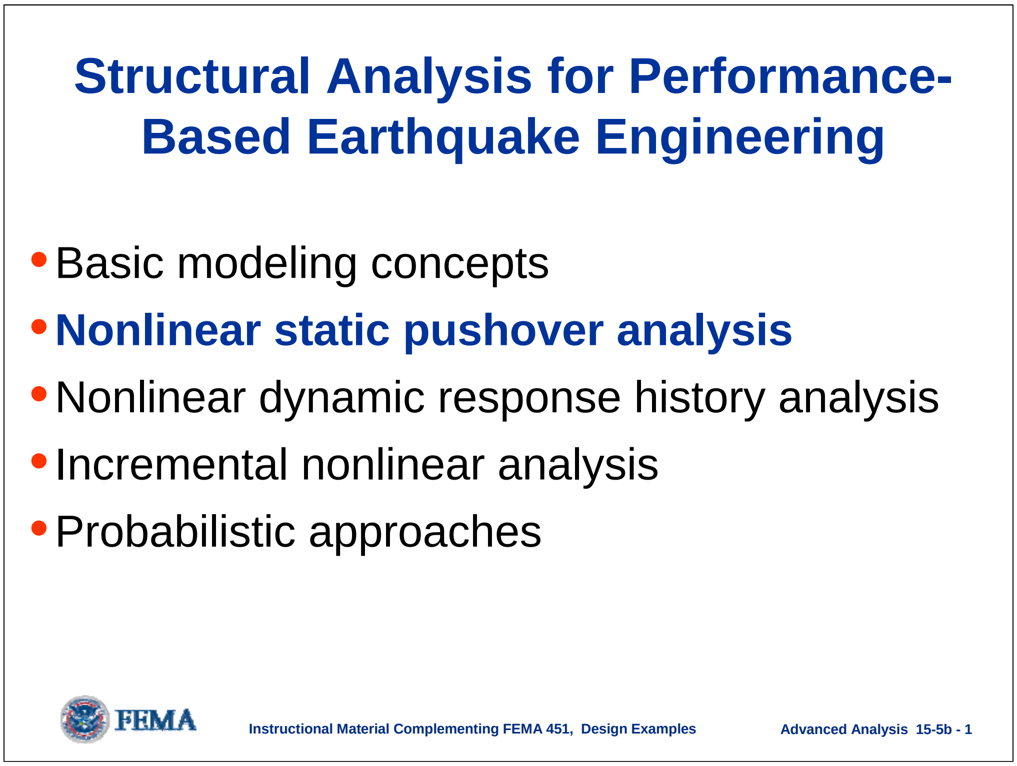

Structural Analysis for Performance-Based Earthquake Engineering

- Basic modeling concepts
- **Nonlinear static pushover analysis**
- Nonlinear dynamic response history analysis
- Incremental nonlinear analysis
- Probabilistic approaches

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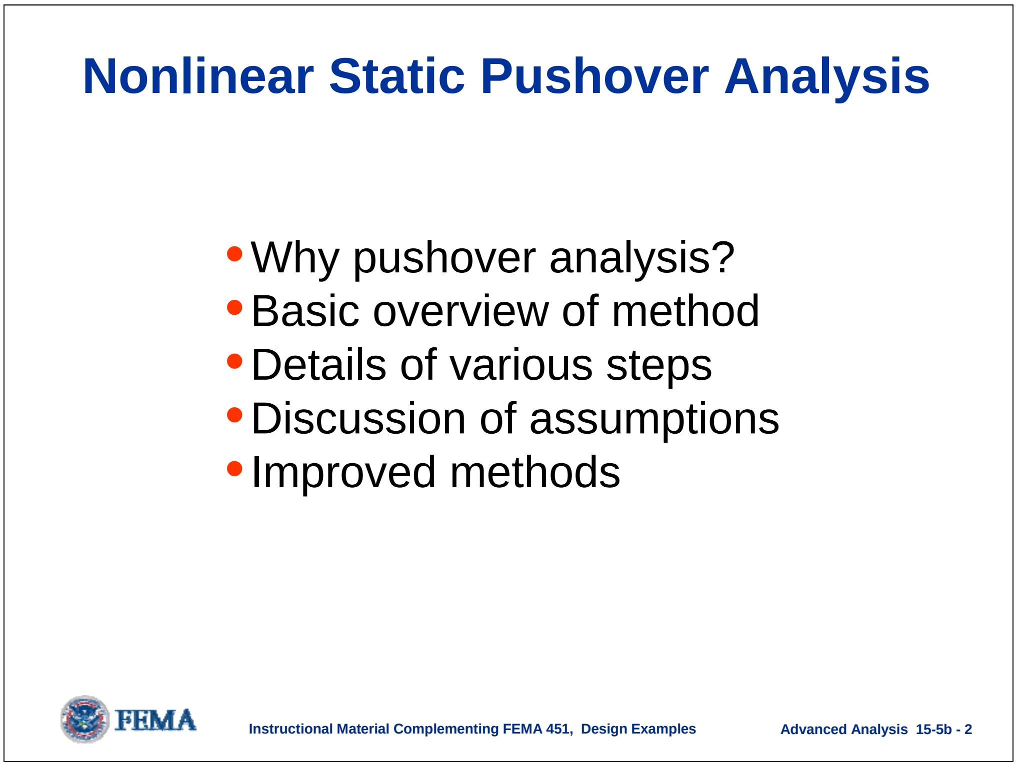
In performance-based engineering it is necessary to obtain realistic estimates of inelastic deformations in structures so that these deformations may be checked against deformation limits as established in the appropriate performance criteria. Two basic methods are available for determining these inelastic deformations: Nonlinear static “pushover” analysis and Nonlinear Dynamic Response History analysis. Pushover analysis is the subject of the next several slides.



Nonlinear Static Pushover Analysis

- Why pushover analysis?
- Basic overview of method
- Details of various steps
- Discussion of assumptions
- Improved methods

These are the basic subtopics discussed in the section on pushover analysis.

Why Pushover Analysis?

- Performance-based methods require reasonable estimates of inelastic deformation or damage in structures.
- Elastic Analysis is not capable of providing this information.
- Nonlinear dynamic response history analysis is capable of providing the required information, but may be very time-consuming.

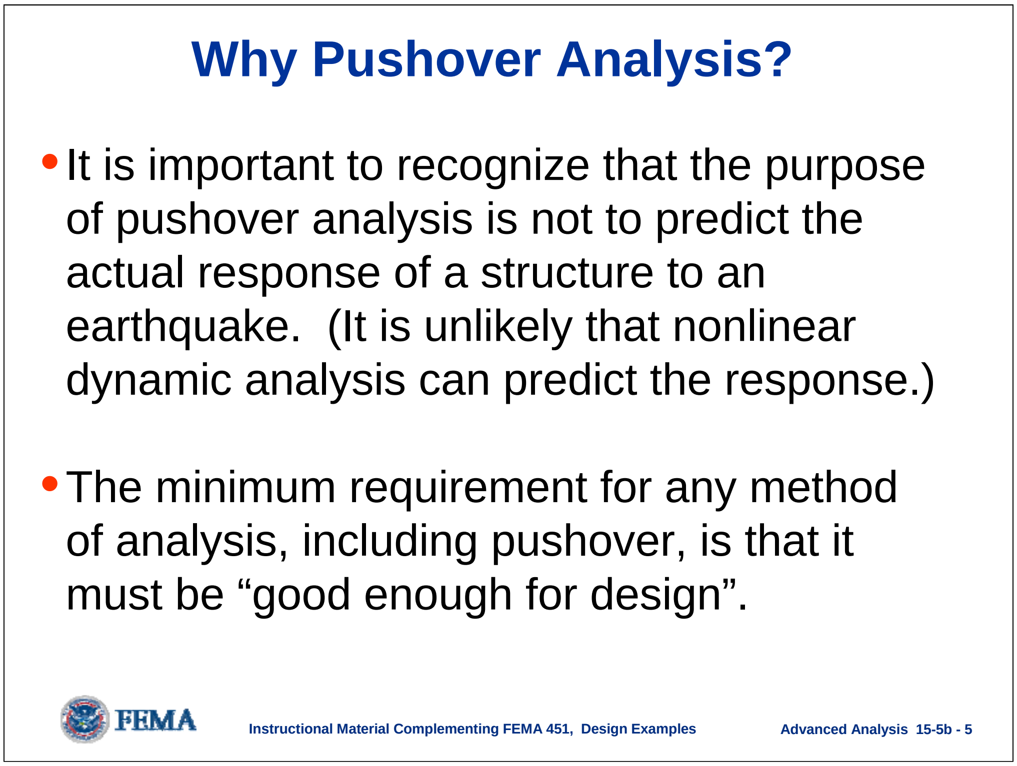
The use of pushover analysis may simply be the lesser of all evils. Elastic analysis does not have the capability to compute inelastic deformations, hence it is out. Nonlinear response history analysis (NRHA) is certainly viable but is very time consuming. Also, NHRA may produce a very wide range of responses for a system subjected to a suite of appropriately scaled ground motions. Computed deformation demands can easily range by an order of magnitude (or more) making it difficult to make engineering decisions. Hence, we are left with Nonlinear Static Pushover Analysis (NSPA) as a reasonable alternative.

Why Pushover Analysis?

- Nonlinear static pushover analysis may provide reasonable estimates of location of inelastic behavior.
- Pushover analysis alone is not capable of providing estimates of maximum deformation. Additional analysis must be performed for this purpose. The fundamental issue is…
  How Far to Push?

NSPA, in addition to providing estimates of deformation demands, provides some useful insight into the pattern of inelastic deformation that may occur. This is very important when assessing desirable behaviors such as strong-column weak-beam behavior.

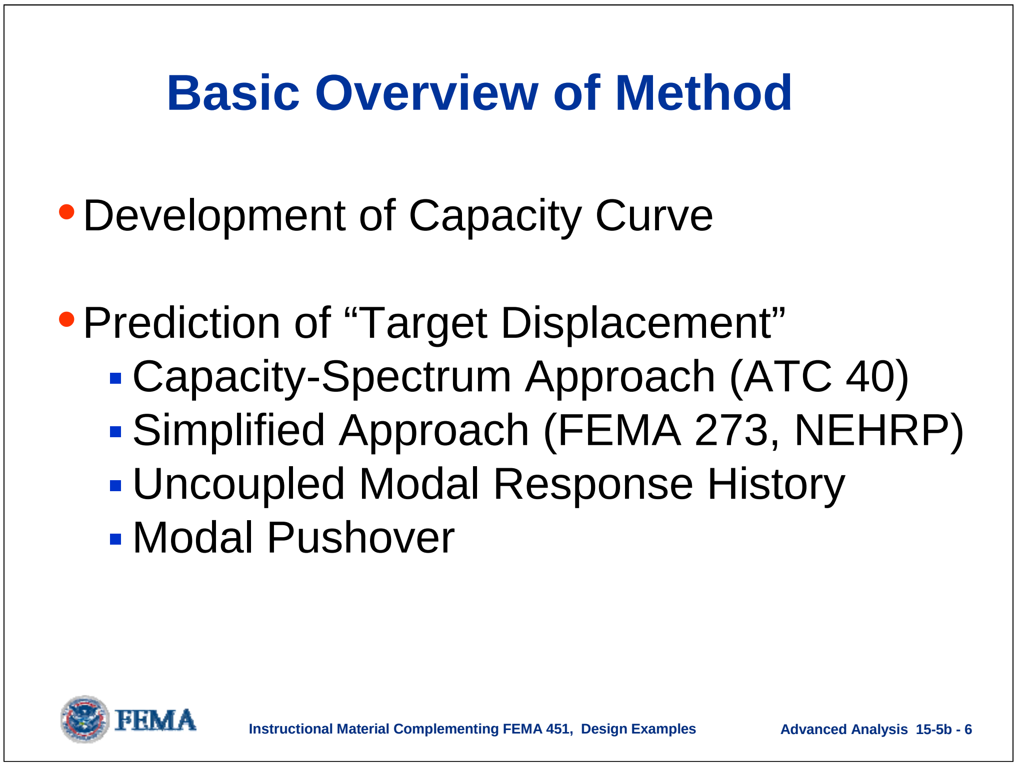
In NSPA an inelastic model is developed and is subjected to gravity load followed by a monotonically increasing static lateral load. While the load pattern is defined, the magnitude of the load is not. The fundamental question in pushover analysis is how far to push? Other computational tools, such as the Capacity Spectrum Approach must be used in concert with NSPA to determine how far to push.



Why Pushover Analysis?

- It is important to recognize that the purpose of pushover analysis is not to predict the actual response of a structure to an earthquake. (It is unlikely that nonlinear dynamic analysis can predict the response.)
- The minimum requirement for any method of analysis, including pushover, is that it must be “good enough for design”.

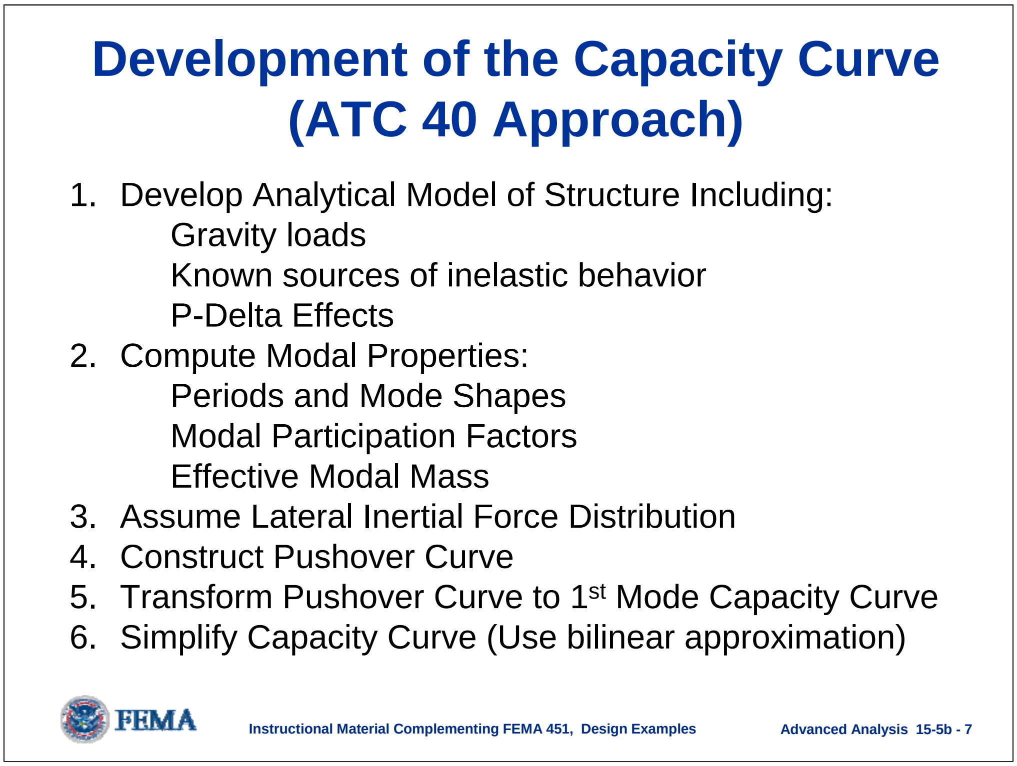
It is very important to note that the purpose of NSPA is not to predict the actual performance of a structure. It is doubtful that even NRHA can do this. The purpose of NSPA is to provide information which may used to assess the adequacy of a design of a new or existing building.



Basic Overview of Method

- Development of Capacity Curve
- Prediction of “Target Displacement”
  - Capacity-Spectrum Approach (ATC 40)
  - Simplified Approach (FEMA 273, NEHRP)
  - Uncoupled Modal Response History
  - Modal Pushover

A pushover analysis consists of two parts. First, the pushover or “Capacity Curve” is determined through application of incremental static loads to an inelastic model of the structure. Second, this curve is used with some other “Demand” tool to determine the target displacement. A variety of demand tools are available, four of which are presented on this slide. In this course emphasis is placed on the first two approaches.



Development of the Capacity Curve (ATC 40 Approach)

1. Develop Analytical Model of Structure Including:
    - Gravity loads
    - Known sources of inelastic behavior
    - P-Delta Effects
2. Compute Modal Properties:
    - Periods and Mode Shapes
    - Modal Participation Factors
    - Effective Modal Mass
3. Assume Lateral Inertial Force Distribution
4. Construct Pushover Curve
5. Transform Pushover Curve to 1st Mode Capacity Curve
6. Simplify Capacity Curve (Use bilinear approximation)

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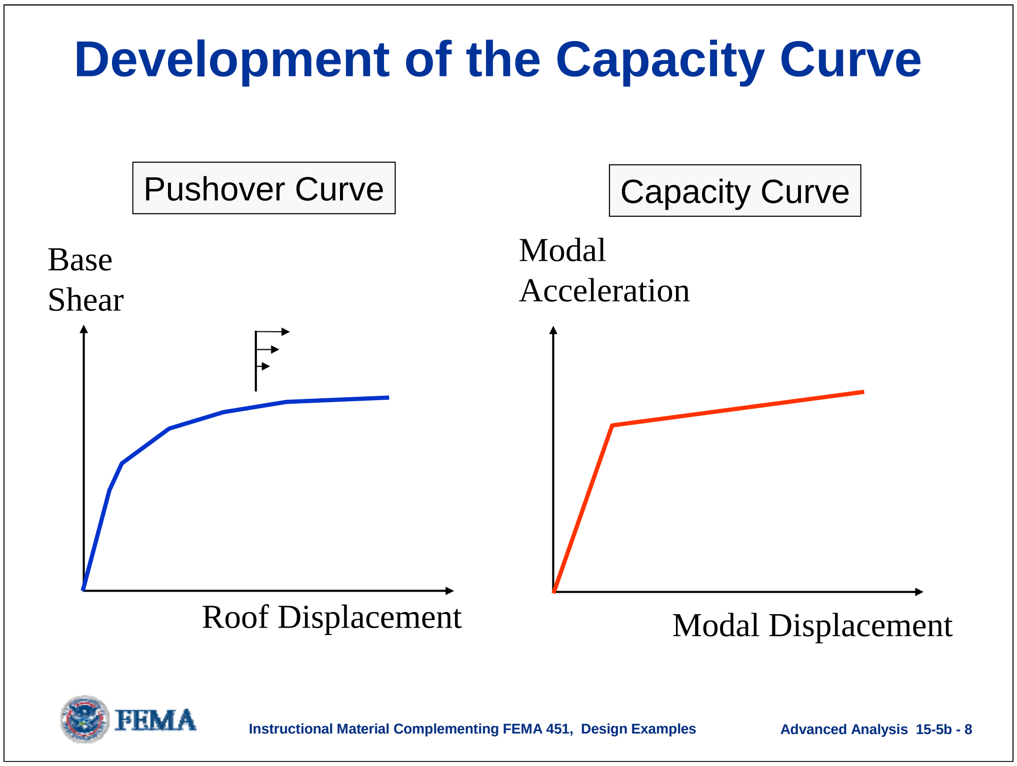
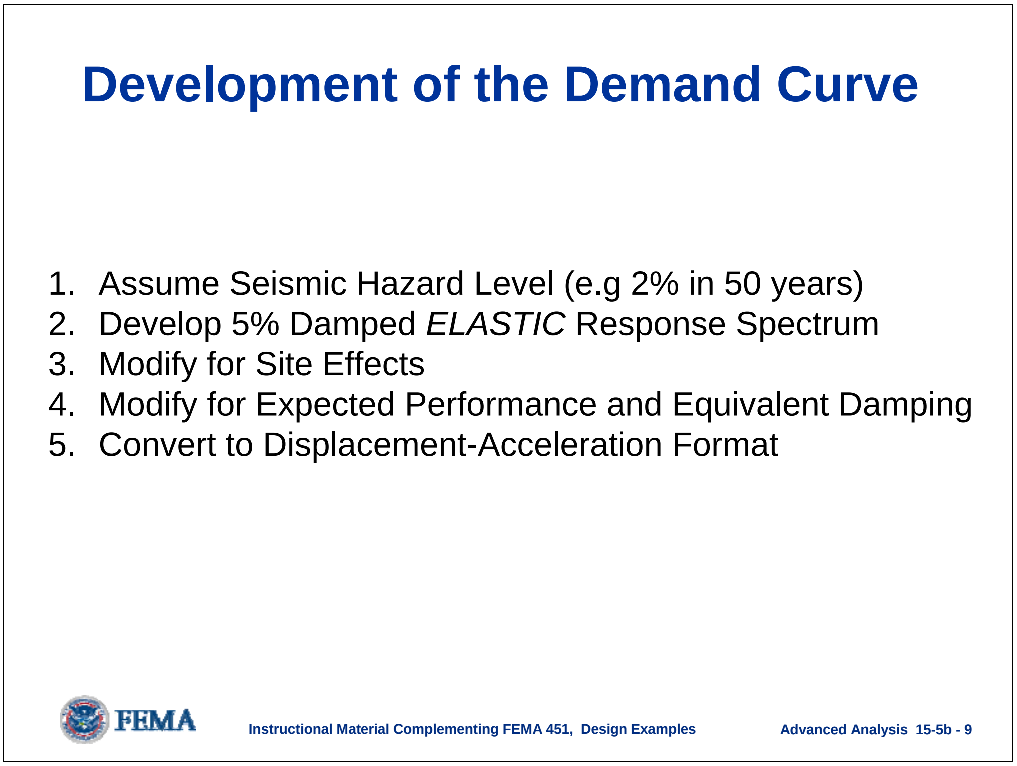
The first approach covered is the so-called Demand Capacity Spectrum approach. This method is described in detail in the ATC 40 document.



Development of the Capacity Curve

The nonlinear static analysis of the structure produces a “pushover curve” as shown at the left. The symbol above the curve indicates that for this curve the lateral load pattern was upper triangular. Other load patterns, such as uniform or proportional to first mode shape will produce different pushover curves.

The curve at the right is a simplified first mode bilinear version of the pushover curve. This curve is called a “Capacity Curve”, or “Capacity Spectrum”. Note that the quantities on the X and Y axes of the capacity curve are modal acceleration and modal displacement. Details on the development of the Capacity Curve are provided later.



Development of the Demand Curve

1. Assume Seismic Hazard Level (e.g 2% in 50 years)
2. Develop 5% Damped *ELASTIC* Response Spectrum
3. Modify for Site Effects
4. Modify for Expected Performance and Equivalent Damping
5. Convert to Displacement-Acceleration Format

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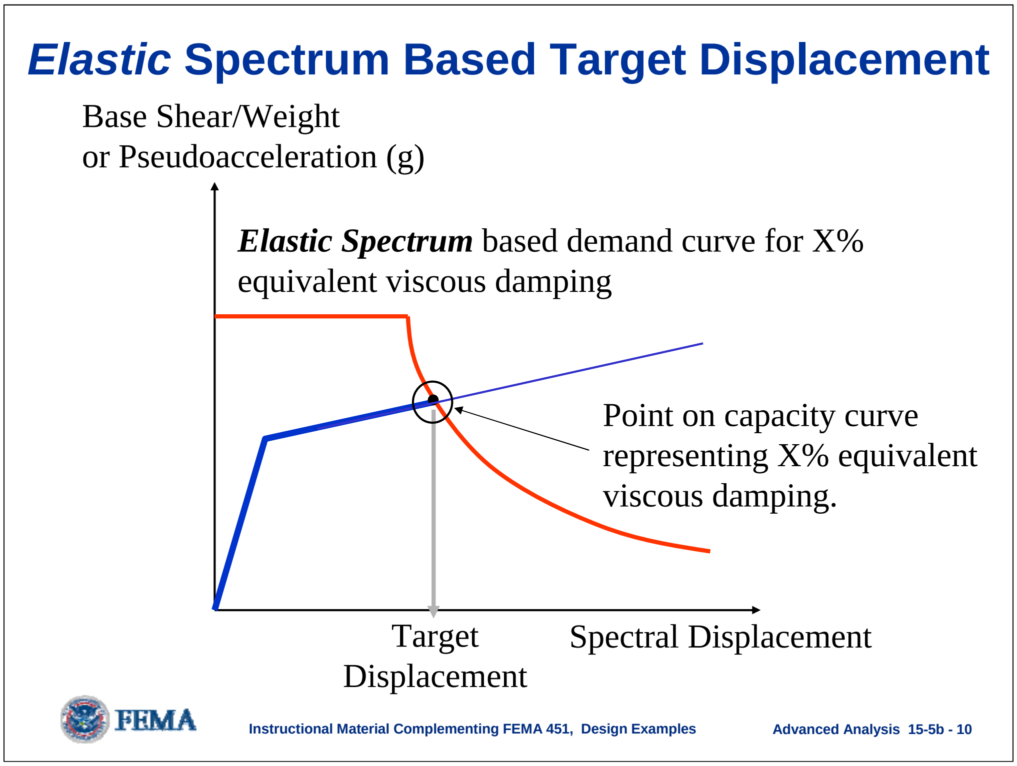
The next step in the analysis is to compute the Demand Curve. This is basically an elastic response spectrum that has been modified for expected performance and equivalent viscous damping. The modifications are HIGHLY EMPIRICAL. The various steps in the development of the demand curve are given here. Details are provided later.



Elastic Spectrum Based Target Displacement

Base Shear/Weight
or Pseudoacceleration (g)

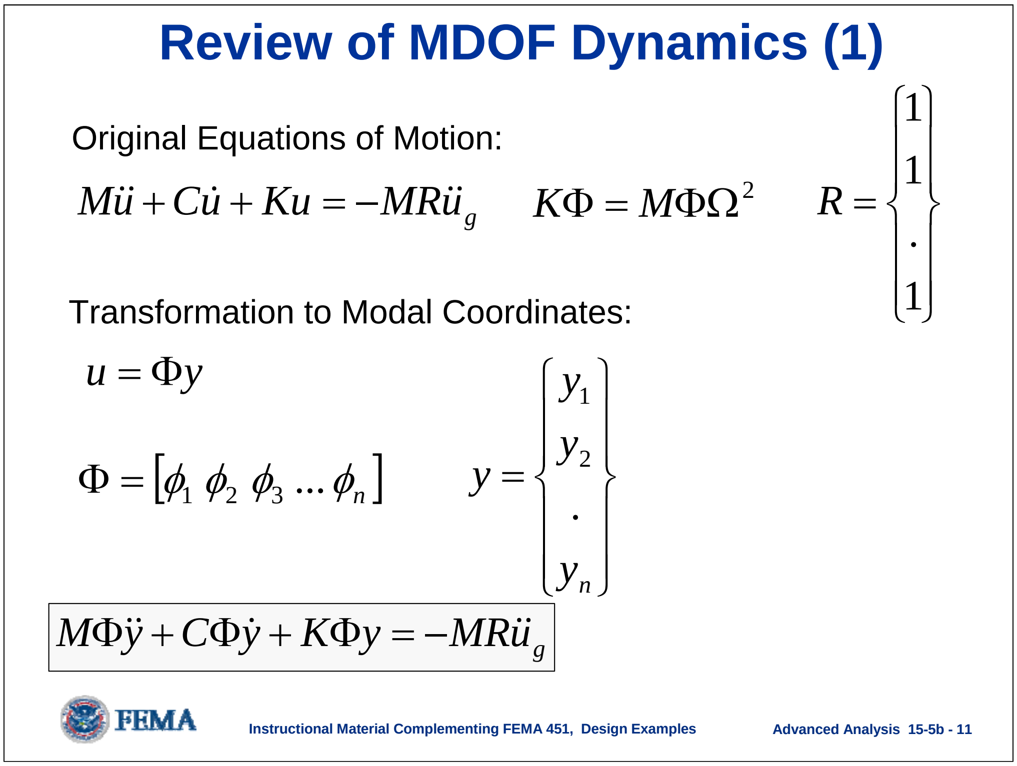
Elastic Spectrum based demand curve for X% equivalent viscous damping

Point on capacity curve representing X% equivalent viscous damping.

Target Displacement

Spectral Displacement

The Demand Curve is used in concert with the Capacity Curve to predict the target displacement. A trial-an-error procedure is typically used to compute the target displacement.



Review of MDOF Dynamics (1)

Original Equations of Motion:

$$M\ddot{u} + C\dot{u} + Ku = -MR\ddot{u}_g \qquad K\Phi = M\Phi\Omega^2 \qquad R = \begin{Bmatrix} 1 \\ 1 \\ \cdot \\ 1 \end{Bmatrix}$$

Transformation to Modal Coordinates:

$$u = \Phi y$$

$$\Phi = \left[\phi_1 \; \phi_2 \; \phi_3 \; \ldots \; \phi_n\right] \qquad y = \begin{Bmatrix} y_1 \\ y_2 \\ \cdot \\ y_n \end{Bmatrix}$$

$$\boxed{M\Phi\ddot{y} + C\Phi\dot{y} + K\Phi y = -MR\ddot{u}_g}$$

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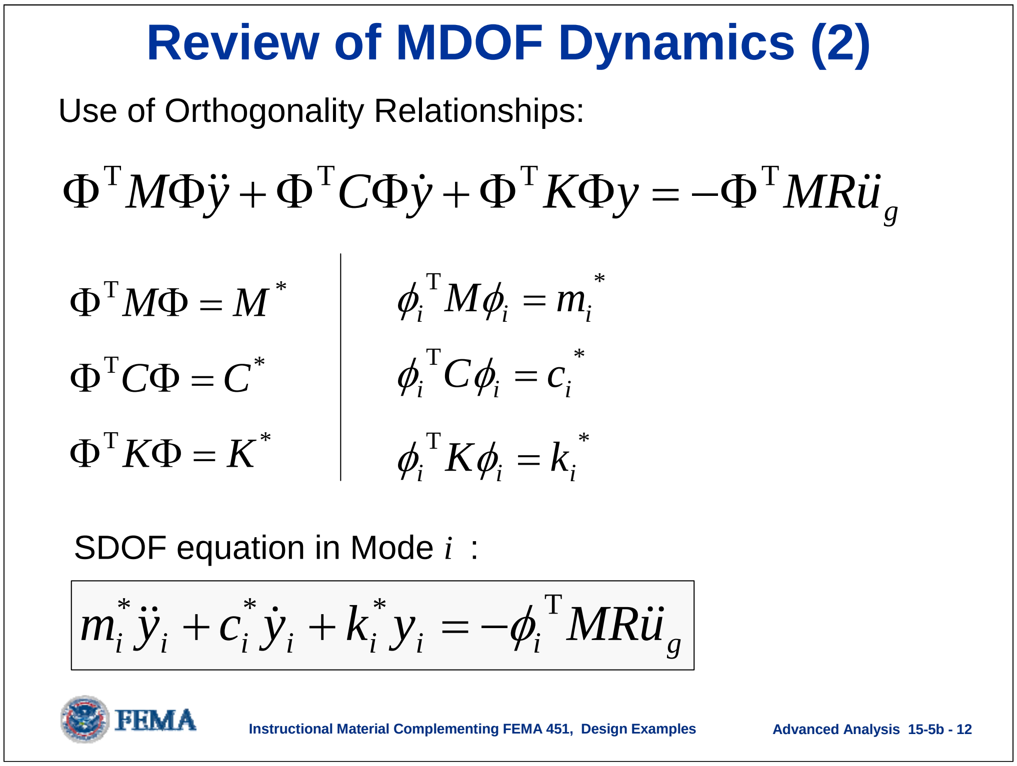
Recall that a main step in the NSPA procedure is the conversion of the pushover curve (in the force vs. displacement domain) to the capacity curve (in the spectral acceleration vs. spectral displacement domain). To facilitate an explanation of this conversion, a review of MDOF dynamics is provided.

