Strain and Stress: Measurement and Analysis

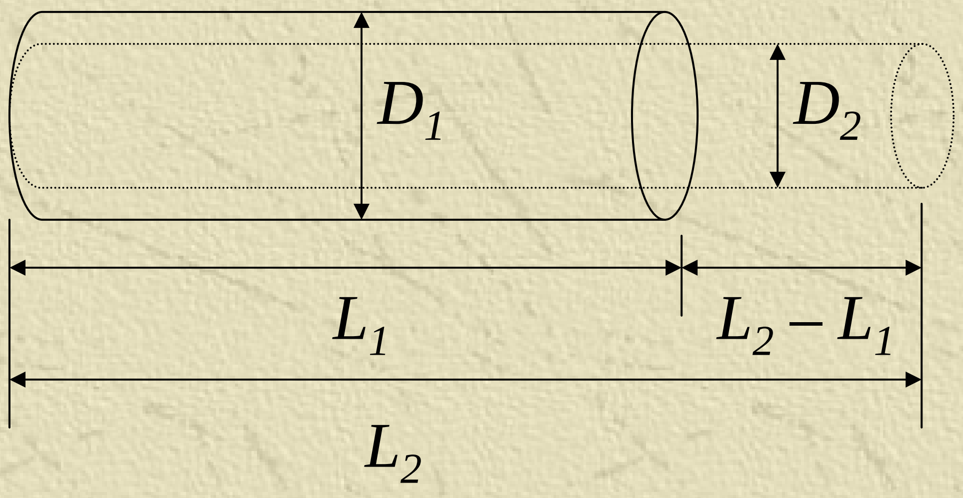
Hsin-yu Shan
Dept. of Civil Eng., NCTU

$$\varepsilon_a = dL / L \approx (L_2 - L_1) / L_1 = \Delta L / L$$

ε_a = axial strain

L_1 = linear dimension or gage length

L_2 = final strained linear dimension



$$E = \frac{\sigma_a}{\varepsilon_a}$$

E = Young's modulus

σ_a = uniaxial stress

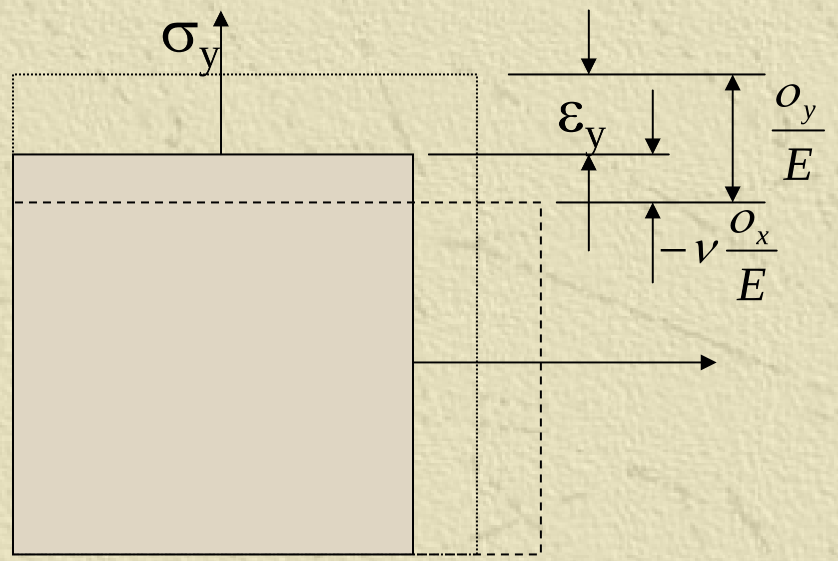
$$\nu = \frac{-\varepsilon_L}{\varepsilon_a}$$

ν = Poisson's ratio

ε_L = lateral strain

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} \qquad \varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E}$$

$$\sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2} \qquad \sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_x)}{1-\nu^2}$$



Strain Measurement

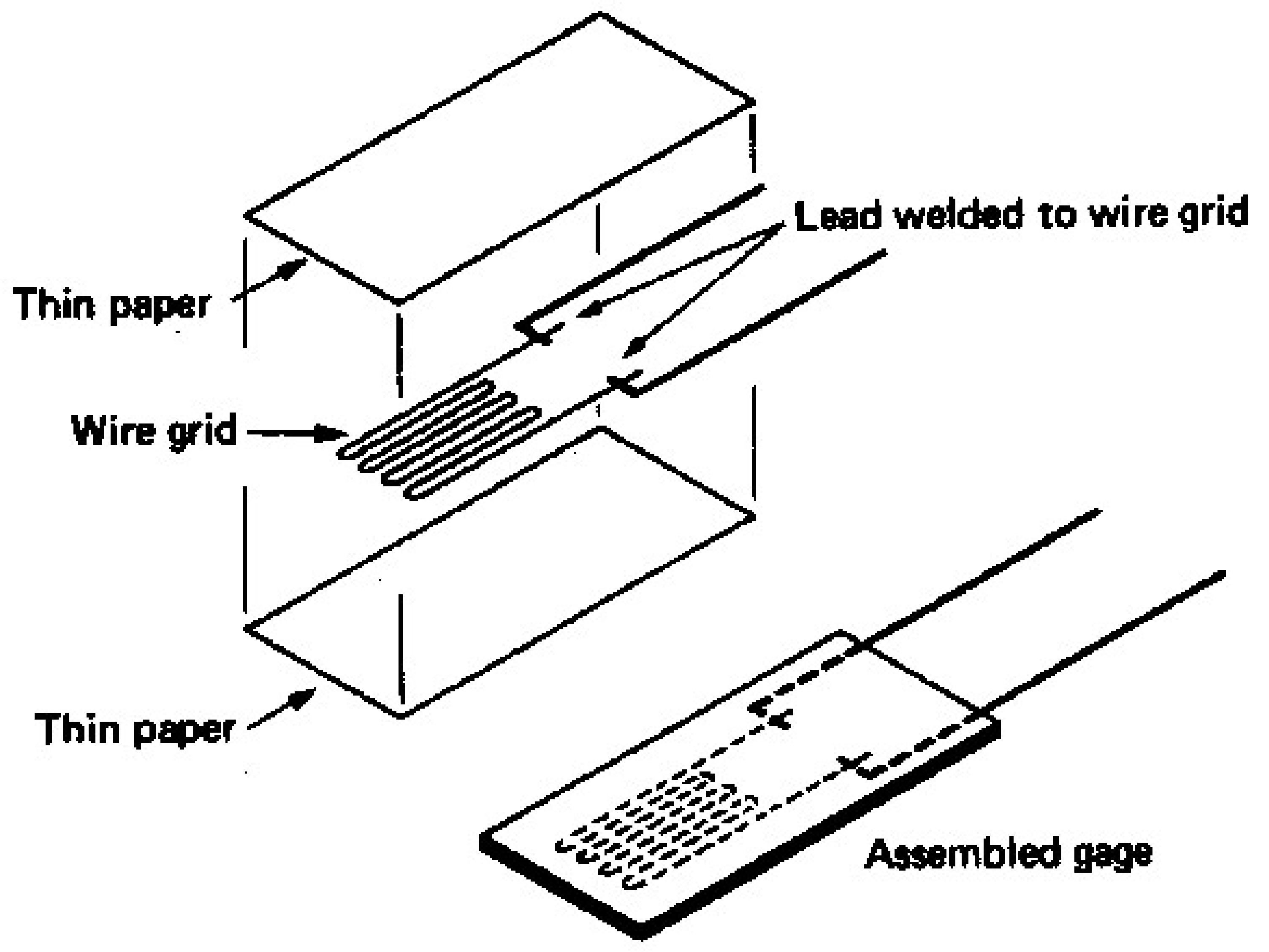
- Extensometer
  - Optical
  - Mechanical
- Electrical strain gage
  - Resistive – the most common
  - Capacitive
  - Inductive
  - Photoelectric

Electrical Resistance Strain Gage

- 1856 Lord Kelvin demonstrated that the resistances of copper wire and iron wire change when the wires are subjected to mechanical strain
  - He used a Wheatstone bridge circuit with a galvanometer as the indicator

- The first resistance strain gage is made by Carlson in 1931
  - Pillars were mounted, separated by the gage length, with wires stretched between them
- The first bonded strain gage was used by Bloach
  - It consisted of a carbon film resistance element applied directly to the surface of the strained member

