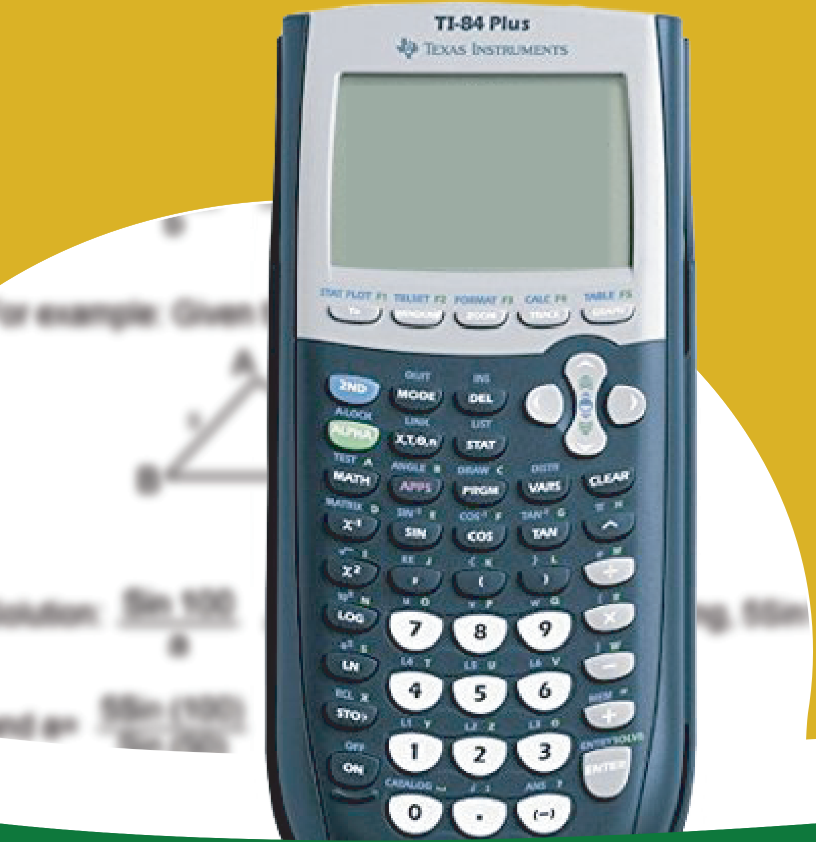
Focus ON THE ACT



Student Workbook for Math & Science

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Preface

The purpose of this workbook is to provide specific test taking strategies relevant to the math and science sections of the ACT. Chapter 1 provides general information about the math test. Chapter 2 contains a review of the specific content addressed by the math test. Chapter 3 contains specific strategies for doing the problems you are most likely to encounter on the test. Chapter 4 provides general information regarding the science test. Chapter 5 has specific strategies for the science test.

Included throughout this workbook are “real” ACT items for you to practice. Practice will enable you to “connect” what you know with how it is asked by the ACT.

Problems or sections marked with a  are appropriate for calculator use.

Chapter 1

Introduction to the Math Test

I. GENERAL INFORMATION

A. There will always be 60 questions to be completed in 60 minutes.
B. Questions address basic content through pre-calculus.

II. CONTENT OF THE MATH TEST

A. The following is the breakdown of the content of any ACT math test:

1. Pre-Algebra (14 questions)
   Fractions, decimals, percents, etc.

2. Elementary Algebra (10 questions)
   Questions from Algebra I.

3. Intermediate Algebra and Coordinate Geometry (18 questions)
   Questions from Algebra II and dealing with X and Y axes.

4. Plane Geometry (14 questions)
   Triangles, polygons, area, etc.

5. Trigonometry (4 questions)
   If you don't know what this is about, don't mess with it.

B. See Appendix C: Content Guide to the ACT Math Test.

III. THE ABCs OF PREPARING FOR THE MATH TEST

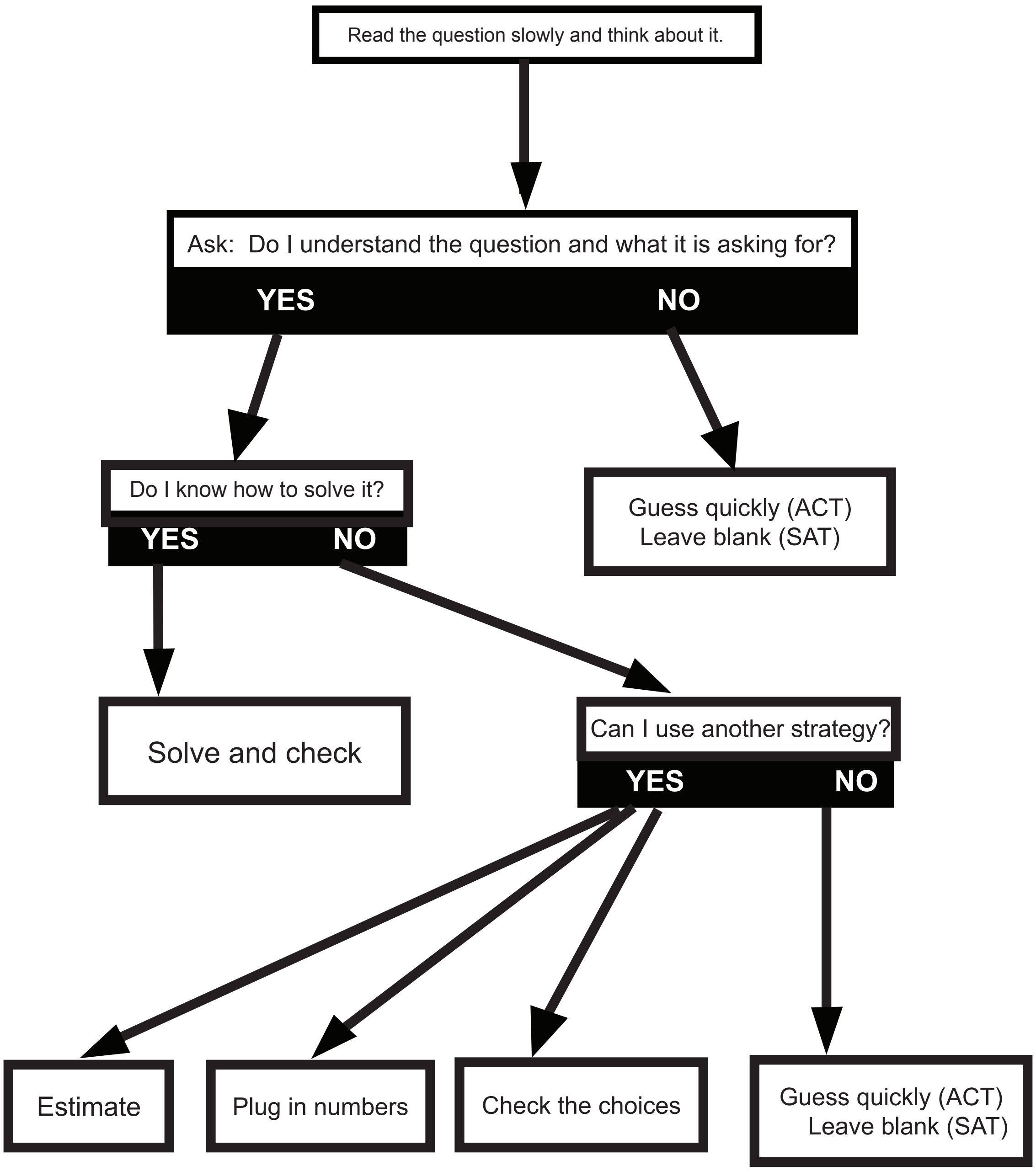
A. Review the relevant math topics.
B. Learn specific strategies that apply to the ACT.
C. Connect what you know to the test by doing problems from actual ACT tests.

How to Use the Math Skill Review

Most questions on the ACT math test require you to remember specific details from courses you may have had years ago. Consequently, a review is essential. Every problem in this review has been specifically designed to focus on a skill required by the ACT. This review should provide you with a foundation that will make practicing with real ACT questions more beneficial.

This skill review is intended to review material you have already studied – to refresh your memory. The best way to use this review is to skim each section, and then work the “Try These” problems. If you answer them correctly, move on to the next section. If you miss one or more, go back and reread the section more carefully. Finally, if you discover something you wish to remember, make sure to make a note of it. I believe you will find keeping track of “Things to Remember” helpful.

Math Question Strategy



Chapter 2

Content Review for the Math Subtest

I. *BASIC SKILLS*

A. *Converting Among Fractions, Decimals, and Percents*

1. *Fractions to decimals*
   To change a fraction to a decimal,
   simply divide the bottom into the top.

Ex. 1 $\frac{3}{4}$

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2.8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Ex. 2 $\frac{2}{3}$

$$\begin{array}{r} .66 \\ 3\overline{)2.00} \\ \underline{1.8} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

2. *Decimals to fractions*
   To change from a terminating decimal to a fraction,
   read the decimal, then write as a fraction, then reduce.
   (*Some calculators will do this for you*.)

Ex. Change .305 to a fraction.

Solution:

read: three hundred-five, thousandths

write: $\frac{305}{1000}$

reduce: $\frac{305 / 5}{1000 / 5} = \frac{61}{200}$

3. *Decimals to percents*
   To change a decimal into a percent,
   move the decimal two places to the right.
   (*Just remember 1 = 100%, then you'll know which direction to move.*)

Ex.1 .32 = 32%

Ex.2 5.4 = 540%

4. *Percents to decimals*
   To change a percent into a decimal, move the decimal point two places to the left.

