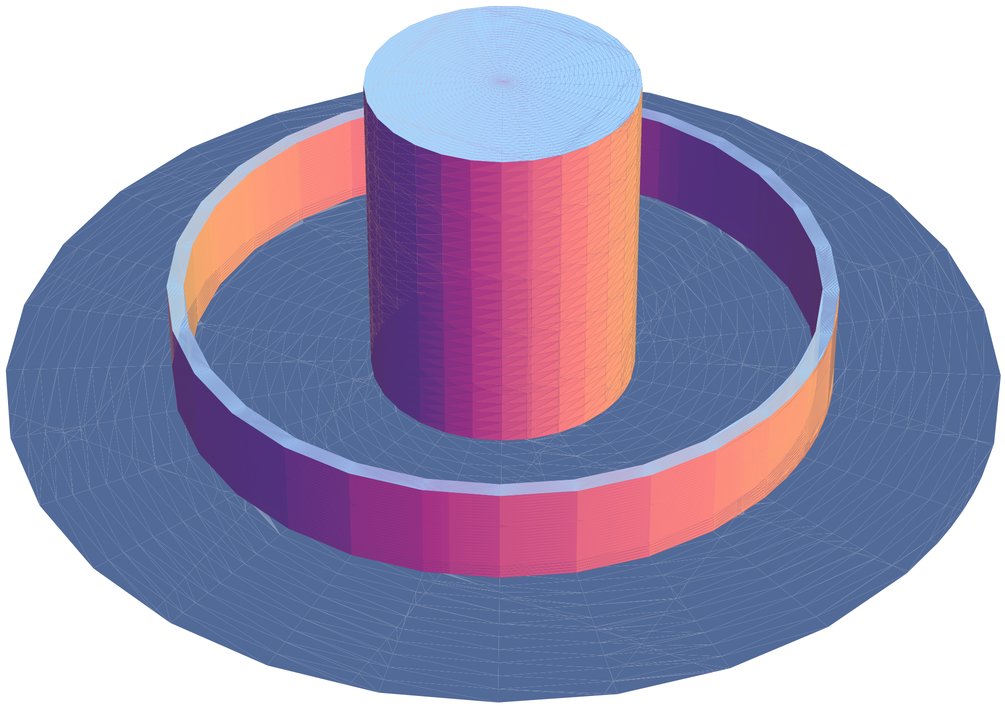
Storage Tank Dike Design



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Purdue University
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Tank Rupture Consequences

- Molasses tank, Boston MA, 1919, 2.5×10^6 gallons, 21 fatalities
- Water tank, Juarez, MX, 1986, 7.5×10^5 gallons, 4 fatalities
- Oil tank, Floreffe PA, 1988, 3.8×10^6 gallons
- Molasses tank, Loveland CO, 1990, 6.2×10^5 gallons
- HCl tank, McDonald, PA, 2002, 10^4 gallons

Floreffe Aftermath

Regulatory Requirements

- Secondary containment volume must be sufficient to hold the contents of the tank (EPA/OSHA).

- Fire code constraints

- Dynamics <u>not</u> considered!

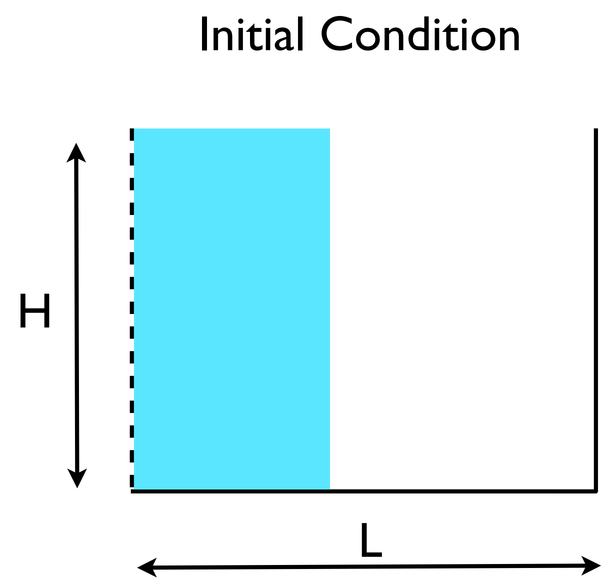
Design Considerations

- Dike height
- Dike diameter
- Dike shape

- Complete tank rupture
- Small hole in tank wall

Rupture Flow Scales

- Height: $H \sim 10$ m
- Distance: $L \sim 10$ m
- Velocity: $(gH)^{1/2} \sim 10$ m/s
- Time: $L/(gH)^{1/2} \sim 1$ s
- Reynolds number: $Re \sim 10^8$