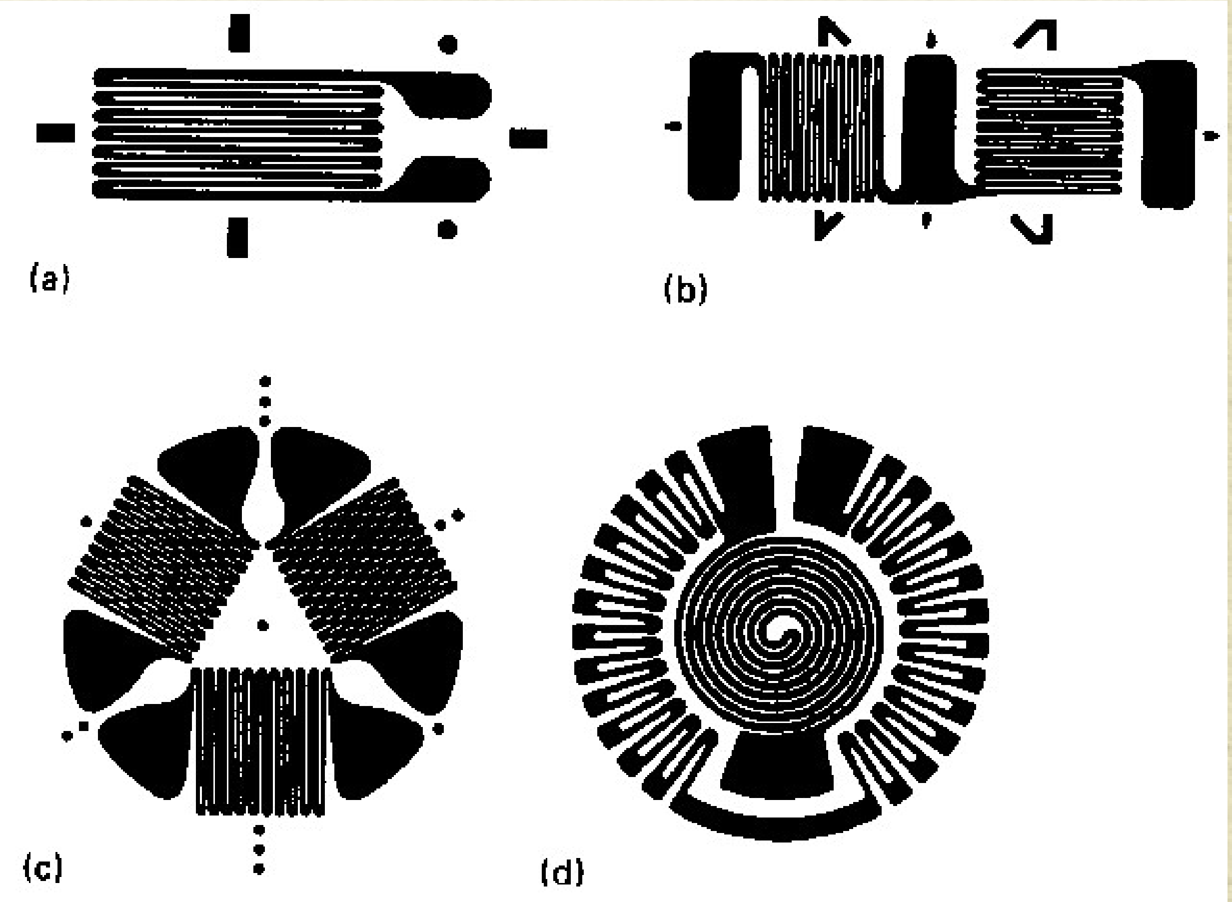
- 1938 Ruge of M.I.T. conceived the idea of making a preassembly by mounting wire between thin pieces of paper



Construction of bonded-wire-type strain gage

Foil-Type Gage

- During the 1950s the foil-type gage replaced the the wire gage
- The common form consists of a metal foil element on a thin epoxy support
  - Manufactured using printed-circuit techniques
- Major advantage – almost unlimited pane configurations are possible



Typical foil-type gages illustrating the following types: (a) single element, (b) two-element rosette, (c) three-element rosette, (d) one example of many different special purpose gages (for pressurized diaphragms)

$$R = \frac{\rho L}{A} = \frac{\rho L}{CD^2}$$

The *gage factor*

$$F = \frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a} = 1 + 2\nu + \frac{d\rho/\rho}{dL/L}$$

$$\varepsilon = \frac{1}{F}\frac{\Delta R}{R}$$

The values of *F* and *R* are supplied by the gage manufacturer, and the user determines ΔR corresponding to the input situation being measured.

Selection and Installation Factors for Bonded Metallic Strain Gages

- Grid material and configuration
- Backing material
- Bonding material and method
- Gage protection
- Associated electrical circuitry

Desirable Properties of Grid Material

- High gage factor, F
- High sensitivity, ρ
- Low temperature sensitivity
- High electrical stability
- High yield strength
- High endurance limit

- Good workability
- Good solderability or weldability
- Low hysteresis
- Low thermal emf when joined to other materials
- Good corrosion resistance

Temperature Sensitivity

- In many applications, compensation is provided in the electrical circuitry – this does not always eliminate the problem
- Two factors are involved:
  - The differential expansion existing between the grid support and the grid proper, resulting in a strain that the gage is unable to distinguish from load strain
  - The change in resistivity ρ with temperature change

Thermal emf Effect

- Thermal emf superimposed on gage output must be avoided if dc circuitry is used
- For ac circuitry this factor would be of little importance

Common Grid Materials

- Constantan*; Copel* 45% Ni, 55% Cu
- Isoelastic* 36% Ni, 8% Cr, 0.5% Mo, Fe remainder

*Trade names

Common Backing Materials

- Thin paper
- Phenolic-impregnated paper
- Epoxy-type plastic films
- Epoxy-impregnated fiberglass

- Most foil gages use an epoxy film backing

Protecting the Strain Gage

- The strain gages must be protected from ambient conditions:
  - Mechanical abuse, moisture, oil, dust and dirt
- Protection material:
  - Petroleum waxes, silicone resins, epoxy preparations, rubberized brushing compounds

Temperature Compensation

- The adjacent-arm compensating gage – bridge circuitry
- Self-temperature compensation
  - Selected-melt gage
    - Through proper manipulation of alloy and processing, grid materials may be prepared to show very low apparent strain versus temperature change
  - Dual-element gage
    - Use two wire elements connected in series in one gage assembly
    - The two elements have different temperature characteristics and are selected so that the net temperature-induced strain is minimized

Strain-Measuring Systems

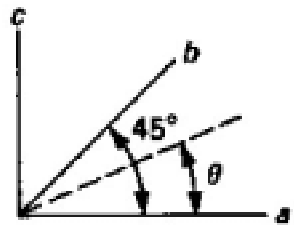
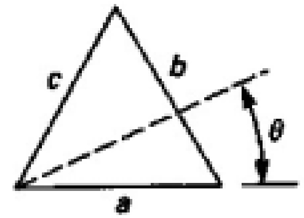
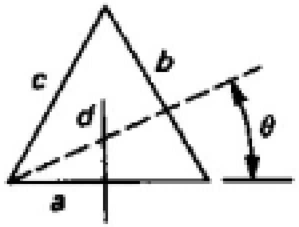
- Basic strain indicator, useful for static: single-channel readings
- Single-channel system either external to or an integral part of a cathode-ray oscilloscope
- Oscillographic systems incorporating either a stylus-and-paper or lightbeam and phtographic paper readout

- Data acquisition systems:
  - Displayed (digitally and/or by a video terminal)
  - Recorded (magnetic tape or hard-copy printout)
  - Fed back into the system for control purpose

Stress-Strain Relationships

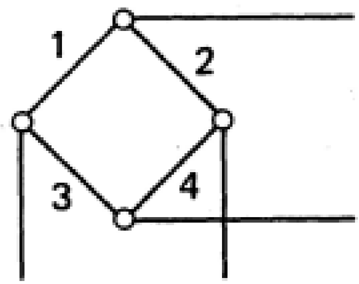
- Simple uniaxial stress situation
- Biaxial stress situation

Stress-strain relations for Rosette gages (T 12.4)

Type of Rosette:	 Rectangular	 Equiangular (delta)	 T-delta
Principal strains, $\varepsilon_1, \varepsilon_2$	$\frac{1}{2}\left[\varepsilon_a + \varepsilon_c \pm \sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2}\right]$	$\frac{1}{3}\left[\varepsilon_a + \varepsilon_b + \varepsilon_c \pm \sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2 + 2(\varepsilon_c - \varepsilon_a)^2}\right]$	$\frac{1}{2}\left[\varepsilon_a + \varepsilon_d \pm \sqrt{(\varepsilon_a - \varepsilon_d)^2 + \frac{4}{3}(\varepsilon_b - \varepsilon_c)^2}\right]$
Principal stresses, σ_1, σ_2	$\frac{E}{2}\left[\frac{\varepsilon_a + \varepsilon_c}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2}\right]$	$\frac{E}{3}\left[\frac{\varepsilon_a + \varepsilon_b + \varepsilon_c}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2 + 2(\varepsilon_c - \varepsilon_a)^2}\right]$	$\frac{E}{2}\left[\frac{(\varepsilon_a + \varepsilon_d)}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{(\varepsilon_a - \varepsilon_d)^2 + \frac{4}{3}(\varepsilon_b - \varepsilon_c)^2}\right]$
Maximum shear, τ_{max}	$\frac{E}{2(1+\nu)}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2}$	$\frac{E}{3(1+\nu)}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2 + 2(\varepsilon_c - \varepsilon_a)^2}$	$\frac{E}{2(1+\nu)}\sqrt{(\varepsilon_a - \varepsilon_d)^2 + \frac{4}{3}(\varepsilon_b - \varepsilon_c)^2}$
$\tan 2\theta$	$\frac{2\varepsilon_b - \varepsilon_a - \varepsilon_c}{\varepsilon_a - \varepsilon_c}$	$\frac{\sqrt{3}(\varepsilon_c - \varepsilon_b)}{(2\varepsilon_a - \varepsilon_b - \varepsilon_c)}$	$\frac{2}{\sqrt{3}}\frac{(\varepsilon_c - \varepsilon_b)}{(\varepsilon_a - \varepsilon_d)}$
$0 < \theta < +90°$	$\varepsilon_b > \frac{\varepsilon_a + \varepsilon_c}{2}$	$\varepsilon_c > \varepsilon_b$	$\varepsilon_c > \varepsilon_b$

