Strategies for Maintaining State of Good Repair for Mixed Transit Fleet

By

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ABSTRACT

Maintaining state of good repair is one of the imperative goals of Moving Ahead for Progress in the 21st Century (MAP-21) vision of U.S Department of Transportation (USDOT). The aging transit fleet if not adequately preserved may have adverse effect on public transportation. Many state DOTs and local agencies are concerned about the escalating costs of new buses and lack of funds to keep up with the replacement needs of the aging fleet. Lack of adequate resource allocation mechanism creates challenges for agencies to determine whether to purchase or rehabilitate/rebuild existing buses in an era of constrained budgets. This paper presents a novel approach that maximizes the total remaining life of buses at the state level consisting of many transit agencies with varying bus types. A branch and bound integer programing is proposed to optimize total remaining life of buses subjected to policy, budget and other constraints. Two scenarios are presented as options to assist transit agencies to assess better strategies. The first scenario analyzes improvement options applicable to buses with zero years of remaining life. The second scenario is mileage based improvement which allows for buses to be eligible for improvement after they reach a certain mileage. The proposed model is applied for transit agencies in the state of Michigan. The results show that mileage based improvement produces better outcomes than replacement when buses reach service life of zero years. The proposed approach is generalized for use by any transit agency to efficiently allocate resources and preserve the aging transit fleet.

INTRODUCTION

In an effort to implement goals set by Moving Ahead for Progress in the 21st Century (MAP-21), transit agencies are required to measure the condition of their assets, and set targets for improvement as a benchmark of advancement. One of the critical assets of public transportation agencies is transit buses. In constrained budget conditions transit agencies are facing challenges in maintaining the bus system in a state of good repair (SGR) *(1)*. MAP-21 defines SGR as "a state that results from application of transportation asset management concepts in which a transit agency maintains its physical assets according to a policy that minimizes asset life-cycle costs while avoiding negative impacts to transit service" *(2)*. The goal of improved transit asset management is to preserve and maintain the bus system in a good condition through strategic approach for assessing needs and prioritizing investments to sustain a SGR.

Fleet management programs in a state Department of Transportation (DOT) in the United States (U.S) are designed to ensure that the buses are kept in productive operation for a minimum normal service life (MNSL). MNSL is defined as the number of years or miles of service that the vehicle must provide before it "qualifies" for federal funds for replacement. Each agency, at any given time, has fleet of varying spans of remaining life (RL), where RL is the difference between the MNSL and vehicle age, expressed either as the number of years, or number of miles (kilometers). A bus that completes its service life should ideally be replaced. However, lack of capital funds often prevents states from procuring new buses for their constituent agencies. While the state DOT's may not have enough capital funds to procure new buses for its constituent agencies, it may be possible for them to allocate capital funds partly for the purchase of new buses, and partly for rebuilding of existing buses, and to distribute the funds among transit agencies in an equitable manner. The question of resource allocation becomes:

"When a state has limited budget, how to allocate funds to constituent agencies to maintain SGR when each agency have variety of buses (by age and size) and how to decide whether to replace, rehabilitate or to rebuild the existing bus with service life of zero years"

Considering the statewide transit fleet as a major public investment, resolving the question of allocation and distribution of funds for new buses and rebuilding of existing buses would require the development of an asset management strategy. Unfortunately, very little research is reported in the literature on management strategies to allocate scarce resources to meet the fleet requirements by a combination of new and rebuilt buses. The problem addressed in this paper relates to the question of allocation of resources among transit agencies for fleet management purposes when each agency carries buses with varying MNSL. Specifically the problem can be stated as follows.

"How to best allocate resources to transit agencies carrying buses with varying MNSL in a state so that the remaining life can be maximum while maintaining same number of fleet in a planning period subjected to budget and other policy constraints"

Transit agencies generally carry three types of buses with more than 20 passenger capacity: medium buses (MNSL 7 years), medium-heavy buses (MNSL 10 years), and large-heavy buses (MNSL 12 years). Buses with higher MNSL involve larger capital cost. Improvements given to the buses when they reach MNSL are different for each bus type. The problem defined is of four dimensions in nature: (1) each state has number of transit agencies, (2) each transit agencies have variety of bus types with each bus has a remaining life associated with it, (3) based on the type of bus and remaining life, different improvement options can be chosen when a fleet reaches MNSL of zero years (4) the budget available for each year to be distributed to each agency and further to be used for a particular bus. The challenge is to track each and every individual bus through the planning period and obtain the type of improvement needed when a bus of specific size reaches MNSL of zero years. Another question remains as do agencies need to wait till the bus reach service life of zero years or it will be helpful to allow the bus to become eligible for improvement before it reaches zero year of service life. While transit resource allocation is a challenging task, this paper attempts to analyze the aforementioned questions using optimization. The paper is organized as follows. In the next section literature review is presented followed by methodology and model formulation. The data section describes fleet composition, budget and policies, used for fleet management. The result section describes findings from the proposed approach. The summary and conclusions are presented in the final section.

LITERATURE REVIEW

Literature review section is organized into three areas: (i) transit fleet improvement options, (ii) quantitative measures for fleet management, (iii) modeling approaches in resource allocation. The review is not intended to be exhaustive, but to highlight some of the general trends in addressing the allocation problem.

Transit Fleet Improvement Options

Transit improvement options drew significant attention in the 1980s, and received renewed research interest in the late 1990s. A number of studies found replacement, rehabilitation, and remanufacturing are the preferred options *(3–5)*. The studies mentioned above stressed the importance of proper preventive maintenance as a primary factor contributing to the success of rehabilitation programs. These studies emphasized that rehabilitation, "if properly done," can be a successful strategy, clearly referring to the quality of maintenance and steps taken by the agency to prevent major breakdowns in machine components or bus body infrastructure. For the purpose of this paper, the following terms are adapted from the literature.

i. Replacement (REPL): Process of retiring an existing vehicle and procuring a completely new vehicle. Vehicles replaced using federal dollars must have completed their MNSL requirements.

ii. Rehabilitation (REHAB): Process by which an existing vehicle is rebuilt to the original manufacturer's specification, with primary focus on the vehicle interior and mechanical system.

iii. Remanufacturing (REBUILD): Process by which the structural integrity of the vehicle is restored to original design standards. This includes remanufacturing the body, the chassis, the drive train, and the vehicle interior and mechanical system.

Quantitative Measures for Fleet Management

Service life maximization is the most commonly adopted measure of effectiveness (MOE) for the resource allocation in conjunction with budgetary constraints. Some prominent bus maintenance management studies include: bus maintenance programs for cost-effective reliable transit service *(6, 7)*, a generalized framework for transit bus maintenance operation *(8)*, manpower allocation for transit bus maintenance program *(9)*, framework for evaluating a transit agency's maintenance program *(10)*, a simulation model for comparing a bus maintenance system's performance under various repair policies *(11)*, and performance indicators for maintenance management *(12)*. These problems primarily cater to an operator, who is concerned with the day to day maintenance for an efficient fleet operation.