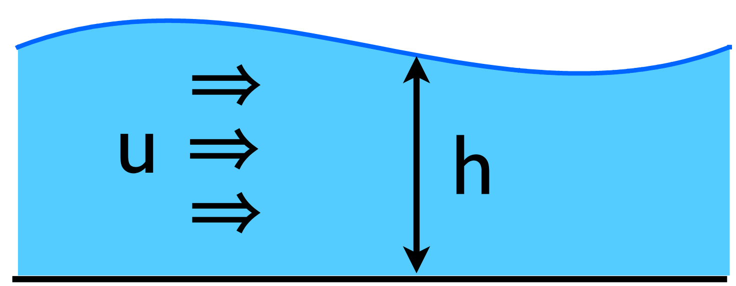


Shallow-Water Theory

- Depth-averaged inviscid equations of motion
- Exact mass balance
- Hydrostatic pressure
- Analogous to compressible gas dynamics

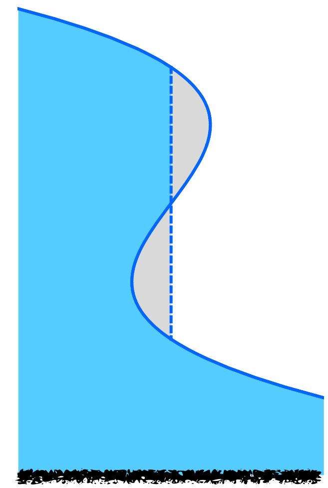
$$\dot{h} + \nabla \bullet (hu) = 0$$

$$\dot{u} + u \bullet \nabla u + \nabla h = 0$$



Shock Conditions

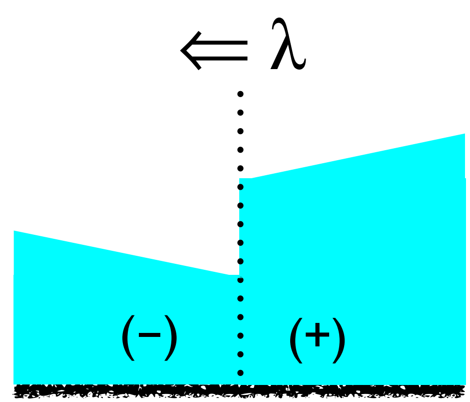
- Multi-valued SWT solutions



- Supplemental jump equations

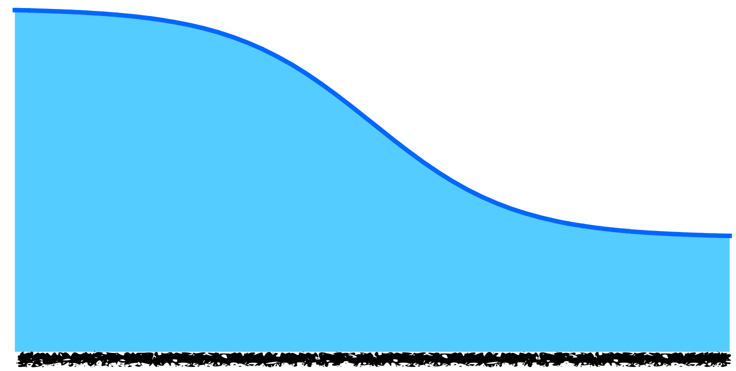
$$\left(u^{+}-\lambda\right)h^{+}-\left(u^{-}-\lambda\right)h^{-}=0$$

$$\left(u^{+}-\lambda\right)^{2}h^{+}-\left(u^{-}-\lambda\right)^{2}h^{-}+\frac{1}{2}\left((h^{+})^{2}-(h^{-})^{2}\right)=0$$

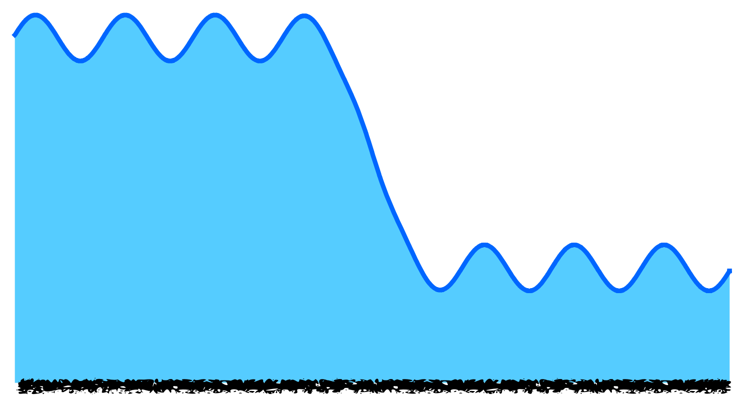


Numerical Pitfalls

- Diffusive Errors



- Dispersive Errors



Computational Methods

- Method of Characteristics
- Explicit Finite-Difference (Lax, FCT, WENO)
- Implicit Finite Element (DG, SUPG)
- Projection Methods (POD)

