Structure and dynamics of multiplex networks

Federico Battiston

School of Mathematical Sciences, Queen Mary University of London, UK
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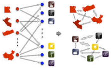
Center for Network Science @ Central European University - February 13, 2017 - Budapest, Hungary

A very short presentation...

ROME

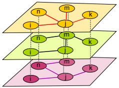
SAPIENZA
UNIVERSITÀ DI ROMA



Statistical mechanics / economic complexity

LONDON

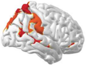
Queen Mary
University of London



Multi-layer networks

PARIS

ARAMIS LAB
BRAIN DATA SCIENCE

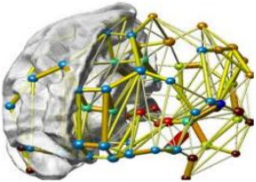


ICM
Institut du Cerveau
et de la Moelle épinière

Brain networks

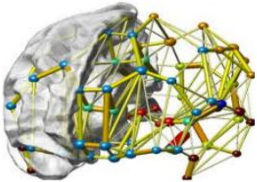
Towards richer architectures: multiplex networks

Many systems, one framework



adjacency matrix $A = \{a_{ij}\}$

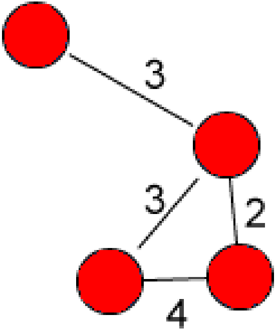
Many systems, one framework



adjacency matrix $A = \{a_{ij}\}$

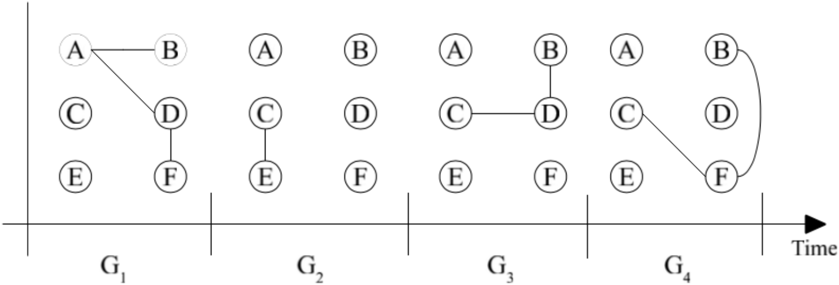
node degree $k_i = \sum_j a_{ij}$

Towards a richer architecture: weighted networks



Weighted adjacency matrix $W = \{w_{ij}\}$

Weights are used to represent strength, distance, cost, time, ...



Temporal networks: connections can change over time

General formalism for multiplex networks

A multiplex is a system whose basic units are connected through a variety of different relationships. Links of different kind are embedded in different layers.

- Node index $i = 1, \ldots, N$
- Layer index $\alpha = 1, \ldots, M$

General formalism for multiplex networks

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- Layer index $\alpha = 1, \ldots, M$

For each layer α:

- adjacency matrix $A^{[\alpha]} = \{a_{ij}^{[\alpha]}\}$
- node degree $k_i^{[\alpha]} = \sum_j a_{ij}^{[\alpha]}$

General formalism for multiplex networks

A multiplex is a system whose basic units are connected through a variety of different relationships. Links of different kind are embedded in different layers.

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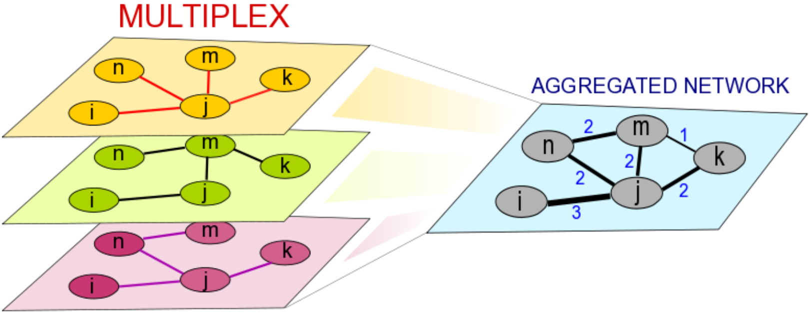
For the multiplex:

- vector of adjacency matrices $\mathbf{A} = \{A^{[1]}, ..., A^{[M]}\}$.
- vector of degrees $\mathbf{k}_i = (k_i^{[1]}, ..., k_i^{[M]})$.

Do we really need to preserve all this information?.

What are we losing collapsing all the information into a single network?



Two early reviews...

Journal of Complex Networks Advance Access published July 14, 2014

Journal of Complex Networks (2014) Page 1 of 69
doi:10.1093/comnet/cnu016

Multilayer networks

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Physics Reports 544 (2014) 1–122

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

The structure and dynamics of multilayer networks

S. Boccaletti[a,b,*], G. Bianconi[c], R. Criado[d,e], C.I. del Genio[f,g,h], J. Gómez-Gardeñes[i], M. Romance[d,e], I. Sendiña-Nadal[j,e], Z. Wang[k,l], M. Zanin[m,n]

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… and more recent material

BOOKS

INTRODUCTIONS

Eur. Phys. J. B (2015) 88: 48
DOI: 10.1140/epjb/e2015-50742-1
THE EUROPEAN PHYSICAL JOURNAL B
Colloquium

Towards real-world complexity: an introduction to multiplex networks

The new challenges of multiplex networks: measures and models

Federico Battiston,[1] Vincenzo Nicosia,[1] and Vito Latora[1,2]
[1] *School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom*
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(Dated: September 15, 2016)

Spreading processes in Multilayer Networks

Mostafa Salehi, Rajesh Sharma, Moreno Marzolla, Matteo Magnani, Payam Siyari, Danilo Montesi