Here the MDOF equations are shown. Terminology follows that in the textbook by Clough and Penzien. The first step is to transform from natural coordinates (displacements at the various DOF) to modal coordinates (amplitudes of mode shapes).



Review of MDOF Dynamics (2)

Use of Orthogonality Relationships:

$$\Phi^{\mathrm{T}} M \Phi \ddot{y} + \Phi^{\mathrm{T}} C \Phi \dot{y} + \Phi^{\mathrm{T}} K \Phi y = -\Phi^{\mathrm{T}} M R \ddot{u}_g$$

$$\Phi^{\mathrm{T}} M \Phi = M^* \qquad \phi_i^{\mathrm{T}} M \phi_i = m_i^*$$

$$\Phi^{\mathrm{T}} C \Phi = C^* \qquad \phi_i^{\mathrm{T}} C \phi_i = c_i^*$$

$$\Phi^{\mathrm{T}} K \Phi = K^* \qquad \phi_i^{\mathrm{T}} K \phi_i = k_i^*$$

SDOF equation in Mode i :

$$\boxed{m_i^* \ddot{y}_i + c_i^* \dot{y}_i + k_i^* y_i = -\phi_i^{\mathrm{T}} M R \ddot{u}_g}$$

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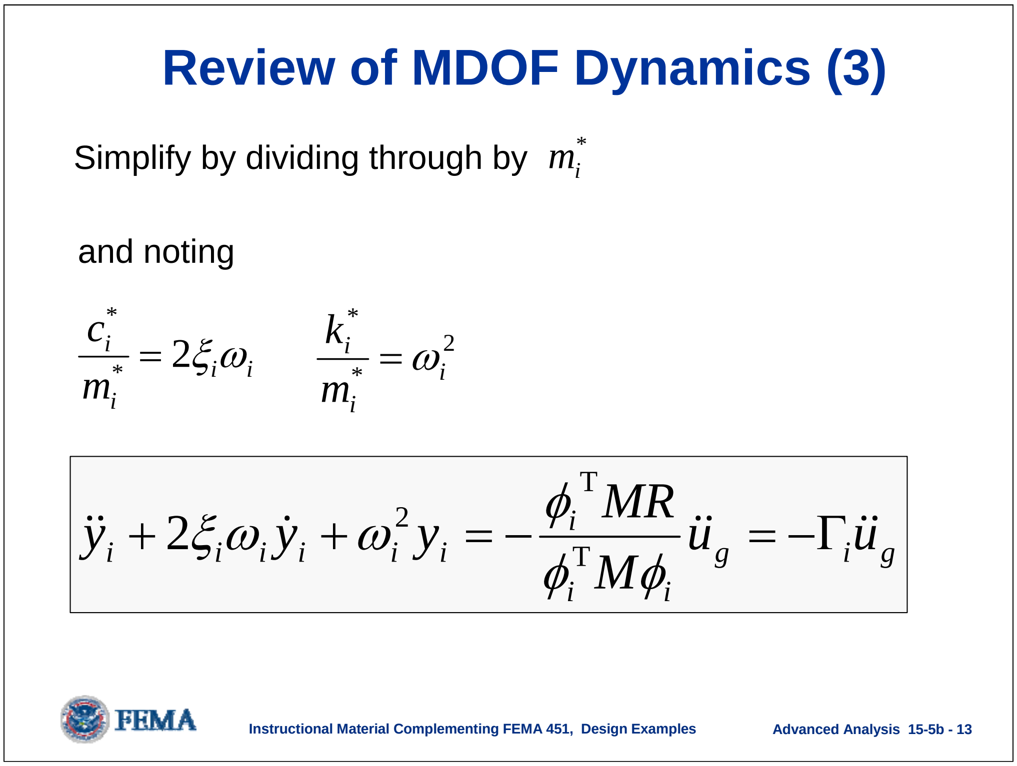
The orthogonality conditions are used to decouple the equations, resulting in one equation for each mode.



Review of MDOF Dynamics (3)

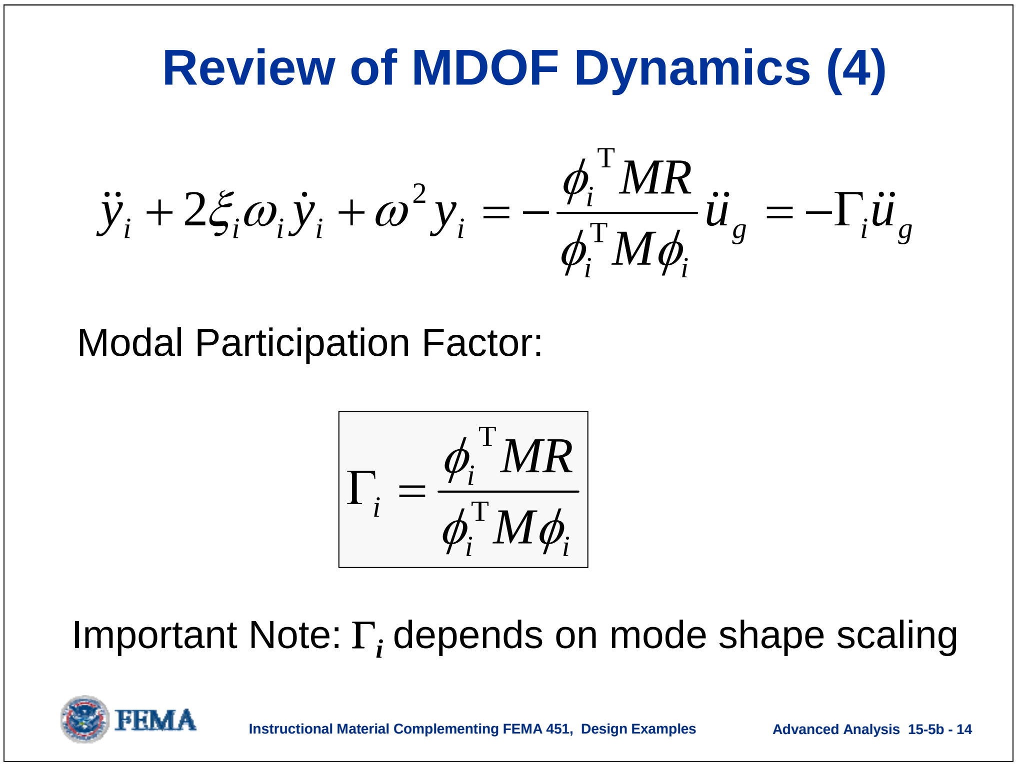
Simplify by dividing through by m_i^*

and noting

$$\frac{c_i^*}{m_i^*} = 2\xi_i\omega_i \qquad \frac{k_i^*}{m_i^*} = \omega_i^2$$

$$\boxed{\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}\ddot{u}_g = -\Gamma_i\ddot{u}_g}$$

Dividing through by the generalized mass in each mode produces the “standard” modal equation as shown. Note that this is identical to the standard SDOF equation except for the presence of the Gamma term Γ which is referred to as the modal participation factor of the mode.



Review of MDOF Dynamics (4)

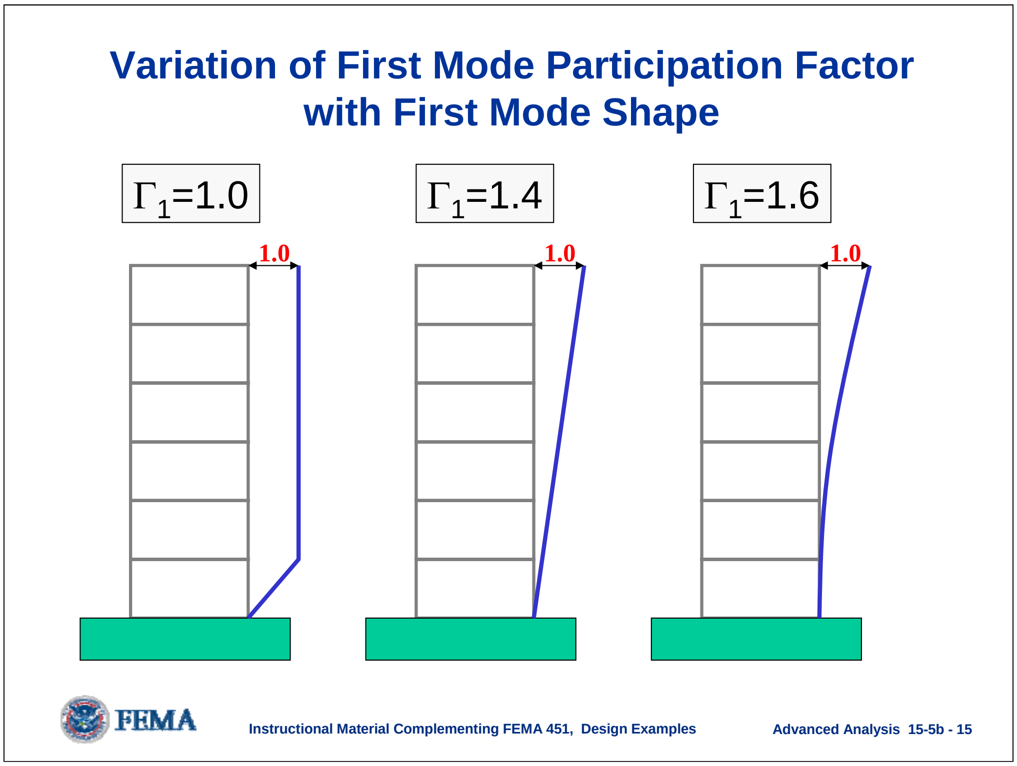
$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega^2 y_i = -\frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}\ddot{u}_g = -\Gamma_i\ddot{u}_g$$

Modal Participation Factor:

$$\boxed{\Gamma_i = \frac{\phi_i^{\mathrm{T}} MR}{\phi_i^{\mathrm{T}} M\phi_i}}$$

Important Note: Γ_i depends on mode shape scaling

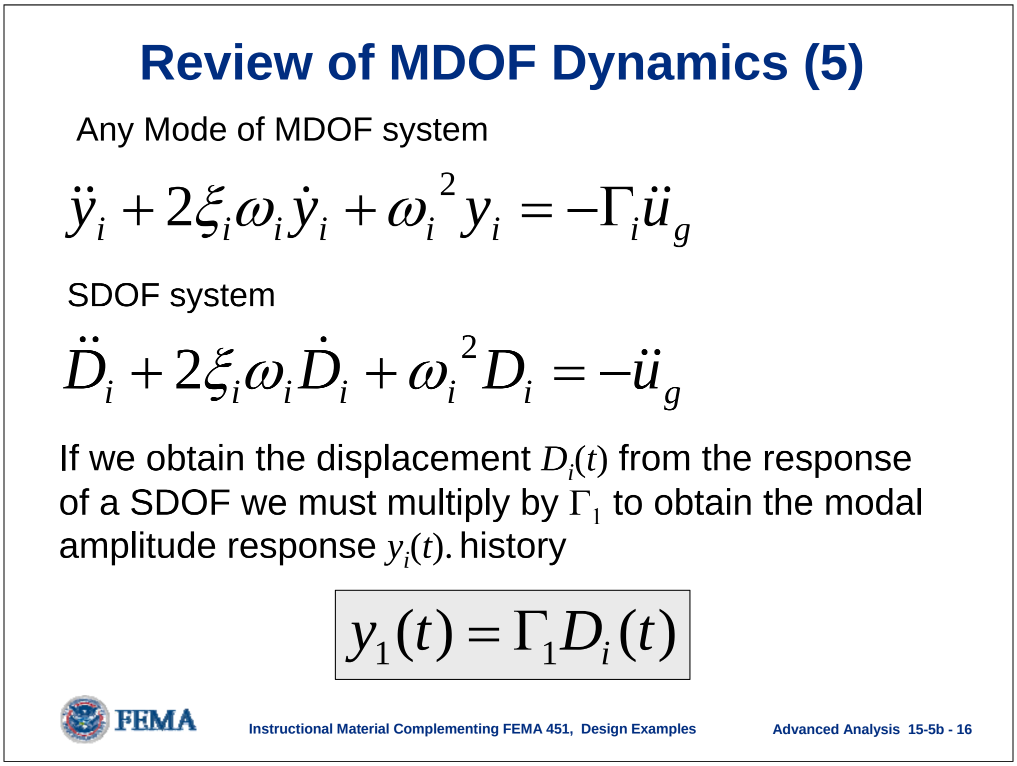
It is important to note that the amplitude of the modal participation factor is dependent on the (arbitrary) modal scaling factor. This is evident from the fact that one ϕ appears in the numerator and two ϕ terms appear in the denominator.



Variation of First Mode Participation Factor with First Mode Shape

$\Gamma_1=1.0$ $\Gamma_1=1.4$ $\Gamma_1=1.6$

This slide shows how the modal participation factor is dependent on the *shape* of the mode (which is independent of the scale factor). Note that the mode shapes have been normalized such that the top level displacement is 1.0.



Review of MDOF Dynamics (5)

Any Mode of MDOF system

$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^{\,2} y_i = -\Gamma_i\ddot{u}_g$$

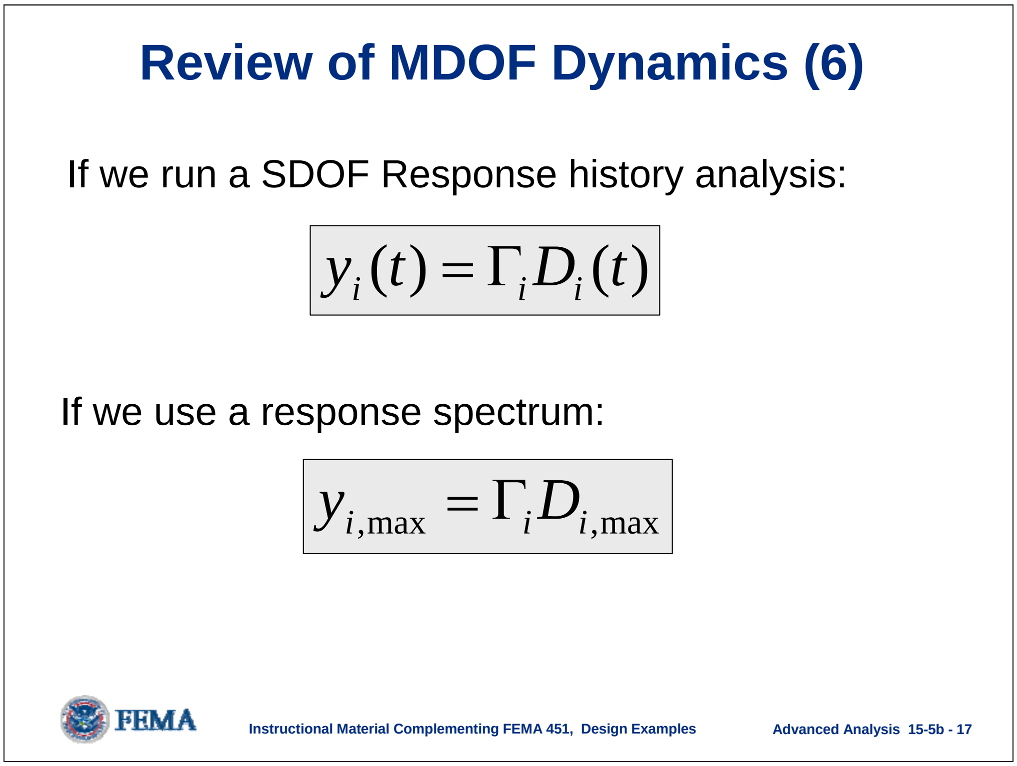
SDOF system

$$\ddot{D}_i + 2\xi_i\omega_i\dot{D}_i + \omega_i^{\,2} D_i = -\ddot{u}_g$$

If we obtain the displacement $D_i(t)$ from the response of a SDOF we must multiply by Γ_1 to obtain the modal amplitude response $y_i(t)$. history

$$\boxed{y_1(t) = \Gamma_1 D_i(t)}$$

Note that the only real difference between the a single mode of the MDOF system and the SDOF system is the modal participation factor on the RHS of the individual mode of the MDOF. Code based response spectra (used in determining the target displacement) DO NOT have the modal participation factor built in.



Review of MDOF Dynamics (6)

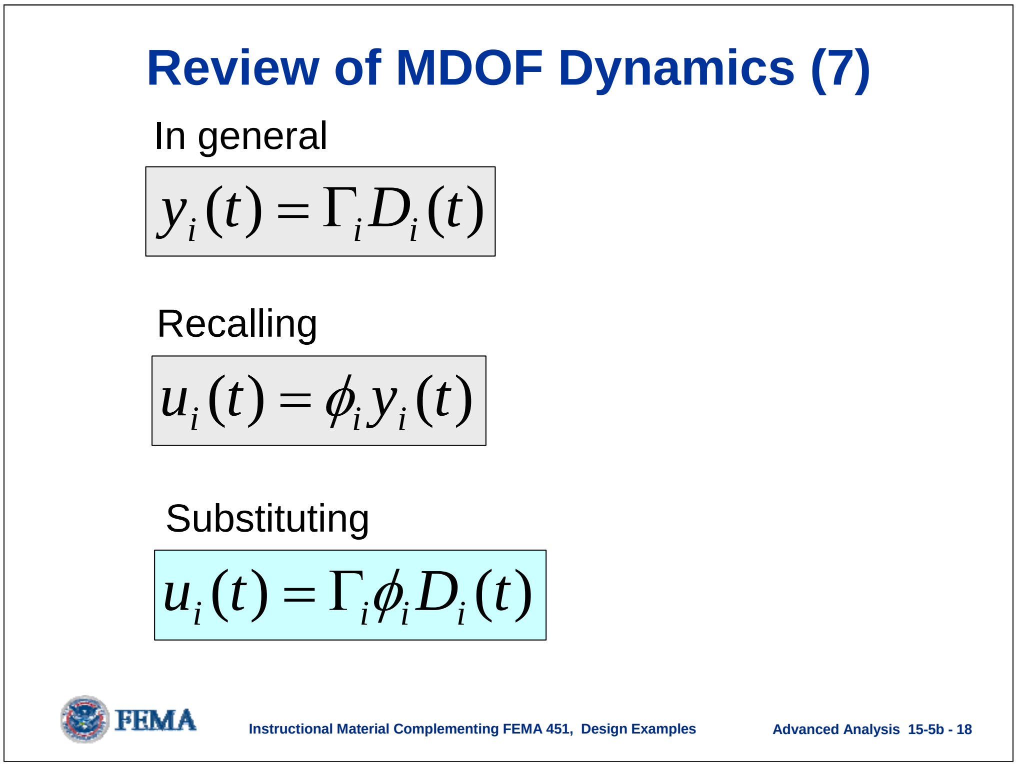
If we run a SDOF Response history analysis:

$$\boxed{y_i(t) = \Gamma_i D_i(t)}$$

If we use a response spectrum:

$$\boxed{y_{i,\max} = \Gamma_i D_{i,\max}}$$

The response history or response spectrum ordinate of a single mode of a MDOF system is easily obtained from the equivalent SDOF system.



Review of MDOF Dynamics (7)

In general

$$y_i(t) = \Gamma_i D_i(t)$$

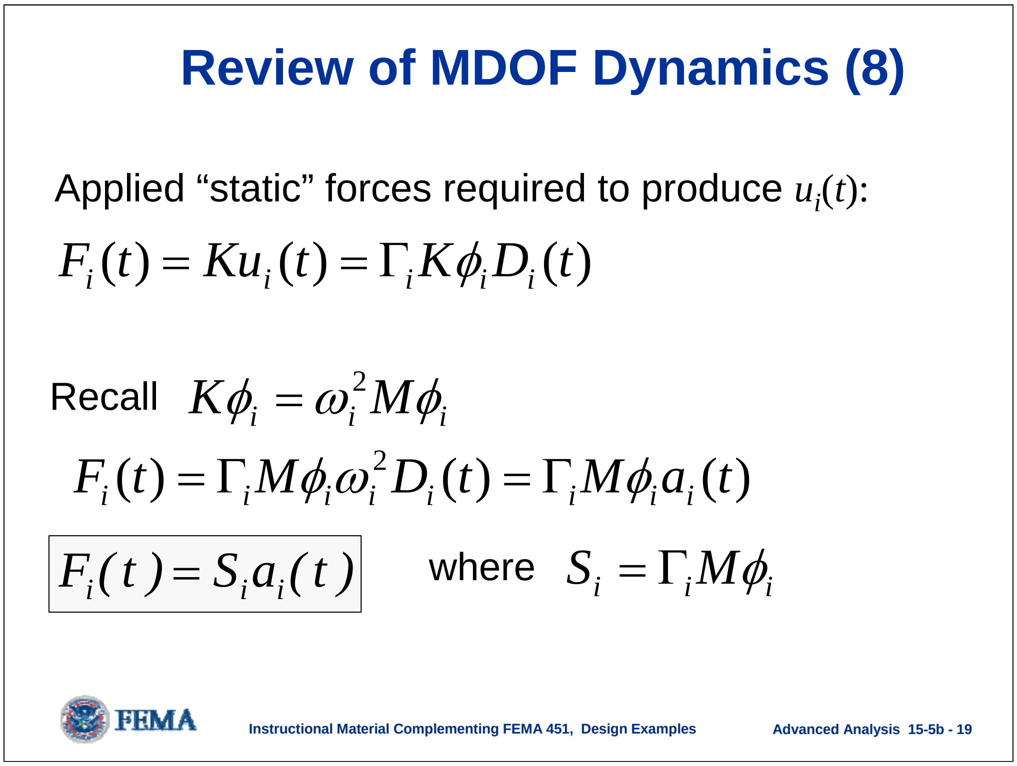
Recalling

$$u_i(t) = \phi_i y_i(t)$$

Substituting

$$u_i(t) = \Gamma_i \phi_i D_i(t)$$

The natural displacement vector (e.g. nodal displacements) in any mode is given by the lower equation. This is obtained by simple algebraic manipulation of two previous equations.



Review of MDOF Dynamics (8)

Applied “static” forces required to produce $u_i(t)$:

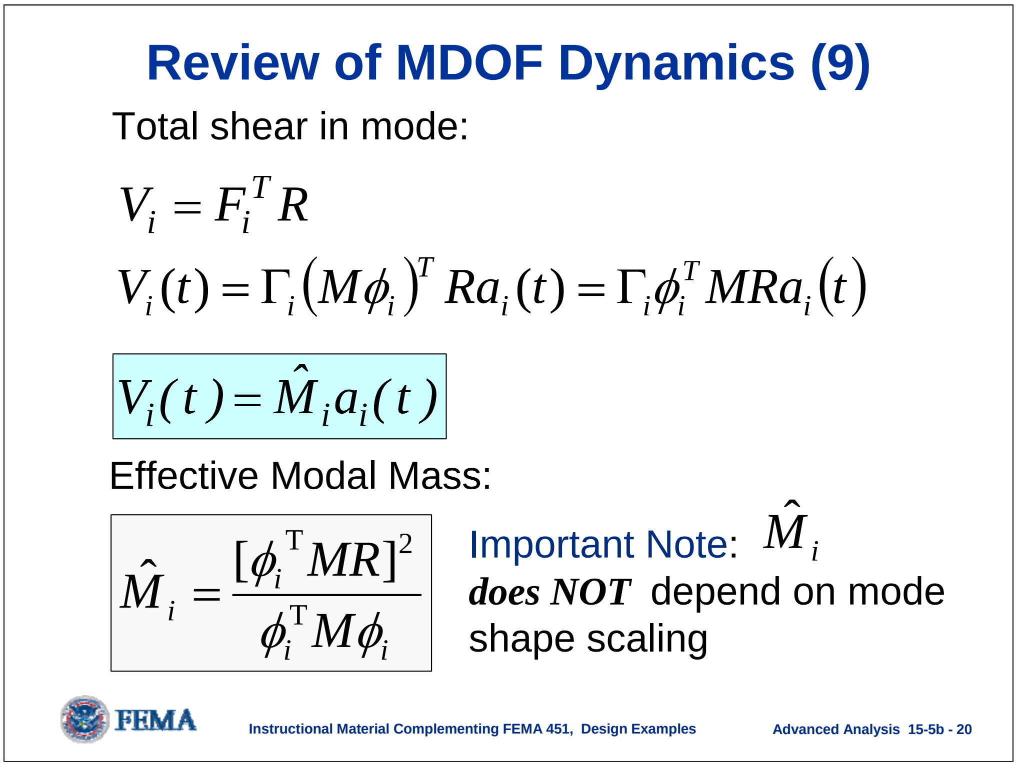
$$F_i(t) = Ku_i(t) = \Gamma_i K \phi_i D_i(t)$$

Recall $K\phi_i = \omega_i^2 M \phi_i$

$$F_i(t) = \Gamma_i M \phi_i \omega_i^2 D_i(t) = \Gamma_i M \phi_i a_i(t)$$

$\boxed{F_i(t) = S_i a_i(t)}$ where $S_i = \Gamma_i M \phi_i$

Given the displacements and the elastic stiffness *K*, the equivalent static forces *F* required to produce the displacements can be obtained for any mode. The equation is manipulated to obtain the equivalent static forces in terms of pseudoacceleration and a force distribution vector S.



Review of MDOF Dynamics (9)

Total shear in mode:

$$V_i = F_i^T R$$

$$V_i(t) = \Gamma_i \left(M\phi_i\right)^T R a_i(t) = \Gamma_i \phi_i^T M R a_i\left(t\right)$$

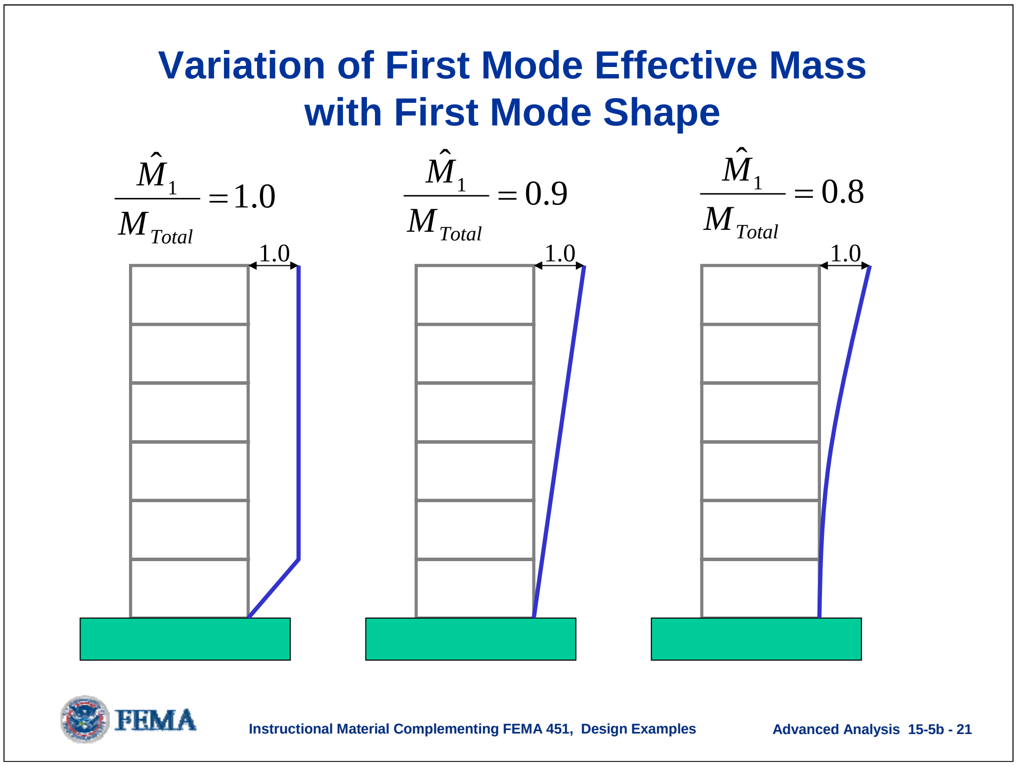
$$\boxed{V_i(t) = \hat{M}_i a_i(t)}$$

Effective Modal Mass:

$$\boxed{\hat{M}_i = \frac{[\phi_i^{\mathrm{T}} M R]^2}{\phi_i^{\mathrm{T}} M \phi_i}}$$

Important Note: $\hat{M}_i$ ***does NOT*** depend on mode shape scaling

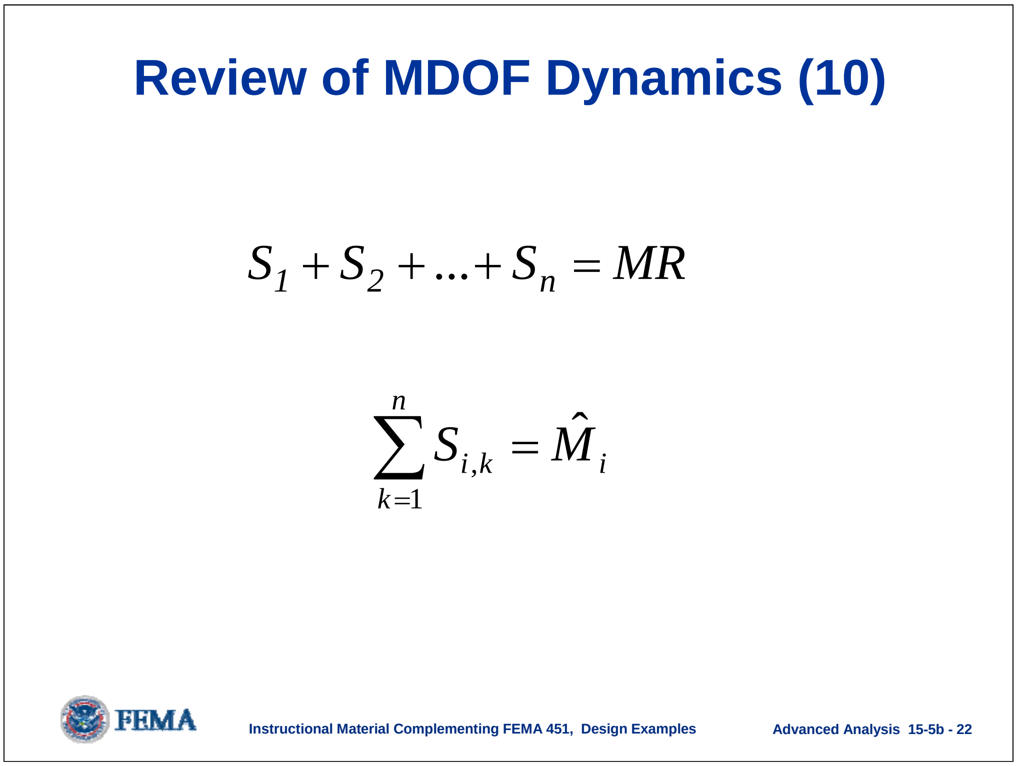
The total shear in each mode is obtained as shown in the top equation. Some algebraic manipulation results in the effective modal mass for each mode. Note that this quantity is NOT dependent on mode shape scaling (as a pair of φ□s appear in the numerator and the denominator). Though not evident from this slide the sum of the effective mass in all of the modes is equal to the total mass of the system.



