MIDDLESEX
COMMUNITY COLLEGE

BEDFORD · MASSACHUSETTS · LOWELL

Strategies for Success
COURSE GUIDE

Intermediate **Algebra** (MAT 100)

MIDDLESEX COMMUNITY COLLEGE
Strategies for Success
achievement ★ persistence ★ retention ★ engagement

Title III Strengthening Institutions Project
Strategies for Success: Increasing Achievement, Persistence, Retention and Engagement

The Strategies for Success Title III initiative is a major, five-year project (2009-2013) funded by a two million dollar grant from the U.S. Department of Education. This initiative is intended to transform Middlesex Community College by improving the academic achievement, persistence, retention, and engagement of its students.

The project focuses on reformed curricula and comprehensive advising. *Reformed Curriculum* involves the design of developmental and college Gateway courses and learning communities embedded with Core Student Success Skills related to critical thinking, communication, collaboration, organization, and self-assessment. Overall, 45 courses will be impacted over the five years of the project. *Comprehensive Advising* involves the design of integrated advising services to include identification of academic and career goals, creation of realistic educational plans, and continuous tracking and intervention with an emphasis on the Core Student Success Skills. Comprehensive Advising Services will be specifically tailored to each program of study. Cross-division curriculum and advising design teams composed of faculty and staff are designing, piloting, and assessing the curriculum and advising initiatives.

The Title III grant provides resources to support faculty professional development related to designing and piloting new curriculum and advising students. The grant also supports the purchase of advising software programs and the hiring of a Pedagogical Instructional Designer, Learning Engagement Specialist, Advising Coordinator, and two academic advisors. The resources provided by the grant offer an exciting opportunity for the college community to work together to develop the strong programs and services that will increase student success.

Table of Contents

Introduction

Intermediate Algebra is a 3-credit course that introduces students to more advanced algebra skills, building upon the basic algebra skills learned in Algebra I and Algebra II. This course is typically taken by students who are working towards a STEM degree and who will be continuing with the pre-calculus/calculus sequence or by students who will be taking MAT 177, Statistics.

This course has been designed to incorporate the following Core Student Success Skills (CSSS) as a result of a Title III grant: Critical Thinking, Collaboration, Communication, Organization, and Self-Assessment. Development of these skills accounts for twenty-five percent of this course. The concept is to lead students to apply these skills as they learn the course content. The expectation is that by practicing these skills in this course, they will develop into more successful college students overall.

The following resources in this Guide are provided to help instructors promote active learning by students both in class and outside of class.

- **Guided Readings in Module Workbook**
  Organization; Collaboration; Communication; Critical thinking; Self-Assessment
  The Guided Readings promote active student learning by having students take notes, answer questions, and do problems as they study a text section. The Guided Readings help students organize their note taking and provide formative assessment. The students will be able to identify concepts that they find difficult and will allow them to focus their questions during the classes in which the concepts are discussed.

  The Guided Readings can be assigned to students to complete *before* a particular section is covered in class. They can also be used as a collaborative, in-class activity by having students work in pairs to complete the readings. It is certainly possible to assign some Guided Readings as homework to be done outside of class and to assign others as an in-class activity.

- **MyMathLab**
  Self-assessment; Organization
  MyMathLab (MML) is used extensively in this course. All homework assignments are completed online using MML. In addition, MML provides lecture videos for each text section, animations showing solutions of select problems, and an online interactive textbook with built-in links to video clips and animations. An overview of how to use MML in the course is included.

- **Technology Guides/Calculator Assignments**
  Critical thinking; Communication; Collaboration; Self-assessment
  The TI-83+ and TI-84+ graphing calculators are introduced in this course as a tool students can use to more easily connect algebraic and graphical concepts and to perform calculations that are burdensome to perform "by hand". For each major topic in the course, Technology Guides and corresponding Calculator Assignments have been developed to cover key concepts. These Guides and Assignments are designed so that they can be used independently by students or collaboratively by groups of 2 to 3 students.

A sample syllabus is also included in this Resource Guide.

RAMP-Up Class Format

It is important to note that the course has been modularized so that it can also be used by students who have completed through Module 12 in RAMP-Up Math and wish to take *Intermediate Algebra* in the RAMP-Up learning format. The Guided Readings, the MyMathLab homework assignments and technology resources, and the Technology Guides/Calculator Assignments can be used in both traditional classroom sections and in RAMP-Up sections.

Module Workbooks

Activity: Guided Readings in Module Workbooks

Learning Objectives:
1. Students will be able to read and understand the textbook.
2. Students will be able to explain mathematical definitions and concepts in their own words.
3. Students will be able to understand and explain the steps in the examples given in their textbook.
4. Students will be able to identify the concepts that they don't fully understand, make a note of them, and ask the appropriate questions during class.
5. Students will be able to organize their note taking as they read the textbook.
6. Students will be able to manage the time they spend studying the textbook.

Core Student Success Skills: Organization, Collaboration, Communication, Critical thinking, and Self-Assessment

Materials: Textbook (or e-text if in RAMP-Up format), Module Workbook, notebook, pencil, and TI-83+ or TI-84+ graphing calculator.

Context within the course: This activity extends across the entire course.

Procedure in a lecture section:
1. Students will open the Module Workbook to the guided reading for the current text section.

2. Students will open the textbook to the current text section

3. Students will follow the detailed instructions in the guided reading answering questions, writing notes and definitions, and doing problems as they progress through the text section.

4. Students will complete a section in the Module Workbook:
   a. before the section is covered in class **and/or**
   b. individually or in pairs as a collaborative activity during class

5. Students will formulate any questions they have as they study the section to ask during class.

6. Students will complete all of the text sections covered in the Module Workbook to prepare for an exam on the Module.

Procedure in a RAMP-Up section:

1. Students will open the Module Workbook to the guided reading for the current text section.

2. Students will open the e-text to the current text section in MyMathLab.

3. Students will follow the detailed instructions in the Module Workbook answering questions, writing notes and definitions, and doing problems as they progress through the e-text section.

4. Students will complete the sections in the Module Workbook during class and outside of class

5. Students will ask questions during class about concepts that are unclear.

6. Students will complete all of the e-text sections covered in the Module Workbook to prepare for an exam on the Module.

Notes for the Instructors:

1. Lecture sections
   a. Check Module Workbooks regularly to encourage students to stay up to date with their readings.
   b. You are encouraged to pair students to work collaboratively on the Module Workbooks as an in-class activity during some class periods.

2. RAMP-Up sections
   a. Students' primary activity during class will be completion of Module Workbooks.
   b. During class, "circulate" among students to see if they have questions as they are studying a text section and completing a Module Workbook.
   c. Check Module Workbooks regularly to encourage students to stay up to date with their studying.

Guided Readings in the Module Workbooks: Instructions for Students in Lecture Sections

Your course is divided into parts called *modules*, each one consisting of one to five textbook sections. Each module has a Module Workbook, which guides you through, and helps you study each textbook section in the Module. In a Workbook, you will be asked to answer questions, write definitions, explain steps used to solve problems, and write out problems for practice. Many of the questions, examples, and problems are straightforward; some require critical thinking, but all of the activities are designed so that you actively study the textbook in order to understand the concepts presented, instead of just skimming the section without lifting a pencil.

The Module Workbooks do contribute towards your course grade. Working on the guided readings in the Workbooks consistently can significantly improve your grade. On the other hand, not doing them can significantly lower your grade. The guided readings are designed to show you what is important in each textbook section and will help you learn the concepts presented in this course more efficiently. The Workbooks are not difficult to do, but doing them requires discipline and organization on your part. You will learn the math in the course more easily if you work for shorter amounts of time (45 minutes to 1 hour) each day than if you work on the course just once a week for a long period of time.

Many of the guided readings in the Module Workbooks will be completed outside of class. However, there will be times when working on the guided readings will be an in-class activity, either working individually or in pairs. So, be sure to bring your Module Workbook and textbook to each class.

To use the Module Workbooks most effectively for learning, you should do the following:

- Open your Module Workbook and textbook to the same section.
- Follow all of the detailed instructions in the Workbook and carefully study the steps used to solve each worked-out example.
- Try to answer all of the questions and do all of the problems in the Workbook.
- Mark those parts of the Workbook that you don't understand so you can return to them later.
- Once you reach the end of the Workbook, return to the earlier problems that you did not understand and try them again. If you still don't understand a concept, mark it in your Workbook so you can ask the question in class and/or can discuss it with one of your classmates.
- Each Module Workbook will be collected after the exam on the Module. A grade will be assigned to the Workbook.

INTERMEDIATE ALGEBRA
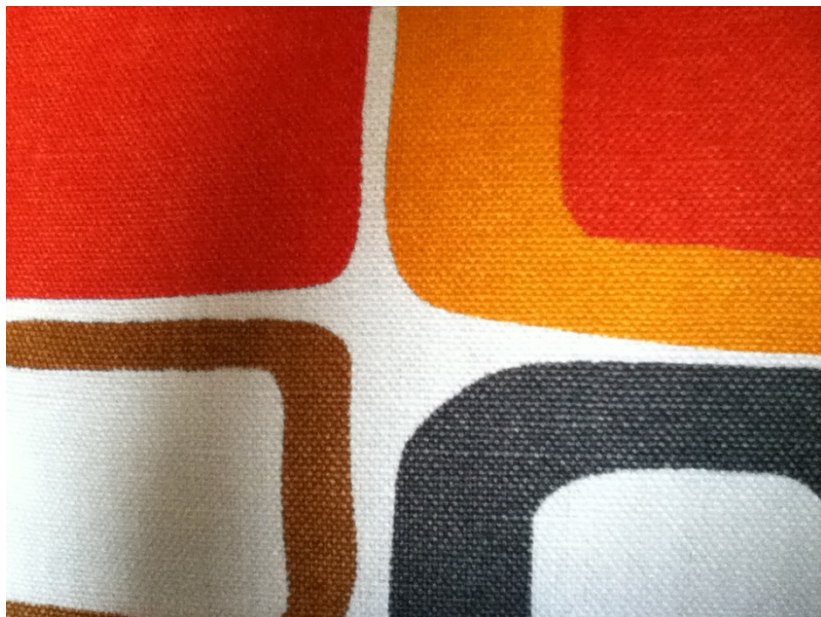
MODULE 1   *Introduction to Functions*

Middlesex Community College

Intermediate Algebra
Middlesex Community College
Module 1: Introduction to Functions

Intermediate Algebra through Applications,
2th edition, 2009, by Akst and Bragg

Name _____

Class Days and Time _____

Lab Day and Time _____

Instructor _____

Section 3.6 Introduction to Functions

Objective 1: To identify a function

Study pages 257 - 258. Then, fill in the missing words in the following sentences.

a) A **relation** is a _____.

b) A **function** is a relation in which no two ordered pairs have the same _____ coordinates.

c) The first coordinates of the pairs of a function refer to the _____ variable.

Study the top half of page 258. Then answer the following question.

If we write the ordered pairs of a function in the reverse order, how will the dependent and independent variables be affected?

_____
_____
_____
_____

Study the bottom half of page 258. Then complete the following exercise.

Determine whether each of the following relations represents a function. Explain.

a. {(-5, 1), (4, 3), (-2, 4), (4, 7), (-3, 6)}

_____
_____

b.

x	3	0	-1	4	2
y	1	5	4	5	5

_____
_____

Objective 2: To determine the domain and range of a function

Study the definitions on page 259. Then determine the **domain** and **range** of each of the following functions.

   a. The function defined by the set of ordered pairs: { (1,4), (3, 6), (6, -9), (0, -4)}

        Domain: { _____ }   Range: { _____ }

   b. The function defined by the following graph:
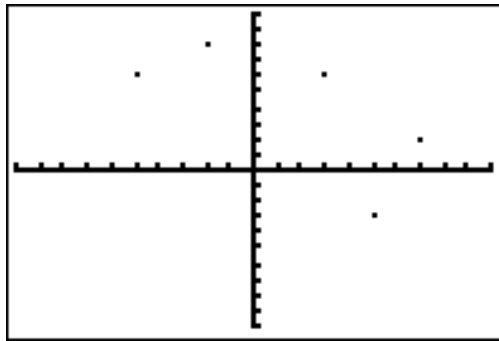
        Domain: { _____ }   Range: { _____ }

Objective 3: To evaluate a function written in function notation

Study page 260. Then complete the following sentences.

   a) The expression *f(x)* is read _____

   b) The ordered pair (*x, f(x)*) corresponds to a _____ with coordinates (x, y) on the graph of the function *f(x)*

   c) If a function *g(x)* is defined as *g(x)* = 3x -5, we will obtain the pair (4, __) and the pair (__ , 1)

   d) If a function *h(x)* is defined as *h(x)* = -5x + 2, then h(k) will be _____

Given the function *h(x)* = 7- 4x, evaluate each of the following.

   a. *h(-3)* _____   **b.** *h(0)* _____   **c.** *h(1.5)* _____

Objective 4: To identify various types of functions

Study page 261 - 263. Then complete the following sentences.

    a) The _____ function *f(x)* can be defined by *f(x)* = *mx* + *b*

    b) The function *f(x)* = *1/x* is undefined for the value *x* = _____

Given the function f(x) = 3x+ 2, complete the following table and plot the points.

x	-2	-1	0	1	2
f(x)					
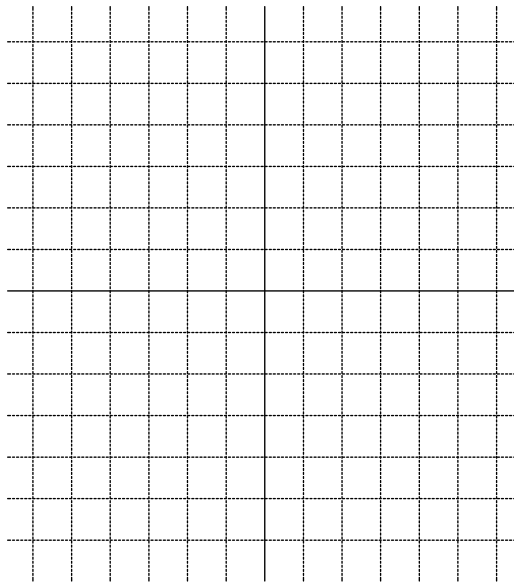
Objective 5: To recognize the graph of a function

Study page 264. Then explain the Vertical – Line Test in your own words.

_____

_____

_____

- For each of the following graphs, indicate whether it represents a function.

_____   _____   _____
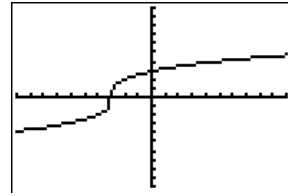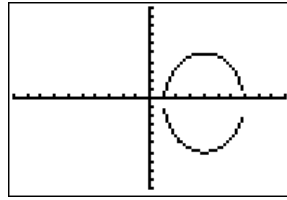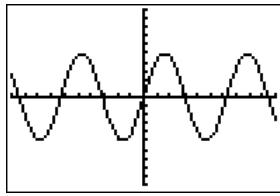
- The following graph shows the number of new AIDS cases in the US from 1980 to 1995 (source: www.nlreg.com/aids.htm)

New Cases of AIDS in The United States
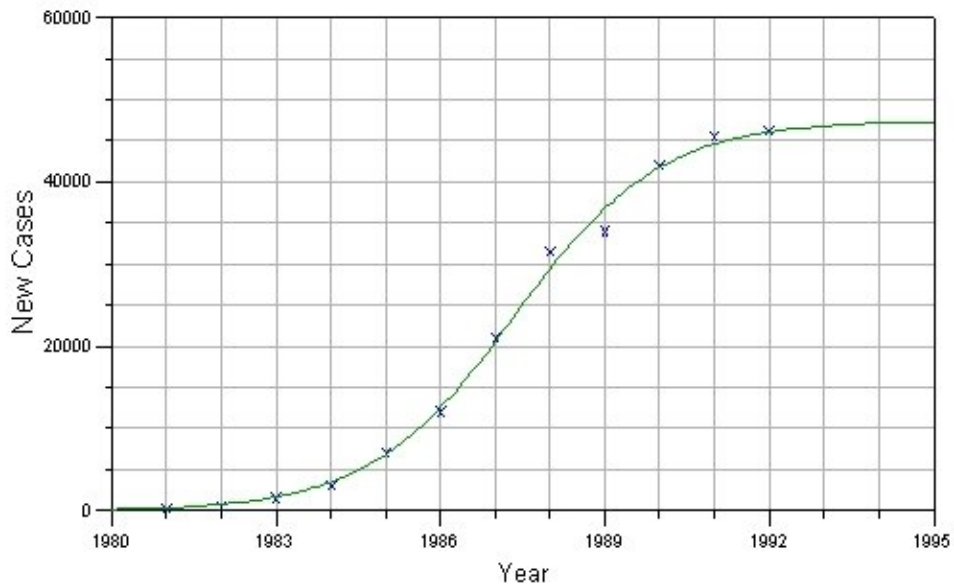
a. Does this graph represent a function?

  ....

b. If we use the name **f(x)** to represent the number of new cases of AIDS in the US as a function of the years, use the graph to find f(1991)

  ....

c. Use the graph to find the value of **x** for which f(x) = 12,500.

  ....

13

- Use the graph below to find the value of x for which *f(x) = g(x)*?
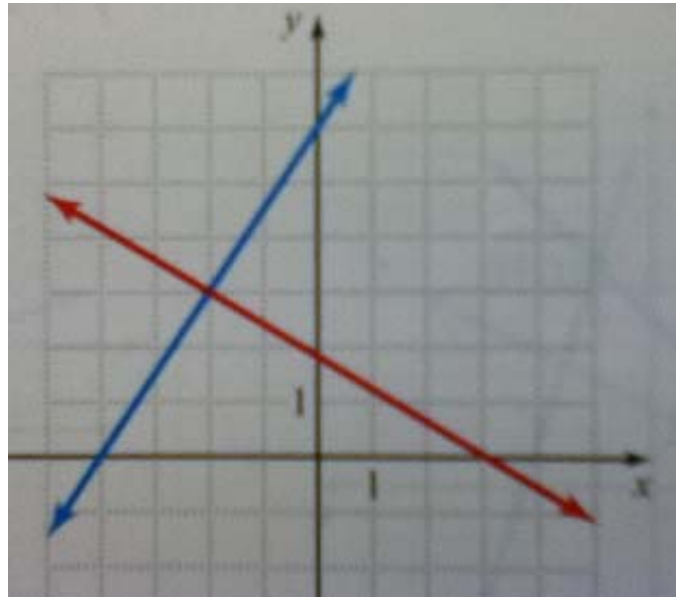
The value of x for which f(x) = g(x) is x = _____

TECHNOLOGY SKILLS. Read the *Technology Guides* 1 and 2. Then, complete the *Technology Skills Assignment 1*.

☺ A car is purchased new for $19,800 and depreciates in value by $1300 per year.

a) Use function notation to express the value *V(t)* of the car as a function of the years, *t*, after it is purchased.

_____

b) What is the meaning of *V(7)*? Find its value.

_____
_____

c) Use your graphing calculator to obtain a graph of the function *V(t)*.

Do the *Mathematically Speaking Animation Exercise* in the multimedia library. (The exercise is shown on page 267 of your textbook). Then, write your answers below.

1. _____   *2.* _____

3. _____   *4.* _____

5. _____

6. _____

MyMathLab   Do the Section 3.6 Homework problems in MyMathLab.

Do the <u>Mindstretcher</u> exercise on page 276 of your textbook. Then, write your answers below.

1.

a.   **Domain:** _____

Range: _____

b.   **Domain:** _____

Range: _____

c.   **Domain:** _____

Range: _____

d.   **Domain:** _____

Range: _____

2.

a. _____

b. _____

c.   **i.** ___   **ii.** ___   **iii.** ___   **iv.** ___

3.

_____

_____

_____

_____

STOP This is the last section in this module. Check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers

Date assignments completed:_____

INTERMEDIATE ALGEBRA

MODULE 1 | Introduction to Functions

Middlesex Community College

Intermediate Algebra
Middlesex Community College
Module 2: Solving Systems of Linear Equations

Intermediate Algebra through Applications,
2ᵗʰ edition, 2009, by Akst and Bragg

Name _____

Class Days and Time _____

Lab Day and Time _____
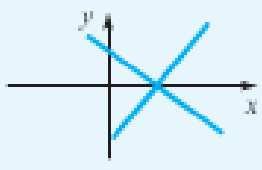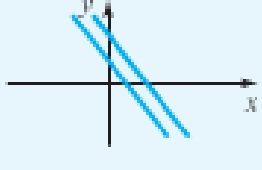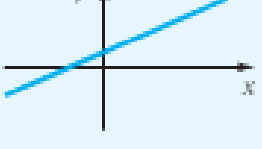
Instructor _____

Section 4.1 Solving Linear Equations by Graphing

Objective 1: Solving linear equations by graphing

Review pages 298-302, Examples 1 – 4 (try practice problems as needed).

Recall that you can visually inspect a graph of the lines to determine the solution(s). Review the following table from page 302 which summarizes these possible solutions.

Number of Solutions	Description of the System and Its Equations	Description of the System's Graphs	Possible Graphs
One solution	The system is **consistent** and the equations are **independent**.	The graphs intersect at exactly one point.	
No solution	The system is **inconsistent** and the equations are **independent**.	The graphs are parallel lines.	
Infinitely many solutions	The system is **consistent** and the equations are **dependent**.	The graphs are the same line.	

Objective 2: To solve applied problems involving systems of linear equations.

Study example 5 on page 303

Do Practice 5 from page 303 shown below:

Two video stores offer a special annual discount membership for video rentals. Uptown Video offers a discount membership for $10 plus $2.50 for each video rental. Midtown Video offers a discount membership for $20 plus $2 per rental.

    a) Express these relationships as a system of equations, where **C** represents the annual cost for **v** video rentals.

       _____

b) Use graphing to solve the system and interpret the results.
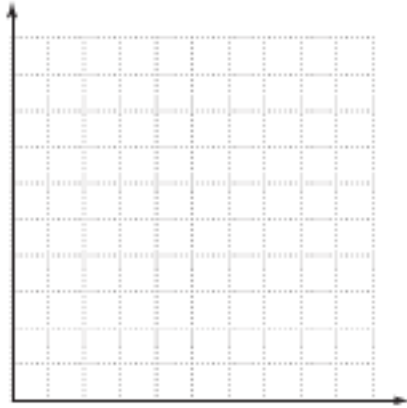
Study example 6 on page 304

Notice:

R(x)   = $ *   X

Revenue = (sale price per item) * (number of **items sold**)

C(x)   =   fixed $ +   unit $ *   X

Cost   = (fixed cost) + ((Cost per Unit) * (number of **units produced**))

Break-even point is when R(x) = C(x)

Do Practice 6 from page 304 shown below:

1) A musical production costs $80,000 plus $5900 per performance. A sold-out performance makes $7500.

a) What is the revenue function **R(x)** from **x** sold-out performances?

_____

b) What is the cost function **C(x)** for the production of **x** sold-out performances?

_____

c) Graph the system to determine the number of sold-out performances the production will need in order to break even.
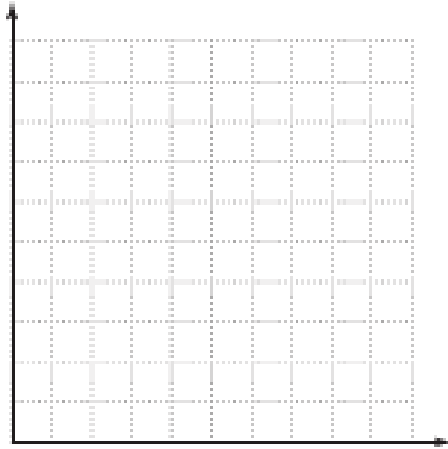
 TECHNOLOGY SKILLS. . Read the Technology Guide 3. Then complete the Technology Skills Assignment 3.

 A vote on a proposition to allocate funds for a new school was held. The number of votes in favor of allocating the funds was 1500 more than the number of votes against allocating the funds. The total number of votes cast for this proposition was 13,754.

    d) Express the given information as a system of equations.

_____
_____
_____

    .
    e) Use your graphing calculator to obtain a graph of this system. (Rewrite system in terms of y1=, y2= and use information from Technology Guide 3)

    f) Use your graphing calculator to obtain the intersection point for the graph you found in step (b).

    g) How many votes were cast for and against this proposition?

_____
_____

Do the *Mathematically Speaking* Exercise on page 306 of your textbook. Then, write your answers below.

1. _____   *2.* _____

3. _____   *4.* _____

5. _____
                                 6. _____

MyMathLab   Do the Section 4.1 Homework problems in MyMathLab.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers

Date assignments completed:_____   Move to the next section ⇨

Section 4.2 Solving Systems of Linear Equations Algebraically by Substitution or Elimination

Objective 1: Review solving a system of equations by substitution.

Study pages 316 - 319.

Review the procedure for solving systems of linear equations by substitution in the box on page 317.

Solve by the substitution method:

-2x + y = 4
 4x = -7 + 2y

Objective 2: Review solving a system of linear equations by elimination.

Study pages 319 – 321.

Review the procedure for solving system of linear equations by elimination in the box on page 321.

Solve by the elimination method.

7x + 2y = 10
-7x + y = -16

Objective 3: To solve applied problems involving systems of linear equations using substitution or elimination.

Study examples 9 – 11 on pages 322 - 324.

Try practice problem 9:
   An outlet store is having a sale on women's shoes. Some shoes are selling for $20 a pair and others for $25 a pair. At the end of the day, the total receipts for the sale of 65 pairs of shoes were $1500.

   a) Write the system of equations for this problem.

   b) Calculate how many pairs of $20 shoes were sold?

Try practice problem 10:
A Coast Guard cutter in New York Harbor is sent information that a boat, which left the harbor 3 hr earlier traveling east at 20 mph, has run aground. The Coast Guard cutter, traveling at 32 mph east, leaves the harbor to reach the boat.

a) Write a system of equations to represent this situation.

b) How long after the boat leaves the harbor will it take the Coast Guard cutter to reach the boat?

_____

Try practice problem 11:
A chef combines two sauces, a sauce that contains 70% tomato paste and a sauce that contains 40% tomato paste. How much of each sauce should be mixed to produce 5 L of sauce that is 60% tomato paste?

_____

TECHNOLOGY SKILLS. Read the Technology Guide 4 Then complete the Technology Skills Assignment 4.

A student takes out two loans for tuition totaling $8000. One loan charges 8% simple interest, the other charges 7.25% simple interest. After one year, the interest owed on the loans was $621.25

a) Write the system of equations needed to solve this problem.

_____
_____

b) Rewrite the equations as functions (Y1, Y2)

_____
_____

c) Use your graphing calculator to graph the system and find the solution.

_____

MyMathLab   Do the Section 4.2 Homework problems in MyMathLab.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Online homework in MyMathLab (with a grade of 80 or better).

Date assignments completed:_____   Move to the next section. ⇨

Section 4.3 Solving Systems of Linear Equations in Three Variables

Objective 1: To determine whether an ordered triple is a solution of a system of linear equations

Study pages 330 – 332. Then, fill in the missing words in the following sentences.