Modeling Approaches in Resource Allocation

Similar to the diverse objectives and requirements of the allocation problem, the modeling and solution approaches also vary. However, the most common modeling approach is optimization, with linear programming as the most popular tool because of its faster convergence feature *(13, 14)*. Non-linear programming can be used to model different systems realistically. However, convergence to unique solution is computationally intensive *(15, 16)*. Some of these problems require complex non-convex formulation; and Branch and Bound Algorithm (BBA) has been used to solve specifically large scale allocation problems. Examples include forecasting of energy consumption in multi-facility locations *(17)*, generating signal timing plans to maximize bandwidth for traffic networks *(18)*, a single-track train timetabling problem to minimize the total train travel time *(19)*, and journey planning procedures for multi modal passenger transport services *(20)*, and optimum allocation of resources for replacement and rehabilitation of transit fleet *(21, 22)*.

METHODOLOGY

In the methodology section we propose mathematical formulation of the optimization model, the objective function, and associated constraints. A quality measure can be considered as the remaining life of the fleet system. An agency's objective is to maximize the quality measure. Recent studies attempted to improve upon the original model by suggesting both structural and methodological changes *(21, 22)*.

These studies attempted to maximize the quality of the bus fleet by optimizing different surrogates of Remaining Life (RL)[1]. In this context, Mean Remaining Life (MRL) for one agency can be defined as:

$$MRL = \frac{\sum_{j_\emptyset} f_{ij_\emptyset}^m r_{ij_\emptyset}^m}{\sum_{j_\emptyset} f_{ij_\emptyset}^m} \tag{1}$$

where, $f_{ij_\emptyset}^m$ is the number of buses for an agency *i* with remaining life of *j* years for bus type ∅ (e.g. medium, medium-heavy, and large-heavy) on m^{th} planning year; $r_{ij_\emptyset}^m$ is the remaining life of *j* years for bus type ∅ for an agency *i* on m^{th} planning year for a corresponding bus; *i* is the agency, *j* is the remaining life, ∅ is the bus type, and *m* is the planning year in consideration. In a state, the total mean remaining life (TMRL) is obtained by sum of MRL across all agencies (*i*)

$$TMRL = \sum_i \frac{\sum_{j_\emptyset} f_{ij_\emptyset}^m r_{ij_\emptyset}^m}{\sum_{j_\emptyset} f_{ij_\emptyset}^m} \tag{2}$$

States often plan for preserving fleet for a planning period. Total System Mean Remaining Life (TSMRL) for a planning period is given by:

$$TSMRL = \sum_m \sum_i \frac{\sum_{j_\emptyset} f_{ij_\emptyset}^m r_{ij_\emptyset}^m}{\sum_{j_\emptyset} f_{ij_\emptyset}^m} \tag{3}$$

Where, *m* refers to the number of years in a planning period. Both TMRL and TSMRL can be looked upon as surrogates of the quality of the fleet and need to be *maximized*. The following subsections present the objective function and constraints used to analyze the transit fleet management to obtain SGR.

Objective Function

The objective function (4) represents the sum total of the mean remaining life of the mixed fleet of all the constituent agencies for the whole planning period, designated as *TSMRL* or *Z(x)*. The decision variable $x_{i,j_\emptyset}^m$ is defined in equation (5) with the help of an auxiliary variable $y_{ik_\emptyset}^m$.

$$Z_x = \sum_{m=1}^{N} \sum_{i=1}^{A} \frac{\sum_{j_\emptyset=1}^{J_\emptyset} (r_{i,j_\emptyset}^m + x_{i,j_\emptyset}^m) * j_\emptyset}{\sum_{j=1}^{J} (r_{i,j_\emptyset}^m + x_{i,j_\emptyset}^m)} \ \forall\, j_\emptyset \tag{4}$$

$$x_{ij_\emptyset}^m = \begin{cases} y_{ik_\emptyset}^m, if j_\emptyset = l_{k_\emptyset}, l_{k_\emptyset} \in \{\alpha_\emptyset, \beta_\emptyset, \gamma_\emptyset, \theta_\emptyset\} \\ 0, \quad Otherwise \end{cases} \tag{5}$$

This definitional constraint ensures that the life of the buses is improved by either $\alpha_\emptyset, \beta_\emptyset, \gamma_\emptyset,$ and $\theta_\emptyset$ years. $\alpha_\emptyset$ and $\beta_\emptyset$ represents two types of REHAB options, $\gamma_\emptyset$ represents REBUILD options, and $\theta_\emptyset$ represents REPL option; ∅ represents a specific bus type.

[1]RL can be defined as the difference between the minimum normal service life (MNSL) and the age of the bus. The MNSL of a medium-sized bus, the subject matter of this study is taken as seven years per guidelines of the U. S. Department of Transportation.

Constraint: Budget

The equations (6) and (7) represent the budget constraints for the planning period. The former (6) represents a fixed total budget (B) that cannot be exceeded in the planning period. Equation (7) states that total budget is the sum of annual budgets (b_m) for each year. It is assumed that the planner has flexibility of borrowing funds from the future years without exceeding the total budget for the planning period.

$$\sum_{m=1}^{N}\sum_{i=1}^{A}\sum_{k_{\emptyset}=1}^{P} y_{ik_{\emptyset}}^{m} * c_{k_{\emptyset}}^{m} < B \tag{6}$$

$$B = \sum_{m=1}^{N} b_m \,, \forall m \tag{7}$$

Constraint: Eligibility for Improvement

Eligibility of improvement is suggested in two ways. The equation (8) ensures that all the buses that have completed their Minimum Normal Service Life (MNSL) requirements will be eligible for improvement as per Federal Transit Administration (FTA) standards. MNSL can be defined as the number of years or miles of service that the vehicle must provide before it "qualifies" for federal funds for rehabilitation, remanufacturing and replacement. However agencies also prefer mileage based improvement. The advantage of mileage based continual improvement is that buses do need to wait for MNSL of zero years rather are eligible for improvement when they reach a certain threshold of mileage as shown in equation (9).

$$\sum_{k_{\emptyset}=1}^{P} y_{ik_{\emptyset}}^{m} = r_{ij_{\emptyset}}^{m} \,, \forall i, m, j_{\emptyset} \tag{8}$$

$$\sum_{k_{\emptyset}=1}^{P} y_{ik_{\emptyset}}^{m} = \omega_{ij_{\emptyset}}^{m} \,, \forall i, m, j_{\emptyset} \tag{9}$$

Constraint: Restriction for Multiple Rehabilitation

Equation (10) represents policy constraints which ensure that the buses that have been rehabilitated twice or remanufactured once will be replaced. The two terms in this constraint are defined in equations (11) and (11). These three constraints are specific to the case study presented in this paper, and can be revised at the discretion of the user. Thus, equations (10) and (11) ensure that a bus that was rebuilt twice (each time its life is increased by $\alpha_{\emptyset}$ or $\beta_{\emptyset}$ years is replaced. This policy is applicable only after $\alpha_{\emptyset} + \beta_{\emptyset}$ years. Similarly, a bus that is remanufactured resulting in an increase in life by $\gamma_{\emptyset}$ years must be replaced (equations 10 and 12) and is applicable only after $\gamma_{\emptyset}$ years. This constraint presented in equations (10, 11, and 12) is specific to the case study presented in this paper, and can be revised at the discretion of the user. Equation (13) is a non-negativity constraint to ensure that the number of buses chosen for improvement is never negative. The formulation involves non-linear functions, non-differentiable functions, step functions, and integer variables. Although the step function can be generalized to linear forms, the formulation will require additional variables which may result in variable explosion rendering the model unsuitable for large/real world problems.