Variation of First Mode Effective Mass with First Mode Shape

$$\frac{\hat{M}_1}{M_{Total}} = 1.0 \qquad \frac{\hat{M}_1}{M_{Total}} = 0.9 \qquad \frac{\hat{M}_1}{M_{Total}} = 0.8$$

This slide shows how the effective mass is dependent on mode shape. Again the modes have been normalized to a value of 1.0 at the top level. It should be noted that the first case is actually impossible for an MDOF system as all of the effective mass is in the first mode (leaving none for the higher modes).



Review of MDOF Dynamics (10)

$$S_1 + S_2 + \ldots + S_n = MR$$

$$\sum_{k=1}^{n} S_{i,k} = \hat{M}_i$$

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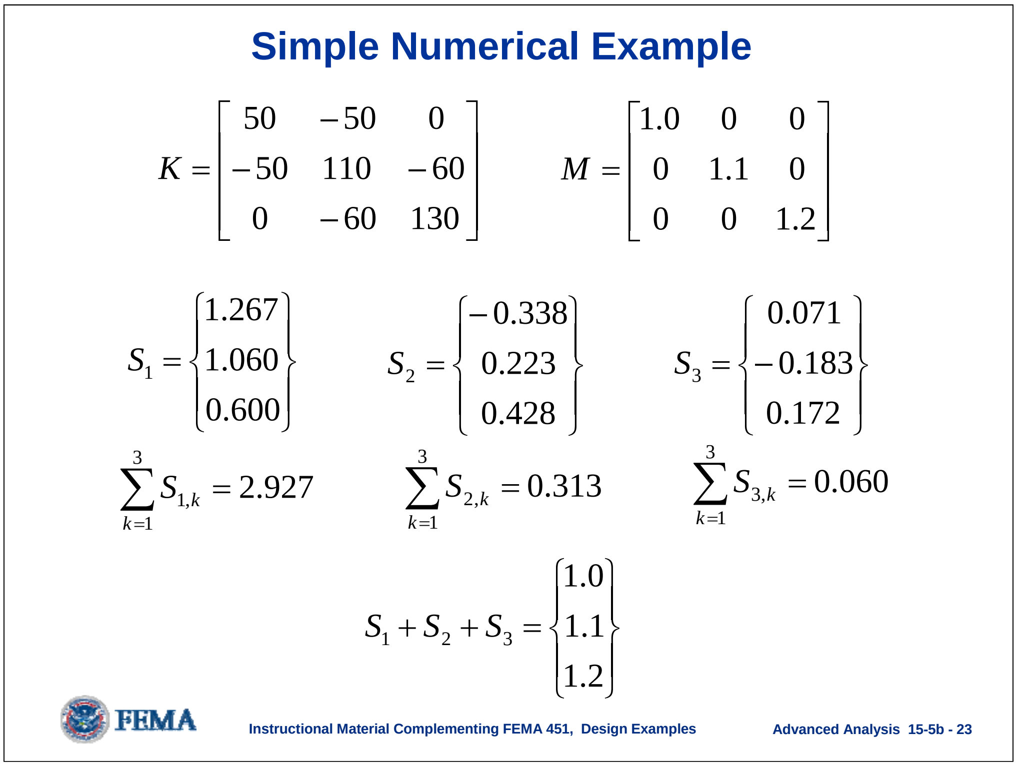
In may be shown that the sum of the S vectors is equal to the product of M and R. The sum of the entries in each row of each S vector is the effective modal mass in that mode. Note that i is an index over modes and k is an index over DOF.



Simple Numerical Example

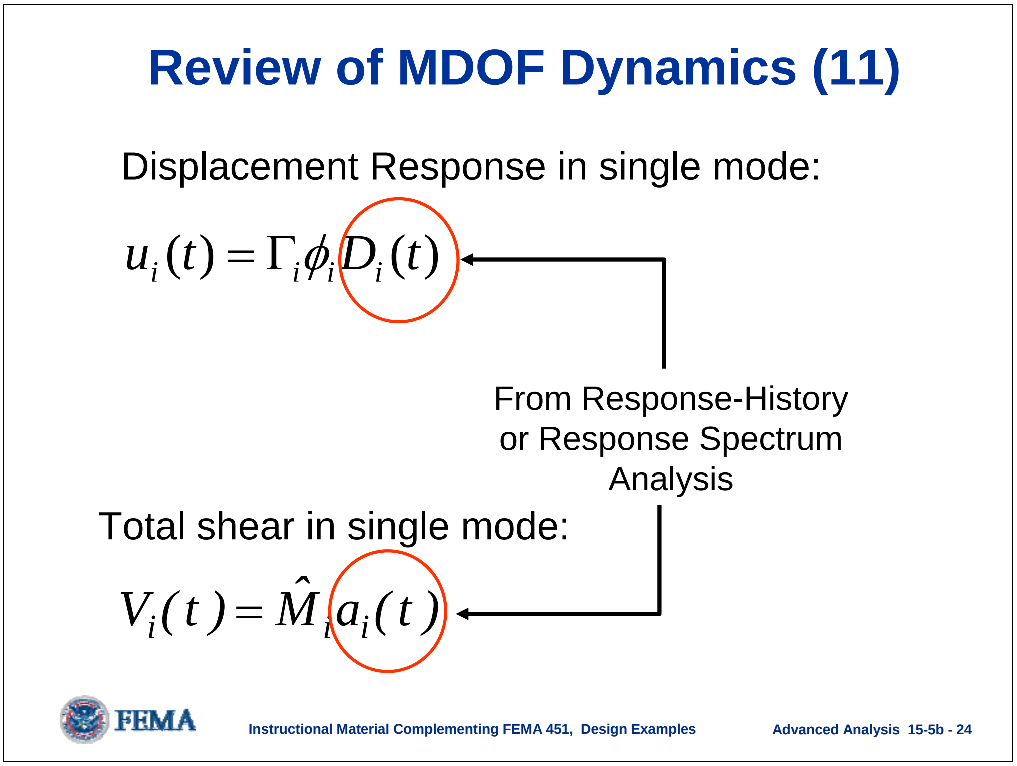
$$K = \begin{bmatrix} 50 & -50 & 0 \\ -50 & 110 & -60 \\ 0 & -60 & 130 \end{bmatrix} \qquad M = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}$$

$$S_1 = \begin{Bmatrix} 1.267 \\ 1.060 \\ 0.600 \end{Bmatrix} \qquad S_2 = \begin{Bmatrix} -0.338 \\ 0.223 \\ 0.428 \end{Bmatrix} \qquad S_3 = \begin{Bmatrix} 0.071 \\ -0.183 \\ 0.172 \end{Bmatrix}$$

$$\sum_{k=1}^{3} S_{1,k} = 2.927 \qquad \sum_{k=1}^{3} S_{2,k} = 0.313 \qquad \sum_{k=1}^{3} S_{3,k} = 0.060$$

$$S_1 + S_2 + S_3 = \begin{Bmatrix} 1.0 \\ 1.1 \\ 1.2 \end{Bmatrix}$$

This example illustrates some of the properties of the S vectors.



Review of MDOF Dynamics (11)

Displacement Response in single mode:

$$u_i(t) = \Gamma_i \phi_i D_i(t)$$

From Response-History
or Response Spectrum
Analysis

Total shear in single mode:

$$V_i(t) = \hat{M}_i a_i(t)$$

We are now ready to make the appropriate transformations. The quantities D_i and a_i would come from a linear RHA or response spectrum analysis of the equivalent SDOF system for the i-th mode. The u_i and V_i terms are the equivalent structural displacements and forces in the i-th mode. If the structural forces and displacements are known (as from a pushover analysis), the modal equivalents, D_i and a_i, may be determined.

First Mode Response as Function of System Response

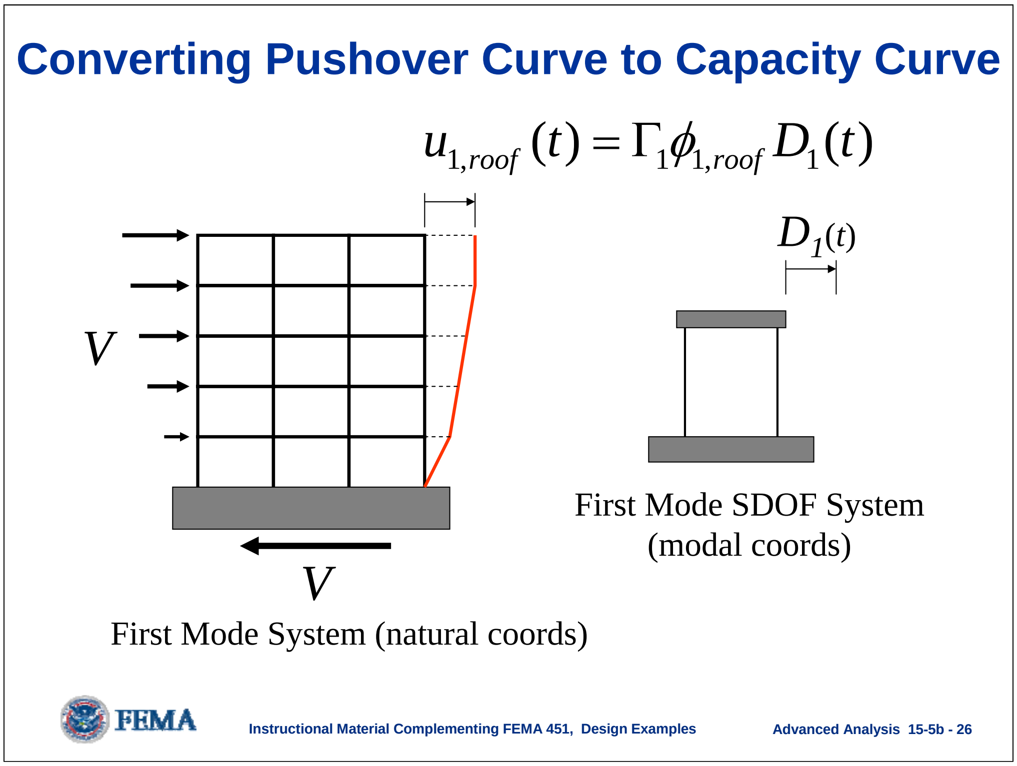
Modal Displacement:

$$D_1(t) = \frac{u_{1,roof}(t)}{\Gamma_1 \phi_{1,roof}}$$

Modal Acceleration:

$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$

Here are the final equations used to make the transformation from the pushover curve (in the base shear vs roof displacement domain) to the capacity curve (in the modal acceleration vs modal displacement domain).



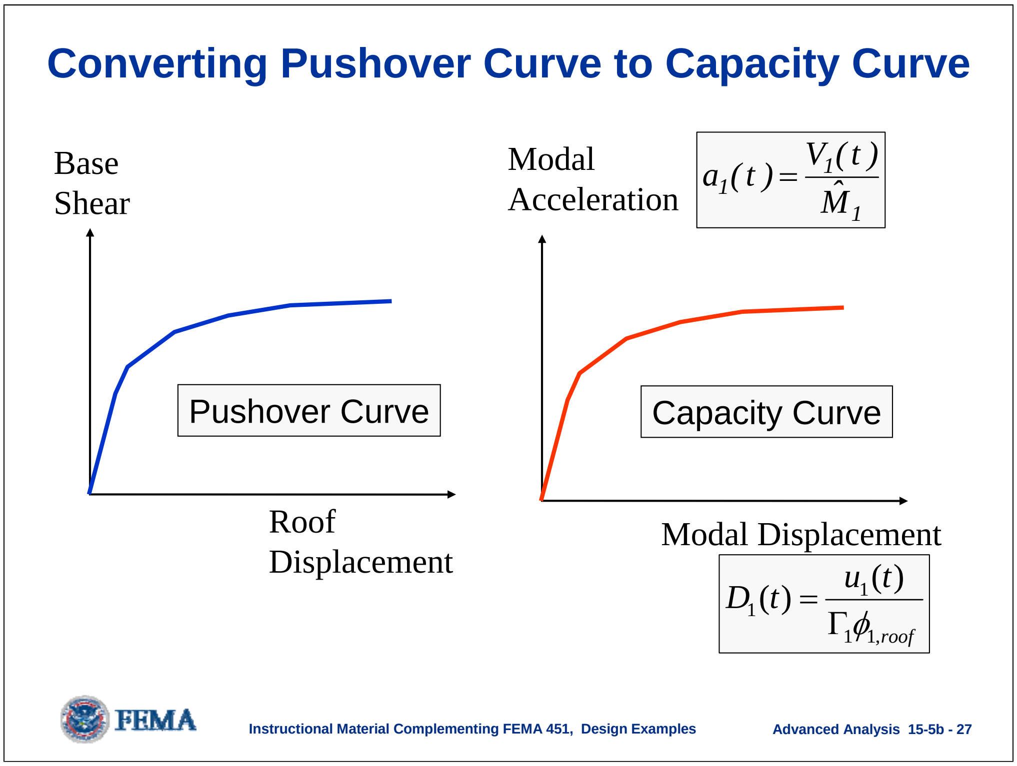
Converting Pushover Curve to Capacity Curve

$$u_{1,roof}(t) = \Gamma_1 \phi_{1,roof} D_1(t)$$

First Mode SDOF System (modal coords)

First Mode System (natural coords)

Here the transformation from first mode system natural displacement coordinates to first mode modal amplitude coordinates is shown.

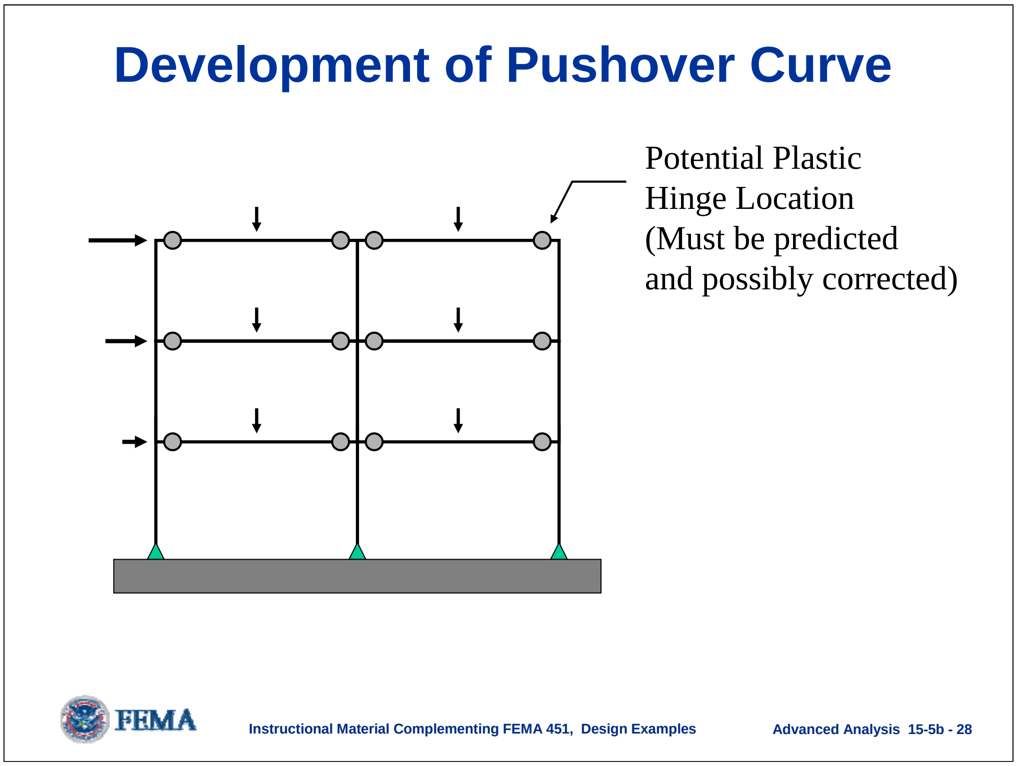


Converting Pushover Curve to Capacity Curve

$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$

$$D_1(t) = \frac{u_1(t)}{\Gamma_1 \phi_{1,roof}}$$

Finally, the first mode capacity curve is obtained from the pushover curve through the use of the transformation equations determined on the last several slides. We will get back to the use of the capacity curve later.



Development of Pushover Curve

Potential Plastic
Hinge Location
(Must be predicted
and possibly corrected)

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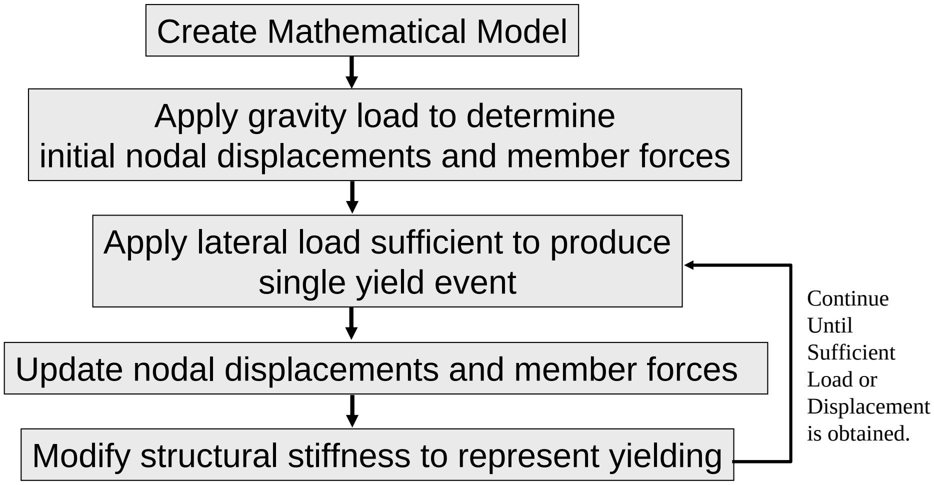
Now it is necessary to discus the development of the pushover curve itself. In the development of the curve it is first necessary to develop a realistic nonlinear model of the system. All possible sources of inelastic deformation should be included in the analytical model. If it is found during analysis that sections that were not modeled inelastically develop forces or moments in excess of yield capacity the model should be modified to include such behavior and the analysis should be rerun.

Event-to-Event Pushover Analysis



Create Mathematical Model

Apply gravity load to determine initial nodal displacements and member forces

Apply lateral load sufficient to produce single yield event

Update nodal displacements and member forces

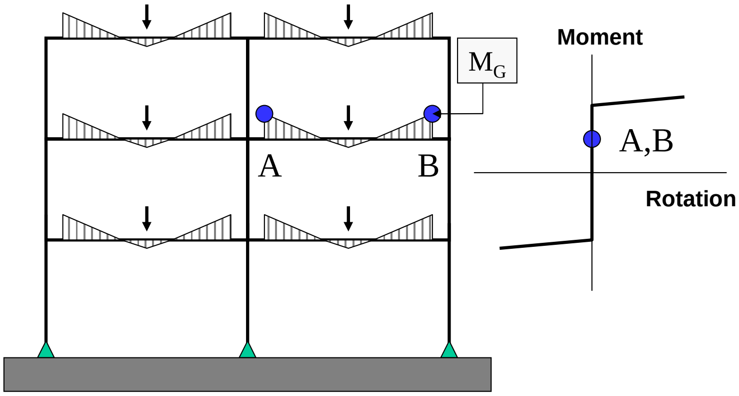
Modify structural stiffness to represent yielding

Continue Until Sufficient Load or Displacement is obtained.

This is the basic flowchart for event-to-event pushover analysis. Each step will be explained in more detail in later slides. Note that the analysis may be performed under force control or under displacement control. Displacement control is required if the tangent stiffness matrix of the structure is not positive definite at any step (usually the latter steps).

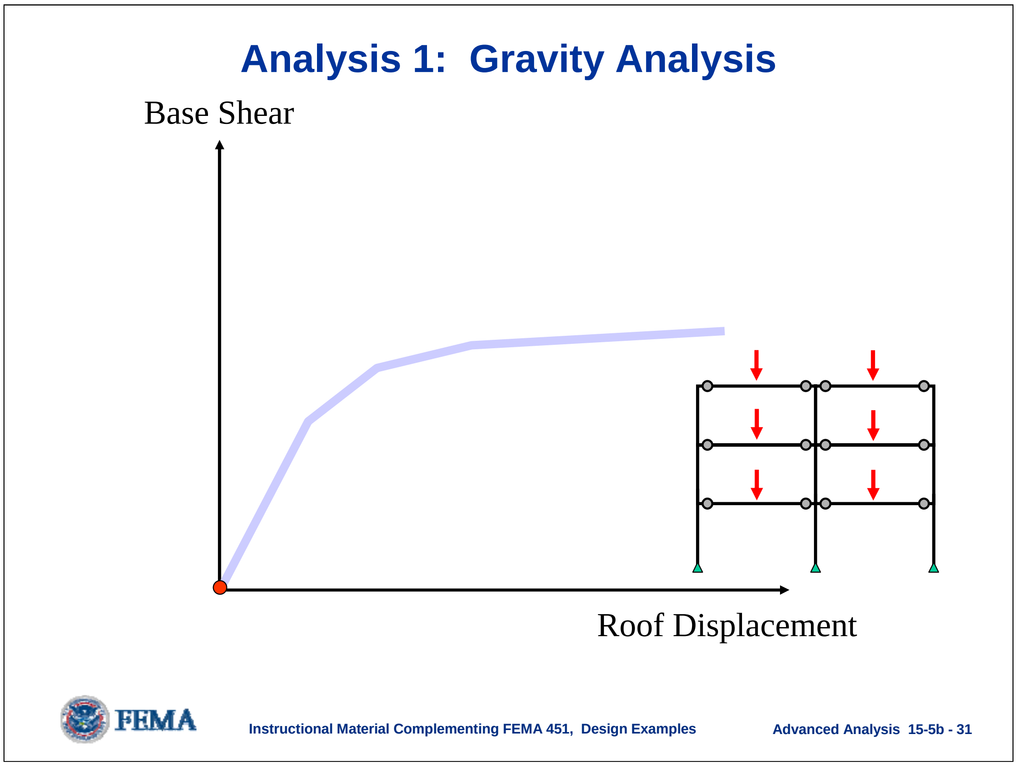
Note that this sequence assumes that no yielding occurs under gravity load. (If it does, the structure should be redesigned!)

Initial Gravity Load Analysis



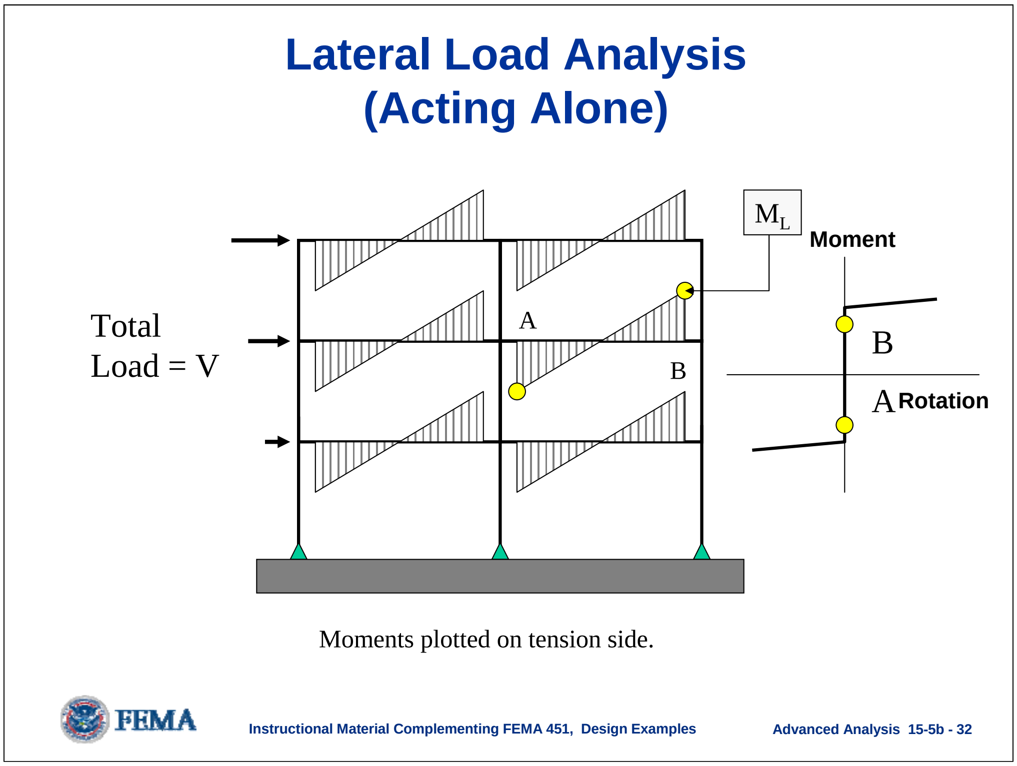
Moments plotted on tension side.

The first step in any pushover analysis is to run a gravity analysis. It is extremely rare that yielding will occur in the gravity analysis, however the pattern of moment and forces that develop in the individual structural components will have an effect on the location of and sequencing of hinges in the lateral load phase of the analysis. The gravity load analysis will also cause gravity related P-Delta effects to be activated (if such effects are explicitly included in the analytical model).



Analysis 1: Gravity Analysis

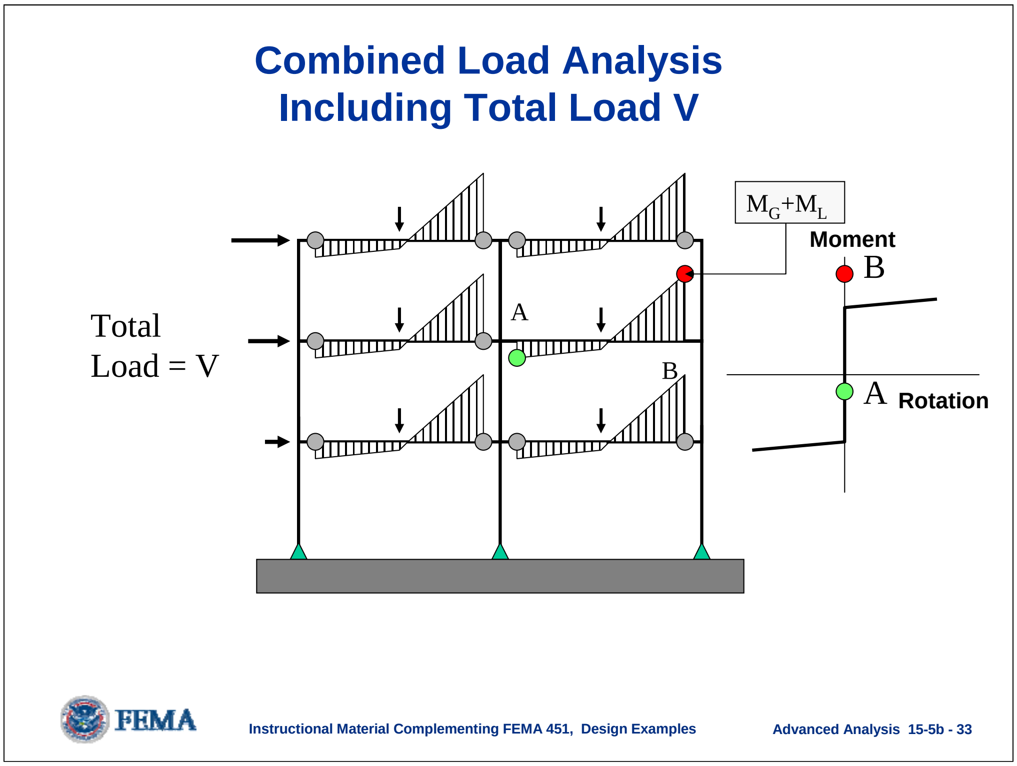
This slide shows the state of the structure just after gravity loads are applied but before any lateral load has been applied.



Lateral Load Analysis (Acting Alone)

Moments plotted on tension side.

Now the lateral load is applied. The idealized moments in two potential hinging regions are shown for the lateral load only. Insufficient lateral load has been applied to cause yielding.



