Structure and Complexity of Grammar-Based Machine Translation

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Introduction

- State of the art machine translation systems are based on mathematical **translation models**, which account for all the elementary operations that rule the translation process
- Translation models are usually enriched with statistical parameters to drive the search
- Translation models are also exploited in word/phrase alignment, multilingual document retrieval, automatic dictionary construction, bilingual corpora annotation, etc.

Introduction (cont'd)

- Early translation models based on finite-state machinery :
  - IBM model, word to word [Brown et al. 1993]
  - Phrase-based [Och et al. 1999, Och and Ney 2002]
- Finite state techniques cannot easily model translations between languages with strong differences in word ordering

Introduction (cont'd)

- Recent shift towards more powerful hierarchical translation models :
  - Inversion Transduction Grammars [Wu 1997]
  - Head Transducer Grammars [Alshawi et al. 2000]
  - Tree-to-string models [Yamada and Knight 2001], [Galley et al. 2004]
  - Loosely tree-based model [Gildea 2003]
  - Multi-Text Grammars [Melamed 2003]
  - Hierarchical phrase-based models [Chiang 2005]

Introduction (cont'd)

- Most of the translation models above can be abstractly viewed as **synchronous context-free grammars**
- Synchronous context-free grammars are rooted in the theory of compilers, where they are called **syntax-directed translation schemata** (SDTS) [Lewis and Stearns 1968], [Aho and Ullman 1969]

Synchronous context-free grammars

- A **synchronous context-free grammar** (SCFG) is based on three components :
  - Context free grammar (CFG) for source language
  - CFG for target language
  - **Pairing relation** (bijection) on the productions of the two grammars and their nonterminals
- Each rule pair called **synchronous production**
- Pairing relation between nonterminals represented by superscript integers called **indices**

Example

Fragment SCFG (English to Japanese, [Yamada and Knight 2001])

$$
\begin{array}{lll}
s_1 : & [\mathrm{VB} \rightarrow \mathrm{PRP}^{(1)}\ \mathrm{VB1}^{(2)}\ \mathrm{VB2}^{(3)}, & \mathrm{VB} \rightarrow \mathrm{PRP}^{(1)}\ \mathrm{VB2}^{(3)}\ \mathrm{VB1}^{(2)}] \\
s_2 : & [\mathrm{VB2} \rightarrow \mathrm{VB}^{(1)}\ \mathrm{TO}^{(2)}, & \mathrm{VB2} \rightarrow \mathrm{TO}^{(2)}\ \mathrm{VB}^{(1)}\ \mathrm{ga}] \\
s_3 : & [\mathrm{TO} \rightarrow \mathrm{TO}^{(1)}\ \mathrm{NN}^{(2)}, & \mathrm{TO} \rightarrow \mathrm{NN}^{(2)}\ \mathrm{TO}^{(1)}] \\
s_4 : & [\mathrm{PRP} \rightarrow \mathrm{he}, & \mathrm{PRP} \rightarrow \mathrm{kare\ ha}] \\
s_5 : & [\mathrm{VB1} \rightarrow \mathrm{adores}, & \mathrm{VB1} \rightarrow \mathrm{daisuki\ desu}] \\
s_6 : & [\mathrm{VB} \rightarrow \mathrm{listening}, & \mathrm{VB} \rightarrow \mathrm{kiku\ no}] \\
s_7 : & [\mathrm{TO} \rightarrow \mathrm{to}, & \mathrm{TO} \rightarrow \mathrm{wo}] \\
s_8 : & [\mathrm{NN} \rightarrow \mathrm{music}, & \mathrm{NN} \rightarrow \mathrm{ongaku}]
\end{array}
$$

Derivations

- A SCFG generates pairs of strings/trees, representing the desired translation
- The **derive** relation applies a synchronous production to simultaneously rewrite two paired nonterminals (nonterminals with same index)
- Pairing relation must be updated after each application of a synchronous production