Ex.1 63% = 63.% = .63

Ex.2 .3% = .003

Try These Finish the chart by doing all necessary conversions

Fraction	Decimal	Percent
¼		
	.35	
		150%
⅜		
	.004	
		6.5%

B. *Percent Problems*

1. *Base-rate percentage problems*
   There are many ways to do these. Perhaps the easiest way is to recognize the following:

   is means **equals (=)**
   of means **multiply (x)**
   what means **variable**

   Using these facts we can write, then solve the appropriate equation.

Ex.1 What number is 10% of 500?

$$n = .1 \cdot 500$$
$$n = 50$$

Ex. 2 16 is what percent of 80?

$$16 = n \cdot 80$$

$$\frac{16}{80} = n$$

$$.2 = n$$

$$20\% = n$$

2. *Percent increase and decrease*
   The key to these problems is recognizing that percent increase or decrease will always equal the change divided by the original.

$$\frac{\text{Change}}{\text{Original}}$$

> Ex.1 The price of a bicycle goes from \$40 to \$50.
> What is the percent increase?

Solution: The "change" = \$50 – \$40 = \$10
The "original" = \$40
So, the percent increase = $\frac{10}{40}$ = .25 = 25%

3. *Three types of Sale Price Problems*

> Ex. 1a The original price of a shirt is \$30.
> It's on sale for 20% off. What's the sale price?

Solution: Let "original" = 30
Making the "change" = 30 - (.20)30 = \$24.00

> Ex. 1b The sale price of a shirt is \$30.
> If this is 20% <u>off</u> the original price,
> what was the original price?

Solution: Let "original" = x
Making the equation: x - .20x = 30
.8x = 30
x = 30/.8
x = \$37.50 is the original price.

> Ex. 1c The sale price of a shirt is \$30.
> If this is 20% <u>of</u> the original price,
> what is the original price?

Solution: Let "original" = x
Making the equation: .20x = 30
x = 30/.20
x = \$150.00

Try These

1. The value of a house in 2010 was $80,000. If its value in 2011 is $85,000, by what % did the value of the house increase?

2. A television is on sale at a 15% discount. If the original price was $300, what is the sale price?

3. The cost of a used car was $8,000. If this was 40% <u>of</u> the original price, what was the original price of the car?

C. *Operations with Fractions*

1. *Multiplying and dividing fractions*
   With either multiplication or division, the first step is to write all fractions in "improper" form (not mixed). Second, cancel any top with any bottom. Finally, multiply straight across. (Most calculators will compute with fractions.)

Ex. $5\frac{1}{4} \bullet 3\frac{1}{7}$

Step 1: $5\frac{1}{4} = \frac{(5\bullet 4)+1}{4} = \frac{21}{4}$ and $3\frac{1}{7} = \frac{(3\bullet 7)+1}{7} = \frac{22}{7}$

Step 2: $\frac{\cancel{21}^{3}}{\cancel{4}_{2}} \bullet \frac{\cancel{22}^{11}}{\cancel{7}_{1}} = \frac{3}{2} \bullet \frac{11}{1}$

Step 3: $\frac{3}{2} \bullet \frac{11}{1} = \frac{33}{2}$

Division is the same. **Just make sure you flip the number after the division sign before you multiply.**

2. *Adding and subtracting fractions*
   This is a little more complicated, because now is when you need a common denominator. Here are two methods.

<u>Method 1</u> First, convert all fractions into improper form, then place all fractions over the common denominator, and finally add across the top.

Ex. $4\frac{1}{4} + 10\frac{1}{3}$

Step 1: $4\frac{1}{4} = \frac{(4\bullet 4)+1}{4} = \frac{17}{4}$ and $10\frac{1}{3} = \frac{(10\bullet 3)+1}{3} = \frac{31}{3}$

Step 2: $\frac{17}{4} + \frac{31}{3} = \frac{17\bullet 3}{4\bullet 3} + \frac{31\bullet 4}{3\bullet 4} = \frac{51}{12} + \frac{124}{12} = \frac{175}{12}$

<u>Method 2</u> Add the whole numbers and find the common denominator for only the fractional part.

Ex. 2 $4\frac{1}{4} + 10\frac{1}{3}$

$$\begin{array}{r} 4\frac{1}{4} \\ +\ 10\frac{1}{3} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{3}{12} \\ +\ 10\frac{4}{12} \\ \hline 14\frac{7}{12} \end{array}$$

Use <u>Method 2</u> when dealing with large numbers.

Ex. 3

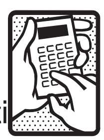
$$100\frac{1}{5} + 152\frac{1}{4} \qquad \begin{array}{r} 100\frac{1}{5} \\ +\ 152\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} 100\frac{4}{20} \\ +\ 152\frac{5}{20} \\ \hline 252\frac{9}{20} \end{array}$$

Use <u>Method 1</u> when you have to carry or borrow.

Ex. 4

$$5\frac{3}{4} - 3\frac{7}{8} = \frac{23}{4} - \frac{31}{8} = \frac{46}{8} - \frac{31}{8} = \frac{15}{8}$$

Try These



Perform the indicated operati

(1) $\frac{3}{4} \bullet \frac{4}{5} \bullet 4\frac{1}{6}$

(2) $1\frac{1}{4} \div \frac{3}{4}$

(3) $4\frac{5}{8} - 2\frac{3}{4}$

(4) $5\frac{1}{4} + 6\frac{3}{8} + \frac{7}{2}$

II. *PROPERTIES OF NATURAL NUMBERS AND BASIC STATISTICS*

A. *Divisors and Factors*

(A calculator can determine if a number is a divisor.)

1. A divisor of a number is any number which will divide it evenly (that is, without a remainder).

Ex. 10 is a divisor of 30, since 30 ÷ 10 = 3 with no remainder

2. The factors of a number are all the numbers which divide into it evenly.

Ex. The factors of 50 are: 1, 50, 2, 25, 5, 10
Note: You can find these in pairs.

B. *Prime, Composite, and Unit*

1. *Definitions*

The ACT uses these 3 words to describe natural numbers {1, 2, 3 . . .}. 1 is called a <u>unit</u>. Any number with only itself and 1 as factors is called <u>prime</u>.
For example: 2, 3, 5, 7, 11 . . . are prime numbers.

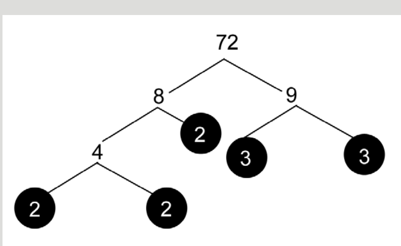
Note: All prime numbers after 2 are odd (why?) and there is no highest prime (they go on forever).

Numbers with more than 2 factors are called <u>composite</u>.
For example: 4, 6, 8, 9, 10 . . . are composite numbers.

2. *Prime factorization*

Any composite number can be written as a product of prime numbers. This is accomplished using a factor tree.

Ex. What is the prime factorization of 72?



Thus, the prime factorization of 72 = 2 • 2 • 2 • 3 • 3 = $2^3 \cdot 3^2$

Note: It is possible to have different "trees" for the same number, but the end results are always the same.

3. *Greatest Common Factor vs. Least Common Multiple*

The key is understanding the difference between **factors** and **multiples**.

The factors of 12 are 1, 2, 3, 4, 6, and 12. Factors are <u>smaller</u> than (or equal to) the number itself.

The multiples of 12 are 12, 24, 36, 48 … Multiples are <u>bigger</u> than (or equal to) the number itself.

The **greatest common factor** (GCF) is the greatest factor that is common to two or more numbers.

The **least common multiple** (LCM) of two numbers is the smallest number (excluding zero) that is a multiple of both of the numbers.

Ex. 1 Find the GCF of 24 and 36.

a. 6
b. 12
c. 48
d. 72
e. 144

Solution: Of the 5 choices given, which are too big to be factors of 24 and 36?

48, 72, and 144 are too big to be factors, so only 6 and 12 are possible solutions. Check the greatest one first. The answer is 12.

Ex. 2 Find the LCM of 24 and 36.

a. 6
b. 12
c. 48
d. 72
e. 144

Solution: Of the 5 choices given, which are too small to be multiples of 24 and 36?

6 and 12 are too small to be multiples, so only 48, 72 and 144 are possible solutions. Check the smallest one first. 48 is NOT a multiple of 36 so it is not the correct answer. 72 is a multiple of 24 and 36 so the answer is 72.

Try These

1. Determine all the factors of 120.

2. Write the prime factorization of 100.

3. Find the Least Common Multiple (LCM):
   (remember your divisibility tests)

   a) 12, 32 b) 8, 50, 120

   c) x^2y, xy^3z d) $5ab^2$, $10abc^3$, $15a^2d$

C. *Basic Probability and Statistics*

1. *Probability*
   The idea of probability will hopefully seem somewhat intuitive. Basically, the probability of something occurring is

$$\frac{\text{the number of ways it could occur}}{\text{the total number of possible occurrences}}$$

Ex. 1 If there are 10 red balls, 3 green balls, and 2 blue balls in a jar, what is the probability that a green ball will be chosen?

Solution: Number of green balls = 3
Total number of balls = 10+2+3 = 15
Probability = 3/15 = 1/5

Note: This procedure assumes that the selection is random — all balls are equally likely to be chosen.

Ex. 2 If you are rolling 2 different colored dice, what is the probability you roll a sum of 8?

Solution: Number of possible outcomes for the first die = 6
Number of possible outcomes for second die = 6
Therefore, the total possible outcomes = 6 • 6 = 36
(this is your denominator)

Now, what are the ways to get a sum of 8?
2 & 6, 3 & 5, 4 & 4, 5 & 3, 6 & 2 which is 5 ways
(this is your numerator)

So your answer is $\frac{5}{36}$

Note: All probabilities must be between (or equal to) 0 and 1.

2. *The arithmetic mean (average)*
The mean of a set of numbers is the average. To compute, add up all the scores and then divide by the number of scores.

Ex. 1 Find the average of 10, 12, & 23.

