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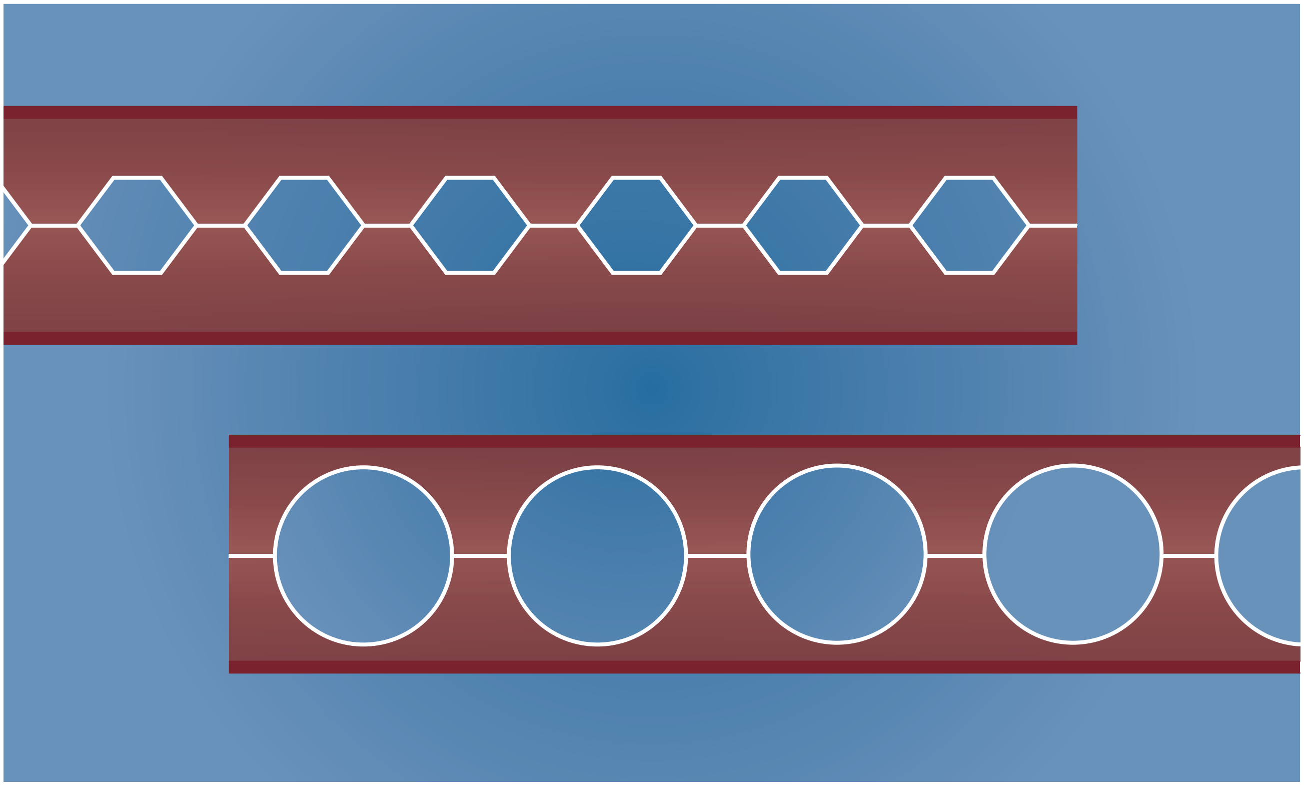
Steel Design Guide

Castellated and Cellular Beam Design



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Castellated and Cellular Beam Design

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Preface

This Design Guide provides guidance for the design of castellated and cellular beams based on structural principles and adhering to the 2016 AISC *Specification for Structural Steel Buildings* and the 14th Edition AISC *Steel Construction Manual*. Both load and resistance factor design and allowable strength design methods are employed in the design examples.

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Chapter 1
Introduction

1.1 HISTORY

The idea of creating single web openings in wide-flange steel beams in order to pass service lines through the beam stems back to the early use of steel sections. The design of beams with web openings is addressed in AISC Design Guide 2, *Design of Steel and Composite Beams with Web Openings*, which explicitly notes that the design provisions do not apply to castellated beams—beams with expanded web sections that included repeating openings (Darwin, 1990). In this document, castellated beams are defined as steel beams with expanded sections containing hexagonal openings. Cellular beams are defined as expanded steel sections with circular openings.

Beams with expanded web sections with repeating web openings were first used in 1910 by the Chicago Bridge and Iron Works (Das and Srimani, 1984). This idea was also developed independently by G.M. Boyd in Argentina in 1935 and was later patented in the United Kingdom (Knowles, 1991). In the 1940s, the use of castellated and cellular beams increased substantially, in part due to the limited number of structural sections that the steel mills could fabricate in Europe. Steel mills could efficiently produce a number of larger section sizes by manually expanding beams because of low labor-to-material cost ratios. However, steel mills in the United States did not experience the same section limitations and low labor costs as the mills in Europe; consequently, the fabrication of such beams was not economically efficient. As a result, the use of castellated and cellular beams diminished until automated manufacturing techniques became available. The improved automation in fabrication, coupled with the need for architects and structural engineers to search for more efficient and less costly ways to design steel structures, has resulted in the use of castellated and cellular beams in the United States. An increase in use of expanded sections has occurred around the world and contributed to the formation of the International Institute of Cellular Beam Manufacturers in 1994 to develop, establish and maintain standards for the design and manufacturing of castellated and cellular beams worldwide.

1.2 MANUFACTURING

Castellated and cellular beams are custom designed for a specific location on a specific project. The process by which castellated and cellular beams are fabricated is similar, but not identical. Castellated beams are fabricated by using a computer operated cutting torch to cut a zigzag pattern along the web of a wide-flange section. The step-by-step process of manufacturing a castellated beam is presented in Figure 1-1. Once the section has been cut in the appropriate pattern (a), the two halves are offset (b). The waste at the ends of the beam is removed (c), and the two sections are welded back together to form the castellated section (d). A full or partial penetration butt weld is then typically made from one side of the web, without prior beveling of the edges if the web thickness is relatively small. A photograph of the manufacturing process of a castellated beam is shown in Figure 1-2.

Cellular beams are fabricated in a similar manner using a nested semicircular cutting pattern. In order to achieve the repeating circular pattern, two cutting passes are required, as shown in Figure 1-3. The two cutting passes increase the handling of the steel during the manufacturing process; consequently, the time to produce a cellular beam is slightly greater than that of a castellated beam. The cuts are made in a circular pattern instead of the zigzag used for the castellated beams. The circular cutting produces additional waste as compared to castellated beams, as shown in Figure 1-3(b). Once the two cuts have been made, the two halves that have been created are offset and welded back together to form a cellular beam. A photograph of the manufacturing process of a cellular beam is presented in Figure 1-4.

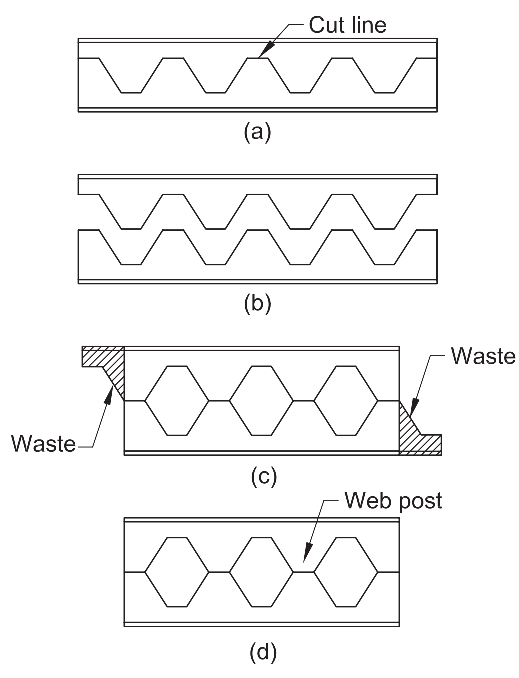


Fig. 1-1. Manufacturing of a castellated beam.

Fig. 1-2. Cutting of a castellated pattern.

1.3 NOMENCLATURE

Typical nomenclature for a steel section indicates the shape type, the approximate depth, and the approximate weight of the shape per linear foot. For example, a W8×10 represents a wide-flange section with a depth of approximately 8 in. and a nominal weight of 10 lb/ft. A similar nomenclature is used for castellated and cellular beams. Castellated beams are represented by CB, while cellular beams are noted as LB. The number representations are identical to those of standard steel sections. For example a castellated and cellular beam constructed from a W8×10 root beam is called out as a CB12×10 and LB12×10, respectively, as the depth is approximately one and half times that of the root beam and the weight is the same as the root beam. Under certain conditions, it is beneficial to produce an asymmetric section. In this case, the nomenclature for these sections is based on the two different root beams used to make the castellated or cellular section. For example, if the root beam for the top tee of the castellated or cellular beam is a W21×44 and the root beam for the bottom is a W21×57, then the castellated and cellular beam call outs would be CB30×44/57 and LB30×44/57, respectively. The first number presents the approximate depth and the second pair of numbers provides

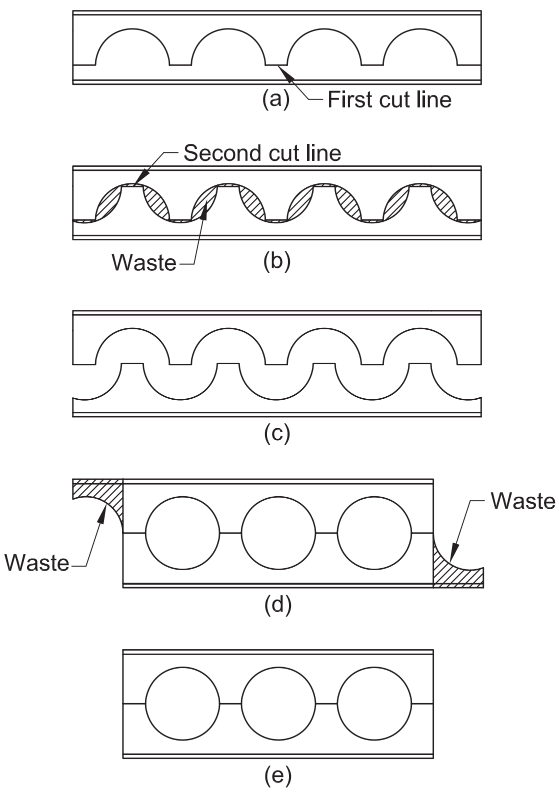


Fig. 1-3. Manufacturing of a cellular beam.

Fig. 1-4. Second cutting of a cellular pattern.

the nominal weight of the root beam used for the top of the section followed by a forward slash and the nominal weight of the root beam used for the bottom of the section. The weight per foot of the resulting asymmetric beam is the average of the two numbers. The use of asymmetric sections is discussed in further detail in Section 2.2.6.

1.4 INTRODUCTION OF DESIGN GUIDE

Although the use of castellated and cellular beams around the world is becoming more commonplace and there is a growing body of literature on the topic, there are very few publications that include comprehensive design recommendations. This Design Guide presents the state of the practice for the design of castellated and cellular beams in the United States. The Guide provides a unified approach to the design of castellated and cellular beams for noncomposite and composite applications. Chapter 2 presents information pertaining to appropriate applications for castellated and cellular beams, including advantages, efficiencies, and limitations of use. The differences between designing traditional beams versus those with web openings are identified in Chapter 3, along with the detailed procedures for designing castellated and cellular beams in accordance with the 2016 AISC *Specification for Structural Steel Buildings*, hereafter referred to as the AISC *Specification* (AISC, 2016). The procedures presented include both noncomposite and composite design for both castellated and cellular beams. Chapter 4 presents detailed design examples conforming to the procedures presented in Chapter 3. A detailed listing of the symbols used throughout the Design Guide is supplied at the end of the document, as is a complete list of references cited in the Design Guide and a bibliography of publications for further reading.

Chapter 2
Use of Castellated and Cellular Beams

2.1 GENERAL

In comparison to their root beams, castellated and cellular beams offer many design and construction advantages. As a result of expanding the web and introducing web openings, these members have an increased depth-to-weight ratio, an increased section modulus, S_x, and increased strong-axis moment of inertia, I_x. These increases not only make longer spans possible, but their increased efficiency also provides the potential for significant cost savings when used in long spans. These advantages come at the expense of a more complex analysis. This chapter presents ideal applications for the use of castellated and cellular beams, discusses some of the advantages and limitations of their use, and highlights some special considerations for designers.

2.2 APPLICATIONS AND ADVANTAGES

The primary use for castellated and cellular beams is in spanning long distances utilizing a lighter weight section. In general, they are practical for spans greater than 30 ft and prove to be a very economical alternative for spans greater than 40 ft (Estrada et al., 2006). Utilizing the long span capability of castellated and cellular beams can provide a more open floor plan. This allows the end-user of the building more flexibility in space planning, which can be an advantage to the building owner in the event of future tenant changes. Advantages of castellated and cellular beams include the ability to use fewer columns and footings to support the longer-spanning sections, thereby creating additional column-free space and floor space flexibility. The ability to use longer, lighter spans makes fewer members necessary for a given system, saving erection costs for the structure. Castellated and cellular beams are ideal for structures with long open space requirements, such as parking garages, industrial and warehouse facilities, office buildings, schools, and hospitals.

2.2.1 Parking Structures

The use of castellated and cellular beams in parking structures has increased dramatically in the past 15 years. Parking structures have spans that are typically in the 60-ft range, and serviceability often controls the design over strength. For typical garage loading requirements, a 30-in.-deep castellated or cellular beam that weighs approximately 60 lb/ft used in composite construction will meet or exceed the strength and serviceability requirements. Compared to a W30×90, which is the lightest 30-in.-deep wide-flange beam available, the use of a castellated or cellular beam will net a weight savings of approximately 33%. This will provide substantial direct material costs savings and will also reduce the overall mass of the structure, resulting in reduced lateral design forces and reduced foundation loads. Open-web sections allow light transmission through the web openings, brightening the building interior. For parking garages with low interstory heights, the brighter interior gives the appearance of being more spacious than similar structures with solid-web beams (Churches et al., 2004). Figures 2-1 and 2-2 show examples of parking structures utilizing cellular beams.

It should be noted that when utilizing steel sections for a parking structure, the choice of the coating system should be carefully considered. It is recommended that either a hot-dipped galvanized coating or a high-performance epoxy paint system be applied to the steel. For more information on this topic, refer to Section 2.4.5 and AISC Design Guide 18, *Steel-Framed Open-Deck Parking Structures* (Churches et al., 2004).

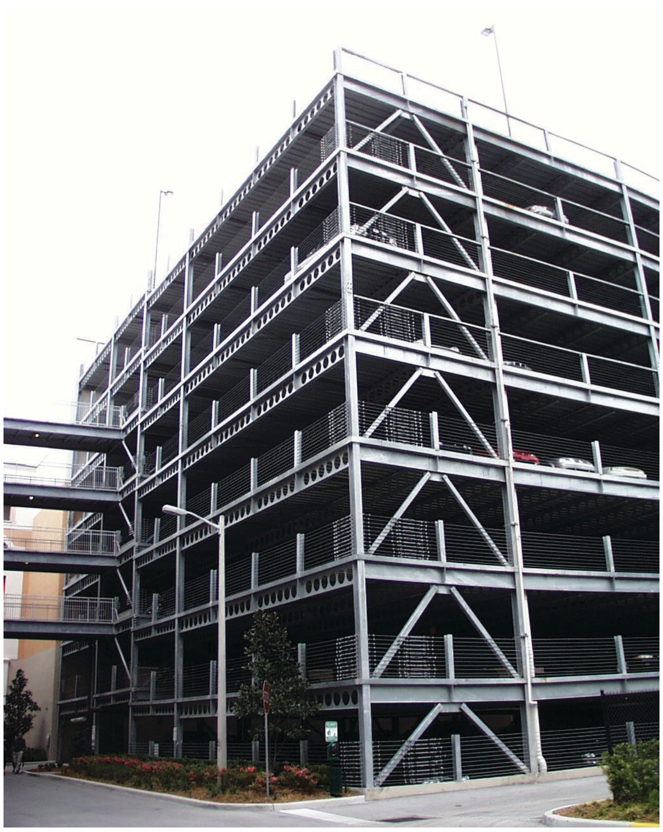


Fig. 2-1. Exterior of parking garage utilizing cellular beams.

2.2.2 Industrial Facilities

Industrial facilities often utilize mezzanines to provide additional square footage for things such as overhead conveyors, air handling equipment, process equipment, and even office space. The long-span capabilities of castellated and cellular beams perform very well for this application. They help to minimize the number of columns supporting the mezzanine and have excellent vibration characteristics, which are addressed in Section 2.2.5. An industrial facility is shown in Figure 2-3.

2.2.3 Service/HVAC Integration

One of the greatest advantages of castellated and cellular beams is the ability to run utilities directly through the web openings. Figure 2-4 illustrates that several inches of height per floor of the structure can be saved by using open web sections. The openings in the beams may be utilized for the installation of conduit, HVAC and sprinkler piping, and any other utility system, as shown in Figure 2-5. Use of castellated and cellular beams in office buildings provides the owner and future tenants the flexibility to install additional wiring for telephones, computers, or other office equipment.

The integration of conduits within the beam depth is also advantageous in medical buildings where gas lines, data lines and other services are often installed or relocated after the building construction is complete. Additionally, when all utilities are housed within the depth of the beam, the ceiling can be directly affixed to the structure, as shown in Figure 2-6. Power generating facilities and other structures with significant piping could also benefit from the use of castellated and cellular beams for service integration.

2.2.4 Construction Efficiency

There are potentially many construction efficiencies that can be directly and indirectly realized by using castellated and cellular beams. Long-span construction minimizes the number of columns required and results in a reduced number of foundations, which is a direct cost savings. Additionally, the number of pieces to be erected in the field can be greatly reduced, thereby saving fabrication and erection costs. Indirect cost savings in general conditions and financing resulting from a shorter construction schedule can also be realized

Fig. 2-2. Parking structure during the day.

Fig. 2-3. Industrial facility with mezzanine.

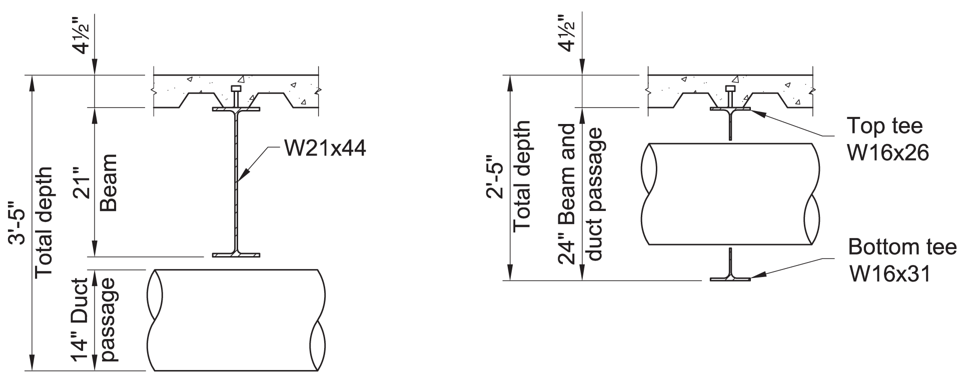


Fig. 2-4. 14-in.-diameter HVAC duct.

if the steel erection is on the critical path of the construction schedule. Finally, and perhaps more importantly, a shorter construction schedule can allow for earlier revenue recognition by the owner.

2.2.5 Vibration Resistance

Castellated and cellular beams are approximately 1.5 times deeper than the equivalent-weight wide-flange beams. Consequently, castellated and cellular beams have excellent vibration resistance in comparison to their root beams. Increasing stiffness will decrease the effects of vibration on a floor or roof system. AISC Design Guide 11, *Vibrations of Steel-Framed Structural Systems Due to Human Activity,* covers the vibrations of steel floor systems (Murray et al., 2016). Full-scale testing was conducted on composite castellated beams and it was verified that Design Guide 11 can be used for castellated flooring systems. It is reasonable to extrapolate that the document could be used for composite and noncomposite cellular beams as well.

2.2.6 Asymmetric Sections

As described in Section 1.2, the manufacturing process used to fabricate castellated and cellular beams involves cutting the root beam into two sections. Provided that the opening spacing is identical, the halves of two different root beams can be welded together to create an asymmetric shape. This is especially advantageous for composite applications where the top tee works in conjunction with the concrete slab. For composite design, it is typically cost effective to specify an asymmetric section, using a smaller root beam for the top tee and a larger one for the bottom tee. Examples 4.3 and 4.4 use asymmetric sections for the design of a composite castellated and composite cellular beam, respectively.

2.2.7 Aesthetics

The process of fabricating a castellated or cellular beam provides the designer great flexibility in creating architecturally significant structural elements. The beams can be easily cambered. Unlike traditional wide-flange beams that are cambered through mechanical methods or by applying heat, camber is built in to castellated and cellular beams during the fabrication process. This process also allows beams to be curved as shown in Figure 2-7 and Figure 2-8, by curving the tees prior to welding the top and bottom halves of the beam together. Additionally, exposed tapered elements can be utilized as well, as highlighted in Figure 2-9. A tapered beam

Fig. 2-5. Openings utilized for conduit and HVAC routing.

Fig. 2-7. Curved cellular beam.

Fig. 2-6. Service integration with ceiling attached to structure.

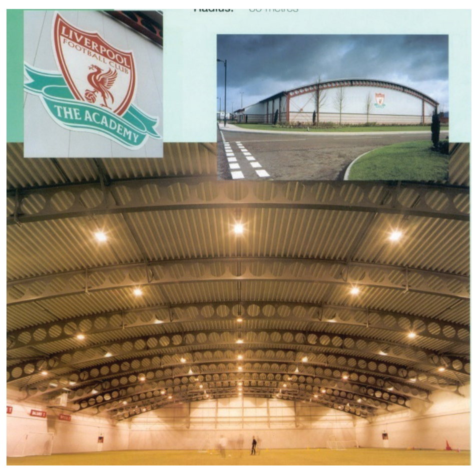


Fig. 2-8. Curved cellular beam roof structure.

is formed by sloping the cutting pattern along the length of the web and rotating one half of the beam 180° prior to completing the web post welds. Castellated and cellular beams are generally considered aesthetically pleasing structural elements, providing an architectural advantage in structures where the beams are exposed.

2.3 WEB OPENING SIZE AND SPACING AND TYPICAL CONNECTIONS

There are geometric limits on web opening size and spacing described in Section 3.3. These limits can be used to select a preliminary web opening size and spacing. The opening size and spacing will define the geometry of the web post, the web material between two openings, and the depth of the beam. Any opening size or spacing that meets these guidelines is acceptable for applying the strength calculations described in Chapter 3. Due to the fabrication process of staggering the halves and aligning web posts, the opening size and spacing chosen must be consistent throughout the length of the beam.

Fig. 2-9. Tapered roof support beams.

2.3.1 End Connections

The types of end connections used for castellated and cellular beams are no different than those used for wide-flange beams. Typical connections used are shear tabs, double angles and single angles. It is standard practice to adjust the opening pattern when possible to allow for a full web post width at the end of each beam. In cases where this cannot be achieved, a partial or complete opening fill will be shop installed by the castellated beam supplier to allow the end connection to be made.

2.3.2 Infilling of Openings

The most economical opening pattern is one that provides a full web post width at each end of the beam without the need for infills. An infill, also referred to as a partial or complete opening fill, is a piece of plate that is the same grade and thickness as the beam web that is cut to the shape of the opening and welded into the opening to allow a connection to be made. Typical types of infills are shown in Figure 2-10. In cases where other beams frame to a cellular or castellated beam web, the preferred connection type would be a shear tab. If the connection does not fall at a web post, then a partial or complete opening fill must be shop installed by the beam supplier to allow the end connection to be made. A photograph of this detail is shown in Figure 2-11. During the design process, opening spacing should be optimized

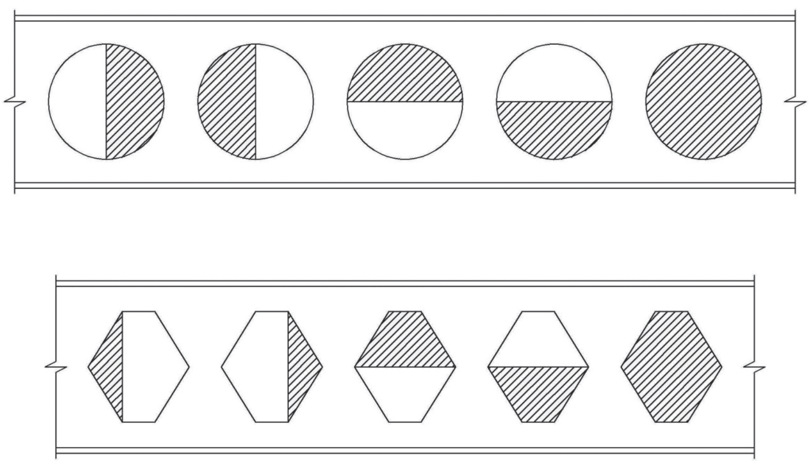


Fig. 2-10. Typical infill patterns for cellular (top) and castellated beams (bottom).

to avoid infills when possible, to minimize the costs of the added material and labor.

2.3.3 Large Copes

At the location of the opening nearest the end connection is an area of special consideration for castellated and cellular beams. The minimum diagonal distance from the corner of the cope to the side of the first opening is designated as e'. This dimension is shown in Figure 2-12. In atypical instances when large copes are required in the beam, the dimension e' can become quite small. Experimental testing and analytical modeling have demonstrated that large reductions in the e' dimension are required before large reduction in strength occurs. As compared to a state with no cope, a reduction in the e' distance of 50% for noncomposite sections results in a reduction of maximum strength of 7% for castellated beams and 3% for cellular beams (Hoffman et al., 2005). It is recommended for beams with e' values 40% or less than the diagonal distance from the corner of the beam in an uncoped state to the side of the first opening, that partial infills be used. Since the hole spacing and end conditions can be controlled in design, this e' consideration does not typically control the design.

Fig. 2-11. Infill of cellular beam to accommodate a secondary beam.

2.4 SPECIAL CONSIDERATIONS

2.4.1 Concentrated Loads

Although cellular and castellated beams are typically used for uniform load applications, they may also be used to support concentrated loads or in a combined case with uniform and concentrated loads. Guidance for this loading condition is presented in Section 3.8.

2.4.2 Depth-Sensitive Projects

The overall depth of the castellated or cellular beam is dependent on the depth of the root beam, the opening diameter, and the opening spacing The use of castellated or cellular beams provides the designer with the flexibility to adjust the opening size and spacing to achieve a beam of virtually any depth with an upper limit of 66 in. For example, if the design engineer has a special situation that requires a castellated beam that is exactly 36-in. deep, they can specify that depth on the design documents. In contrast, for sizes commonly used for beams, the actual depth of a W36 rolled beam ranges from 35½ in. to 37⅜ in. (not including over- and under-rolling), and the size that is exactly 36 in. may not be adequate. Hence it is often difficult, if not impossible, to select rolled shapes with exactly the depth desired as commonly used for beams. Furthermore, the castellation process allows the fabricator to build a beam to nonstandard depths such as 34 in., or any other depth for custom applications.

2.4.3 Erection Stability

During the transportation, lifting and handling procedures, laterally unbraced castellated and cellular beams may exhibit instability under the relatively light loading of structure self-weight combined with the weight of an erector. Caution must be exercised during the installation process. Therefore, it is recommended that erection bracing be used for the

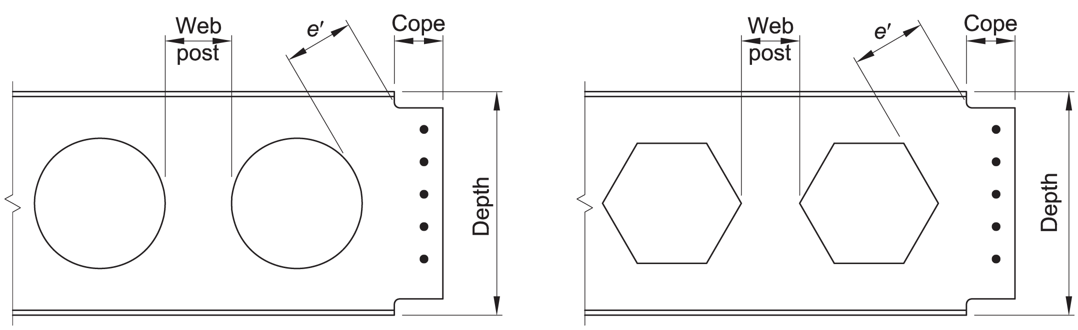


Fig. 2-12. Minimum distance between the end cope and the first opening, e′

installation of cellular and castellated beams. The bracing requirements vary depending on span and depth and are typically determined by the beam manufacturer. Typical erection bracing is shown in Figure 2-13.

The failure of a castellated beam by lateral-torsional buckling is similar to that of the equivalent solid-webbed beams with the holes having little influence (Bradley, 2003). Provisions for determining the lateral-torsional buckling strength of plain-webbed beams can be used for castellated and cellular beams if the cross-sectional properties are calculated at the gross section of the beam. When analyzing deep castellated beams, the gross cross-sectional properties should be used when considering lateral-torsional buckling beam stability due to erection loading (Bradley, 2003). The classical lateral-torsional buckling solution presented in the AISC *Specification* (AISC, 2016), with the addition of the effective length factors, should be used to assure a safe and efficient design when checking for lateral-torsional buckling. The lateral-torsional buckling check is presented in Section 3.2.2.

2.4.4 Fireproofing

There are three UL fire-rated assemblies for use of castellated and cellular beams. UL assemblies N784 and N831 are fire-rated assemblies for a slab on deck system using composite cellular or castellated beams. These assemblies are beam-only assemblies and can be used to achieve up to a 4-hour rating and utilize spray-on fireproofing. According to AISC Design Guide 19, *Fire Resistance of Structural Steel Framing*, Section VII.11, the fact that they are beam-only assemblies allows them to be substituted into other assemblies with an equivalent or better heat dissipating slab above (Ruddy et al., 2003). The third assembly, UL P225, is for use in a roof assembly.

The key to calculating steel fire-resistance protection is understanding the weight-to-heated-perimeter ratio, commonly referred to as the W/D ratio. It is not currently recommended that W/D ratios be used to reduce the thickness of the spray-on fireproofing for cellular and castellated beams. The W/D ratio for a steel shape is determined by dividing the weight per linear foot, W, by the exposed surface area of the steel member, D. The higher the ratio, the greater the member's fire resistance, thus requiring less protection when calculating ratings for the various types of fire protection. Additional testing is under way to determine if W/D ratios, as applied to wide-flange beams, are also applicable to castellated and cellular beams.

Intumescent paint is another option that has been used to fireproof castellated and cellular beams. The thickness of the intumescent coating required varies per manufacturer and rating requirements and must be handled on a case-by-case basis.

2.4.5 Coating Systems

The coating systems used to protect castellated and cellular beams are no different than those used to protect conventional structural steel. The system used should be determined based on the type of exposure the steel will be subject to and the desired maintenance schedule. However, in the case of parking structures, or any other structure exposed to weather, it is recommended that an epoxy-based paint system be applied or the steel should be hot dip galvanized. For hot-dipped galvanized structures, the sharp corners around the openings in the castellated shapes could be notch risers and initiate cracking. Cellular profiles with their smooth transitions are therefore preferred in galvanized applications. The use of an alkyd-based system is not recommended for structures exposed to weather.

Fig. 2-13. Typical erection bracing for cellular beams (similar bracing used for castellated beams).

Chapter 3
Design Procedures

3.1 INTRODUCTION

Due to the presence of the significant number of web openings, castellated and cellular beams cannot be treated as solid-web members or members with web openings. These structural members are highly indeterminate elements, which do not lend themselves to a simple method of analysis. The presence of web openings introduces many additional failure modes not present in solid web members (Kerdal and Nethercot, 1984). Design checks on the web posts and tee-sections that form the opening are required. Additionally, shear deformations with the top and bottom tees in the beams can be significant, thereby increasing the difficulty of deflection analysis. The re-entrant corners at the openings of castellated beams provide a location of stress concentration that may limit their use in applications where dynamic effects are significant (Dougherty, 1993).

Localized forces develop in open web beams both around the openings and at the web posts; consequently, additional modes of failure must be investigated beyond those which are normal for solid web flexural members. Research has shown that castellated and cellular beams behave similar to Vierendeel trusses. The design theory for castellated beams is based largely on *Design of Welded Structures* (Blodgett, 1966), and additional research focused on web post buckling (Aglan and Redwood, 1974; Redwood and Shrivastava, 1980). The design theory for cellular beams has been developed by the Steel Construction Institute of the United Kingdom (Ward, 1990). The design procedures have many similarities, but because the procedures were developed by different parties, there are areas where slightly different approaches are used, such as investigation of horizontal and vertical shear.

The following limit states should be investigated when designing castellated or cellular beams:

1. Compactness and local buckling
2. Overall beam flexural strength
3. Vierendeel bending of tees
4. Web post buckling
5. Axial tension/compression
6. Horizontal shear
7. Vertical shear
8. Lateral-torsional buckling

The first step in designing both castellated and cellular beams is to calculate the overall bending moment and shear force at each opening and web post caused by external loads. These forces will be referred to as global forces. The global forces will be used to compute localized forces in the top and bottom tees, the web posts, and the gross section. The components (tees and web posts) of the beams will then be examined for failure under the localized forces.

3.2 VIERENDEEL BENDING IN NONCOMPOSITE BEAMS

Vierendeel bending is caused by the transfer of shear force across the openings in order to be consistent with the rate of change of bending moment along the beam. Vierendeel failure occurs by the formation of plastic hinges at four locations around the opening in the regions of high shear.

The global shear force passing through the opening creates a localized bending moment in the top and bottom tees, known as the Vierendeel moment. The global bending moment and shear forces change along the length of the beam; consequently, it is necessary to examine the interaction of the global shear and moment at each web opening along the entire length of the beam. By examining the interaction of the global moment and shear at each opening, a critical opening can be identified. The global shear and moment interact to produce an overall stress; therefore, the most efficient use of the beam is in a situation where the maximum global shear and moment occur away from each other such as a simply supported beam with a uniform load.

There are three steps in examining Vierendeel bending:

1. Calculate axial forces due to the global moment and Vierendeel moment due to the global shear in top and bottom tees at each opening that result from external loads.
2. Calculate the available axial and flexural strength of the top and bottom tees using Chapters D, E and F of the AISC *Specification for Structural Steel Buildings* (AISC, 2016).
3. Check the interaction of the axial force and Vierendeel moment using the equations in Chapter H of the AISC *Specification*.

3.2.1 Calculation of Axial Force and Vierendeel Moment at Each Opening

3.2.1.1 Calculation of Axial Forces in Beam

The axial (tensile/compressive) force is a function of the global bending moment in the beam. It is similar to a chord force in a truss. The axial force is calculated by dividing the global moment in the beam by the distance between the centroids of the top and bottom tees, d_{effec}. The axial force is assumed to act uniformly over the cross section. See Figure 3-1 for terminology and variables used to calculate the axial forces.

Required axial compressive strength:

$$P_r = \frac{M_r}{d_{effec}} \tag{3-1}$$

where

M_r = required flexural strength, kip-in. (N-mm)

d_{effec} = distance between centroids of top and bottom tees, in. (mm)

3.2.1.2 Calculation of Vierendeel Moment in Beam

The Vierendeel moment is calculated by dividing the global shear force in the beam between the top and bottom tees and multiplying that shear force by a moment arm. If the top and bottom tees are identical, then the shear can be divided equally between the top and bottom tees. If the top and bottom tees are not identical (as is the case in asymmetric sections), the shear force should be proportioned between the top and bottom tees based on the areas of the tees relative to each other. For castellated sections, the moment arm for calculating the Vierendeel moment is one-half the width of the top of the opening, $e/2$. For cellular beams, the moment arm should be taken as $D_o/4$ (Bjorhovde, 2000).

Vierendeel required flexural strength, M_{vr}

Castellated Beams: $$M_{vr} = V_r\left(\frac{A_{tee}}{A_{net}}\right)\left(\frac{e}{2}\right) \tag{3-2}$$

Cellular Beams: $$M_{vr} = V_r\left(\frac{A_{tee}}{A_{net}}\right)\left(\frac{D_o}{4}\right) \tag{3-3}$$

where

A_{net} = combined area of top and bottom tees, in.^{2} (mm^2)

A_{tee} = area of tee section, in.^{2} (mm^2)

D_o = opening diameter, in. (mm)

V_r = required shear strength, kips (N)

e = length of solid-web section along centerline, in. (mm)

The axial forces (compression or tension) and the flexural forces act concurrently on the top and bottom tees. In the case of a positive global moment, the top tee is in compression and the bottom tee is in tension. Both tees undergo flexure from the global shear forces in the beam.

3.2.2 Calculation of Axial (Tensile/Compressive) and Flexural Strength of Top and Bottom Tees

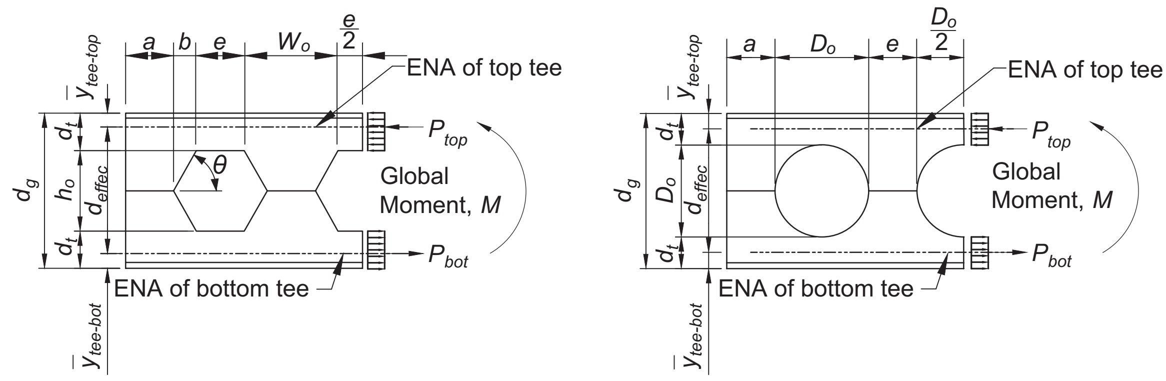
The available axial strength of the top and bottom tees can be calculated using AISC *Specification* Chapters D and E, and the available flexural strength can be calculated using AISC *Specification* Chapter F (AISC, 2016). Chapter H can then be used to check the interaction of the two forces acting concurrently. For simplicity, it is acceptable (and slightly conservative) to treat the tension force on the bottom tee as a compression force, thereby reducing the number of calculations.