The European Physical Journal B
May 2015, 88:124

Evolutionary games on multilayer networks: a colloquium

Authors | Authors and affiliations
Zhen Wang, Lin Wang, Attila Szolnoki, Matjaž Perc

ESS ARTICLE

The physics of spreading processes in multilayer networks

Manlio De Domenico, Clara Granell, Mason A. Porter & Alex Arenas
Affiliations | Corresponding authors

Nature Physics **12**, 901–906 (2016) | doi:10.1038/nphys3865
Received 06 April 2016 | Accepted 22 July 2016 | Published online 22 August 2016

THEMATIC REVIEWS

MULTIPLEX NETWORKS

STRUCTURE

Basic measures

Motif analysis

Community structure

Core-periphery structure

DYNAMICS

Random walks

Opinion formation

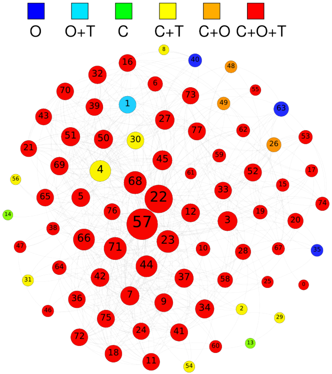
Cultural dynamics

Evolutionary game theory

Structure of multiplex networks

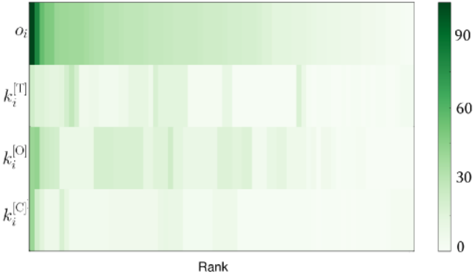
The multi-layer network of Indonesian terrorists

LAYER	CODE	N	K
MULTIPLEX	M	78	911
Trust	T	70	259
Operations	O	68	437
Communications	C	74	200
Businness	B	13	15



A layer-by-layer exploration of node properties: the case of the degree distribution.

overlapping degree: $o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}$



Different layers show different patterns.

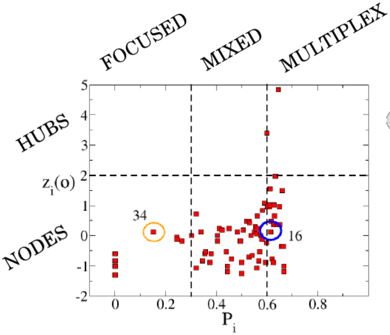
Basic node properties: cartography of a multiplex

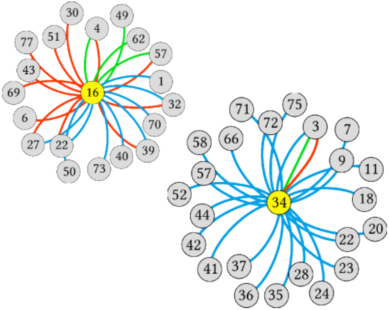
Z-score of the overlapping degree: $z_i(o) = \frac{o_i - <o>}{\sigma_o}$ $o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}$

1. Simple nodes $-2 \leq z_i(o) \leq 2$
2. Hubs $z_i(o) > 2$

Participation coefficient: $P_i = \frac{M}{M-1}\left[1 - \sum_{\alpha=1}^{M}\left(\frac{k_i^{[\alpha]}}{o_i}\right)^2\right]$

1. Focused nodes $0 \leq P_i \leq 1/3$
2. Mixed-pattern nodes $1/3 < P_i \leq 2/3$
3. Truly multiplex nodes $2/3 < P_i \leq 1$





Multiplex analysis successfully distinguishes node 16 from node 34.

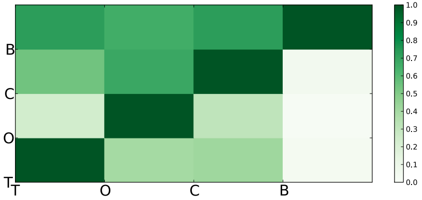
F. Battiston, V. Nicosia, V. Latora (2014)

Edge overlap

o_{ij}	Percentage of edges (%)
1	46
2	27
3	23
4	4

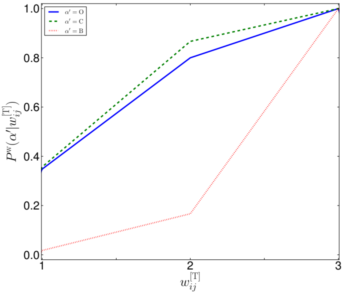
Conditional probability to have overlap:

$$P(a_{ij}^{[\alpha']}|a_{ij}^{[\alpha]}) = \frac{\sum_{ij} a_{ij}^{[\alpha']} a_{ij}^{[\alpha]}}{\sum_{ij} a_{ij}^{[\alpha]}} \quad (1)$$

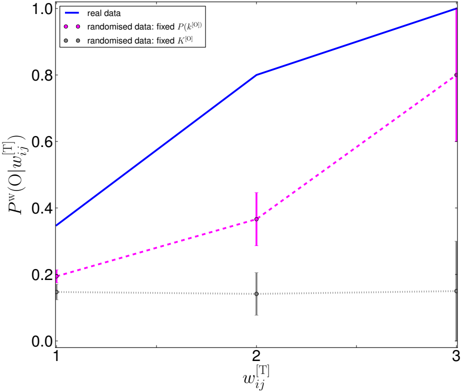


Edge overlap and social reinforcement

$$P(a_{ij}^{[\alpha']}|a_{ij}^{[\alpha]}) \to P^{\mathrm{w}}(a_{ij}^{[\alpha']}|w_{ij}^{[\alpha]})$$