Combined Load Analysis Including Total Load V

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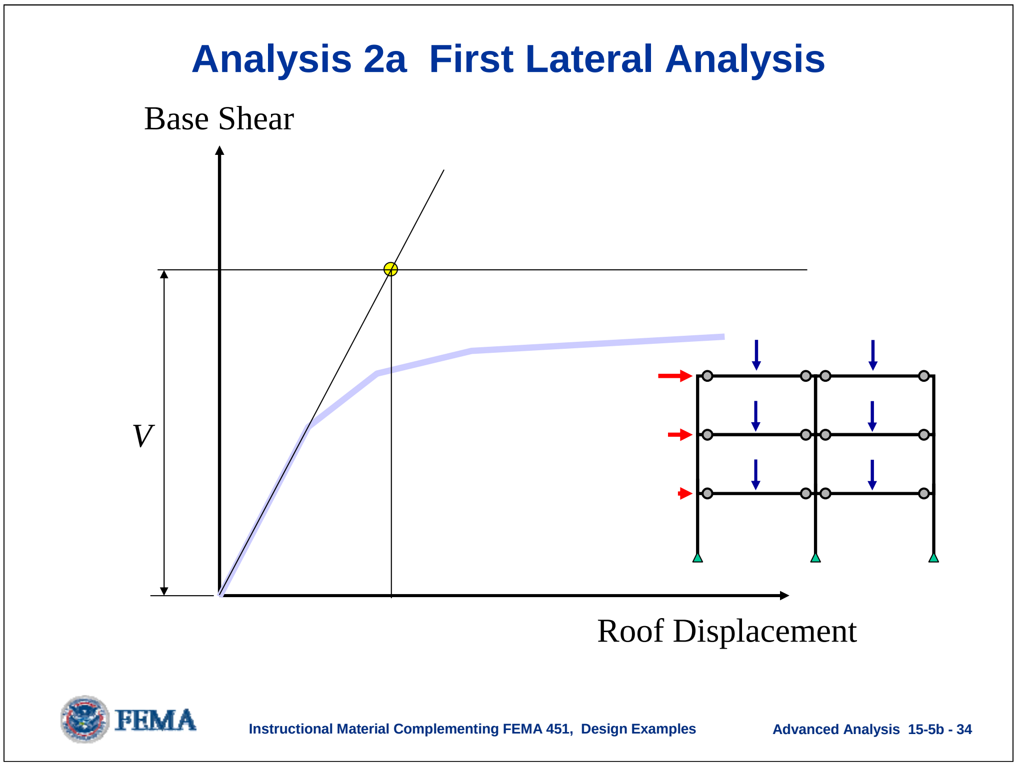
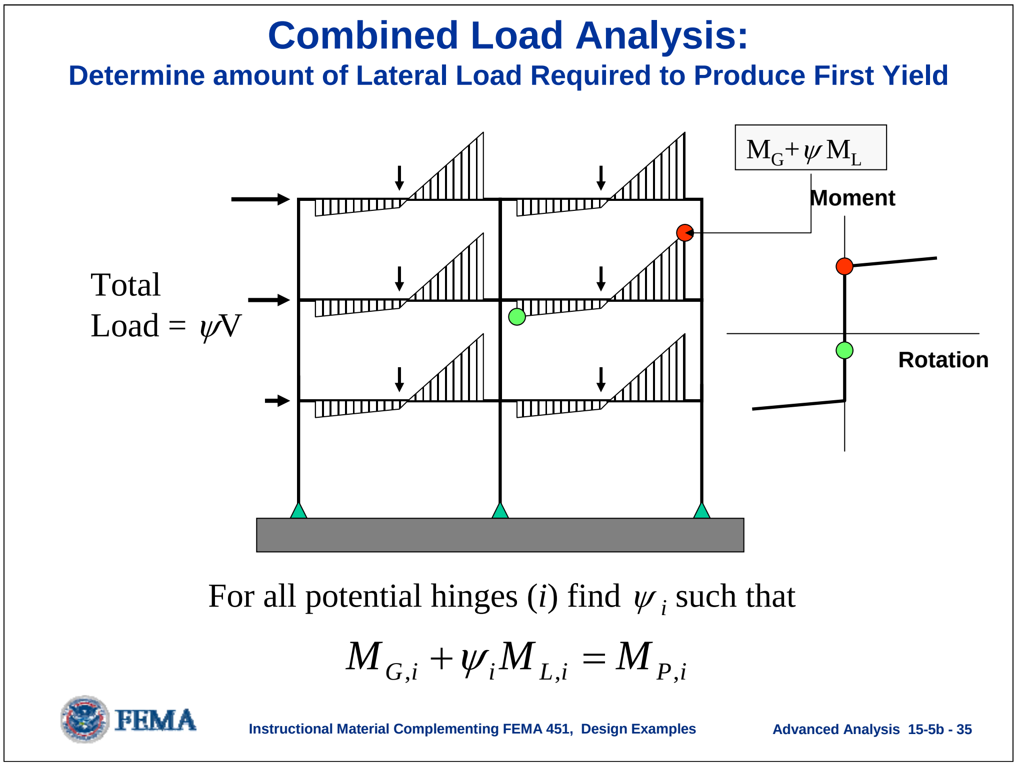
Advanced Analysis 15-5b - 33

If the member forces from gravity load are added to the member forces from the lateral loads it is seen that the moment computed at the right span, right hinge is well in excess of the capacity. The program performing the analysis will then compute the fraction of the lateral load, that when added to the gravity load, causes first yielding in the structure.



Analysis 2a First Lateral Analysis

Here the total load V is applied to the structure which has not yet yielded.



Combined Load Analysis:

Determine amount of Lateral Load Required to Produce First Yield

For all potential hinges (i) find ψ_i such that

$$M_{G,i} + \psi_i M_{L,i} = M_{P,i}$$

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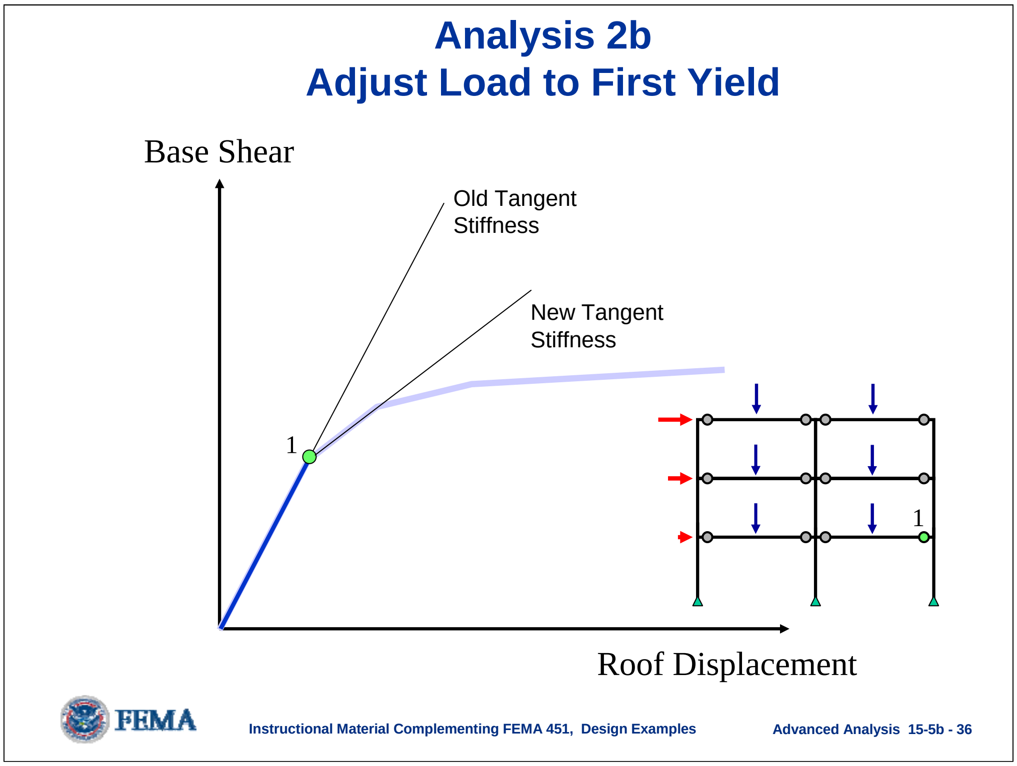
Advanced Analysis 15-5b - 35

We have applied too much lateral load. Hence, we want to compute the portion of load, ψV, that just causes the first yielding.



Analysis 2b
Adjust Load to First Yield

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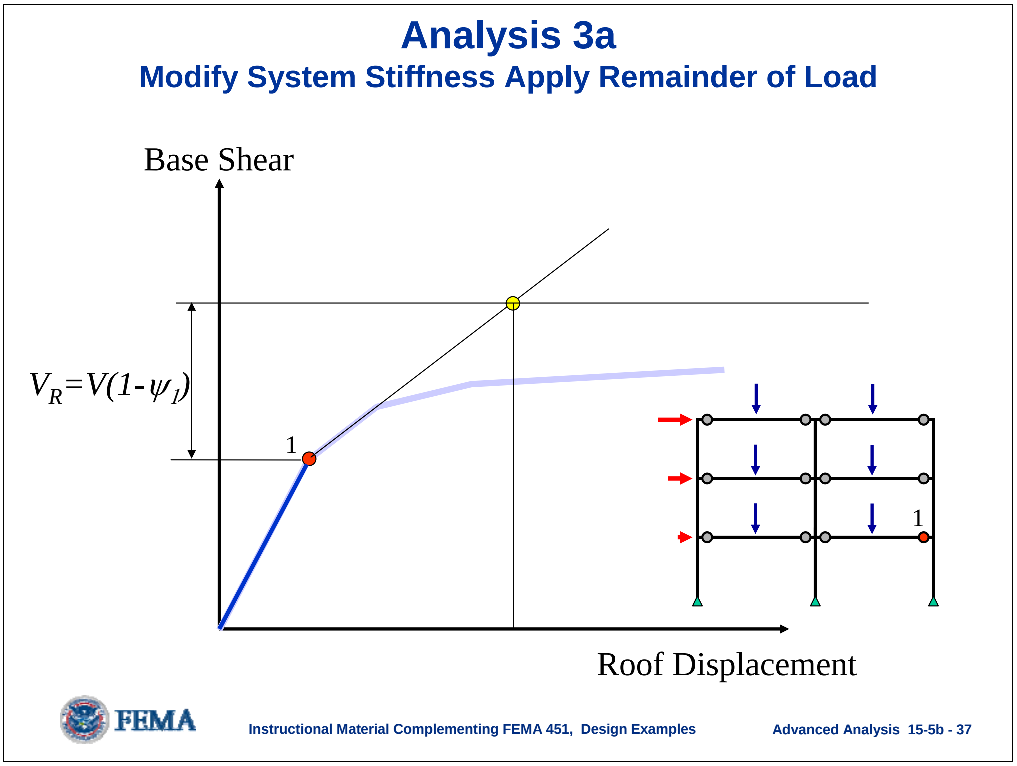
The pushover curve is not at the state shown, with only one hinge present.



Analysis 3a

Modify System Stiffness Apply Remainder of Load

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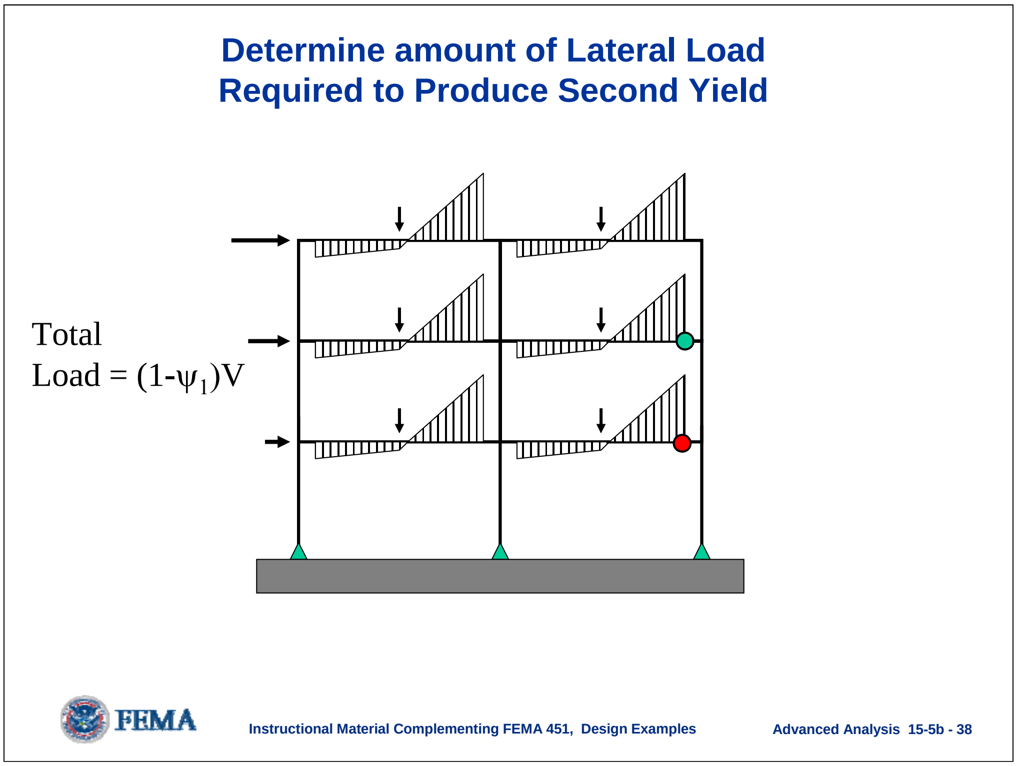
Advanced Analysis 15-5b - 37

We now apply the remainder of the load $V_R = V(1-\psi)$. We will want to determine how much of the remaining load causes the *next* hinge to form.



Determine amount of Lateral Load Required to Produce Second Yield

Total
Load = $(1-\psi_1)V$

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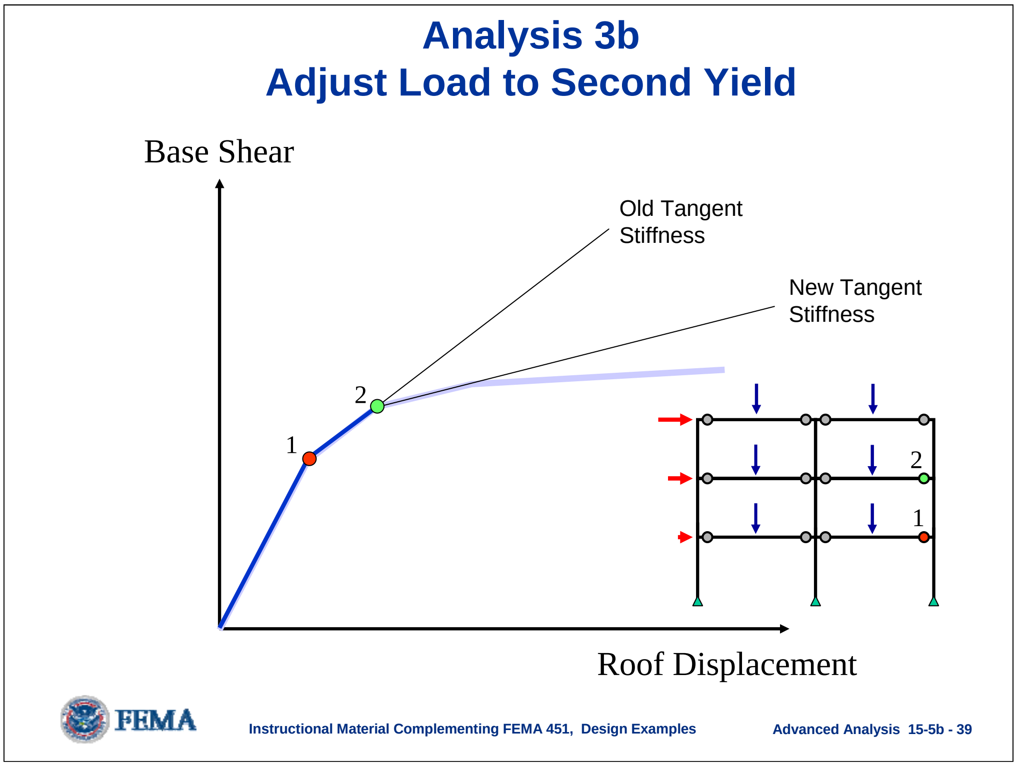
Advanced Analysis 15-5b - 38

The next hinge will occur at the right of the second story girder of the right bay.



Analysis 3b
Adjust Load to Second Yield

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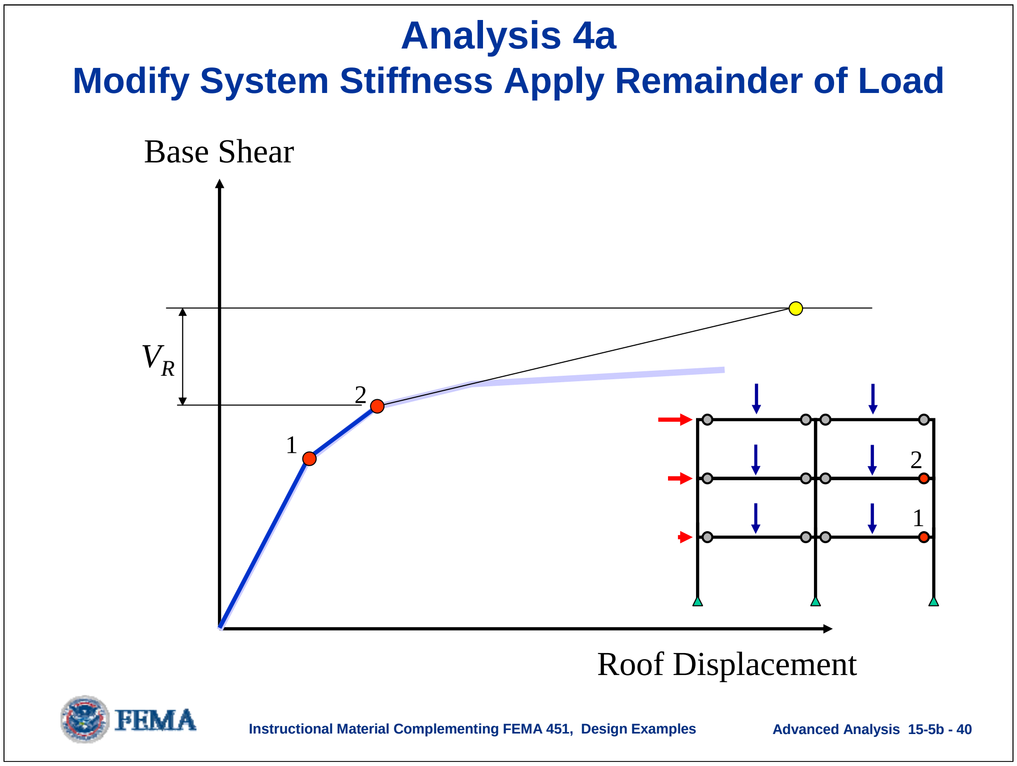
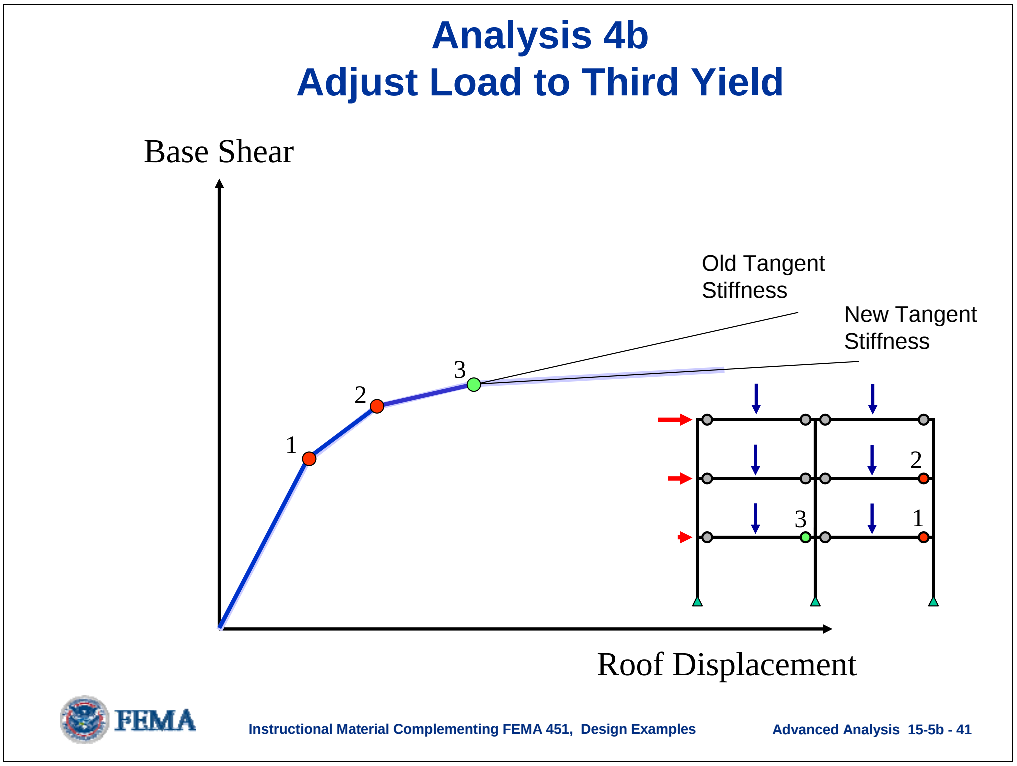
The second hinge is formed and the stiffness is changed.



Analysis 4a

Modify System Stiffness Apply Remainder of Load

The remaining load is applied, and the next hinge location is found.



Analysis 4b
Adjust Load to Third Yield

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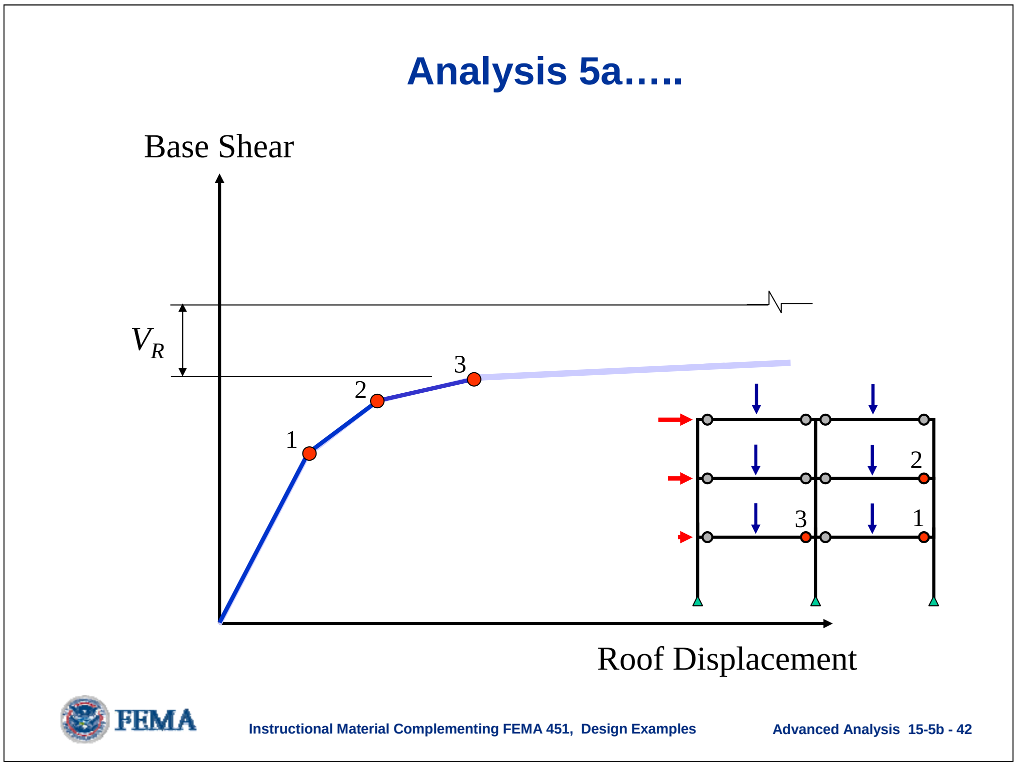
It appears that the next hinge will form at the right hand side of the first story girder in the left bay.



Analysis 5a…..

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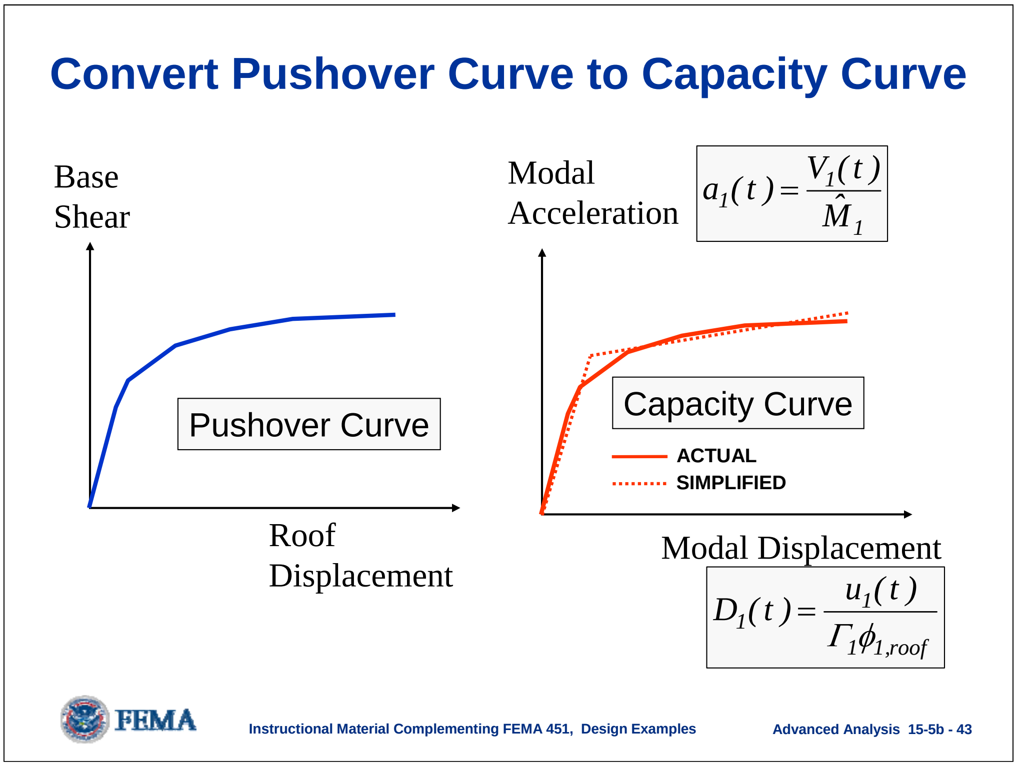
Advanced Analysis 15-5b - 42

The procedure is continued until adequate displacement has been obtained. A maximum expected displacement would be 3% of the height of the structure (as this is in excess of the seismic drift limit in most codes). The *Provisions* and a few other documents require that the pushover curve be extended to 1.5 times the Target Displacement, where the Target Displacement is determined empirically. The empirical expressions for computing Target Displacement are discussed later.



Convert Pushover Curve to Capacity Curve

Base Shear

Roof Displacement

Pushover Curve

Modal Acceleration

$$a_1(t) = \frac{V_1(t)}{\hat{M}_1}$$

Capacity Curve

ACTUAL

SIMPLIFIED

Modal Displacement

$$D_1(t) = \frac{u_1(t)}{\Gamma_1 \phi_{1,roof}}$$

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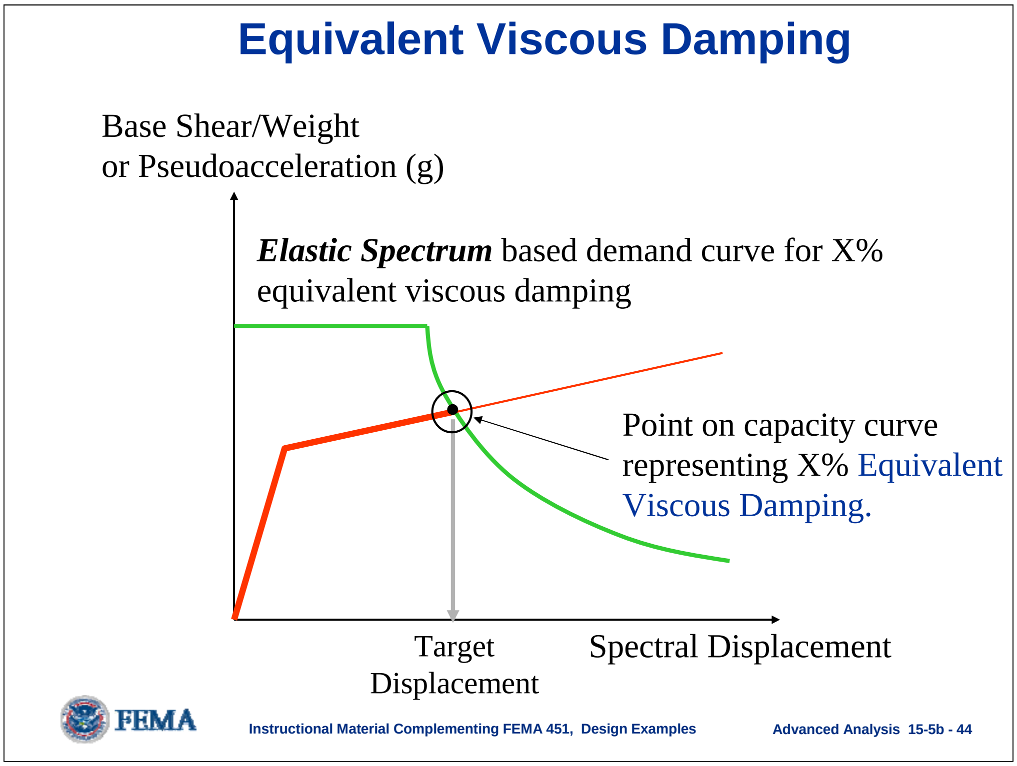
Advanced Analysis 15-5b - 43

In the capacity-spectrum approach it is necessary to transform the pushover curve (in Force-Displacement space) into a Capacity Curve (in Modal Acceleration-Modal Displacement Space). The transformation equations developed earlier are used for this purpose. In many cases it is convenient to replace the capacity curve by a simplified bilinear version as shown. We will use the bilinear version in subsequent discussions.



Equivalent Viscous Damping

Base Shear/Weight
or Pseudoacceleration (g)

Elastic Spectrum based demand curve for X% equivalent viscous damping

Point on capacity curve representing X% Equivalent Viscous Damping.

Target Displacement

Spectral Displacement

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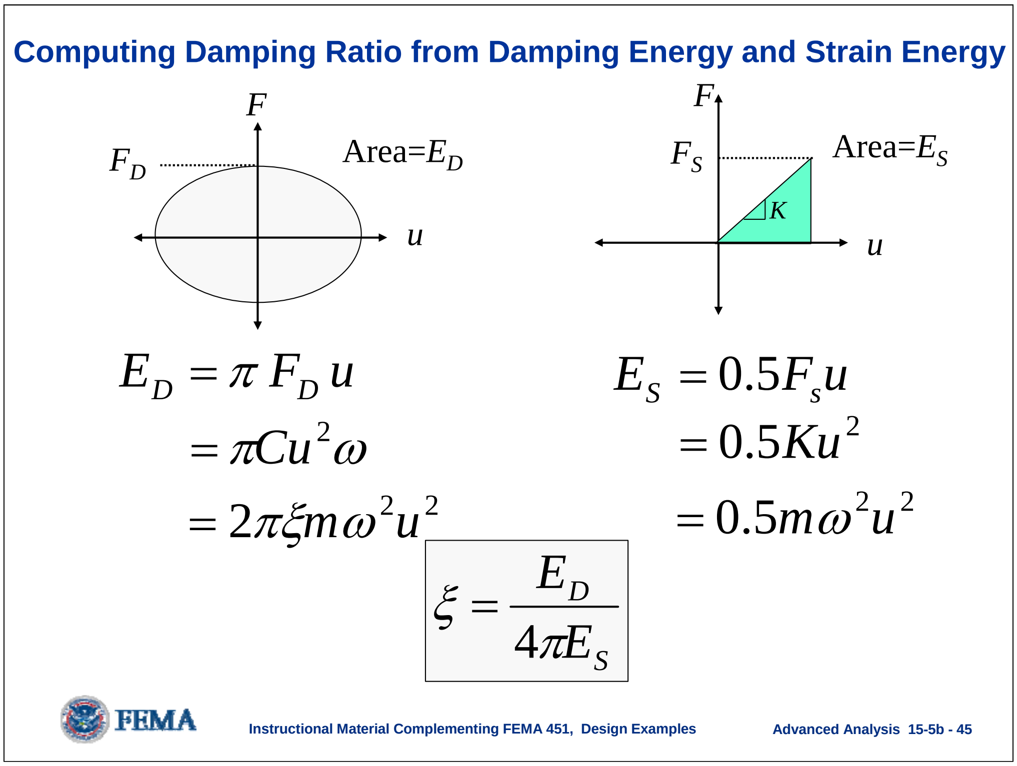
Advanced Analysis 15-5b - 44

The next step is to determine the target displacement. Here, we are using the Capacity Curve in association with a Demand Curve which is an elastic response spectrum modified for site effects, and then modified for an amount of equivalent viscous damping, X, which is consistent with an amount of hysteretic energy dissipated by the system.