3.2.2.1 Calculation of Nominal Axial Strength of Top and Bottom Tees

The nominal compressive strength, P_n, is the lowest value obtained based on the applicable limit states of flexural buckling and flexural-torsional buckling.

Design Assumptions

1. $K_x = 0.65$ (assumes rotation and translation are fixed at the ends of the tee section)



Castellated beam *Cellular beam*

Fig. 3-1. Terminology used for calculating axial forces in noncomposite beams.

2. $K_y = 1.0$
3. E = 29,000 ksi (200 000 MPa)
4. L = laterally unbraced length of the member, in. (mm)
   = e for castellated beams
   = $D_o/2$ for cellular beams
5. $L_c = KL$ = effective length, in. (mm)
6. $L_c = K_x L$
   $= K_y L$
7. G = 11,200 ksi (77 200 MPa)

Compressive Strength for Flexural Buckling—AISC Specification Section E3

For members without slender elements, the nominal compressive strength for flexural buckling is calculated using AISC *Specification* Section E3,

$$P_n = F_{cr} A_g \qquad (Spec.\ \text{Eq. E3-1})$$

$$\phi_c = 0.90 \text{ (LRFD)} \qquad \Omega_c = 1.67 \text{ (ASD)}$$

where
$A_g = A_{tee}$

The critical stress, F_{cr}, is determined as follows:

(a) When $\dfrac{L_c}{r} \le 4.71\sqrt{\dfrac{E}{F_y}}$ $\left(\text{or } \dfrac{F_y}{F_e} \le 2.25\right)$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad (Spec.\ \text{Eq. E3-2})$$

(b) When $\dfrac{L_c}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ $\left(\text{or } \dfrac{F_y}{F_e} > 2.25\right)$

$$F_{cr} = 0.877 F_e \qquad (Spec.\ \text{Eq. E3-3})$$

where
r = minimum of r_x and r_y of the tee section, in.

The elastic buckling stress, F_e, is determined as follows:

$$F_e = \frac{\pi^2 E}{\left(\dfrac{L_c}{r}\right)^2} \qquad (Spec.\ \text{Eq. E3-4})$$

If the tees include slender compression elements, AISC *Specification* Section E7 is used to calculate F_{cr}.

Compressive Strength for Flexural-Torsional Buckling—AISC Specification Section E4

The nominal compressive strength is determined based on the limit state of flexural-torsional buckling as follows:

$$P_n = F_{cr} A_g \qquad (Spec.\ \text{Eq. E4-1})$$

$$\phi_c = 0.90 \text{ (LRFD)} \qquad \Omega_c = 1.67 \text{ (ASD)}$$

The critical stress, F_{cr}, is determined according to Equation E3-2 or E3-3, using the torsional or flexural-torsional elastic buckling stress, F_e, determined from:

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + F_{ez})^2}}\right]$$

(*Spec*. Eq. E4-3)

where

C_w = warping constant, in.^{6} (mm^6)

$F_{ey} = \dfrac{\pi^2 E}{\left(\dfrac{L_{cy}}{r_y}\right)^2}$ (*Spec*. Eq. E4-6)

$F_{ez} = \left[\dfrac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\dfrac{1}{A_g \bar{r}_o^2}$ (*Spec*. Eq. E4-7)

G = shear modulus of elasticity of steel = 11,200 ksi (77 200 MPa)

$H = 1 - \dfrac{x_o^2 + y_o^2}{\bar{r}_o^2}$ (*Spec*. Eq. E4-8)

I_x, I_y = moment of inertia about the principal axes, in.^{4} (mm^4)

J = torsional constant, in.^{4} (mm^4)

K_x = effective length factor for flexural buckling about x-axis

K_y = effective length factor for flexural buckling about y-axis

K_z = effective length factor for flexural buckling about the longitudinal axis

L_{cx} = effective length of member for buckling about x-axis, in. (mm)
$= K_x L_x$

L_{cy} = effective length of member for buckling about y-axis, in. (mm)
$= K_y L_y$

L_{cz} = effective length of member for buckling about the longitudinal axis, in. (mm)
$= K_z L_z$

$\bar{r}_o$ = polar radius of gyration about the shear center, in. (mm)

$\bar{r}_o^2 = x_o^2 + y_o^2 + \dfrac{I_x + I_y}{A_g}$ (*Spec*. Eq. E4-9)

r_x = radius of gyration about x-axis, in. (mm)

r_y = radius of gyration about y-axis, in. (mm)

x_o, y_o = coordinates of the shear center with respect to the centroid, in. (mm)

Tensile Strength—AISC Specification Section D2

The nominal tensile strength is determined based on the limit state of tensile yielding in the gross section:

$$P_n = F_y A_g \qquad (Spec.\ \text{Eq. D2-1})$$

$$\phi_c = 0.90\ \text{(LRFD)} \qquad \Omega_c = 1.67\ \text{(ASD)}$$

3.2.2.2 Calculation of Nominal Flexural Strength, M_n

The flexural strength of the top and bottom tee sections must be determined and compared to the required flexural strength to support the Vierendeel design moment, with an unbraced length equal to the length of the tee section (reference Figure 3-2).

Design Assumptions

1. E = 29,000 ksi (200 000 MPa)
2. $L_b = e$ for castellated beams
   $= D_o/2$ for cellular beams
3. G = 11,200 ksi (77 200 MPa)

The nominal flexural strength is the lowest value obtained according to the limit states of yielding (plastic moment), lateral–torsional buckling, flange local buckling, and local buckling of tee stems.

Nominal Flexural Strength for Yielding—AISC Specification Section F9.1

The nominal flexural strength for yielding, M_n, is determined as follows:

$$M_n = M_p \qquad (Spec.\ \text{Eq. F9-1})$$

$$M_p = M_y \qquad (Spec.\ \text{Eq. F9-4})$$

where

$M_y = F_y S_{x\text{-}tee}$
= yield moment about the axis of bending, kip-in. (N-mm)

$S_{x\text{-}tee}$ = elastic section modulus of tee about the x-axis, in.^{3} (mm^3)

Note: Because the stem will be in compression, M_p is limited to the minimum section modulus of the tee, $S_{x\text{-}tee}$, multiplied by the yield strength of the steel.

Lateral-Torsional Buckling—AISC Specification Section F9.2

When $L_b \le L_p$ the limit state of lateral-torsional buckling does not apply.

When $L_p < L_b \le L_r$

$$M_n = M_p - \left(M_p - M_y\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)$$

(*Spec*. Eq. F9-6)

when $L_b > L_r$

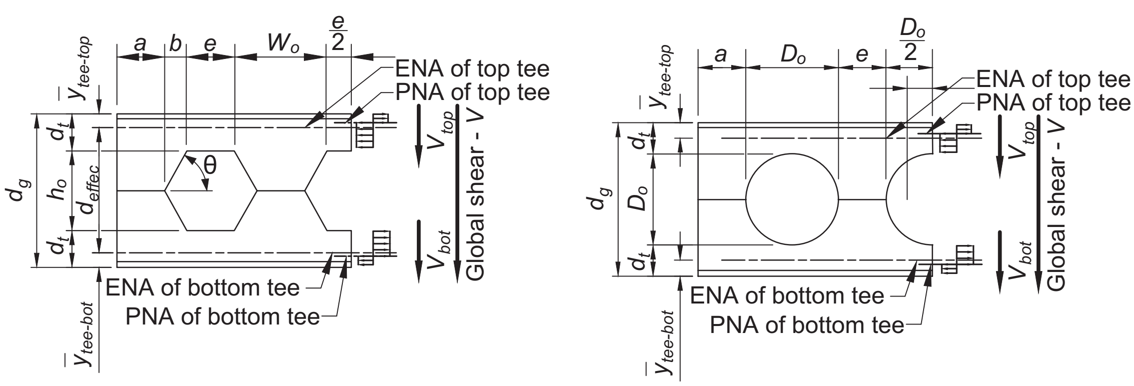
$$M_n = M_c \qquad (Spec.\ \text{Eq. 9-7})$$

where

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad (Spec.\ \text{Eq. F9-8})$$

$$L_r = 1.95\left(\frac{E}{F_y}\right)\frac{\sqrt{I_y J}}{S_x}\sqrt{2.36\left(\frac{F_y}{E}\right)\frac{dS_x}{J} + 1}$$

(*Spec*. Eq. F9-9)



Castellated Beams *Cellular Beams*

Fig. 3-2. Terminology used for calculating Vierendeel moment in noncomposite beams.

$$M_{cr} = \frac{1.95E}{L_b}\sqrt{I_y J}\left(B + \sqrt{1 + B^2}\right) \qquad \text{(Spec. Eq. F9-10)}$$

$$B = 2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \qquad \text{(Spec. Eq. F9-11)}$$

d = depth of tee in tension, in. (mm)

For stems in compression anywhere along the unbraced length, M_{cr} is given by Equation F9-10 with

$$B = -2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \qquad \text{(Spec. Eq. F9-12)}$$

where

d = depth of tee in compression, in. (mm)

For tee stems

$$M_n = M_{cr} \le M_y \qquad \text{(Spec. Eq. F9-13)}$$

Flange Local Buckling of Tees—AISC Specification Section F9.3

(a) For sections with a compact flange in flexural compression, the limit state of flange local buckling does not apply.

(b) For sections with a noncompact flange in flexural compression

$$M_n = \left[M_p - \left(M_p - 0.7F_y S_{xc}\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \le 1.6M_y$$

(*Spec*. Eq. F9-14)

(c) For sections with a slender flange in flexural compression

$$M_n = \frac{0.7ES_{xc}}{\left(\dfrac{b_f}{2t_f}\right)^2} \qquad \text{(Spec. Eq. F9-15)}$$

where

S_{xc} = elastic selection modulus referred to the compressions flange, in.^{3} (mm^3)

$\lambda = \dfrac{b_f}{2t_f}$

$\lambda_{pf} = \lambda_p$, the limiting slenderness for a compact flange, AISC *Specification* Table B4.1b

$\lambda_{rf} = \lambda_r$, the limiting slenderness for a noncompact flange, AISC *Specification* Table B4.1b

Local Buckling of Tee Stems in Flexural Compression—AISC Specification Section F9.4

The nominal flexural strength for local buckling in flexural compression, M_n, is determined as follows:

$$M_n = F_{cr} S_x \qquad \text{(Spec. Eq. F9-16)}$$

where

S_x = elastic section modulus about the x-axis, in.^{3} (mm^3)

The critical stress, F_{cr}, is determined as follows:

(a) When $\dfrac{d}{t_w} \le 0.84\sqrt{\dfrac{E}{F_y}}$

$$F_{cr} = F_y \qquad \text{(Spec. Eq. F9-17)}$$

(b) When $0.84\sqrt{\dfrac{E}{F_y}} < \dfrac{d}{t_w} \le 1.52\sqrt{\dfrac{E}{F_y}}$

$$F_{cr} = \left(1.43 - 0.515\frac{d}{t_w}\sqrt{\frac{F_y}{E}}\right)F_y$$

(*Spec*. Eq. F9-18)

(c) When $\dfrac{d}{t_w} > 1.52\sqrt{\dfrac{E}{F_y}}$

$$F_{cr} = \frac{1.52E}{\left(\dfrac{d}{t_w}\right)^2} \qquad \text{(Spec. Eq. F9-19)}$$

where

$d = d_t$

3.2.3 Check of Top and Bottom Tees Subjected to Combined Flexural and Axial Forces

The interaction of flexure and axial forces in top and bottom tees constrained to bend about a geometric axis (x and/or y) is limited by AISC *Specification* Equations H1-1a and H1-1b.

For combined flexure and compression:

(a) For $\dfrac{P_r}{P_c} \ge 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

(*Spec*. Eq. H1-1a)

(b) For $\dfrac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \le 1.0$$

(*Spec.* Eq. H1-1b)

where

P_c = available axial strength, kips (N)
P_r = required axial strength, kips (N)
M_c = available flexural strength, kip-in. (N-mm)
M_r = required flexural strength, kip-in. (N-mm)
x = subscript relating symbol to major axis bending
y = subscript relating symbol to minor axis bending

A check for combined flexure and tension can be made using the same equations; however the available compressive strength will always be less than the available tensile strength and therefore the available compressive strength may be used to simplify the calculations. This may not be true in asymmetric sections, in which case the available tensile strength may need to be investigated as well.

3.3 VIERENDEEL BENDING IN COMPOSITE BEAMS

Vierendeel bending in composite beams is similar to that in noncomposite beams. The shear force resisted by the concrete slab is first deducted from the applied shear force. The net shear is apportioned between the tees. It cannot be assumed that the highest internal forces occur at the midspan of the beam; each opening along the length of the beam must be examined for shear and flexural interaction.

Composite action results in smaller forces in the tees, resulting in a more favorable design. However, at web openings near the ends of the beams, composite action provides less strength because of the limited number of studs between the end of the beam and the opening being investigated. As a result, less force is carried by the concrete and more force is carried by the steel tee. The contribution of the composite section must consider the actual available studs between the beam end and the opening. An additional benefit of composite construction is that concrete helps resist the global shear forces, thereby reducing the Vierendeel moment in the tees.

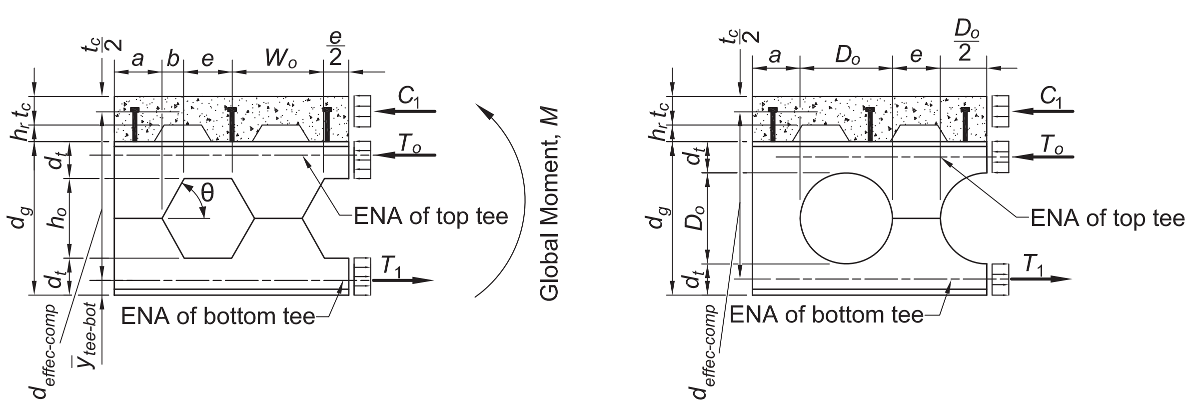
There are five steps to checking Vierendeel bending in composite beams:

1. Calculate the shear strength of the concrete deck—this will be subtracted from the global shear to calculate the net shear, which will be used to compute the Vierendeel moment in the top and bottom tees.
2. Calculate the net shear and moment at each.
3. Calculate the axial forces and Vierendeel moments in the top and bottom tees.
4. Calculate the available axial compressive and flexural strength of the top and bottom tees using Chapters E and F of the AISC *Specification*.
5. Check the interaction of the available axial compressive and flexural strength using the equations in Chapter H of the AISC *Specification*.

3.3.1 Calculation of Axial Force and Vierendeel Moment at Each Opening

3.3.1.1 Calculation of Axial Forces in Beam

It is assumed that a sufficient number of steel anchors exist to develop enough of the concrete such that all of the compression force is resisted by the concrete section and that the bottom tee resists all the tension force. Therefore, when the beam at this section is fully composite, T_o shown in Figure 3-3 is zero, and $C_1 = T_1$. The compression/tension



Castellated Beams *Cellular Beams*

Fig. 3-3. Terminology used for calculating axial forces in composite beams.

couple force is produced by the global moment. After this initial assumption it is necessary to recalculate Y_c, which is the thickness of concrete that will be used to resist the global moment. In the iterative calculations, the term Y_c will replace t_c in the equations that follow. Figure 3-3 presents the dimensions and forces used to calculate the axial forces in the composite beam.

3.3.1.2 Effective Width

The effective width, b_{effec}, of the concrete slab is not to exceed:

1. For an interior beam,

$$b_{effec} \leq \frac{\text{beam span}}{4} \tag{3-4a}$$

$$b_{effec} \leq \text{beam spacing} \quad \text{(for equal beam spacing)} \tag{3-4b}$$

2. For an exterior beam,

$$b_{effec} \leq \frac{\text{beam span}}{8} + \text{distance from beam center to edge of slab} \tag{3-5}$$

$$b_{effec} \leq \frac{\text{beam spacing}}{2} + \text{distance from beam center to edge of slab} \tag{3-6}$$

The net area, A_{net}, is

$$A_{net} = A_{tee\text{-}top} + A_{tee\text{-}bot} \tag{3-7}$$

The effective depth of the composite section, $d_{effec\text{-}comp}$, is

$$d_{effec\text{-}comp} = d_g - \bar{y}_{tee\text{-}bot} + h_r + 0.5t_c \tag{3-8}$$

where

d_g = depth of the expanded beam, in. (mm)

h_r = height of deck ribs, in. (mm)

t_c = thickness of concrete above the deck ribs, in. (mm)

$\bar{y}_{tee\text{-}bot}$ = distance from the bottom fiber to the centroid of the bottom tee, in. (mm)

Calculate the axial force in the bottom tee, T_1, and the axial force in the concrete, C_1, assuming full composite action:

$$T_1 = C_1 = \frac{M_r}{d_{effec\text{-}comp}} \tag{3-9}$$

Calculate the depth of concrete used to resist the global moment, Y_c:

$$Y_c = \frac{T_1}{0.85 f_c' b_{effec}} \tag{3-10}$$

where

f_c' = compressive strength of concrete, ksi (MPa)

The term Y_c is similar to "*a*," which represents the depth of the stress block in concrete design. It will be used to replace the term t_c in the calculation of d_{effec} in subsequent calculation iterations. This process may take several iterations to converge. Note that this process occurs at every opening.

Using AISC *Specification* Section I.2d.1, calculate the total horizontal shear force, V', between the point of maximum positive moment and the point of zero moment. V' is taken as the lowest value according to the limit states of concrete crushing or tensile yielding of the steel section.

$$V' = 0.85 f_c' A_c \qquad \textit{(Spec. Eq. I3-1a)}$$

$$V' = F_y A_s \qquad \textit{(Spec. Eq. I3-1b)}$$

where

A_c = area of concrete slab within effective width, in.^{2} (mm^2)

A_s = area of steel cross section, in.^{2} (mm^2)
= A_{net}

The distance from the end of the beam to the center of the opening being investigated is defined as X_i and the total number of studs across the beam is defined as N_s.

The shear stud density, q, is defined as:

$$q = \left(\frac{N_s Q_n}{Span}\right) \tag{3-11}$$

where

Q_n = nominal shear strength of one steel stud, kips (N)

The total number of studs between the end of the beam and the opening under consideration is defined as N_1, and the total stud strength between the end of the beam and the opening under consideration is determined by multiplying the shear stud density, q, by X_i.

If T_1 at the opening being investigated exceeds the total stud strength provided by the studs up to that opening, then partial composite action exists; if not, then the section is fully composite, and the assumption that the concrete can resist all the compressive force is valid. In other words, if there are not enough studs to develop the force T_1, then the section must be considered partially composite. If the section is partially composite, calculate the force T_o that acts on the top tee and bottom tee. T_o represents the additional axial force that must be resisted by the steel beam tees due to lack of composite action.

If the composite beam section at the web opening is partially composite, calculate T_o, $T_{1\text{-}new}$ and $C_{1\text{-}new}$ as shown in Figure 3-4.

If the web opening is partially composite, calculate T_o and revise T_1.

The axial force in the top tee, T_o, is calculated as follows:

$$T_o = M_r \left[\frac{1 - \dfrac{(q)(X_i)}{T_1}}{d_{effec}} \right] \tag{3-12}$$

The axial force in the bottom tee, $T_{1\text{-}new}$, is then calculated as:

$$\begin{aligned} T_{1\text{-}new} &= qX_i + T_o \\ &= C_{1\text{-}new} + T_o \end{aligned} \tag{3-13}$$

3.3.2 Calculation of Vierendeel Bending Moment of the Upper and Lower Tees

The Vierendeel moments in the top and bottom tees are calculated in composite beams in the same manner as noncomposite sections. Many times in composite sections it makes sense to use asymmetric sections (a larger bottom than top section) since the top tee usually does not carry significant axial tensile forces. If asymmetric sections are used, the global shear force should be divided between the top and bottom tees based on the area of the tees relative to each other. Also, for composite sections allow the concrete deck to take some of the global shear force to reduce the Vierendeel bending moment. The nominal strength of the concrete deck is V_c. Figure 3-5 presents the dimensions and forces used to calculate the Vierendeel moments in the beam.

Calculate the concrete deck punching shear strength, V_{nc}:

$$V_{nc} = 3(h_r + t_c)(t_c)\left(4\sqrt{f_c'}\right) \tag{3-14}$$

The available concrete shear strength, V_c, can then be determined as:

$$V_c = \phi_{cv} V_{nc} \text{ (LRFD)} \tag{3-15a}$$

$$V_c = V_{nc}/\Omega_{cv} \text{ (ASD)} \tag{3-15b}$$

$$\phi_{cv} = 0.75 \qquad \Omega_{cv} = 2.00$$

Determine the net shear force as:

$$V_{r\text{-}net} = V_r - V_c \tag{3-16}$$

Vierendeel Required Flexural Strength

For castellated beams, the Vierendeel required flexural strength is determined as follows:

$$M_{vr} = V_{r\text{-}net}\left(\frac{A_{tee}}{A_n}\right)\left(\frac{e}{2}\right) \tag{3-17}$$

And for cellular beams:

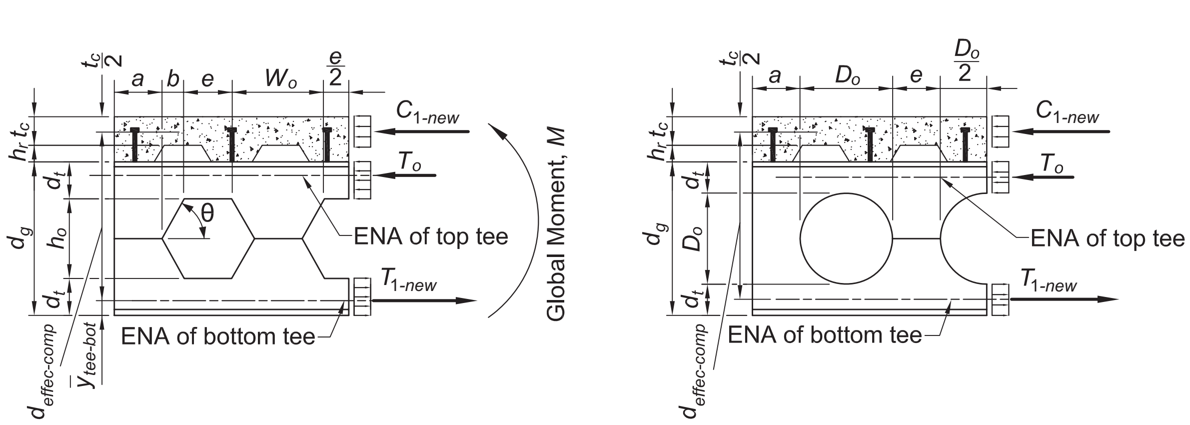
$$M_{vr} = V_{r\text{-}net}\left(\frac{A_{tee\text{-}crit}}{A_{crit}}\right)\left(\frac{D_o}{4}\right) \tag{3-18}$$

where

A_{crit} = sum of the top and bottom areas of the critical section, in.^{2} (mm^2)

A_{tee} = area of the tee section, in.^{2} (mm^2)

$A_{tee\text{-}crit}$ = area of the critical section which is located $0.225D_o$ away from center of the opening, in.^{2} (mm^2)



Castellated Beams *Cellular Beams*

Fig. 3-4. Terminology used for calculating axial forces in partially composite beams.

3.3.3 Calculation of Available Axial and Flexural Strength of Top and Bottom Tees

The nominal strength of the top and bottom tees should be calculated as in noncomposite sections. The only difference is that the bottom tee should be examined as a member subject to true tension (axial stress) and Vierendeel moment (bending stress) rather than assuming that the axial force is compressive, as is done in noncomposite sections for simplicity. If the T_o force does exist, the top tee should be examined as a compression member. Also, as in noncomposite sections, the combined forces should be checked in accordance with AISC *Specification* Chapter H.

3.4 WEB POST BUCKLING

Web post buckling is caused by the horizontal shear force passing through the web post. The ultimate strength of the web post is governed by one of two modes:

1. Flexural failure caused by the development of a plastic hinge in the web post.
2. Buckling failure of the web post.

The mode of failure is dependent on the geometry and the thickness of the web post. Separate checks are made for the top and bottom tees, which may have different thicknesses and available strengths.

For castellated beams, the buckling capacity of the web posts is calculated using equations that have been developed through destructive testing (Aglan and Redwood, 1974). This set of equations defines the buckling capacity of the web post as a percentage of its plastic bending moment, M_p, which is a function of e, b, t_w and F_y. The buckling capacity, as a percentage of M_p, is a function of $2h/e$; the value of $2h/e$ need not be taken as more than 2. The destructive testing that was completed was for beams with the angle of the hexagonal cut, $\theta = 45°$ and $60°$; i.e., one set of equations was developed for $\theta = 45°(\pm 2°)$ and another set for $\theta = 60°(\pm 2°)$. It is not permissible to use these equations for $\theta < 43°$ or for $\theta > 62°$. It is, however, acceptable to design beams with web posts having an angle, θ, between 45° and 60° by interpolating between the two equation sets, and applying a larger factor of safety to the allowable web post bending moment. Also, the equations are only applicable for $10 \le e/t_w \le 30$ and $2h/e \le 8$. It is typically most efficient to maintain an angle, θ, of between 58° and 62°.

For cellular beams, a similar set of equations was developed through destructive testing by the Steel Construction Institute of the United Kingdom (Ward, 1990). There are three values ($C1$, $C2$ and $C3$), which are functions of the properties of the web post that are used to calculate the buckling capacity of the web post as a function of the web post elastic capacity at a critical section $0.9R$. These equations are the result of detailed nonlinear finite element studies, and only applicable for $1.08 \le S/D_o \le 1.50$ and $1.25 \le d_g/D_o \le 1.75$.

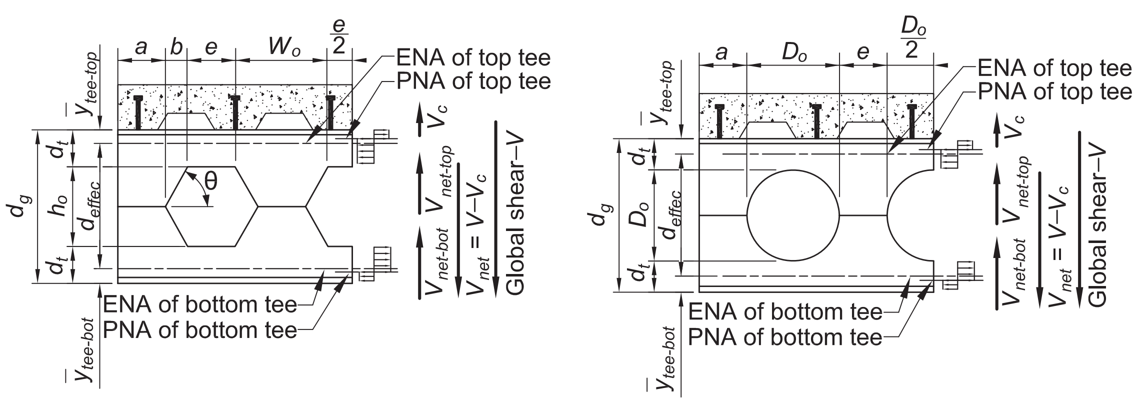
Although the methods for calculating the web post buckling strength are different for castellated and cellular beams, the results are generally comparable. Figure 3-6 presents the terminology used to calculate web post buckling for castellated and cellular beams.

3.4.1 Web Post Buckling in Castellated Beams

3.4.1.1 Calculation of Horizontal Shear and Resulting Moment on Web Post

Horizontal Shear, V_{rh}*—Noncomposite*

Treat a segment of the beam as a free body acted upon by the global bending moment force. The difference in this axial force from one end of the segment to the other is transferred out as horizontal shear along the web post. See Figure 3-7.



Castellated Beams *Cellular Beams*

Fig. 3-5. Terminology used for calculating Vierendeel moments in composite beams.

Calculate the horizontal shear, V_{rh}:

$$V_{rh} = \left| \frac{M_{r(i+1)} - M_{r(i)}}{d_{effec}} \right| = \left| T_{r(i)} - T_{r(i+1)} \right| \tag{3-19}$$

Horizontal Shear, $\mathrm{V_{rh}}$—*Composite*

Consider a segment of composite castellated beam as shown in Figure 3-6(c).

The horizontal shear of a composite section, V_{rh}, is calculated from Equation 3-19:

$$V_{rh} = \left| T_{r(i)} - T_{r(i+1)} \right|$$

Required flexural strength in the web post

Top tee:

$$M_{rh} = V_{rh} h_{top} \tag{3-20}$$

Bottom tee:

$$M_{rh} = V_{rh} h_{bot} \tag{3-21}$$

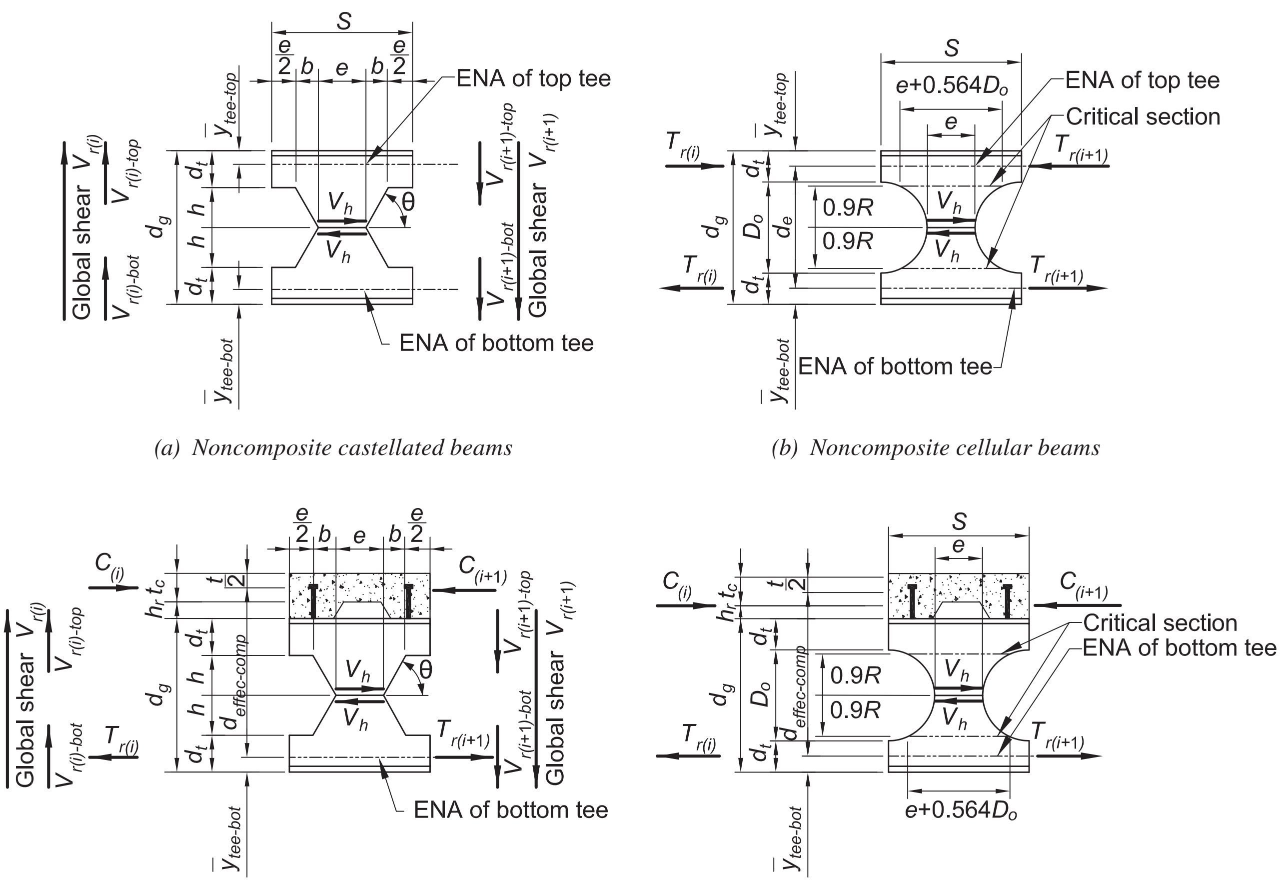
For symmetrical sections (most noncomposite sections), the design moment on the top and bottom web posts will be the same.

3.4.1.2 Calculation of Available Flexural Strength of Web Post

Calculate the plastic bending moment, M_p

$$M_p = 0.25 t_w \left(e + 2b\right)^2 F_y \tag{3-22}$$

Calculate M_{ocr}/M_p, where M_{ocr} is the critical moment for web post lateral buckling.



(a) Noncomposite castellated beams

(b) Noncomposite cellular beams

(c) Composite castellated beams

(d) Composite cellular beams

Fig. 3-6. Terminology used for calculating web post buckling.

For $\theta = 45°$, where θ is the angle of the hexagonal cut:

$e/t_w = 10$

$$\frac{M_{ocr}}{M_p} = 0.351 - 0.051\left(\frac{2h}{e}\right) + 0.0026\left(\frac{2h}{e}\right)^2 \leq 0.26 \tag{3-23}$$

$e/t_w = 20$

$$\frac{M_{ocr}}{M_p} = 3.276 - 1.208\left(\frac{2h}{e}\right) + 0.154\left(\frac{2h}{e}\right)^2 - 0.0067\left(\frac{2h}{e}\right)^3 \tag{3-24}$$

$e/t_w = 30$

$$\frac{M_{ocr}}{M_p} = 0.952 - 0.30\left(\frac{2h}{e}\right) + 0.0319\left(\frac{2h}{e}\right)^2 - 0.0011\left(\frac{2h}{e}\right)^3 \tag{3-25}$$

Note: The value of M_{ocr}/M_p is limited to 0.26, which is M_{ocr}/M_p at $e/t_w = 10$ with $2h/e = 2$. Interpolate between equations 1 through 3 based on actual e/t_w for M_{ocr}/M_p at $\theta = 45°$.

Calculate M_{ocr}/M_p

For $\theta = 60°$:

$e/t_w = 10$

$$\frac{M_{ocr}}{M_p} = 0.587(0.917)^{\frac{2h}{e}} \leq 0.493 \tag{3-26}$$

$e/t_w = 20$

$$\frac{M_{ocr}}{M_p} = 1.96(0.699)^{\frac{2h}{e}} \tag{3-27}$$

$e/t_w = 30$

$$\frac{M_{ocr}}{M_p} = 2.55(0.574)^{\frac{2h}{e}} \tag{3-28}$$

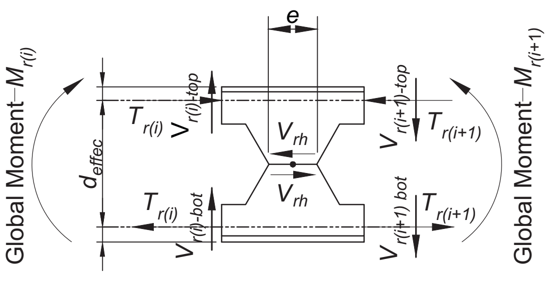


Fig. 3-7. Terminology used for calculating web post horizontal shear in noncomposite castellated beam.

Note: The value of M_{ocr}/M_p is limited to 0.493, which is M_{ocr}/M_p at $e/t_w = 10$ with $2h/e = 2$. Interpolate between equations 1 through 3 based on actual e/t_w for M_{ocr}/M_p at $\theta = 60°$.

Resistance factor (ϕ)—LRFD

$\theta = 43°$ to $47°$	$\phi_b = 0.90$
$\theta = 52.5°$	$\phi_b = 0.60$
$\theta = 58°$ to $62°$	$\phi_b = 0.90$

Note: The value of ϕ_b decreases linearly from 0.90 (at θ = 58° to 62°) to 0.60 at $\theta = 52.5°$. Linear interpolation should be used based on actual angle of the hexagonal cut, θ. See Figure 3-8.

Factor of safety (Ω)—ASD

$\theta = 43°$ to $47°$	$\Omega_b = 1.67$
$\theta = 52.5°$	$\Omega_b = 2.50$
$\theta = 58°$ to $62°$	$\Omega_b = 1.67$

Note: The value of Ω_b increases linearly from 1.67 (at θ = 58° to 62°) to 2.5 at $\theta = 52.5°$. Linear interpolation should be used based on actual angle of the hexagonal cut, θ.

