The Dublin School of Grinds

3rd Year

Maths

Higher Level

Strand 5 of 5

[Topics:

Functions & Graphs]

The Dublin School of Grinds

6-HOUR CRASH COURSES

MAY & JUNE 2016

Pictured: Economics teacher Rónán Murdock

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Crash Courses Timetable

6th Year

Subject	Level	Date	Time
Accounting	H	Sunday 29th May	9am - 3pm
Biology	H	Saturday 28th May	9am - 3pm
Business	H	Sunday 29th May	2pm - 8pm
Chemistry	H	Saturday 4th June	9am - 3pm
Economics	H	Saturday 28th May	9am - 3pm
English	H	Sunday 29th May	9am - 3pm
English	H	Saturday 4th June	9am - 3pm
French	H	Saturday 4th June	9am - 3pm
Geography	H	Saturday 28th May	9am - 3pm
Irish	H	Saturday 4th June	9am - 3pm
Maths Paper 1	H	Saturday 4th June	9am - 3pm
Maths Paper 2	H	Sunday 5th June	9am - 3pm
Maths	O	Saturday 28th May	9am - 3pm
Maths	O	Saturday 4th June	9am - 3pm
Physics	H	Saturday 28th May	9am - 3pm
Spanish	H	Sunday 5th June	9am - 3pm

3rd Year

Subject	Level	Date	Time
Business Studies	H	Sunday 5th June	9am - 3pm
English	H	Sunday 5th June	9am - 3pm
French	H	Sunday 29th May	9am - 3pm
Irish	H	Sunday 29th May	9am - 3pm
Maths	H	Sunday 29th May	9am - 3pm
Science	H	Saturday 4th June	9am - 3pm

H = Higher O = Ordinary

Please note that all courses will take place at our Learning Centre at The Primary School in Oatlands, Stillorgan, Co. Dublin.

Strand 5 is worth 5 % to 16% of The Junior Cert.

It appears on paper 1.

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Functions and Graphs

Functions and Graphs is worth 5 % to 16% of The Junior Cert.

It appears on Paper 1.

1) Understanding functions

A function is a rule that produces one output for each input.

For example, if I say “pick a number, then add 3, then multiply by 5”:

If you start with an input of 10, then your output would be 65.

If you start with an input of 6, your output would be 45.

If you start with an input of 100, your output would by 515

and so on.

The set of inputs in called the ‘domain’.

The set of outputs is called the ‘range’.

The fancy way to write the above rule is:

$$f(x) = 5(x+3)$$
$$= 5x + 15$$

Instead of using $f(x) =$, we can use $f: x \to$ or $y =$

Example 1

If $f(x) = 3x - 7$, find

i) $f(2)$

ii) $f(0)$

iii) $f(-8)$

i) Well, if $f(x) = 3x - 7$

$$\Rightarrow f(2) = 3(2) - 7$$
$$= 6 - 7$$
$$= -1$$

ii) Well, if $f(x) = 3x - 7$

$$\Rightarrow f(0) = 3(0) - 7$$
$$= 0 - 7$$
$$= -7$$

iii) Well, if $f(x) = 3x - 7$

$$\Rightarrow f(-8) = 3(-8) - 7$$
$$= -24 - 7$$
$$= -31$$

Example 2

If $f(x) = 6x - 4$, solve $f(x) = 38$.

Now, be careful here, we aren’t asked for $f(38)$, we’re actually being told $f(x) = 38$

$$\Rightarrow 6x - 4 = 38$$
$$6x = 38 + 4$$
$$6x = 42$$
$$x = \frac{42}{6}$$
$$x = 7$$

If there is more than one function in a question, the Examiner usually calls the second one $g(x)$ or $g: x$, although he can actually use any letter.

Example 3

f and g are two functions such that $f: x \to x^2 + 2$ and $g: x \to 17 - 2x$.
Find the values of x for which $f(x) = g(x)$.

Well, we're told they're equal, so we put them equal:

$$x^2 + 2 = 17 - 2x$$
$$x^2 + 2x + 2 - 17 = 0$$
$$x^2 + 2x - 15 = 0$$
$$a = 1 \quad b = 2 \quad c = -15$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{64}}{2}$$
$$x = \frac{-2 \pm 8}{2}$$
$$x = \frac{-2 + 8}{2} \quad or \quad x = \frac{-2 - 8}{2}$$
$$x = \frac{6}{2} \quad or \quad x = \frac{-10}{2}$$
$$x = 3 \quad or \quad x = -5$$

Example 4

If $f: x \to 1 + \frac{2}{x}$, , find the value of k if $f\left(\frac{1}{3}\right) = kf(2)$

Solution

$$1 + \frac{2}{\frac{1}{3}} = k\left[1 + \frac{2}{2}\right]$$
$$1 + 6 = k[1 + 1]$$
$$7 = k[2]$$
$$7 = 2k$$
$$\frac{7}{2} = k$$

Question 1.1

$g(x) = \sqrt{5x - 2}, x \in \mathbb{N}$. Find $g(2)$.

Give your answer in the form $a\sqrt{a}, a \in \mathbb{N}$.

Question 1.2

Here is a number machine.

Input → Add 5 → Multiply by 4 → Output

a) Find the **output** when the **input** is 10

b) Work out the **input** when the **output is 28**

c) Work out the **input** when the **output is -8**

d) Find an expression, in terms of x, for the **output** when the **input** is x.

Question 1.3

If $f(x) = x^2 + 3x - 7$, show that the $f(x+1) + 3 = x^2 + 5x$.

2) Graphing linear and quadratic functions

Functions without squared things in them represent lines. These are called 'linear functions'.

For example $f(x) = 3x - 7$ would represent a line and is called a linear function.

Functions with squared things in them represent U or ∩ shaped curves. These are called quadratic functions.

For example $f(x) = x^2 + 2x - 3$ would be U shaped and is called a quadratic function.

$f(x) = -x^2 + 2x - 3$ would be ∩ shaped and can be called a quadratic function.

How do I know if it's U shaped or ∩ shaped?!

Well, if the x^2 is positive we get a happy face: U

If the x^2 is negative we get a sad face: ∩

<u>Example 1</u>

Let f be the function $f : x \to -2x^2 + 140x$.

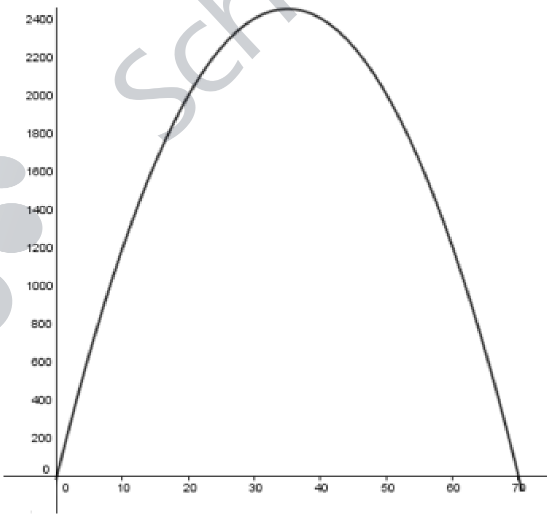
Evaluate $f(x)$ when $x = 0, 10, 20, 30, 40, 50, 60, 70$.

Hence, draw the graph of f for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

It's handy to make out a table when you're asked to work with numerous values. On a graph $f{:}\,x$ or $f(x)$ or $g(x)$ stand for y.

X	$y = -2x^2 + 140x$	
0	$y = -2(0)^2 + 140(0)$	=> $y = 0$
10	$y = -2(10)^2 + 140(10)$	=> $y = 1200$
20	$y = -2(20)^2 + 140(20)$	=> $y = 2000$
30	$y = -2(30)^2 + 140(30)$	=> $y = 2400$
40	$y = -2(40)^2 + 140(40)$	=> $y = 2400$
50	$y = -2(50)^2 + 140(50)$	=> $y = 2000$
60	$y = -2(60)^2 + 140(60)$	=> $y = 1200$
70	$y = -2(70)^2 + 140(70)$	=> $y = 0$

Note: There is a shortcut way to create the above table on your calculator. However, students usually make mistakes so we will do it using the table as above.



Question 2.1

Let f be the function $f : x \to 7x - x^2$.

Draw the graph of f for $0 \leq x \leq 7$, $x \in \mathbb{R}$.

Question 2.2

Let *f* be the function $f: x \to 5x - 4$ and g be the function $g: x \to 3x + 1$. Draw the graph of *f* and the graph of g, for $0 \leq x \leq 5,\ x \in \boldsymbol{R}$.

3) Finding coefficients of functions

One of the Examiners favourite Junior Cert questions is to give you a graph and ask you to find values. Just follow these steps:

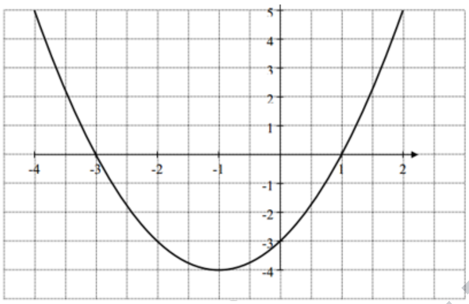
Step 1: Sub in the points given.

Step 2: Do simultaneous equations

Example 1

The diagram below shows part of the graph of the function

$f: x \to x^2 + bx + c$, where $x \in \mathbb{R}$ and $b, c \in \mathbb{Z}$.



The graph cuts the x axis at $(-3, 0)$ and $(1, 0)$, as shown in the diagram.

Calculate the value of b and the value of c.

Solution

Now we know $f: x \to x^2 + bx + c$ can be re-written as $y = x^2 + bx + c$.

Step 1: We know two points on the graph:

$(-3, 0)$ $0 = (-3)^2 + b(-3) + c$ $0 = 9 - 3b + c$ $3b - c = 9$ ①	$(1, 0)$ $0 = (1)^2 + b(-1) + c$ $0 = 1 + 1b + c$ $b + c = -1$ ②

Step 2: Now this is just an Algebra question where we can solve by using simultaneous equations:

① $3b - c = 9$

② $\underline{b + c = -1}$

$4b = 8$

$b = \frac{8}{4}$

$b = 2$

Subbing back into ①:

$3(2) - c = 9$

$6 - c = 9$

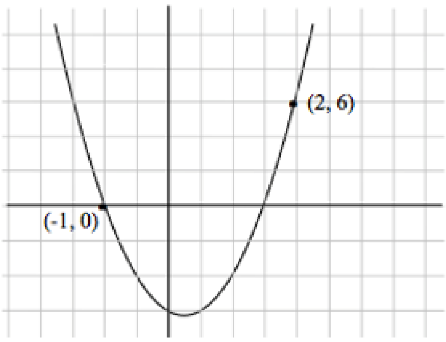
$6 - 9 = c$

$-3 = c$

Question 3.1

Let f be the function $f: x \to 4x^2 + bx + c, x \in \mathbb{R}$ and $b, c \in \mathbb{Z}$.

The points $(2, 6)$ and $(-1, 0)$ lie on the graph of f, as shown in the diagram.

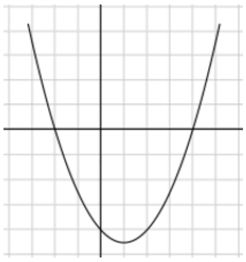


Find the value of b and the value of c.

Question 3.2

The diagram shows part of the graph of the function

$f : x \to x^2 + bx + c$, where $x \in \mathbf{R}$ and $b, c \in \mathbf{Z}$.



The graph intersects the x-axis at $(-1, 0)$ and $(2, 0)$.

(i) Calculate the value of b and the value of c.

(ii) $(k, -k+14)$ is a point on the graph, where $k \in \mathbf{Z}$.

Find the values of k.

4) Graphs crossing the x and y axes

The Examiner can ask you where a graph crosses the x-axis or y-axis.

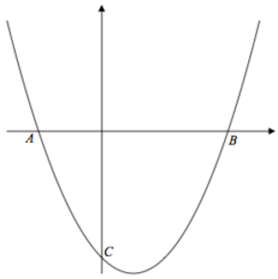
Rule: On the x-axis: y = 0

On the y-axis: x = 0

Note: Where a graph crosses the x-axis is known as the root(s) of the function.

Example 1

The diagram shows part of the graph of the function $f : x \to x^2 - 2x - 8,\ x \in \mathbb{R}$.



The graph intersects the x axis at A and B and the y axis at C.

Find the co-ordinates of A, B and C.

Solution

Well, A and B are on the x-axis, so y = 0

$$=> 0 = x^2 - 2x - 8$$
$$a = 1 \quad b = -2 \quad c = -8$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{36}}{2}$$
$$x = \frac{2 \pm 6}{2}$$
$$x = \frac{2+6}{2} \quad or \quad x = \frac{2-6}{2}$$
$$x = \frac{8}{2} \quad or \quad x = \frac{-4}{2}$$
$$x = 4 \quad or \quad x = -2$$
$$=> B = (4, 0) \quad and \quad A = (-2, 0)$$

Note: You can just tell which one is which by looking at the diagram.

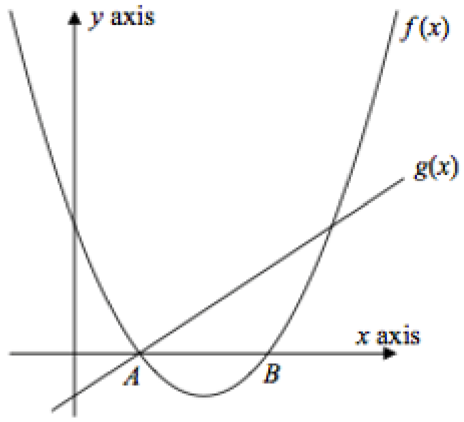
Now, C is on the y-axis, so x = 0

$$=> y = (0)^2 - 2(0) - 8$$
$$y = 0 - 0 - 8$$
$$y = -8$$
$$=> C = (0, -8)$$

Question 4.1

The diagram below shows part of the graphs of the functions

$$f(x) = x^2 - 4x + 3 \text{ and } g(x) = x + k.$$



The graph of $f(x)$ cuts the x axis at A and B.

The graphs of $f(x)$ and $g(x)$ intersect at A.

(i) Find the coordinates of A and the coordinates of B.

(ii) Find the value of k.

Note: If you're asked where 2 graphs intersect simply put them equal to each other and tidy up the algebra to find the x-value (or values if there's more than one intersection).To find the y-values, you simply sub the x-values you found back into either of the two functions.

Question 4.2

Let f be the function $f : x \rightarrow 5 - 3x - 2x^2$ and g be the function $g : x \rightarrow -2x - 1$.

Using the same axes and scales, draw the graph of f

and the graph of g, for $-3 \leq x \leq 2$, $x \in \mathbf{R}$.

Find where the graph intersects the graph by:

i) Reading your graph.

ii) Using algebra.

State one advantage and one disadvantage of each method.

Question 4.3

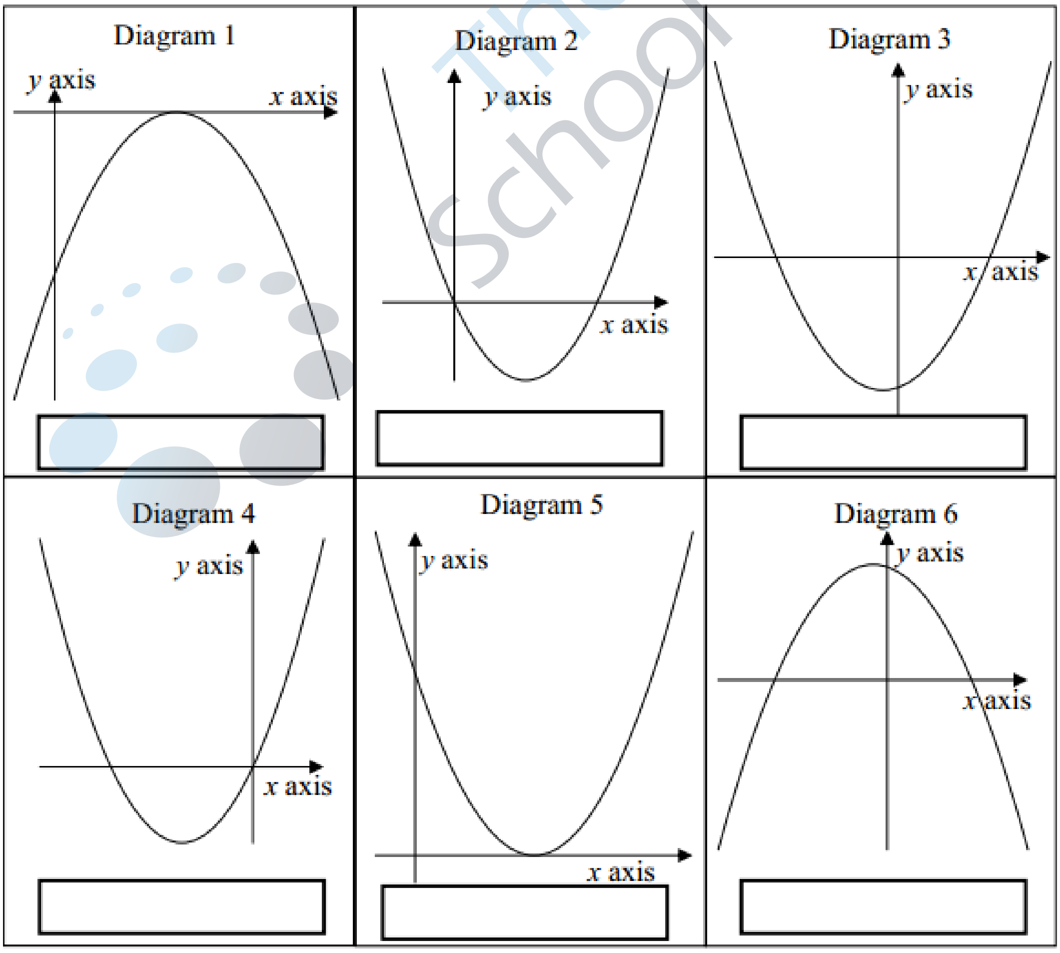
(a) Three functions: $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$f(x) = 2x^2 + x - 6$, $\quad g(x) = x^2 - 6x + 9$ and $\quad h(x) = x^2 - 2x$.