$$y_{ik_\emptyset}^m = \sum_{\alpha_\emptyset \beta_\emptyset} \delta_{i,(\alpha_\emptyset \beta_\emptyset)}^m + \delta_{i,(\gamma_\emptyset)}^m \forall i, m, j_\emptyset;\ \forall \alpha_\emptyset, \beta_\emptyset, \gamma_\emptyset \tag{10}$$

$$\delta_{i,(\alpha_\emptyset \beta_\emptyset)}^m = \begin{cases} min\left\{y_{i\alpha_\emptyset}^{m-(\alpha_\emptyset+\beta_\emptyset)}, y_{i\beta_\emptyset}^{m-\alpha_\emptyset}\right\} if m \geq \alpha_\emptyset + \beta_\emptyset, \forall m > \alpha_\emptyset + \beta_\emptyset, \\ 0\,, \quad Otherwise \end{cases} \tag{11}$$

$$\delta_{i,(\gamma_\emptyset)}^m = \begin{cases} y_{i\gamma_\emptyset}^{m-\gamma_\emptyset}, \forall m > \gamma_\emptyset, \\ 0\,,\ Otherwise \end{cases} \tag{12}$$

$$y_{ik_\emptyset}^m > 0 \tag{13}$$

SOLUTION APPROACH

A Branch and Bound Algorithm (BBA) is used in this paper because of the integer nature of decision variables in transit fleet resource allocation. The BBA approach is applied in three steps. The first step involves coding of the decision variable cells. The second step involves model initialization, where the convexity and the size of the problem in terms of number of variables, integers, bounds and surface nature are determined. A diagnosis of the model is performed to check the nature of the desired model (linear, quadratic, conic, non-linear, etc.). Finally, the third step involves the development of constraint coded cells. Budget constraints (Equation 6 and 7), mandatory replacement constraints (Equation (10)), and REBUILD constraints are coded. The BBA approach is explained below.

Let $y_{ik_\emptyset}^m$ is the number of buses to be added to a fleet when it reaches a zero remaining life for $k_\emptyset$ type of improvement for agency *i*, on *m*th year. If $y_{ik_\emptyset}^m$ is not an integer, we can always find an integer $[y_{ik_\emptyset}^m]$ such that:

$$\left[y_{ik_\emptyset}^m\right] < y_{ik_\emptyset}^m < \left[y_{ik_\emptyset}^m\right] + 1 \tag{14}$$

Equation (12) results in the formulation of two sub problems, with an additional upper bound constraint

$$y_{ik_\emptyset}^m \leq \left[y_{ik_\emptyset}^m\right] \tag{15}$$

and another with lower bound constraint

$$y_{ik_\emptyset}^m < \left[y_{ik_\emptyset}^m\right] + 1 \tag{16}$$

If the decision variables with integer constraints already have integer solutions, no further action is required. If one or more integer variables have non-integer solutions, the Branch and Bound method chooses one such variable and creates two new sub-problems where the value of that variable is more tightly constrained. These sub problems are solved and the process is repeated, until a solution is found where all of the decision variables have integer values (within a small tolerance). The complete methodology is implemented in a VBA based solver platform *(23)*, on an Intel (R) Xeon (R) Core 2 Quad, 4GB memory, 2.0 GHz under Windows 7 operating system. A precision value of 1.0E-6 is used to determine how closely the estimated constraints match with the given values. Each optimization run requires approximately 20,000 iterations to find the optimal value. Each iteration requires 0.10 seconds, and one complete optimization run requires approximately 33 minutes.

DATA

Transit fleet in the state of Michigan is considered to demonstrate the model application. Public Transportation Management System (PTMS) is a database developed by Michigan Department of Transportation (MDOT) containing fleet data with details on number of buses by agency and remaining life, year purchased. For this research buses which have not received any improvement and have not reached their MNSL of zero years is considered for further analysis. Data on three bus types (1) medium, (2) medium-heavy and (3) large-heavy, are extracted and vans and small buses of capacity 10 or less is ignored. In the formulation equations 1 through 16, ∅= 1 represents medium bus, ∅=2 represents medium-heavy and ∅ =3 represents large-heavy bus. The data extraction process resulted in 59 agencies with medium size buses and 20 agencies of medium-heavy and large-heavy buses. To be consistent 20 agencies for each bus type (60 agencies in total) is extracted and used in the analysis and considered to present in the demonstration of results.

TABLE 1 Input Data

Bus Type	Fleet Distribution Remaining Life														Total Fleet	Before TMRL
	Location ID	0	1	2	3	4	5	6	7	8	9	10	11	12		
Medium	386004856	3	0	6	0	6	7	4	0						26	3.65
	382113333	1	3	14	0	5		0	0						23	2.22
	386005853	0	8	0	0	0	0	5	2						15	3.47
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	383236264	0	0	7	0	0	1	1	1						10	3.00
	386000098	0	0	0	4	1	2	4							11	4.55
	386000191	3	1	0	0	5	3	0		.	.	.	.	.	12	3.00
Medium-Heavy	382161440	1	0	0	0	0	0	0	2	0	0	1			4	6.00
	382113333	0	0	6	0	0	14	0	4	0	0	0			24	4.58
	382117978	3	2	0	0	0	0	0	0	0	0	0			5	0.40
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	382272528	3	0	1	5	0	0	0	0	6	0	0			15	4.33
	386004606	0	0	0	0	0	0	0	0	0	1	0			1	9.00
	386004647	0	0	0	0	0	0	0	0	7	0	1	.	.	8	8.25
Large-Heavy	381985122	5	0	0	13	9	14	14	7	40	0	12	22	0	136	7.06
	381966169	19	8	0	1	0	0	5	0	0	1	0	10	0	44	3.64
	382117978	0	0	19	0	0	0	15	5	7	4	10	0	16	76	7.20
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	382575895	1	1	0	2	0	0	0	0	0	1	0	0	0	5	3.20
	386004647	4	0	0	1	1	0	0	0	0	5	0	0	0	11	4.73
	386006063	0	0	0	0	0	4	0	0	5	0	3	1	0	13	11.00
Total		**228**	**200**	**251**	**99**	**208**	**73**	**98**	**47**	**93**	**75**	**38**	**101**	**21**	**1532**	**243.34**

Note: Shaded potion represents fleet life is shorter

Table 1 shows distribution of remaining life of buses for each agency by bus type. Total fleet and MRL for each agency are also shown in Table 1. For example for the first location ID, MRL is estimated as (3*0+0*1+6*2+…. +4*6+0*7)/26 = 3.65. MRL of 3.65 represents as the mean remaining life of the first agency is 3.65 years. Similarly, MRL is estimated for all agencies. TMRL for all agencies for the year 2013 is 243.34 years.