The available flexural strength of the web post is calculated from Equations 3-29a and 3-29b:

$$\phi M_n = \phi_b\left(\frac{M_{ocr}}{M_p}\right)M_p \quad \text{(LRFD)} \tag{3-29a}$$

$$\frac{M_n}{\Omega_b} = \left(\frac{M_{ocr}}{M_p}\right)\left(\frac{M_p}{\Omega_b}\right) \quad \text{(ASD)} \tag{3-29b}$$

3.4.2 Web Post Buckling in Cellular Beams

Calculate the horizontal shear, V_{rh}

$$V_{rh} = \left|T_{r(i)} - T_{r(i+1)}\right| \tag{3-30}$$

The required flexural strength in the web post can then be determined by:

$$M_{rh} = 0.90\frac{D_o}{2}V_{rh} \tag{3-31}$$

The elastic bending moment M_e, at $0.9R$ is then:

$$M_e = \frac{t_w\left(S - D_o + 0.564D_o\right)^2}{6}F_y \tag{3-32}$$

Calculate $C1$, $C2$ and $C3$

$$C1 = 5.097 + 0.1464\left(\frac{D_o}{t_w}\right) - 0.00174\left(\frac{D_o}{t_w}\right)^2 \tag{3-33}$$

$$C2 = 1.441 + 0.0625\left(\frac{D_o}{t_w}\right) - 0.000683\left(\frac{D_o}{t_w}\right)^2 \quad \text{(3-34)}$$

$$C3 = 3.645 + 0.0853\left(\frac{D_o}{t_w}\right) - 0.00108\left(\frac{D_o}{t_w}\right)^2 \quad \text{(3-35)}$$

Calculate M_{allow}/M_e

$$\frac{M_{allow}}{M_e} = C1\left(\frac{S}{D_o}\right) - C2\left(\frac{S}{D_o}\right)^2 - C3 \quad \text{(3-36)}$$

The available flexural strength of the web post is:

$$\phi M_n = \phi_b\left(\frac{M_{allow}}{M_e}\right)M_e \quad \text{(LRFD)} \quad \text{(3-37a)}$$

$$\frac{M_n}{\Omega_b} = \left(\frac{M_{allow}}{M_e}\right)\left(\frac{M_e}{\Omega_b}\right) \quad \text{(ASD)} \quad \text{(3-37b)}$$

$\phi_b = 0.90$ (LRFD) $\qquad \Omega_b = 1.67$ (ASD)

3.5 HORIZONTAL AND VERTICAL SHEAR

As in all flexural members, horizontal and vertical shear forces are resisted by the web of castellated and cellular beams. In open web beams, the shear becomes more critical for two reasons. First, the vertical shear must be resisted by the net section of the member. Second, horizontal shear that passes down the midline of the beam web becomes magnified at each web post due to the adjacent web openings. Vertical shear should be checked using the global shear force calculated at each opening and resisted by the net section at web openings, or the gross section at web posts. The horizontal shear force, V_{rh}, from the web post buckling calculation can be used to check horizontal shear.

3.5.1 Calculation of Available Horizontal Shear Strength

The available horizontal shear strength of the web post is calculated based on AISC *Specification* Section J4.2. For castellated and cellular beams, a practical limit is reached well before the code limits would be approached that require further calculations.

Calcuate the nominal shear strength, V_n

$$V_n = 0.6F_yA_w \quad (\textit{Spec.}\text{ Eq. J4-3})$$

where

$A_w = et_w$, in.^{2} (mm^2)

The available horizontal shear strength is then:

$$V_c = \phi_v V_n \quad \text{(LRFD)}$$

$$V_c = \frac{V_n}{\Omega_v} \quad \text{(ASD)}$$

$\phi_v = 1.00 \qquad \Omega_v = 1.50$

3.5.2 Calculation of Available Vertical Shear Strength

The available vertical shear strength must be calculated at the net section as well as the gross section. In both cases, AISC *Specification* Section G2 should be used. At the net section, the shear force should be proportioned between the top and bottom tees based on the areas of the tees relative to each other. For the gross section, h/t_w should be calculated using the clear distance between flanges less the fillet, and d_t used for the net section. The term k_v should be taken as 5.34 for the gross section and as 1.2 for the stem of the tee shape at the net section.

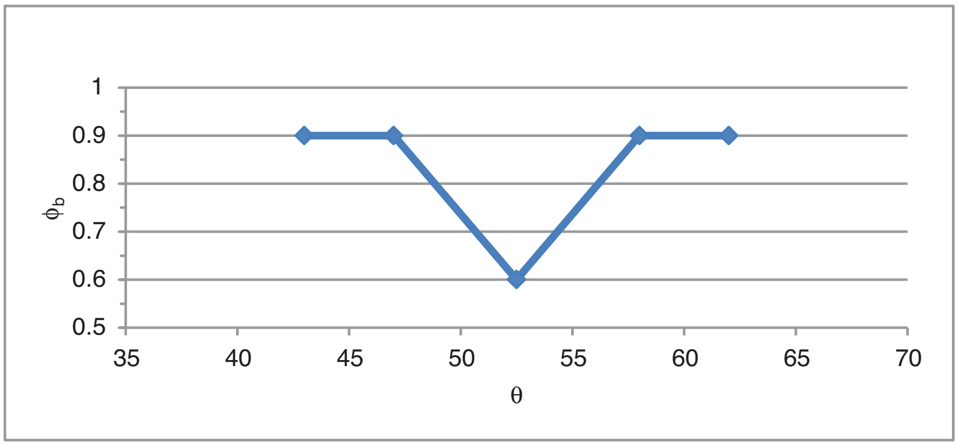


Fig. 3-8. Resistance factor by hexagonal cut angle.

At the gross section:

$$\frac{h}{t_w} = \frac{d_g - k_{top} - k_{bot}}{t_w}$$

Calculate C_{v1}:

When $\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_y}}$

$$C_{v1} = 1.0 \qquad (\textit{Spec.}\ \text{Eq. G2-3})$$

When $\frac{h}{t_w} > 1.10\sqrt{\frac{k_v E}{F_y}}$

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad (\textit{Spec.}\ \text{Eq. G2-4})$$

At the net section:

$$\frac{h}{t_w} = \frac{d_t}{t_w}$$

Calculate C_{v2}:

When $\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_y}}$

$$C_{v2} = 1.0 \qquad (\textit{Spec.}\ \text{Eq. G2-9})$$

When $1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \le 1.37\sqrt{\frac{k_v E}{F_y}}$

$$C_{v2} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad (\textit{Spec.}\ \text{Eq. G2-10})$$

When $\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}}$

$$C_{v2} = \frac{1.51 k_v E}{\left(h/t_w\right)^2 F_y} \qquad (\textit{Spec.}\ \text{Eq. G2-11})$$

For the gross and net sections, determine ϕ_v and Ω_v:

For $\frac{h}{t_w} \le 2.24\sqrt{\frac{E}{F_y}}$ $\quad \phi_v = 1.00$ and $\Omega_v = 1.50$

For $\frac{h}{t_w} > 2.24\sqrt{\frac{E}{F_y}}$ $\quad \phi_v = 0.90$ and $\Omega_v = 1.67$

For a castellated beam, the nominal shear strength, V_n, is:
Net section, top tee

$$V_n = 0.6F_y(d_{t\text{-}top} + d_{t\text{-}bot})t_w C_{v2}$$

(from *Spec.* Eq. G3-1)

Gross section

$$V_n = 0.6F_y d_g t_w C_{v1}$$

(from *Spec.* Eq. G2-1)

The available vertical shear strength is:

$$V_c = \phi_v V_n \qquad \text{(LRFD)}$$

$$V_c = \frac{V_n}{\Omega_v} \qquad \text{(ASD)}$$

3.6 LATERAL-TORSIONAL BUCKLING

Lateral-torsional buckling, flange local buckling, and tension flange yielding should be checked in castellated and cellular beams in accordance with AISC *Specification* Chapter F, Sections F2 through F5, similar to ordinary wide-flange beams. The gross section properties can be used when checking for lateral-torsional buckling. For composite sections, it may be assumed that the deck stabilizes the top flange.

3.7 DEFLECTION

Castellated and cellular beams typically have higher span-to-depth ratios than ordinary wide-flange sections; consequently, deflection typically does not govern the design. In most cases, castellated and cellular beams behave like prismatic sections as it relates to deflection. However, additional deflection due to shear deformation around the openings does occur. The magnitude of this deflection is usually only significant in very short spans or when heavy concentrated loads are present. For most applications, it is not necessary to do any rigorous deflection calculations beyond what is typically done for prismatic sections. The deflection for both composite and noncomposite castellated and cellular beams can be approximated by using 90% of the moment of inertia at the net section and treating it as a prismatic section. Transformed section properties, at a section cut through an opening, may be used for deflection calculations in the composite condition. More complex methods of predicting deflection have also been developed (Hosain et al., 1974; Altfillisch et al., 1957).

3.8 CONCENTRATED LOADING

Castellated and cellular beams with concentrated loads applied normal to one flange are to have a flange and web proportioned to satisfy the flange local bending, web local yielding, web local crippling, and sidesway web buckling criteria listed in AISC *Specification* Section J10. When the required strength exceeds the available strength as determined for the limit states listed in this section, stiffeners and/or doublers should be provided.

Chapter 4
Design Examples

This chapter contains four examples; a noncomposite castellated roof beam, a noncomposite cellular roof beam, a composite castellated floor beam, and composite cellular floor beam. All examples are presented using LRFD and ASD.

Example 4.1—Noncomposite Castellated Beam Design

Given:

A 40-ft-long roof beam with simple supports, as shown in Figure 4-1, will be evaluated as a noncomposite castellated section subject to uniform loading.

Beam span: 40 ft
Beam spacing: 5 ft
Trial beam: W12×14 → CB18×14
Loading: Live load = 20 psf
Dead load = 25 psf (not including beam self-weight)
Total load = 100 lb/ft + 125 lb/ft + 14 lb/ft
= 239 lb/ft
Deflection limits: $L/240$ live load, $L/180$ total load
Bracing: Beam is fully braced by roof deck, $L_b = 0$ in.
Material: ASTM A992
Connections: Assume that connections exist on either end to provide stability during construction (prior to deck being attached) and that the connections are sufficiently rigid to prevent web post buckling at the first web post on each end.

Solution:

From the AISC *Steel Construction Manual* (AISC, 2011), hereafter referred to as the AISC *Manual*, Table 2-4, the material properties are as follows:

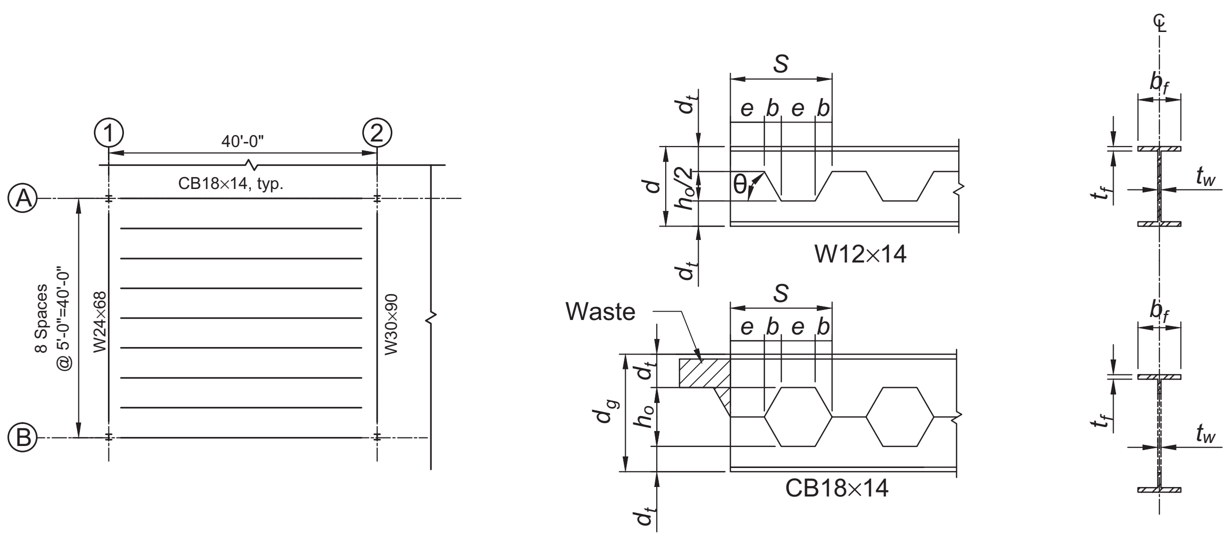


Fig. 4-1. Structural framing layout and castellated beam nomenclature for Example 4.1.

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam:
W12×14
$A = 4.16\text{ in.}^2$ $d = 11.9$ in. $t_w = 0.200$ in. $b_f = 3.97$ in. $t_f = 0.225$ in.
$S_x = 14.9\text{ in.}^3$ $Z_x = 17.4\text{ in.}^3$ $I_x = 88.6\text{ in.}^4$

Calculate section properties of tee and beam

Resultant shape section properties for the CB18×14 are determined as follows:

The values of e, b and d_t are designated based on the depth of the root beam section and a trial opening size.

$$e = 3.00\text{ in.}$$

$$b = 3.50\text{ in.}$$

$$d_t = 3.00\text{ in.}$$

$$\begin{aligned} h &= d - 2d_t \qquad (4\text{-}1) \\ &= 11.9\text{ in.} - 2(3.00\text{ in.}) \\ &= 5.90\text{ in.} \end{aligned}$$

$$\begin{aligned} h_o &= 2h \qquad (4\text{-}2) \\ &= 2(5.90\text{ in.}) \\ &= 11.8\text{ in.} \end{aligned}$$

$$\begin{aligned} d_g &= h_o + 2d_t \qquad (4\text{-}3) \\ &= 11.8\text{ in.} + 2(3.00\text{ in.}) \\ &= 17.8\text{ in.} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{h}{b}\right) \qquad (4\text{-}4) \\ &= \tan^{-1}\left(\frac{5.90\text{ in.}}{3.50\text{ in.}}\right) \\ &= 59.3^\circ \end{aligned}$$

$$\begin{aligned} S &= 2e + 2b \qquad (4\text{-}5) \\ &= 2(3.00\text{ in.}) + 2(3.50\text{ in.}) \\ &= 13.0\text{ in.} \end{aligned}$$

Figure 4-2 and Table 4-1 present relevant cross-sectional dimensions and properties.

Beam net section properties

$$\begin{aligned} A_{net} &= 2A_{tee} \qquad (4\text{-}6) \\ &= 2\left(1.45\text{ in.}^2\right) \\ &= 2.90\text{ in.} \end{aligned}$$

$$\begin{aligned} y &= \bar{x} \qquad (4\text{-}7) \\ &= \frac{d_g}{2} \\ &= \frac{17.8\text{ in.}}{2} \\ &= 8.90\text{ in.} \end{aligned}$$

Table 4-1. Top and Bottom Tee Section Properties at Center of Opening			
A_{tee} = 1.45 in.^{2}	x = 2.82 in.	r_x = 0.883 in.	r_y = 0.901 in.
$\bar{y}_{tee}$ = 2.32 in.	$S_{x\text{-}top}$ = 1.64 in.^{3}	$S_{x\text{-}bot}$ = 0.489 in.^{3}	Z_x = 0.863 in.^{3}
I_x = 1.13 in.^{4}	I_y = 1.18 in.^{4}	J = 0.022 in.^{4}	y_o = 2.20 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

$$d_{effec} = d_g - 2\left(d_t - \bar{y}_{tee}\right) \tag{4-8}$$

$$= 17.8 \text{ in.} - 2(3.00 \text{ in.} - 2.32 \text{ in.})$$

$$= 16.4 \text{ in.}$$

$$I_{x\text{-}net} = 2I_{x\text{-}tee} + 2A_{tee}\left(\frac{d_{effec}}{2}\right)^2 \tag{4-9}$$

$$= 2\left(1.13 \text{ in.}^4\right) + 2(1.45 \text{ in.})\left(\frac{16.4 \text{ in.}}{2}\right)^2$$

$$= 197 \text{ in.}^4$$

$$S_{x\text{-}net} = \frac{I_{x\text{-}net}}{\left(\frac{d_g}{2}\right)} \tag{4-10}$$

$$= \frac{197 \text{ in.}^4}{\left(\frac{17.8 \text{ in.}}{2}\right)}$$

$$= 22.1 \text{ in.}^3$$

$$Z_{x\text{-}net} = 2A_{tee}\left(\frac{d_{effec}}{2}\right) \tag{4-11}$$

$$= 2\left(1.45 \text{ in.}^2\right)\left(\frac{16.4 \text{ in.}}{2}\right)$$

$$= 23.8 \text{ in.}^3$$

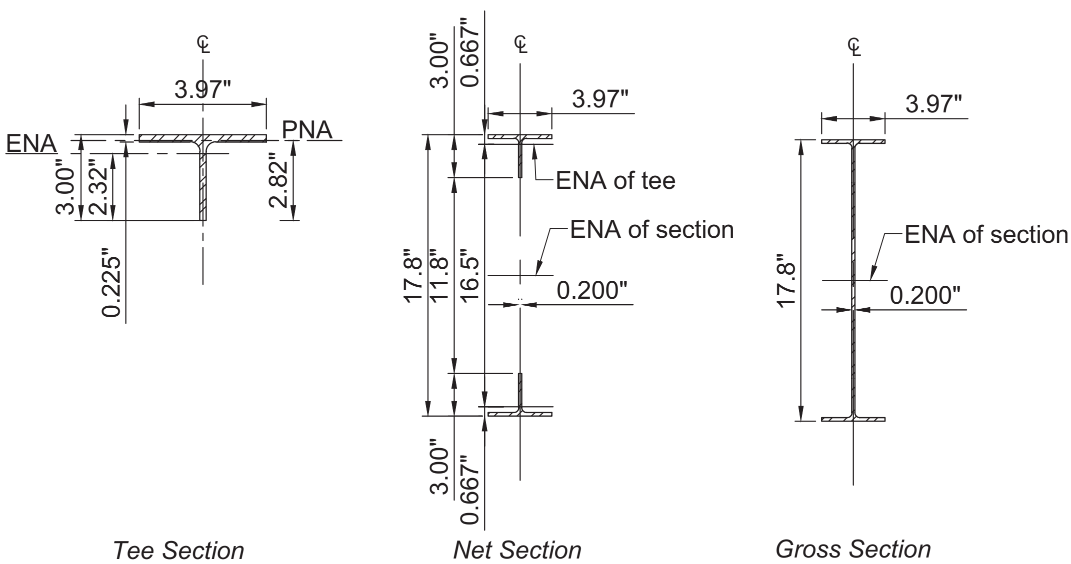


Fig. 4-2. Tee, net and gross sections of castellated beam for Example 4.1.

Beam gross section properties

$$A_{gross} = A_{net} + h_o t_w \tag{4-12}$$

$$= 2.90 \text{ in.}^2 + 11.8 \text{ in.}(0.200 \text{ in.})$$

$$= 5.26 \text{ in.}^2$$

$$I_{x\text{-}gross} = I_{x\text{-}net} + \left(\frac{t_w h_o^3}{12}\right) \tag{4-13}$$

$$= 197 \text{ in.}^4 + \left[\frac{(0.200 \text{ in.})(11.8 \text{ in.})^3}{12}\right]$$

$$= 224 \text{ in.}^4$$

$$S_{x\text{-}gross} = \frac{I_{x\text{-}gross}}{\left(\dfrac{d_g}{2}\right)} \tag{4-14}$$

$$= \frac{224 \text{ in.}^4}{\left(\dfrac{17.8 \text{ in.}}{2}\right)}$$

$$= 25.2 \text{ in.}^3$$

$$Z_{x\text{-}gross} = Z_{x\text{-}net} + 2t_w h\left(\frac{h}{2}\right) \tag{4-15}$$

$$= 23.8 \text{ in.}^3 + 2(0.200 \text{ in.})(5.90 \text{ in.})\left(\frac{5.90 \text{ in.}}{2}\right)$$

$$= 30.8 \text{ in.}^3$$

Check Vierendeel bending

The governing load cases are:

LRFD	ASD
Load case 1: $w = 1.4D$ $= 1.4(139 \text{ lb/ft})$ $= 195 \text{ lb/ft}$ Load case 2: $w = 1.2D + 1.6L$ $= 1.2(139 \text{ lb/ft}) + 1.6(100 \text{lb/ft})$ $= 327 \text{ lb/ft}$ **governs**	$w = D + L$ $= 139 \text{ lb/ft} + 100 \text{ lb/ft}$ $= 239 \text{ lb/ft}$

Calculate global shear and moment at each opening to be used to calculate local internal forces (axial and flexural) at each opening. The results are presented in Table 4-2.

Calculate the axial force and Vierendeel moment in the top and bottom tees resulting from the global shear and global moment respectively. The results are shown in Table 4-3.

Local axial force:

$$P_r = \frac{M_r}{d_{effec}} \tag{3-1}$$

Table 4-2. Global Shear and Moment at Each Opening									
		Global Shear				Global Moment			
				V_r, kips				M_r, kip-ft	
Opening No.	X_i, ft	D, kips	L, kips	ASD	LRFD	D, kip-ft	L, kip-ft	ASD	LRFD
End	0.000	2.78	2.00	4.78	6.54	0.00	0.00	0.00	0.00
1	0.667	2.69	1.93	4.62	6.32	1.82	1.31	3.13	4.29
2	1.75	2.54	1.83	4.36	5.96	4.65	3.35	8.00	10.9
3	2.83	2.39	1.72	4.10	5.61	7.32	5.27	12.6	17.2
4	3.92	2.24	1.61	3.84	5.26	9.82	7.07	16.9	23.1
5	5.00	2.09	1.50	3.59	4.90	12.2	8.75	20.9	28.6
6	6.08	1.93	1.39	3.33	4.55	14.3	10.3	24.7	33.7
7	7.17	1.78	1.28	3.07	4.19	16.4	11.8	28.1	38.4
8	8.25	1.63	1.18	2.81	3.84	18.2	13.1	31.3	42.8
9	9.33	1.48	1.07	2.55	3.49	19.9	14.3	34.2	46.8
10	10.4	1.33	0.958	2.29	3.13	21.4	15.4	36.8	50.4
11	11.5	1.18	0.850	2.03	2.78	22.8	16.4	39.2	53.6
12	12.6	1.03	0.742	1.77	2.42	24.0	17.3	41.2	56.4
13	13.7	0.880	0.633	1.51	2.07	25.0	18.0	43.0	58.8
14	14.8	0.730	0.525	1.26	1.72	25.9	18.6	44.5	60.9
15	15.8	0.579	0.417	0.996	1.36	26.6	19.1	45.7	62.5
16	16.9	0.429	0.308	0.737	1.01	27.1	19.5	46.7	63.8
17	18.0	0.278	0.200	0.478	0.654	27.5	19.8	47.3	64.7
18	19.1	0.127	0.092	0.219	0.300	27.7	20.0	47.7	65.2
Bm. CL	20.0	0.000	0.000	0.000	0.000	27.8	20.0	47.8	65.4

Local Vierendeel moment:

$$M_{vr} = \frac{V_r}{2}\left(\frac{e}{2}\right) \tag{3-2}$$

Calculate the available shear and flexural strength of top and bottom tees

Determine the limiting flange width-to-thickness ratio from AISC *Specification* Table B4.1b, Case 10:

$$\begin{aligned}\lambda_p &= 0.38\sqrt{\frac{E}{F_y}}\\ &= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}\\ &= 9.15\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{b}{t}\\ &= \frac{b_f}{2t_f}\\ &= \frac{3.97 \text{ in.}}{2(0.225 \text{ in.})}\\ &= 8.82 < 9.15\end{aligned}$$

Table 4-3. Local Axial Force and Vierendeel Moment at Each Opening									
		Axial Forces				Vierendeel Moments			
Opening No.	X_i, ft	Global Moment M_r, kip-ft		Local Axial Force P_r, kips		Global Shear V_r, kips		Local Vierendeel Moment M_{vr}, kip-in.	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
End	0.000	0.000	0.000	0.000	0.000	4.78	6.54	3.59	4.91
1	0.667	3.13	4.29	2.29	3.13	4.62	6.32	3.47	4.74
2	1.75	8.00	10.9	5.84	7.99	4.36	5.96	3.27	4.47
3	2.83	12.6	17.2	9.19	12.6	4.10	5.61	3.08	4.21
4	3.92	16.9	23.1	12.3	16.9	3.84	5.26	2.88	3.94
5	5.00	20.9	28.6	15.3	20.9	3.59	4.90	2.69	3.68
6	6.08	24.7	33.7	18.0	24.6	3.33	4.55	2.50	3.41
7	7.17	28.1	38.4	20.5	28.1	3.07	4.19	2.30	3.15
8	8.25	31.3	42.8	22.9	31.3	2.81	3.84	2.11	2.88
9	9.33	34.2	46.8	25.0	34.2	2.55	3.49	1.91	2.61
10	10.4	36.8	50.4	26.9	36.8	2.29	3.13	1.72	2.35
11	11.5	39.2	53.6	28.6	39.1	2.03	2.78	1.52	2.08
12	12.6	41.2	56.4	30.1	41.2	1.77	2.42	1.33	1.82
13	13.7	43.0	58.8	31.4	43.0	1.51	2.07	1.14	1.55
14	14.8	44.5	60.9	32.5	44.5	1.26	1.72	0.941	1.29
15	15.8	45.7	62.5	33.4	45.7	0.996	1.36	0.747	1.02
16	16.9	46.7	63.8	34.1	46.6	0.737	1.01	0.553	0.756
17	18.0	47.3	64.7	34.6	47.3	0.478	0.654	0.359	0.490
18	19.1	47.7	65.2	34.8	47.7	0.219	0.300	0.164	0.225
Bm. CL	20.0	47.8	65.4	34.9	47.8	0.000	0.000	0.000	0.000

Because $\lambda < \lambda_p$, the flanges of the tee are compact; therefore, it is not necessary to check flange local buckling when calculating the available flexural strength.

Determine the limiting stem width-to-thickness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\begin{aligned}\lambda_r &= 0.75\sqrt{\frac{E}{F_y}} \\ &= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 18.1\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{d_t}{t_w} \\ &= \frac{3.00 \text{ in.}}{0.200 \text{ in.}} \\ &= 15.0 < 18.1\end{aligned}$$

Because $\lambda < \lambda_r$, the tee stem is nonslender; therefore, it is not necessary to consider AISC *Specification* Section E7 when calculating the available compressive strength.

Calculate available axial (compression) strength of tee

Flexural buckling

Determine which L_c/r ratio controls:

From Section 3.2.2.1, $L = e$ for castellated beams.

$$\frac{L_c}{r_x} = \frac{K_x e}{r_x} = \frac{0.65(3.00 \text{ in.})}{0.883 \text{ in.}} = 2.21$$

$$\frac{L_c}{r_y} = \frac{K_y e}{r_y} = \frac{1.0(3.00 \text{ in.})}{0.901 \text{ in.}} = 3.33 \quad \textbf{governs}$$

Using AISC *Specification* Section E3, calculate the elastic buckling stress, F_e:

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \qquad \text{(\textit{Spec}. Eq E3-4)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(3.33)^2} = 25{,}800 \text{ ksi}$$

From AISC *Specification* Section E3:

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 113$$

Because $\frac{L_c}{r} = 3.33 < 113$, AISC *Specification* Equation E3-2 is used to calculate F_{cr}:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$

$$= \left(0.658^{\frac{50 \text{ ksi}}{25{,}800 \text{ ksi}}}\right)(50 \text{ ksi}) = 50.0 \text{ ksi}$$

$$P_n = F_{cr} A_{tee} \qquad \text{(from \textit{Spec}. Eq. E3-1)}$$

$$= (50.0 \text{ ksi})(1.45 \text{ in.}^2) = 72.5 \text{ kips}$$

Flexural-torsional buckling

The nominal compressive strength is determined based on the limit state of flexural-torsional buckling using AISC *Specification* Equation E4-1:

$$P_n = F_{cr} A_{tee} \qquad \text{(from \textit{Spec}. Eq. E4-1)}$$

The critical stress, F_{cr}, is determined according to Equation E3-2, using the torsional or flexural-torsional elastic buckling stress, F_e, determined from:

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \quad \text{(\textit{Spec}. Eq. E4-3)}$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \quad \text{(\textit{Spec}. Eq. E4-6)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{\left(\frac{3.00 \text{ in.}}{0.901 \text{ in.}}\right)^2}$$

$$= 25{,}800 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{A_{tee}\bar{r}_o^2} \quad \text{(from \textit{Spec}. Eq. E4-7)}$$

From the User Note in AISC *Specification* Section E4, for tees, C_w is omitted when calculating F_{ez} and x_o is taken as 0.

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \quad \text{(\textit{Spec}. Eq. E4-9)}$$

$$= y_o^2 + \frac{I_x + I_y}{A_{tee}}$$

$$= (2.20 \text{ in.})^2 + \frac{1.13 \text{ in.}^4 + 1.18 \text{ in.}^4}{1.45 \text{ in.}^2}$$

$$= 6.43 \text{ in.}^2$$

$$F_{ez} = \left[\frac{\pi^2 (29{,}000 \text{ ksi})}{(3.00 \text{ in.})^2} + (11{,}200 \text{ ksi})(0.022 \text{ in.}^4)\right]\frac{1}{(1.45 \text{ in.}^2)(6.43 \text{ in.}^2)}$$

$$= 3{,}440 \text{ ksi}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \quad \text{(\textit{Spec}. Eq. E4-8)}$$

$$= 1 - \frac{(2.20 \text{ in.})^2}{(6.43 \text{ in.})^2}$$

$$= 0.880$$

$$F_e = \left[\frac{25{,}800 \text{ ksi} + 3{,}440 \text{ ksi}}{2(0.880)}\right]\left[1 - \sqrt{1 - \frac{4(25{,}800 \text{ ksi})(3{,}440 \text{ ksi})(0.880)}{(25{,}800 \text{ ksi} + 3{,}440 \text{ ksi})^2}}\right]$$

$$= 3{,}880 \text{ ksi}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \quad \text{(\textit{Spec}. Eq. E3-2)}$$

$$= \left(0.658^{\frac{50 \text{ ksi}}{3{,}880 \text{ ksi}}}\right)(50 \text{ ksi})$$

$$= 49.7 \text{ ksi}$$

$$P_n = F_{cr} A_{tee}$$
$$= (49.7 \text{ ksi})(1.45 \text{ in.}^2)$$
$$= 72.1 \text{ kips}$$

The available compressive strength of the tee is:

LRFD	ASD
From Table 4-3, $P_r = 47.8$ kips $P_u = \phi_c P_n$ $= 0.90(72.1 \text{ kips})$ $= 64.9 \text{ kips} > 47.8 \text{ kips}$ **o.k.**	From Table 4-3, $P_r = 34.9$ kips $P_a = \dfrac{P_n}{\Omega_c}$ $= \dfrac{72.1 \text{ kips}}{1.67}$ $= 43.2 \text{ kips} > 34.9 \text{ kips}$ **o.k.**

Calculate available flexural strength of tee

Yielding

Yielding of the tee with the stem in compression is calculated using AISC *Specification* Section F9.1

$$M_p = M_y \qquad \text{(}Spec\text{. Eq. F9-4)}$$
$$M_y = F_y S_{x\text{-}bot} \qquad \text{(from }Spec\text{. Eq. F9-3)}$$
$$= (50 \text{ ksi})(0.489 \text{ in.}^3)$$
$$= 24.5 \text{ kip-in.}$$

Lateral-torsional buckling

For lateral torsional buckling of the tee:

Because $L_b = 0$, the limit state of lateral-torsional buckling does not apply.

Flange local buckling

Per AISC *Specification* Section F9.3(a), the limit state of flange local buckling does not apply because the flange is compact.

Local buckling of tee stems

The nominal flexural strength for local buckling of the tee stem in flexural compression, M_n, is determined using AISC *Specification* Section F9.4:

$$M_n = F_{cr} S_{x\text{-}bot} \qquad \text{(from }Spec\text{. Eq. F9-16)}$$

Because $d/t_w < 0.84\sqrt{\dfrac{E}{F_y}}$, the critical stress, F_{cr}, is determined using AISC *Specification* Equation F9-17:

$$F_{cr} = F_y \qquad \text{(}Spec\text{. Eq. F9-17)}$$

And thus,

$$M_n = (50 \text{ ksi})(0.489 \text{ in.}^3)$$
$$= 24.5 \text{ kip-in.}$$

Table 4-4. LRFD Interaction Check							
		Local Forces on Tee		LRFD Interaction Check			
Opening No.	**X_i, ft**	**P_r, kips**	**M_{vr}, kip-in.**	**P_r/P_c**	***Spec.* Eq. H1-1a**	***Spec.* Eq. H1-1b**	**Interaction***
End	0.000	0.000	4.90	0.000	0.000	0.000	0.000
1	0.667	3.12	4.74	0.048	0.238	**0.238**	0.238
2	1.75	7.96	4.47	0.123	0.302	**0.263**	0.263
3	2.83	12.5	4.21	0.192	0.361	**0.286**	0.286
4	3.92	16.8	3.94	0.257	**0.416**	0.307	0.416
5	5.00	20.8	3.68	0.318	**0.466**	0.326	0.466
6	6.08	24.5	3.41	0.375	**0.513**	0.342	0.513
7	7.17	28.0	3.15	0.428	**0.555**	0.356	0.555
8	8.25	31.1	2.88	0.477	**0.593**	0.369	0.593
9	9.33	34.0	2.61	0.521	**0.626**	0.379	0.626
10	10.4	36.6	2.35	0.561	**0.655**	0.387	0.655
11	11.5	39.0	2.08	0.596	**0.680**	0.392	0.680
12	12.6	41.0	1.82	0.628	**0.701**	0.396	0.701
13	13.7	42.8	1.55	0.655	**0.717**	0.398	0.717
14	14.8	44.3	1.29	0.678	**0.730**	0.397	0.730
15	15.8	45.5	1.02	0.696	**0.737**	0.394	0.737
16	16.9	46.4	0.756	0.711	**0.741**	0.390	0.741
17	18.0	47.0	0.490	0.721	**0.740**	0.383	0.740
18	19.1	47.4	0.225	0.726	**0.735**	0.373	0.735
Bm. CL	20.0	47.5	0.000	0.728	**0.728**	0.364	0.728
* Reflects bold face value of controlling interaction equation.						I_{max}:	**0.741**

The available flexural strength of the tee is:

LRFD	ASD
From Table 4-3, $M_r = 4.74$ kip-in. $M_u = \phi_b M_n$ $= 0.90(24.5 \text{ kip-in.})$ $= 22.1 \text{ kip-in.} > 4.74 \text{ kip-in.}$ **o.k.**	From Table 4-3, $M_r = 3.47$ kip-in. $M_a = \dfrac{M_n}{\Omega_b}$ $= \dfrac{24.5 \text{ kip-in.}}{1.67}$ $= 14.7 \text{ kip-in.} > 3.47 \text{ kip-in.}$ **o.k.**

Check tees for combined axial and flexural loads

LRFD results are presented in Table 4-4, and ASD results are presented in Table 4-5.

From Tables 4-4 and 4-5, the Vierendeel bending is summarized as follows:

LRFD	ASD
$I_{max} = 0.741 < 1.0$ **o.k.**	$I_{max} = 0.815 < 1.0$ **o.k.**

Table 4-5. ASD Interaction Check							
		Local Forces on Tee		ASD Interaction Check			
Opening No.	X_i, ft	P_r, kips	M_{vr}, kip-in.	P_r/P_c	*Spec.* Eq. H1-1a	*Spec.* Eq. H1-1b	Interaction*
End	0.000	0.000	3.59	0.000	0.000	0.000	0.000
1	0.667	2.28	3.47	0.053	0.262	**0.262**	0.262
2	1.75	5.82	3.27	0.135	0.332	**0.290**	0.290
3	2.83	9.15	3.08	0.212	**0.397**	0.315	0.397
4	3.92	12.3	2.88	0.283	**0.457**	0.338	0.457
5	5.00	15.2	2.69	0.350	**0.513**	0.358	0.513
6	6.08	17.9	2.50	0.413	**0.564**	0.376	0.564
7	7.17	20.5	2.30	0.471	**0.610**	0.392	0.610
8	8.25	22.8	2.11	0.525	**0.652**	0.406	0.652
9	9.33	24.9	1.91	0.573	**0.689**	0.417	0.689
10	10.4	26.8	1.72	0.617	**0.721**	0.425	0.721
11	11.5	28.5	1.52	0.656	**0.748**	0.432	0.748
12	12.6	30.0	1.33	0.691	**0.771**	0.436	0.771
13	13.7	31.3	1.14	0.721	**0.789**	0.438	0.789
14	14.8	32.4	0.941	0.746	**0.803**	0.437	0.803
15	15.8	33.3	0.747	0.766	**0.811**	0.434	0.811
16	16.9	33.9	0.553	0.782	**0.815**	0.429	0.815
17	18.0	34.4	0.359	0.793	**0.815**	0.421	0.815
18	19.1	34.7	0.164	0.799	**0.809**	0.411	0.809
Bm. CL	20.0	34.8	0.000	0.801	**0.801**	0.401	0.801
* Reflects bold face value of controlling interaction equation.						I_{max}:	**0.815**

Check web post buckling

From Section 3.4.1, use Equation 3-19 to calculate the horizontal shear, V_{rh}:

$$V_{rh} = \left| T_{r(i)} - T_{r(i+1)} \right| \tag{3-19}$$

Table 4-6 presents the horizontal shear and resultant moment at each gross section for web post buckling.