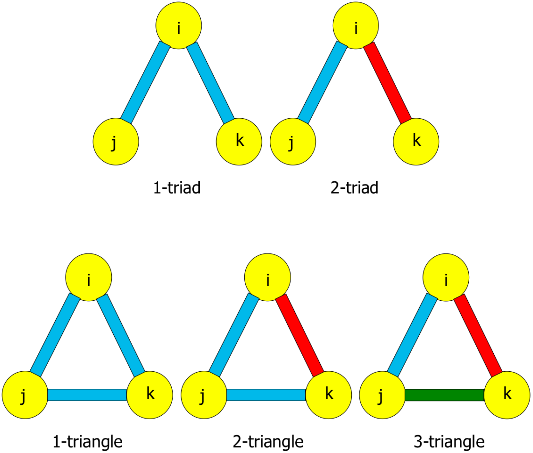


The existence of strong connections in the Trust layer, which represents the strongest relationships between two people, actually fosters the creation of links in other layers.



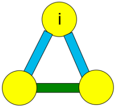
Social reinforcement obtained in real data can not simply be explained by inter-layer degree-degree correlation.

Triads and triangles



F. Battiston, V. Nicosia, V. Latora (2014)

$C_{i,1}$ =

of  centered in i

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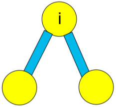
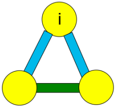
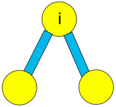
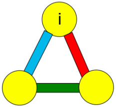
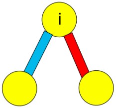
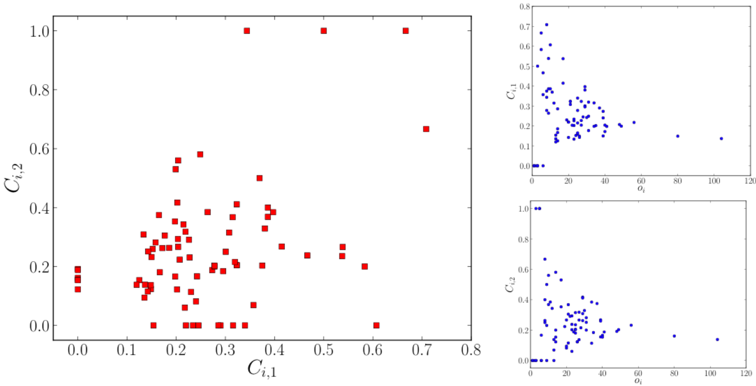
of  centered in i

Clustering

$C_{i,1} =$ (# of  centered in i) / (# of  centered in i)

$C_{i,2} =$ (# of  centered in i) / (# of  centered in i)

$C_{i,1}$ and $C_{i,2}$ show different patterns of multi-clustering and are not correlated with o_i.



Our **structural measures** for multiplex networks have been used in **different disciplines**

MEDICINE

Briefings in Bioinformatics, 17(3), 2016, 527–540

doi: 10.1093/bib/bbv062
Advance Access Publication Date: 24 August 2015
Paper

Systems medicine of inflammaging

Gastone C. Castellani*, Giulia Menichetti*, Paolo Garagnani, Maria Giulia Bacalini, Chiara Pirazzini, Claudio Franceschi, Sebastiano Collino, Claudia Sala, Daniel Remondini, Enrico Giampieri, Ettore Mosca, Matteo Bersanelli, Silvia Vitali, Italo Faria do Valle, Pietro Liò and Luciano Milanesi

ECOLOGY

Recommend Journal of Complex Networks to your librarian

Parasite spreading in spatial ecological multiplex networks

Massimo Stella†
†Corresponding author. Email: massimo.stella@inbox.com
Cecilia S. Andreazzi
Sanja Selakovic
Alireza Goudarzi
Alberto Antonioni

NEUROSCIENCE

Loss of inter-frequency brain hubs in Alzheimer's disease

J. Guillon[b,a], Y. Attal[c], O. Colliot[b,a], V. La Corte[b], B. Dubois[d], D. Schwartz[b], M. Chavez[b,a], F. De Vico Fallani[a,b,*]

SOCIOLOGY

Information Sciences
Volume 351, 10 July 2016, Pages 30–47

Network structure and resilience of Mafia syndicates

Santa Agreste[a], Salvatore Catanese[a,b], Pasquale De Meo[c], Emilio Ferrara[d], Giacomo Fiumara[a]
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ENERGY

Applied Energy
Volume 156, 15 October 2015, Pages 542–554

Features and evolution of international fossil energy trade relationships: A weighted multilayer network analysis

Cuixia Gao[a], Mei Sun[a], Bo Shen[b]

MOBILITY

Chapter
Interconnected Networks
Part of the series Understanding Complex Systems pp 149-164

Date: 05 February 2016

Several Multiplexes in the Same City: The Role of Socioeconomic Differences in Urban Mobility

Laura Lotero, Alessio Cardillo, Rafael Hurtado, Jesús Gómez-Gardeñes

Interconnected Networks
Look Inside

ECONOMICS

Physica A: Statistical Mechanics and its Applications
Volume 467, 1 February 2017, Pages 380–394

Measuring the dissimilarity of multiplex networks: An empirical study of international trade networks

Xiaohang Zhang[a, b], Huiyuan Cui[a], Ji Zhu[c], Yu Du[a], Qi Wang[a], Wenhua Shi[a]
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SYSTEMIC RISK

Risk journals

Research Paper

A multiplex network analysis of the Mexican banking system: link persistence, overlap and waiting times

José-Luis Molina-Borboa[1], Serafin Martínez-Jaramillo[1], Fabrizio López-Gallo[1] and Marco van der Leij[2,3,4]