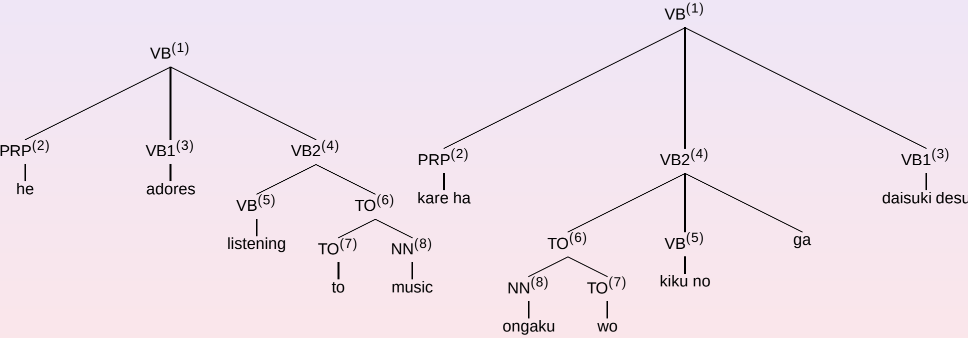
Example (cont'd)

Fragment derivation:

$$
\begin{aligned}
& [\mathrm{VB}^{(1)},\ \ \mathrm{VB}^{(1)}] \\
& \Rightarrow_G^{s_1} \ [\mathrm{PRP}^{(2)}\ \mathrm{VB1}^{(3)}\ \mathrm{VB2}^{(4)},\ \ \mathrm{PRP}^{(2)}\ \mathrm{VB2}^{(4)}\ \mathrm{VB1}^{(3)}] \\
& \Rightarrow_G^{s_4} \ [\text{he } \mathrm{VB1}^{(3)}\ \mathrm{VB2}^{(4)},\ \ \text{kare ha } \mathrm{VB2}^{(4)}\ \mathrm{VB1}^{(3)}] \\
& \Rightarrow_G^{s_5} \ [\text{he adores } \mathrm{VB2}^{(4)},\ \ \text{kare ha } \mathrm{VB2}^{(4)} \text{ daisuki desu}] \\
& \Rightarrow_G^{s_2} \ [\text{he adores } \mathrm{VB}^{(5)}\ \mathrm{TO}^{(6)},\ \ \text{kare ha } \mathrm{TO}^{(6)}\ \mathrm{VB}^{(5)} \text{ ga daisuki desu}]
\end{aligned}
$$

Example (cont'd)

Parse trees :



Translation

- Let G be a SCFG and w a string
- **Translation relation** : Set of all string pairs generated by G

$$T(G) \quad = \quad \{[u, v] \mid [S^{(1)}, S^{(1)}] \Rightarrow_G^* [u, v]\}$$

- **Image** of w : Set of strings that are translations of w

$$T(w, G) \quad = \quad \{v \mid [w, v] \in T(G)\}$$

Probabilistic SCFGs

- In a **Probabilistic SCFG**, each synchronous production associated with a probability

$$p_G([A_1 \rightarrow \alpha_1,\ A_2 \rightarrow \alpha_2])$$

- Normalization conditions for each pair $[A_1, A_2]$

$$\sum_{\alpha_1,\alpha_2} p_G([A_1 \rightarrow \alpha_1,\ A_2 \rightarrow \alpha_2]) \ = \ 1$$

PSCFGs (cont'd)

- In PSCFG we can define several joint distributions (t_i trees, w_i strings, y = yield)

$$
\begin{aligned}
p_G([t_1,\ t_2] &= \prod_{i=1}^{n} p_G(s_i) \\
p_G([w_1,\ w_2]) &= \sum_{y([t_1,t_2])=[w_1,w_2]} p_G([t_1,t_2]) \\
p_G([w_1,\ t_2]) &= \sum_{y(t_1)=w_1} p_G([t_1,t_2]) \\
&\vdots
\end{aligned}
$$

Computational Problems

- **Translation problem** : given SCFG G and string w, compute **parse forest** for strings in $T(w, G)$
- Size of parse forest for $T(w, G)$ can be a double exponential function in the size of w
- Highly compressed representation of parse forest is needed; we consider context-free grammars [Lang 1994] or, equivalently, hyper-graphs [Klein and Manning 2001]

Computational Problems (cont'd)

- **Recognition/Parsing problem** : given SCFG G and string pair $[u, v]$
  - decide whether $[u, v] \in T(G)$
  - construct parse forest for all derivations of $[u, v]$ by G
- The parsing problem is used in word/phrase alignment applications, bilingual dictionary construction, parallel corpora annotations, etc.