The following improvement options are used in the case study:

- Replacement (REPL)—process of retiring an existing vehicle and procuring a completely new vehicle. Buses proposed to be replaced using federal dollars are expected to be at the end of their MNSLs, as described above.
- Rehabilitation (REHAB)—process by which an existing bus is rebuilt to the original manufacturer's specification. The focus of rehabilitation is on the vehicle interior and mechanical systems, including rebuilding engines, transmission, brakes, and so on. Two types of rehabilitation: REHAB1 and REHAB2 with moderate to higher levels of engine rebuilds are considered in this study.
- Remanufacturing (REBUILD)—process by which the structural integrity of the bus is restored to original design standards. This includes remanufacturing the bus chassis as well as the drivetrain, suspension system, steering components, engine, transmission, and differential with new and manufactured components and a new bus body.
- Further, it was assumed that a vehicle may be rehabilitated (REHAB1 or REHAB2) only up to two consecutive terms, and then must be replaced (REPL) with a new bus. A vehicle with REHAB1 and REHAB2 (or vice versa) in two consecutive terms also should be replaced. A vehicle may be remanufactured (REBUILD) only one time, and then must be replaced (REPL) with a new bus. A vehicle rehabilitated (REHAB1 and REHAB2) once can be eligible for remanufacturing (REBUILD) before it is replaced (REPL).

Table 1 also shows that 228 buses have reached MNSL of zero years and eligible for improvement options. The cost of different improvement options by fleet type is shown in Table 2. The cost of each improvement option is increased by two percent every alternate year. A planning period of 11 years is considered. The purpose of considering planning period of 11 years is to capture the tracking of buses over a longer period. However, the duration of planning period can be increased or decreased as preferred by the agency in the analysis. If minimum improvement options are considered for the 228 buses then minimum budget needed for the first year is $15.26 million, and if all the buses will be considered to be purchased then the budget will be $69.90 million. These two budgets are extreme cases, and in reality budget availability is somewhere in between minimum and maximum. A budget of $35 million is considered for the first year.

MODEL APPLICATION RESULTS

The results are presented in two sub-sections. First a single year analysis is shown to demonstrate the model working principle and advantage. Second, 11 year planning period is considered to show the allocation of fleet to different improvement options in the future years. In the planning period analysis two policies are considered (1) buses become eligible for improvement only when they reach service life of zero years, (2) a mileage based improvement which suggests that buses become eligible when they reach a certain threshold of mileage and need not wait to reach zero years of service life. It is assumed that there is flexibility in borrowing funds from the future years without exceeding the total budget for the planning period.

TABLE 2 Cost of Improvement Options and Budget

Year	Mid Size ($)				Mid Size- Heavy ($)				Large-Heavy ($)				Annual Budget ($)
	Rehab-I	Rehab-II	REBUILD	Replace	Rehab-I	Rehab-II	REBUILD	Replace	Rehab-I	Rehab-II	REBUILD	Replace	
	Life=2	Life=3	Life=4	Life=7	Life=3	Life=4	Life=6	Life=10	Life=4	Life=5	Life=7	Life=12	
2013	17,800	24,500	30,320	81,540	43,660	60,093	74,368	200,000	76,404	105,163	130,145	350,000	35,000,000
2014	17,800	24,500	30,320	81,540	43,660	60,093	74,368	200,000	76,404	105,163	130,145	350,000	35,000,000
2015	18,690	25,725	31,836	85,617	45,843	63,098	78,087	210,000	80,224	110,421	136,652	367,500	36,750,000
2016	18,690	25,725	31,836	85,617	45,843	63,098	78,087	210,000	80,224	110,421	136,652	367,500	36,750,000
2017	19,625	27,011	33,428	89,898	48,135	66,253	81,991	220,500	84,236	115,942	143,485	385,875	38,587,500
2018	19,625	28,362	35,099	94,393	50,541	69,565	86,091	231,525	88,447	121,739	150,659	405,169	38,587,500
2019	20,606	29,780	36,854	99,112	53,068	73,044	90,395	243,101	92,870	127,826	158,192	425,427	40,516,875
2020	20,606	29,780	36,854	99,112	53,068	73,044	90,395	243,101	92,870	127,826	158,192	425,427	40,516,875
2021	21,636	31,269	38,697	104,068	55,722	76,696	94,915	255,256	97,513	134,218	166,101	446,699	42,542,719
2022	21,636	31,269	38,697	104,068	55,722	76,696	94,915	255,256	97,513	134,218	166,101	446,699	42,542,719
2023	22,718	32,832	40,632	109,271	58,508	80,531	99,661	268,019	102,389	140,929	174,406	469,033	44,669,855
Total													137,983,970

One Year Analysis

Results of the first year optimization are shown in Table 3. 19 medium buses, 32 medium-heavy buses and 177 large-heavy buses have zero years of service life (Table 3, column 3). These buses are eligible for improvement in different options. If these buses are to be replaced then a total cost of $69,899,260 is needed (i.e. 19*$81,540 + 32*200,000 + 177*350,000). A budget of $35 million is available for allocation (Table 2). The need for an optimization model is very helpful in this situation to provide optimal allocation of buses to different improvement options. The objective function for one year analysis is maximization of TMRL, subjected to budget constraint. All the fleet eligible for improvement must be allocated.

Table 3 shows results for a subset of agencies. For example, for agency (ID= 386004856), 3 medium size buses needs to be replaced, optimization model allocates these three buses to REHAB-I option. Because of this improvement MRL for this agency has increased from 3.65 to 4.46 years. The cost for this improvement is $244,620. For all agencies in medium size, 19 buses are chosen for REPL option. TMRL for medium size buses have improved from 72.55 years to 81.98 years at an expenditure of $1.549 million. Similarly, for medium-heavy buses, all 32 buses are chosen for REPL improvement option resulting in TMRL improvement from 74.43 years to 138.77 years at an expense of $6.4 million. For large-heavy buses, 100 in REHAB-I, 34 in REBUILD, and 43 in REPL options are chosen. The TMRL has improved from 96.36 years to 113.76 years at an expense of $27.05 million. . For all types of buses in one year analysis total TMRL has improved from 243.34 years to 334.51 years at an expense of $35 million. A detailed analysis showing all the agencies are not shown in Table 3 for brevity. The allocation for the first year shows that 100 buses are chosen for REHAB-I, none for REHAB-II, 34 for REBUILD, and 43 for REPL option. The advantage of the optimization model is that it allows an optimal distribution of fleet to various improvement options in such a way that TMRL is maximized. It is nearly impossible to do such an analysis without an optimization model.

Multiple Year Analysis

Case-1: Zero Remaining Life Based Improvement (ZRLBI)

Table 4 shows resource allocation for 11 year planning period (2013 to 2023) for the mixed transit fleet in Michigan. Results are presented by each type of transit fleet over the planning period. For example, for medium buses in the year 2013, 19 buses are chosen for REPL category, resulting in a TMRL of 81.98 years with allocated cost of $1.549 million. For 2014, 14 buses are chosen for REPL category resulting in TMRL of 67.77 years. Similar allocations can be observed for other years in the planning period. At the end of the planning period a total of 431 buses were chosen for improvement. Among these, 37 buses for REHAB-I, 18 buses for REHAB-II, none for REBUILD, and 376 for REPL are chosen with a TSMRL of 840.52 at expense of $36.59 million.

