PEARSON EDEXCEL INTERNATIONAL A LEVEL

DECISION MATHEMATICS 1

Student Book

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ABOUT THIS BOOK

The following three themes have been fully integrated throughout the Pearson Edexcel International Advanced Level in Mathematics series, so they can be applied alongside your learning.

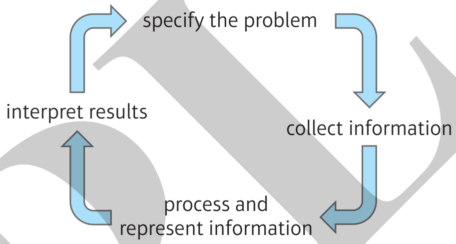
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols

2. Mathematical problem-solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Challenge questions provide extra stretch

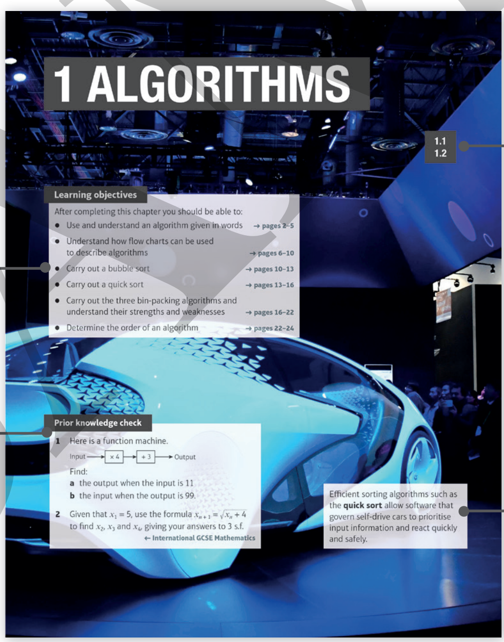
The Mathematical Problem-Solving Cycle



3. Transferable skills

- Transferable skills are embedded throughout this book, in the exercises and in some examples
- These skills are signposted to show students which skills they are using and developing

Finding your way around the book



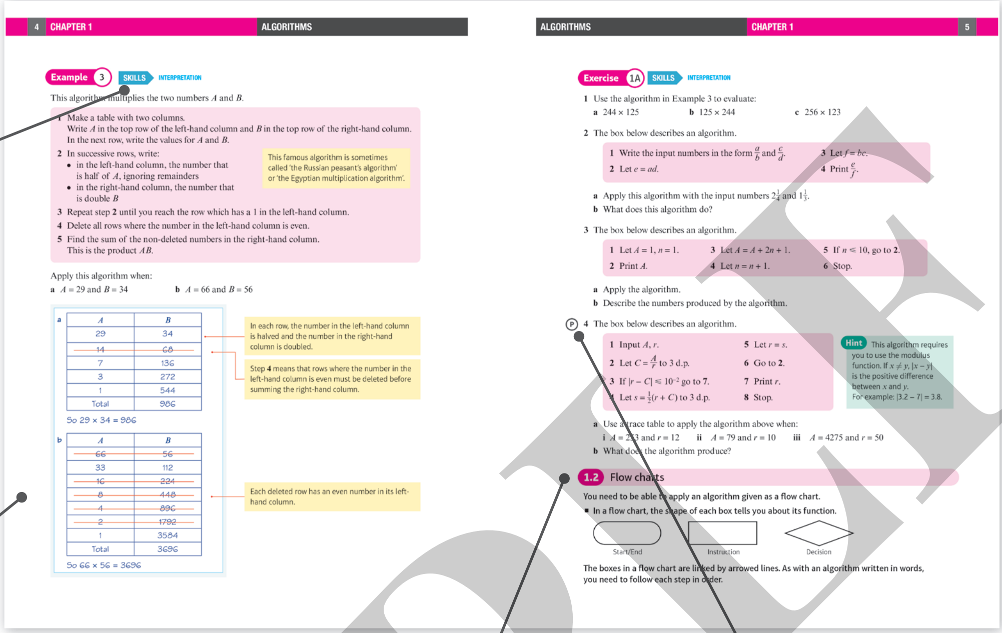
Each chapter starts with a list of *Learning objectives*

The *Prior knowledge check* helps make sure you are ready to start the chapter

Glossary terms will be identified by bold blue text on their first appearance.

Each chapter is mapped to the specification content for easy reference

The real world applications of the mathematics you are about to learn are highlighted at the start of the chapter.



Transferable skills are signposted where they naturally occur in the exercises and examples

Challenge boxes give you a chance to tackle some more difficult questions

Step-by-step worked examples focus on the key types of questions you'll need to tackle

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Each section begins with explanation and key learning points

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

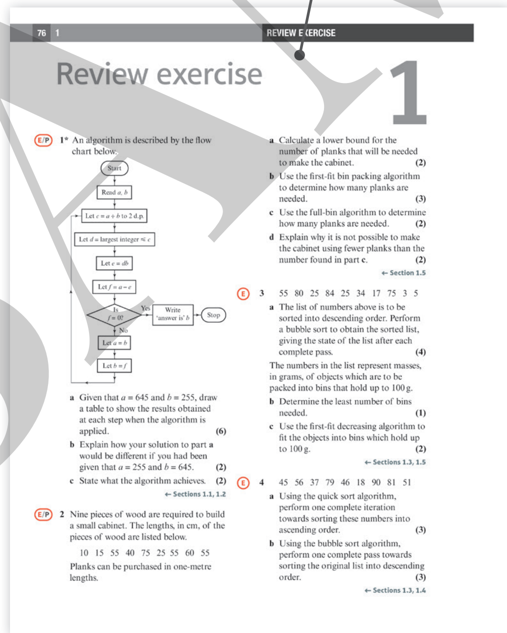
Exam-style questions are flagged with (E)

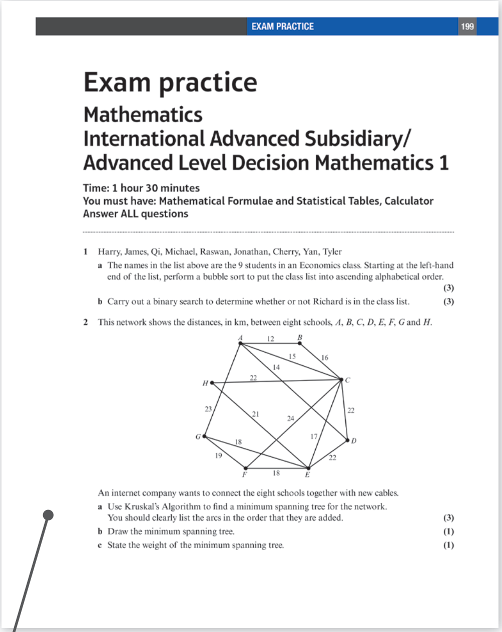
Problem-solving questions are flagged with (P)

Exercises are packed with exam-style questions to ensure you are ready for the exams

Each chapter ends with a *Chapter review* and a *Summary of key points*

After every few chapters, a *Review exercise* helps you consolidate your learning with lots of exam-style questions





A full practice paper at the back of the book helps you prepare for the real thing

QUALIFICATION AND ASSESSMENT OVERVIEW

Qualification and content overview

Decision Mathematics 1 (D1) is an **optional** unit in the following qualifications:

International Advanced Subsidiary in Mathematics

International Advanced Subsidiary in Further Mathematics

International Advanced Level in Mathematics

International Advanced Level in Further Mathematics

Assessment overview

The following table gives an overview of the assessment for this unit.

We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

Unit	Percentage	Mark	Time	Availability
D1: Decision Mathematics 1 Paper code WDM11/01	$33\frac{1}{3}$% of IAS $16\frac{2}{3}$% of IAL	75	1 hour 30 mins	January and June First assessment June 2019

IAS: International Advanced Subsidiary, IAL: International Advanced A Level.

Assessment objectives and weightings

		Minimum weighting in IAS and IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%
AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Relationship of assessment objectives to units

D1	Assessment objective				
	AO1	AO2	AO3	AO4	AO5
Marks out of 75	20–25	20–25	15–20	5–10	5–10
%	$26\frac{2}{3}$–$33\frac{1}{3}$	$26\frac{2}{3}$–$33\frac{1}{3}$	20–$26\frac{2}{3}$	$6\frac{2}{3}$–$13\frac{1}{3}$	$6\frac{2}{3}$–$13\frac{1}{3}$

Calculators

Students may use a calculator in assessments for these qualifications. Centres are responsible for making sure that calculators used by their students meet the requirements given in the table below.

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, ln x, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory.

Prohibitions

Calculators with any of the following facilities are prohibited in all examinations:

- databanks
- retrieval of text or formulae
- built-in symbolic algebra manipulations
- symbolic differentiation and/or integration
- language translators
- communication with other machines or the internet

Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides a full worked solution for questions in the book.
Download all the solutions as a PDF or quickly find the solution you need online.

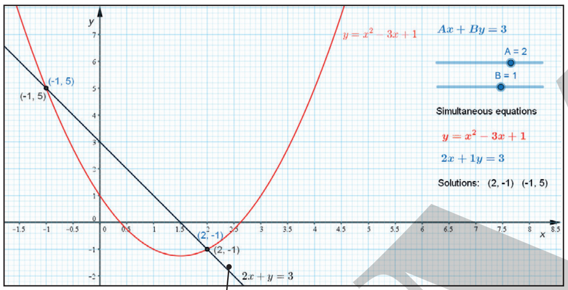
Use of technology

Explore topics in more detail, visualise problems and consolidate your understanding. Use pre-made GeoGebra activities or Casio resources for a graphic calculator.

