Stress Strain Diagram:

For Ductile Materials:

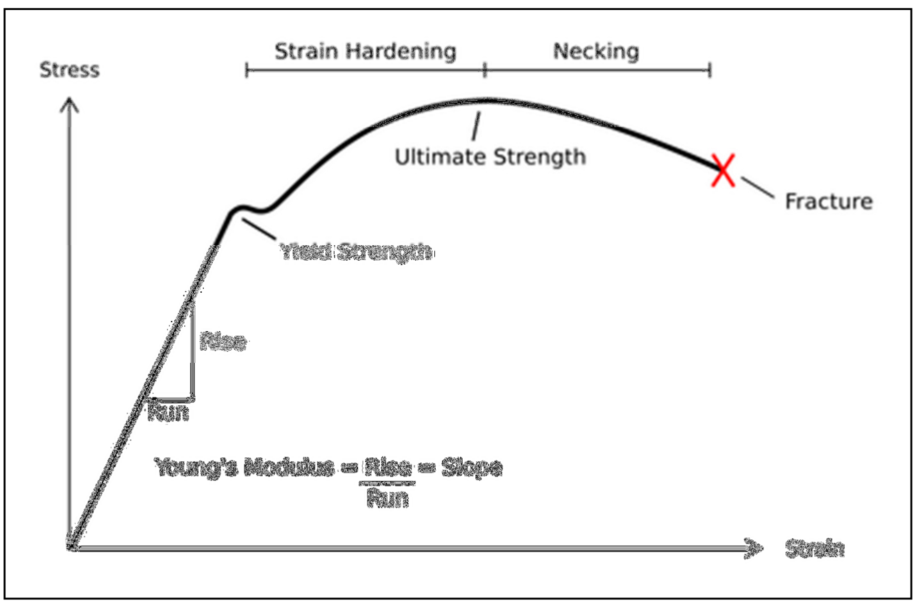
Ductile Materials:

Ductile materials are those which are capable of having large strains before they are fractured. Ductile materials can withstand high stress and are also capable of absorbing large amount of energy before their failure. A ductile material has a large Percentage of elongation before failure.

Some examples of ductile materials are aluminum, mild steel and some of its alloys i.e. copper, magnesium, brass, nickel, bronze and many others.

Stress Strain Diagram For Ductile Material:

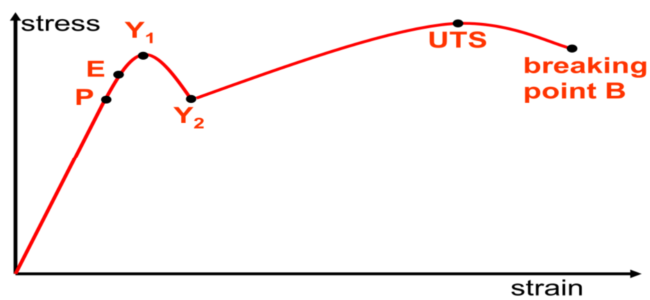
We have taken annealed mild steel as a ductile material.



Different Points On Stress Strain Curve:

- **Proportional Limit (σ_{PL})**

Proportional limit is the point on stress strain curve which shows the highest stress at which Stress and Strain are linearly proportional to each other where the proportionality constant is E known as modulus of elasticity. Above this point, stress is no longer linearly proportional to strain. On stress strain curve, proportional limit is shown by P. It is denoted by $\boldsymbol{\sigma_{PL}}$. For annealed mild steel the limit of proportionality occurs at **230 MPa**.



The above graph shows that the length of graph up to proportional limit (P) is a straight line which means that up to proportional limit stress is linearly proportional to strain.