Solve $f(x) = 0$	Solve $g(x) = 0$	Solve $h(x) = 0$

(b) The table below shows the sketches of six different functions. Three of the sketches belong to the three functions from part **(a)**.

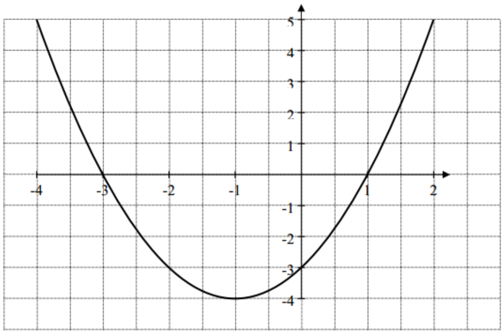
Write $f(x)$, $g(x)$ or $h(x)$ into the box underneath the correct sketch for each of the three functions.



5) Maximum/minimum values

The maximum (or minimum) value of a graph is the highest (or lowest) y-value.

For example, in the following graph:



... the minimum value is −4 (it has no maximum.)

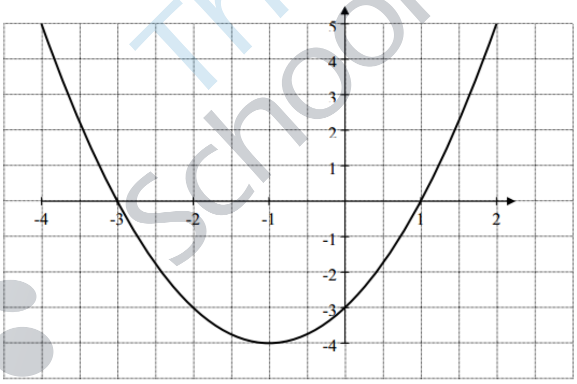
Note the way we talk about y-values here , not x-values.

6) Increasing/ Decreasing functions

A function is increasing when the graph is 'going uphill'.

A function is decreasing when the graph is 'going downhill'

In the graph below:



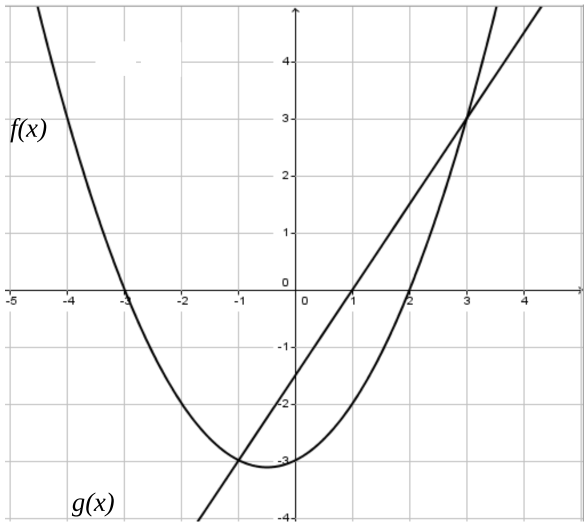
... the function is increasing from x = −1 to x = 2.

The function is decreasing from x = −4 to x = −1.

Note the way we talk about x-values here, not y-values.

7) One graph below the other

If you were given the following graph:



and asked where is $f(x) < g(x)$?

This simply means: where is f(x) below g(x)?

i.e.: Where is the curve below the line?

The answer is "between x = −1 and x = 3."

The fancy way you might see this written is $-1 < x < 3$, but you can just write it in words like we did above, as this gets full marks.

What if you were asked where is f(x)<0?

This simply means: where is f(x) lower than the x-axis?

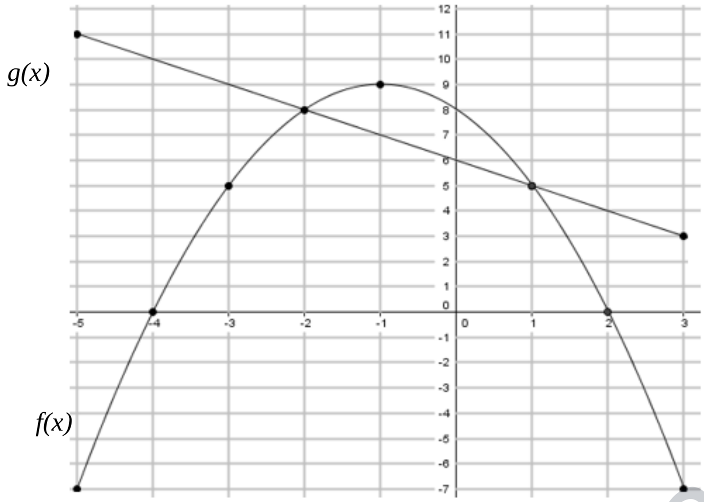
The answer is "between −3 and 2", which can be written as $-3 < x < 2$.

Note: If you're not sure whether to use < or ≤, just use what's given in the question.

Question 7.1

Write the range of the values of x for which

i) $g(x) < f(x)$
ii) $f(x) > 0$

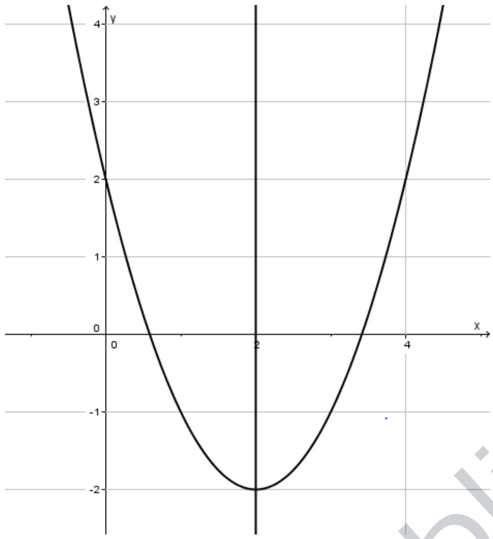


8) Axis of symmetry

The axis of symmetry is simply a straight line that the graph could fold over onto itself.

The Examiner can ask you to draw or write the equation of the axis of symmetry.

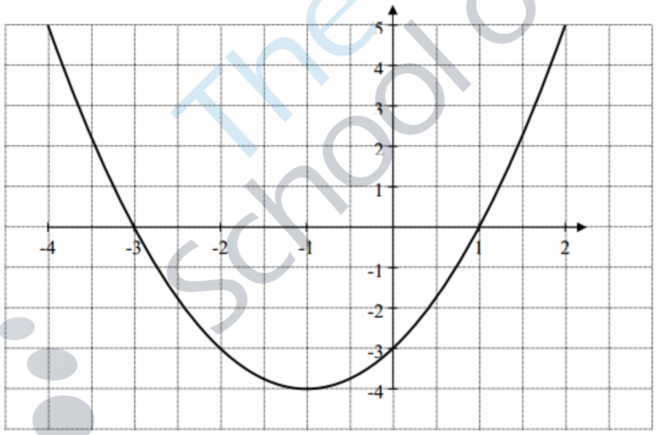
For example, the straight line below shows the axis of symmetry:



Question 8.1

The diagram below shows part of the graph of the function

$f : x \rightarrow x^2 + bx + c$, where $x \in \mathbb{R}$ and $b, c \in \mathbb{Z}$.



The graph has a minimum point at $(-1, -4)$.

Write down the equation of the axis of symmetry of the function in the form $x = k$, where $k \in \mathbb{Z}$.

9) Quadratic real life graphs

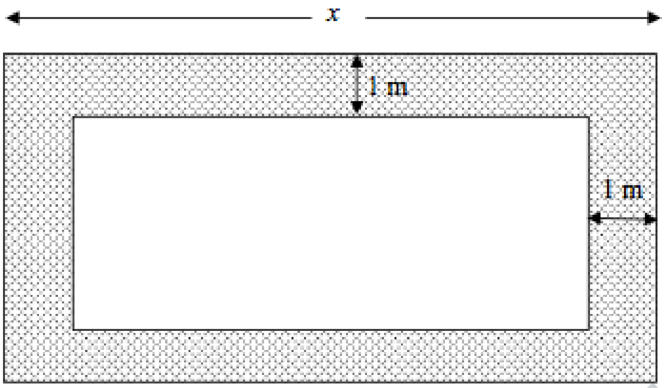
The Examiner loves to relate quadratic graph questions to real life.

Example 1

(a) The diagram shows a rectangular garden of perimeter 24 m.

The length of the garden is x m.

Write down an expression in x for the width of the garden.



(b) Paving of width 1 m is placed around the garden as shown.

(i) Write expressions in x for the length and width of the inner section.

(ii) ✍ Show that the area, in m^2, of the inner section is $-x^2 + 12x - 20$.

(c) The area of the inner section is represented by the function:

$f : x \to -x^2 + 12x - 20.$

(i) ✍ Draw the graph of f for $2 \le x \le 10,\ x \in \mathbf{R}$.

(ii) Write down the maximum possible area of the inner section.

Solution

a) We're asked to find the width.
Let's call it y.
Now, we're told the perimeter is = 24metres

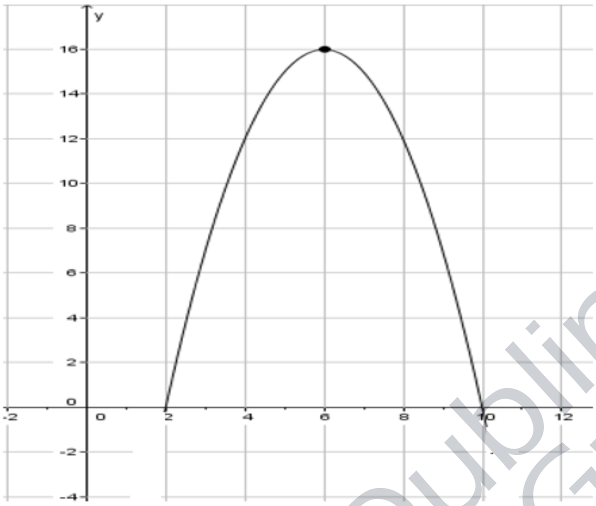
$$=> x + y + x + y = 24$$
$$2x + 2y = 24$$
$$2y = 24 - 2x$$
$$y = \frac{24 - 2x}{2}$$
$$y = 12 - x$$

b) (i) Inner length = $x - 2$
Inner width = $y - 2$
$= (12 - x) - 2$
$= 10 - x$

(ii) Area $= (length)(width)$
$= (x - 2)(10 - x)$
$= 10x - x^2 - 20 + 2x$
$= -x^2 + 12x - 20$

c) (i)

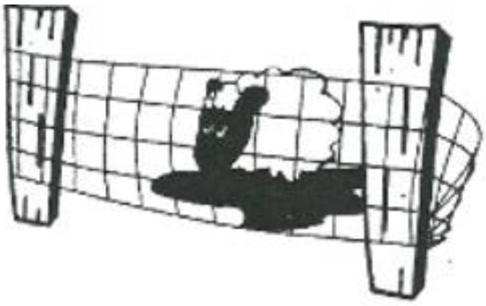
x	$y = -x^2 + 12x - 20$	
2	$y = -(2)^2 + 12(2) - 20$	=> $y = 0$
3	$y = -(3)^2 + 12(3) - 20$	=> $y = 7$
4	$y = -(4)^2 + 12(4) - 20$	=> $y = 12$
5	$y = -(5)^2 + 12(5) - 20$	=> $y = 15$
6	$y = -(6)^2 + 12(6) - 20$	=> $y = 16$
7	$y = -(7)^2 + 12(7) - 20$	=> $y = 15$
8	$y = -(8)^2 + 12(8) - 20$	=> $y = 12$
9	$y = -(9)^2 + 12(9) - 20$	=> $y = 7$
10	$y = -(10)^2 + 12(10) - 20$	=> $y = 0$



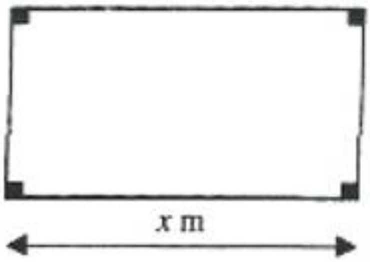
(ii) We're asked to find the max possible area. But area $= -x^2 + 12x - 20$ (from part (b))
But $f: x \rightarrow -x^2 + 12x - 20$ (from part (c))
=> area = f:x
But f:x is y
=> area = y
So, really, we're being asked to find the max possible y-value. From the graph this is clearly 16.
i.e.: max possible area = 16m²

Question 9.1

A person has 18 metres of wire fencing and four posts with which to make a rectangular enclosure in the middle of a field.



One side of the rectangle is x metres long.



(a) Show that the area enclosed will be $9x - x^2$ square metres.

(b) Let f be the function $f : x \rightarrow 9x - x^2$.

Evaluate $f(x)$ when $x = 1, 2, 3, 4, 5, 6, 7$.

Hence, draw the graph of f for $1 \leq x \leq 7$, $x \in \mathbf{R}$.

(c) Use your graph from part **(b)** to estimate:

(i) the area of the enclosure when $x = 2{\cdot}7$ metres

(ii) the two possible values of x for which the area is 19 square metres

(iii) the maximum possible area

(iv) the length and breadth of the enclosure of maximum area.

Question 9.2

The perimeter of a rectangle is 14 m. The width of the rectangle is x m.

(a) ✍ Write an expression in x for the length of the rectangle.

(b) (i) ✍ Show that the area, in m^2, of the rectangle is $7x - x^2$.

(ii) ✍ Let f be the function $f : x \to 7x - x^2$.

Draw the graph of f for $0 \le x \le 7$, $x \in \mathbf{R}$.

(c) Use your graph from part **(b)** to estimate:

(i) the area of the rectangle when the width is $1 \cdot 5$ m

(ii) the maximum possible area of the rectangle

(iii) the two possible values of the width of the rectangle when the area is $4\ m^2$.

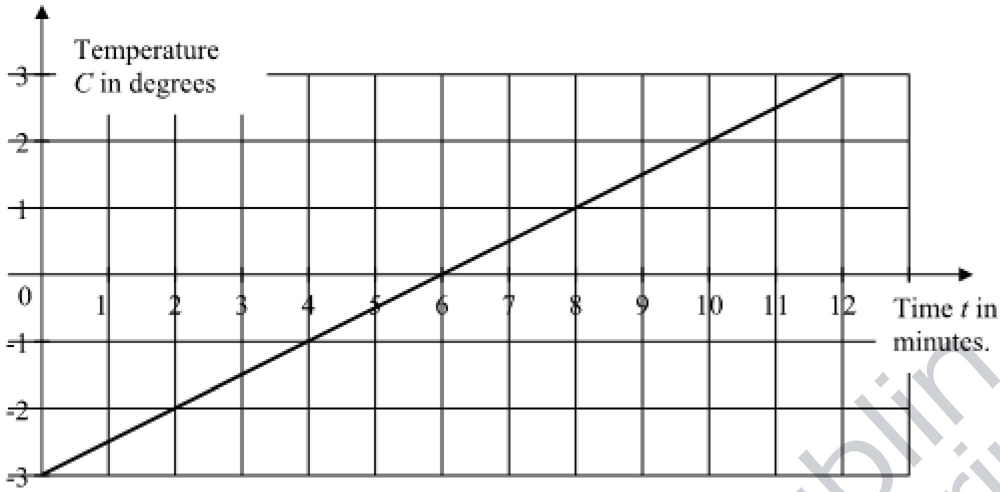
10) Other real life graphs

We meet 'distance-time' graphs in the Arithmetic chapter.

However, the Examiner may use a different scenario.

Example 1

A cold object is placed in a warm room.
Its temperature C degrees after time t minutes is shown in the following graph.



(i) After what time interval is the temperature of the object 0 degrees?

(ii) What is the rise in temperature of the object in the first 10 minutes?

(iii) The relationship between the temperature C and the time t is given by

$$C = \frac{1}{2}(t + k).$$

Find the value of k.

Solution

i) We can see the temperature is 0 degrees after 6 minutes.

ii) The temperature was -3 degrees at the start. At 10 minutes it was 2 degrees, so therefore the rise in the first 10 minutes was 5 degrees.

iii) Here, we're being asked to find a missing coefficient, so, according to Section 3, we must sub in a point on the graph.
Now, there are loads of points we could pick, let's just pick (6, 0), where 6 is the time (t) and 0 is the temperature (C)

$$C = \frac{1}{2}(t + k)$$

$$0 = \frac{1}{2}(6 + k)$$

$$\times 2: \quad 0 = 6 + k$$

$$-6 = k$$

Question 10.1

At a certain point during the flight of a space shuttle, the booster rockets separate from the shuttle and fall back to earth. The altitude of these booster rockets (their height above sea level) is given by the following formula:

$$h = 45 + \frac{7}{10}t - \frac{1}{200}t^2$$

where h is the altitude in kilometres, and t is the time in seconds after separation from the shuttle.