TABLE 3 One Year Improvement

Bus Type	Location ID	MNSL (0 Years)	Optimized Improvement Option				Total Improved	Before MRL	After MRL	Cost ($)
			Rehab-I	Rehab-II	REBUILD	Replace				
	386004856	3	0	0	0	3	3	3.65	4.46	244,620
	382113333	1	0	0	0	1	1	2.22	2.52	81,540
	386005853	0	0	0	0	0	0	3.47	3.47	0
	.	.	.	.	.	.	.		.	.
	.	.	.	.	.	.	.		.	.
	383236264	0	0	0	0	0	0	3.20	3.20	0
	386000098	0	0	0	0	0	0	4.55	3.00	0
	386000191	3	0	0	0	3	3	3.00	4.75	244,620
Medium Size	**Sub-total**	**19**	**0**	**0**	**0**	**19**	**19**	**72.55**	**81.98**	**1,549,260**
	382161440	1	0	0	0	1	1	6.00	8.50	200,000
	382113333	0	0	0	0	0	0	4.58	4.58	0
	382117978	3	0	0	0	3	3	0.40	6.40	600,000
	.	.	.	.	.	.	.		.	.
	.	.	.	.	.		.		.	.
	382272528	3	0	0	0	3	3	4.33	6.33	600,000
	386004606	0	0	0	0	0	0	9.00	9.00	0
	382588506	2	0	0	0	2	0	0	10.00	400,000
Medium Size Heavy	**Sub-total**	**32**	**0**	**0**	**0**	**32**	**32**	**74.43**	**138.77**	**6,400,000**
	381985122	5	0	0	5	0	5	7.06	7.32	650,725
	381966169	19	0	0	0	19	19	3.64	8.82	6,650,000
	382117978	0	0	0	0	0	0	7.20	7.20	0
	.	.	.	.	.	.	.		.	.
	.	.	.	.	.	.	.		.	.
	382575895	1	0	0	0	1	1	3.20	5.60	350,000
	386004647	4	0	0	0	4	4	4.73	9.09	1,400,000
	386006063	0	0	0	0	0	0	11.00	11.00	0
Large Size Heavy	**Sub-total**	**177**	**100**	**0**	**34**	**43**	**177**	**96.36**	**113.76**	**27,050,740**
Total		**228**	**100**	**0**	**34**	**94**	**228**	**243.34**	**334.51***	**35,000,000**

Note: * Objective function in the optimization

For medium-heavy buses in 2013, 26 buses are chosen for REHAB-I and six buses for REPL with a resulting TMRL of 96.96 years. For the planning period, a total of 164 buses (34 for REHAB-I and 130 for REPL) are chosen for improvement resulting in TSMRL of 1191.10 years at an expense of $31.11 million. Similarly, for large-heavy buses, for the first year 177 buses were chosen for REPL option resulting in TMRL of 117.74 years. For the planning period, 36 buses were chosen for REHAB-I, none for REHAB-II, six for REBUILD, and 987 for REPL option resulting in TSMRL of 1363.72 years at an expense of $363.75 million. Overall for including all bus types, 107 buses were chosen for REHAB-I, 18 for REHAB-II, six for REBUILD, and 1582 in REPL option were chosen. Overall, TSMRL of 3395.35 is attained with an expenditure of $431.46 million.

In a comparison, it is noted that the TMRL achieved in one year analysis is 334.51 years (Table 3) and the corresponding value for multi-year analysis is 296.68 years (Table 4, sum of TMRL for first year). This difference is because of the fact that in multi-year analysis the optimization works in such a way that TMRL is maximized while in one year analysis the model is blind for future years. A number of constraints come into play for multiyear analysis such as a bus that receives twice REHAB-I and REHAB-II, or REBUILD or a combination of REHAB and REBUILD must be replaced (constraints as shown in equation 8-10). These constraints do not apply to one year analysis. One has an option of conducting optimization one year at a time at the discretion of obtaining sub-optimal or infeasible solution. Previous research has already established that multi-year analysis has several benefits over annual allocation *(24)*.

Case-2: Continual Mileage Based Improvement (CMBI)

In mileage based continual replacement buses are eligible for improvement when they reach a threshold of mileage. It is assumed that each bus reach a mileage threshold in every three years. In practice each bus is used for a fixed route and also mileage travelled by buses at the end of every year is also nearly constant *(25–27)*. With the mileage threshold these assumptions are made based on recent statewide practices:

- Only new buses whose life is three years or more become eligible for continual improvement for a maximum of three times.
- Buses as they reach normal service life of zero years also become eligible for improvement
- Cost of buses continual improvement is 50% of that of an option REHAB-I, REHAB-II, and REBUILD as the improvement is applied not at the end of the service life but when the bus has completed a threshold of mileage.

Table 5 shows result of CMBI strategy for all types of fleet for the planning period. For medium-buses, in the year 2013, 5 buses are chosen for REHAB-I, one for REHAB-II, none for REBUILD, and 13 for REPL, resulting in TMRL of 79.80 and allocated cost of $1.186 million. During the planning period for medium size buses 114 buses are chosen for REHAB-I, 9 for REHAB-II, 282 for REBUILD, and 632 for REPL option. The resulting TSMRL is 825.80 years, and the corresponding cost is $34.785 million.

TABLE 4 Planning Period Allocation Results for ZRLBI

Fleet Type	Year	REHAB-I	REHAB -II	REBUILD	REPL	Total	TMRL	Allocated Cost
Medium	2013	0	0	0	19	19	81.98	1,549,260
	2014	0	0	0	14	14	67.77	1,141,560
	2015	0	0	0	52	52	69.95	4,452,084
	2016	37	18	0	0	55	60.82	1,154,580
	2017	0	0	0	61	61	66.15	5,483,778
	2018	0	0	0	66	66	82.28	6,229,938
	2019	0	0	0	70	70	98.44	6,937,840
	2020	0	0	0	28	28	92.85	2,775,136
	2021	0	0	0	14	14	78.64	1,456,952
	2022	0	0	0	52	52	80.82	5,411,536
	2023	0	0	0	0	0	60.82	0
	Sub-Total	**37**	**18**	**0**	**376**	**431**	**840.52**	**36,592,664**
Medium-Heavy	2013	26	0	0	6	32	96.96	2,335,160
	2014	0	0	0	4	4	82.50	800,000
	2015	0	0	0	12	12	81.38	2,520,000
	2016	0	0	0	38	38	144.90	7,980,000
	2017	0	0	0	8	8	137.40	1,764,000
	2018	0	0	0	14	14	123.23	3,241,350
	2019	0	0	0	9	9	114.11	2,187,909
	2020	0	0	0	11	11	104.63	2,674,111
	2021	0	0	0	23	23	106.29	5,870,888
	2022	0	0	0	5	5	108.60	1,276,280
	2023	8	0	0	0	8	91.10	468,064
	Total	**34**	**0**	**0**	**130**	**164**	**1191.10**	**31,117,762**
Large-Heavy	2013	0	0	0	177	177	117.74	61,950,000
	2014	0	0	0	182	182	160.47	63,700,000
	2015	0	0	0	187	187	153.90	68,722,500
	2016	0	0	0	32	32	142.73	11,760,000
	2017	0	0	0	139	139	131.92	53,636,625
	2018	0	0	0	30	30	121.88	12,155,070
	2019	0	0	0	37	37	108.18	15,740,799
	2020	0	0	0	27	27	110.97	11,486,529
	2021	0	0	0	70	70	105.80	31,268,930
	2022	0	0	6	64	70	113.18	29,647,159
	2023	36	0	0	0	36	96.93	3,686,004
	Total	**36**	**0**	**6**	**945**	**987**	**1363.72**	**363,753,616**
Total		**107**	**18**	**6**	**1451**	**1582**	**3395.35**	**431,464,042**

