Structure Analysis I

Chapter 7

Approximate Analysis of Statically Indeterminate Structures

Introduction

- Using approximate methods to analyse statically indeterminate trusses and frames
- The methods are based on the way the structure deforms under the load
- Trusses
- Portal frames with trusses
- Vertical loads on building frames
- Lateral loads on building frames
  - Portal method
  - Cantilever method

Approximate Analysis

- Statically determinate structure – the force equilibrium equation is sufficient to find the support reactions
- Approximate analysis – is to develop a simple model of the structure which is statically determinate to solve a statically indeterminate problem
- The method is based on the way the structure deforms under loads
- Their accuracy in most cases compares favourably with more exact methods of analysis (the statically indeterminate analysis)

Determinacy - Trusses

$$b + r = 2j$$

Statically determinate

$$b + r > 2j$$

Statically indeterminate

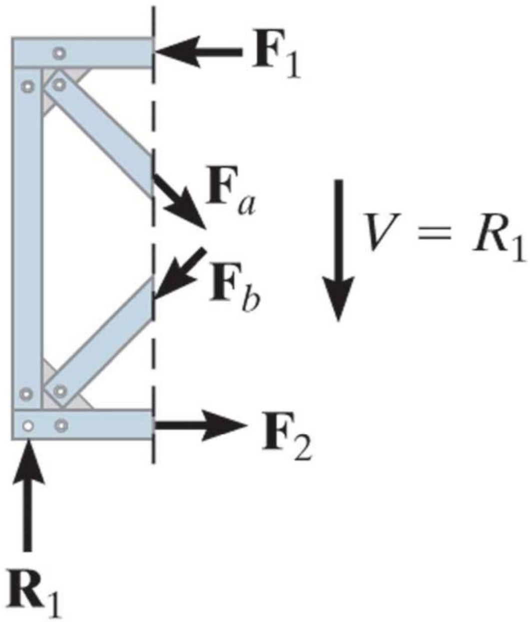
b – total number of bars

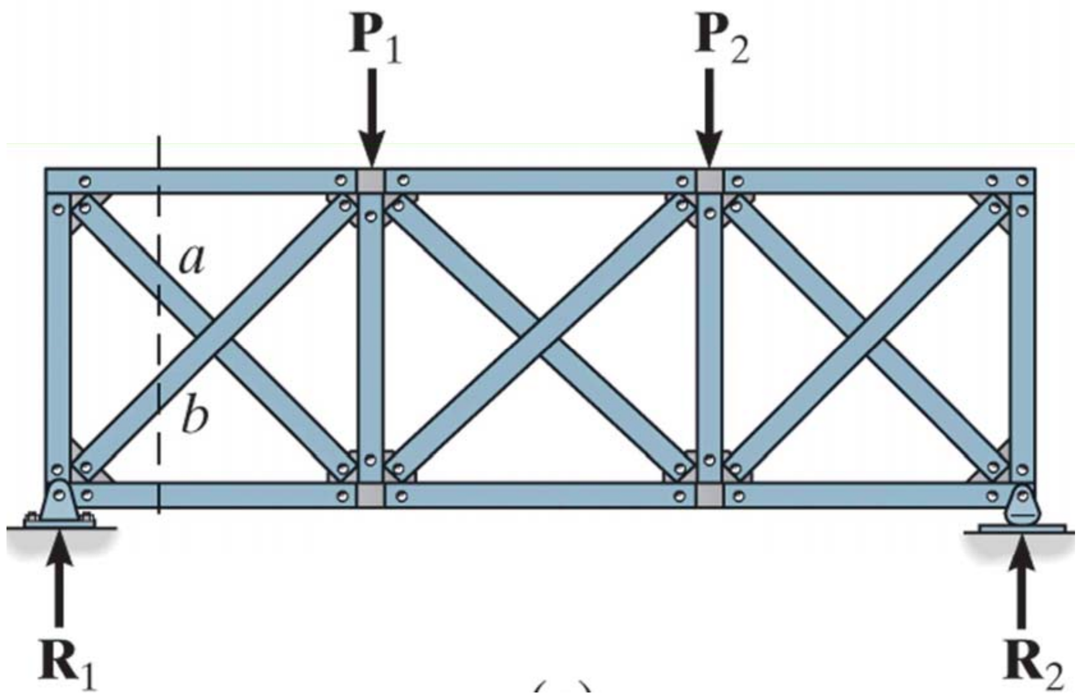
r – total number of external support reactions

j – total number of joints

Trusses

Real Structure





b=16, r=3, j=8

b+r = 19 > 2j=16

The truss is statically indeterminate to the third degree

Trusses

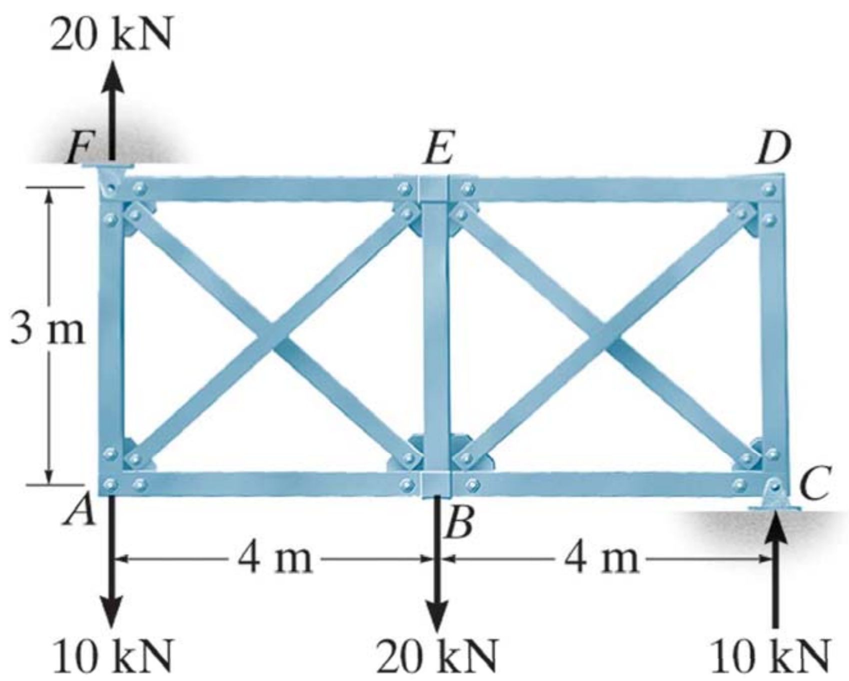
Approximation Method

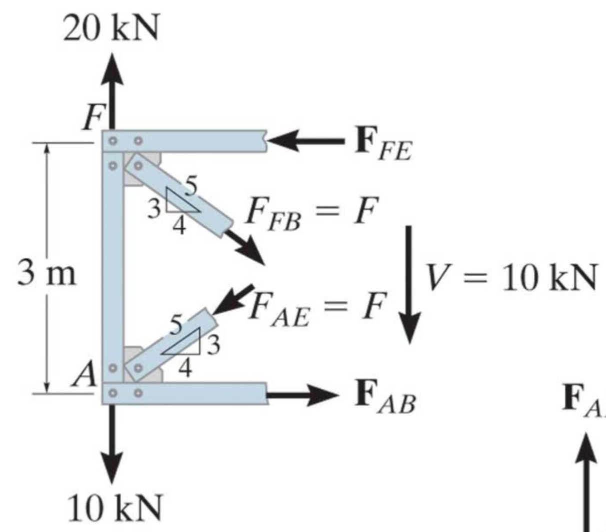
Assumption 1: If the diagonals are designed to be long and slender, it is assumed that diagonals can not support any compressive force and all panel shear are resisted only by tensile diagonals.

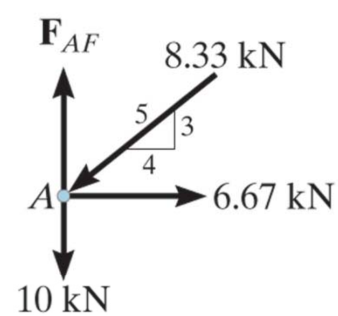
Assumption 2: If the diagonals are designed to be large rolled sections such as angles or channels, they are assumed to support both tensile and compressive forces. Each diagonal is assumed to carry half the panel shear.

Example 1

Determine (approximately) the forces in the members of the truss shown in Fig. 7–2*a*. The diagonals are to be designed to support both tensile and compressive forces, and therefore each is assumed to carry half the panel shear. The support reactions have been computed.







$$+\uparrow \Sigma F_y = 0; \qquad 20 - 10 - 2\left(\tfrac{3}{5}\right)F = 0 \qquad F = 8.33 \text{ kN} \qquad \textit{Ans.}$$

so that

$$F_{FB} = 8.33 \text{ kN (T)} \qquad \textit{Ans.}$$

$$F_{AE} = 8.33 \text{ kN (C)} \qquad \textit{Ans.}$$

$$\circlearrowright + \Sigma M_A = 0; \quad -8.33\left(\tfrac{4}{5}\right)(3) + F_{FE}(3) = 0 \quad F_{FE} = 6.67 \text{ kN (C)} \quad \textit{Ans.}$$

$$\circlearrowright + \Sigma M_F = 0; \quad -8.33\left(\tfrac{4}{5}\right)(3) + F_{AB}(3) = 0 \quad F_{AB} = 6.67 \text{ kN (T)} \quad \textit{Ans.}$$

From joint A, Fig. 7–2c,

$$+\uparrow \Sigma F_y = 0; \quad F_{AF} - 8.33\left(\tfrac{3}{5}\right) - 10 = 0 \qquad F_{AF} = 15 \text{ kN (T)} \quad \textit{Ans.}$$

A vertical section through the right panel is shown in Fig. 7–2d. Show that

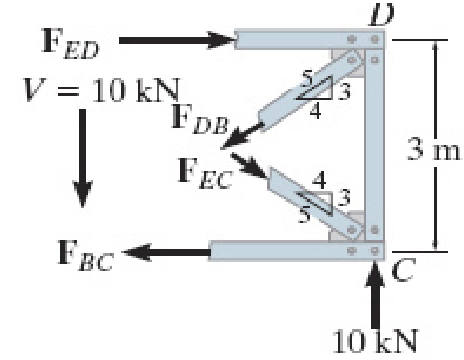
$$F_{DB} = 8.33 \text{ kN (T)}, \quad F_{ED} = 6.67 \text{ kN (C)} \qquad \textit{Ans.}$$

$$F_{EC} = 8.33 \text{ kN (C)}, \quad F_{BC} = 6.67 \text{ kN (T)} \qquad \textit{Ans.}$$

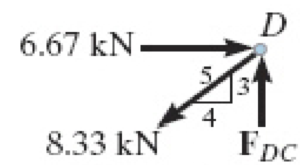
Furthermore, using the free-body diagrams of joints D and E, Figs. 7–2e and 7–2f, show that

$$F_{DC} = 5 \text{ kN (C)} \qquad \textit{Ans.}$$

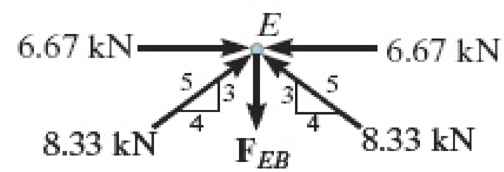
$$F_{EB} = 10 \text{ kN (T)} \qquad \textit{Ans.}$$



(d)



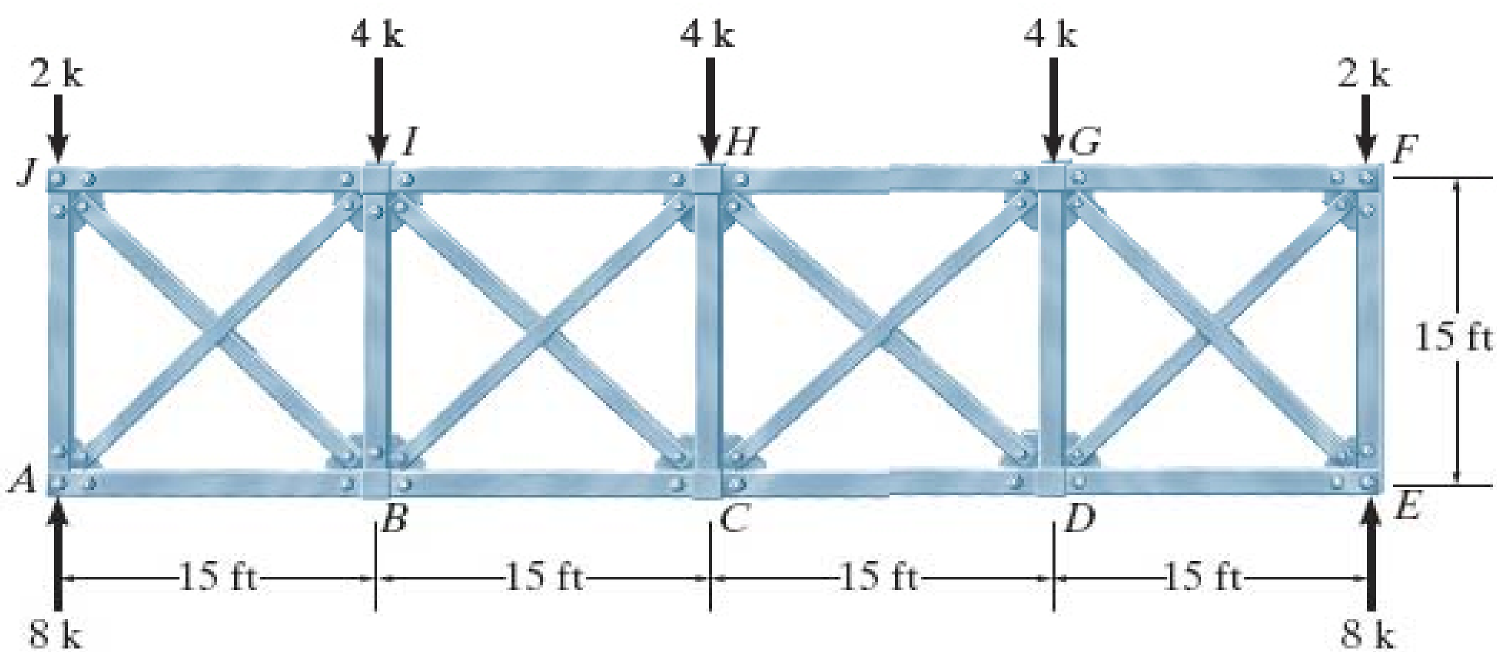
(e)



(f)

Example 2

Determine (approximately) the forces in the members of the truss shown in Fig. 7–3*a*. Assume the diagonals are slender and therefore will not support a compressive force. The support reactions have been computed.