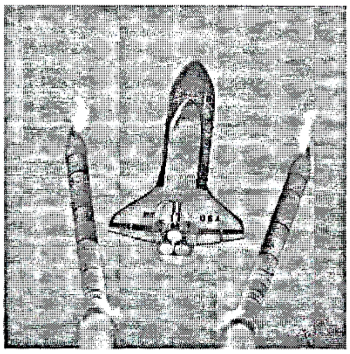


Image: NASA

(a) Complete the table below, showing the altitude of the rockets at the indicated times.

time in seconds, t	0	20	40	60	80	100
altitude in km, h						

(b) On the opposite page, draw a graph of the altitude of the rockets for the first 100 seconds after separation from the shuttle.

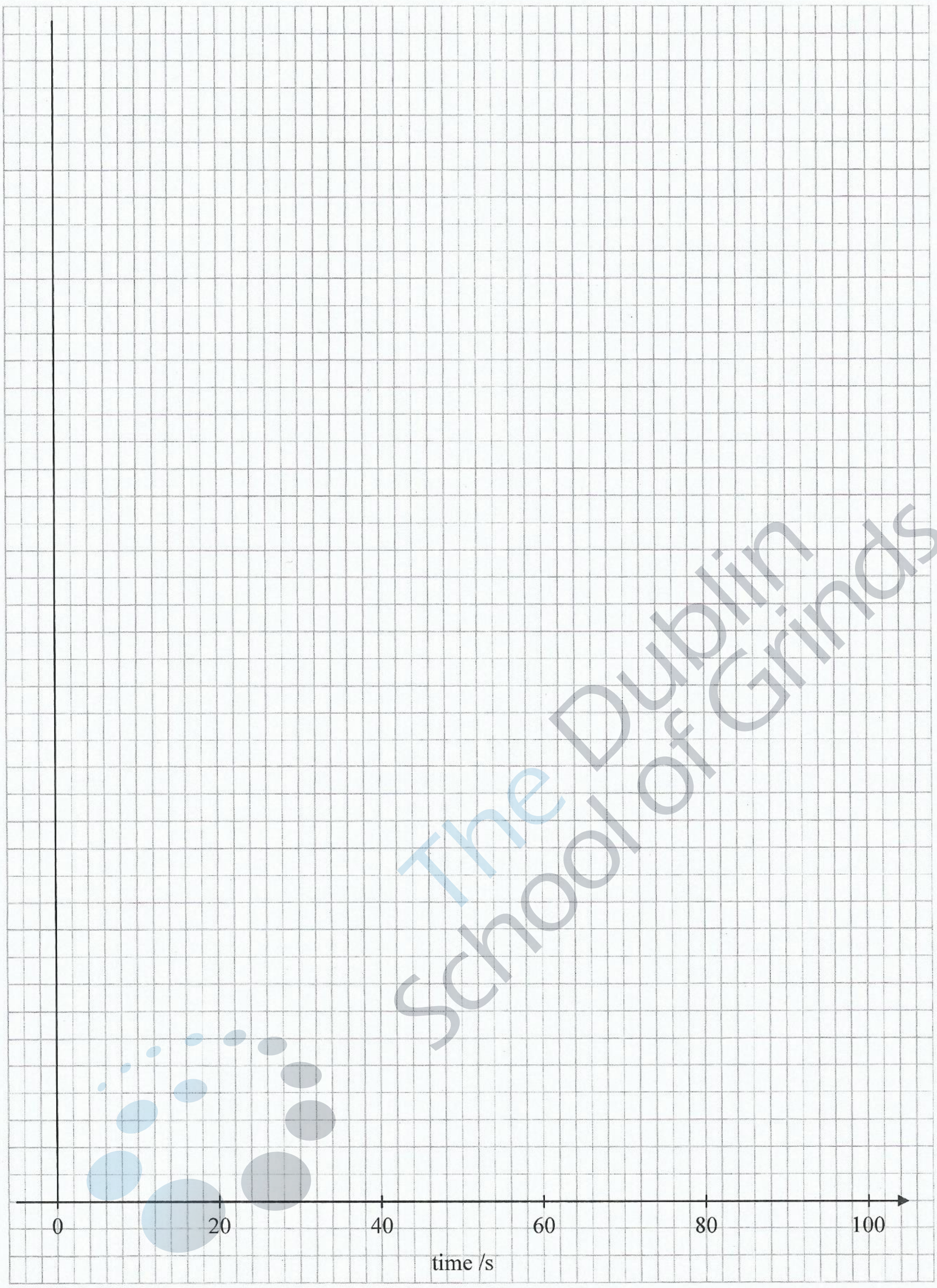
(c) Use your graph to estimate the greatest altitude reached by the rockets.

Answer: __________________

(d) Use the graph to estimate **one** time at which the altitude is 60 km. Show your work clearly on the graph.

Answer: __________________

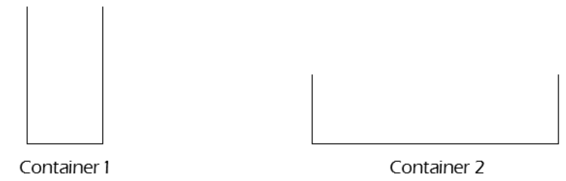
Graph of altitude over time.



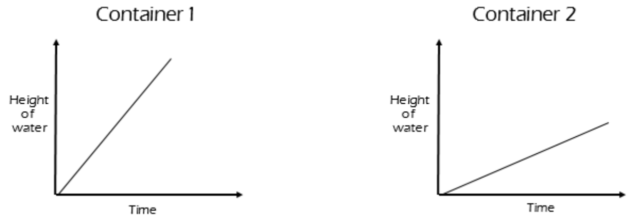
(e) Check your answer to part **(d)** using the formula for the altitude.

(f) By solving an equation, find the value of t at which the altitude of the rockets is 9 km.

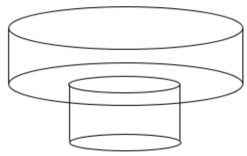
Another type of potential exam question involves water filling containers.
If I took two containers:



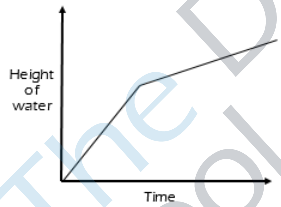
... and I put a water hose in each of them. The height of the water in container 1 would increase far quicker than container 2:



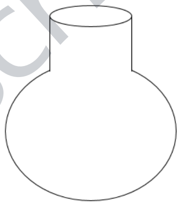
Now, the increase in height may not always be linear (a straight line)
For example, the following shape:



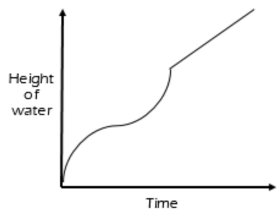
would have a graph like this:



What about if you fill up a spherical flask:

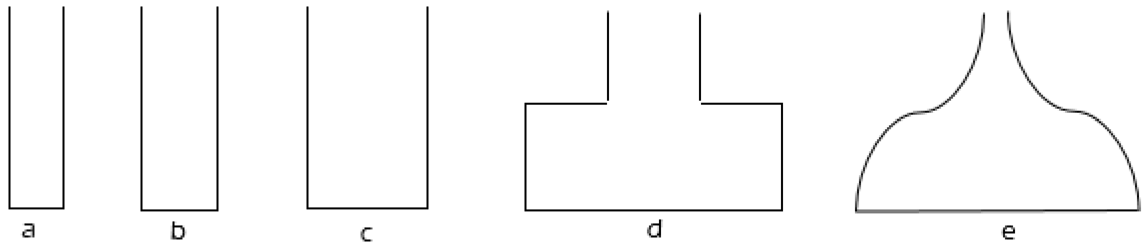


The graph for this would be:



Question 10.2

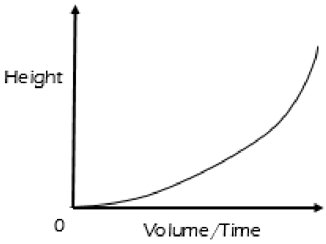
The following shapes are all containers that are to be filled with water from a hose pipe. The water flows at a steady rate all of the time:

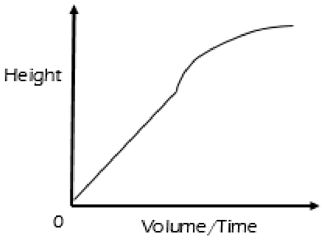


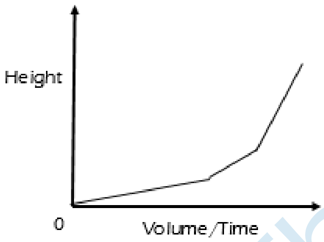
Sketch a graph for each container, representing the height of the water over time

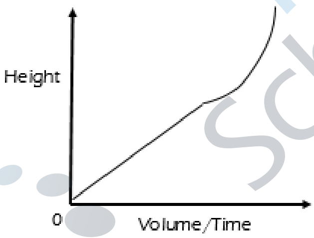
Question 10 .3

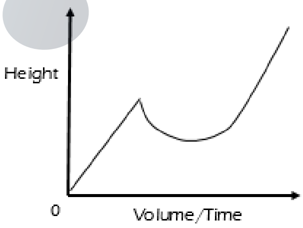
For each graph of height versus volume/time below, sketch a container that could result in such a graph when filled at a constant rate. It may be that it's not possible to match a container to some graphs – in which case you should explain why a match can't be found.











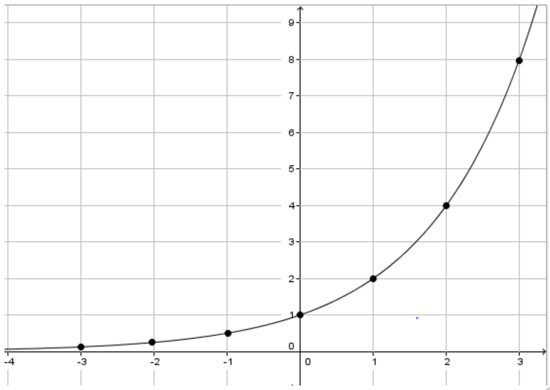
11) Exponential functions

Functions which have a variable power are called exponential functions.

For example $f(x) = 2^x$ is an exponential function.

When graphed, these have weird shapes, similar to a skateboard ramp.

$f(x) = 2^x$ would look like this:



Example 1

Graph the function $f(x) = 4.2^x$ in the domain $-4 \leq x \leq 1$, where $x \in R$.

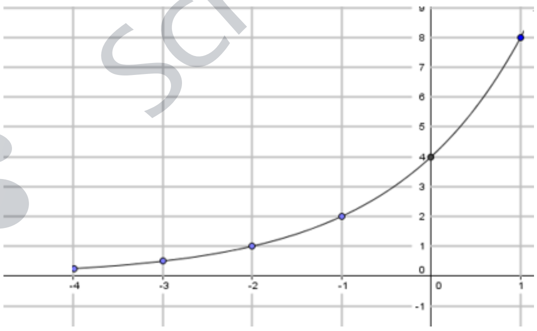
Note: The dot here means multiply, not decimal.

Solution

Just like other graphs, we do up a table to help us:

x	$\boldsymbol{y = (4).2^x}$	
-4	$y = (4).2^{-4}$	$=> y = 0.25$
-3	$y = (4).2^{-3}$	$=> y = 0.5$
-2	$y = (4).2^{-2}$	$=> y = 1$
-1	$y = (4).2^{-1}$	$=> y = 2$
0	$y = (4).2^{0}$	$=> y = 4$
1	$y = (4).2^{1}$	$=> y = 8$

So our graph would look like:



Note that the graph crosses the y-axis at 4. This is no coincidence! The graph will always cut the y-axis at whatever number your function is multiplied by.

For example **7**.2^x would cross at y = 7

-8.3^x would cross at y = -8

2^x would cross at y = 1 (because it is 1.2^x)

... and so on.

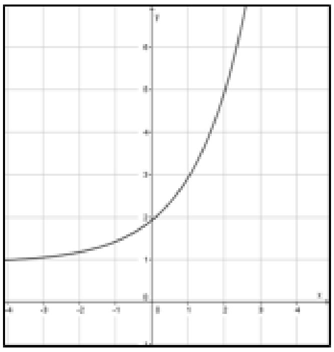
- In the above diagram, what if I asked you to find f(-2)? This simply means: find the height of the graph at x = -2. The answer is 1.
  Similarly $f(-3) = \frac{1}{2}$, $f(-1) = 2$, and so on.
- And if I asked you to estimate the value of x for which $f(x) = 1.1$?
  This simply means: find which x value gives a height of 1.1?
  The answer is ≈ - 1.9.

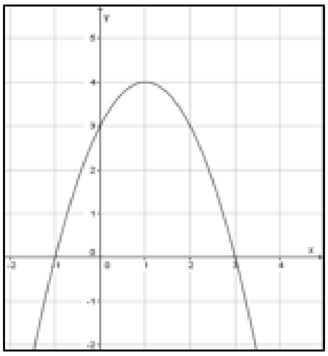
Question 11.1

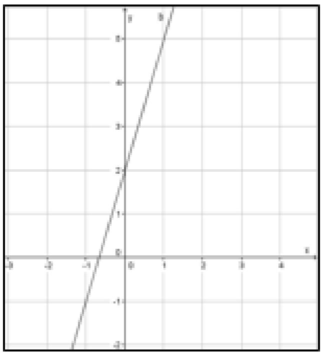
a) Sketch the graph of $f(x) = 3^x$ in the domain $-2 \leq x \leq 2$, where $x \in R$.
b) Where is $f(x) = 9$?
c) On the same diagram, sketch $g(x) = 2.3^x$

Question 11.2

Identify each function as linear, quadratic, or exponential







Question 11.3

Give an example of

i) A linear relationship
ii) A quadratic relationship
iii) An exponential relationship

12) Past and probable exam questions

Question 1

(a)

Let f be the function $f : x \to 10 - x - 2x^2$.

(i) ✍ Draw the graph of f for $-3 \leq x \leq 3,\ x \in \mathbb{R}$.

(ii) Use your graph to estimate the maximum value of $f(x)$.

(iii) Use your graph to estimate the values of x for which $f(x) = 6$.

(b)

Given that $f(x) = 3x - 4$ and that $f(k) = 11$, find the value of k.

Question 2

(a)

Let f be the function $f: x \to 7x - x^2$.

Draw the graph of f for $0 \leq x \leq 7, x \in \mathbb{R}$.

(b)

The formula for the height, y metres, of a golf ball above ground level x seconds after it is hit, is given by $7x - x^2$.

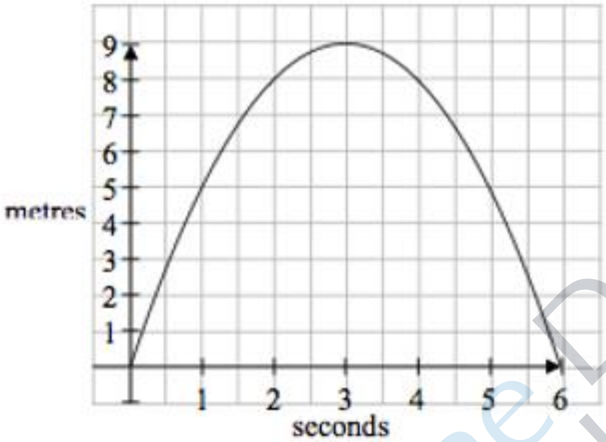
Use your graph from part (a):

(i) ✍ to find the maximum height reached by the golf ball

(ii) ✍ to estimate the number of seconds the golf ball was more than 2 metres above the ground.

The graph below represents the flight of another golf ball.

The flight of the golf ball is given by the formula $ax - x^2$, $x \in \mathbb{R}$.



(iii) ✍ Find the value of a.

Question 3

(a)

Let f be the function $f : x \to -x^2 - 4x + 5,\ x \in \mathbf{R}$.

(i) ✍ Find the co-ordinates of the points where the graph of $f(x)$ cuts the x-axis.

(ii) ✍ Solve $f(x) = f(x+1)$.

(b)

Given that $f(x) = 5x - 12$ and that $f(a) = a$, find the value of a.

(c)

Given that $f(x) = kx + 8$ and that $f(9) = 44$, find the value of k.

Question 4

(i) Let f be the function $f : x \to 2x^2 - 4x + 5$.

✍ Draw the graph of f for $-2 \leq x \leq 4$, $x \in \mathbf{R}$.

(ii) ✍ Use your graph to find the values of x for which $f(x) = 7$.

Question 5

(a) Let f be the function $f : x \to 35x - 5x^2$.

Draw the graph of f for $0 \le x \le 7$, $x \in \mathbf{R}$.

(b) The formula for the height, y metres, of a ball above ground level, x seconds after it is fired vertically into the air, is given by:

$$y = 35x - 5x^2.$$

Use your graph from part (a) to estimate

(i) the maximum height reached by the ball

(ii) the height of the ball after $5 \cdot 5$ seconds.