Online Find the point of intersection graphically using technology.

GeoGebra

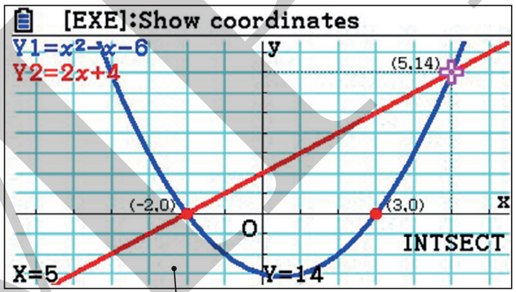
GeoGebra-powered interactives



Interact with the mathematics you are learning using GeoGebra's easy-to-use tools

CASIO

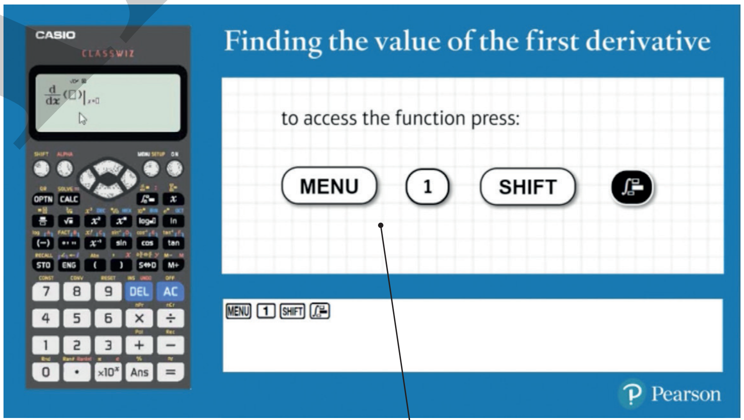
Graphic calculator interactives



Explore the mathematics you are learning and gain confidence in using a graphic calculator

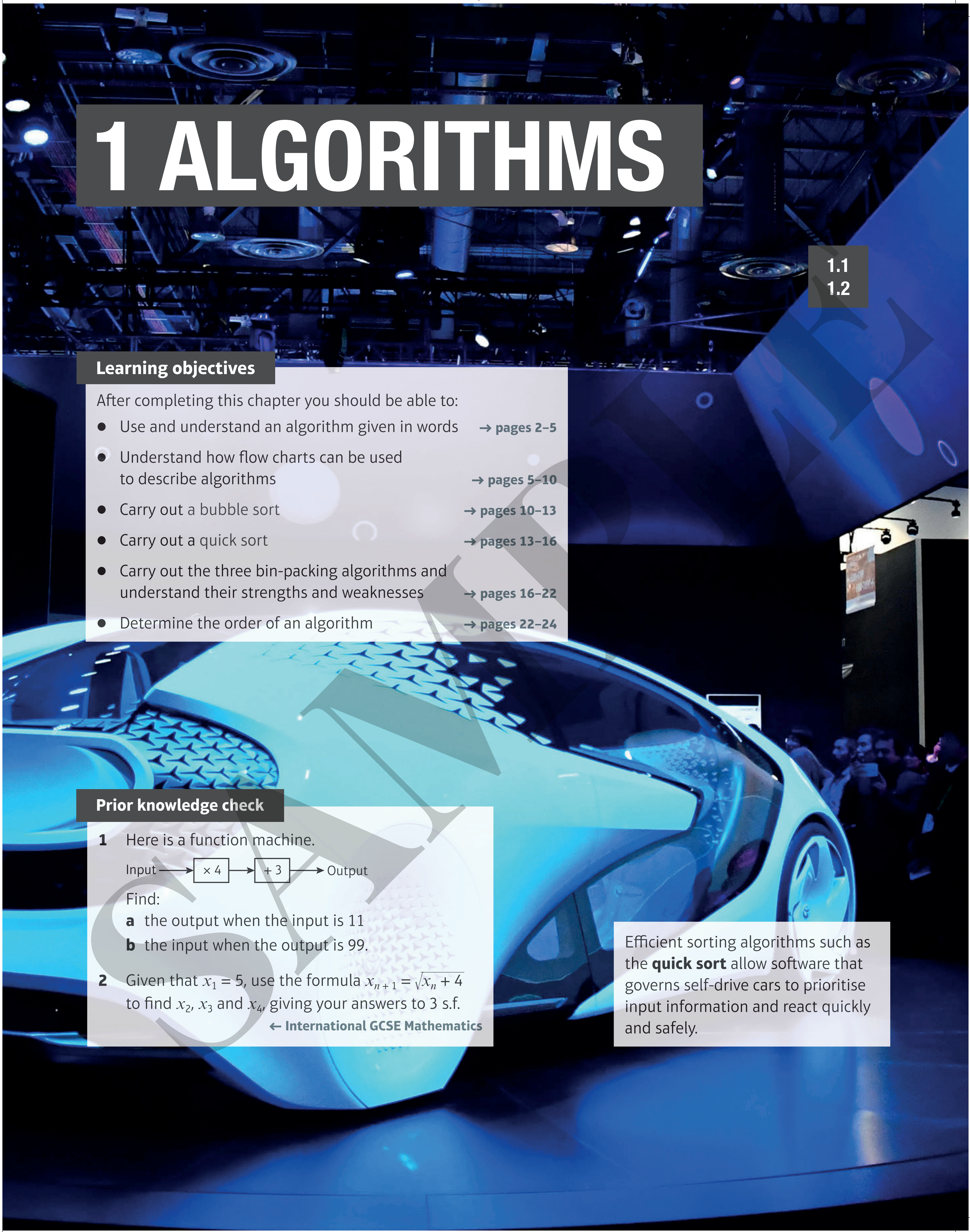
Calculator tutorials

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio's scientific and colour graphic calculators.



Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator's screen

Online Work out each coefficient quickly using the ${}^nC_r$ and power functions on your calculator.



1 ALGORITHMS

1.1
1.2

Learning objectives

After completing this chapter you should be able to:

- Use and understand an algorithm given in words → pages 2–5
- Understand how flow charts can be used to describe algorithms → pages 5–10
- Carry out a bubble sort → pages 10–13
- Carry out a quick sort → pages 13–16
- Carry out the three bin-packing algorithms and understand their strengths and weaknesses → pages 16–22
- Determine the order of an algorithm → pages 22–24

Prior knowledge check

1 Here is a function machine.

Input → × 4 → + 3 → Output

Find:

a the output when the input is 11

b the input when the output is 99.

2 Given that $x_1 = 5$, use the formula $x_{n+1} = \sqrt{x_n + 4}$ to find x_2, x_3 and x_4, giving your answers to 3 s.f.

← International GCSE Mathematics

Efficient sorting algorithms such as the **quick sort** allow software that governs self-drive cars to prioritise input information and react quickly and safely.

1.1 Using and understanding algorithms

- An **algorithm** is a **finite** sequence of step-by-step instructions carried out to solve a problem.

Algorithms can be given in words or in **flow charts**.

You need to be able to understand and use an algorithm given in words.

You have been using algorithms since you started school. Some examples of mathematical algorithms that you will be familiar with are:

- how to add several two-digit numbers
- how to multiply two two-digit numbers
- how to add, subtract, multiply or divide fractions.

It can be quite challenging to write a sequence of instructions for someone else to follow accurately.

Here are some more examples:

> At the end of the street turn right and go straight over the crossroads, take the third left after the school, then ...

> Affix base (B) to leg (A) using screw (F) and ...

> Dice two large onions.
> Slice 100 g mushrooms.
> Grate 100 g cheese.

Example 1 SKILLS ANALYSIS

A 'happy' number is defined by the algorithm:

- write down any **integer**
- square its digits and find the sum of the squares
- repeat until either the answer is 1 (in which case the number is 'happy') or until you get trapped in a **cycle** (in which case the number is 'unhappy')

Show that:

a 70 is happy **b** 4 is unhappy

Watch out You will need to be able to understand, describe and apply specific algorithms in your exam. You do not need to learn any of the algorithms in this section.

a $7^2 + 0^2 = 49$
$4^2 + 9^2 = 97$
$9^2 + 7^2 = 130$
$1^2 + 3^2 + 0^2 = 10$
$1^2 + 0^2 = 1$
so 70 is happy

b $4^2 = 16$
$1^2 + 6^2 = 37$
$3^2 + 7^2 = 58$
$5^2 + 8^2 = 89$
$8^2 + 9^2 = 145$
$1^2 + 4^2 + 5^2 = 42$
$4^2 + 2^2 = 20$
$2^2 + 0^2 = 4$
$4^2 = 16$
so 4 is unhappy

As soon as the sum of the squares matches a previous result, all of the steps in-between will be repeated, creating a cycle.

Example 2 SKILLS INTERPRETATION

a Apply this algorithm.

