + 30^2} = 50$ m/s^2

39. $p = mv$; $F = \dfrac{mv^2}{r}$

$\therefore$ $\dfrac{F}{p} = \dfrac{mv^2}{r} \times \dfrac{1}{mv} = \dfrac{v}{r}$

40. Using, $F_s = \dfrac{mv^2}{r}$

$\therefore$ $v^2 = \dfrac{F_s r}{m} = \dfrac{10^5 \times 10}{10^2} = 10^4$

$\therefore$ $v = 100$ m/s

41. $F = \dfrac{mv^2}{r}$

If m and v are constants, then $F \propto \dfrac{1}{r}$

$\therefore$ $\dfrac{F_1}{F_2} = \left(\dfrac{r_2}{r_1}\right)$

42. Using, $F = \frac{mv^2}{r}$

$\therefore$ $r = \frac{mv^2}{F} = \frac{10 \times (5)^2}{125} = \frac{250}{125} = 2$ m

43. Using, $v^2 = \frac{Tr}{m}$

Breaking tension $T = \frac{mv^2}{r}$

(r = length of the string)

$\therefore$ $v^2 = \frac{50 \times 1}{1}$

$\therefore$ $v = 5\sqrt{2}$ m/s

44. Using, $F = mr\omega^2 = m \times 4\pi^2 n^2 r$

$\therefore$ $m \times 4\pi^2 n^2 r = 6 \times 10^{-14}$

$\therefore$ $n^2 = \frac{6 \times 10^{-14}}{4 \times 1.6 \times 10^{-27} \times 3.14^2 \times 0.12}$

$\therefore$ $n \approx 5 \times 10^6$ cycles/s

53. Centripetal acceleration,

$a_{cp} = \omega^2 r = \frac{gl\sin\theta}{l\cos\theta} = g\tan\theta$

$= 10 \times \tan 60° = 17.3$ m/s^2

54. Using,

$mr\omega^2 = T$ and $\omega = 2\pi n$

$n = \frac{1}{2\pi}\sqrt{\frac{T}{mr}} = 2\,\text{Hz}$

59. For looping the loop, minimum velocity at the lowest point should be $\sqrt{5gl}$.

60. Thrust at the lowest point of concave bridge

$= mg + \frac{mv^2}{r}$

61. $N = mg\cos\theta - \frac{mv^2}{R}$, θ = angle with vertical

As vehicle descends, angle increases, its cosine decreases, hence N decreases.

64. $\mu\, mr\omega^2 \geq mg$;

$\omega \geq \sqrt{\frac{g}{\mu\, r}}$

65. $v_1 = \sqrt{rg}$

$v_2 = \sqrt{5rg} = \sqrt{5} \times \sqrt{rg} = \sqrt{5} \times v_1$

66. Using,

$\alpha = \frac{\omega - \omega_0}{t} = \frac{2\pi(n - n_0)}{t}$

$= \frac{2 \times 3.14 \times (350 - 0)}{220} \approx 10$ rad/s^2

67. Using,

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 4 \times 10 + \frac{1}{2} \times 2 \times (10)^2 = 140$ rad

$n = \frac{\theta}{2\pi} = \frac{140}{2 \times 3.142} \approx 22$

68. $v = 72$ km/hr $= 72 \times \frac{5}{18} = 20$ m/s,

$d = 0.5$ m $\Rightarrow r = \frac{0.5}{2}$ m

$\therefore$ $\omega_0 = \frac{v}{r} = \frac{20}{0.5/2} = 80$ rad/s

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$0 = (80)^2 + 2\alpha(2\pi \times 20)$

$-6400 = 80\pi\alpha$

$\alpha = \frac{-80}{\pi} = -25.48$ rad/s^2

69. Difference in tensions = 6 mg = 6 × 2 × 9.8 = 12 kg wt

70. $F = m\omega^2 R$

$\therefore$ $R \propto \frac{1}{\omega^2}$ (m and F are constant)

If ω is doubled, then radius will become 1/4 times i.e., R/4

Critical Thinking

1. Frequency of wheel, $n = \frac{300}{60} = 5$ r.p.s.

Angle described by wheel in one rotation

$= 2\pi$ rad.

Therefore, angle described by wheel in 1 sec

$\theta = 2\pi \times 5$ radians $= 10\pi$ rad

2. In non-uniform circular motion, particle possesses both centripetal as well as tangential accelerations.

3. $n = 2000$, distance $= 9500$ m
Distance covered in 'n' revolutions $= n(2\pi r)$
$= n\pi D$
$\therefore$ $2000\pi D = 9500$
$\therefore$ $D = \dfrac{9500}{2000 \times \pi} = 1.5$ m

4. Period of second hand $= T_s = 60$ s and
Period of minute hand $= T_m = 60 \times 60 = 3600$ s
Angular speed of second hand $\omega_s = \dfrac{2\pi}{T_s} = \dfrac{2\pi}{60}$
Angular speed of minute hand
$\omega_m = \dfrac{2\pi}{T_m} = \dfrac{2\pi}{3600}$
$\therefore$ $\dfrac{\omega_s}{\omega_m} = \dfrac{2\pi}{60} \times \dfrac{3600}{2\pi} = 60 : 1$

5. For minute hand, $T = 60$ min $= 60 \times 60$ s
Angular speed, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{60 \times 60}$ rad/s
$= \dfrac{\pi}{1800} \times \dfrac{180}{\pi} = 0.1$
….[$\because$ 1 rad $= \dfrac{180^\circ}{\pi}$]

6. $\omega = \dfrac{\text{angle described}}{\text{time taken}} = \dfrac{2\pi}{2} = \pi$ rad/s

7. $n = \dfrac{540}{60} = 9$ r.p.s., $\omega = 2\pi n = 18\pi$ rad/s
Angular acceleration
$= \dfrac{\text{Gain in angular velocity}}{\text{time}} = \dfrac{18\pi}{6} = 3\pi \text{ rad s}^{-2}$

8. Using, $\alpha = \dfrac{d\omega}{dt}$
$\therefore$ $\alpha = \dfrac{15\pi - 10\pi}{4 - 2} = \dfrac{5\pi}{2} = 2.5\ \pi$ rad/s^2

9. Using,
$\theta = 2t + 3t^2$
$\therefore$ $\omega = \dfrac{d\theta}{dt} = 2 + 6t$
$\alpha = \dfrac{d\omega}{dt} = 6$ rad/s^2

10. $v = r.\omega$
where r is distance from axis of rotation.
At the north-pole, $r = 0 \Rightarrow v = 0$

11. A particle will describe a circular path if the angle between velocity, $\vec{v}$ and acceleration $\vec{a}$ is 90°.

12. Frequency $= \dfrac{n}{60}$ r.p.s., $t = 1$ min $= 60$ s
Angular velocity, $\omega = 2\pi \dfrac{n}{60}$
$\therefore$ Linear velocity, $v = \omega r = \dfrac{2\pi n \times \pi}{60}$
$= \dfrac{2\pi^2 n}{60}$ cm/s

13. Speed of $C_1 = \omega R_1 = \dfrac{2\pi}{T} R_1$
Speed of $C_2 = \omega R_2 = \dfrac{2\pi}{T} R_2$
$\therefore$ $\dfrac{\text{Speed of } C_1}{\text{Speed of } C_2} = \dfrac{2\pi R_1 / T}{2\pi R_2 / T} = \dfrac{R_1}{R_2}$

14. $r = 0.25$ m, $n = 15$ r.p.m. $= \dfrac{15}{60}$ r.p.s.
$\omega = 2\pi n = \dfrac{2 \times \pi \times 15}{60} = \dfrac{\pi}{2}$ rad/s
$v = r\omega = 0.25 \times \dfrac{\pi}{2} = \dfrac{\pi}{8}$ m/s

15. $T = \dfrac{20}{40} = \dfrac{1}{2} = 0.5$ s
$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.5} = 4\pi$ rad/s
Let $r = 50$ cm $= 0.5$ m
$v = r\omega = 0.5 \times 4\pi = 2\pi$ m/s

16. $T = 24$ hr, $r = 6400$ km
$v = \omega r = \dfrac{2\pi}{T} \times r = \dfrac{2\pi}{24} \times 6400$
$= \dfrac{2 \times 3.14 \times 6400}{24}$
$v \approx 1675$ km/hr

17. $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

18. $\theta = 2t^3 + 0.5$

$\therefore$ $\omega = \dfrac{d}{dt}(2t^3 + 0.5) = 6t^2$

At t = 2 s, $\omega = 6 \times 2^2 = 24$ rad/s

21. While moving along a circle, the body has a constant tendency to regain its natural straight line path.
This tendency gives rise to a force called centrifugal force. The centrifugal force does not act on the body in motion, the only force acting on the body in motion is centripetal force. The centrifugal force acts on the source of centripetal force to displace it radially outward from centre of the path.