On two occasions the ball is 20 metres above the ground.

(iii) Use your graph from part **(b)** to estimate the two times when this occurred.

Question 6

Let f be the function $f : x \to x^2 + bx + c,\ x \in \mathbf{R}$ and $b, c \in \mathbf{Z}$.

The points (2, -6) and (0, 6) lie on the graph of f.

(i) Find the value of b and the value of c.

(ii) k is a positive real number and $(k, -k)$ is a point on the graph.

Find the two possible values of k.

Question 7

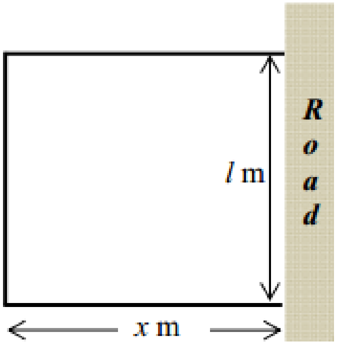
A rectangular site, with one side facing a road,

is to be fenced off.

The side facing the road, which does not require fencing, is l m in length.

The sides perpendicular to the road are x m in length.

The length of fencing that will be used to enclose the rest of the site is 140 m.



(a) ✍ Write an expression, in terms of x, for the length (l) of the side facing the road.

(b) **(i)** ✍ Show that the area of the site, in m^2, is $-2x^2 + 140x$.

(ii) Let f be the function $f : x \to -2x^2 + 140x$.

✍ Evaluate $f(x)$ when $x = 0, 10, 20, 30, 40, 50, 60, 70$.

Hence, draw the graph of f for $0 \le x \le 70$, $x \in \mathbb{R}$.

(c) Use your graph from part **(b)** to estimate:

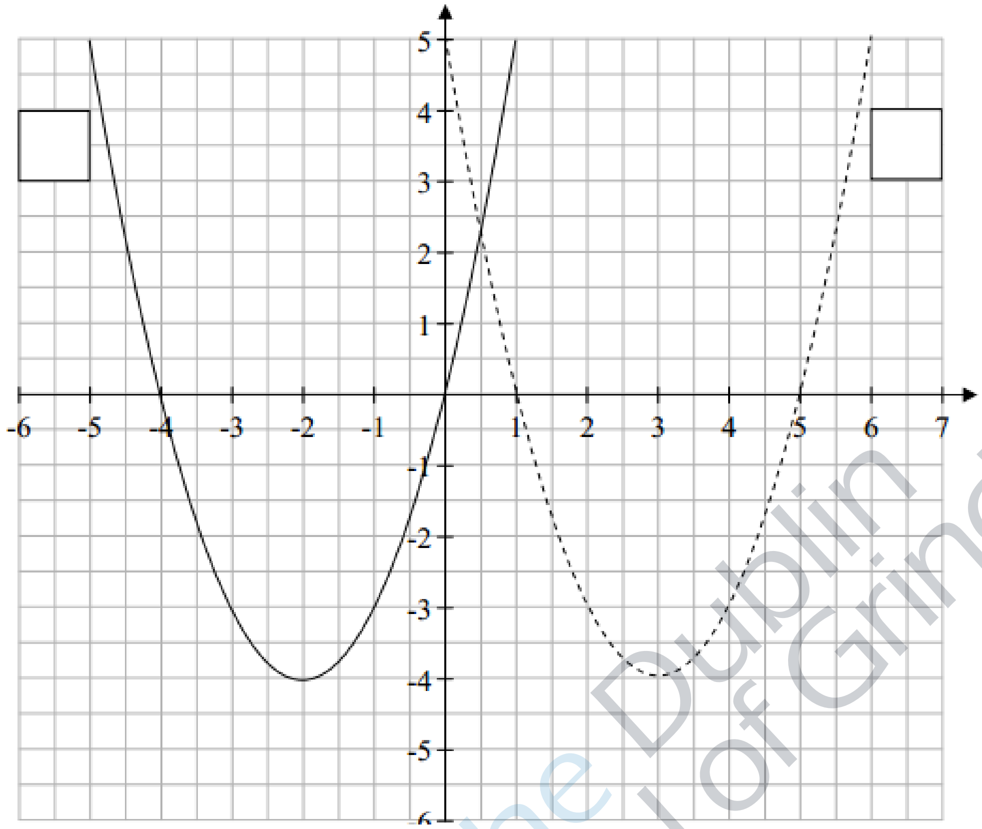
(i) ✍ the maximum possible area of the site

(ii) ✍ the area of the site when the road frontage (l) is 30 m long.

Question 8

The graphs of two functions *f* and *g* are shown on the grid below. The functions are:

$$f(x) = (x+2)^2 - 4$$
$$g(x) = (x-3)^2 - 4$$



(a) Match the graphs to the functions by putting $f(x)$ or $g(x)$ beside the corresponding graphs on the grid.

(b) Write down the roots of $f(x)$ and the roots of $g(x)$.

$f(x)$:	
$g(x)$:	

(c) Sketch the graph of $h : x \mapsto (x-1)^2 - 4$ on the grid above.

(d) If $(x-h)^2 - 2 = x^2 - 10x + 23,\ h \in \mathbb{N}$. Find the value of h.

(e) Write down the equation of the axis of symmetry of the graph of the function $f : x \mapsto x^2 - 10x + 23$.

____

Question 9

Let g be the function $g : x \mapsto 2^{x-3}$.

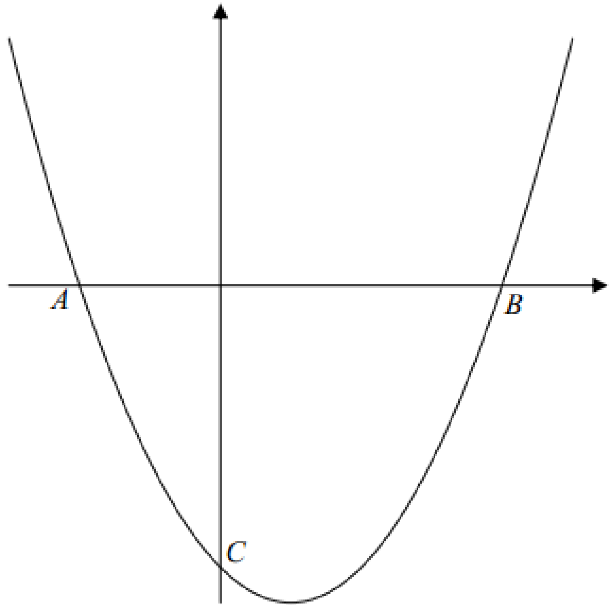
(a) Find the value of $g(3)$.

(b) Let h be the function $h : x \mapsto x^2 - 3x$.

(i) Express $h(t)$ and $h(2t+1)$ in terms of t.

(ii) Hence, find the values of t for which $h(t) = h(2t+1)$.

(c) The diagram shows part of the graph of the function $f : x \mapsto x^2 - 2x - 8,\ x \in \mathbb{R}$.



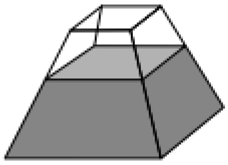
The graph intersects the x-axis at A and B, and the y-axis at C.

(i) Find the co-ordinates of A, B and C.

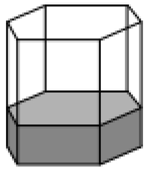
(ii) Hence, write down the range of values of x for which $x^2 - 2x - 8 \leq 0$.

Question 10

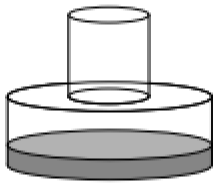
Below are three containers, labelled **1**, **2**, and **3**.
Water is poured into each container at a constant rate, until it is full.



1

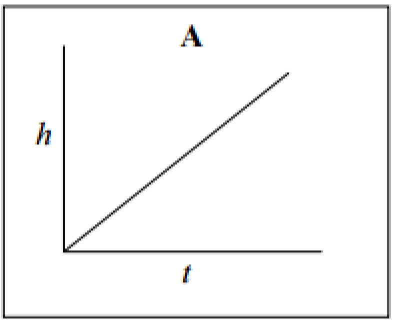


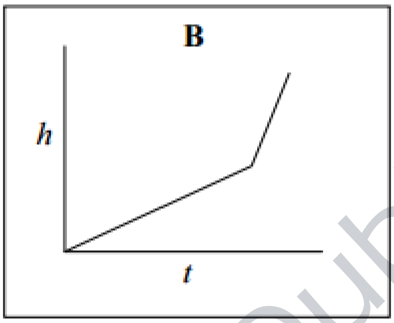
2

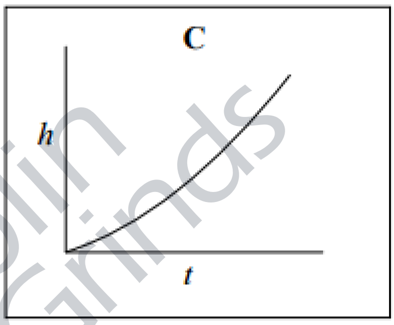


3

The three graphs, **A**, **B**, and **C**, show the height of the water, *h*, in the containers after time *t*.



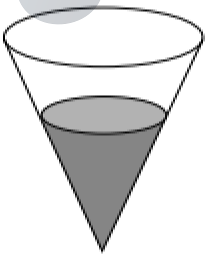


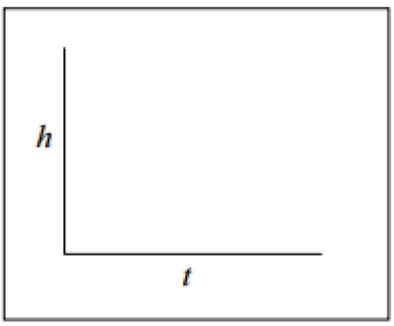


(a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	**1**	**2**	**3**
Graph			

(b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height (*h*) against time (*t*) for this container.





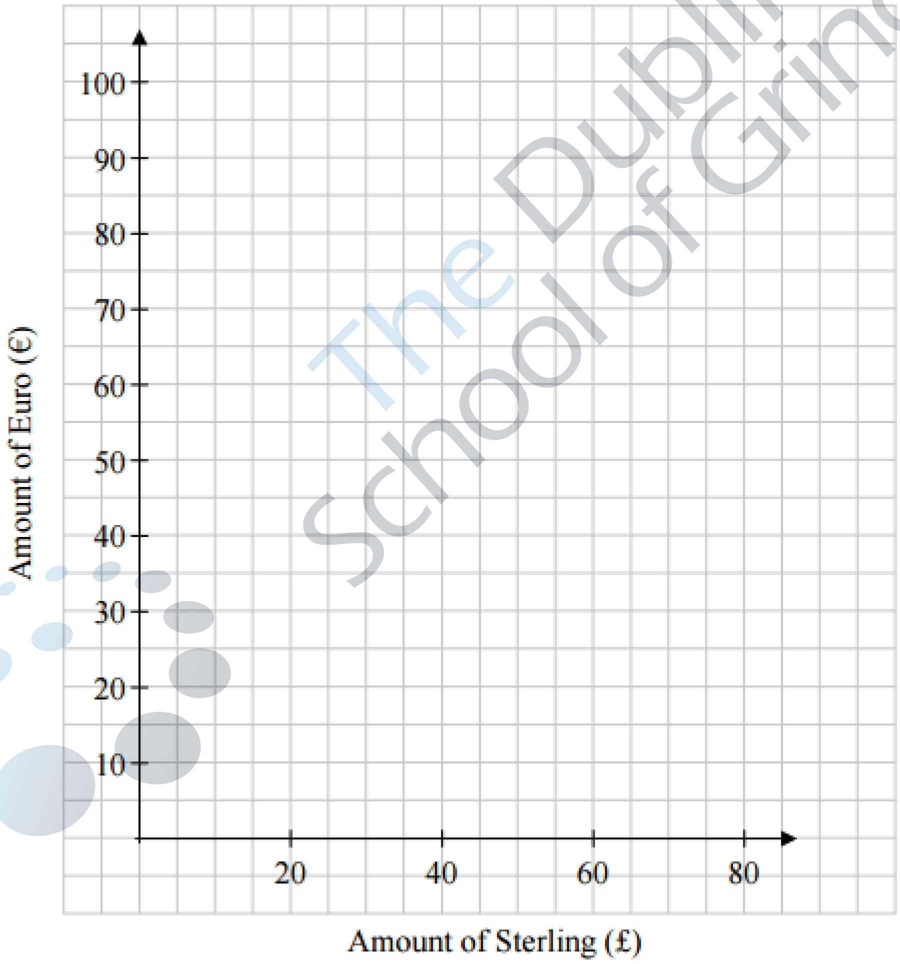
Question 11

Jack and Sarah are going on a school tour to England. They investigate how much different amounts of sterling (£) will cost them in euro (€). They each go to a different bank.

Their results are shown in the table below.

Amount of sterling (£)	Cost in euro (€) for Jack	Cost in euro (€) for Sarah
20	33	24
40	56	48
60	79	72
80	102	96

(i) On the grid below, draw graphs to show how much the sterling will cost Jack and Sarah, for up to £80.



(ii) Using the table, or your graph, find the slope (rate of change) of Jack's graph. Explain what this value means. Refer to both euro and sterling in your explanation.

Slope:

Explanation:

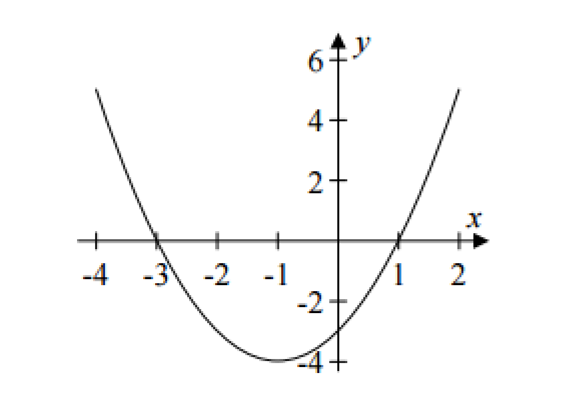
(iii) Write down a formula to represent what Jack must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

(iv) Write down a formula to represent what Sarah must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

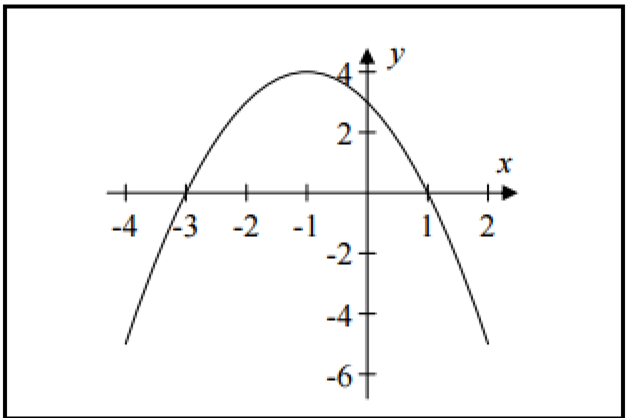
(v) Using your formulas from **(iii)** and **(iv)**, or otherwise, find the amount of sterling Jack and Sarah could buy that would cost them the same amount each in euro.

Question 12

(a) The graphs of the functions $f(x)=x^2+2x-3$ and $g(x)=-x^2-2x+3$ are shown below. Identify each graph by writing $f(x)$ or $g(x)$ in the space provided below the graph.

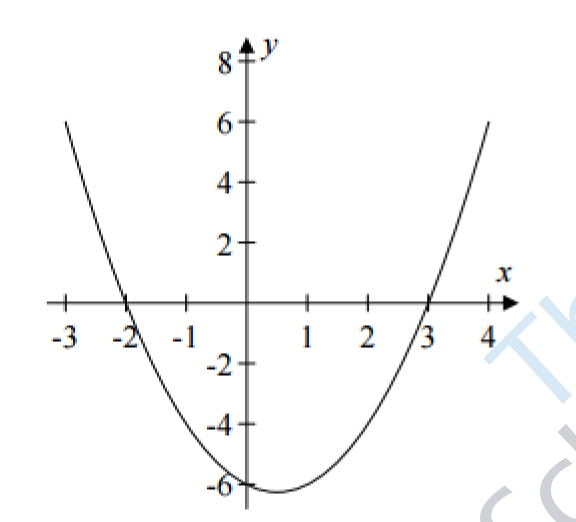


Answer: ________

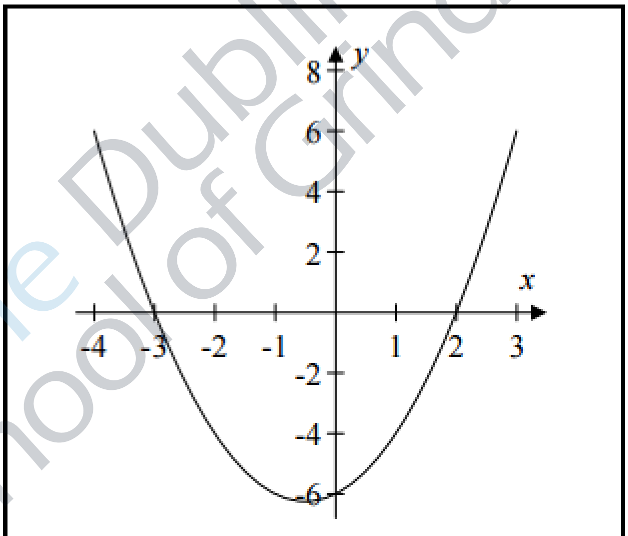


Answer: ________

(b) The graphs of the functions $y=h(x)$ and $y=k(x)$ are shown below.