1 Let $n = 1$, $A = 1$, $B = 1$.

2 Print A and B.

3 Let $C = A + B$.

4 Print C.

5 Let $n = n + 1$, $A = B$, $B = C$.

6 If $n < 5$, go to step **3**.

7 If $n = 5$, stop.

These are not equations.
They are instructions that mean:
- replace n by $n + 1$ (add 1 to n)
- A takes B's current value
- B takes C's current value

b Describe the numbers that are generated by this algorithm.

a Use a trace table.

A **trace table** is used to record the values of each variable as the algorithm is run.

Step	n	A	B	C	Print
1	1	1	1		
2					1, 1
3				2	
4					2
5	2	1	2		
6	Go to step 3				
3				3	
4					3
5	3	2	3		
6	Go to step 3				
3				5	
4					5
5	4	3	5		
6	Go to step 3				
3				8	
4					8
5	5	5	8		
6	Continue to step 7				
7	Stop				

You may be asked to complete a printed trace table in your exam. Just obey each instruction, in order.

b This algorithm produces the first few numbers in the Fibonnacci sequence.

You may be asked what the algorithm does.

Example 3 SKILLS INTERPRETATION

This algorithm multiplies the two numbers *A* and *B*.

1 Make a table with two columns.
Write *A* in the top row of the left-hand column and *B* in the top row of the right-hand column.
In the next row, write the values for *A* and *B*.

2 In successive rows, write:
- in the left-hand column, the number that is half of *A*, ignoring remainders
- in the right-hand column, the number that is double *B*

3 Repeat step **2** until you reach the row which has a 1 in the left-hand column.

4 Delete all rows where the number in the left-hand column is even.

5 Find the sum of the non-deleted numbers in the right-hand column.
This is the product *AB*.

This famous algorithm is sometimes called 'the Russian peasant's algorithm' or 'the Egyptian multiplication algorithm'.

Apply this algorithm when:

a $A = 29$ and $B = 34$ **b** $A = 66$ and $B = 56$

a

A	*B*
29	34
~~14~~	~~68~~
7	136
3	272
1	544
Total	986

So 29 × 34 = 986

In each row, the number in the left-hand column is halved and the number in the right-hand column is doubled.

Step **4** means that rows where the number in the left-hand column is even must be deleted before summing the right-hand column.

b

A	*B*
~~66~~	~~56~~
33	112
~~16~~	~~224~~
~~8~~	~~448~~
~~4~~	~~896~~
~~2~~	~~1792~~
1	3584
Total	3696

So 66 × 56 = 3696

Each deleted row has an even number in its left-hand column.

Exercise 1A SKILLS INTERPRETATION

1 Use the algorithm in Example 3 to evaluate:

a 244×125 **b** 125×244 **c** 256×123

2 The box below describes an algorithm.

1 Write the input numbers in the form $\frac{a}{b}$ and $\frac{c}{d}$.	**3** Let $f = bc$.
2 Let $e = ad$.	**4** Print $\frac{e}{f}$.

a Apply this algorithm with the input numbers $2\frac{1}{4}$ and $1\frac{1}{3}$.

b What does this algorithm do?

3 The box below describes an algorithm.

1 Let $A = 1$, $n = 1$.	**3** Let $A = A + 2n + 1$.	**5** If $n \leqslant 10$, go to **2**.
2 Print A.	**4** Let $n = n + 1$.	**6** Stop.

a Apply the algorithm.

b Describe the numbers produced by the algorithm.

(P) **4** The box below describes an algorithm.

1 Input A, r.	**5** Let $r = s$.
2 Let $C = \frac{A}{r}$ to 3 d.p.	**6** Go to **2**.
3 If $\lvert r - C \rvert \leqslant 10^{-2}$ go to **7**.	**7** Print r.
4 Let $s = \frac{1}{2}(r + C)$ to 3 d.p.	**8** Stop.

Hint This algorithm requires you to use the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y.
For example: $|3.2 - 7| = 3.8$.

a Use a trace table to apply the algorithm above when:

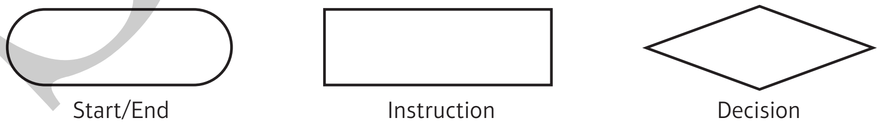
i $A = 253$ and $r = 12$ **ii** $A = 79$ and $r = 10$ **iii** $A = 4275$ and $r = 50$

b What does the algorithm produce?

1.2 Flow charts

You need to be able to apply an algorithm given as a flow chart.

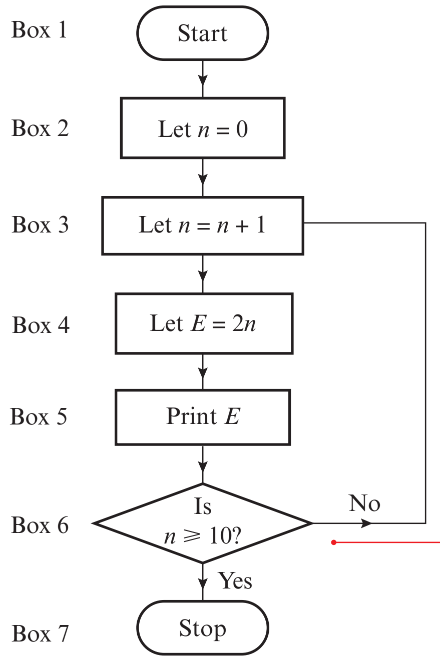
- In a flow chart, the shape of each box tells you about its function.



The boxes in a flow chart are linked by arrowed lines. As with an algorithm written in words, you need to follow each step in order.

Example 4 SKILLS ANALYSIS

The flow chart below describes an algorithm.



n is acting as a counter. It ensures that the algorithm stops after this **loop** has been completed 10 times.

A decision box will contain a question to which the answer is either 'yes' or 'no'.

a Apply this algorithm using a trace table.

b Alter box 4 to read 'Let $E = 3n$' and apply the algorithm again. How does this alter the output of the algorithm?

a

n	E	Box 6
0		
1	2	No
2	4	No
3	6	No
4	8	No
5	10	No
6	12	No
7	14	No
8	16	No
9	18	No
10	20	Yes

Output is 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

b

n	E	Box 6
0		
1	3	No
2	6	No
3	9	No
4	12	No
5	15	No
6	18	No
7	21	No
8	24	No
9	27	No
10	30	Yes

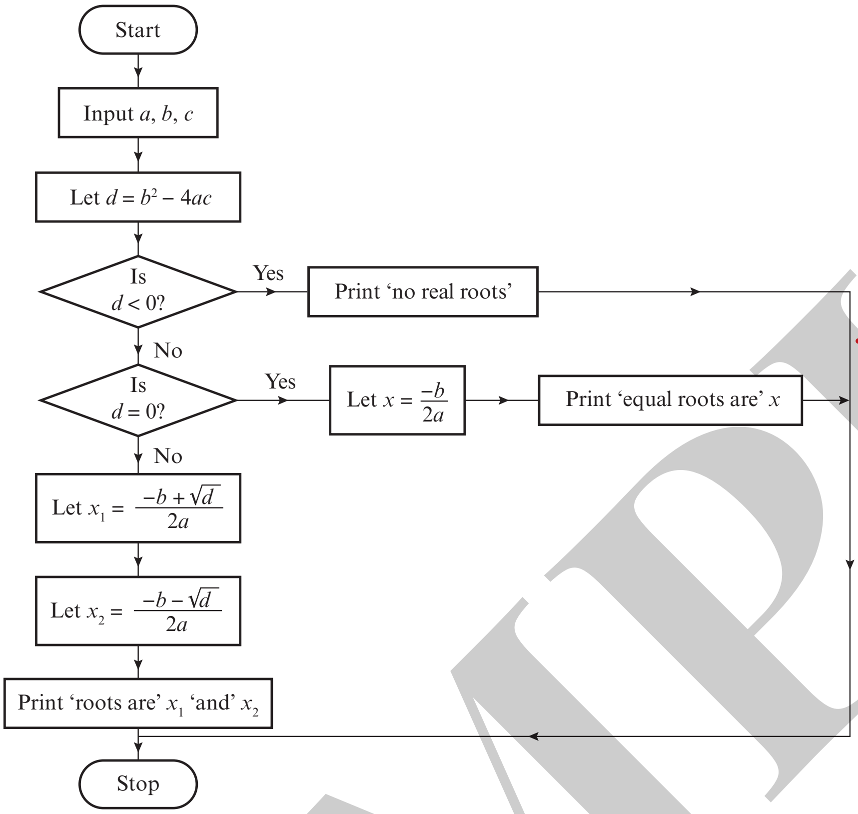
Output is 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

This gives the first ten multiples of 3 rather than the first ten multiples of 2.

In a trace table each step must be made clear.

Example 5 SKILLS INTERPRETATION

This flow chart can be used to find the roots of an equation of the form $ax^2 + bx + c = 0$.



You should recognise d as the discriminant of the equation.