- **Elastic Limit (σ_{EL})**

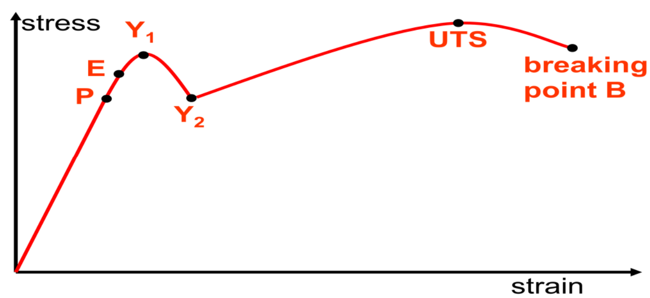
Elastic limit is the point which shows the maximum stress that can be applied to the body without resulting in permanent deformation when stress is removed. At elastic limit when the load is removed from the body, it returns to original size and shape. At elastic limit stress is no longer linearly proportional to strain. It is denoted by $\boldsymbol{\sigma_{EL}}$. For stress strain graph of mild steel, elastic limit is just close to proportional limit.

- **Yield point (σ_Y)**

Yield point is the point which shows the stress at which a little or no increase in stress results to large increase in strain that is material continues to deform without increase in load. At this point the material will have permanent deformation. It is denoted by σ_Y. For steel, yield point is also just above proportional limit. Yield point is of two types:

- Upper yield point.
- Lower yield point.

Upper yield point is shown by Y_1 and lower yield point is shown by Y_2 as in diagram given below:



Among the common materials, only steel exhibits yield point. For annealed mild steel, upper yield point occurs at 260 MPa and lower yield point occurs at 230 MPa.

- **Ultimate Tensile Strength** **(σ_U)**

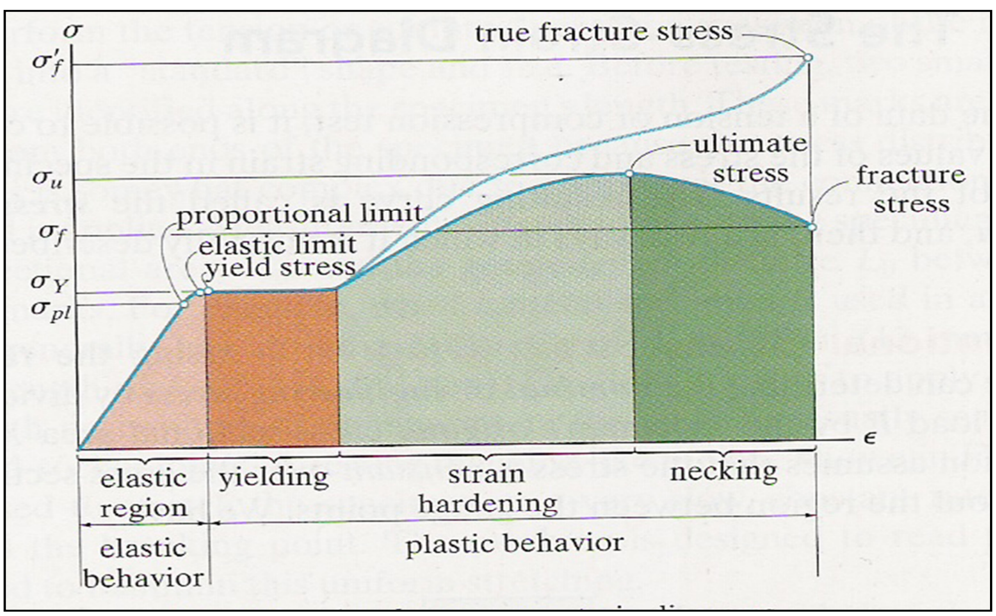
As the stress on material is increased further, the stress and the strain increases from yield point to a point called ultimate tensile strength (UTS) where stress applied is maximum. Thus ultimate tensile strength can be defined as the highest stress on the specimen which it can withstand. For annealed mild steel, ultimate tensile strength occurs at **400 MPa**. It is denoted by $\boldsymbol{\sigma_U}$.

- **Fracture Stress (σ_F)**

After ultimate tensile strength, the applied stress decreases until the stress is obtained where material fractures called fracture stress. Fracture stress is also called breaking strength. It is denoted by $\boldsymbol{\sigma_F}$.

Different Regions under Area of Stress Strain Curve:

This is the general diagram of stress strain curve, which elaborates different regions under stress strain curve.



- **Elastic region:**

Elastic region is the area under the curve from initial point to elastic limit. In this region material will return to its original size and shape when load is removed from the body.