$h(x)$



$k(x)$

Write down the roots of each function.
Hence, or otherwise, write down an equation for each function.

Roots of $h(x)$:

Equation: $h(x)=$

Roots of $k(x)$:

Equation: $k(x)=$

Question 13

(i) g is the function $g : x \mapsto x - 1$, where $x \in \mathbb{R}$. Find the value of each of the following.

$g(3)$

$g(-2)$

(ii) f is the function $f : x \mapsto 2x^2 - x - 6$, where $x \in \mathbb{R}$.

Using the same axes and scales, draw the graphs of the functions $y = f(x)$ and $y = g(x)$ in the domain $-2 \le x \le 3$.

Use your graphs from **(ii)** to estimate:

(iii) the minimum value of $f(x)$

(iv) the range of values of x for which $f(x) < 0$

(v) the range of values of x for which $g(x) \geq 0$.

Question 14

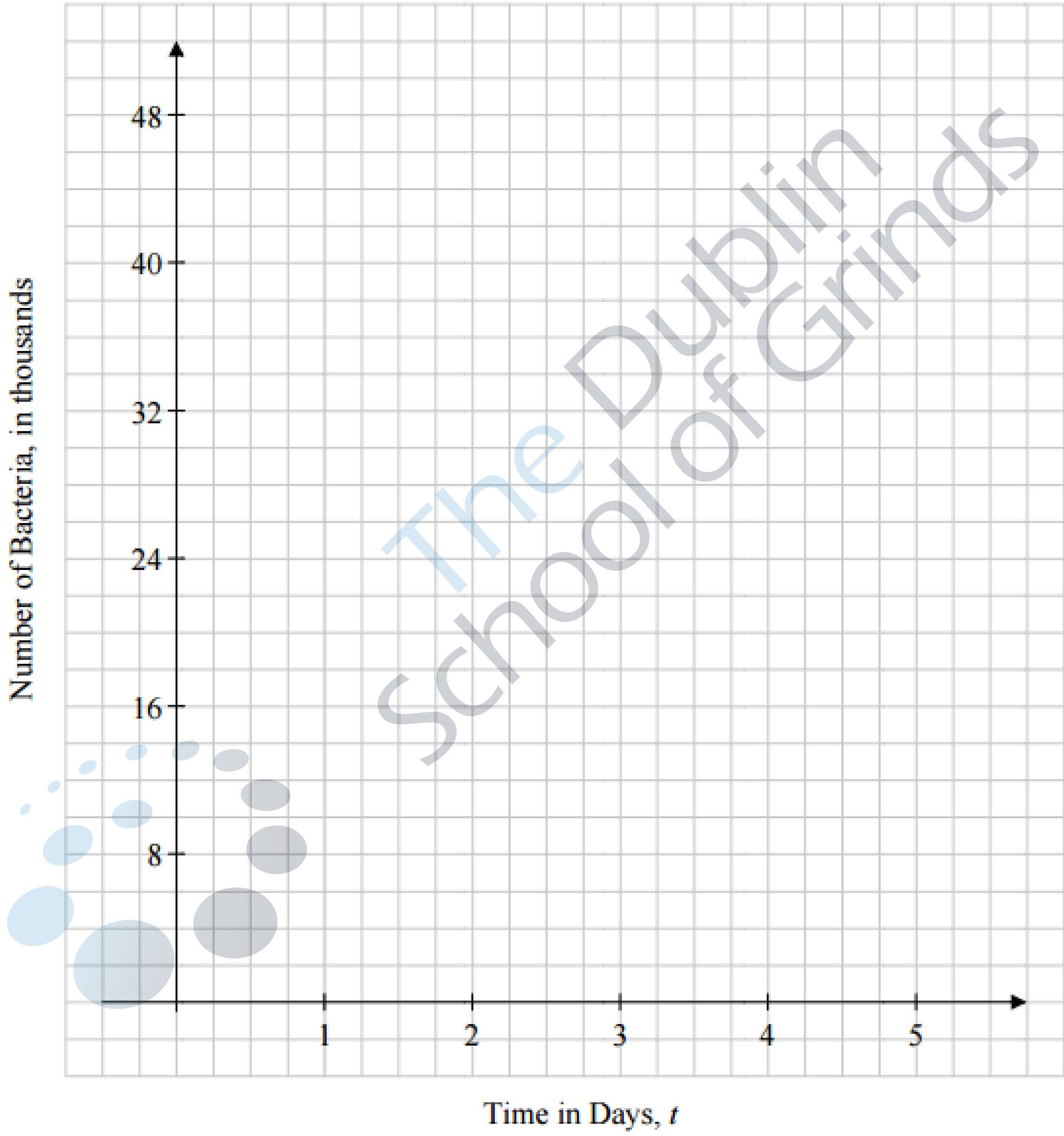
Paul and Marie have been studying the growth of a particular bacterium in school.
They each come up with a function to predict the number of bacteria in a colony, in thousands, after t days. They both assume that there are 1000 bacteria in the colony at the beginning ($t = 0$).

Paul comes up with the function: $f : t \mapsto 2^t$.

Marie comes up with the function: $g : t \mapsto t^2 + 2t + 1$.

(i) On the grid below, draw the graphs of $y = f(t)$ and $y = g(t)$ in the domain $0 \leq t \leq 5$, $t \in \mathbb{R}$.

There is room for working out on the next page.



For parts **(ii)**, **(iii)**, and **(iv)**, you **must** show your working out on the diagram on the previous page.

(ii) Use your graphs to find the difference in the number of bacteria predicted by Paul and the number of bacteria predicted by Marie after 2·5 days.

(iii) Use your graphs to estimate the range of values of t for which **both** Paul and Marie predict that there will be at least 20 000 bacteria in the colony.

Answer: $t \geq$

(iv) By extending your graphs, estimate the value of t (other than $t = 0$) for which the number of bacteria predicted by Paul and the number of bacteria predicted by Marie will be the same.

(v) The actual number of bacteria after two weeks (14 days) is roughly $1 \cdot 6 \times 10^7$.
Based on this, which formula would you say gives the better prediction for the number of bacteria? Explain your answer.

Answer:

Reason:

Question 15

Let $f(x) = 3x + 5$, for $x \in \mathbb{R}$.

(a) Find the value of $f(7)$.

(b) Write $f(k)$ in terms of k.

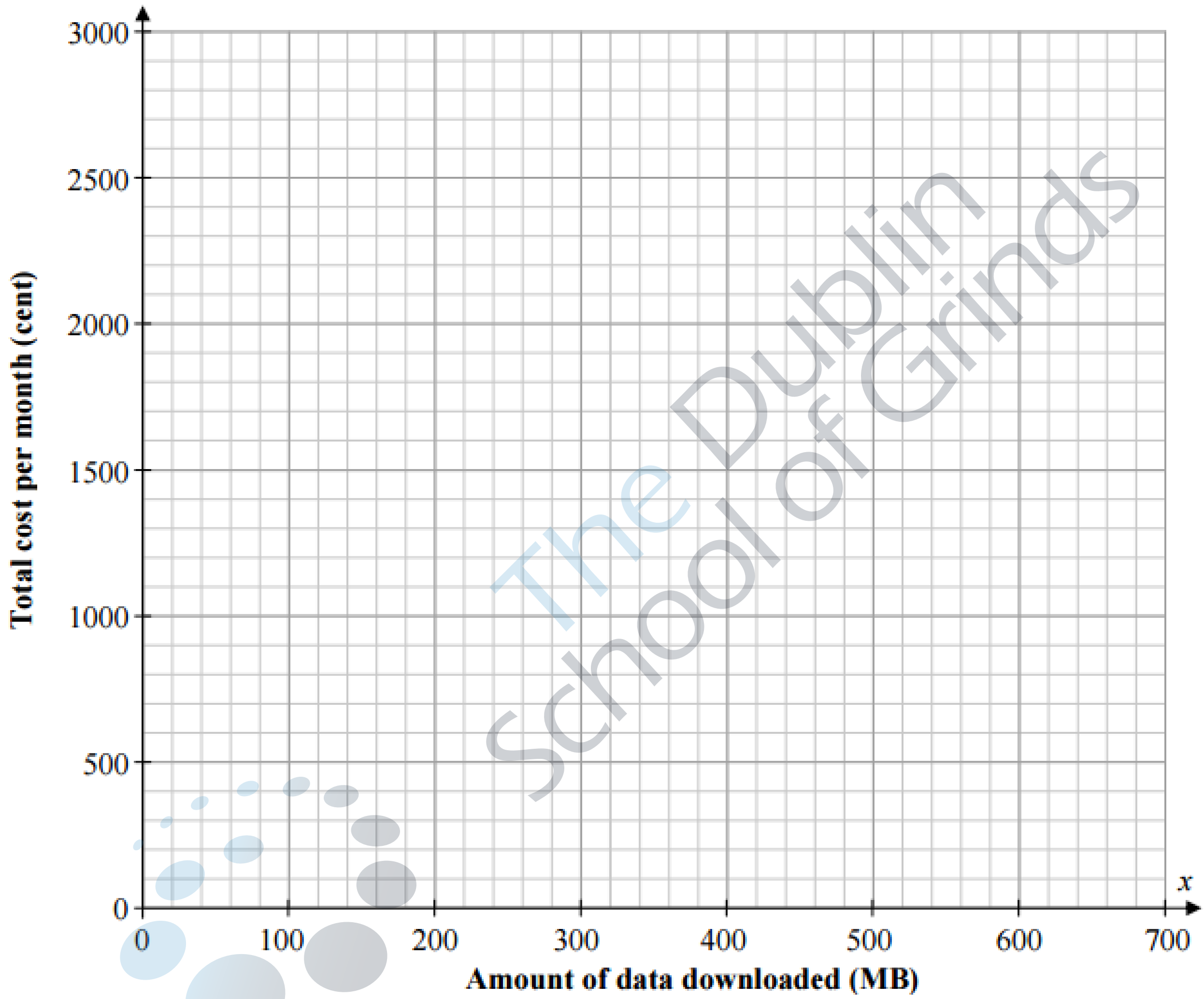
(c) Using your answer to part **(b)**, or otherwise, find the value of k for which $f(k) = k$.

Question 16

Two mobile phone companies, *Cellulon* and *Mobil*, offer price plans for mobile internet access. A formula, in x, for the total cost per month for each company is shown in the table below. x is the number of MB of data downloaded per month.

Phone company	Total cost per month (cent)
Cellulon	$c(x) = 4x$
Mobil	$m(x) = 1000 + 2x$

(a) **Draw** the graphs of $c(x)$ and $m(x)$ on the co-ordinate grid below to show the total cost per month for each phone company, for $0 \le x \le 700$. **Label** each graph clearly.



(b) Which company charges **no** fixed monthly fee?
Justify your answer, with reference to the relevant **formula** or **graph**.

Answer:

Reason:

(c) Write down the **point of intersection** of the two graphs.

Fergus wants to buy a mobile phone from one of these two companies, and wants his mobile internet bill to be as low as possible.

(d) **Explain** how your answer to part **(c)** would help Fergus choose between *Cellulon* and *Mobil*.

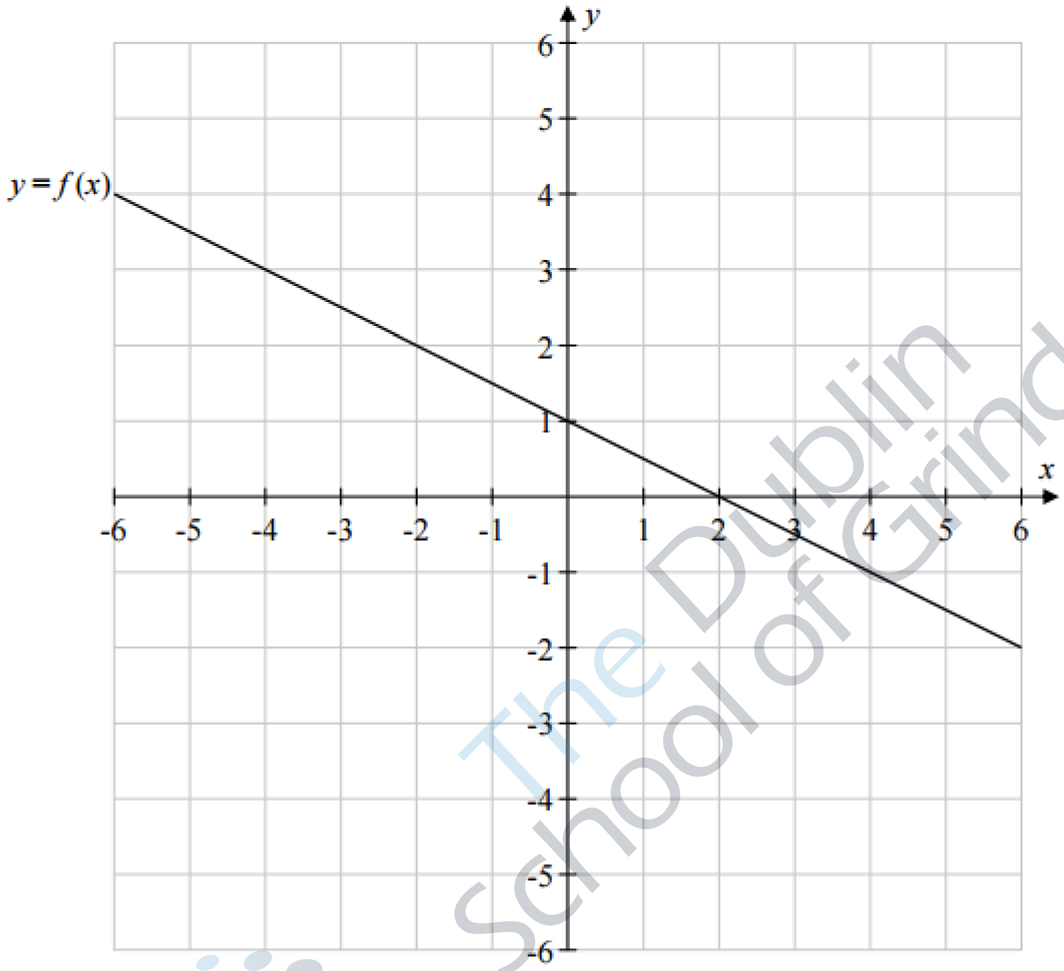
Question 17

The graph of the linear function $y = f(x)$ is drawn on the co-ordinate grid below.

Using the same axes, draw the graph of each of the following functions, where $-6 \le x \le 6$, $x \in \mathbb{R}$. Label each graph clearly.