22. Tangential force acting on the car increases with the magnitude of its speed.

$\therefore$ a_t = time rate of change of its speed
= change in the speed of the car per unit time which is 3 m/s

$\therefore$ Tangential acceleration = 3 m /s^2

23. There is no relation between centripetal and tangential acceleration. Centripetal acceleration is a must for circular motion but tangential acceleration may be zero.

24. When a body is moving with constant speed, the tangential acceleration developed in a body is zero.

25. Radius of horizontal loop, r = 1 km = 1000 m

$v = 900 \text{ km/h} = \dfrac{900 \times 10^3}{3600} = 250$ m/s

$\therefore$ $a = \dfrac{v^2}{r} = \dfrac{250 \times 250}{1000} = 62.5$ m/s^2

$\therefore$ $\dfrac{a}{g} = \dfrac{62.5}{10} = 6.25$

26. Velocity, $v = \omega r$

$\therefore$ $v' = \omega r' = \dfrac{\omega r}{2} = \dfrac{v}{2} = 10$ cm/s

$\therefore$ $a = \omega^2 r$

$\therefore$ $a' = \omega^2 r' = \omega^2 \times \dfrac{r}{2} = \dfrac{a}{2} = 10$ cm/s^2

27. In uniform circular motion, acceleration is caused due to change in direction and is directed radially towards centre.

28. As ω is constant, acceleration is due to the change in direction of velocity = $\omega^2 r$
As $r_A > r_B \Rightarrow a_A > a_B$

29. In half a circle, the direction of acceleration is reversed.

It goes from $\dfrac{v^2}{r}$ to $\dfrac{-v^2}{r}$

Hence, change in centripetal acceleration

$= \dfrac{v^2}{r} - \left(\dfrac{-v^2}{r}\right) = \dfrac{2v^2}{r}$

30. If $a_r = 0$, there is no radial acceleration and circular motion is not possible
So $a_r \neq 0$
If $a_t \neq 0$ the motion is not uniform as angular velocity will change
So $a_r \neq 0$ and $a_t = 0$ for uniform circular motion

31. Centripetal force = $\dfrac{mv^2}{r}$ and is directed always towards the centre of circle. Sense of rotation does not affect magnitude and direction of this centripetal force.

32. This is due to centrifugal force.

33. Distance covered, $s = \dfrac{\theta}{360^\circ} \times 2\pi r$

$660 = \dfrac{90}{360} \times 2\pi r$

r = 420 m

$F = \dfrac{mv^2}{r} = \dfrac{840 \times 10 \times 10}{420} = 200$ N

34. Using,

$F_{cp} = m\omega^2 r = m\left(\dfrac{2\pi}{T}\right)^2 r$

$= 500 \times 10^{-3} \times \left(2 \times \dfrac{22}{7} \times \dfrac{1}{11}\right)^2 \times 0.49$

$= \dfrac{500 \times 10^{-3} \times 16 \times 0.49}{49} = 0.08$ N

35. $F \propto \dfrac{1}{r^2}$ and $r \propto n^2$ where n is principal quantum no.

$\therefore$ $\dfrac{F_1}{F_2} = \dfrac{n_2^4}{n_1^4} = \left(\dfrac{3}{2}\right)^4 = \dfrac{81}{16}$

36. m = 2 kg, r = 1 m, F = 32 N
Force, F = $m\omega^2 r$
$\therefore$ $\omega^2 = \dfrac{32}{2\times1} = 16$ $\therefore$ $\omega = 4$ rad/s
$\therefore$ Frequency of revolution per minute
$n = \dfrac{\omega}{2\pi}\times 60 = \dfrac{4\times7}{2\times22}\times 60$
≈ 38 rev / min

37. r = 20 cm = 20×10^{-2} m = 0.2 m
Using, $F = \dfrac{mv^2}{r} = 10$
$\therefore$ $\dfrac{1}{2}mv^2 = 10\times\dfrac{r}{2} = 10\times\dfrac{0.20}{2} = 1$ J

38. $r_1 = 9$ cm
In the given condition, friction provides the required centripetal force and that is constant. i.e. $m\omega^2 r$ = constant.
$\therefore$ $r \propto \dfrac{1}{\omega^2}$ $\therefore$ $r_2 = r_1\left(\dfrac{\omega_1}{\omega_2}\right)^2 = 9\left(\dfrac{1}{3}\right)^2 = 1$ cm

39. Using,
$\mu mg = m\omega^2 r$
$\therefore$ $\omega = \sqrt{\dfrac{\mu g}{r}} = \sqrt{\dfrac{0.4\times10}{1}} = \sqrt{4} = 2$ rad/s

40. Using,
$\mu_s mg \le mr\omega^2$
$\mu_s g = r\omega^2$ (For minimum angular speed)
$\omega^2 = \dfrac{\mu_s g}{r} = \dfrac{0.25\times9.8}{5\times10^{-2}} = \dfrac{25}{5}\times 9.8$
$= 9.8\times5 = 49.0$
$\therefore$ $\omega = 7$ rad/s

41. Breaking tension = $4\times10 = 40$ N
$\therefore$ $T = mr\omega^2$
$\therefore$ $\omega^2 = \dfrac{T}{mr} = \dfrac{40}{200\times10^{-3}\times1} = 200$
$\therefore$ $\omega \approx 14$ rad/s

42. Using,
$v^2 = \mu rg = 0.8\times100\times9.8 = 784$
$\therefore$ v = 28 m/s

43. $v = \sqrt{\mu gr}$
When μ becomes $\dfrac{\mu}{2}$, v becomes $\dfrac{v}{\sqrt{2}}$ i.e. $\dfrac{10}{\sqrt{2}}$
$= \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$ ms^{-1}

44. v = 36 km/hr = $\dfrac{36\times10^3}{3600}$ = 10 m/s
The speed with which the car turns is
$v^2 \ge \mu Rg$
$\therefore$ $R \le (10)^2\times\dfrac{1}{0.8\times10} = 12.5$ m
$R \le 12.5$ m
$\therefore$ R = 12 m

45. $v = 12\,m/s, v' = 4\sqrt{2}\,m/s$
$v = \sqrt{\mu rg}$
$\therefore$ $12 = \sqrt{\mu rg}$, $4\sqrt{2} = \sqrt{\mu' rg}$
$\dfrac{12}{4\sqrt{2}} = \sqrt{\dfrac{\mu}{\mu'}} \Rightarrow \dfrac{3}{\sqrt{2}} = \sqrt{\dfrac{\mu}{\mu'}}$
$\therefore$ $\mu' = \dfrac{2}{9}\mu$

46. Using,
$v = \sqrt{\mu rg} = \sqrt{0.4\times50\times9.8} = \sqrt{196}$
v = 14 m/s
$\omega = \dfrac{v}{r} = \dfrac{14}{50} = 0.28$ rad/s

47. For the crate not to slide, the centripetal force should be $\dfrac{mv^2}{r} = \mu mg$
$\therefore$ $v^2 = \mu rg = 0.6\times35\times9.8 = 205.8$
$\therefore$ v = 14.3 m/s

48. Using,
$\mu mg = \dfrac{mv^2}{r}$
$\therefore$ $0.5\text{ mg} = \dfrac{mv^2}{r}$
$v^2 = 0.5\times r\times g = 0.5\times10\times9.8 = 49$
$\therefore$ v = 7 m/s

49. Since car turns through 90° after travelling 471 m on the circular road, the distance 471 m is quarter of the circumference of the circular path. If R is the radius of the circular path, then
$\dfrac{1}{4}(2\pi R) = 471$
$\therefore$ $R = \dfrac{471\times2}{\pi} = \dfrac{471\times2}{3.14} = 300$ m
v = 12 m/s, m = 1000 kg
$\therefore$ Centripetal force,
$F_{cp} = \dfrac{mv^2}{R} = \dfrac{1000\times(12)^2}{300} = 480$ N