It is important to note here that the demand spectrum is plotted as pseudoacceleration vs displacement, not pseudoacceleration vs period as is traditional. We will get back to this later. Before doing so it is instructive to discuss the concept of equivalent viscous damping which is a key (yet dubious) ingredient in the procedure.



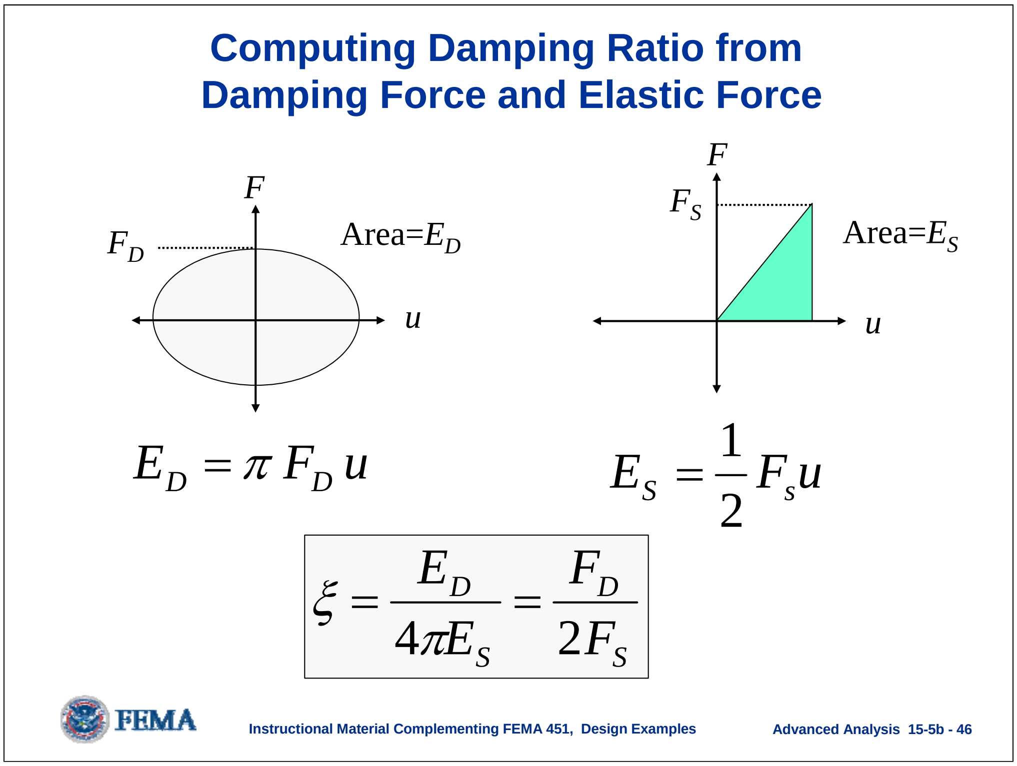
Computing Damping Ratio from Damping Energy and Strain Energy

$$\begin{aligned} E_D &= \pi F_D u \\ &= \pi C u^2 \omega \\ &= 2\pi \xi m \omega^2 u^2 \end{aligned}$$

$$\begin{aligned} E_S &= 0.5 F_s u \\ &= 0.5 K u^2 \\ &= 0.5 m \omega^2 u^2 \end{aligned}$$

$$\boxed{\xi = \frac{E_D}{4\pi E_S}}$$

This is the derivation of damping ratio as computed on the basis of damping energy and strain energy. It is applicable only to systems under harmonic resonance.



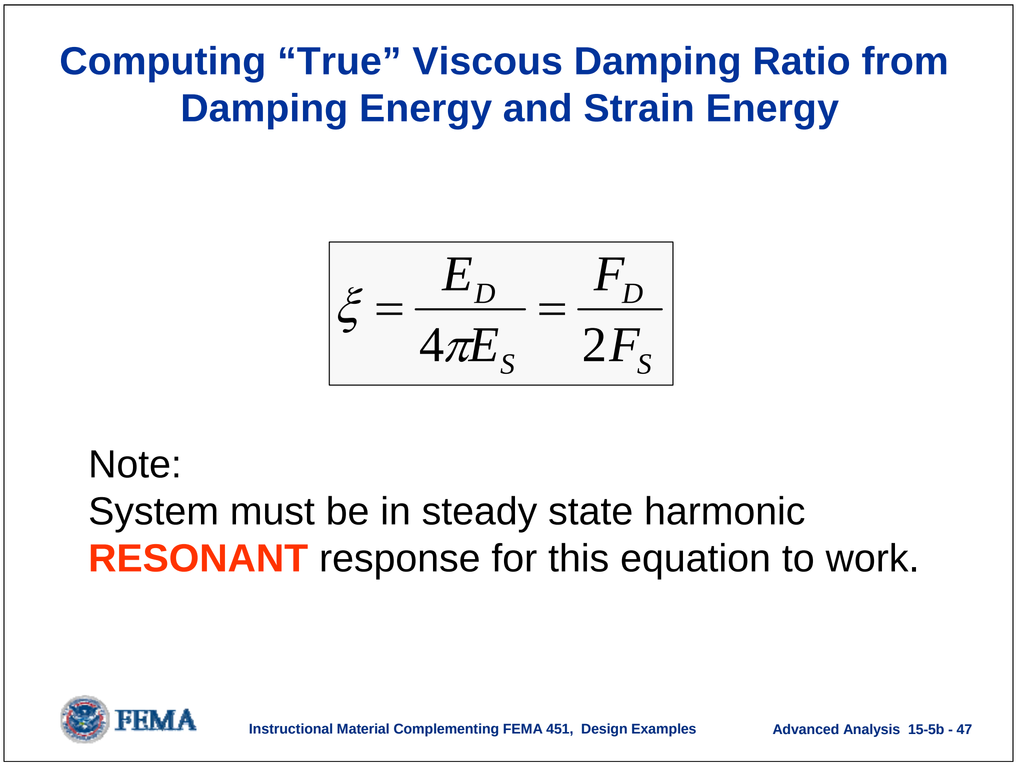
Computing Damping Ratio from Damping Force and Elastic Force

$$E_D = \pi F_D u$$

$$E_S = \frac{1}{2} F_s u$$

$$\boxed{\xi = \frac{E_D}{4\pi E_S} = \frac{F_D}{2F_S}}$$

A manipulation of terms in the previous slide shows that for a system under harmonic resonance, the damping ratio may be expressed as 0.5 times the ratio of the damping force to the elastic spring force. This equation holds at any time in the response history *after* the transients have fully damped out.

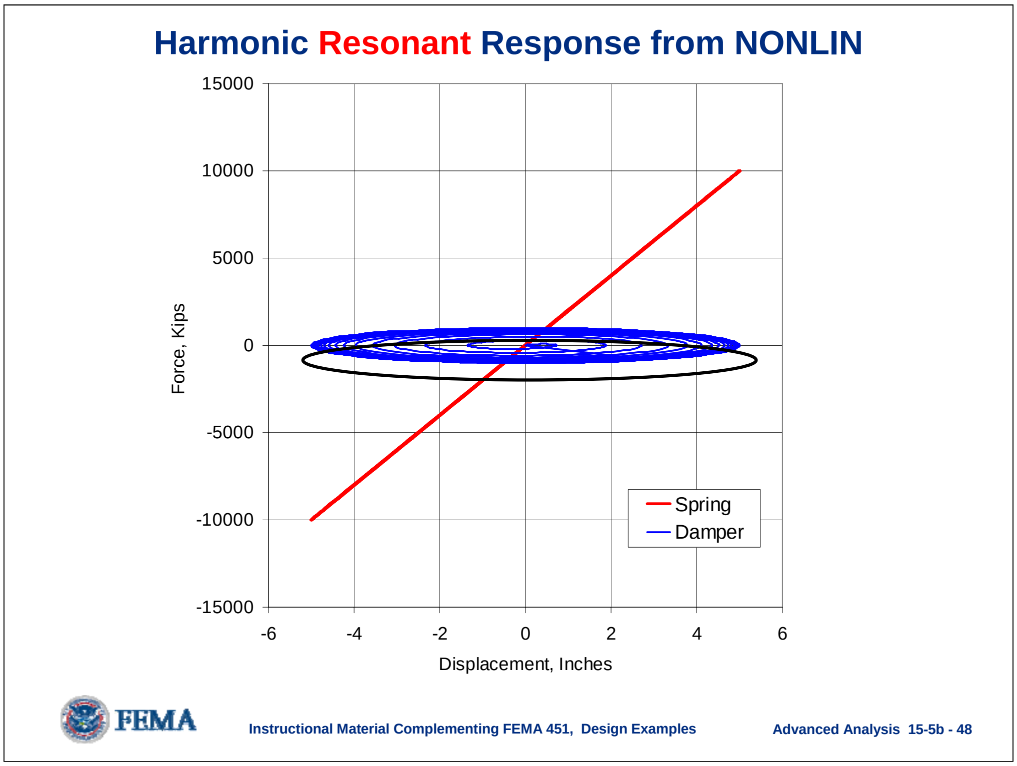


Computing “True” Viscous Damping Ratio from Damping Energy and Strain Energy

$$\xi = \frac{E_D}{4\pi E_S} = \frac{F_D}{2F_S}$$

Note:
System must be in steady state harmonic **RESONANT** response for this equation to work.

This slide is here to emphasize the key restriction on the previous derivations.



Harmonic Resonant Response from NONLIN

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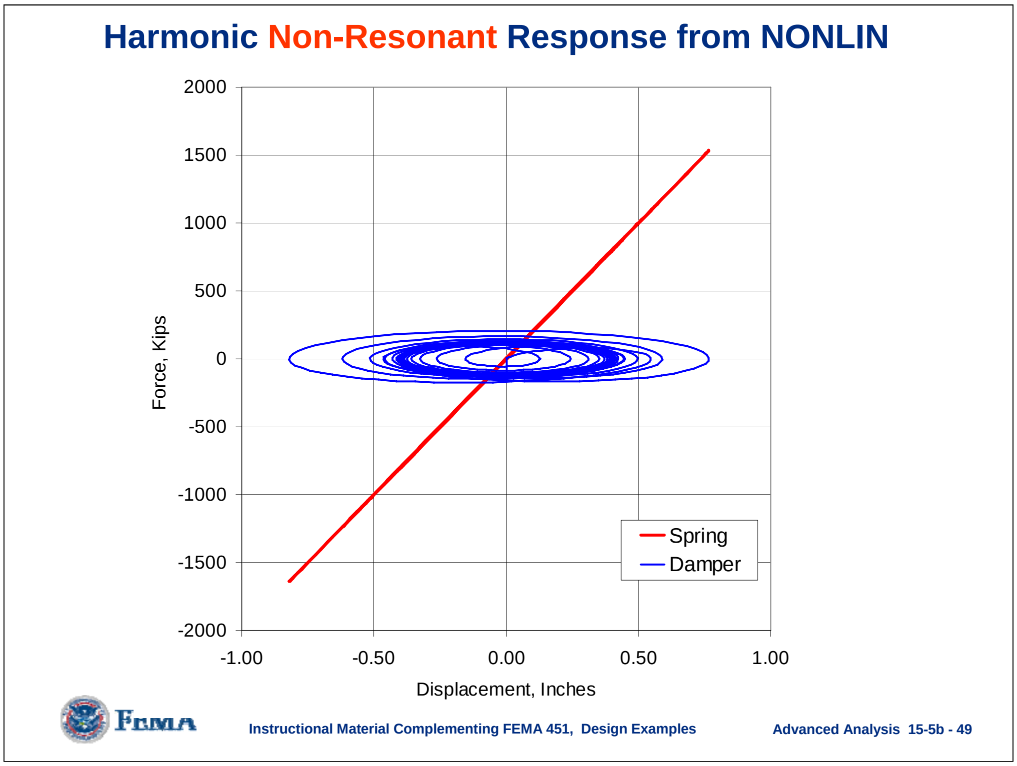
This is an example of a system response at harmonic resonance. The damping energy is computed from the perimeter of the ellipse (shown in black).



Harmonic Non-Resonant Response from NONLIN

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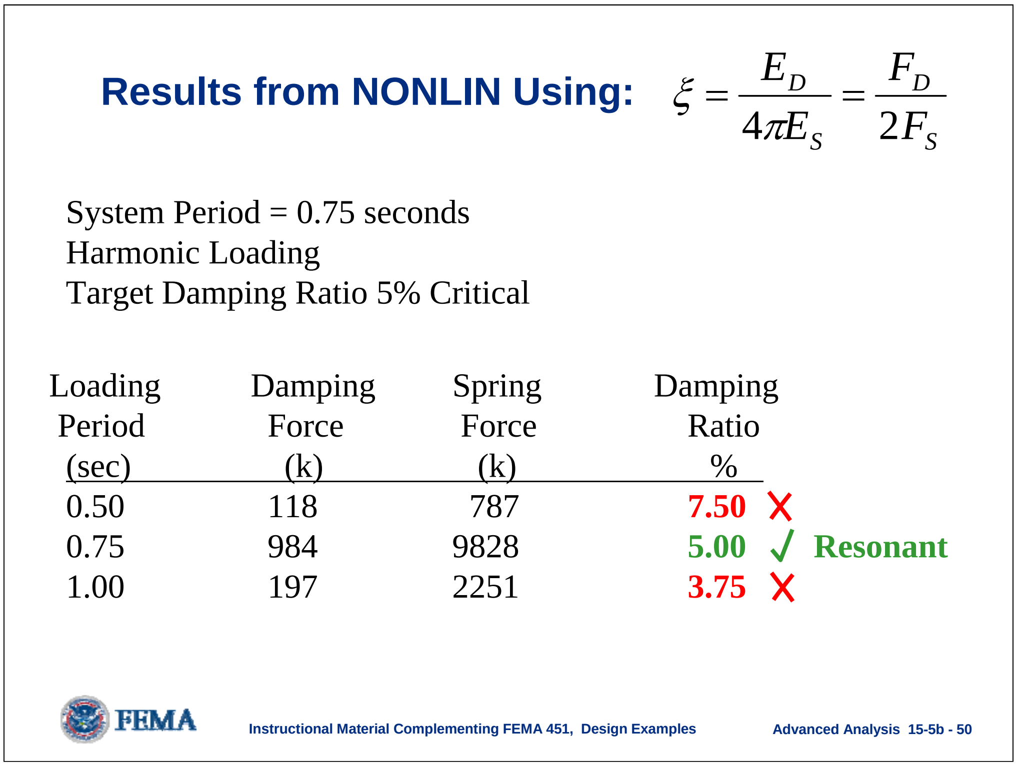
This is a similar plot for a nonresonant system. There is no true elliptical steady state response.

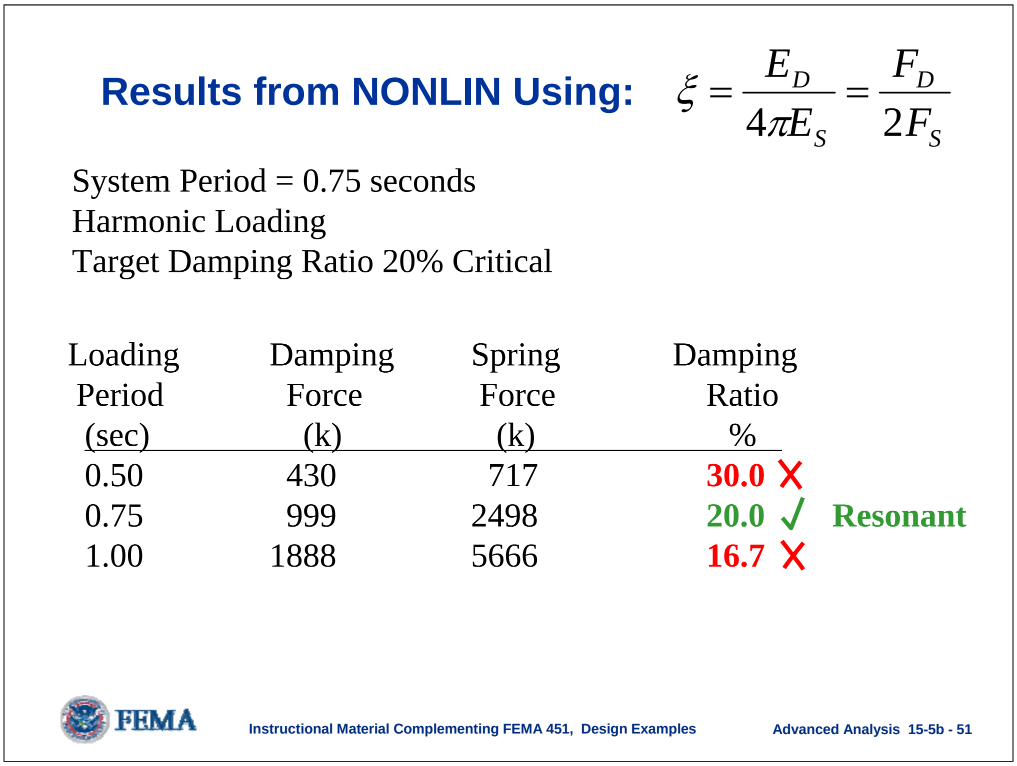


Results from NONLIN Using: $\xi = \dfrac{E_D}{4\pi E_S} = \dfrac{F_D}{2F_S}$

System Period = 0.75 seconds
Harmonic Loading
Target Damping Ratio 5% Critical

Loading Period (sec)	Damping Force (k)	Spring Force (k)	Damping Ratio %	
0.50	118	787	**7.50** ✗	
0.75	984	9828	**5.00** ✓	**Resonant**
1.00	197	2251	**3.75** ✗	

Here is a numerical example for a system with 5% damping at resonance. The use of the upper equation produces various results, depending on the ratio of the loading period to the system period. Only the resonant response produces the correct damping ratio.

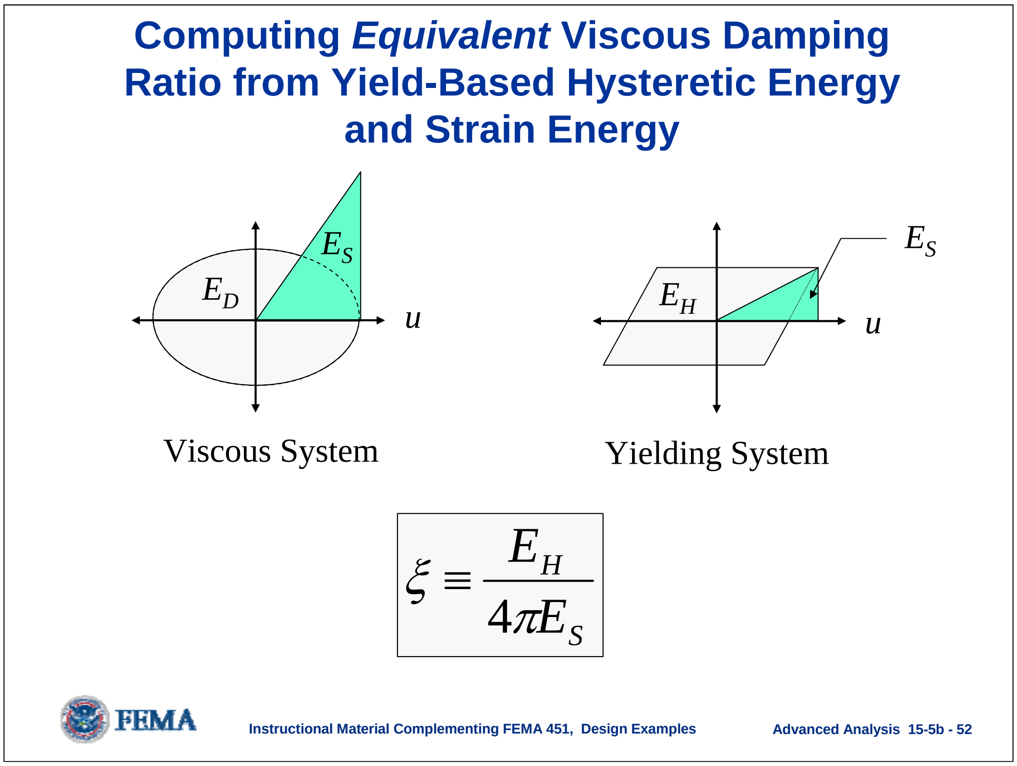


Results from NONLIN Using: $\xi = \dfrac{E_D}{4\pi E_S} = \dfrac{F_D}{2F_S}$

System Period = 0.75 seconds
Harmonic Loading
Target Damping Ratio 20% Critical

Loading Period (sec)	Damping Force (k)	Spring Force (k)	Damping Ratio %	
0.50	430	717	**30.0** ✗	
0.75	999	2498	**20.0** ✓	**Resonant**
1.00	1888	5666	**16.7** ✗	

Here, a similar analysis was performed, except the target damping ratio was 20% of critical. Again, only the resonant response produces the correct damping ratio.



Computing *Equivalent* Viscous Damping Ratio from Yield-Based Hysteretic Energy and Strain Energy

Viscous System

Yielding System

$$\boxed{\xi \equiv \frac{E_H}{4\pi E_S}}$$

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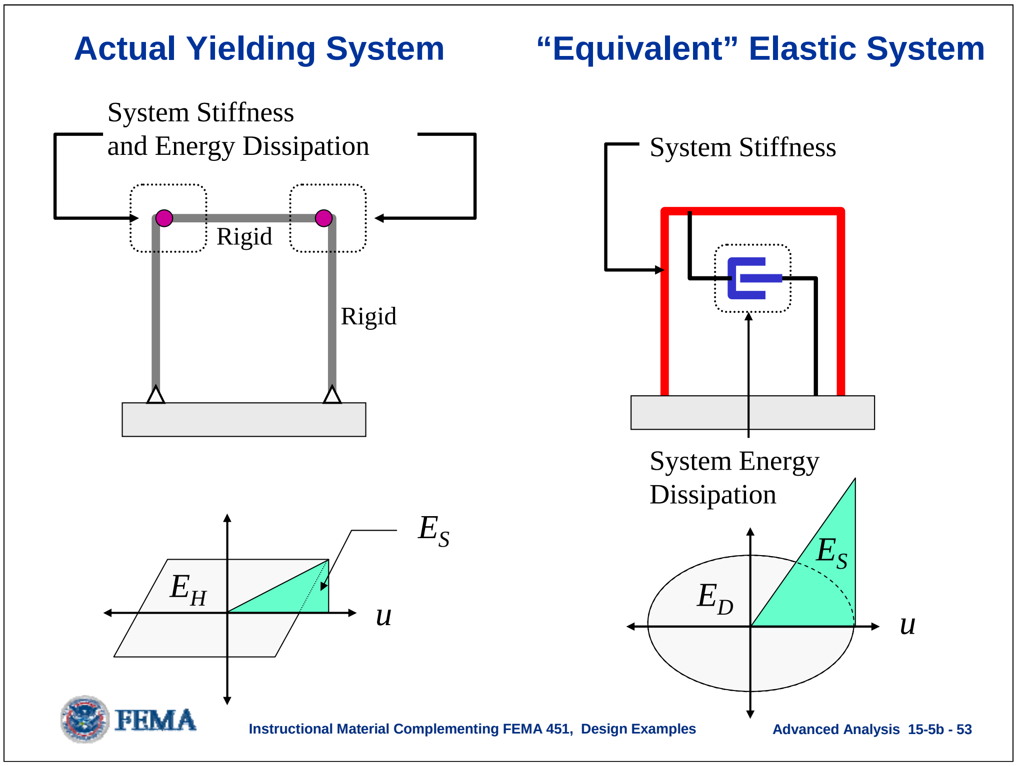
Now, an attempt is made to compute an equivalent viscous damping ratio for a system whose energy dissipation is hysteretic, rather than viscous. It is very important to note that in the viscous system the elastic energy is based on the initial stiffness of the system, whereas in the yielding system the secant stiffness is used.



Actual Yielding System

“Equivalent” Elastic System

System Stiffness
and Energy Dissipation

Rigid

Rigid

E_H

E_S

u

System Stiffness

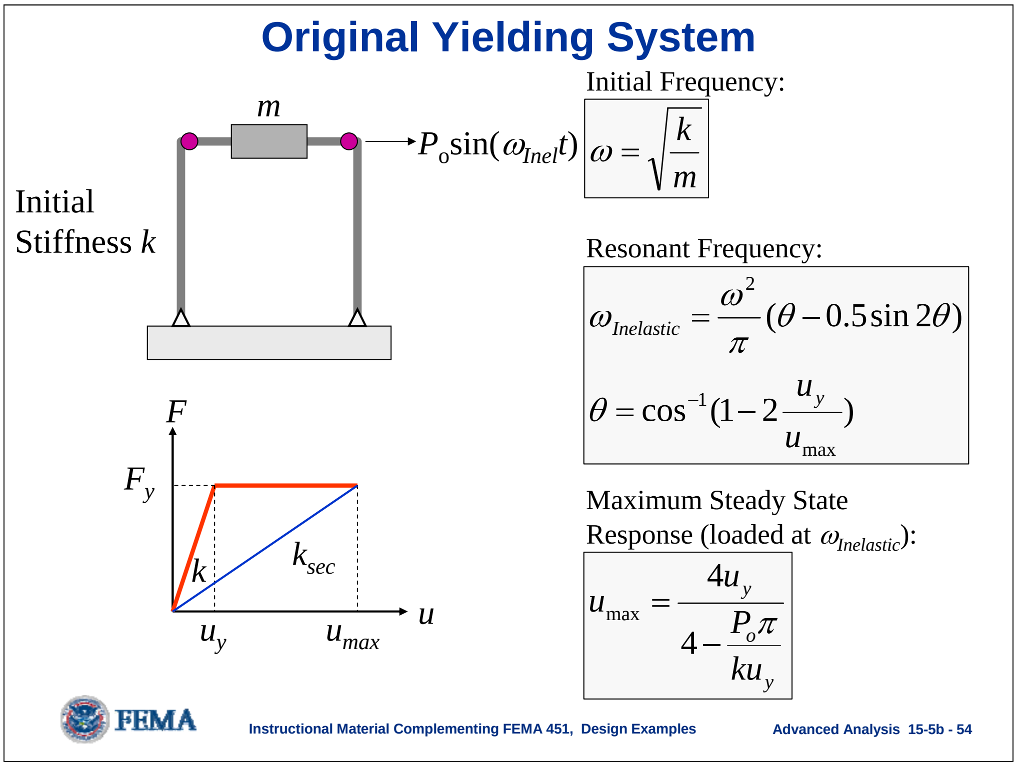
System Energy
Dissipation

E_D

E_S

u

This slide compares the conceptual differences between equivalent viscous damping for hysteretic systems, and equivalent viscous damping in elastic systems.



Original Yielding System

Initial Frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

Resonant Frequency:

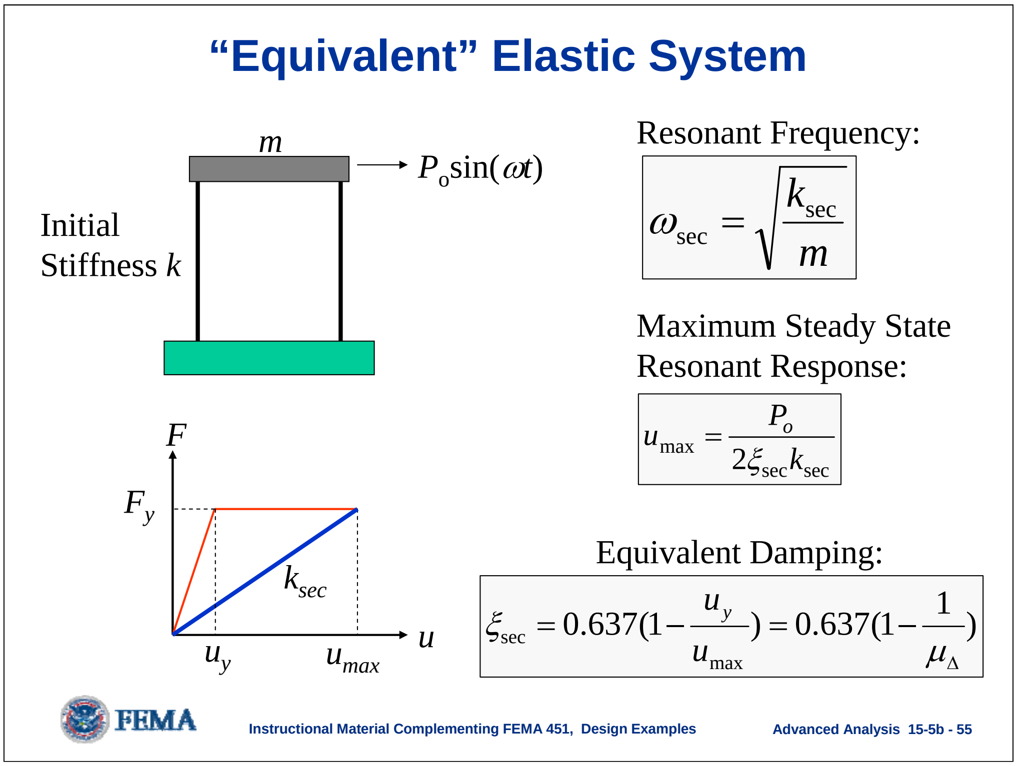
$$\omega_{Inelastic} = \frac{\omega^2}{\pi}(\theta - 0.5\sin 2\theta)$$

$$\theta = \cos^{-1}(1 - 2\frac{u_y}{u_{\max}})$$

Maximum Steady State Response (loaded at $\omega_{Inelastic}$):

$$u_{\max} = \frac{4u_y}{4 - \frac{P_o\pi}{ku_y}}$$

Here we show some actual properties of the yielding system. The formulas shown here are from the 1968 article by Jennings.