Method of Characteristics

$$\phi = \begin{Bmatrix} h \\ u \end{Bmatrix} \quad \Rightarrow \quad \mathbf{A} \bullet \frac{\partial \phi}{\partial t} + \mathbf{B} \bullet \frac{\partial \phi}{\partial x} + \psi = 0$$

Eigenvalue: $\lambda \left(\xi \bullet \mathbf{A} \right) = \xi \bullet \mathbf{B}$

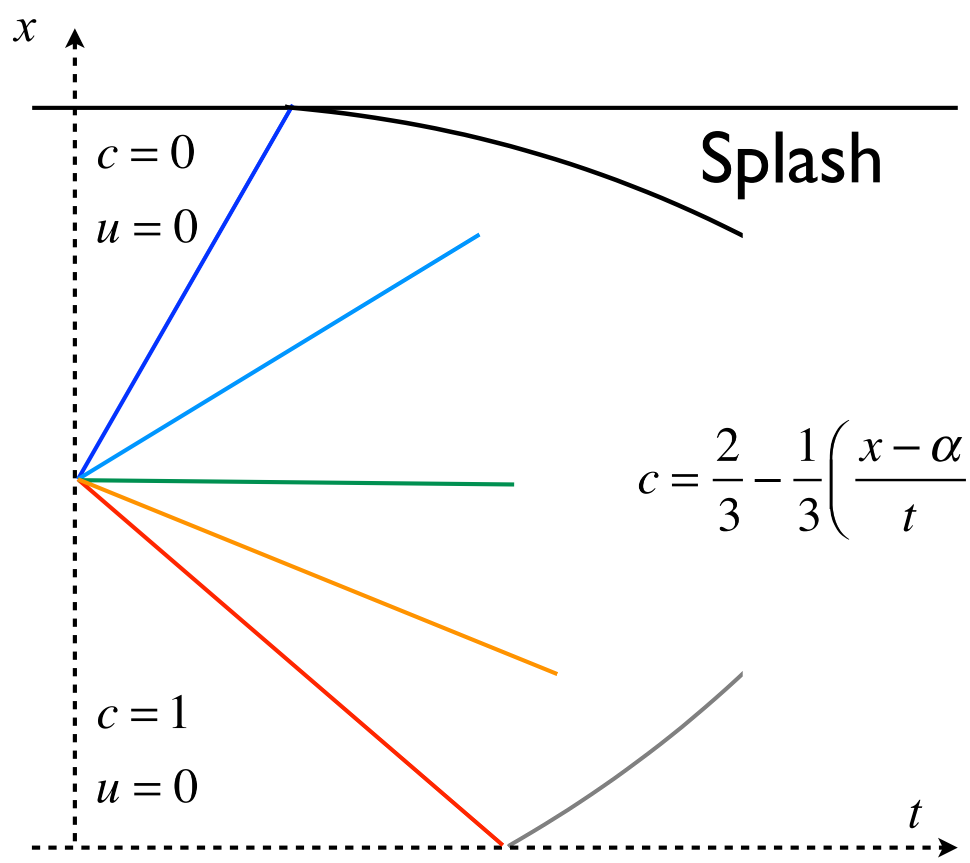
$$\left(\xi \bullet \mathbf{A} \right) \bullet \left\{ \frac{\partial \phi}{\partial t} + \lambda \frac{\partial \phi}{\partial x} \right\} + \xi \bullet \psi = 0$$

$$\left(\xi \bullet \mathbf{A} \right) \bullet \frac{d\phi}{dt} + \xi \bullet \psi = 0 \quad : \quad \frac{dx}{dt} = \lambda$$

Simple Planar Wave

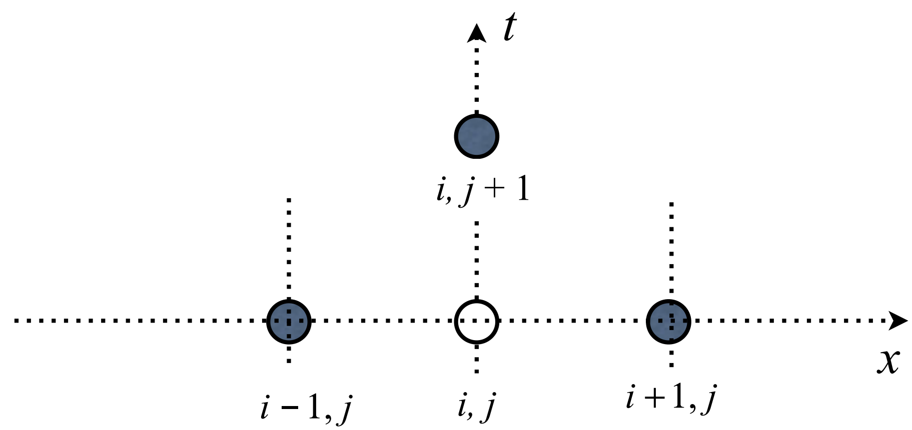
$$\text{Wavespeed}: \ c = \sqrt{h}$$

$$\frac{dh}{dt} \pm c\frac{du}{dt} = 0 \quad : \quad \frac{dx}{dt} = u \pm c$$



$$c = \frac{2}{3} - \frac{1}{3}\left(\frac{x-\alpha}{t}\right) \quad : \quad u = \frac{2}{3} + \frac{2}{3}\left(\frac{x-\alpha}{t}\right)$$