Computational Problems (cont'd)

- We introduce a new problem called the **intersection problem**; this generalizes the translation and the recognition/parsing problems, and several others
- We provide an abstract framework for the solution of the intersection problem
- Many of the (superficially different) translation and parsing algorithms proposed in the literature can be viewed as special cases of the above framework
- Similar attempts to define abstract frameworks for translation algorithms in [Bertsch and Nederhof 2001] and [Melamed and Wang 2005]

SCFG Projection

- We can project SCFG G into its **left and right grammar** components

$$\text{proj}(G, 1), \qquad \text{proj}(G, 2)$$

which are both CFGs

- We can similarly project the translation $T(G)$ into its **left and right language** components ($i = 1, 2$)

$$\text{proj}(T(G), i) \quad = \quad \{w_i \mid [w_1,\ w_2] \in T(G)\}$$

SCFG Projection (cont'd)

- In general the left grammar and the left language are not equivalent

$$L(\text{proj}(G, 1)) \quad \neq \quad \text{proj}(T(G), 1)$$

  (similarly for right case)
- This is because in synchronous derivations the left and right grammars interact; this is called mutual **controlled rewriting**

SCFG auto-projection

- We can efficiently construct the left and right **auto-projection** of SCFG G

$$\text{auto-proj}(G, 1), \qquad \text{auto-proj}(G, 2)$$

- The left auto-projection grammar and the left language are equivalent (similarly for right case)
- $\text{auto-proj}(G, 1)$ and $\text{auto-proj}(G, 2)$ are CFGs; this proves the **weak language preservation property** [Rambow and Satta 1996]

Intersection construction

- Let M_1, M_2 be Finite Automata (FAs); define the **Cartesian product**

$$L(M_1) \times L(M_2) \quad = \quad \{[u,\ v] \mid u \in L(M_1),\ v \in L(M_2)\}.$$

- Given SCFG G and FAs M_1, M_2, the **intersection construction** provides a new SCFG $G_\cap$ such that

$$T(G_\cap) \quad = \quad T(G) \cap (L(M_1) \times L(M_2))$$

- Parse trees are also preserved (modulo node relabeling)
- $G_\cap$ is called the **intersection** SCFG

Intersection construction (cont'd)

- $G_\cap$ has nonterminals of the form

$$(q_1, A, q_2)$$

for q_1, q_2 states of the source FAs and A a nonterminal of the source SCFG

- $G_\cap$ has productions of the form

$$\begin{array}{rcl}[(q_{10}, A_{10}, q_{1r}) & \rightarrow & (q_{10}, A_{11}, q_{11})^{(t_1)} \cdots (q_{1r-1}, A_{1r}, q_{1r})^{(t_r)}, \\ (q_{20}, A_{20}, q_{2r}) & \rightarrow & (q_{20}, A_{21}, q_{21})^{(t_{\pi(1)})} \cdots (q_{2r-1}, A_{2r}, q_{2r})^{(t_{\pi(r)})}]\end{array}$$

Translation algorithm

- Input: SCFG G, string w
- Algorithm:
  - construct M_1 such that $L(M_1) = \{w\}$
  - construct M_2 such that $L(M_2) = V_T^*$
  - construct $G_\cap$ by intersection of G with M_1 and M_2
  - output parse forest (CFG) auto-proj$(G_\cap, 2)$

Parsing algorithm

- Input: SCFG G, strings u, v
- Algorithm:
  - construct M_1 such that $L(M_1) = \{u\}$
  - construct M_2 such that $L(M_2) = \{v\}$
  - construct $G_\cap$ by intersection of G with M_1 and M_2
  - output parse forests (CFG) auto-proj$(G_\cap, 1)$, auto-proj$(G_\cap, 2)$ and synchronous parse forest (SCFG) $G_\cap$

Computational analysis

- Parameters:
  - SCFG G with maximum right-hand side length r, called **rank**
  - FA M_1 with states Q_1 and transitions δ_1
  - FA M_2 with states Q_2 and transitions δ_2
- Auto-projection can be constructed in time $\mathcal{O}(|G|)$
- In the worst case, construction of intersection grammar takes time

$$\Theta(|G| \cdot (|Q_1|^{r+1} + |\delta_1|) \cdot (|Q_2|^{r+1} + |\delta_2|))$$