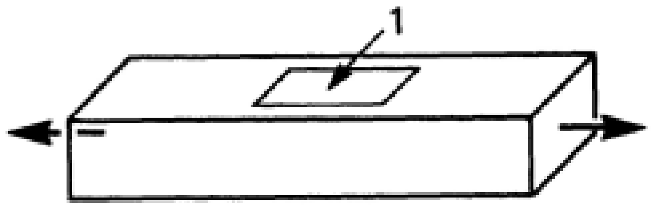
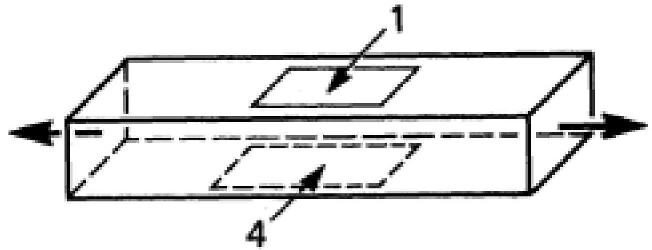
Gage Orientation and Interpretation of Results

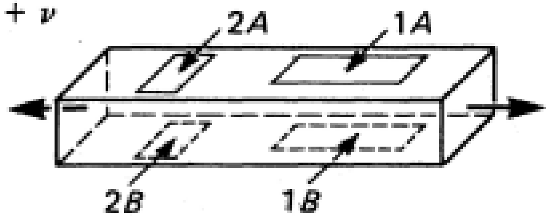
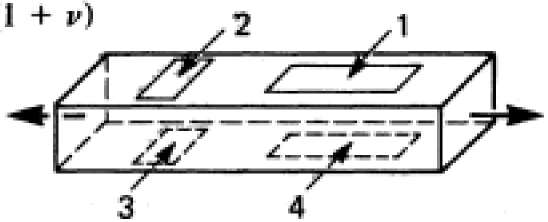
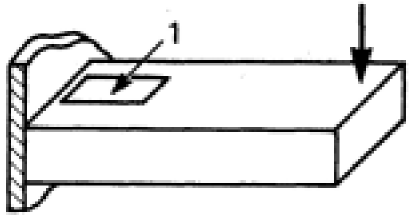
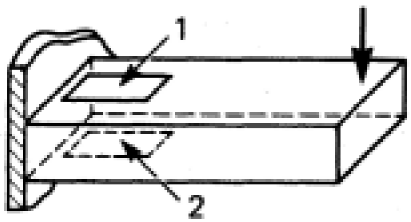
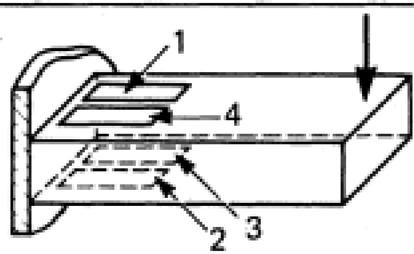
Standard Bridge Configuration



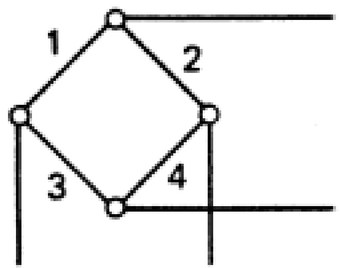
Requirement for null: $R_1/R_2 = R_3/R_4$

$$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$$

A	$K = 1$ 	Compensates for temperature if "dummy" gage is used in arm 2 or arm 3. Does not compensate for bending.
B	$K = 2$ 	Compensates for bending. Two-arm bridge does not provide temperature compensation. Four-arm bridge ("dummy" gages in arms 2 and 3) provides temperature compensation.

C	$K = 1 + \nu$ 	Two-arm bridge compensates for temperature and bending.
D	$K = 2(1 + \nu)$ 	Four-arm bridge compensates for temperature and bending.
E	$K = 1$ 	Temperature compensation accomplished when "dummy" gage is used in arm 2 or arm 3. Bridge is also sensitive to axial and torsional components of loading.
F	$K = 2$ 	Temperature effects and axial and torsional components are compensated.
G	$K = 4$ 	Four-arm bridge. Temperature effects and axial and torsional components are compensated.

Standard Bridge Configuration

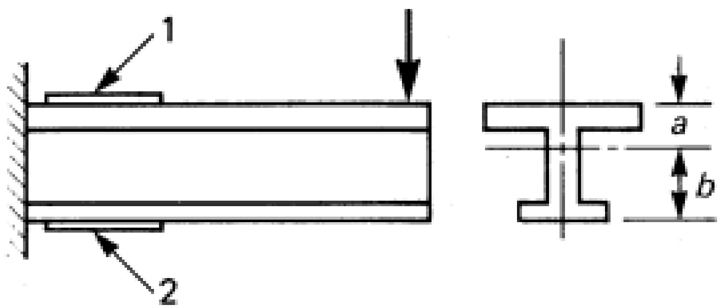


Requirement for null: $R_1/R_2 = R_3/R_4$

$$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$$

H

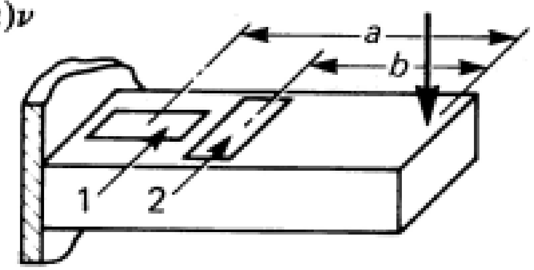
$K = (a + b)/a$



Temperature effects and axial and torsional components are compensated.

I

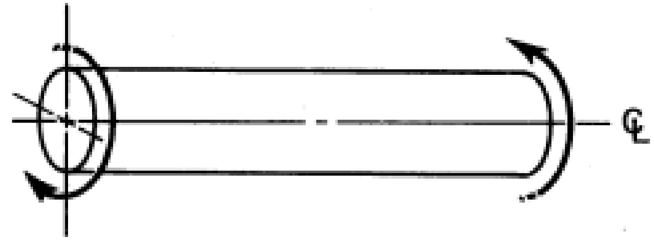
$K = 1 + (b/a)\nu$

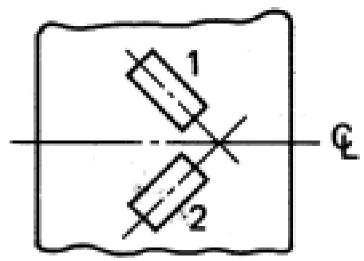
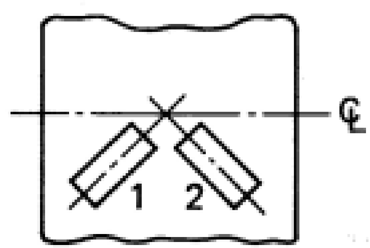
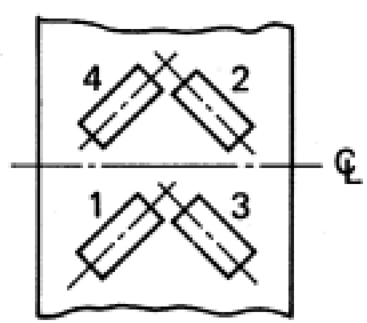


Temperature effects are compensated.

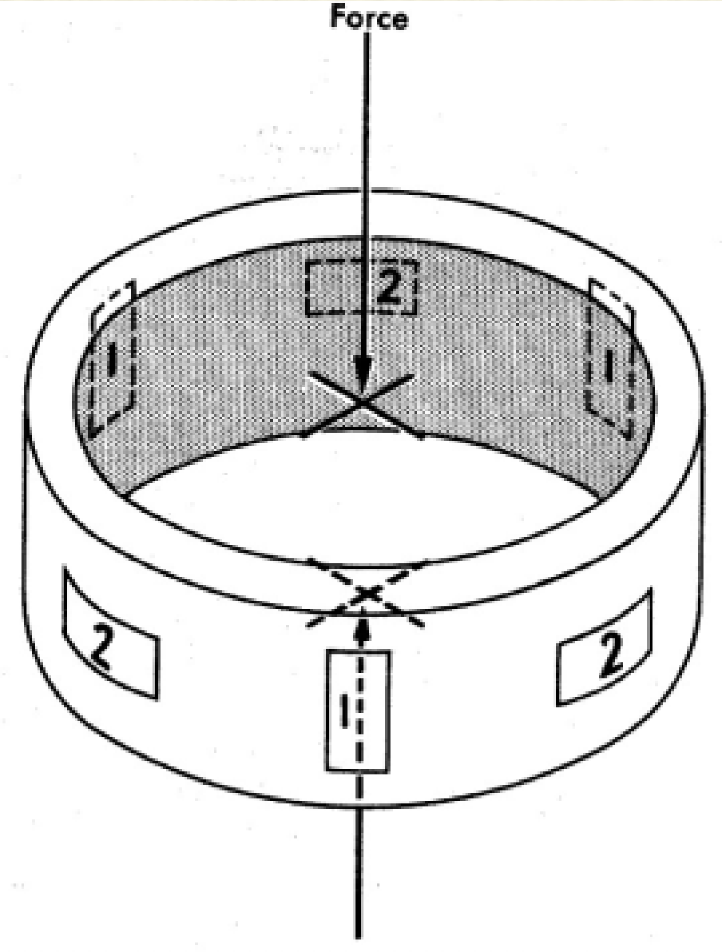
Axial and torsional load components are not compensated.