“Equivalent” Elastic System

Resonant Frequency:

$$\omega_{\sec} = \sqrt{\frac{k_{\sec}}{m}}$$

Maximum Steady State Resonant Response:

$$u_{\max} = \frac{P_o}{2\xi_{\sec} k_{\sec}}$$

Equivalent Damping:

$$\xi_{\sec} = 0.637(1 - \frac{u_y}{u_{\max}}) = 0.637(1 - \frac{1}{\mu_\Delta})$$

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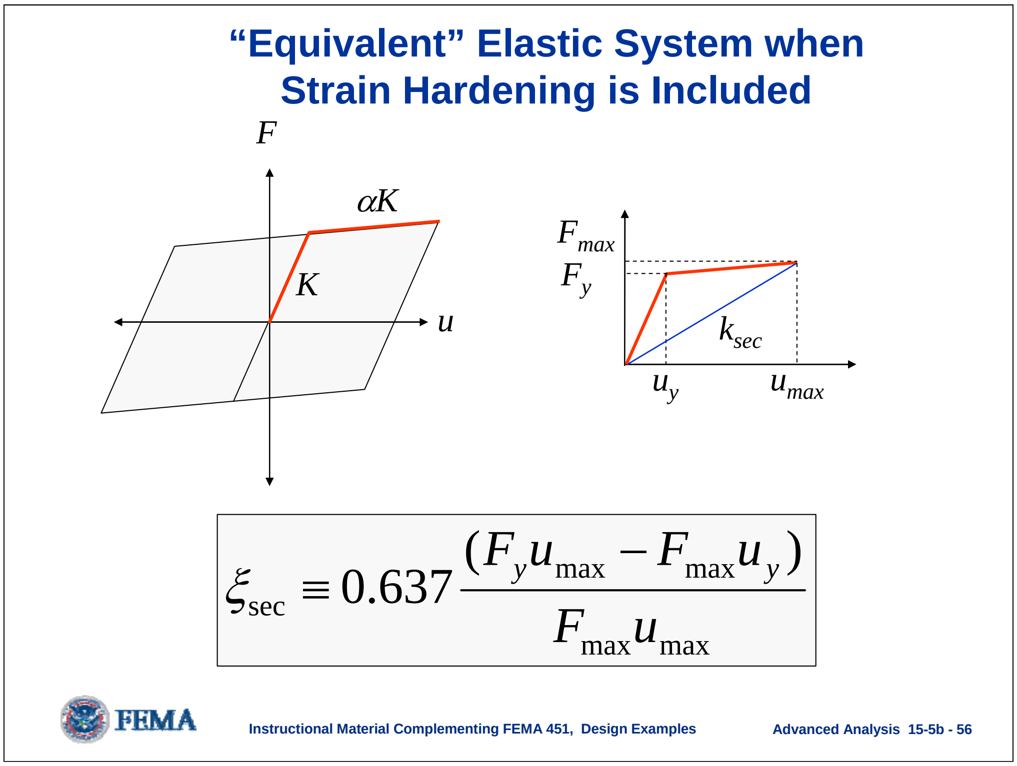
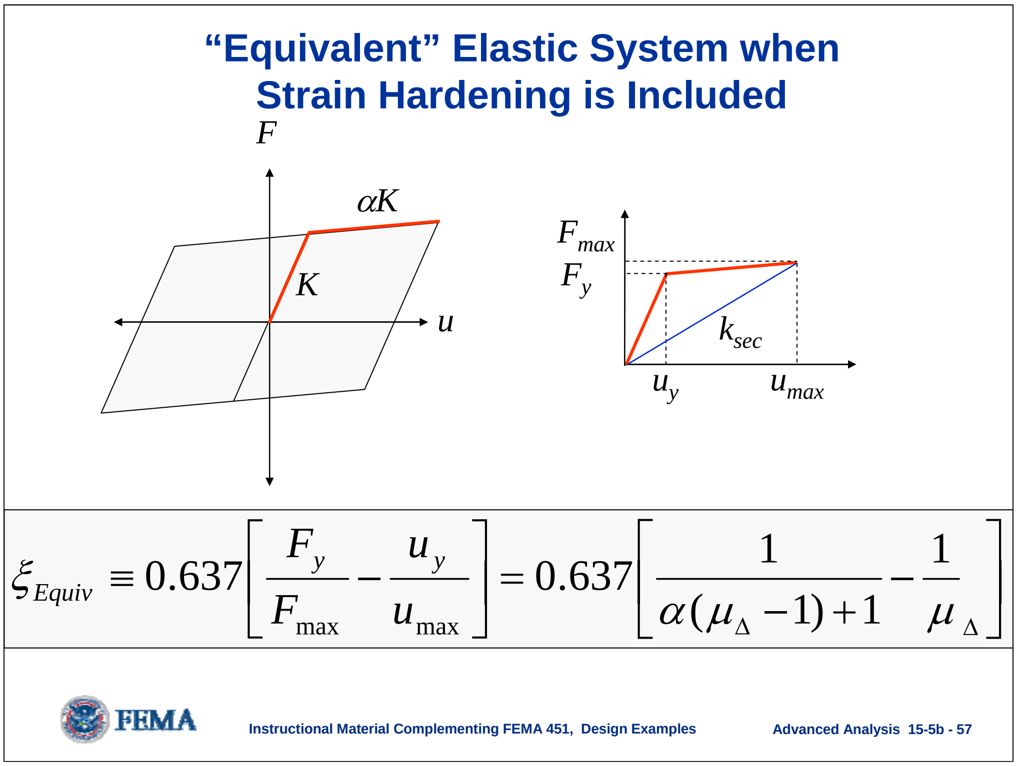
These are the properties of the equivalent system. Note the relationship between equivalent damping and ductility demand.



“Equivalent” Elastic System when Strain Hardening is Included

$$\xi_{\text{sec}} \equiv 0.637\,\frac{(F_y u_{\max} - F_{\max} u_y)}{F_{\max} u_{\max}}$$

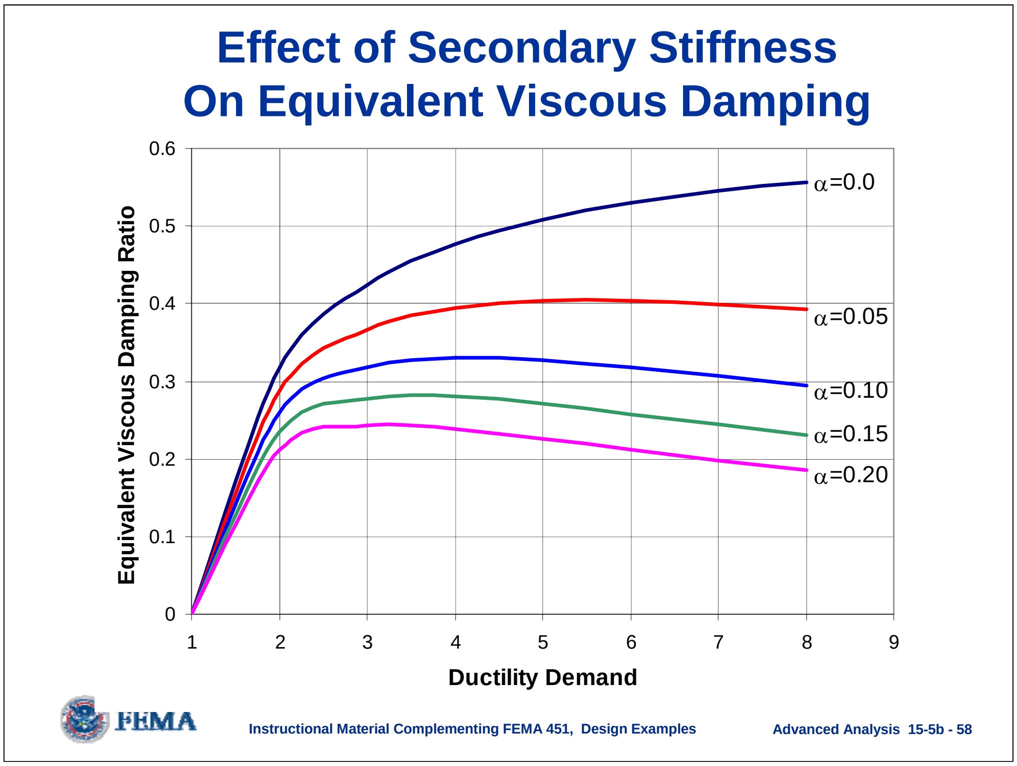
For a system with strain hardening, the equivalent viscous damping is computed as shown here. Note that the quantity α is the strain hardening ratio.



“Equivalent” Elastic System when Strain Hardening is Included

$$\xi_{Equiv} \equiv 0.637\left[\frac{F_y}{F_{\max}} - \frac{u_y}{u_{\max}}\right] = 0.637\left[\frac{1}{\alpha(\mu_\Delta - 1) + 1} - \frac{1}{\mu_\Delta}\right]$$

This is a recast of the previous formula in terms of ductility and strain hardening ratio.



Effect of Secondary Stiffness On Equivalent Viscous Damping

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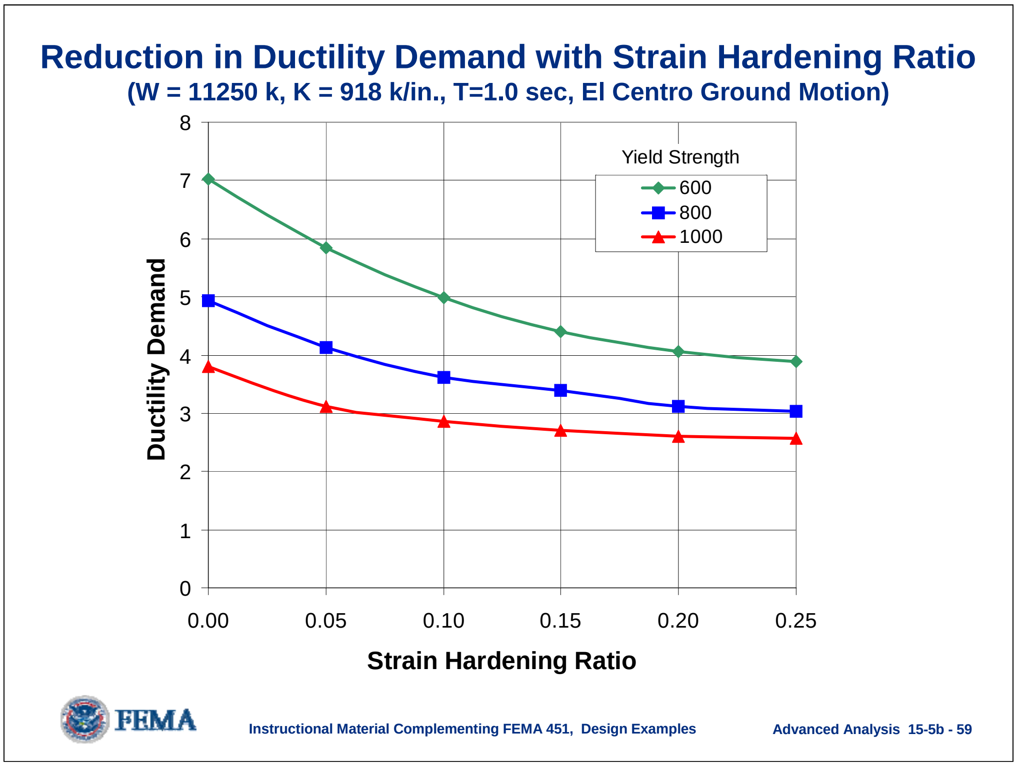
Advanced Analysis 15-5b - 58

This is a plot of equivalent viscous damping ratios versus ductility demand for systems with different strain hardening ratios. Note that systems with high strain hardening ratios actually have reduced equivalent damping ratios at larger ductility demands.



Reduction in Ductility Demand with Strain Hardening Ratio

(W = 11250 k, K = 918 k/in., T=1.0 sec, El Centro Ground Motion)

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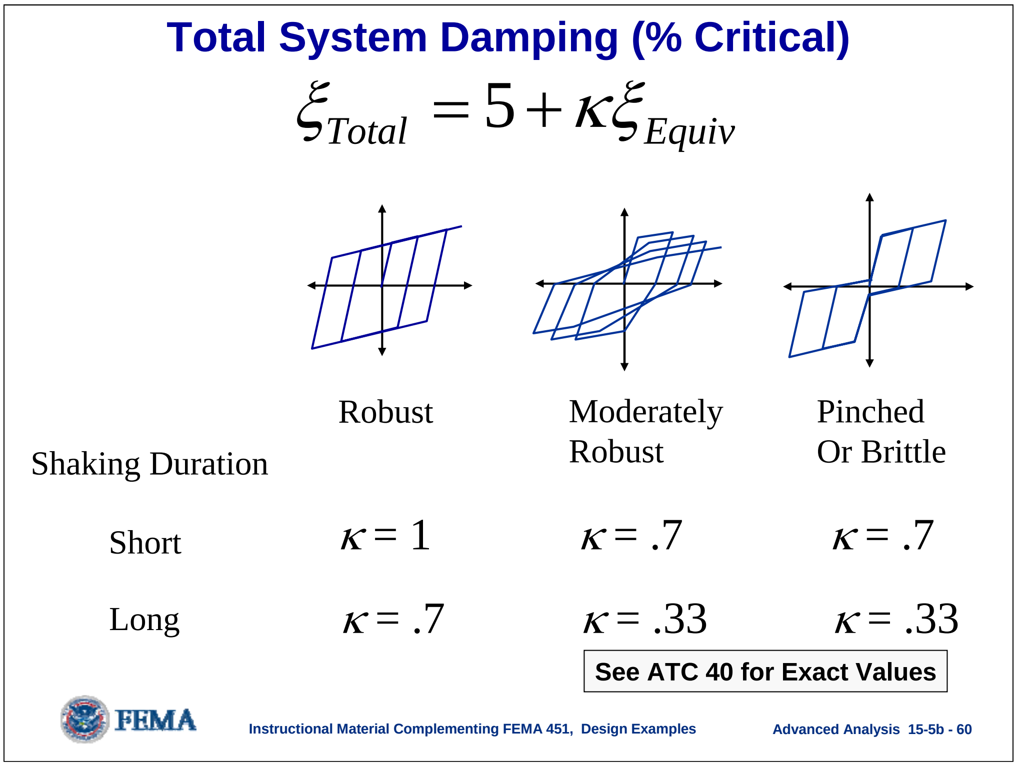
Advanced Analysis 15-5b - 59

In practical cases the reduction in damping with larger ductility ratios and strain hardening ratios is not likely a problem due to the fact that (as expected) the ductility demand reduces with strain hardening ratio. For systems with high strain hardening it is unlikely that the ductility demand will be high enough to indicate "decreasing damping" with increased ductility demand.



Total System Damping (% Critical)

$$\xi_{Total} = 5 + \kappa \xi_{Equiv}$$

Shaking Duration	Robust	Moderately Robust	Pinched Or Brittle
Short	$\kappa = 1$	$\kappa = .7$	$\kappa = .7$
Long	$\kappa = .7$	$\kappa = .33$	$\kappa = .33$

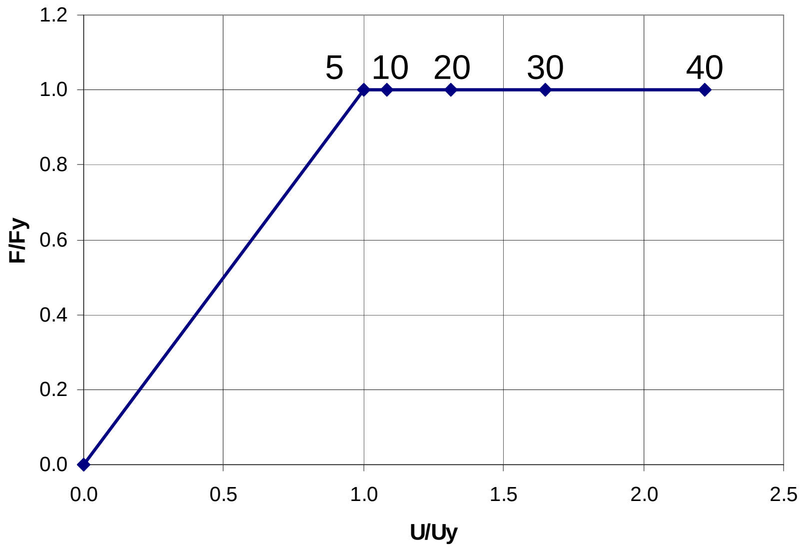
See ATC 40 for Exact Values

In the previous derivations it was assumed that the inelastic hysteretic behavior is “robust”. For systems with less robust behavior the energy dissipated per cycle will be reduced. Also, the longer the duration of strong shaking, the more the likelihood of reduced stiffness and strength.

The kappa term of ATC 40 compensates for this effect. The total damping in a system is the 5% inherent damping plus kappa times the additional equivalent viscous damping from inelastic energy dissipation.

Equivalent Viscous Damping Values for EPP System

(Values Shown are Percent Critical)



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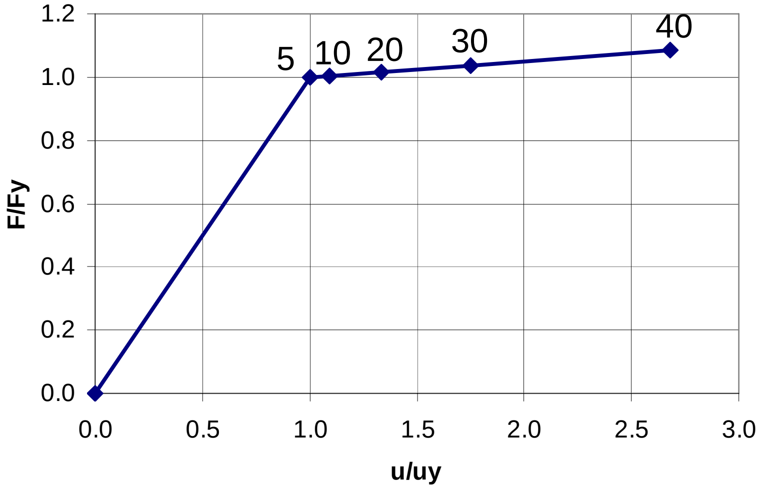
This plot shows the equivalent viscous damping of 5 to 40 percent superimposed in a normalized bilinear capacity curve. This follows the formula that damping=5+63.7(1-1/mu). Note that the equivalent damping at yield is 5% (as expected for an elastic system). Note also that equivalent damping increases very rapidly with ductility demand. At a ductility demand of only 2, equivalent viscous damping is about 35% of critical.

Equivalent Viscous Damping Values for System

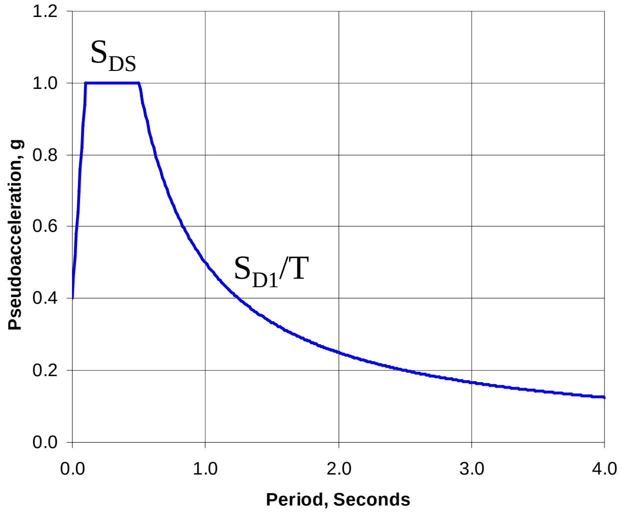
With 5% Strain Hardening Ratio

(Values Shown are Percent Critical)



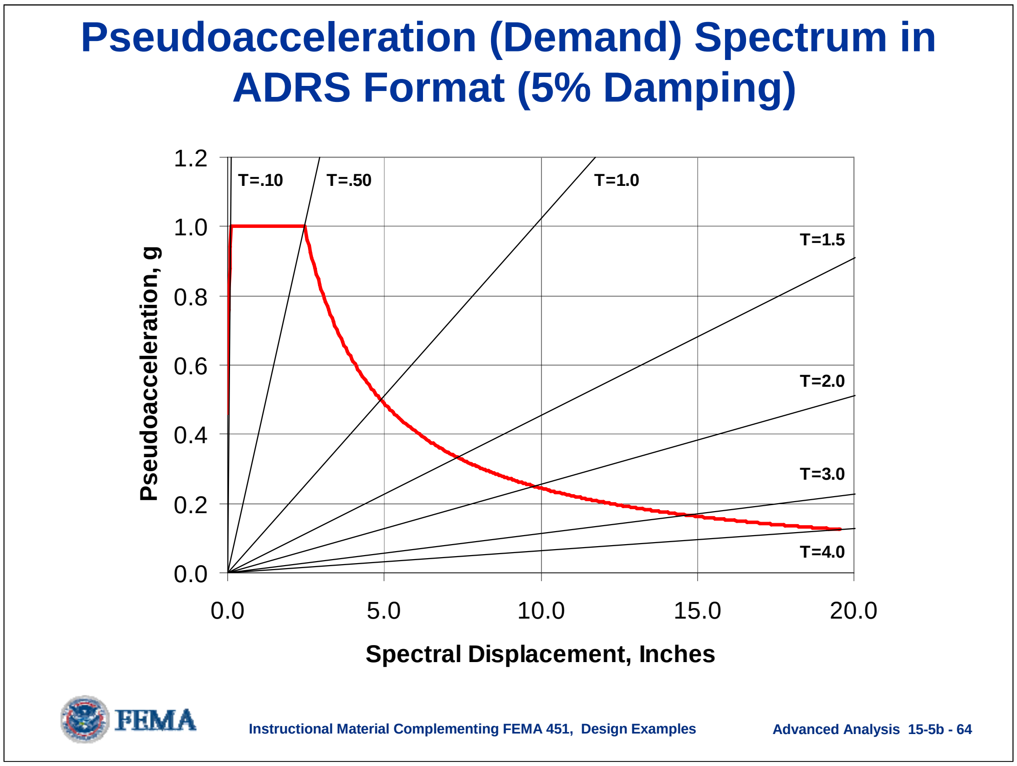
This is similar to the previous plot, but strain hardening is 5% of the initial stiffness. Note that equivalent viscous damping is somewhat less than 35% critical when the ductility demand is 2.0.

Pseudoacceleration Spectrum in Traditional Format



Now we are ready to discuss the demand curve. Shown is a the NEHRP response spectrum normalized to a maximum value of 1.0 g. Recall that this spectrum includes site effects as well as the 2/3 factor to account for “expected good behavior”. In the western U.S. this is equivalent to a 10% in 50 year earthquake. In the eastern U.S. it is closer to a 5% in 50 year earthquake.

Note that the response spectrum is plotted in the traditional manner of pseudoacceleration vs period of vibration.



Pseudoacceleration (Demand) Spectrum in ADRS Format (5% Damping)

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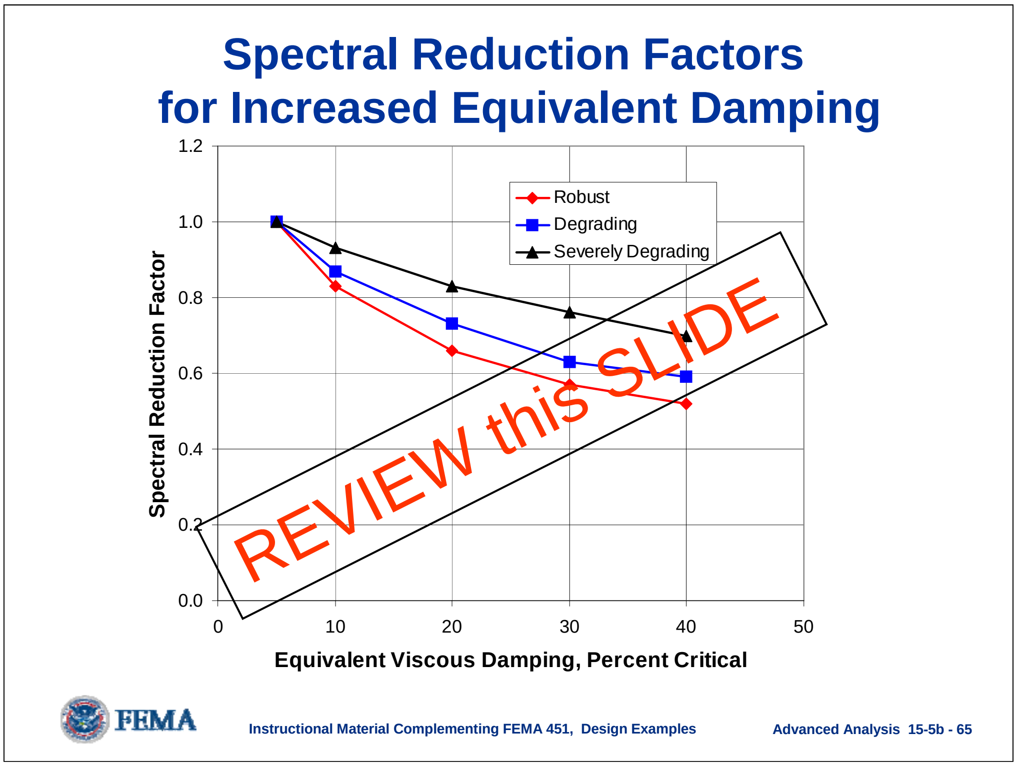
Advanced Analysis 15-5b - 64

In this plot the response spectrum is plotted in so-called Acceleration-Displacement space, hence the name Acceleration Displacement Response Spectrum (ADRS). Here, period values are shown as diagonal lines.



Spectral Reduction Factors for Increased Equivalent Damping

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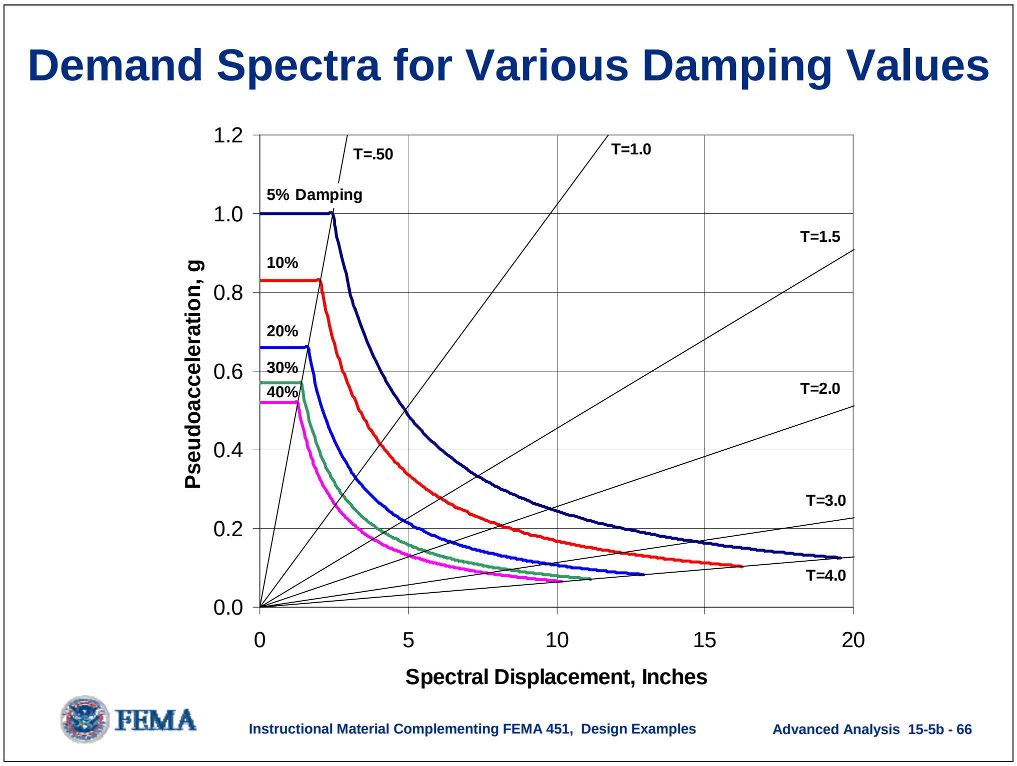
Advanced Analysis 15-5b - 65

The previous spectrum was computed for a system with 5% damping. For higher levels of viscous damping the resulting displacements and accelerations will be lower. It is unclear why different curves are provided for systems with degrading and severely degrading response… a response spectrum is by definition based on a linear elastic analysis of a system with a certain level of damping. Also, the effect of hysteretic behavior on damping is already included in the kappa factor shown on a previous slide.



Demand Spectra for Various Damping Values

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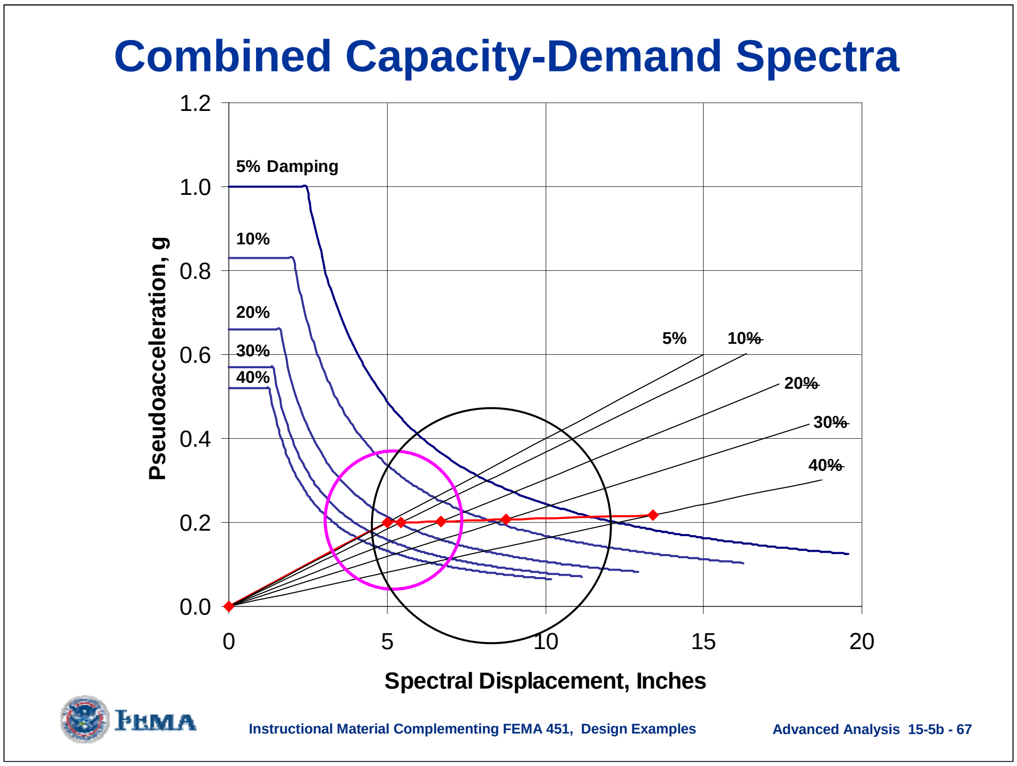
Advanced Analysis 15-5b - 66

Here we show a family of demand spectra for various damping values ranging from 5 to 40% of critical. At any period value the displacement and the pseudoaccelerations are significantly reduced as damping increases.