Solution: Average $= \frac{10 + 12 + 23}{3} = \frac{45}{3} = 15$

Note: Many times problems on the ACT will require going from an average back to the total.

Ex. 2 A student needs to average 80% on 5 tests to get a B. If the first 4 tests were 70, 75, 86, and 84, what does this student need on the 5th test?

Solution:
(**Method 1 –conceptual**) Since the average of those 5 tests is 80%, the total of the 5 tests must be 5 • 80 = 400. The total of the 4 tests already taken is 70+75+86+84=315. So, the student needs the difference on the 5th test to be 400 – 315 = 85.

(**Method 2 –algebraic**)
Let x = 5th test. So, $\frac{70 + 75 + 86 + 84 + x}{5} = 80$

$\frac{315 + x}{5} = 80 \qquad 315 + x = 80(5) \qquad 315 + x = 400 \qquad x = 85$

3. *Median and mode*

The median of a set of scores is the score in the middle. To determine the median, place scores in order (high to low, or low to high) and pick the score in the middle. The mode is the score that occurs most often.

Ex. 1: Find the median and mode of the following set of numbers: 8, 15, 9, 3, 15, 10

Solutions:

First, to find the **median,** arrange numbers in order:

3, 8, 9, 10, 15, 15.

Next, since there is an even number of scores, (in this case 6), we must "average" the 2 scores in the middle:

$$\frac{9 + 10}{2} = \frac{19}{2} = 9.5 \quad \textbf{(median)}$$

Note: For an odd number of scores, there will be one score in the middle.

The **mode** is 15 because 15 occurs twice, and all other numbers occur once.

Note: If more than one score occurs the most, each of these scores is the mode. If no score occurs more than once, there is no mode.

4. *Counting problems (series of choices)*

Whenever you have to count how many ways to make a series of choices, do the following:

1. Determine how many choices have to be made and put down a blank for each choice.
2. Fill in each blank with the number of ways that a choice can be made.
3. Multiply the numbers together.

Note: This assumes all combinations are possible.

Ex. 2: How many different sandwiches consisting of one cheese, one meat, and one bread can be made from a choice of 4 cheeses, 3 meats, and 5 breads?

Solution: Since there are 3 choices to be made, put down 3 blanks:

______	______	______
Cheese	Meat	Bread

Since there are 4 cheeses, we put a 4 in the cheese blank; similarly we put a 3 and a 5 in the meat and bread blanks. We multiply 4•3•5 = 60 different sandwich combinations.

Ex. 3A: Fourteen kids are running in the student council election. Five kids are running for president, three kids are running for vice president, and six kids are running for secretary/treasurer. How many different ways can the spots be filled?

Solution: Put down 3 blanks:

5	3	6
President	Vice-President	Sec/Treasurer

We multiply 5•3•6 = 90 possible ways the spots can be filled.

Ex. 3B: 12 kids are trying for 3 positions as student council representatives. How many different ways can the spots be filled?

Solution:

______	______	______
1st	2nd	3rd

Since there are 3 positions, we put a 12 in the first blank; similarly we put an 11 in the second blank, and a 10 in the third blank, as a result of the number of possible combinations.

We multiply 12•11•10 = 1,320 different possible combinations.

Try These

1. Each of the numbers 12 – 20 is placed in a hat. If a number is selected at random, find the probability that it is an:

   (a) 8

   (b) even number

   (c) integer greater than 12

2. Find the mean, median and mode of the following set of numbers: {10,12, 8, 7, 22, 8, 10}

3. On an algebra test, the 10 boys in the class averaged 80%, while the 15 girls averaged 76%.
   What was the average of all 25 students?

4. How many different ice cream cones can be made consisting of a cone (waffle or sugar), an ice cream (10 flavors) and a topping (5 different toppings)?

III. BASIC ALGEBRA

A. *Simplifying Algebraic Expressions*

1. *Combining similar terms*
   Terms (pieces) are alike if the letter parts are the same.

> Ex.1 $6x^2$ and $15x^2$ are similar terms.
> $5y$ and $6y^2$ are not similar terms.

Combine similar terms by adding the coefficients (the numbers in front).

> Ex.2 $3x-2(x+3y)-4y = 3x-2x-6y-4y = x-10y$

2. *Multiplying binomials*
   A binomial is composed of two terms. Multiply binomials by using the "FOIL" method.

<u>F</u>irst <u>O</u>utside <u>I</u>nside <u>L</u>ast

> Ex.1 $(x - 3)(2x + 4) = 2x^2 + 4x - 6x -12 = 2x^2 - 2x -12$

> Ex.2 $(2x + 5)^2 = (2x + 5)(2x + 5) = 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25$

Note: A similar procedure works when you have more than two terms. Multiply each term in the first group by each term of the second group, then combine similar terms.

Try These

1. Combine similar terms: $5ab^2 - 3a^2b - 6ab^2 + 4a^2b$

2. Simplify: $3(x - 2) - 5(2x + 4)$

3. Multiply: $(3x - 2)(x + 5)$

4. Multiply: $(2x + 5)(x^2 - 3x -10)$

B. *Solving Equations and Inequalities*

1. *Equations*

To solve an equation, do the following procedure:

Step 1: Remove all fractions by multiplying by the LCD.
Step 2: Distribute (get rid of parentheses).
Step 3: Combine similar terms.
Step 4: Get the variable on one side.
Step 5: Simplify by adding and subtracting.
Step 6: Simplify by multiplication and division.

> Note:
> Steps 1 and 2 may be reversed from time to time!

> Ex. Solve: $\dfrac{3(x - 2)}{4} + 5 = \dfrac{(x + 2)}{2}$

Solution:
Step 1: Multiply both sides by 4 and cancel:

$$\frac{12(x - 2)}{4} + 20 = \frac{4(x + 2)}{2} \Rightarrow 3(x - 2) + 20 = 2(x + 2)$$

Step 2: Distribute: $3x - 6 + 20 = 2x + 4$
Step 3: Combine similar terms: $3x + 14 = 2x + 4$
Step 4: Get the variable on one side: $3x - 2x + 14 = 4$, $x + 14 = 4$
Step 5: Simplify by adding and subtracting: $x = -10$

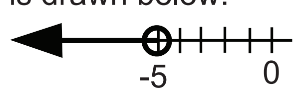
Note: As shown here, not all steps will be needed every time.

2. *Inequalities*

Solve these using the exact same procedures as you would with equations. **EXCEPT, when you multiply or divide by a negative number, you must reverse the inequality sign.**

> Ex. Solve and graph: $3(2x - 1) > 8x + 7$

Graphically, the solution is drawn below:



$x < -5$

Solution:

$$6x - 3 > 8x + 7$$
$$-8x \qquad -8x$$ subtract 8x from both sides

$$-2x - 3 > 7$$
$$+3 \qquad +3$$ add 3 to both sides

$$\frac{-2x}{-2} > \frac{10}{-2}$$ divide by -2 and reverse the inequality sign

$$x < -5$$

Note: If the variable drops out and you are left with a false statement, there is no solution.
If you are left with a true statement, there are an infinite number of solutions.

Ex.1 Solve: $3x + 4 \leq 2(x+2) + x$

Solution:

$$3x + 4 \leq 2x + 4 + x$$
$$3x + 4 \leq 3x + 4$$
$$-3x \qquad -3x$$
$$4 \leq 4$$

True statement. Infinite number of solutions (all real numbers).

Ex.2 Solve: $2x + 3 < 2x - 2$

Solution:

$$2x + 3 < 2x - 2$$
$$-2x \qquad -2x$$
$$3 < -2$$

False statement. (No value of x can make this true.)
No solution.

Try These

1. Solve: $\dfrac{3(y + 4)}{2} = \dfrac{2(y + 3)}{3} + 9$

2. Solve and graph: $3x + 4 < 4(x + 2)$

C. *Solving Rational Equations (equations with variables in the denominator)*

These are solved the same as any other equation, but with these **you must "check" your answer**. If your answer makes a denominator zero, you can't use it.

Ex. 1 Solve: $\dfrac{(x - 3)}{(x + 4)} = \dfrac{(x - 2)}{(x + 2)}$

Note: A fraction is undefined when its denominator equals 0.
So, with this example, x cannot equal -4 or -2.

Solution: Multiply both sides by the common denominator: $(x + 4)(x + 2)$:

$$\frac{(x + 4)(x + 2)}{1} \bullet \frac{(x - 3)}{(x + 4)} = \frac{(x + 4)(x + 2)}{1} \bullet \frac{(x - 2)}{(x + 2)}$$

$$(x + 2)(x - 3) = (x + 4)(x - 2)$$

$$\begin{aligned} x^2 - 3x + 2x - 6 &= x^2 - 2x + 4x - 8 \\ -x - 6 &= 2x - 8 \\ +x \quad &\quad +x \\ -6 &= 3x - 8 \\ +8 \quad &\quad +8 \\ 2 &= 3x \\ \frac{2}{3} &= x \end{aligned}$$

Check:
Since 2/3 does not make the denominator = 0, then it can be assumed that it works.

Ex. 2 Solve: $\frac{1}{(x - 4)} + \frac{1}{(x + 2)} = \frac{6}{(x - 4)(x + 2)}$

Solution: Multiply both sides by the common denominator.

$$\frac{(x - 4)(x + 2)}{1} \cdot \frac{1}{(x - 4)} + \frac{1}{(x + 2)} = \frac{6}{(x - 4)(x + 2)} \cdot \frac{(x-4)(x+2)}{1}$$

$$(x + 2) + (x - 4) = 6$$

$$2x - 2 = 6$$

$$2x = 8$$

$$x = 4$$

Check:
Since 4 makes $x - 4 = 0$, we cannot use it. Therefore, there is ***no solution***.