1. A **solution** of a system of linear equations in three variables is an _____ _____ of numbers that makes ALL three equations in the system true.

2. To determine if an ordered triple is a solution of a system _____ for each variable and _____ if all three equations are true.

Determine whether the ordered triple is a solution of the system.

$$x + 3y - 2Z = 9$$
$$-2x + 4z = 0$$
$$3x - 5y + z = -29$$

a. (6, 1, 3)   Yes   or   No

b. (-4, 3, -2)   Yes   or   No

c. (0, 0, -4)   Yes   or   No

Review the diagram from page 332 in the book to help you to visualize solutions to these systems.

Notice that although planes could intersect in a variety of ways, systems with three equations and three variables will still have ONE SOLUTION, NO SOLUTION or INFINITELY MANY SOLUTIONS.

Objective 2: To solve systems of linear equations in three variables.

Study pages 332 – 337. Then, fill in the missing words in the following sentences.

1. To use the elimination method with three linear equations, we eliminate one of the variables in order to get _____ in two variables.

2. Steps to use elimination method for three variables and three equations:

   a) The first step to eliminate one of the three variables is to choose _____ equations.
   b) Next step is to use _____ pair of equations and eliminate _____ variable.
   c) Now solve using the new equations containing only 2 variables. Solve for _____ variable.
   d) Now solve by substituting the variable you found in either onr or your new equations. Finally substitute 2 variables you found into any _____ equation.

The table on page 334 generalizes the process of using the Elimination method for three variables.

> **To Solve a System of Linear Equations in Three Variables by Elimination**
>
> - Write all equations in the general form $Ax + By + Cz = D$.
> - Choose a pair of equations and use them to eliminate a variable, getting an equation in two variables.
> - Next, use a different pair of equations to eliminate the *same* variable that was eliminated in the previous step, getting another equation in the same two variables.
> - Solve the system of equations formed by the equations found in the previous two steps, getting the values of two of the variables.
> - Substitute the values found in the previous step in any of the original equations to find the value of the third variable.
> - Check by substituting the values found in the previous two steps in all three equations of the original system.

Try practice 2 from page 332 shown here.

Solve by the elimination method.

$$x + y + z = 2$$
$$2x - y + 5z = -5$$
$$-x + 2y + 2z = 1$$

Try practice 4 from page 335 shown here.

Solve by the elimination method. Note since some equations have missing terms, one elimination step can be omitted.

$$3x + 4z = 5$$
$$2x - 5y = 8$$
$$2y + 3z = 2$$

Objective 3: To solve applied problems involving systems of linear equations in three variables.

Study pages 337 - 338   Example 7.

Try practice problem 7 shown here.

A multimedia student saved $3200 working part-time as a Web designer. She invested her savings in a growth fund, an income fund, and a money market fund with return rates of 10%, 7%, and 5%, respectively. To maximize her return, she placed twice as much money in the growth fund as in the money market fund. How should she invest the $3200 to get a return of $250 after one year?

MyMathLab Do the Section 4.3 Homework problems in MyMathLab.

MINDSTRETCHERS

Mathematical Reasoning

1. The method for solving a system of equations in three variables can be extended to four linear equations in four variables. Solve the following system:

$$w + x - y + z = -5$$
$$2w - 3x + 4y + z = 6$$
$$w - 2x - y + 3z = 0$$
$$x + 2y + 2z = 5$$

Writing

2. Write a word problem that can be solved using the following system of equations:

$$x + y + z = 100$$
$$5x + 3y + 2z = 365$$
$$x - y = 15$$

Critical Thinking

3. Find values of A, B, and C so that the following system has the solution $(4, 1, -3)$:

$$Ax + By + Cz = 13$$
$$Bx + Cy + Az = -18$$
$$Cx + Ay + Bz = 5$$

STOP This is the last section in this module. Check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers

Date assignments completed:_____

INTERMEDIATE ALGEBRA

MODULE 1	Introduction to Functions

Middlesex Community College

Intermediate Algebra
Middlesex Community College
Module 3: Solving and Graphing Linear Inequalities

Intermediate Algebra through Applications,
2th edition, 2009, by Akst and Bragg

Name _____

Class Days and Time _____

Lab Day and Time _____

Instructor _____

Section 2.3 Solving Linear Inequalities

Objective 1: To determine whether a given number is a solution of a given inequality

Study page 125. . Then, fill in the missing words in the following sentences.

*d) An **inequality** is* _____.

*e) A **solution of an inequality** is* _____ *that makes the*

    _____.

f) To _____ *is to find all of its solutions.*

Study the top half of page 126. Then determine whether each value is a solution to the inequality by substituting the value for **x**. If the resulting inequality is true, the value is a solution.
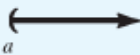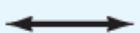
Is 2 a solution of	$3x - 7 > -2$?	Yes	No
Is -7 a solution of	$4x + 15 > 6x + 20$?	Yes	No
Is -1 a solution of	$5x + 1 < -4$?	Yes	No

Objective 2: Interval Notation, Inequality Notation and Graphs

There are three ways to express inequalities in one variable. Read page 126-127 and practice graphing the inequality and expressing it in interval notation.

INTERVALS, INEQUALITY NOTATION, AND GRAPHS

Interval Notation	Inequality Notation	Graph*	Meaning
(a, ∞)	$x > a$		All real numbers greater than a, excluding a
$[a, \infty)$	$x \geq a$		All real numbers greater than or equal to a
$(-\infty, b)$	$x < b$		All real numbers less than b, excluding b
$(-\infty, b]$	$x \leq b$		All real numbers less than or equal to b
$(-\infty, \infty)$			All real numbers

*Some textbooks use the graphical representations ⚬—▸ and •—▸ instead of ⊢—▸ and ⊢—▸ , respectively.

x < -1
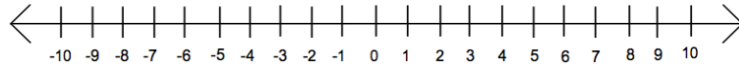
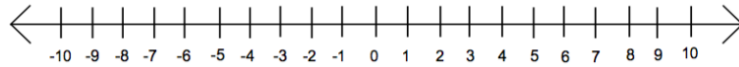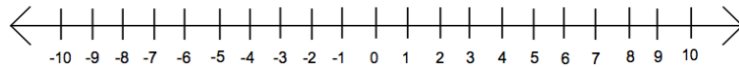**x > 5**
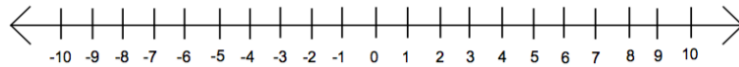
x ≥ 2

x ≤ -4

Objective 3: Solving Inequalities Using the Addition Property

Two inequalities are equivalent if they have the same solution. Explain why x < 3 and 3 > x are equivalent.

_____

_____

Addition Property of Inequalities

For any real numbers a, b, and c,

• if $a < b$, then $a + c < b + c$.

• if $a > b$, then $a + c > b + c$.

Similar statements hold for $\leq$ and $\geq$.

Show that you understand this property by expressing it in your own words.

_____

_____

This property also works for subtraction. Rewrite the property using the subtraction symbol.

_____

_____

_____

_____

Solve the following inequalities by isolating the variable to one side. Do this by adding or subtracting the same number from each side of the inequality. Then graph.

a. x + 6 ≥ 10
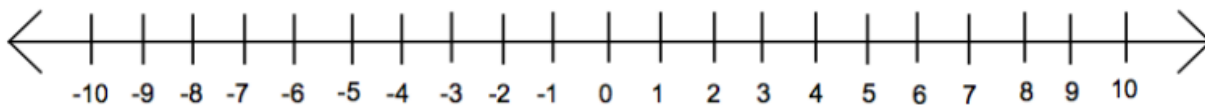
b. 7x + 3 > 6x - 1
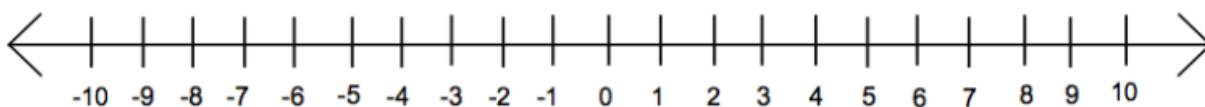
c. 2x − 9 < x - 6
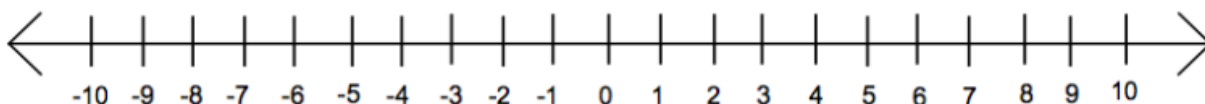
Objective 4: Solving inequalities Using the Multiplication Property

VERY IMPORTANT PROPERTY!!
If you multiply each side of an inequality by a negative number, you should reverse the direction of the inequality

Multiplication Property of Inequalities

For any real numbers a, b, and c,

• if a < b and c is positive, then ac < bc.

• if a < b and c is negative, then ac > bc.

Similar statements hold for >, ≤, and ≥.

Read pages 130 and 131 and do the four practice problems. Remember to reverse the inequality if you multiply by a negative number!

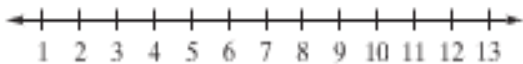List each step as you are solving!

PRACTICE 6

Solve and graph: $\dfrac{2n}{5} > 1$

Multiply by 5 and then Divide by 2

PRACTICE 7

Solve and graph: $-12x < -108$

Divide by -12 and reverse the inequality!!

PRACTICE 8

Solve and graph:
$4x - 5x - 10 \geq 10 + x$

Combine Like terms first

PRACTICE 9
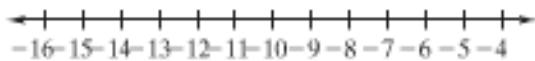
Solve and graph:
$2 - (4y + 5) \leq 2y + 1$

Use the Distributive property

Combine Like terms
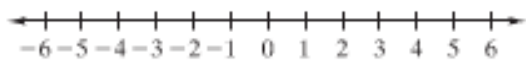
Objective 5: Word Problems

Many word problems can be solved using inequalities when the answer is not a single number but a number greater or less than a value.

Read Example 10 on Page 132 and fill in the values in the inequality.

Weight of Crates		Weight of Worker		Weight of Cart	At Most	Maximum Load Elevator Holds
_____	+	_____	+	_____	≤	4000

Since *x* represents the number of crates, *60x* will be the weight of the crates.

Solve the inequality and check your answer on page 133.

Do Problems 85 and 86 on page 137 using the same setup.

85. Let x be the number of times you park in a month

Cost at Daily Rate   <   Monthly Rate

_____ < _____

Solve the inequality

86. Let x be the number of times you ride the transit system in a month

Cost at Per Ride Rate   <   Monthly Rate

_____ < _____

Solve the inequality

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TECHNOLOGY SKILLS. There are no specific Technology Skills for this section.

Do the *Mathematically Speaking Animation Exercise* in the multimedia library. (The exercise is shown on page 134 of your textbook).

1. _____   2. _____

3. _____   4. _____

5. _____   6. _____

7. _____   8. _____

MyMathLab   Do the Section 2.3 Homework problems in MyMathLab.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).

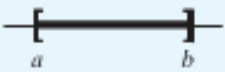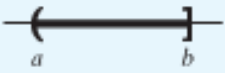Date assignments completed:_____   Move to the next section ⟹

Section 2.4 Solving Compound Inequalities

Two Inequalities that are joined by the word **OR** or the word **AND** form a
_____ inequality.

Review Intervals, Inequalities, and graphs by filling in the missing parts of this table.

INTERVALS, INEQUALITIES, AND GRAPHS

Interval Notation	Inequality Notation	Graph*	Meaning
	$a < x < b$		All real numbers between a and b, excluding a and b
$[a, b]$			All real numbers between a and b, including a and b
$[a, b)$	$a \le x < b$		All real numbers between a and b, including a and excluding b
$(a, b]$	$a < x \le b$		

*Some textbooks use the graphical representations —O———O— and •———• instead of, respectively,
(———) and [———] .

Objective 1: Solving Inequalities with the word AND

Read the definition on Page 141 and fill in the blanks.

The solution of a compound inequality joined by AND is the _____
of the solutions of the individual inequalities. In other words, a solution is any value of
the variable that makes _____ inequalities true.

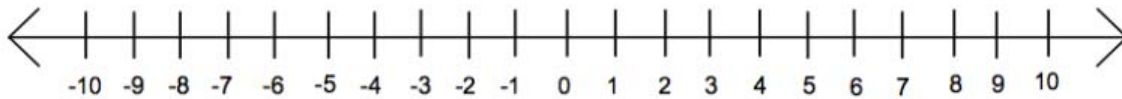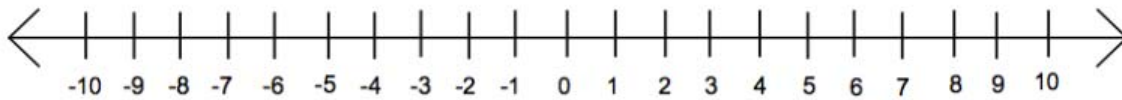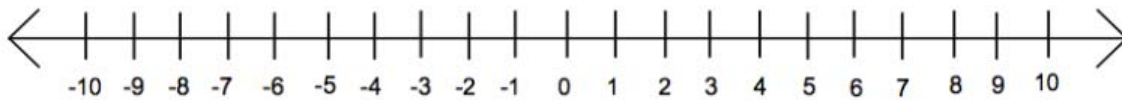Study Example 1 on page 141. Then use the same method to solve and graph the Practice 1 and 2.

PRACTICE 1

Solve and graph $8x \leq 32$ AND $5x + 3 \geq 3$

$$8x \leq 32 \qquad\qquad \text{AND} \qquad 5x + 3 \geq 3$$

Now graph each solution separately and then graph the intersection.

Write the solution in your own words and also in Interval Notation

_____

_____

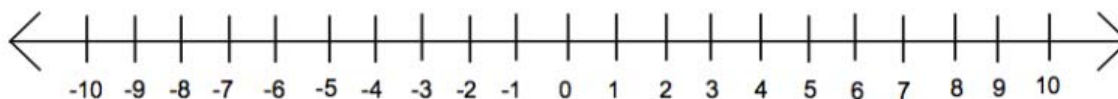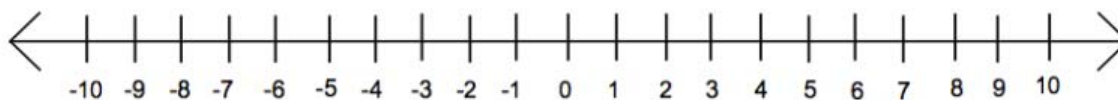_____

PRACTICE 2.

Solve and graph 2x > -10 AND 3x – 1 < 8

$$2x > -10 \qquad \text{AND} \qquad 3x - 1 < 8$$

Now graph each solution separately and then graph the intersection.

Write the solution in your own words and also in Interval Notation

_____

_____

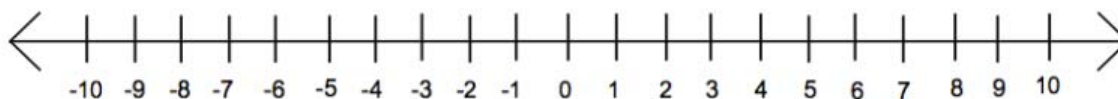_____

Look at Example 4 on page 143. The solution is that x ≤ .5 and x ≥ 2. Since there is no number that satisfies both of these conditions we say that there is no solution.

Alternative Method for expressing and solving a compound inequality

Sometimes instead of using the word AND the compound Inequality will be written with two inequality symbols.

For example: -5 < 6x + 1 < 7

This is still the intersection of the two Inequalities and the answer will be the numbers which are solutions to both.

You can solve this another way; by adding, subtracting, dividing, or multiplying the same value to all parts of the Inequality. Look at examples 2 and 3 on pages 142 – 143.

Solve the inequality and graph it on the number line.

Objective 2: Solving Inequalities with the word OR.

Read the definition on Page 144 and fill in the blanks.

The solution of a compound inequality joined by OR is the _____
of the solutions of the individual inequalities. In other words, a solution is any variable that makes _____ inequality true.

Read Example 5 on page 144. The solution to the compound inequality is x ≤ 2 or x ≥ 4.

Because this is an OR problem, we show both graphs on the number line even though they have no values in common. The lines are moving in the opposite direction of each other.

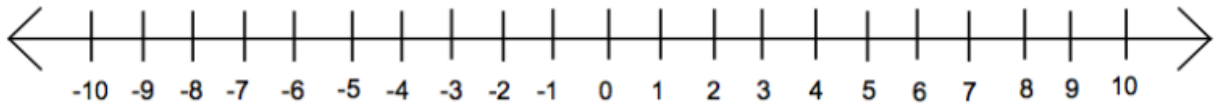The Interval notation for this answer is as follows: (-∞, 2] U [4,∞) The symbol U denotes the union of these two sets.

If this problem were x ≤ 2 AND x ≥ 4, there would be NO solution. Explain why.
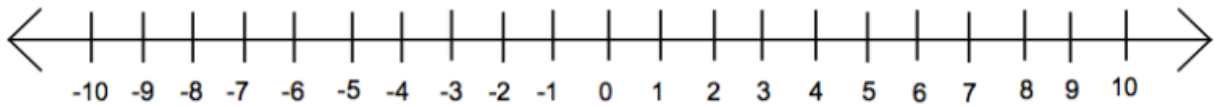
_____
_____

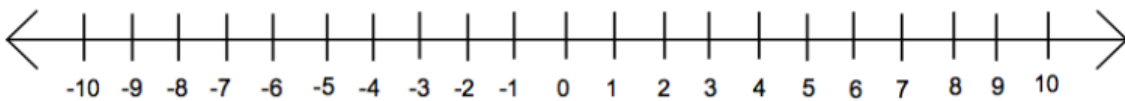Solve and graph the following inequalities.

5x - 7 ≥ 8 and 4x < 14

x < -4   or x > 3

7 ≤ 6x + 1 and -4x ≥28

Comparing results of inequalities using AND with those using OR.

Graph the following inequalities and explain the difference in your own words.
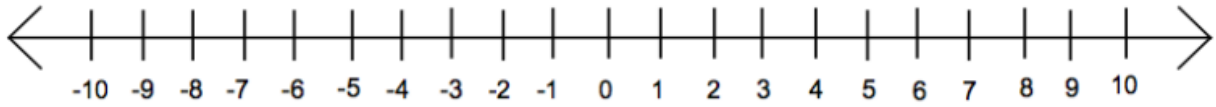
a) x ≤ 4 and x ≥ -3

x ≤ 4 or x ≥ -3

_____

_____

b) x ≥ 5 and x ≤ -2

x ≥ 5 or x ≤ -2
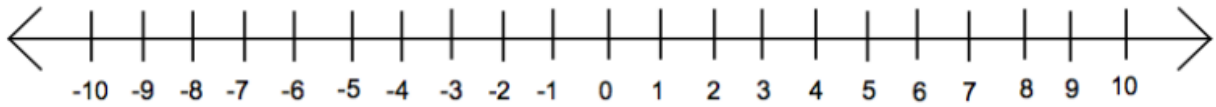
_____

_____

TECHNOLOGY SKILLS. There are no specific Technology Skills for this section.

Do the *Mathematically Speaking Animation Exercise* in the multimedia library. (The exercise is shown on page 146 of your textbook). Then, write your answers below.

1. _____   *2.* _____

3. _____   *4.* _____

MyMathLab   Do the Section 2.4 Homework problems in MyMathLab.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).

Date assignments completed:_____   Move to the next section

Section 3.5 Graphing Linear Inequalities

Objective 1: To graph a linear Inequality in two variables

Study the definitions on page 245 and complete/answer the following sentences/questions.

   a) A **linear inequality in two variables** can be written in the form

   _____.

   b) What are the four inequality symbols?_____

   c) What does it mean to be a solution of a linear Inequality ? _____

   _____

Is (2, 3) a solution to $y > 5 - 2x$?   Yes   No

Is (4, 1) a solution to $8y - 5x \geq -11$?   Yes   No

Is (-4,-6) a solution to $6y - 9x \leq 13$?   Yes   No

How to graph Linear Inequalities in two variables?

1 – Graph the Inequality as if it were an equation. Use a dotted line if the symbols are < or > and a solid line if the symbols are ≤ or ≥.
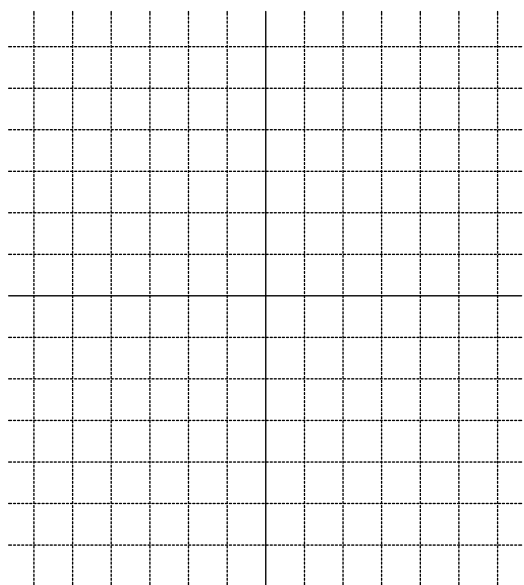
2 – Test the origin (0,0) to see if it makes the original Inequality true or false.
   If it is true, shade the area where the origin lies.
   If it is false, shade the area on the other side of the line.

   (Note that you can use any point to test the inequality, but (0,0) will be the easiest values to plug in.)

45
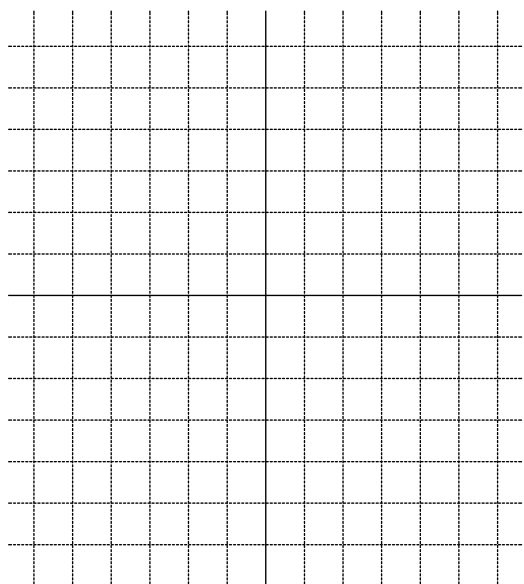
Study Example 2 on page 247 and then try Practice 2 on the same page.

$$y - x \geq 6$$

Did you use a dotted line or a solid line?   Why? _____

Did you test the origin in the original inequality? Was it true or false?_____

$$2x+4y < 8$$

Did you use a dotted line or a solid line?   Why? _____

Did you test the origin in the original inequality? Was it true or false?_____

Objective 2: Graphing Horizontal or Vertical Lines

How do you graph inequalities such as *x≥ 5?, x < -1?,   x ≤ 6?, or x ≥ -2?*

First graph the line as if it were an equality. Remember that this will be a vertical line. For example for x = 5, the line is vertical 5 units to the right of the origin. Here is where x always = 5, but y can be anything.
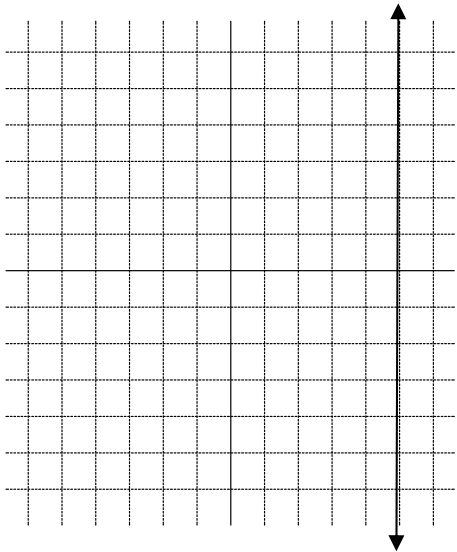
Now where are the values of x ≥ 5? Of course this is the area to the right of the line. So shade the area to the right.

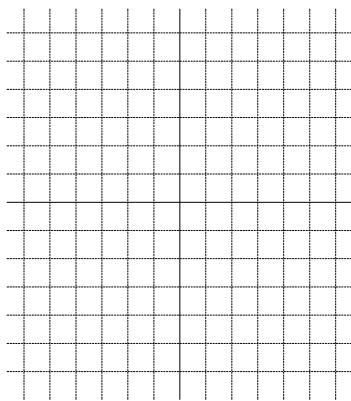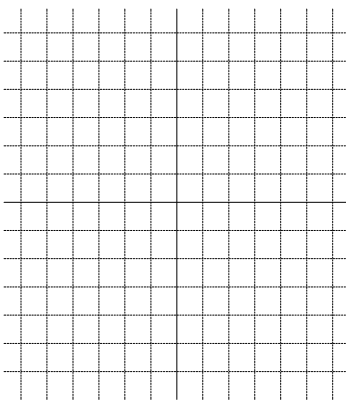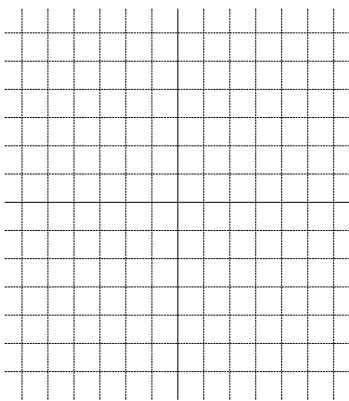Try the other three Inequalities remembering that they will all be vertical lines.

How do you graph inequalities such as *y ≥ 5?, y < -1?, y ≤ 6?, or y ≥ -2?*

First graph the line as if it were an equality. Remember that this will be a horizontal line. For example for y = 5, the line is horizontal 5 units to the top of the origin. Here is where y always = 5, but x can be anything.

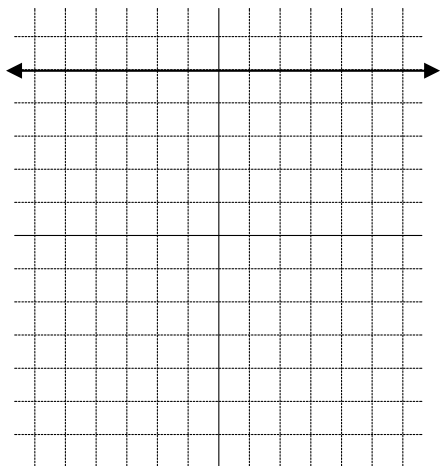Now where are the values of y ≥ 5? Of course this is the area above the line. So shade the area above the line.