(a) $y = f(x) + 2$

(b) $y = -f(x)$



13) Solutions to Functions and Graphs

Question 1.1

$g(x) = \sqrt{5x - 2}$
$g(2) = \sqrt{5(2) - 2}$
$= \sqrt{10 - 2}$
$= \sqrt{8}$
$= 2\sqrt{2}$

Question 1.2

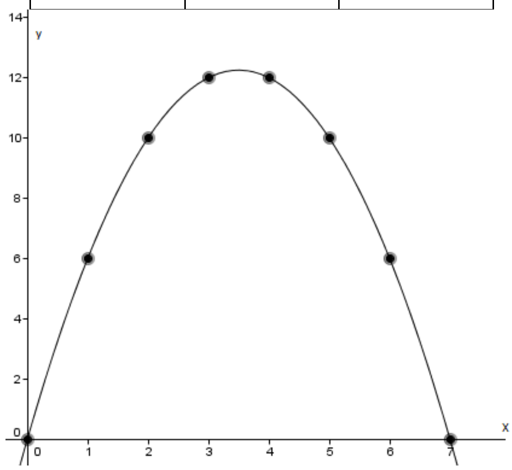
a) $10 + 5 = 15$
$15 \times 4 = 60$
Output = 60
b) $(x + 5) \times 4 = 28$
$\Rightarrow x + 5 = \frac{28}{4} = 7$
$\Rightarrow x + 5 = 7$
$x = 2$
c) $(x + 5) \times 4 = -8$
$\Rightarrow x + 5 = -2$
$\Rightarrow x = -7$
d) $4(x + 5)$

Question 1.3

Substitute $(x + 1)$ for x
$(x + 1)^2 + 3(x + 1) - 7 + 3$
$= x^2 + 2x + 1 + 3x + 3 - 7 + 3$
$= x^2 + 5x$

Question 2.1

x	$7x - x^2$	y
0	$7(0) - 0^2$	0
1	$7(1) - 1^2$	6
2	$7(2) - 2^2$	10
3	$7(3) - 3^2$	12
4	$7(4) - 4^2$	12
5	$7(5) - 5^2$	10
6	$7(6) - 6^2$	6
7	$7(7) - 7^2$	0



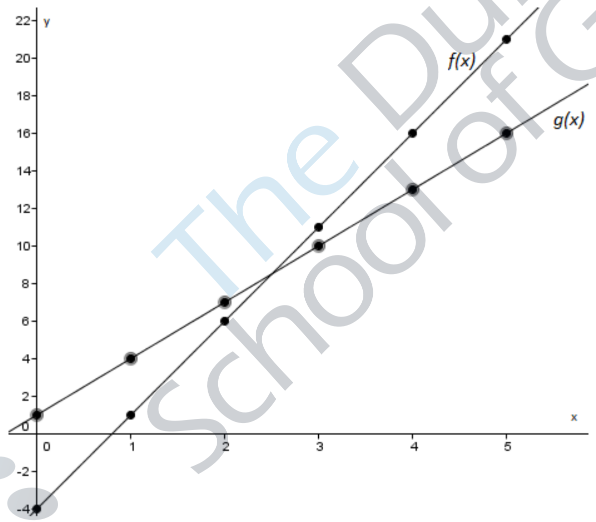
Question 2.2

$f: x \rightarrow 5x - 4$

x	5x – 4	y
0	5(0) – 4	– 4
1	5(1) – 4	1
2	5(2) – 4	6
3	5(3) – 4	11
4	5(4) – 4	16
5	5(5) – 4	21

$g(x) = 3x + 1$

x	3x + 1	y
0	3(0) + 1	1
1	3(1) + 1	4
2	3(2) + 1	7
3	3(3) + 1	10
4	3(4) + 1	13
5	3(5) + 1	16



Question 3.1

$f(x) \rightarrow 4x^2 + bx + c$

(2,6) $4(2)^2 + b(2) + c = 6$

$16 + 2b + c = 6$

$\Rightarrow 2b + c = -10$

(−1,0) $4(-1)^2 + b(-1) + c = 0$

$4 - b + c = 0$

$\Rightarrow -b + c = -4$

$2b + c = -10\ (\times 1)$ $\qquad$ $2b + c = -10$

$-b + c = -4\ (\times 2)$ $\qquad$ $\underline{-2b + 2c = -8}$

$3c = -18$

$\Rightarrow c = -6$

$-b + c = -4$

$-b - 6 = -4$

$-b = 2$

$\Rightarrow b = -2$

Question 3.2

i)

(−1,0) $(-1)^2 + b(-1) + c = 0$

$1 - b + c = 0$

$\Rightarrow -b + c = -1$

(2,0) $(2)^2 + b(2) + c = 0$

$\Rightarrow 2b + c = -4$

$-b + c = -1 \ (\times 2)$ $\quad$ $-2b + 2c = -2$

$2b + c = -4 \ (\times 1)$ $\quad$ $\underline{2b + c = -4}$

$3c = -6$

$\Rightarrow c = -2$

$-b + c = -1$

$-b - 2 = -1$

$-b = 1$

$\Rightarrow b = -1$

$= x^2 - x - 2 = 0$

ii) $k^2 - k - 2 = -k + 14$

$k^2 - 16 = 0$

$(k + 4)(k - 4) = 0$

$\Rightarrow k = 4$ or $k = -4$

Question 4.1

i) $f(x) = x^2 - 4x + 3$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$\Rightarrow x = 3$ or $x = 1$

A = (1,0) and B = (3,0)

ii) A = (1,0) which is also a point on g(x)

$g(x) = x + k$ (sub in (1,0))

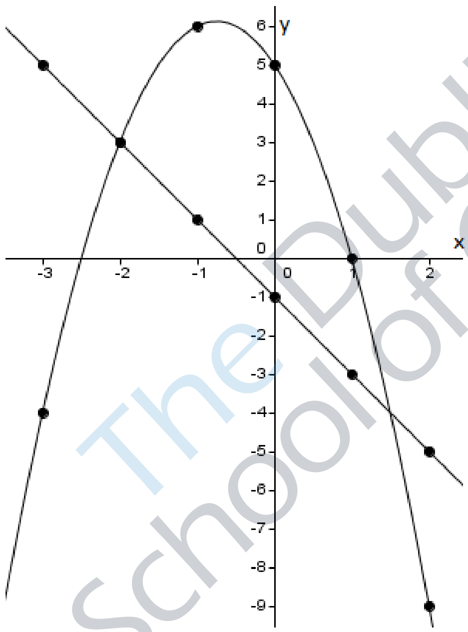
$0 = 1 + k$

$\Rightarrow k = -1$

Question 4.2

x	$5 - 3x - 2x^2$	y
−3	$5 - 3(-3) - 2(-3)^2$	−4
−2	$5 - 3(-2) - 2(-2)^2$	3
−1	$5 - 3(-1) - 2(-1)^2$	6
0	$5 - 3(0) - 2(0)^2$	5
1	$5 - 3(1) - 2(1)^2$	0
2	$5 - 3(2) - 2(2)^2$	−9

x	$-2x - 1$	y
−3	$-2(-3) - 1$	5
−2	$-2(-2) - 1$	3
−1	$-2(-1) - 1$	1
0	$-2(0) - 1$	−1
1	$-2(1) - 1$	−3
2	$-2(2) - 1$	−5



i) Using Algebra

$5 - 3x - 2x^2 = -2x - 1$

$\Rightarrow 2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ or $x = -2$

$x = \frac{3}{2}$ or $x = -2$

If $x = \frac{3}{2}$, then $y = -2(\frac{3}{2}) - 1 = -4$

$(\frac{3}{2}, -4)$

If $x = -2$, then $y = -2(-2) - 1 = 3$

$(-2, 3)$

Points of intersection are $(-2, 3)$ and $(\frac{3}{2}, -4)$.

Graph:

Advantage → It's easy to read straight from the graph

Disadvantage → It may not be accurate

Algebra

Advantage → It will give precise points of intersection

Disadvantage → It is more difficult

Question 4.3

a) $f(x) = 2x^2 + x - 6 = 0$
$(2x - 3)(x + 2) = 0$
$2x - 3 = 0$ or $x + 2 = 0$
$\Rightarrow x = \frac{3}{2}$ or $x = -2$

$g(x) = x^2 - 6x + 9 = 0$
$(x - 3)(x - 3) = 0$
$\Rightarrow x = 3$

$h(x) = x^2 - 2x = 0$
$x^2 - 2x = 0$
$x(x - 2) = 0$
$\Rightarrow x = 0$ or $x = 2$

b) h(x) = Diagram 2
f(x) = Diagram 3
g(x) = Diagram 5

Question 7.1

i) $-2 < x < 1$
ii) $-4 < x < 2$

Question 8.1

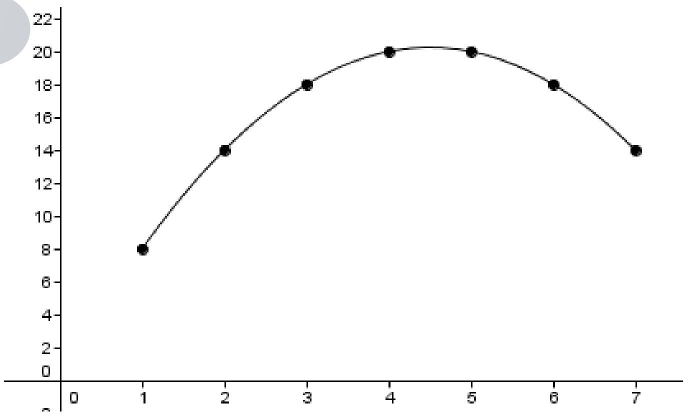
Axis of Symmetry
$x = -1$

Question 9.1

a) $2x + 2y = 18$
$\Rightarrow 2y = 18 - 2x$
$\Rightarrow y = 9 - x$
Area = length × width = $x(9 - x) = 9x - x^2$

b) Area function:

x	$9x - x^2$	y
1	$9(1) - (1)^2$	8
2	$9(2) - (2)^2$	14
3	$9(3) - (3)^2$	18
4	$9(4) - (4)^2$	20
5	$9(5) - (5)^2$	20
6	$9(6) - (6)^2$	18
7	$9(7) - (7)^2$	14



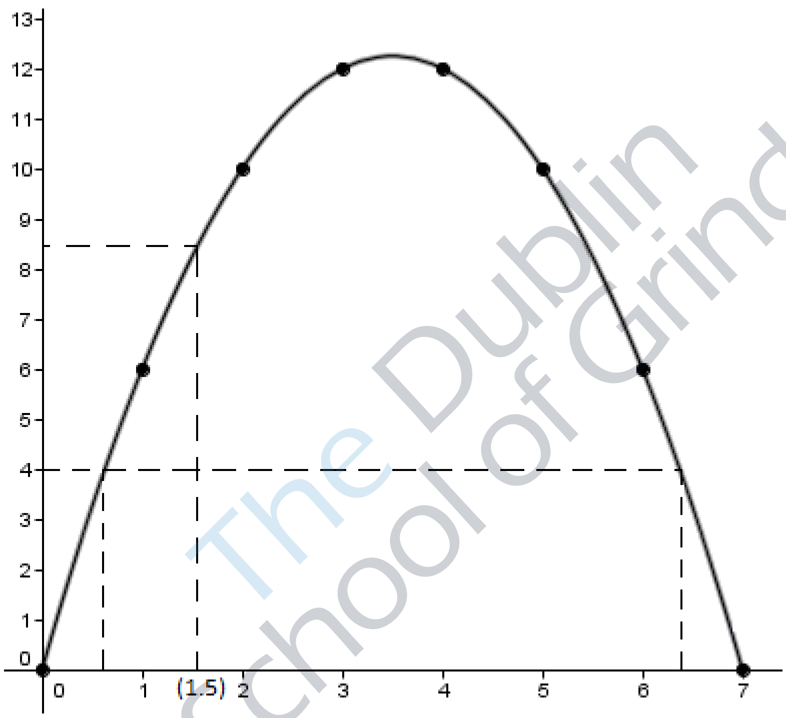
c)
i) Area when $x = 2.7 = 17\text{m}^2$
ii) $x = 3.3$ and 5.4
iii) Max possible area = 20.25 m^2
iv) Length = Breadth = 4.5

Question 9.2

a) $2x + 2y = 14$
$\Rightarrow 2y = 14 - 2x$
$\Rightarrow y = 7 - x$

b) $x(7 - x) = 7x - x^2$

x	$7x - x^2$	y
0	$7(0) - (0)^2$	0
1	$7(1) - (1)^2$	6
2	$7(2) - (2)^2$	10
3	$7(3) - (3)^2$	12
4	$7(4) - (4)^2$	12
5	$7(5) - (5)^2$	10
6	$7(6) - (6)^2$	6
7	$7(7) - (7)^2$	0



c)
i) Area when width is 1.5 = 8.5 m^2
ii) Max possible area = 12.2 m^2
iii) 0.5m or 6.4m

Question 10.1

a)

0 : $45 + \frac{7}{10}(0) - \frac{1}{200}(0)^2 = 45$

20 : $45 + \frac{7}{10}(20) - \frac{1}{200}(20)^2 = 57$

40 : $45 + \frac{7}{10}(40) - \frac{1}{200}(40)^2 = 65$

60 : $45 + \frac{7}{10}(60) - \frac{1}{200}(60)^2 = 69$

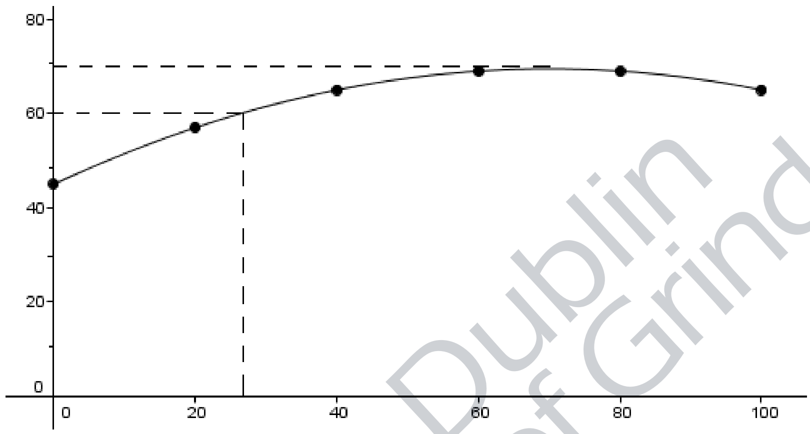
80 : $45 + \frac{7}{10}(80) - \frac{1}{200}(80)^2 = 69$

100 : $45 + \frac{7}{10}(100) - \frac{1}{200}(100)^2 = 65$

Table

Time is seconds, *t*	0	20	40	60	80	100
Altitude in km, *h*	45	57	65	69	69	65

b)



c) 69.5 km

d) 26 sec

e) $h = 45 + \frac{7}{10}(26) - \frac{1}{200}(26)^2 = 59.82$

f) $45 + \frac{7}{10}t - \frac{1}{200}t^2 = 9$ (multiply everything by 200)

$\Rightarrow 200(45) + 200(\frac{7}{10})t - 200\left(\frac{1}{200}\right)t^2 = 9(200)$

$9{,}000 + 140t - t^2 = 1800$

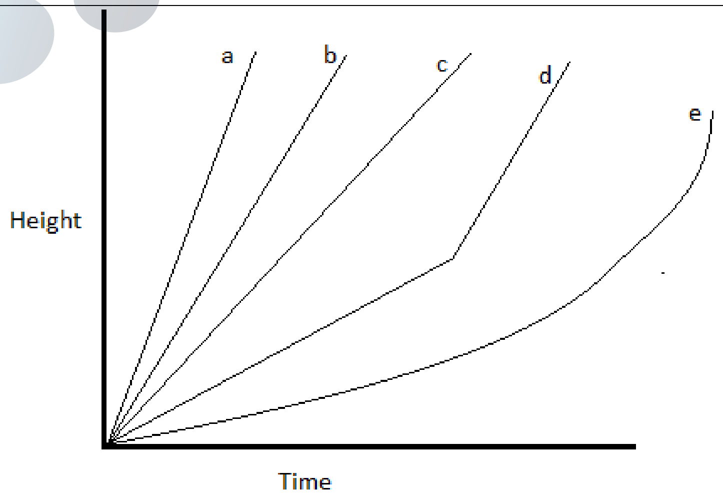
$\Rightarrow t^2 - 140t - 7200 = 0$

$(t - 180)(t + 40) = 0$

$\Rightarrow t = 180$ or $t = -40$ (not possible since t must be greater than 0)

$\therefore t = 180$ sec

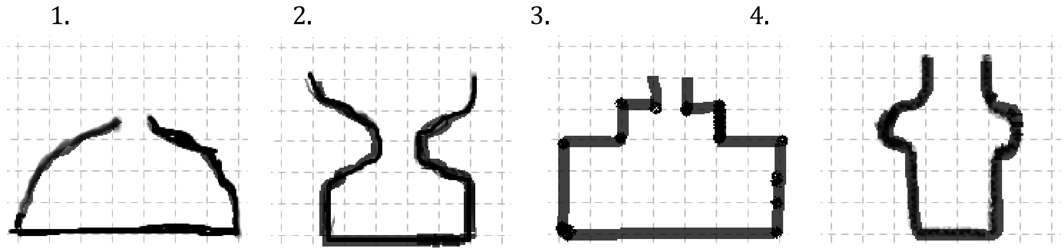
Question 10.2



N.B. 2nd part of d is parallel to b

Question 10.3

Types of containers:



Container no. 5 is impossible to draw since the height of water reduces. This is only possible if liquid is removed which is not the case.