← Pure 1 Section 2.5

Demonstrate this algorithm for these equations:

a $6x^2 - 5x - 11 = 0$ **b** $x^2 - 6x + 9 = 0$ **c** $4x^2 + 3x + 8 = 0$

a

a	b	c	d	$d < 0$?	$d = 0$?	x_1	x_2
6	–5	–11	289	No	No	$\frac{11}{6}$	–1

roots are $\frac{11}{6}$ and –1

b

a	b	c	d	$d < 0$?	$d = 0$?	x
1	–6	9	0	No	Yes	3

equal roots are 3

c

a	b	c	d	$d < 0$?
4	3	8	–119	Yes

no real roots

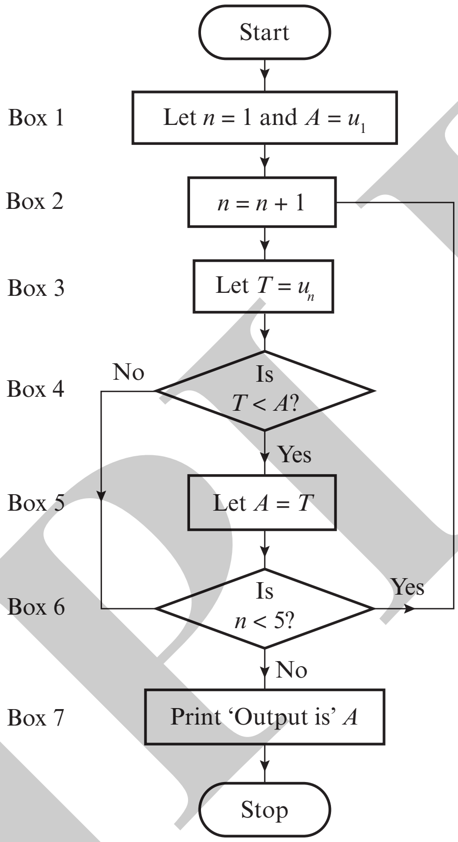
Example 6

SKILLS INTERPRETATION

Apply the algorithm shown by the flow chart on the right to the data:

$u_1 = 10, u_2 = 15, u_3 = 9, u_4 = 7, u_5 = 11$.

What does the algorithm do?



	n	A	T	$T < A$?	$n < 5$?
Box 1	1	10			
Box 2	2				
Box 3			15		
Box 4				No	
Box 6					Yes
Box 2	3				
Box 3			9		
Box 4				Yes	
Box 5		9			
Box 6					Yes
Box 2	4				
Box 3			7		
Box 4				Yes	
Box 5		7			
Box 6					Yes
Box 2	5				
Box 3			11		
Box 4				No	
Box 6					No
Box 7	Output is 7				

The algorithm selects the smallest number from a list.

This is quite complicated because it has questions and a list of data. Tackle one step at a time.

The box numbers have been included to help you to follow the algorithm. You do not need to include them in your exam.

Exercise 1B

SKILLS PROBLEM-SOLVING

1 Apply the flow chart in Example 5 to the following equations.

a $4x^2 - 12x + 9 = 0$ **b** $-6x^2 + 13x + 5 = 0$ **c** $3x^2 - 8x + 11 = 0$

2 **a** Apply the flow chart in Example 6 to the following sets of data.

i $u_1 = 28, u_2 = 26, u_3 = 23, u_4 = 25, u_5 = 21$

ii $u_1 = 11, u_2 = 8, u_3 = 9, u_4 = 8, u_5 = 5$

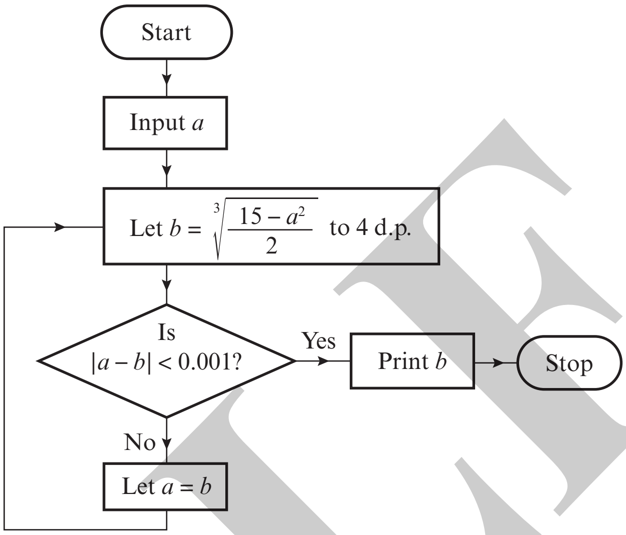
b If box 4 is altered to (Is $T > A$?), how will this affect the output?

c Which box would need to be altered if the algorithm was to be applied to a list of 8 numbers?

3 The flow chart describes an algorithm that can be used to find the roots of the equation $2x^3 + x^2 - 15 = 0$.

a Use $a = 2$ to find a root of the equation.

b Use $a = 20$ to find a root of the equation. Comment on your answer.



E/P **4** The flow chart on the right describes how to apply Euclid's algorithm to two non-zero integers, a and b.

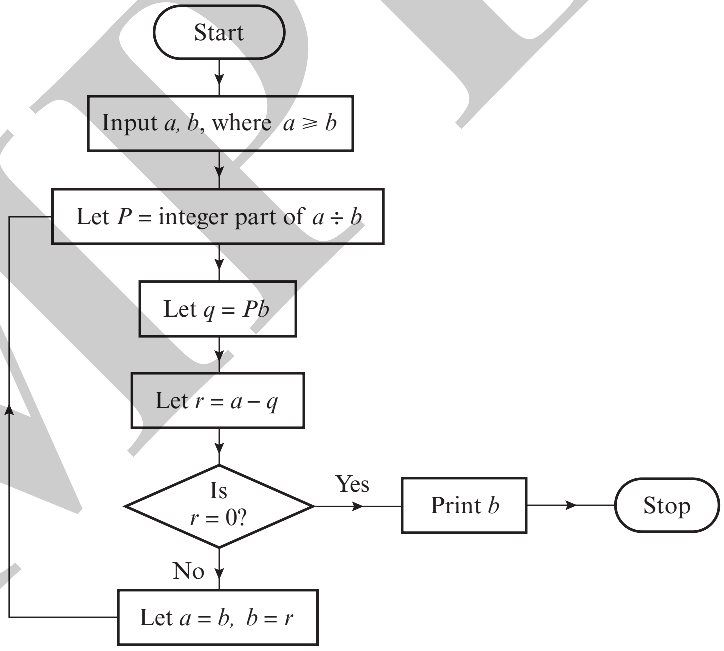
a Apply Euclid's algorithm to:

i 507 and 52 **(2 marks)**

ii 884 and 85 **(2 marks)**

iii 4845 and 3795 **(2 marks)**

b Explain what Euclid's algorithm does. **(2 marks)**



E/P **5** The flow chart describes an algorithm.

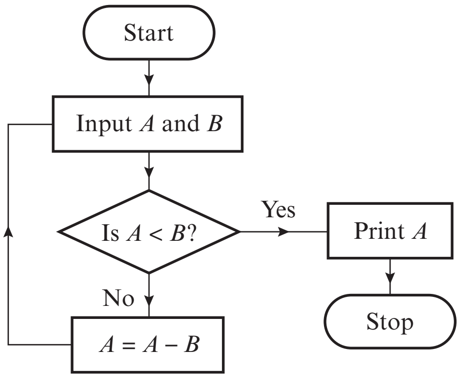
a Copy and complete this table, using the flow chart with $A = 18$ and $B = 7$.

A	B	$A < B$?	Output

(4 marks)

b Explain what is achieved by this flow chart. **(2 marks)**

c Given that $A = kB$ for some positive integer k, write down the output of the flow chart. **(1 mark)**



1.3 Bubble sort

A common data processing task is to sort an unordered list (a list which is not in order) into alphabetical or numerical order.

Lists can be put into ascending (increasing) or descending (decreasing) order.

- Unordered lists can be sorted using a **bubble sort** or a **quick sort**.
- In a bubble sort, we work through the list by comparing pairs of adjacent items (items that are next to each other) in the list.
  - If the items are in the correct order, leave them
  - If the items are not in the correct order, swap them

Once we have done this to all of the items in the list, we have completed the first pass.

If sorting the list into ascending order, the first pass will place the largest item in its correct position in the list.

If sorting the list into descending order, the first pass will place the smallest item in its correct position in the list.

We then repeat this until no swaps are made in a pass. If no swaps are made then the list is in order. You will need to write that no swaps have been made.

As the bubble sort develops, it is helpful to consider the original list as being divided into a **working list**, where comparisons are made, and a **sorted list** containing the items that are in their final positions. To start with, all items are in the working list.

This is the bubble sort algorithm:

1 Start at the beginning of the working list and move from left to right, comparing adjacent items.
  a If they are in order, leave them.
  b If they are not in order, swap them.

2 When you get to the end of the working list, the last item will be in its final position. This item is then no longer in the working list.

3 If you have made some swaps in the last pass, repeat step **1**.

4 When a pass is completed without any swaps, every item is in its final position and the list is in order.

Notation Each time you get to the end of the working list you complete one **pass** of the algorithm. The length of the working list reduces by 1 with each pass.

Notation The elements in the list 'bubble' to the end of the list in the same way that bubbles in a fizzy drink rise to the top. This is how the algorithm got its name.

You need to learn the bubble sort algorithm.

Example 7 SKILLS ANALYSIS

Use a bubble sort algorithm to arrange the list below into ascending order.

