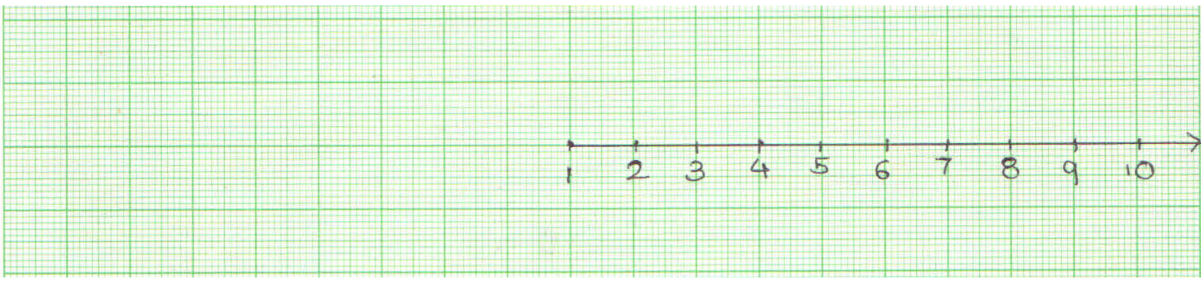
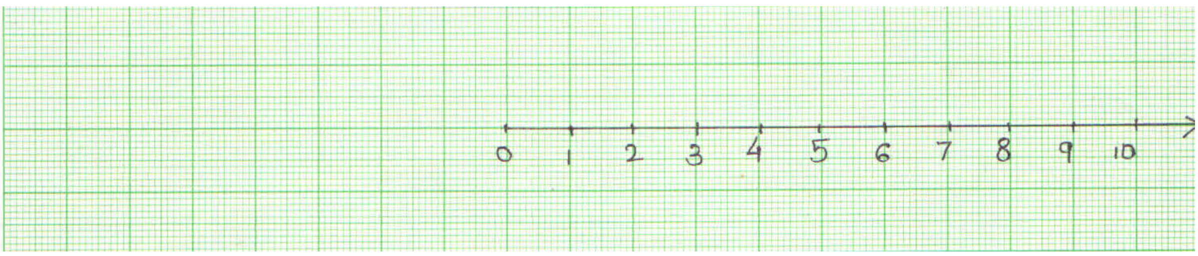
Stepping stones for Number systems

1) **Concept of a number line :**
    - Marking using sticks on the floor. (1 stick length = 1 unit)

2) **Counting numbers: 1,2,3,... Natural numbers**
    - Represent natural numbers on a number line.
    - Set of natural numbers = N = {1,2,3,4,5,6,...}
    - Discuss : Which operations done on two natural numbers give the answer as a natural number. (Addition and Multiplication)

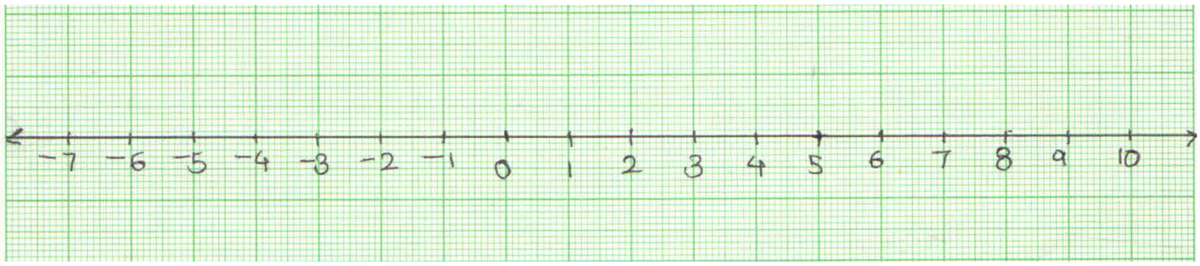


3) Add zero to the set of natural numbers : **0,1,2,3,... : Whole numbers**
    - What happens when we subtract a natural number from itself? (Zero)
    - Add this to the set of natural numbers and you get Whole numbers.
    - Set of whole numbers = W = {0,1,2,3,4,...}
    - Represent whole numbers on a number line.
    - Discuss : Which operations done on two whole numbers give the answer as a whole number. (Addition and Multiplication)