Calculate web post buckling flexural strength

From Section 3.4.1, use Equation 3-20 to calculate the required flexural strength in the web post:

LRFD	ASD
From Table 4-6, $V_{uh} = 4.86$ kips $M_u = V_{uh}h$ (from Eq. 3-20) $= (4.86\text{ kips})(5.90\text{ in.})$ $= 28.7$ kip-in.	From Table 4-6, $V_{ah} = 3.56$ kips $M_a = V_{ah}h$ (from Eq. 3-20) $= (3.56\text{ kips})(5.90\text{ in.})$ $= 21.0$ kip-in.

Table 4-6. ASD and LRFD Web Post Buckling Check

Opening No.	X_i, ft	ASD $M_{r(i)}$, kip-ft	ASD $M_{r(i+1)}$, kip-ft	ASD ΔM_r, kip-ft	ASD V_{ah}, kips	LRFD $M_{r(i)}$, kip-ft	LRFD $M_{r(i+1)}$, kip-ft	LRFD ΔM_r, kip-ft	LRFD V_{uh}, kips
End	0.000	0.000				0.000			
1	0.667	3.13	8.00	4.87	3.56	4.29	10.9	6.65	4.86
2	1.75	8.00	12.6	4.59	3.35	10.9	17.2	6.27	4.58
3	2.83	12.6	16.9	4.31	3.15	17.2	23.1	5.89	4.30
4	3.92	16.9	20.9	4.02	2.94	23.1	28.6	5.50	4.02
5	5.00	20.9	24.7	3.74	2.74	28.6	33.7	5.12	3.74
6	6.08	24.7	28.1	3.46	2.53	33.7	38.4	4.74	3.46
7	7.17	28.1	31.3	3.18	2.33	38.4	42.8	4.35	3.18
8	8.25	31.3	34.2	2.90	2.12	42.8	46.8	3.97	2.90
9	9.33	34.2	36.8	2.62	1.92	46.8	50.4	3.58	2.62
10	10.4	36.8	39.2	2.34	1.71	50.4	53.6	3.20	2.34
11	11.5	39.2	41.2	2.06	1.51	53.6	56.4	2.82	2.06
12	12.6	41.2	43.0	1.78	1.30	56.4	58.8	2.43	1.78
13	13.7	43.0	44.5	1.50	1.10	58.8	60.9	2.05	1.50
14	14.8	44.5	45.7	1.22	0.891	60.9	62.5	1.67	1.22
15	15.8	45.7	46.7	0.939	0.686	62.5	63.8	1.28	0.938
16	16.9	46.7	47.3	0.658	0.481	63.8	64.7	0.900	0.657
17	18.0	47.3	47.7	0.378	0.276	64.7	65.2	0.516	0.377
18	19.1	47.7	47.8	0.100	0.073	65.2	65.4	0.137	0.100
Bm. CL	20.0	47.8				65.4			
				Max	3.56			Max	4.86

Calculate available flexural strength of web post

From Section 3.4.1b, use Equation 3-22 to calculate the plastic moment, M_p

$$\begin{aligned} M_p &= 0.25t_w(e+2b)^2 F_y \qquad (3\text{-}22) \\ &= 0.25(0.200 \text{ in.})\left[3.00 \text{ in.} + 2(3.50 \text{ in.})\right]^2 (50 \text{ ksi}) \\ &= 250 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} \frac{2h}{e} &= \frac{2(5.90 \text{ in.})}{3.00 \text{ in.}} \\ &= 3.93 \end{aligned}$$

$$\begin{aligned} \frac{e}{t_w} &= \frac{3.00 \text{ in.}}{0.200 \text{ in.}} \\ &= 15.0 \end{aligned}$$

For $e/t_w = 10$

$$\begin{aligned} \frac{M_{ocr}}{M_p} &= 0.587(0.917)^{\frac{2h}{e}} \qquad (3\text{-}26) \\ &= 0.587(0.917)^{3.93} \\ &= 0.418 \end{aligned}$$

For $e/t_w = 20$

$$\frac{M_{ocr}}{M_p} = 1.96(0.699)^{\frac{2h}{e}} \tag{3-27}$$

$$= 1.96(0.699)^{3.93}$$

$$= 0.480 > 0.418$$

Interpolate for $e/t_w = 15$

$$\frac{M_{ocr}}{M_p} = 0.418$$

The available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b \left(\frac{M_{ocr}}{M_p}\right) M_p$ (3-29a) $= 0.90(0.418)(250 \text{ kip-in.})$ $= 94.1$ kip-in. $> M_u = 28.7$ kip-in. **o.k.**	$\frac{M_n}{\Omega_b} = \frac{1}{\Omega_b}\left(\frac{M_{ocr}}{M_p}\right) M_p$ (3-29b) $= \frac{1}{1.67}(0.418)(250 \text{ kip-in.})$ $= 62.6$ kip-in. $> M_a = 21.0$ kip-in. **o.k.**

Check horizontal and vertical shear

The available horizontal shear strength is calculated using AISC *Specification* Section J4.2.

LRFD	ASD
From Table 4-6, $V_{uh} = 4.86$ kips From *Spec*. Eq. J4-3, $\phi_v V_{n\text{-}horiz} = \phi_v 0.6F_y(et_w)$ (from *Spec*. Eq. J4-3) $= 0.6(50 \text{ ksi})[(3.00 \text{ in.})(0.200 \text{ in.})]$ $= 18.0 \text{ kips} > 4.86 \text{ kips}$ **o.k.**	From Table 4-6, $V_{ah} = 3.56$ kips From *Spec*. Eq. J4-3, $\frac{V_{n\text{-}horiz}}{\Omega_v} = \frac{0.6F_y(et_w)}{\Omega_v}$ (from *Spec*. Eq. J4-3) $= \frac{0.6(50 \text{ ksi})[(3.00 \text{ in.})(0.200 \text{ in.})]}{1.50}$ $= 12.0 \text{ kips} > 3.56 \text{ kips}$ **o.k.**

Check vertical shear at beam net section

From AISC *Specification* Section G3,

$$\frac{h}{t_w} = \frac{d_t}{t_w}$$

$$= \frac{3.00 \text{ in.}}{0.200 \text{ in.}}$$

$$= 15.0$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$

$$= 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$

$C_{v2} = 1.0$ (*Spec*. Eq. G2-9)

The available vertical shear strength at the net section is calculated using AISC *Specification* Equation G3-1.

LRFD	ASD
From Table 4-2, $V_u = 6.32\text{ kips}$ From *Spec*. Eq. G3-1, $\phi_v V_{n\text{-}net} = \phi 0.6F_y(2d_t t_w)C_{v2}$ $= 1.00(0.6)(50\text{ ksi})[2(3.00\text{ in.})(0.200\text{ in.})](1.0)$ $= 36.0\text{ kips} > 6.32\text{ kips}$ **o.k.**	From Table 4-2, $V_a = 4.62\text{ kips}$ From *Spec*. Eq. G3-1, $\dfrac{V_{n\text{-}net}}{\Omega_v} = \dfrac{0.6F_y(2d_t t_w)C_{v2}}{\Omega_v}$ $= \dfrac{0.6(50\text{ ksi})[2(3.00\text{ in.})(0.200\text{ in.})](1.0)}{1.50}$ $= 24.0\text{ kips} > 4.62\text{ kips}$ **o.k.**

Check vertical shear at beam gross section

From AISC *Specification* Section G2.1(b)(1)

$$\frac{h}{t_w} = \frac{17.8\text{ in.} - 2(0.525\text{ in.})}{0.200\text{ in.}}$$
$$= 83.8$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5.34(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 61.2$$

Because $h/t_w > 61.2$,

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \quad (\textit{Spec}.\text{ Eq. G2-4})$$
$$= \frac{1.10\sqrt{5.34(29{,}000\text{ ksi})/(50\text{ ksi})}}{83.8}$$
$$= 0.731$$

From AISC *Specification* Section G1, because $h/t_w > 2.24\sqrt{E/F_y} = 53.9$,

$\phi_v = 0.90$ (LRFD) $\quad \Omega_v = 1.67$ (ASD)

LRFD	ASD
From Table 4-2, $V_u = 6.54\text{ kips}$ From *Spec*. Eq. G2-1, $\phi_v V_{n\text{-}gross} = \phi_v 0.6F_y(d_g t_w)C_{v1}$ $= 0.90(0.6)(50\text{ ksi})(17.8\text{ in.})(0.200\text{ in.})(0.731)$ $= 70.3\text{ kips} > 6.54\text{ kips}$ **o.k.**	From Table 4-2, $V_a = 4.78\text{ kips}$ From *Spec*. Eq. G2-1, $\dfrac{V_{n\text{-}gross}}{\Omega_v} = \dfrac{0.6F_y(d_g t_w)C_{v1}}{\Omega_v}$ $= \dfrac{0.6(50\text{ ksi})(17.8\text{ in.})(0.200\text{ in.})(0.731)}{1.67}$ $= 46.7\text{ kips} > 4.78\text{ kips}$ **o.k.**

The following is a summary of the beam shear strengths:

LRFD	ASD
Horizontal shear $$V_{uh}/\phi_v V_{n\text{-}horiz} = 4.86 \text{ kips}/18.0 \text{ kips} = 0.270 < 1.0 \quad \textbf{o.k.}$$	*Horizontal shear* $$V_{ah}\Omega_v/V_{n\text{-}horiz} = 3.56 \text{ kips}/12.0 \text{ kips} = 0.297 < 1.0 \quad \textbf{o.k.}$$
Vertical shear—net section $$V_u/\phi_v V_{n\text{-}net} = 6.32 \text{ kips}/36.0 \text{ kips} = 0.176 < 1.0 \quad \textbf{o.k.}$$	*Vertical shear—net section* $$V_a\Omega_v/V_{n\text{-}net} = 4.62 \text{ kips}/24.0 \text{ kips} = 0.193 < 1.0 \quad \textbf{o.k.}$$
Vertical shear—gross section $$V_u/\phi_v V_{n\text{-}gross} = 6.54 \text{ kips}/70.3 \text{ kips} = 0.093 < 1.0 \quad \textbf{o.k.}$$	*Vertical shear—gross section* $$V_a\Omega_v/V_{n\text{-}gross} = 4.78 \text{ kips}/46.7 \text{ kips} = 0.102 < 1.0 \quad \textbf{o.k.}$$

Check Deflection

Deflections are calculated using 90% of the moment of inertia per Section 3.7.

From AISC *Manual* Table 3-23, Case 1, the live load and dead load deflections are:

$$\Delta_{LL} = \frac{5wL^4}{384EI_{x\text{-}net}(0.90)}$$

$$= \frac{5(0.1 \text{ kip/ft})(1 \text{ ft}/12 \text{ in.})\left[(40 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(197 \text{ in.}^4)(0.90)}$$

$$= 1.12 \text{ in.}$$

$$= \frac{L}{430} < \frac{L}{240} \quad \textbf{o.k.}$$

$$\Delta_{DL} = \frac{5wL^4}{384EI_{x\text{-}net}(0.90)}$$

$$= \frac{5(0.139 \text{ kip/ft})(1 \text{ ft}/12 \text{ in.})\left[(40 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(197 \text{ in.}^4)(0.90)}$$

$$= 1.56 \text{ in.}$$

Total load deflection is:

$$\Delta_{TL} = \Delta_{LL} + \Delta_{DL}$$

$$= 1.12 \text{ in.} + 1.56 \text{ in.}$$

$$= 2.68 \text{ in.}$$

$$= \frac{L}{180} \leq \frac{L}{180} \quad \textbf{o.k.}$$

Because $\Delta_{DL} = 1.56$ in., a 1½-in. camber is used.

Example 4.2—Noncomposite Cellular Beam Design

Given:

This example presents the evaluation of a cellular beam for the same design presented in Example 4.1. For cellular beams, there is no obvious lever arm to calculate the Vierendeel moment as in castellated sections ($e/2$). Therefore, assume that there is a critical section at which Vierendeel bending is examined. The critical section is located $0.225D_o$ away from the center of the opening. The distance from the center of the circle to the horizontal line passing through the point at $0.225D_o$ is defined as "y". Additionally, it is assumed that the effective length for investigating column buckling on the tee sections is two times the moment arm length. Figure 4-3 presents the nomenclature of the cellular beam.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W12×14
$A = 4.16$ in.^{2} $\quad d = 11.9$ in. $\quad t_w = 0.200$ in. $\quad b_f = 3.97$ in. $\quad t_f = 0.225$ in.
$S_x = 14.9$ in.^{3} $\quad Z_x = 17.4$ in.^{3} $\quad I_x = 88.6$ in.^{4}

Resultant shape section properties for the LB18×14 are as follows:

The values of D_o and S are designated based on the depth of the original beam section and a trial opening size.

$$D_o = 12.3 \text{ in.}$$

$$S = 16.8 \text{ in.}$$

$$\begin{aligned} e &= S - D_o \\ &= 16.8 \text{ in.} - 12.3 \text{ in.} \\ &= 4.50 \text{ in.} \end{aligned} \qquad (4\text{-}16)$$

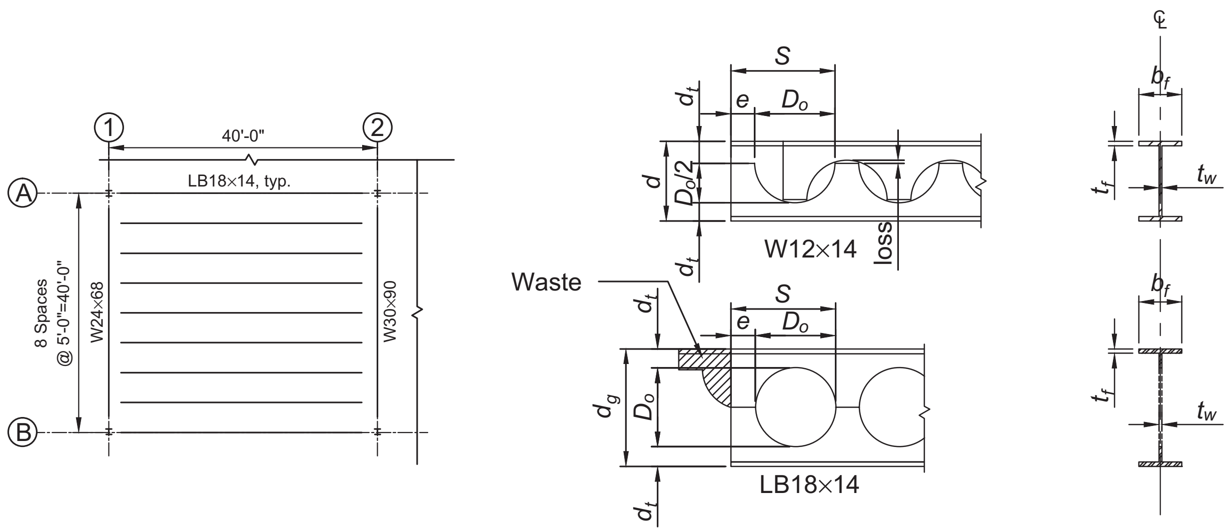


Fig. 4-3. Structural framing layout and cellular beam nomenclature for Example 4.2.

$$loss = \frac{D_o}{2} - \sqrt{\left(\frac{D_o}{2}\right)^2 - \left(\frac{S - D_o}{2}\right)^2} \tag{4-17}$$

$$= \frac{12.3 \text{ in.}}{2} - \sqrt{\left(\frac{12.3 \text{ in.}}{2}\right)^2 - \left(\frac{16.8 \text{ in.} - 12.3 \text{ in.}}{2}\right)^2}$$

$$= 0.426 \text{ in.}$$

$$d_g = d + \frac{D_o}{2} - loss \tag{4-18}$$

$$= 11.9 \text{ in.} + \frac{12.3 \text{ in.}}{2} - 0.426 \text{ in.}$$

$$= 17.6 \text{ in.}$$

$$d_{t\text{-}net} = \frac{d_g - D_o}{2} \tag{4-19}$$

$$= \frac{17.6 \text{ in.} - 12.3 \text{ in.}}{2}$$

$$= 2.65 \text{ in.}$$

$$y = \sqrt{(0.5D_o)^2 - (0.225D_o)^2} \tag{4-20}$$

$$= \sqrt{[0.5(12.3 \text{ in.})]^2 - [0.225(12.3 \text{ in.})]^2}$$

$$= 5.49 \text{ in.}$$

$$d_{t\text{-}crit} = \frac{D_o}{2} - y + d_{t\text{-}net} \tag{4-21}$$

$$= \frac{12.3 \text{ in.}}{2} - 5.49 \text{ in.} + 2.65 \text{ in.}$$

$$= 3.31 \text{ in.}$$

Check limits of applicability

According to Section 3.4, the design procedures for web post buckling are only applicable if the following conditions concerning the cutting pattern are met: $1.08 < S/D_o < 1.5$ and $1.25 < d_g/D_o < 1.75$.

$$\frac{S}{D_o} = \frac{16.8 \text{ in.}}{12.3 \text{ in.}}$$

$$= 1.37 \qquad \textbf{o.k.}$$

$$\frac{d_g}{D_o} = \frac{17.6 \text{ in.}}{12.3 \text{ in.}}$$

$$= 1.43 \qquad \textbf{o.k.}$$

Calculate section properties of tee and beam

Relevant cross sections of the cellular beam are presented in Figure 4-4 and the corresponding section properties for center of opening and critical section are provided in Tables 4-7 and 4-8, respectively.

Beam net section properties

$$A_{net} = 2A_{tee\text{-}net} \qquad \text{(from Eq. 4-6)}$$

$$= 2(1.38 \text{ in.}^2)$$

$$= 2.76 \text{ in.}^2$$

Table 4-7. Top and Bottom Tee Section Properties at Center of Opening			
$A_{tee\text{-}net}$ = 1.38 in.^{2}	x = 2.50 in.	r_x = 0.767 in.	r_y = 0.922 in.
$\bar{y}_{tee\text{-}net}$ = 2.09 in.	$S_{x\text{-}top}$ = 1.39 in.^{3}	$S_{x\text{-}bot}$ = 0.39 in.^{3}	Z_x = 0.690 in.^{3}
$I_{x\text{-}tee\text{-}net}$ = 0.814 in.^{4}	I_y = 1.18 in.^{4}	J = 0.021 in.^{4}	y_o = 1.97 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

Table 4-8. Top and Bottom Tee Section Properties at Critical Section			
$A_{tee\text{-}crit}$ = 1.51 in.^{2}	x = 3.14 in.	r_x = 1.00 in.	r_y = 0.881 in.
$\bar{y}_{tee\text{-}crit}$ = 2.53 in.	$S_{x\text{-}top}$ = 1.91 in.^{3}	$S_{x\text{-}bot}$ = 0.598 in.^{3}	Z_x = 1.06 in.^{3}
$I_{x\text{-}tee\text{-}crit}$ = 1.52 in.^{4}	I_y = 1.18 in.^{4}	J = 0.023 in.^{4}	y_o = 2.42 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

$$\begin{aligned}\bar{y} &= \bar{x} \\ &= \frac{d_g}{2} \\ &= \frac{17.6\text{ in.}}{2} \\ &= 8.80\text{ in.}\end{aligned} \tag{4-7}$$

$$\begin{aligned}d_{effec\text{-}net} &= d_g - 2\left(d_{t\text{-}net} - \bar{y}_{tee\text{-}net}\right) \\ &= 17.6\text{ in.} - 2\left(2.65\text{ in.} - 2.09\text{ in.}\right) \\ &= 16.5\text{ in.}^4\end{aligned} \tag{4-22}$$

$$\begin{aligned}I_{x\text{-}net} &= 2I_{x\text{-}tee\text{-}net} + 2A_{tee\text{-}net}\left(\frac{d_{effec\text{-}net}}{2}\right)^2 \\ &= 2\left(0.814\text{ in.}^4\right) + 2\left(1.38\text{ in.}^2\right)\left(\frac{16.5\text{ in.}}{2}\right)^2 \\ &= 190\text{ in.}^4\end{aligned} \tag{from Eq. 4-9}$$

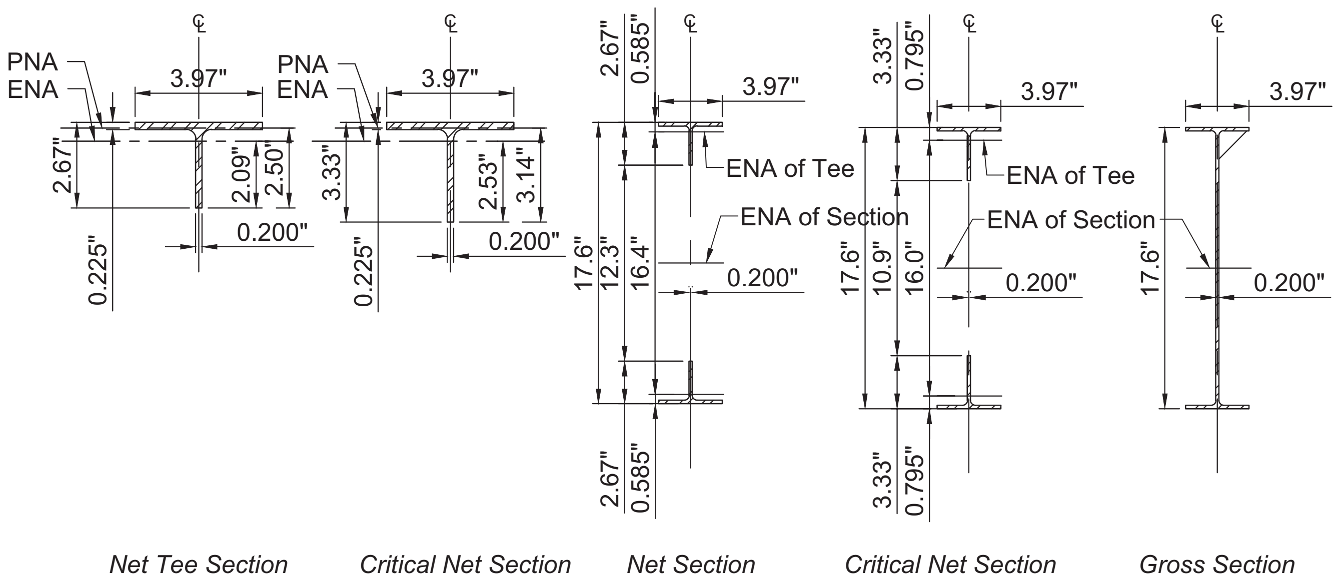


Fig. 4-4. Tee, net and gross sections of cellular beam for Example 4.2.

$$S_{x\text{-}net} = \frac{I_{x\text{-}net}}{\left(\frac{d_g}{2}\right)} \tag{4-10}$$

$$= \frac{190 \text{ in.}^4}{\left(\frac{17.6 \text{ in.}}{2}\right)}$$

$$= 21.6 \text{ in.}^3$$

$$Z_{x\text{-}net} = 2A_{tee\text{-}net}\left(\frac{d_{effec\text{-}net}}{2}\right) \qquad \text{(from Eq. 4-11)}$$

$$= 2(1.38 \text{ in.})\left(\frac{16.5 \text{ in.}}{2}\right)$$

$$= 22.8 \text{ in.}^3$$

Beam critical net section properties

$$A_{crit} = 2A_{tee\text{-}crit} \qquad \text{(from Eq. 4-6)}$$

$$= 2\left(1.51 \text{ in.}^2\right)$$

$$= 3.02 \text{ in.}^2$$

$$\bar{y} = \bar{x} \tag{4-7}$$

$$= \frac{d_g}{2}$$

$$= \frac{17.6 \text{ in.}}{2}$$

$$= 8.80 \text{ in.}$$

$$d_{effec\text{-}crit} = d_g - 2\left(d_{t\text{-}crit} - \bar{y}_{tee\text{-}net}\right) \qquad \text{(from Eq. 4-22)}$$

$$= 17.6 \text{ in.} - 2(3.31 \text{ in.} - 2.53 \text{ in.})$$

$$= 16.0 \text{ in.}$$

$$I_{x\text{-}crit} = 2I_{x\text{-}tee\text{-}crit} + 2A_{tee\text{-}crit}\left(\frac{d_{effec\text{-}crit}}{2}\right)^2 \qquad \text{(from Eq. 4-9)}$$

$$= 2\left(1.52 \text{ in.}^4\right) + 2\left(1.51 \text{ in.}^2\right)\left(\frac{16.0 \text{ in.}}{2}\right)^2$$

$$= 196 \text{ in.}^4$$

$$S_{x\text{-}crit} = \frac{I_{x\text{-}crit}}{\left(\frac{d_g}{2}\right)} \qquad \text{(from Eq. 4-10)}$$

$$= \frac{196 \text{ in.}^4}{\left(\frac{17.6 \text{ in.}}{2}\right)}$$

$$= 22.3 \text{ in.}^3$$

$$Z_{x\text{-}net} = 2A_{tee\text{-}crit}\frac{d_{effec\text{-}crit}}{2} \quad \text{(from Eq. 4-11)}$$

$$= 2\left(1.51\text{ in.}^2\right)\left(\frac{16.0\text{ in.}}{2}\right)$$

$$= 24.2\text{ in.}^3$$

Beam gross section properties

$$A_{gross} = A_{net} + D_o t_w \quad (4\text{-}23)$$

$$= 2.76\text{ in.}^2 + (12.3\text{ in.})(0.200\text{ in.})$$

$$= 5.22\text{ in.}^2$$

$$I_{x\text{-}gross} = I_{x\text{-}net} + \frac{t_w D_o^3}{12} \quad (4\text{-}24)$$

$$= 189\text{ in.}^3 + \frac{(0.200\text{ in.})(12.3\text{ in.})^3}{12}$$

$$= 220\text{ in.}^4$$

$$S_{x\text{-}gross} = \frac{I_{x\text{-}gross}}{\left(\frac{d_g}{2}\right)} \quad (4\text{-}14)$$

$$= \frac{220\text{ in.}^4}{\left(\frac{17.6\text{ in.}}{2}\right)}$$

$$= 25.0\text{ in.}^3$$

Check Vierendeel bending

The governing load cases are:

LRFD	ASD
Load case 1: $w = 1.4D$ $= 1.4(139\text{ lb/ft})$ $= 195\text{ lb/ft}$ Load case 2: $w = 1.2D + 1.6L$ $= 1.2(139\text{ lb/ft}) + 1.6(100\text{ lb/ft})$ $= 327\text{ lb/ft}$ **governs**	$w = D + L$ $= 139\text{ lb/ft} + 100\text{ lb/ft}$ $= 239\text{ lb/ft}$

Calculate global shear and moment at each opening to be used to calculate local internal forces (axial and flexural) at each opening. The results are presented in Table 4-9.

Calculate the axial force and Vierendeel moment in the top and bottom tees resulting from the global shear and global moment, respectively. The results are shown in Table 4-10.

Local axial force:

$$P_r = \frac{M_r}{d_{effec\text{-}crit}} \quad \text{(from Eq. 3-1)}$$

Table 4-9. Global Shear and Moment at Each Opening									
		Global Shear				Global Moment			
				V_r, kips				M_r, kip-ft	
Opening No.	X_i, ft	D, kips	L, kips	ASD	LRFD	D, kip-ft	L, kip-ft	ASD	LRFD
End	0.00	2.78	2.00	4.78	6.54	0.00	0.00	0.00	0.00
1	0.885	2.66	1.91	4.57	6.25	2.41	1.73	4.14	5.66
2	2.28	2.46	1.77	4.24	5.79	5.98	4.30	10.3	14.1
3	3.68	2.27	1.63	3.90	5.33	9.28	6.68	16.0	21.8
4	5.07	2.08	1.49	3.57	4.88	12.3	8.86	21.2	29.0
5	6.47	1.88	1.35	3.23	4.42	15.1	10.8	25.9	35.4
6	7.87	1.69	1.21	2.90	3.97	17.6	12.6	30.2	41.3
7	9.26	1.49	1.07	2.57	3.51	19.8	14.2	34.0	46.5
8	10.7	1.30	0.934	2.23	3.05	21.7	15.6	37.4	51.1
9	12.1	1.11	0.795	1.90	2.60	23.4	16.8	40.3	55.0
10	13.4	0.911	0.655	1.57	2.14	24.8	17.9	42.7	58.3
11	14.8	0.717	0.516	1.23	1.69	26.0	18.7	44.6	61.0
12	16.2	0.523	0.376	0.899	1.23	26.8	19.3	46.1	63.0
13	17.6	0.329	0.236	0.565	0.773	27.4	19.7	47.1	64.4
14	19.0	0.135	0.097	0.232	0.317	27.7	20.0	47.7	65.2
Bm. CL	20.0	0.000	0.000	0.000	0.000	27.8	20.0	47.8	65.4

Local Vierendeel moment:

$$M_{vr} = \frac{V_r}{2}\left(\frac{D_o}{4}\right) \qquad \text{(from Eq. 3-3)}$$

Calculate the available shear and flexural strength of top and bottom tees at critical section

Determine the limiting flange width-to-thickness ratio from AISC *Specification* Table B4.1b, Case 10:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

$$= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 9.15$$

$$\lambda = \frac{b}{t}$$

$$= \frac{b_f}{2t_f}$$

$$= \frac{3.97 \text{ in.}}{2(0.225 \text{ in.})}$$

$$= 8.82 < 9.15$$

Because $\lambda < \lambda_p$, the flanges of the tee are compact; therefore, it is not necessary to check flange local buckling when calculating the available flexural strength.

Table 4-10. Local Axial Force and Vierendeel Moment at Each Opening									
		Axial Forces				Vierendeel Moments			
		Global Moment M_r, kip-ft		Local Axial Force P_r, kips		Global Shear V_r, kips		Local Vierendeel Moment M_{vr}, kip-in.	
Opening No.	X_i, ft	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
End	0.000	0.000	0.000	0.000	0.000	4.78	6.54	7.35	10.1
1	0.885	4.14	5.66	3.10	4.24	4.57	6.25	7.00	9.57
2	2.28	10.3	14.1	7.71	10.5	4.24	5.79	6.49	8.87
3	3.68	16.0	21.8	12.0	16.4	3.90	5.33	5.97	8.17
4	5.07	21.2	29.0	15.9	21.7	3.57	4.88	5.46	7.47
5	6.47	25.9	35.4	19.4	26.6	3.23	4.42	4.95	6.77
6	7.87	30.2	41.3	22.6	31.0	2.90	3.97	4.44	6.07
7	9.26	34.0	46.5	25.5	34.9	2.57	3.51	3.93	5.37
8	10.7	37.4	51.1	28.0	38.3	2.23	3.05	3.42	4.68
9	12.1	40.3	55.0	30.2	41.3	1.90	2.60	2.91	3.98
10	13.4	42.7	58.3	32.0	43.7	1.57	2.14	2.40	3.28
11	14.8	44.6	61.0	33.5	45.7	1.23	1.69	1.89	2.58
12	16.2	46.1	63.0	34.6	47.3	0.899	1.23	1.38	1.88
13	17.6	47.1	64.4	35.3	48.3	0.565	0.773	0.865	1.18
14	19.0	47.7	65.2	35.8	48.9	0.232	0.317	0.355	0.485
Bm. CL	20.0	47.8	65.4	35.8	49.0	0.000	0.000	0.000	0.000

Determine the limiting stem width-to-thickness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\begin{aligned}\lambda_r &= 0.75\sqrt{\frac{E}{F_y}}\\ &= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}\\ &= 18.1\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{d_{t\text{-}crit}}{t_w}\\ &= \frac{3.31 \text{ in.}}{0.200 \text{ in.}}\\ &= 16.6 < 18.1\end{aligned}$$

Because $\lambda < \lambda_r$ the tee stem is nonslender; therefore, it is not necessary to consider AISC *Specification* Section E7 when calculating the available compressive strength.

Calculate available axial (compression) strength of tee

Flexural buckling

Determine which L_c/r ratio controls

From Section 3.2.2.1, $L = D_o/2$ for cellular beams.

$$\frac{L_c}{r_x} = \frac{K_x\left(D_o/2\right)}{r_x}$$
$$= \frac{0.65(6.15\text{ in.})}{1.00\text{ in.}}$$
$$= 4.00$$

$$\frac{L_c}{r_y} = \frac{K_y\left(D_o/2\right)}{r_y}$$
$$= \frac{1.0(6.15\text{ in.})}{0.881\text{ in.}}$$
$$= 6.98 \qquad \textbf{governs}$$

Calculate the elastic buckling stress, F_e, from AISC *Specification* Section E3:

$$F_e = \frac{\pi^2 E}{\left(\dfrac{L_c}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(6.98)^2}$$
$$= 5{,}870\text{ ksi}$$

From AISC *Specification* Section E3:

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113$$

Because, $\dfrac{L_c}{r} = 6.98 < 113$, AISC *Specification* Equation E3-2 is used to calculate F_{cr}:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(}\textit{Spec}\text{. Eq. E3-2)}$$
$$= \left[0.658^{\left(\frac{50\text{ ksi}}{5{,}870\text{ ksi}}\right)}\right]50\text{ ksi}$$
$$= 49.8\text{ ksi}$$

$$P_n = F_{cr}A_{tee\text{-}crit} \qquad \text{(from }\textit{Spec}\text{. Eq. E3-1)}$$
$$= (49.8\text{ ksi})(1.51\text{ in.}^2)$$
$$= 75.2\text{ kips}$$

Flexural-torsional buckling

The nominal compressive strength is determined based on the limit state of flexural-torsional buckling using AISC *Specification* Equation E4-1:

$$P_n = F_{cr}A_{tee\text{-}crit} \qquad \text{(from }\textit{Spec}\text{. Eq. E4-1)}$$

The critical stress, F_{cr}, is determined according to AISC *Specification* Equation E3-2, using the torsional or flexural-torsional elastic buckling stress, F_e, determined from:

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(}\textit{Spec}\text{. Eq. E4-3)}$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E4-6)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{\left(\frac{6.15 \text{ in.}}{0.881 \text{ in.}}\right)^2}$$

$$= 5{,}870 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 EC_w}{(L_{cz})^2} + GJ\right]\frac{1}{A_{tee\text{-}crit}\,\bar{r}_o^2} \qquad \text{(from }\textit{Spec}\text{. Eq. E4-7)}$$

From the User Note in AISC *Specification* Section E4, for tees, C_w, is omitted when calculating F_{ez} and x_o is taken as 0.

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \qquad \text{(}\textit{Spec}\text{. Eq. E4-9)}$$

$$= y_o^2 + \frac{I_{x\text{-}tee\text{-}crit} + I_y}{A_{tee\text{-}crit}}$$

$$= (2.42 \text{ in.})^2 + \frac{1.52 \text{ in.}^4 + 1.18 \text{ in.}^4}{1.51 \text{ in.}^2}$$

$$= 7.64 \text{ in.}^2$$

$$F_{ez} = \left[\frac{\pi^2 (29{,}000 \text{ ksi})}{(6.15 \text{ in.})^2} + (11{,}200 \text{ ksi})(0.023 \text{ in.}^4)\right]\frac{1}{(1.51 \text{ in.}^2)(7.64 \text{ in.}^2)}$$

$$= 678 \text{ ksi}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \qquad \text{(}\textit{Spec}\text{. Eq. E4-8)}$$

$$= 1 - \frac{(2.42 \text{ in.})^2}{7.65 \text{ in.}^2}$$

$$= 0.233$$

$$F_e = \left[\frac{5{,}870 \text{ ksi} + 678 \text{ ksi}}{2(0.233)}\right]\left[1 - \sqrt{1 - \frac{4(5{,}870 \text{ ksi})(678 \text{ ksi})(0.233)}{(5{,}870 \text{ ksi} + 678 \text{ ksi})^2}}\right]$$

$$= 622 \text{ ksi}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(}\textit{Spec}\text{. Eq. E3-2)}$$

$$= \left(0.658^{\frac{50 \text{ ksi}}{622 \text{ ksi}}}\right)(50 \text{ ksi})$$

$$= 48.3 \text{ ksi}$$

$$P_n = F_{cr} A_{tee\text{-}crit}$$

$$= (48.3 \text{ ksi})(1.51 \text{ in.}^2)$$

$$= 72.9 \text{ kips}$$

The available compressive strength of the tee is:

LRFD	ASD
From Table 4-10, $P_r = 49.0$ kips $P_u = \phi_c P_n$ $= 0.90(72.9 \text{ kips})$ $= 65.6 \text{ kips} > 49.0 \text{ kips}$ **o.k.**	From Table 4-10, $P_r = 35.8$ kips $P_a = \dfrac{P_n}{\Omega_c}$ $= \dfrac{72.9 \text{ kips}}{1.67}$ $= 43.7 \text{ kips} > 35.8 \text{ kips}$ **o.k.**

Calculate available flexural strength of tee

Yielding

Yielding of the tee with the stem in compression is calculated using AISC *Specification* Section F9.1

$$M_p = M_y \qquad \text{(\textit{Spec.} Eq. F9-4)}$$

$$M_y = F_y S_{x\text{-}bot} \qquad \text{(from \textit{Spec.} Eq. F9-3)}$$

$$= (50 \text{ ksi})(0.598 \text{ in.}^3)$$

$$= 29.9 \text{ kip-in.}$$

Lateral-torsional buckling

Because $L_b = 0$, the limit state of lateral-torsional buckling does not apply.

Flange local buckling

Per AISC *Specification* Section F9.3(a), the limit state of flange local buckling does not apply because the flange is compact.

Local buckling of tee stems

The nominal flexural strength for local buckling of the tee stem in flexural compression , M_n, is determined using AISC *Specification* Section F9.4:

$$M_n = F_{cr} S_x \qquad \text{(\textit{Spec.} Eq. F9-16)}$$

Because $d/t_w < 0.84\sqrt{\dfrac{E}{F_y}}$, the critical stress, F_{cr}, is determined using AISC *Specification* Equation F9-17:

$$F_{cr} = F_y$$

And thus,

$$M_n = (50 \text{ ksi})(0.598 \text{ in.}^3)$$

$$= 29.9 \text{ kip-in.}$$

The available flexural strength of the tee is:

LRFD	ASD
From Table 4-10, $M_{vr} = 8.17$ kip-in. $M_u = \phi_b M_n = 0.90(29.9 \text{ kip-in.}) = 26.9 \text{ kip-in.} > 8.17 \text{ kip-in.}$ **o.k.**	From Table 4-10, $M_{vr} = 5.97$ kip-in. $M_a = \dfrac{M_n}{\Omega_b} = \dfrac{29.9 \text{ kip-in.}}{1.67} = 17.9 \text{ kip-in.} > 5.97 \text{ kip-in.}$ **o.k.**

Check tees for combined axial and flexural loads

LRFD results are presented in Table 4-11, and the ASD results are presented in Table 4-12.