(a)

$$F_{AI} = 0$$

$+\uparrow \Sigma F_y = 0;$ $$8 - 2 - F_{JB} \cos 45^\circ = 0$$

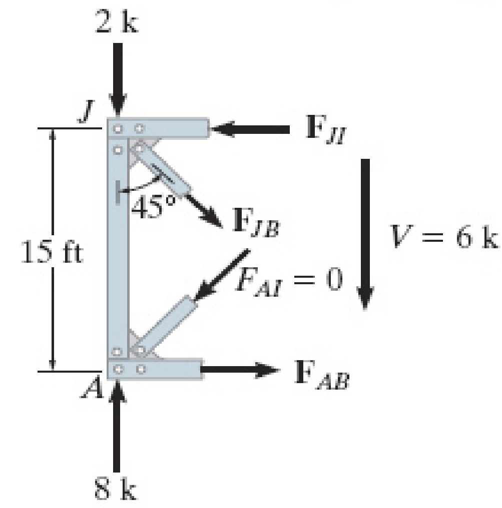
$$F_{JB} = 8.49 \text{ k (T)}$$

$\circlearrowright + \Sigma M_A = 0;$ $$-8.49 \sin 45^\circ(15) + F_{JI}(15) = 0$$

$$F_{JI} = 6 \text{ k (C)}$$

$\circlearrowright + \Sigma M_J = 0;$ $$-F_{AB}(15) = 0$$

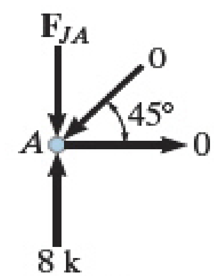
$$F_{AB} = 0$$



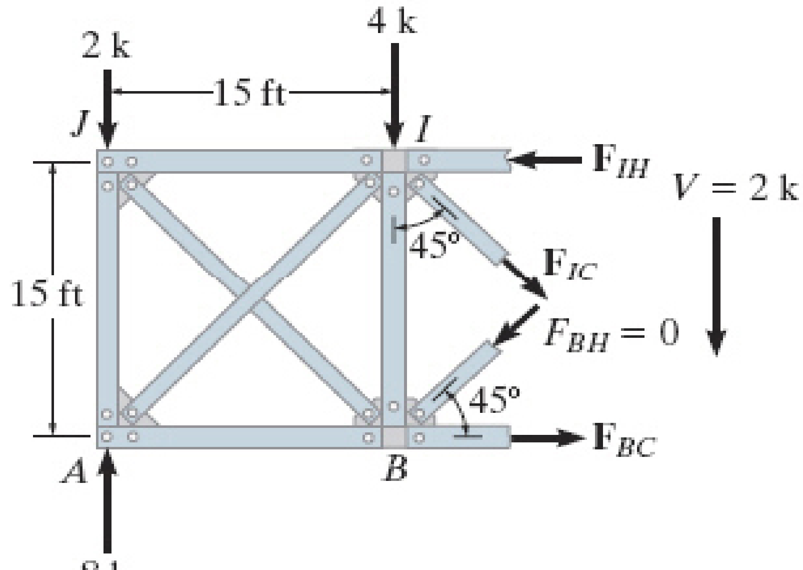
(b)

From joint A, Fig. 7–3c,

$$F_{JA} = 8 \text{ k (C)}$$



(c)



(d)

A vertical section of the truss through members IH, IC, BH, and BC is shown in Fig. 7–3d. The panel shear is $V = \Sigma F_y = 8 - 2 - 4 = 2$ k. We require

$$F_{BH} = 0 \qquad \textit{Ans.}$$

$+\uparrow \Sigma F_y = 0;$ $$8 - 2 - 4 - F_{IC} \cos 45^\circ = 0$$

$$F_{IC} = 2.83 \text{ k (T)} \qquad \textit{Ans.}$$

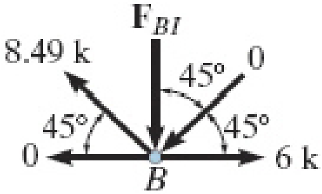
$\circlearrowright + \Sigma M_B = 0; \quad -8(15) + 2(15) - 2.83 \sin 45°(15) + F_{IH}(15) = 0$

$$F_{IH} = 8 \text{ k (C)} \qquad \textit{Ans.}$$

$\circlearrowright + \Sigma M_I = 0; \quad -8(15) + 2(15) + F_{BC}(15) = 0$

$$F_{BC} = 6 \text{ k (T)} \qquad \textit{Ans.}$$

From joint B, Fig. 7–3e,

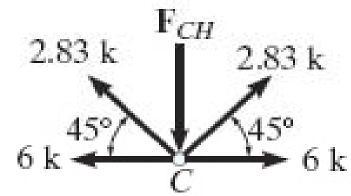


(e)

$+\uparrow \Sigma F_y = 0; \quad 8.49 \sin 45° - F_{BI} = 0$

$$F_{BI} = 6 \text{ k (C)} \qquad \textit{Ans.}$$

The forces in the other members can be determined by symmetry, except F_{CH}; however, from joint C, Fig. 7–3f, we have



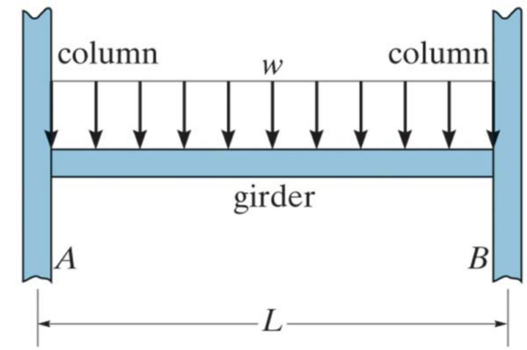
(f)

$+\uparrow \Sigma F_y = 0; \quad 2(2.83 \sin 45°) - F_{CH} = 0$

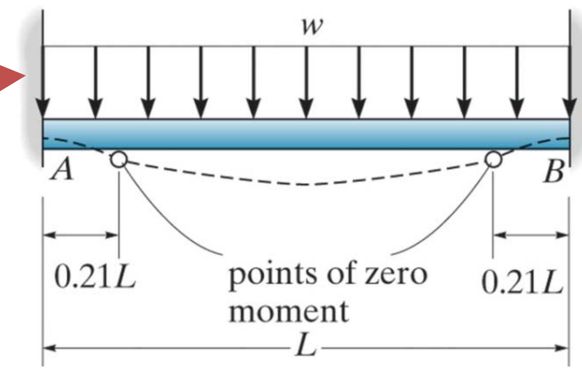
$$F_{CH} = 4 \text{ k (C)} \qquad \textit{Ans.}$$

Vertical Loads on Building Frames

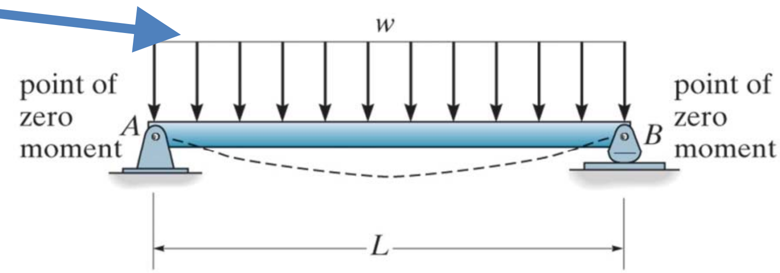
- Building frames often consist of girders that are rigidly connected to columns



If the columns are extremely stiff



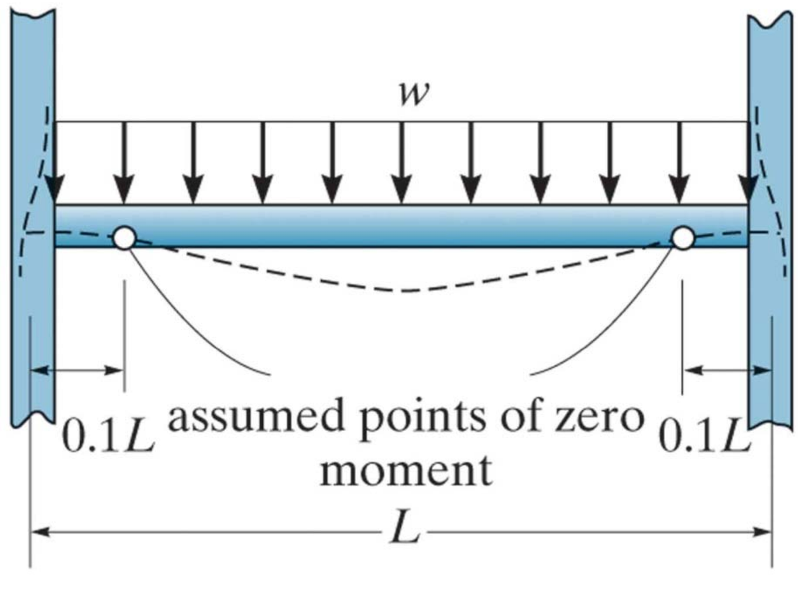
If the columns are extremely flexible



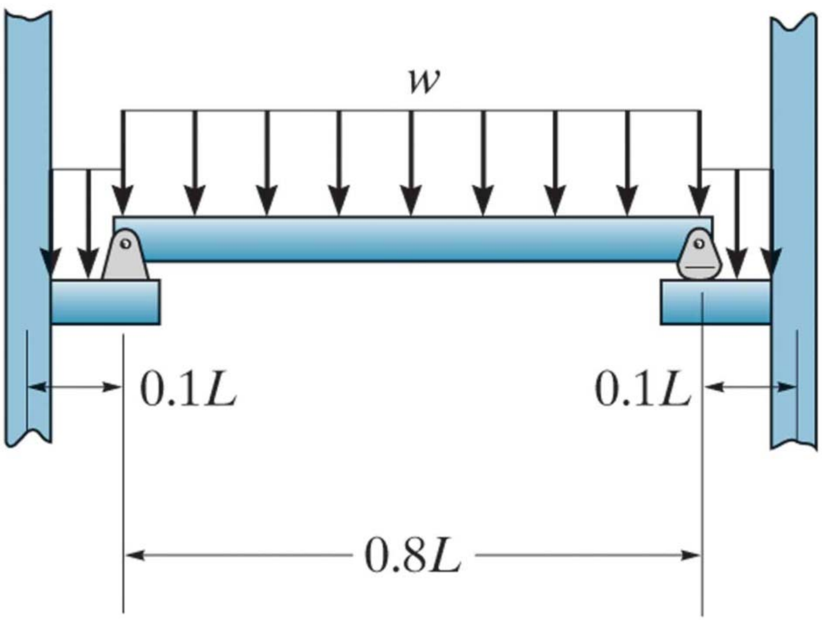
Average point between the two extremes = (0.21L+0)/2 ≈ 0.1L

Vertical Loads on Building Frames

Average point between the two extremes = (0.21L+0)/2 $\approx$ 0.1L

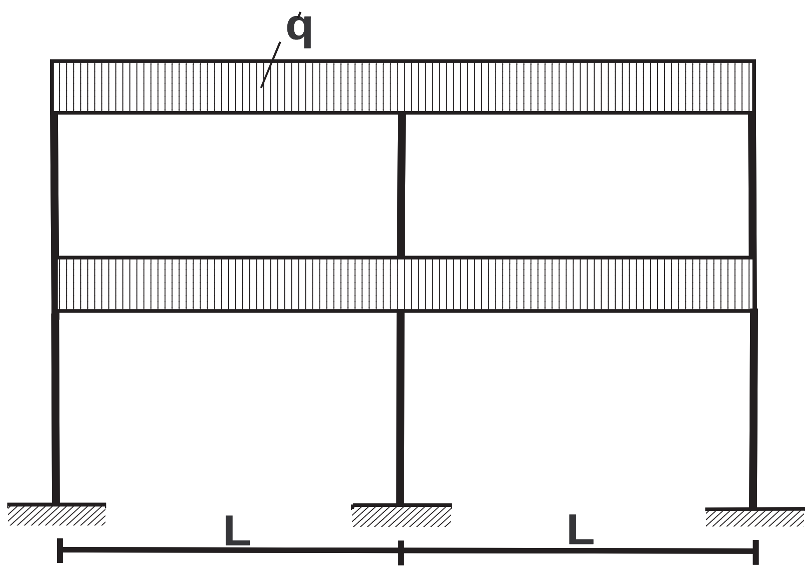
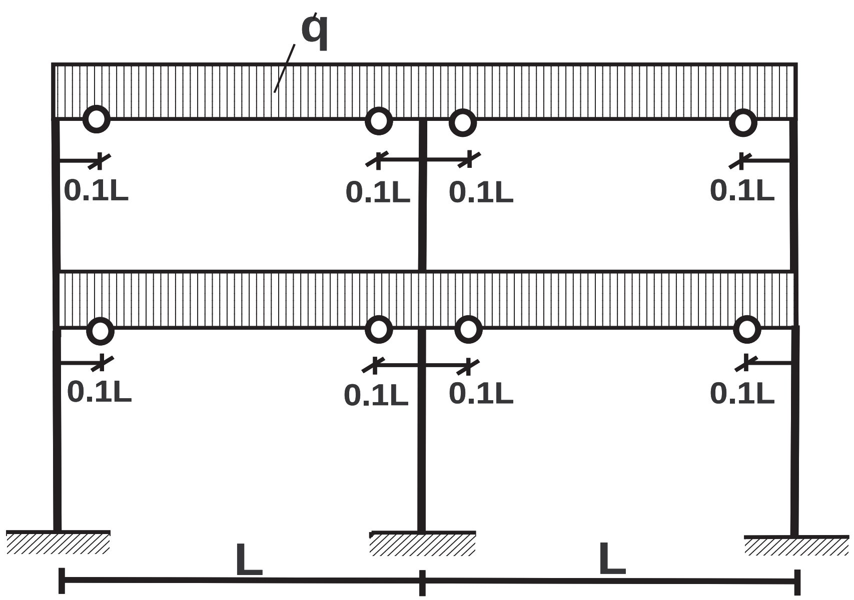


approximate case



model

Building Frames – Vertical Loads

Real structure	Approximation
	

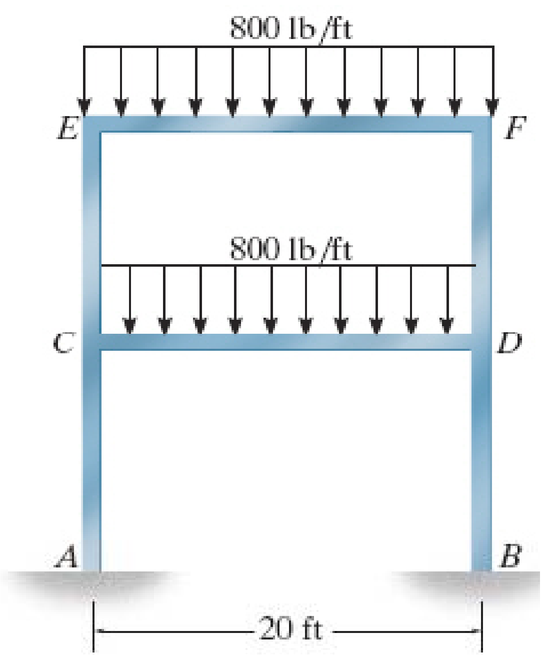
1.There is zero moment (hinge) in the girder 0.1L from the left support

2. There is zero moment (hinge) in the girder 0.1L from the right support

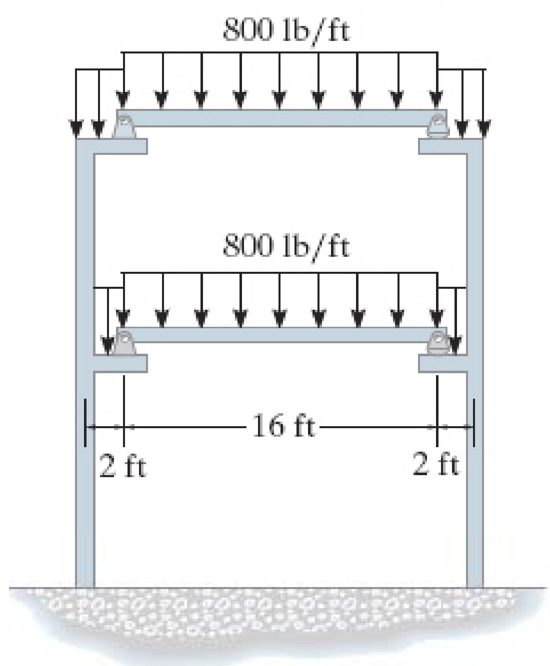
3.The girder does not support an axial force.