For medium-heavy buses, in the year 2013, two buses were chosen for REHAB-II and 30 buses for REPL option, resulting in a TMRL of 137.67 years at an expense of $6.11 million. In the planning period, two buses were chosen for REHAB-I, 29 for REHAB-II, 219 for REBUILD, and 79 for REPL. A total of 328 buses were eligible for improvement resulting in TMRL of 1,832.75 years at an expense of $38.97 million. For large-heavy buses in the year 2013, 43 buses were chosen for REHAB-I, none for REHAB-II, 64 for REBUILD and 70 for REPL option. A total of 177 buses were improved for year 2013 resulting in TMRL of 113.47 years at an expense of $36.024 million. In the planning period a total of 2,170 buses were improved (309 for REHAB-I, 1442 for REHAB-II, 130 for REBUILD, and 289 for REPL) with resulting TSMRL of 1382.68 years and allocated cost of $357.41 million. When allocation of all buses is combined, 424 buses for REHAB-I, 1480 REHAB-II, 631 for REBUILD, and 594 REPL are used. A total of 3130 buses were improved using different options resulting in TSMRL of 4041.22 years at cost of $431.169 million.

Annual Expenditures

Table 6 shows budget allocation for each year and for both cases (ZRLBI and CMBI). Year and given budget is shown in the first and second column of Table 6. For each case allocated budget, surplus/deficit, and cumulative surplus/deficit is shown. For ZRLBI case, in the first year allocated budget is $65.83 million resulting in a deficit of $30.83 million. In the multiple year analysis it is assumed that there is flexibility in borrowing funds from the future years ensuring that budget in the planning period is not exceeded (equation 6). Similarly deficit and surplus for other years for the base case is shown in Table 6. It is seen that for base case model large deficits are incurred in the beginning years of the planning period but surplus remained in the later years ensuring the rule of not violating total budget constraint. At the end of the planning period, the allocated budget $431.464 million is used leaving no surplus or deficit. During the planning period deficit are incurred in the year 2013, 2014, 2015, and 2017.

Similarly annual allocated costs for all years in CMBI strategy suggest that deficits were incurred in multiple years in the planning period but in lesser amount. For example, in the year 2013, allocated cost is $43.32 million resulting in a deficit of 8.32 million (compared to $30.83 million deficit for the base case). For the planning period total allocated cost is $431.16 million resulting in a surplus of $294,392. It is observed that deficit is incurred from 2013 through 2016 and 2019.

TABLE 5 Planning Period Allocation Results for CMBI

Fleet Type	Year	REHAB-I	REHAB -II	REBUILD	REPL	Total	TMRL	Allocated Cost
Mid Size	2013	5	1	0	13	19	79.80	1,186,239
	2014	0	0	0	14	14	65.59	1,141,560
	2015	5	2	0	51	57	68.90	4,468,078
	2016	3	0	164	52	220	80.33	6,639,358
	2017	0	0	14	5	19	94.89	1,629,305
	2018	0	0	51	5	56	80.01	5,131,076
	2019	0	0	52	0	52	69.68	5,093,214
	2020	20	3	1	37	61	81.89	3,625,992
	2021	7	4	0	18	29	72.83	1,747,264
	2022	40	0	0	32	72	73.60	3,510,686
	2023	33	0	0	0	33	58.28	612,636
	Sub-Total	**114**	**9**	**282**	**226**	**632**	**825.80**	**34,785,409**
Mid Size-Heavy	2013	0	2	0	30	32	137.67	6,117,922
	2014	0	0	0	4	4	123.20	800,000
	2015	2	0	0	10	12	121.61	2,251,672
	2016	0	2	102	10	114	124.26	10,758,720
	2017	0	1	14	1	16	150.36	1,477,174
	2018	0	0	51	2	52	146.72	5,070,404
	2019	0	0	52	0	52	171.23	5,093,214
	2020	0	3	0	7	10	234.85	1,937,231
	2021	0	6	0	9	15	220.33	2,804,893
	2022	0	3	0	6	9	209.15	1,773,817
	2023	0	11	0	0	11	193.36	885,841
	Total	**2**	**29**	**219**	**79**	**328**	**1832.75**	**38,970,888**
Large-Heavy	2013	43	0	64	70	177	113.47	36,024,918
	2014	92	0	4	86	182	149.51	37,746,204
	2015	0	146	0	41	187	138.89	31,162,756
	2016	18	600	0	14	632	125.77	94,782,589
	2017	10	127	13	20	171	143.77	10,549,957
	2018	44	549	26	22	641	141.93	21,297,575
	2019	0	0	0	0	0	124.30	93,415,236
	2020	8	7	10	15	39	133.53	9,422,976
	2021	3	3	3	9	18	125.93	5,249,261
	2022	11	11	11	12	44	101.50	9,617,845
	2023	80	0	0	0	80	84.08	8,144,036
	Total	**309**	**1442**	**130**	**289**	**2170**	**1382.68**	**357,413,353**
Total		**424**	**1480**	**631**	**594**	**3130**	**4041.22**	**431,169,650**

TABLE 6 Available Budget and Allocated Cost For Each Year

Year	Given Budget	ZRLBI			CMBI		
		Allocated Cost	Surplus/Deficit	Cumulative	Allocated Cost	Surplus/Deficit	Cumulative
2013	35,000,000	65,834,420	-30,834,420	-30,834,420	43,329,079	-8,329,079	-8,329,079
2014	35,000,000	65,641,560	-30,641,560	-61,475,980	39,687,764	-4,687,764	-13,016,843
2015	36,750,000	75,694,584	-38,944,584	-100,420,564	37,882,506	-1,132,506	-14,149,350
2016	36,750,000	20,894,580	15,855,420	-84,565,144	112,180,667	-75,430,667	-89,580,017
2017	38,587,500	60,884,403	-22,296,903	-106,862,047	13,656,435	24,931,065	-64,648,952
2018	38,587,500	21,626,358	16,961,142	-89,900,905	31,499,056	7,088,444	-57,560,508
2019	40,516,875	24,866,548	15,650,327	-74,250,578	103,601,663	-63,084,788	-120,645,296
2020	40,516,875	16,935,776	23,581,099	-50,669,479	14,986,199	25,530,676	-95,114,621
2021	42,542,719	38,596,770	3,945,949	-46,723,530	9,801,418	32,741,301	-62,373,320
2022	42,542,719	36,334,975	6,207,744	-40,515,787	14,902,349	27,640,370	-34,732,950
2023	44,669,855	4,154,068	40,515,787	0	9,642,512	35,027,342	294,392
Total	431,464,042	431,464,042			431,169,650		

