CHAPTER 4 AXIAL LOAD

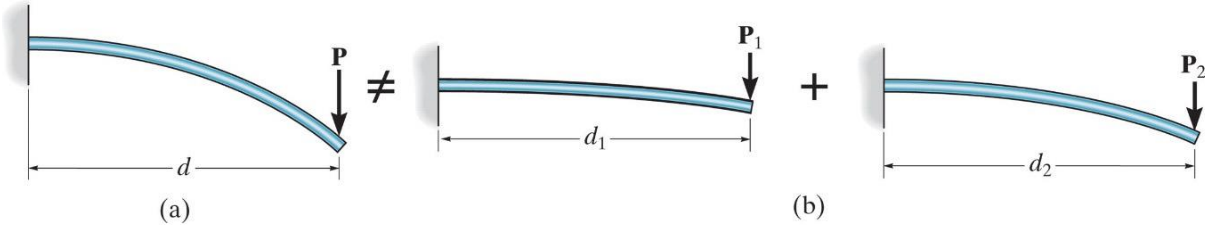
3. PRINCIPLE OF SUPERPOSITION

The principle of superposition states that the resultant stress or displacement at a point can be determined by algebraically summing the stress or displacement caused by each load component applied separately to the member.

The principle of superposition is valid only when:

•The loading is linearly related to the stress or displacement that is to be determined.

•The loading does not change significantly the original geometry or configuration of the member.



4. STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

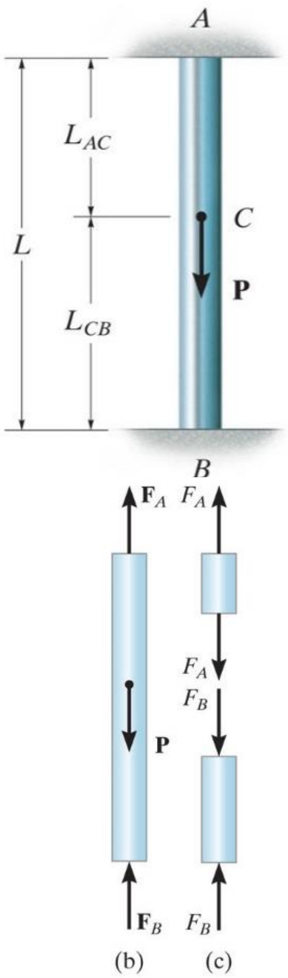
A member is statically indeterminate if the number of unknown reactions is greater than the number of equilibrium equations.

$\Sigma F_y = 0$: FA + FB – P = 0

• Additional equations can be determined by considering the displacements along the bar.

• The equation that specifies the conditions for displacement is known as ***compatibility*** or ***kinematic condition***.

• For example, in an axially loaded bar shown below, ends A and B are fix–supported, thus the total axial displacement of the bar should equal to zero ($\delta_{A/B} = 0$).



If linear–elastic behavior occurs $\delta = PL/AE$

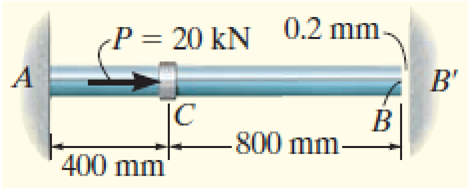
This results in the following relation:

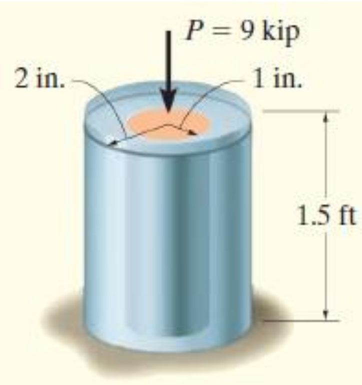
$\delta_{AC} + \delta_{CB} = 0$ Or $$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

Solve

$$F_A = P\left(\frac{L_{CB}}{L}\right) \quad \text{and} \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

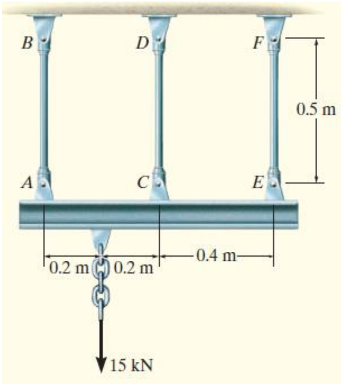
Example 5: The steel rod shown in Figure has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B' and the rod. Determine the reactions at A and B' if the rod is subjected to an axial force of P = 20 KN as shown. Neglect the size of the collar at C. Take E_{st} = 200 GPa.



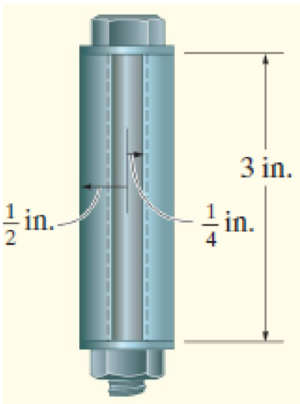


Example 6: The aluminum post shown in Figure is reinforced with a brass core. If this assembly supports an axial compressive load of P = 9 kip, applied to the rigid cap, determine the average normal stress in the aluminum and the brass. Take $E_{al} = 10(10^3)$ ksi and $E_{br} = 15(10^3)$ ksi.