From Tables 4-11 and 4-12, the Vierendeel bending is summarized as follows:

LRFD	ASD
$I_{max} = 0.759 < 1.0$ **o.k.**	$I_{max} = 0.835 < 1.0$ **o.k.**

Check web post buckling

From Section 3.4.2, use Equation 3-30 to calculate the horizontal shear, V_{rh}:

$$V_{rh} = \left| T_{r(i)} - T_{r(i+1)} \right| \tag{3-30}$$

Table 4-13 presents the horizontal shear at each gross section for web post buckling.

Calculate web post buckling flexural strength

From Section 3.4.2, use Equation 3-31 to calculate the required flexural strength in the web post.

LRFD	ASD
From Table 4-13, $V_{uh} = 6.26$ kips $M_u = 0.90 \dfrac{D_o}{2} V_{uh}$ (3-31) $= 0.90\left(\dfrac{12.3 \text{ in.}}{2}\right)(6.26 \text{ kips}) = 34.6$ kip-in.	From Table 4-13, $V_{ah} = 4.61$ kips $M_a = 0.90 \dfrac{D_o}{2} V_{ah}$ (3-31) $= 0.90\left(\dfrac{12.3 \text{ in.}}{2}\right)(4.61 \text{ kips}) = 25.5$ kip-in.

Calculate available flexural strength of web post

From Section 3.4.2, use Equation 3-32 to calculate the elastic moment, M_e:

$$M_e = \frac{t_w \left(S - D_o + 0.564 D_o\right)^2}{6} F_y \tag{3-32}$$

$$= \frac{(0.200 \text{ in.})\left[16.8 \text{ in.} - 12.3 \text{ in.} + 0.564(12.3 \text{ in.})\right]^2}{6} (50 \text{ ksi})$$

$$= 218 \text{ kip-in.}$$

Table 4-11. LRFD Interaction Check							
		Local Forces on Tee		LRFD Interaction Check			
Opening No.	X_i, ft	P_r, kips	M_{vr}, kip-in.	P_r/P_c	*Spec.* Eq. H1-1a	*Spec.* Eq. H1-1b	Interaction*
End	0.000	0.000	10.0	0.000	0.000	0.000	0.000
1	0.885	4.24	9.57	0.062	0.378	**0.387**	0.387
2	2.28	10.5	8.87	0.155	0.448	**0.407**	0.407
3	3.68	16.4	8.17	0.241	**0.511**	0.424	0.511
4	5.07	21.7	7.47	0.320	**0.566**	0.437	0.566
5	6.47	26.6	6.77	0.391	**0.615**	0.447	0.615
6	7.87	31.0	6.07	0.456	**0.657**	0.454	0.657
7	9.26	34.9	5.37	0.514	**0.691**	0.457	0.691
8	10.7	38.3	4.68	0.564	**0.719**	0.456	0.719
9	12.1	41.3	3.98	0.608	**0.739**	0.452	0.739
10	13.4	43.7	3.28	0.644	**0.753**	0.444	0.753
11	14.8	45.7	2.58	0.674	**0.759**	0.433	0.759
12	16.2	47.3	1.88	0.696	**0.758**	0.418	0.758
13	17.6	48.3	1.18	0.712	**0.751**	0.400	0.751
14	19.0	48.9	0.485	0.720	**0.736**	0.378	0.736
Bm. CL	20.0	49.0	0.000	0.722	**0.722**	0.361	0.722
						I_{max}:	0.759

* Reflects bold face value of controlling interaction equation.

Table 4-12. ASD Interaction Check							
		Local Forces on Tee		ASD Interaction Check			
Opening No.	X_i, ft	P_r, kips	M_{vr}, kip-in	P_r/P_c	*Spec.* Eq. H1-1a	*Spec.* Eq. H1-1b	Interaction*
End	0.000	0.000	7.35	0.000	0.000	0.000	0.000
1	0.885	3.10	7.02	0.069	0.417	**0.427**	0.427
2	2.28	7.71	6.51	0.171	0.494	**0.449**	0.449
3	3.68	12.0	6.00	0.265	**0.563**	0.468	0.563
4	5.07	15.9	5.49	0.351	**0.624**	0.482	0.624
5	6.47	19.4	4.97	0.430	**0.677**	0.493	0.677
6	7.87	22.6	4.46	0.502	**0.723**	0.500	0.723
7	9.26	25.5	3.95	0.565	**0.761**	0.503	0.761
8	10.7	28.0	3.42	0.622	**0.792**	0.502	0.792
9	12.1	30.2	2.90	0.670	**0.814**	0.497	0.814
10	13.4	32.0	2.43	0.707	**0.828**	0.489	0.828
11	14.8	33.5	1.91	0.740	**0.835**	0.477	0.835
12	16.2	34.6	1.40	0.765	**0.834**	0.461	0.834
13	17.6	35.3	0.882	0.782	**0.826**	0.440	0.826
14	19.0	35.8	0.367	0.792	**0.810**	0.416	0.810
Bm. CL	20.0	35.8	0.000	0.794	**0.794**	0.397	0.794
						I_{max}:	0.835

* Reflects bold face value of controlling interaction equation.

Table 4-13. ASD and LRFD Web Post Buckling Check							
		ASD			LRFD		
Post No.	X_i, ft	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	V_{ah}, kips	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	V_{uh}, kips
1	1.58	3.10	7.71	4.61	10.5	4.24	6.26
2	2.98	7.71	12.0	4.29	16.4	10.5	5.90
3	4.38	12.0	15.9	3.90	21.7	16.4	5.30
4	5.77	15.9	19.4	3.50	26.6	21.7	4.90
5	7.17	19.4	22.6	3.30	31.0	26.6	4.40
6	8.56	22.6	25.5	2.80	34.9	31.0	3.90
7	9.96	25.5	28.0	2.50	38.3	34.9	3.40
8	11.4	28.0	30.2	2.20	41.3	38.3	3.00
9	12.8	30.2	32.0	1.60	43.7	41.3	2.40
10	14.1	32.0	33.5	1.50	45.7	43.7	2.00
11	15.5	33.5	34.6	1.20	47.3	45.7	1.60
12	16.9	34.6	35.3	0.700	48.3	47.3	1.00
13	18.3	35.3	35.8	0.500	48.9	48.3	0.600
14	19.7	35.8	35.8	0.000	49.0	48.9	0.100
			Maximum	4.61		Maximum	6.26

$$C1 = 5.097 + 0.1464\left(\frac{D_o}{t_w}\right) - 0.00174\left(\frac{D_o}{t_w}\right)^2 \tag{3-33}$$

$$= 5.097 + 0.1464\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right) - 0.00174\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right)^2$$

$$= 7.54$$

$$C2 = 1.441 + 0.0625\left(\frac{D_o}{t_w}\right) - 0.000683\left(\frac{D_o}{t_w}\right)^2 \tag{3-34}$$

$$= 1.441 + 0.0625\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right) - 0.000683\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right)^2$$

$$= 2.70$$

$$C3 = 3.645 + 0.0853\left(\frac{D_o}{t_w}\right) - 0.00108\left(\frac{D_o}{t_w}\right)^2 \tag{3-35}$$

$$= 3.645 + 0.0853\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right) - 0.00108\left(\frac{12.3\text{ in.}}{0.200\text{ in.}}\right)^2$$

$$= 4.81$$

$$\frac{M_{allow}}{M_e} = C1\left(\frac{S}{D_o}\right) - C2\left(\frac{S}{D_o}\right)^2 - C3 \tag{3-36}$$

$$= 7.54\left(\frac{16.8\text{ in.}}{12.3\text{ in.}}\right) - 2.70\left(\frac{16.8\text{ in.}}{12.3\text{ in.}}\right)^2 - 4.81$$

$$= 0.450$$

The available flexural strength is:

LRFD	ASD
From Equation 3-37a, $\phi_b\left(\frac{M_{allow}}{M_e}\right)M_e = 0.90(0.450)(218\text{ kip-in.})$ $= 88.3\text{ kip-in.} > M_u = 34.6\text{ kip-in.}$ **o.k.**	From Equation 3-37b, $\frac{M_{allow}}{M_e}\left(\frac{M_e}{\Omega_b}\right) = 0.450\left(\frac{218\text{ kip-in.}}{1.67}\right)$ $= 58.7\text{ kip-in.} > M_a = 25.5\text{ kip-in.}$ **o.k.**

Check horizontal and vertical shear

The available horizontal shear strength is calculated using AISC *Specification* Section J4.2.

LRFD	ASD
From Table 4-13, $V_{uh} = 6.26\text{ kips}$ $\phi_v V_{n\text{-}horiz} = \phi_v 0.6F_y(et_w)$ (from *Spec*. Eq. J4-3) $= 0.6(50\text{ ksi})\left[(4.50\text{ in.})(0.200\text{ in.})\right]$ $= 27.0\text{ kips} > 6.26\text{ kips}$ **o.k.**	From Table 4-13, $V_{ah} = 4.61\text{ kips}$ $\frac{V_{n\text{-}horiz}}{\Omega_v} = \frac{0.6F_y(et_w)}{\Omega_v}$ (from *Spec*. Eq. J4-3) $= \frac{0.6(50\text{ ksi})\left[(4.50\text{ in.})(0.200\text{in.})\right]}{1.50}$ $= 18.0\text{ kips} > 4.61\text{ kips}$ **o.k.**

Check vertical shear at beam net section

From AISC *Specification* Section G3:

$$\frac{h}{t_w} = \frac{d_{t\text{-}net}}{t_w} = \frac{2.65\text{ in.}}{0.200\text{ in.}} = 13.3$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{1.2(29{,}000\text{ ksi})}{50\text{ ksi}}} = 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$

$C_{v2} = 1.0$ (*Spec*. Eq. G2-9)

The available vertical shear strength at the net section is calculated using AISC *Specification* Equation G2-3.

LRFD	ASD
From Table 4-10, $V_u = 6.25\text{ kips}$ From *Spec*. Eq. G2-3, $\phi V_{n\text{-}net} = \phi 0.6F_y(2d_{t\text{-}net}t_w)C_{v2}$ $= 1.00(0.6)(50\text{ ksi})(2)(2.65\text{ in.})(0.200\text{ in.})(1.0)$ $= 31.8\text{ kips} > 6.25\text{ kips}$ **o.k.**	From Table 4-10, $V_a = 4.57\text{ kips}$ From *Spec*. Eq. G2-3, $\frac{V_{n\text{-}net}}{\Omega_v} = \frac{0.6F_y(2d_{t\text{-}net}t_w)C_{v2}}{\Omega_v}$ $= \frac{0.6(50\text{ ksi})(2)(2.65\text{ in.})(0.200\text{ in.})(1.0)}{1.50}$ $= 21.2\text{ kips} > 4.57\text{ kips}$ **o.k.**

Check vertical shear at beam gross section

From AISC *Specification* Section G2.1(b)(1)

$$\frac{h}{t_w} = \frac{17.6 \text{ in.} - 2(0.525 \text{ in.})}{0.200 \text{ in.}}$$
$$= 82.8$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5.34(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 61.2$$

Because $h/t_w > 61.2$,

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(Spec. Eq. G2-4)}$$
$$= \frac{1.10\sqrt{5.34(29{,}000 \text{ ksi})/50 \text{ ksi}}}{82.8}$$
$$= 0.739$$

From AISC *Specification* Section G1, because $h/t_w > 2.24\sqrt{E/F_y} = 53.9$

$\phi_v = 0.90$ (LRFD) $\qquad \Omega_v = 1.67$ (ASD)

LRFD	ASD
From Table 4-10, $V_u = 6.54 \text{ kips}$ From *Spec*. Eq. G2-1, $\phi_v V_{n\text{-}gross} = \phi_v 0.6F_y(d_g t_w)C_{v1}$ $= 0.90(0.6)(50 \text{ ksi})(17.6 \text{ in.})(0.200 \text{ in.})(0.739)$ $= 70.2 \text{ kips} > 6.54 \text{ kips}$ **o.k.**	From Table 4-10, $V_a = 4.78 \text{ kips}$ From *Spec*. Eq. G2-1, $\frac{V_{n\text{-}gross}}{\Omega_v} = \frac{0.6F_y(d_g t_w)C_{v1}}{\Omega_v}$ $= \frac{0.6(50 \text{ ksi})(17.6 \text{ in.})(0.200 \text{ in.})(0.739)}{1.67}$ $= 46.7 \text{ kips} > 4.78 \text{ kips}$ **o.k.**

The following is a summary of the beam shear strengths:

LRFD	ASD
Horizontal shear $V_{uh}/\phi_v V_{n\text{-}horiz} = 6.26 \text{ kips}/27.0 \text{ kips}$ $= 0.232 < 1.0$ **o.k.**	*Horizontal shear* $V_{ah}\Omega_v/V_{n\text{-}horiz} = 4.61 \text{ kips}/18.0 \text{ kips}$ $= 0.256 < 1.0$ **o.k.**
Vertical shear—net section $V_u/\phi_v V_{n\text{-}net} = 6.25 \text{ kips}/31.8 \text{ kips}$ $= 0.197 < 1.0$ **o.k.**	*Vertical shear—net section* $V_a\Omega_v/V_{n\text{-}net} = 4.57 \text{ kips}/21.2 \text{ kips}$ $= 0.216 < 1.0$ **o.k.**

LRFD	ASD
Vertical shear—gross section $V_u/\phi_v V_{n\text{-}gross} = 6.54 \text{ kips}/70.2 \text{ kips}$ $= 0.093 < 1.0$ **o.k.**	*Vertical shear—gross section* $V_a\Omega_v/V_{n\text{-}gross} = 4.78 \text{ kips}/46.7 \text{ kips}$ $= 0.102 < 1.0$ **o.k.**

Check Deflection

Deflections are calculated using 90% of the moment of inertia as discussed in Section 3.7.

From AISC *Manual* Table 3-23, Case 1, the live load and dead load deflections are:

$$\Delta_{LL} = \frac{5wl^4}{384EI_{x\text{-}net}(0.90)}$$

$$= \frac{5(0.1 \text{ kip/ft})(1 \text{ ft/12 in.})[(40 \text{ ft})(12 \text{ in./ft})]^4}{384(29{,}000 \text{ ksi})(190 \text{ in.}^4)(0.90)}$$

$$= 1.16 \text{ in.}$$

$$= \frac{L}{410} < \frac{L}{240} \quad \textbf{o.k.}$$

$$\Delta_{DL} = \frac{5wl^4}{384EI_{x\text{-}net}(0.90)}$$

$$= \frac{5(0.139 \text{ kip/ft})(1 \text{ ft/12 in.})[(40 \text{ ft})(12 \text{ in./ft})]^4}{384(29{,}000 \text{ ksi})\left(190 \text{ in.}^4\right)(0.90)}$$

$$= 1.61 \text{ in.}$$

Because Δ_{DL} = 1.61 in., a 1½-in. camber is required.

Total load deflection is:

$$\Delta_{TL} = \Delta_{LL} + \Delta_{DL}$$

$$= 1.16 \text{ in.} + 1.61 \text{ in.}$$

$$= 2.77 \text{ in.}$$

$$= \frac{L}{172} > \frac{L}{180} \quad \textbf{n.g.}$$

This beam does not meet the deflection criteria. Either a larger section (LB18×16) should be considered, or the cutting pattern could be modified to increase the stiffness of the section.

Example 4.3—Composite Castellated Beam Design

Given:

A 50-ft-long floor beam with simple supports, shown in Figure 4-5, will be evaluated as a composite castellated section subject to uniform loading.

Beam span: 50 ft
Beam spacing: 8 ft
Trial beam: Asymmetric Section: W21×44 (top) + W21×57 (bottom) → CB30×44/57

Loading: Live load = 100 psf
Dead load = 75 psf (not including beam self-weight)
Metal deck and concrete weight = 55 psf
Total load = 800 lb/ft + 600 lb/ft + 51 lb/ft
= 1,450 lb/ft
Deflection limits: $L/360$ live load, $L/240$ total load
Bracing: Beam is fully braced by concrete deck, $L_b = 0$
Material: ASTM A992
Metal deck: depth = 2 in., rib width = 6 in., flutes perpendicular to beam
Studs: diameter = ¾ in., height = 4 in., $F_u = 65$ ksi
Concrete: $f'_c = 3{,}000$ psi, $w_c = 145$ lb/ft^3, $t_c = 3$ in. (5 in. total deck thickness)
Connections: Assume that connections exist on either end to provide stability during construction (prior to deck being attached) and that the connections are sufficiently rigid to prevent web post buckling at the first web post on each end. Assume that the beam has been checked in its pre-composite stage for the wet concrete weight and construction loads.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Top Root Beam:
W21×44
$A = 13.0$ in.^{2} $d_{top} = 20.7$ in. $t_w = 0.350$ in. $b_f = 6.50$ in. $t_f = 0.450$ in.
$S_x = 81.6$ in.^{3} $Z_x = 95.4$ in.^{3} $I_x = 843$ in.^{4}

Bottom Root Beam:
W21×57
$A = 16.7$ in.^{2} $d_{bot} = 21.1$ in. $t_w = 0.405$ in. $b_f = 6.56$ in. $t_f = 0.650$ in.
$S_x = 111$ in.^{3} $Z_x = 129$ in.^{3} $I_x = 1{,}170$ in.^{4}

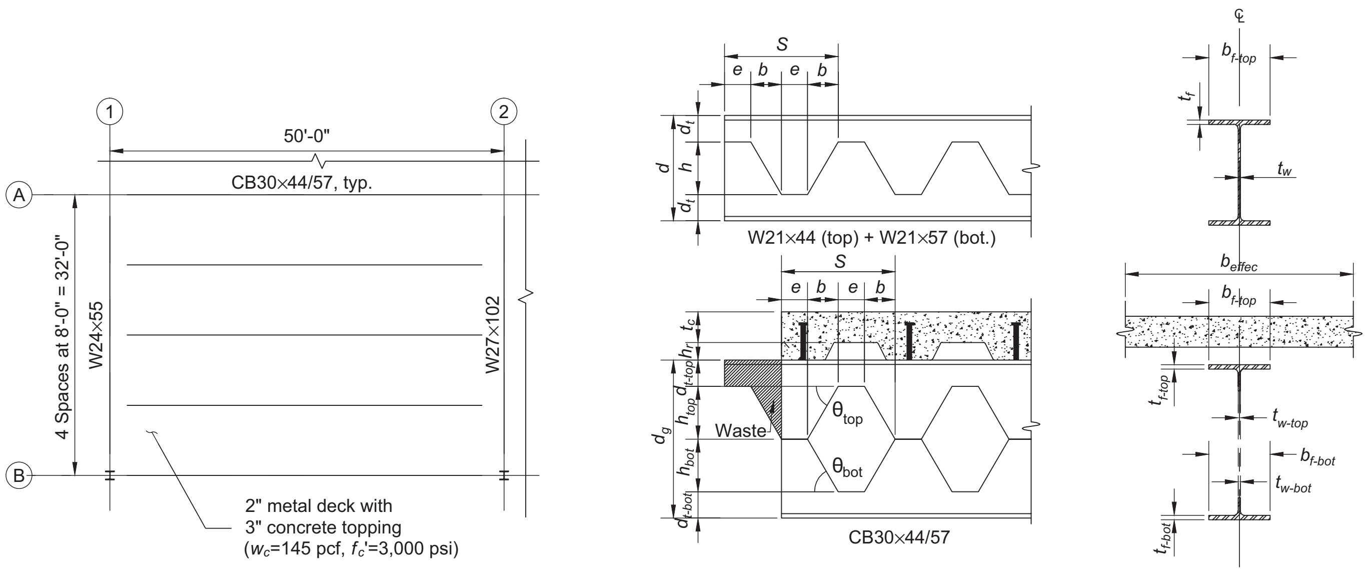


Fig. 4-5. Structural framing layout and composite castellated beam nomenclature for Example 4.3.

Resultant shape section properties for the CB30×44/57 are determined as follows:

The values of e, b and d_t are designated based on the depth of the root beam section and a trial opening size.

$$e = 8.00 \text{ in.}$$

$$b = 5.50 \text{ in.}$$

$$d_t = 5.50 \text{ in.}$$

$$h_{top} = d_{top} - 2d_t \qquad \text{(from Eq. 4-1)}$$
$$= 20.7 \text{ in.} - 2(5.50 \text{ in.})$$
$$= 9.70 \text{ in.}$$

$$h_{bot} = d_{bot} - 2d_t \qquad \text{(from Eq. 4-1)}$$
$$= 21.1 \text{ in.} - 2(5.50 \text{ in.})$$
$$= 10.1 \text{ in.}$$

$$h_o = h_{top} + h_{bot} \qquad \text{(from Eq. 4-2)}$$
$$= 9.70 \text{ in.} + 10.1 \text{ in.}$$
$$= 19.8 \text{ in.}$$

$$d_g = h_o + 2d_t \qquad \text{(4-3)}$$
$$= 19.8 \text{ in.} + 2(5.50 \text{ in.})$$
$$= 30.8 \text{ in.}$$

$$\theta_{top} = \tan^{-1}\left(\frac{h_{top}}{b}\right) \qquad \text{(from Eq. 4-4)}$$
$$= \tan^{-1}\left(\frac{9.70 \text{ in.}}{5.50 \text{ in.}}\right)$$
$$= 60.4°$$

$$\theta_{bot} = \tan^{-1}\left(\frac{h_{bot}}{b}\right) \qquad \text{(from Eq. 4-4)}$$
$$= \tan^{-1}\left(\frac{10.1 \text{ in.}}{5.50 \text{ in.}}\right)$$
$$= 61.4°$$

$$S = 2e + 2b \qquad \text{(4-5)}$$
$$= 2(8.00 \text{ in.}) + 2(5.50 \text{ in.})$$
$$= 27.0 \text{ in.}$$

Calculate section properties of top and bottom tee and beam

Relevant cross sections are provided in Figure 4-6, and the section properties for the top and bottom tees are reported in Tables 4-14 and 4-15, respectively.

Beam net section properties

$$A_{net} = A_{tee\text{-}top} + A_{tee\text{-}bot} \qquad \text{(3-7)}$$
$$= 4.70 \text{ in.}^2 + 6.22 \text{ in.}^2$$
$$= 10.9 \text{ in.}^2$$

$$\bar{y}_{bs} = \frac{A_{tee\text{-}top}\left(d_t + h_o + \bar{y}_{tee\text{-}top}\right) + A_{tee\text{-}bot}\bar{y}_{tee\text{-}bot}}{A_{net}} \qquad \text{(4-25)}$$
$$= \frac{4.70 \text{ in.}^2(5.50 \text{ in.} + 19.8 \text{ in.} + 4.24 \text{ in.}) + (6.22 \text{ in.}^2)(1.19 \text{ in.})}{10.9 \text{ in.}^2}$$
$$= 13.4 \text{ in.}$$

Table 4-14. Top Tee Section Properties at Center of Opening			
$A_{tee\text{-}top}$ = 4.70 in.^{2}	x = 5.14 in.	r_x = 1.61 in.	r_y = 1.48 in.
$\bar{y}_{tee\text{-}top}$ = 4.24 in.	$S_{x\text{-}top}$ = 9.63 in.^{3}	$S_{x\text{-}bot}$ = 2.86 in.^{3}	Z_x = 5.07 in.^{3}
$I_{x\text{-}tee\text{-}top}$ = 12.1 in.^{4}	I_y = 10.3 in.^{4}	J = 0.266 in.^{4}	y_o = 4.01 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

Table 4-15. Bottom Tee Section Properties at Center of Opening			
$A_{tee\text{-}bot}$ = 6.22 in.^{2}	x = 0.475 in.	r_x = 1.51 in.	r_y = 1.57 in.
$\bar{y}_{tee\text{-}bot}$ = 1.19 in.	$S_{x\text{-}top}$ = 3.29 in.^{3}	$S_{x\text{-}bot}$ = 11.9 in.^{3}	Z_x = 5.95 in.^{3}
$I_{x\text{-}tee\text{-}bot}$ = 14.2 in.^{4}	I_y = 15.3 in.^{4}	J = 0.685 in.^{4}	y_o = 0.870 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

$$\bar{y}_{ts} = d_g - \bar{y}_{bs} \quad (4\text{-}26)$$
$$= 30.8 \text{ in.} - 13.4 \text{ in.}$$
$$= 17.4 \text{ in.}$$

$$d_{effec} = d_g - \left[\left(d_t - \bar{y}_{tee\text{-}top}\right) + \bar{y}_{tee\text{-}bot}\right] \quad (4\text{-}27)$$
$$= 30.8 \text{ in.} - \left[(5.50 \text{ in.} - 4.24 \text{ in.}) + 1.19 \text{ in.}\right]$$
$$= 28.4 \text{ in.}$$

$$I_{x\text{-}net} = I_{x\text{-}tee\text{-}top} + A_{tee\text{-}top}\left[\bar{y}_{ts} - \left(d_t - \bar{y}_{tee\text{-}top}\right)\right]^2 + I_{x\text{-}tee\text{-}bot} + A_{tee\text{-}bot}\left(\bar{y}_{bs} - \bar{y}_{tee\text{-}bot}\right)^2 \quad (4\text{-}28)$$
$$= 12.1 \text{ in.}^4 + \left(4.70 \text{ in.}^2\right)\left[17.4 \text{ in.} - (5.50 \text{ in.} - 4.24 \text{ in.})\right]^2 + 14.2 \text{ in.}^4 + \left(6.22 \text{ in.}^2\right)(13.4 \text{ in.} - 1.19 \text{ in.})^2$$
$$= 2{,}180 \text{ in.}^4$$

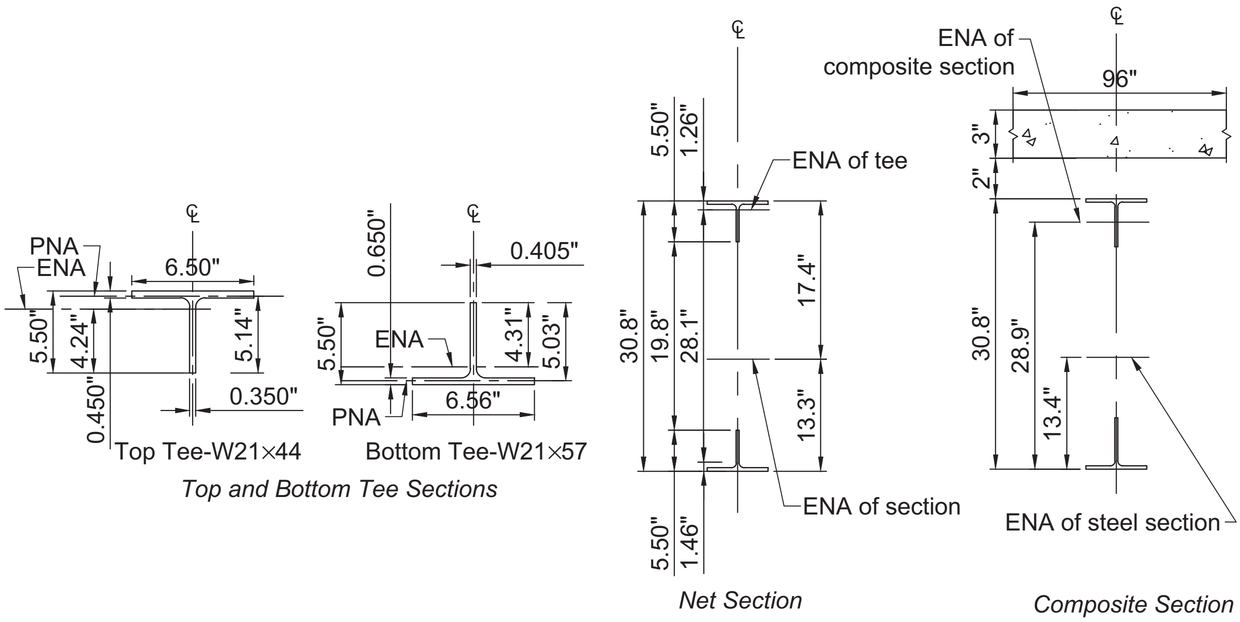


Fig. 4-6. Tee, net and composite sections for castellated beam for Example 4.3.

$$S_{x\text{-}net\text{-}top} = \frac{I_{x\text{-}net}}{\bar{y}_{ts}} = \frac{2{,}180 \text{ in.}^4}{17.4 \text{ in.}} = 125 \text{ in.}^3 \quad (4\text{-}29)$$

$$S_{x\text{-}net\text{-}bot} = \frac{I_{x\text{-}net}}{\bar{y}_{bs}} = \frac{2{,}180 \text{ in.}^4}{13.4 \text{ in.}} = 163 \text{ in.}^3 \quad (4\text{-}30)$$

Composite section properties in accordance with The Structural Engineer's Handbook *(Gaylord and Gaylord, 1992)*

$$n = \frac{E_s}{E_c} = \frac{29{,}000{,}000 \text{ psi}}{33(145 \text{ pcf})^{1.5}\sqrt{3{,}000 \text{ psi}}} = 9.19 \quad (4\text{-}31)$$

$$b_{effec} = \min\{Span/4,\ Spacing\} = \min\left\{\frac{50 \text{ ft}}{4}, \frac{8 \text{ ft} + 8 \text{ ft}}{2}\right\}(12 \text{ in./ft}) = 96.0 \text{ in.} \quad (3\text{-}4)$$

$$A_c = b_{effec}t_c = (96.0 \text{ in.})(3.00 \text{ in.}) = 288 \text{ in.}^2 \quad (4\text{-}32)$$

$$A_{ctr} = \frac{A_c}{n} = \frac{288 \text{ in.}^2}{9.19} = 31.3 \text{ in.}^2 \quad (4\text{-}33)$$

$$K_c = \frac{A_{ctr}}{A_{ctr} + A_{net}} = \frac{31.3 \text{ in.}^2}{31.3 \text{ in.}^2 + 10.9 \text{ in.}^2} = 0.741 \quad (4\text{-}34)$$

$$e_c = h_r + \frac{t_c}{2} = 2.00 \text{ in.} + \frac{3.00 \text{ in.}}{2} = 3.50 \text{ in.} \quad (4\text{-}35)$$

Assuming that the neutral axis is in the concrete.

$$y_{cc} = \left(\frac{A_{net}t_c}{A_{ctr}}\right)\left[\sqrt{1+\frac{2A_{ctr}}{A_{net}\,t_c}\left(\bar{y}_{ts}+e_c+\frac{t_c}{2}\right)}-1\right] \tag{4-36}$$

$$= \left[\frac{\left(10.9\text{ in.}^2\right)\left(3.00\text{ in.}\right)}{31.3\text{ in.}^2}\right]\left[\sqrt{1+\frac{2\left(31.3\text{ in.}^2\right)}{\left(10.9\text{ in.}^2\right)\left(3.00\text{ in.}\right)}\left(17.4\text{ in.}+3.50\text{ in.}+\frac{3.00\text{ in.}}{2}\right)}-1\right]$$

$$= 5.87\text{ in.}$$

$$t_c + h_r = 3.00\text{ in.} + 2.00\text{ in.}$$

$$= 5.00\text{ in.} < 5.87\text{ in.}$$

Because $t_c + h_r < y_{cc}$, the neutral axis is in the steel.

$$\bar{y}_c = \left(\bar{y}_{ts}+e_c\right)K_c \tag{4-37}$$

$$= \left(17.4\text{ in.}+3.50\text{ in.}\right)0.741$$

$$= 15.5\text{ in.}$$

$$I_{x\text{-}comp} = \left(\bar{y}_{ts}+e_c\right)\bar{y}_c A_{net} + I_{x\text{-}net} + \frac{A_{ctr}t_c^2}{12} \tag{4-38}$$

$$= \left(17.4\text{ in.}+3.50\text{ in.}\right)\left(15.5\text{ in.}\right)\left(10.9\text{ in.}^2\right) + 2{,}180\text{ in.}^4 + \frac{\left(31.3\text{ in.}^2\right)\left(3.50\text{ in.}\right)^2}{12}$$

$$= 5{,}740\text{ in.}^4$$

$$S_{x\text{-}comp\text{-}conc} = \frac{I_{x\text{-}comp}}{\bar{y}_{ts}-\bar{y}_c+e_c+0.5t_c} \tag{4-39}$$

$$= \frac{5{,}740\text{ in.}^4}{17.4\text{ in.}-15.5\text{ in.}+3.50\text{ in.}+0.5\left(3.00\text{ in.}\right)}$$

$$= 832\text{ in.}^3$$

$$S_{x\text{-}comp\text{-}steel} = \frac{I_{x\text{-}comp}}{\bar{y}_{bs}+\bar{y}_c} \tag{4-40}$$

$$= \frac{5{,}740\text{ in.}^4}{13.4\text{ in.}+15.5\text{ in.}}$$

$$= 199\text{ in.}^3$$

For the first iteration,

$$d_{effec\text{-}comp} = d_g - \bar{y}_{tee\text{-}bot} + h_r + 0.5t_c \tag{3-8}$$

$$= 30.8\text{ in.} - 1.19\text{ in.} + 2.00\text{ in.} + 0.5\left(3.00\text{ in.}\right)$$

$$= 33.1\text{ in.}$$

Check Vierendeel bending

The governing load cases are:

Table 4-16. Global Shear and Moment at Each Opening									
		Global Net Shear				Global Moment			
Opening No.	X_i, ft	*D*, kips	*L*, kips	$V_{r\text{-}net}$, kips		*D*, kips	*L*, kips	M_r, kip-ft	
				ASD	LRFD			ASD	LRFD
End	0.000	16.3	20.0	31.3	44.1	0.000	0.000	0.000	0.000
1	1.46	15.3	18.8	29.2	41.1	23.0	28.3	51.3	72.9
2	3.71	13.9	17.0	26.0	36.5	55.8	68.7	125	177
3	5.96	12.4	15.2	22.7	31.8	85.4	105	190	270
4	8.21	10.9	13.4	19.4	27.2	112	137	249	353
5	10.5	9.46	11.6	16.2	22.6	135	165	300	426
6	12.7	8.00	9.83	12.9	17.9	154	190	344	488
7	15.0	6.53	8.03	9.64	13.3	170	210	380	540
8	17.2	5.07	6.23	6.38	8.66	184	226	409	581
9	19.5	3.61	4.43	3.11	4.03	193	238	431	612
10	21.7	2.14	2.63	0.000	0.000	200	246	445	633
11	24.0	0.678	0.833	0.000	0.000	203	250	452	643
Bm. CL	25.0	0.000	0.000	0.000	0.000	203	250	453	644

Note: The shear force shown is the net shear force; i.e., the shear strength of the concrete has been subtracted from the global shear force on the beam.

LRFD	ASD
Load case 1: $w = 1.4D$ $= 1.4(651 \text{ lb/ft})$ $= 911 \text{ lb/ft}$ Load case 2: $w = 1.2D + 1.6L$ $= 1.2(651 \text{ lb/ft}) + 1.6(800 \text{ lb/ft})$ $= 2{,}060 \text{ lb/ft}$ **governs**	$w = D + L$ $= 651 \text{ lb/ft} + 800 \text{ lb/ft}$ $= 1{,}450 \text{ lb/ft}$

Calculate the available shear strength of the concrete deck:

LRFD	ASD
$V_c = \phi_{cv} V_{nc}$ (3-15a) $V_{nc} = 4\sqrt{f_c'}(3)(h_r + t_c)t_c$ (3-14) $= \dfrac{4\sqrt{3{,}000 \text{ psi}}(3)(2.00 \text{ in.} + 3.00 \text{ in.})(3.00 \text{ in.})}{1{,}000 \text{ lb/kip}}$ $= 9.85 \text{ kips}$ $V_c = 0.75(9.85 \text{ kips})$ $= 7.39 \text{ kips}$	$V_c = \dfrac{V_{nc}}{\Omega_{cv}}$ (3-15b) (3-14) $V_{nc} = 4\sqrt{f_c'}(3)(h_r + t_c)t_c$ $= \dfrac{4\sqrt{3{,}000 \text{ psi}}(3)(2.00 \text{ in.} + 3.00 \text{ in.})(3.00 \text{ in.})}{1{,}000 \text{ lb/kip}}$ $= 9.85 \text{ kips}$ $V_c = \dfrac{9.85 \text{ kips}}{2.00}$ $= 4.93 \text{ kips}$

Calculate the global shear and moment at each opening to be used to calculate local internal forces (axial and flexural) at each opening. These values are presented in Table 4-16.