- **Plastic region:**

Plastic region is the area under curve which starts from elastic limit to fracture point. Under the area body shows plastic behavior i.e. when the load is removed from body, it does not come back to its original size and shape.

- **Yielding region:**

This region starts from elastic limit to yield point where the body produces strain with a little or no increase in load.

- **Strain Hardening;**

Area from upper yield point to ultimate tensile stress is called strain hardening. Under this area the body will elongate only with increasing the stress until the stress is at maximum point whereas the cross sectional area will decrease uniformly.

- **Necking:**

Necking covers the area from ultimate tensile stress to fracture point. It is the region where cross sectional area of material will decrease in a localized spot and capacity of material to carry load will decrease. In necking region, stress strain curve has neck like curve.

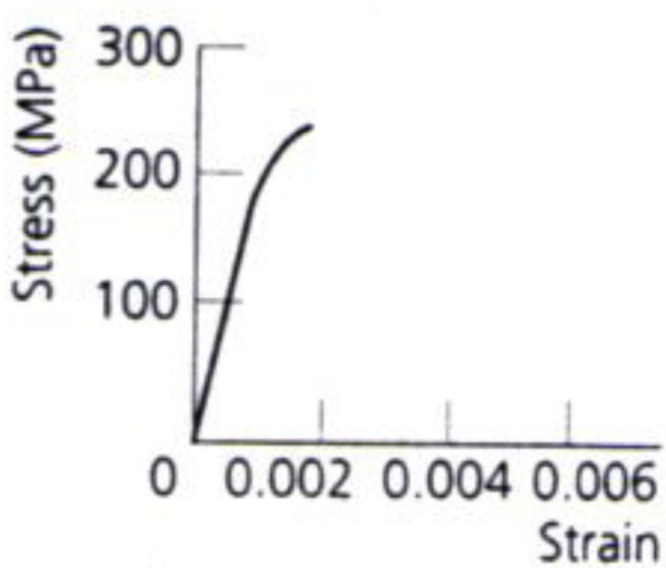
For Brittle Materials:

Brittle Materials:

Brittle materials are those which break suddenly under stress at a point just beyond elastic limit. They have little or no yielding before failure and their percentage of elongation is very low. If percentage elongation is equal to or less than 5%, we consider that material brittle.

Brittle materials include glass, concrete, cast iron and plaster.

Stress Strain Diagram for Brittle Materials:



Above graph shows that gray cast iron exhibit less plastic region i.e it fractures just after elastic limit so it is a brittle material.

Different Points On Stress Strain Curve:

- **Proportional Limit (σ_{PL})**

Like ductile material, proportional limit of brittle material (gray cast iron) is the point on stress strain curve which shows the highest stress at which Stress and Strain are linearly proportional to each other where the proportionality constant is E known as modulus of elasticity. Above this point, stress is no longer linearly proportional to strain. It is difficult to determine the point at which the limit of proportionality occurs, but it is approximately **200 MPa**. It is denoted by σ_{PL}.

- **Elastic Limit (σ_{EL})**

Like ductile materials, elastic limit of brittle material is the point which shows the maximum stress that can be applied to the body without resulting in permanent deformation when stress is removed.
As shown in graph when the load is removed from gray cast iron (at elastic limit), it returns to original size and shape. At elastic limit stress is no longer linearly proportional to strain. It is denoted by σ_{EL}.

- **Ultimate Yield Stress (σ_Y)**

Ultimate tensile stress is the ratio of ultimate load to original area of cross-section. At this point gray cast iron will have little permanent deformation and just after this point, gray cast iron fracture. We can say that yield point is the ultimate stress or breaking stress for gray cast iron. It is denoted by σ_Y. For gray cast iron, it occurs at **250 MPa**. For brittle materials yield point is not well defined.

Mechanical Properties of Materials:

Mechanical Properties:

The properties which determine the applications and behavior of materials are called mechanical properties of the materials. Mechanical properties are helpful in identification of materials because every material has its own identical properties and also tell the usefulness of materials. Mechanical properties are determined through a series of standardized mechanical tests such that Hardness and Tensile testing, Torque testing, Shear testing, Fatigue testing, micro hardness testing, Bend testing and many more.