50. $r = 50$ m, $l = 10$ m, $h = 1.5$ m

$$\frac{v^2}{rg} = \frac{h}{l}$$

$\therefore$ $v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50\times9.8\times1.5}{10}} = 8.6$ m/s

51. $l = 1$ m, $g = 110$ m/s^2

$r = 400$ m, $v = 72$ km/hr $= 72 \times \frac{5}{18} = 20$ m/s,

$$\frac{v^2}{rg} = \frac{h}{l}$$

$\therefore$ $h = \frac{v^2 l}{rg} = \frac{20\times20\times1}{400\times10} = 0.1$ m $= 10$ cm

52. This horizontal inward component provides required centripetal force to negotiate the curve safely.

53. $\tan\theta = \frac{v^2}{rg} \Rightarrow \tan\theta \propto v^2$

$\therefore$ $\frac{\tan\theta_1}{\tan\theta_2} = \frac{v_1^2}{v_2^2} = \frac{v^2}{4v^2} = \frac{1}{4}$

$\therefore$ $\tan\theta_2 = 4\tan\theta_1$

54. $\sin\theta = \frac{h}{l}$ and $\tan\theta = \frac{v^2}{rg}$

$\therefore$ $\tan\left\{\sin^{-1}\left(\frac{h}{l}\right)\right\} = \frac{v^2}{rg}$

55. Reaction on inner wheel, $R_1 = \frac{1}{2}M\left[g - \frac{v^2h}{ra}\right]$

Reaction on outer wheel, $R_2 = \frac{1}{2}M\left[g + \frac{v^2h}{ra}\right]$

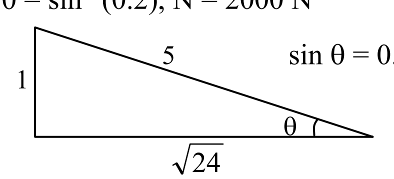
where, r = radius of circular path, 2a = distance between two wheels and h = height of centre of gravity of car.

56. Using,

$\tan\theta \approx \theta = \frac{h}{l}$

$h = l\,\theta = 1.5 \times 0.01 = 0.015$ m

57. $\theta = \sin^{-1}(0.2)$, $N = 2000$ N



$\sin\theta = 0.2 = \frac{1}{5}$

$mg = N\cos\theta$

$\therefore$ Weight $= N\cos\theta = \frac{\sqrt{24}}{5} \times 2000 = 1959.6$ N

.... $\left[\because \cos\theta = \sqrt{1-\left(\frac{1}{5}\right)^2} = \frac{\sqrt{24}}{5}\right]$

58. Using,

$v = \sqrt{rg\tan\theta} = \sqrt{10\times10\times\tan\theta}$

$10 = 10\sqrt{\tan\theta}$

$\tan\theta = 1$

$\therefore$ $\theta = 45°$

59. Using, $h = l\sin\theta$

$\therefore$ $\sin\theta \approx \tan\theta = \frac{h}{l} = \frac{1.2}{8} = 0.15$

$\therefore$ $\tan\theta = 0.15$

Now, $v = \sqrt{rg\tan\theta} = \sqrt{40\times9.8\times0.15} \approx 8$ m/s

60. The maximum velocity for a banked road with friction,

$$v^2 = gr\left(\frac{\mu + \tan\theta}{1-\mu\tan\theta}\right)$$

$\therefore$ $v^2 = 9.8 \times 1000 \times \left(\frac{0.5+1}{1-0.5\times1}\right)$

.... [$\because \tan 45 = 1$]

$\therefore$ $v \approx 172$ m/s

61. Using,

$\tan\theta = \frac{v^2}{rg}$

$\therefore$ $v = \sqrt{\tan\theta\ rg}$

$= \sqrt{\tan 30^\circ \times 17.32 \times 10}$

$= \sqrt{\frac{1}{\sqrt{3}}\times17.32\times10}$

$= 10$ m/s

62. Using,

$\tan\theta = \frac{v^2}{rg} = \frac{20\times20}{20\times9.8} = \frac{20}{9.8} = 2.04$

$\theta = \tan^{-1}(2.04) = 63.90°$

63. $v = 60$ km/h $= 60 \times \frac{5}{18} = \frac{50}{3}$ m/s,

$r = 0.1$ km $= 0.1 \times 1000 = 100$ m

$\tan\theta = \frac{v^2}{rg} = \left(\frac{50}{3}\right)^2 \times \frac{1}{0.1\times10^3\times9.8}$

$\therefore \theta = \tan^{-1}\left[\frac{(50/3)^2}{100\times9.8}\right] \approx \tan^{-1}\left(\frac{1}{5}\right)$

64. $v = 180$ km/hr $= \frac{5}{18}\times 180 = 50$ m/s

Using,

$\tan\theta = \frac{v^2}{rg} = \frac{50\times50}{500\times10} = \frac{5}{10} = \frac{1}{2}$

$\therefore \theta = \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(0.5)$

65. $m = 80$ kg, $v = 20$ m/s, $\theta = \tan^{-1}(0.5)$

In order for the cyclist to turn,

frictional force = centripetal force

$\therefore \mu mg = m\left(\frac{v^2}{r}\right) = mg\frac{v^2}{rg}$

But $\frac{v^2}{rg} = \tan\theta$

$\therefore \mu mg = mg\tan\theta = 80 \times 10 \times 0.5 = 400$ N

66. Let initial velocity $= v_1$

New velocity $v_2 = v\left(1+\frac{20}{100}\right) = \frac{6v}{5}$

$r_1 = 30$ m, $\tan\theta_1 = \frac{v_1^2}{r_1 g}$, $\tan\theta_2 = \frac{v_2^2}{r_2 g}$

As there is no change in angle of banking,

$\theta_1 = \theta_2 \;\therefore \tan\theta_1 = \tan\theta_2$

$\therefore \frac{v_1^2}{r_1 g} = \frac{v_2^2}{r_2 g}$

$\therefore \frac{r_1}{r_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{v_1}{\frac{6}{5}v_1}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$\therefore r_2 = \frac{36}{25}r_1 = \frac{36}{25}\times 30 = \frac{216}{5} = 43.2$ m

67. Using,

$F_s = \frac{mv^2}{r}$ But, $\tan\theta = \frac{v^2}{rg}$

$\frac{v^2}{r} = g\tan\theta$

$F_s = mg\tan\theta = 90 \times 10 \times \tan 30° \approx 520$ N

68. For banking of road, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$\theta = \tan^{-1}(0.24)$

$\therefore \tan\theta = 0.24$

Also, $\tan\theta = \frac{v^2}{rg} = \mu \Rightarrow \mu = 0.24$

69. $T = ma = mr\omega^2$

$T \propto \omega^2$

$\frac{\omega'^2}{\omega^2} = \frac{T'}{T} = \frac{4T}{T} = 4$

$\therefore \omega'^2 = 4\omega^2$

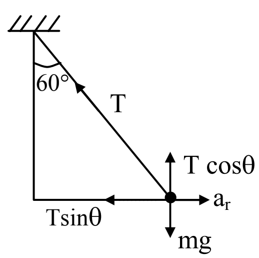
$\therefore \omega' = 2\omega$

$n' = 2n = 2 \times 5 = 10$ r.p.m.