Table 4-17. Local Axial Force at Each Opening

ASD									
Opening Number	X_i, ft	M_r, kip-ft	$T_{1(i)}$, kips	$X_{c(i+1)}$, in.	$T_{1(i+1)}$, kips	$T_{1(i)}/T_{1(i+1)}$	$X_{c(i+2)}$, in.	$T_{1(i+2)}$, kips	$T_{1(i+1)}/T_{1(i+2)}$
End	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	1.46	51.3	18.6	0.076	17.8	1.05	0.073	17.8	1.00
2	3.71	125	45.2	0.184	43.2	1.04	0.177	43.2	1.00
3	5.96	190	69.0	0.282	66.2	1.04	0.270	66.2	1.00
4	8.21	249	88.2	0.369	86.6	1.04	0.354	86.6	1.00
5	10.5	300	109	0.444	105	1.04	0.427	105	1.00
6	12.7	344	125	0.509	120	1.04	0.490	120	1.00
7	15.0	380	138	0.563	133	1.04	0.543	133	1.00
8	17.2	409	148	0.606	143	1.04	0.585	143	1.00
9	19.5	431	156	0.639	151	1.04	0.616	151	1.00
10	21.7	445	162	0.660	156	1.04	0.637	156	1.00
12	24.0	452	164	0.670	158	1.04	0.647	158	1.00
Bm. CL	25.0	453	164	0.672	159	1.04	0.649	159	1.00
LRFD									
Opening Number	X_i, ft	M_r, kip-ft	$T_{1(i)}$, kips	$X_{c(i+1)}$, in.	$T_{1(i+1)}$, kips	$T_{1(i)}/T_{1(i+1)}$	$X_{c(i+2)}$, in.	$T_{1(i+2)}$, kips	$T_{1(i+1)}/T_{1(i+2)}$
End	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	1.46	72.9	26.5	0.108	25.3	1.05	0.103	25.3	1.00
2	3.71	177	64.1	0.262	61.4	1.04	0.251	61.4	1.00
3	5.96	270	98.0	0.401	94.1	1.04	0.384	94.1	1.00
4	8.21	353	128	0.524	123	1.04	0.504	123	1.00
5	10.5	426	155	0.631	149	1.04	0.608	149	1.00
6	12.7	488	177	0.723	171	1.04	0.698	171	1.00
7	15.0	540	196	0.800	189	1.04	0.773	189	1.00
8	17.2	581	211	0.861	204	1.03	0.833	204	1.00
9	19.5	612	222	0.907	215	1.03	0.878	215	1.00
10	21.7	633	230	0.937	222	1.03	0.908	222	1.00
12	24.0	643	233	0.952	226	1.03	0.923	226	1.00
Bm. CL	25.0	644	234	0.954	226	1.03	0.925	226	1.00

Calculate the local axial force in the top and bottom tees resulting from the global moment. These values are shown in Table 4-17.

In the calculations for composite beams, assume that the concrete flange takes all the compression and that the bottom tee takes all the tension force. This is a valid assumption, assuming that sufficient studs exist at a given opening to have developed the concrete flange. This assumption must be checked as part of the design.

Local axial force:

For the first iteration, recalculate $d_{effec\text{-}comp}$ each time,

$$T_{1(i)} = \frac{M_{r(i)}}{d_{effec\text{-}comp}} \tag{3-9}$$

Recalculate effective concrete depth,

$$X_c = \frac{T_{1(i)}}{0.85 f_c' b_{effec}} \tag{3-10}$$

Recalculate $d_{effec\text{-}comp}$

$$d_{effec\text{-}comp} = d_g - \bar{y}_{tee\text{-}bot} + t_c + h_r - \frac{X_c}{2} \quad \text{(from Eq. 3-8)}$$

Recalculate until the difference ≤ 1%

$$T_{1(i+1)} = \frac{M_{r(i+1)}}{d_{effec\text{-}comp}} \tag{3-9}$$

The next step is to calculate the number of studs for full composite action and shear stud density along the length of the beam. This will be used to calculate composite percentage at each web opening. If sufficient studs are not present to resist the compression force in the concrete ($T_{1(i+2)}$ in Table 4-17), an additional force, T_o, will be resisted by the top tee section.

From AISC *Specification* Section I.2d.1, consider the limit states of concrete crushing and tensile yielding of the steel section to determine the number of studs for full composite action.

Concrete crushing:

$$\begin{aligned} V' &= 0.85 f_c' A_c \quad \text{(}\textit{Spec}\text{. Eq. I3-1a)} \\ &= 0.85(3\text{ ksi})\left(288\text{ in.}^2\right) \\ &= 734\text{ kips} \end{aligned}$$

Tensile yielding of the steel section:

$$\begin{aligned} V' &= F_y A_{net} \quad \text{(from }\textit{Spec}\text{. Eq. I3-1b)} \\ &= (50\text{ ksi})\left(10.9\text{ in.}^2\right) \\ &= 545\text{ kips} \quad \textbf{controls} \end{aligned}$$

From AISC *Manual* Table 3-21 for a ¾-in.-diameter shear stud,

$$\begin{aligned} Q_n &= 21.0\text{ kips/stud} \\ N &= \frac{V'}{Q} \\ &= \frac{545\text{ kips}}{21.0\text{ kips/stud}} \\ &= 26\text{ studs} \end{aligned}$$

Between the point of maximum moment and the end of the beam, 26 studs need to be provided. Because the point of maximum moment is at the center of the beam, 52 studs are required over the length of the beam. The flutes of the deck are 12 in. on center; therefore, one additional stud will be placed in the first flute of the deck at each end. When two studs per rib are present, Q_n = 18.3 kips. Investigate if providing an additional stud in the first flute provides adequate shear resistance.

$$\begin{aligned} V_{provided} &= (1\text{ stud})(2\text{ studs/rib})Q_n + (24\text{ studs})(1\text{ stud/rib})Q_n \\ &= (1)(2)(18.3\text{ kips}) + (24)(1)(21.0\text{ kips}) \\ &= 541\text{ kips} < 545\text{ kips} \quad \textbf{n.g.} \end{aligned}$$

Double the number of studs in the second flute from each end.

$$\begin{aligned} V_{provided} &= (2\text{ studs})(2\text{ studs/rib})Q_n + (23\text{ studs})(1\text{ stud/rib})Q_n \\ &= (2)(2)(18.3\text{ kips}) + (23)(1)(21.0\text{ kips}) \\ &= 556\text{ kips} > 545\text{ kips} \quad \textbf{o.k.} \end{aligned}$$

Therefore, 54 studs are required over the entire length of the beam.

Next, calculate an average stud density over the length of the beam, q , and use this value to determine the amount of concrete that has been developed at the web opening that is being examined for Vierendeel bending.

$$q = \frac{(2)V_{provided}}{Beam\ span}$$
$$= \frac{(2)(556\text{ kips})}{50\text{ ft}}$$
$$= 22.2\text{ kips/ft}$$

The next step is to calculate the amount of concrete that has been developed by the studs between the end of the beam and the opening under consideration and to then determine if the stud strength is greater than the force $T_{1(i+2)}$ noted in Table 4-17. If the force $T_{1(i+2)}$ is less than the amount of concrete developed, consider the beam fully composite at that opening—i.e., the concrete has the strength to resist the chord force $T_{1(i+2)}$ and the previous assumption is valid. If this is not the case, take the difference between T_{i+2} and NQ_n as a force, T_o, in the top tee of the castellated section and recalculate the force on the bottom tee as $T_{1\text{-}new}$ to account for the fact that the section is not acting fully composite.

The compression force to be resisted by the top tee at its centroid is then

$$T_o = M_r \left[\frac{1 - \dfrac{q(X_i)}{T_{1(i+2)}}}{d_{effec}} \right] \tag{3-12}$$

The revised tension force to be resisted by bottom tee at its centroid is then

$$T_{1\text{-}new} = qX_i + T_o \tag{3-13}$$

The revised local axial forces at each opening are reported in Table 4-18. In this case, all the web openings were fully composite because enough concrete was developed at each opening to fully resist the global moment. The assumption that the concrete takes all the compression and the bottom tee resists all the tension is valid; therefore, use the forces $T_{1(i)}$ from Table 4-17.

If fewer than 54 studs had been used, the results would have been different. In the case of 30 studs, the shear stud density, q, is 12.6 kip/ft. The results would require that the first seven holes be considered as partially composite and the revised top and bottom tee forces be accounted for. These results are shown in Table 4-19 but will not be used in the rest of the example.

Calculate the local moment on the top and bottom tees resulting from the net shear force passing through the web opening. The local moments at each opening are presented in Table 4-20.

Top tee local Vierendeel moment:

$$M_{vr\text{-}top} = V_{net} \frac{A_{tee\text{-}top}}{A_{net}} \left(\frac{e}{2} \right) \qquad \text{(from Eq. 3-2)}$$

Bottom tee local Vierendeel moment:

$$M_{vr\text{-}bot} = V_{net} \frac{A_{tee\text{-}bot}}{A_{net}} \left(\frac{e}{2} \right) \qquad \text{(from Eq. 3-2)}$$

Calculate the available shear and flexural strength of top and bottom tees

Determine the limiting flange width-to-thickness ratio from AISC *Specification* Table B4.1b, Case 10:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$
$$= 0.38\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 9.15$$

Table 4-18. Revised Local Axial Force at Each Opening (LRFD)						
Opening Number	**X_i, ft**	**$T_1 = T_{1(i+2)}$, kips**	**qX_i, kips**	**Composite Status**	**T_o, kips**	**$T_{1\text{-}new}$, kips**
End	0.000	0.000	0.000	N/A	N/A	N/A
1	1.46	25.3	32.4	Full	0.000	25.3
2	3.71	61.4	82.4	Full	0.000	61.4
3	5.96	94.1	132	Full	0.000	94.1
4	8.21	123	182	Full	0.000	123
5	10.5	149	233	Full	0.000	149
6	12.7	171	282	Full	0.000	171
7	15.0	189	333	Full	0.000	189
8	17.2	204	382	Full	0.000	204
9	19.5	215	433	Full	0.000	215
10	21.7	222	482	Full	0.000	222
11	24.0	226	533	Full	0.000	226
Bm. CL	25.0	226	555	Full	0.000	226

Table 4-19. Local Axial Force at Each Opening for 30 Studs (LRFD)						
Opening Number	**X_i, ft**	**T_1, kips**	**qX_i, kips**	**Composite Status**	**T_o, kips**	**$T_{1\text{-}new}$, kips**
End	0.000	0.000	0.000	N/A	0.000	0.000
1	1.46	25.3	18.4	Partial	8.43	26.8
2	3.71	61.4	46.7	Partial	17.9	64.6
3	5.96	94.1	75.1	Partial	23.1	98.2
4	8.21	123	103	Partial	24.0	127
5	10.5	149	132	Partial	20.7	152
6	12.7	171	160	Partial	13.0	173
7	15.0	189	189	Partial	0.885	189
8	17.2	204	217	Full	0.000	204
9	19.5	215	246	Full	0.000	215
10	21.7	222	273	Full	0.000	222
11	24.0	226	302	Full	0.000	226
Bm. CL	25.0	226	315	Full	0.000	226

The width-to-thickness ratio for the top flange is:

$$\lambda = \frac{b}{t}$$

$$= \frac{b_f}{2t_f}$$

$$= \frac{6.50 \text{ in.}}{2(0.450 \text{ in.})}$$

$$= 7.22 < 9.15$$

Table 4-20. Local Vierendeel Moment at Each Opening							
		ASD			LRFD		
Opening Number	X_i, ft	V_a, kips	$M_{va\text{-}top}$, kip-in.	$M_{va\text{-}bot}$, kip-in.	V_u, kips	$M_{vu\text{-}top}$, kip-in.	$M_{vu\text{-}bot}$, kip-in.
End	0.000	31.3	54.0	71.4	44.1	76.1	101
1	1.46	29.2	50.3	66.6	41.1	70.7	93.7
2	3.71	26.0	44.6	59.2	36.5	62.7	83.2
3	5.96	22.7	39.0	51.7	31.8	54.8	72.6
4	8.21	19.4	33.4	44.3	27.2	46.8	62.0
5	10.5	16.2	27.8	36.9	22.6	38.8	51.5
6	12.7	12.9	22.2	29.4	17.9	30.8	40.9
7	15.0	9.64	16.6	22.0	13.3	22.9	30.3
8	17.2	6.38	11.0	14.5	8.66	14.9	19.7
9	19.5	3.11	5.35	7.10	4.03	6.92	9.18
10	21.7	0.000	0.000	0.000	0.000	0.000	0.000
11	24.0	0.000	0.000	0.000	0.000	0.000	0.000
Bm. CL	25.0	0.000	0.000	0.000	0.000	0.000	0.000

The width-to-thickness ratio for the bottom flange is:

$$\begin{aligned}\lambda &= \frac{b}{t} \\ &= \frac{b_f}{2t_f} \\ &= \frac{6.56 \text{ in.}}{2(0.650 \text{ in.})} \\ &= 5.05 < 9.15\end{aligned}$$

Because $\lambda < \lambda_p$, the flanges of both the top and bottom tees are compact; therefore, it is not necessary to check flange local buckling when calculating the available flexural strength.

Determine the limiting stem width-to-thickness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\begin{aligned}\lambda_r &= 0.75\sqrt{\frac{E}{F_y}} \\ &= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 18.1\end{aligned}$$

The width-to-thickness ratio for the top stem is:

$$\begin{aligned}\lambda &= \frac{d_t}{t_w} \\ &= \frac{5.50 \text{ in.}}{0.350 \text{ in.}} \\ &= 15.7 < 18.1\end{aligned}$$

The width-to-thickness ratio for the bottom stem is:

$$\lambda = \frac{d_t}{t_w}$$
$$= \frac{5.50 \text{ in.}}{0.405 \text{ in.}}$$
$$= 13.6 < 18.1$$

Because $\lambda < \lambda_r$, both top and bottom tee stems are nonslender, therefore, it is not necessary to consider AISC *Specification* Section E7 when calculating the available compressive strength.

It is not necessary to calculate the available compressive strength of the top or bottom tee in this example because all openings are fully composite, and therefore, all compression is taken by the concrete flange. If compression did exist in the top or bottom tee, the available compressive strength would be calculated as shown in Example 4.1.

Calculate available tensile strength of bottom tee

$$P_n = F_y A_{tee\text{-}bot} \qquad \text{(from } \textit{Spec.} \text{ Eq. D2-1)}$$
$$= (50 \text{ ksi})\left(6.22 \text{ in.}^2\right)$$
$$= 311 \text{ kips}$$

Calculate available flexural strength of tee

Yielding

For tee stems in compression:

$$M_{p\text{-}top} = M_y \qquad \text{(from } \textit{Spec.} \text{ Eq. F9-4)}$$
$$M_y = F_y S_{x\text{-}bot} \qquad \text{(from } \textit{Spec.} \text{ Eq. F9-3)}$$
$$= (50 \text{ ksi})\left(2.86 \text{ in.}^3\right)$$
$$= 143 \text{ kip-in.}$$

$$M_{p\text{-}bot} = M_y \qquad \text{(from } \textit{Spec.} \text{ Eq. F9-4)}$$
$$M_y = F_y S_{x\text{-}top} \qquad \text{(from } \textit{Spec.} \text{ Eq. F9-3)}$$
$$= (50 \text{ ksi})\left(3.29 \text{ in.}^3\right)$$
$$= 165 \text{ kip-in.}$$

In both cases, the stem is assumed to be in compression; this will be conservative for the bottom tee. It is possible to take advantage of this to calculate a higher value for the available flexural strength of the bottom tee because the stem is in tension.

Lateral-torsional buckling

For lateral-torsional buckling of the top tee:

$$B_{top} = -2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \qquad (\textit{Spec.} \text{ Eq. F9-12})$$
$$= -2.3\left(\frac{5.50 \text{ in.}}{8.00 \text{ in.}}\right)\sqrt{\frac{10.3 \text{ in.}^4}{0.266 \text{ in.}^4}}$$
$$= -9.84$$

$$M_{cr\text{-}top} = \frac{1.95E}{L_b}\sqrt{I_y J}\left[B + \sqrt{1+B^2}\right] \quad (Spec. \text{ Eq. F9-10})$$

$$= \frac{1.95(29{,}000 \text{ ksi})}{8.00 \text{ in.}}\sqrt{(10.3 \text{ in.}^4)(0.266 \text{ in.}^4)}\left[-9.84 + \sqrt{1+(-9.84)^2}\right]$$

$$= 596 \text{ kip-in.}$$

For lateral-torsional buckling of the bottom tee:

$$B_{bot} = -2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \quad (Spec. \text{ Eq. F9-12})$$

$$= -2.3\left(\frac{5.50 \text{ in.}}{8.00 \text{ in.}}\right)\sqrt{\frac{15.3 \text{ in.}^4}{0.685 \text{ in.}^4}}$$

$$= -7.47$$

$$M_{cr\text{-}bot} = \frac{1.95E}{L_b}\sqrt{I_y J}\left[B + \sqrt{1+B^2}\right] \quad (Spec. \text{ Eq. F9-10})$$

$$= \frac{1.95(29{,}000 \text{ ksi})}{8.00 \text{ in.}}\sqrt{(15.3 \text{ in.}^4)(0.685 \text{ in.}^4)}\left[-7.47 + \sqrt{1+(-7.47)^2}\right]$$

$$= 1{,}550 \text{ kip-in.}$$

Flange local buckling

Per AISC *Specification* Section F9.3(a), the limit state of flange local buckling does not apply because the flanges are compact.

Local buckling of tee stems

The nominal flexural strength for local buckling of the tee stem in flexural compression, M_n, is determined using AISC *Specification* Section F9.4:

$$M_n = F_{cr}S_x \quad (Spec. \text{ Eq. F9-16})$$

Because $d/t_w < 0.84\sqrt{\frac{E}{F_y}}$, the critical stress, F_{cr}, is determined using AISC *Specification* Equation F9-17:

$$F_{cr} = F_y \quad (Spec. \text{ Eq. F9-17})$$

And thus,

For the top tee:

$$M_{n\text{-}top} = F_y S_{x\text{-}bot}$$

$$= (50 \text{ ksi})(2.86 \text{ in.}^3)$$

$$= 143 \text{ kip-in.}$$

For the bottom tee:

$$M_{n\text{-}bot} = F_y S_{x\text{-}top}$$

$$= (50 \text{ ksi})(3.29 \text{ in.}^3)$$

$$= 165 \text{ kip-in.}$$

The available tensile and flexural strengths of the tee are:

LRFD	ASD
Available tensile strength—bottom tee	*Available tensile strength—bottom tee*
$P_u = \phi_c P_n$ $= 0.90(311 \text{ kips})$ $= 280 \text{ kips}$	$P_a = \dfrac{P_n}{\Omega_c}$ $= \dfrac{311 \text{ kips}}{1.67}$ $= 186 \text{ kips}$
Available flexural strength—top tee	*Available flexural strength—top tee*
$M_u = \phi_b M_{p\text{-}top}$ $= 0.90(143 \text{ kip-in.})$ $= 129 \text{ kip-in.}$	$M_a = \dfrac{M_{p\text{-}top}}{\Omega_b}$ $= \dfrac{143 \text{ kip-in.}}{1.67}$ $= 85.6 \text{ kip-in.}$
Available flexural strength—bottom tee	*Available flexural strength—bottom tee*
$M_u = \phi_b M_{p\text{-}bot}$ $= 0.90(165 \text{ kip-in.})$ $= 149 \text{ kip-in.}$	$M_a = \dfrac{M_{p\text{-}bot}}{\Omega_b}$ $= \dfrac{165 \text{ kip-in.}}{1.67}$ $= 98.8 \text{ kip-in.}$

Check tees for combined axial and flexural loads

The interaction values for each opening are presented in Table 4-21.

From Table 4-21, the composite Vierendeel bending is summarized as follows:

LRFD	ASD
Top tee	*Top tee*
$I_{max} = 0.549 < 1.0$ **o.k.**	$I_{max} = 0.586 < 1.0$ **o.k.**
Bottom tee	*Bottom tee*
$I_{max} = 0.858 < 1.0$ **o.k.**	$I_{max} = 0.911 < 1.0$ **o.k.**.

Check web post buckling

It is necessary to check both the top and bottom web posts for buckling in this case. Although the top web post is thinner and is therefore more likely to buckle first, the value of $2h/e$ is different for the top and bottom web posts, and it is therefore necessary to investigate both web posts.

Calculate vertical and horizontal shear and resultant moment at each gross section for web post buckling check

Table 4-22 presents the vertical shear force at each opening, and Table 4-23 presents the horizontal shear force at each web post.

From Section 3.4.1a, calculate the horizontal shear, V_{rh}, using Equation 3-19:

$$V_{rh} = \left|T_{r(i)} - T_{r(i+1)}\right| \tag{3-19}$$

Table 4-21. Interaction Values at Each Opening for LRFD and ASD

LRFD

Opening Number	X_i, ft	Top Tee P_r, kips	Top Tee $M_{vr\text{-}top}$, kip-in.	Top Tee $\frac{M_{vr}}{M_c}$	Bottom Tee P_r, kips	Bottom Tee $M_{vr\text{-}bot}$, kip-in.	Bottom Tee $\frac{P_r}{P_c}$	Spec. Eq. H1-1a	Spec. Eq. H1-1b	Interaction*
End	0.000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
1	1.46	0.000	70.7	0.549	25.3	93.7	0.09	0.653	**0.678**	0.678
2	3.71	0.000	62.7	0.487	61.6	83.2	0.220	**0.719**	0.672	0.719
3	5.96	0.000	54.8	0.425	94.3	72.6	0.336	**0.772**	0.659	0.772
4	8.21	0.000	46.8	0.363	123	62.0	0.440	**0.813**	0.639	0.813
5	10.5	0.000	38.8	0.301	149	51.5	0.532	**0.841**	0.613	0.841
6	12.7	0.000	30.8	0.239	171	40.9	0.610	**0.856**	0.581	0.856
7	15.0	0.000	22.9	0.178	189	30.3	0.676	**0.858**	0.543	0.858
8	17.2	0.000	14.9	0.116	204	19.7	0.728	**0.847**	0.497	0.847
9	19.5	0.000	6.92	0.054	215	9.18	0.767	**0.822**	0.446	0.822
10	21.7	0.000	0.000	0.000	222	0.000	0.793	**0.793**	0.397	0.793
11	24.0	0.000	0.000	0.000	226	0.000	0.806	**0.806**	0.403	0.806
Bm. CL	25.0	0.000	0.000	0.000	226	0.000	0.807	**0.807**	0.404	0.807
			I_{max}:	**0.549**					I_{max}:	**0.858**

ASD

Opening Number	X_i, ft	Top Tee P_r, kips	Top Tee $M_{vr\text{-}top}$, kip-in.	Top Tee $\frac{M_{vr}}{M_c}$	Bottom Tee P_r, kips	Bottom Tee $M_{vr\text{-}bot}$, kip-in.	Bottom Tee $\frac{P_r}{P_c}$	Spec. Eq. H1-1a	Spec. Eq. H1-1b	Interaction*
End	0.000	N/A	N/A	NA	N/A	N/A	N/A	N/A	N/A	N/A
1	1.46	0.000	50.3	0.586	17.8	66.6	0.096	0.697	**0.724**	0.724
2	3.71	0.000	44.6	0.521	43.3	59.2	0.232	**0.766**	0.717	0.766
3	5.96	0.000	39.0	0.455	66.2	51.7	0.355	**0.822**	0.703	0.822
4	8.21	0.000	33.4	0.390	86.7	44.3	0.465	**0.865**	0.682	0.865
5	10.5	0.000	27.8	0.324	105	36.9	0.561	**0.894**	0.655	0.894
6	12.7	0.000	22.2	0.259	120	29.4	0.644	**0.909**	0.621	0.909
7	15.0	0.000	16.6	0.193	133	22.0	0.712	**0.911**	0.579	0.911
8	17.2	0.000	11.0	0.128	143	14.5	0.767	**0.899**	0.531	0.899
9	19.5	0.000	5.35	0.062	151	7.10	0.809	**0.873**	0.476	0.873
10	21.7	0.000	0.000	0.000	156	0.000	0.836	**0.836**	0.418	0.836
11	24.0	0.000	0.000	0.000	158	0.000	0.849	**0.849**	0.425	0.849
Bm. CL	25.0	0.000	0.000	0.000	159	0.000	0.851	**0.851**	0.425	0.851
			I_{max}:	**0.586**					I_{max}:	**0.911**

* Reflects bold face value of controlling interaction equation.

Table 4-22. Vertical Shear Force at Each Opening							
		ASD			LRFD		
Post Number	X_i, ft	$V_{r(i-1)}$, kips	$V_{r(i+1)}$, kips	$V_{r(i)}$, kips	$V_{r(i-1)}$, kips	$V_{r(i+1)}$, kips	$V_{r(i)}$, kips
1	2.58	29.2	26.0	21.5	41.1	36.5	38.8
2	4.83	26.0	22.7	19.2	36.5	31.8	34.2
3	7.08	22.7	19.4	16.8	31.8	27.2	29.5
4	9.33	19.4	16.2	14.4	27.2	22.6	24.9
5	11.6	16.2	12.9	12.1	22.6	17.9	20.3
6	13.8	12.9	9.64	9.72	17.9	13.3	15.6
7	16.1	9.64	6.38	7.35	13.3	8.66	11.0
8	18.3	6.38	3.11	4.99	8.66	4.03	6.34
9	20.6	3.11	0.000	2.63	4.03	0.000	2.01
10	22.8	0.000	0.000	0.339	0.000	0.000	0.000
11	24.5	0.000	0.000	0.000	0.000	0.000	0.000
	Maximum:	29.2		Maximum:	41.1	Maximum:	38.8

Table 4-23. Horizontal Shear Force at Each Web Post							
		ASD			LRFD		
Opening Number	X_i, ft	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	$V_{ah} = \Delta T_r$, kips	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	$V_{uh} = \Delta T_r$, kips
End	0.000	0.000					
1	1.46	17.8	43.3	25.5	25.3	61.6	36.3
2	3.71	43.3	66.2	22.9	61.6	94.3	32.7
3	5.96	66.2	86.7	20.5	94.3	123	28.7
4	8.21	86.7	105	18.3	123	149	26.0
5	10.5	105	120	15.0	149	171	22.0
6	12.7	120	133	13.0	171	189	18.0
7	15.0	133	143	10.0	189	204	15.0
8	17.2	143	151	8.00	204	215	11.0
9	19.5	151	156	5.00	215	222	7.00
10	21.7	156	158	2.00	222	226	4.00
11	24.0	158.4	158.7	0.300	225.9	226.3	0.400
Bm. CL	25.0	159			226		

Calculate web post buckling flexural strength

LRFD	ASD
From Table 4-23, $V_{uh} = 36.3$ kips $M_{u\text{-}top} = V_{uh} h_{top}$ (from Eq. 3-20) $= (36.3 \text{ kips})(9.70 \text{ in.})$ $= 352$ kip-in. $M_{u\text{-}bot} = V_{uh} h_{bot}$ (from Eq. 3-21) $= (36.3 \text{ kips})(10.1 \text{ in.})$ $= 367$ kip-in.	From Table 4-23, $V_{ah} = 25.5$ kips $M_{a\text{-}top} = V_{ah} h_{top}$ (from Eq. 3-20) $= (25.5 \text{ kips})(9.70 \text{ in.})$ $= 247$ kip-in. $M_{a\text{-}bot} = V_{ah} h_{bot}$ (from Eq. 3-21) $= (25.5 \text{ kips})(10.1 \text{ in.})$ $= 258$ kip-in.

Calculate available flexural strength of web post

Top web post

$$M_p = 0.25 t_w (e + 2b)^2 F_y \tag{3-22}$$

$$= 0.25(0.350 \text{ in.})\left[8.00 \text{ in.} + 2(5.50 \text{ in.})\right]^2 (50 \text{ ksi})$$

$$= 1{,}580 \text{ kip-in.}$$

$$\frac{2h_{top}}{e} = \frac{2(9.70 \text{ in.})}{8.00 \text{ in.}}$$

$$= 2.43$$

$$\frac{e}{t_w} = \frac{8.00 \text{ in.}}{0.350 \text{ in.}}$$

$$= 22.9$$

For $\theta = 60^\circ$

For $e/t_w = 10$

$$\frac{M_{ocr}}{M_p} = 0.587(0.917)^{\frac{2h_{top}}{e}} \tag{3-26}$$

$$= 0.587(0.917)^{2.43}$$

$$= 0.476 < 0.493$$

For $e/t_w = 20$

$$\frac{M_{ocr}}{M_p} = 1.96(0.699)^{\frac{2h_{top}}{e}} \tag{3-27}$$

$$= 1.96(0.699)^{2.43}$$

$$= 0.821 > 0.493$$

For $e/t_w = 30$

$$\frac{M_{ocr}}{M_p} = 2.55(0.574)^{\frac{2h_{top}}{e}} \tag{3-28}$$

$$= 2.55(0.574)^{2.43}$$

$$= 0.662 > 0.493$$

For $e/t_w = 22.9$, use

$$\frac{M_{ocr}}{M_p} = 0.476$$

Bottom web post

$$M_p = 0.25 t_w (e + 2b)^2 F_y \tag{3-22}$$

$$= 0.25(0.405 \text{ in.})\left[8.00 \text{ in.} + 2(5.50 \text{ in.})\right]^2 (50 \text{ ksi})$$

$$= 1{,}830 \text{ kip-in.}$$

$$\frac{2h_{bot}}{e} = \frac{2(10.1 \text{ in.})}{8.00 \text{ in.}}$$

$$= 2.53$$

$$\frac{e}{t_w} = \frac{8.00 \text{ in.}}{0.405 \text{ in.}}$$

$$= 19.8$$

For $e/t_w = 10$

$$\frac{M_{ocr}}{M_p} = 0.587(0.917)^{\frac{2h_{top}}{e}} \tag{3-26}$$

$$= 0.587(0.917)^{2.53}$$

$$= 0.471 < 0.493$$

For $e/t_w = 20$

$$\frac{M_{ocr}}{M_p} = 1.96(0.699)^{\frac{2h_{top}}{e}} \tag{3-27}$$

$$= 1.96(0.699)^{2.53}$$

$$= 0.792 > 0.493$$

For $e/t_w = 30$

$$\frac{M_{ocr}}{M_p} = 2.55(0.574)^{\frac{2h_{top}}{e}} \tag{3-28}$$

$$= 2.55(0.574)^{2.53}$$

$$= 0.626 > 0.493$$

For $e/t_w = 19.8$, use

$$\frac{M_{ocr}}{M_p} = 0.471$$

From Equation 3-29a and 3-29b, the available flexural strength is

LRFD	ASD
Top web post	*Top web post*
$\phi_b\left(\frac{M_{ocr}}{M_p}\right)M_{p\text{-}top} = 0.90(0.476)(1{,}580 \text{ kip-in.})$	$\frac{1}{\Omega_b}\left(\frac{M_{ocr}}{M_p}\right)M_{p\text{-}top} = \frac{1}{1.67}(0.476)(1{,}580 \text{ kip-in.})$
$= 677 \text{ kip-in.}$	$= 450 \text{ kip-in.}$
$I_{max\text{-}top} = \frac{352 \text{ kip-in.}}{677 \text{ kip-in.}}$	$I_{max} = \frac{247 \text{ kip-in.}}{450 \text{ kip-in.}}$
$= 0.520 < 1.0$ **o.k.**	$= 0.549 < 1.0$ **o.k.**
Bottom web post	*Bottom web post*
$\phi_b\left(\frac{M_{ocr}}{M_p}\right)M_{p\text{-}bot} = 0.90(0.471)(1{,}830 \text{ kip-in.})$	$\frac{1}{\Omega_b}\left(\frac{M_{ocr}}{M_p}\right)M_{p\text{-}bot} = \frac{1}{1.67}(0.471)(1{,}830 \text{ kip-in.})$
$= 776 \text{ kip-in.}$	$= 516 \text{ kip-in.}$
$I_{max\text{-}bot} = \frac{366 \text{ kip-in.}}{776 \text{ kip-in.}}$	$I_{max} = \frac{257 \text{ kip-in.}}{516 \text{ kip-in.}}$
$= 0.472 < 1.0$ **o.k.**	$= 0.498 < 1.0$ **o.k.**

Check horizontal shear

The available horizontal shear strength is calculated using AISC *Specification* Section J4.2. By inspection, the top section will control because the web is thinner.

LRFD	ASD
From Table 4-23,	From Table 4-23,
$V_u = 36.3 \text{ kips}$	$V_a = 25.5 \text{ kips}$
From *Spec*. Eq. J4-3,	From *Spec*. Eq. J4-3,
$\phi_v V_{n\text{-}horiz} = \phi_v 0.6F_y(et_w)$	$\frac{V_{n\text{-}horiz}}{\Omega_v} = \frac{0.6F_y(et_w)}{\Omega_v}$
$= 1.00(0.6)(50 \text{ ksi})\left[(8.00 \text{ in.})(0.350 \text{ in.})\right]$	$= \frac{0.6(50 \text{ ksi})\left[(8.00 \text{ in.})(0.350 \text{ in.})\right]}{1.50}$
$= 84.0 \text{ kips} > 36.3 \text{ kips}$ **o.k.**	$= 56.0 \text{ kips} > 25.5 \text{ kips}$ **o.k.**

Check vertical shear

The concrete shear strength will be disregarded when checking vertical shear for the net and gross sections. The concrete shear strength will be added to the net shear force.

Check vertical shear at the beam net section:

LRFD	ASD
From Table 4-16, $V_{u\text{-}net} = 41.1 \text{ kips}$ $V_{u\text{-}global} = V_{u\text{-}net} + \textit{concrete shear strength}$ $= 41.1 \text{ kips} + 7.39 \text{ kips}$ $= 48.5 \text{ kips}$	From Table 4-16, $V_{a\text{-}net} = 29.2 \text{ kips}$ $V_{a\text{-}global} = V_{a\text{-}net} + \textit{concrete shear strength}$ $= 29.2 \text{ kips} + 4.93 \text{ kips}$ $= 34.1 \text{ kips}$

The shear force between the top and bottom tees will be divided based on their relative areas.

LRFD	ASD
$V_{u\text{-}top} = V_{u\text{-}global}\left(\frac{A_{tee\text{-}top}}{A_{net}}\right)$ $= (48.5 \text{ kips})\left(\frac{4.70 \text{ in.}^2}{10.9 \text{ in.}^2}\right)$ $= 20.9 \text{ kips}$ $V_{u\text{-}bot} = V_{u\text{-}global}\left(\frac{A_{tee\text{-}bot}}{A_{net}}\right)$ $= (48.5 \text{ kips})\left(\frac{6.22 \text{ in.}^2}{10.9 \text{ in.}^2}\right)$ $= 27.7 \text{ kips}$	$V_{a\text{-}top} = V_{a\text{-}global}\left(\frac{A_{tee\text{-}top}}{A_{net}}\right)$ $= (34.1 \text{ kips})\left(\frac{4.70 \text{ in.}^2}{10.9 \text{ in.}^2}\right)$ $= 14.7 \text{ kips}$ $V_{a\text{-}bot} = V_{a\text{-}global}\left(\frac{A_{tee\text{-}bot}}{A_{net}}\right)$ $= (34.1 \text{ kips})\left(\frac{6.22 \text{ in.}^2}{10.9 \text{ in.}^2}\right)$ $= 19.5 \text{ kips}$

Check vertical shear at top and bottom tees

From AISC *Specification* Section G3,

Top tee:

$$\frac{h}{t_w} = \frac{d_{t\text{-}top}}{t_{w\text{-}top}}$$
$$= \frac{5.50 \text{ in.}}{0.350 \text{ in.}}$$
$$= 15.7 < 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} = 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$,

$$C_{v2} = 1.0 \qquad \text{(}\textit{Spec}\text{. Eq. G2-9)}$$
$$V_{n\text{-}top} = 0.60F_y\left(d_{t\text{-}top}t_{w\text{-}top}\right)C_{v2} \qquad \text{(from }\textit{Spec}\text{. Eq. G3-1)}$$
$$= 0.60(50 \text{ ksi})(5.50 \text{ in.})(0.350 \text{ in.})(1.0)$$
$$= 57.8 \text{ kips}$$

Bottom tee:

$$\frac{h}{t_w} = \frac{d_{t\text{-}bot}}{t_{w\text{-}bot}}$$
$$= \frac{5.50 \text{ in.}}{0.405 \text{ in.}}$$
$$= 13.6 < 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} = 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$,

$$C_{v2} = 1.0 \qquad \text{(}\textit{Spec}\text{. Eq. G2-9)}$$
$$V_{n\text{-}bot} = 0.60F_y(d_{t\text{-}bot}t_{w\text{-}bot})C_{v2} \qquad \text{(from }\textit{Spec}\text{. Eq. G3-1)}$$
$$= 0.60(50 \text{ ksi})(5.50 \text{ in.})(0.405 \text{ in.})(1.0)$$
$$= 66.8 \text{ kips}$$

Available vertical shear strength at top and bottom tees

LRFD	ASD
$\phi_v V_{n\text{-}top} = 1.00(57.8 \text{ kips})$ $= 57.8 \text{ kips}$ $\phi_v V_{n\text{-}bot} = 1.00(66.8 \text{ kips})$ $= 66.8 \text{ kips}$	$\dfrac{V_{n\text{-}top}}{\Omega_v} = \dfrac{57.8 \text{ kips}}{1.50}$ $= 38.5 \text{ kips}$ $\dfrac{V_{n\text{-}bot}}{\Omega_v} = \dfrac{66.8 \text{ kips}}{1.50}$ $= 44.5 \text{ kips}$

Check vertical shear at beam gross section

LRFD	ASD
$V_{u\text{-}net} = 44.1 \text{ kips (see Table 4-16)}$ $V_{u\text{-}global} = V_{u\text{-}net} + \textit{concrete shear strength}$ $= 44.1 \text{ kips} + 7.39 \text{ kips}$ $= 51.5 \text{ kips}$	$V_{a\text{-}net} = 31.3 \text{ kips (see Table 4-16)}$ $V_{a\text{-}global} = V_{a\text{-}net} + \textit{concrete shear strength}$ $= 31.3 \text{ kips} + 4.93 \text{ kips}$ $= 36.2 \text{ kips}$

From AISC *Specification* Section G2.1(b)(1):

$$\frac{h}{t_{w\text{-}min}} = \frac{30.8 \text{ in.} - (0.950 \text{ in.} + 1.15 \text{ in.})}{0.350 \text{ in.}}$$
$$= 82.0$$

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(}\textit{Spec}\text{. Eq. G2-4)}$$
$$= \frac{1.10\sqrt{5.34(29{,}000 \text{ ksi})/(50 \text{ ksi})}}{82.0}$$
$$= 0.747$$

$$V_{n\text{-}gross} = 0.60F_y(d_g t_{w\text{-}min})C_{v1} \qquad \text{(from }\textit{Spec}\text{. Eq. G2-1)}$$
$$= 0.60(50 \text{ ksi})(30.8 \text{ in.})(0.350 \text{ in.})(0.747)$$
$$= 242 \text{ kips}$$

From AISC *Specification* Section G1:

$$\frac{h}{t_w} = 82.0 > 2.24\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}} = 53.9$$

Therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

Available vertical shear strength at gross section

LRFD	ASD
$\phi_v V_{n\text{-}gross} = 0.90(242\text{ kips})$ $= 218\text{ kips}$	$\dfrac{V_{n\text{-}gross}}{\Omega_v} = \dfrac{242\text{ kips}}{1.67}$ $= 145\text{ kips}$

The following is a summary of the beam shear strengths:

LRFD	ASD
Horizontal shear $V_u/\phi_v V_{n\text{-}horiz} = 36.3\text{ kips}/84.0\text{ kips}$ $= 0.432$ **o.k.**	*Horizontal shear* $V_a\Omega_v/V_{n\text{-}horiz} = 25.5\text{ kips}/56.0\text{ kips}$ $= 0.455$ **o.k.**
Vertical shear–top tee $V_{u\text{-}top}/\phi_v V_{n\text{-}top} = 20.9\text{ kips}/57.8\text{ kips}$ $= 0.362$ **o.k.**	*Vertical shear–top tee* $V_{a\text{-}top}\Omega_v/V_{n\text{-}top} = 14.7\text{ kips}/38.5\text{ kips}$ $= 0.382$ **o.k.**
Vertical shear–bottom tee $V_{u\text{-}bot}/\phi_v V_{n\text{-}bot} = 27.7\text{ kips}/66.8\text{ kips}$ $= 0.415$ **o.k.**	*Vertical shear–bottom tee* $V_{a\text{-}bot}\Omega_v/V_{n\text{-}bot} = 19.5\text{ kips}/44.5\text{ kips}$ $= 0.438$ **o.k.**
Vertical shear–gross section $V_{u\text{-}global}/\phi_v V_{n\text{-}gross} = 51.5\text{ kips}/218\text{ kips}$ $= 0.236$ **o.k.**	*Vertical shear–gross section* $V_{a\text{-}global}\Omega_v/V_{n\text{-}gross} = 36.2\text{ kips}/145\text{ kips}$ $= 0.250$ **o.k.**

Check deflection

Deflections are calculated using 90% of the moment of inertia per Section 3.7.