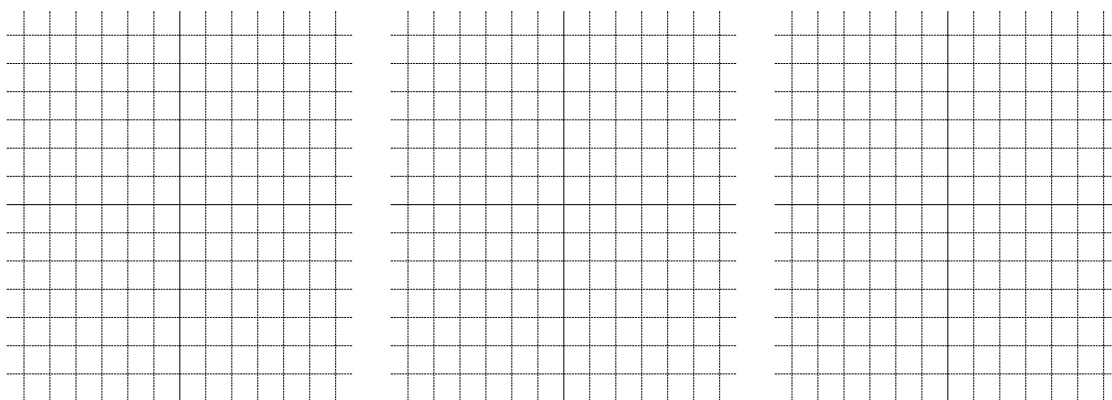Try the other three Inequalities remembering that they will all be areas above or below horizontal lines.

Objective 3: Graphing Linear Inequalities using MyMathLab

Go to Page 251 in the online Multimedia textbook and click on the Exercises button. This will take you to MyMathLab where you can learn to graph linear Inequalities. The method is slightly different.

   a. Graph the line as an equality using a dotted line for < and >, and using a solid line for ≤ and ≥.
   b. Test the origin to see which area to shade. If (0,0) makes the original inequality true, then shade that side. Otherwise shade the other side.
   c. Use the Paint Bucket icon to shade!!

MULTIMEDIA TEXTBOOK EXERCISE SET

Select an exercise to practice. When you have finished the exercise, either click Similar Exercise to try the same problem with new values, or return to this menu to select a different exercise. Your work on these exercises will not be tracked.

Click your browser's Back button to return to your textbook exercise set.

Exercise 19	Exercise 27	Exercise 33
Exercise 21	Exercise 29	Exercise 35
Exercise 23	Exercise 31	

This is what you will see when you click the button but use the text to actually get to the exercises. Do at least 4 of these exercises and you will see if you are graphing them correctly.

Objective 4: To solve applied problems involving a graph of a linear inequality

 Click on the video in your online text on page 248. Look for this symbol and watch the video to see how an application problem using inequalities is set up and solved. After you watch the video, click the You Try it Button to do one yourself.

Try some similar problems in your text. Problems 45 and 47 are similar to the one solved above. All of these problems have two variables which can vary, but together when added they cannot exceed a limit. This is a common decision we make all the time. If you have $20 for example, a movie costs $10, popcorn costs $5, and soda costs $2.50, you can buy various combinations of these items.
        1 movie, 1 popcorn, and 2 sodas
        1 movie and 2 popcorns
        1 movie and 4 sodas
        2 popcorn and 2 sodas
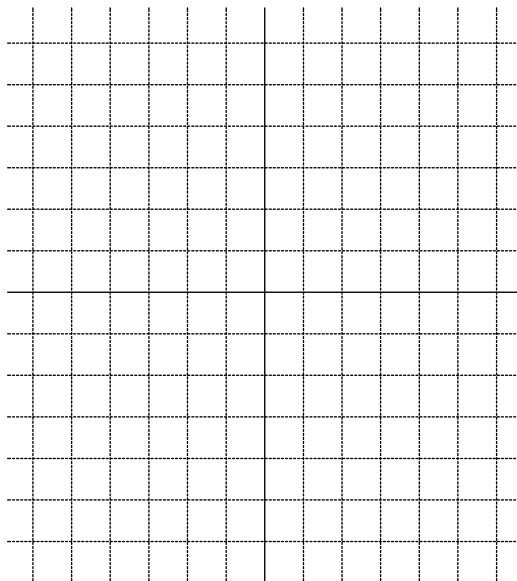All of these combinations are at most $20.

Use these three steps as outlined in the text.

     a. Express the inequality using the 2 variables and the maximum amount of the sum
     b. Graph the Inequality using the x and y intercept
     c. Test a point to see if it is in the shaded part or not.

45. (on page 254)
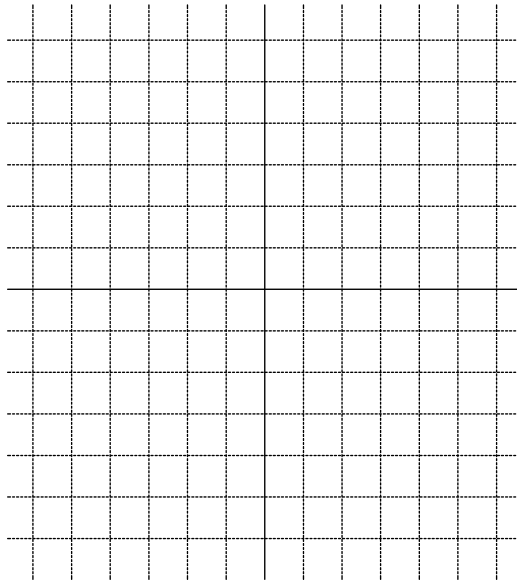
47 (on Page 254)

TECHNOLOGY SKILLS. Read the *Technology Guide* 5 on how to graph linear inequalities using the graphing calculator. Then, complete the *Technology skills Assignment 5*.

Use the graphing calculator to graph inequalities from the text on page 251.   See if your answers match the graphs in the answer key page A-15 in the back of the text.

21. $y < x - 4$

23. $y \geq \frac{1}{2}x + 3$

* Put these in slope y-intercept form first

31. $4x + 2y \geq -2$

33. $2x - 3y \leq -6$

29. $y \leq 4$

Note that you cannot graph x < 2 on the graphing calculator because this is not a function. The inequality has to start with y= to use the calculator.

Do the *Mathematically Speaking Animation Exercise* in the multimedia library. (The exercise is shown on page 250 of your textbook). Then, write your answers below.

1. _____   2. _____

3. _____   *4.* _____

5. _____   6. _____

MyMathLab   Do the Section 3.5 Homework problems in MyMathLab.
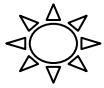
Do the <u>Mindstretcher</u> Writing exercise on page 256 of your textbook.

2.

_____
_____
_____
_____

_____
_____
_____
_____

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers

Date assignments completed:_____   Move to the next section   ⟹
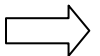
Section 4.5 Solving Systems of Linear Inequalities by Graphing

Objective 1: To solve a system of linear inequalities by graphing

How is this different from solving a system of linear equations?

   The solution to a system of linear equations is ONE POINT (or the empty set) where the solution to a system of linear Inequalities is a SHADED REGION.

How is this different from graphing a linear inequality?

   You actually have already learned all the skills you need for this section. You just have to graph BOTH lines on the same coordinate plane and learn to read the solution after you shade each region.

   There will be three ways to learn this: Graphing it by hand, graphing it using MyMathLab, and graphing it using the graphing calculator. Try to master all three methods.

   Graphing by hand. Go to the online textbook and click on the symbol Video on page 356. The video shows step by step how to graph a system of linear inequalities by hand.

    After watching the video, solve these systems:

-x – y < 2

y – 2x > 1
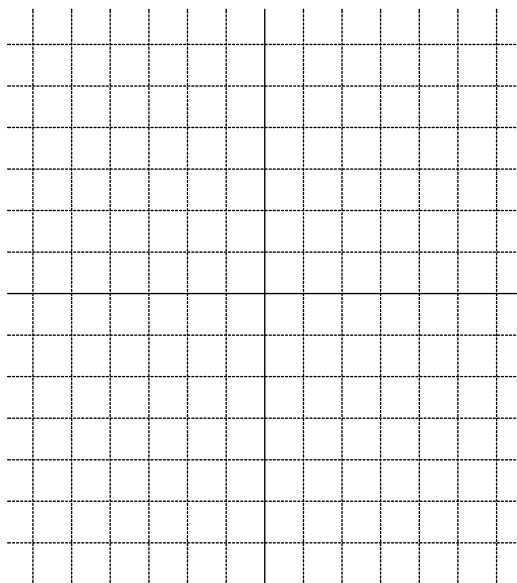
$y > 2x - 1$
$y < -x + 3$
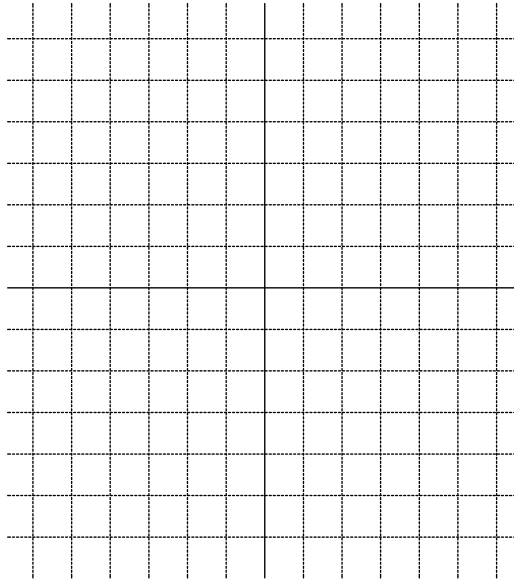
Check your answers on page 357.

Graphing a system of linear Inequalities in MyMathLab

    a. Graph the line as an equality using a dotted line for < and > , and using a solid line for ≤ and ≥.

    b. Graph the second line as an equality using a dotted line for < and > , and using a solid line for ≤ and ≥.

    c. This part is tricky now, because MyMathLab only wants to see the area shaded where BOTH are true. So work the areas out on scrap and then just shade the triangle area which is the solution.

    d. Test the origin to see which area to shade. If (0,0) makes the original Inequality true, then shade that side. Otherwise shade the other side.

    e. Use the Paint Bucket icon to shade!!

Find this icon on page 359 and click on the exercises to try a few in MyMathLab. These are just for practice. You can graph the line either by putting it in slope y-intercept form or by graphing the x and y intercepts.

Graphing a system of linear Inequalities with the Graphing Calculator. This will explained in the Technology Guides corresponding to this section.

TECHNOLOGY SKILLS. The easiest method to solve a system of linear Inequalities is to use your graphing calculator. Read the *Technology Guide* 6 on how to graph systems of linear inequalities using the graphing calculator. Then, complete the *Technology skills Assignment 6*.

Do the Section 4.5 Homework problems in MyMathLab.

This is the last section in this module. Check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Online homework in MyMathLab (with a grade of 80 or better).

Date assignments completed:_____

INTERMEDIATE ALGEBRA

MODULE 1	Introduction to Functions

Middlesex Community College

Intermediate Algebra
Middlesex Community College
Module 1: Introduction to Functions

Intermediate Algebra through Applications,
2th edition, 2009, by Akst and Bragg

Name _____

Class Days and Time _____

Lab Day and Time _____

Instructor _____

Section 5.6: Special Factoring

Important Note: You will study only Factoring the Sum and Difference of Cubes in this text section, beginning at the bottom of page 439.

Objective 1: Factoring the Sum and Difference of Cubes

At the bottom of page 439, begin studying "The Sum and Difference of Cubes" and continue until the middle of page 440.

Write the formula for Factoring the Sum of Two Perfect Cubes.

Write the formula for Factoring the Difference of Two Perfect Cubes.

Very Important! These formulas MUST be memorized!
For the other factoring techniques and formulas you have learned, it is possible to visualize how the factors multiply to make the original expression. For example, most students can see how the difference of squares, $x^2 - 25$ factors to $(x + 5)(x - 5)$. However, it is not possible for most students (and instructors!) to visualize the factorizations of the sum and difference of cubes. Therefore, the formulas for factoring the Sums and Differences of Cubes must be memorized.

Study Example 6, which begins on the bottom half of page 440.

Indicate whether each binomial is a sum or difference of cubes.

Binomial	Sum or Difference of squares? (Write "Yes" or "No") (If the binomial is not a sum or difference of cubes, explain why it is not)
$p^3 - q^3$	
$8 + x^6$	
$x^4 + 27y^9$	

On page 441, watch the Video that accompanies Example 7. Only watch the first two examples presented in the Video. Write the steps used to factor the first two expressions given in the Video:

Factor: $125x^3 + y^3$

Factor: $8p^{12} - q^9$

Factor the following expressions. Show your work.

Factor: $125 - y^3$

Write the factoring formula that will be used to factor this expression.

Formula: _____

Show how to factor the expression:

Factor: $27m^3n^6 + 1$

Write the factoring formula that will be used to factor this expression.

Formula: _____

Show how to factor the expression:

TECHNOLOGY SKILLS. Read the *Technology Guide* 7. Then, complete the *Technology Skills Assignment 7*.

MyMathLab Do the Section 5.6 Homework problems in MyMathLab.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment
☐ Online homework in MyMathLab (with a grade of 80 or better).
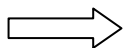
Solving Application Problems involving Polynomials

Now that we have a "tool box" of factoring techniques, we can solve a variety of application problems.

Read Example 4 on page 451 of the textbook. Then try Practice 4:

Suppose that you invested $16,000 in a high – risk fund that after 2 years was worth $25,000. Your broker used the equation
$$16{,}000(1 + r)^2 = 25{,}000$$
to find the average annual rate of return, **r**. What is that rate?

   a. Re - write the equation so the polynomial is set equal to 0

   b. Factor the polynomial and solve the equation.

Now try this problem:

The height of a ball thrown into the air is measured by the equation $h(t) = h_0 + v_0 t - 16t^2$
where

   - $h(t)$ is the height at time t measured in feet,
   - t is the time measured in seconds,
   - h_0 is the initial height of the ball when released by the pitcher,
   - v_0 is the initial velocity of the ball, and
   - $-16 \ feet/\sec^2$ is the acceleration due to gravity.

A small child threw a small ball *straight* up into the air. The child is 3 feet tall.

a. How many seconds did it take the ball to return and hit him on the top of his head if he initially released the ball $3\frac{1}{4}$ feet above the ground with a velocity of 50 feet/sec?

b. How high was the ball at $t = 0$ seconds?

c. How high was the ball after 2 second?

d. After how many seconds did the ball hit the child on the top of his head?

TECHNOLOGY SKILLS. Read the *Technology Guide* 8. Then, complete the *Technology Skills Assignment 8*.

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Technology Skills Assignment

Date assignments completed:_____   Move to the next section ⟹

Section 6.1: Multiplication and Division of Rational Expressions

Objective 1: To identify values for which a rational expression is undefined.

Study page 473.

*Write the definition of a **rational expression**.*

_____
_____
_____
_____

Study the top half of page 474.

*Write 3 examples of rational expressions. Do **not** use the examples given on page 474.*

1.	2.	3.

Study Example 1 on page 474; then watch the Video that accompanies Practice 1.
In the box below, write the steps used in the example given in the Video:

Find the values of the variable for which the rational expression $\dfrac{n+6}{n^2-8n+12}$ is undefined:

Find the values of the variable for which each rational expression is undefined:

$\dfrac{2m-7}{m+4}$
$\dfrac{5x}{x^2-36}$

Objective 2: To Simplify Rational Expressions

Study page 475.

Write the rule for simplifying rational expressions.

To Simplify a Rational Expression

Study Example 2 on page 476.

Write each of the following rational expressions in simplest form:

$\dfrac{-72m^2n}{-12m^3n^2}$	$\dfrac{5a^4}{25a^2-15a^3}$

Study Example 3 on page 476; then watch the Animation that accompanies Practice 3. In the box below, write steps used to simplify the first example given in the Animation:

Write the rational expression $\dfrac{4x^2 - 9}{6 - 4x}$ in lowest terms.

Simplify each of the following rational expressions, if possible. If an expression cannot be simplified, explain why it can't be simplified.

$\dfrac{5x+15}{9+3x}$	$\dfrac{5-x}{x-5}$	$\dfrac{x+1}{x-1}$

Read the top of page 477. Write the Tip given at the top of the page.

Tip _____

_____

_____

_____

Study Example 4 on page 477; then watch the Video that accompanies Example 4. In the box below, write the steps used to simplify the rational expression given in the Video:

Simplify the rational expression $\dfrac{x^2 - x - 42}{x^2 + 8x + 12}$.

Write each of the following rational expressions in simplest form:

$\dfrac{n^2 + 11n + 18}{n^2 + 7n + 10}$

$\dfrac{y - x}{y^2 - xy + y - x}$

Objective 3: To Multiply Rational Expressions

Study the top half of page 478.

Write the rule for multiplying rational expressions.

> **To Multiply Rational Expressions**
>
> _____
>
> _____
>
> _____
>
> _____
>
> _____

Study Example 6 on pages 478-479; then watch the Video that accompanies Practice 6 on page 478. In the box below, write the steps used to perform the multiplication given in the Video:

> Multiply and express the answer in lowest terms: $\dfrac{2n^2+11n+12}{n^2+3n-4} \cdot \dfrac{n^2-9n+8}{6n^2+5n-6}.$

Do the following multiplications and express each answer in lowest terms:

$$\frac{7x-14}{-20x^2} \cdot \frac{40x}{3x^2-6x}$$

$$\frac{a^2-a-2}{4a^2+16a-20} \cdot \frac{a^2+11a+24}{16-6a-a^2}$$

Objective 4: To Divide Rational Expressions

Study from the bottom half of page 479, starting with "Dividing Rational Expressions", to the top of page 480.

   Write the rule for dividing rational expressions.

To Divide Rational Expressions
_____
_____
_____
_____

Study Example 7 on pages 480-481; then watch the Video that accompanies Practice 7 on page 480. In the box below, write the steps used to perform the division given in the Video:

Divide. Express the answer in lowest terms: $\dfrac{a^2 - 2a}{3a^3 + 9a^2} \div \dfrac{a^2 - 4a + 4}{a^2 + a - 6}$.

Study Example 8 on page 481.

Perform the indicated operations:

$$\frac{2r^2 + 3rs + s^2}{r^2 - 49s^2} \div \frac{2r + s}{r - 7s} \cdot \frac{r + 7s}{2r - 3s}$$

Study the last paragraph on page 481 and Example 9 on page 482.

Do the following problem:

If $g(t) = \dfrac{t+3}{t-3}$ and $h(t) = \dfrac{t+3}{t}$ find:

a. $g(t) \cdot h(t)$	b. $g(t) \div h(t)$

Do the <u>Mathematically Speaking</u> Exercise on page 483. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

|			
rational number	rational expression	multiply	factoring
divide	denominator	no common factor	has an asymptote
equivalent	simplified rational expression	common factors	intersects the y-axis
multiplying		equivalent rational expression	
numerator	opposites		

1. $A(n)$ _____, $\dfrac{P}{Q}$, is an algebraic expression that can be written as the quotient of two polynomials, P and Q, where $Q \neq 0$.

2. When the _____ of a rational expression is equal to 0, the expression is undefined.

3. The graph of $y = \dfrac{4}{x-2}$ _____ at $x = 2$.

4. A rational expression is simplified when its numerator and denominator have _____ other than 1.

5. Multiplying or dividing the numerator and denominator of a rational expression by the same nonzero polynomial always results in a(n) _____.

6. A rational expression can be simplified by _____ the numerator and denominator and then dividing out any common factors.

7. If the terms in the numerator and denominator of a rational expression are _____, the expression simplifies to –1.

8. Helpful first steps in multiplying rational expressions are to factor the numerators and denominators and then to _____ them by all common factors.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

5. _____   6. _____

7. _____   8. _____

MyMathLab Do the Section 6.1 Homework problems in MyMathLab.
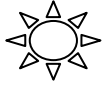
Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

MINDSTRETCHERS

Find a rational expression that is undefined at $x = 0$ and $x = -1$ and has a value of 3 at $x = 6$.
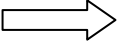
Show your work here:

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking

□ Online homework in MyMathLab (with a grade of 80 or better).
□ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

Section 6.2: Addition and Subtraction of Rational Expressions

Objective 1: Adding and Subtracting Rational Expressions with the Same Denominator

Read the opening paragraphs on the top half of page 489.

Write the steps for adding and subtracting rational expressions with the same denominator.

To Add or Subtract Rational Expressions with the Same Denominator
_____
_____
_____

Study Example 1 on the bottom half of page 489. Then do the following two problems.

Find the sum

$\dfrac{3y}{x+y} + \dfrac{2y}{x+y}$	$\dfrac{5x+1}{2x-3} + \dfrac{x-10}{2x-3}$

Study Example 2 at the top of page 490. Then, watch the Video that accompanies Practice 2. Write the steps used to do the subtraction given in the Video:

Subtract: $\dfrac{7x+2}{x+9} - \dfrac{8x+3}{x+9}$

Watch the Animation that accompanies Practice 2. Write the steps used to do the subtraction given in the Animation:

Subtract: $\dfrac{x^3+4x-3}{5x^2-10x} - \dfrac{4x+5}{5x^2-10x}$

Find the difference

$\dfrac{r-5s}{s+4r} - \dfrac{r+11s}{s+4r}$	$\dfrac{3n-1}{n^2+5} - \dfrac{2-n}{n^2+5}$

Objective 2: The Least Common Denominator (LCD) of Rational Expressions

Read the bottom half of page 490, beginning at "The Least Common Denominator of Rational Expressions" to the top of page 491.

Write the steps used to Find the LCD of Rational Expressions.

To Find the LCD of Rational Expressions

_____

_____

_____

_____

_____

Study Example 3 at on page 491. Then, watch the Animation that accompanies Example 3. Write the steps used to find the LCD of the rational expressions given in the Animation:

Find the LCD for: $\dfrac{1}{r^2 - 4r - 5}$, $\dfrac{1}{r^2 - r - 20}$, and $\dfrac{1}{r^2 - 10r + 25}$

Find the LCD for each of the following:

$\dfrac{5}{6x^2 y}$ and $\dfrac{3}{4xy^3}$	$\dfrac{m}{m - n}$ and $\dfrac{m}{m + n}$

$$\frac{2y}{x^2 - 2xy + y^2}, \frac{x}{x^2 - y^2}, \text{ and } \frac{x - y}{x^2 + 2xy + y^2}$$

Objective 3: Adding and Subtracting Rational Expressions with Different Denominators

Read paragraph on "Adding and Subtracting Rational Expressions with Different Denominators" at the bottom half of page 491.

Study Example 4 on page 492.

Watch the Video that accompanies *Example 3 back on page 491*. Write the steps used to do the subtraction given in the Video:

Find the LCD of these rational expressions. Then, write each expression in terms of the LCD: $\dfrac{3}{2t^2 + 12t}$ and $\dfrac{4}{3t^3 + 18t}$
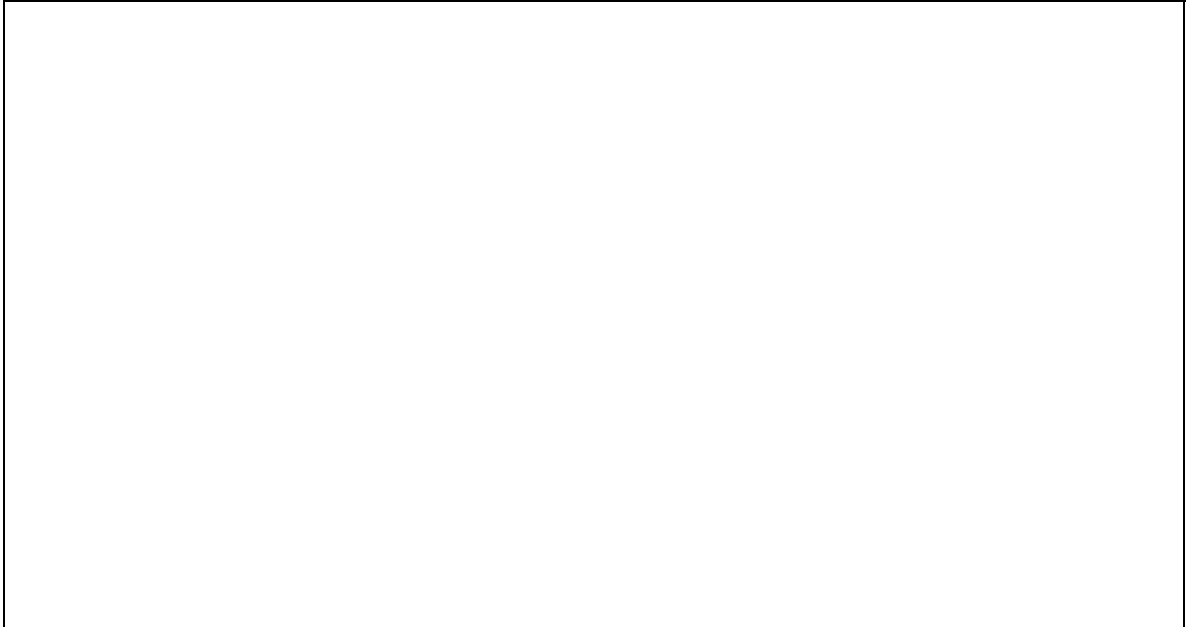
Write the following rational expressions in terms of their LCD.

$$\frac{3n}{6n+8} \text{ and } \frac{5}{n-4}$$

$$\frac{7}{3x-1} \text{ and } \frac{2x+3}{3x^2+11x-4}$$

Write the steps used to Add (or Subtract Rational Expressions with Different Denominators given at the top of page 493.