Torsion



J	$K = 2$		Two-arm bridge. Temperature and axial load components are compensated. Bending components are accentuated.
K	$K = 2$		Two-arm bridge. Temperature effects and axial load components are compensated. Relatively insensitive to bending.
L	$K = 4$		Four-arm bridge. Sensitive to torsion only. (Gages 1 and 3 are on opposite sides of the shaft from gages 2 and 4.)

Gages Connected in Series

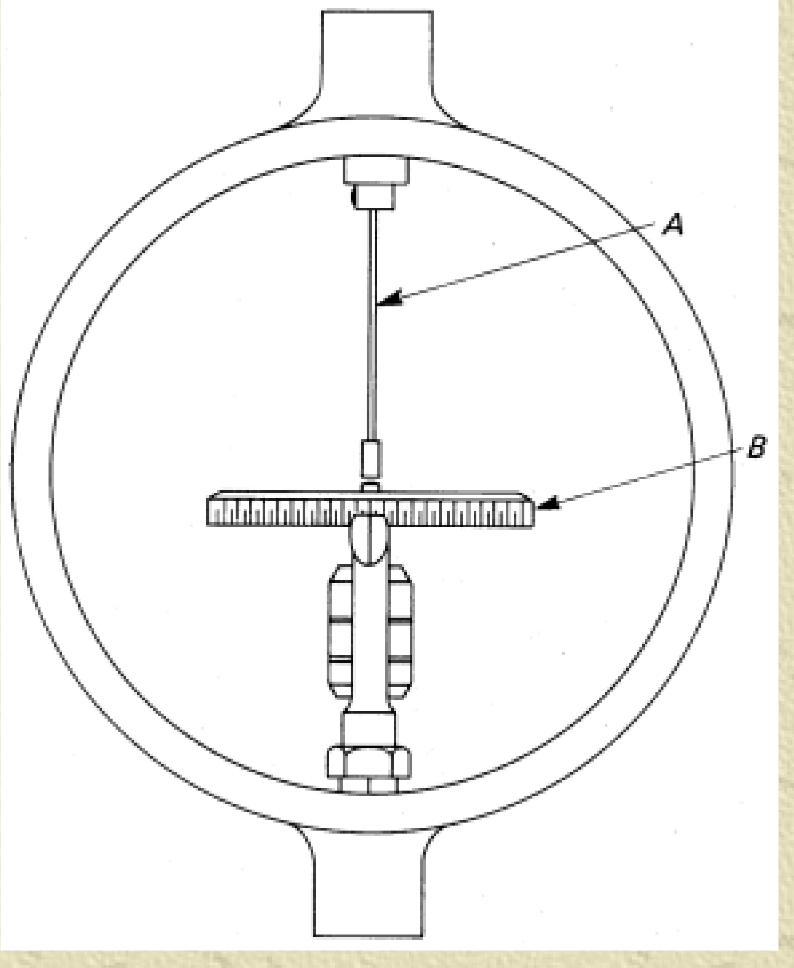


Load cell employing three series-connected axial gages and three series-connected Poisson-ratio gages

Measurement of Force

- Mechanical weighing systems
  - Balance, multiple-lever system, pendulum force-measuring mechanism
- Elastic transducers
  - Proving ring
- Strain-gage load cells
- Piezoelectric-type load cells
- Hydraulic and pneumatic systems

Proving Ring

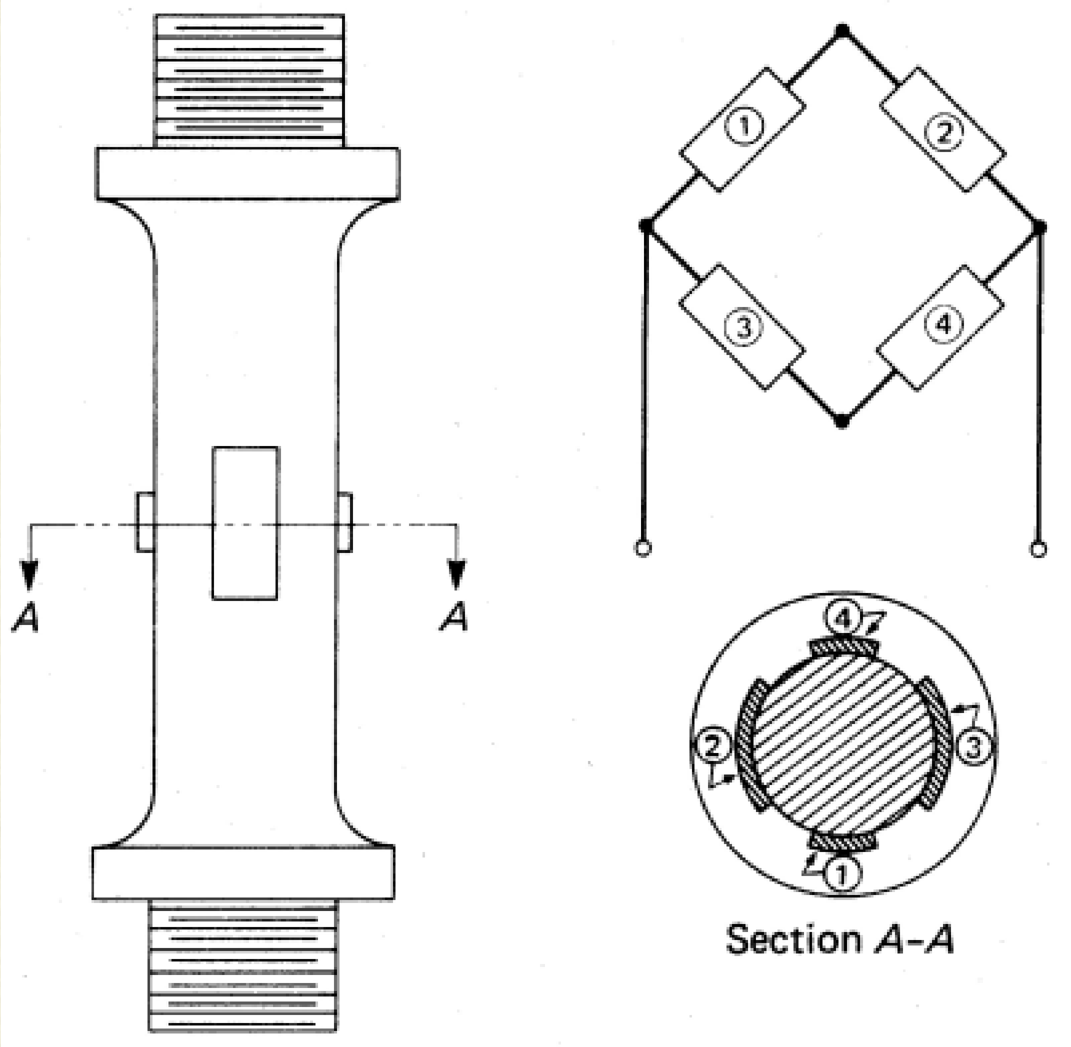


Compression-type proving ring with vibrating reed

- Deflection is used as the measure of applied load, with the deflection measured by means of a precision micrometer