Try These

1. For what values of x is $\dfrac{x + 3}{(x - 3)(x + 4)}$ undefined?

2. Solve: $\dfrac{2x + 1}{x - 5} = \dfrac{x + 5}{5 - x}$

D. *Evaluating and Solving Formulas*

Many problems on the ACT require you to plug numbers into formulas.

(***This will not be hard if you go slowly.***)

To do this, replace each variable with its given value, then simplify using the correct order of operations.

Make sure you use parentheses when you put a number in for a letter or variable--

Sometimes they are unnecessary, but they never hurt.

> Ex.1 Given $a = -1$, $b = 2$, $c = -2$, what is the value of $a^2b - b^2c$?

Solution:

$$\begin{aligned} a^2b - b^2c &= (-1)^2(2) - (2)^2(-2) \\ &= (1)(2) - (4)(-2) \\ &= 2 - (-8) \\ &= 2 + 8 \\ &= 10 \end{aligned}$$

Note: It helps to keep things straight if you use parentheses.

Other problems on the ACT require you to solve a formula for a given variable (letter). Do this by pretending that letter is "x" and then solve as always.

> Ex. 2 Solve for *a*: $d = 1/2 \bullet at^2$

Solution:

$$d = \frac{1}{2} \bullet at^2$$

$$2d = at^2 \qquad \textit{(multiply both sides by 2)}$$

$$\frac{2d}{t^2} = a \qquad \textit{(divide both sides by } t^2\textit{)}$$

Ex. 3 If $f(x) = 2x^2 - 5x$, *what's* $f(-4)$?

Solution: First, substitute the value -4 into the given function for each variable, using parentheses:

$f(-4) = 2(-4)^2 - 5(-4)$

Now simplify using order of operations:

$f(-4) = 2(16) - (-20)$

$f(-4) = (32) + (20)$

$f(-4) = 52$

Ex. 4a $f(x) = x^2 + 1$ & $g(x) = 3x - 1$
Find $f(g(2))$

Solution: $f(g(2))$: $g(2) = 3(2) - 1 = 5$
$f(5) = 5^2 + 1 = 26$

Ex. 4b $f(x) = x^2 + 1$ & $g(x) = 3x - 1$
Find $f(g(x))$

Solution: $f(g(x)) = f(3x - 1) = (3x - 1)^2 + 1$

Try These

1. Given $x = -2$, $y = 3$, and $z = -4$, evaluate:

a) $xy^2 - z$

b) $\dfrac{x(y-z)^2}{y}$

2. Given $f(x) = 3x^2 - 2x + 1$, find $f(-1)$

3. Find $f(g(-3))$ if $f(x) = x + 2$ and $g(x) = 2x^2 - 1$

4. Solve $A = 2\pi rh + 2\pi r^2$ *for h.*

5. Translate into an expression:
Car 1 is going r miles/hr. Car 2 is travelling 10 miles/hr faster.
In terms of r, what is the total number of miles travelled
by the two cars in 4 hours?

E. *Factoring*

There are 4 main types of factoring:

1. *Taking out the greatest common factor (GCF)*

Ex. 1 Factor: $6x^2 - 3x$

Solution: Step 1: The GCF = $3x$
Step 2: Divide each term by $3x$
Step 3: Place the GCF on the outside

$$6x^2 - 3x = 3x(2x - 1)$$

Note: You can check your answer by multiplying it together. You should get back to the original problem.

2. *Difference of squares and the sum or difference of cubes*

a) If you have two terms, see if they are the difference between two perfect squares.

Factor $a^2 - b^2$ into $(a - b)(a + b)$

Ex.1 Factor: $4x^2 - 9$

Note: Check by "FOILing"

Solution: $4x^2$ is the square of $2x$, it's our "a"
9 is the square of 3, it's our "b"
So, $4x^2 - 9 = (2x - 3)(2x + 3)$

Ex. 2 Factor: $25x^2 + 64$

Solution: Cannot be factored because there is no way, using real numbers, to make the middle term drop out.

b) If you have two terms and they're not the difference of squares, maybe they're the <u>sum</u> or <u>difference</u> of cubes.

Factor $a^3 - b^3$ into $(a - b)(a^2 + ab + b^2)$
$a^3 + b^3$ into $(a + b)(a^2 - ab + b^2)$

Ex.3 Factor: $27x^3 - 8$

Solution: $27x^3$ is the cube of $3x$, it's our "a"
8 is the cube of 2, it's our "b"
$a^3 - b^3$ into $(a - b)(a^2 + ab + b^2)$
So, $27x^3 - 8 = (3x - 2)(9x^2 + 6x + 4)$

3. *With three terms, try to unFOIL.*
This is a trial and error process. (There are other methods, but most people can't remember them.)

Ex. 1 Factor: $x^2 - 3x - 10$

Solution:
The first term is x^2 so we have $(x \quad)(x \quad)$
The 10 is negative so we know the signs are different.
So, now we have $(x - \quad)(x + \quad)$
Finally, since $2 \bullet (-5) = -10$ and $2 + (-5) = -3$
we know that $x^2 - 3x - 10 = (x - 5)(x + 2)$

Ex. 2 Factor: $2x^2 + 11x - 6$

Solution: First, $2x^2$ must have come from $2x$ and x, so we have $(2x \quad)(x \quad)$. Next, by trial and error we check pairs of numbers which multiply to -6 until we have a middle term of $11x$.

Remember, you only have to check the middle term.

Finally, $2x^2 + 11x - 6 = (2x - 1)(x + 6)$

Ex. 3 Factor: $2x^2 + 4x - 30$ (with a GCF)

Solution: $2x^2 + 4x - 30$
$2(x^2 + 2x - 15)$
$2(x + 5)(x - 3)$

4. *With 4 terms, factor by grouping*

Ex. Factor: $xy + 7x + 3y + 21$

Solution:

Step 1: Group the first two terms together and take out the x. Group the last two terms together and take out the 3.

$xy + 7x + 3y + 21 = x(y + 7) + 3(y + 7)$

Step 2: Take out the GCF $(y + 7)$. The factor $(x + 3)$ will be "left" on the inside.

$(x + 3)(y + 7)$

5. Sometimes you will have to do more than one step.

Always remember to apply the "Yogi Berra Theorem":

"A problem's not over until it's over."

Translation:
Keep factoring as long as you can...

Ex.1 Factor: $6x^5 - 6x$

Solution:

Step 1: Take out GCF $6x^5 - 6x = 6x(x^4 - 1)$

Step 2: Factor by difference between squares.

$6x(x^2 - 1)(x^2 + 1)$

Step 3: Finish by again factoring the difference between squares:

$6x(x - 1)(x + 1)(x^2 + 1)$

Ex.2 Factor: $x^6 - 1$

Solution:

Step 1: Apply difference between squares:

$(x^6 - 1) = (x^3 - 1)(x^3 + 1)$

Step 2: Apply difference & sum of cubes:

$(x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$

Try These

Change the following expanded form(s) into factored form(s):

1. $4x^3 - 25x$

2. $x^2 + 3x - 4$

3. $2x^2 - 5x - 3$

4. $3x^2 - 7xy + 2y^2$

F. *Solving Quadratic Equations*

1. *Square root method*
   If the side of the equation with the variables is a perfect square, then you can solve the equation by taking the square root of both sides.

Ex. $(x-3)^2 = 25$

Solution: $(x - 3)^2 = 25$

$x - 3 = \pm\sqrt{25}$

note + or -

$x - 3 = 5$ **or** $x - 3 = -5$

$+ 3$ $\quad$ $+ 3$

$x = 8$ $\quad$ $x = -2$

2. *Factoring*
   Most quadratic equations cannot be easily solved with the square root method, but if the equation factors, then this is usually the best approach.

Ex. $x^2 - 3x = 10$

Solution:

First, set the equation equal to zero.

So, $x^2 – 3x = 10$ becomes $x^2 – 3x – 10 = 0$

Next, factor $(x – 5)(x + 2) = 0$

Finally, set each factor = 0

$x – 5 = 0$ or $x + 2 = 0$

$x = 5$ $\quad$ $x = -2$

3. *Quadratic Formula*
   A final way to solve a quadratic equation is by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: Solutions to quadradic equations are sometimes called "roots" or " zeros".

Ex. Solve: $x^2 + 6x + 4 = 0$

Solution:

For this example a = 1, b = 6, and c = 4

Therefore,

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm \sqrt{4 \cdot 5}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2\left(-3 \pm \sqrt{5}\right)}{2}$$

$$x = -3 \pm \sqrt{5}$$

Try These

1. $(x + 3)^2 = 16$

2. $x^2 - 6x = -8$

3. $x^2 + 10x + 3 = 0$

G. *Exponents*

1. *Multiplying*
   To multiply "like" bases, just add the exponents.

   Ex. 1 $x^2 \cdot x^3 = (x \cdot x)(x \cdot x \cdot x) = x^{2+3} = x^5$

   Ex. 2 $3x^3y \cdot 4xy^5 = (3 \cdot 4)(x^3 \cdot x^1)(y^1 \cdot y^5) = 12x^4y^6$

   Note: Multiply coefficients as always, and remember $y = y^1$

   Ex. 3 $2^5 \cdot 2^4 = 2^9$

2. *Dividing*
   To divide "like" bases, just subtract exponents.

   Ex.1 $\dfrac{x^4}{x^3} = x^{4-3} = x^1 = x$

   Ex.2 $\dfrac{10x^2y^5}{5xy^2} = 2xy^3$

   Note: Divide coefficients as always.

   Ex. 3 $\dfrac{6xy^2z}{8x^3yz^4} = \dfrac{6 \cdot x \cdot y \cdot y \cdot z}{8 \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z \cdot z \cdot z} = \dfrac{3y}{4x^2z^3}$

   Note: If the larger exponent is on the bottom, the answer will be on the bottom.