To Add (or Subtract) Rational Expressions with Different Denominators

_____

_____

_____

_____

_____

_____

Study Example 5 on page 493. Then, watch the Video that accompanies Example 5. Write the steps used to add the rational expressions given in the Video:

Perform the indicated operation: $\dfrac{1}{4-2r} + \dfrac{7}{3r^2 - 6r}$

Watch the Animation that accompanies Example 5. Write the steps used to add the rational expressions given in the Animation:

Add: $\dfrac{3}{x^2-4x} + \dfrac{2}{x^2-5x+4}$

Find the sum:

$$\frac{x}{x-1} + \frac{3}{x+1}$$

Find the sum:

$$\frac{t+5}{t^2-9t+20} + \frac{6}{t-4}$$

Study Example 6 on page 494. Then, watch the Animation that accompanies Example 6. Write the steps used to subtract the rational expressions given in the Animation:

Subtract and write the difference in lowest terms: $\dfrac{9}{x-2} - \dfrac{3}{x}$
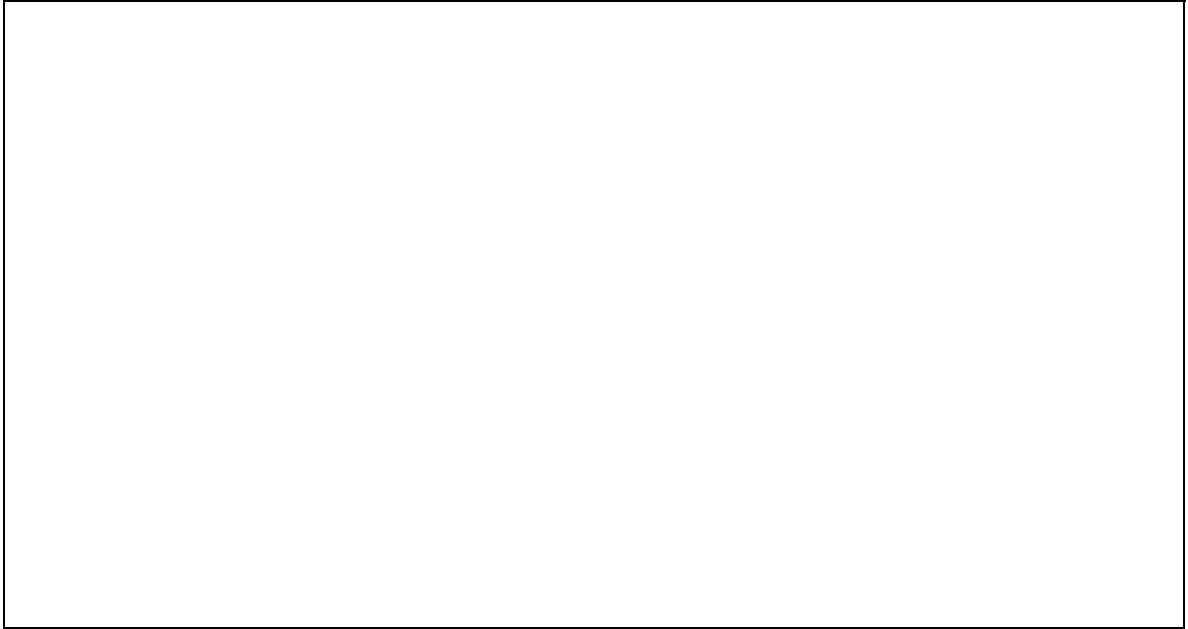
Find the difference:

$$\frac{2}{x-3} - \frac{x-1}{3-x}$$
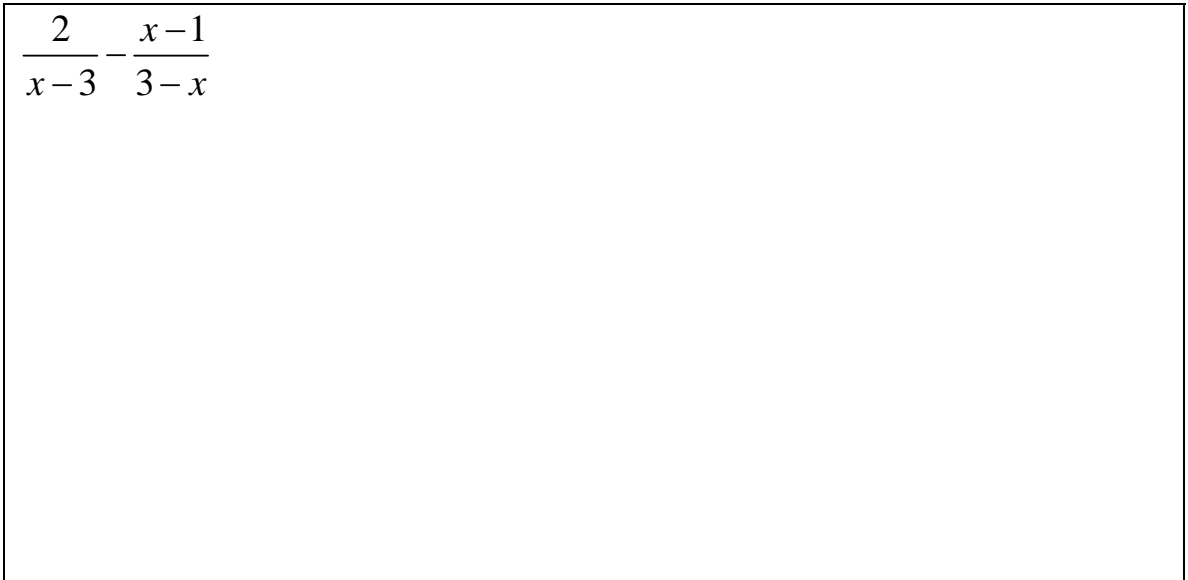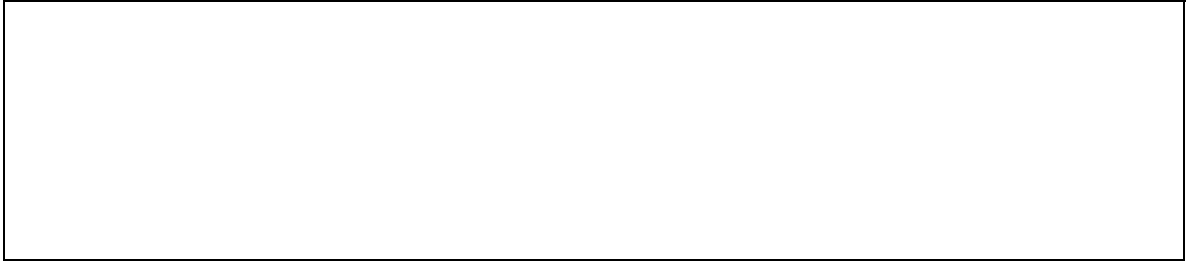
Find the difference:

$$\frac{7y+1}{y^2+4y+3} - \frac{7y-2}{y^2+2y+1}$$
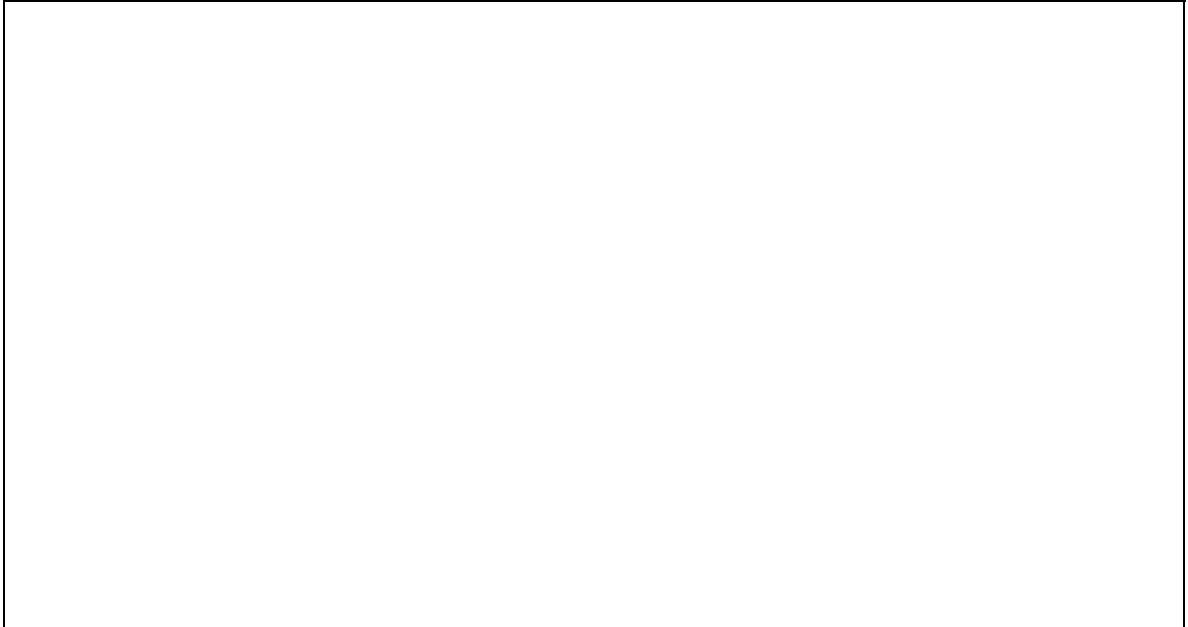
Study Example 7 beginning at the bottom of page 494 and continuing onto the top half of page 495.

Perform the indicated operations:

$$\frac{4}{2x^2 + x - 6} - \frac{2}{2x^2 - 3x} + \frac{1}{x^2 - 4}$$

Study Example 8 on page 495.

Do the following problems:

Given that $f(x) = \dfrac{2x}{x^2 - 25}$ and $g(x) = \dfrac{3}{x+5}$

a. Find $f(x) + g(x)$

b. Find $f(x) - g(x)$

On page 496, watch the Video that accompanies Practice 9. Write the solution to the problem given in the Video.

An airplane flies m miles with a wind whose speed is w mph. On the return flight, the airplane flies against the same wind. The expression $\dfrac{m}{s+w} + \dfrac{m}{s-w}$, where s is the speed of the airplane in still air, represents the total time in hours that it takes to make a round-trip flight.
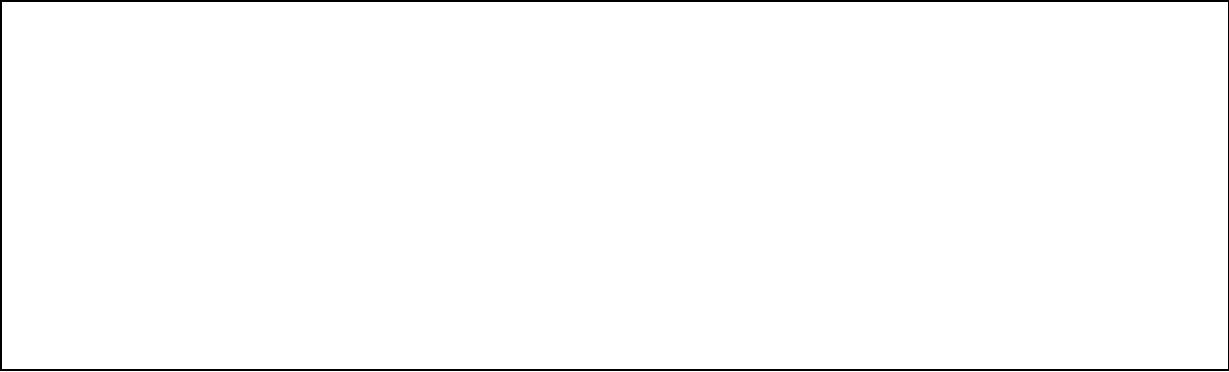
Study Example 9 on page 496. Write the Solution to parts a. and b. below.

Solution to Example 9,part **a.**

Solution to Example 9, part **b.**

Do the <u>Mathematically Speaking</u> Exercise on page 497. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

different denominators	least	greatest
find the product of the denominators	the same denominator	factor each denominator completely

1. To add (or subtract) rational expressions with the same denominator, add (or subtract) the numerators and keep _____.

2. To find the LCD of rational expressions, first _____.

3. To add (or subtract) rational expressions with _____, first find the LCD of the rational expressions.

4. The LCD of rational expressions is the product of the different factors in the denominators, where the power of each factor is the _____ number of times that it occurs in any single denominator.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

MyMathLab Do the Section 6.2 Homework problems in MyMathLab.

Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

MINDSTRETCHERS

Write the following expression as a single rational expression: $(x^2 - 9)^{-1} + 2(x - 3)^{-1}$

Show your work here:

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section

Section 6.3: Complex Rational Expressions
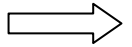
Objective 1: Simplifying Complex Rational Expressions Using the Division Method

Study the top part of page 503.

Write steps used to Simplify a Complex Rational Expression: The Division Method

**To Simplify a Complex Rational Expression: The Division Method**

_____

_____

_____

_____

_____

Study Example 1, which begins at the bottom of page 503 and continues to the top of page 504.

Simplify:

$$\dfrac{\dfrac{n-1}{n}}{\dfrac{n-1}{n^2}}$$

Study Example 2 on page 504.

Simplify:

$$\dfrac{\dfrac{1}{y}+\dfrac{3}{y^2}}{2y}$$

Study Example 3, which begins at the bottom of page 504 and continues on page 505.

Simplify:

$$\frac{\dfrac{4}{m}+\dfrac{4}{n}}{\dfrac{1}{m^2}-\dfrac{1}{n^2}}$$

Read the last paragraph and the box at the bottom of page 505.

Write steps used to Simplify a Complex Rational Expression: The LCD Method

To Simplify a Complex Rational Expression: The LCD Method

_____

_____

_____

_____

_____

Study Example 4 on page 506.

Simplify:

$$\frac{\dfrac{y^2-9}{5y}}{\dfrac{y+3}{6y^2}}$$

Study Example 5 on page 506. Then, watch the animation that accompanies Practice 5. Write the steps used to simplify the complex rational expression given in the Animation:

Simplify the complex rational expression: $\dfrac{\dfrac{1}{y}+\dfrac{2}{y+2}}{\dfrac{4}{y}-\dfrac{3}{y+2}}$
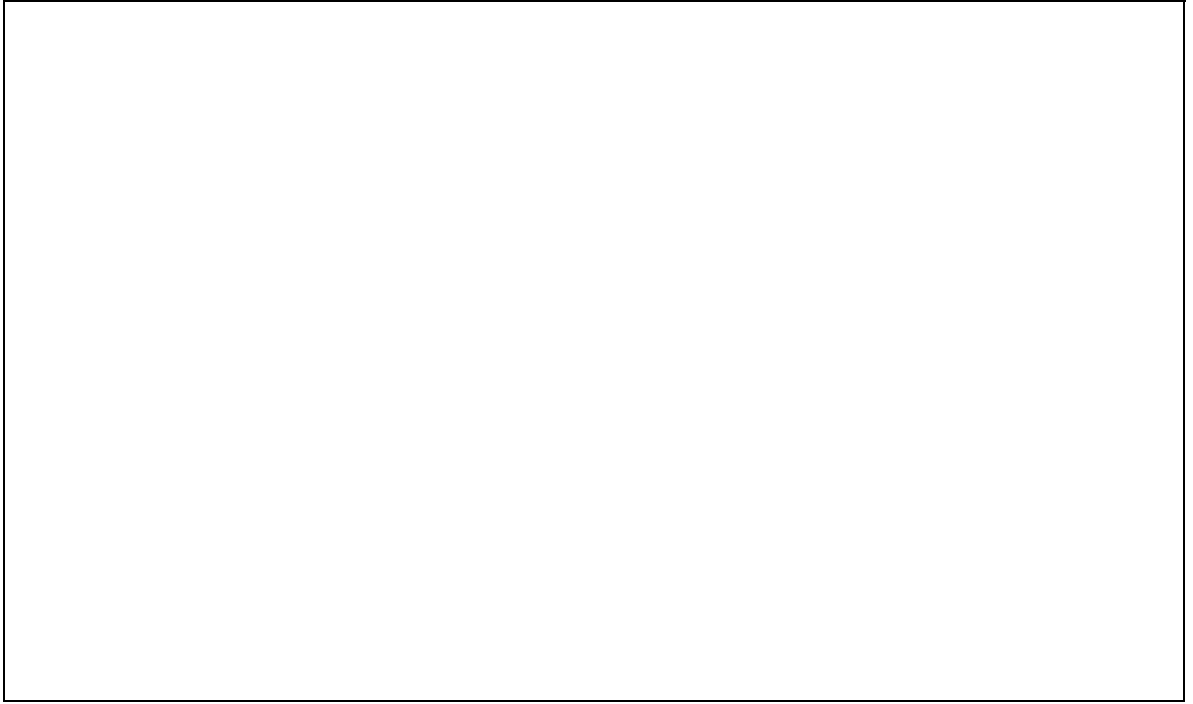
Simplify:

$$\frac{3 - \dfrac{2}{n}}{9 - \dfrac{4}{n^2}}$$

Study Example 6 on page 507.

Simplify:

$$\frac{\dfrac{1}{p^3} + \dfrac{1}{q^3}}{\dfrac{1}{p} + \dfrac{1}{q}}$$

Do the <u>Mathematically Speaking</u> Exercise on page 508. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

reciprocal	LCD method
algebraic	have no common factors other than 1
are factored	division method
rational	

1. A rational expression whose numerator, denominator, or both contain one or more _____ expressions is called a complex rational expression or a complex algebraic fraction.

2. A complex rational expression is simplified when it is in the form $\dfrac{P}{Q}$, where P and Q are polynomials that _____.

3. To simplify a complex rational expression using the _____, first write both the numerator and the denominator as single rational expressions in simplified form.

4. To simplify a complex rational expression using the _____, first find the LCD of all rational expressions within the complex rational expression.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

MyMathLab Do the Section 6.3 Homework problems in MyMathLab.

☼ Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

Investigation

1. Consider the rational expression $\frac{1}{x}$.

   a. Complete the table.

x	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	5	10	100	1000
$\frac{1}{x}$											

Calculation space for part **a.**, if needed:

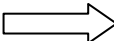   b. What happens to the value of $\frac{1}{x}$ as x gets larger? What happens as x gets smaller?

Write your explanation:

97

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook

☐ Mathematically Speaking

☐ Online homework in MyMathLab (with a grade of 80 or better).

☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

Section 6.4: Solving Rational Equations

Objective 1: Solving Rational Equations

Study from the top of page 512 through the top of page 513.

Write steps used to Solve a Rational Equation

To Solve a Rational Equation

_____

_____

_____

_____

_____

Study Example 1 on page 513.

Solve and check:

$$\frac{1}{2y} - \frac{y+1}{y} = 2$$

Read the paragraph at the top of page 514.

Fill in the blanks to make the following sentences true.

When multiplying each side of a rational equation by a _____ expression, the resulting equation may have a solution that does not satisfy the _____ equation. If such a number makes a denominator in the original equation become _____, then the rational expression with that denominator of 0 is _____. Such numbers, called _____ _____, are not solutions of the _____ equation. So, in solving a rational equation, it is particularly important to _____ for _____ _____.

Study Example 2 on page 514.

Solve and check:

$$\frac{y}{y-3} = \frac{3}{y-3} - 1$$

Watch the animation that accompanies Example 3. Write the steps used to solve the rational Equation given in the Animation:

Solve: $\dfrac{5}{x-3} = \dfrac{x}{x-2} + \dfrac{x}{x^2-5x+6}$

Study Example 3 on page 515.

Solve: $\dfrac{m}{m+3} - \dfrac{2}{3-m} = \dfrac{2m+6}{m^2-9}$

Study Example 4 on page 516.

Solve: $\dfrac{n-3}{3} = \dfrac{3}{n+5}$

Read the bottom portion of page 516 starting with the paragraph following Example 4.

Study Example 5 at the top of page 517.

Solve using the cross-product method:

$\dfrac{5}{t-2} = \dfrac{3}{2t+3}$

Study Example 6 on page 517.

Given that $g(x) = \dfrac{x(x+2)}{x+6}$, find all values of x for which $g(x) = 1$.

Study Example 7 on page 517 and the top of page 518.

Do the following problem:

A certain iceberg in Antarctica moves about 2 feet per year. At this rate, how long will the iceberg take to move 1½ feet?

Study Example 8 on page 518.

Let's use diagrams to see why the expressions for rate are written the way they are in Example 8.

Downriver
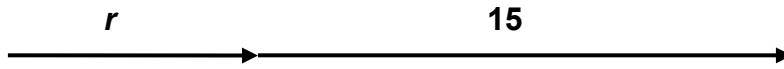
Speed of current is unknown, so we will call it *r*.

Speed of boat in still water is **15** mph

For the downriver trip, the current pushes *with* the boat so it *adds to* the boat's speed. Therefore, the downriver rate will be **15 + *r***

Upriver

Speed of current is unknown, so we will call it *r*.

Speed of boat in still water is **15** mph

For the upriver trip, the current pushes *against* the boat so it *subtracts from* the boat's speed. Therefore, the downriver rate will be **15 – *r***

Do the following problem:

A cargo ship travels 7 kilometers per hour in still water. It travels 45 km up the river and the same distance down the river in a total time of 14 hours. Find the speed of the river's current. *Hint:* Let *r* = the speed of the current.

To organize the information given in the problem, complete the following table. Recall that $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

	d (distance)	*r* (rate or speed)	*t* (time) $\dfrac{\text{distance}}{\text{rate}}$
Downriver			
Upriver			

Read the sentence at the top of page 519. Then study Example 9.

Do the following problem:

A local bus travels 9 mph more slowly than the express bus. The express bus travels 90 miles in the same time it takes the local bus to travel 75 miles. Find the speed of the local bus. *Hint:* Let r = speed of the express bus

To organize the information given in the problem, complete the following table. Recall that $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

	d (distance)	r (rate or speed)	t (time) $\dfrac{\text{distance}}{\text{rate}}$
Express Bus			
Local Bus			

Read the paragraph at the bottom of page 519. Then watch the Animation that accompanies Example 10 on page 520. *Note:* This Animation is lengthy, but gives a thorough explanation of how to solve work problems. You may want to view it twice in order to understand all of the concepts presented in this Animation.

Study Example 10 on page 220.

Do the following problem:

One printing press can print an order of college admission brochures in 12 hours. Another press can complete the same job in 18 hours. How long would it take if the college used both presses?

To help you start this problem, complete the following table.

	Rate of work	• Time worked	= Part of the task completed
First printing press			
Second printing press			

Read the sentence at the top of page 521. Then read Example 11. The Solution to Example 11, part **a.** does not tell what is occurring in each step when solving the equation for *x*. The Solution to part **a.** is re-done below with a description of what is occurring in each step.

Solution to Example 11, part **a.**, on page 521:

The directions state to "Solve equation $C = \dfrac{20x + 3000}{x}$ for *x* in terms of *C*." This

means we need to rewrite the equation so *x* is isolated.

	Description
$C = \dfrac{20x + 3000}{x}$	• Original equation
$x \cdot C = \dfrac{20x + 3000}{\cancel{x}} \cdot \cancel{x}$	• Multiply both sides of the equation by the LCD, *x*, to cancel the denominator.
$Cx = 20x + 3000$	• Equation after cancelling

$Cx - 20x = 20x - 20x + 3000$	• Subtract 20x from both sides of the equation so the two terms that contain x are on the same side.
$Cx - 20x = 3000$	• Equation with Cx – 20x on the same side.
$x(C - 20) = 3000$	• Factor out the common factor of x on the left side of the equation.
$\dfrac{x(C - 20)}{(C - 20)} = \dfrac{3000}{(C - 20)}$	• Isolate the x by dividing both sides of the equation by (C – 20)
$\dfrac{x\,\cancel{(C-20)}}{\cancel{(C-20)}} = \dfrac{3000}{(C - 20)}$	• Cancel
$x = \dfrac{3000}{(C - 20)}$	• x is isolated

Do the following problem:

Recall from geometry that a trapezoid is a four-sided geometric figure, two of whose sides (the bases) are parallel. Suppose we draw the two diagonals of a trapezoid and also the line parallel to the bases that passes through the point of intersection of the diagonals (see figure below).
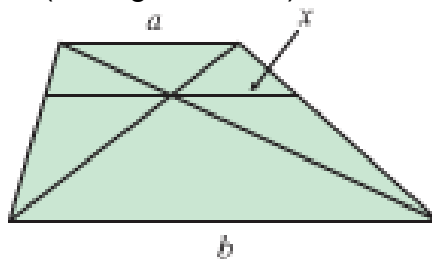
It can be shown that the three lengths x, a, and b are related by the following equation:

$$\frac{2}{x} = \frac{1}{a} + \frac{1}{b}$$

a. Solve the equation for x in terms of a and b.

b. Using the equation found in part (**a**) above, determine the value of x if the lengths of the two bases are 6 in. and 4 in.

💬 Do the <u>Mathematically Speaking</u> Exercise on page 522. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

GCF	rate	equal
division	proportion	extraneous
cross-product	LCD	ratio

1. To solve a rational equation, first find the _____ of all the rational expressions.

2. In solving a rational equation, always check for _____ solutions, which make a denominator in the original equation equal to 0.

3. A(n) _____ is a comparison of two numbers or two quantities expressed as a quotient.

4. A(n) _____ is a statement that two ratios are equal.

5. If two rational expressions are equal to each other, the resulting equation can be solved by using either the LCD method or the _____ method.

6. A(n) _____ is the ratio of unlike quantities, that is, quantities with different units.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

5. _____   6. _____

MyMathLab Do the Section 6.4 Homework problems in MyMathLab.

Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class

MINDSTRETCHERS

Solve the following system of equations:

$$\frac{4}{x} + \frac{3}{y} = 2$$

$$\frac{5}{x} + \frac{2}{y} = -1$$
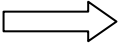
Show your work:

Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

INTERMEDIATE ALGEBRA

| MODULE 1 | Introduction to Functions |

Middlesex Community College

Intermediate Algebra
Middlesex Community College
Module 5: Radical Expressions and Equations

Intermediate Algebra through Applications,
2th edition, 2009, by Akst and Bragg

Name _____

Class Days and Time _____

Lab Day and Time _____

Instructor _____

Section 7.1: Radical Expressions and Rational Exponents

Objective 1: To evaluate radical expressions

Study page 551 to the top of page 552.

Fill in the boxes to label the parts that make up the square root.

The square root of *a*

$$\sqrt{a}$$
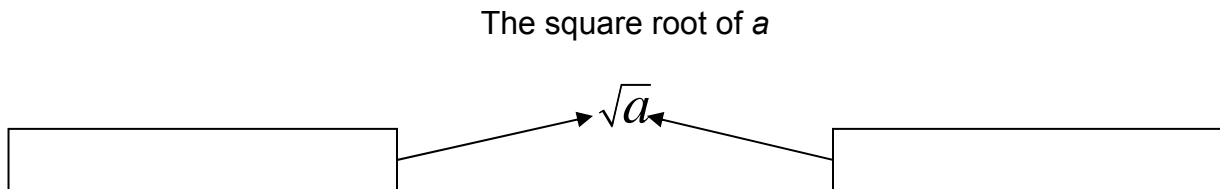
Study Example 1 on page 552.

Evaluate the following square roots, if possible.

$\sqrt{25}$	$\sqrt{\dfrac{9}{100}}$	$-4\sqrt{81}$	$\sqrt{-1}$

Study the paragraph in the middle of page 552; then study Example 2.

Use a calculator to find the value of $\sqrt{6}$ rounded to the nearest thousandth.
Answer: _____

Study the paragraph at the bottom of page 552 and continue onto page 553. Stop when you get to Example 3.

Write the Squaring a Square Root property given at the top of page 553.

Squaring a Square Root

Write the Taking the Square Root of a Square property given near the top of page 553.

Taking the Square Root of a Square

Study the paragraphs in the middle of page 553.

Watch the Video that accompanies Example 3 on page 553. Then, study Example 3.

Simplify each of the following expressions.

$-\sqrt{4y^2}$	$\sqrt{36x^6y^6}$

Read the two sentences about "Cube Roots" at the bottom of page 553. Then, continue studying page 554. Stop when you get to Example 4.

Write the definition of a cube root given at the top of page 554.

Definition

Write the Cubing a Cube Root property given on page 554.

Cubing a Cube Root

Write the Taking the Cube Root of a Cube property given on page 554.

Taking the Cube Root of a Cube

Study Example 4 on page 554.

Find the following cube roots.

$\sqrt[3]{216}$	$\sqrt[3]{-27}$	$\sqrt[3]{\dfrac{1}{125}}$	$\sqrt[3]{-64x^6}$

Study page 555.

Write the definition of an nth root.

Definition

Fill in the boxes to label the parts that make up the nth root.