Lax FD Method

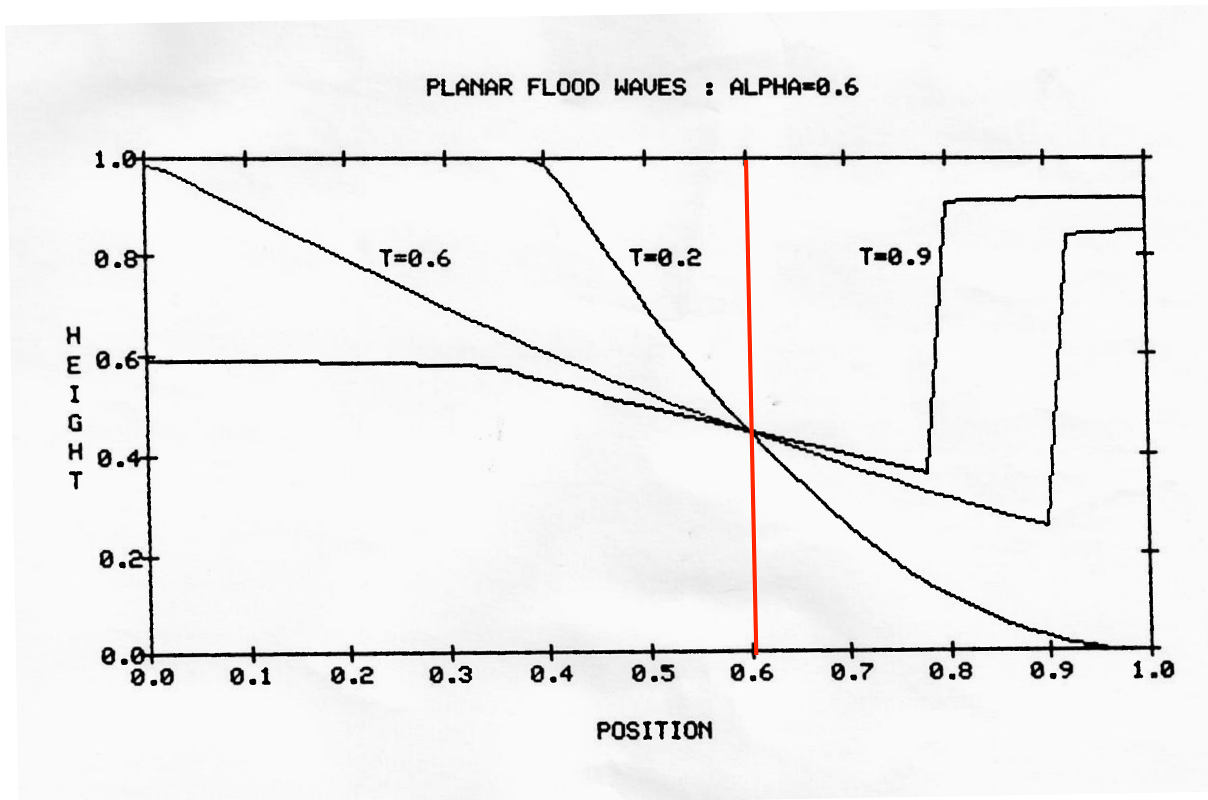


$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 : \; q \equiv uh$$

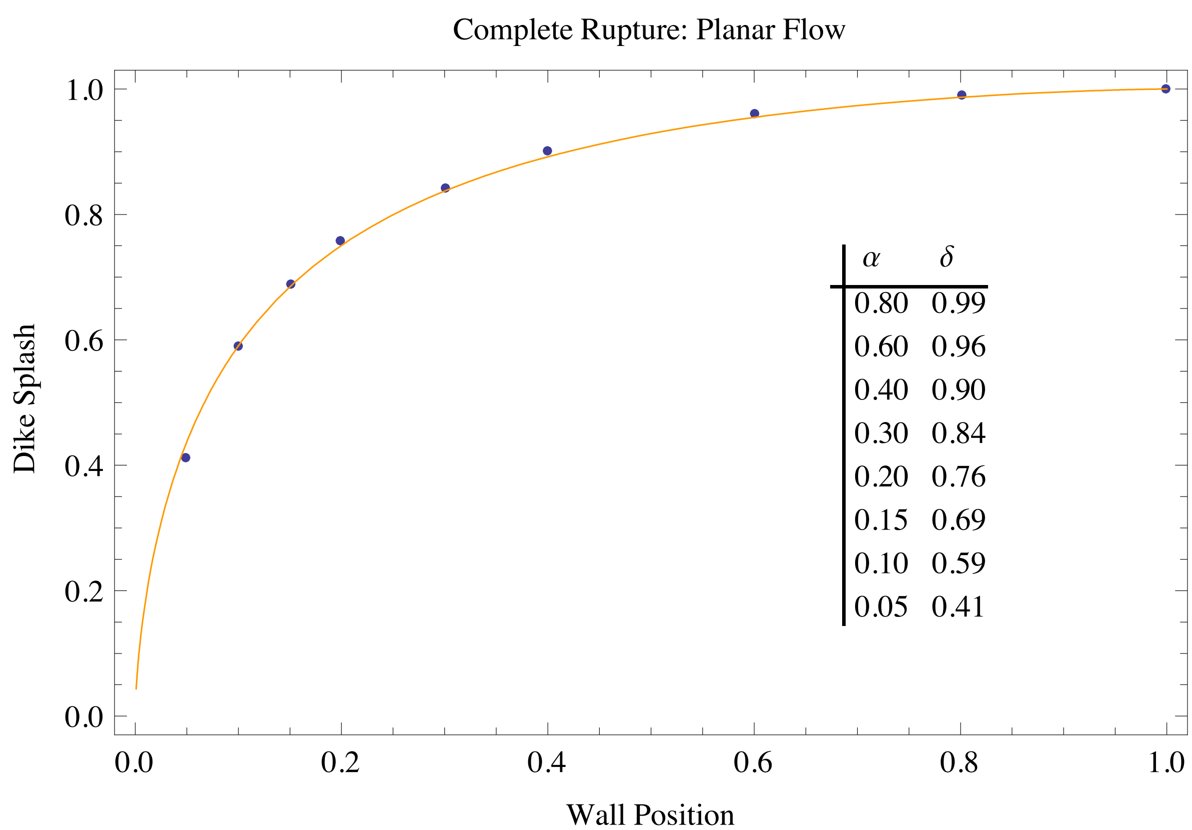
$$\Downarrow$$

$$h_{i,j+1} = \frac{1}{2}\left(h_{i-1,j} + h_{i+1,j}\right) - \left(\frac{\Delta t}{2\Delta x}\right)\left(q_{i+1,j} - q_{i-1,j}\right)$$

