Mario Paz • Young Hoon Kim

Structural Dynamics

Theory and Computation

Sixth Edition

Springer

Contents

Part I Structures Modeled as a Single-Degree-of-Freedom System

1 Undamped Single Degree-of-Freedom System 3
1.1 Degrees of Freedom 3
1.2 Undamped System 5
1.3 Springs in Parallel or in Series 6
1.4 Newton's Law of Motion 7
1.5 Free Body Diagram 8
1.6 D'Alembert's Principle 9
1.7 Solution of the Differential Equation of Motion 10
1.8 Frequency and Period 12
1.9 Amplitude of Motion 14
1.10 Response of SDF Using MATLAB Program 18
1.11 Summary 20
1.12 Problems 21

2 Damped Single Degree-of-Freedom System 29
2.1 Viscous Damping 29
2.2 Equation of Motion 29
2.3 Critically Damped System 30
2.4 Overdamped System 31
2.5 Underdamped System 32
2.6 Logarithmic Decrement 33
2.7 Response of SDF Using MATLAB Program 39
2.8 Summary 40
2.9 Problems 41

3 Response of One-Degree-of-Freedom System to Harmonic Loading 45
3.1 Harmonic Excitation: Undamped System 45
3.2 Harmonic Excitation: Damped System 47
3.3 Evaluation of Damping at Resonance 54
3.4 Bandwidth Method (Half-Power) to Evaluate Damping . 55
3.5 Energy Dissipated by Viscous Damping 57
3.6 Equivalent Viscous Damping 58

3.7 Response to Support Motion 60
3.7.1 Absolute Motion 60
3.7.2 Relative Motion 65
3.8 Force Transmitted to the Foundation 69
3.9 Seismic Instruments 71
3.10 Response of One-Degree-of-Freedom System to Harmonic Loading Using MATLAB 73
3.11 Summary 76
3.12 Analytical Problem 78
3.13 Problems 79

4 Response to General Dynamic Loading 85
4.1 Duhamel's Integral – Undamped System 85
4.1.1 Constant Force 86
4.1.2 Rectangular Load 88
4.1.3 Triangular Load 90
4.2 Duhamel's Integral-Damped System 95
4.3 Response by Direct Integration 95
4.4 Solution of the Equation of Motion 97
4.5 Summary 104
4.6 Analytical Problems 105
4.7 Problems 108

5 Response Spectra 115
5.1 Construction of Response Spectrum 115
5.2 Response Spectrum for Support Excitation 118
5.3 Tripartite Response Spectra 119
5.4 Response Spectra for Elastic Design 123
5.5 Influence of Local Soil Conditions 126
5.6 Response Spectra for Inelastic Systems 128
5.7 Response Spectra for Inelastic Design 131
5.8 Seismic Response Spectra Using MATLAB 135
5.9 Summary 139
5.10 Problems 140

6 Nonlinear Structural Response 143
6.1 Nonlinear Single-Degree-of-Freedom Model 143
6.2 Integration of the Nonlinear Equation of Motion 145
6.3 Constant Acceleration Method 146
6.4 Linear Acceleration Step-by-Step Method 148
6.5 The Newmark: β Method 150
6.6 Elastoplastic Behavior 156
6.7 Algorithm for Step-by-Step Solution for Elastoplastic Single-Degree-of-Freedom System 158
6.8 Response for Elastoplastic Behavior Using MATLAB . . 163
6.9 Summary 166
6.10 Problems 167

Part II Structures Modeled as Shear Buildings

7 Free Vibration of a Shear Building 173
7.1 Stiffness Equations for the Shear Building 173
7.2 Natural Frequencies and Normal Modes 176
7.3 Orthogonality Property of the Normal Modes 181
7.4 Rayleigh's Quotient 186
7.5 Summary 187
7.6 Problems 188

8 Forced Motion of Shear Buildings 193
8.1 Modal Superposition Method 193
8.2 Response of a Shear Building to Base Motion 199
8.3 Response by Modal Superposition Using MATLAB . . . 205
8.4 Harmonic Force Excitation 207
8.5 Harmonic Response: MATLAB Program 211
8.6 Combining Maximum Values of Modal Response 214
8.7 Summary 214
8.8 Problems 215

9 Reduction of Dynamic Matrices 219
9.1 Static Condensation 219
9.2 Static Condensation Applied to Dynamic Problems 222
9.3 Dynamic Condensation 233
9.4 Modified Dynamic Condensation 241
9.5 Summary 245
9.6 Problems 246

Part III Framed Structures Modeled as Discrete Multi-Degree-of-Freedom Systems

10 Dynamic Analysis of Beams 251
10.1 Shape Functions for a Beam Segment 251
10.2 System Stiffness Matrix 256
10.3 Inertial Properties-Lumped Mass 259
10.4 Inertial Properties—Consistent Mass 260
10.5 Damping Properties 264
10.6 External Loads 265
10.7 Geometric Stiffness 266
10.8 Equations of Motion 270
10.9 Element Forces At Nodal Coordinates 276
10.10 Program 13—Modeling Structures as Beams 278
10.11 Summary 284
10.12 Problems 285