70. Using,

$T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$ ....(i)

$T\cos\theta = mg$ ....(ii)



$\therefore$ From (i) and (ii), $\omega^2 = \frac{g}{l\cos\theta}$

$\therefore \omega = \sqrt{\frac{g}{l\cos\theta}}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$= 2 \times 3.14 \times \sqrt{\frac{1\times\cos 60°}{10}}$

$= 1.4$ s

71. Using,

$r = l\sin\theta$

$r = 10\sin 30° \Rightarrow r = 5$ m, $T = 3$ s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

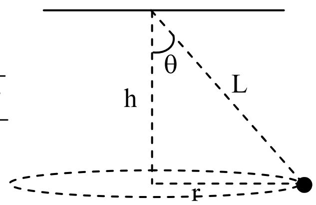
Centripetal force $= m\omega^2 r$

$= 5 \times 10^{-2} \times \frac{4\pi^2}{9} \times 5$

$= 25 \times 10^{-2} \times 4$

$= 100 \times 10^{-2} \approx 1$ N

72. $T = \dfrac{mg}{\cos\theta}$



$\cos\theta = \dfrac{h}{L} = \dfrac{\sqrt{L^2 - r^2}}{L}$

$\therefore \quad T = \dfrac{mg\,L}{\sqrt{L^2 - r^2}}$

73. At the highest point,

$mg = \dfrac{mv^2}{r}$

$\therefore \quad v = \sqrt{rg} = \sqrt{4000 \times 10} = 200$ m/s

74. r = 6.4 m

Minimum velocity at the bottom,

$v = \sqrt{5gr} = \sqrt{5\times9.8\times6.4}$

$= \sqrt{313.6} = 17.7$ m/s

75. Using,

$F = \dfrac{mv^2}{r} = m\omega^2 r = mg$

$\therefore \quad \omega = \sqrt{\dfrac{g}{r}} \Rightarrow \dfrac{2\pi}{T} = \sqrt{\dfrac{9.8}{4}}$

$\therefore \quad T = \dfrac{2\pi\times2}{\sqrt{9.8}} \approx 4$ s

76. $T_L - T_H = \dfrac{m}{r}(u^2 + gr) - \dfrac{m}{r}(u^2 - 5gr)$

$= \dfrac{m}{r}(u^2 + gr - u^2 + 5gr) = \dfrac{m}{r}(6gr) = 6$ mg

77. Using,

$\dfrac{mv^2}{r} = \dfrac{2\times(4)^2}{1} = 32$ N

It is clear that tension will be 52 N at the bottom of the circle because we know that,

$T_{Bottom} = mg + \dfrac{mv^2}{r}$

78. $T_L = 350$ N

Using,

$\dfrac{mv^2}{r} = T_L - mg = (2 \times 350 - 40 \times 10) = 300$

$\therefore \quad v^2 = \dfrac{300\times3}{40} = 22.5$ m/s

$v \approx 4.7$ m/s

79. At the highest point of the circle,

$F = \dfrac{mv^2}{r} - mg = 70 \times \left[\dfrac{4\times10^4}{400} - 10\right]$

$= 6300$ N

80. At the lowest point of the circle,

$F = \dfrac{mv^2}{r} + mg = 70 \times \left[\dfrac{4\times10^4}{400} + 10\right]$

$= 7700$ N

81. Using,

$\dfrac{mv^2}{r} = mg$

$\therefore \quad v^2 = gr$

$v = \sqrt{gr} = \sqrt{10\times12.1} = \sqrt{121} = 11$ m/s

82. Using,

$(K.E.)_L - (K.E.)_H = \dfrac{1}{2}m\left[v_L^2 - v_H^2\right] = \dfrac{1}{2}m[5\,rg - rg]$

$= 2$ mrg $= 2 \times 1 \times 1 \times 10$

$= 20$ J

83. Even though particle is moving in a vertical loop, its speed remain constant.

Tension at lowest point, $T_{max} = \dfrac{mv^2}{r} + mg$

Tension at highest point, $T_{min} = \dfrac{mv^2}{r} - mg$

$\dfrac{T_{max}}{T_{min}} = \dfrac{\dfrac{mv^2}{r} + mg}{\dfrac{mv^2}{r} - mg} = \dfrac{5}{3}$

By solving we get, $v = \sqrt{4gr} = \sqrt{4\times9.8\times2.5}$

$= \sqrt{98}$ m/s

84. Using,

$mg - N_1 = \dfrac{mv_1^2}{r}$

$\therefore \quad \dfrac{mv_1^2}{r} = 667 - 556 = 111$

Let $v_2 = 2v_1$

$\therefore \quad \dfrac{mv_2^2}{r} = \dfrac{4mv_1^2}{r} = 4 \times 111 = 444$

$mg - N_2 = \dfrac{mv_2^2}{r}$

$\therefore \quad N_2 = 667 - 444 = 223$ N

85. By conservation of energy,

$\frac{1}{2}mv^2 = mgh$

$v = \sqrt{2gh}$ ....(i)

For looping the loop, the lower velocity must be greater than $\sqrt{5gr}$

$v_{min} = \sqrt{5gr} = \sqrt{\frac{5gD}{2}}$ ....(ii)

From (i) and (ii),

$2gh = \frac{5gD}{2}$

$h = \frac{5D}{4}$

86. According to law of conservation of energy,

$mgh = \frac{1}{2}mv^2 = \frac{1}{2}m \times 5 \times Rg$

$\therefore$ $R = \frac{2}{5}h = \frac{2}{5} \times 5 = 2$ cm

88. Using,

$\alpha = \frac{\omega - \omega_0}{t} = \frac{36-0}{6} = 6$ rad/s^2

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 6 \times 6 \times 6 = 108$ rad

89. $n_2 = 1200$ r.p.m. $= \frac{1200}{60} = 20$ r.p.s.

$n_1 = 600$ r.p.m. $= \frac{600}{60} = 10$ r.p.s., t = 5 s

$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(20-10)}{5}$

$= \frac{20\pi}{5} = 4\pi$ rad/s^2

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 20\pi \times 5 + \frac{1}{2} \times 4\pi \times 25$

$= 100\pi + 50\pi = 150\pi$

Number of revolutions $= \frac{\theta}{2\pi} = \frac{150\pi}{2\pi} = 75$

90. $\alpha = \frac{\omega}{t}$ and $\omega = \frac{\theta}{t}$

$\therefore$ $\alpha = \frac{\theta}{t^2}$

But α = constant $\Rightarrow \theta \propto t^2$

So, $\frac{\theta_1}{\theta_1 + \theta_2} = \frac{(2)^2}{(2+3)^2}$

or $\frac{\theta_1}{\theta_1 + \theta_2} = \frac{4}{25}$

or $\frac{\theta_1 + \theta_2}{\theta_1} = \frac{25}{4}$

or $1 + \frac{\theta_2}{\theta_1} = \frac{25}{4}$

$\therefore$ $\frac{\theta_2}{\theta_1} = \frac{21}{4}$

91. By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n)$

$\therefore$ $\alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi \times 36}$ ....(i)

Now let fan complete total n′ revolutions from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n')$

$\therefore$ $n' = \frac{\omega_0^2}{4\alpha\pi}$

Substituting the value of α from equation (i),

$n' = \frac{\omega_0^2}{4\pi}\frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48$ revolutions

Number of rotations = 48 – 36 = 12

92. Let velocity at A = v_1

Velocity at B = v_2

$\because$ Velocity is constant,

$\therefore$ $v_1 = v_2 = v$ (say)

$\angle AOB = 60°$

$\therefore$ Change in velocity,

$|v_1 - v_2| = \sqrt{{v_1}^2 + {v_2}^2 - 2v_1 v_2 \cos\theta}$

$= \sqrt{v^2 + v^2 - 2v^2 \times \cos\theta}$

$= \sqrt{2v^2(1-\cos\theta)}$

$= v\sqrt{2 \times 2\sin^2\frac{\theta}{2}} = 2v\sin\frac{\theta}{2}$

$= 2v \sin 30°$

(Note: Refer Shortcut 2.)

93. Using,

$v = \frac{2\pi r}{T}$

$\therefore$ $T = \frac{2\pi r}{v} = \frac{2\pi}{80} \times \frac{20}{\pi} = \frac{1}{2}$ s

$\because$ T = Time taken for one revolution

There are 2 revolutions $\Rightarrow$ total time taken = 1 s

$\omega = \frac{2\pi}{T} = 4\pi$ ....($\because$ T = 1)

$\alpha = \frac{d\omega}{dt} = \frac{4}{2}\pi = 2\pi$

$a_t = \alpha \cdot r$ i.e. $= 2\pi \times \frac{20}{\pi} = 40\ m/s^2$

94. Using,

Maximum tension, $T_{max} = \frac{mv_1^2}{r} + mg$

Minimum tension, $T_{min} = \frac{mv_2^2}{r} - mg$

Using the law of conservation of energy,

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 2mgr$

$\therefore$ $v_1^2 = v_2^2 + 4rg$

Hence $\frac{T_{max}}{T_{min}} = \frac{\frac{v_1^2}{r} + g}{\frac{v_2^2}{r} - g} = \frac{v_1^2 + rg}{v_2^2 - rg}$

$= \frac{v_2^2 + 5rg}{v_2^2 - rg} = \frac{4}{1}$

....[$\because$ $v_1^2 = v_2^2 + 4rg$]