Strain-Gage Load Cells

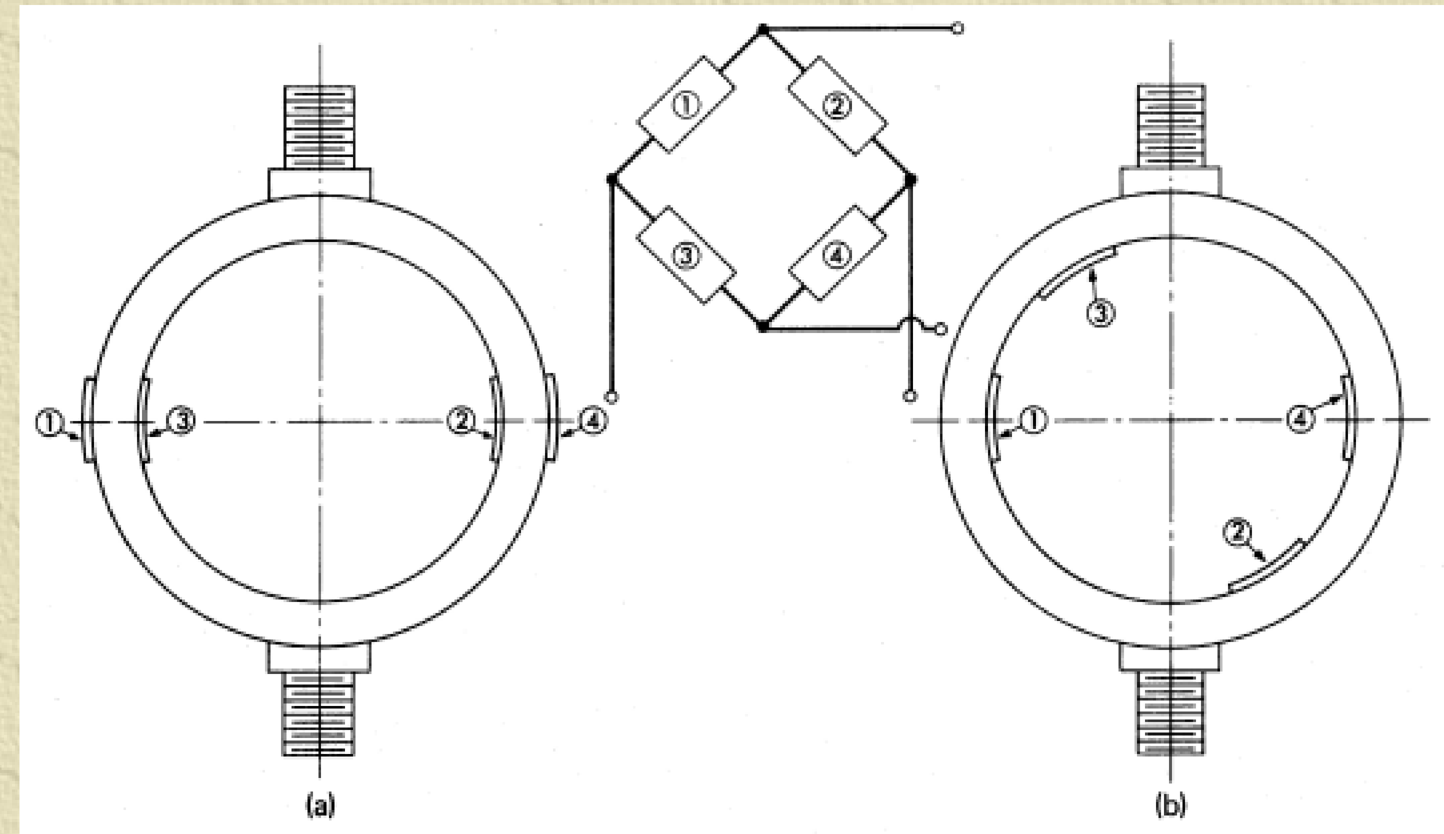
- Measures load in terms of unit strain
- One of the possible forms of elastic member is selected, and the gages are mounted to provide maximum output



Tension-compression resistance strain-gage load cell

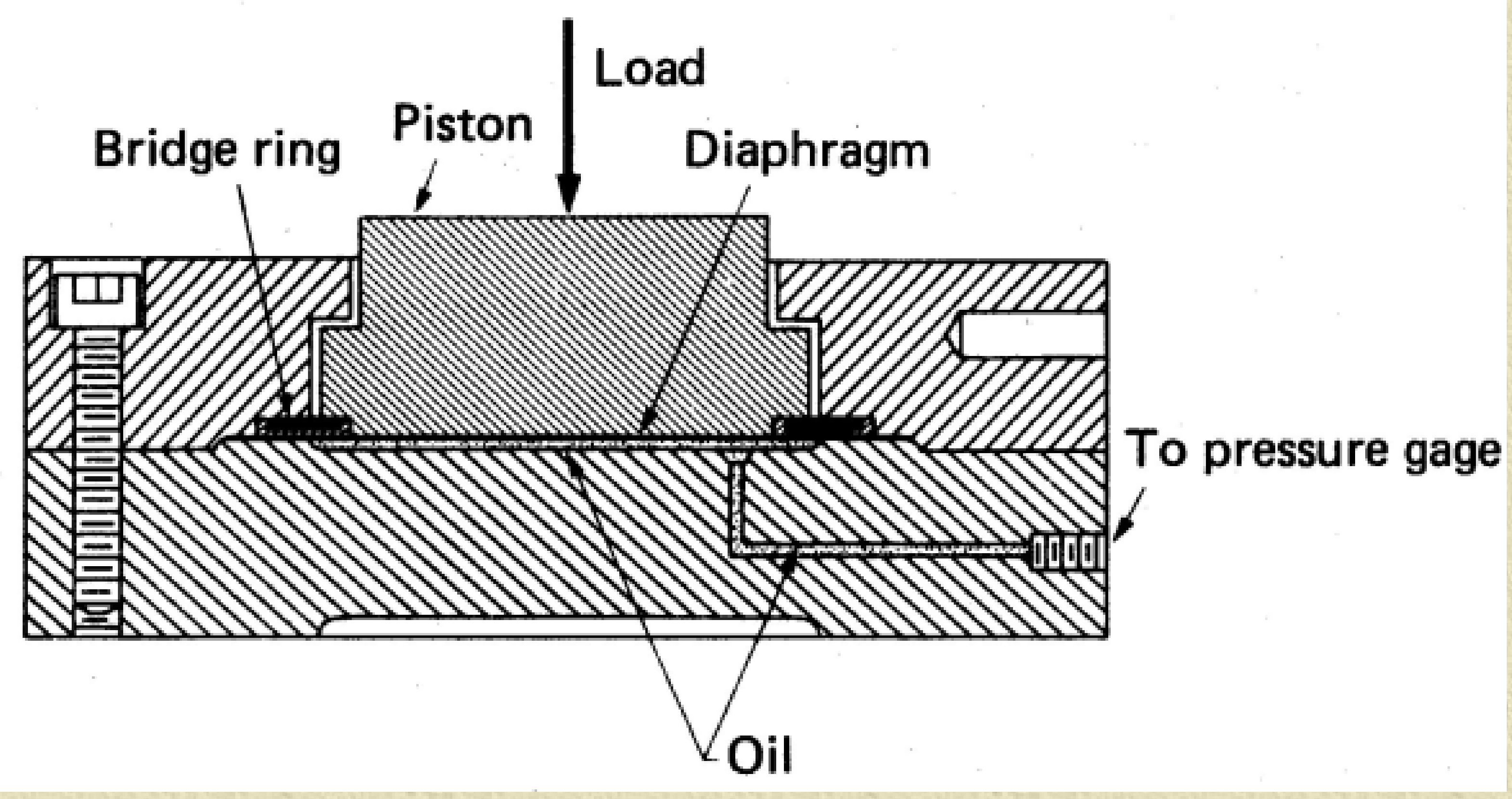
(a) The bridge output is a function of the bending strains only, the axial components being canceled in the bridge arrangement

(b) Greater sensitivity may be obtained because the output includes both the bending and the axial components sensed by gages 1 and 4

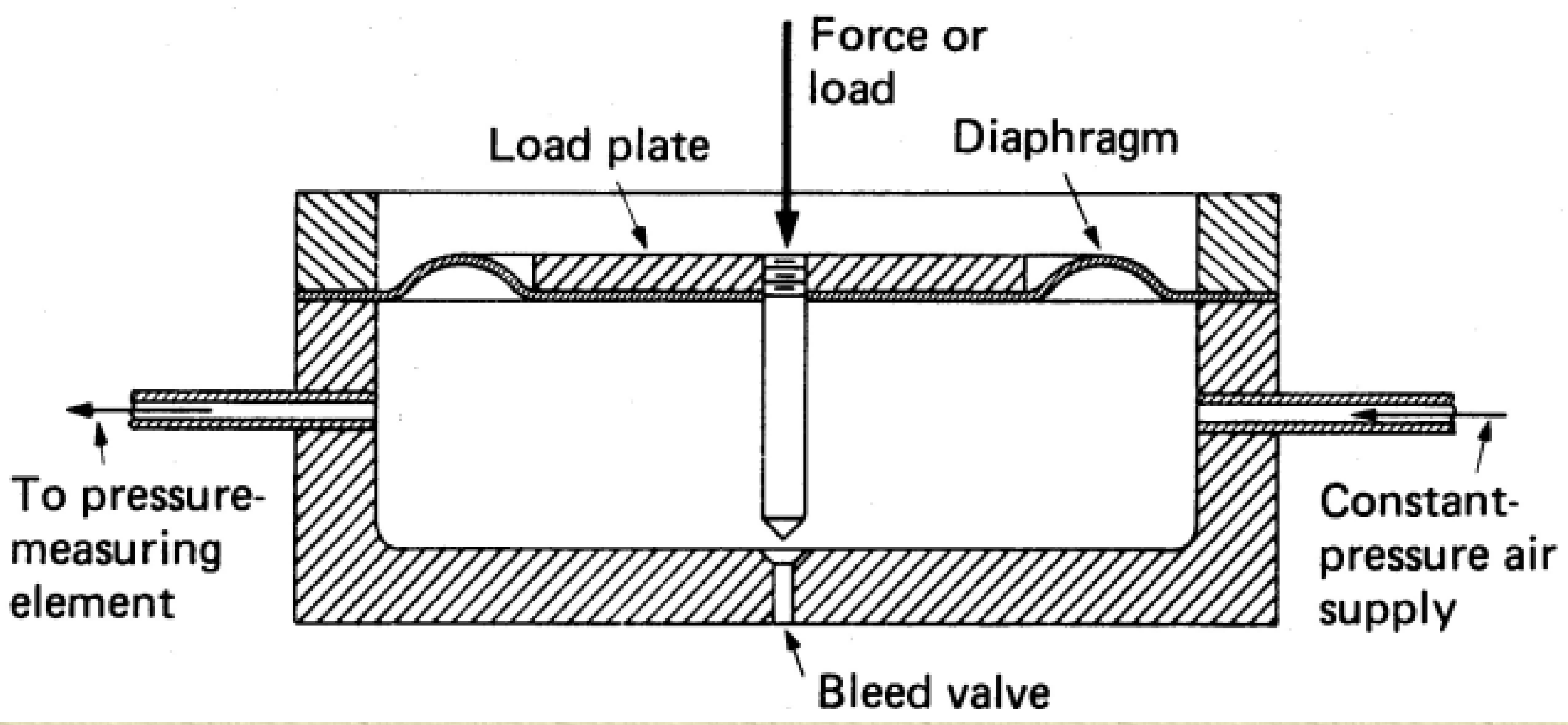


Two arrangements of circular-shaped load cells employing resistance strain gages as secondary transducers