EPIDEMIOLOGY

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RESEARCH ARTICLE

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Rossana Mastrandrea, Alain Barrat

LINGUISTIC

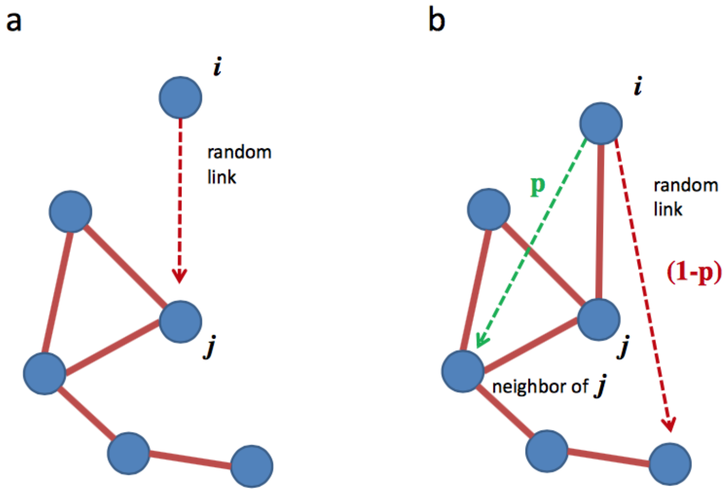
Mental Lexicon Growth Modelling Reveals the Multiplexity of the English Language

Massimo Stella* and Markus Brede

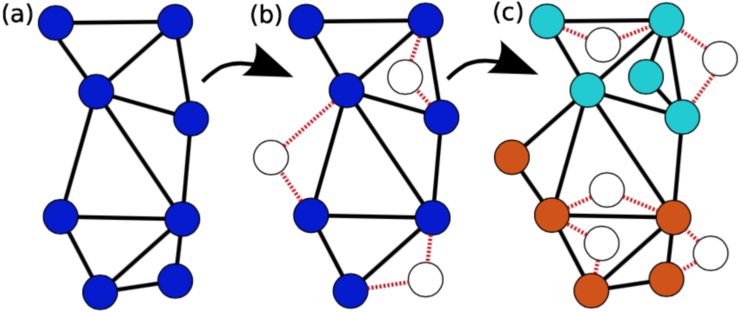
Not only local node and edge properties!

Real-world systems are characterised by non-trivial structures at the micro- and the meso-scale, such as motifs, communities and cores

At each time step a new node attaches with 2 links:
a) the first link is at random
b) the second link closes a triangle with probability p



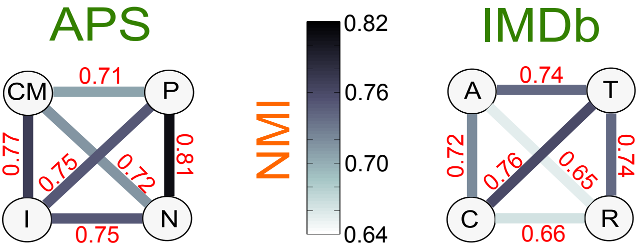
Communities and triadic closure



G. Bianconi et al., *Physical Review E* (2014)

APS: Particle (P), Nuclear (N), Condensed Matter (CM) and Interdisciplinary (I) physics

IMDb: Action (A), Crime (C), Thriller (T) and Romance (R) genres



Different layers may have more or less similar community structure

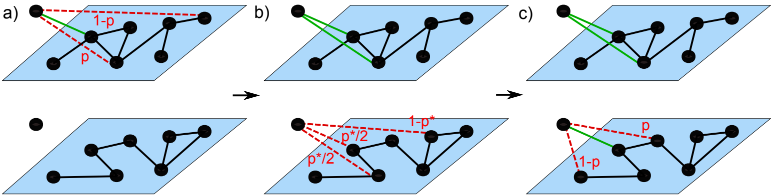
APS: Particle (P), Nuclear (N), Condensed Matter (CM) and Interdisciplinary (I) physics

IMDb: Action (A), Crime (C), Thriller (T) and Romance (R) genres

$$NMI(\mathcal{P}_\alpha, \mathcal{P}_\beta) = \frac{-2 \sum_{m=1}^{M_\alpha} \sum_{m'=1}^{M_\beta} N_{mm'} \log \left(\frac{N_{mm'} N}{N_m N_{m'}} \right)}{\sum_{m=1}^{M_\alpha} N_m \log \left(\frac{N_m}{N} \right) + \sum_{m'=1}^{M_\beta} N_{m'} \log \left(\frac{N_{m'}}{N} \right)}$$

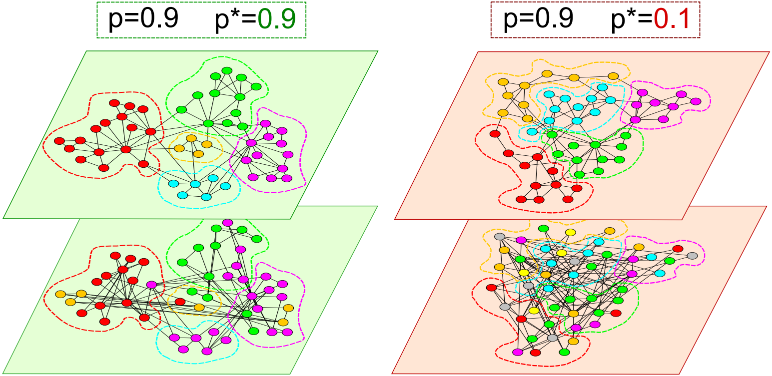
L. Danon et al., *Journal of Statistical Mechanics: Theory and Applications* (2015)