Example 3

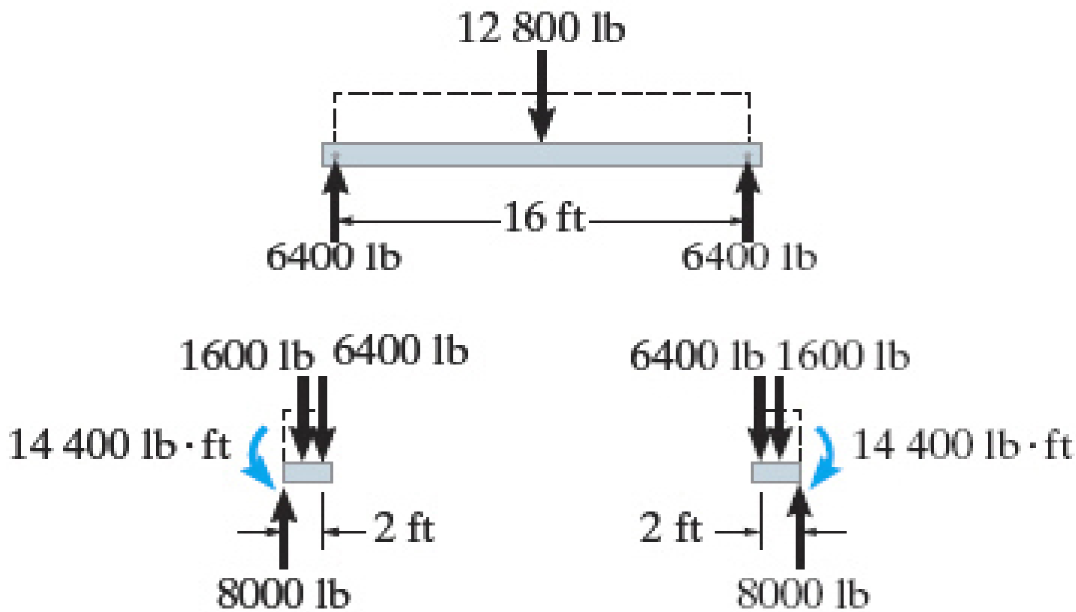
Determine (approximately) the moment at the joints E and C caused by members EF and CD of the building bent in Fig. 7–6a.



(a)



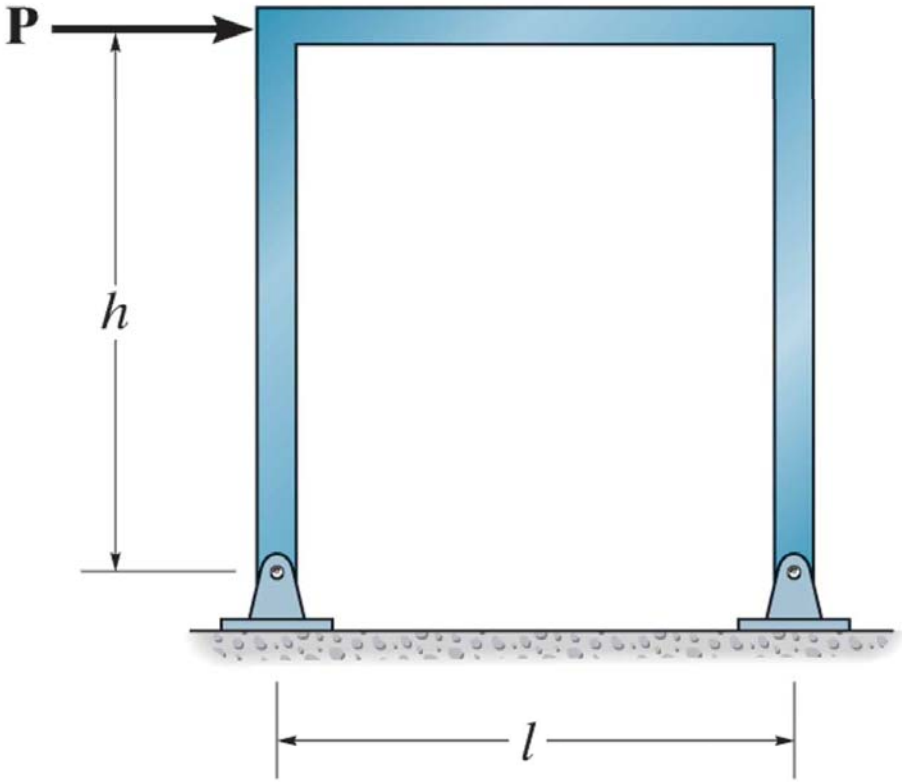
(b)

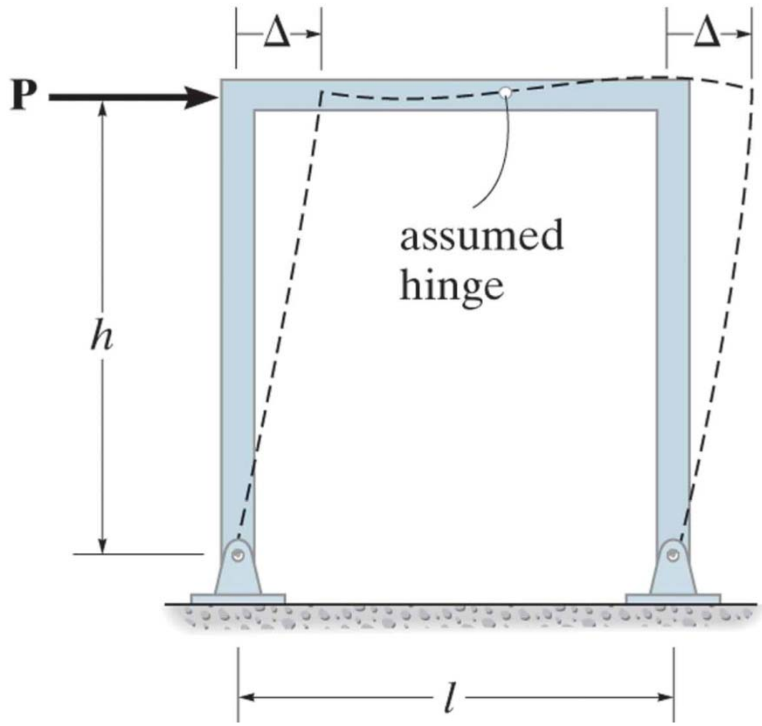

12 800 lb
16 ft
6400 lb
6400 lb
1600 lb
6400 lb
14 400 lb · ft
2 ft
8000 lb
6400 lb
1600 lb
14 400 lb · ft
2 ft
8000 lb

Portal Frames – Lateral Loads

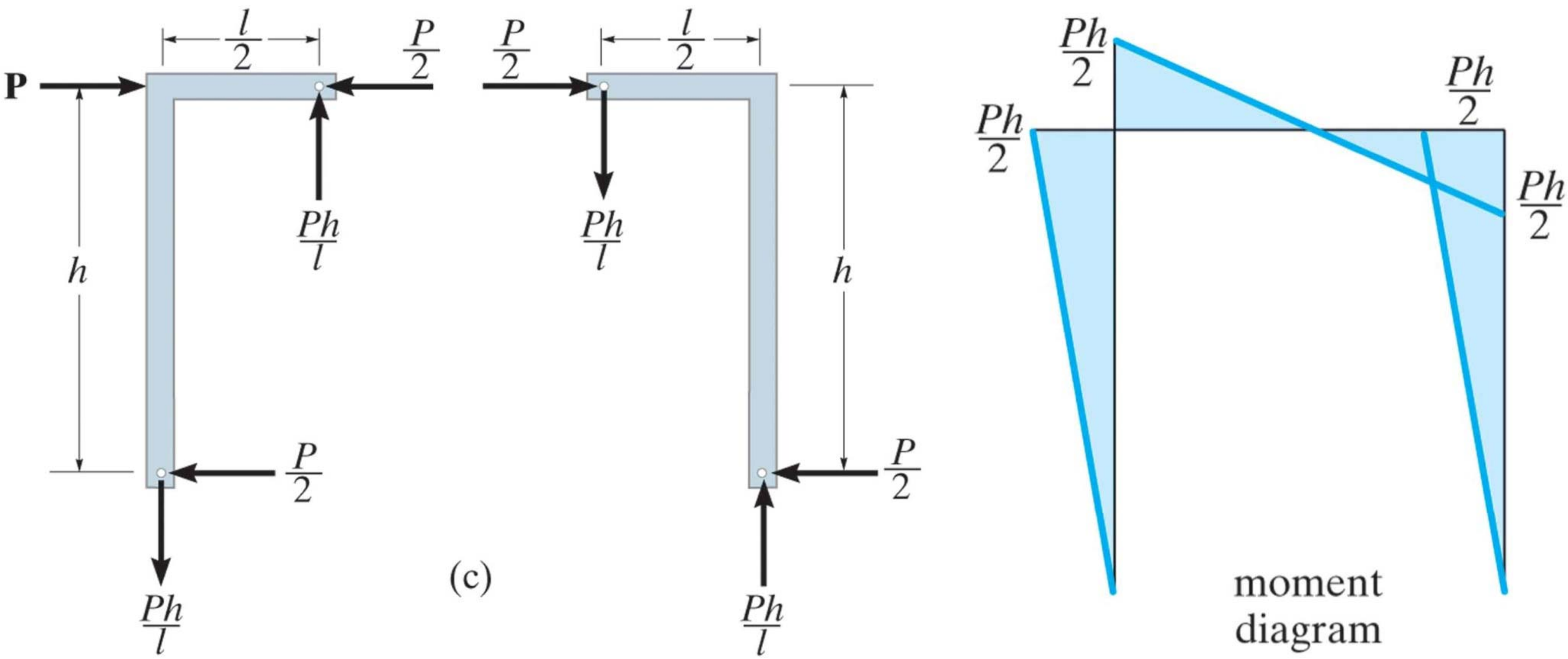
- ✓ Portal frames are frequently used over the entrance of a bridge or in industrial buildings.
- ✓ Portals can be pin supported, fixed supported or supported by partial fixity