11 Dynamic Analysis of Plane Frames 291
11.1 Element Stiffness Matrix for Axial Effects 291
11.2 Element Mass Matrix for Axial Effects 293
11.3 Coordinate Transformation 297
11.4 Modeling Structures as Plane Frames Using MATLAB . 304

11.5 Dynamic Analysis of Plane Frames Using MATLAB . . . 307
11.6 Summary . . . 312
11.7 Problems . . . 312

12 Dynamic Analysis of Grid Frames . . . 317
12.1 Local and Global Coordinate Systems . . . 317
12.2 Torsional Effects . . . 318
12.3 Stiffness Matrix for a Grid Element . . . 320
12.4 Consistent Mass Matrix for a Grid Element . . . 320
12.5 Lumped Mass Matrix for a Grid Element . . . 321
12.6 Transformation of Coordinates . . . 321
12.7 Modeling Structures as Grid Frames Using MATLAB . . 327
12.8 Summary . . . 330
12.9 Problems . . . 331

13 Dynamic Analysis of Three-Dimensional Frames . . . 335
13.1 Element Stiffness Matrix . . . 335
13.2 Element Mass Matrix . . . 337
13.3 Element Damping Matrix . . . 337
13.4 Transformation of Coordinates . . . 338
13.5 Differential Equation of Motion . . . 342
13.6 Dynamic Response . . . 342
13.7 Modeling Structures as Space Frames Using MATLAB . . . 342
13.8 Summary . . . 347

14 Dynamic Analysis of Trusses . . . 349
14.1 Stiffness and Mass Matrices for the Plane Truss . . . 349
14.2 Transformation of Coordinates . . . 351
14.3 Stiffness and Mass Matrices for Space Trusses . . . 361
14.4 Equation of Motion for Space Trusses . . . 363
14.5 Modeling Structures as Space Trusses Using MATLAB . 364
14.6 Summary . . . 368
14.7 Problems . . . 368

15 Dynamic Analysis of Structures Using the Finite Element Method . . . 371
15.1 Plane Elasticity Problems . . . 372
15.1.1 Triangular Plate Element for Plane Elasticity Problems . . . 373
15.2 Plate Bending . . . 378
15.2.1 Rectangular Element for Plate Bending . . . 379
15.3 Summary . . . 384
15.4 Problems . . . 385

16 Time History Response of Multi-Degree-of-Freedom Systems . . . 389
16.1 Incremental Equations of Motion . . . 389
16.2 The Wilson-θ Method . . . 391

16.3 Algorithm for Step-by-Step Solution of a Linear System Using the Wilson-θ Method 393
16.3.1 Initialization 393
16.3.2 For Each Time Step 393
16.4 Response by Step Integration Using MATLAB 397
16.5 The Newmark Beta Method 401
16.6 Elastoplastic Behavior of Framed-Structures 402
16.7 Member Stiffness Matrix 402
16.8 Member Mass Matrix 405
16.9 Rotation of Plastic Hinges 407
16.10 Calculation of Member Ductility Ratio 408
16.11 Summary 409
16.12 Problems 409

Part IV Structures Modeled with Distributed Properties

17 Dynamic Analysis of Systems with Distributed Properties . . . 415
17.1 Flexural Vibration of Uniform Beams 415
17.2 Solution of the Equation of Motion in Free Vibration . . 417
17.3 Natural Frequencies and Mode Shapes for Uniform Beams 418
17.3.1 Both Ends Simply Supported 418
17.3.2 Both Ends Free (Free Beam) 421
17.3.3 Both Ends Fixed 422
17.3.4 One End Fixed and the Other End Free (Cantilever Beam) 424
17.3.5 One End Fixed and the Other Simply Supported 425
17.4 Orthogonality Condition Between Normal Modes 426
17.5 Forced Vibration of Beams 427
17.6 Dynamic Stresses in Beams 432
17.7 Summary 433
17.8 Problems 434

18 Discretization of Continuous Systems 437
18.1 Dynamic Matrix for Flexural Effects 437
18.2 Dynamic Matrix for Axial Effects 439
18.3 Dynamic Matrix for Torsional Effects 441
18.4 Beam Flexure Including Axial-Force Effect 443
18.5 Power Series Expansion of the Dynamic Matrix for Flexural Effects 445
18.6 Power Series Expansion of the Dynamic Matrix for Axial and for Torsional Effects 447
18.7 Power Series Expansion of the Dynamic Matrix Including the Effects of Axial Forces 447
18.8 Summary 448

Part V Special Topics: Fourier Analysis, Evaluation of Absolute Damping, Generalized Coordinates