4) Add negative numbers to the set of whole numbers: ...,-3,-2,-1,0,1,2,3... : **Integers**

- What happens when we subtract a larger whole number from a smaller whole number, or we go below zero, or we create loan etc (negative numbers)
- Set of Integers =Z = {...,-4,-3,-2,-1,0,1,2,3,4...}
- Represent integers on the number line.

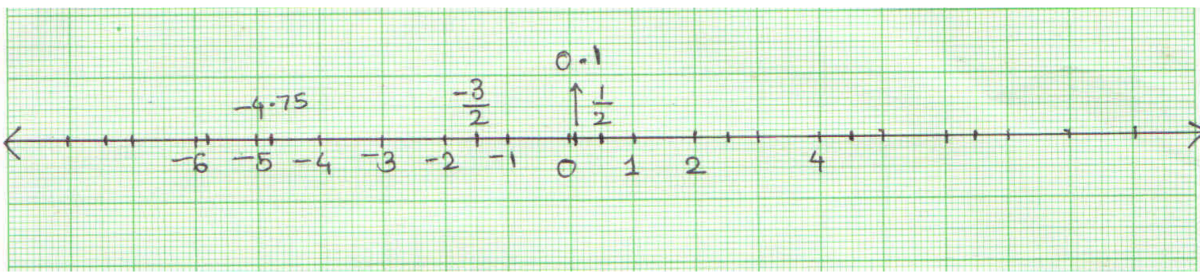


5) Add fractions to the set of Integers : **Rational numbers :**
   - When we divide an integer by another integer, sometimes we get a fraction.
   - Add fractions to the set of integers and you get rational numbers.
   - Defined as p/q where p and q are integers and q is non zero.
   - Is 3 a rational number? Yes, can be written in p/q as 3/1 or 6/2 etc.
   - Is -25 a rational number? Yes, can be written as -25/1 or -50/2 etc.
   - Is 0.3333... (3 repeating) a rational number? Yes, can be written as 1/3.
   - Rational numbers, when written in decimal form, either terminate or repeat.

     e.g. ¾ = 0.75 (terminates)

     2/3 = 0.666.. (repeats)
   - Rational numbers on a number line :

     Here we show some of the rational numbers.



Some numbers are shown. Writing the numbers at each of the marking.

Mark the following numbers :

1) 5 ½ 2) 7.5 3) 8.3 4) 12/2 5) -14/2

6) **Write the following numbers in the form of p/q :**

2 = 2/1 , 2 = 4/2 , 2 = 6/3 ….

1 = 1/1 1 = 5/5 1 = 6/6…

1.5 = 3/2 1.5 = 15/10

-5 = -5/1 -5 = -10/2 -5 = -15/3

0 = 0/1 0 = 0/2 0 = 0/5

How do you convert 0.3333... into fractional representation?

x = 0.3333...
10x= 3.3333...
10x – x = 3
9x = 3
x = 3/9 or 1/3

How do you convert 0.121212... into fractional form?
x = 0.121212...
100x = 12.121212...
99x = 12
X = 12/99 = 4/33

7) **Activity on graph paper –**

- Draw A number line (X axis)
- Mark points 0, 1, 3
- Find the midpoints of following line segments
    - 1 and 3 → 2

- 1 and 5 → 3
- 5 and 7 → 6

- What is the relationship? Is that true always?
- Find the midpoint of 1 and 2? 1.5
- What is the rule? How will you put it in formula? (a+b)/2
- This means that between any two numbers we have another number.
- Let's take two rational numbers 999/1000 and 998/1000 are very close to each other. Find a number between these two numbers.
- Does it mean that whole line is made up of rational numbers?