SYNTHESIS OF RESULTS

A summary of results is shown in Figure 2. Figure 2(a) presents TSMRL of ZRLBI and CMBI. In almost all bus types CMBI results in higher TSMRL compared to ZRLBI. Specifically medium-heavy buses in CMBI resulted in significantly higher TSMRL. Figure 2 (b) shows TMRL for each year in the planning period. There is no trend observed between years. But for all years' CMBI resulted in higher TMRL except for the year 2016. Figure 2(c) shows type of improvement chosen by each case. For ZRLBI type of improvement maximum buses received REPL and REBUILD improvement has the highest and least numbers respectively. In contrast, for CMBI maximum buses received REHAB-II improvement, and other improvement options received more or less equal number buses. Overall, for CMBI higher number of buses was improved. This is logical because buses were eligible for improvement even before they reached their MNSL of zero years. Figure 2(d) shows surplus or deficit for each year for the two cases. For both cases deficits are incurred in the early years. But during initial years, for ZRLBI case, higher deficits are incurred compared to CMBI. In contrast for CMBI two large deficits are experienced in the year 2015 and 2019. In the later years both approaches produced surpluses balancing the deficits from the initial years to ensure that the total allocated cost is within the planning period budget.

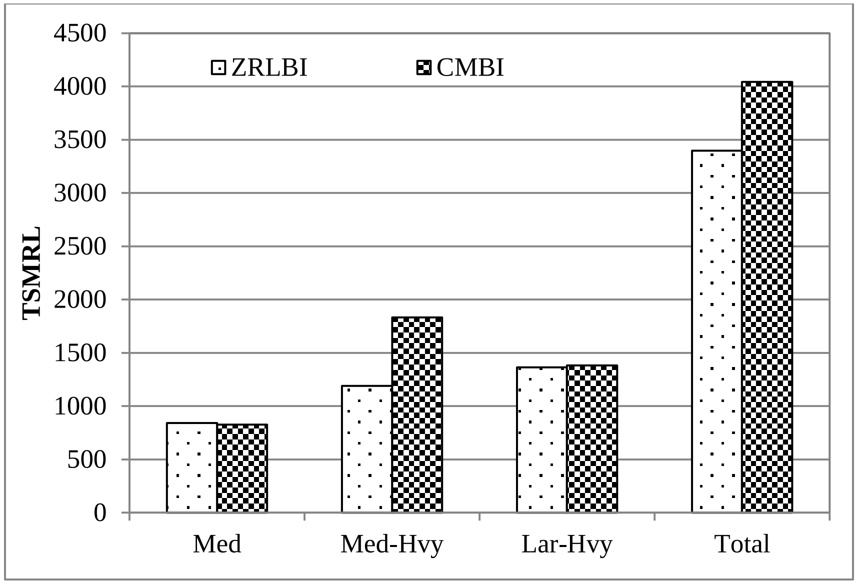

Figure 2(a): TSMRL By Fleet Type

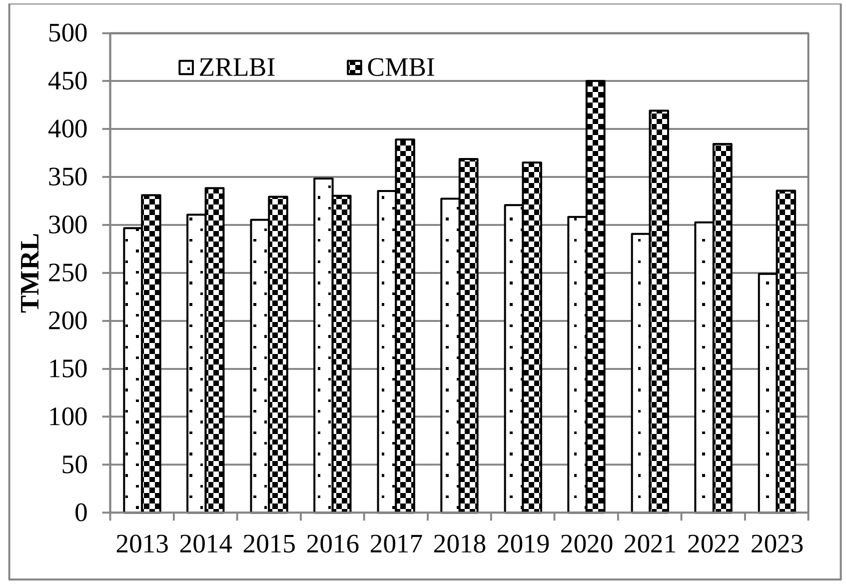

Figure 2(b): TMRL by Strategies

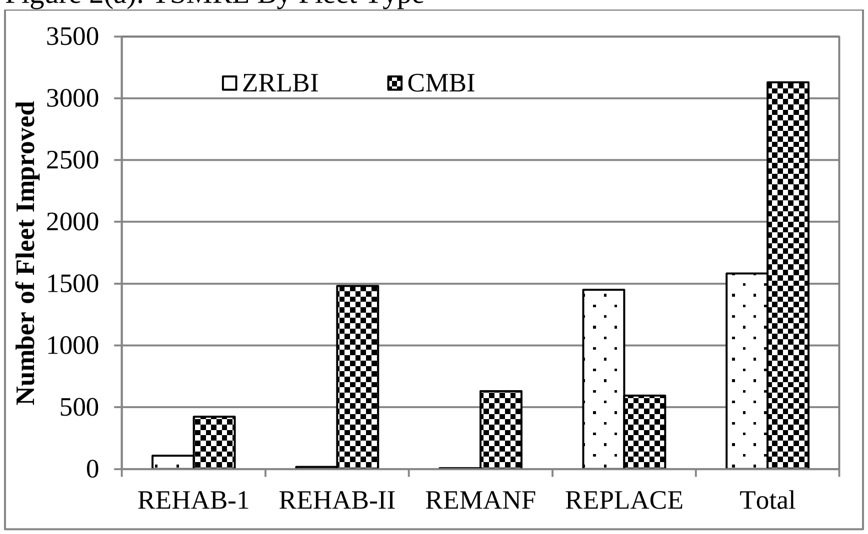

Figure 2(c): Improvement Options Selected by Fleet Type

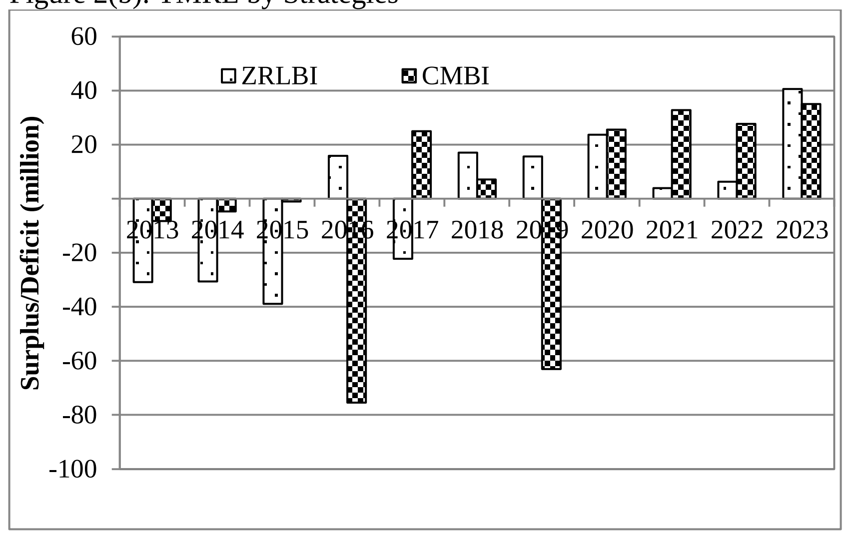

Figure 2(d): Cumulative Surplus/Deficit by Strategies

FIGURE 2: Synthesis of Results by Strategies by Fleet Type and by Year

Clearly CMBI provided better results than ZRLBI in terms of the TSWRL values (objective function) and the budget (constraint). However, a careful examination of the results shows that CMBI case arrived at larger TSMRL by adopting large investments in the interim years. On the other hand, the ZRLBI based improvement results that tend to use all the available options at somewhat moderate spending rate over the years (with deficits in the initial years), may be the preferred alternative for some agencies. Overall, both approaches provide intuitive results, but CMBI surpasses its counterpart because of the ability to make buses eligible for improvement before they reach service life of zero years at a lower cost.

CONCLUSION

MAP-21 clearly identifies SGR as one of the critical aspects to maintain and preserve the current public transit system. Each state has number of transit agencies, and each transit agencies have variety of bus types. The remaining life associated with each bus is different. The challenge is to find optimal improvement strategy through an allocation process when each bus is eligible for improvement in such a way that the total remaining life of the bus system is maximized subjected to budget and other constraints. The proposed approach can be considered to have four dimensions, namely the choice of fleet improvement program, allocation of funds among the constituent transit agencies, and allocation of funds over the planning period, and fleet type. The model is formulated as a mathematical program that maximizes total weighted average of remaining life of the fleet, subject to budget, demand, rebuild and non-negativity constraints. Two variants of the model are formulated: Zero Remaining Life Based Improvement (ZRLBI) and Continual Mileage Based Improvement (CMBI). ZRLBI model is based on the principle that a fleet is eligible for improvement when it reaches zero years of service life. In contrast, CMBI model allows a bus to be eligible for improvement after it reaches a threshold and not necessarily waiting for zero years of service life.

Model application is demonstrated using mixed fleet data from state of Michigan. Three types of buses (medium, medium-heavy, large-heavy) with varying service life, capital and O&M cost is considered in model application. First, one year allocation of funds is shown to demonstrate the model working principle and then a planning period of 11 years is considered to show a multi-year allocation of resources for the transit bus system in Michigan. Results show that CMBI performs better in obtaining a higher mean remaining life but involves in very large borrowing for one to two years within the planning period. ZRLBI also involves in borrowing but in lesser amount compared to CMBI. Both models incur deficit in the initial years of the planning period and have surplus in the later years ensuring that the total amount committed in improvement is not exceeded than the total budget.