3. *When raising a "power to a power," multiply exponents.*

   Ex. 1 $(x^2)^4 = (x^2)(x^2)(x^2)(x^2) = x^{2 \cdot 4} = x^8$

   Note: Treat numbers like letters.

   Ex. 2 $(3x^3y^2)^4 = 3^{1 \cdot 4} \cdot x^{3 \cdot 4} \cdot y^{2 \cdot 4} = 81x^{12}y^8$

4. *Negative and zero exponents*

Since $\frac{x^5}{x^5} = x^{5-5} = x^0 \; and \; \frac{x^5}{x^5} = 1$, anything divided by itself = 1.

So, $x^0 = 1$. (That is, anything raised to the zero power = 1.)

Note: $-x^0 = -(x^0) = -(1) = -1$

Next, since $\frac{x^5}{x^8} = x^{5-8} = x^{-3} \; and \; \frac{x^5}{x^8} = \frac{1}{x^{8-5}} = \frac{1}{x^3}$; x^{-3} is defined to equal $\frac{1}{x^3}$ Or, in general $x^{-n} = \frac{1}{x^n}$

So, anytime you see a negative exponent, just move the base to the other side of the division bar and make the exponent positive.

Ex. 1 $\frac{-8x^{-2}y^{3}z^{-3}}{2x^{-3}y^{-2}z^{4}}$

$-4x^{(-2\,-\,-3)}y^{(3\,-\,-2)}z^{(-3\,-\,4)}$

$-4xy^5z^{(-7)} = \frac{-4xy^5}{z^7}$

Ex. 2 $3x^{-2} = \frac{3}{x^2}$

Ex. 3 $(3x^2y^3)^0(-4xy)$

$(1)(-4xy)$

$-4xy$

Ex. 4 $(3x^2y^0)(-4xy)$

$(3x^2)(-4xy)$

$-12x^3y$

Try These

Simplify

1. $(3x^2y^{-3})(4xy^4)$

2. $(x^2y)^3(2xy^2)^2$

3. $5x^0(-2xy)^2$

4. $\frac{(2x^2y)^3}{4xy^{-2}}$

H. Square Roots

1. *Approximating*
   By knowing that $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, etc., we can approximate the square root of smaller numbers.

*Ex.*1 What is an approximation for $\sqrt{50}$?

Solution:

Since $7^2 = 49$, we know that $\sqrt{50} > 7$

Secondly, since 49 is close to 50, we know that it is just a little over 7.

2. *Simplifying*
   To simplify a square root, remove (pull out) anything you can.

Ex. 1a	Using a calculator, which of the following is equivalent to $\sqrt{192}$

a. $2\sqrt{48}$
b. $4\sqrt{12}$
c. $8\sqrt{3}$
d. $16\sqrt{3}$
e. $24\sqrt{2}$

In this case a, b, and c are correct. On the ACT, there will only be ONE choice equivalent to $\sqrt{192}$. It, however, may not be in simplest form.

Ex. 1b Simplify without a calculator $\sqrt{192}$

Solution:
$$\sqrt{192}$$
$$\sqrt{64} \bullet \sqrt{3}$$
$$8\sqrt{3}$$

3. *Adding & subtracting*
The key here is to simplify first.

Ex. 1 Add $3\sqrt{20}+\sqrt{45}$

Solution:

$$3\sqrt{20}+\sqrt{45}=3\sqrt{4\bullet 5}+\sqrt{9\bullet 5}=6\sqrt{5}+3\sqrt{5}=9\sqrt{5}$$

Note: To add square roots, the values inside each radical must be the same.

Watch Out! There will only be one correct answer, but it can be written in many equivilent forms ...

4. *Multiplying & dividing*
The key here is to multiply “outsides” with “outsides” and “insides” with “insides”.

Ex. 1 Simplify $6\sqrt{20}\bullet 3\sqrt{30}$

Solution:

$$6\sqrt{20}\bullet 3\sqrt{30}=(6\bullet 3)(\sqrt{20\bullet 30})$$
$$=18\sqrt{600}$$
$$=18\sqrt{100}\sqrt{6}$$
$$=18\bullet 10\sqrt{6}$$
$$=180\sqrt{6}$$

Ex. 2 Simplify $\dfrac{5\sqrt{27}}{10\sqrt{3}}$

Solution:

$$\frac{5\sqrt{27}}{10\sqrt{3}}=\frac{5\sqrt{9}\sqrt{3}}{10\sqrt{3}}=\frac{5\bullet 3\sqrt{\cancel{3}}}{10\sqrt{\cancel{3}}}=\frac{15}{10}=\frac{5\bullet 3}{5\bullet 2}=\frac{3}{2}$$

5. *Rationalizing*

In simple form, a square root cannot be in a denominator. Remove square roots from a denominator by multiplying the top and bottom (on the inside) by whatever it takes to get the radical (square root) to go away.

Ex. 1 Simplify $\sqrt{\frac{2}{3}}$

Solution:

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2\bullet 3}}{\sqrt{3\bullet 3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3}$$

Ex. 2 Simplify $\frac{5}{3-\sqrt{2}}$

Solution:

$$\frac{5}{(3-\sqrt{2})}\bullet\frac{(3+\sqrt{2})}{(3+\sqrt{2})} = \frac{5(3+\sqrt{2})}{9+3\sqrt{2}-3\sqrt{2}-\sqrt{4}} = \frac{5(3+\sqrt{2})}{9-2}$$

$$= \frac{15+5\sqrt{2}}{7}$$

Note: Since the bottom has 2 terms with only one piece under the root sign, we multiply top and bottom by the "conjugate."
(That is, the same thing only with the sign in the middle changed.)

Try These

Simplify

1. $\sqrt{72}$

2. $3\sqrt{40} - 2\sqrt{90} + \sqrt{10}$

3. $\dfrac{5\sqrt{80}}{4\sqrt{5}}$

4. $4\sqrt{6} \bullet 3\sqrt{12}$

I. *Systems of Equations*

1. *Solving by using the addition method*
   The goal of the addition method is to eliminate one of the variables. To accomplish this, multiply one of the equations by a number (on both sides) so that the coefficients of one of the variables are equal yet opposite in sign.

Ex. 1 Solve: $3x + 2y = 10$ $x - 5y = {}^{-}8$

Solution:

Multiply bottom equation by -3:
-3(x - 5y) = -8(-3) or -3x + 15y = 24
Next, add the 2 equations:

$$\begin{array}{r} 3x + 2y = 10 \\ \underline{-3x + 15y = 24} \\ 17y = 34 \end{array}$$

Solve for y: **y = 2**

Now, substitute 2 in for y into either original equation:
x – 5(2) = -8
x – 10 = -8
Solve for x: **x = 2**

Finally, write the answer as an ordered pair: (**2,2**)

Note: Sometimes you will need to multiply both equations by a number.

Ex.2 Solve: 3x + 5y = 13   2x – 2y = -2

One possible way is to multiply the top equation by 2 and the bottom equation by 5:

$$\begin{array}{lcr} 2(3x + 5y = 13) & \qquad & 6x + 10y = 26 \\ 5(2x - 2y = -2) & & \underline{10x - 10y = -10} \\ & & 16x \qquad = 16 \end{array}$$

Solve for x: **x = 1**
Substitute 1 for x: 3(1) + 5y = 13, 3 + 5y = 13, 5y = 10, **y = 2**
Write the answer as an ordered pair: **(1,2)**

2. *Solving by substitution*

To use this method, first solve for one of the variables in either equation. Next, substitute this expression for the variable into the other equation. Finally, solve as always.

> Ex.1 Solve: $3x + 2y = 10$
> $x - 5y = -8$

Solution:

By solving the second equation for x (it's the easiest since its coefficient is 1), we have:

$x - 5y = -8 \qquad x = (-8 + 5y)$

By substituting (-8 + 5y) into the first equation in place of x, we can now solve for y:

$$3(-8 + 5y) + 2y = 10$$
$$-24 + 15y + 2y = 10$$
$$-24 + 17y = 10$$
$$17y = 34$$
$$\mathbf{y = 2}$$

Now, substitute 2 in for y: $x = -8 + 5(2)$

$$x = -8 + 10$$
$$\mathbf{x = 2}$$

Write the answer as an ordered pair: **(2,2)**

Note: As before, if all variables drop out, then there is no solution (false statement remains) or infinite solutions (true statement remains).

> Ex.2 Solve: $2x + 4y = 6$
> $x + 2y = -4$

Solution:

Multiply bottom equation by -2, then add.

$$\begin{array}{lcr} 2x + 4y = 6 & \Rightarrow & 2x + 4y = 6 \\ -2(x + 2y = -4) & \Rightarrow & (+)\ \underline{-2x - 4y = 8} \\ & & 0 = 14 \end{array}$$

Since all the variables dropped out and a **false statement** remains, there is **no solution.**

Try These

1. $2y = -3x + 10$
   $x - 4y = 8$

2. $-4b + 3a = 2$
   $b + 2a = 5$

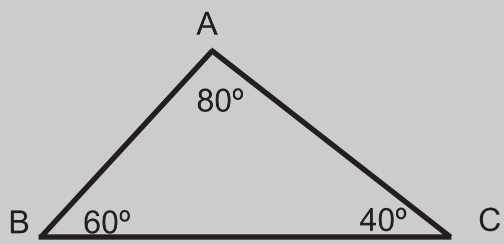
3. $3x - y = 4$
   $6x = 8 + 2y$

4. $2c + d = 10$
   $\frac{1}{2}c + \frac{1}{4}d = 5$

IV. GEOMETRY

A. Triangle Theorems and Definitions

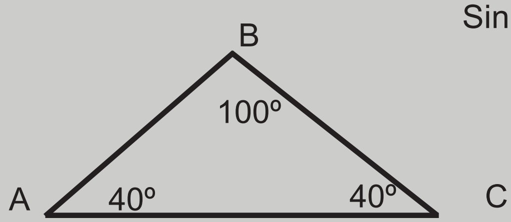
1. The sum of the interior angles of a triangle is 180°.
2. An acute angle has measure < 90°.
   A right angle has measure = 90°.
   An obtuse angle has measure > 90°.
   An isosceles triangle has 2 (or more) congruent sides.
   An equilateral triangle has 3 congruent sides. (All angles are congruent, too.)
3. In any triangle, the longest side will be opposite the biggest angle; the "second" longest side will be opposite the "second" biggest angle, etc.