ALL RATIONAL NUMBERS CAN BE DRAWN ON A NUMBER LINE.

8) Irrational numbers :

Geometrically draw √2. (Use Pythagoras theorem)

- Draw a number line.
- Plot 0 and 1
- You have a segment of length 1 cm.
- Draw a vertical 1 cm segment at point 1
- Join the hypotenuse to make a triangle.
- Length of hypotenuse is √2
- With the help of compass, take this distance and mark it on number line from 0.
- You get a point which is representing √2
- It means √2 exists on a number line.

What is special about numbers like √2, √3, √5 ?

When we try to write them in the form of decimal, they neither terminate, nor repeat. It means they cannot be expressed as p/q where p and q both are integers and q is non-zero.

If we try to write $\sqrt{2}$ as $\sqrt{2}$ /1 for writing as p/q, the p is not integer.

We may also think that $\sqrt{2}$ is 1.41, so we can write it as 141/100, that is not right.

To find why, ask students to find the value of $\sqrt{2}$ by using a calculator, without using √ function on the calculator. So, we have to find a number and multiply by itself to get 2.

e.g. Try 1.5 x 1.5

It is 2.25, which is more that 2. Therefore take a smaller number.

Try 1.4 x 1.4 = 1.96

Smaller than 2. So take a bigger number.

1.45 x 1.45 = 2.1025

1.42 x 1.42 = 2.0164

1.41 x 1.41 = 1.9881

Go on trying.. you reach close to 2, but not exactly at 2.

Therefore $\sqrt{2}$ is not a rational number.

It is called irrational number.

Examples of irrational numbers :

e. g $\sqrt{2}$, 0.30300300030000... , π, $-\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ इ.

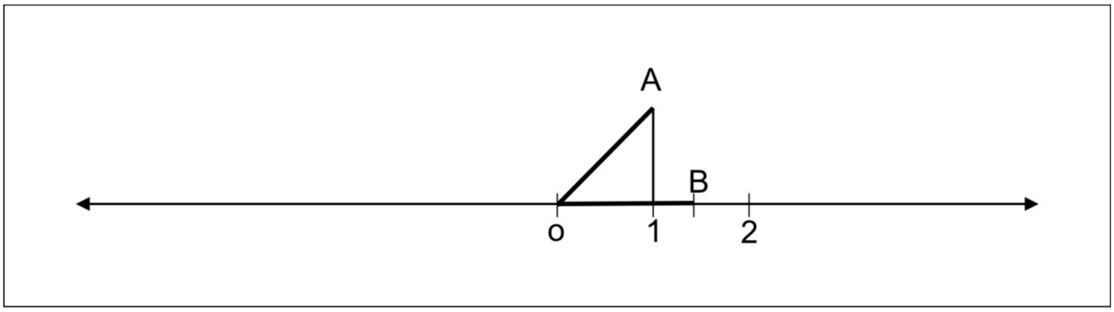
Such numbers are called irrational numbers.

π is another such irrational number.

There are numbers on number line which are irrational.

9) Showing irrational numbers on a number line geometrically :

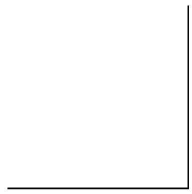
$\sqrt{2}$



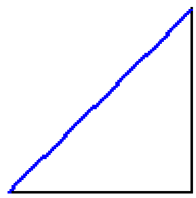
It might seem that the rational numbers would cover any possible number. After all, if I measure a length with a ruler, it is going to come out to some fraction—maybe 2 and 3/4 inches. Suppose I then measure it with more precision. I will get something like 2 and 5/8 inches, or maybe 2 and 23/32 inches. It seems that however close I look it is going to be *some* fraction. However, this is not always the case.