24 18 37 11 15 30

(24 18) 37 11 15 30	1st comparison: swap
18 (24 37) 11 15 30	2nd comparison: leave
18 24 (37 11) 15 30	3rd comparison: swap
18 24 11 (37 15) 30	4th comparison: swap
18 24 11 15 (37 30)	5th comparison: swap
18 24 11 15 30 37	End of first pass

After the second pass the list becomes

18 11 15 24 30 37

After the third pass the list is

11 15 18 24 30 37

After the fourth pass the list is

11 15 18 24 30 37

No swaps were made in the fourth pass, so the list is in order.

Hint In your exam you may be asked to show each comparison for one pass, but generally you will only be required to give the state of the list after each pass.

37 is already in its final position. It is now not in the working list. We now return to the start of the working list for the second pass.

After the third pass, the last three items are guaranteed to be in their final positions. In this example, the list is fully ordered but the algorithm requires another pass to be made.

Example 8 SKILLS REASONING/ARGUMENTATION

A list of n letters is to be sorted into alphabetical order, starting at the left-hand end of the list.

a Describe how to carry out the first pass of a bubble sort on the letters in the list.

b Carry out the first pass of a bubble sort to arrange the letters in the word ALGORITHM into alphabetical order, showing every step of the working.

c Show the order of the letters at the end of the second pass.

a Starting at the beginning of the list, compare the first two letters. If they are in alphabetical order, leave them in position, otherwise swap them. Continue through the list, to the end, comparing every pair of letters in the same way.

b

A L G O R I T H M	1st comparison: leave
(A L) G O R I T H M	2nd comparison: swap
A (G L) O R I T H M	3rd comparison: leave
A G (L O) R I T H M	4th comparison: leave
A G L (O R) I T H M	5th comparison: swap
A G L O (I R) T H M	6th comparison: leave
A G L O I (R T) H M	7th comparison: swap
A G L O I R (H T) M	8th comparison: swap
A G L O I R H (M T)	

c A G L I O H M R T

At the end of the first pass, the last letter is guaranteed to be in its correct place.

Example 9 SKILLS INNOVATION

Use a bubble sort to arrange these numbers into descending order.

39 57 72 39 17 24 48

(39 57) 72 39 17 24 48 — $39 < 57$ so swap
57 (39 72) 39 17 24 48 — $39 < 72$ so swap
57 72 (39 39) 17 24 48 — $39 \not< 39$ so leave
57 72 39 (39 17) 24 48 — $39 \not< 17$ so leave
57 72 39 39 (17 24) 48 — $17 < 24$ so swap
57 72 39 39 24 (17 48) — $17 < 48$ so swap
57 72 39 39 24 48 17

After 1st pass: 57 72 39 39 24 48 17
After 2nd pass: 72 57 39 39 48 24 17
After 3rd pass: 72 57 39 48 39 24 17
After 4th pass: 72 57 48 39 39 24 17
After 5th pass: 72 57 48 39 39 24 17
No swaps in 5th pass, so the list is in order.

Watch out Read the question carefully. You need to sort the list into **descending** order.

Note that the 48 is now between the two 39s. Do not treat the two 39s as one term.

Make sure that you make a statement like this to show that no swaps have been made and you have completed the algorithm.

Exercise 1C SKILLS REASONING/ARGUMENTATION

1 Apply a bubble sort to arrange each list below into:

a ascending order **b** descending order

i	23	16	15	34	18	25	11	19
ii	N	E	T	W	O	R	K	S
iii	A5	D3	D2	A1	B4	C7	C2	B3

For each part, you need to show the state of the list only at the **end** of each pass.

Hint For part **iii,** order alphabetically then numerically. So C2 comes after A5 but before C7.

2 Perform a bubble sort to arrange these place names into alphabetical order.

Chester York Stafford Bridlington Burton Cranleigh Evesham

(P) **3** A list of n items is to be written in ascending order using a bubble sort.

a State the minimum number of passes needed.

b Describe the circumstances in which this number of passes would be sufficient.

c State the maximum number of passes needed.

d Describe the circumstances in which this number of passes would be needed.

E **4** Here is a list of exam scores:

63 48 57 55 32 48 72 49 61 39

The scores are to be put in order, highest first, using a bubble sort.

a Describe how to carry out the first pass. **(2 marks)**

b Apply a bubble sort to put the scores in the required order.
Only show the state of the list at the end of each pass. **(4 marks)**

1.4 Quick sort

The quick sort algorithm can be used to arrange a list into alphabetical or numerical order. In many cases, a quick sort is faster to perform than a bubble sort. We can thus say that it is more efficient.

In a quick sort, we choose an item which we call a **pivot**, and split the items into two sublists:

- One sublist contains items less than the pivot.
- The other sublist contains items greater than the pivot.

Hint If an item is equal to the pivot it can go in either sublist.

Once we have done this we have completed the first pass.

In doing the quick sort, the first pass will place the pivot item in its correct position in the list.

We then repeat this until all items are chosen as pivots, and then the list is in order. You will need to write that all items are chosen as pivots, which means that they are in order.

Here is the quick sort algorithm, used to sort a list into ascending order.

1 Choose the item at the midpoint of the list to be the first pivot.
2 Write down all the items that are less than the pivot, keeping their order, in a sublist.
3 Write down the pivot.
4 Write down the remaining items (those greater than the pivot) in a sublist.
5 Apply steps **1** to **4** to each sublist.
6 When all items have been chosen as pivots, stop.

Use the formula $\frac{(n+1)}{2}$ and round up, if needed, to find the midpoint of the list. For example, if there are 10 items in the list, $\frac{(10+1)}{2} = 5.5$ and so the 6th item in the list is the midpoint.

Do not sort the items as you write them down.

The number of pivots could double at each pass. There is 1 pivot at the first pass, there could be 2 at the second, 4 at the third, 8 at the fourth, and so on.

Example 10 SKILLS ANALYSIS

Use the quick sort algorithm to arrange the numbers below into ascending order.

21 24 42 29 23 13 8 39 38

21	24	42	29	23	13	8	39	38
21	13	8						
21	13	8	23					
21	13	8	23	24	42	29	39	38
21	13	8	23	24	42	29	39	38
8	13	21	23	24	29	42	39	38
8	13	21	23	24	29	38	39	42
8	13	21	23	24	29	38	39	42

For n items, the pivot will be the $\frac{n+1}{2}$th item, rounding up if necessary. There are 9 numbers in the list so the midpoint will be $\frac{9+1}{2} = 5$, so the pivot is the 5th number in the list. Circle it.

Write all the numbers less than 23.

Write the pivot in a box , then write the remaining numbers.

Now select a pivot in each sublist.

There are now four sublists so we choose four pivots (circled).

We can choose only two pivots this time. Each number has been chosen as a pivot, so the list is in order.

Example 11 SKILLS PROBLEM-SOLVING

Use the quick sort algorithm to arrange the list below into descending order.

37 20 17 26 44 41 27 28 50 17

37	20	17	26	44	41	27	28	50	17
44	50	41	37	20	17	26	27	28	17
50	44	41	37	27	28	26	20	17	17
50	44	41	37	28	27	26	20	17	17
50	44	41	37	28	27	26	20	17	17
50	44	41	37	28	27	26	20	17	17

There are 10 items in the list so the midpoint will be $\frac{(10+1)}{2} = 5.5$, and so the pivot is the 6th number in the list. Circle it. Numbers greater than the pivot are to the left of the pivot, those smaller than the pivot are to the right, keeping the numbers in order. Numbers equal to the pivot may go either side, but must be dealt with in the same way each time you do a pass.

Two pivots are chosen, one for each sublist.

Now three pivots are selected.

We now choose the next two pivots, even if the sublist is in order.

The final pivots are chosen to give the list in order.

Watch out Colour is used here to make the method clear, but colours should not be used in your exam.

Exercise 1D

SKILLS REASONING/ARGUMENTATION

1 Use a bubble sort to arrange the list of numbers below into:

a ascending order **b** descending order

8 3 4 6 5 7 2

2 Use the quick sort algorithm to arrange the list below into:

a ascending order **b** descending order

22 17 25 30 11 18 20 14 7 29

3 Sort the letters below into alphabetical order using:

a a bubble sort **b** a quick sort

N H R K S C J E M P L

4 The list shows the test results of a group of students.

Alex	33	Hetal	9
Alison	56	Janelle	89
Amy	93	Josh	37
Annie	51	Lucy	57
Dewei	77	Masingur	19
Greg	91	Sam	29
Harry	49	Sophie	77

Produce a list of students, in descending order of their marks, using:

a a bubble sort **b** a quick sort

E/P **5** A list of n items is to be written in ascending order using the bubble sort algorithm.

a Find an expression, in terms of n, for the maximum number of comparisons to be made. **(2 marks)**

b Describe a situation where a bubble sort might be quicker than a quick sort. **(2 marks)**

c Decide whether a bubble sort or a quick sort will be quicker in the following cases:

i 1 2 3 7 4 5 6

ii 2 3 4 5 6 7 1

Explain how you made your decisions. **(4 marks)**

E **6** The table shows a list of nine names of students in a dance class.