This gives, $4v_2^2 - 4rg = v_2^2 + 5rg$

$\therefore$ $3v_2^2 = 9\ rg = 9 \times \frac{10}{3} \times 10$

$\therefore$ $v_2^2 = \frac{9}{3} \times \frac{10}{3} \times 10$

$\therefore$ $v_2^2 = 100$

$\therefore$ $v_2 = 10$ m/s

Competitive Thinking

2. $T_E = 24$ hr, $T_H = 12$ hr

$\therefore$ $\frac{\omega_E}{\omega_H} = \frac{2\pi / T_E}{2\pi / T_H} = \frac{T_H}{T_E} = \frac{12}{24} = \frac{1}{2}$

3. $n_1 = 600$ r.p.m., $n_2 = 1200$ r.p.m.,

Using,

Increment in angular velocity, $\omega = 2\pi(n_2 - n_1)$

$\omega = 2\pi(1200 - 600)$ rad/min $= \frac{2\pi \times 600}{60}$ rad/s

$\omega = 20\pi$ rad/s

4. For an hour hand, T = 12 hr = 12 × 3600 s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600}$ rad/s

6. Angular acceleration $= \frac{d^2\theta}{dt^2} = 2\theta_2$

7. $v = r\omega$

$\therefore$ $\omega = \frac{v}{r}$ = constant [As v and r are constant]

8. $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$\omega = \frac{v}{r} \Rightarrow \frac{v}{r}$ = constant

$\therefore$ $\frac{v_1}{r_1} = \frac{v_2}{r_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{R}{r}$

9. For seconds hand, T = 60 s,

$r = 3\ cm = 3 \times 10^{-2}$ m

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047$ rad/s

and $v = \omega r = 0.1047 \times 3 \times 10^{-2} = 0.00314$ m/s

10. n = 600 r.p.m. $= \frac{600}{60}$ r.p.s. = 10 r.p.s.

$v = r\omega = r \times 2\pi n = 10 \times 2 \times 3.142 \times 10$

$= 628.4$ cm/s.

11. Using,

$v = r\omega = 0.5 \times 70 = 35$ m/s

12. Using,

$v = r\omega = r \times \frac{2\pi}{T} = 60 \times \frac{2 \times 3.14}{60} = 6.28$ mm/s

$\Delta v = 6.28\sqrt{2}$ mm/s ≈ 8.88 mm/s

13. No.of revolutions $= \frac{\text{Total time}}{\text{Time period}} = \frac{140\,s}{40\,s}$

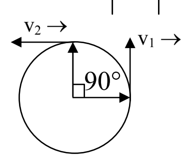
= 3.5 Rev.

So, distance $= 3.5 \times 2\pi R = 3.5 \times 2\pi \times 10$

≈ 220 m

14. In 15 seconds hand rotates through 90°

Change in velocity $\left|\Delta \vec{v}\right| = 2v \sin\left(\frac{\theta}{2}\right)$



$= 2(r\omega)\sin\left(\frac{90°}{2}\right) = 2 \times 1 \times \frac{2\pi}{T} \times \frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\frac{cm}{s}$

(Note: Refer Shortcut 2.)

15. In circular motion,
Centripetal force ⊥ Displacement
∴ work done is zero.

16. $L = I\omega$. In U.C.M., ω = constant
∴ L = constant

17. Work done by centripetal force in uniform circular motion is always equal to zero.

19. Angular momentum is an axial vector. It is directed always in a fixed direction (perpendicular to the plane of rotation either outward or inward), if the sense of rotation remains same.

20. The instantaneous velocity of a body in U.C.M. is always perpendicular to the radius or along the tangent to the circle at the point.

21. In one complete revolution, total displacement is zero. So average velocity is zero

22. $r = \pi$, $n = \left(\frac{p}{t}\right)$ r.p.s.

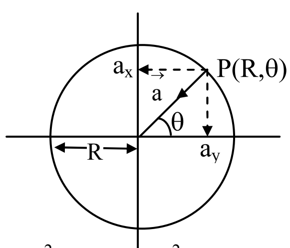
$v = r\omega = r \times 2\pi n = \pi \times 2\pi \times \frac{p}{t} = \frac{2\pi^2 p}{t}$

23. $E = \frac{1}{2} mv^2 \Rightarrow v^2 = \frac{2E}{m}$

$a = \frac{v^2}{r} = \frac{2E}{mr}$

26. The radius vector points outwards while the centripetal acceleration points inwards along the radius.

27.



$\vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

28. They have same angular speed ω.
Centripetal acceleration = $\omega^2 r$

$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

29. $a = \omega^2 R = \left(\frac{2\pi}{0.2\pi}\right)(5 \times 10^{-2}) = 5\ m/s^2$

30. Using,
$\omega = 2\pi n = 2\pi \times 1 = 2\pi$ rad/s
$a = r\omega^2 = 0.4 \times (2\pi)^2 = 0.4 \times 4\pi^2$
$a = 1.6\pi^2\ m/s^2$

31. Since, n = 2, $\omega = 2\pi \times 2 = 4\pi\ rad/s^2$
So acceleration = $\omega^2 r = (4\pi)^2 \times \frac{25}{100}\ m/s^2 = 4\pi^2$

32. Using,
$a = \omega^2 r = 4\pi^2 n^2 r = 4(3.14)^2 \times 1^2 \times 20 \times 10^3$
∴ $a \approx 8 \times 10^5\ m/s^2$

33. Net acceleration in non-uniform circular motion

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} \approx 2.7\ m/s^2$

36. Radial force = $\frac{mv^2}{r} = \frac{m}{r}\left(\frac{p}{m}\right)^2 = \frac{p^2}{mr}$

….[∵ p = mv]

38. Using,
$T = mr\omega^2 \Rightarrow \omega^2 = \frac{T}{mr}$

∴ $\omega = \sqrt{\frac{6.4}{0.1 \times 6}} = \sqrt{14} \approx 3$ rad/s

39. $F = \frac{mv^2}{r}$
∴ $F \propto v^2$. If v becomes double, then F (tendency to overturn) will become four times.

40. $L = r\,p\sin\theta = r\,p$ for U.C.M. [∵ θ = 90°]

∴ $\frac{L^2}{mr^3} = \frac{r^2 m^2 v^2}{mr^3} = \frac{mv^2}{r}$

41. Using,
$T = m\omega^2 r$
∴ $10 = 0.25 \times \omega^2 \times 0.1$
∴ ω = 20 rad/s

42. v = 36 km/h = 10 m/s
Using,
∴ $F = \frac{mv^2}{r} = \frac{500 \times 100}{50} = 1000$ N

43. m = 100 kg, v = 9 m/s, r = 30 m
Maximum force of friction = centripetal force
$\frac{mv^2}{r} = \frac{100 \times (9)^2}{30} = 270$ N

44. Using, $F = mr\omega^2 = m\,4\pi^2 n^2 r$
∴ $m\,4\pi^2 n^2 r = 4 \times 10^{-13}$

∴ $n = \sqrt{\frac{4 \times 10^{-13}}{1.6 \times 10^{-27} \times 4 \times 3.14^2 \times 0.1}}$

∴ $n = 0.08 \times 10^8$ cycles/second

45. The centripetal force, $F = \frac{mv^2}{r}$

$\therefore$ $r = \frac{mv^2}{F}$

$\therefore$ $r \propto v^2$ or $v \propto \sqrt{r}$

(If m and F are constant), $\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}}$

46. $r_1 = 4$ cm, $\omega_2 = 2\omega_1$

$r\omega^2 = $ constant

$\therefore$ $r_1\omega_1^2 = r_2\omega_2^2$

$\therefore$ $r_1\omega_1^2 = r_1(2\omega_1)^2 = r_1 = 4\,r_2$

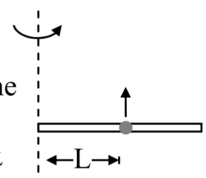
$\therefore$ $r_2 = \frac{r_1}{4} = \frac{4}{4} = 1$ cm

47. $F = \frac{mv^2}{r}$

$F \propto v^2$ i.e. force will become 4 times.