The pre-composite dead load deflection is:

$$\begin{aligned}\Delta_{PDL} &= \frac{5wL^4}{384EI_{x\text{-}net}(0.90)}\\ &= \frac{5\left(\dfrac{0.44\text{ kip/ft}}{12\text{ in./ft}}\right)\left[(50\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})\left(2{,}180\text{ in.}^4\right)(0.90)}\\ &= 1.09\text{ in.}\end{aligned}$$

Live load deflection is:

$$\Delta_{LL} = \frac{5wL^4}{384EI_{x\text{-}comp}(0.90)}$$

$$= \frac{5\left(\dfrac{0.8\text{ kip/ft}}{12\text{ in./ft}}\right)\left[(50\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})\left(5{,}740\text{ in.}^4\right)(0.90)}$$

$$= 0.749\text{ in.}$$

$$= \frac{L}{800}$$

Dead load deflection is:

$$\Delta_{DL} = \frac{5wL^4}{384EI_{x\text{-}comp}\phi}$$

$$= \frac{5\left(\dfrac{0.16\text{ kip/ft}}{12\text{ in./ft}}\right)\left[(50\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})\left(5{,}740\text{ in.}^4\right)(0.90)}$$

$$= 0.150\text{ in.}$$

$$= \frac{L}{4{,}000}$$

Total load deflection is:

$$\Delta_{TL} = \Delta_{LL} + \Delta_{DL}$$

$$= 0.749\text{ in.} + 0.150\text{ in.}$$

$$= 0.899\text{ in.}$$

$$= \frac{L}{667}$$

Deflection summary

$\Delta_{PDL} = 1.09$ in.; therefore, camber 1 in.

$\Delta_{LL} \le \dfrac{L}{360}$ **o.k.**

$\Delta_{TL} \le \dfrac{L}{240}$ **o.k.**

Example 4.4—Composite Cellular Beam Design

Given:

Evaluate the same beam from Example 4.3 using a cellular beam instead of a castellated beam, as shown in Figure 4-7. As in the noncomposite cellular beam, a rectangular opening will be approximated for Vierendeel bending.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1–1, the geometric properties are as follows:

Top root beam
W21×44

$A = 13.0 \text{ in.}^2$ $\quad d_{top} = 20.7 \text{ in.}$ $\quad t_w = 0.350 \text{ in.}$ $\quad b_f = 6.50 \text{ in.}$ $\quad t_f = 0.450 \text{ in.}$ $\quad I_x = 843 \text{ in.}^4$

$S_x = 81.6 \text{ in.}^3$ $\quad Z_x = 95.4 \text{ in.}^3$

Bottom root beam
W21×57

$A = 16.7 \text{ in.}^2$ $\quad d_{bot} = 21.1 \text{ in.}$ $\quad t_w = 0.405 \text{ in.}$ $\quad b_f = 6.56 \text{ in.}$ $\quad t_f = 0.650 \text{ in.}$ $\quad I_x = 1{,}170 \text{ in.}^4$

$S_x = 111 \text{ in.}^3$ $\quad Z_x = 129 \text{ in.}^3$

Resultant shape properties for the LB30×44/57 are determined as follows:

The values of D_o and S are designated based on the depth of the original beam section and a trial opening size.

$$D_o = 20.8 \text{ in.}$$
$$S = 28.8 \text{ in.}$$
$$\begin{aligned} e &= S - D_o \qquad (4\text{-}16) \\ &= 8.00 \text{ in.} \end{aligned}$$

$$loss = \frac{D_o}{2} - \sqrt{\left(\frac{D_o}{2}\right)^2 - \left(\frac{S - D_o}{2}\right)^2} \qquad (4\text{-}17)$$
$$= \frac{20.8 \text{ in.}}{2} - \sqrt{\left(\frac{20.8 \text{ in.}}{2}\right)^2 - \left(\frac{28.8 \text{ in.} - 20.8 \text{ in.}}{2}\right)^2}$$
$$= 0.802 \text{ in.}$$

$$d_{t\text{-}top\text{-}net} = \frac{1}{2}\left[d_{top} - \left(\frac{D_o}{2} + loss\right)\right] \qquad (4\text{-}41)$$
$$= \frac{1}{2}\left[20.7 \text{ in.} - \left(\frac{20.8 \text{ in.}}{2} + 0.802 \text{ in.}\right)\right]$$
$$= 4.75 \text{ in.}$$

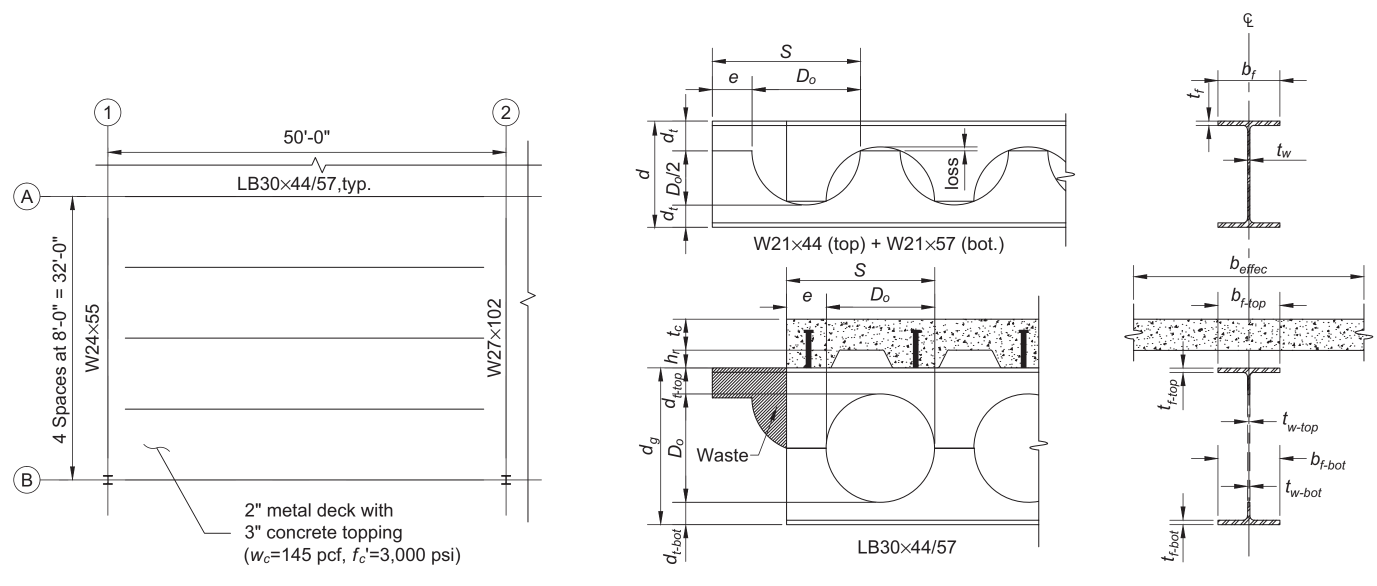


Fig. 4-7. Structural framing layout and composite cellular beam nomenclature for Example 4.4.

$$d_{t\text{-}bot\text{-}net} = \frac{1}{2}\left[d_{bot} - \left(\frac{D_o}{2} + loss\right)\right] \tag{4-42}$$

$$= \frac{1}{2}\left[21.1 \text{ in.} - \left(\frac{20.8 \text{ in.}}{2} + 0.802 \text{ in.}\right)\right]$$

$$= 4.95 \text{ in.}$$

$$d_g = d_{t\text{-}top\text{-}net} + D_o + d_{t\text{-}bot\text{-}net} \tag{4-43}$$

$$= 4.75 \text{ in.} + 20.8 \text{ in.} + 4.95 \text{ in.}$$

$$= 30.5 \text{ in.}$$

$$y = \sqrt{(0.5D_o)^2 - (0.225D_o)^2} \tag{4-20}$$

$$= \sqrt{[(0.5)(20.8 \text{ in.})]^2 - [(0.225)(20.8 \text{ in.})]^2}$$

$$= 9.29 \text{ in.}$$

$$d_{t\text{-}top\text{-}crit} = \frac{D_o}{2} - y + d_{t\text{-}top\text{-}net} \qquad \text{(from Eq. 4-21)}$$

$$= \frac{20.8 \text{ in.}}{2} - 9.29 \text{ in.} + 4.75 \text{ in.}$$

$$= 5.86 \text{ in.}$$

$$d_{t\text{-}bot\text{-}crit} = \frac{D_o}{2} - y + d_{t\text{-}bot\text{-}net} \qquad \text{(from Eq. 4-21)}$$

$$= \frac{(20.8 \text{ in.})}{2} - 9.29 \text{ in.} + 4.95 \text{ in.}$$

$$= 6.06 \text{ in.}$$

Check limits of applicability

According to Section 3.4, the design procedures for web post buckling are only applicable if the following conditions concerning the cutting pattern are met: $1.08 < S/D_o < 1.5$ and $1.25 < d_g/D_o < 1.75$.

$$\frac{S}{D_o} = \frac{28.8 \text{ in.}}{20.8 \text{ in.}}$$

$$= 1.38 < 1.5 \qquad \textbf{o.k.}$$

$$\frac{d_g}{D_o} = \frac{30.5 \text{ in.}}{20.8 \text{ in.}}$$

$$= 1.47 < 1.75 \qquad \textbf{o.k.}$$

Calculate section properties of top and bottom tees and beam

Relevant cross sections are provided in Figure 4-8, and the section properties for the top and bottom tees are reported in Tables 4-24 through 4-27.

Beam net section properties at center of opening

$$A_{net} = A_{tee\text{-}top} + A_{tee\text{-}bot} \tag{3-7}$$

$$= 444 \text{ in.}^2 + 6.01 \text{ in.}^2$$

$$= 10.5 \text{ in.}^2$$

$$\bar{y}_{bs} = \frac{A_{tee\text{-}top}\left(d_g - \bar{y}_{tee\text{-}top}\right) + A_{tee\text{-}bot}\,\bar{y}_{tee\text{-}bot}}{A_{net}} \qquad \text{(from Eq. 4-25)}$$

$$= \frac{\left(4.44 \text{ in.}^2\right)(30.5 \text{ in.} - 3.73 \text{ in.}) + \left(6.01 \text{ in.}^2\right)(1.05 \text{ in.})}{10.5 \text{ in.}^2}$$

$$= 11.9 \text{ in.}$$

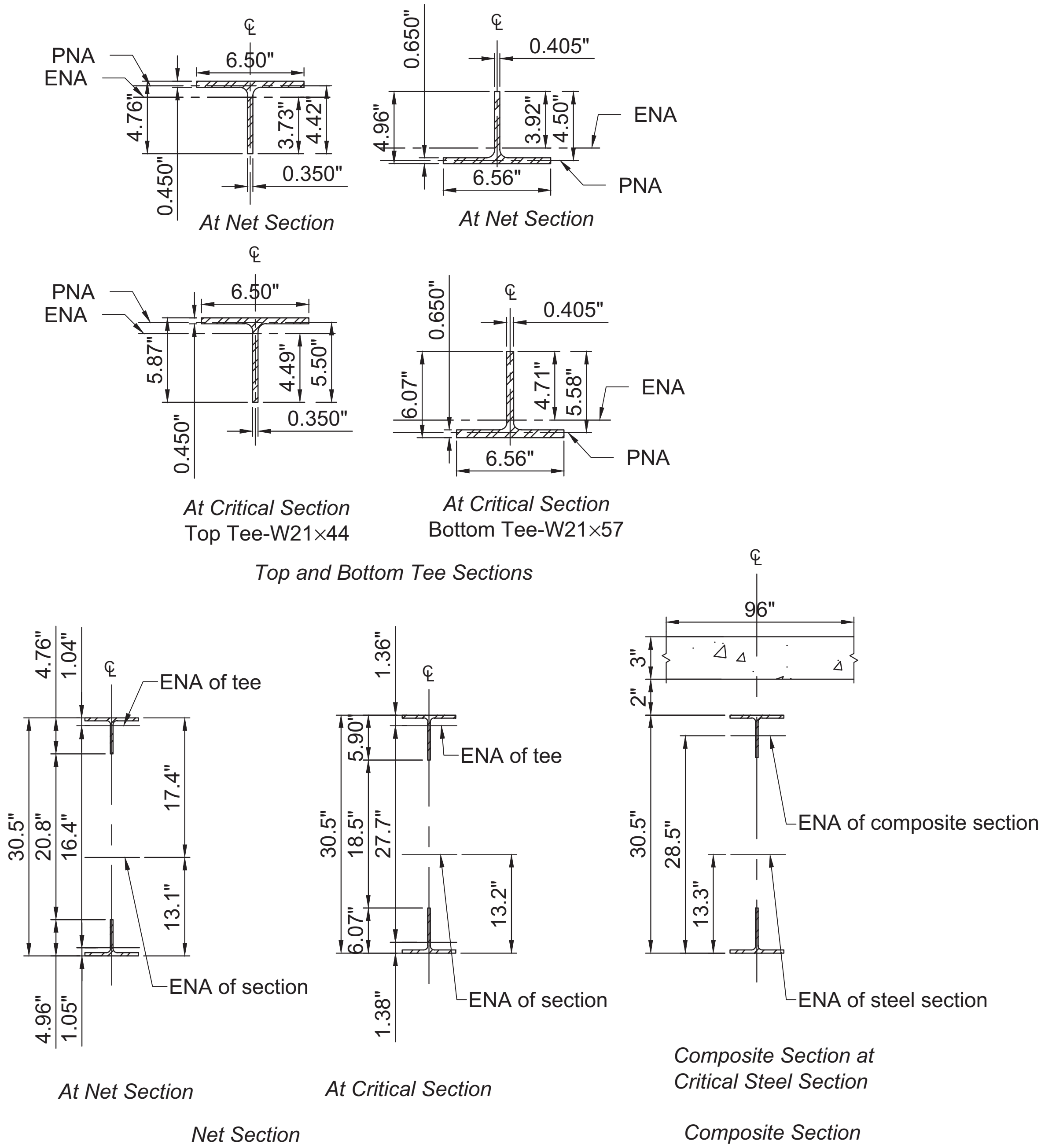


Fig. 4-8. Tee, net and composite section for cellular beam for Example 4.4.

Table 4-24. Top Tee Section Properties at Center of Opening			
$A_{tee\text{-}top}$ = 4.44 in.^{2}	x = 4.42 in.	r_x = 1.35 in.	r_y = 1.53 in.
$\bar{y}_{tee\text{-}top}$ = 3.73 in.	$S_{x\text{-}top}$ = 7.76 in.^{3}	$S_{x\text{-}bot}$ = 2.16 in.^{3}	Z_x = 3.83 in.^{3}
$I_{x\text{-}tee\text{-}top}$ = 8.03 in.^{4}	I_y = 10.3 in.^{4}	J = 0.255 in.^{4}	y_o = 3.50 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

Table 4-25. Bottom Tee Section Properties at Center of Opening			
$A_{tee\text{-}bot}$ = 6.01 in.^{2}	x = 0.458 in.	r_x = 1.32 in.	r_y = 1.60 in.
$\bar{y}_{tee\text{-}bot}$ = 1.05 in.	$S_{x\text{-}top}$ = 2.68 in.^{3}	$S_{x\text{-}bot}$ = 10.0 in.^{3}	Z_x = 4.91 in.^{3}
$I_{x\text{-}tee\text{-}bot}$ = 10.5 in.^{4}	I_y = 15.3 in.^{4}	J = 0.673 in.^{4}	y_o = 0.72 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

Table 4-26. Top Tee Section Properties at Critical Section			
$A_{crit\text{-}top}$ = 4.82 in.^{2}	x = 5.50 in.	y = 4.49 in.	$\bar{y}_{crit\text{-}top}$ = 1.38 in.
$S_{x\text{-}top}$ = 10.6 in.^{3}	$S_{x\text{-}bot}$ = 3.25 in.^{3}	Z_x = 5.76 in.^{3}	J = 0.271 in.^{4}
$I_{x\text{-}crit\text{-}top}$ = 14.6 in.^{4}	I_y = 10.3 in.^{4}	r_x = 1.74 in.	r_y = 1.46 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

Table 4-27. Bottom Tee Section Properties at Critical Section			
$A_{crit\text{-}bot}$ = 6.46 in.^{2}	x = 0.492 in.	y = 4.71 in.	$\bar{y}_{crit\text{-}bot}$ = 1.36 in.
$S_{x\text{-}top}$ = 4.01 in.^{3}	$S_{x\text{-}bot}$ = 13.9 in.^{3}	Z_x = 7.17 in.^{3}	J = 0.698 in.^{4}
$I_{x\text{-}crit\text{-}bot}$ = 18.9 in.^{4}	I_y = 15.3 in.^{4}	r_x = 1.71 in.	r_y = 1.54 in.
Note: The fillet radius is assumed to be zero in the section properties calculations.			

$$\begin{aligned}\bar{y}_{ts} &= d_g - \bar{y}_{bs} \\ &= 30.5\text{ in.} - 11.9\text{ in.} \\ &= 18.6\text{ in.}\end{aligned} \tag{4-26}$$

$$\begin{aligned}d_{effec} &= d_g - \left(\bar{y}_{tee\text{-}top} + \bar{y}_{tee\text{-}bot}\right) \\ &= 30.5\text{ in.} - (3.73\text{ in.} + 1.05\text{ in.}) \\ &= 25.7\text{ in.}\end{aligned} \quad \text{(from Eq. 4-27)}$$

$$\begin{aligned}I_{x\text{-}net} &= I_{x\text{-}tee\text{-}top} + A_{tee\text{-}top}\left(d_g - \bar{y}_{bs} - \bar{y}_{crit\text{-}top}\right)^2 + I_{x\text{-}tee\text{-}bot} + A_{tee\text{-}bot}\left(\bar{y}_{bs} - \bar{y}_{tee\text{-}bot}\right)^2 \\ &= 8.03\text{ in.}^4 + \left(4.44\text{ in.}^2\right)(30.5\text{ in.} - 11.9\text{ in.} - 3.73\text{ in.})^2 + 10.5\text{ in.}^4 + \left(6.01\text{ in.}^2\right)(11.9\text{ in.} - 1.05\text{ in.})^2 \\ &= 1{,}710\text{ in.}^4\end{aligned} \quad \text{(from Eq. 4-28)}$$

$$\begin{aligned}S_{x\text{-}net\text{-}top} &= \frac{I_{x\text{-}net}}{\bar{y}_{ts}} \\ &= \frac{1{,}710\text{ in.}^4}{18.6\text{ in.}} \\ &= 91.9\text{ in.}^3\end{aligned} \tag{4-29}$$

$$S_{x\text{-}net\text{-}bot} = \frac{I_{x\text{-}net}}{\bar{y}_{bs}} \tag{4-30}$$

$$= \frac{1{,}710 \text{ in.}^4}{11.9 \text{ in.}}$$

$$= 144 \text{ in.}^3$$

Beam critical net section properties at center of opening

$$A_{net\text{-}crit} = A_{crit\text{-}top} + A_{crit\text{-}bot} \qquad \text{(from Eq. 4-6)}$$

$$= 4.83 \text{ in.}^2 + 6.46 \text{ in.}^2$$

$$= 11.3 \text{ in.}^2$$

$$\bar{y}_{bs} = \frac{A_{crit\text{-}top}\left(d_g - \bar{y}_{crit\text{-}top}\right) + A_{crit\text{-}bot}\,\bar{y}_{crit\text{-}bot}}{A_{net}} \qquad \text{(from Eq. 4-25)}$$

$$= \frac{\left(4.83 \text{ in.}^2\right)(30.5 \text{ in.} - 1.38 \text{ in.}) + \left(6.46 \text{ in.}^2\right)(1.36 \text{ in.})}{11.3 \text{ in.}^2}$$

$$= 13.2 \text{ in.}$$

$$\bar{y}_{ts} = d_g - \bar{y}_{bs} \tag{4-26}$$

$$= 30.5 \text{ in.} - 13.2 \text{ in.}$$

$$= 17.3 \text{ in.}$$

$$d_{effec} = d_g - \left(\bar{y}_{crit\text{-}top} + \bar{y}_{crit\text{-}bot}\right) \qquad \text{(from Eq. 4-27)}$$

$$= 30.5 \text{ in.} - (1.38 \text{ in.} + 1.36 \text{ in.})$$

$$= 27.8 \text{ in.}$$

$$I_{x\text{-}net\text{-}crit} = I_{x\text{-}crit\text{-}top} + A_{crit\text{-}top}\left(d_g - \bar{y}_{bs} - \bar{y}_{crit\text{-}top}\right)^2 + I_{x\text{-}crit\text{-}bot} + A_{crit\text{-}bot}\left(\bar{y}_{bs} - \bar{y}_{crit\text{-}bot}\right)^2 \tag{4-46}$$

$$= 14.6 \text{ in.}^4 + \left(4.82 \text{ in.}^2\right)(30.5 \text{ in.} - 13.2 \text{ in.} - 1.38 \text{ in.})^2 + 18.9 \text{ in.}^4 + \left(6.46 \text{ in.}^2\right)(13.2 \text{ in.} - 1.36 \text{ in.})^2$$

$$= 2{,}160 \text{ in.}^4$$

$$S_{x\text{-}crit\text{-}top} = \frac{I_{x\text{-}net\text{-}crit}}{\bar{y}_{ts}} \qquad \text{(from Eq. 4-29)}$$

$$= \frac{2{,}160 \text{ in.}^4}{17.3 \text{ in.}}$$

$$= 125 \text{ in.}^3$$

$$S_{x\text{-}crit\text{-}bot} = \frac{I_{x\text{-}net\text{-}crit}}{\bar{y}_{bs}} \qquad \text{(from Eq. 4-30)}$$

$$= \frac{2{,}160 \text{ in.}^4}{13.2 \text{ in.}}$$

$$= 163 \text{ in.}^3$$

Composite section properties at critical section in accordance with The Structural Engineer's Handbook

$$n = \frac{E_s}{E_c} \tag{4-31}$$

$$= \frac{29{,}000{,}000 \text{ psi}}{33(145 \text{ pcf})^{1.5}\sqrt{3{,}000} \text{ ksi}}$$

$$= 9.19$$

$$b_{effec} = \min\{Span/4, Spacing\} \tag{3-4}$$

$$= \min\left\{\frac{50 \text{ ft}}{4}, \frac{8 \text{ ft} + 8 \text{ ft}}{2}\right\}(12 \text{ in./ft})$$

$$= 96.0 \text{ in.}$$

$$A_c = b_{effec} t_c \tag{4-32}$$

$$= (96.0 \text{ in.})(3.00 \text{ in.})$$

$$= 288 \text{ in.}^2$$

$$A_{ctr} = \frac{A_c}{n} \tag{4-33}$$

$$= \frac{288 \text{ in.}^2}{9.19}$$

$$= 31.3 \text{ in.}^2$$

$$K_c = \frac{A_{ctr}}{A_{ctr} + A_{net\text{-}crit}} \qquad \text{(from Eq. 4-34)}$$

$$= \frac{31.3 \text{ in.}^2}{31.3 \text{ in.}^2 + 11.3 \text{ in.}^2}$$

$$= 0.735$$

$$e_c = h_r + \frac{t_c}{2} \tag{4-35}$$

$$= 2.00 \text{ in.} + \frac{3.00 \text{ in.}}{2}$$

$$= 3.50 \text{ in.}$$

Assuming that the neutral axis is in the concrete,

$$y_{cc} = \left(\frac{A_{net\text{-}crit} t_c}{A_{ctr}}\right)\left[\sqrt{1 + \frac{2A_{ctr}}{A_{net\text{-}crit}\, t_c}\left(\overline{y}_{ts} + e_c + \frac{t_c}{2}\right)} - 1\right] \qquad \text{(from Eq. 4-36)}$$

$$= \left[\frac{(11.3 \text{ in.}^2)(3.00 \text{ in.})}{31.3 \text{ in.}^2}\right]\left[\sqrt{1 + \frac{2(31.3 \text{ in.}^2)}{(11.3 \text{ in.}^2)(3.00 \text{ in.})}\left(17.3 \text{ in.} + 3.50 \text{ in.} + \frac{3.00 \text{ in.}}{2}\right)} - 1\right]$$

$$= 5.94 \text{ in.}$$

Because $t_c + h_r = 5.00 \text{ in.} < y_{cc}$, the neutral axis is in the steel.

$$\overline{y}_c = (\overline{y}_{ts} + e_c) K_c \tag{4-37}$$

$$= (17.3 \text{ in.} + 3.50 \text{ in.})(0.735)$$

$$= 15.3 \text{ in.}$$

$$I_{x\text{-}comp\text{-}crit} = (\overline{y}_{ts} + e_c)\overline{y}_c A_{net\text{-}crit} + I_{x\text{-}net\text{-}crit} + \frac{A_{ctr} t_c^2}{12} \qquad \text{(from Eq. 4-38)}$$

$$= (17.3 \text{ in.} + 3.50 \text{ in.})(15.3 \text{ in.})(11.3 \text{ in.}^2) + 2{,}160 \text{ in.}^4 + \frac{(31.3 \text{ in.}^2)(3.50 \text{ in.})^2}{12}$$

$$= 5{,}790 \text{ in.}^4$$

$$S_{x\text{-}comp\text{-}conc} = \frac{I_{x\text{-}comp\text{-}crit}}{\overline{y}_{ts} - \overline{y}_c + e_c + 0.5t_c} \qquad \text{(from Eq. 4-39)}$$

$$= \frac{5{,}790 \text{ in.}^4}{17.3 \text{ in.} - 15.3 \text{ in.} + 3.50 \text{ in.} + 0.5(3.00 \text{ in.})}$$

$$= 827 \text{ in.}^3$$

$$S_{x\text{-}comp\text{-}steel} = \frac{I_{x\text{-}comp\text{-}crit}}{\bar{y}_{bs} + \bar{y}_c} \quad \text{(from Eq. 4-40)}$$

$$= \frac{5{,}790 \text{ in.}^4}{13.2 \text{ in.} + 15.3 \text{ in.}}$$

$$= 203 \text{ in.}^3$$

For the first iteration,

$$d_{effec\text{-}comp} = d_g - \bar{y}_{crit\text{-}bot} + h_r + 0.5t_c \quad \text{(Eq. 3-8)}$$

$$= 30.5 \text{ in.} - 1.36 \text{ in.} + 2.00 \text{ in.} + 0.5(3.00 \text{ in.})$$

$$= 32.6 \text{ in.}$$

Composite section properties at net section per the Structural Engineer's Handbook

$$y_{cc} = \left(\frac{A_{net} t_c}{A_{ctr}}\right)\left[\sqrt{1 + \frac{2A_{ctr}}{A_{net} t_c}\left(\bar{y}_{ts} + e_c + \frac{t_c}{2}\right)} - 1\right] \quad \text{(from Eq. 4-36)}$$

$$= \left[\frac{(10.5 \text{ in.}^2)(3.00 \text{ in.})}{31.3 \text{ in.}^2}\right]\left[\sqrt{1 + \frac{2(31.3 \text{ in.}^2)}{(10.5 \text{ in.}^2)(3.00 \text{ in.})}\left(17.4 \text{ in.} + 3.50 \text{ in.} + \frac{3.00 \text{ in.}}{2}\right)} - 1\right]$$

$$= 5.76 \text{ in.}$$

$$t_c + h_r = 3.00 \text{ in.} + 2.00 \text{ in.}$$

$$= 5.00 \text{ in.} < 5.76 \text{ in.}$$

$$\bar{y}_c = (\bar{y}_{ts} + e_c)K_c \quad \text{(4-37)}$$

$$= (17.4 \text{ in.} + 3.50 \text{ in.})(0.735)$$

$$= 15.3 \text{ in.}$$

$$I_{x\text{-}comp} = (\bar{y}_{ts} + e_c)\bar{y}_c A_{net} + I_{x\text{-}net} + \frac{A_{ctr} t_c^2}{12} \quad \text{(4-38)}$$

$$= (17.4 \text{ in.} + 3.50 \text{ in.})(15.3 \text{ in.})(10.5 \text{ in.}^2) + 1{,}710 \text{ in.}^4 + \frac{(31.3 \text{ in.}^2)(3.50 \text{ in.})^2}{12}$$

$$= 5{,}100 \text{ in.}^4$$

Check Vierendeel bending

The governing load cases are:

LRFD	ASD
Load case 1: $w = 1.4D$ $= 1.4(651 \text{ lb/ft})$ $= 911 \text{ lb/ft}$ Load case 2: $w = 1.2D + 1.6L$ $= 1.2(651 \text{ lb/ft}) + 1.6(800 \text{ lb/ft})$ $= 2{,}060 \text{ lb/ft}$ **governs**	$w = D + L$ $= 651 \text{ lb/ft} + 800 \text{ lb/ft}$ $= 1{,}450 \text{ lb/ft}$ **governs**

Table 4-28. Global Shear and Moment at Each Opening									
		Global Shear				Global Moment			
				$V_{r\text{-}net}$, kips				M_r, Kip-ft	
Opening No.	X_i, ft	D, kips	L, kips	ASD	LRFD	D, kips	L, kips	ASD	LRFD
End	0.000	16.3	20.0	31.3	44.1	0.000	0.000	0.000	0.000
1	1.53	15.3	18.8	29.1	41.0	24.1	29.7	53.8	76.5
2	3.93	13.7	16.9	25.6	36.0	58.8	72.4	131	186
3	6.32	12.1	14.9	22.2	31.1	89.8	110	200	285
4	8.72	10.6	13.0	18.7	26.2	117	144	261	371
5	11.1	9.03	11.1	15.2	21.2	141	173	313	445
6	13.5	7.47	9.19	11.7	16.3	160	197	358	508
7	15.9	5.92	7.28	8.26	11.3	176	217	393	559
8	18.3	4.36	5.36	4.79	6.41	189	232	421	598
9	20.7	2.80	3.44	1.31	1.47	197	243	440	625
10	23.1	1.24	1.53	0.000	0.000	202	249	451	640
Bm. CL	25.0	0.000	0.000	0.000	0.000	203	250	453	644

Note: The shear force shown is the net shear force; i.e., the shear strength of the concrete has been subtracted from the global shear force on the beam.

Calculate the available shear strength of the concrete deck:

LRFD	ASD
$V_c = \phi_{cv} V_{nc}$ (3-15a) $V_{nc} = 4\sqrt{f_c'}(3)(h_r + t_c)t_c$ (3-14) $= \dfrac{4\sqrt{3{,}000 \text{ psi}}(3)(2.00 \text{ in.} + 3.00 \text{ in.})(3.00 \text{ in.})}{1{,}000 \text{ lb/kip}}$ $= 9.85$ kips $V_c = 0.75(9.85 \text{ kips})$ $= 7.39$ kips	$V_c = \dfrac{V_{nc}}{\Omega_{cv}}$ (3-15b) $V_{nc} = 4\sqrt{f_c'}(3)(h_r + t_c)t_c$ (3-14) $= \dfrac{4\sqrt{3{,}000 \text{ psi}}(3)(2.00 \text{ in.} + 3.00 \text{ in.})(3.00 \text{ in.})}{1{,}000 \text{ lb/kip}}$ $= 9.85$ kips $V_c = \dfrac{9.85 \text{ kips}}{2.00}$ $= 4.93$ kips

Calculate the global shear and moment at each opening to be used to calculate local internal forces (axial and flexural) at each opening. These results are presented in Table 4-28.

Calculate the local axial force in the top and bottom tees resulting from the global moment. These values are presented in Table 4-29.

As in castellated composite beams, assume that the concrete flange takes all the compression and that the bottom tee takes all the tension force. Once again, this is a valid assumption assuming that sufficient studs exist at a given opening to have developed the concrete flange. It is necessary to check the validity of this assumption.

Local axial force:

For the first iteration, recalculate $d_{effec\text{-}comp}$ each time

$$T_{1(i)} = \frac{M_{r(i)}}{d_{effec\text{-}comp}} \tag{3-9}$$

Table 4-29. Local Axial Force at Each Opening									
ASD									
Opening Number	**X_i, ft**	**M_r, kip-ft**	**$T_{1(i)}$, kips**	**$X_{c(i+1)}$, in.**	**$T_{1(i+1)}$, kips**	**$\frac{T_{1(i)}}{T_{1(i+1)}}$**	**$X_{c(i+2)}$, in.**	**$T_{1(i+2)}$, kips**	**$\frac{T_{1(i)}}{T_{1(i+2)}}$**
End	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	1.53	53.8	19.8	0.081	18.9	1.05	0.077	18.9	1.00
2	3.93	131	48.4	0.198	46.3	1.05	0.189	46.3	1.00
3	6.32	200	73.7	0.301	70.6	1.04	0.289	70.6	1.00
4	8.72	261	96.1	0.393	92.3	1.04	0.377	92.2	1.00
5	11.1	313	115	0.471	111	1.04	0.453	111	1.00
6	13.5	358	132	0.538	127	1.04	0.517	127	1.00
7	15.9	393	145	0.591	139	1.04	0.570	139	1.00
8	18.3	421	155	0.633	149	1.04	0.610	149	1.00
9	20.7	440	162	0.662	156	1.04	0.638	156	1.00
10	23.1	451	166	0.678	160	1.04	0.654	160	1.00
Bm. CL	25.0	453	167	0.682	161	1.04	0.658	161	1.00
LRFD									
Opening Number	**X_i, ft**	**M_r, kip-ft**	**$T_{1(i)}$, kips**	**$X_{c(i+1)}$, in.**	**$T_{1(i+1)}$, kips**	**$\frac{T_{1(i)}}{T_{1(i+1)}}$**	**$X_{c(i+2)}$, in.**	**$T_{1(i+2)}$, kips**	**$\frac{T_{1(i)}}{T_{1(i+2)}}$**
End	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	1.53	76.5	28.1	0.115	26.9	1.05	0.110	26.9	1.00
2	3.93	186	68.7	0.281	65.7	1.05	0.269	65.7	1.00
3	6.32	285	105	0.428	100	1.05	0.410	100	1.00
4	8.72	371	137	0.558	131	1.05	0.536	131	1.00
5	11.1	445	164	0.669	158	1.04	0.645	158	1.00
6	13.5	508	187	0.764	180	1.04	0.737	180	1.00
7	15.9	559	206	0.840	199	1.04	0.812	199	1.00
8	18.3	598	220	0.899	213	1.03	0.870	213	1.00
9	20.7	625	230	0.940	223	1.03	0.910	223	1.00
10	23.1	640	236	0.963	228	1.03	0.933	228	1.00
Bm. CL	25.0	644	237	0.969	230	1.03	0.939	230	1.00

Recalculate the effective concrete depth

$$X_c = \frac{T_{1(i)}}{0.85 f_c' \, b_{effec}} \tag{3-10}$$

Recalculate $d_{effec\text{-}comp}$

$$d_{effec\text{-}comp} = d_g - \bar{y}_{tee\text{-}bot\text{-}crit} + t_c + h_r - \frac{X_c}{2} \qquad \text{(from Eq. 3-8)}$$

Recalculate until the difference ≤ 1%

$$T_{1(i+1)} = \frac{M_{r(i+1)}}{d_{effec\text{-}comp}} \tag{3-9}$$

Table 4-30. Revised Local Axial Force at Each Opening (LRFD)						
Opening Number	**X_i, ft**	**$T_1 = T_{1(i+2)}$, kips**	**$(q)(X_i)$, kips**	**Composite Status**	**T_o, kips**	**$T_{1\text{-}new}$, kips**
End	0.000	0.000	0.000	N/A	N/A	N/A
1	1.53	26.9	34.1	Full	0.000	26.9
2	3.93	65.8	87.4	Full	0.000	65.8
3	6.32	101	141	Full	0.000	101
4	8.72	131	194	Full	0.000	131
5	11.1	158	247	Full	0.000	158
6	13.5	181	301	Full	0.000	181
7	15.9	199	354	Full	0.000	199
8	18.3	213	407	Full	0.000	213
9	20.7	223	461	Full	0.000	223
10	23.1	228	514	Full	0.000	228
CL	25.0	230	556	Full	0.000	230

The same number of studs as those used in Example 4.3 has been selected; therefore, the same number of studs and stud density is applicable. The number of studs for full composite action is 54 across the length of the beam and the shear stud density = 22.3 kip/ft. Also, as in Example 4.3, the next step is to calculate the amount of concrete that has been developed by the studs between the end of the beam and the opening under consideration and determine whether or not that section of the beam is fully or partially composite. If it is determined to be partially composite, calculate the added force that the steel section is required to resist, T_o, and $T_{1\text{-}new}$ (refer to Example 4.3 for further explanation). Table 4-30 shows the axial force at each opening.