Hassler	Sauver	Finch	Giannini	Mellor	Clopton	Miranti	Worth	Argi
H	S	F	G	Me	C	Mi	W	A

a Explain how to carry out the first pass of a quick sort algorithm to order the list alphabetically. **(2 marks)**

b Carry out the first two passes of a quick sort on this list, writing down the pivots used in each pass. **(3 marks)**

Challenge

SKILLS INNOVATION

You will need a pack of ordinary playing cards, with any jokers removed.
A pack of playing cards has 52 cards, split into 4 suits:

Hearts ♥ Diamonds ♦ Clubs ♣ Spades ♠

There are 13 cards in each suit, as follows:
Ace (1), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (11), Queen (12), King (13)

a Use the quick sort algorithm to sort the cards into ascending order, from Ace to King within each suit and with the suits in the order: Hearts, Clubs, Diamonds, Spades. Follow these steps:

1 Shuffle the pack thoroughly and hold it face up.

2 Remove the 27th card and place it face up. This is your pivot card.

3 Go through the pack from the top. Place the cards into two piles depending on whether they are lower or higher than the pivot card.

4 Repeat these steps with each new pile, choosing the card halfway through the pile as the pivot card.

Record the total number of passes needed to sort the deck completely.

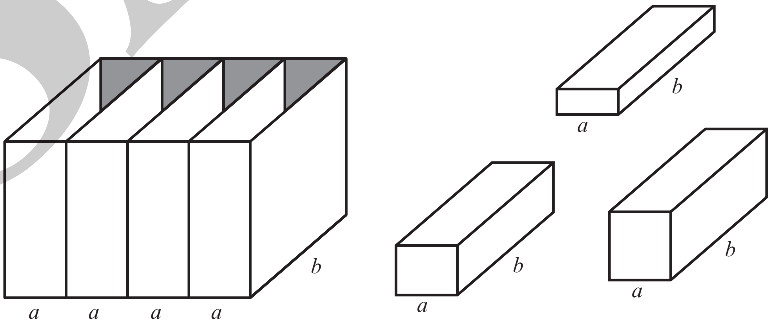
b Once the cards are in order, what single change could be made so that a bubble sort would require 51 passes to put the cards back in order?

Hint The final order should be:
A♥, 2♥, …, K♥, A♣, 2♣, …, K♣, A♦, …, K♦, A♠, …, K♠

1.5 Bin-packing algorithms

Bin packing refers to a whole class of problems.

The easiest way of thinking about this algorithm is to imagine boxes of fixed width a and length b, but varying heights, and stacking them into bins of width a and length b, using the minimum number of bins.



Similar problems are: loading cars of different lengths onto a ferry with several lanes of equal length, a plumber needing to cut sections from lengths of copper pipe, or recording music tracks onto a set of CDs.

You need to be able to apply three different **bin-packing** algorithms, and be aware of their strengths and weaknesses.

- The three bin-packing algorithms are: **first-fit**, **first-fit decreasing** and **full-bin**.

It is useful to first find a **lower bound** for the number of bins needed. There is no guarantee that you will be able to pack the items into this number of bins, but it will tell you if you have found an optimal solution.

Notation An **optimal solution** is one that cannot be improved upon. For bin packing, an optimal solution will use the smallest possible number of bins.

Example 12 SKILLS ANALYSIS

Nine boxes of fixed cross-section have heights, in metres, as follows.

0.3 0.7 0.8 0.8 1.0 1.1 1.1 1.2 1.5

They are to be packed into bins with the same fixed cross-section and height 2 m. Determine the lower bound for the number of bins needed.

$0.3 + 0.7 + 0.8 + 0.8 + 1.0 + 1.1 + 1.1 + 1.2 + 1.5 = 8.5\text{ m}$

$\frac{8.5}{2} = 4.25$ bins

So a minimum of 5 bins will be needed.

Sum the heights and divide by the bin size. You must always round **up** to determine the lower bound.

Watch out In practice, it may not be possible to pack these boxes into 5 bins. The lower bound simply tells us that **at least** 5 bins will be needed.

With small amounts of data it is often possible to 'spot' an optimal answer.

The algorithms you will learn in this chapter will not necessarily find an **optimal solution**, but can be applied quickly.

- The first-fit algorithm works by considering items in the order they are given.

First-fit algorithm

1. Take the items **in the order given**.
2. Place each item in the first available bin that can take it. Start from bin 1 each time.

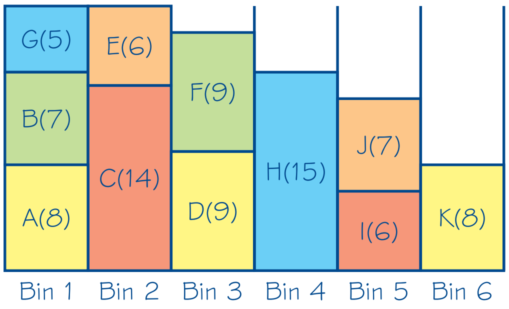
Advantage: It is quick to apply.
Disadvantage: It is not likely to lead to a good solution.

Online See the operation of the first-fit algorithm using GeoGebra.

Example 13 SKILLS EXECUTIVE FUNCTION

Use the first-fit algorithm to pack the following items into bins of size 20. (The numbers in brackets are the size of the item.) State the number of bins used and the amount of wasted space.

A(8) B(7) C(14) D(9) E(6) F(9) G(5) H(15) I(6) J(7) K(8)



This used 6 bins and there are
2 + 5 + 7 + 12 = 26 units of waste of space.

A(8) goes into bin 1, leaving space of 12.
B(7) goes into bin 1, leaving space of 5.
C(14) goes into bin 2, leaving space of 6.
D(9) goes into bin 3, leaving space of 11.
E(6) goes into bin 2, leaving space of 0.
F(9) goes into bin 3, leaving space of 2.
G(5) goes into bin 1, leaving space of 0.
H(15) goes into bin 4, leaving space of 5.
I(6) goes into bin 5, leaving space of 14.
J(7) goes into bin 5, leaving space of 7.
K(8) goes into bin 6, leaving space of 12.

- The first-fit decreasing algorithm requires the items to be in descending order before applying the algorithm.

First-fit decreasing algorithm

1 Sort the items so that they are in descending order.
2 Apply the first-fit algorithm to the reordered list.

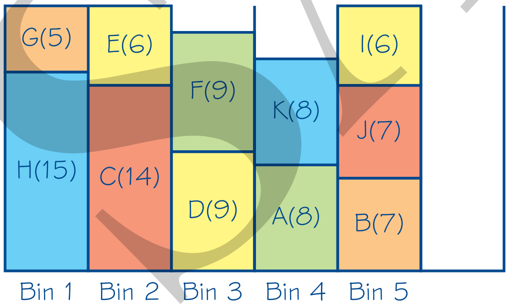
Advantages: You usually get a fairly good solution.
It is easy to apply.
Disadvantage: You may not get an optimal solution.

Online See the operation of the first-fit decreasing algorithm using GeoGebra.

Example 14 SKILLS ANALYSIS

Apply the first-fit decreasing algorithm to the data given in Example 13.

Sort the data into descending order:
H(15) C(14) D(9) F(9) A(8) K(8) B(7) J(7) E(6) I(6) G(5)



This used 5 bins and there are
2 + 4 = 6 units of wasted space.

H(15) goes into bin 1, leaving space of 5.
C(14) goes into bin 2, leaving space of 6.
D(9) goes into bin 3, leaving space of 11.
F(9) goes into bin 3, leaving space of 2.
A(8) goes into bin 4, leaving space of 12.
K(8) goes into bin 4, leaving space of 4.
B(7) goes into bin 5, leaving space of 13.
J(7) goes into bin 5, leaving space of 6.
E(6) goes into bin 2, leaving space of 0.
I(6) goes into bin 5, leaving space of 0.
G(5) goes into bin 1, leaving space of 0.

- Full-bin packing uses **inspection** to select items that will combine to fill bins. Remaining items are packed using the first-fit algorithm.

Full-bin packing

1 Use observation to find combinations of items that will fill a bin. Pack these items first.
2 Any remaining items are packed using the first-fit algorithm.

Advantage: You usually get a good solution.
Disadvantage: It is difficult to do, especially when the numbers are plentiful and awkward.

Example 15 SKILLS EXECUTIVE FUNCTION

A(8) B(7) C(10) D(11) E(13) F(17) G(4) H(6) I(12) J(14) K(9)

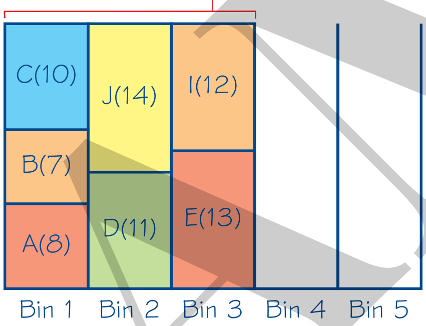
The items above are to be packed in bins of size 25.

a Determine the lower bound for the number of bins.
b Apply the full-bin algorithm.
c Is your solution optimal? Give a reason for your answer.

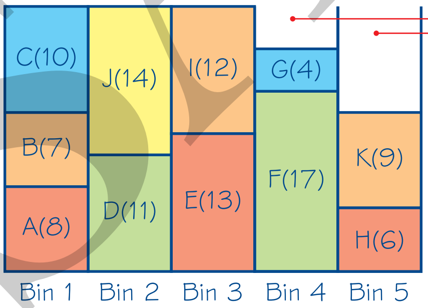
a 111 ÷ 25 = 4.44
So lower bound is 5 bins.

b Three groupings of numbers that sum to 25 can be made as follows:
8 + 7 + 10 = 25
11 + 14 = 25
13 + 12 = 25

The first three bins are full bins.