Applications

- One of the very first translation algorithms has been proposed in [Wu and Wong 1998] for Stochastic Inversion Transduction Grammars (SITG)
- Translates an English sentence w into Chinese, using a filtering 2-gram language model for target language
- Algorithm runs in time $\mathcal{O}(|w|^7)$ (grammar size ignored here)
- Improved to $\mathcal{O}(|w|^6)$ in [Huang et al. 2005]

Application (cont'd)

- We can provide a very simple account of previous upper bound within our framework
  - SITG have rank $r = 2$
  - M_1 encodes w in $|w| + 1$ states
  - M_2 encodes Chinese 2-gram model in $\mathcal{O}(|w|)$ states; this is restricted to Chinese words that are image of English words in w
- Intersection algorithm then runs in time

$$\mathcal{O}(|Q_1|^{r+1} \cdot |Q_2|^{r+1}) \quad = \quad \mathcal{O}(|w|^6)$$

Application (cont'd)

- We can provide a similar polynomial time upper bound for Head Transducer Grammars [Alshawi et al. 2000]
- Polynomial time also holds if
  - SCFG is fixed; or else
  - there is a constant upper bound on the rank of the SCFG
- Otherwise, intersection construction runs in **exponential** time in the size of the input

Rank

- **Result**: SCFGs do not admit canonical forms with bounded rank [Aho and Ullman 1969] (contrast with Chomsky normal form for CFGs)
- Higher rank (flat structure) used when language pair does not satisfy **direct correspondence assumption** [Hwa et al. 2002]
- Question : Is constant upper bound on rank a plausible hypothesis for natural language translation?
  - If you need unbounded rank, your translation relation may be out of the reach of CFG analysis (scrambling, etc.)

Rank (cont'd)

- Synchronous productions that cannot be reduced in rank implement so-called **simple** permutations
- Percentage of the $r!$ permutations that are simple approaches e^{-2} [Albert et al. 2003]
- How many simple permutations are observed in real data?
- **Result**: One can decompose a rank r synchronous production into smallest rank components in time $\mathcal{O}(|r|)$ [Gildea et al. 2006]
- Above algorithm can also be used to decide whether a permutation is simple

Parsing

- **Result**: Parsing problem for SCFGs is NP-hard [Satta and Peserico 2005]
- Proof: Reduction from 3SAT; complexity comes from complex permutations
- Result transfers to translation models in [Yamada and Knight 2001], [Gildea 2003], [Melamed 2003]

Translation

- **String-to-tree** (1-best) translation problem :
  - Input a probabilistic SCFG G and a string w
  - Output the parse tree with highest probability that translates w

$$\underset{t}{argmax}\ p_G([w,\ t])$$

- **Result**: String-to-tree problem is NP-hard [Satta and Peserico 2005]
- Proof: Reduction from the consensus problem [Casacuberta and de la Higuera 2000]; complexity comes from hidden layer of source parse trees

Translation (cont'd

- String-to-tree problem remains hard even in case of constant upper bound on rank of SCFG
- Becomes polynomial time if paired nonterminals are always equal
  - Algorithm: Intersection construction + Viterbi search on right auto-projection
- Becomes undecidable if infinite ambiguity is allowed, even for a fixed SCFG !!

Parsing

- Parsing problem for SCFGs usually solved through tabular methods (chart parsing)
- if we parse left-to-right on the source sentence, we end up with **discontinuous constituents** on the target sentence
- Discontinuous constituents (multiple edges) increase the time complexity of the parser
- Are there better strategies for tabular methods?

Parsing (cont'd)

- In the worst case, tabular methods require time

$$\Theta(|G| \cdot |w|^{k(G)})$$

- We know that, unless P $=$ NP, $k(G)$ cannot be a constant
- **Result**: In the worst case, standard tabular methods for the SCFG parsing problem require an amount of time $\Omega(|G|\, n^{c \cdot \sqrt{r}})$, with r the rank of G and c some constant [Satta and Peserico 2005]
- Proof: combinatorial argument

Conclusions

- All hardness and lower bound results exploit constructions that are quite artificial
- If unbounded rank is needed, then the translation is probably out of the reach of CFG analysis
- Efficient algorithms exist for reducing rank to a minimum (expected low)
- Intersection construction extends to
  - specialized and efficient parsing strategies
  - estimation algorithm based on frequency count of synchronous productions

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