Imagine a line segment exactly one unit long:

Now draw another line one unit long, perpendicular to the first one, like this:



Now draw the diagonal connecting the two ends:

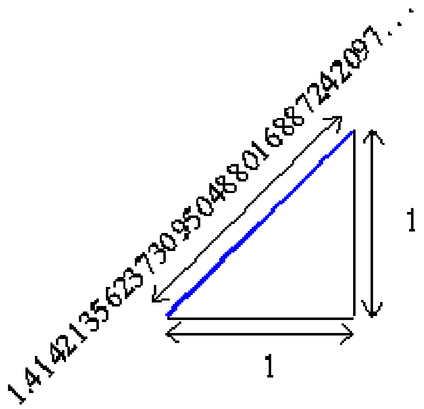


Congratulations! You have just drawn a length that cannot be measured by any rational number. According to the Pythagorean Theorem, the length of this diagonal is the square root of 2; that is, the number which when multiplied by itself gives 2.

According to my calculator,

$$\sqrt{2} = 1.41421356237$$

But my calculator only stops at eleven decimal places because it can hold no more. This number actually goes on forever past the decimal point, without the pattern ever terminating or repeating.



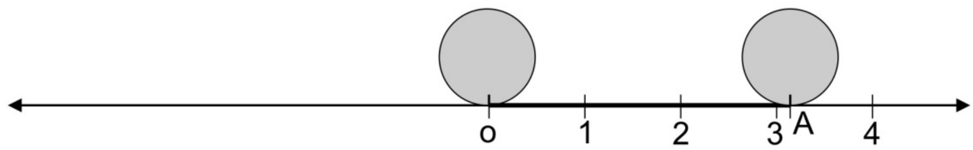
This is because if the pattern ever stopped or repeated, you could write the number as a fraction—and it can be proven that the square root of 2 can never be written as

$$\sqrt{2} = \frac{a}{b}$$

for *any* choice of integers for *a* and *b*. The proof of this was considered quite shocking when it was first demonstrated by the followers of Pythagoras 26 centuries ago.

Drawing pi on number line

Take a circle. Assume that its diameter is 1 unit. Draw a number line with that 1 as a unit. Now make a mark at one point on the circumference of the circle. Place this mark at 0 of number line. Rotate the circle till the marked point again meets the number line. This point is pi.



10) Add irrational numbers to the set of Rational numbers : **Real numbers**

Real numbers represent all numbers on number line.

$\frac{3}{4} = 0.75$	Rational (terminates)
$\frac{2}{3} = 0.66666\overline{6}$	Rational (repeats)
$\frac{5}{11} = 0.454545\overline{45}$	Rational (repeats)
$\frac{5}{7} = 0.714285\overline{714285}$	Rational (repeats)
$\sqrt{2} = 1.41421356...$	Irrational (never repeats or terminates)
$\pi = 3.14159265...$	Irrational (never repeats or terminates)