Combined Capacity-Demand Spectra

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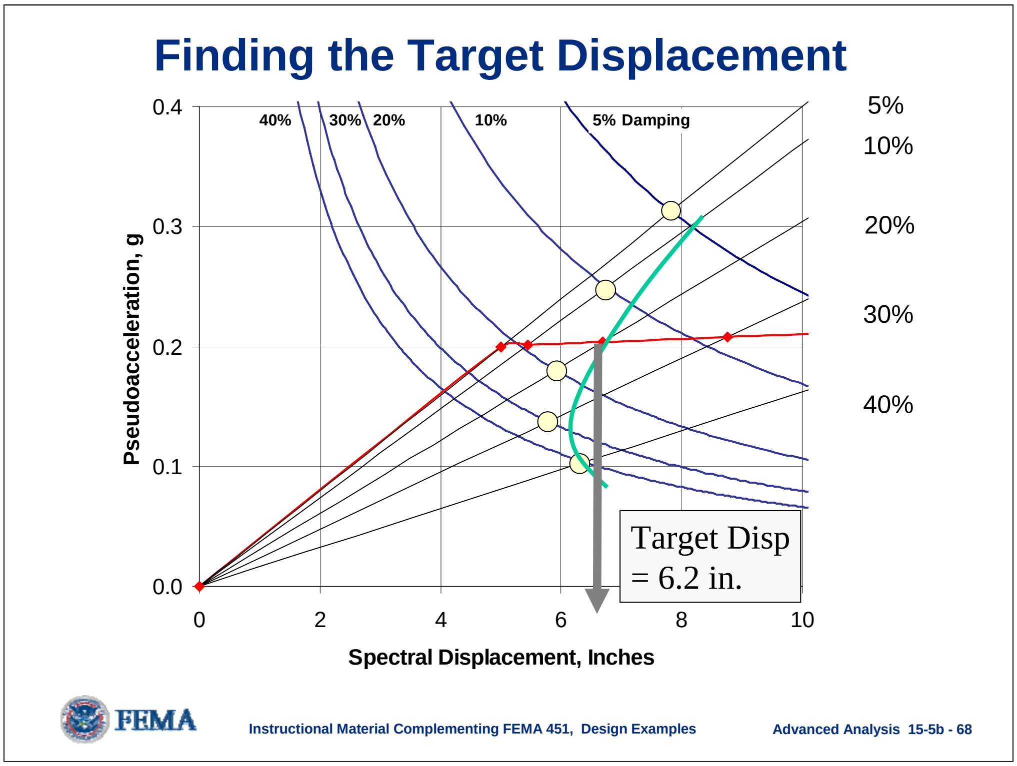
Advanced Analysis 15-5b - 67

Now the demand spectrum and the capacity spectrum are plotted on the same graph. This is the advantage of making the initial pushover curve transformation from force-displacement to modal acceleration- modal displacement. Here, diagonal lines are used to label the damping values on the capacity plot. We will zoom in on the area in the circle on the next slide.



Finding the Target Displacement

Target Disp
= 6.2 in.

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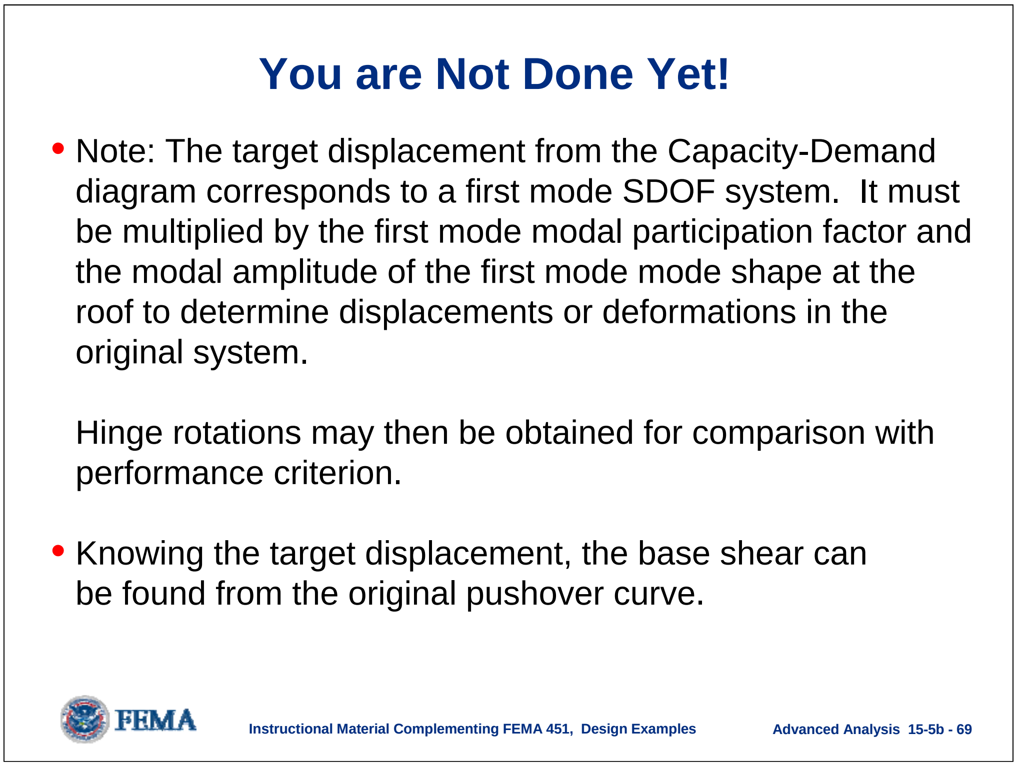
Finally, we are able to determine the target displacement. It is the displacement corresponding to the point where the X percent demand spectra and the X percent damping point on the capacity spectrum intersect. This point is found graphically on this plot. It may also be found by iteration or for simple capacity and demand curves, a closed-form solution may be found. Recall that this displacement corresponds to the first mode SDOF system and must therefore be transformed back to the displacement within the MDOF system.



You are Not Done Yet!

- Note: The target displacement from the Capacity-Demand diagram corresponds to a first mode SDOF system. It must be multiplied by the first mode modal participation factor and the modal amplitude of the first mode mode shape at the roof to determine displacements or deformations in the original system.

  Hinge rotations may then be obtained for comparison with performance criterion.

- Knowing the target displacement, the base shear can be found from the original pushover curve.

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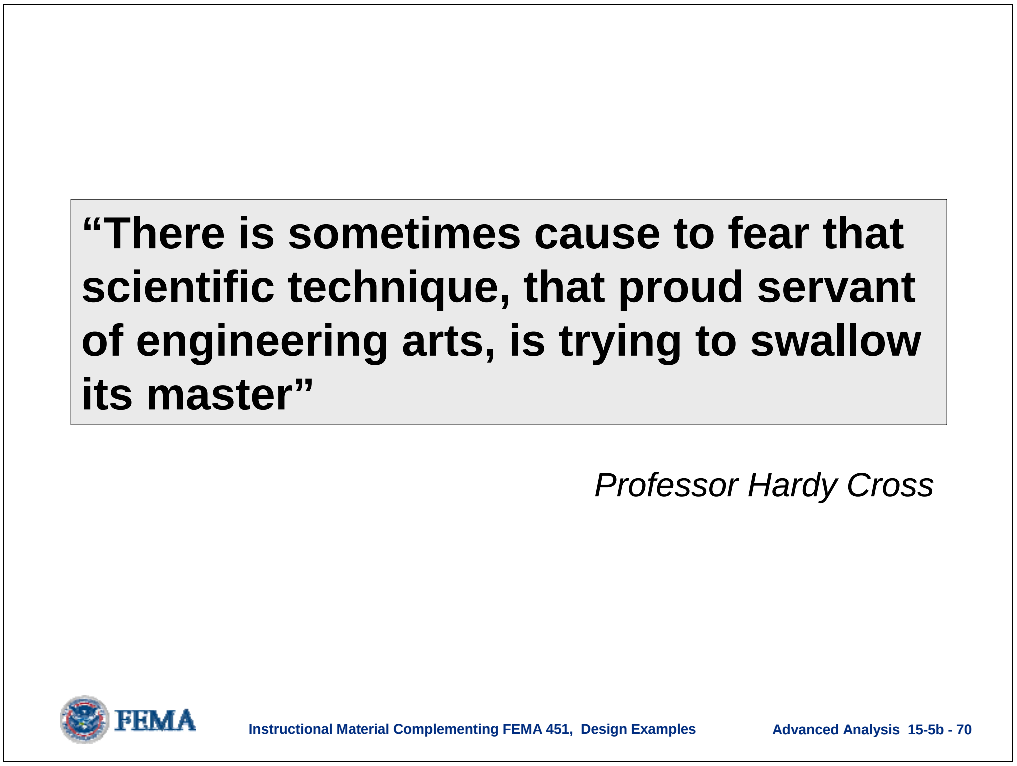
These are the final steps in the process.



> **“There is sometimes cause to fear that scientific technique, that proud servant of engineering arts, is trying to swallow its master”**

Professor Hardy Cross

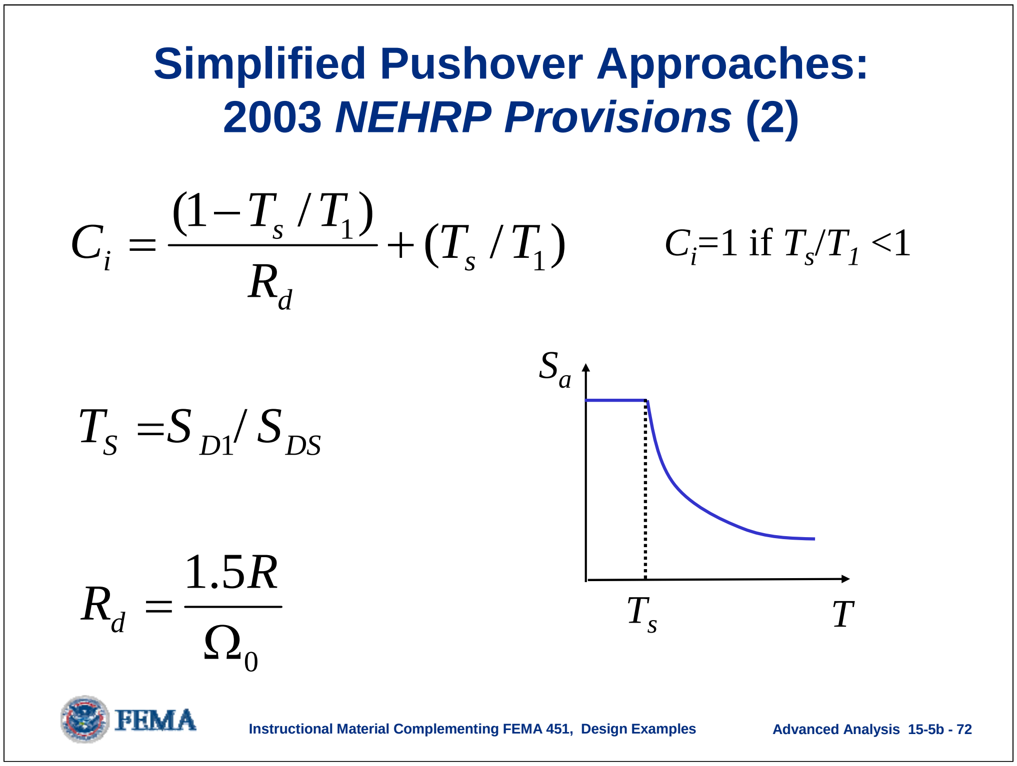
It has taken a lot of work to find the one target displacement. Such an analysis would need to be repeated for different lateral load patterns, and possibly for various perturbations in hysteretic behavior. Fortunately, several commercial programs (e.g SAP2000, RAM PERFORM) make the process relatively simple. However, there are many many assumptions and simplifications involved in the process, and one might wonder if a more simple approach could be used without tremendous loss in accuracy. Two such simplified approaches are given in the next several slides.

Simplified Pushover Approaches: *2003 NEHRP Recommended Provisions*

- Procedure is presented in Appendix to Chapter 5
- Gravity Loads include 25% of live load (but Provisions are not specific on P-Delta Modeling Requirements)
- Lateral Loads Applied in a “First Mode Pattern”
- Structure is pushed to 150% of target displacement
- Target displacement is assumed equal to the displacement computed from a first mode response spectrum analysis, multiplied by the factor C_i
- C_i adjusts for “error” in equal displacement theory when structural period is low

The first of the simplified procedures is given by the 2003 NEHRP Provisions. A list of the basic assumptions is presented on this slide.

The use of the first mode displacement follows the “equal displacement” observation… that is the displacements predicted from an elastic and inelastic analysis of the same structure are approximately the same. The NEHRP response spectrum is used in the displacement part of the analysis.



Simplified Pushover Approaches: 2003 *NEHRP Provisions* (2)

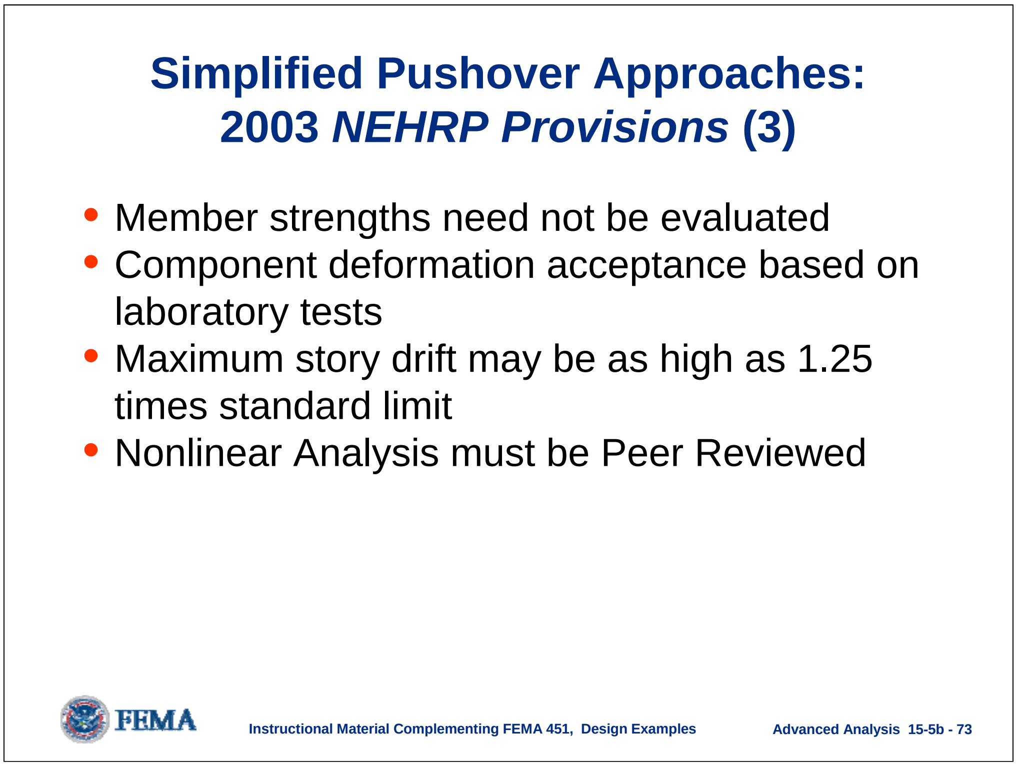
$$C_i = \frac{(1-T_s/T_1)}{R_d} + (T_s/T_1)$$

$C_i = 1$ if $T_s/T_1 < 1$

$$T_S = S_{D1}/S_{DS}$$

$$R_d = \frac{1.5R}{\Omega_0}$$

C_i is a correction factor for very short period structures for which it has been observed that the equal displacement approach is not particularly reliable.



Simplified Pushover Approaches: 2003 *NEHRP Provisions* (3)

- Member strengths need not be evaluated
- Component deformation acceptance based on laboratory tests
- Maximum story drift may be as high as 1.25 times standard limit
- Nonlinear Analysis must be Peer Reviewed

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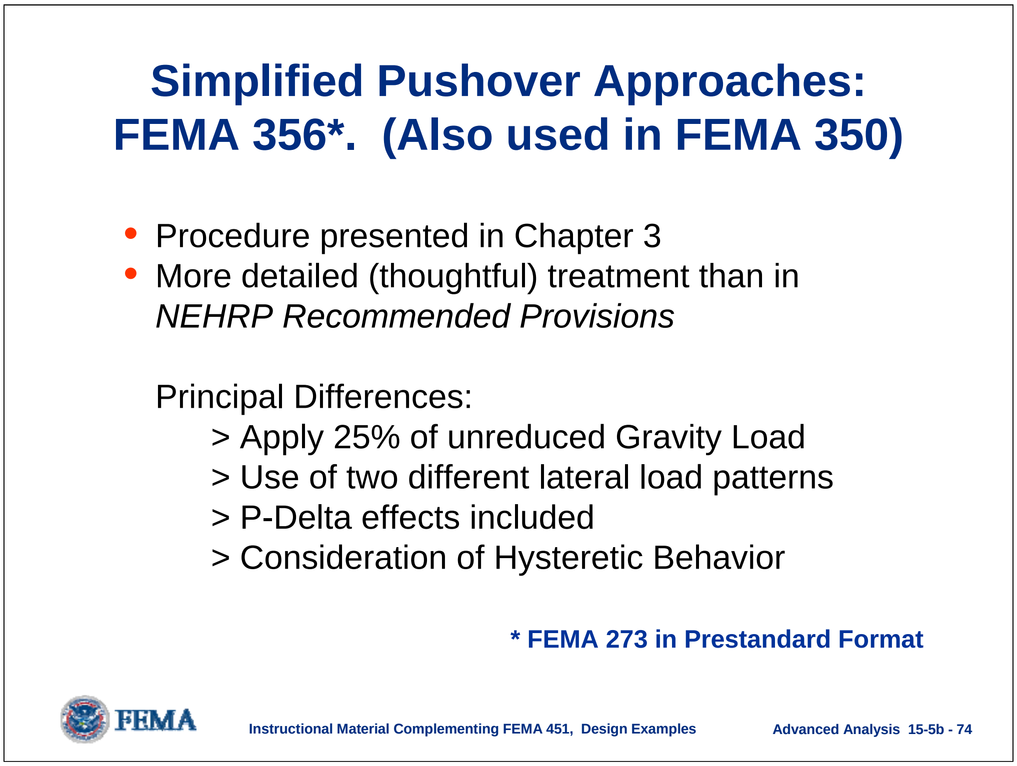
The first statement is a testament to the old seismic design adage “strength is essential but otherwise unimportant”.

The *Provisions* do not provide acceptance criteria for component deformations. It is suggested by the Provisions that such limits be based on available test data. Instead, performance criteria from ATC40 or FEMA 356 may be used.

An “Advantage” to performing pushover analysis (or nonlinear response history analysis) is that the allowable story drifts are increased by 25%.



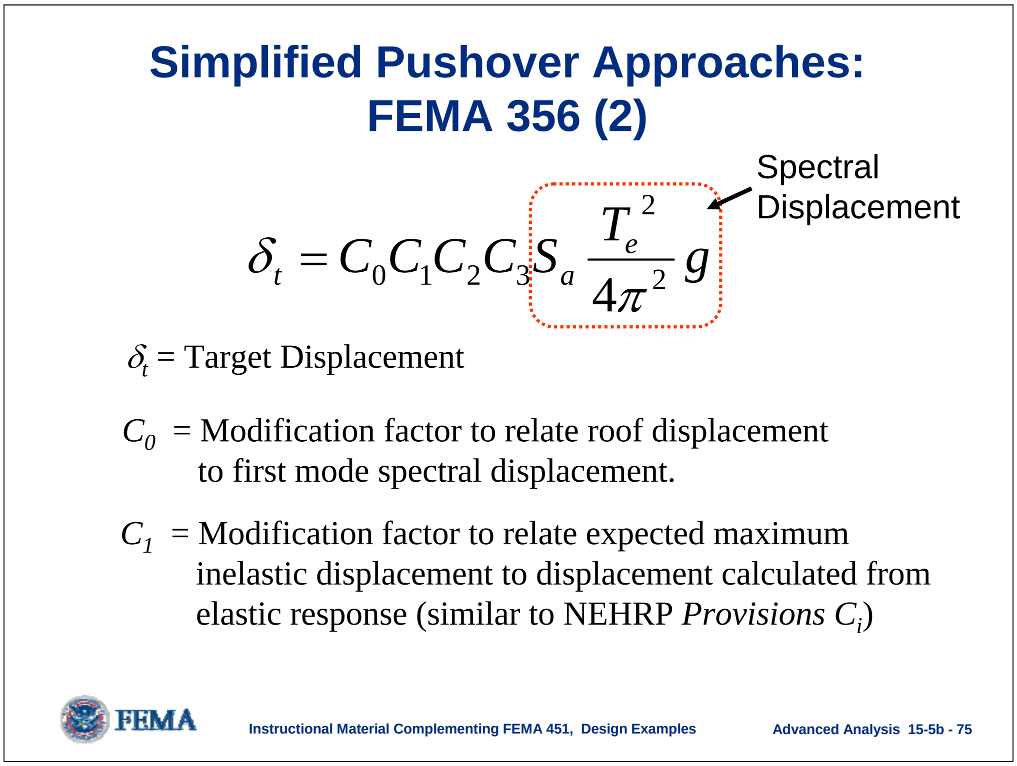
Simplified Pushover Approaches: FEMA 356*. (Also used in FEMA 350)

- Procedure presented in Chapter 3
- More detailed (thoughtful) treatment than in *NEHRP Recommended Provisions*

Principal Differences:
> Apply 25% of unreduced Gravity Load
> Use of two different lateral load patterns
> P-Delta effects included
> Consideration of Hysteretic Behavior

*** FEMA 273 in Prestandard Format**

FEMA 356 gives a similar but somewhat more detailed procedure. The principal differences are shown here.



Simplified Pushover Approaches: FEMA 356 (2)

Spectral Displacement

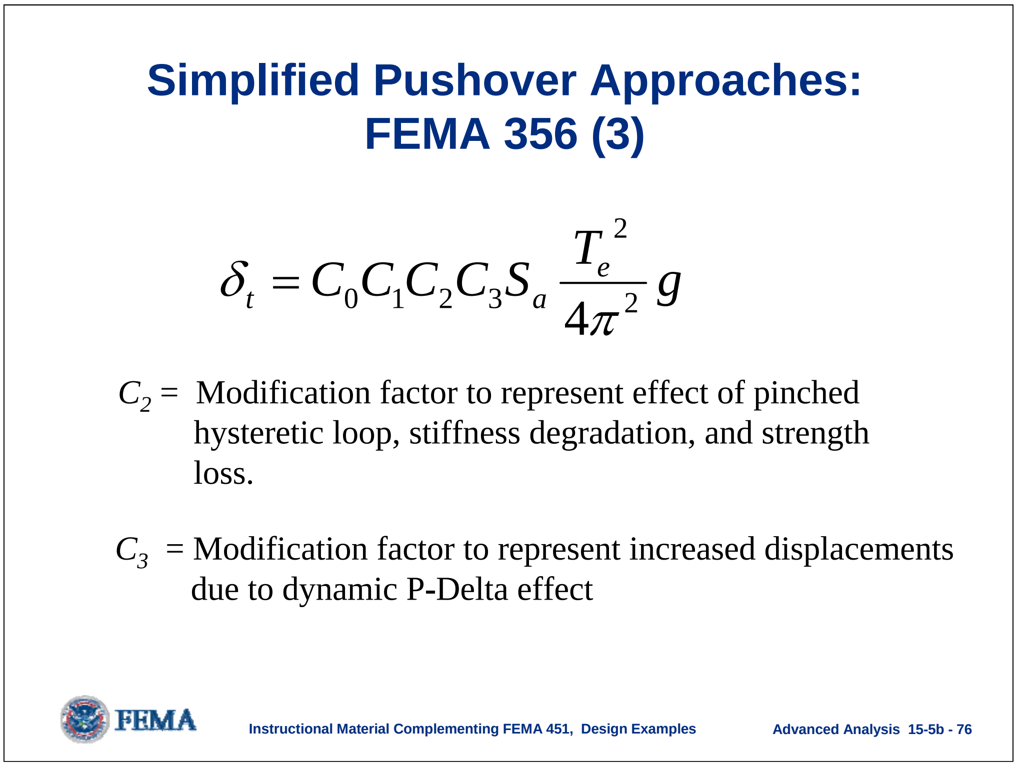
$$\delta_t = C_0 C_1 C_2 C_3 \boxed{S_a \frac{T_e^{\,2}}{4\pi^2} g}$$

δ_t = Target Displacement

C_0 = Modification factor to relate roof displacement to first mode spectral displacement.

C_1 = Modification factor to relate expected maximum inelastic displacement to displacement calculated from elastic response (similar to NEHRP *Provisions* C_i)

In FEMA 356, the target displacement is computed from a product of multipliers on the spectral displacement. The first multiplier, C_o, is effectively the first mode participation factor times the ordinate of the mode shape at the roof of the structure. The second multiplier, C_1, accounts for “errors” in the equal displacement concept for low period buildings.



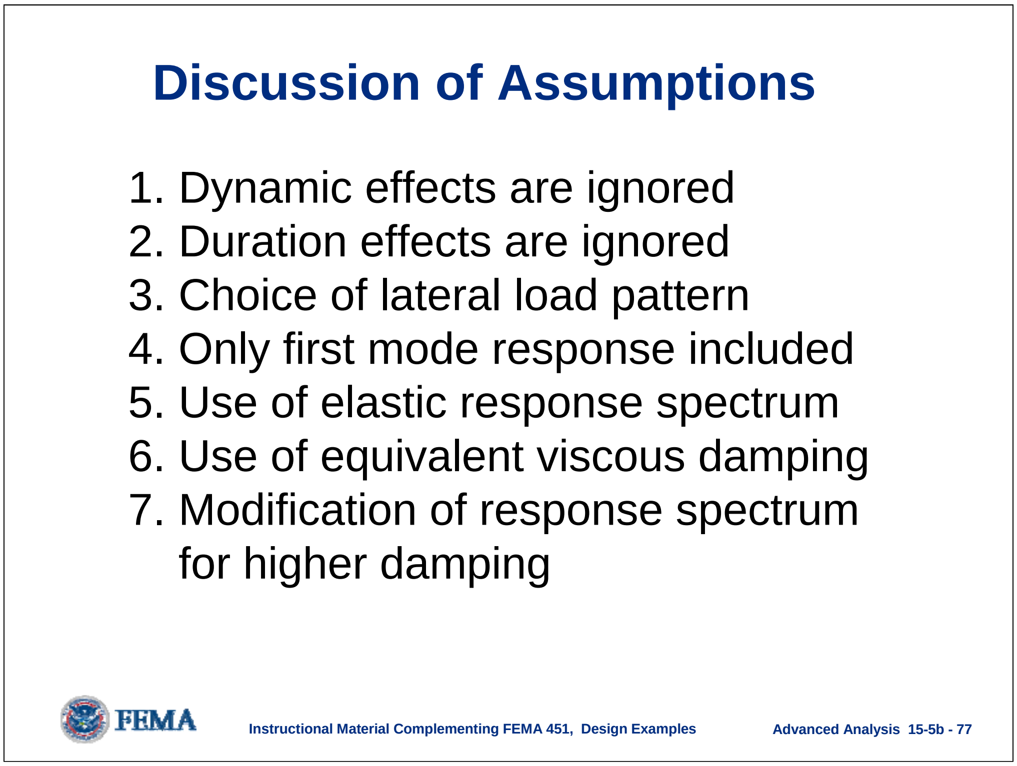
Simplified Pushover Approaches: FEMA 356 (3)

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^{\,2}}{4\pi^2} g$$

C_2 = Modification factor to represent effect of pinched hysteretic loop, stiffness degradation, and strength loss.

C_3 = Modification factor to represent increased displacements due to dynamic P-Delta effect

Multiplier C_2 is an adjustment for hysteretic behavior, and C_3 is a modifier for P-Delta effects.

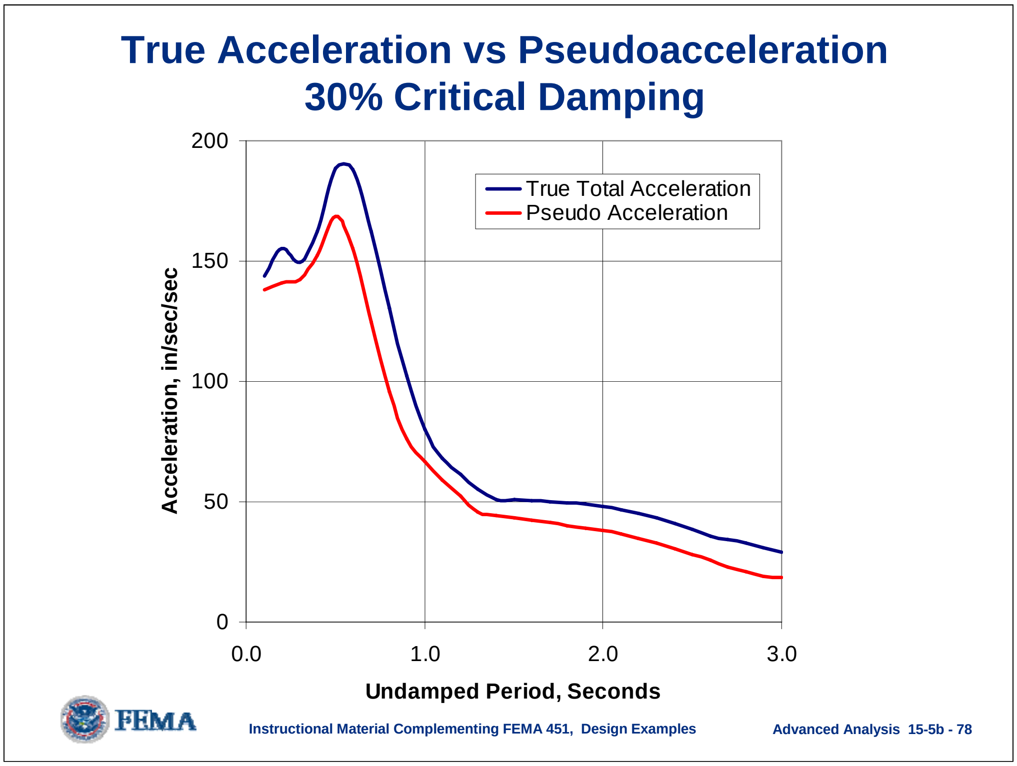


Discussion of Assumptions

1. Dynamic effects are ignored
2. Duration effects are ignored
3. Choice of lateral load pattern
4. Only first mode response included
5. Use of elastic response spectrum
6. Use of equivalent viscous damping
7. Modification of response spectrum for higher damping

This is a list of the most pertinent (glaring) assumptions in pushover analysis. The list is quite long, and because of these issues, many engineers and researchers believe that pushover analysis is not a viable analysis/design tool. This is difficult to argue with. However, a pushover analysis does provide more information than does a purely elastic analysis. In particular it is beneficial to know the sequencing of hinging and to develop a true estimate of overstrength. Pushover analysis can be considered as a useful evaluation tool, not to be used alone, but used in concert with other tools to assess the likely performance of a structure.

This being said, some of the potential problems with pushover analysis are illustrated in the following slides.