Since $\measuredangle$ A is the biggest angle,
$\overline{BC}$ is the longest side.
$\measuredangle$ B is the next largest,
so, $\overline{AC}$ is the second longest side.
$\measuredangle$ C is the smallest angle,
so, $\overline{AB}$ is the shortest side.

4. If 2 sides of a triangle are equal, then the angles opposite those sides are also equal (and vice versa).

Since $m\measuredangle A = m\measuredangle C = 40^\circ$, $\overline{AB} = \overline{BC}$



5. In any triangle, the sum of the lengths of any two sides must be greater than the length of the third side.

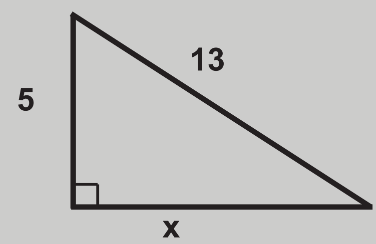
Ex. Two sides of a triangle measure 3 and 5.
The length of the third side must be between what 2 integers?

Solution: Assume the third side is the longest. Therefore, it must be less than 3 + 5 = 8. Next, assume the "5" side is the longest. Therefore, (the third side + 3) must be greater than 5. That is, (x + 3 > 5), so the third side must be > 2.

6. *Pythagorean Theorem*
In short, this theorem states that the sum of the squares of the 2 legs of a right triangle must equal the square of the hypotenuse.

(That is, leg^2 + leg^2 = hypotenuse^2)

Ex. 1 Given the following right triangle, what is the length of the missing side?



Solution:

Since the 13 is the hypotenuse, it goes by itself.

So we have: $5^2 + x^2 = 13^2$

$$25 + x^2 = 169$$
$$-25 \qquad -25$$
$$x^2 = 144$$
$$x = 12$$

7. *Pythagorean Triples*
Some sets of 3 integers will always form a right triangle (for example, 3,4, and 5). It can be proven that any multiple of a set of integers that forms a right triangle (a Pythagorean triple) also forms a right triangle.

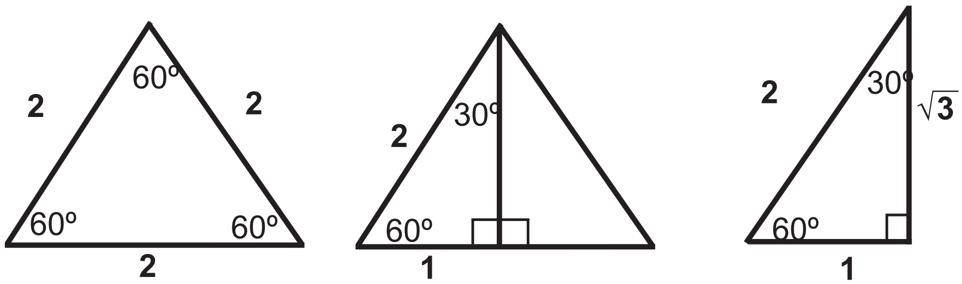
Ex.1 7, 24, and 25 are a Pythagorean triple.
Would the lengths 70, 240, and 250 form a right triangle?

Solution:

The set 70, 240, and 250 is a multiple of 7, 24, and 25.

So, yes, they would form a right triangle.

8. *Special right triangles*
The first special right triangle is splitting an equilateral triangle in half.



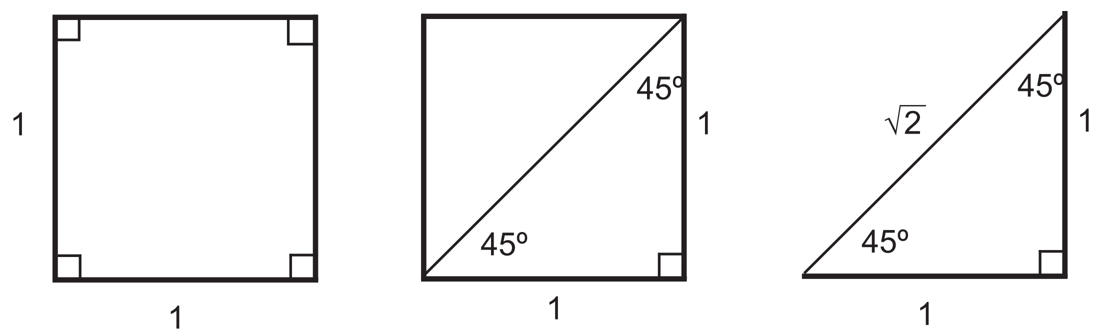
For convenience, let each side of the original triangle have length 2. Therefore, the "short leg" has length 1, and by the Pythagorean theorem, the "long leg" has length $\boxed{\sqrt{3}}$. Using these proportions, missing lengths can be calculated.

Ex.1 Given the following 30° - 60° - 90° triangle, what are the lengths of the missing sides?

Solution:

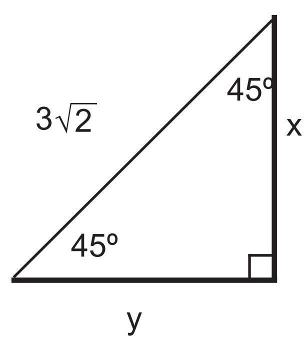
First draw and label a 30°– 60°– 90° triangle next to the original, and use it as a reference. Next, recognize that since the short leg is always half the hypotenuse it must have length 5. Finally, the long leg always equals the short leg times $\sqrt{3}$. So, it must be $5\sqrt{3}$.

Another special triangle is the right isosceles (45 – 45 – 90) triangle.
It can be formed by cutting a square in half diagonally.



For convenience, let each side of the square have length 1. By the Pythagorean theorem, the hypotenuse must have length $\boxed{\sqrt{2}}$.

Ex. 2 Find the length of sides x & y in the right triangle below:



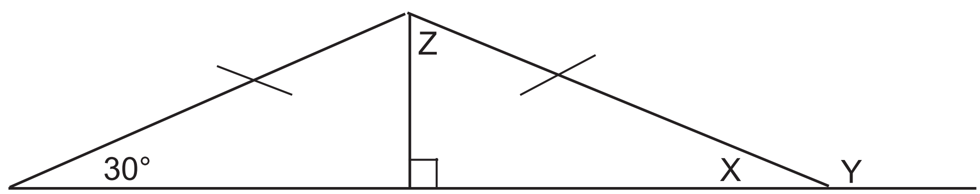
Solution:

Notice that the short leg of a 45 - 45 - 90 triangle is always equal to the hypotenuse length divided by $\sqrt{\mathbf{2}}$.

So, the length of the leg(s) must be 3.

Try These

1. Find the missing angles.



2. Would the following lengths form a triangle? If so, is it a right triangle?

a. 6, 8, 10

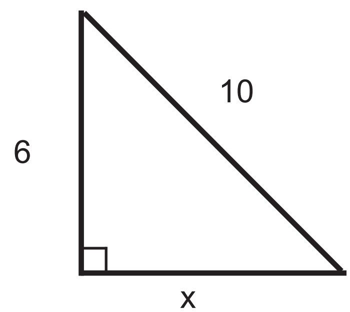
b. 2, 2, 4

c. 8, 10, 12

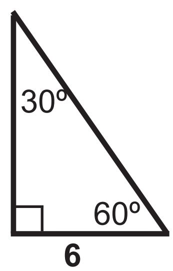
Try These (continued)

3. Find the missing side(s).

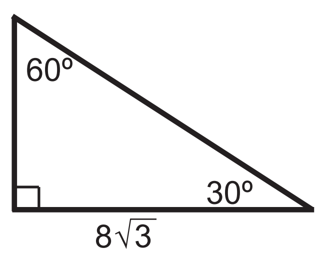
a.



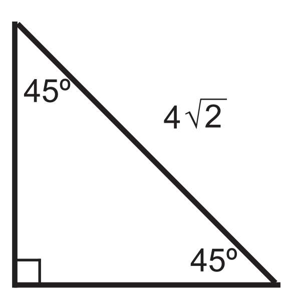
b.



c.



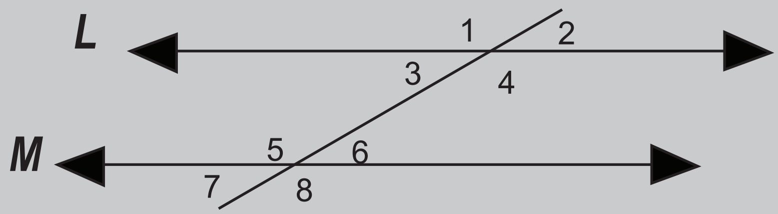
d.



B. *Parallel Line Theorems*

1. When 2 parallel lines are cut by a third line (usually called a transversal), the angles that look alike are equal.

Ex. If $L \parallel M$, what angles are equal to $\measuredangle 1$?

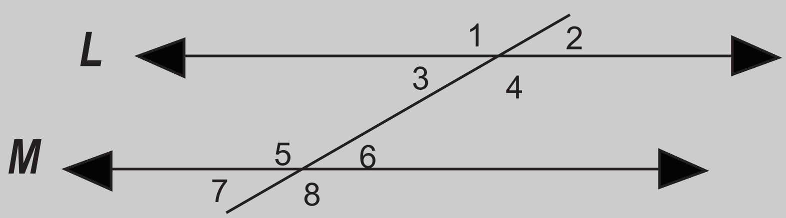


Solution:
$\measuredangle 1$, $\measuredangle 4$, $\measuredangle 5$ and $\measuredangle 8$ all look alike, and in fact are equal.