Rational + Irrational = Real numbers

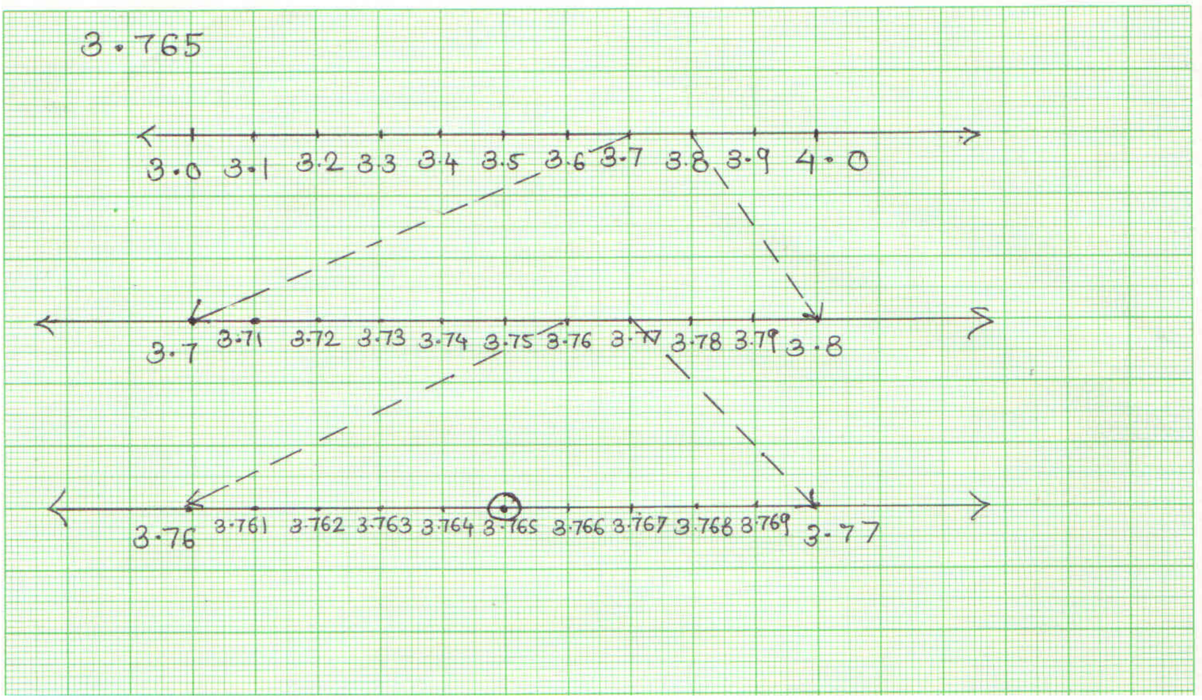
All points on a number line. All distances on the number line.

The Real Numbers

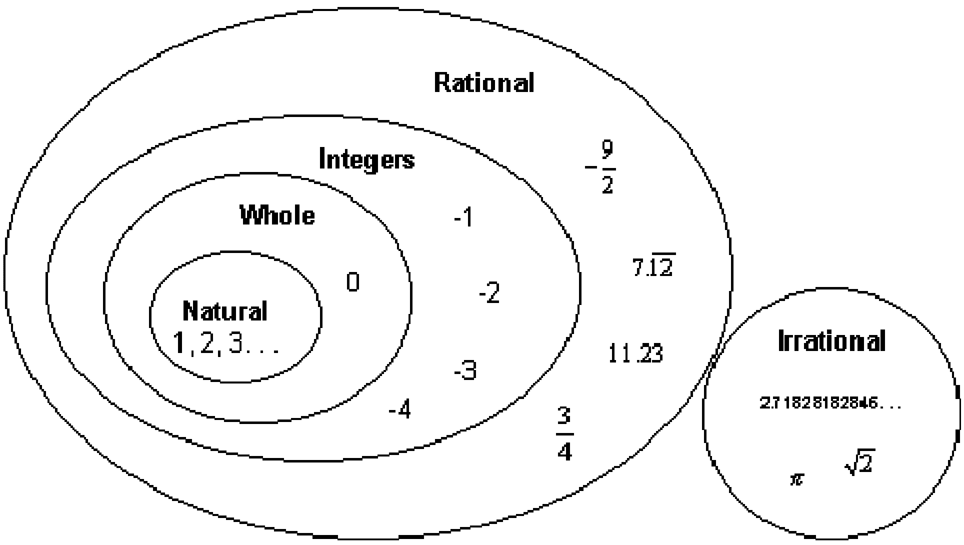
- Rationals + Irrationals
- All points on the number line
- Or all possible distances on the number line

When we put the irrational numbers together with the rational numbers, we finally have the complete set of real numbers. Any number that represents an amount of something, such as a weight, a volume, or the distance between two points, will always be a real number. The following diagram illustrates the relationships of the sets that make up the real numbers.

Q 11. Visualise 3.765 on the number line using successive magnification.



Summary chart :



Question Bank - Understanding of Number System

Rational and Irrational numbers:

Q 1. Write 1.2 in the form p/q

Q 2. Write 1.25 in the form p/q

Q 3 Write 1.3333 in the form p/q

Q 4 Write 1/3 as a repeating decimal

Q 5 Write 1/7 as a repeating decimal

Q 6 Which of the following is a rational number ?