Pin-Supported Portal Frames

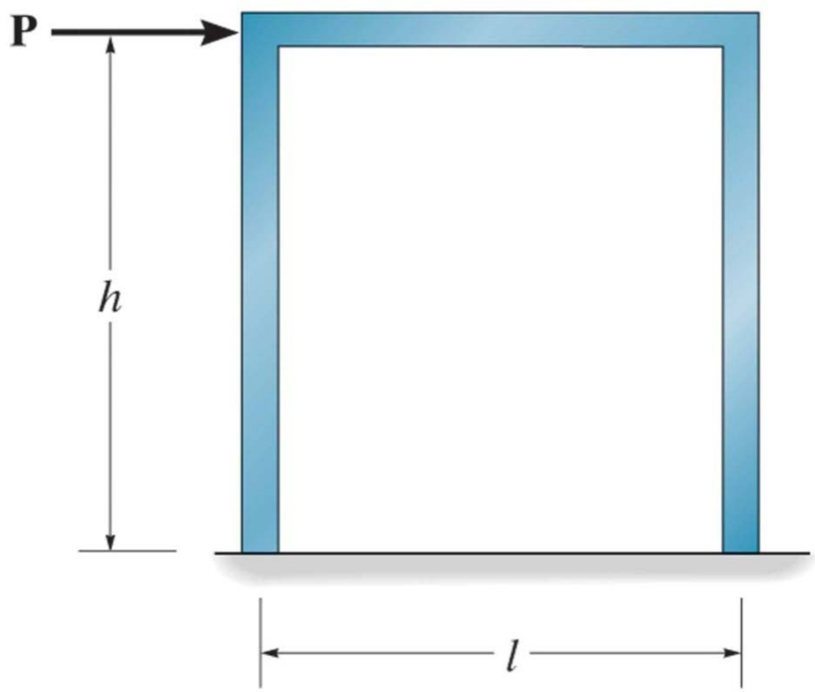


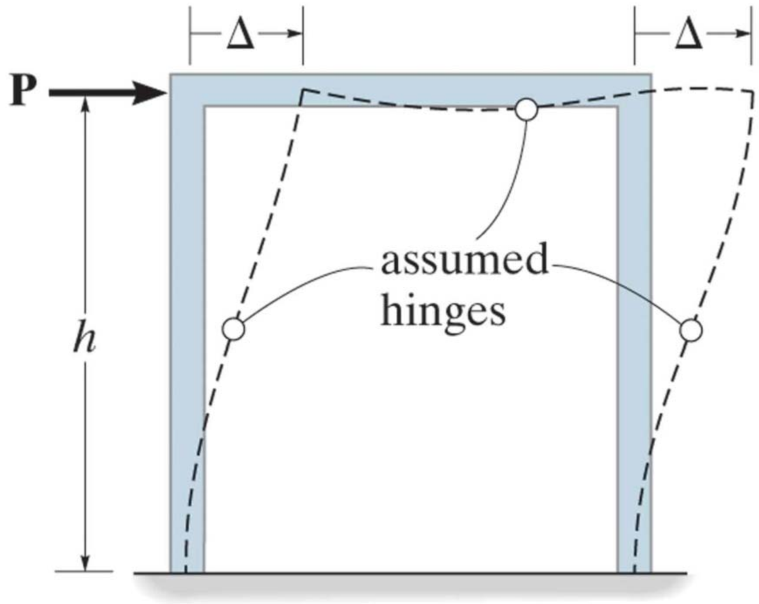


Pin-Supported Portal Frames

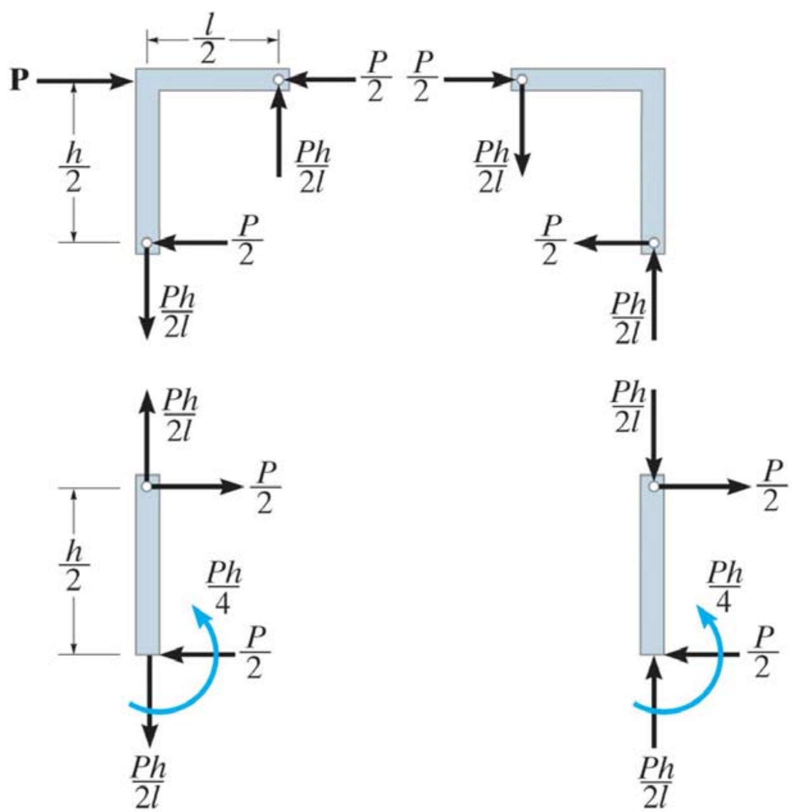


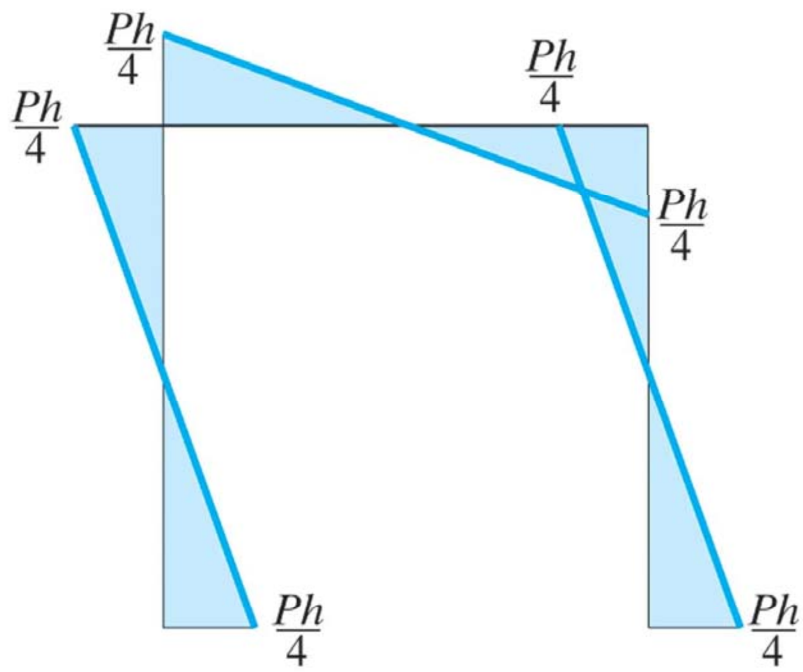
Fixed-Supported Portal Frames





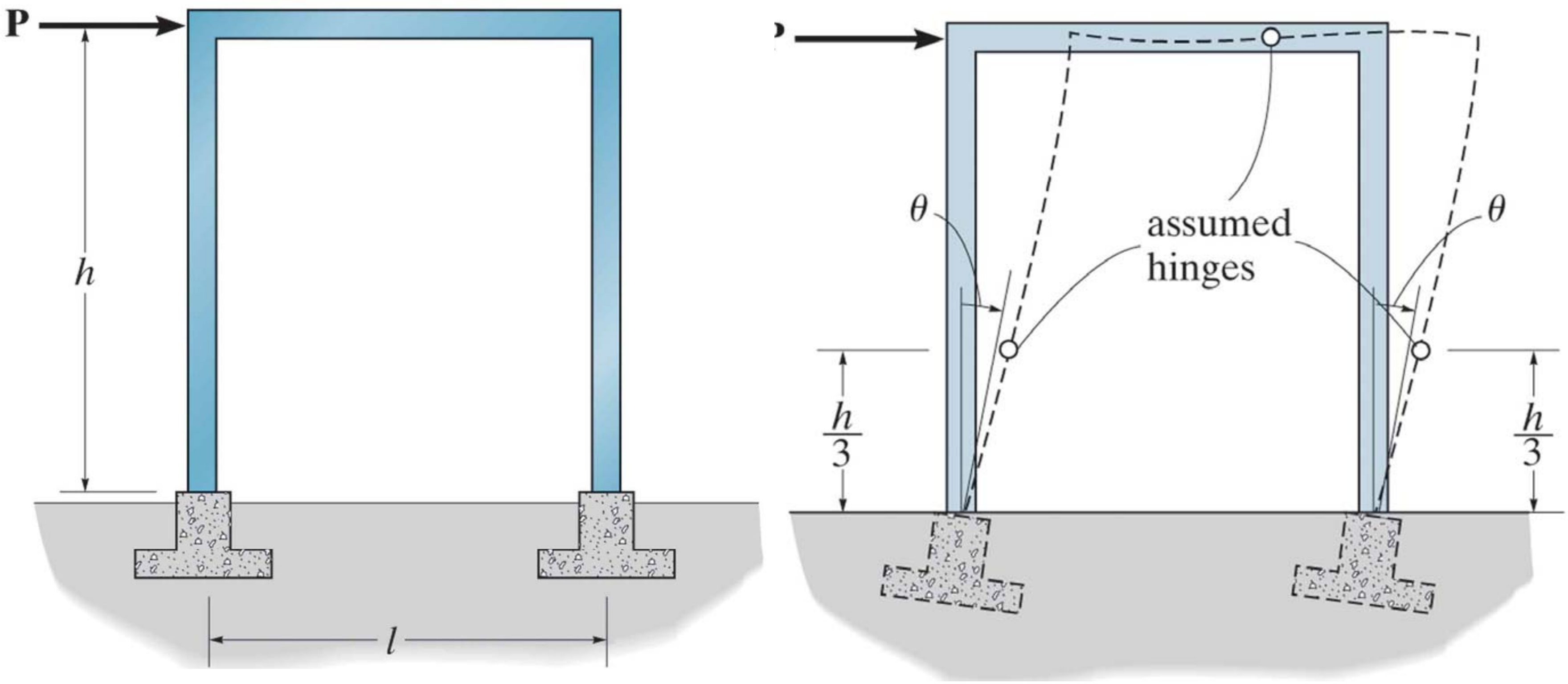
Fixed-Supported Portal Frames





moment diagram

Partial Fixed-Supported Portal Frames

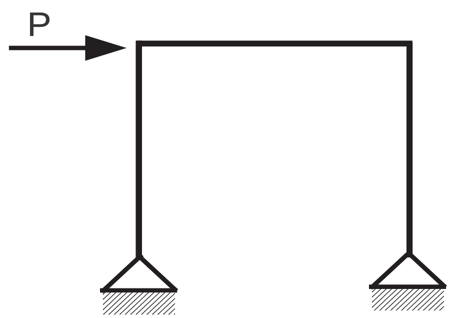


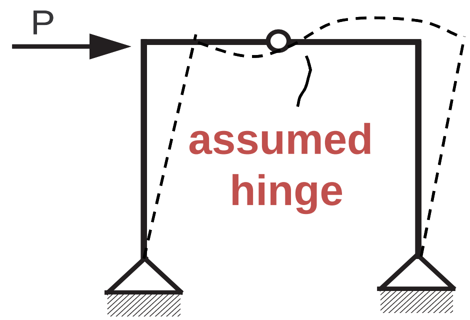
Portal Frames – Lateral Loads

Real Structure	Approximation

Pin-supported

One assumption must be made

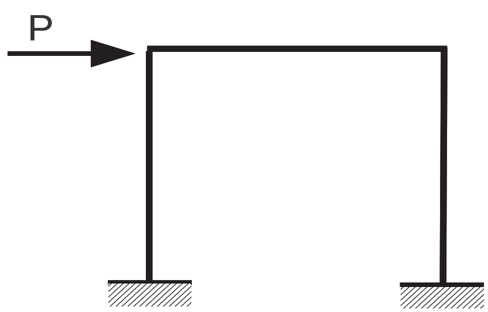


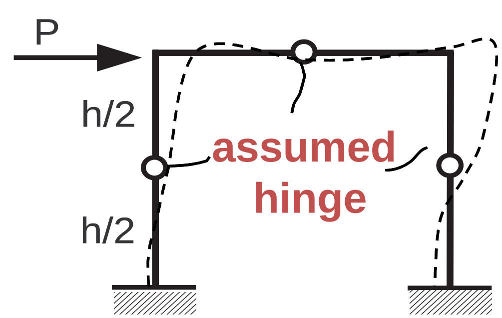


A point of inflection is located ***approximately*** at the girder's midpoint

fixed -supported

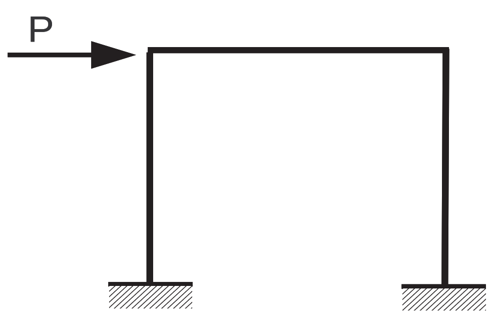
Three assumption must be made

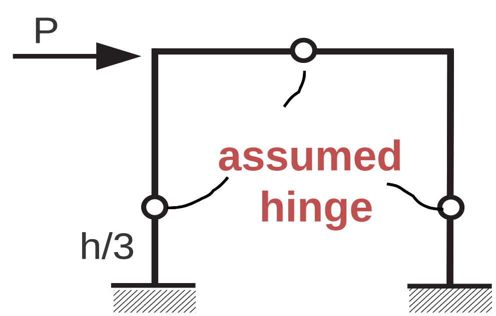




Points of inflection are located ***approximately*** at the midpoints of all three members

Partial fixity

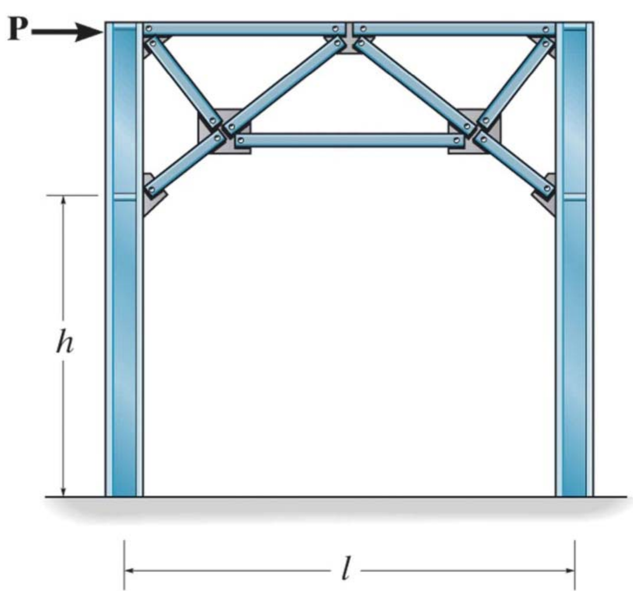


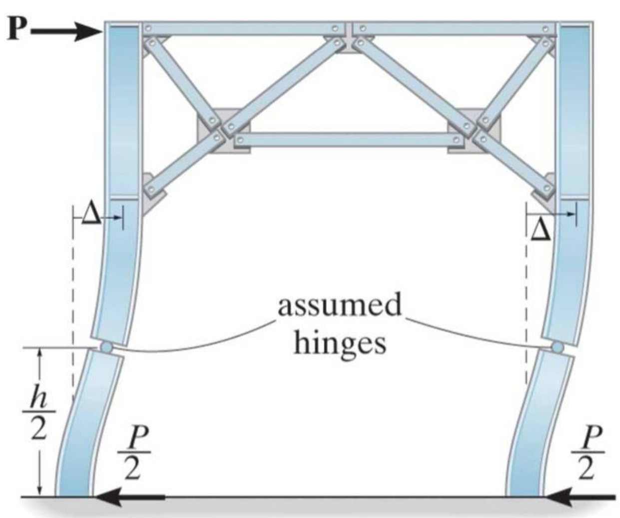


Points of inflection for columns are located ***approximately*** at h/3 and the centre of the girder

Frames with Trusses

- When a portal frame is used to span large distance, a truss may be used in place of the horizontal girder
- The suspended truss is assumed to be pin connected at its points of attachment to the columns
- Use the same assumptions as those used for simple portal frames





Frames with Trusses

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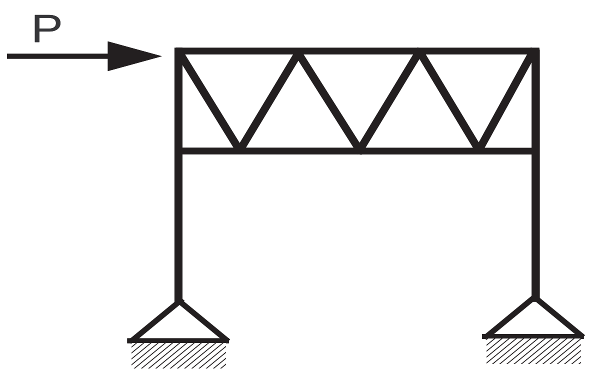
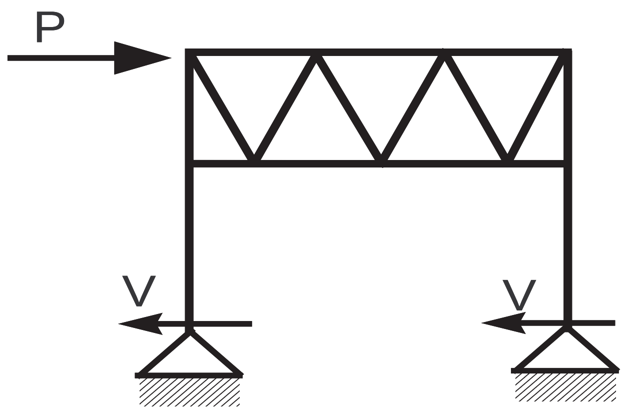
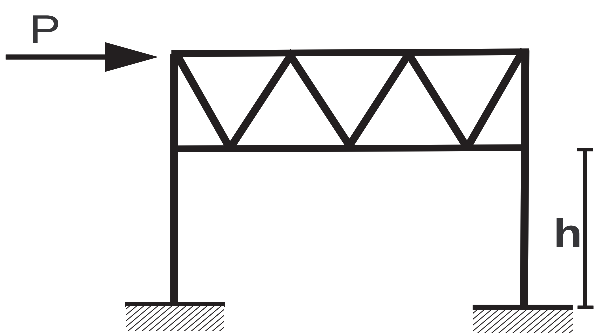
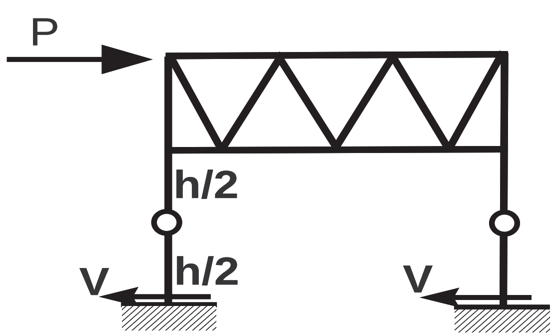
<tr><td>