Overall the results show efficiency of proposed transit fleet resource allocation model. The proposed model is compact and offers flexibility for an agency to choose between both short-term and long-term planning. This study clearly demonstrates the benefit of the new model in providing a better solution and comparing results between two variants of improvement options. Further, this model is expected to be a rational tool in deciding the federal/state fund allocation to competing transit agencies by maintaining SGR and to keep a healthy transit fleet. In future research the model can be structured to analyze annual budget constraint where agencies might not have the flexibility of borrowing funds from the future years. Also the model ignores unexpected need of improvement because of uncertain events and bus failures.

ACKNOWLEDGEMENTS

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NOTATIONS

The formulation notations are given below followed by their explanation:

Variables		Explanation
b_m	:	budget available for m^{th} planning year
$c_{k_\emptyset}^m$	:	cost of implementation of the improvement program *k* for fleet type ∅ on m^{th} year
$f_{i,j_\emptyset}^m$	:	Number of buses for an agency *i* with remaining life of *j* years fleet type ∅ on m^{th} planning year
$l_{k_\emptyset}$	:	additional year added to the life of the bus due to improvement program *k* for fleet type ∅, $l_{k_\emptyset} \in \{\alpha_\emptyset, \beta_\emptyset, \gamma_\emptyset, \theta_\emptyset\}$
$r_{ij_\emptyset}^m$	:	number of existing buses with remaining life of *j* years for fleet type ∅ for an agency *i* on m^{th} planning year
$x_{ij_\emptyset}^m$	:	number of buses which received remaining life of *j* years for fleet type ∅ for an agency *i* on m^{th} planning year due to the improvement program
$y_{ik_\emptyset}^m$	:	number of buses chosen for the improvement program *k* for fleet type ∅ adopted for an agency *i* on m^{th} planning year
$\delta_{i,(\alpha_\emptyset\beta_\emptyset)}^m$	:	number of buses already improved by $\alpha_\emptyset$, $\beta_\emptyset$ years for fleet type ∅ due to rehabilitation in the m^{th} planning year for agency *i*
$\delta_{i,(\gamma_\emptyset)}^m$	:	number of buses already improved by $\gamma_\emptyset$ years for fleet type ∅ due to remanufacture in the m^{th} planning year for agency *i*
$\omega_{ij_\emptyset}^m$	:	number of miles on a bus with remaining life of *j* years for fleet type ∅ for an agency *i* on m^{th} planning year
$\emptyset$	:	Fleet type
$\alpha_\emptyset$	:	REHAB-I improvement option for fleet type ∅
$\beta_\emptyset$	:	REHAB-II improvement option for fleet type ∅
$\gamma_\emptyset$	:	REBUILD improvement option for fleet type ∅
$\theta_\emptyset$	:	REPL improvement option for fleet type ∅
A	:	total number of agencies
B	:	total budget available for the project for all planning years
i	:	1, 2, …,A, the subscript for a transit agency
j	:	1, 2, …,Y, the subscript for remaining life
k	:	1,2,…., P the subscript used for improvement program
m	:	1, 2, …,N, the subscript used planning year
N	:	number of years in the planning period
P	:	number of improvement programs
REHAB-I	:	the first improvement program- rehabilitation of bus yielding $\alpha_\emptyset$ additional years
REHAB-II	:	the second improvement program- rehabilitation of bus yielding $\beta_\emptyset$additional years
REBUILD	:	the third improvement program- rehabilitation of bus yielding $\gamma_\emptyset$ additional years
REPLACE	:	the last improvement program-replacement of bus yielding $\theta_\emptyset$ additional years
TSMRL	:	Total System Mean Remaining Life , $TSMRL = \sum_m TWARL$
TMRL	:	Total Mean Remaining Life=$TMRL = \sum_i WARL_i$
MRL	:	Mean Remaining Life for agency *i*=$MRL_i = \frac{\sum_j f_{ij_\emptyset}^m r_{ij_\emptyset}^m}{\sum_j f_{ij_\emptyset}^m}$
Y	:	minimum service life of buses
Z_x	:	The objective function as minimization of NPV for the resource allocation in the planning period

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