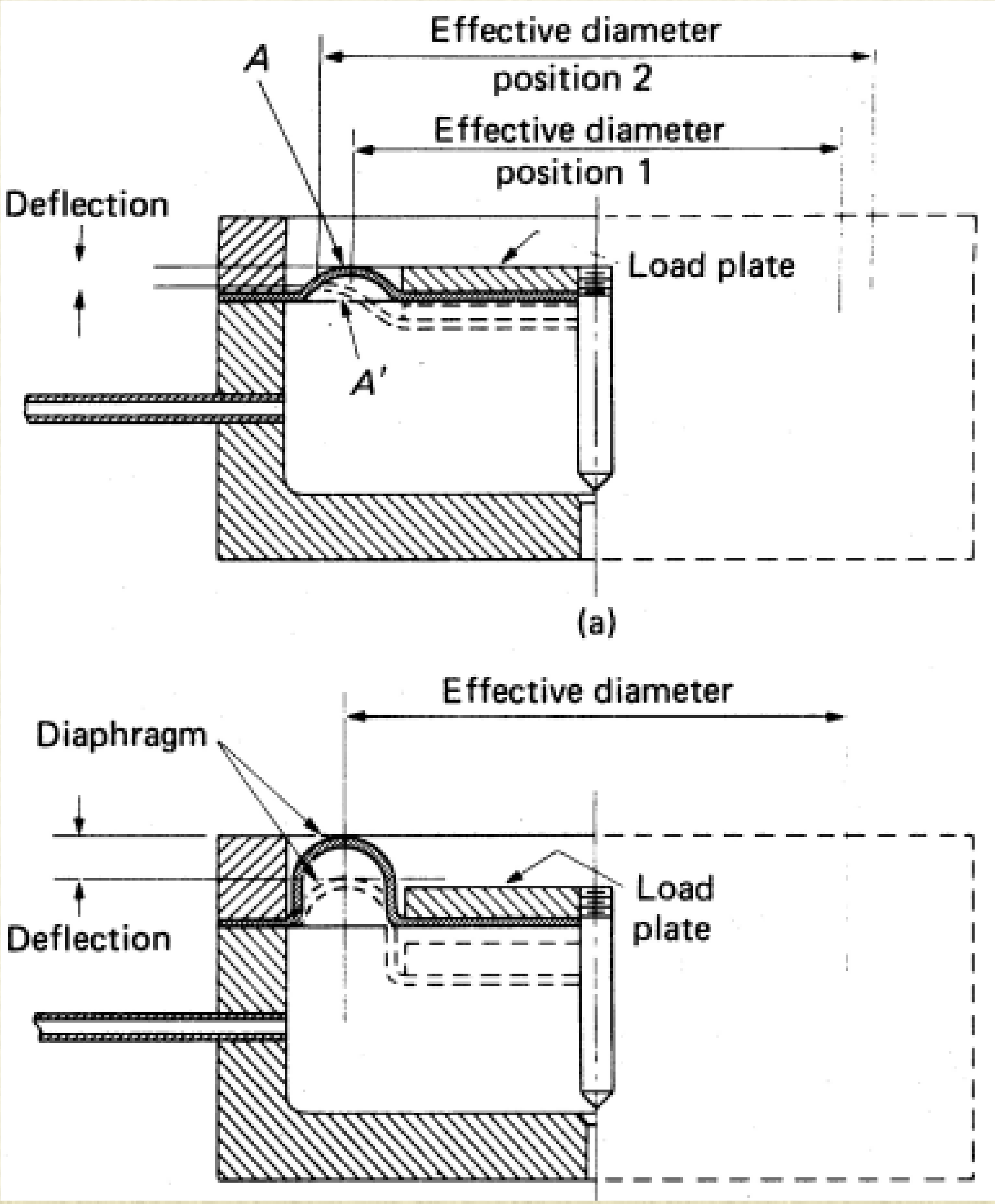
Hydraulic Load Cell



Section through a hydraulic load cell



Section through a pneumatic load cell



(a) Section through a diaphragm showing how a change in effective area may take place.

(b) When sufficient “roll” is provided, the effective area remains constant

Pneumatic Load Cell

- Use air rather than liquid as the pressurized medium
- Use diaphragms of a flexible material rather than pistons

- Designed to regulate the balancing pressure automatically
  - Air pressure is supplied to one side of the diaphragm and allowed to escape through a position-controlling bleed valve
  - The pressure under the diaphragm is controlled both by source pressure and bleed valve position
  - The diaphragm seeks the position that will result in just the proper air pressure to support the load

Measurement of Pressure

- Absolute pressure
- Gage pressure
- Vacuum

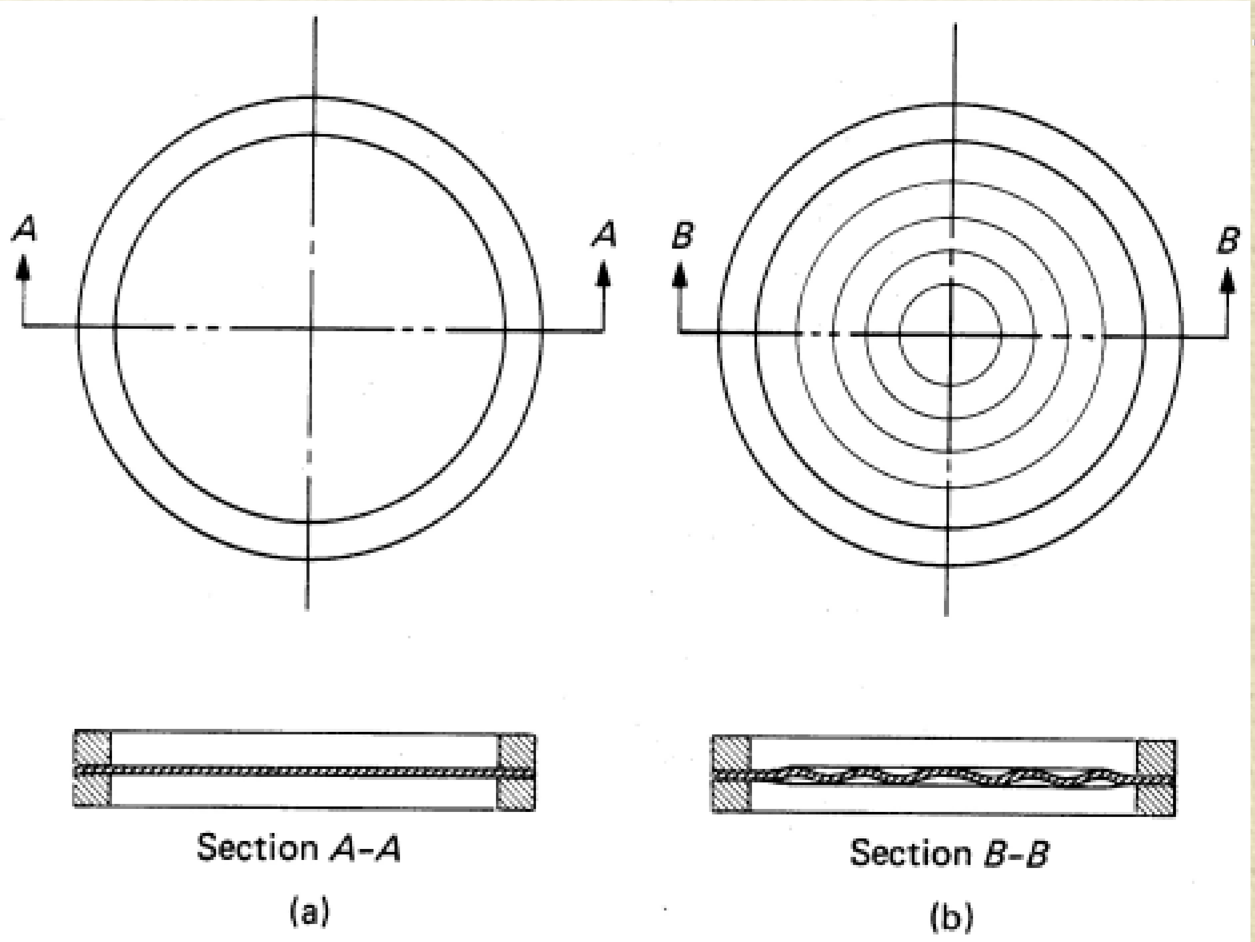
Pressure-Measuring Transducers

- Gravitational types
  - Liquid columns
  - Pistons or loose diaphragm, and weights