pin supported columns
pin connection truss-column</td><td>

the horizontal reactions (shear) are equal</td></tr>
<tr><td>

fixed supported columns
pin connection truss-column</td><td>

horizontal reactions (shear) are equal
there is a zero moment (hinge) on each column</td></tr>
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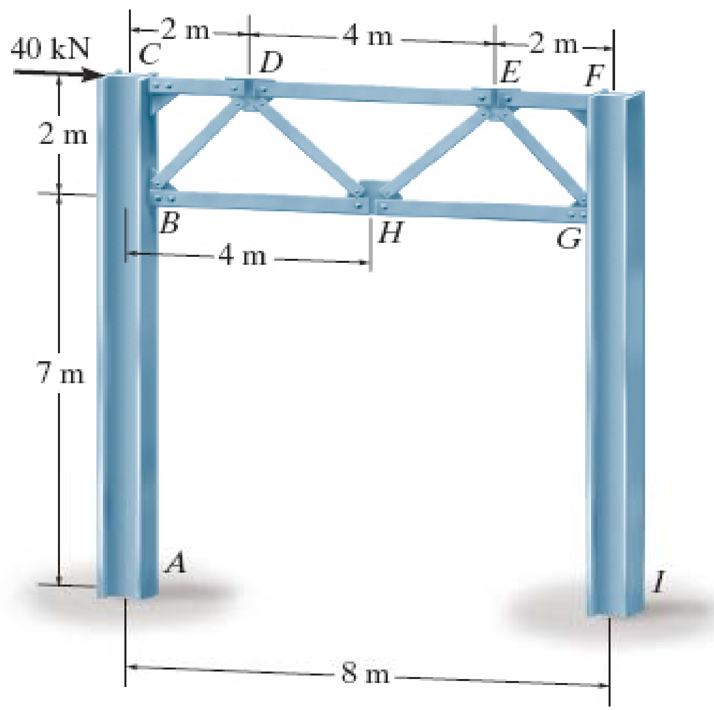
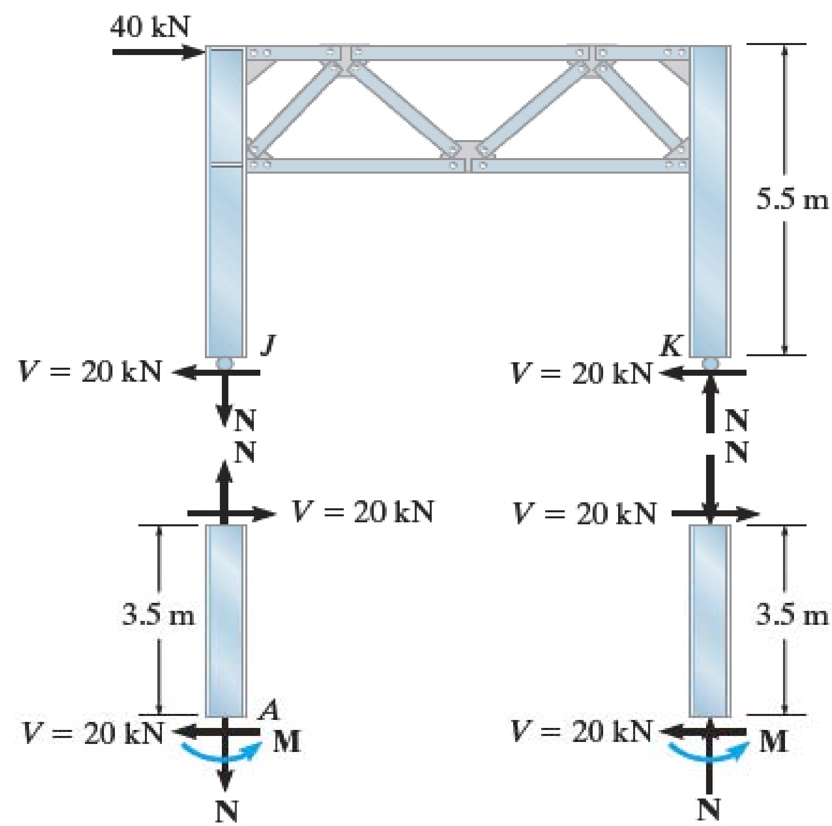
Example 4

Determine by approximate methods the forces acting in the members of the Warren portal shown in Fig. 7–11*a*.



(a)



(b)

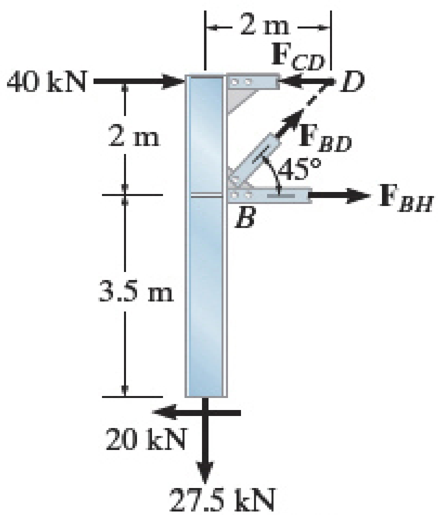
Lower Half of Column

$\circlearrowleft + \Sigma M_A = 0; \qquad M - 3.5(20) = 0 \qquad M = 70 \text{ kN} \cdot \text{m}$

Upper Portion of Column

$\circlearrowleft + \Sigma M_J = 0; \qquad -40(5.5) + N(8) = 0 \qquad N = 27.5 \text{ kN}$

Using the method of sections, Fig. 7–11*c*, we can now proceed to obtain the forces in members *CD*, *BD*, and *BH*.



(c)

$+\uparrow \Sigma F_y = 0; \quad -27.5 + F_{BD} \sin 45° = 0 \qquad F_{BD} = 38.9 \text{ kN (T)}$ ***Ans.***

$\circlearrowleft + \Sigma M_B = 0; \quad -20(3.5) - 40(2) + F_{CD}(2) = 0 \quad F_{CD} = 75 \text{ kN (C)}$ ***Ans.***

$\circlearrowleft + \Sigma M_D = 0; \quad F_{BH}(2) - 20(5.5) + 27.5(2) = 0 \quad F_{BH} = 27.5 \text{ kN (T)}$ ***Ans.***

In a similar manner, show that one obtains the results on the free-body diagram of column *FGI* in Fig. 7–11*d*. Using these results, we can now find the force in each of the other truss members of the portal using the method of joints.

Joint D, Fig. 7–11e

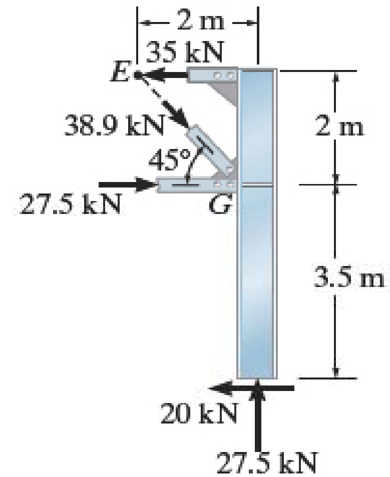
$$+\uparrow \Sigma F_y = 0; \quad F_{DH} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \quad F_{DH} = 38.9 \text{ kN (C)} \quad \textit{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 75 - 2(38.9 \cos 45^\circ) - F_{DE} = 0 \quad F_{DE} = 20 \text{ kN (C)} \quad \textit{Ans.}$$

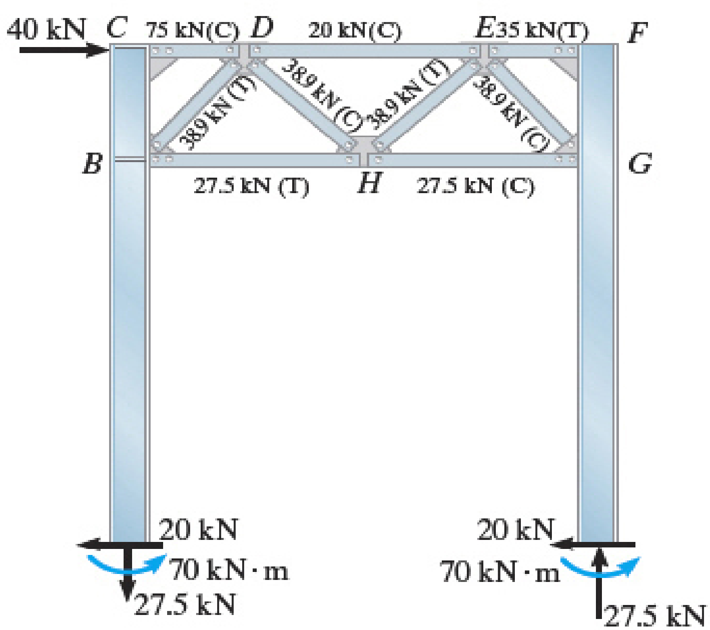
Joint H, Fig. 7–11f

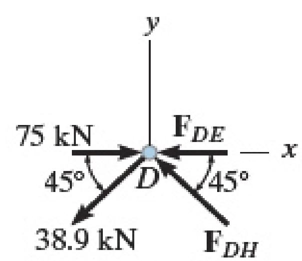
$$+\uparrow \Sigma F_y = 0; \quad F_{HE} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \quad F_{HE} = 38.9 \text{ kN (T)} \quad \textit{Ans.}$$

These results are summarized in Fig. 7–11*g*.

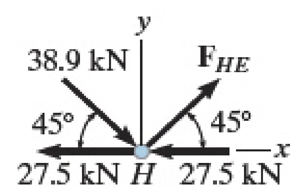


(d)





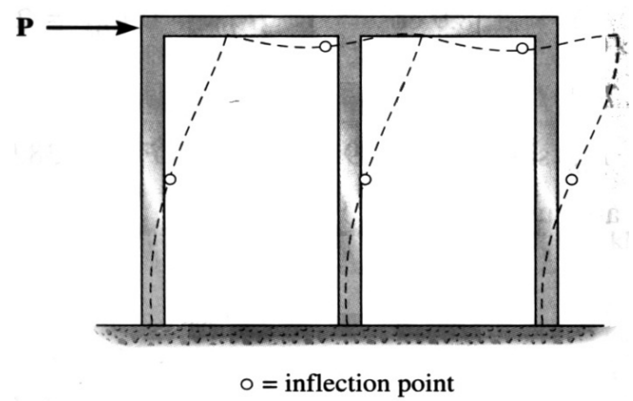
(e)

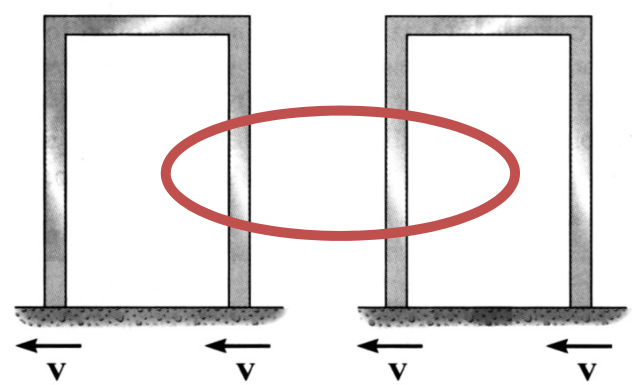


(f)

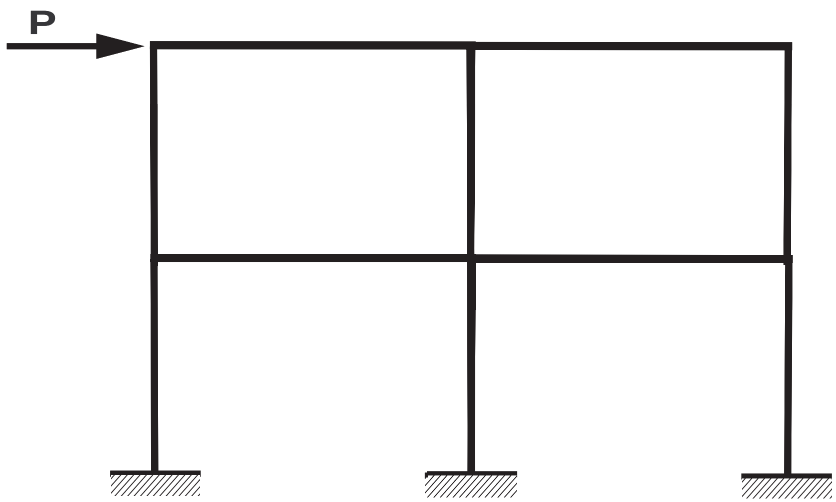
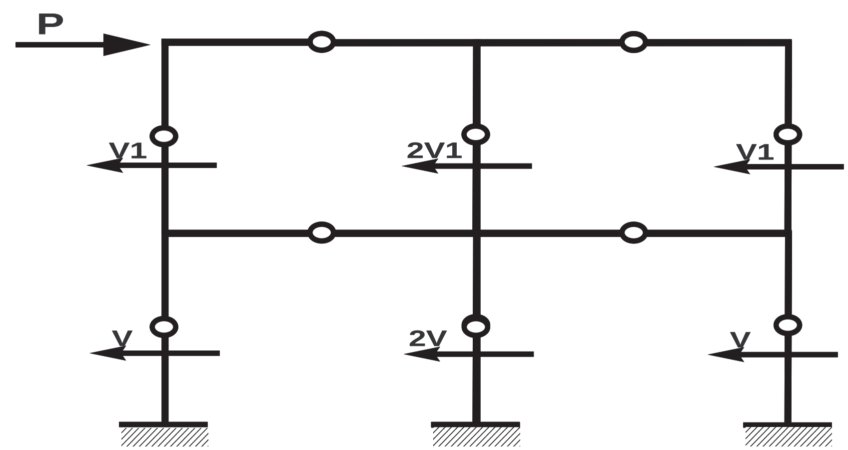
Building frames – Lateral Loads
Portal Method

- A building bent deflects in the same way as a portal frame
- The assumptions would be the same as those used for portal frames
- The *interior columns* would represent the effect of *two portal columns*



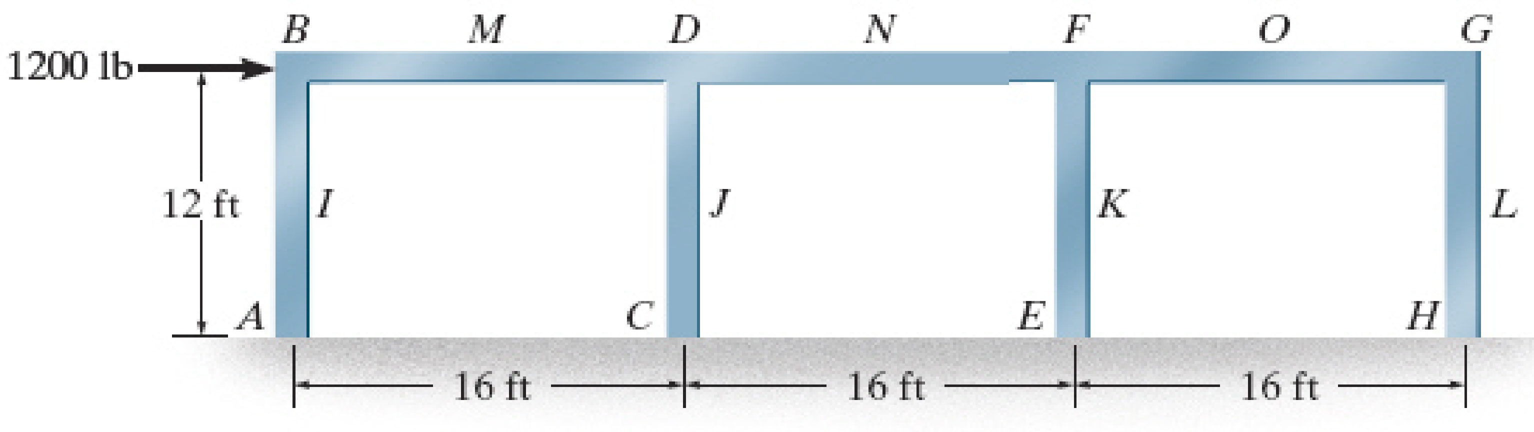


Building frames – Lateral loads
Portal Method

real structure	approximation
 The method is most suitable for buildings having low elevation and uniform framing	 1. A hinge is placed at the centre of each girder, this is assumed to be a point of zero moment. 2. A hinge is placed at the centre of each column, this to be a point of zero moment. 3. At the given floor level the shear at the interior column hinges is twice that at the exterior column hinges

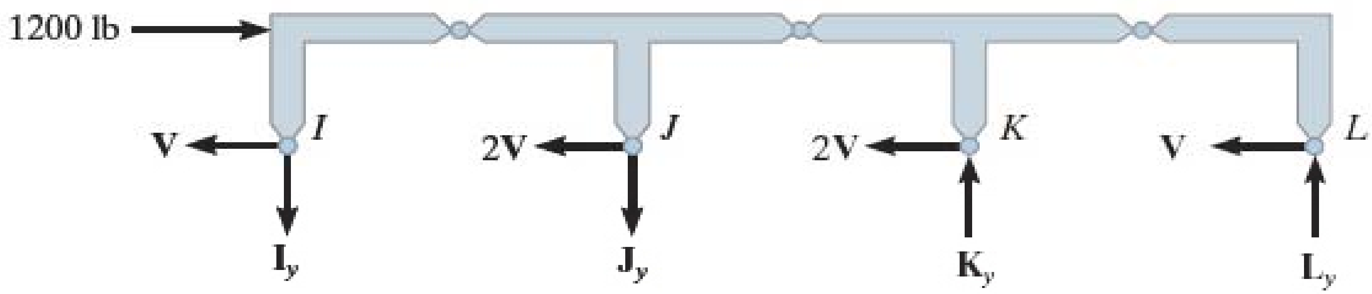
Example 5

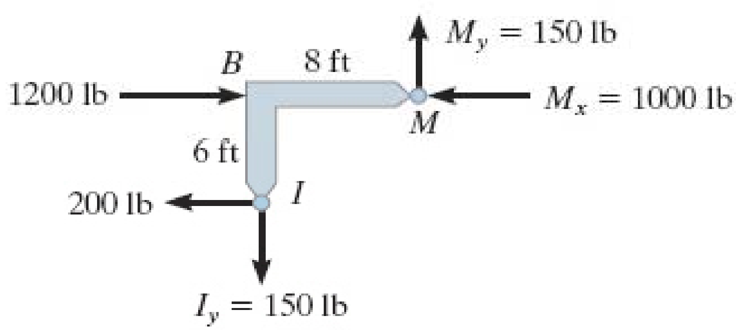
Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7–13*a*. Use the portal method of analysis.