True Acceleration vs Pseudoacceleration
30% Critical Damping

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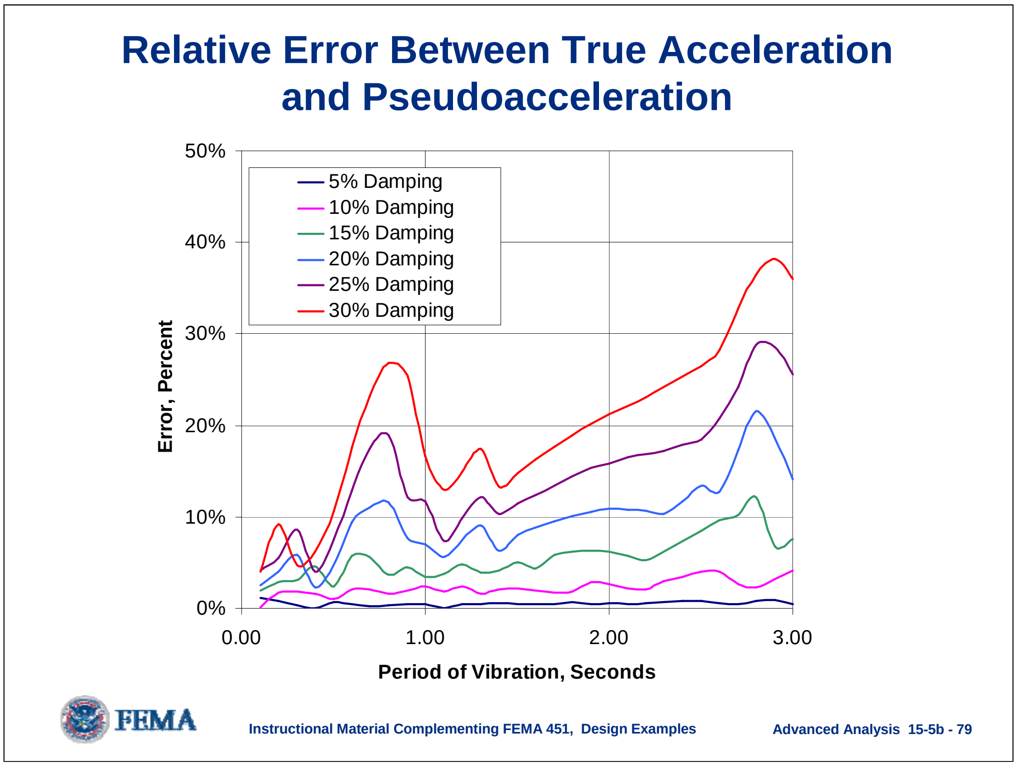
The main issue with pushover analysis is the use of the equivalent viscous damping to predict the response of a yielding system. One reason for concern is that we use a pseudo acceleration spectrum as the basis for predicting displacements in a SDOF system. Recall that the pseudoacceleration spectrum is derived from the true displacement spectrum by dividing each displacement value by the frequency squared. For low damping values, there is negligible error in this assumption. For higher damping values the error can be quite large, particularly when the system has a long period of vibration. This plot shows the comparison of a pseudoacceleration spectrum and a true acceleration spectrum for a system with 30% critical damping. At the higher period values the error is 30%, on the unconservative side.



Relative Error Between True Acceleration and Pseudoacceleration

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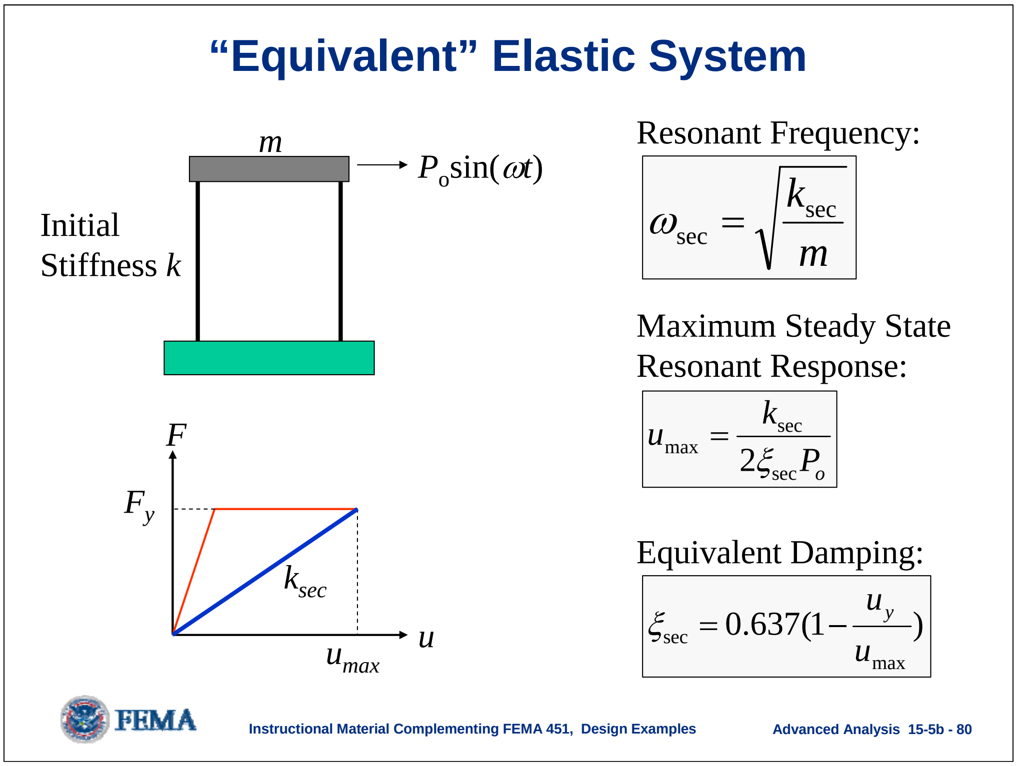
This plot shoes the error in computed pseudoacceleration and true acceleration for a range of different damping values. In general, the higher the damping value the larger the error, and the longer the period of vibration, the higher the error.



"Equivalent" Elastic System

Resonant Frequency:

$$\omega_{\mathrm{sec}} = \sqrt{\frac{k_{\mathrm{sec}}}{m}}$$

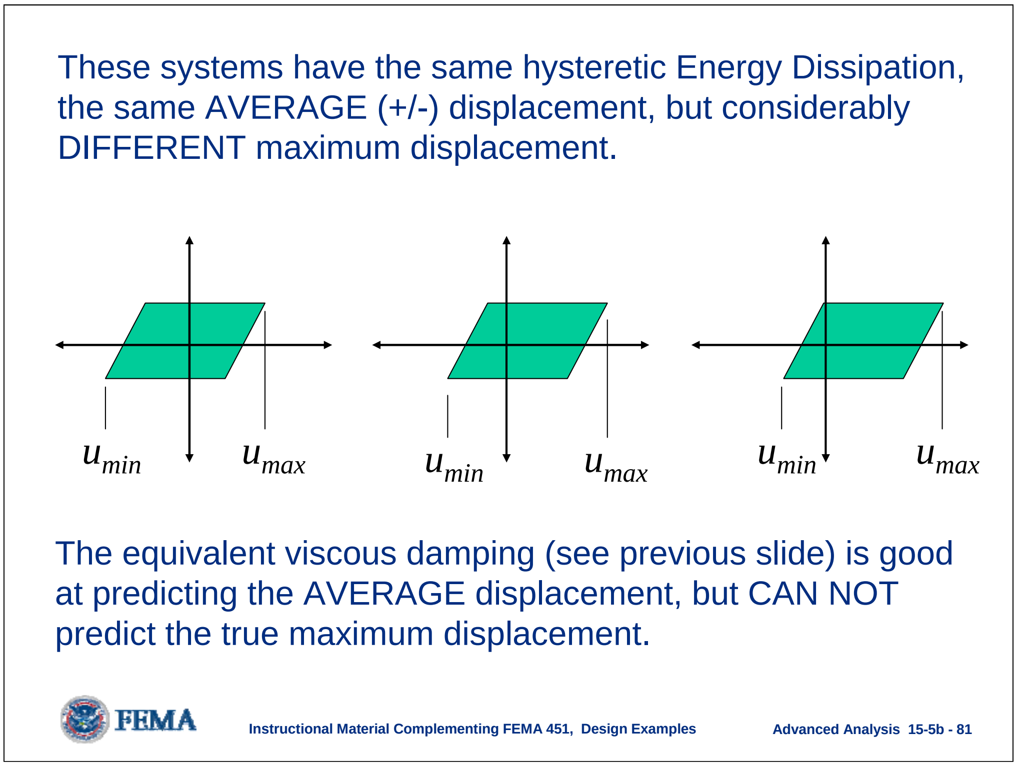
Maximum Steady State
Resonant Response:

$$u_{\max} = \frac{k_{\mathrm{sec}}}{2\xi_{\mathrm{sec}} P_o}$$

Equivalent Damping:

$$\xi_{\mathrm{sec}} = 0.637(1 - \frac{u_y}{u_{\max}})$$

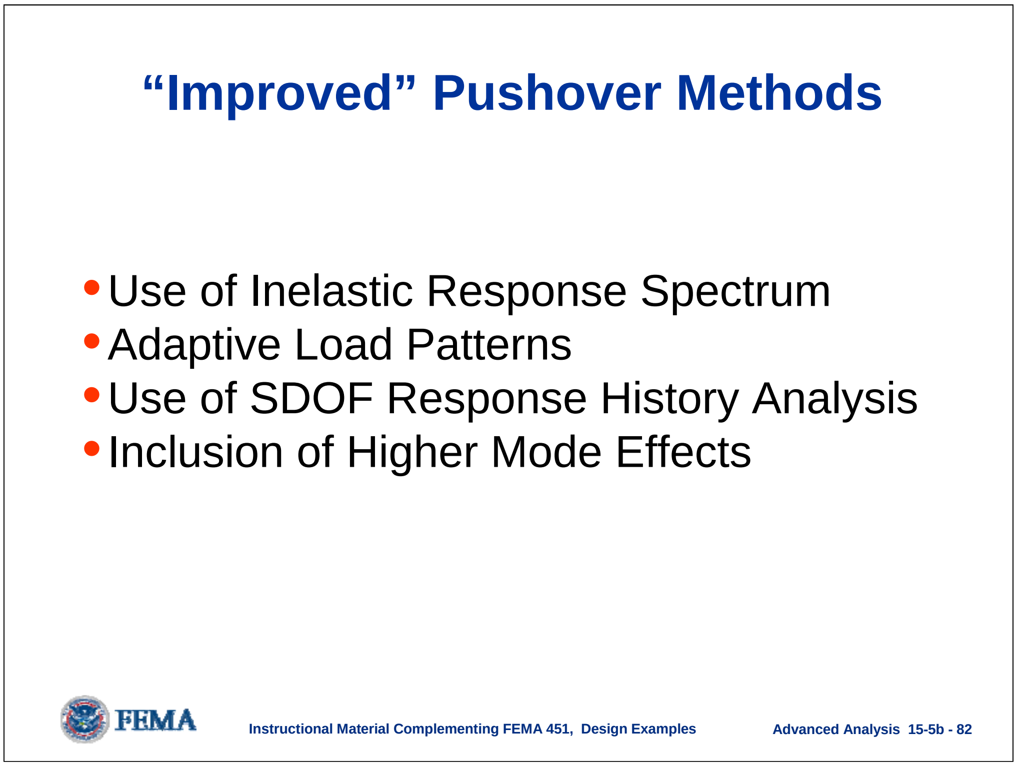
Another big concern is the use of equivalent viscous damping itself. This is a review of how the equivalent damping is computed.



These systems have the same hysteretic Energy Dissipation, the same AVERAGE (+/-) displacement, but considerably DIFFERENT maximum displacement.

The equivalent viscous damping (see previous slide) is good at predicting the AVERAGE displacement, but CAN NOT predict the true maximum displacement.

Here, three systems that have the same hysteretic energy have considerably different maximum displacements due to residual deformation. This effect can occur for a variety of reasons, some having to do with ground motion, and others with structural properties. Jennings pointed this out in his original paper that forwarded the use of equivalent damping. To date, this effect is not included in pushover analysis. It is included automatically in response history analysis.



“Improved” Pushover Methods

- Use of Inelastic Response Spectrum
- Adaptive Load Patterns
- Use of SDOF Response History Analysis
- Inclusion of Higher Mode Effects

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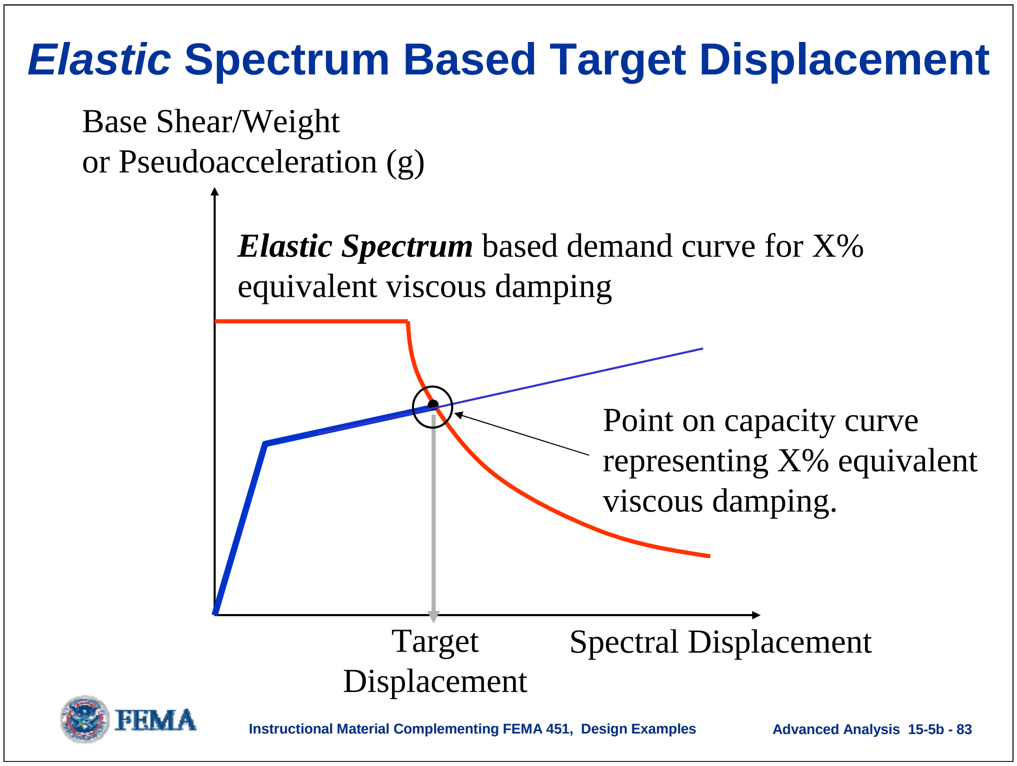
These are several of the ideas that have been used to improve pushover analysis. In most cases the desire is to produce a predicted response that more closely matched that from response history analysis.



Elastic Spectrum Based Target Displacement

Base Shear/Weight
or Pseudoacceleration (g)

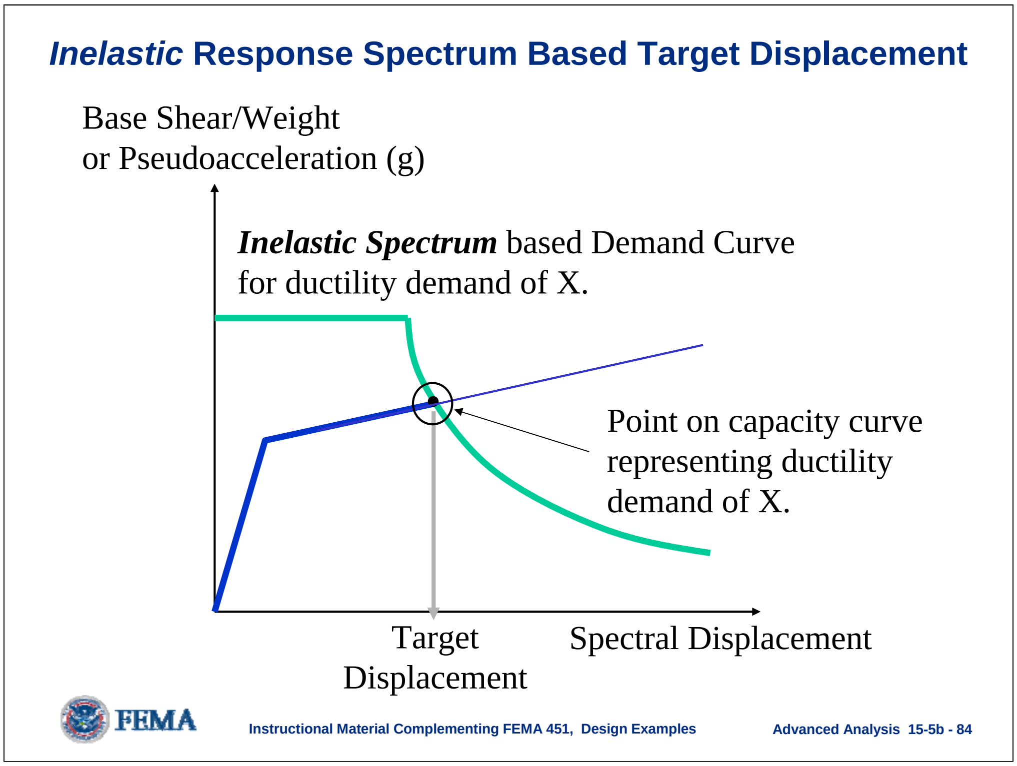
Elastic Spectrum based demand curve for X% equivalent viscous damping

Point on capacity curve representing X% equivalent viscous damping.

Target Displacement

Spectral Displacement

One approach is to use ductility, not equivalent damping, to determine the target displacement. Here the concept of using equivalent damping is reviewed.



Inelastic Response Spectrum Based Target Displacement

Base Shear/Weight
or Pseudoacceleration (g)

Inelastic Spectrum based Demand Curve
for ductility demand of X.

Point on capacity curve
representing ductility
demand of X.

Target
Displacement

Spectral Displacement

Instead of using equivalent viscous damping, one can use ductility as a modifier for the elastic spectrum. The target displacement is now found as the projection on the displacement axis of the point at which a spectrum with a ductility of X meets the ductility of X on the capacity curve. It has been shown by Chopra and others that this produces more consistent results when compared to nonlinear response history analysis.

Inelastic Spectrum Based Target Displacement

- Gives the same results as the equal displacement theory for (longer period) EPP systems
- When compared to inelastic response history analysis, the use of inelastic spectra gives better results than ATC 40 procedure.

This is a summary of the advantages of using the inelastic spectrum.

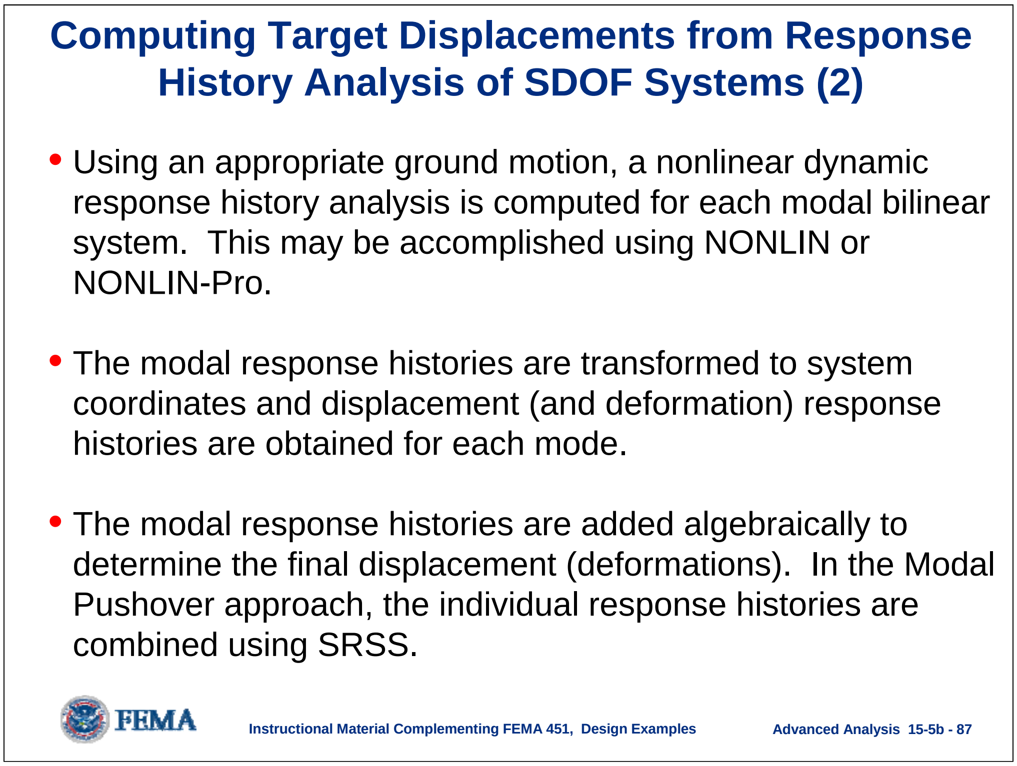
Computing Target Displacements from Response History Analysis of SDOF Systems

- Method called “Uncoupled Modal Response History Analysis” (UMRHA) is described by Chopra and Goel. See, for example, Appendix A of PEER Report 2001/03, entitled *Modal Pushover Analysis Procedure to Estimate Seismic Demands for Buildings*.

- In the UMHRA method, the undamped mode shapes are used to determine a static load pattern for each mode.

- Using these static lateral loads, a series of pushover curves and corresponding bilinear capacity curves are obtained for the first few modes. This is done using the procedures described earlier for the ATC 40 approach.

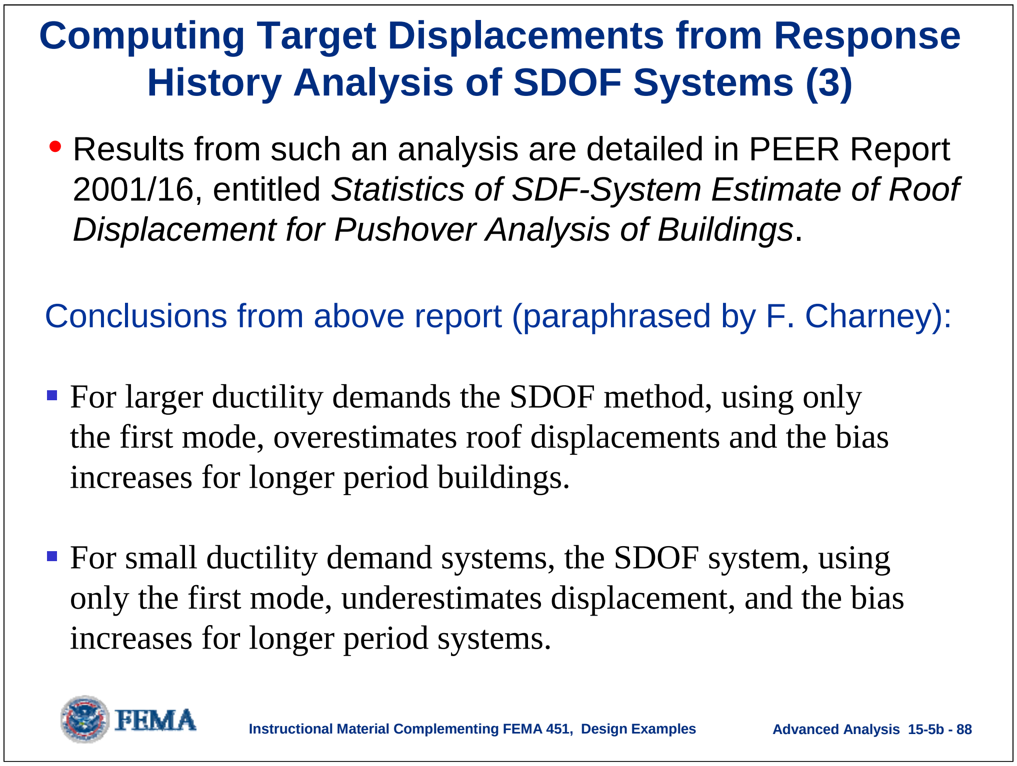
Another approach which uses inelastic response history analysis of “uncoupled” SDOF inelastic systems has been suggested by Chopra. Again, this procedure appears to produce better results that do the more basic approaches. The steps in the procedure are outlined in this and the next slide.



Computing Target Displacements from Response History Analysis of SDOF Systems (2)

- Using an appropriate ground motion, a nonlinear dynamic response history analysis is computed for each modal bilinear system. This may be accomplished using NONLIN or NONLIN-Pro.
- The modal response histories are transformed to system coordinates and displacement (and deformation) response histories are obtained for each mode.
- The modal response histories are added algebraically to determine the final displacement (deformations). In the Modal Pushover approach, the individual response histories are combined using SRSS.

Continuation of previous series.



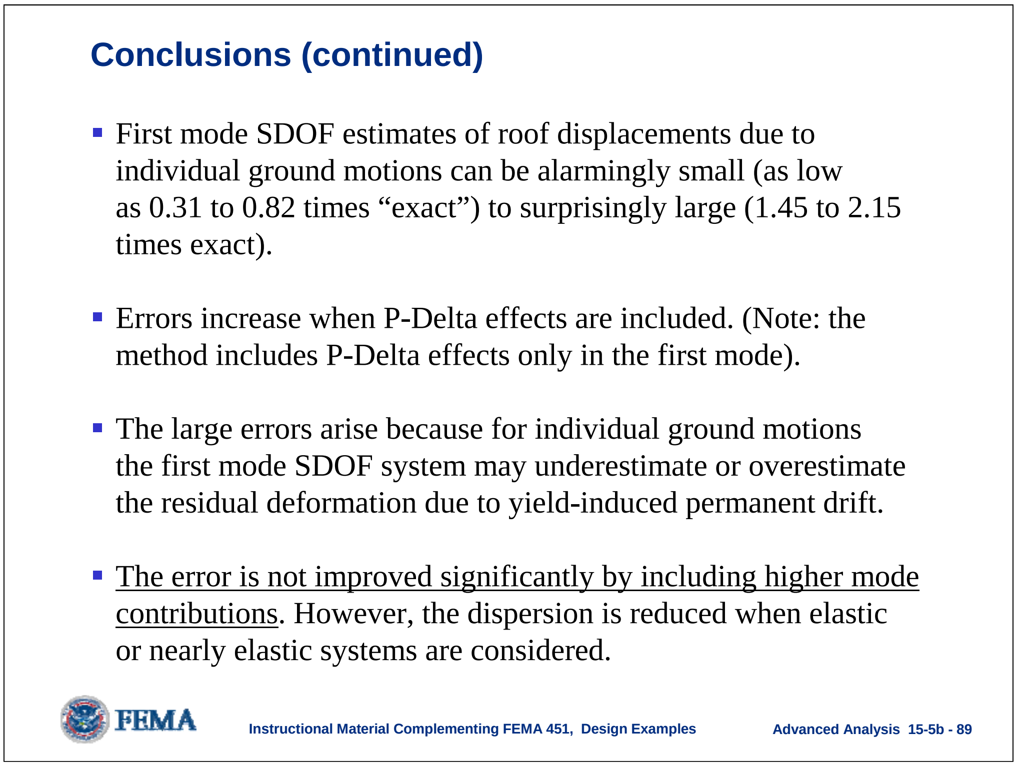
Computing Target Displacements from Response History Analysis of SDOF Systems (3)

- Results from such an analysis are detailed in PEER Report 2001/16, entitled *Statistics of SDF-System Estimate of Roof Displacement for Pushover Analysis of Buildings*.

Conclusions from above report (paraphrased by F. Charney):

- For larger ductility demands the SDOF method, using only the first mode, overestimates roof displacements and the bias increases for longer period buildings.
- For small ductility demand systems, the SDOF system, using only the first mode, underestimates displacement, and the bias increases for longer period systems.

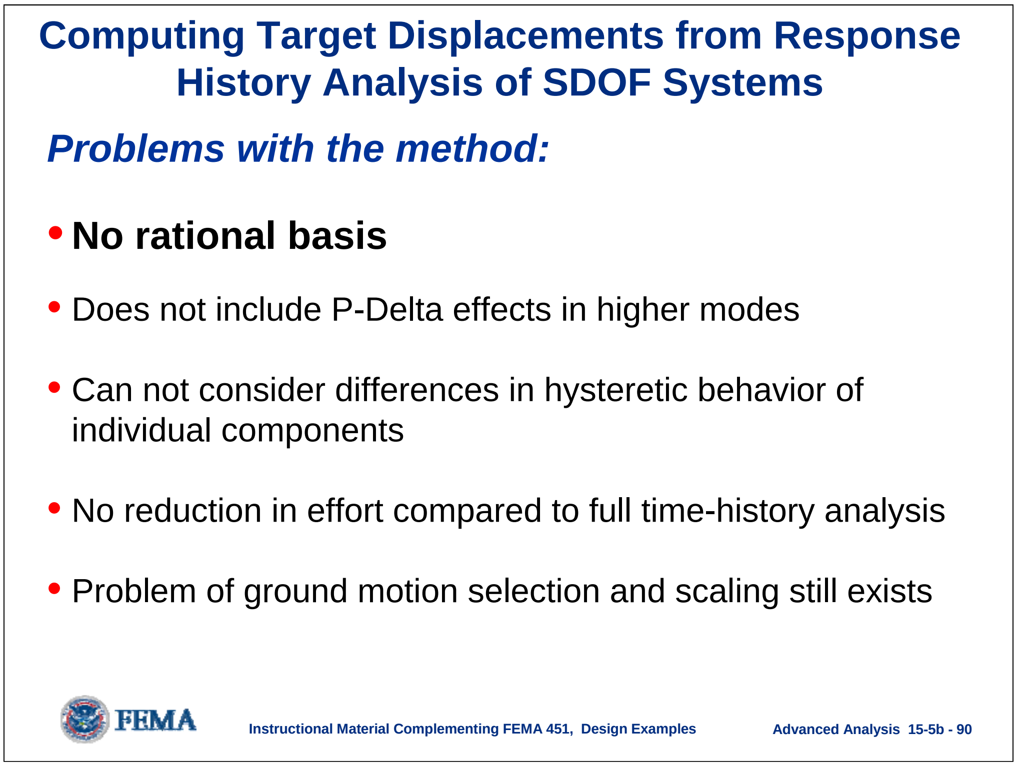
These are the conclusions from the report describing the method.



Conclusions (continued)

- First mode SDOF estimates of roof displacements due to individual ground motions can be alarmingly small (as low as 0.31 to 0.82 times “exact”) to surprisingly large (1.45 to 2.15 times exact).
- Errors increase when P-Delta effects are included. (Note: the method includes P-Delta effects only in the first mode).
- The large errors arise because for individual ground motions the first mode SDOF system may underestimate or overestimate the residual deformation due to yield-induced permanent drift.
- The error is not improved significantly by including higher mode contributions. However, the dispersion is reduced when elastic or nearly elastic systems are considered.

Continuation of the conclusions.



Computing Target Displacements from Response History Analysis of SDOF Systems

Problems with the method:

- **No rational basis**
- Does not include P-Delta effects in higher modes
- Can not consider differences in hysteretic behavior of individual components
- No reduction in effort compared to full time-history analysis
- Problem of ground motion selection and scaling still exists

The problem with the “improved approach” is that it has absolutely no rational basis. If one is going to go through all the effort indicated, one might as well use response history analysis which is the subject of the following set of slides.

Proceed to Topic 15-5c.