- Direct-acting elastic types
  - Unsymmetrically loaded tubes
  - Symmetrically loaded tubes
  - Elastic diaphragms
  - Bellows
  - Bulk compression
- Direct-acting elastic type, a piston with elastic restraining member

Elastic Diaphragms

- Use elastic diaphragm as primary pressure transducer
- Flat type –
  - Used in conjunction with electrical secondary transducers whose sensitivity permits quite small diaphragm deflections
- Corrugated type is particularly useful when larger deflections are required



(a) Flat diaphragm (b) Corrugated diaphragm

Diaphragm Design Requirements

- Dimensions and total load must be compatible with physical properties of the material used
- Flexibility must be such as to provide the sensitivity required by the secondary transducer
- Volume of displacement should be minimized to provide reasonable dynamic response

- Diaphragm displacement may be transmitted by mechanical means to some form of indicators
- For engineering measurements, diaphragm motion is sensed by some form of electrical secondary transducer
  - Resistive, capacitive, inductive, or piezoelectric

- Natural frequency of the diaphragm should be sufficiently high to provide satisfactory frequency response
- Output should be linear

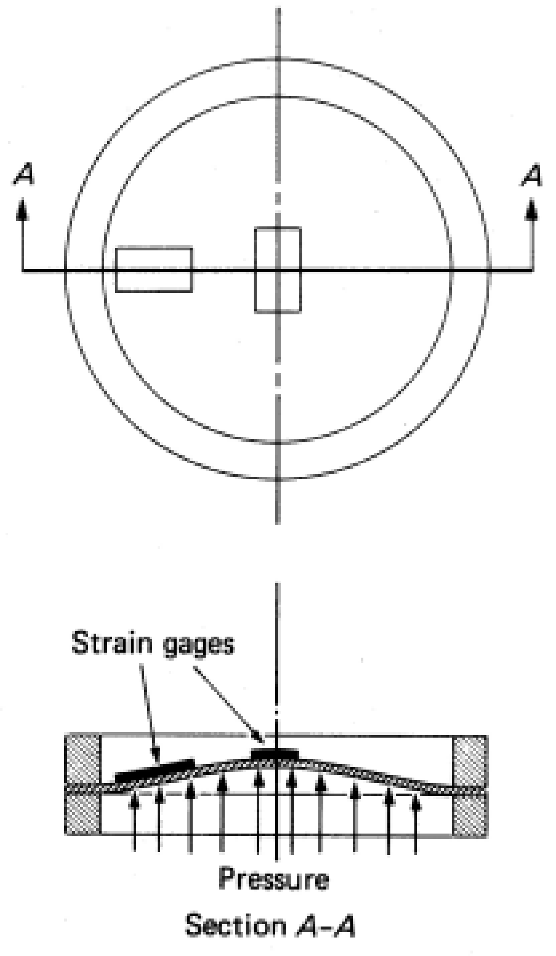
Flat Metal Diaphragm

- Deflection of flat metal diaphragm is limited either by stress requirements or deviation from linearity
- As a general rule the maximum deflection that can be tolerated maintaining a linear pressure-displacement relation is about 30% of the diaphragm thickness

Secondary Transducers Used with Diaphragms

- Resistance strain gages with flat diaphragms
- Inductive types
- Piezoelectric-type pressure cells

Resistance Strain Gages

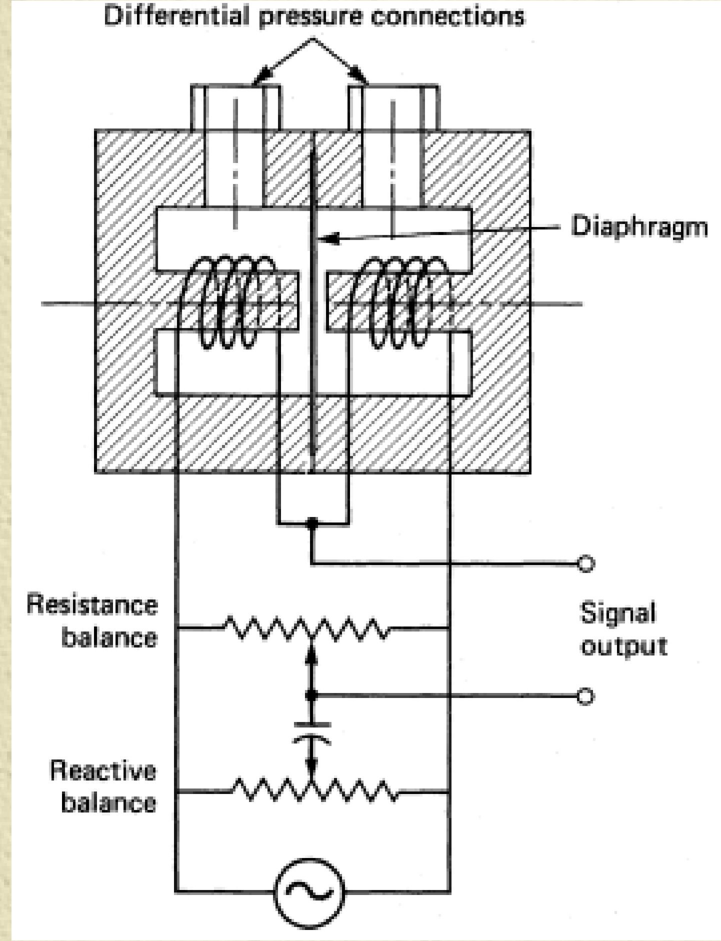


Location of strain gages on flat diaphragm

The central gage is subjected to tension while the outer gage senses compression

- Apply strain gage directly to a diaphragm surface and calibrate the measured strain in terms of pressure
- Drawback – the small physical area available for mounting the gages

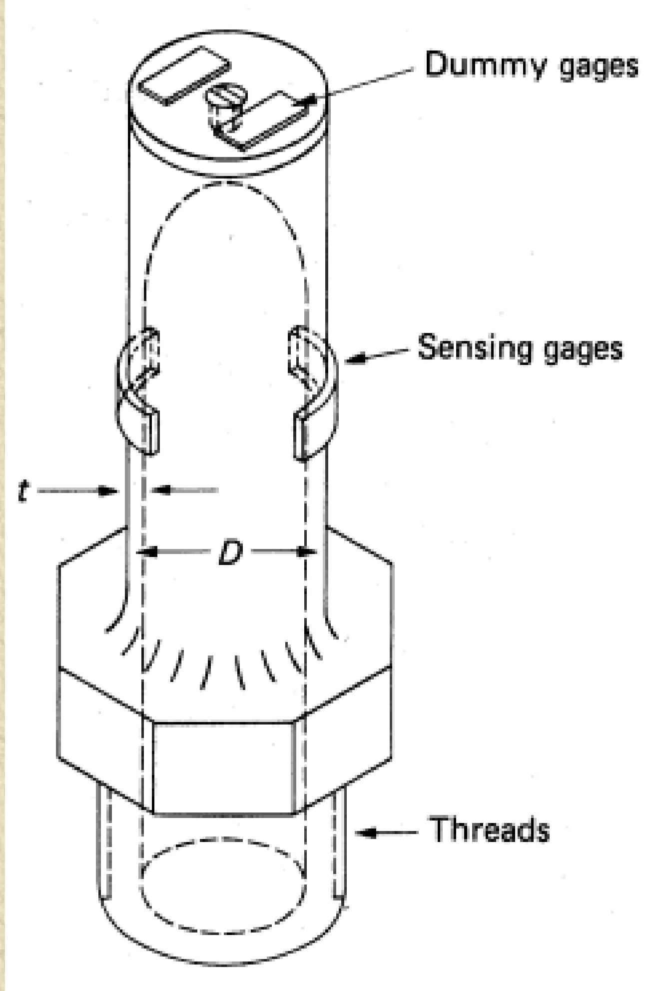
Inductive Types



Differential pressure cell with inductance-type secondary transducer

- Flexing the diaphragm due to applied pressure causes it to move toward on pole piece and away from the other →
- Altering the relative inductances
- Standard laboratory equipment, such as an oscilloscope or electronic voltmeter, as well as recorders, may be used to display the gage output