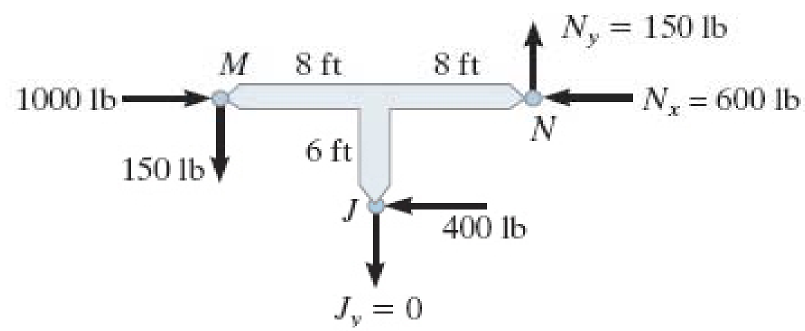


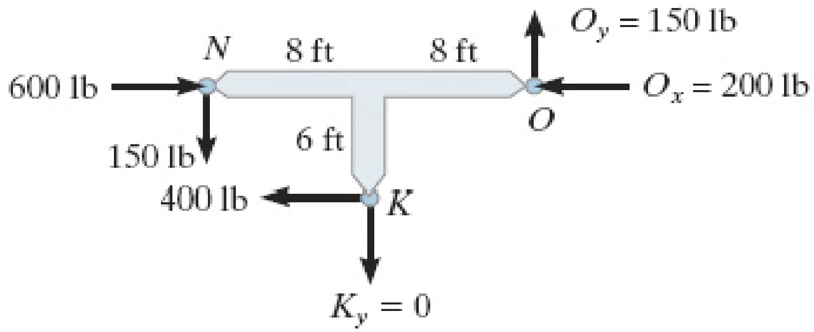
(a)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 1200 - 6V = 0 \qquad V = 200 \text{ lb}$$



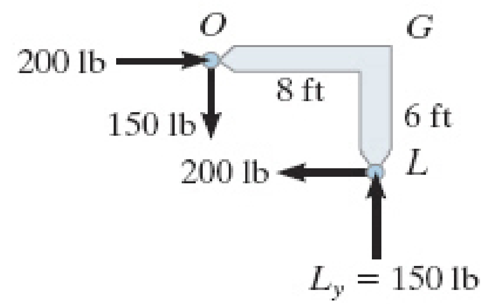






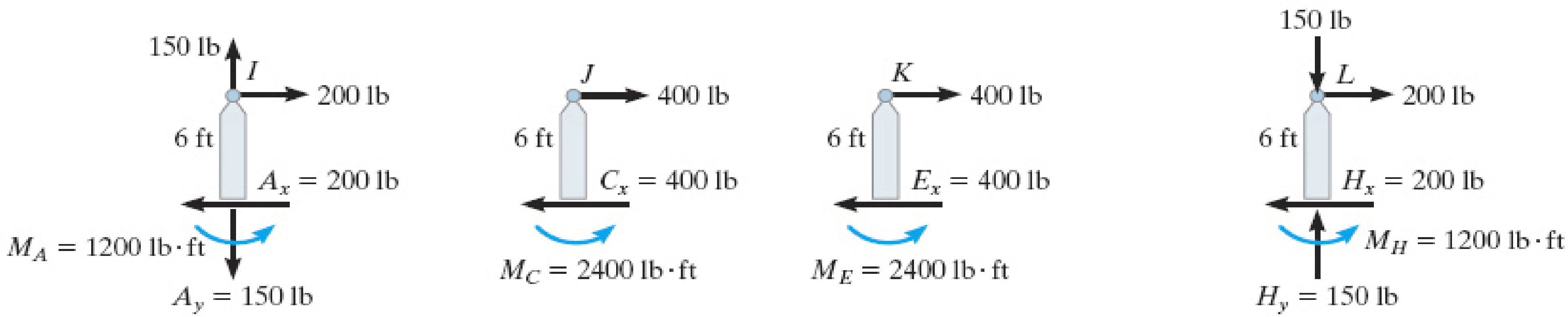
N
8 ft
8 ft
$O_y = 150$ lb
600 lb
$O_x = 200$ lb
O
6 ft
150 lb
400 lb
K
$K_y = 0$

(e)



O
G
200 lb
8 ft
6 ft
150 lb
200 lb
L
$L_y = 150$ lb

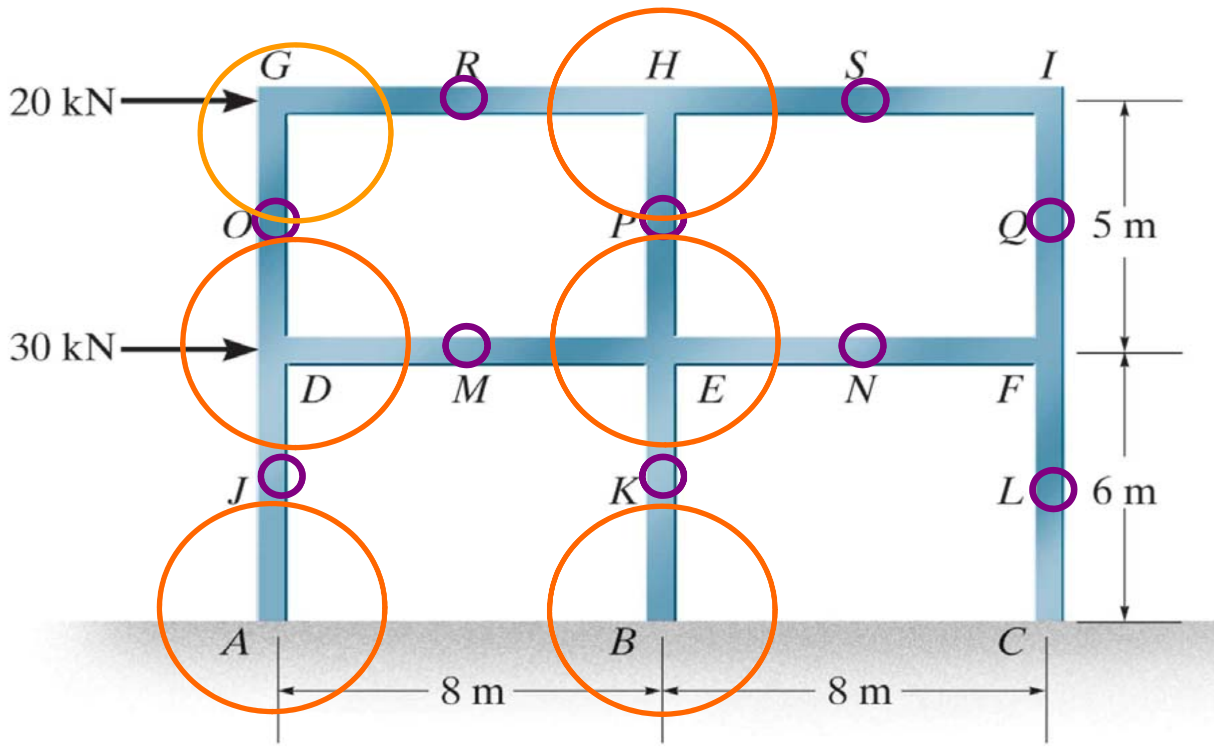
(f)



150 lb
I
200 lb
6 ft
$A_x = 200$ lb
$M_A = 1200$ lb · ft
$A_y = 150$ lb
J
400 lb
6 ft
$C_x = 400$ lb
$M_C = 2400$ lb · ft
K
400 lb
6 ft
$E_x = 400$ lb
$M_E = 2400$ lb · ft
150 lb
L
200 lb
6 ft
$H_x = 200$ lb
$M_H = 1200$ lb · ft
$H_y = 150$ lb

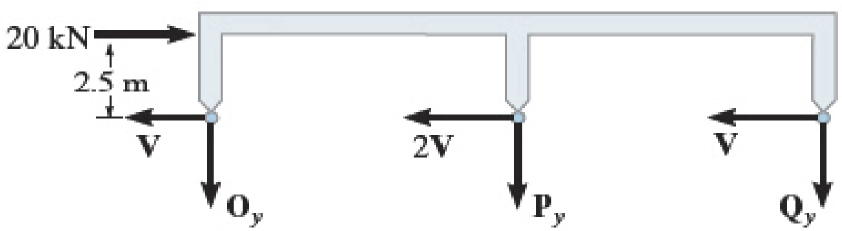
Example 6

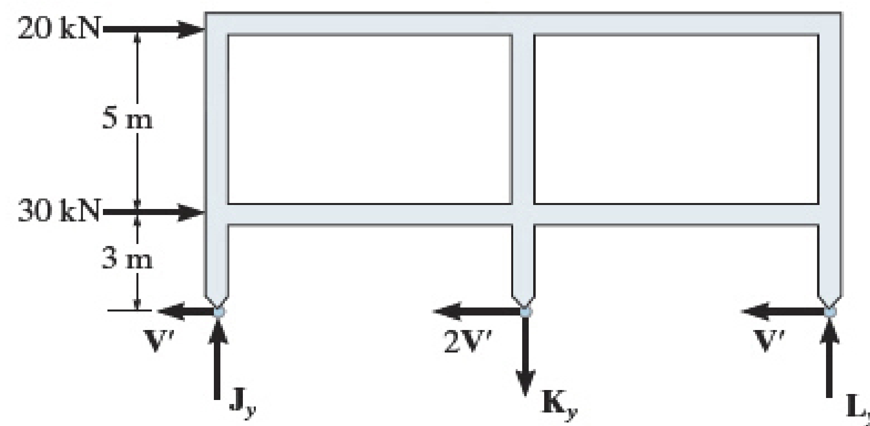
Determine (approximately) the reactions at the base of the columns of the frame.

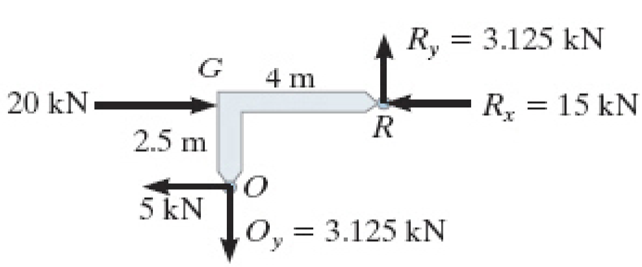


$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 20 - 4V = 0 \qquad V = 5\text{ kN}$$

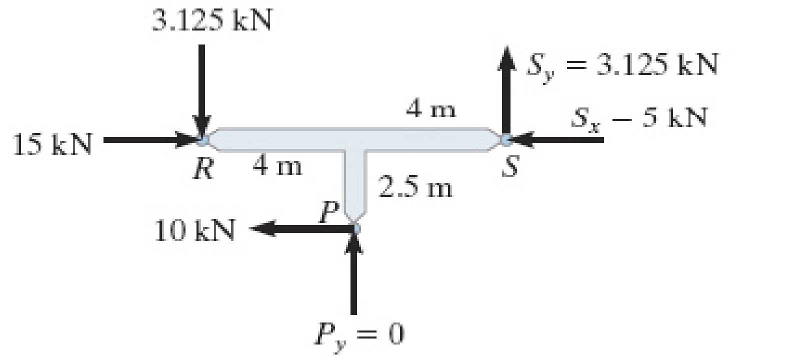
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 20 + 30 - 4V' = 0 \qquad V' = 12.5\text{ kN}$$



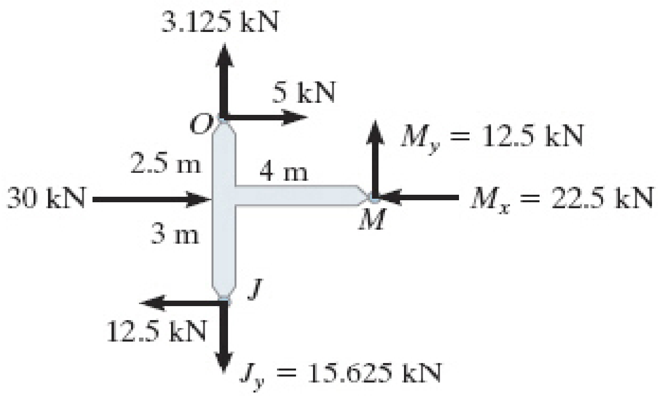




R_y = 3.125 kN
G
4 m
20 kN
R_x = 15 kN
R
2.5 m
O
5 kN
O_y = 3.125 kN

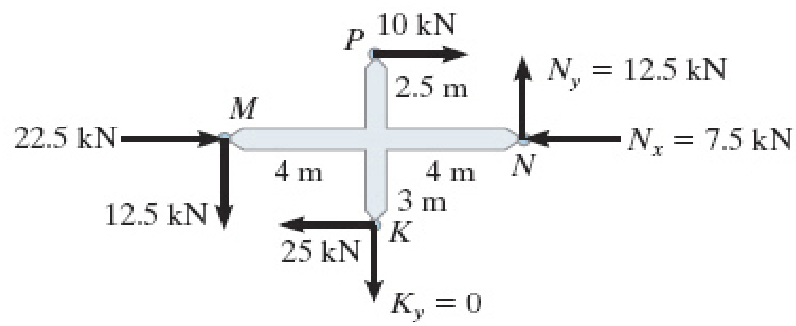


3.125 kN
S_y = 3.125 kN
4 m
S_x – 5 kN
15 kN
R
4 m
S
2.5 m
P
10 kN
P_y = 0



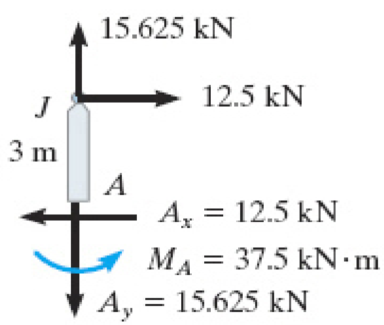
3.125 kN
5 kN
O
M_y = 12.5 kN
2.5 m
4 m
30 kN
M_x = 22.5 kN
M
3 m
J
12.5 kN
J_y = 15.625 kN