48. Let the bead starts slipping after time t

For critical condition, frictional force provides the centripetal force

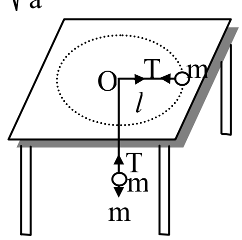


$m\omega^2L = \mu R = \mu m \times a_1 = \mu Lm\alpha$

$\Rightarrow m(\alpha t)^2L = \mu mL\alpha$

$\Rightarrow t = \sqrt{\frac{\mu}{a}}$ ....[$\because \omega = \alpha t$]

49.



Tension T in the string will provide centripetal force $\Rightarrow \frac{mv^2}{l} = T$ ....(i)

Also, tension T is provided by the hanging ball of mass m,

$\Rightarrow T = mg$ ....(ii)

$mg = \frac{mv^2}{l} \Rightarrow g = \frac{v^2}{l}$

50. Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first.

54. Using,

$v_{max} = \sqrt{\mu rg} = \sqrt{0.2 \times 100 \times 9.8} = 14$ m/s

55. Using,

$v = \sqrt{\mu rg} = \sqrt{0.4 \times 30 \times 9.8} = 10.84$ m/s

56. $C = 34.3$ m $\Rightarrow r = \frac{34.3}{2 \times \pi}$,

$T = \sqrt{22}$ s $\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{\sqrt{22}}$ s

$\therefore$ $\theta = \tan^{-1}\left(\frac{r\omega^2}{g}\right) = \tan^{-1}\left(\frac{34.3}{2\pi} \times \frac{2\pi \times 2\pi}{22} \times \frac{1}{9.8}\right)$

$= \tan^{-1}\left(34.3 \times 2 \times \frac{22}{7 \times 22} \times \frac{1}{9.8}\right) = \tan^{-1}\left(\frac{4.9 \times 2}{9.8}\right)$

$= \tan^{-1}(1) = 45°$

57. Using, $\tan\theta = \frac{v^2}{rg}$

$\therefore$ $\tan 12° = \frac{(150)^2}{r \times 10}$

$\therefore$ $r = 10.6 \times 10^3$ m $= 10.6$ km

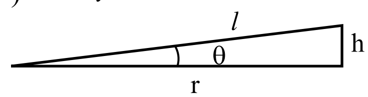
58. For banking, $\tan\theta = \frac{v^2}{Rg}$

$\tan 45 = \frac{v^2}{90 \times 10} = 1$

$v = 30$ m/s

59. $\tan\theta = \frac{h}{(l^2 - h^2)^{1/2}} \approx \frac{h}{l}$

$(l^2 >> h^2)$



$\tan\theta = \frac{v^2}{rg}$

$\frac{h}{l} = \frac{v^2}{rg}$ $\therefore$ $h = \frac{v^2 l}{rg}$

60. The inclination of person from vertical is given by,

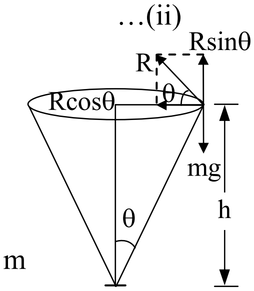
$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50 \times 10} = \frac{1}{5}$

$\therefore$ $\theta = \tan^{-1}(1/5)$

61. The particle is moving in circular path.

From the figure, $mg = R\sin\theta$ ...(i)

$\frac{mv^2}{r} = R\cos\theta$ ...(ii)



From equation (i) and (ii) we get

$\tan\theta = \frac{rg}{v^2}$ but $\tan\theta = \frac{r}{h}$

$\therefore$ $h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025$ m

$= 2.5$ cm

62. Because tension is maximum at the lowest point.

63. When body is released from the position (inclined at angle θ from vertical), then velocity at mean position,
$v = \sqrt{2gl(1-\cos\theta)}$

$\therefore$ Tension at the lowest point = $mg + \dfrac{mv^2}{l}$

$= mg + \dfrac{m}{l}[2gl(1-\cos60°)] = mg + mg = 2mg$

64. Tension at mean position, $mg + \dfrac{mv^2}{r} = 3mg$

$v = \sqrt{2gl}$ ....(i)

and if the body displaces by angle θ with the vertical then $v = \sqrt{2gl(1-\cos\theta)}$ ....(ii)

Comparing (i) and (ii), $\cos\theta = 0$

$\therefore$ $\theta = 90°$

67. Tension, $T = \dfrac{mv^2}{r} + mg\cos\theta$

For, $\theta = 30°$, $T_1 = \dfrac{mv^2}{r} + mg\cos30°$

$\theta = 60°$, $T_2 = \dfrac{mv^2}{r} + mg\cos60°$

$\therefore$ $T_1 > T_2$

68. $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$ ....$[\omega = 2\pi n]$

$= m\left[g + \left(4\pi^2\left(\dfrac{n}{60}\right)^2 r\right)\right] = m\left[g + \left(\dfrac{\pi^2 n^2 r}{900}\right)\right]$

69. Minimum angular velocity,

$\omega_{min} = \sqrt{\dfrac{g}{R}}$

$\therefore$ $T_{max} = \dfrac{2\pi}{\omega_{min}} = 2\pi\sqrt{\dfrac{R}{g}} = 2\pi\sqrt{\dfrac{2}{10}}$

$= 2\sqrt{2} \approx 3$ s

70. Using, $mr\omega^2 = mg$

$\therefore$ $r\left(\dfrac{2\pi}{T}\right)^2 = g \Rightarrow T^2 = \dfrac{4\pi^2 r}{g}$

$\therefore$ $T = 2\pi\sqrt{\dfrac{r}{g}} = 2 \times 3.14 \times \sqrt{\dfrac{4}{9.8}} \approx 4$ s

71. Critical velocity at highest point $= \sqrt{gR}$

$= \sqrt{10 \times 1.6}$

$= 4$ m/s

72. $v = \sqrt{3gr}$ and $a = \dfrac{v^2}{r} = \dfrac{3gr}{r} = 3g$

73. $T_{max} = 30$ N

Using,

$T_{max} = m\omega^2_{max} r + mg$

$\therefore$ $\dfrac{T_{max}}{m} = \omega^2 r + g$

$\dfrac{30}{0.5} - 10 = \omega^2_{max} r$

$\omega_{max} = \sqrt{\dfrac{50}{r}} = \sqrt{\dfrac{50}{2}} = 5$ rad/s

74. Max. tension that string can bear = 3.7 kg-wt

= 37 N

Tension at lowest point of vertical loop

$= mg + m\omega^2 r = 0.5 \times 10 + 0.5 \times \omega^2 \times 4$

$= 5 + 2\omega^2$

$\therefore$ $37 = 5 + 2\omega^2$

$\therefore$ $\omega = 4$ rad/s

75. Using,

$T_L = \dfrac{mv_L^2}{r} + mg = 6\,mg = 6 \times 5 \times 10 = 130$ N

$\therefore$ The mass is at the bottom position.

76. $a_c = k^2rt^2$

Here the tangential acceleration also exits which requires power.

$a_c = \dfrac{v^2}{r}$

$\therefore$ $\dfrac{v^2}{r} = k^2rt^2$

$\therefore$ $v^2 = k^2r^2t^2$ or $v = kt$

Tangential acceleration $a_t = \dfrac{dv}{dt} = kr$

$F = m \times a = mkr$

So power, $P = F \times v = mkr \times krt = mk^2r^2t$

77. $n = \dfrac{2}{\pi}$ r.p.s.

$T\sin\theta = M\omega^2 R$ ....(i)

$T\sin\theta = M\omega^2 L\sin\theta$ ....(ii)

From (i) and (ii),

$T = M\omega^2 L$

$= M\,4\pi^2 n^2 L = M\,4\pi^2\left(\dfrac{2}{\pi}\right)^2 L = 16\,ML$

78. $\frac{mv^2}{r} = \frac{k}{r^2}$

$\therefore$ $mv^2 = \frac{k}{r}$

$\therefore$ K.E.$= \frac{1}{2} mv^2 = \frac{k}{2r}$

P.E. $= \int F\,dr = \int \frac{k}{r^2} dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E + P.E $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$

79. Linear velocity, $v = \omega r = 2\pi nr$
$= 2 \times 3.14 \times 3 \times 0.1$
$= 1.88$ m/s

Acceleration, $a = \omega^2 r = (6\pi)^2 \times 0.1 = 35.5$ m/s^2

Tension in string, $T = m\,\omega^2 r = 1 \times (6\pi)^2$
$= 1 \times (6\pi)^2 \times 0.1$
$= 35.5$ N

82. Centripetal acceleration,

$a_c = \omega^2 r = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2}{(0.2\pi)^2} \times 5 \times 10^{-2} = 5$ ms^{-2}

As particle is moving with constant speed, its tangential acceleration, $a_T = 0$.