- ## Ductility:

Ductility is the measure of deformation in a material which it can withstand before fracturing. The most common measure of ductility is the percentage of change in Ductility is generally measured by percentage elongation.

$$\textbf{Percentage elongation} = \frac{l_1 - l_o}{l_o} \times 100$$

High values of percentage elongation indicate that material is very ductile whereas low values indicate that material is brittle and has low ductility. For mild steel, the percentage elongation usually is 20%. Gold is most ductile material.

- ## Brittleness:

Brittleness is the opposite of ductility. It is the property of material to fracture just after elastic limit when stress is applied on it. It is also measured by percentage elongation. Clay, glass and ceramics are some brittle materials.

- **Malleability:**

The ability of material to bend or to be hammered in all the directions without any fracture is called malleability. Malleable materials can be deformed to thin and flat sheets. Most of the malleable materials are also ductile. Gold, iron, aluminum, lead and copper (to some extent) are some examples of malleable material.

- **Toughness:**

The ability of material to absorb energy before rupturing is called toughness. Total area under stress strain curve represents toughness. It is measured by impact test. Unit of toughness is joule per cubic meter **(J/m^3).**

- **Strength:**

The capability of material to withstand the load being applied on it without failure is called strength of material. Strength is of many types i.e. fatigue strength which is capability of material to withstand cyclic loading and impact strength, which is capability of material to withstand suddenly applied load.

- **Elasticity:**

It is the ability of material to produce strain in all directions under the action of applied stress without permanent deformation. The body returns to its original size and shape when unloaded. In stress strain curve, elasticity is the region from zero to elastic limit.

- **Plasticity:**

The ability of material to change in size and shape permanently under the action of applied force is called plasticity. In stress strain curve, plasticity starts from elastic limit and ends at breaking stress. This property of materials is used to mold different materials to desired form.

- **Resilience:**

The ability of material to absorb energy when it is deformed elastically is called resilience. When the body will be unloaded, it'll release energy. It can also be defined as the maximum energy that can be absorbed per unit volume without creating permanent distortion. The energy absorbed in area under stress strain curve from zero to elastic limit is resilience.

$$\boldsymbol{U_r = \frac{\sigma_y^2}{2E}}$$

Where U_r is the modulus of resilience, σ_y is the yield strength and E is the young's modulus.

- **Poisson's Ratio:**

Poisson's Ratio is defined as the ratio of liner strain to the lateral strain. Mathematically it is written as:

$$\textbf{Poisson's ratio} = \frac{\textbf{Lateral strain}}{\textbf{Linear strain}}$$

$$\nu = \frac{-\epsilon_y}{\epsilon_x}$$

Poisson's ratio has no unit.

- **Hardness:**

The ability of material to resist scratch, bending, aberration, wear, tear and indentation is called hardness. A hard material also offers resist the penetration of another bodies in it. Diamond is the hardest naturally occurring material.

- **Flexural Strength:**

The flexural strength is the maximum stress experienced by a body at its moment of fracture. Flexural Strength is also known modulus of rupture or bends strength. It is a mechanical parameter for brittle material. It is taken as a stress so $\boldsymbol{\sigma}$ is used to represent it.

- **Fatigue Ratio:**

Fatigue ratio is the ratio of fatigue strength to tensile strength. The material having high fatigue ratio shows that it will crack down during loading. It is also called "Endurance ratio".

- **Young's Modulus:**

Young's modulus is the ratio of stress applied to the strain under elastic limit. It is given as:

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \frac{\sigma}{\varepsilon}$$

It is also known as tensile modulus or modulus of elasticity. By young's modulus, we can measure the stiffness of elastic material.

- **Tensile Strength:**

Tensile Strength or ultimate tensile strength is the measure of maximum stress that a body can withstand while being stretched or pulled away before necking. It is opposite of compressive strength in which body is compressed.

Relations b/w Elastic Constants:

Following are elastic constants:

E = Young's modulus.