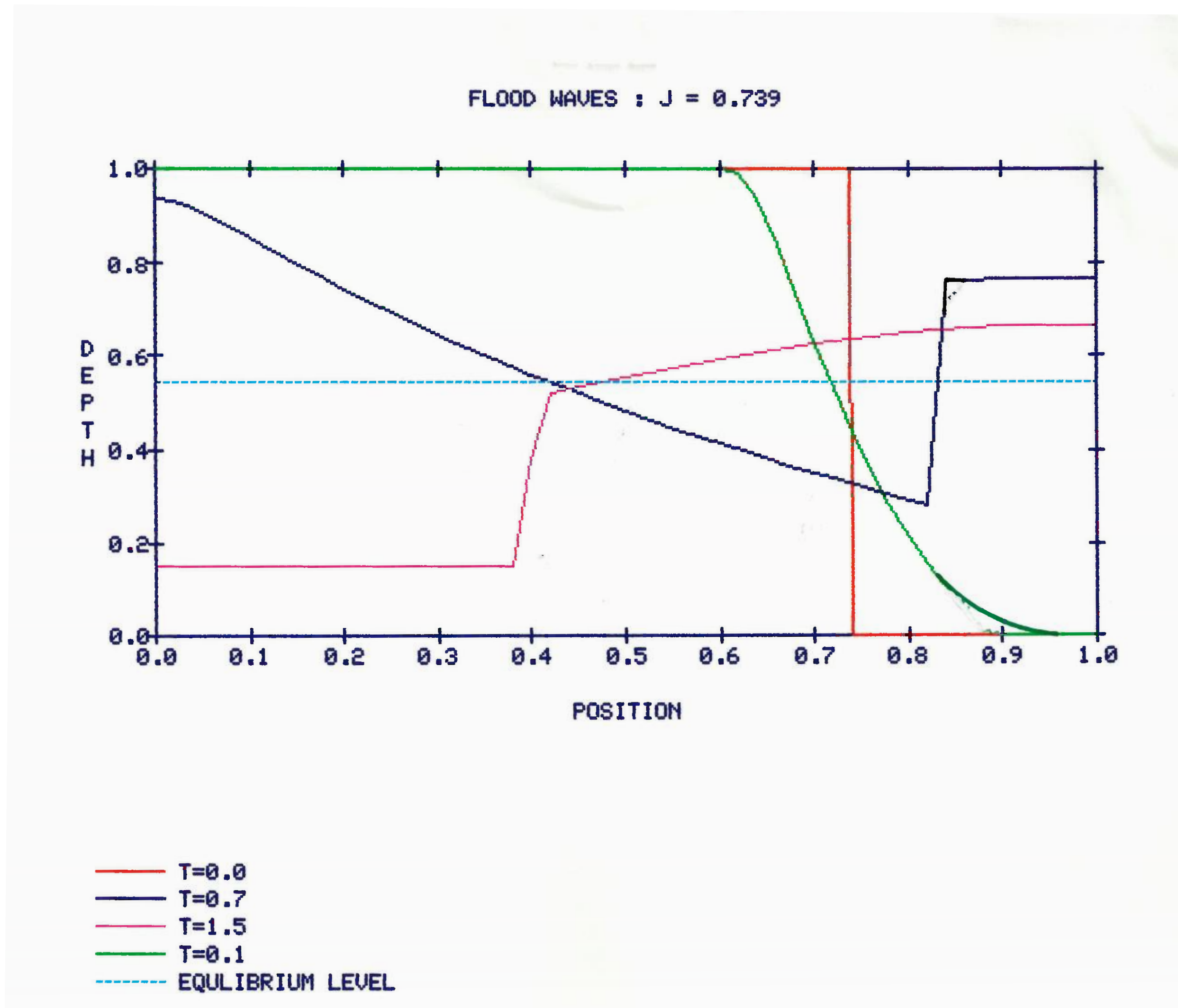
Planar Rupture Splash



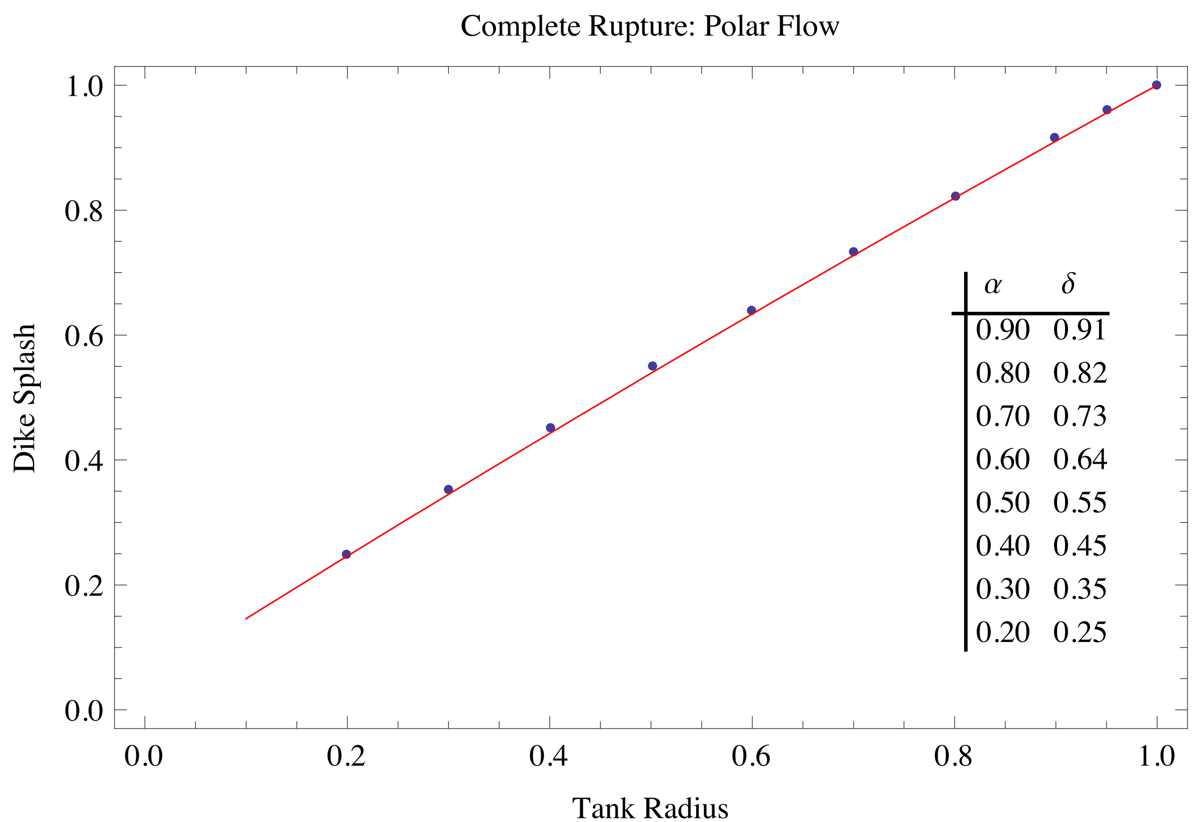
Complete Rupture
Symmetric Planar Flow



Polar Rupture Splash