The acceleration of the particle,

$a = \sqrt{a_c^2 + a_T^2} = \sqrt{5^2 + 0^2} = 5$ m/s^2

83. At an instant, speed of P = v, going in clockwise direction

Speed of Q = v, going in anticlockwise direction

Relative angular velocity of P w.r.t.

Q = $\omega - (-\omega) = 2\,\omega$

Relative angular separation of P and Q in time t

$\theta = 2\,\omega t$.

Relative speed between the points P and Q at time t

$|\vec{v}_r| = \sqrt{v^2 + v^2 - 2vv\cos(2\omega r)}$

$= \sqrt{2v^2(1 - \cos 2\omega r)}$

$= \sqrt{2v^2 \times 2\sin^2 \omega r}$

$= 2v \sin \omega r$

Since, $|\vec{v}_r|$ will not have any negative value so the lower part of the sine wave will come upper side.

The Answers to Physics of…..

1. ***A trapeze act in a circus***

When the man and his partner are stationary, the man's arms must support his partner's weight. When the two are swinging, however, the man's arms must do an additional job. Then, the partner is moving on a circular arc and has a centripetal acceleration. The man's arms must exert an additional pull so that there will be sufficient centripetal force to produce this acceleration. Because of the additional pull, it is harder for the man to hold his partner while swinging than while stationary.

2. ***Riding the bicycle in a loop the loop***

A key idea in analyzing the stunt is to assume that rider and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration $\vec{a}$ *of this particle must have the magnitude* $a = v^2/R$ *and be directed downwards, toward the centre of the circular loop.*

The gravitational force $\vec{F}_g$ *is directed downward along a y-axis. The normal force* $\vec{N}$ *on the particle from the loop is also directed downward. Thus, Newton's second law for y-axis components* ($F_{net,y} = ma_y$) *gives us*

$-N - F_g = m(-a)$

$\therefore$ $-N - mg = m\left(\frac{v^2}{R}\right)$ …(1)

Another Key idea is that if the particle has the least speed v *needed to remain in contact, then it is on the verge of losing contact with the loop (falling away from the loop), which means that N = 0. Substituting this value for N into Equation (1) gives,* $v = \sqrt{gR}$

The rider has to make certain that his speed at the top of the loop is greater than $\sqrt{gR}$ *so that he does not lose contact with the loop and fall away from it. Note that this speed requirement is independent of mass rider and his bicycle.*

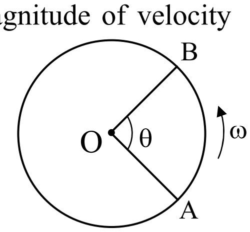
Evaluation Test

1. In children's park, there was a slide to be made by contract. By mistake, the person who had taken the contract made the coefficient of friction of the slide as high as 1/3. Now, the fun is that the child expecting to slide down the incline will stop somewhere in between. Find the angle θ with the horizontal at which he will stop on the incline. (Assume negligible frictional losses)
   (A) 45° (B) 37°
   (C) 53° (D) 60°

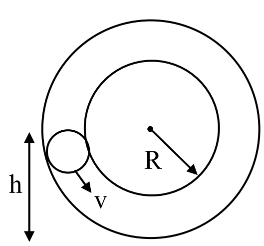
2. A particle is moving parallel to X-axis such that y-component of its position vector is constant and equal to 'b'. The angular velocity about the origin will be
   (A) $\frac{v}{b}\sin^2\theta$ (B) $\frac{v}{b}$
   (C) $\frac{v}{b}\sin\theta$ (D) vb

3. A wire which is bent in the shape of a curve given by, $y = a^3 x^4$. A bead of mass m is located at point P(x, y). If the wire is smooth, find ω with which wire needs to be rotated for bead to be static.
   (A) $a\sqrt{x^3 g}$ (B) $2a\sqrt{x^3 g}$
   (C) $2x\sqrt{a^3 g}$ (D) $x\sqrt{a^3 g}$

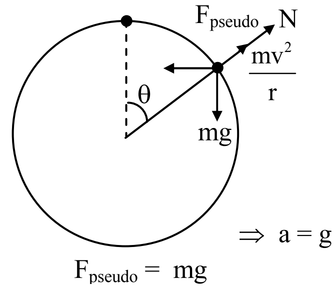
4. A circular disc of radius R is rotating about its axis through O with uniform angular velocity ω rad/s as shown. The magnitude of velocity of A relative to B is
   (A) zero
   (B) Rω sin (θ/2)
   (C) 2Rω sin (θ/2)
   (D) $\sqrt{3}$ Rω sin (θ/2)



5. With what minimum speed v must a small ball be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube?
   (A) $\sqrt{2g(h+2R)}$
   (B) $\frac{5}{2}R$
   (C) $\sqrt{g(5R-2h)}$
   (D) $\sqrt{2g(2R-h)}$

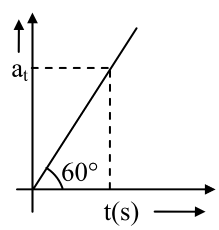


6. A smooth sphere of radius R is made to translate in a straight line with constant acceleration g. A particle kept on top of the sphere is released from there with zero velocity w.r.t. sphere. The speed of particle w.r.t sphere as a function of θ is,

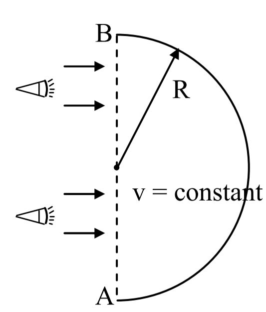


   (A) $\sqrt{Rg\left(\frac{\sin\theta+\cos\theta}{2}\right)}$
   (B) $\sqrt{Rg(1+\cos\theta-\sin\theta)}$
   (C) $\sqrt{4Rg\sin\theta}$
   (D) $\sqrt{2Rg(1+\sin\theta-\cos\theta)}$

7. Tangential acceleration of a particle moving on a circle of radius 1m varies with time t as shown in the graph (initial velocity of particle is zero). Time after which total acceleration of particle makes an angle of 30° with radial acceleration is,
   (A) 4 s
   (B) $\frac{4}{3}$ s
   (C) $2^{2/3}$ s
   (D) $\sqrt{2}$ s

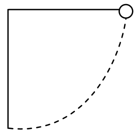


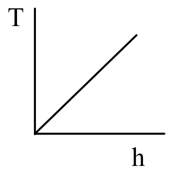
8. A small block can move in a straight horizontal line along AB. Flash lights from one side project its shadow on a vertical wall which has a horizontal cross-section as a circle. Find radial acceleration as a function of time.
   (A) $\frac{vR^2}{(3R^2t - Rvt^2)}$
   (B) $\frac{vR}{(2Rt - vt^2)}$
   (C) $\frac{R.t^{-2/3}}{(2R^3t - vR^2t^2)^{2/3}}$
   (D) $\frac{\frac{3}{2}vRt}{\left(t-\frac{R}{v}\right)}$

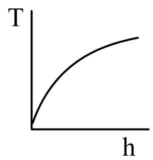


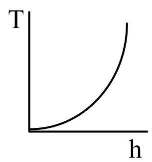
Top view

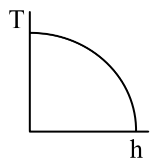
9. A bob attached to a string is held horizontal and released. The tension and vertical distance from point of suspension can be represented by



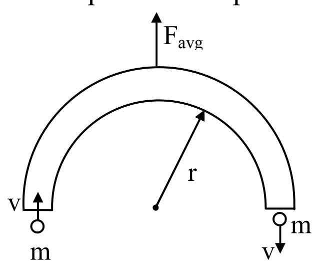
(A) 

(B) 

(C) 

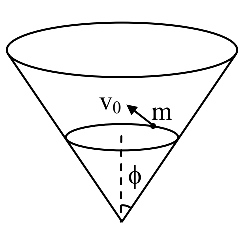
(D) 

10. A ball enters a semicircular tube with velocity v and maintains a constant velocity. Find the average force required to keep the tube stable.