Example 7: The three A992 steel bars shown in Figure are pin connected to a rigid member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars AB and EF each has a cross-sectional area of 50 mm^2, and bar CD has a cross-sectional area of 30 mm^2.

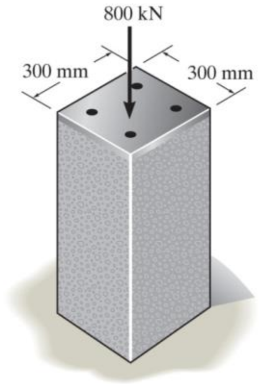


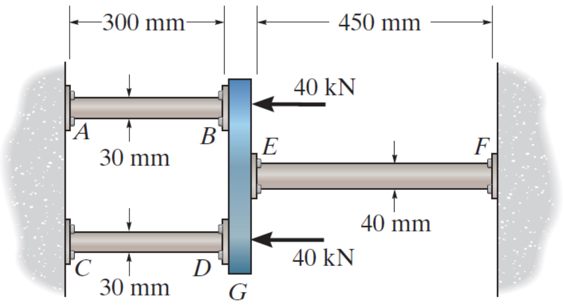
Example 8: The bolt shown in Figure is made of 2014-T6 aluminum alloy, and it passes through the cylindrical tube made of Am 1004-T61 magnesium alloy. The tube has an outer radius of 1/2 in., and it is assumed that both the inner radius of the tube and the radius of the bolt are ¼ in. When the bolt is snug against the tube it produces negligible force in the tube. Using a wrench, the nut is then further tightened one-half turn. If the bolt has 20 threads per inch, determine the stress in the bolt. Take $E_{al} = 10.6(10^3)$ ksi and $E_{mag} = 6.48(10^3)$ ksi.



Sheet No. 2

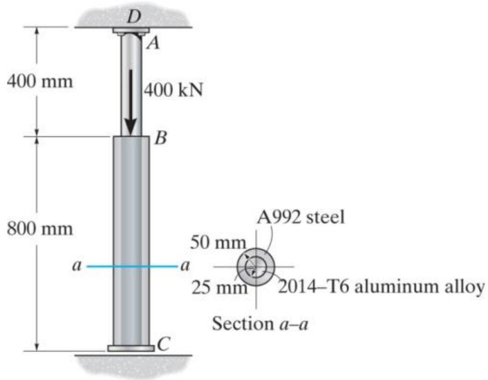
Q 1: The concrete column is reinforced using four steel reinforcing rods, each having a diameter of 18 mm. Determine the stress in the concrete and the steel if the column is subjected to an axial load of 800 kN. E_{st} = 200 GPa, E_c = 25 GPa.

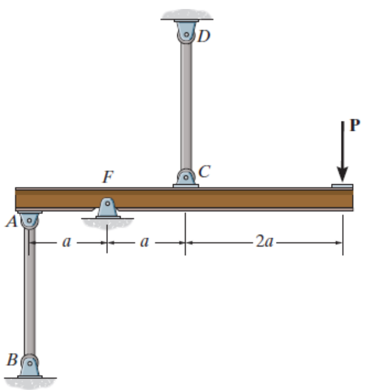




Q 2: The assembly consists of two red brass C83400 copper rods AB and CD having a diameter of 30 mm, a 304 stainless steel rod EF having a diameter of 40 mm, and a rigid member G. If the supports at A, C, and F are fixed, determine the average normal stress developed in the rods when the load is applied.

Q 3: The 2014-T6 aluminum rod AC is reinforced with the firmly bonded A992 steel tube BC. When no load is applied to the assembly, the gap between end C and the rigid support is 0.5 mm. Determine the support reactions when the axial force of 400 kN is applied.





Q 4: Two identical rods AB and CD each have a length L and diameter d, and are used to support the rigid beam, which is pinned at F. If a vertical force P is applied at the end of the beam, determine the angle of rotation of the beam. The rods are made of material that has a modulus of elasticity of E.

5. THERMAL STRESS

Changes in temperature can produce changes in the dimensions of a body.

• In general, a temperature increase will produce expansion in the body.

• Experiments show that for a homogeneous and isotropic body the displacement of a body of length L due to a temperature change is given by:

$$\delta_T = \alpha \, \Delta T L$$

α is the linear coefficient of thermal expansion ($1/^{o}F$, $1/^{o}C$, 1/K)

ΔT is the change in the temperature of the element.

L is the original length of the element.

δT is the change in the length of the member.

• The change in length of a statically determinate members can easily be calculated using the above equation, since the member is free to expand or contract when it undergoes a temperature change.

• When a member is constrained and the temperature is increased, significant reaction forces may be developed in order to keep the material from expanding or contracting due to the temperature change. Therefore, thermal stresses will be produced that must be considered in design.