Complete Rupture Symmetric Polar Flow



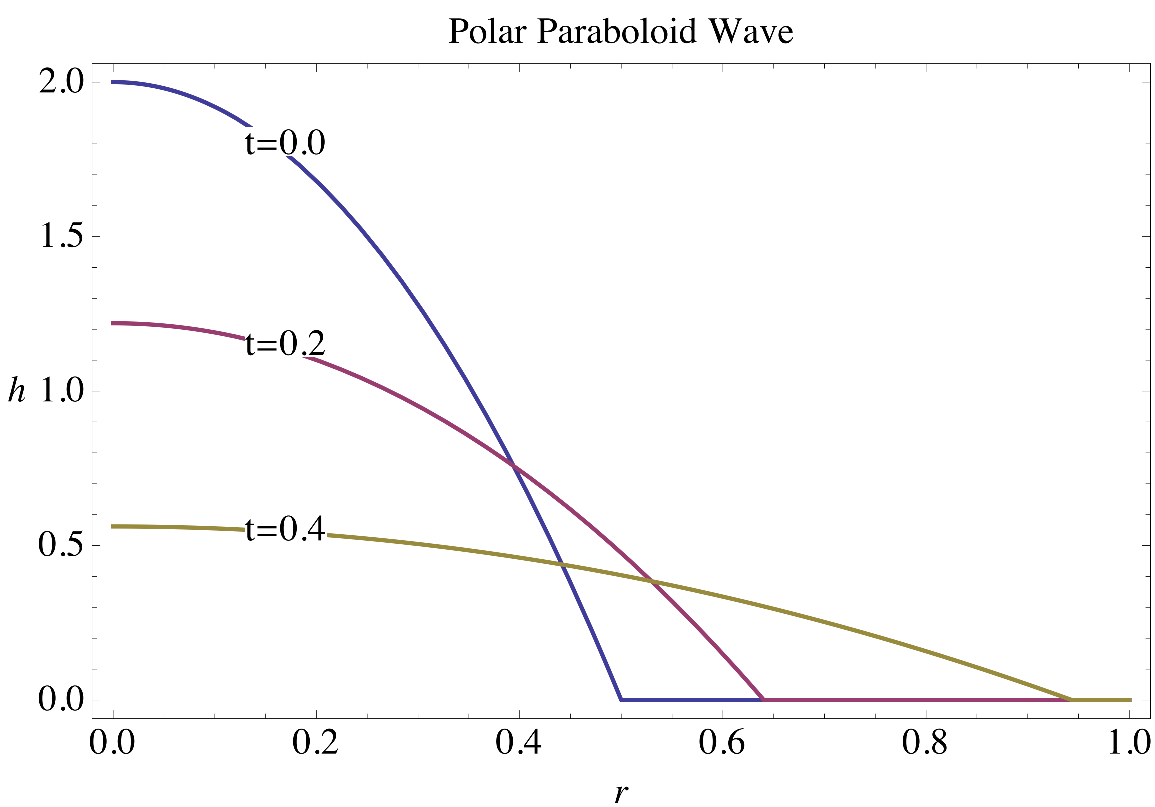
α	δ
0.90	0.91
0.80	0.82
0.70	0.73
0.60	0.64
0.50	0.55
0.40	0.45
0.30	0.35
0.20	0.25