(A) $\frac{mv^2}{\pi r}$ (B) $\frac{2mv^2}{\pi r}$
(C) $\frac{2mv^2}{r}$ (D) $\frac{2\pi}{r}mv^2$

11. Consider an object of mass m that moves in a circular orbit with constant velocity v_0 along the inside of a cone. Assume the wall of the cone to be frictionless. Find radius of the orbit.

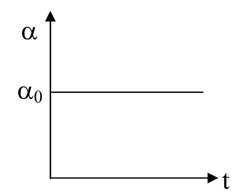


(A) $\frac{v_0^2}{g}\tan^2\phi$ (B) $\frac{v_0^2}{g}\cos^2\phi$
(C) $\frac{v_0^2}{g}\tan\phi$ (D) $\frac{v_0^2}{g}$

12. A bullet is moving horizontally with certain velocity. It pierces two paper discs rotating co-axially with angular speed ω separated by a distance *l*. If the hole made by bullet on second disc is shifted by an angle θ with respect to the first, find velocity of bullet.

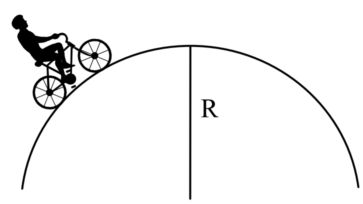
(A) ωl (B) $\frac{l\theta}{\omega}$
(C) $\omega\frac{l}{\theta}$ (D) $\omega l(\theta)^2$

13. Given is the α-t graph for a car wheel, where brakes produce an acceleration α. Which of the following can be the form of θ – t graph ?



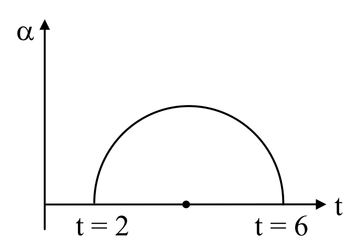
(A) Straight line (B) Parabola
(C) Circle (D) Hyperbola

14. A cyclist is going on an overbridge with constant speed. The value of frictional force acting on the cycle



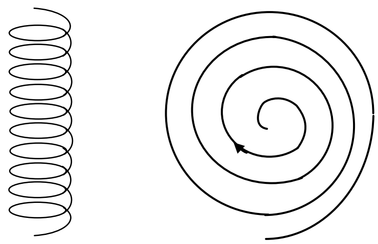
(A) first decreases and then increases.
(B) first increases and then decreases.
(C) decreases continuously.
(D) increases continuously.

15. For the given situation as shown in the graph, the initial angular velocity of the particle is 2π rad/s. What will be the final angular velocity if the particle follows the given α – t graph?



(A) 3π rad/s (B) 4π rad/s
(C) 5π rad/s (D) 6π rad/s

16. A particle is going in a uniform helical and spiral path separately below with constant speed as shown in figure.

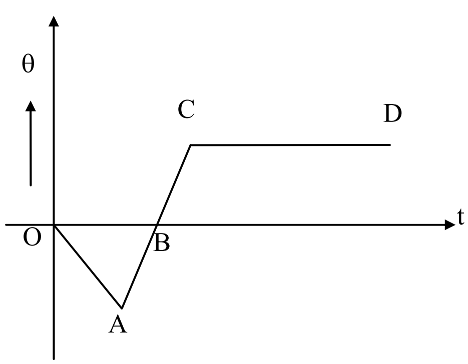


(A) (B)

(A) The velocity of particle is constant in both cases.
(B) The acceleration of particle is constant in both cases.
(C) The magnitude of acceleration is constant in (A) and decreasing in (B).
(D) The magnitude of acceleration is decreasing continuously in both cases.

17. Figure shows the plot of angular displacement and time of rotating disc. Corresponding to the segments marked in the plot, the direction of motion is as:
(A) Disc rotates in clockwise direction in segments OA and AB.
(B) Disc rotates in clockwise direction in the segment OA but in anticlockwise direction in the segment AB.
(C) Disc rotates in anticlockwise direction in segment BC.
(D) At point C, the disc stops rotating suddenly.

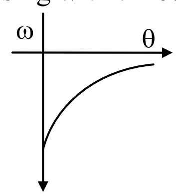
Which of the following is a false statement?

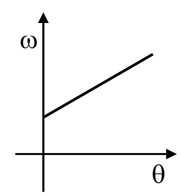


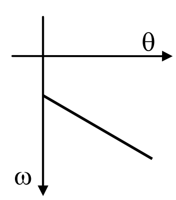
18. The metro which has been recently introduced in Mumbai, encounters a sharp turn between Andheri and Chakala. To avoid any derailing issues, the authorities thought of banking the rails. The turn is of a radius of 400 m and the maximum speed attained by Mumbai Metro is 72 km/hr. If the distance between the rails is 1 m then through what height should the outer rail be raised?
(A) 2.5 cm (B) 0.5 cm
(C) 10 cm (D) 15 cm

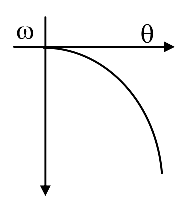
19. A clown is exhibiting a magic trick on the streets wherein he rotates a bucket in a vertical plane without allowing the water in it to spill out. Here, clearly the clown uses centrifugal force to balance the weight of water. This will be possible when,
(A) the bucket has r.p.m. = $\sqrt{\dfrac{400g}{\pi^2 R}}$
(B) the bucket has maximum speed = $\sqrt{2gR}$
(C) the bucket has r.p.m. = $\sqrt{\dfrac{900\,g}{\pi^2 R}}$
(D) the bucket has r.p.m = $\sqrt{\dfrac{3600\,g}{\pi^2 R}}$

20. The graphs below show angular velocity as a function of θ. In which one of these is the magnitude of angular velocity constantly decreasing with time?

(A)


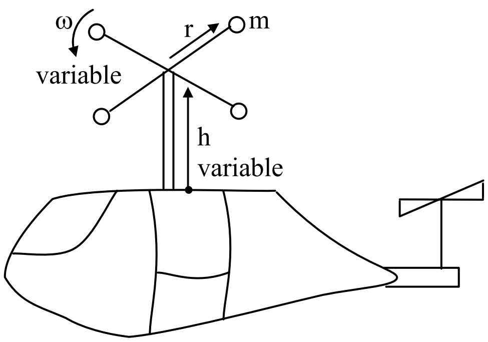
(B)


(C)


(D)


21. For a particle moving in a circle,
(A) the resultant force on the particle must be towards the centre.
(B) the cross product of tangential acceleration and angular velocity will be zero.
(C) direction of angular acceleration and angular velocity must be same.
(D) the resultant force must be away from the centre.

22. A helicopter was designed at MIT in which there was a different system to keep the helicopter stable at a height. The radius of the masses and the angular velocity of rotation can be varied (see figure). The mass of each bob is 5 kg and the number of bobs is 8.



During the test run, empty helicopter of mass 400 kg is to be kept stable. Due to small malfunction, the radius had to be fixed at 1 m and could not be changed. Find the value of ω required to successfully complete the test run. [Take $g = 10\ m/s^2$]

(A) 5 rad/s (B) 2.5 rad/s
(C) 10 rad/s (D) 20 rad/s

23. A chain of mass m and radius R placed on a smooth table is revolving with v about a vertical axis coinciding with the symmetry axis of the chain. Find tension in the chain.

(A) $\frac{Mv^2}{2R}$ (B) $\frac{Mv^2}{R}$
(C) $\frac{Mv^2}{2\pi R}$ (D) $\frac{3}{2}\frac{Mv^2}{R}$

24. A ball suspended by a thread swings in a vertical plane so that its acceleration values at the extreme and lowest positions are equal. Find the thread deflection angle in extreme position.

(A) 53° (B) 37°
(C) 45° (D) 47°

25. A swing moving in a children's garden is observed to move with an angular velocity given by, $\omega = a(t^2)\hat{i} + b(e^{-t})\hat{j}$. What will be the angle between angular acceleration and angular velocity at t = 1 s given that a = b = 1 unit?

(A) 72° (B) 36°
(C) 18° (D) 9°

Answers to Evaluation Test

1. (A)	2. (A)	3. (C)	4. (C)
5. (D)	6. (D)	7. (C)	8. (B)
9. (A)	10. (B)	11. (C)	12. (C)
13. (B)	14. (A)	15. (D)	16. (C)
17. (A)	18. (C)	19. (C)	20. (B)
21. (A)	22. (C)	23. (C)	24. (A)
25. (A)			