The *n*th root of *a*

$$\sqrt[n]{a}$$
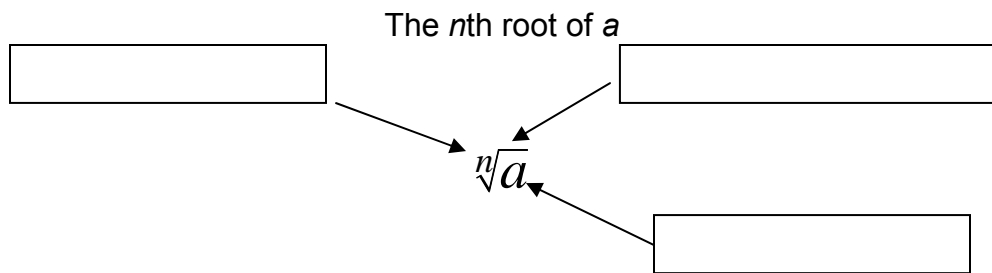
Fill in the missing word to make each statement true.

- If the radicand is negative and the index is _____, the radical expression is **not** a real number.

- If the radicand is negative and the index is _____, the radical expression is a real number.

Write the Taking the nth Root of an nth Power property.

Taking the *n*th Root of an *n*th Power

Watch the Video that accompanies **Practice 4 back on page 554**. Write the steps used to simplify the expressions given in the Video:

_____
_____
_____
_____
_____

Simplify. Assume all variable represent positive numbers.

$\sqrt[4]{16t^{12}}$

$\sqrt[5]{p^5 q^{15}}$

Study Example 5 on page 556.

Express each of the following without a radical sign.

$\sqrt[6]{64}$	$\sqrt[5]{-32}$	$\sqrt[4]{256 y^8}$

Objective 2: To write exponential expressions as radical expressions and vice versa

Study the bottom half of page 556, beginning at "Rational Exponents", and continue to the top of page 557, stopping before Example 6.

Write the definition of $a^{1/n}$.

Definition

Watch the Video that accompanies Example 6 on page 557. Write the steps used to simplify the expression given in the Video:

Rewrite using radical notation. Then simplify, if possible. $(-64)^{1/3}$

Study Example 6 on page 557.

Express each of the following as a radical expression. Simplify.

$36^{1/2}$	$27^{1/3}$

Express each of the following as a radical expression. Simplify.

$-n^{1/4}$	$(-125y^9)^{1/3}$

Study the bottom half of page 557, beginning after Example 6.

Write the definition of $a^{m/n}$.

Definition

Study Example 6 on page 558.

Express each of the following in radical notation. Simplify if possible.

$81^{3/4}$	$(-64)^{2/3}$	$\left(\dfrac{4}{9}y^4\right)^{5/2}$

Study the remainder of page 558 following Example 7.

Write the definition of $a^{-m/n}$.

Definition

Study Example 8 on page 559.

Simplify using the laws of exponents.

$-81^{-1/4}$	$(27a^6)^{-4/3}$

Study the laws of exponents summarized in the table in the middle of page 559.

Use the table on page 559 to complete the right-hand column in this table:

Raising a number to a negative exponent	
The product rule of exponents	
The quotient rule of exponents	
The power rule of exponents	
Raising a product to a power	
Raising a quotient to a power	

Watch the Video that accompanies Example 9 on page 559. Write the steps used to simplify the expressions given in the Video:

Simplify each of the following using the laws of exponents. Then, write the answer in radical notation.	
(a) $64^{2/3} \cdot 64^{1/6}$	**(b)** $(x^{3/8})^2$

Study Example 9 on pages 559 and 360.

Simplify using the laws of exponents.

$81^{1/4} \cdot 81^{1/2}$	$\dfrac{n^{3/5}}{n^{2/5}}$
$(r^{1/6})^3$	$\left(\dfrac{x^4}{4y^2}\right)^{1/2}$

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Objective 3: To solve applied problems involving radical expressions or rational exponents

Study Example 10 on page 560.

Do the following problem:

PRACTICE 10	Show your work here:
In our solar system, the *period* of a planet, that is, the time that it takes to make a complete orbit around the Sun, can be approximated by the expression $2\pi\sqrt{\dfrac{d^3}{Gm}}$, where d is the average distance of the planet from the Sun, G is a constant, and m is the mass of the planet, in appropriate units. Write this radical expression as an exponential expression in simplest form. (*Source:* Eric Chaisson and Steve McMillan, *Astronomy, A Beginner's Guide to the Universe*, 2001)	
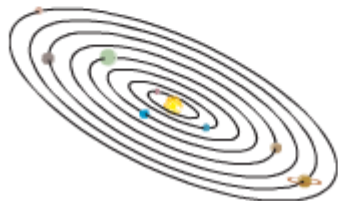
💬 Do the <u>Mathematically Speaking</u> Exercise on page 561. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

is not	square root	raised to the power
irrational	is	radicand
multiplied by	cube root	principal
square	index	rational

1. The number b is a(n) ———— of a if $b^2 = a$, for any real numbers a and b and for a nonnegative.

2. The symbol $\sqrt{}$ stands for the positive or ———— square root.

3. The number under the radical sign is called the ————.

4. The square root of a negative number ———— a real number.

5. The square root of a number that is not a perfect square is a(n) ———— number.

6. The number b is the ———— of a if $b^3 = a$, for any real numbers a and b.

7. The cube root of a is written $\sqrt[3]{a}$, where 3 is called the ———— of the radical.

8. A number ———— $\frac{1}{n}$ is the nth root of that number.

Answers for "Mathematically Speaking":

1. _____ 2. _____

3. _____ 4. _____

5. _____ 6. _____

7. _____ 8. _____

MyMathLab Do the Section 7.1 Homework problems in MyMathLab.
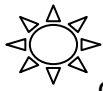
☀ Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

MINDSTRETCHERS

Consider the equation $y = \sqrt{x}$. Explain why only Quadrant I is used in graphing this equation.
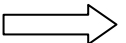
Write your explanation here:

STOP Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Module Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

Section 7.2: Simplifying Radical Expressions

Objective 1: To simplify radical expressions

Read the opening paragraphs at the top of page 565.

Watch the Video that accompanies Example 1 on page 565. Write the steps used to simplify the expressions given in the Video:

Simplify each radical expression. Then, write in radical notation.	
(a) $\sqrt[8]{x^4}$	**(b)** $\sqrt[4]{49}$

Study Example 1.

If possible, simplify by using rational exponents. Write your answer as a radical expression.

$\sqrt[8]{y^2}$	$\sqrt[6]{25}$
$\sqrt{n} \cdot \sqrt[5]{n}$	$\dfrac{\sqrt{t}}{\sqrt[4]{t}}$

Watch the Animation that accompanies Example 2 on page 565.

Write the steps used to simplify the expressions given in the Animation:

Simplify each of the following. Write the answers using rational exponents.
(a) $\sqrt[3]{x^2} \cdot \sqrt[4]{x}$
(b) $\dfrac{\sqrt{x^3}}{\sqrt[3]{x^2}}$
(c) $\sqrt{\sqrt[4]{z}}$

Study Example 2.

If possible, simplify by using rational exponents. Write your answer as a radical expression.

(a) $\sqrt[3]{\sqrt[4]{n}}$

Continues on the next page

(b) $\sqrt[8]{a^2 b^6}$
(c) $\sqrt[5]{5} \cdot \sqrt{7}$

Study the paragraph near the bottom of page 565, following the Solution to Example 2. Then study the Product Rule of Radicals at the top of page 566.

Write the Product Rule of Radicals.

Product Rule of Radicals

Watch the Video that accompanies Practice 3 on page 566. Write the steps used to multiply the expressions given in the Video:

Multiply each of the following.	
(a) $\sqrt{3x} \cdot \sqrt{2y}$	**(b)** $\sqrt[4]{3n} \cdot \sqrt[4]{7n^2}$

Study Example 3 on page 566.
Find the product for each of the following.

$\sqrt{7} \cdot \sqrt{3}$	$\sqrt[5]{2} \cdot \sqrt[5]{y^3}$
$\sqrt[3]{4p} \cdot \sqrt[3]{7p}$	$\sqrt{\dfrac{3}{v}} \cdot \sqrt{\dfrac{5}{u}}$

Study the paragraphs following Example 3 on page 566. Then study Example 4.

Watch the Video that accompanies Practice 5 on page 567. Write the steps used to simplify the expressions given in the Video:

(a) $-3\sqrt{80}$	**(b)** $\sqrt[4]{96}$

Write each of the following in simplified form.

$\sqrt{72}$	$-7\sqrt{18}$
$\sqrt[3]{56}$	$\sqrt[4]{48}$

Read the paragraph at the top of page 567.
Study Example 5.

Simplify each of the following.

$\sqrt{y^7}$	$\sqrt{18x^5}$
$\sqrt[3]{81a^4b^6}$	

Study the remainder of page 567 following Example 5.

Write the Quotient Rule of Radicals.

> **Quotient Rule of Radicals**

Watch the Video that accompanies Practice 6 on page 568. Write the steps used to simplify the expression given in the Video:

> $$\dfrac{\sqrt{12x^3y}}{\sqrt{3x}}$$

Study Example 6 on page 568.

Find the quotient for each of the following.

$$\frac{\sqrt{42n}}{\sqrt{6}}$$

$$\frac{\sqrt{72}}{\sqrt{2}}$$

$$\frac{\sqrt[3]{10x^2y^2}}{\sqrt[3]{5x^2y}}$$

Read the paragraph in between Example 6 and Example 7 on page 568.

Watch the Animation that accompanies Practice 7. View **only** Examples 3 and 4 in this Animation. Write the steps used to simplify Examples 3 and 4 in the Animation:

$$\frac{\sqrt{x^9y^{11}}}{\sqrt{36}}$$

$$\sqrt[3]{\frac{x^{16}y^7}{8}}$$

Study Example 7 on page 568.

Express each of the following in simplest terms.

$\sqrt{\dfrac{16}{25}}$	
$\sqrt{\dfrac{n}{64}}$	
$\sqrt[3]{\dfrac{64x}{125y^9}}$	
$\sqrt{\dfrac{7p^6q^2}{36r^4}}$	

Read the paragraphs following Example 7 on page 568. Study the paragraphs at the top of page 569.

Write the **distance formula** *for finding the distance between two points on a coordinate plane.*

Distance Formula

Study Example 8 on page 569.

Find the distance between the points $(5, -7)$ *and* $(3, -1)$ *on the coordinate plane.*

Show your work here:

Objective 2: To solve applied problems involving radical expressions or rational exponents

Study Example 9 on page 570.

PRACTICE 9

One truck driver traveled west for 20 mi and then north for 50 mi. A second driver traveled 10 mi east and then 60 mi north. Both drivers had started from the same point.

(figure: map with two truck routes)

N
W —+— E
S

60 mi

50 mi

20 miles | 10 miles
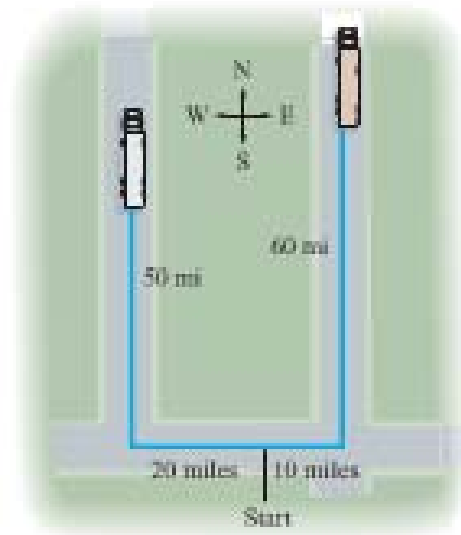
Start

a. Using their starting point as the origin of a coordinate plan, find the coordinates of the two destinations.

b. How far apart were the drivers at the end of their travels? Express the answer as a radical in simplified form.

Do the Mathematically Speaking Exercise on page 571. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

radicands	the same index	distance
square of the distance	perfect nth power	
multiple of n	different indices	

1. The product rule of radicals states that to multiply radicals with _____, multiply the radicands.

2. In general, the expression $\sqrt[n]{a}$ is not simplified if the radicand a has a factor that is a(n) _____.

3. The quotient rule of radicals states that to divide radicals with the same index, divide the _____.

4. The _____ between two points (x_1, y_1) and (x_2, y_2) on a coordinate plane is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Answers for "Mathematically Speaking":

1. _____

2. _____

3. _____

4. _____

MyMathLab   Do the Section 7.2 Homework problems in MyMathLab.
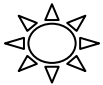
Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

MINDSTRETCHERS
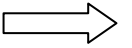
Use a grapher to display the graphs of $y = \sqrt{\dfrac{1}{x^2 + 1}}$ and $y = \dfrac{1}{\sqrt{x^2 + 1}}$. Compare the two graphs. What conclusion can you draw from this comparison?

Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Module Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

Section 7.3: Addition and Subtraction of Radical Expressions

Objective 1: To combine like radical expressions

Study page 575. Stop before Example 1.

Fill in the blanks to complete the sentence.
   Radicals are said to be ***like*** if they have the same _____ *and* the same _____.

Tell if the following pairs of radicals are like or unlike. If they are unlike, explain why they are unlike.

	Like or Unlike?	If unlike, explain why.
$5\sqrt[3]{6}$ and $3\sqrt{6}$		
$4\sqrt{10}$ and $4\sqrt{6}$		
$7\sqrt[4]{12}$ and $3\sqrt[4]{12}$		

Watch the Video that accompanies Example 1 on page 575. Write the steps used to do the problems given in the Video:

Combine, if possible.

(a) $7\sqrt[3]{10}+4\sqrt[3]{10}$	**(b)** $8\sqrt{x}-3\sqrt{x}-2\sqrt{x}$
(c) $5\sqrt{2y+1}-\sqrt{2y+1}$	**(d)** $\sqrt[3]{x}+3\sqrt{x}$

Study Example 1 at the bottom of page 575 and the top of page 576.

Add or subtract, as indicated (if possible).

(a) $-\sqrt[4]{9}+3\sqrt[4]{9}$	**(b)** $6\sqrt{p}+\sqrt{p}-2\sqrt{p}$
(c) $-5\sqrt{t^2-4}+\sqrt{t^2-4}$	**(d)** $5\sqrt[3]{a}-2\sqrt[3]{b}$

Turn to page 576. Write the **Tip** given near the top of the page.

Tip

Objective 2: To combine unlike radical expressions

Some *unlike* radicals become *like* when they are simplified and are able to be combined.

Watch the Video that accompanies Practice 2 on page 576. Write the steps used to do the problems given in the Video:

Simplify, if possible.

(a) $3\sqrt{18} - 4\sqrt{2} + \sqrt{50}$

(b) $-5\sqrt[3]{24} + \sqrt[3]{-81} + 9\sqrt[3]{3}$

Study Example 2 on page 576.

Combine.

(a) $\sqrt{75} + \sqrt{27}$

(b) $\sqrt{96} - 3\sqrt{54} - 7\sqrt{24}$

(c) $9\sqrt[4]{2} - 4\sqrt[4]{162} - \sqrt[4]{32}$

(d) $\sqrt{64} - \sqrt{28} + \sqrt{63}$

Study Example 3 on page 577.

Combine.

(a) $4\sqrt{25n} + \sqrt{n}$

(b) $\sqrt[3]{27ab^3} - 11\sqrt[3]{a}$

(c) $\sqrt{50x} - \sqrt{32x^3}$

Study Example 4 on page 577.

If $p(x) = x\sqrt[3]{81x}$ *and* $q(x) = \sqrt[3]{24x^4}$ *, find each of the following:*

(a) $p(x) + q(x)$	**(b)** $p(x) - q(x)$

Objective 3: To solve applied problems involving the addition or subtraction of radical expressions

Read the paragraph about the Pythagorean Theorem at the bottom of page 577. Then Study Example 5 on page 578.

Do the following problem:

The size of a television set is commonly given by the length of its diagonal.
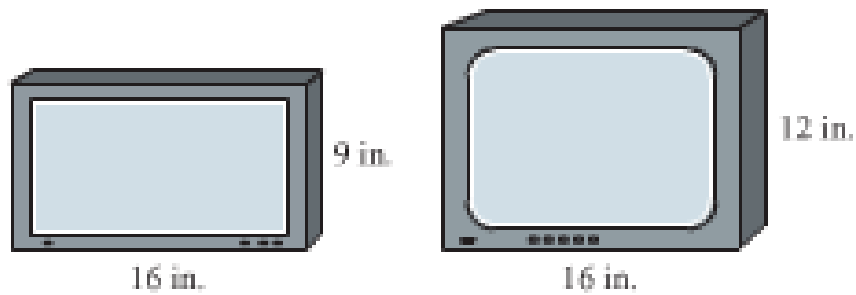
a) Find the length of the diagonal for each of the sets.

Small set:	Large set:

b) What is the difference of the lengths of the diagonals?

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Do the <u>Mathematically Speaking</u> Exercise on page 579. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

unlike	simplified	reduced
indices	coefficients	like
area of a triangle formula	Pythagorean theorem	

1. Radicals that have the same index and the same radicand are called _____ radicals.

2. Like radicals can be added by combining their _____ and then multiplying this sum by the radical.

3. Some unlike radicals can become like radicals when they are _____.

4. The _____ can be expressed as $c = \sqrt{a^2 + b^2}$.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

MyMathLab Do the Section 7.3 Homework problems in MyMathLab.
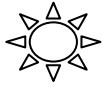
☀ Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.

MINDSTRETCHERS

Find two radical expressions whose sum is $4\sqrt{2}$ and whose difference is $7\sqrt{2}$.

Show your work

STOP   Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Module Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____   Move to the next section ⟹

Section 7.4: Multiplication and Division of Radical Expressions

Objective 1: To multiply radical expressions

Read the opening paragraphs at the top of page 582.

Write the Product Rule of Radicals.

Product Rule of Radicals

Watch the Video that accompanies Practice 1 on page 582. Write the steps used to do the problems given in the Video:

Simplify, if possible.

(a) $\sqrt{8n^3} \cdot \sqrt{3n}$	(b) $\left(-5\sqrt[3]{7}\right)\left(4\sqrt[3]{2}\right)$

Study Example 1.

Find the product.

(a) $\sqrt{12y^7} \cdot \sqrt{6y}$	(b) $\left(-2\sqrt[3]{3}\right)\left(7\sqrt[3]{5}\right)$

Read the paragraph in between Example 1 and Example 2 on page 582. Then study Example 2 at the bottom of page 582 and the top of page 583.

Multiply and simplify:

(a) $\sqrt[4]{5}\left(\sqrt[4]{7} + 6\sqrt[4]{2}\right)$
(b) $\left(3\sqrt{x} - 1\right)\left(2\sqrt{x} + 1\right)$

Do the following problems.

Find the product.

(a) $\left(\sqrt{2} - \sqrt{5}\right)\left(\sqrt{2} + \sqrt{5}\right)$	**(b)** $\left(\sqrt{2x} + 3\right)\left(\sqrt{2x} - 3\right)$
(c) $\left(\sqrt{p} - 4\sqrt{q}\right)\left(\sqrt{p} + 4\sqrt{q}\right)$	

Read the paragraph in between Example 3 and Example 4 on page 583.

Finish writing each of the formulas for squaring a binomial:

$(a+b)^2 = $ _____

$(a-b)^2 = $ _____

Study Example 4 at the bottom of page 583 and the top of page 584.

Do the following problems.

Simplify

(a) $\left(\sqrt{3} + 2b \right)^2$

(b) $\left(\sqrt{n+1} - 3 \right)^2$

Study Example 5 on page 584

Do the following problem.

Find the area of the rectangle shown, expressed as a simplified radical. Recall, the formula for area of a rectangle is $A = L \cdot W$.

$\sqrt{6}$ cm

$\sqrt{20}$ cm

Show your work:

Objective 2: To divide radical expressions

Read the paragraph under "Dividing Radical Expressions" on page 584.

Write the Quotient Rule of Radicals.

Quotient Rule of Radicals

Watch the Video that accompanies Practice 6. Write the steps used to perform the division given in the Video:

Divide and simplify, if possible: $\dfrac{3\sqrt{50}}{\sqrt{2}}$

Study Example 6 at the bottom of page 584 and the top of page 585.

Find the quotient. Simplify, if possible.

(a) $\dfrac{5\sqrt{12}}{\sqrt{3}}$

(b) $\dfrac{\sqrt[3]{18p^7}}{-5p\sqrt[3]{2p^2}}$

Read the paragraph in the middle of page 585.

Fill in the boxes to make the following statement true:

$$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[\square]{\square}}{\sqrt[\square]{\square}}$$

Study Example 7 on page 585.

Simplify each of the following

$\sqrt{\dfrac{y^2}{9x^8}}$	$\sqrt[3]{\dfrac{2n^5}{27}}$

Objective 3: To rationalize denominators in radical expressions

Study the paragraph under "Rationalizing the Denominator" near the bottom of page 585. Then study Example 8 on page 586.

Rationalize the denominator:

$\dfrac{4}{\sqrt{7}}$	$\dfrac{\sqrt{y}}{\sqrt{32}}$

continues on the next page

Rationalize the denominator:

$\dfrac{\sqrt{4p^4}}{\sqrt{18}}$	$\dfrac{\sqrt[3]{5pq}}{\sqrt[3]{2r^2}}$

Study Example 9 at the bottom of page 586 and the top of page 587.

Rationalize the denominator:

$\sqrt{\dfrac{x}{3y}}$	$\sqrt[3]{\dfrac{5}{a}}$

Write the Tip given near the top of page 587.

Tip Remember that a radical expression is considered simplified if
•
•
•

Read the sentence between the Tip and Example 10 on page 587. Then study Example 10.

Rationalize the denominator:

$\dfrac{4\sqrt{10}-\sqrt{6}}{\sqrt{2}}$	$\dfrac{5+\sqrt{b}}{\sqrt{b}}$

Study the paragraphs at the top of page 588.

Write the conjugate for each of the following:

$\sqrt{x}+5$	$\sqrt{6}-\sqrt{7}$

Fill in the boxes to make a true statement:

$$(2-\sqrt{3})(2+\sqrt{3}) = \boxed{}^{\boxed{}} - \boxed{}^{\boxed{}}$$

Watch the Video that accompanies Practice 11 on page 588. Write the steps used to rationalize the denominator of the fraction given in the Video:

Rationalize the denominator: $\dfrac{1}{2+\sqrt{2}}$

Study Example 11.

Rationalize the denominator for each of the following fractions:

$\dfrac{4}{2+\sqrt{2}}$

$\dfrac{a}{\sqrt{b}-\sqrt{c}}$

Study Example 12 on page 589.

If $p(x) = x\sqrt[3]{81x}$ and $q(x) = \sqrt[3]{24x^4}$ determine each of the following:

a. $p(x) \cdot q(x)$	b. $\dfrac{p(x)}{q(x)}$

Objective 4: To solve applied problems involving the multiplication or division of radical expressions

Study Example 13 on page 589.

Do the following problem:

The following diagram shows a sphere of volume V. The radius of the sphere can be modeled by the expression $\dfrac{\sqrt[3]{3V}}{\sqrt[3]{4\pi}}$.	Rewrite this expression, rationalizing the denominator.
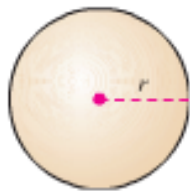
💬 Do the <u>Mathematically Speaking</u> Exercise on page 590. Do the problem as an animation in the multimedia textbook and write in the answers below.

Mathematically Speaking

Fill in each blank with the most appropriate term or phrase from the given list.

numerator	denominator	squaring a binomial
perfect power	constant	multiply
product	rationalize	
commutative	distributive	
the sum and difference of the same two terms	quotient	

1. To multiply radical expressions containing more than one term, we use the _____ property.

2. When squaring binomials that contain radical terms, apply the formulas for _____.

3. To simplify a radical that has its radicand in the form of a fraction, we can use the _____ rule of radicals "in reverse."

4. To _____ the denominator means to rewrite an expression that has a radical in its denominator as an equivalent expression that does not have one.

5. A denominator can be rationalized by multiplying the numerator and denominator by a factor that makes the radicand in the denominator a(n)

6. To rationalize a denominator with two terms, multiply both the numerator and the denominator by the conjugate of the _____.

Answers for "Mathematically Speaking":

1. _____   2. _____

3. _____   4. _____

5. _____   6. _____

MyMathLab Do the Section 7.4 Homework problems in MyMathLab.

Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.
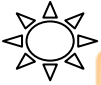
MINDSTRETCHERS

Some equations have solutions that contain radicals. Show that $3 - 2\sqrt{2}$ is a solution of the equation $x^2 - 6x + 1 = 0$.
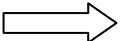
Show your work:

Before moving on to the next section, check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Mathematically Speaking
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mathematically Speaking

Date assignments completed:_____   Move to the next section ⟹

Section 7.5: Solving Radical Equations

Objective 1: To Solve Racial Equations

Study page 595, stopping before Example 1.

Write the definition of a radical equation

Definition
_____
_____

*Explain why $x = \sqrt{6}$ is **NOT** a radical equation*

_____
_____

Fill in the boxes to make the following property of equality true:

If $a = b$, then $a^{\square} = b^{\square}$, where $\square$ is a positive integer.

Fill in the blanks to complete the following sentence.

To solve radical equations, we first _____ the radical and then raise

_____ _____ of the equation to the _____ _____ in

order to produce a new equation that contains _____ _____.

Fill in the blanks to complete the following sentences.

When raising each side of the equation to the same power, the resulting equation

may have one or more solutions that ____ _____ satisfy the _____

equation. When solving a radical equation, it is particularly important to _____

all possible solutions in the _____ equation.

Show each of the steps used to solve the following equation.

$\sqrt{x} + 3 = 8$	• Original equation
	• Isolate the radical
	• Square both sides of the equation
	• Possible solution
	• Check the possible solution in the ***original*** equation.

Study Example 1, which begins at the bottom of page 595 and continues to the top of page 596.

Solve and check:

$\sqrt{y} + 2 = 10$

Watch the Video that accompanies Practice 2 on page 596. Show how the equation given in the Video is solved. Then, show the check of the possible solution.

Solve. $\sqrt[3]{3y+10} = -2$	Show the Check.

Study Example 2 on page 596.

Solve and check:

Solve $\sqrt[3]{2x+7} = 3$.	Show the Check.

*Write the **Tip** given after Example 2.*

Tip
_____
_____
_____

Study Example 3, which begins at the bottom of page 596 and continues on page 597.

Solve and check:

Solve $\sqrt{3t+1}+4=0$.	Show the Check.

Study Example 4 on page 597. In the table below, show each of the steps used to solve Example 4.

Note: Do **not** watch the Video that accompanies Example 4. This Video is not a good fit with Example 4.

$\sqrt{2x+11}-\sqrt{4x+1}=0$	• Original equation
	• Isolate one of the radicals
	• Square both sides of the equation
	• Simplify to find the possible solution

Show the check of the possible solution on the next page

	• Check the possible solution in the ***original*** equation, $\sqrt{2x+11} - \sqrt{4x+1} = 0$.

Watch the Video that accompanies Example 5 on page 598. Show how the equation given in the Video is solved.

Solve $\sqrt{2x} - \sqrt{x-2} = 2$.

Use the space provided on the next page to check the possible solutions.

Check the possible solutions in the original equation, $\sqrt{2x} - \sqrt{x-2} = 2$.	

Solve and Check $\sqrt{2y+3} = \sqrt{y-2} + 2$:

Use the space provided on the next page to check the possible solutions.