Question 11.1

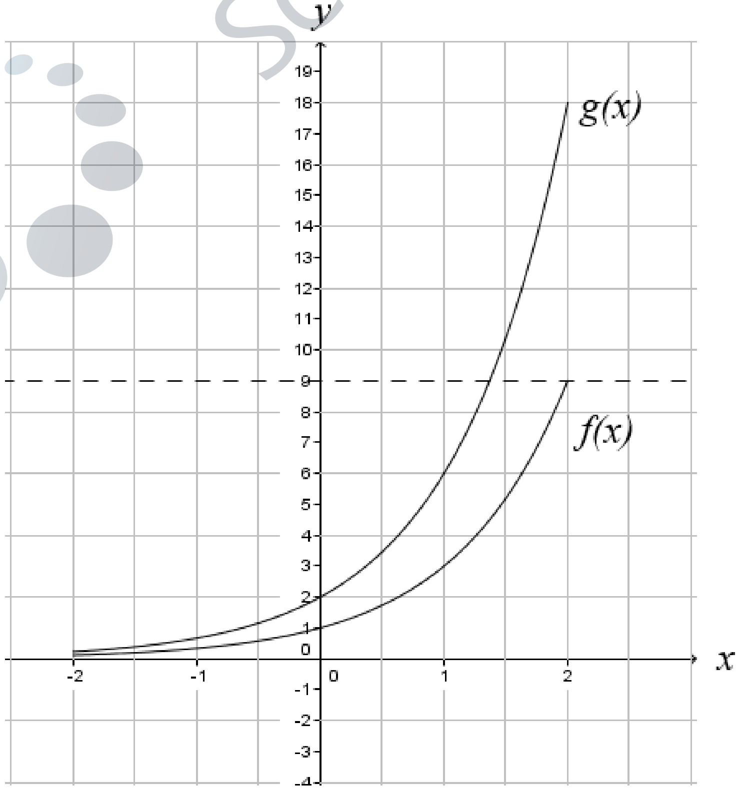
a)

x	$y = 3^x$	
-2	$y = 3^{-2}$	$\Rightarrow y = \frac{1}{9}$
-1	$y = 3^{-1}$	$\Rightarrow y = \frac{1}{3}$
0	$y = 3^0$	$\Rightarrow y = 1$
1	$y = 3^1$	$\Rightarrow y = 3$
2	$y = 3^2$	$\Rightarrow y = 9$

b) $f(x) = 9$ when $x = 2$.

c)

x	$y = (2).3^x$	
-2	$y = (2).3^{-2}$	$\Rightarrow y = \frac{2}{9}$
-1	$y = (2).3^{-1}$	$\Rightarrow y = \frac{2}{3}$
0	$y = (2).3^0$	$\Rightarrow y = 2$
1	$y = (2).3^1$	$\Rightarrow y = 6$
2	$y = (2).3^1$	$\Rightarrow y = 18$



Question 11.2

exponential	quadratic	linear

Question 11.3

i) Age in years
ii) Angle of the sun in the sky
iii) Bacterial growth

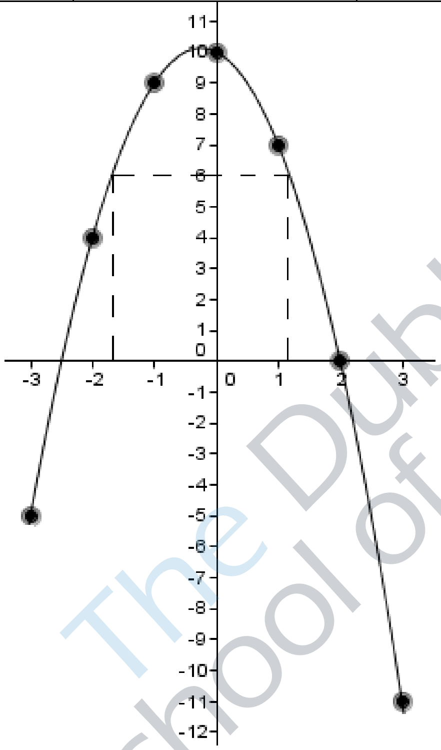
12) Past & Probable Exam Questions

Question 1

(a)

i)

x	$10 - x - 2^x$	y
−3	$10 - (-3) - 2(-3)^2$	−5
−2	$10 - (-2) - 2(-2)^2$	4
−1	$10 - (-1) - 2(-1)^2$	9
0	$10 - (0) - 2(0)^2$	10
1	$10 - (1) - 2(1)^2$	7
2	$10 - (2) - 2(2)^2$	0
3	$10 - (3) - 2(3)^2$	−11



ii) Max value = 10.1

iii) $x = -1.8$ or $x = 1.2$

(b)

$f(x) = 3x - 4$

$3k - 4 = 11$

$\Rightarrow 3k = 11 + 4$

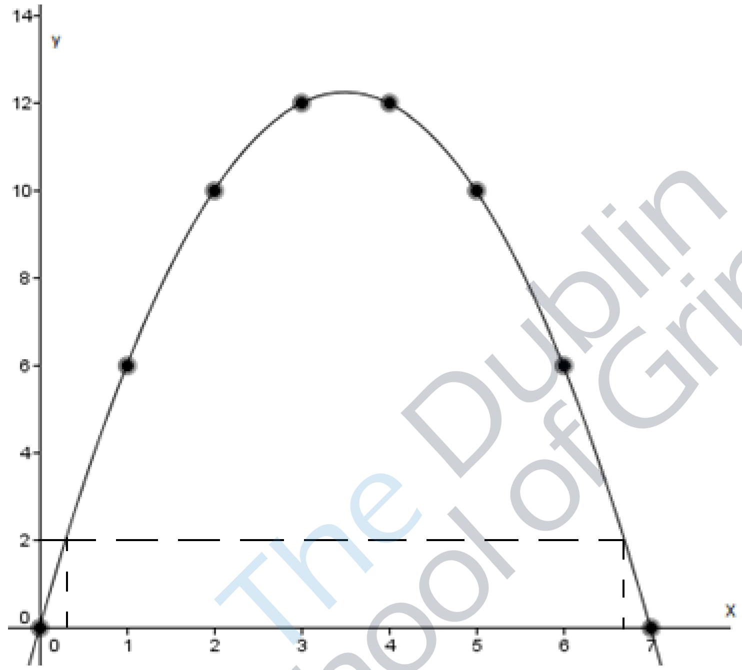
$\Rightarrow 3k = 15$

$\Rightarrow k = 5$

Question 2

(a)

x	$7x - x^2$	y
0	$7(0) - 0^2$	0
1	$7(1) - 1^2$	6
2	$7(2) - 2^2$	10
3	$7(3) - 3^2$	12
4	$7(4) - 4^2$	12
5	$7(5) - 5^2$	10
6	$7(6) - 6^2$	6
7	$7(7) - 7^2$	0



(b)

i) Max height = 12.2m

ii) 6.8 – 0.3 = 6.5 seconds

iii) Take a point on the graph, for example (6,0)

$f(x) = ax - x^2$ and substitute in (6,0)

$0 = a(6) - 6^2$

$0 = 6a - 36$

$\Rightarrow 6a = 36$

$\Rightarrow a = 6$

Question 3

i) $-x - 4x + 5 = 0$

$\Rightarrow 0 = x + 4x - 5$

$(x + 5)(x - 1) = 0$

$x = -5$ or $x = 1$

ii) $x^2 + 4x - 5 = (x + 1)^2 + 4(x + 1) - 5$

$x^2 + 4x - 5 = x^2 + 2x + 1 + 4x + 4 - 5$

$x^2 + 2x + 1 + 4x + 4 - 5 - x^2 - 4x + 5 = 0$

$2x + 5 = 0$

$2x = -5$

$\Rightarrow x = -\frac{5}{2}$

iii) $f(x) = 5x - 12$

$5a - 12 = a$

$\Rightarrow 5a - a = 12$

$4a = 12$

$\therefore a = 3$

iv) $9k + 8 = 44$

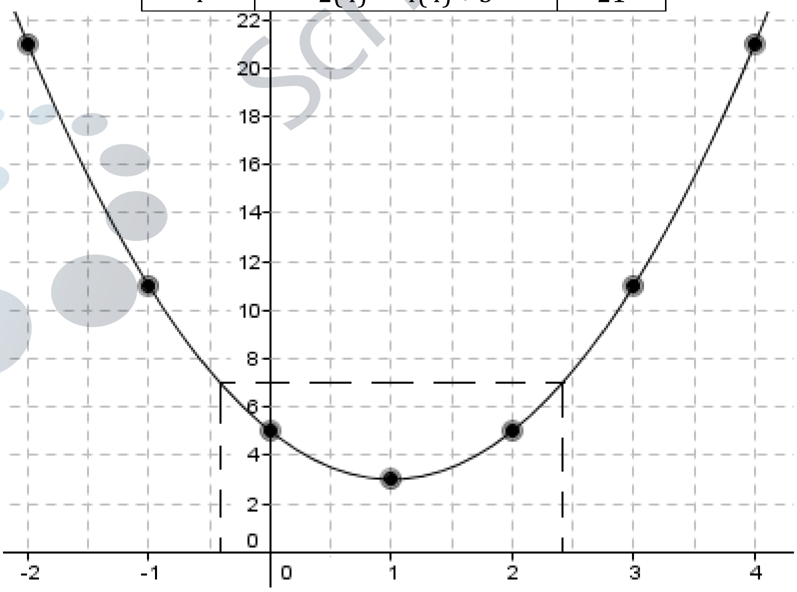
$9k = 36$

$\Rightarrow k = 4$

Question 4

i)

x	$2x^2 - 4x + 5$	y
-2	$2(-2)^2 - 4(-2) + 5$	21
-1	$2(-1)^2 - 4(-1) + 5$	11
0	$2(0)^2 - 4(0) + 5$	5
1	$2(1)^2 - 4(1) + 5$	3
2	$2(2)^2 - 4(2) + 5$	5
3	$2(3)^2 - 4(3) + 5$	11
4	$2(4)^2 - 4(4) + 5$	21

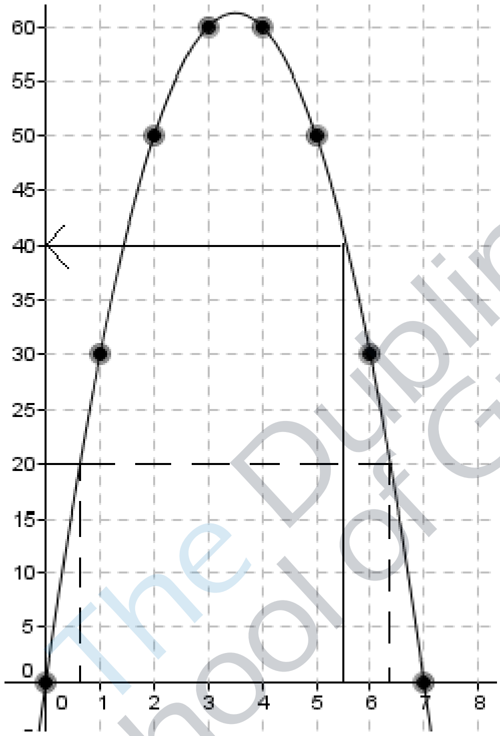


ii) $x = -0.4$ or $x = 2.4$

Question 5

a)

x	$35x - 5x^2$	y
0	$35(0) - 5(0)^2$	0
1	$35(1) - 5(1)^2$	30
2	$35(2) - 5(2)^2$	50
3	$35(3) - 5(3)^2$	60
4	$35(4) - 5(4)^2$	60
5	$35(5) - 5(5)^2$	50
6	$35(6) - 5(6)^2$	30
7	$35(7) - 5(7)^2$	0



b)

i) Max height = 61m

ii) Height after 5.5seconds = 40m

iii) 0.7 seconds and 6.3 seconds

Question 6

i) $f: x \to x^2 + bx + c$

(2, −6) $2^2 + 2b + c = -6$

$\Rightarrow 2b + c = -10$

(0,6) $0^2 + b(0) + c = 6$

$\Rightarrow c = 6$

If c = 6 : $2b + 6 = -10$

$\Rightarrow 2b = -16$

$\Rightarrow b = -8$

$x^2 - 8x + 6 = 0$

ii) Sub (k, −k) into $x^2 - 8x + 6$

$k^2 - 8k + 6 = -k$

$k^2 - 7k + 6 = 0$

$(k - 1)(k - 6) = 0$

$\therefore k = 1$ or $k = 6$

Question 7

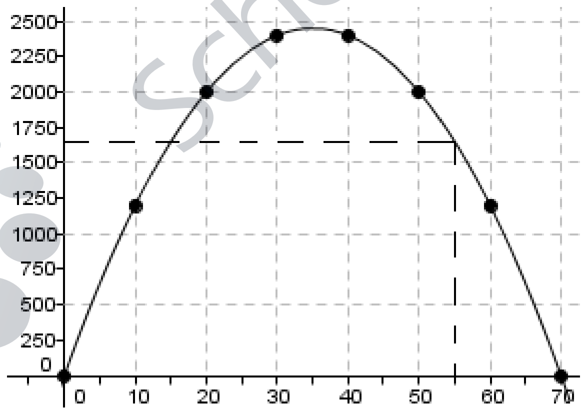
a) $2x + l = 140$

$l = 140 - 2x$

b)

i) Length × width = $(140 - 2x)(x) = 140x - 2x^2 = -2x^2 + 140x$

x	$-2x^2 + 140x$	y
0	$-2(0)^2 + 140(0)$	0
10	$-2(1)^2 + 140(1)$	1200
20	$-2(2)^2 + 140(2)$	2000
30	$-2(3)^2 + 140(3)$	2400
40	$-2(4)^2 + 140(4)$	2400
50	$-2(5)^2 + 140(5)$	2000
60	$-2(6)^2 + 140(6)$	1200
70	$-2(7)^2 + 140(7)$	0



c)

i) Max possible area = 2450m^2

ii) Area when road frontage is 30m long

$L = 30 \Rightarrow 30 = 140 - 2x$

$30 - 140 = -2x$

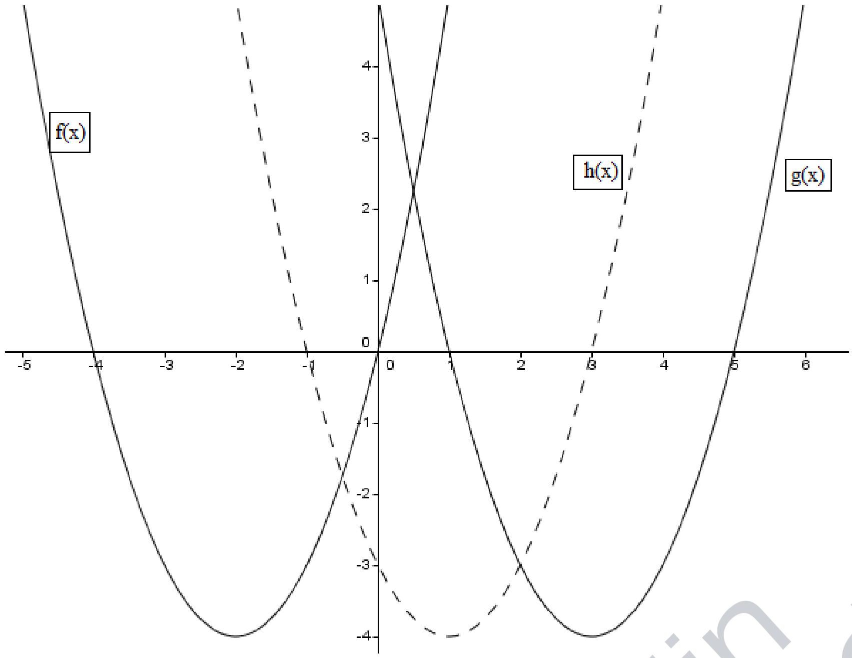
$110 = 2x$

$\Rightarrow x = 55$

When x = 55, y = Area = 1600m^2

Question 8

a)



b) f(x) : Roots = 0 and −4 (just find where the graph cuts the x–axis)
g(x) : Roots = 1 and 5

c) Find the roots
$(x - 1)(x - 1) - 4$
$= x^2 - 2x + 1 - 4 = x^2 - 2x - 3$
$(x - 3)(x + 1) = 0$
$x = 3$ or $x = -1$
∴ it cuts the x–axis at −1 and 3
(−1,0) and (3,0) are on the graph of h(x)

Looking for other points:
If $x = 0 \Rightarrow y = 0^2 - 2(0) - 3 = -3$
(0, −3) is on the graph
If $x = 1 \Rightarrow y = 1^2 - 2(1) - 3 = -4$
(1, −4) is on the graph
If $x = 2 \Rightarrow y = 2^2 - 2(2) - 3 = -3$
(2, −3) is on the graph

d) $(x - h)(x - h) - 2 = x^2 - 10x + 23$
$x^2 - 2xh + h^2 - 2 = x^2 - 10x + 23$
$\Rightarrow -2xh = -10x$
$\Rightarrow h = 5$

e) In this Question $f(x) = x^2 - 10x + 23 = (x - 5)^2 - 2$
We can see from f(x), g(x) and h(x) that the axis of symmetry can be read from the function if it is in the form $(x - h)^2 + a$, and it is $x = h$.
∴ the axis of symmetry of f(x) is $x = 5$

Another way to find this is as follows:
Axis of symmetry is $x = -\frac{b}{2a}$
Using $x^2 - 10x + 23$
$\Rightarrow$ Axis of symmetry $= -\frac{-10}{2}$
$x = 5$ is axis of symmetry

Question 9

a) $g(3) = 2^{3-3} = 2^0 = 1$

b)

i) $h(t) = t^2 - 3t$

$h(2t + 1) = (2t + 1)^2 - 3(2t + 1)$

$= 4t^2 + 4t + 1 - 6t - 3$

$= 4t^2 - 2t - 2$

ii) $t^2 - 3t = 4t^2 - 2t - 2$

$\Rightarrow 3t^2 + t - 2 = 0$

$(3t - 2)(t + 1) = 0$

$3t = 2$ or $t = -1$

$\Rightarrow t = \frac{2}{3}$ or $t = -1$

c)

i) To find A and B just solve the equation:

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4$ or $x = -2$

$\Rightarrow (4,0)$ and $(-2,0)$ are A and B

To find C, let $x = 0$

$0^2 - 2(0) - 8 = -8$

$\Rightarrow C = (0,-8)$

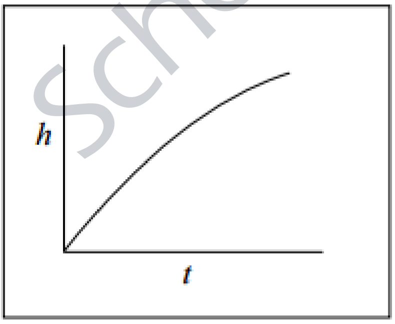
ii) $-2 \le x \le 4$

Question 10

(a)

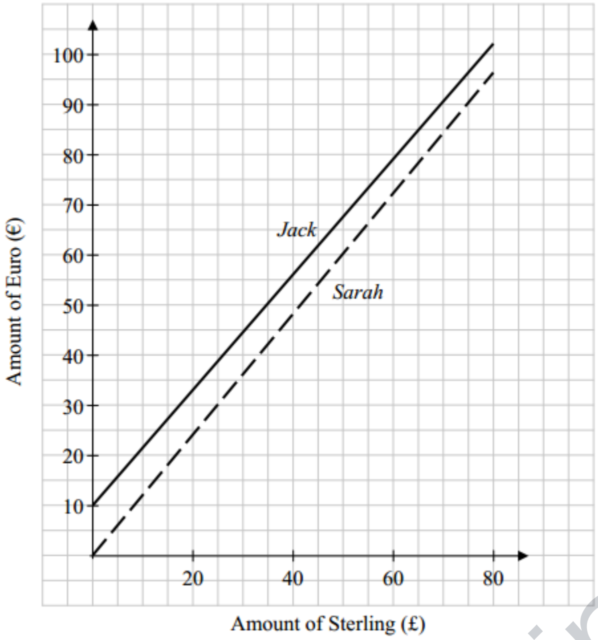
Container	1	2	3
Graph	C	A	B

(b)



Question 11

(i)



(ii)

$$\text{Slope} = \frac{56-33}{40-20} = \frac{23}{20}, \text{ or } 1\cdot15.$$

Explanation: Each extra £1 costs Jack an extra €1·15.