Similarity Solution

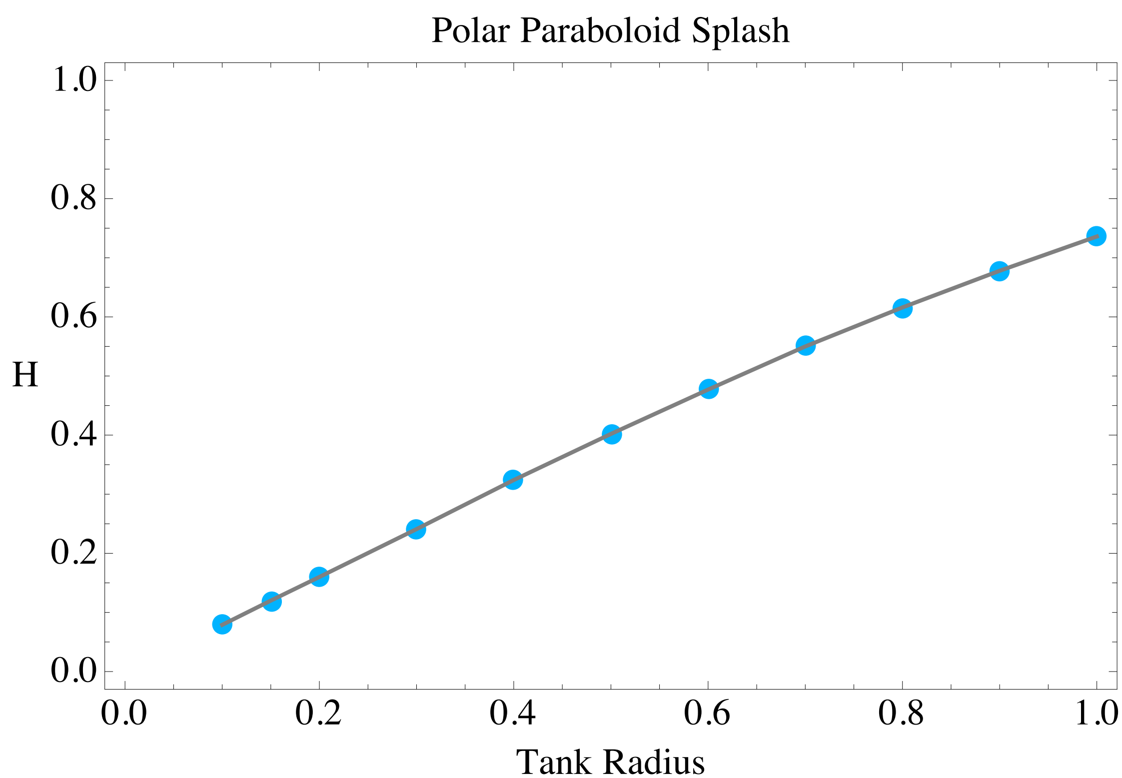
$$0 \leq r \leq R(t) = \sqrt{\alpha^2 + 4t^2}$$

$$h(r,t) = 2\frac{\alpha^2}{R^2}\left(1 - \frac{r^2}{R^2}\right)$$

$$u(r,t) = \frac{4rt}{R^2}$$



Polar Parabaloid Splash



Rupture Flow Experiments

- Greenspan & Young (1978)

  (a) planar flow, validated SWT

  (b) sloped dikes particularly ineffective

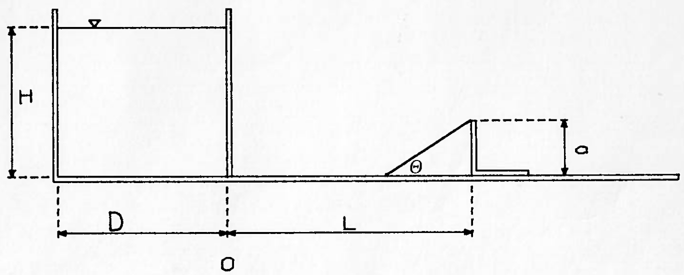
  (c) initial splash exceeds tank height

- Greenspan & Johansson (1981)

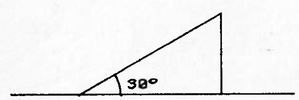
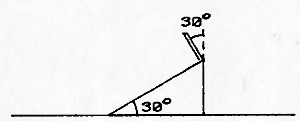
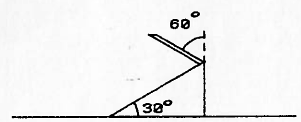
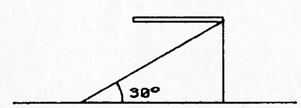
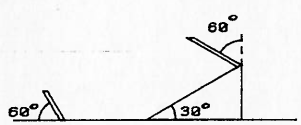
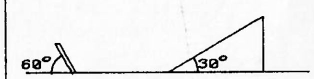
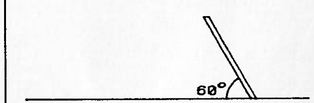
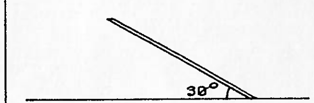
  (a) polar flow

  (b) dike shape study

  (c) proposed trip rings, deflectors



(a)

No.	dyke configuration	a/H	Q (%)
1		.5 .6	37.2 30.8
2		.5 .6	24.6 20.6
3		.5 .6	14.0 10.6
4		.5 .6	12.9 7.9
5		.5	8.5
6		.5	17.2
7		.5	20.7
8		.5	14.4
9		.5	9.5
10		.5	14.0

(b)

Figure 6. (a) Schematic of the channel flow experiment. The removable wall is at O; the dike of height a is inclined at an angle θ with the horizontal. (b) The ten dike designs tested with measured overflows. The maximum height is a in each case, and the condition of equal storage and containment volume was maintained in all tests.

Summary

- Low (equilibrium) dikes do not contain rupture flow

- Shallow Water Theory can be applied to address:

  (a) small hole splash

  (b) dike overflow

  (c) multiple dike interaction

- Small-scale experiments are representative