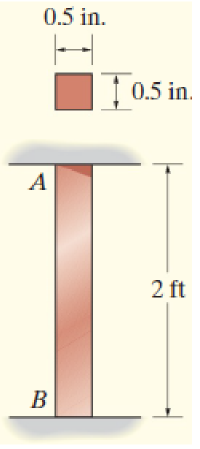
If a member is exposed to an axial force and a temperature change then:

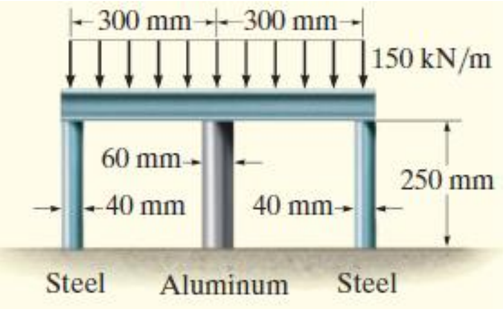
$$\delta = \frac{PL}{EA} + \alpha \Delta T \, L$$

For a member with multiple sections having constant P, A, E, and ΔT:

$$\delta = \sum_i \left[\frac{PL}{EA} + \alpha \Delta T \, L\right]_i$$

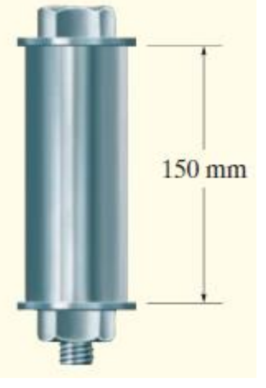
Example 9: The A-36 steel bar shown in Figure is constrained to just fit between two fixed supports when T_1 = 60 ^oF. If the temperature is raised to T_2 = 120 ^oF, determine the average normal thermal stress developed in the bar. Take $E_{st} = 29(10^3)$ kip/in^2 and $\alpha = 6.60(10^{-6})/^oF$.





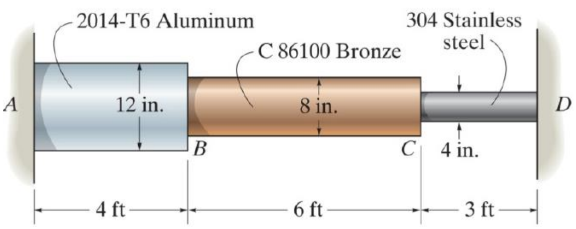
Example 10: The rigid beam shown in Figure is fixed to the top of the three posts made of A992 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the beam, and the temperature is T_1 = 20 ^oC. Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to T_2 = 80 ^oC. Take E_{st} = 200 Gpa, E_{al} = 73.1 Gpa, $\alpha_{st} = 12(10^{-6})/^oc$ and $\alpha_{al} = 23(10^{-6})/^oc$.

Example 11: A 2014-T6 aluminum tube having a cross-sectional area of 600 mm^2 is used as a sleeve for an A-36 steel bolt having a cross-sectional area of 400 mm^2, Figure below. When the temperature is T_1 = 15 ^oC, the nut holds the assembly in a snug position such that the axial force in the bolt is negligible. If the temperature increases to T_2 = 80 ^oC, determine the force in the bolt and sleeve. Take E_{st} = 200 Gpa, E_{al} = 73.1 Gpa, $\alpha_{st} = 12(10^{-6})/^oc$ and $\alpha_{al} = 23(10^{-6})/^oc$.



Sheet No. 3

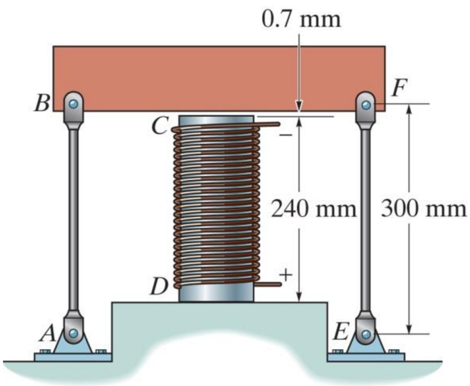
Q 1: The assembly has the diameters and material make-up indicated. If it fits securely between its fixed supports when the temperature is T_1 = 70°F, determine the average normal stress in each material when the temperature reaches T_2 = 110°F.



2014-T6 Aluminum: E_{AL} = 10.6×103 ksi, αAL = 12.8×10^{-6} /°F

C86100 Bronze: E_{BR} = 15×103 ksi, αBR = 9.6×10^{-6} /°F

304 Stainless steel: E_{ST} = 28×103 ksi, αST = 9.6×10^{-6} /°F



Q 2: The center rod CD of the assembly is heated from T_1 = 30°C to T_2 = 180°C using electrical resistance heating. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2.

E_{st} = 200 GPa, E_{al} = 70 GPa, and αal = 23(10^{-6})/°C.

Q 3: The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$).This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, $E = [(E_2 - E_1)/w]\, y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.