Real mechanisms by which collaborations grow:
1) 'intra-layer' triadic closure (with prob. p)
2) 'inter-layer' proximity bias (with prob. p^*)

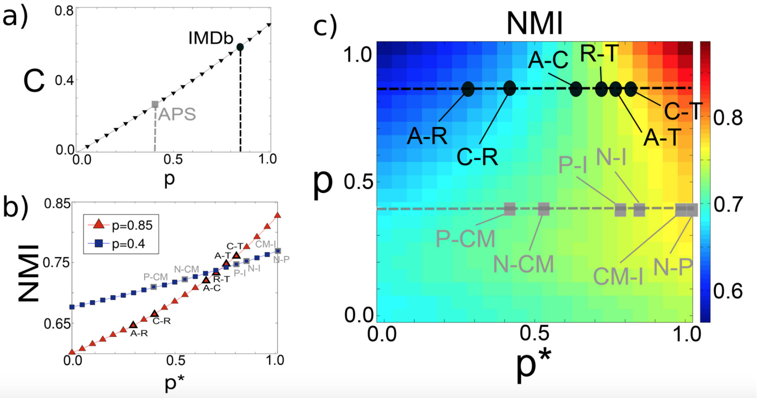


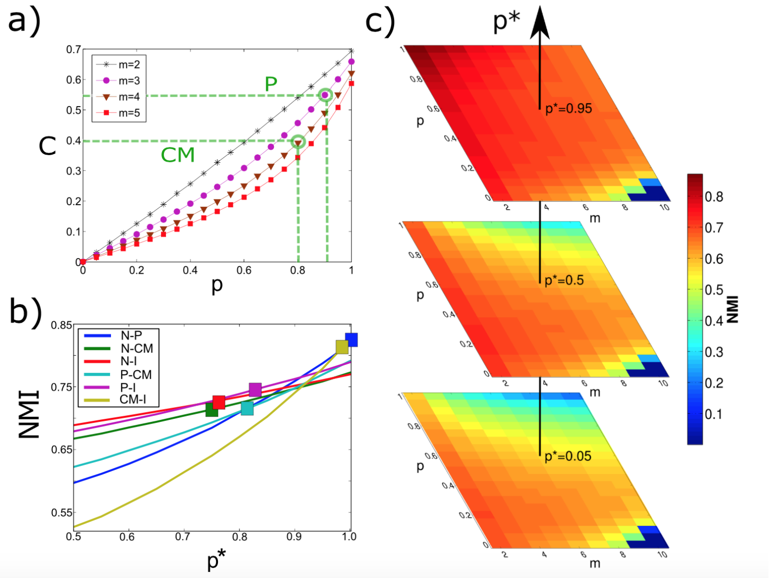
F. Battiston, J. Iacovacci et al., (2016)

Growing models for multiplexes with communities



By tuning the strength of the 'inter-layer' proximity bias mechanism we can obtain similar ($p^* = 0.9$) or different ($p^* = 0.1$) community structures

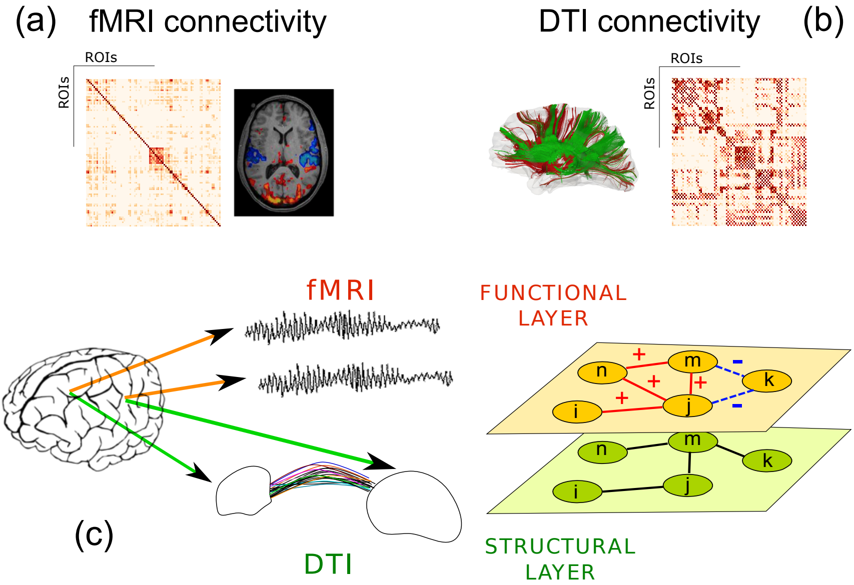

a)
C
0.8
0.4
0.0
IMDb
APS
0.0
0.5
1.0
p
b)
NMI
0.85
0.75
0.65
p=0.85
p=0.4
C-T
CM-I
A-T
N-P
N-CM
P-CM
P-I
N-I
R-T
A-C
C-R
A-R
0.0
0.5
1.0
p*
c)
NMI
1.0
0.5
0.0
p
A-C
R-T
C-T
A-R
C-R
A-T
P-I
N-I
P-CM
N-CM
CM-I
N-P
0.8
0.7
0.6
0.0
0.5
1.0
p*

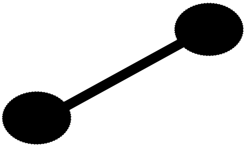

a)
C
p
P
CM
m=2
m=3
m=4
m=5
b)
NMI
p*
N-P
N-CM
N-I
P-CM
P-I
CM-I
c)
p*
p*=0.95
p*=0.5
p*=0.05
p
m
NMI