We now apply the first-fit algorithm to the remainder.
F(17) goes into bin 4, leaving space of 8.
G(4) goes into bin 4, leaving space of 4.
H(6) goes into bin 5, leaving space of 19.
K(9) goes into bin 5, leaving space of 10.



c The lower bound is 5, and 5 bins were used, so the solution is optimal.

Example 16 SKILLS EXECUTIVE FUNCTION

A plumber needs to cut the following lengths of copper pipe. (Lengths are in metres.)

A(0.8) B(0.8) C(1.4) D(1.1) E(1.3) F(0.9) G(0.8) H(0.9) I(0.8) J(0.9)

The pipe comes in lengths of 2.5 m.

a Calculate the lower bound of the number of lengths of pipe needed.

b Use the first-fit decreasing algorithm to determine how the required lengths may be cut from the 2.5 m lengths.

c Use full-bin packing to find an optimal solution.

a $\frac{0.8 + 0.8 + 1.4 + 1.1 + 1.3 + 0.9 + 0.8 + 0.9 + 0.8 + 0.9}{2.5}$

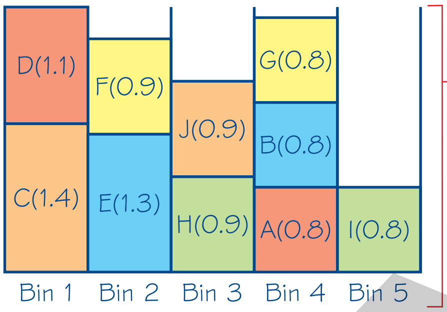
$= 3.88$

So at least 4 lengths are required.

b Sorting into descending order,

C(1.4), E(1.3), D(1.1), F(0.9), H(0.9), J(0.9), A(0.8), B(0.8), G(0.8), I(0.8)

Since a sort was not asked for, this can be done by inspection.



C goes into bin 1, leaving space of 1.1.
E goes into bin 2, leaving space of 1.2.
D goes into bin 1, leaving space of 0.
F goes into bin 2, leaving space of 0.3.
H goes into bin 3, leaving space of 1.6.
J goes into bin 3, leaving space of 0.7.
A goes into bin 4, leaving space of 1.7.
B goes into bin 4, leaving space of 0.9.
G goes into bin 4, leaving space of 0.1.
I goes into bin 5, leaving space of 1.7.

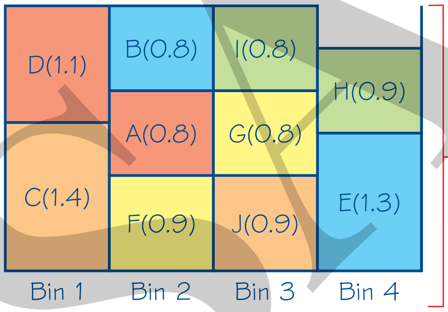
c By inspection,

C(1.4) + D(1.1) = 2.5

F(0.9) + A(0.8) + B(0.8) = 2.5

J(0.9) + G(0.8) + I(0.8) = 2.5

A full-bin solution is:



In part **a** we found that at least 4 lengths would be needed, so this solution is optimal since it uses 4 lengths.

Exercise 1E

SKILLS REASONING/ARGUMENTATION

1 18 4 23 8 27 19 3 26 30 35 32

The above items are to be packed in bins of size 50.

a Calculate the lower bound for the number of bins.

b Pack the items into the bins using:
i the first-fit algorithm **ii** the first-fit decreasing algorithm **iii** the full-bin algorithm

2 Laura hosts an internet music channel and wishes to play the 13 pieces of music listed below. Each day, she hosts a session which is at most 3 hours long.

Piece of music	A	B	C	D	E	F	G	H	I	J	K	L	M
Length (minutes)	30	30	30	45	45	60	60	60	60	75	90	120	120

a Apply the first-fit algorithm, in the order A to M, to determine the number of days that need to be used. State which music is played on each day.

b Repeat part **a** using the first-fit decreasing algorithm.

c Is your answer to part **b** optimal? Give a reason for your answer.

Laura finds that her session time is now reduced to only 2 hours.

d Use the full-bin algorithm to determine the number of days she needs to use. State which music is played on each day.

E **3** A small ferry loads vehicles into 30 m lanes. The vehicles are loaded bumper to bumper.

	Vehicle	**Length (m)**
A	car	4 m
B	car and trailer	7 m
C	lorry	13 m
D	van	6 m
E	lorry	13 m

	Vehicle	**Length (m)**
F	car	4 m
G	lorry	12 m
H	lorry	14 m
I	van	6 m
J	lorry	11 m

a Describe one difference between the first-fit and full-bin methods of bin packing. **(1 mark)**

b Use the first-fit algorithm to determine the number of lanes needed to load all the vehicles onto the ferry. **(4 marks)**

c Use a full-bin method to obtain an optimal solution using the minimum number of lanes. Explain why your solution is optimal. **(4 marks)**

E **4** The ground floor of an office block is to be fully recarpeted, with specially made carpet incorporating the firm's logo. The carpet comes in rolls of 15 m.

The following lengths are required.

A 3 m	D 4 m	G 5 m	J 7 m
B 3 m	E 4 m	H 5 m	K 8 m
C 4 m	F 4 m	I 5 m	L 8 m

The lengths are arranged in **ascending** order of size.

a Obtain a lower bound for the number of rolls of carpet needed. **(2 marks)**

b Use the first-fit decreasing bin-packing algorithm to determine the number of rolls needed. State the length of carpet that is wasted using this method. **(3 marks)**

c Give one disadvantage of the first-fit decreasing bin-packing algorithm. **(1 mark)**

d Use a full-bin method to obtain an optimal solution, and state the total length of wasted carpet using this method. **(4 marks)**

E/P **5** Eight computer programs need to be copied onto 40 GB USB sticks. The size of each program is given below.

Program	A	B	C	D	E	F	G	H
Size (GB)	8	16	17	21	22	24	25	25

a Use the first-fit decreasing algorithm to determine which programs should be recorded onto each USB stick. **(3 marks)**

b Calculate a lower bound for the number of USB sticks needed. **(2 marks)**

c Explain why it is not possible to record these programs on the number of USB sticks found in part **b**. **(1 mark)**

Problem-solving

Consider the programs over 20 GB in size.

1.6 Binary search

You need to be able to carry out a binary search.

A binary search will look through an **ordered** list to find out whether or not an item you are trying to find is in the list. If the item is in the list, the binary search will locate its position within the list.

If the list is not in order, then you may need to use a bubble sort or quick sort to put the items into order first.

- In a binary search, we look at halving the size of the list each time we perform a **pass**.
  - In a binary search, we locate the midpoint of the list using $\frac{n+1}{2}$. We call this the **pivot**. Like with the quick sort, we round this up if it is not an integer.
  - We compare this midpoint with the item we are trying to locate; this will help us decide which half of the list to choose.
  - Eventually we will get to one item – it will either be the item we are trying to locate, or it will not be. In this case we can say that the item we were trying to locate is not in the list.

Here is the binary search algorithm to locate an item in a list:

1 Select the midpoint of the list using $\frac{n+1}{2}$ and round up if necessary. This is the pivot.

2 **a** If the pivot is the item we are locating, then the search is complete.

b If the pivot is after the item we are locating, then we look in the first half of the list.

c If the pivot is before the item we are locating, then we look in the second half of the list.

3 Repeat steps 1 and 2 to each remaining list until the item is located. If the item is not found, then it is not in the list.

Example 17

Use the binary search algorithm to try to locate these names in the list that follows.

a Robinson **b** Davies

1	Acharya	6	Laing
2	Blackstock	7	Leung
3	Cheung	8	Robinson
4	Coetzee	9	Saludo
5	Fowler	10	Xiao

Watch out Remember that a search can be unsuccessful. You may be asked to try to locate something that is not in the list. You must be able to show that the item is not in the list.

a The middle name is the $\left(\frac{n+1}{2} = 5.5\right)$ 6th name:
6 Laing
Robinson is after Laing, so the list reduces to
1 Leung
2 Robinson
3 Saludo
4 Xiao
The middle name in this sublist is the $\left(\frac{4+1}{2} = 2.5\right)$ 3rd name:
3 Saludo
Robinson is before Saludo, so the list reduces to:
1 Leung
2 Robinson
The middle name in this sublist is the $\left(\frac{2+1}{2} = 1.5\right)$ 2nd name:
2 Robinson
The search is complete.
Robinson has been found in the list.

b The middle name is the $\left(\frac{10+1}{2} = 5.5\right)$ 6th name:
6 Laing
Davies is before Laing so the list reduces to:
1 Acharya
2 Blackstock
3 Cheung
4 Coetzee
5 Fowler
The middle name is the $\left(\frac{5+1}{2} = 3\right)$ 3rd name:
3 Cheung
Davies is after Cheung so the list reduces to
1 Coetzee
2 Fowler

Remember to round up if $\frac{n+1}{2}$ is not an integer.

Since Robinson is after Laing, Robinson cannot be in the first part of the list and so we consider the list after the pivot.