(d)



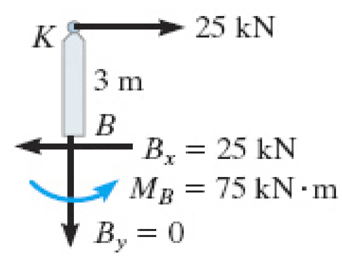
10 kN
P
2.5 m
N_y = 12.5 kN
M
22.5 kN
N_x = 7.5 kN
4 m
4 m
N
3 m
12.5 kN
K
25 kN
K_y = 0

(g)



15.625 kN
12.5 kN
J
3 m
A
A_x = 12.5 kN
M_A = 37.5 kN·m
A_y = 15.625 kN

(e)

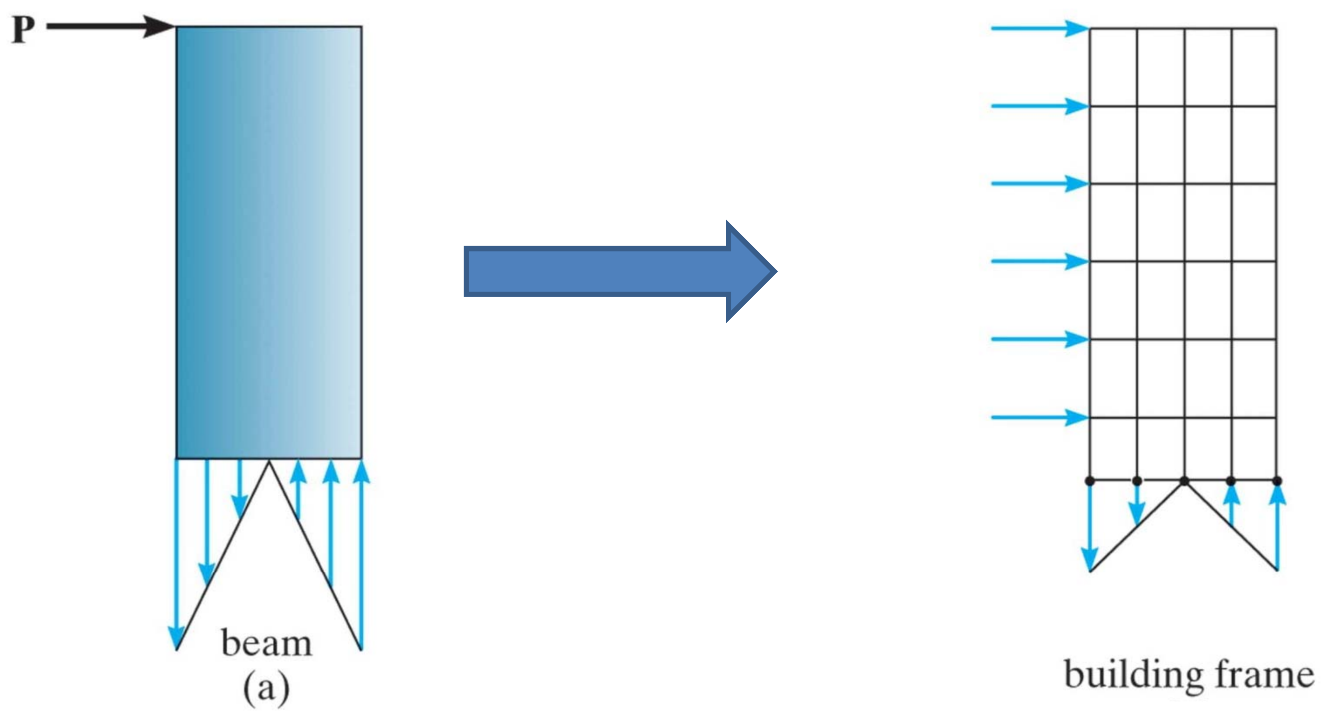


K
25 kN
3 m
B
B_x = 25 kN
M_B = 75 kN·m
B_y = 0

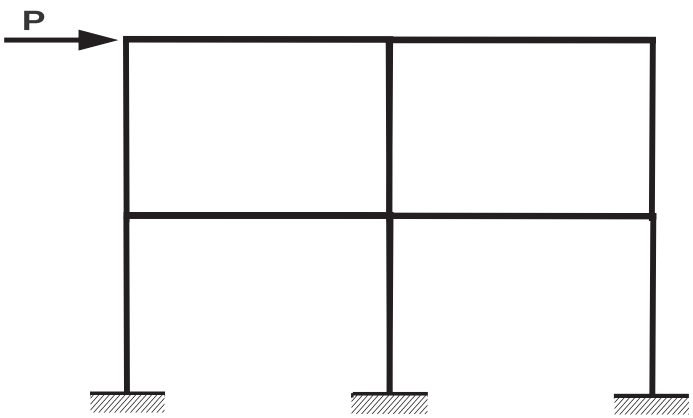
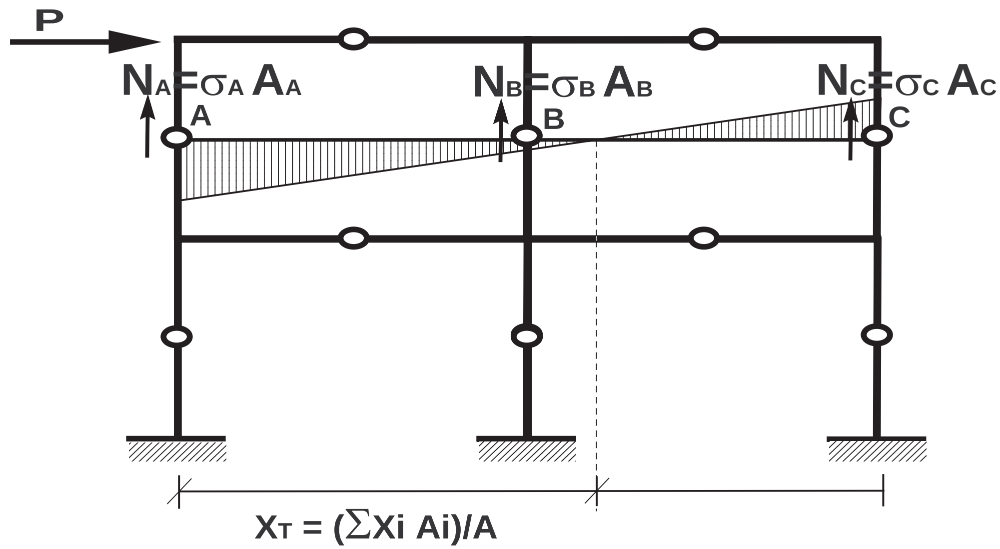
(h)

Building frames – Lateral loads
Cantilever method

- The method is based on the same action as a long cantilevered beam subjected to a transverse load
- It is reasonable to assume the axial stress has a linear variation from the centroid of the column areas

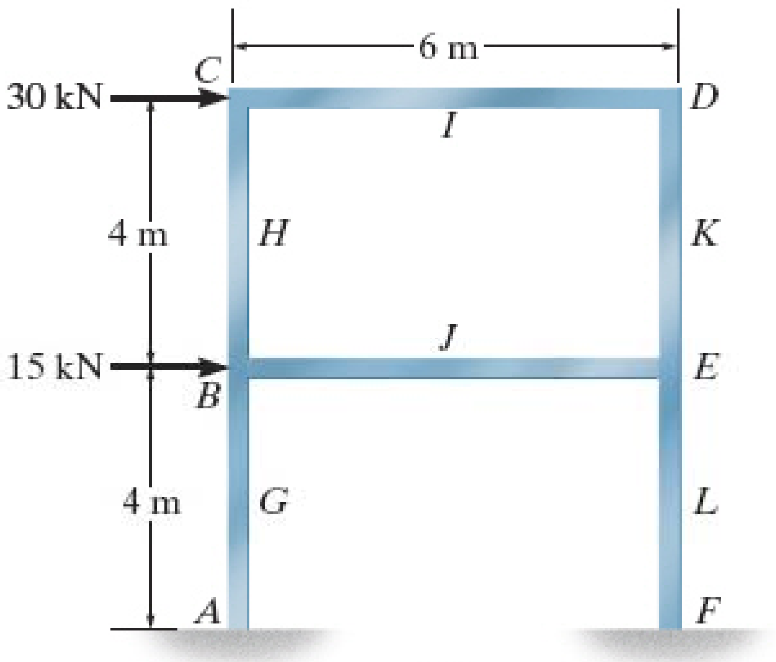


Building frames – lateral loads: Cantilever method

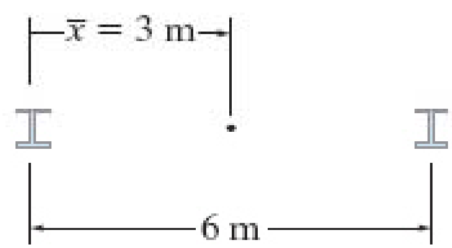
Real structure	Approximation
 	
The method is most suitable if the frame is tall and slender, or has columns with different cross sectional areas.	1. Zero moment (hinge) at the centre of each girder 2. Zero moment (hinge) at the centre of each column 3. The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level

Example 7

Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7–16*a*. The columns are assumed to have equal cross-sectional areas. Use the cantilever method of analysis.



(a)



(b)

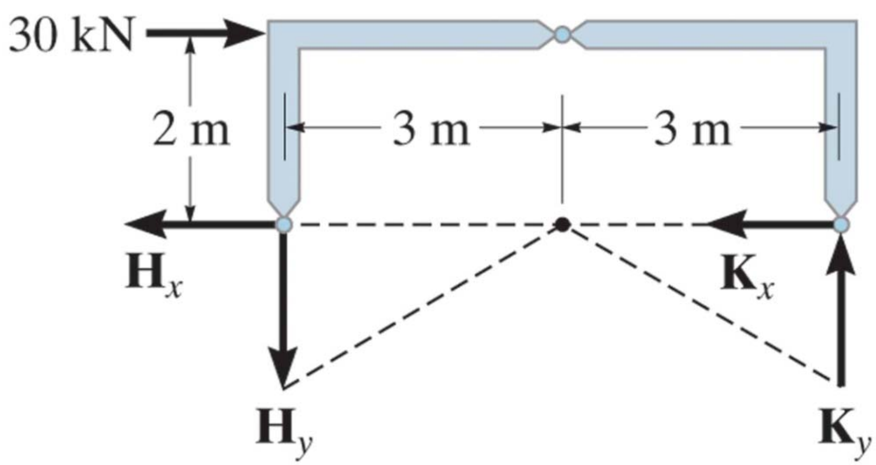
$\circlearrowright + \Sigma M = 0; \qquad -30(2) + 3H_y + 3K_y = 0$

The unknowns can be related by proportional triangles, Fig. 7–16*c*, that is,

$$\frac{H_y}{3} = \frac{K_y}{3} \qquad \text{or} \qquad H_y = K_y$$

Thus,

$$H_y = K_y = 10\text{ kN}$$

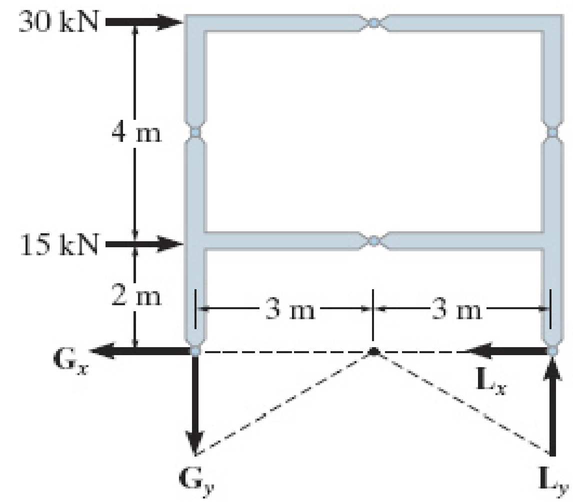


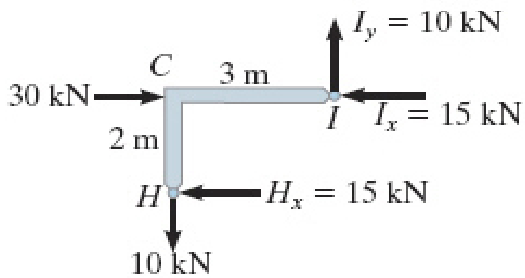
In a similar manner, using a section of the frame through the hinges at G and L, Fig. 7–16d, we have

$$\circlearrowright + \Sigma M = 0; \qquad -30(6) - 15(2) + 3G_y + 3L_y = 0$$

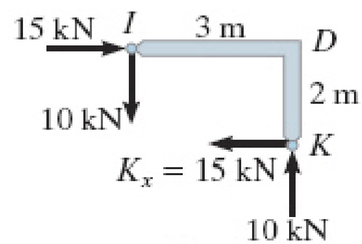
Since $G_y/3 = L_y/3$ or $G_y = L_y$, then

$$G_y = L_y = 35 \text{ kN}$$

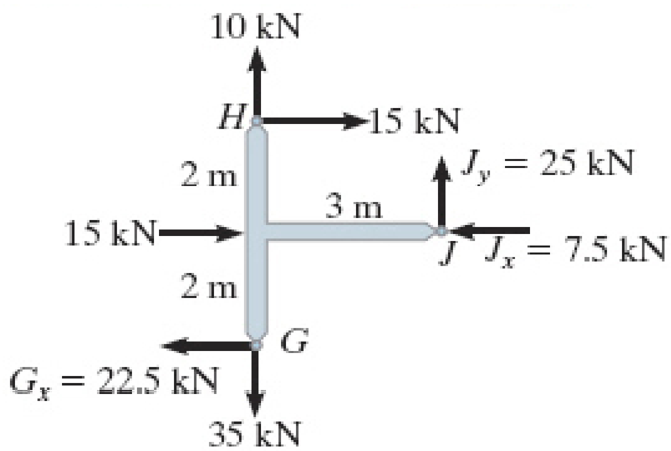




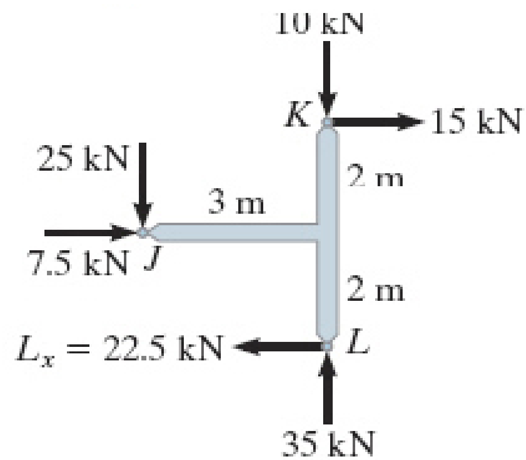
(e)