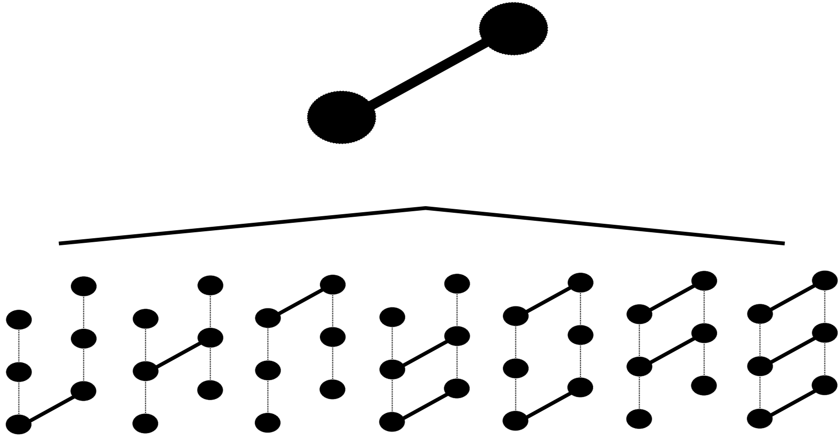
Why the **brain**?

Because it can be naturally mapped in a multiplex network

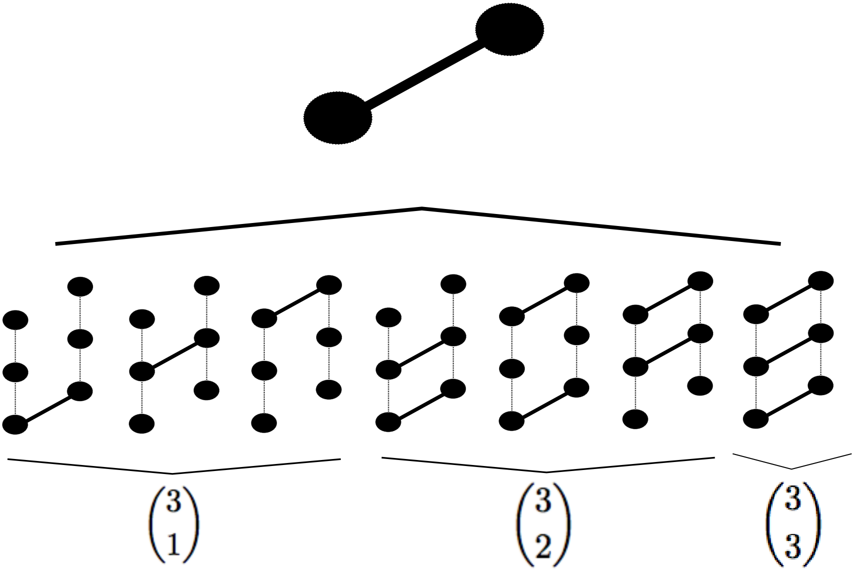
$N = 264$ Regions of Interests (ROIs) - Nodes
Different types of links

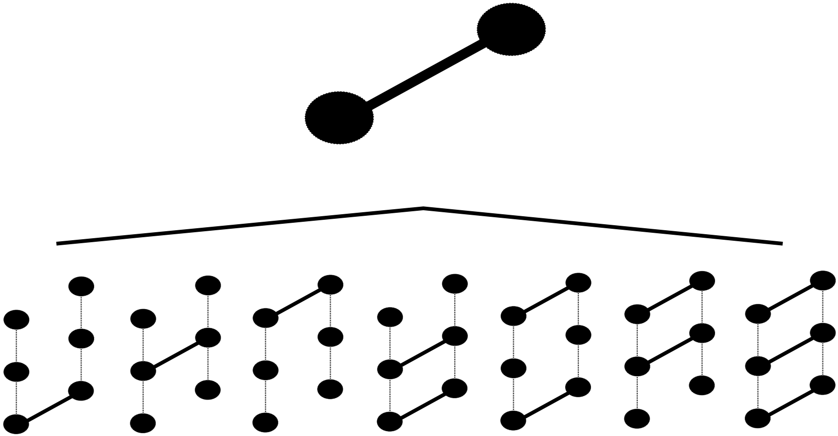

(a)
fMRI connectivity
DTI connectivity
(b)
ROIs
ROIs
ROIs
ROIs
fMRI
FUNCTIONAL
LAYER
+
+
+
+
-
-
n
m
k
i
j
n
m
k
i
j
(c)
DTI
STRUCTURAL
LAYER



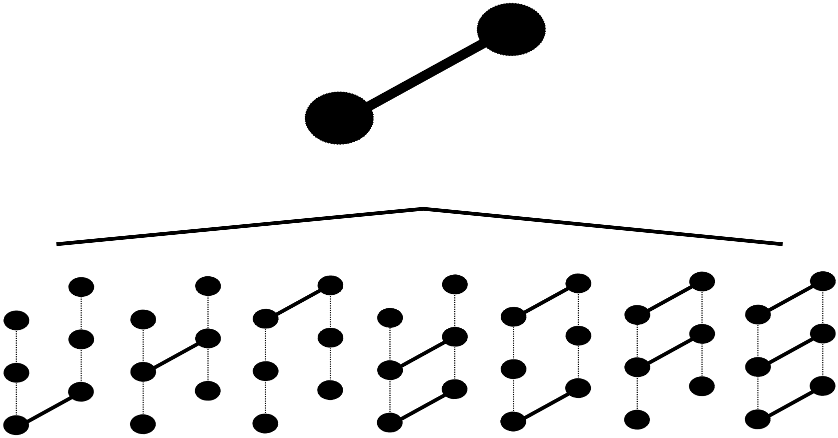


The concept of multilink was introduced by G. Bianconi in *Statistical mechanics of multiplex networks: entropy and overlap*, PRE, 2013