K = Bulk modulus

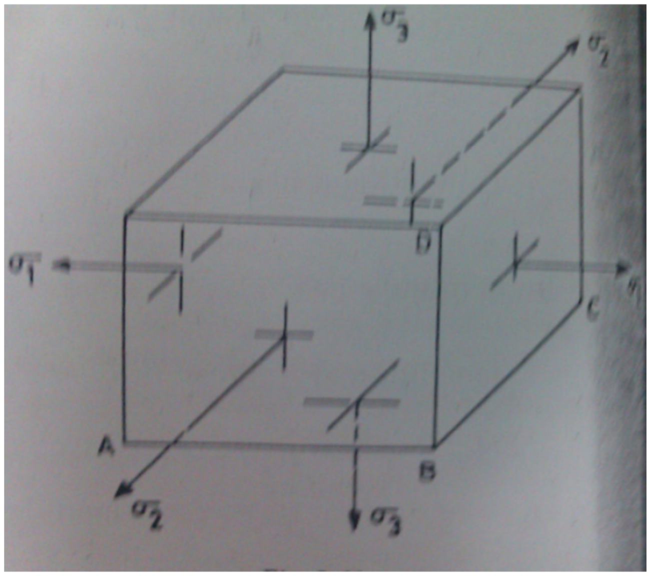
G = Shear modulus

N = Poison's ratio

Relation between *E, K* and *ν*:

Let σ_1, σ_2 and σ_3 be three stresses acting on a rectangular box as shown below:

Let ϵ_1, ϵ_2 and ϵ_3 are three respective strains in three principle directions.



In order to find ϵ_1, consider each of the stresses acting separately and find the strain in the direction of stress σ_1 due to each stress. Then superimpose the results and get net strainϵ_1.

Since σ_1 is acting alone in its direction, the strain produced in the direction of σ_1 is given as:

$$\textbf{Strain} = \frac{\sigma_1}{E}$$

Similarly:

Strain produced due to σ_2 = $\frac{-\nu\sigma_2}{E}$ (negative sign indicate that strain is compressive)

Strain produced due to σ_3 = $\frac{-\nu\sigma_3}{E}$

Therefore:

$$\epsilon_1 = \textbf{net strain in direction of } \sigma_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$$

Hence:

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \qquad \text{............ Eq. (1)}$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_3 + \sigma_1)] \qquad \text{............ Eq. (2)}$$

$$\epsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)] \qquad \text{............ Eq. (3)}$$

Adding Eq. (1), Eq. (2) and Eq. (3), we get:

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = \frac{1}{E}[(\sigma_1 + \sigma_2 + \sigma_3) - 2\nu(\sigma_1 + \sigma_2 + \sigma_3)]$$

$$= (\sigma_1 + \sigma_2 + \sigma_3)\frac{(1-2\nu)}{E}$$

But: $\epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_\nu$

Hence:

$$\epsilon_\nu = \frac{(1-2\nu)}{E}(\sigma_1 + \sigma_2 + \sigma_3) \qquad \text{............ Eq. (4)}$$

From Eq. (4), putting

$\sigma_1 = \sigma_2 = \sigma_3 = -p$, we get:

$$\epsilon_\nu = \frac{(1-2\nu)}{E}(-3p)$$

$$\epsilon_\nu = \frac{3(1-2\nu)}{E}p$$

$$\frac{\epsilon_\nu}{P} = \frac{3(1-2\nu)}{E}$$

$$\frac{P}{\epsilon_\nu} = \frac{E}{3(1-2\nu)}$$

Where:

$$\frac{P}{\epsilon_\nu} = K \text{ (Bulk modulus)}$$

Therefore:

$$K = \frac{E}{3(1-2\nu)}$$ Hence proved:

Relation between E, G and K:

With the reference of above relations:

$$E = 2G\ (1+\nu)$$

$$E = 3K\ (1-2\ \nu)$$

To eliminate ν from the two expressions for E, we have:

$$\nu = \frac{E}{2G} - 1$$

And

$$E = 3K\ [1-2(\frac{E}{2G} - 1)]$$

Solving this equation, we get:

$$E = 9K - \frac{3KE}{G}$$

$$E + \frac{3KE}{G} = 9K$$

$$E = \frac{9KG}{G+3k}$$ Hence proved