Note: $m\measuredangle 1 = m\measuredangle 4$ because they are across from each other (vertical). This will always be true.

2. When two parallel lines are cut by a third line, the inside (interior) angles on the same side of the transversal add to 180°.

Ex. If $L \parallel M$, which angles will add to equal 180°?



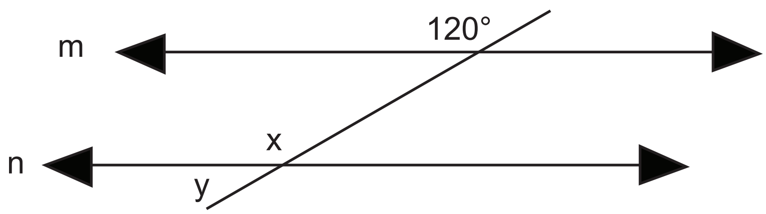
Solution:
$m\measuredangle 3 + m\measuredangle 5 = 180°$ (Interior angles add to 180°.)
$m\measuredangle 4 + m\measuredangle 6 = 180°$
Incidentally, $m\measuredangle 1 + m\measuredangle 7 = 180°$ (Exterior angles add to 180°.)
$m\measuredangle 2 + m\measuredangle 8 = 180°$

Note: $m\measuredangle 1 + m\measuredangle 2 = 180°$ Anytime two angles form a straight line together they will add to 180°.

3. Another related fact is that the opposite angles of a parallelogram are congruent. (So are the opposite sides.)

Try These

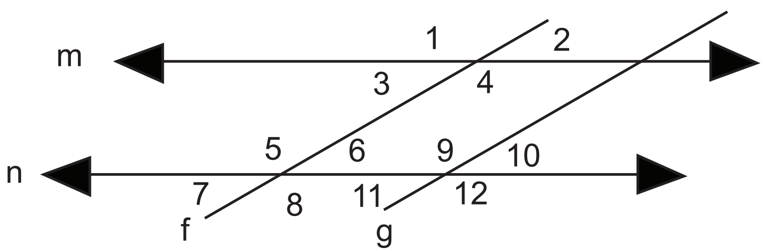
1. Find the value of the missing angles ($m \parallel n$).



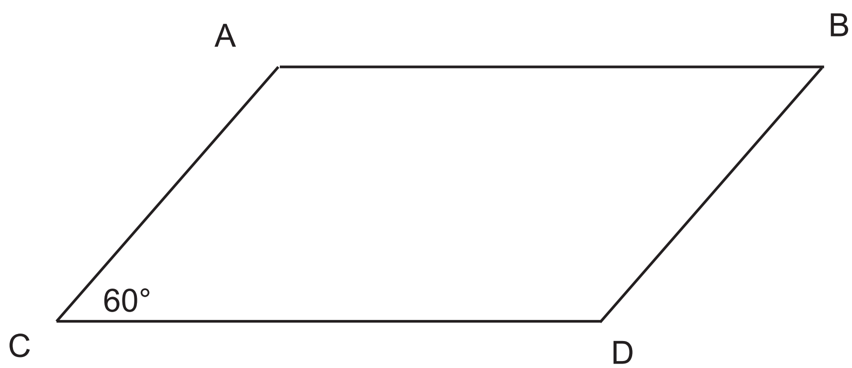
x =

y =

2. Name all angles equal to $m\sphericalangle 1$ (assume $m \parallel n$, and $f \parallel g$).



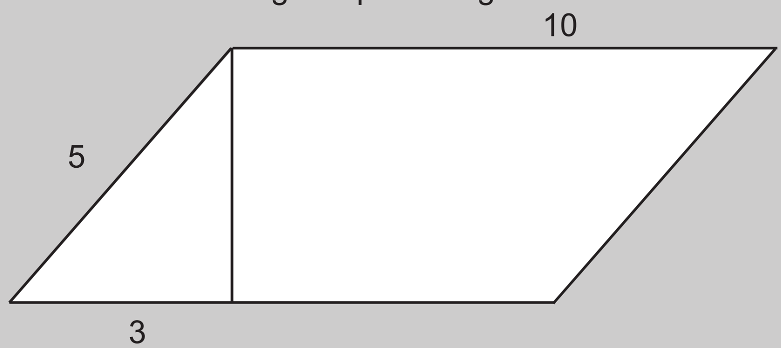
3. Find the missing angles of parallelogram ABCD.



C. *Area of Polygons*

1. The area of any parallelogram (including rectangles and squares) is the base times the height (A = bh).

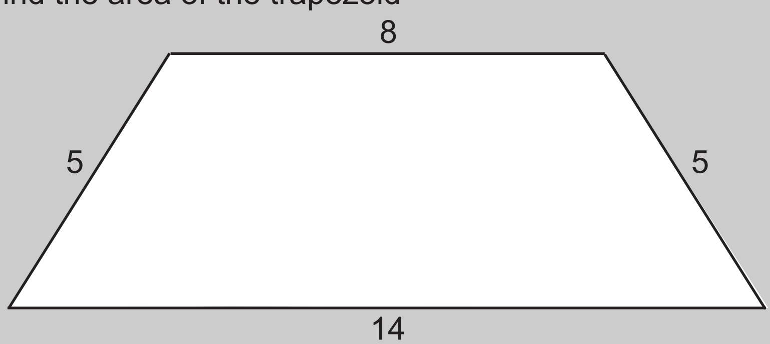
Ex. 1 Find the area of the given parallelogram.



Solution: Since the height is unknown we must find it. Using the Pythagorean Theorem, we write $h^2 + 3^2 = 5^2$. So, $h^2 + 9 = 25$, $h^2 = 16$, $h = 4$. Now, the area of the parallelogram can be found by taking base x height, $10 \times 4 = 40$.

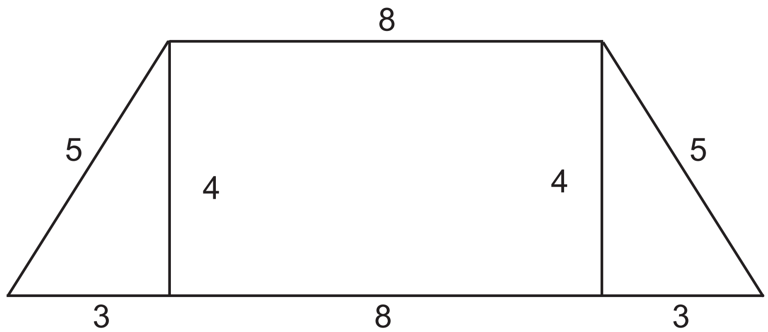
2. The area of a trapezoid can always be found by taking the height times the average of the bases.

Ex. 1 Find the area of the trapezoid



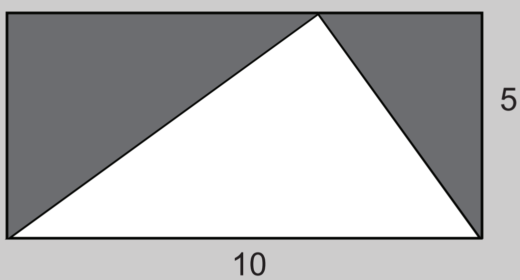
Solution:

By the Pythagorean Theorem, the height of the trapezoid can be found by $5^2 = 3^2 + b^2$. $b^2 = 16$ $b = 4$. The average of the bases = $\frac{(8+14)}{2} = 11$
So, the area of the trapezoid = 4 • 11 = 44



3. The area of any triangle is exactly ½ the area of a rectangle with its same base and height. Therefore, the area of a triangle = (base • height)/ 2, or (A = ½ bh).

Ex. 1 Find the area of the shaded triangles.



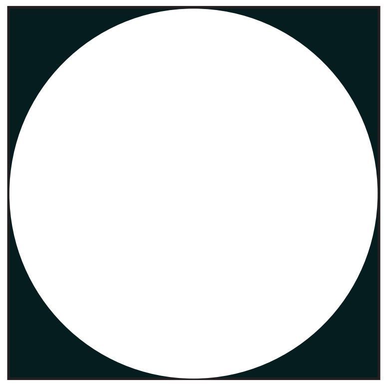
Solution: View the composite image as two separate objects. Now find the area of the "outside" shape minus the area of the "inside" shape.

The rectangle must have an area of 50, while the triangle has an area of
$\frac{1}{2}(10)(5) = 25$.

This leads to the answer:

outside - inside = 50 - 25 = 25

Ex. 2 Given a circle with radius 5, inscribed within a square, find the area of the shaded region as illustrated below:



Solution: View the composite image as two separate objects. Now find the area of the "outside" shape minus the area of the "inside" shape. The square must have an area of 100, while the circle has an area of 25 π. This leads to the answer:

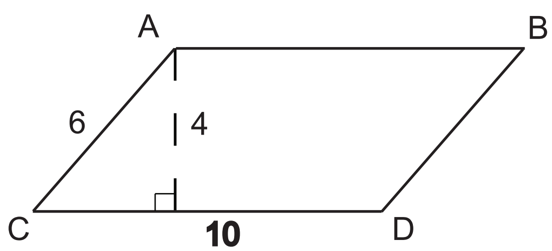
$$\text{Outside} - \text{Inside} = 100 - 25\pi$$

Note: The ACT may leave the solution in this form, or may reduce the final answer to a decimal. 100 - 25 (3.14) = 21.5

Try These

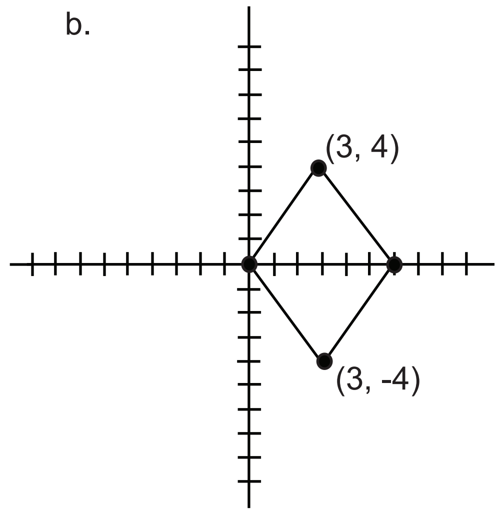
1. Find the area and perimeter of each figure.

a.