Robinson is before Saludo so it cannot be in the second list and so we consider the list before the pivot.

It is important to write this down.

Consider the list **before** the pivot.

Consider the list **after** the pivot.

The middle name is the $\left(\frac{n+1}{2} = 1.5\right)$ 2nd name:
2 Fowler
Davies is before Fowler so the list reduces to:
1 Coetzee
The list has only one item which is not Davies.
Therefore Davies is not in the list.

Consider the list **before** the pivot.

It is important to write this down.

Exercise 1F

SKILLS ANALYSIS

1 Use the binary search algorithm to try to locate these names in the list that follows:

a Connock **b** Walkey **c** Peabody

1 Berry
2 Connock
3 Li
4 Sully
5 Tapner
6 Walkey
7 Wilson
8 Wu

2 Use the binary search algorithm to try to locate these numbers in the list that follows:

a 21 **b** 5

1 3, **2** 4, **3** 7, **4** 9, **5** 10, **6** 13, **7** 15, **8** 17, **9** 18, **10** 20, **11** 21, **12** 24

(P) **3** The binary search algorithm is applied to an ordered list of n items.
Find the maximum number of times the algorithm is run when n is equal to:

a 100 **b** 1000 **c** 10 000

4 a Use the quick sort algorithm to put the list below into ascending order.

1 Adam
2 Ed
3 Lei
4 Lottie
5 Saul
6 Ramin
7 Alex
8 Emily
9 Felix
10 Leo
11 Oli
12 Lotus
13 Des
14 George
15 Jess
16 Miranda
17 Matt
18 Katie
19 Doug
20 Hongmei

b Use the binary search algorithm to try to locate:

i George **ii** David **iii** Jess

Chapter review 1

(E) **1** Use the bubble-sort algorithm to sort, in ascending order, the list

27 15 2 38 16 1

giving the state of the list at each stage. **(4 marks)**

E/P **2** **a** Use the bubble-sort algorithm to sort, in descending order, the list

25 42 31 22 26 41

giving the state of the list on each occasion when two or more values are interchanged (swapped). **(4 marks)**

b Find the **maximum** number of interchanges needed to sort a list of six pieces of data using the bubble-sort algorithm. **(2 marks)**

E **3** 8 4 13 2 17 9 15

This list of numbers is to be sorted into ascending order.
Perform a quick sort to obtain the sorted list, giving the state of the list after each rearrangement. **(5 marks)**

E **4** 111 103 77 81 98 68 82 115 93

a The list of numbers above is to be sorted into descending order. Perform a quick sort to obtain the sorted list, giving the state of the list after each rearrangement and indicating the pivot elements used. **(5 marks)**

b **i** Use the first-fit decreasing bin-packing algorithm to fit the data into bins of size 200. **(3 marks)**

ii Explain how you decided in which bin to place the number 77. **(1 mark)**

E **5** Trishna wishes to record eight television programmes. The lengths of the programmes, in minutes, are:

75 100 52 92 30 84 42 60

Trishna decides to use 2-hour (120 minute) DVDs only to record all of these programmes.

a Explain how to apply the first-fit decreasing bin-packing algorithm. **(2 marks)**

b Use this algorithm to fit these programmes onto the smallest number of DVDs possible, stating the total amount of unused space on the DVDs. **(3 marks)**

Trishna wants to record an additional two 25-minute programmes.

c Determine whether she can do this using only 5 DVDs, giving reasons for your answer. **(3 marks)**

E **6** A DIY enthusiast requires the following 14 pieces of wood as shown in the table.

Length in metres	0.4	0.6	1	1.2	1.4	1.6
Number of pieces	3	4	3	2	1	1

The DIY store sells wood in 2 m and 2.4 m lengths. He considers buying six 2 m lengths of wood.

a Explain why he will not be able to cut all of the lengths he requires from these six 2 m lengths. **(2 marks)**

b He eventually decides to buy 2.4 m lengths. Use a first-fit decreasing bin-packing algorithm to show how he could use six 2.4 m lengths to obtain the pieces he requires. **(4 marks)**

c Obtain a solution that requires only five 2.4 m lengths. **(4 marks)**

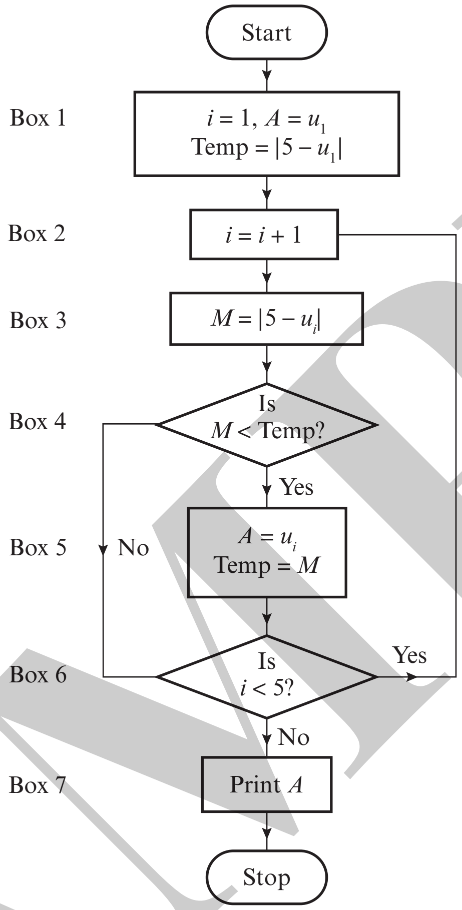
E/P **7** The algorithm described by the flow chart below is to be applied to the five pieces of data below.

$u_1 = 6.1$, $u_2 = 6.9$, $u_3 = 5.7$, $u_4 = 4.8$, $u_5 = 5.3$

Hint This question uses the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y, e.g. $|5 - 6.1| = 1.1$.

a Obtain the final output of the algorithm using the five values given for u_1 to u_5. **(4 marks)**

b In general, for any set of values u_1 to u_5, explain what the algorithm achieves. **(2 marks)**



c If Box 4 in the flow chart is altered to
'Is $M >$ Temp?'
state what the algorithm achieves now. **(1 mark)**

E **8** A plumber is cutting lengths of PVC pipe for a bathroom.
The lengths needed, in metres, are:

0.3   2.0   1.3   1.6   0.3   1.3   0.2   0.1   2.0   0.5

The pipe is sold in 2 m lengths.

a Carry out a bubble sort to produce a list of the lengths needed in **descending** order. Give the state of the list after each pass. **(4 marks)**

b Apply the first-fit decreasing bin-packing algorithm to your ordered list to determine the total number of 2 m lengths of pipe needed. **(3 marks)**

c Does the answer to part **b** use the minimum number of 2 m lengths? You must justify your answer. **(2 marks)**

E/P **9** Here are the names of eight students in an A level group:

Manisha, Vivien, Cath, Alex, Da Ming, Beth, Kandis, Sze-To

Use a quick sort to put the names in alphabetical order. Show the result of each pass and identify the pivots. **(5 marks)**

Challenge

E/P **10** A binary search is to be performed on a list of names to try to locate Kim.

1	Jenny	**6**	Hyo
2	Merry	**7**	Kim
3	Charles	**8**	Richard
4	Ben	**9**	Greg
5	Toby	**10**	Freya

a Explain why a binary search cannot be performed with the list in its present form. **(1 mark)**

b Using an appropriate algorithm, alter the list so that a binary search can be performed, showing the state of the list after each complete iteration. State the name of the algorithm you have used. **(4 marks)**

c Use the binary search algorithm to locate the name Kim in the list you obtained in **b**. You must make your method clear. **(4 marks)**

Summary of key points

1 An **algorithm** is a finite sequence of step-by-step instructions carried out to solve a problem.

2 In a **flow chart**, the shape of each box tells you about its function.

3 Unordered lists can be sorted using a bubble sort or a quick sort.

4 In a **bubble sort**, you compare adjacent items in a list:
- If they are in order, leave them.
- If they are not in order, swap them.
- The list is in order when a pass is completed without any swaps.

5 In a **quick sort**, you select a pivot and then split the items into two sublists:
- One sublist contains items less than the pivot.
- The other sublist contains items greater than the pivot.
- You then select further pivots from within each sublist and repeat the process.

6 The three bin-packing algorithms are first-fit, first-fit decreasing, and full-bin:
- The **first-fit** algorithm works by considering items in the order they are given.
- The **first-fit decreasing** algorithm requires the items to be in descending order before applying the algorithm.
- **Full-bin packing** uses inspection to select items that combine to fill bins completely. Remaining items are packed using the first-fit algorithm.

7 The three bin-packing algorithms have the following advantages and disadvantages:

Type of algorithm	Advantage	Disadvantage
First-fit	Quick to apply	Not likely to lead to a good solution
First-fit decreasing	Usually a good solution; easy to apply	May not get an optimal solution
Full-bin	Usually a good solution	Difficult to do, especially when the numbers are plentiful or awkward

8 A **binary search** will search an ordered list to find out whether an item is in the list. If it is in the list, it will locate its position in the list.

In a binary search, the pivot is the middle item of the list. If the target item is not the pivot, the pivot and half of the list are discarded. The list length halves at each pass.

The middle of n items is found by $\frac{n+1}{2}$, rounding up if necessary.