(iii)

$e = 1\cdot15s + 10$, where s is the amount, in sterling, and e is the amount, in euro.

(iv)

$$\text{Slope} = \frac{48-24}{40-20} = \frac{6}{5}, \text{ or } 1\cdot2. \qquad y\text{-intercept} = 0$$

$e = 1\cdot2s$, where s is the amount, in sterling, and e is the amount, in euro.

(v)

Using formulas:

$e = 1\cdot15s + 10$ and $e = 1\cdot2s$, so $1\cdot15s + 10 = 1\cdot2s$,

i.e. $s = 200$ and $e = 240$.

Amount of sterling: £200.

From table:

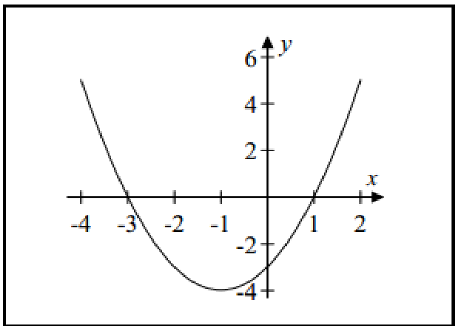
Each time the amount of sterling goes up by 20, the difference between the costs decreases by €1.

This difference is €9 for £20.

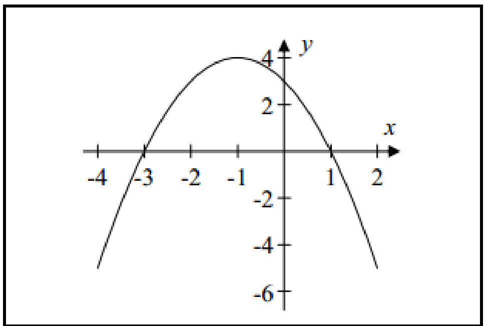
So after 9 increases, i.e. increase of $9 \times 20 = £180$, the costs are the same, i.e. for £200.

Question 12

(a)

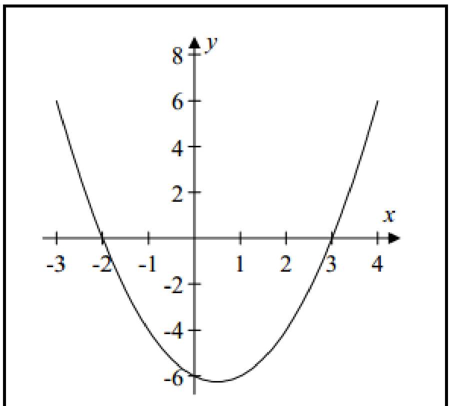


Answer: $f(x)$

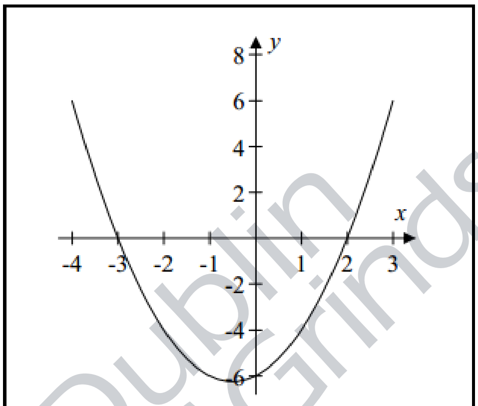


Answer: $g(x)$

(b)



$h(x)$



$k(x)$

Roots of $h(x)$:	$x=-2$ and $x=3$.
Equation:	$h(x)=(x+2)(x-3)$, or $h(x)=x^2-x-6$.
[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $h(0)=-6$, which corresponds to the graph.]	
Roots of $k(x)$:	$x=-3$ and $x=2$.
Equation:	$k(x)=(x+3)(x-2)$, or $k(x)=x^2+x-6$.
[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $k(0)=-6$, which corresponds to the graph.].	

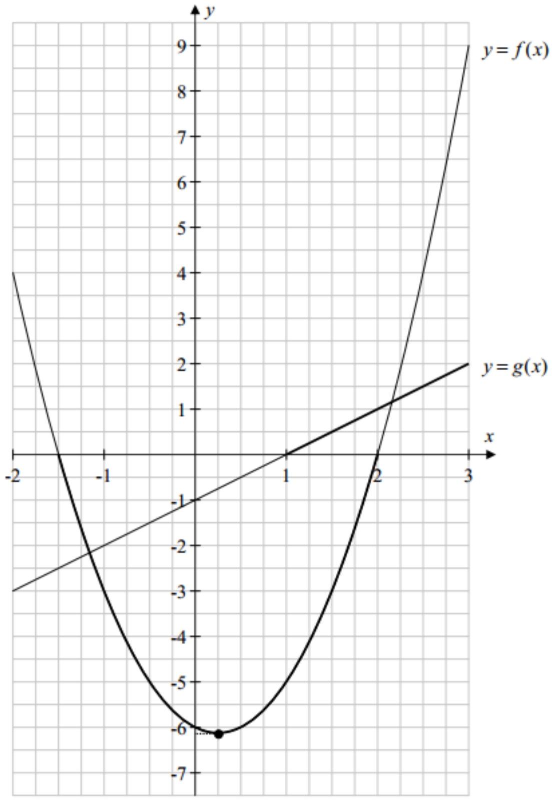
Question 13

(i)

$g(3) = 3 - 1 = 2.$

$g(-2) = -2 - 1 = -3.$

(ii)



Graphing g:

Straight line, so only need the two points from **(i)**:

$(3,2)$ and $(-2,-3)$.

Or:

$g(x) = x - 1$

x	x	- 1	y
-2	-2	- 1	-3
-1	-1	- 1	-2
0	0	- 1	-1
1	1	- 1	0
2	2	- 1	1
3	3	- 1	2

Graphing f:

$f(-2) = 4$

$f(-1) = -3$

$f(0) = -6$

$f(1) = -5$

$f(2) = 0$

$f(3) = 9$

Or:

$f(x) = 2x^2 - x - 6$

x	$2x^2$	$-x$	- 6	y
-2	8	+2	- 6	4
-1	2	+ 1	- 6	- 3
0	0	0	- 6	- 6
1	2	- 1	- 6	- 5
2	8	- 2	- 6	0
3	18	- 3	- 6	9

(iii)

$f_{\min}(x) = -6 \cdot 1$

(iv)

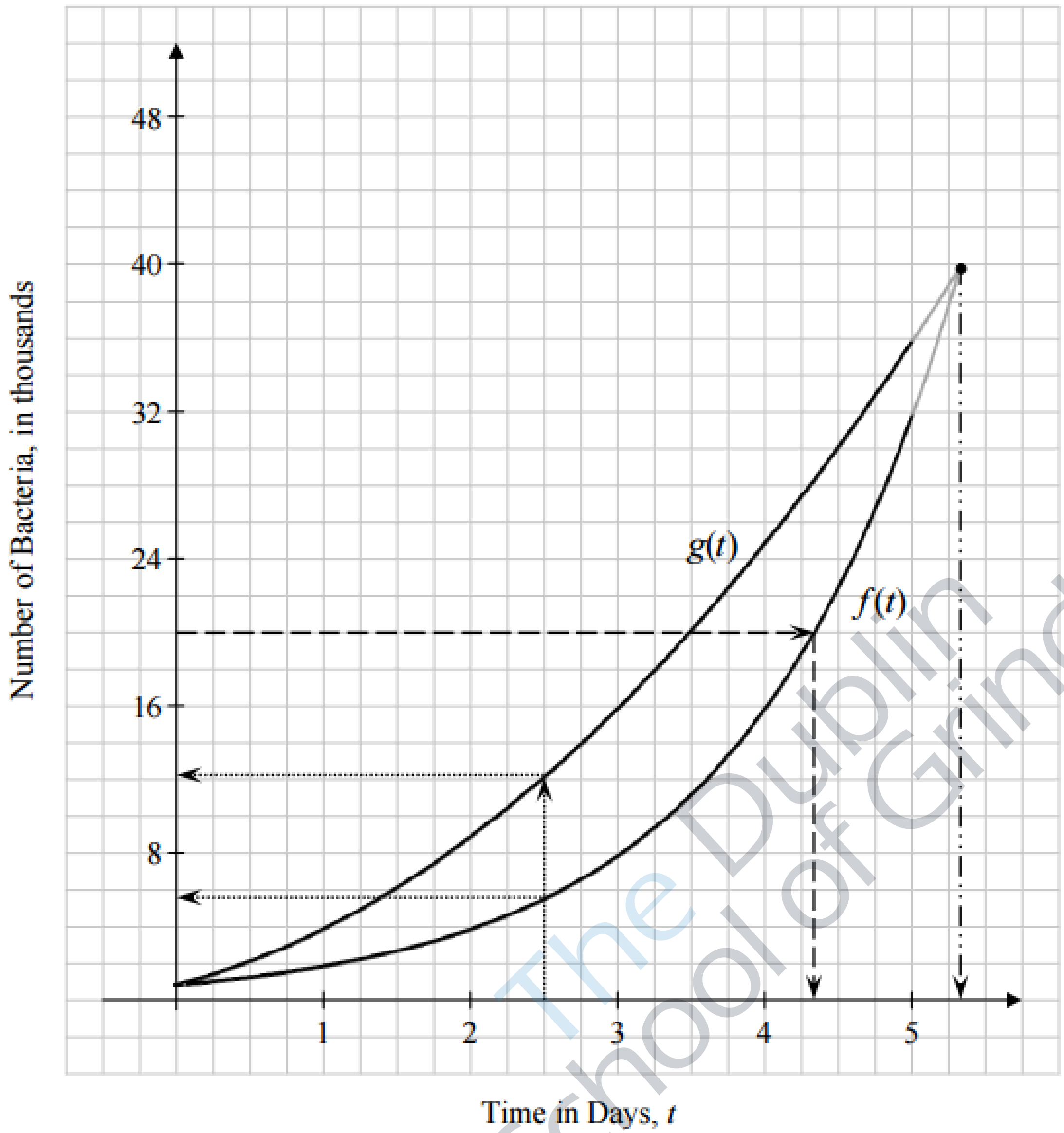
$-1 \cdot 5 < x < 2$ ... *see graph*

(v)

$x \geq 1$ ... *see graph*

Question 14

(i)



$g(t) = t^2 + 2t + 1$

$g(0) = (0)^2 + 2(0) + 1 = 0 + 0 + 1 = 1$

$g(1) = (1)^2 + 2(1) + 1 = 1 + 2 + 1 = 4$

$g(2) = (2)^2 + 2(2) + 1 = 4 + 4 + 1 = 9$

$g(3) = (3)^2 + 2(3) + 1 = 9 + 6 + 1 = 16$

$g(4) = (4)^2 + 2(4) + 1 = 16 + 8 + 1 = 25$

$g(5) = (5)^2 + 2(5) + 1 = 25 + 10 + 1 = 36$

(ii)

Marie after $2 \cdot 5$ days: 12 000 bacteria, approximately.

Paul after $2 \cdot 5$ days: 6000 bacteria, approximately.

Difference: 12 000 – 6000 = 6000 bacteria.

(iii)

$t \geq 4 \cdot 3$ days.

(iv)

$t = 5 \cdot 3$ days.

(v)

Answer: Paul, i.e. $f(t)$.

Reason: $f(14) = 16\ 384 \approx 1 \cdot 6 \times 10^4$, so Paul predicts $1 \cdot 6 \times 10^4 \times 1000 = 1 \cdot 6 \times 10^7$.

$g(14) = 225 \approx 2 \cdot 3 \times 10^2$, so Marie predicts $2 \cdot 3 \times 10^2 \times 1000 = 2 \cdot 3 \times 10^5$.

Question 15

(a)

$$f(7) = 3(7) + 5$$
$$= 26$$

(b)

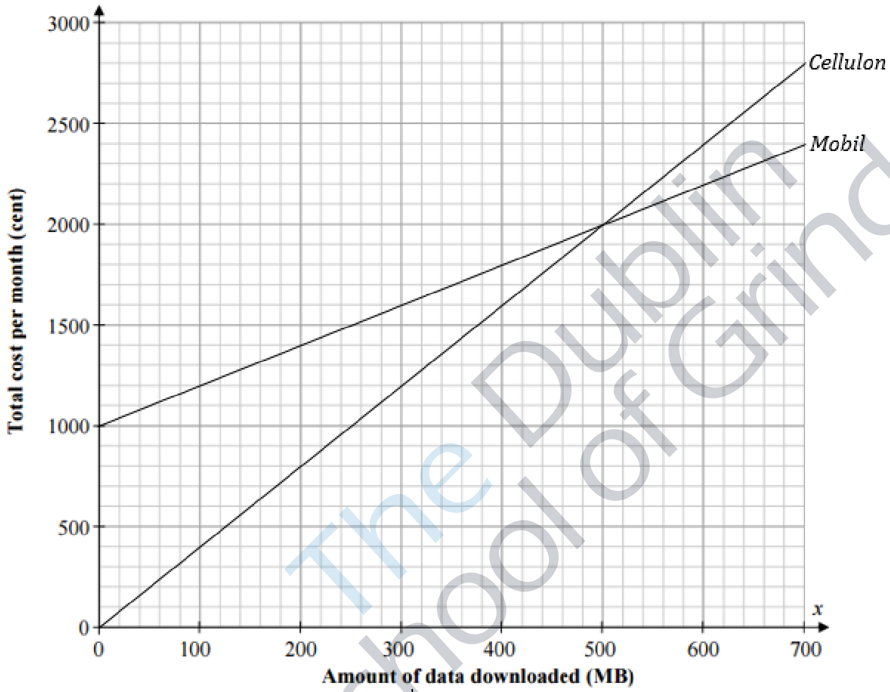
$$f(k) = 3k + 5$$

(c)

$$3k + 5 = k$$
$$2k = -5$$
$$k = -\frac{5}{2}$$

Question 16

(a)



Cellulon	Mobil
$c(x) = 4x$	$m(x) = 1000 + 2x$
$c(0) = 4(0) = 0$	$m(0) = 1000 + 2(0) = 1000$
$c(700) = 4(700) = 2800$	$m(700) = 1000 + 2(700) = 2400$

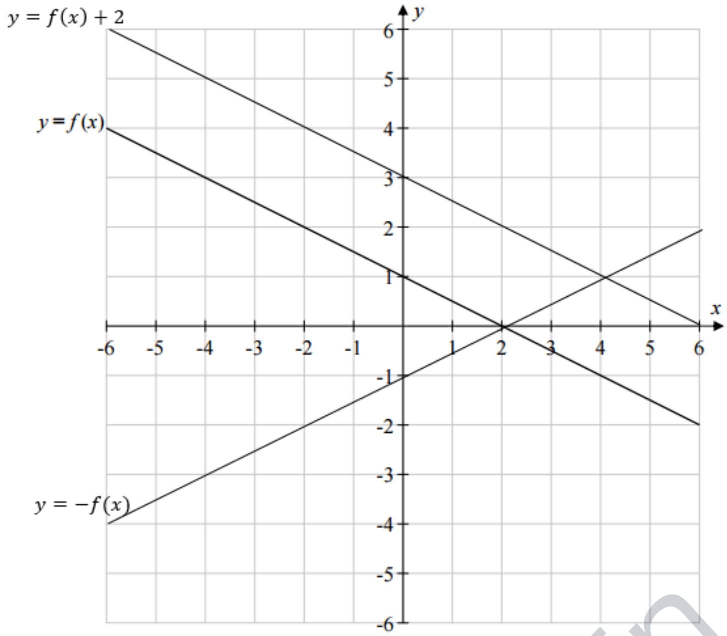
(b) Answer: Cellulon
Reason: From the graph you can see that the cost is zero when no data is downloaded.

(c) $(500, 200)$

(d) If Fergus is going to use less than 500MB of data he should choose Cellulon, but if he is going to use more he should choose Mobil.

Question 13

(a) & (b)



$y = f(x) + 2$ we need to add 2 to the y-axis.
$y = -f(x)$ we need to invert the graph.

Carl Brien

3rd Year Maths Higher Level

Having worked for The State Examinations Commission, Carl brings his reputation as an authority on the Maths Syllabus to The Dublin School of Grinds.

As a member of the Irish Mathematics Teachers Association, he is a popular teacher amongst students due to his mastery of the Project Maths Syllabus and is well-known for his ability to stimulate students interest in Maths.

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