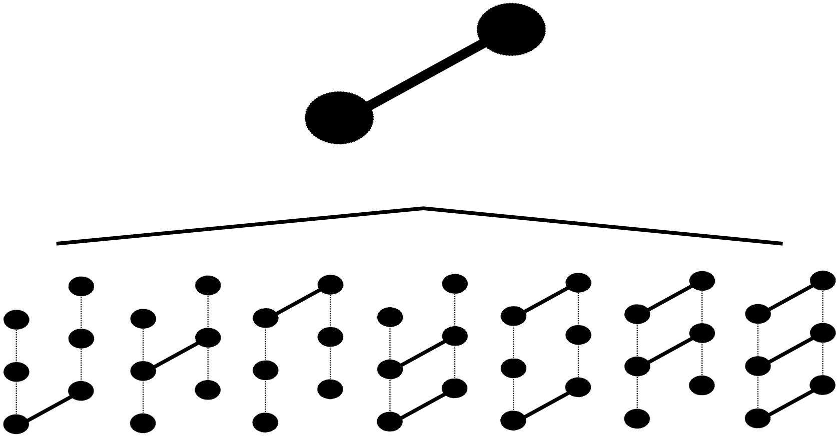

$\binom{3}{1}$
$\binom{3}{2}$
$\binom{3}{3}$



$$c = \sum_{m=1}^{M} \binom{M}{m} = 2^M - 1,$$



$$c = \sum_{m=1}^{M} g^m \binom{M}{m} = (g+1)^M - 1$$



$$c = \left[\prod_{\alpha=1}^{M} (g^{[\alpha]} + 1) \right] - 1$$

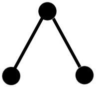
We suggest to categorise motifs at three different levels:

- first level: number of nodes
- second level: pattern on the aggregated network
- third level: specific multi-layer connectivity

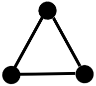
F. Battiston, V. Nicosia, M. Chavez, V. Latora, *Multilayer motifs analysis of brain networks*, arXiv: 1606.09115

Related work on *Isomorphisms on multi-layer networks* by M. Kivela and M. Porter, arXiv:1506.00508

3-node subgraphs

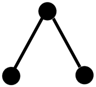


$$\binom{c+1}{2}$$

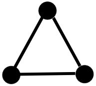


$$\binom{c+2}{3}$$

3-node subgraphs

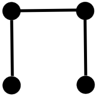


$$\binom{c+1}{2}$$

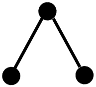


$$\binom{c+2}{3}$$

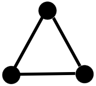
4-node subgraphs



3-node subgraphs

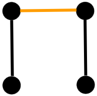


$\binom{c+1}{2}$



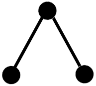
$\binom{c+2}{3}$

4-node subgraphs

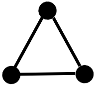


$c\binom{c+1}{2}$

3-node subgraphs

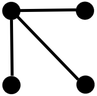


$$\binom{c+1}{2}$$

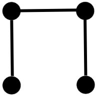


$$\binom{c+2}{3}$$

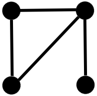
4-node subgraphs



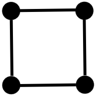
$$\binom{c+2}{3}$$



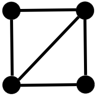
$$c\binom{c+1}{2}$$



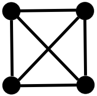
$$c\binom{c+2}{3}$$



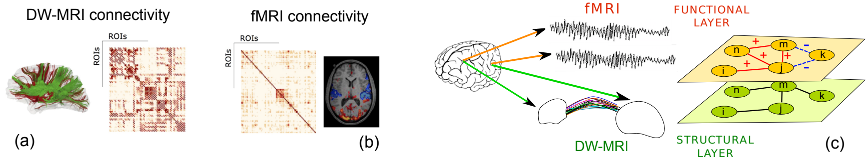
$$\binom{\binom{c+1}{2}+1}{2}$$

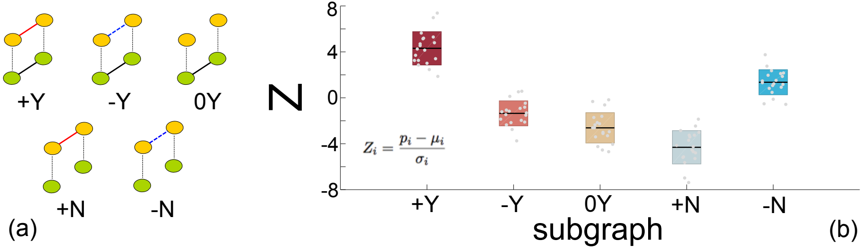


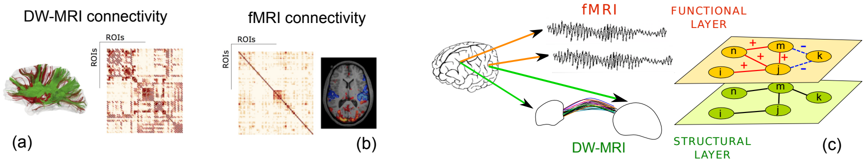
$$c\binom{\binom{c+1}{2}+1}{2}$$

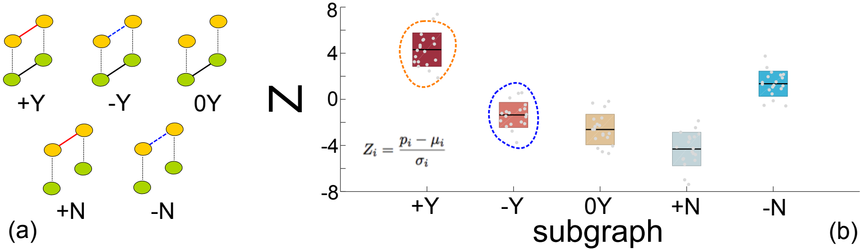


$$\binom{\binom{c+1}{2}+2}{3}$$


DW-MRI connectivity
ROIs
ROIs
(a)
fMRI connectivity
ROIs
ROIs
(b)
fMRI
FUNCTIONAL LAYER
DW-MRI
STRUCTURAL LAYER
n
m
k
i
j
(c)