The compression force to be resisted by the top tee at its centroid is:

$$T_o = M_r \left[\frac{1 - \dfrac{(q)(X_i)}{T_{1(i+2)}}}{d_{effec}} \right] \tag{3-12}$$

The revised tensile force to be resisted by the bottom tee at its centroid is then:

$$T_{1\text{-}new} = qX_i + T_o \tag{3-13}$$

Calculate the local moment on the top and bottom tees resulting from the net shear force passing through the web opening. These results are presented in Table 4-31.

Top tee local Vierendeel moment:

$$M_{vr\text{-}top} = V_{net}\left(\frac{A_{crit\text{-}top}}{A_{net\text{-}crit}}\right)\frac{D_o}{4} \qquad \text{(from Eq. 3-2)}$$

Bottom tee local Vierendeel moment:

$$M_{vr\text{-}bot} = V_{net}\left(\frac{A_{crit\text{-}bot}}{A_{net\text{-}crit}}\right)\frac{D_o}{4} \qquad \text{(from Eq. 3-3)}$$

Table 4-31. Local Vierendeel Moment at Each Opening							
		ASD			LRFD		
Opening Number	X_i, ft	V_a, kips	$M_{va\text{-}top}$, kip-in.	$M_{va\text{-}bot}$, kip-in.	V_u, kips	$M_{vu\text{-}top}$, kip-in.	$M_{vu\text{-}bot}$, kip-in.
End	0.000	31.3	69.4	93.0	44.1	97.8	131
1	1.53	29.1	65.7	87.0	41.0	92.4	122
2	3.93	25.6	57.9	76.6	36.0	81.3	108
3	6.32	22.2	50.0	66.2	31.1	70.2	92.9
4	8.72	18.7	42.2	55.8	26.2	59.0	78.2
5	11.1	15.2	34.3	45.5	21.2	47.9	63.4
6	13.5	11.7	26.5	35.1	16.3	36.7	48.6
7	15.9	8.26	18.6	24.7	11.3	25.6	33.9
8	18.3	4.79	10.8	14.3	6.41	14.5	19.1
9	20.7	1.31	2.97	3.93	1.47	3.32	4.40
10	23.1	0.000	0.000	0.000	0.000	0.000	0.000
CL	25.0	0.000	0.000	0.000	0.000	0.000	0.000

Calculate the available shear and flexural strength of top and bottom tees at the critical section

Determine the limiting flange width-to-thickness ratio from AISC *Specification* Table B4.1b, Case 10:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 9.15$$

The width-to-thickness ratio for the top flange is:

$$\lambda = \frac{b}{t}$$

$$= \frac{b_f}{2t_f}$$

$$= \frac{6.50 \text{ in.}}{2(0.450 \text{ in.})}$$

$$= 7.22 < 9.15$$

The width-to-thickness ratio for the bottom flange is:

$$\lambda = \frac{b}{t}$$

$$= \frac{b_f}{2t_f}$$

$$= \frac{6.56 \text{ in.}}{2(0.650 \text{ in.})}$$

$$= 5.05 < 9.15$$

Because $\lambda < \lambda_p$, the flanges of both the top and bottom tees are compact; therefore, it is not necessary to check flange local buckling when calculating the available flexural strength.

Determine the limiting stem width-to-thickness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\begin{aligned}\lambda_r &= 0.75\sqrt{\frac{E}{F_y}}\\ &= 0.75\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}\\ &= 18.1\end{aligned}$$

The width-to-thickness ratio for the top stem is:

$$\begin{aligned}\lambda &= \frac{dt_{top\text{-}crit}}{t_w}\\ &= \frac{5.87\text{ in.}}{0.35\text{ in.}}\\ &= 16.8 < 18.1\end{aligned}$$

The width-to-thickness ratio for the bottom stem is:

$$\begin{aligned}\lambda &= \frac{dt_{bot\text{-}crit}}{t_w}\\ &= \frac{6.07\text{ in.}}{0.405\text{ in.}}\\ &= 15.0 < 18.1\end{aligned}$$

Because $\lambda < \lambda_r$, both top and bottom tee stems are nonslender; therefore, it is not necessary to consider AISC *Specification* Section E7 when calculating the available compressive strength.

It is not necessary to calculate the available compressive strength of top or bottom tee in this example because all openings are fully composite, and therefore, all compression is taken by the concrete flange. If compression did exist in top or bottom tee, the available compressive strength would be calculated as shown in Example 4.2.

Calculate available tensile strength of bottom tee

$$\begin{aligned}P_n &= F_y A_{crit\text{-}bot} \qquad \text{(from } \textit{Spec}\text{. Eq. D2-1)}\\ &= (50\text{ ksi})\left(6.46\text{ in.}^2\right)\\ &= 323\text{ kips}\end{aligned}$$

Calculate available flexural strength of tee

Yielding

For tee stems in compression:

$$M_{p\text{-}top} = M_y \qquad \text{(from } \textit{Spec}\text{. Eq. F9-4)}$$

$$\begin{aligned}M_y &= F_y S_{x\text{-}bot} \qquad \text{(from } \textit{Spec}\text{. Eq. F9-3)}\\ &= (50\text{ ksi})\left(3.25\text{ in.}^3\right)\\ &= 163\text{ kip-in.}\end{aligned}$$

$$M_{p\text{-}bot} = M_y \qquad \text{(from } \textit{Spec}\text{. Eq. F9-4)}$$

$$\begin{aligned}M_y &= F_y S_{x\text{-}top} \qquad \text{(from } \textit{Spec}\text{. Eq. F9-3)}\\ &= (50\text{ ksi})\left(4.01\text{ in.}^3\right)\\ &= 201\text{ kip-in.}\end{aligned}$$

In both cases the stem is assumed to be in compression, this will be conservative for the bottom tee. It is possible to take advantage of this to calculate a higher value for the available flexural strength of the bottom tee because the stem is in tension.

Lateral-Torsional Buckling

For lateral-torsional buckling of the top tee:

$$B_{top} = -2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \qquad (Spec.\ \text{Eq. F9-12})$$

$$= -2.3\left(\frac{5.86\text{ in.}}{10.4\text{ in.}}\right)\sqrt{\frac{10.3\text{ in.}^4}{0.255\text{ in.}^4}}$$

$$= -8.24$$

$$M_{cr\text{-}top} = \frac{1.95E}{L_b}\sqrt{I_y J}\left(B + \sqrt{1+B^2}\right) \qquad (Spec.\ \text{Eq. F9-10})$$

$$= \frac{1.95(29{,}000\text{ ksi})}{10.4\text{ in.}}\sqrt{\left(10.3\text{ in.}^4\right)\left(0.255\text{ in.}^4\right)}\left[-8.24 + \sqrt{1+(-8.24)^2}\right]$$

$$= 534\text{ kip-in.}$$

For lateral-torsional buckling of the bottom tee:

$$B_{bot} = -2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} \qquad (Spec.\ \text{Eq. F9-12})$$

$$= -2.3\left(\frac{6.06\text{ in.}}{10.4\text{ in.}}\right)\sqrt{\frac{15.3\text{ in.}^4}{0.673\text{ in.}^4}}$$

$$= -6.39$$

$$M_{cr\text{-}bot} = \frac{1.95E}{L_b}\sqrt{I_y J}\left(B + \sqrt{1+B^2}\right) \qquad (Spec.\ \text{Eq. F9-10})$$

$$= \frac{1.95(29{,}000\text{ ksi})}{10.4\text{ in.}}\sqrt{\left(15.3\text{ in.}^4\right)\left(0.673\text{ in.}^4\right)}\left[-6.39 + \sqrt{1+(-6.39)^2}\right]$$

$$= 1{,}350\text{ kip-in.}$$

Flange local buckling

According to AISC *Specification* Section F9.3(a), the limit state of flange local buckling does not apply because the flanges are compact.

Local buckling of tee stems

The nominal flexural strength for local buckling of the tee stem in flexural compression, M_n, is determined using AISC *Specification* Section F9.4:

$$M_n = F_{cr}S_x \qquad (Spec.\ \text{Eq. F9-16})$$

Because $d/t_w < 0.84\sqrt{\frac{E}{F_y}}$, the critical stress, F_{cr}, is determined using AISC *Specification* Equation F9-17:

$$F_{cr} = F_y \qquad (Spec.\ \text{Eq. F9-17})$$

And thus:

For the top tee:

$$M_{n\text{-}top} = F_y S_{x\text{-}bot}$$

$$= (50\text{ ksi})\left(3.25\text{ in.}^3\right)$$

$$= 163\text{ kip-in.}$$

For the bottom tee:

$$M_{n\text{-}bot} = F_y S_{x\text{-}top}$$
$$= (50 \text{ ksi})\left(4.01 \text{ in.}^3\right)$$
$$= 201 \text{ kip-in.}$$

The available tensile and flexural strengths of the tee are:

LRFD	ASD
Available tensile strength—bottom tee $P_c = \phi_c P_n = 0.90(323 \text{ kips}) = 291 \text{ kips}$	*Available tensile strength—bottom tee* $P_c = \dfrac{P_n}{\Omega_c} = \dfrac{323 \text{ kips}}{1.67} = 193 \text{ kips}$
Available flexural strength—top tee $M_c = \phi_b M_n = 0.90(163 \text{ kip-in.}) = 147 \text{ kip-in.}$	*Available flexural strength—top tee* $M_c = \dfrac{M_n}{\Omega_b} = \dfrac{163 \text{ kip-in.}}{1.67} = 97.6 \text{ kip-in.}$
Available flexural strength—bottom tee $M_c = \phi_b M_n = 0.90(201 \text{ kip-in.}) = 181 \text{ kip-in.}$	*Available flexural strength—bottom tee* $M_c = \dfrac{M_n}{\Omega_b} = \dfrac{201 \text{ kip-in.}}{1.67} = 120 \text{ kip-in.}$

Check tees for combined axial and flexural loads

The interaction values for each opening are presented in Table 4-32.

From Table 4-32, the composite Vierendeel bending is summarized as follows:

LRFD	ASD
Top tee $I_{max} = 0.631 < 1.0$ **o.k.**	*Top tee* $I_{max} = 0.674 < 1.0$ **o.k.**
Bottom tee $I_{max} = 0.861 < 1.0$ **o.k.**	*Bottom tee* $I_{max} = 0.911 < 1.0$ **o.k.**

Check web post buckling

Calculate horizontal shear and resultant moment at each gross section for web post buckling

Table 4-33 presents the horizontal shear force at each opening.

Table 4-32. Interaction Values at Each Opening for LRFD and ASD										
LRFD										
		Top Tee			**Bottom Tee**					
Opening Number	**X_i, ft**	**P_r, kips**	**$M_{vr\text{-}top}$, kip-in.**	**$\frac{M_{vr}}{M_c}$**	**P_r, kips**	**$M_{vr\text{-}bot}$, kip-in.**	**$\frac{P_r}{P_c}$**	***Spec.* Eq. H1-1a**	***Spec.* Eq. H1-1b**	**Interaction***
End	0.000	N/A	N/A	NA	N/A	N/A	N/A	N/A	N/A	N/A
1	1.53	0.000	92.4	0.631	26.9	122	0.093	0.696	**0.726**	0.726
2	3.93	0.000	81.3	0.555	65.8	108	0.226	**0.757**	0.711	0.757
3	6.32	0.000	70.2	0.479	101	92.9	0.346	**0.805**	0.689	0.805
4	8.72	0.000	59.0	0.403	131	78.2	0.452	**0.838**	0.660	0.838
5	11.1	0.000	47.9	0.327	158	63.4	0.544	**0.857**	0.624	0.857
6	13.5	0.000	36.7	0.251	181	48.6	0.621	**0.861**	0.581	0.861
7	15.9	0.000	25.6	0.175	199	33.9	0.684	**0.851**	0.530	0.851
8	18.3	0.000	14.5	0.099	213	19.1	0.733	**0.827**	0.473	0.827
9	20.7	0.000	3.32	0.023	223	4.40	0.766	**0.788**	0.408	0.788
10	23.1	0.000	0.000	0.000	228	0.000	0.785	**0.785**	0.393	0.785
Bm. CL	25.0	0.000	0.000	0.000	230	0.000	0.790	**0.790**	0.395	0.790
			I_{max}:	**0.631**					***I_{max}:***	**0.861**
ASD										
		Top Tee			**Bottom Tee**					
Opening Number	**X_i, ft**	**P_r, kips**	**$M_{vr\text{-}top}$, kip-in.**	**$\frac{M_{vr}}{M_c}$**	**P_r, kips**	**$M_{vr\text{-}bot}$, kip-in.**	**$\frac{P_r}{P_c}$**	***Spec.* Eq. H1-1a**	***Spec.* Eq. H1-1b**	**Interaction***
End	0.00	N/A	N/A	NA	N/A	N/A	N/A	N/A	N/A	N/A
1	1.53	0.000	65.7	0.674	19.0	87.0	0.098	0.743	**0.775**	0.775
2	3.93	0.000	57.9	0.594	46.3	76.6	0.239	**0.807**	0.759	0.807
3	6.32	0.000	50.0	0.513	70.8	66.2	0.366	**0.857**	0.735	0.857
4	8.72	0.000	42.2	0.433	92.3	55.8	0.477	**0.891**	0.704	0.891
5	11.1	0.000	34.3	0.352	111	45.5	0.574	**0.911**	0.666	0.911
6	13.5	0.000	26.5	0.272	127	35.1	0.655	**0.915**	0.620	0.915
7	15.9	0.000	18.6	0.191	140	24.7	0.721	**0.904**	0.567	0.904
8	18.3	0.000	10.8	0.111	149	14.3	0.772	**0.878**	0.505	0.878
9	20.7	0.000	2.97	0.03	156	3.93	0.807	**0.837**	0.436	0.837
10	23.1	0.000	0.000	0.000	160	0.000	0.827	**0.780**	0.360	0.780
Bm. CL	25.0	0.000	0.000	0.000	161	0.000	0.832	**0.723**	0.293	0.723
			I_{max}:	**0.674**					***I_{max}:***	**0.915**

* Reflects bold face value of controlling interaction equation.

Table 4-33. Horizontal Shear Force at Each Opening							
		ASD			LRFD		
Post Number	X_i, ft	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	$V_{ah} = \Delta T_r$, kips	$T_{r(i)}$, kips	$T_{r(i+1)}$, kips	$V_{uh} = \Delta T_r$, kips
1.00	2.73	19.0	46.3	27.3	26.9	65.8	38.9
2.00	5.13	46.3	70.8	24.5	65.8	101	35.2
3.00	7.52	70.8	92.3	21.5	101	131	30.0
4.00	9.92	92.3	111	18.7	131	158	27.0
5.00	12.3	111	127	16.0	158	181	23.0
6.00	14.7	127	140	13.0	181	199	18.0
7.00	17.1	140	149	9.00	199	213	14.0
8.00	19.5	149	156	7.00	213	223	10.0
9.00	21.9	156	160	4.00	223	228	5.00
			Maximum:	27.3		Maximum:	38.9

Calculate web post buckling flexural strength

LRFD	ASD
From Table 4-33, $V_{uh} = 38.9$ kips $M_u = 0.90\dfrac{D_o}{2}V_{uh}$ (from Eq. 3-31) $= 0.90\left(\dfrac{20.8\text{ in.}}{2}\right)(38.9\text{ kips})$ $= 363$ kip-in.	From Table 4-33, $V_{ah} = 27.3$ kips $M_a = 0.90\dfrac{D_o}{2}V_{ah}$ (from Eq. 3-31) $= 0.90\left(\dfrac{20.8\text{ in.}}{2}\right)(27.3\text{ kips})$ $= 255$ kip-in.

Calculate available flexural strength of web post

By inspection the top web post will control because the diameter of the web opening is the same as the bottom web post, but the web is thinner.

$$S_{x\text{-}webpost\text{-}top} = \frac{t_w\left(S - D_o + 0.564D_o\right)^2}{6} \tag{3-32}$$

$$= \frac{(0.350\text{ in.})\left[28.8\text{ in.} - 20.8\text{ in.} + 0.564(20.8\text{ in.})\right]^2}{6}$$

$$= 22.6\text{ in.}^3$$

$$M_e = S_{x\text{-}webpost\text{-}top}F_y \qquad \text{(from } Spec. \text{ Eq. F9-3)}$$

$$= \left(22.6\text{ in.}^3\right)(50\text{ ksi})$$

$$= 1{,}130\text{ kip-in.}$$

$$C1 = 5.097 + 0.1464\left(\frac{D_o}{t_w}\right) - 0.00174\left(\frac{D_o}{t_w}\right)^2 \tag{3-33}$$

$$= 5.097 + 0.1464\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right) - 0.00174\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right)^2$$

$$= 7.68$$

$$C2 = 1.441 + 0.0625\left(\frac{D_o}{t_w}\right) - 0.000683\left(\frac{D_o}{t_w}\right)^2 \tag{3-34}$$

$$= 1.44 + 0.0625\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right) - 0.000683\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right)^2$$

$$= 2.75$$

$$C3 = 3.645 + 0.0853\left(\frac{D_o}{t_w}\right) - 0.00108\left(\frac{D_o}{t_w}\right)^2 \tag{3-35}$$

$$= 3.645 + 0.0853\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right) - 0.00108\left(\frac{20.8\text{ in.}}{0.350\text{ in.}}\right)^2$$

$$= 4.91$$

$$\frac{M_{allow}}{M_e} = C1\left(\frac{S}{D_o}\right) - C2\left(\frac{S}{D_o}\right)^2 - C3 \tag{3-36}$$

$$= 7.68\left(\frac{28.8\text{ in.}}{20.8\text{ in.}}\right) - 2.75\left(\frac{28.8\text{ in.}}{20.8\text{ in.}}\right)^2 - 4.91$$

$$= 0.466$$

The available flexural strength is:

LRFD	ASD
From Equation 3-37a, $\phi_b\left(\frac{M_{allow}}{M_e}\right)M_e = 0.90(0.466)(1{,}130\text{ kip-in.})$ $= 474\text{ kip-in.}$	From Equation 3-37b, $\left(\frac{M_{allow}}{M_e}\right)\frac{M_e}{\Omega_b} = 0.466\left(\frac{1{,}130\text{ kip-in.}}{1.67}\right)$ $= 315\text{ kip-in.}$

Web Post Buckling Summary

LRFD	ASD
$\frac{M_u}{\phi_b\left(\frac{M_{allow}}{M_e}\right)M_e} = \frac{363\text{ kip-in.}}{474\text{ kip-in.}}$ $= 0.766 < 1.0$ **o.k.**	$\frac{M_a}{\left(\frac{M_{allow}}{M_e}\right)\frac{M_e}{\Omega_b}} = \frac{255\text{ kip-in.}}{315\text{ kip-in.}}$ $= 0.810 < 1.0$ **o.k.**

Check horizontal and vertical shear

The available horizontal shear strength is calculated using AISC *Specification* Section J4.2. By inspection, the top section will control because the web is thinner.

LRFD	ASD
From Table 4-33, $V_{uh} = 38.9\text{ kips}$ From *Spec*. Eq. J4-3, $\phi_v V_{n\text{-}horiz} = \phi_v 0.60F_y(et_w)$ $= 0.60(50\text{ ksi})[(8.00\text{ in.})(0.350\text{ in.})]$ $= 84.0\text{ kips} > 38.9\text{ kips}$ **o.k.**	From Table 4-33, $V_{ah} = 27.3\text{ kips}$ From *Spec*. Eq. J4-3, $\frac{V_{n\text{-}horiz}}{\Omega_v} = \frac{0.60F_y(et_w)}{\Omega_v}$ $= \frac{0.60(50\text{ ksi})[(8.00\text{ in.})(0.350\text{ in.})]}{1.50}$ $= 50.3\text{ kips} > 27.3\text{ kips}$ **o.k.**

Check vertical shear

The concrete shear strength will be disregarded when checking vertical shear for the net and gross sections. The concrete shear strength will be added to the net shear force.

Check vertical shear at the beam net section:

LRFD	ASD
From Table 4-31, $$V_{u\text{-}net} = 41.0 \text{ kips}$$ $$V_{u\text{-}global} = V_{u\text{-}net} + concrete\ shear\ strength$$ $$= 41.0 \text{ kips} + 7.39 \text{ kips}$$ $$= 48.4 \text{ kips}$$	From Table 4-31, $$V_{a\text{-}net} = 29.1 \text{ kips}$$ $$V_{a\text{-}global} = V_{a\text{-}net} + concrete\ shear\ strength$$ $$= 29.1 \text{ kips} + 4.93 \text{ kips}$$ $$= 34.0 \text{ kips}$$

The shear force between the top and bottom tees will be divided based on their relative areas.

LRFD	ASD
$$V_{u\text{-}top} = V_{u\text{-}global}\left(\frac{A_{tee\text{-}top}}{A_{net}}\right)$$ $$= (48.4 \text{ kips})\left(\frac{4.44 \text{ in.}^2}{10.5 \text{ in.}^2}\right)$$ $$= 20.5 \text{ kips}$$ $$V_{u\text{-}bot} = V_{u\text{-}global}\left(\frac{A_{tee\text{-}bot}}{A_{net}}\right)$$ $$= (48.4 \text{ kips})\left(\frac{6.01 \text{ in.}^2}{10.5 \text{ in.}^2}\right)$$ $$= 27.7 \text{ kips}$$	$$V_{a\text{-}top} = V_{a\text{-}global}\left(\frac{A_{tee\text{-}top}}{A_{net}}\right)$$ $$= (34.0 \text{ kips})\left(\frac{4.44 \text{ in.}^2}{10.5 \text{ in.}^2}\right)$$ $$= 14.4 \text{ kips}$$ $$V_{a\text{-}bot} = V_{a\text{-}global}\left(\frac{A_{tee\text{-}bot}}{A_{net}}\right)$$ $$= (34.0 \text{ kips})\left(\frac{6.01 \text{ in.}^2}{10.5 \text{ in.}^2}\right)$$ $$= 19.5 \text{ kips}$$

From AISC *Specification* Section G3:

Top tee:

$$\frac{h}{t_w} = \frac{d_{t\text{-}top\text{-}net}}{t_{w\text{-}top}}$$
$$= \frac{4.75 \text{ in.}}{0.350 \text{ in.}}$$
$$= 13.6 < 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} = 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$,

$$C_{v2} = 1.0 \qquad (Spec.\ \text{Eq. G2-9})$$

$$V_{n\text{-}top} = 0.60F_y\left(d_{t\text{-}top\text{-}net}t_{w\text{-}top}\right)C_{v2} \qquad (\text{from } Spec.\ \text{Eq. G3-1})$$
$$= 0.60(50 \text{ ksi})(4.75 \text{ in.})(0.350 \text{ in.})(1.0)$$
$$= 49.9 \text{ kips}$$

Bottom tee:

$$\frac{h}{t_w} = \frac{d_{t\text{-}bot\text{-}net}}{t_{w\text{-}bot}}$$
$$= \frac{4.95 \text{ in.}}{0.405 \text{ in.}}$$
$$= 12.2 < 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} = 29.0$$

Because $h/t_w < 1.10\sqrt{k_v E/F_y}$,

$$C_{v2} = 1.0 \qquad (\textit{Spec}. \text{ Eq. G2-9})$$

$$V_{n\text{-}bot} = 0.60F_y\left(d_{t\text{-}bot\text{-}net}t_{w\text{-}bot}\right)C_{v2} \qquad (\text{from } \textit{Spec}. \text{ Eq. G3-1})$$
$$= 0.60(50 \text{ ksi})(4.95 \text{ in.})(0.405 \text{ in.})(1.0)$$
$$= 60.1 \text{ kips}$$

Available vertical shear strength at top and bottom tees

LRFD	ASD
$\phi_v V_{n\text{-}top} = 1.00(49.9 \text{ kips}) = 49.9 \text{ kips}$ $\phi_v V_{n\text{-}bot} = 1.00(60.1 \text{ kips}) = 60.1 \text{ kips}$	$\frac{V_{n\text{-}top}}{\Omega_v} = \frac{49.9 \text{ kips}}{1.50} = 33.3 \text{ kips}$ $\frac{V_{n\text{-}bot}}{\Omega_v} = \frac{60.1 \text{ kips}}{1.50} = 40.1 \text{ kips}$

Check vertical shear at beam gross section

LRFD	ASD
$V_u = 44.1$ kips (see Table 4-28)	$V_a = 31.3$ kips (see Table 4-28)

From AISC *Specification* Section G2.1(b)(1):

$$\frac{h}{t_{w\text{-}min}} = \frac{30.5 \text{ in.} - (0.950 \text{ in.} + 1.15 \text{ in.})}{0.350 \text{ in.}}$$
$$= 81.1 > 1.10\sqrt{\frac{5.34(29{,}000 \text{ ksi})}{50 \text{ ksi}}} = 61.2$$

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad (\textit{Spec}. \text{ Eq. G2-4})$$
$$= \frac{1.10\sqrt{\frac{5.34(29{,}000 \text{ ksi})}{50 \text{ ksi}}}}{81.1}$$
$$= 0.755$$

$$V_{n\text{-}gross} = 0.60F_y\left(d_g t_{w\text{-}min}\right)C_{v1} \qquad (\textit{Spec}. \text{ Eq. G2-1})$$
$$= 0.60(50 \text{ ksi})(30.5 \text{ in.})(0.350 \text{ in.})(0.755)$$
$$= 242 \text{ kips}$$

LRFD	ASD
From Table 4-28, $V_{u\text{-}net} = 44.1 \text{ kips}$ $V_{u\text{-}global} = V_{u\text{-}net} + \textit{concrete shear strength}$ $= 44.1 \text{ kips} + 7.39 \text{ kips}$ $= 51.5 \text{ kips}$	From Table 4-28, $V_{a\text{-}net} = 31.3 \text{ kips}$ $V_{a\text{-}global} = V_{a\text{-}net} + \textit{concrete shear strength}$ $= 31.3 \text{ kips} + 4.93 \text{ kips}$ $= 36.2 \text{ kips}$

From AISC *Specification* Section G1:

$$\frac{h}{t_w} = 81.1 > 2.24\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 53.9$$

Therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

Available vertical shear strength at gross section

LRFD	ASD
$\phi_v V_{n\text{-}gross} = 0.90(242 \text{ kips})$ $= 218 \text{ kips}$	$\dfrac{V_{n\text{-}gross}}{\Omega_v} = \dfrac{242 \text{ kips}}{1.67}$ $= 145 \text{ kips}$

The following is a summary of the beam shear strengths:

LRFD	ASD
Horizontal shear $V_u/\phi_v V_{n\text{-}horiz} = 38.9 \text{ kips}/84.0 \text{ kips}$ $= 0.463$ **o.k.**	*Horizontal shear* $V_a \Omega_v/V_{n\text{-}horiz} = 27.3 \text{ kips}/50.3 \text{ kips}$ $= 0.543$ **o.k.**
Vertical shear–top tee $V_{u\text{-}top}/\phi_v V_{n\text{-}top} = 20.5 \text{ kips}/49.9 \text{ kips}$ $= 0.411$ **o.k.**	*Vertical shear–top tee* $V_{a\text{-}top}\Omega_v/V_{n\text{-}top} = 14.4 \text{ kips}/33.3 \text{ kips}$ $= 0.432$ **o.k.**
Vertical shear–bottom tee $V_{u\text{-}bot}/\phi_v V_{n\text{-}bot} = 27.7 \text{ kips}/60.1 \text{ kips}$ $= 0.461$ **o.k.**	*Vertical shear–bottom tee* $V_{a\text{-}bot}\Omega_v/V_{n\text{-}bot} = 19.5 \text{ kips}/40.1 \text{ kips}$ $= 0.486$ **o.k.**
Vertical shear–gross section $V_u/\phi_v V_{n\text{-}gross} = 44.1 \text{ kips}/218 \text{ kips}$ $= 0.202$ **o.k.**	*Vertical shear–gross section* $V_a \Omega_v/V_{n\text{-}gross} = 33.3 \text{ kips}/145 \text{ kips}$ $= 0.230$ **o.k.**

Check deflection

Deflections are calculated using 90% of the moment of inertia per Section 3.7.

The pre-composite dead load deflection is:

$$\Delta_{PDL} = \frac{5wL^4}{384EI_{x\text{-}net}(0.90)}$$

$$= \frac{5\left(\dfrac{0.44 \text{ kip/ft}}{12 \text{ in./ft}}\right)\left[(50 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})\left(1{,}710 \text{ in.}^4\right)(0.90)}$$

$$= 1.39 \text{ in.}$$

Live load deflection is:

$$\Delta_{LL} = \frac{5wL^4}{384EI_{x\text{-}comp}(0.90)}$$

$$= \frac{5\left(\dfrac{0.8 \text{ kip/ft}}{12 \text{ in./ft}}\right)\left[(50 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})\left(5{,}100 \text{ in.}^4\right)(0.90)}$$

$$= 0.845 \text{ in.}$$

$$= \frac{L}{710}$$

Dead load deflection is:

$$\Delta_{DL} = \frac{5wL^4}{384EI_{x\text{-}comp}(0.90)}$$

$$= \frac{5\left(\dfrac{0.16 \text{ kip/ft}}{12 \text{ in./ft}}\right)\left[(50 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})\left(5{,}100 \text{ in.}^4\right)(0.90)}$$

$$= 0.169 \text{ in.}$$

$$= \frac{L}{3{,}550}$$

Total load deflection is:

$$\Delta_{TL} = \Delta_{LL} + \Delta_{DL}$$

$$= 0.845 \text{ in.} + 0.169 \text{ in.}$$

$$= 1.01 \text{ in.}$$

$$= \frac{L}{590}$$

Deflection summary

$\Delta_{PDL} = 1.39$ in.; therefore, camber 1 in.

$\Delta_{LL} < \dfrac{L}{360}$ **o.k.**

$\Delta_{TL} < \dfrac{L}{240}$ **o.k.**

SYMBOLS

A Cross-sectional area, in.^{2} (mm^2)

A_c Area of concrete in compression, in.^{2} (mm^2)

A_{net} Combined area of top and bottom tees, in.^{2} (mm^2)

A_{tee} Area of tee section, in.^{2} (mm^2)

B Factor for lateral-torsional buckling in tee

C_v Web shear coefficient

C_1 Axial force in concrete of a composite section, kips (N)

D_o Opening diameter, in. (mm)

E Modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

ENA Elastic neutral axis

G Shear modulus of elasticity of steel = 11,200 ksi (77 200 MPa)

F_{cr} Critical stress, ksi (MPa)

F_{cry} Critical stress about the minor axis, ksi (MPa)

F_{crz} Critical torsional buckling stress, ksi (MPa)

F_e Elastic critical buckling stress, ksi (MPa)

H Flexural constant

I_x Moment of inertia about x-axis, in.^{4} (mm^4)

I_y Moment of inertia about y-axis, in.^{4} (mm^4)

J Torsional constant, in.^{4} (mm^4)

K_x Effective length factor with respect to x-axis

K_y Effective length factor with respect to y-axis

L Length of compression member, in. (mm)

L_b Distance between lateral braces, in. (mm)

M_c Allowable flexural strength (ASD), kip-in. (N-mm)

M_c Design flexural strength (LRFD), kip-in. (N-mm)

M_{cr} Nominal flexural strength based on lateral-torsional buckling limit state, kip-in. (N-mm)

M_e Elastic bending moment of web post, kip-in. (N-mm)

M_m Nominal flexural strength based on flange local buckling limit state, kip-in. (N-mm)

M_n Nominal flexural strength, kip-in. (N-mm)

M_{ocr} Critical moment for lateral buckling, kip-in. (N-mm)

M_p Plastic bending moment, kip-in. (N-mm)

M_r Required flexural strength using load combinations, kip-in. (N-mm)

M_{vr} Required flexural strength in tee, kip-in. (N-mm)

N Number of shear studs between the point of maximum moment and end of beam

N_s Total number of studs across the length of the beam

P_c Allowable axial compressive strength (ASD), kips (N)

P_c Design axial compressive strength (LRFD), kips (N)

P_n Nominal axial compressive strength (LRFD), kips (N)

PNA Plastic neutral axis

P_r Required axial strength of tee using load combinations, kips (N)

Q_n Nominal strength of one stud shear connector, kips (N)

R Radius of cellular opening, in. (mm)

S Spacing of openings, in. (mm)

S_x Elastic section modulus about x-axis, in.^{3} (mm^3)

$S_{x\text{-}tee}$ Section modulus of tee about x-axis, in.^{3} (mm^3)

T_i Axial force at centerline of opening (i), kips (N)

T_{i+1} Axial force at centerline of opening ($i + 1$), kips (N)

$T_{r(i)}$ Required axial force in tee at opening (i), kips (N)

$T_{r(i+1)}$ Required axial force in tee at opening ($i + 1$), kips (N)

T_o Axial force in top tee, kips (N)

T_1 Axial force in bottom tee, kips (N)

$T_{1\text{-}new}$ Axial force in bottom tee for partial composite action, kips (N)

$T_{u(i)}$ Axial force in tee at opening (i) (LRFD), kips (N)

$T_{u(i+1)}$ Axial force in tee at opening ($i + 1$) (LRFD), kips (N)

V Global shear, kips (N)

V' Total horizontal shear force between point of max positive moment and the point of zero moment, kips (N)

V_{allow} Allowable horizontal shear strength (ASD), kips (N)

$V_{a\text{-}global}$ Service shear force (ASD), kips (N)

$V_{a\text{-}global}$ Required global shear strength (ASD), kips (N)

$V_{a\text{-}net}$ Net service shear force resisted by beam (ASD), kips (N)

V_c Shear strength of concrete deck, kips (N)

V_h Horizontal shear force at neutral axis, kips (N)

V_{ha} Required horizontal shear force at neutral axis (ASD), kips (N)

V_{hu} Required horizontal shear force at neutral axis (LRFD), kips (N)

V_i Global shear force at opening (i), kips (N)

V_{i+1} Global shear force at opening ($i + 1$), kips (N)

V_n Nominal shear strength, kips (N)

V_{net} Net shear force resisted by beam, kips (N)

$V_{u\text{-}global}$ Ultimate shear force (LRFD), kips (N)

$V_{u\text{-}global}$ Required global shear strength (LRFD), kips (N)

$V_{u\text{-}net}$ Net ultimate shear force (LRFD), kips (N)

$V_{u(i)}$ Required shear strength at opening (i), kips (N)

$V_{u(i+1)}$ Required shear strength at opening ($i + 1$), kips (N)

X_i Distance from end of beam to center of the opening being analyzed, in. (mm)

Y_c Depth of concrete used to resist global moment, in. (mm)

Z_x Plastic section modulus about x-axis, in.^{3} (mm^3)

a Length of end web post, in. (mm)

b Horizontal length $= 0.5h_o/\tan\theta$, in. (mm)

b_{effec} Effective width of concrete slab, in. (mm)

b_f Flange width, in. (mm)

d Full nominal depth of tee, in. (mm)

d_{effec} Distance between centroids of top and bottom tees, in. (mm)

$d_{effect\text{-}comp}$ Effective depth of composite section, in. (mm)

d_g Depth of expanded beam, in. (mm)

d_t Depth of tee, in. (mm)

e Length of tee section, also length of solid web section along centerline, in. (mm)

e' Minimum diagonal distance from the corner of the cope to the first opening, in. (mm)

f'_c Compressive strength of concrete, ksi (MPa)

h Half height of castellated opening, in. (mm)

h_o Height of opening of castellated beam, in. (mm)

h_r Height of deck ribs, in. (mm)

i Reference number for castellated or cellular opening

k_v Web plate buckling coefficient

q Shear stud density, kip/ft (N/mm)

r_{min} Minimum radius of gyration of tee, in. (mm)

r_o Polar radius of gyration about the shear center, in. (mm)

r_x Radius of gyration about x-axis, in. (mm)

r_y Radius of gyration about y-axis, in. (mm)

t_c Thickness of concrete above deck ribs, in. (mm)

t_f Flange thickness, in. (mm)

y_c Distance from top of concrete to centroid of compression block, in. (mm)

$\bar{y}_{tee\text{-}bot}$ Distance from bottom fiber to centroid of bottom tee, in. (mm)

$\bar{y}_{tee\text{-}top}$ Distance from top fiber to centroid of top tee, in. (mm)

w_o $e + 2b$, in. (mm)

Δ_{DL} Dead load deflection

Δ_{LL} Live load deflection

Δ_{PDL} Pre-dead load deflection

Δ_{TL} Total load deflection

ϕ_b Resistance factor for flexure

ϕ_c Resistance factor for compression

ϕ_t Resistance factor for tension

ϕ_v Resistance factor for shear

Ω_b Safety factor for flexure

Ω_v Safety factor for shear

θ Angle of hexagonal cut, degrees

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