$\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$.

Q 7 a. Is 1/7 a rational number ?

b. Is 2/5 a rational number ?

c. Is 1/7 x 2/5 a rational number ?

d. Is 1/7 / 2/5 a rational number ?

e. Is 1/7 + 2/5 a rational number ?

f. Is 1/7 – 2/5 a rational number ?

Q 8 Say whether true or false

a. Every whole numbers is a natural number.
b. Every integer is a rational number.
c. Every rational number is an integer.
d. The sum of two rational numbers is a rational number.
e. If I divide a rational number by a rational number the answer is always a rational number
f. The square of a rational number is always a rational number.
g. The square root of a rational number is always a rational number.

Q 9 Say whether true or false

a. If lengths of two sides of a rectangle are rational numbers, then the diagonal is also a rational number.
b. If lengths of two sides of a rectangle are rational numbers , then the perimeter length is always rational
c. The perimeter of a circle of radius 4 cm is not a rational number

Q 10 Give two examples of numbers which are not rational numbers.

Q 11 On graph paper, show a length which is a rational number.

Q 12 On the same graph paper show a length which is not a rational number.

Real Numbers

Q 1. Say whether true or false

a. Every point on the number line represents a real number
b. Rational numbers are real numbers
c. Numbers which are not rational numbers are also not real numbers
d. The diagonal of a rectangle which has sides 1 cm and 2 cm is not a real number
e. The length of the perimeter of a circle which has radius length of 4 cm is not a real number.
f. The square root of a real number is always a real number.
g. All numbers which can be represented in the form of a decimal is a real number

Q 2. a. Is root 2 a real number ? b. Is root 2 multiplied by root 3 real number ?

Q 3 Give an example of a real number.

Q 4. Give an example of a number which is not a real number.

Q 5. Some numbers are given in the following table. Decide the types of each of these numbers and put tick marks accordingly.

Number	Natural	Whole	Integer	Rational	Irrational	Real	Not real

5							
-5							
1							
0							
1/2							
-1/2							
6/2							
-6/2							
7/3							
-7/3							
4.2							
-4.2							
— 2.4141							
— -2.4141							
√**4**							
√**3**							

-√3							
π = 3.14159..							
-π = - 3.14159..							
√1							
√-1							
√-3							

Q 6 On a graph paper draw two lengths which represent numbers which are real numbers but which are not integers.

Q 7 On a graph paper draw two lengths which represent numbers which are real numbers which are not rational numbers

Q 8 On a graph paper I can draw a length which represents a length which is not a real number. Is this True or False ? If True, draw the length on graph paper.

Q 9. Express 7/3 as a decimal number.

Q 10. Express 1/3 as a decimal number.

Operations on real numbers :

Basics :

- a + a + a = a taken 3 times = 3 x a = 3a
- y + y + y =
- $x + x + x + x =$
- $5m =$
- $6n =$
- $1x =$
- $\mathcal{X} =$
- Identify like terms that can be added to each other :
  2 x 3y 4 x y 5y
- Mangoes can be added to mangoes and apples can be added to apples.
  - 3 mangoes + 2 mangoes =
  - 3 mangoes + 2 apples =
  - 3 mangoes + 2 apples + 1 mango + 3 apples =
  - 3 x + 2 x =
  - 3 x + 2 y + 2 x + 4 y =
  - $3\sqrt{2} + 2\sqrt{2} =$
  - $3\sqrt{2} + 5\sqrt{2} =$
  - $3\sqrt{2} + 2\sqrt{5} =$
- Revise addition, subtraction, multiplication and division of positive and negative numbers. (from Algebra module)
- Revise addition, subtraction, multiplication and division of positive and negative terms. (from Algebra module)
- Revise rules of indices.

1) Solve the problems on operations of real numbers from class 9 and 10 textbook.

2) Drawing a segment of given irrational length (e.g $\sqrt{3.5}$) should be done with constructions.