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Check the possible solutions in the original equation, $\sqrt{2y+3} = \sqrt{y-2} + 2$.

Write procedure used to Solve a Racial Equation given at the end of page 598.

To Solve a Radical Equation
•
•
•
•
•
•

Study Example 6 on page 599.

Solve and check: $\sqrt[3]{n^2 + 8} + \sqrt[3]{4 - 7n} = 0$

Show the checks of the possible solutions	

Watch the Animation that accompanies Example 7 on page 599. Show how the equation given in the Animation is solved.

Solve and check: $\sqrt{2y-3} = y - 3$

Show the checks of the possible solutions

Study Example 7 beginning at the bottom of page 599 and continuing to the top of page 600.

Solve and check:

Show the checks of the possible solutions

Objective 2: To Solve Applied Problems Involving Racial Equations

Read Example 8 on page 600. The Solution to Example 8 does not tell what is occurring in each step when solving the equation for *L*. The Solution is re-done below with a description of what is occurring in each step.

Solution to Example 8 on page 600.

The directions state to "Rewrite the formula $t = 2\pi\sqrt{\dfrac{L}{32}}$ by solving for *L*." This means we need to rewrite the equation so the *L* is isolated.

	Description
$t = 2\pi\sqrt{\dfrac{L}{32}}$	• Original formula
$\dfrac{t}{2\pi} = \dfrac{\cancel{2\pi}\sqrt{\dfrac{L}{32}}}{\cancel{2\pi}}$	• Divide both sides of the equation by 2π to isolate the radical.
$\dfrac{t}{2\pi} = \sqrt{\dfrac{L}{32}}$	• The radical is now isolated.
$\left(\dfrac{t}{2\pi}\right)^2 = \left(\sqrt{\dfrac{L}{32}}\right)^2$	• Square both sides of the equation to remove the radical
$\dfrac{t^2}{4\pi^2} = \dfrac{L}{32}$	• The radical is now removed.
$32 \cdot \dfrac{t^2}{4\pi^2} = \dfrac{L}{32} \cdot 32$	• Multiply both sides of the equation by 32 to isolate the *L*.
$\overset{8}{\cancel{32}} \cdot \dfrac{t^2}{1\cancel{4}\pi^2} = \dfrac{L}{1\cancel{32}} \cdot \cancel{32}^1$	• Cancel
$\dfrac{8t^2}{\pi^2} = L$	• Multiply remaining factors. *L* is now isolated.

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For the following problem, show the work that is described in each Step Description given in the right-hand column.

For an electrical appliance, the formula $I = \sqrt{\dfrac{P}{R}}$ shows relationship between its resistance, R, the amount of current, I, that it draws, and the power, P, that it consumes. Rewrite this formula by solving for P.
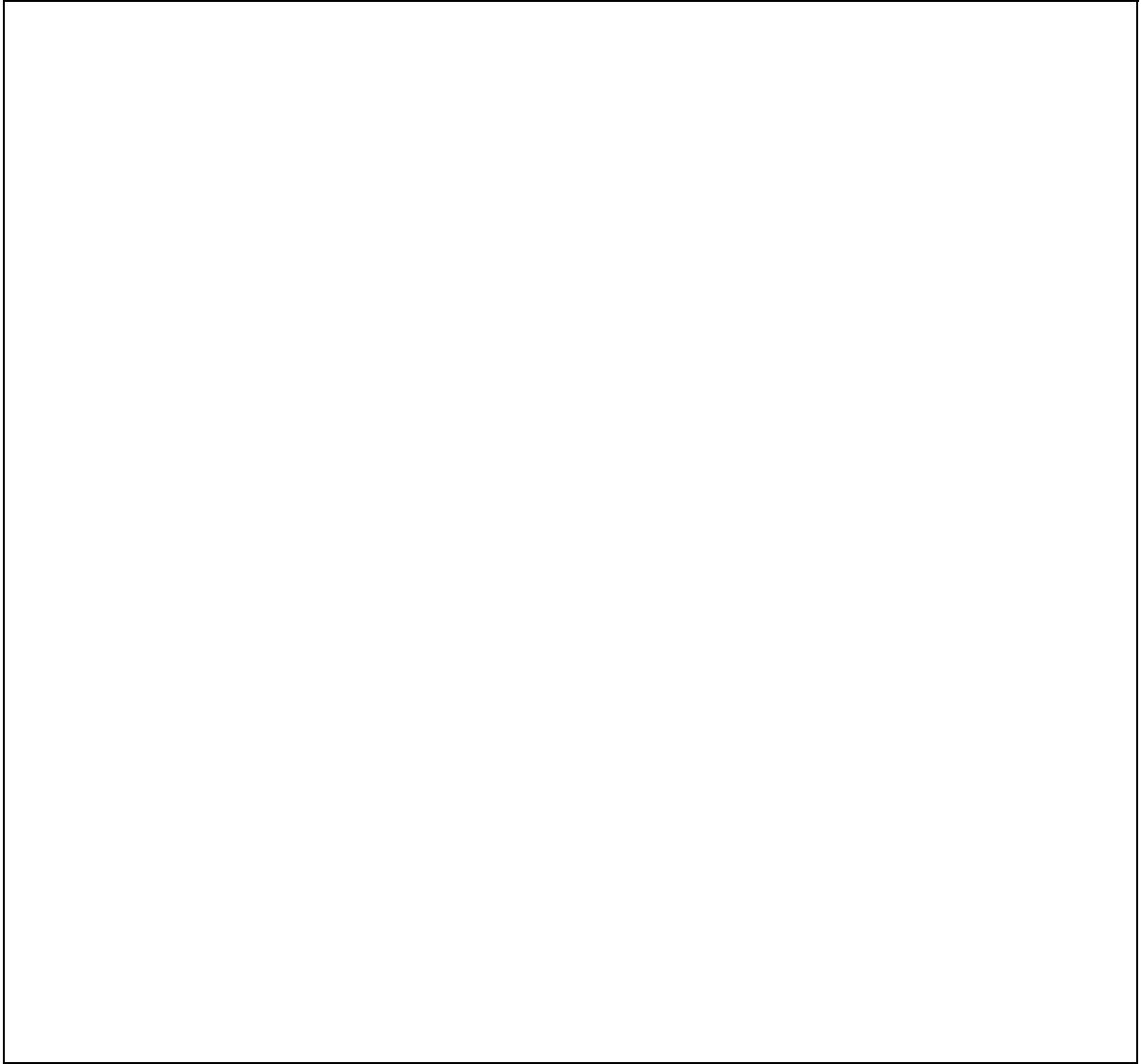
Show the work	Step Description
$I = \sqrt{\dfrac{P}{R}}$	• Original formula
	• Square both sides of the equation to remove the radical
	• The radical is now removed.
	• Multiply both sides of the equation by R to isolate the P.
	• Cancel
	• Multiply remaining factors. P is now isolated.

Study Example 9 on page 601. Create a more detailed solution below by showing the work described in each Step Description given in the right-hand column.
Note: Do *not* watch the Video that accompanies Example 9, at this point. This Video is not a good fit with Example 9.

Show the work	Step Description
	• Original formula
	• Square both sides of the equation to remove the radical
	• The radical is now removed.
	• Multiply both sides of the equation by 4π to isolate the *A*.
	• Cancel
	• Multiply remaining factors. *A* is now isolated.

Solve the formula $V = \sqrt{\dfrac{2K}{m}}$ for K Show your work.

Watch the Video that accompanies Example 9 on page 601. Show how the problem given in the Video is solved.

The distance d to the horizon for an object h miles above Earth's surface is given by the equation $d = \sqrt{8000h + h^2}$. How many miles above Earth's surface is a satellite if the distance to the horizon is 900 miles?

Do the following problem:

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The formula $r = 2\sqrt{5L}$ can be used to approximate the speed r, in miles per hour, of a car that has left a skid mark of length, L, in feet. How far will a car skid at 75 mph? Show your work.

☼ Do the two <u>Mindstretchers</u> exercise given below. This is to be done outside of class.
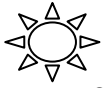
MINDSTRETCHERS

Solve the following equation: $\sqrt{\sqrt{x^2-9}}=2$. Show your work.

STOP   This is the last section in this module. Check to make sure you have done ALL the assignments in this section. Put an X in the box next to those that you have completed. Go back to those assignments which are incomplete.

☐ Student Instructional Workbook
☐ Online homework in MyMathLab (with a grade of 80 or better).
☐ Mindstretchers exercise

Date assignments completed:_____

Technology

Guides

&

Assignments

Technology Guides and Assignments

An important component of Intermediate Algebra is learning how to use a graphing calculator. We have developed ten Technology Guides and ten accompanying Technology Skills Assignments. Each module workbook clearly refers the students to the guide and assignments that correspond to the concepts and skills introduced in the section/s of the textbook. Encourage your students to master these skills on the calculator even if they feel they can work without it. These skills are the basis for more calculator work that they will need to do in higher level math classes.

Core Student Success Skills: Critical thinking, communication, collaboration, self – assessment.

Materials: Module Workbook, notebook, pencil, and TI-83+ or TI-84+ graphing calculator.

Context within the course: This activity extends across the entire course.

Procedure in a lecture section:

1 – Students will read the guide for homework or go over it in class. Encourage your student to do the assignments and note that the answers are on the second page of each assignment.

2 – The assignment won't be collected or graded; students can use the answers provided for self – assessment.

3 – Students should be given time to ask questions about the calculator steps before being asked to complete the In – class exercise included in each assignment.

4 – Students will complete the In - class exercise as a way to show the instructor their understanding of the calculator skill/s presented.

The following are the topics included in the Technology Guides and Assignments

Module 1 – Introduction to Functions
Technology Guide 1: Basic Operations on your Graphing Calculator Technology Skills Assignment 1
Technology Guide 2: Evaluating and Graphing Functions Technology Skills Assignment 2
Module 2 – Solving Systems of Linear Equations
Technology Guide 3: Solving Systems of Linear Equations by Graphing Technology Skills Assignment 3
Technology Guide 4: Checking the solutions of an equation or a systems of Equations Technology Skills Assignment 4
Module 3 – Solving and Graphing Linear Inequalities
Technology Guide 5: Graphing Linear Inequalities in two variables Technology Skills Assignment 5
Technology Guide 6: Graphing Systems of Linear Inequalities in two variables Technology Skills Assignment 6
Module 4 – Rational Expressions and Equations
Technology Guide 7: Using Graphs to check a factorization of a polynomial Technology Skills Assignment 7
Technology Guide 8: Solving Equations using the Intersect and the Zero features Technology Skills Assignment 8
Module 3 – Solving and Graphing Linear Inequalities
Technology Guide 9: Radicals on your Graphing Calculator Technology Skills Assignment 9
Technology Guide 10: Radical Equations on your Graphing Calculator Technology Skills Assignment 10

Technology Guide 1

Basic operations on your graphing calculator.

Order of Operations: You will have to use lots of parentheses to make sure the calculations are done in the right way.

For example, if you want to compute —— you need to enter:

(5 − 3) ÷ (7 − 1) + 4 ENTER

(the answer 4.333333333 will appear on your screen)

Fractions: if you want to get an answer in fraction form, you must press MATH and choose option 1 .

For example, for the answer 4.333333333 that we got in the previous example:

```
(5-3)/(7-1)+4
          4.333333333
Ans►Frac
               13/3
```
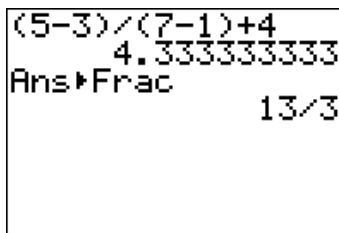
Negative numbers: there is a key for the negative sign (-); it is located to the left of the ENTER key.

Exponents: the ^ key is the one used to raise a value to an exponent. There is a special key x^2 for raising values to the second power.

```
3^4
            81
(-4)^3
            -64
5²
            25
```

Radicals: ▪The square root can be obtained by pressing [2nd] [x^2]

▪The cube root can be obtained by pressing [MATH] [4]

▪Radicals with index greater than 3 can be obtained by entering the index and then pressing [MATH] [5]
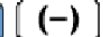
For example: $\overline{}$ will be obtained by entering:

[5] [MATH] [5] [(] [2] [8] [+] [4] [)] [ENTER]

the answer 2 will appear on your screen)

Variables: all letters can be entered by using [2nd] and the corresponding key, but the variable X is used so often that there is a special key for it; it is located to the right of the ALPHA key [X,T,θ,n]

General Actions:

▪To delete a value or symbol, press [DEL]

▪To clear a line, press [CLEAR]. Press twice to clear the screen.

▪To insert a value or symbol, press [2nd] [DEL]

▪To retrieve lines press [2nd] [ENTER] as many times as needed.

▪To retrieve answers press [2nd] [(−)] as many times as needed.

Evaluating and graphing functions

The screen where we enter the functions is the Y – editor. To open it, press [Y=]

```
Plot1 Plot2 Plot3
\Y1=█
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```
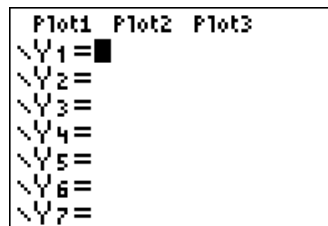
Let's use the function   as an example.

[3] [X,T,θ,n] [−] [5]

```
Plot1 Plot2 Plot3
\Y1=3X-5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```
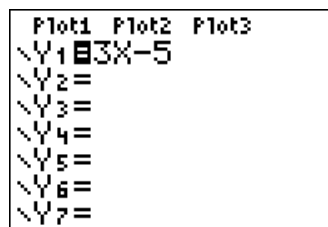
Be careful! If you press [ENTER] when your cursor is on the **=** symbol, it won't appear

boldface any more and the function will become "inactive". (We will find this useful later)

Evaluating a Function:

First, we will open the TABLE SETUP menu.

[2nd] [WINDOW]

```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Next, we have two choices for the *independent variable*. One is the Auto option, which means that the calculator will start at the value indicated in TblStart and automatically generate a table at intervals ΔTbl. The other is the Ask option, in which the user enters specific values of x.

To see the table that has been generated, we need to press [2nd] [GRAPH]

For example, a table can start at the value x = 5 and evaluate the function for other values of x at intervals of 2, like

```
TABLE SETUP
 TblStart=5
 ΔTbl=2
Indpnt: Auto Ask
Depend: Auto Ask
```

X	Y₁	
5	10	
7	16	
9	22	
11	28	
13	34	
15	40	
17	46	

X=5

Or we can evaluate only *f(2), f(7), f(-3),* and *f(0)* by entering each value of *x* and [ENTER]

```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto ASK
Depend: Auto Ask
```

X	Y₁	
2	1	
7	16	
-3	-14	
0	-5	

X=

Graphing a Function:

Choosing a Window: Pressing [WINDOW] will allow you to determine the values of *x* and *y* used in the coordinate system displayed in your calculator window.

The default values are as follows, showing each axis from -10 to 10 and making marks for each unit.

```
WINDOW
  Xmin=-10
  Xmax=10
  Xscl=1
  Ymin=-10
  Ymax=10
  Yscl=1
  Xres=1
```
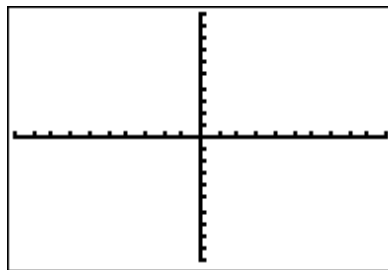
By changing the values, you can modify the limits on both axes as well as the separation between marks. The following settings will show the x-axis from -20 to 50 at intervals of 10 units, and the y – axis from -500 to 600 at intervals of 100 units.

```
WINDOW
  Xmin=-20
  Xmax=50
  Xscl=10
  Ymin=-500
  Ymax=600
  Yscl=100
  Xres=1
```
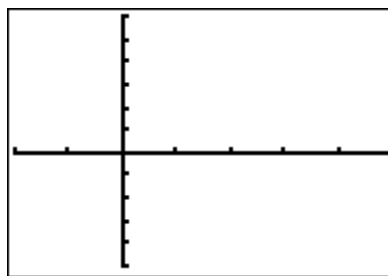
Looking at the Graph: After setting the desired window, press [GRAPH] to see the graph.

```
WINDOW
  Xmin=-5
  Xmax=20
  Xscl=1
  Ymin=-10
  Ymax=20
  Yscl=2■
  Xres=1
```

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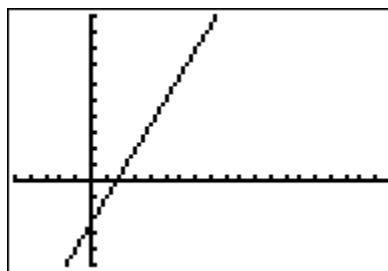
If you want to see a graph in the default window, you can easily do that by pressing ZOOM and choosing the option 6: ZStandard

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```
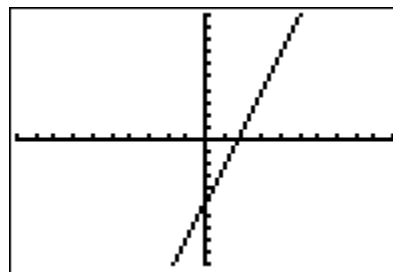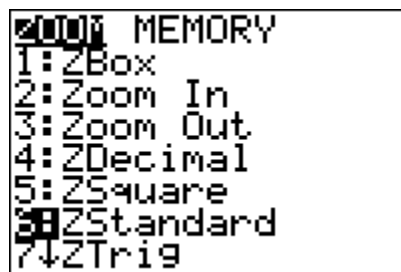
Solving Systems of Linear Equations by Graphing

Consider the system of linear equations $\begin{cases} 3x + 5y = 9 \\ 3x - 2y = -12 \end{cases}$

To find the solution by graphing, the first step is to write each equation in the form $y = mx + b$. Then, the equations are entered in the Y – editor:

```
 Plot1  Plot2  Plot3
\Y1▤(9-3X)/5
\Y2▤(3X+12)/2■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Remember that the calculator will show the graph on the window that is currently defined. You should check the WINDOW settings or choose ZOOM, 6:ZStandard to see the graph in the default window. (Refer to Technology Guide 2). If we choose ZStandard, this is what we see:
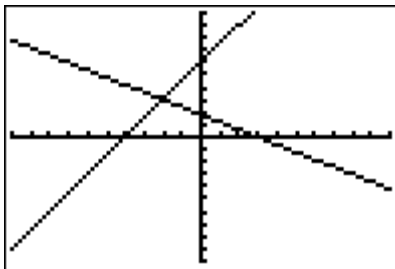
The solution of the system is the ordered pair of the coordinates of the point of intersection, so we must find those coordinates.

If you do not see the point of intersection on the screen, you must change the WINDOW settings or use ⎡ZOOM⎤, option 3: Zoom Out until you see it.(Unless the lines have the same slope! Why?)

The TRACE feature.

This feature allows you to move along the lines and see the coordinates of the points you pass.

While on the graph screen, press ⎡TRACE⎤; the cursor will appear on the screen and the coordinates of the point at which it is placed will be shown on the lower part of the screen.

Y1=(9-3X)/5

X=0   Y=1.8
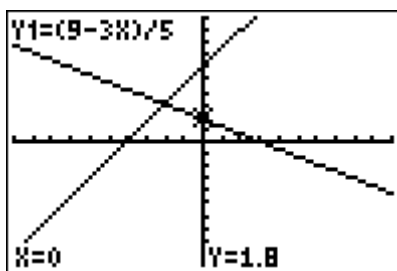
The equation of the line on which the point lies is shown on the upper – left part of the screen. To move along a line, use the ⎡‹⎤ ⎡›⎤ arrows; to "jump" from one line to the other use the ⎡⌄⎤ ⎡⌃⎤ arrows.

You can move towards the point of intersection and read its coordinates on the screen.

Y1=(9-3X)/5
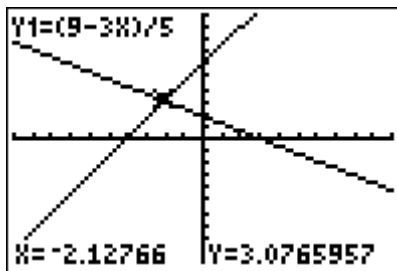
X=-2.12766   Y=3.0765957

This method gives you an approximate solution of the system, the pair (-2.12766, 3.0765957)

The INTERSECT feature.

While on the graph screen, press [2nd] [TRACE] to access the CALCULATE menu. There, choose option 5: intersect. The screen will show the lines while the legend **First curve?** and the coordinates of the point at which the cursor is placed will appear on the bottom of the screen.
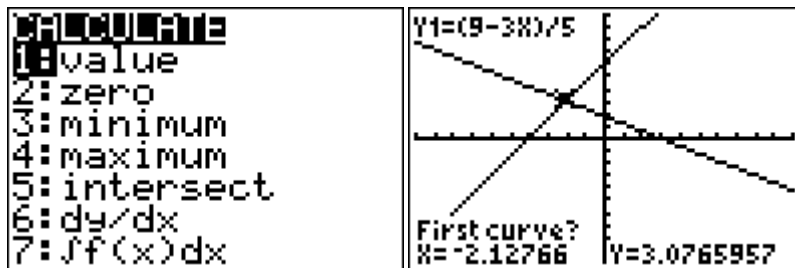
Press [ENTER], the cursor will move to a point on the other line and the legend **Second curve?** will appear. Press [ENTER] again, there will be a **Guess?**; press [ENTER] again and the **Intersection** point's coordinates will be shown.
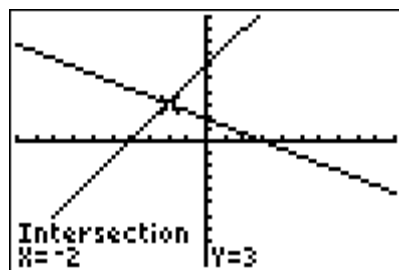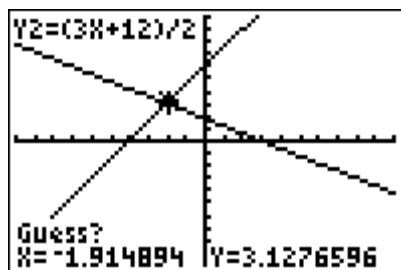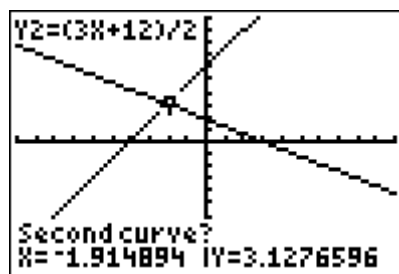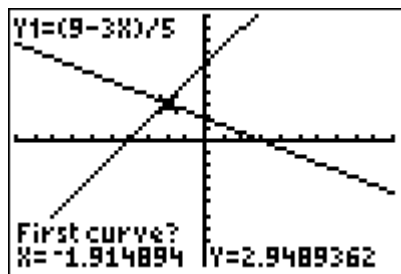
The solution of the system is the pair (-2, 3)

EXTRAS: If you want to enlarge the area around the point of intersection to see things more clearly, you can use [ZOOM], option 1: ZBox or [ZOOM]. option 2: Zoom In

1: ZBox

While on the graph screen, press [ZOOM], 1: ZBox and you will see the cursor flashing on the screen. Move the cursor to a point that will determine one corner of a "box" that you will draw surrounding the point of intersection. Press [ENTER]
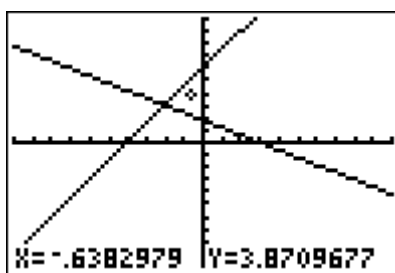
Now, move the cursor to determine the opposite corner of the box. As you move it you will see the sides of the box that are being drawn.
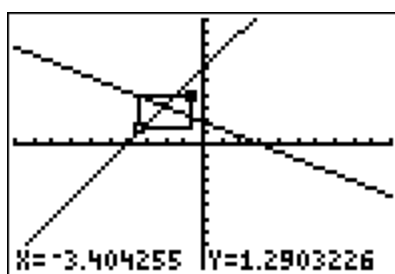
When you are satisfied, press [ENTER] again and the screen will show the area that was inside the box that you created.

X=-2.021277 Y=2.5806452
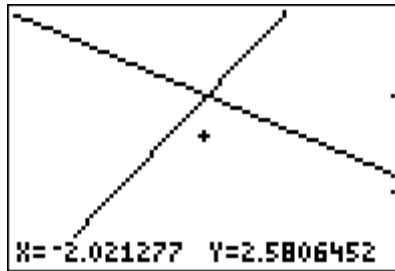
2: Zoom In

While on the graph screen, press `ZOOM`, 2: Zoom In, then move the cursor and place it close to the point of intersection. Press `ENTER` and you will be shown a new screen, with a "close up" of the area around the cursor.

X=-1.702128 Y=1.6129032

X=-1.702128 Y=1.6129032
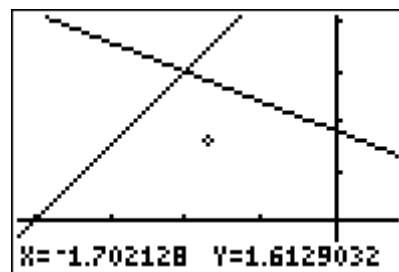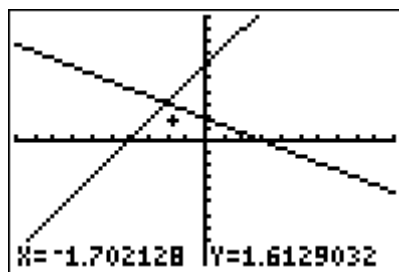
Technology Skills How - To Guide 4

Checking the solutions of an Equation or a Systems of Equations

The STO⟩ key is a very useful feature to check solutions of equations or systems of equations.

Let's assume we have solved the equation   and gotten the solution   . To check this solution, we will "store" the value -1 for the variable x by entering -1, STO⟩, X, ENTER

```
-1→X
              -1
■
```

From now on, whenever we enter X , the calculator will "replace it" by the value -1. That is exactly the way we check an equation! We replace the variable by the value we have found.
So now all we have to do is enter each side of the equation and see if we get the same answer. Enter   , ENTER and the evaluation of that expression will appear (-8 in this case); enter   and the evaluation of that expression will appear (-8 in this case). Since both sides of the equation yielded the same answer, the value -1 is indeed the solution of the equation.

```
-1→X
              -1
5X-3
              -8
9X+1
              -8
```

The [STO►] key can be used to check solutions to a system of equations, too. Each variable will have a value stored in its place and then you will have to enter each equation to check whether it is true.

For example, consider the system of linear equations   and the

potential solution (-2, 3).

 We will store the value -2 for the variable *x*, and the value 3 for the variable *y*.

Then, we enter each equation and check that it is true.

Both equations are verified, so the pair (-2, 3) is indeed the solution of the system.