19 Fourier Analysis and Response in the Frequency Domain . . . 453
19.1 Fourier Analysis . . . 453
19.2 Response to a Loading Represented by Fourier Series . . 454
19.3 Fourier Coefficients for Piecewise Linear Functions . . . 456
19.4 Exponential Form of Fourier Series . . . 457
19.5 Discrete Fourier Analysis . . . 458
19.6 Fast Fourier Transform . . . 461
19.7 Response in the Frequency Domain Using MATLAB . . 463
19.8 Summary . . . 469
19.9 Problems . . . 470

20 Evaluation of Absolute Damping from Modal Damping Ratios . . . 477
20.1 Equations for Damped Shear Building . . . 477
20.2 Uncoupled Damped Equations . . . 478
20.3 Conditions for Damping Uncoupling . . . 479
20.4 Summary . . . 487
20.5 Problems . . . 487

21 Generalized Coordinates and Rayleigh's Method . . . 491
21.1 Principle of Virtual Work . . . 491
21.2 Generalized Single-Degree-of-Freedom System–Rigid Body . . . 492
21.3 Generalized Single-Degree-of-Freedom System–Distributed Elasticity . . . 495
21.4 Shear Forces and Bending Moments . . . 500
21.5 Generalized Equation of Motion for a Multistory Building . . . 504
21.6 Shape Function . . . 506
21.7 Rayleigh's Method . . . 510
21.8 Improved Rayleigh's Method . . . 517
21.9 Shear Walls . . . 520
21.10 Summary . . . 523
21.11 Problems . . . 524

Part VI Random Vibration

22 Random Vibration . . . 531
22.1 Statistical Description of Random Functions . . . 532
22.2 Probability Density Function . . . 534
22.3 The Normal Distribution . . . 536
22.4 The Rayleigh Distribution . . . 537
22.5 Correlation . . . 538
22.6 The Fourier Transform . . . 542
22.7 Spectral Analysis . . . 543
22.8 Spectral Density Function . . . 547

22.9 Narrow-Band and Wide-Band Random Processes 550
22.10 Response to Random Excitation: Single-Degree-of-Freedom System 553
22.11 Response to Random Excitation: Multiple-Degree-of-Freedom System 559
22.11.1 Relationship Between Complex Frequency Response and Unit Impulse Response 559
22.11.2 Response to Random Excitation: Two-Degree-of-Freedom System 561
22.11.3 Response to Random Excitation: *N* Degree of Freedom System 565
22.12 Summary 567
22.13 Problems 568

Part VII Earthquake Engineering

23 Dynamic Method 575
23.1 Modal Seismic Response of Building 575
23.1.1 Modal Equation and Participation Factor 575
23.1.2 Modal Shear Force 576
23.1.3 Effective Modal Weight 578
23.1.4 Modal Lateral Forces 579
23.1.5 Modal Displacements 579
23.1.6 Modal Drift 579
23.1.7 Modal Overturning Moment 580
23.1.8 Modal Torsional Moment 580
23.2 Total Design Values 580
23.3 Scaling of Results 582
23.4 Summary 588
23.5 Problems 588

24 IBC-2018 and ASCE 7-16 589
24.1 Response Spectral Acceleration: S_S, S_1 589
24.2 Soil Modified Response Spectral Acceleration: S_{MS}, S_{M1} 590
24.3 Design Response Spectral Acceleration: S_{DS}, S_{D1} 590
24.4 Site Class Definition: A, B . . .F 591
24.5 Risk Category and Seismic Importance Factor (I_e) 592
24.6 Seismic Design Category (A, B, C, D, E and F) 592
24.7 Design Response Spectral Curve: S_a Vs. T 593
24.8 Determination of the Fundamental Period 595
24.9 Minimum Lateral Force Procedure [ASCE 7-16: Section 1.4.3] 596
24.10 Simplified Lateral Force Analysis Procedure [ASCE 7-16: Section 12.14.8 and IBC 2018 Section 1613.2.5.2] 596
24.10.1 Seismic Base Shear 597
24.10.2 Response Modification Factor *R* 597
24.10.3 Vertical Distribution of Lateral Forces 598

24.11 Equivalent Lateral Force Procedure [ASCE 7-16: Section 12.8] 598
24.11.1 Distribution of Lateral Forces [ASCE 7-16: Section 12.8.3] 600
24.11.2 Overturning Moments [ASCE 7-16: Section 12.8.5] 601
24.11.3 Horizontal Torsional Moment 601
24.11.4 *P*-Delta Effect (P–Δ) [ASCE 7-16: Section 12.8.7] 601
24.11.5 Story Drift [ASCE 7-16: Sect. 12.8.6] 602
24.12 Redundancy [ASCE 7-16: Section 12.3.4] 603
24.13 Earthquake Load Effect [ASCE 7-16 Section 12.4.2] . . . 604
24.14 Building Irregularities [ASCE 7-16 Section 12.3.2.1] . . . 604
24.15 Summary 609

Appendices 611

Selected Bibliography 625

Index 629