

Math 242 Chapter 4/Section 1-3

Topics: Probability Basics and Rules of Probability

Define the following terms:

1. Probability for Equally Likely Outcomes (f/N Rule)

2. Experiment

3. Event

4. Sample Space

5. State the 3 basic properties of probabilities

6. Mutually Exclusive Events

7. State the rule for $P(A \text{ or } B)$

8. State the Complementation Rule

Solve the following problems:

1. Which of the following numbers could not possibly be a probability? Justify your answer.
a. $\frac{3}{4}$ b. 1.2 c. 0 d. 1 e. $\frac{5}{4}$ f. 0.2

2. An experiment has 50 possible outcomes, all equally likely. An event can occur in 3 ways. What is the probability that the event occurs?

3. Given a standard playing cards, find the following probability.
 - a. Getting an ace

 - b. Getting a heart

 - c. Getting an ace and a heart

 - d. Getting an ace or a heart

4. Flipping a coin 3 times, find the following probability.
 - a. Exactly 2 heads

b. At least 2 heads

c. All 3 heads

5. Construct a Venn diagram representing the following event

a. $A \& B$

b. $A \text{ or } B$

c. $A \text{ and } B \text{ and not } C$

d. $(\text{Not } A) \& B$

6. Rolling a dice twice, find the following probability

a. 6 does not appear

b. At least one 6

c. The sum is greater than 10

d. The sum is less than or equal to 10

7. Suppose that A and B are mutually exclusive events such that $P(A)=0.3$ and $P(B)=0.4$. Determine $P(A \text{ or } B)$.

8. Suppose that A and B are events such that $P(A) = 1/5$, $P(A \text{ or } B) = 1/3$ and $P(A \& B) = 1/10$.

a. Find $P(B)$

b. Are events A and B mutually exclusive? Justify your answer.

Math 242 Chapter 4/Section 1-3

Topics: Probability Basics and Rules of Probability

Define the following terms:

1. Probability for Equally Likely Outcomes (f/N Rule)

Suppose an experiment has N possible outcomes, all equally likely. An event that can occur in f ways has probability $\frac{f}{N}$ of occurring. In other words, probability of an event = $\frac{f}{N}$ where f represents number of ways event can occur and N represents total number of possible outcomes.

2. Experiment

An action whose outcome cannot be predicted with certainty.

3. Event

The collection of all possible outcomes for an experiment.

4. Sample Space

A collection of outcomes for the experiment. Any subset of the sample space.

5. State the 3 basic properties of probabilities

Property 1: The probability of an event is always between 0 and 1

Property 2: The probability of an event that cannot occur is 0

Property 3: The probability of an event that must occur is 1

6. Mutually Exclusive Events

Two or more events are mutually exclusive if no two of them have outcomes in common. In other words $P(A \text{ and } B) = 0$ if A and B are mutually exclusive.

7. State the rule for $P(A \text{ or } B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, if A and B are mutually exclusive, then

$P(A \text{ or } B) = P(A) + P(B)$.

8. State the Complementation Rule

For any event E , $P(E) = 1 - P(\text{not } E)$.

Solve the following problems:

1. Which of the following numbers could not possibly be a probability? Justify your answer.

- a. $3/4$ b. 1.2 c. 0 d. 1 e. $5/4$ f. 0.2

B and E are not possible. According to the 3 basic properties of probability, the probability of an event is always between 0 and 1. B and E are both greater than 1 therefore they could not possibly be a probability.

2. An experiment has 50 possible outcomes, all equally likely. An event can occur in 3 ways. What is the probability that the event occurs?

According to f/N rule, probability = $3/50$ since there are 3 ways of an event can occur and there are 50 possible outcomes.

3. Given a standard playing cards, find the following probability.

- a. Getting an ace

$P(\text{Getting an ace}) = 4/52 = 1/13$ since there are 4 aces in a deck of cards and there are 52 cards total.

- b. Getting a heart

$P(\text{Getting a heart}) = 13/52 = 1/4$ since there are 13 hearts in a deck and 52 cards total.

- c. Getting an ace and a heart

Since there is only one ace of hearts, $P(\text{Getting an ace and a heart}) = 1/52$.

- d. Getting an ace or a heart

$P(\text{Getting an ace or a heart}) = P(\text{Getting an ace}) + P(\text{Getting a heart}) - P(\text{Getting an ace and a heart})$ according to the rule for $P(A \text{ or } B)$.

$$\text{Therefore } P(\text{Getting an ace or a heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

4. Flipping a coin 3 times, find the following probability.

- a. Exactly 2 heads

The sample space is $\{TTT, TTH, THT, HTT, HHT, HTH, THH, HHH\}$. There are 8 outcomes with the events exactly 2 heads occurring 3 times (THH, HHT, HTH). Therefore $P(\text{Exactly 2 heads}) = 3/8$.

b. At least 2 heads

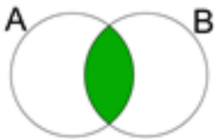
From part a, there are 8 outcomes with the event at least 2 heads occurring 4 times (THH, HHT, HTH, HHH), Therefore $P(\text{At least 2 heads}) = 4/8 = 1/2$

c. All 3 heads

From part a, there are 8 outcomes with the event all 3 heads occurring 1 time. Therefore $P(\text{All 3 heads}) = 1/8$

5. Construct a Venn diagram representing the following event

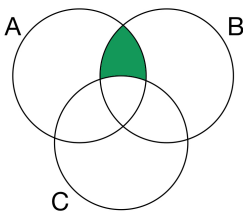
a. A & B



b. A or B



c. A and B and not C



d. (Not A) & B



6. Rolling a dice twice, find the following probability

a. 6 does not appear

There are 36 total outcomes. Since 6 appears 11 times (1-6, 2-6, 3-6, 4-6, 5-6, 6-6, 6-5, 6-4, 6-3, 6-2, 6-1), $P(6 \text{ appears}) = 11/36$. Therefore by complementation rule $P(6 \text{ does not appear}) = 1 - 11/36 = 25/36$

b. At least one 6

From part a, $P(\text{at least one } 6) = 11/36$.

c. The sum is greater than 10

First notice that the largest sum possible is 12 (6+6) so we're finding the probability of sum is either 11 or 12. Therefore there are 3 ways to get the sum that is greater than 10 (5-6, 6-5, 6-6). Therefore $P(\text{Sum is greater than } 10) = 3/36 = 1/12$

d. The sum is less than or equal to 10

By part c, using the complementation rule $P(\text{Sum is less than or equal to } 10) = 1 - P(\text{Sum} > 10) = 1 - 1/12 = 11/12$

7. Suppose that A and B are mutually exclusive events such that $P(A)=0.3$ and $P(B)=0.4$. Determine $P(A \text{ or } B)$.

$P(A \text{ or } B) = P(A) + P(B)$ since A and B are mutually exclusive. Therefore $P(A \text{ or } B) = 0.3 + 0.4 = 0.7$

8. Suppose that A and B are events such that $P(A) = 1/5$, $P(A \text{ or } B) = 1/3$ and $P(A \& B) = 1/10$.

a. Find $P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Therefore

$$\frac{1}{3} = \frac{1}{5} + P(B) - \frac{1}{10}$$

$$P(B) = \frac{1}{3} - \frac{1}{5} + \frac{1}{10} = \frac{7}{30}$$

b. Are events A and B mutually exclusive? Justify your answer.

No, A and B are not mutually exclusive since $P(A \text{ and } B)$ does not equal to 0.