If you are working with more variables and more equations, just store the values for each variable using the [STO►] key and then check each equation. For example, let's check if the triple (3, 2, -3) is a solution of the system:

```
3→X
           3
2→Y
           2
-3→Z
          -3
```

```
2X+3Y+5Z
          -3
5X+2Y+3Z
          10
3X+5Y+2Z
          13
■
```

Clearly, none of the equations is satisfied and the triple (3, 2, -3) is not a solution of the system.

NOTE: You can store the values of all the variables in the problem on a same line. This is done by separating the entries with the colon symbol "**:**" that can be obtained by pressing ALPHA [.]

For example, if you want to store the values (2, 4, 7) for (x, y, z), you will write:

```
2→X:4→Y:7→Z
           7
```

Technology Skills How - To Guide 5

Graphing linear inequalities in two variables.

Press the [APPS] key on your calculator to access the APPLICATIONS menu. There, choose the **:Inequalz** option, and you will see the following screen:

Press any key and you will be taken to the Y – editor screen.

Let's start by graphing the inequality

First, we re-write the inequality by isolating the y – variable:

Then, we enter the equation in the Y-editor.
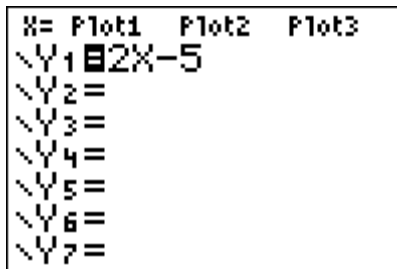
Next, we move the cursor to place it on top of the "=" symbol. When we do that, all five equality and inequality symbols will appear on the bottom of the screen.
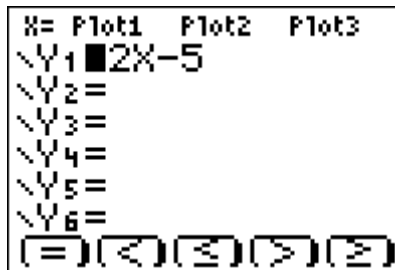
To select the inequality symbol that applies to our problem, we must use the [ALPHA] key and the key that is under the desired symbol. In our case, to obtain the ">" symbol, we will need to press [ALPHA] and the [TRACE] key. The symbol will appear instead of the "=", then press [GRAPH] to see the shaded area that is a solution of the inequality.
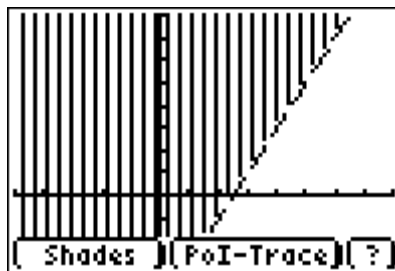
The legends on the bottom of the screen will disappear if you press [ENTER]

Now, you can move the cursor with the arrows and the coordinates of the point where you place it will show on the screen. This may be useful to find pairs   that verify the inequality.

Graphing systems of linear inequalities in two variables.

Press the key on your calculator to access the APPLICATIONS menu. There, choose the **:Inequalz** option, and you will see the following screen:
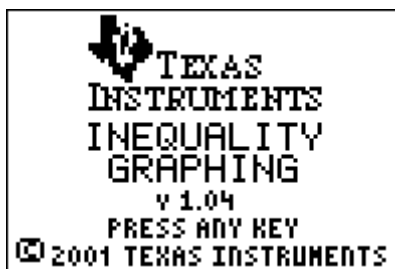
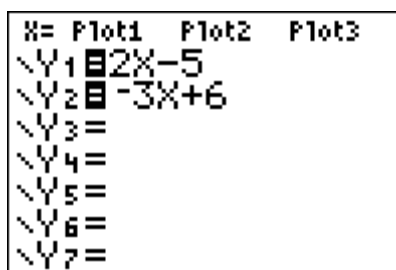Press any key and you will be taken to the Y – editor screen.
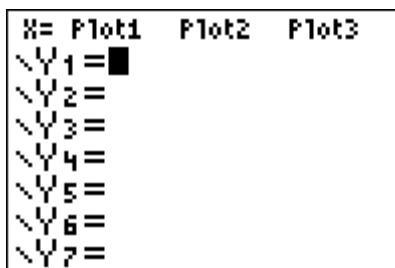
Note: if the Inequal application has already been turned on, you will be shown the following screen; just choose **1:Continue** and you will be taken to the Y – editor screen.
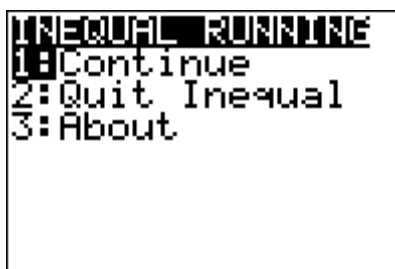
Let's start by graphing the system of linear inequalites
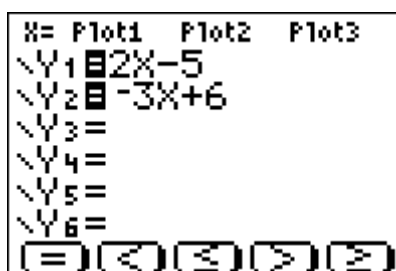
First, we re-write the inequalities by isolating the y – variable:

Then, we enter the equations in the Y-editor.

Next, we move the cursor and place it on top of one of the "=" symbols. When we do that, all five equality and inequality symbols will appear on the bottom of the screen.


To select the inequality symbols that apply to our problem, we must use the ALPHA key and the key that is under each desired symbol. In our case, to obtain the   symbol, we will need to press ALPHA and ZOOM and to obtain the ">" symbol, we will need to press ALPHA and TRACE . The symbols will appear instead of the "=", then press GRAPH to see the shaded areas that are solutions of each inequality.
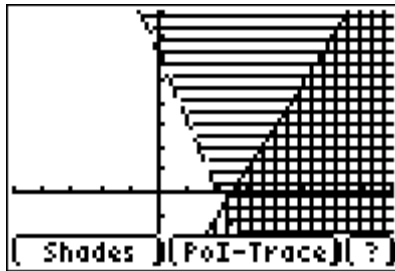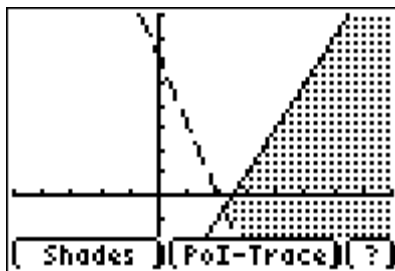
The solution to the system of inequalities is the area that satisfy both inequalities, ,that is the intersection of the two areas shaded. To obtain the solution, we need the **Shades** option that is shown on the bottom of the screen. To open it, press ALPHA Y= or ALPHA WINDOW
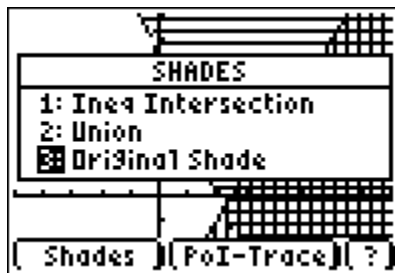
Then, select 1: Ineq Intersection to see the intersection of the areas.

Press ENTER to make the legends disappear and see the area better.Now, you can move the cursor with the arrows and the coordinates of the point where you place it will show on the screen. This may be useful to find pairs   that verify the system of inequalities
.
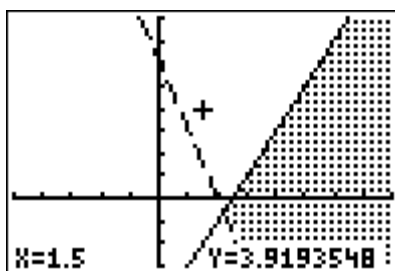
Using Graphs to check a factorization of a polynomial.

We can use our graphing calculator to verify a factorization of a polynomial. Let's consider the polynomial $3x^3 - 12x^2 + 9x$
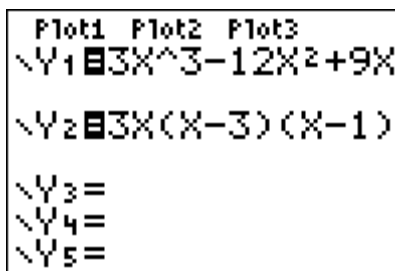
First, we extract common factors 3 and x, to obtain $3x(x^2 - 4x + 3).$

Then, we can factor the trinomial and obtain $3x(x - 3)(x - 1)$

If the factorization is correct, $3x^3 - 12x^2 + 9x$ and $3x(x - 3)(x - 1)$ represent the same polynomial. This means that we will get the same answer when evaluating them for the same value of the variable x, and we will get the same graph when plotting points.

Evaluating Expressions

Enter both, the original polynomial and the factored one, in the Y – editor.


```
 Plot1  Plot2  Plot3
\Y₁=3X^3-12X²+9X
\Y₂=3X(X-3)(X-1)
\Y₃=
\Y₄=
\Y₅=
```

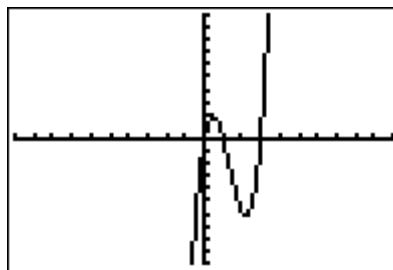Then, set the table in Auto and look at the values on the table.

The tables show the same values for both expressions. The expressions are equivalent.
Graphing Expressions

Enter both, the original polynomial and the factored one, in the Y – editor.

Then, look at the graph (shown here in the standard window).

The screen shows **one** graph, meaning that both expressions are equivalent and result in the same graph.

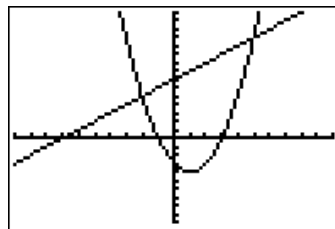## Solving Equations Using the Intersect and the Zero features.

Intersect

When an equation is written in the form "expression 1" = "expression 2", we can enter each expression as a Y – function in the Y – editor, graph, and look at the point/s of intersection of both graphs.
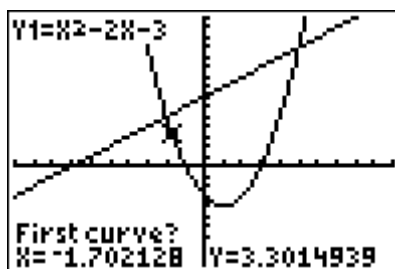
For example, let's solve the equation

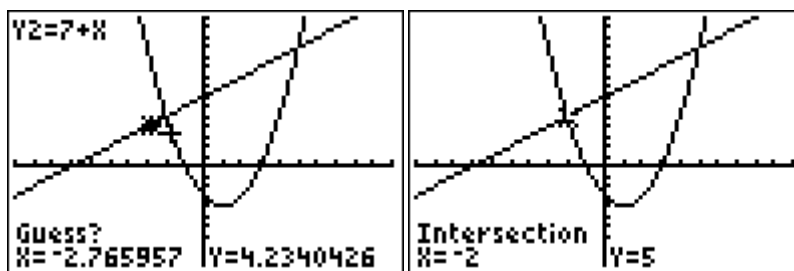In this case there are two points of intersection. To find their coordinates, press <kbd>2nd</kbd> <kbd>TRACE</kbd> and choose option **5: intersect**. Move the cursor to place it near one the points of intersection (use the <kbd>‹</kbd> <kbd>›</kbd> arrows)
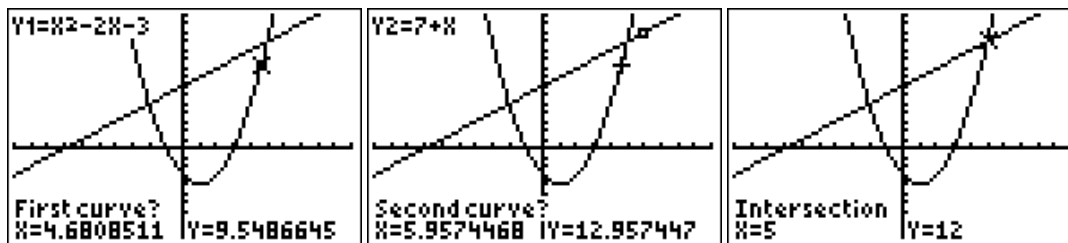
Press [ENTER] . Then move the cursor on the other curve and place it on the other side of the point. Notice that the mark from the first placement and the position of your cursor now must be on different sides of the point, defining a "target zone"; otherwise, the calculator will show you an error message when you press [ENTER] again.


```
Y2=7+X
Second curve?
X=-2.765957   Y=4.2340426
```
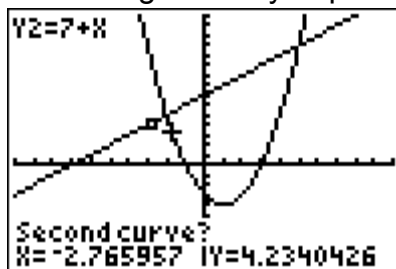
Press [ENTER] twice; you will see the **Guess?** screen first, then the **Intersection** one.


```
Y2=7+X                       

Guess?                      Intersection
X=-2.765957   Y=4.2340426   X=-2         Y=5
```

To find the other point of intersections, we repeat the steps starting with [2nd] [TRACE] , option **5: intersect**


```
Y1=X2-2X-3               Y2=7+X                  

First curve?            Second curve?           Intersection
X=4.6808511 Y=9.5486645 X=5.9574468 Y=12.957447 X=5        Y=12
```
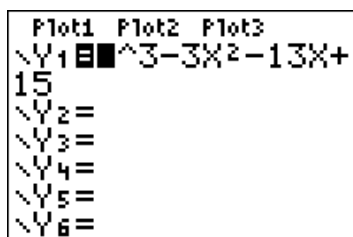
Then, the two points of intersection are: (-2, 5) and (5, 12). Since the equation we were trying to solve was in the variable *x*, the solutions to the equation are the x – coordinates of the points of intersection. That is, the solution set is { -2, 5}
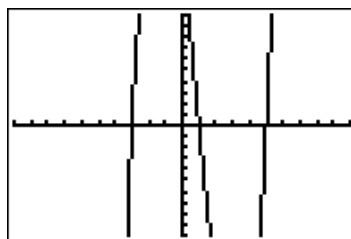
Zero

When an equation is written in the form "expression" = 0, we can enter the expression as a Y – function in the Y – editor, graph, and look at the point/s where the value of the expression is equal to zero. This means that we will be looking at the points with the y – coordinate equal to zero or, in other words, the points on the graph of the function that lie on the x – axis.

For example, let's solve the equation

Enter the polynomial in the Y – editor and graph (shown here in the standard window)

From this graph, we see three points on the graph that have a y – coordinate equal to zero; that is, three points on the graph that lie on the x – axis. To find each of these points we will follow the following steps:

a. Press [2nd] [TRACE] and choose the **2: zero** option. Then, place the cursor to the left of one of the points of interest (use the [‹] [›] arrows). Press [ENTER], then place the cursor to the right of the point.
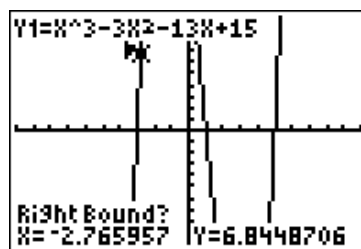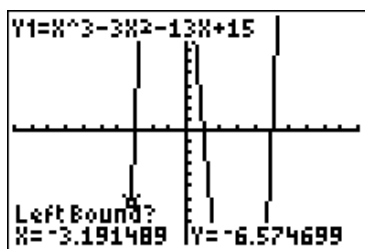
b. Press [ENTER] twice; you will see the **Guess?** screen first, then the **Zero** one.
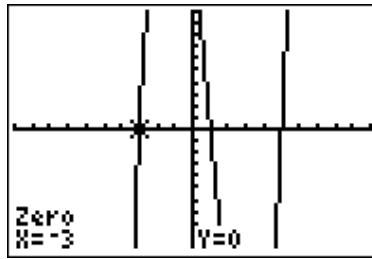
Repeating the steps (a) and (b) for the other two points, we find:
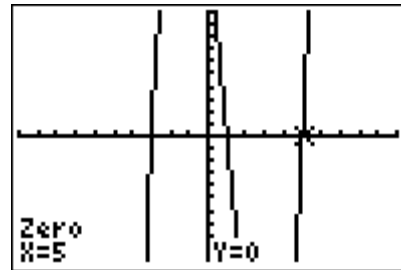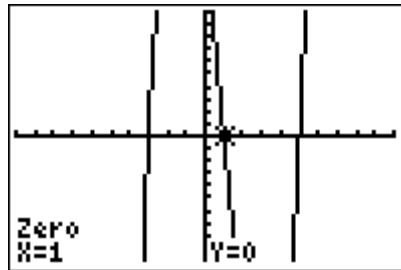
Since the equation we were trying to solve was in the variable x, the solutions to the equation are the x – coordinates of the zeros. That is, the solution set is { -3, 1, 5}
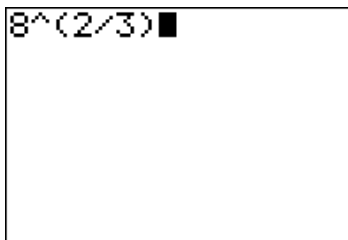
Technology Guide 9

Radicals on your graphing calculator.

Radicals: Please, refer to the Technology Guide 1 to review how to write radicals in your graphing calculator.

Rational Exponents: We mentioned in the Technology Guide 1 that, in general, exponents can be written by using the ⌃ key. When working with radical expressions and equations, we will find that is often necessary to work with rational exponents. These exponents will be also written by using the ⌃ key, but we need to use parentheses to indicate the exponent.
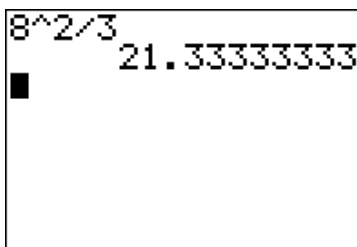
For example, if we want to evaluate ‾, we need to enter:

```
8^(2/3)█
```

to obtain the right answer 4.

If we do not use the parentheses to indicate that the fraction 2/3 is the exponent, we will obtain a wrong answer because, according to the order of operations, the calculator will do   first and then divide it by 3.

```
8^2/3
          21.33333333
█
```
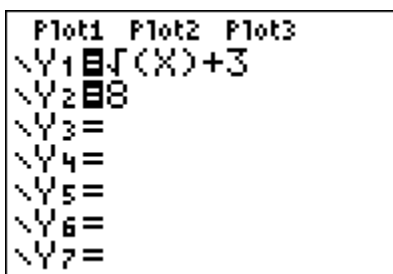
Technology Guide 10

Radicals Equations on your graphing calculator.

Equations: Please, refer to the Technology Guide 8 to review how to solve equations using the Intersect and Zero features of your graphing calculator.

Radical Equations: Let's see how to solve a radical equation with our calculator. For example, the equation ‾

We must first enter each side of the equation as a Y- function in our Y- editor

Secondly, we need to choose a good window where the graphs of both functions can be seen. In this case, the second function represents a horizontal line through the value 8, so the standard window can be a good one to start with.

Then, press GRAPH to see the graphs of both equations and, in particular, to look at the intersection (or lack thereof) of both.
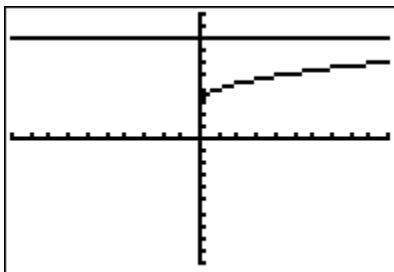
In this case, we notice that that the intersection happens at values of x that are greater than the *x* = 10 which limits the standard window, so we need to modify the window settings to see the graph further to the right. We may also want to change the limits for the variable y, to avoid the point of intersection showing too close to the edge of the screen and make finding its coordinates a bit easier.

For example,
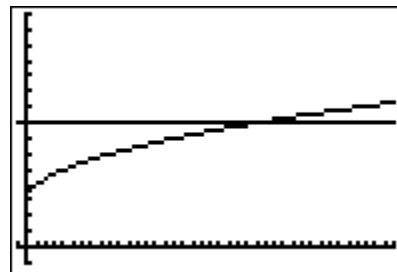
Now, we can use the [2nd] [TRACE] , **5: intersect** option to find the point of intersection.
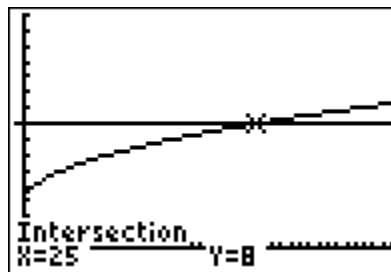
and, since the original equation was on the variable *x*, we find the value *x* = 25 as a *potential* solution. To check that it is indeed a solution, we must replace that value in the equation and verify that it makes it true:

Now, we can say that *x* = 25 is the solution to the equation.

Technology Skills Assignment 1

Basic operations on your graphing calculator.

Use your graphing calculator and the instructions in the Technology Guide 1 to evaluate the following:

    a. $25 - 9 \div 3 + 4$

    b. $\dfrac{20 - 6^2}{9 - 5}$

    c. $\sqrt[3]{64}$

    d. $\sqrt[3]{\dfrac{12}{25}}$

    e. $\dfrac{20 - 3 * 9}{-14}$

<div style="border:1px solid black; padding:10px">

<div align="center">IN CLASS EXERCISE:</div>

Evaluate the following expression

$$\frac{\sqrt[3]{8} - 5}{3^2 + 4}$$

</div>

Answers:

    a. 26

    b. -4

    c. 4

    d. .7829735282

    e. .5

Technology Skills Assignment 2

Evaluating and graphing functions

Use your graphing calculator and the instructions in the Technology Guide 2
.

1) Generate a table of values of the function _____ , starting at x = -2 and at intervals of 3

2) Generate a table that shows the values of *f(0)*, *f(-3)*, and *f(11)* for the function

3) Obtain the graph of the function _____ showing values of *x* from -20 to 20 at intervals of 2, and values of *y* from -100 to 100 at intervals of 10.

4) Obtain the graph of the function _____ in the standard (default) window. (Remember to use the (−) key for the negative slope!)

IN CLASS EXERCISE:

Given the function

Generate a table (with the **Ask** option) to evaluate *f(-1)*, *f(3)*, and *f(0)* and obtain

---

Answers:

1)

```
 Plot1  Plot2  Plot3
\Y1◪4X-7
\Y2=■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
TABLE SETUP
 TblStart=-2
 △Tbl=3■
Indpnt: Auto  Ask
Depend: Auto  Ask
```

```
   X    | Y1   |
 -2     | -15  |
  1     | -3   |
  4     | 9    |
  7     | 21   |
 10     | 33   |
 13     | 45   |
 16     | 57   |
X=-2
```

2)

```
 Plot1  Plot2  Plot3
\Y1◪5-2X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
TABLE SETUP
 TblStart=0
 △Tbl=1
Indpnt: Auto  ███
Depend: Auto  Ask
```

```
   X    | Y1   |
  0     | 5    |
 -3     | 11   |
 11     | -17  |
        |      |
X=
```

3)

```
 Plot1  Plot2  Plot3
\Y1◪5X-6
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=-20
 Xmax=20
 Xscl=2
 Ymin=-100
 Ymax=100
 Yscl=10■
 Xres=1
```
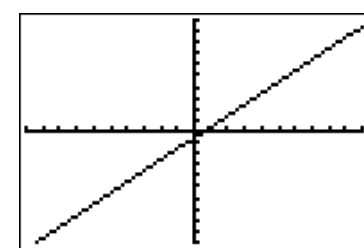

4)

```
 Plot1  Plot2  Plot3
\Y1◪-4X+5■
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```
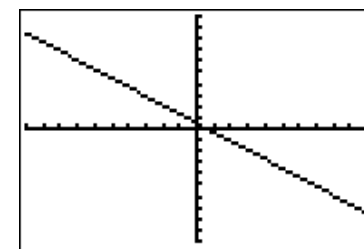
Technology Skills Assignment 3

Solving Systems of Linear Equations by Graphing

Use your graphing calculator and the instructions in the Technology Guide 3.
Find the solution to each of the following systems of equations. Express the solution as a pair (x,y)

1) $\begin{cases} x + y = 6 \\ 3x - y = 2 \end{cases}$

2) $\begin{cases} 3y - 5x = 8 \\ 4y - 4x = 0 \end{cases}$

3) $\begin{cases} 3x + 4y = 1 \\ 2x - 3y = 12 \end{cases}$

IN CLASS EXERCISE:

Solve the following system of linear equations by graphing with your calculator.

1) $\begin{cases} 2x - 3y = -3 \\ -3x + 2y = 3 \end{cases}$

Answers:

1) (2, 4)

2) (-4, -4)

3) (3, -2)

Technology Skills Assignment 4

Checking the solutions of an Equation or a Systems of Equations

Use your graphing calculator and the instructions in the Technology Guide 4.

    I.   Check that the given ordered pair is the solution to the corresponding system of equations.

Pair (3, 0) for the system $\begin{cases} 2x + y = 5 \\ 3x - y = 2 \end{cases}$

Pair (-2, 4) for the system $\begin{cases} 3y - 5x = 8 \\ 4y - x = 1 \end{cases}$

Pair (1, -3) for the system $\begin{cases} 3x + y = 0 \\ 2x - 3y = 11 \end{cases}$

    II.   Is the triple (2, 1, 0) a solution to the system $\begin{cases} 3x - y + 2z = 5 \\ 2x - 3y + 4z = 1 \\ x + 2y - 3z = 4 \end{cases}$?

IN CLASS EXERCISE:

Is the pair (8, -3) a solution to the following system of equations?
Answer using your graphing calculator and the STO feature.

$$\begin{cases} 2x - 3y = -3 \\ -3x + 2y = 3 \end{cases}$$

Answers:

I.

1)

```
3→X
              3
0→Y
              0
2X+Y
              6
```
The pair (3, 0) is not a solution of the system

2)

```
-2→X
             -2
4→Y
              4
3Y-5X
             22
■
```
The pair (-2, 4) is not a solution of the system.

1)

```
1→X: -3→Y
             -3
3X+Y
              0
2X-3Y
             11
```
The pair (1, -3) is a solution of the system.

II.