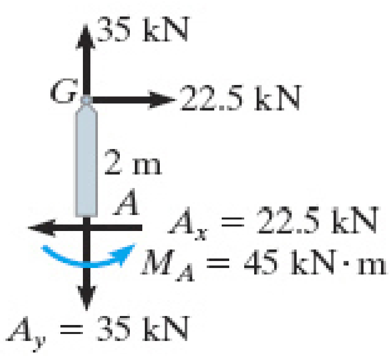
(f)


10 kN
H
15 kN
2 m
J_y = 25 kN
3 m
15 kN
J
J_x = 7.5 kN
2 m
G
G_x = 22.5 kN
35 kN

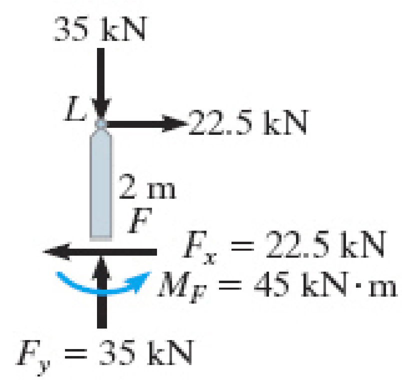
(g)


10 kN
K
15 kN
25 kN
2 m
3 m
7.5 kN
J
2 m
L_x = 22.5 kN
L
35 kN

(h)


35 kN
G
22.5 kN
2 m
A
A_x = 22.5 kN
M_A = 45 kN·m
A_y = 35 kN

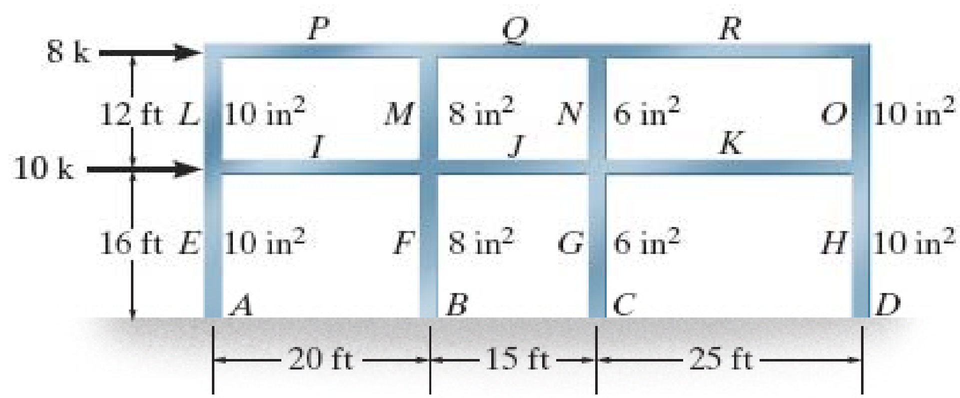
(i)


35 kN
L
22.5 kN
2 m
F
F_x = 22.5 kN
M_F = 45 kN·m
F_y = 35 kN

(j)

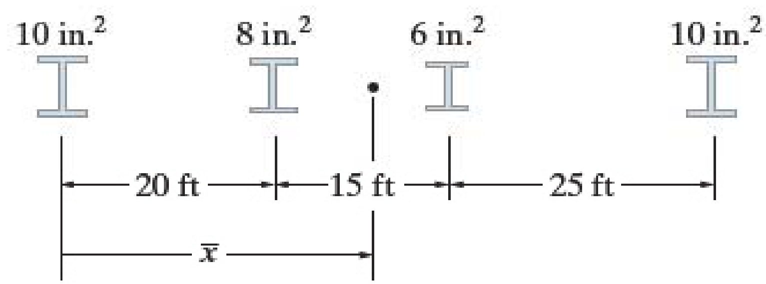
Example 8

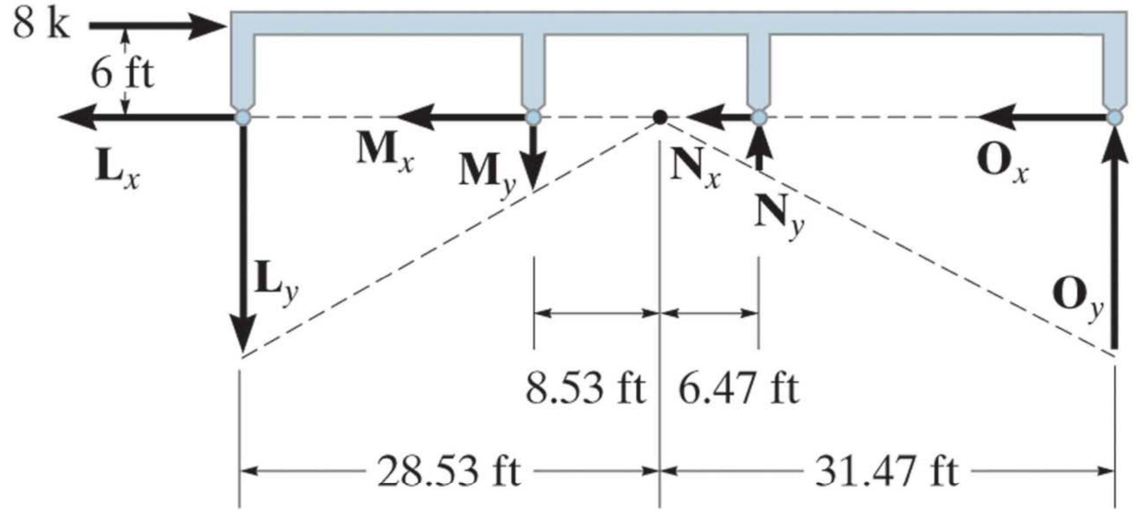
Show how to determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7–17*a*. The columns have the cross-sectional areas shown in Fig. 7–17*b*. Use the cantilever method of analysis.



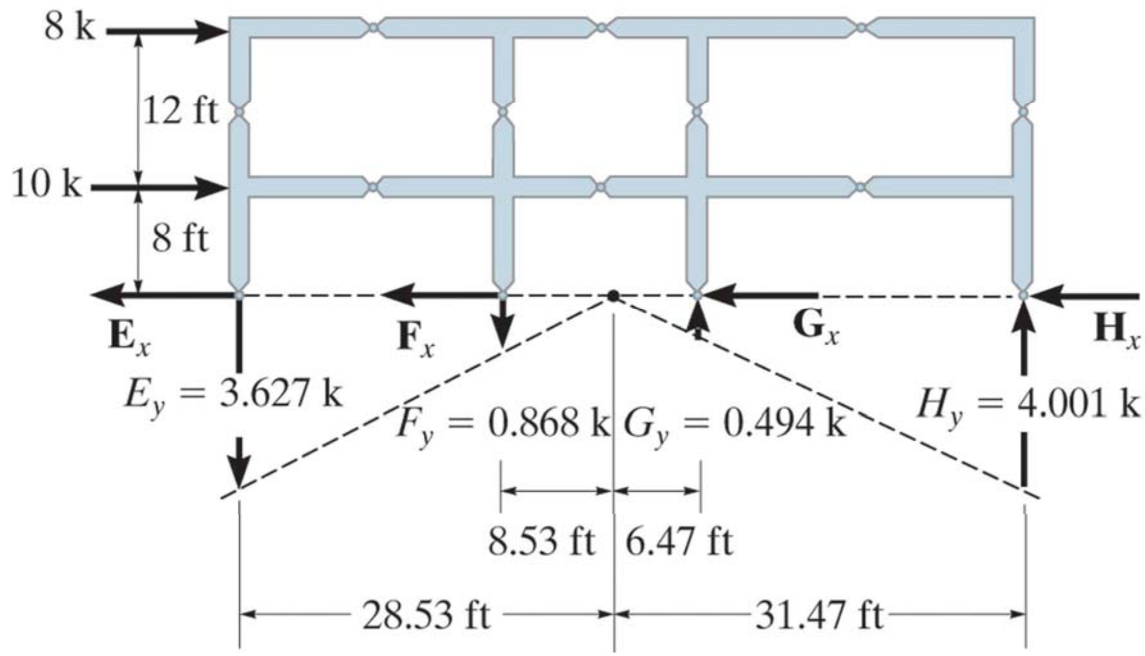
(a)

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0(10) + 20(8) + 35(6) + 60(10)}{10 + 8 + 6 + 10} = 28.53 \text{ ft}$$

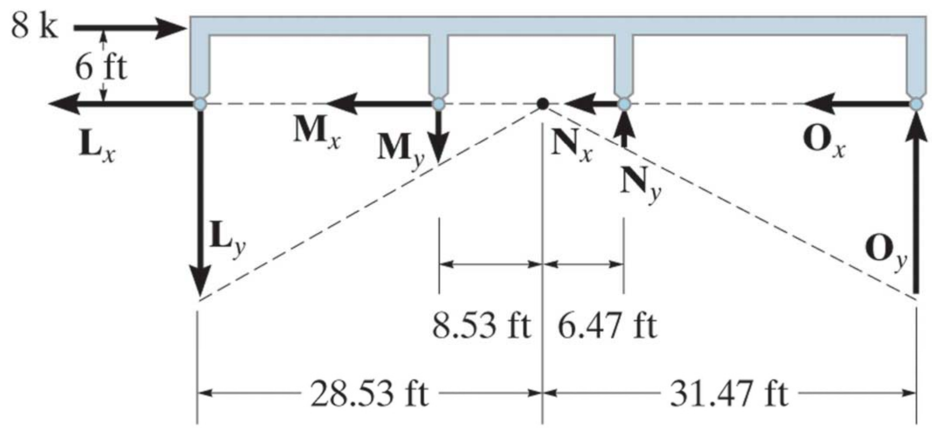




8 k
6 ft
L_x
M_x
M_y
N_x
N_y
O_x
L_y
O_y
8.53 ft
6.47 ft
28.53 ft
31.47 ft



8 k
12 ft
10 k
8 ft
E_x
F_x
G_x
H_x
$E_y = 3.627$ k
$F_y = 0.868$ k
$G_y = 0.494$ k
$H_y = 4.001$ k
8.53 ft
6.47 ft
28.53 ft
31.47 ft



$$\circlearrowright + \Sigma M = 0; \quad -8\text{ k}\,(6\text{ ft}) + L_y(28.53\text{ ft}) + M_y(8.53\text{ ft}) + N_y(6.47\text{ ft}) + O_y(31.47\text{ ft}) = 0 \qquad (1)$$

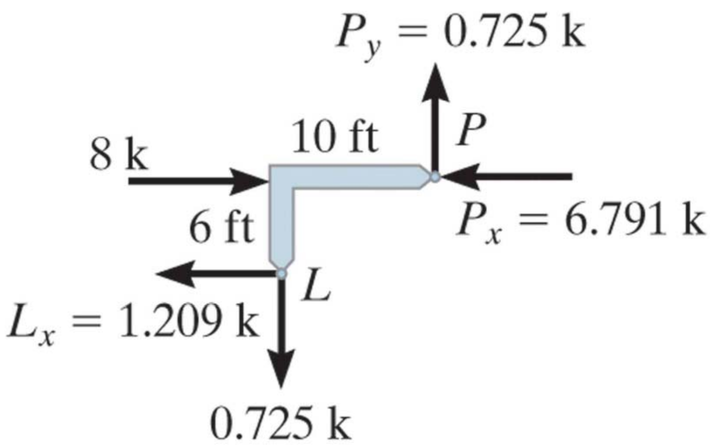
$$\sigma_M = \frac{8.53\text{ ft}}{28.53\text{ ft}}\sigma_L; \qquad \frac{M_y}{8\text{ in}^2} = \frac{8.53}{28.53}\left(\frac{L_y}{10\text{ in}^2}\right) \quad M_y = 0.239L_y \qquad (2)$$

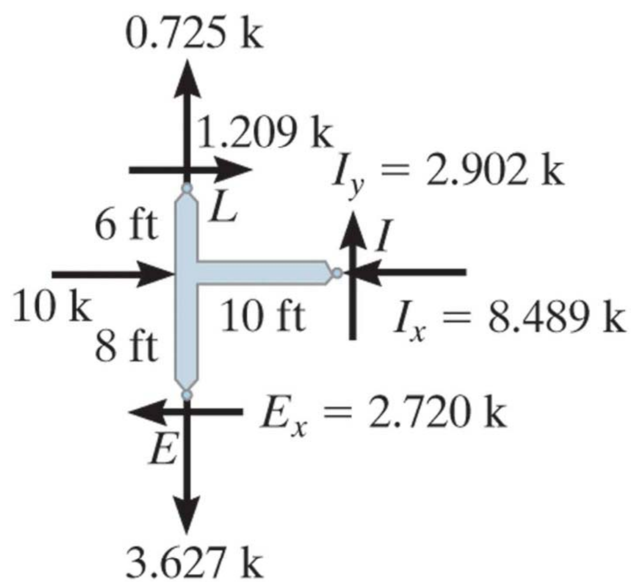
$$\sigma_N = \frac{6.47\text{ ft}}{28.53\text{ ft}}\sigma_L; \qquad \frac{N_y}{6\text{ in}^2} = \frac{6.47}{28.53}\left(\frac{L_y}{10\text{ in}^2}\right) \quad N_y = 0.136L_y \qquad (3)$$

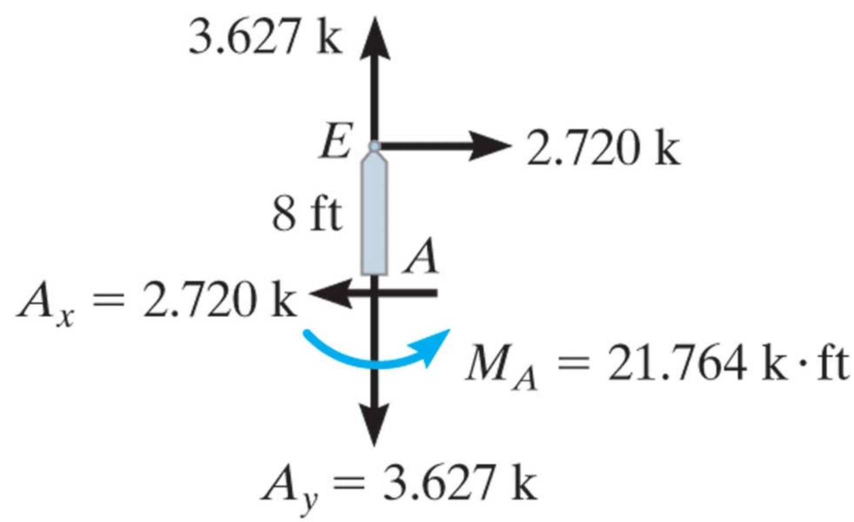
$$\sigma_O = \frac{31.47\text{ ft}}{28.53\text{ ft}}\sigma_L; \qquad \frac{O_y}{10\text{ in}^2} = \frac{31.47}{28.53}\left(\frac{L_y}{10\text{ in}^2}\right) \quad O_y = 1.103L_y \qquad (4)$$

Solving Eqs. (1)–(4) yields

$L_y = 0.725\text{ k} \qquad M_y = 0.174\text{ k} \qquad N_y = 0.0987\text{ k} \qquad O_y = 0.800\text{ k}$


$P_y = 0.725$ k
P
10 ft
8 k
6 ft
$P_x = 6.791$ k
L
$L_x = 1.209$ k
0.725 k


0.725 k
1.209 k
$I_y = 2.902$ k
L
6 ft
I
10 k
10 ft
$I_x = 8.489$ k
8 ft
$E_x = 2.720$ k
E
3.627 k


3.627 k
E
2.720 k
8 ft
A
$A_x = 2.720$ k
$M_A = 21.764$ k · ft
$A_y = 3.627$ k