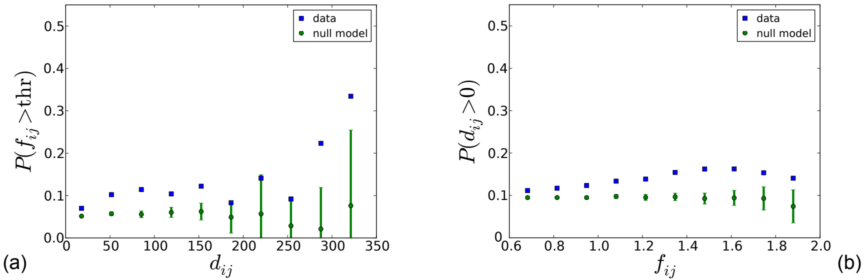

+Y
-Y
0Y
+N
-N
(a)
Z
8
4
0
-4
-8
$Z_i = \frac{p_i - \mu_i}{\sigma_i}$
+Y
-Y
0Y
+N
-N
subgraph
(b)





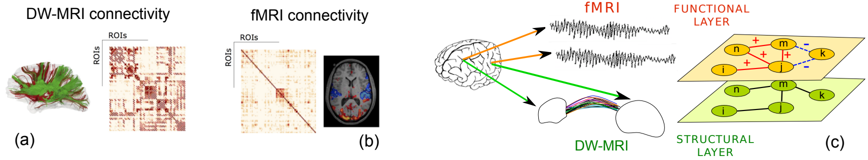
Positive functional connectivity is linked to structural connectivity

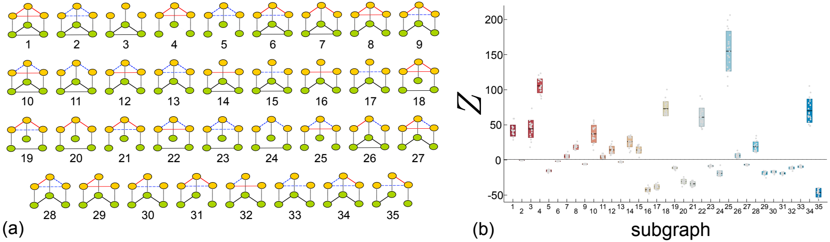
Negative functional connectivity is at random



Reinforcement mechanisms in the structural-functional relationships in the brain

Measure introduced by F. Battiston, V. Nicosia, V. Latora in *Structural measures for multiplex networks*, PRE 2014


DW-MRI connectivity
ROIs
ROIs
(a)
fMRI connectivity
ROIs
ROIs
(b)
fMRI
FUNCTIONAL
LAYER
DW-MRI
STRUCTURAL
LAYER
(c)


(a)
(b)
Z
subgraph

The new challenges of multiplex networks: measures and models

Federico Battiston,[1] Vincenzo Nicosia,[1] and Vito Latora[1,2]

[1]*School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom*
[2]*Dipartimento di Fisica ed Astronomia, Università di Catania and INFN, I-95123 Catania, Italy*

(Dated: June 30, 2016)

What do societies, the Internet, and the human brain have in common? The immediate answer might be "not that much", but in reality they are all examples of complex relational systems, whose emerging behaviours are largely determined by the non-trivial networks of interactions among their constituents, namely individuals, computers, or neurons. In the last two decades, network scientists have proposed models of increasing complexity to better understand real-world systems. Only recently we have realised that multiplexity, i.e. the coexistence of several types of interactions among the constituents of a complex system, is responsible for substantial qualitative and quantitative differences in the type and variety of behaviours that a complex system can exhibit. As a consequence, multilayer and multiplex networks have become a hot topic in complexity science. Here we provide an overview of some of the measures proposed so far to characterise the structure of multiplex networks, and a selection of models aiming at reproducing those structural properties and at quantifying their statistical significance. Focusing on a subset of relevant topics, this brief review is a quite comprehensive introduction to the most basic tools for the analysis of multiplex networks observed in the real-world. The wide applicability of multiplex networks as a framework to model complex systems in different fields, from biology to social sciences, and the colloquial tone of the paper will make it an interesting read for researchers working on both theoretical and experimental analysis of networked systems.

Our handbook for the analysis of multiplex network datasets [EMPIRICAL FOCUS]: measures of multiplexity, models to reproduce empirical findings and to assess their statistical significance (EPJST 2017)

README.md

mammult

Metrics And Models for MULTilayer networks

MAMMULT is a collection of programs for the analysis and modelling of multi-layer networks. Most of the code included in this package was developed during the EU FET project "LASAGNE".

Our open-source software library for the analysis of multiplex networks on GitHub
V. Nicosia & F. Battiston

Still in the making of...

Dynamics of multiplex networks

DIFFUSION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Diffusion Dynamics on Multiplex Networks

S. Gómez, A. Díaz-Guilera, J. Gómez-Gardeñes, C. J. Pérez-Vicente, Y. Moreno, and A. Arenas
Phys. Rev. Lett. **110**, 028701 – Published 8 January 2013

Multiplex networks can be superdiffusive
the time scale associated to the whole system is smaller than that of the single layers considered independently

DIFFUSION


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Nikos E. Kouvaris, Shigefumi Hata & Albert Díaz- Guilera

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