```
2→X:1→Y:0→Z
              0    3X-Y+2Z            0
3X-Y+2Z                               5
              5    2X-3Y+4Z
2X-3Y+4Z                              1
              1    X+2Y-3Z
■                                     4
```
All three equations are verified; the triple (2, 1, 0) is a solution of the system.

Technology Skills Assignment 5

Graphing linear inequalities in two variables.

Use your graphing calculator and the instructions in the Technology Guide to graph the following inequalities.

$$y > 3x + 2$$

$$y \leq -x + 2$$

$$y - 2x > 3$$ (First put the inequality in slope y-intercept form)

$$y > \frac{1}{2}x + 2$$

$$4x + 2y \leq 8$$ (First put the inequality in slope y-intercept form)

IN CLASS EXERCISE:

Graph the inequality using your graphing calculator. Put it in slope y - intercept form first:

$$y + 4 > 3x$$

Answers:

$y > 3x + 2$
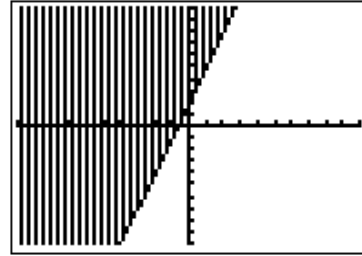
Graph

$y \leq -x + 2$
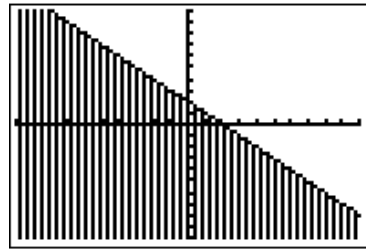
Graph

$y - 2x > -3$ (First put the inequality in slope y-intercept form)
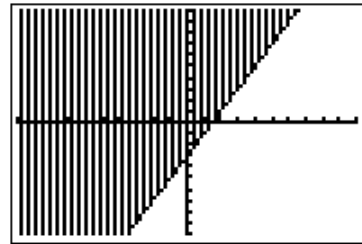
Graph

$y > \dfrac{1}{2}x + 2$
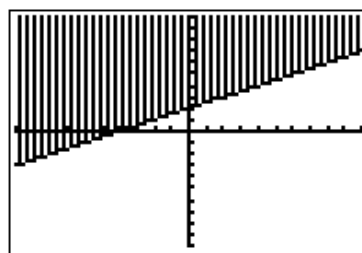
Graph

$4x + 2y \leq 8$ (First put the inequality in slope y-intercept form)
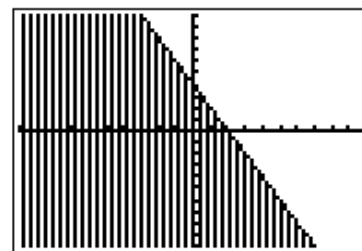
Graph

Technology Skills Assignment 6

Graphing systems of linear inequalities in two variables

Use your graphing calculator and the instructions in the Technology Guide to solve the following inequalities by graphing.

Be sure to put the inequalities in slope y-intercept form first.

$$x - y > 4$$
$$x + y < 6$$

$$2x - y < 4$$
$$3x + 2y > -6$$

$$y < -2x + 4$$
$$y < x - 4$$

IN CLASS EXERCISE:

Graph the system of linear inequalities using your graphing calculator. Put them in slope y_intercept form first:

$$y > 2x - 3$$
$$y < -x + 6$$

Answers:

$x - y > 4$

$x + y < 6$

Graph

$2x - y < 4$

$3x + 2y > -6$
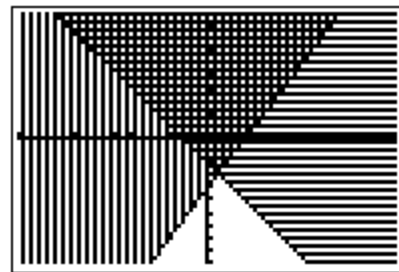
Graph

$y < -2x + 4$

$y < x - 4$
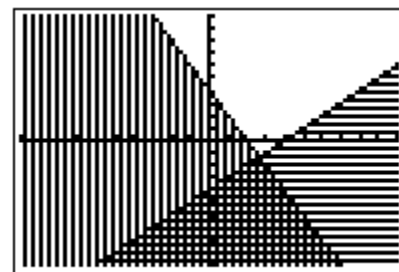
Graph

Using Graphs to check a factorization of a polynomial

Use your graphing calculator and the instructions in the Technology Guide 7.

Factor each polynomial and then enter the original in Y_1 and the factored version in Y_2. If the graphs and the values in the table are the same, the factorization is correct.

$8x^2 + 10x + 3$

$16y^2 - 4y - 2$

$5y^3 - 45y^2 + 70y$

$2x^3 - 5x^2 - 12x$

IN CLASS EXERCISE:

Factor the polynomial and see if your factorization is correct.

$5x^2 - 16x + 3$

Answers:

$8x^2 + 10x + 3$

$(4x + 3)(2x + 1)$

$16y^2 - 4y - 2$

$2(4y + 1)(2y - 1)$

Plot2 Plot3

\Y₁◼8X^2+10X+3
\Y₂◼(4X+3)(2X+1)
\Y₃=
\Y₄=
\Y₅=
\Y₆=

Equation

X	Y₁	Y₂
0	3	3
1	21	21
2	55	55
3	105	105
4	171	171
5	253	253
6	351	351

X=0

Table

Graph

Plot2 Plot3

\Y₁◼16X^2-4X-2
\Y₂◼2(4X+1)(2X-1)
)
\Y₃=
\Y₄=
\Y₅=
\Y₆=

Equation

X	Y₁	Y₂
0	-2	-2
1	10	10
2	54	54
3	130	130
4	238	238
5	378	378
6	550	550

X=0

Table
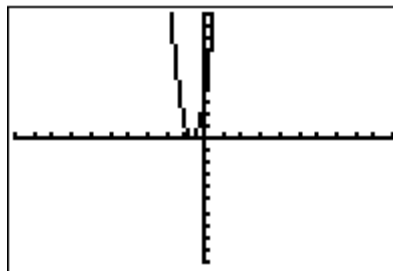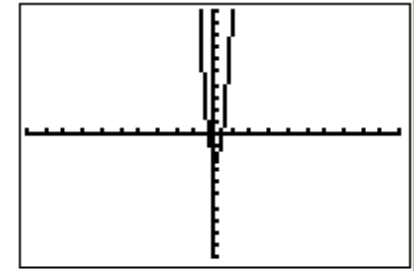
Graph

$5y^3 - 45y^2 + 70y$

$5y(y-2)(y-7)$

$2x^3 - 5x^2 - 12x$

$x(2x+3)(x-4)$

Plot2 Plot3
\Y₁☐5X^3-45X²+70X
\Y₂☐5X(X-2)(X-7)
\Y₃=
\Y₄=
\Y₅=

Equation

X	Y₁	Y₂
0	0	0
1	30	30
2	0	0
3	-60	-60
4	-120	-120
5	-150	-150
6	-120	-120

X=0

Table
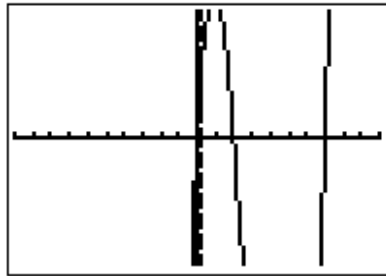
Graph

Plot2 Plot3
\Y₁☐2X^3-5X²-12X
\Y₂☐X(2X+3)(X-4)
\Y₃=
\Y₄=
\Y₅=

Equation

X	Y₁	Y₂
0	0	0
1	-15	-15
2	-28	-28
3	-27	-27
4	0	0
5	65	65
6	180	180

X=0

Table
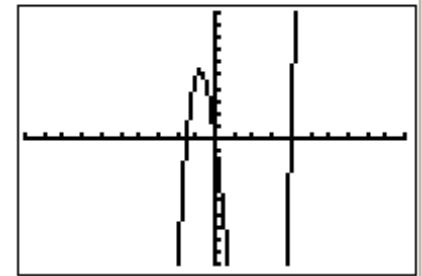
Graph

Technology Skills Assignment 8

Solving Equations Using the Intersect and the Zero features.

Use your graphing calculator and the instructions in the Technology Guide 8.

Enter each side of the equation as an expression in the Y editor and look at the points of intersection.

In some cases you may have to change the YMax and Ymin values to see the entire graph.

You can also check the table to see where the Y_1 value equals the Y_2 value. That is another way to find the solution.

$$x(x-3) = 18$$

$$x^2 + 6x = 7$$

$$x(x+1) = 12$$

$$x(x+9) = 4(2x+5)$$

IN CLASS EXERCISE:

Solve the equation and check the points of intersection using your graphing calculator.

$$(x-1)(x+4) = 14$$

Answers:

$x(x - 3) = 18$ {-3, 6}

$x^2 + 6x = 7$ {-7, 1}

$x(x + 1) = 12$ {-4, 3}

$x(x + 9) = 4(2x + 5)$ {-5, 4}

Technology Skills Assignment 9

Radicals on your graphing calculator.

Use your graphing calculator and the instructions in the Technology Guide 9.

Evaluate the following expressions.

$$\sqrt{32}$$

$$\sqrt{8} - 12$$

$$\sqrt[3]{36}$$

$$12^{4/3}$$

$$-3^{2/5}$$

IN CLASS EXERCISE:

Use your graphing calculator to evaluate

$$36^{2/3}$$

Answers:

$\sqrt{32}$ = 5.656854249

$\sqrt{8-12}$ This problem results in a non – real answer, so your calculator will show the following message:

```
ERR:NONREAL ANS
1 Quit
2:Goto
```

$\sqrt[3]{36}$ = 3.301927249

$12^{4/3}$ = 27.47314182 (remember to use parentheses for the fraction in the exponent!)

$-3^{2/5}$ = -1.551845574

Technology Skills Assignment 10

Radicals Equations on your graphing calculator.

Use your graphing calculator and the instructions in the Technology Guide 10.

Solve the following radical equations.

$4 - \sqrt{x} = 5$

$\sqrt[3]{x} + 5 = 3$

$\sqrt{2x + 5} = x + 1$

$\sqrt{2x + 8} = 5$

IN CLASS EXERCISE:

Use your graphing calculator to solve the following equation

$$\sqrt{3x - 2} + 3 = x + 3$$

Answers:

$4 - \sqrt{x} = 5$
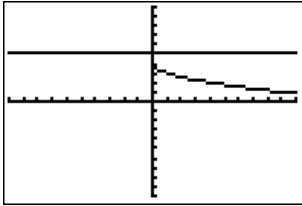
There is no intersection; the equation has no solution.

$\sqrt[3]{x} + 5 = 3$
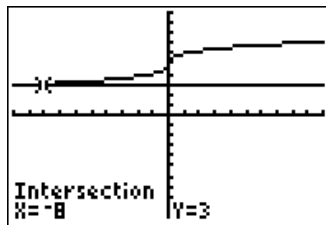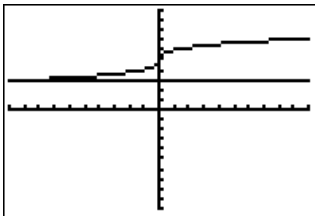
The solution of the equation is the value $x = -8$

$\sqrt{2x + 5} = x + 1$
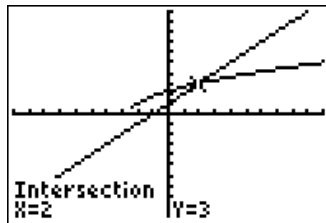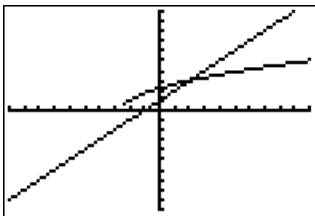
The solution of the equation is the value $x = 2$

$\sqrt{2x + 8} = 5$

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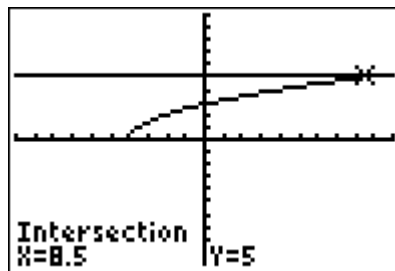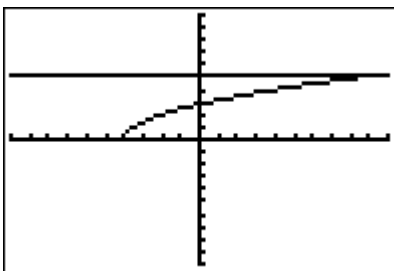
MyMathLab

Homework

Activity: MyMathLab homework assignments

Learning Objectives:

- Students will be able to practice skills learned through readings, lectures, and module workbooks.
- Students will be able to utilize online help to garner further understanding of concepts, while working on practice problems.
- Students will be able to understand and follow the steps in the examples given for each problem that they are working on in MyMathLab.
- Students will be able to identify the concepts that they don't fully understand, and use available resources in MyMathLab to improve their understanding and/or to generate discussions in class.
- Students will be able to increase their ability to work with technology.

Core Student Success Skills: Organization, Communication, Critical thinking, and Self-Assessment

Materials: Textbook (or e-text if in RAMP-Up format), CourseCompass access code, course ID, notebook, and pencil.

Context within the course: This activity extends across the entire course.

Procedure:

The first time MyMathLab is accessed through coursecompass.com the student will need to register as a new user.

a. Go to www.coursecompass.com

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b. Select "Students", and "Register"

c. Follow the prompts on the screen to enter the access code which is provided with the textbook (or the one you can buy online at the Home page of coursecompass.com). The course ID will be provided by your instructor.

d. Use the tutorials provided with coursecompass to become familiar with the product.

After the initial setup, each time the student wants to access MyMathLab, he/she will need to log in at the home page of www.coursecompass.com

Each module workbook includes a reference to MyMathLab exercises at the end of most sections. At that point, the student should log in to his/her account, open the Intermediate Algebra course, and select the Do Homework button to be directed to the list of homework assignments.
Students should work on the problems in a notebook so they can keep the work for future reference, to ask questions to the instructor, and/or to discuss the material in class.

Notes to Instructors:
MyMathLab exercises should be checked regularly to encourage the
students to stay up to date with their homework, and to identify any common areas of difficulty
students may be encountering.

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MyMathLab: Instructions for Students
There are MyMathLab exercises for most sections of each module. The exercises are to be completed by going to www.coursecompass.com and either registering as a new user initially or logging in with your ID. Once you are registered or logged in, you will choose the Intermediate Algebra course and select the Do Homework button.

This will bring you to a screen that lists all the homework assignments available for you to work on.
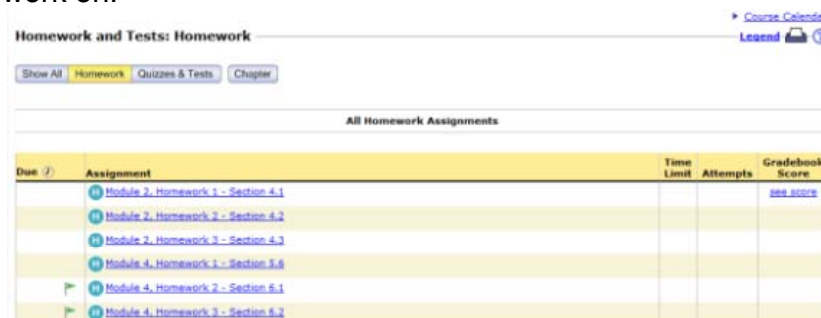
Select the homework for the module and section that you are currently working on. This will take you through the individual problems for that section. Some of the problems will be straight forward; others will require multiple steps and critical thinking. For each problem you will have a variety of buttons displayed on the left-hand side and the right-hand side of the screen to give you immediate access to help.
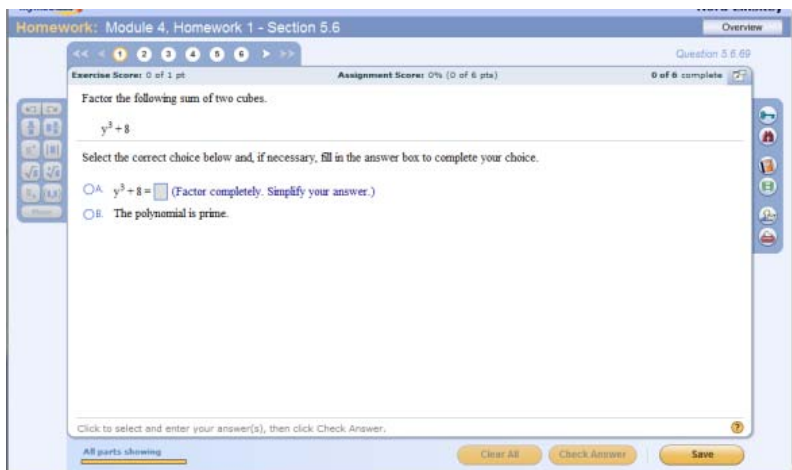
The buttons include:

 Left hand button panel gives you access to special formatting for answers,including fractions, exponents, radicals, decimal point, and ordered pairs.

 "Help me solve this" will take you through the given problem, step by step, then provide you with a new problem for you to work out yourself.

 "View an example" will take you step by step through a similar problem, then allow you to continue working on the problem you were already working on.

 "Textbook" will allow you to access the textbook for the section where the type of problem you are working on is discussed.

 "Ask my instructor" will allow you to send a link to the problem with a question/comment from you to your instructor.

 "Print" will allow you to print out a problem to solve out or bring to class.

Be sure to click the SAVE button to save your answers once exercises are completed!

These exercises are a significant portion of your grade. Doing the MyMathLab exercises can significantly improve your grade; likewise, not doing them or only occasionally doing them can significantly hurt your grade. They are not difficult to do but require discipline and organization on your part. Set aside time every day to complete the assigned exercises and other packet materials.

Review the module packets before, and during your work on these exercises.

Appendices

Appendix A

Letter to Instructors

Dear Intermediate Algebra Instructors:

Welcome to another semester of teaching Intermediate Algebra. This letter, the syllabus, module workbooks, technology guides and assignments, course objectives, and other course materials are available at the Intermediate Algebra Instructor Information Blackboard site.

Textbook: <u>Intermediate Algebra through Applications,</u> 2nd Edition, by Geoffrey Akst and Sadie Bragg, Addison Wesley, 2009. ISBN – 978 – 0 – 321 – 518019

Prerequistes: C or better in MAT 080 or MAT 075 or placement by exam

Graphing Calculator: TI-83 / TI-83+/ TI-84

MyMathLab: MyMathLab (MML) is an online teaching and learning environment that is text specific. All the students will receive MML access codes when they purchase their books; they can also buy access at the website www.coursecompass.com. Each instructor can set up a MML course, but the exercises, quizzes, and tests that support these modules will be available to copy upon request.

Please, feel free to contact me at any time with suggestions or concerns. You can also participate in discussions on the Intermediate Algebra Instructor Information Blackboard site. If you create activities and would like to share them with other instructors, please send them to me and I will make them available on the Blackboard site.

Have a great semester!

Sincerely,

Maria F. Arambel

Maria F. Arambel
Lowell City, 5th Floor, Cubicle # 49
Phone: (978) 656 – 3126
arambelm@middlesex.mass.edu

Sample Syllabus

Middlesex Community College
MAT 100-50 – Intermediate Algebra
FALL 2011
MWF   **9:30AM – 10:20AM**   **Room: LC- 306**

Instructor: **MARIA ARAMBEL**
Office: Lowell City Campus, 5th floor, Cube 49
Office Hours: MTWF 11:30am to 12:30pm (or by appointment)
Phone: (978) 656- 3126
e-mail: arambelm@middlesex.mass.edu

Course Description: MAT 100, Intermediate Algebra, is for students preparing for the Precalculus – Calculus sequence or for Statistics. Topics include graphing and modeling with linear and quadratic functions; solving applications; interpreting data read from graphs; solving absolute value equations and inequalities; solving systems of equations in two and three variables algebraically and graphically; operations with rational expressions; fractional exponents; radicals; complex numbers; and the solving of rational, radical, and quadratic equations.

Prerequisites: C or better in MAT 080 or MAT 075 or placement by exam.

This course was redesigned as part of the Title III grant, Strategies for Success: Increasing Achievement, Persistence, Retention, and Engagement. The course materials will focus on key skills of **critical thinking, communications, collaboration, organization, and self – assessment**

Text: **:** Intermediate Algebra through Applications, 2nd Edition, by Geoffrey Akst and Sadie Bragg, Addison Wesley, 2009. ISBN – 978 – 0 – 321 – 518019

Required Materials:

* Calculator: A TI-83 / TI-83+/ TI-84 graphing calculator is **required**. You must bring your calculator to every class. Calculators cannot be shared during tests.

* MyMathLab student access code. **COURSE CODE:** arambel19411

Course Goals: It is the goal of this course to enable students to develop a proficiency in the algebraic skills needed for the study of advanced math courses and a prerequisite for Precalculus and Statistics. Introduction and utilization of the TI-83/84 graphing calculator is a fundamental component of this course. It is also the goal of this course to develop and extend students' critical thinking skills and students' abilities to interpret, analyze, evaluate, infer, and synthesize concepts studied in Algebra.

Teaching Procedures: Classroom instruction combines lecture, group discussion, and collaborative learning. Students should bring to class the workbook corresponding to the module currently being covered since some of the activities in the workbook will be done at home and some will be done in class.

Communication: I will use Blackboard to communicate with the students. You may find the homework assignments, projects, and general announcements there. I may also e-mail students with notices of class cancellations, changes in schedule and other notifications. I will use MCC e-mail addresses so please check your account daily. When you need to e-mail me, please clearly indicate your name in the message; I can't guess who "soxfan88" is!

Attendance and Behavior: Students are expected to be present at all classes and attendance will be recorded at every class. Attendance counts for 10% of your grade; I will allow a maximum of four (4) absences during the semester. If tardiness becomes a problem I may enforce the policy of closing the door five minutes after the start of the class, and no student will be allowed to enter if the door is closed.
We will work with a lot of handouts and data that may not be found in the textbook. Therefore, it is the responsibility of each student to get all materials given in class when he/she has been absent (even if the absence was a legal one).
Students must be present for all tests. Each student will be allowed ONE MAKE-UP of each test, provided that all homework and workbook activities have been completed thus far and that attendance is at least 90%. Make-ups must be taken outside the regular class meetings and within 7 days of the original test.
Cell phones, pagers, iPods, and other similar gadgets are required to be turned off while students are in the classroom. **No texting or listening to music during class!**
Misconduct of an extreme nature and/or plagiarism will be handled as specified under the Student Handbook rules.

Assignments and Assessment: Homework will be assigned regularly on MyMathLab and in the form of papers to be collected **at the beginning of class** on the due date. Its completion is vital for mastery of the course materials. **All relevant steps must be shown in the paper you turn in or it won't be graded**. Late homework will be accepted only after <u>documented</u> legal absences. The lowest two homework grades will

be dropped, and the rest will be averaged. Homework must be done in a neat manner, in clean paper, and legible handwriting; I will return your paper without grading if it is messy and disorganized.

Each Technology Skills Assignment includes one In – Class problem; each problem will be given 1 point if done correctly so a total of 10 points can be obtained at the end of the semester. The total grade corresponding to these assignments will be counted as one homework grade. (For example, if you get 6 out of 10 points, a grade of 60 will appear as an extra homework grade in your record)

Grading: Your grade will be computed as follows:

Exams	……… 45 %
Homework	……… 25 %
Attendance	……… 10 %
Final Exam	……… 20%

Support Services: Please, do not wait until failing a test to look for help! I'll be happy to work with you outside class time. You can come to my office during office hours or we can set up a time we both find convenient. Other sources for assistance available are:
- The Computer Lab, LC-2nd floor
- The Math Lab, LC- 4th floor in Lowell, or HH-116 in Bedford

* Students with disabilities are encouraged to speak with me about their need for accommodations.

In the event of inclement weather, students can secure notice of class cancellations from television station WCVB Channel 5, WHDH Channel 7, WBZ Channel 4 or radio stations WBZ 1030AM, WCAP 980AM, WRKO 680AM. Other media outlets may also be used to broadcast closing and delayed opening information. Students may also contact the College at 781-280-3200 or 978-656-3200 for an automated announcement for closing or delayed opening information.

The Major Educational Outcomes

For Students Completing the Requirements of the Mathematics Course Sequence

Students who complete the requirements of one or more courses beyond Algebra II or complete Math Connections:

1. Apply mathematical concepts and reasoning skills to model and solve real world problems.

2. Acquire the ability to read, write, listen to and speak mathematics.

3. Use a variety of approaches (e.g. pattern recognition, modeling, logical reasoning, life experiences, etc) to solve problems.

4. Formulate conclusions and judge the reasonableness of the conclusions.

5. Use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results.

6. Demonstrate understanding of the concept of function by several means – verbally, numerically, graphically, and symbolically.

7. Analyze data and use probability and statistical models to make inferences about real-world situations.

8. Gain a sense that mathematics is a growing discipline interrelated with human culture, and connected to other disciplines.

Extracted from The Middlesex Community College, Academic Program Review of the Mathematics Department

Student Learning Outcomes

Upon successful completion of this course, a student should be able to:

1. **Use Graphs and Functions to Model Data**
   - Graph linear and non-linear equations
   - Define a function and correctly use the symbolism and vocabulary associated with functions
   - Use Linear Functions to Model Data
   - Use quadratic functions to model data.

2. *Solve a Variety of Types of Inequalities and Related Equations and Applications*

   - Solve simple inequalities
   - Solve compound inequalities
   - Solve absolute value equations
   - Solve absolute value   inequalities
   - Graph linear inequalities in two dimensions.
   - Solve systems of linear equations in two or three variables

3. **Use Factoring to express a Polynomial in a More Useful or Simpler Form and to Solve Equations and applications**

   - Factor polynomials by various methods
   - Solve selected non-linear polynomial equations and applications modeled by non linear equations by factoring.
   - Determine the domain of rational expressions and perform basic operations on rational expression using factoring when necessary.
   - Solve rational equations with multiple term denominators
   - *Graph rational functions and identify horizontal asymptotes, intercepts and points of discontinuity.**

4. **Apply the Laws of Exponents and Relate Fractional Exponents to Radicals**

   - Apply Laws of Exponents
   - Simplify and perform basic operations with radicals whose index is n
   - Simplify and perform operations with complex numbers
   - *Graph radical functions and identify intercepts and points of discontinuity.**

5. **Apply the quadratic formula.**

- Use quadratic functions to model data. (also under Point 1.)
- *Graph quadratic functions from the standard form, the vertex form, and the factored form where applicable.**

* *Optional material*; these outcomes are sought as time permits within each section.

Graphing Calculator Skills Objectives for MA1106, Intermediate Algebra

At the end of this course the student will be able to perform the following tasks:

1. Perform arithmetic calculations involving multiple operations using parentheses to ensure the proper order of operations.

2. Graph equations of linear, quadratic, absolute value, cubic, rational, and radical functions.

3. Use the storage registers (A through Z) appropriately to verify answers to equations and applications. Select registers that reflect correct mnemonics.

4. Use the TABLSET and TABLE functions to verify answers numerically.

5. Use the ZOOM functions (excluding those related to Trigonometry and Statistics) to efficiently resize graphs when needed.

6. Use the WINDOW function to display all salient features of the functions (from equations and application problems) on the graphing screen.

7. Navigate between the TABLE and GRAPH of a function, utilizing the interconnections between the two.

8. Create paper graphs from the graphing screen that are correctly labeled and appropriately scaled to suit the graphing and/or modeling situation.

9. Select and use the correct function of the CALCULATE screen to solve problems of Values, Zeros, Minimums, Maximums and Intersections.

10. Solve real-life applications using linear functions, inequalities, or quadratic functions as models graphically and numerically.

11. Recognize those circumstances in which the graphing calculator fails to return a reliable answer, such as, points of discontinuity, error messages of division by zero, and simple errors of operation made by the student.

12. Recognize answers and graphs that are obviously wrong due to erroneous inputs, such as, the graph of a straight line when a parabola was expected or an answer of grossly wrong magnitude.

13. Troubleshoot a calculator problem independent of the instructor's help.