Cylindrical-Type Pressure Cell

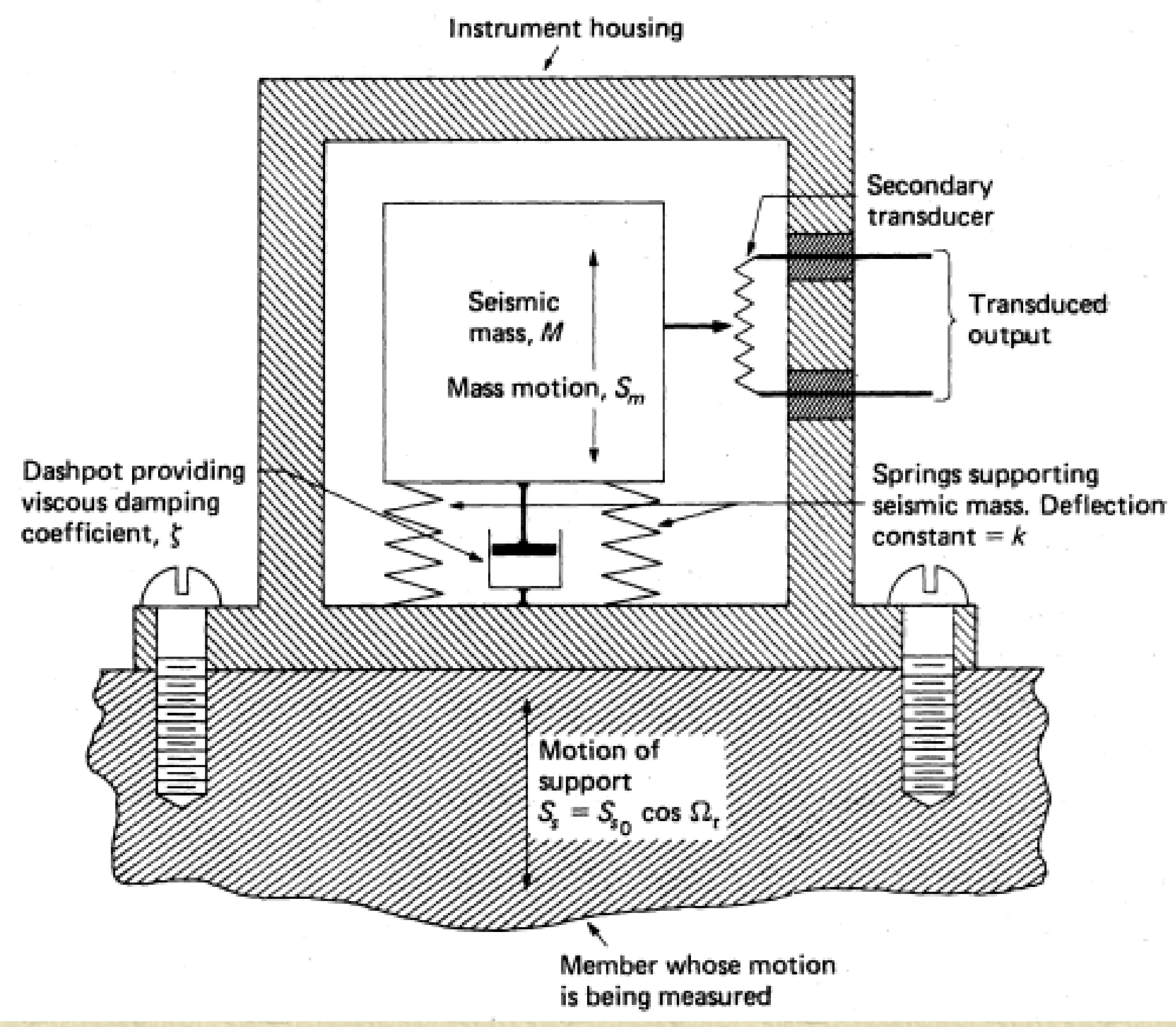


Measurement of Motion

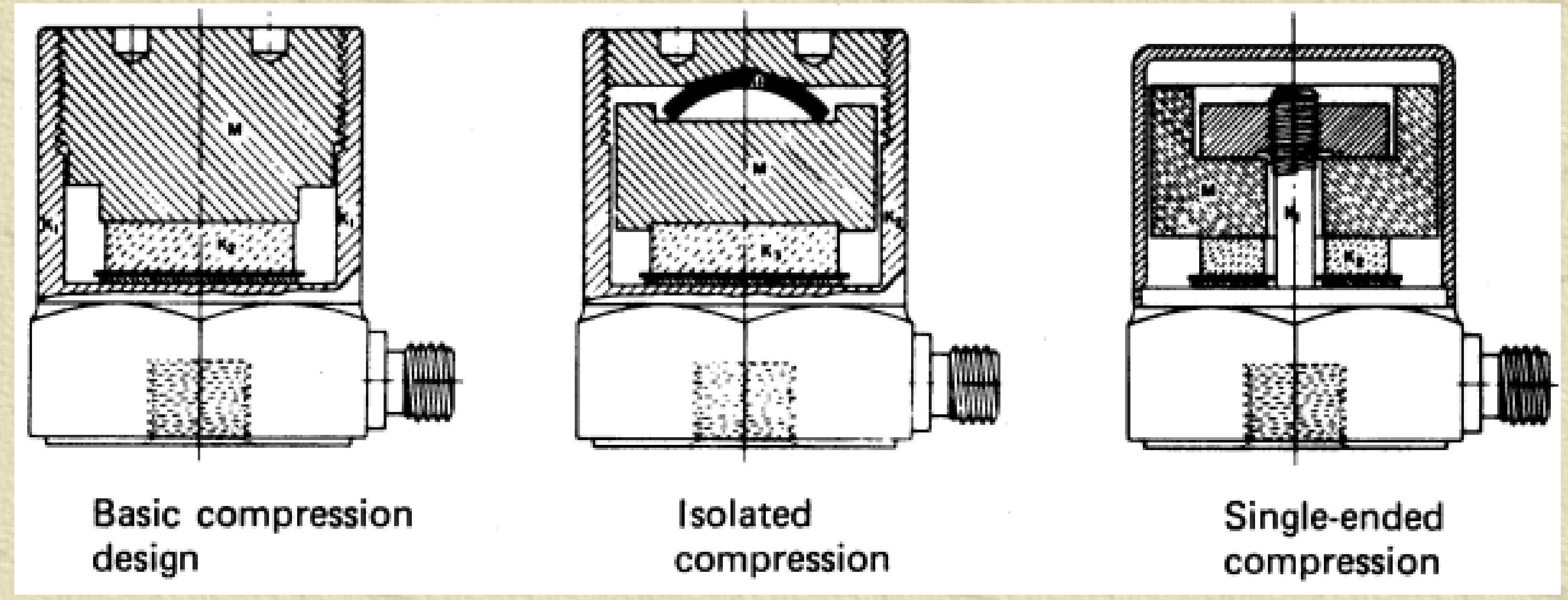
- Displacement
- Velocity
- Acceleration
- Jerk

Human Touch

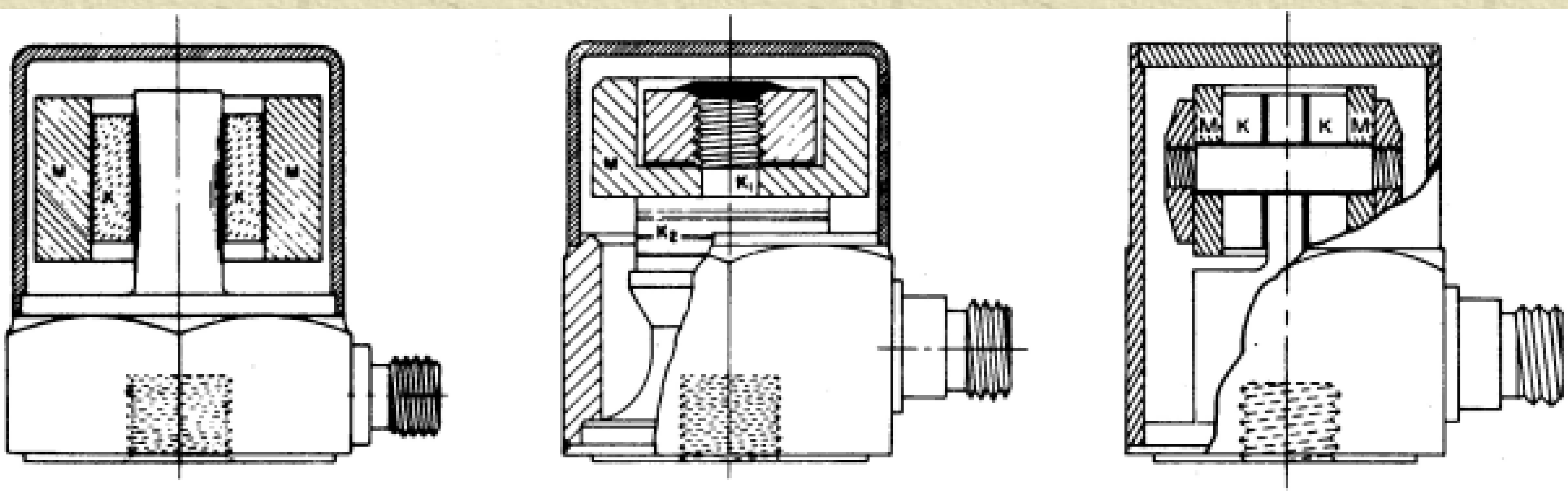
- In spite of the tremendous advances made in vibration-measuring instrumentation, one of the most sensitive vibration detectors is the human touch
- Fingertips – sinusoidal vibrations having amplitudes as low as 12 μ in
- Vibrating member tightly gripped – average minimum detectable amplitude was only slightly greater than 1 μ in
- Greatest sensitivity occurred at a frequency of about 300 Hz



Seismic type of motion-measuring instrument



Typical piezoelectric-type accelerometer designs



Shear design

Isobase® design

Isoshear™ design

Advantages of Piezoelectric-Type Accelerometers

- High sensitivity
- Extreme compactness
- Ruggedness