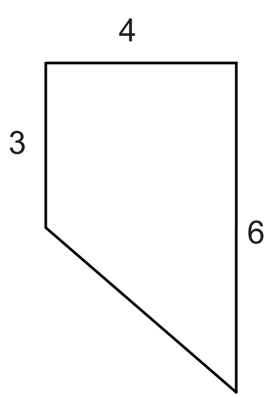
Note: ABCD is a parallelogram

b.

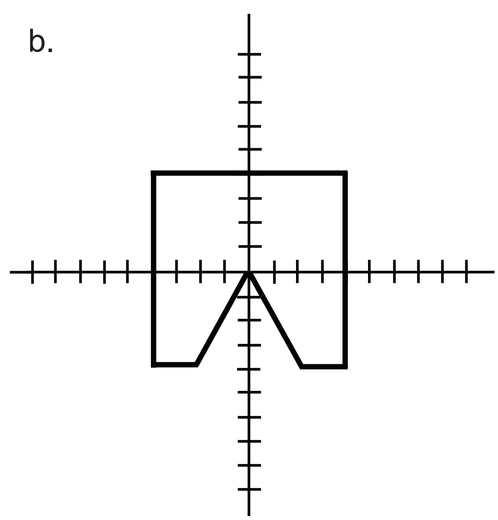


2. Find the area of each composite figure.

a.



b.



D. *Coordinate Geometry*

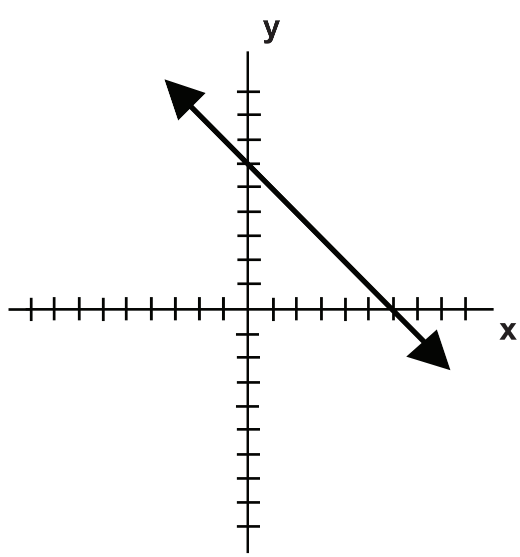
1. *The coordinate plane*
   The coordinate plane is the set of all points which have a 2 dimensional address: a horizontal coordinate and a vertical coordinate.

2. *Linear equations*
   Solutions to first degree equations with 2 variables will be ordered pairs. When graphed, these ordered pairs form a line.

Ex.1 Graph the solution set of $x + y = 6$

Solution:

(0,6), (6,0) and (3,3) are 3 solutions.
Graphing these 3 produces the following line:

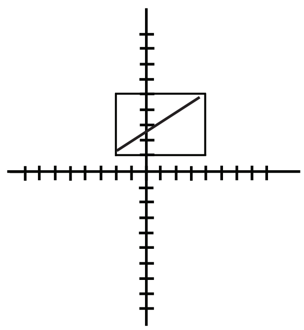


3. *Midpoint*

To find the midpoint between 2 points, find the average of the two x-coordinates and the average of the two y-coordinates.

Ex. 1 The endpoints of the diagonal of a rectangle are (-2,1) and (4,5). Where is the center of the rectangle?

Solution: First draw a picture.



The center is located at the midpoint of the 2 vertices.

The average of the x - coordinates is $\frac{-2+4}{2}=1$.

The average of the y - coordinates is $\frac{1+5}{2}=3$.

So, the midpoint of the 2 vertices (the center of the rectangle) is at (1,3).

4. *Slope*
The slope between two variables represents the relative rate of change of the two variables. It is usually defined to be

$$\frac{\text{how fast the line is changing in the y-direction}}{\text{how fast the line is changing in the x-direction}}$$

For convenience it is usually written

$\frac{\text{change in y}}{\text{change in x}}$ and can be calculated using the formula

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are the coordinates for two points on the line.

Ex.1 What is the slope of the line between (6,9) and (3,4)?

Solution: *To go from (6,9) to (3,4) you must go "down" -5 units in the y direction and "left" -3 units in the x direction.*

Using the formula for "slope":

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-9}{3-6} = \frac{-5}{-3}$$

Thus, the slope is $\frac{5}{3}$

5. *Finding the slope of a line*
The best way is to solve the given equation for y; (that is, put the equation in y = mx + b form). This way, you also determine the y-intercept (b).

Note: The y-intercept is the point where the line crosses the y-axis.

Ex.1 Find the slope and y-intercept of 3x + 4y = 12.

Solution:

$$3x + 4y = 12$$
$$4y = -3x + 12$$
$$y = -\frac{3x}{4} + 3$$

So, the slope is $m = -\frac{3}{4}$ and the y-intercept is $b = 3$

Note: The y-intercept could also be given as the point (0,3)

6. *Parallel and perpendicular lines.*
   If two lines are parallel, they have equal slope.

   Two lines that are perpendicular (meet at a 90° angle) have slopes which are negative reciprocals of each other.

Ex. 1 What is the slope of a line perpendicular to y = 3x + 4?

Solution:
Since the first line has slope 3,
any line perpendicular ($\perp$) to this
will have a slope value which is the
negative reciprocal; $m = -\frac{1}{3}$

7. *Finding the equation of a line*
You can use the equation $y = mx + b$
to write the equation of a line using given information.

> Ex.1 What is the equation of the line parallel to $y = 2x + 5$,
> and passing through the point (3,1)?

Solution:

The slope of $y = 2x + 5$ is $m = 2$, so our new line must also have the same slope, $m = 2$.

Now we know that $y = 2x + b$.

Next, to find the y-intercept, plug in the known point: $1 = 2(3) + b$.

Now, solve for b: $1 = 6 + b$, and $b = -5$.

This makes our equation $\boldsymbol{y = 2x - 5}$.

Try These

1. Find the slope and y-intercept of the line $3x - 5y = 15$.

2. Find the equation of the line which is parallel to $y = \frac{2}{3}x + 4$ with the same y-intercept as $y = 2x - 3$.

3. Find the equation of the line with an x-intercept of (3,0) and a y-intercept of (0,4).

4. Find the equation of the line that is the perpendicular bisector of the line segment that joins (1,1) and (5,3).

E. *Circles*

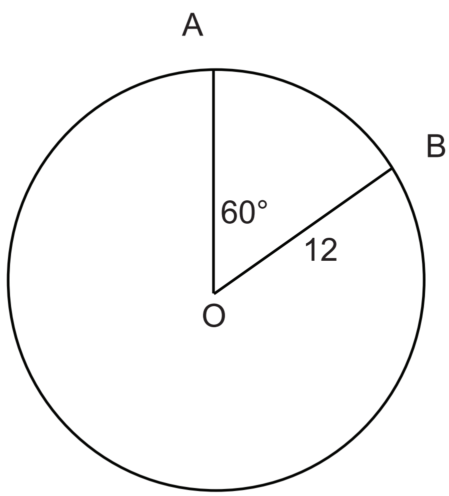
1. First, understand that for any circle pi (π) is the ratio of the circumference to the diameter: $\pi = \dfrac{C}{d}$.

   Consequently, $C = \pi d$ and $C = 2\pi r$.

   To find the area of a circle, use the equation $A = \pi r^2$.

2. To find the length of an arc, first find the circumference of the whole circle, then multiply this by the fraction of the circle your arc represents.

Ex.1 Given *r = 12*, find the length of arc AB.



Solution:

Since $C = 2\pi r$, we know the circumference is 24π.

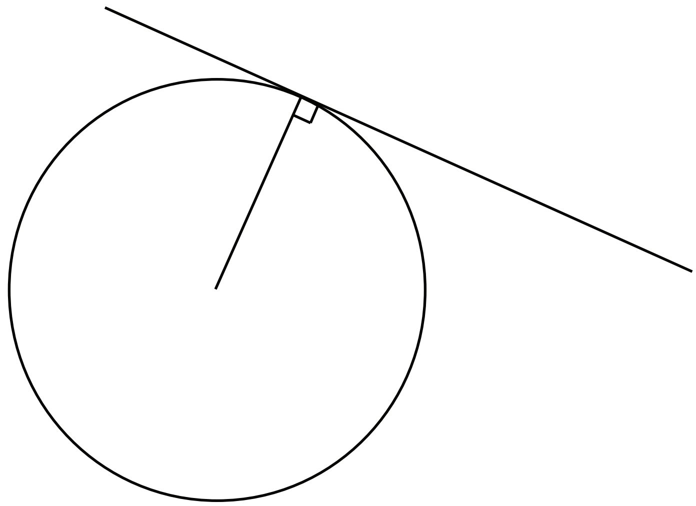
And since $\dfrac{60^\circ}{360^\circ} = \dfrac{1}{6}$, we want $\dfrac{1}{6}$ of the circumference.

Therefore, the measure of arclength $\widehat{AB} = \dfrac{24\pi}{6} = 4\pi$.

Since A = πR^2, we know the area = $\pi\ 12^2$ = 144π.

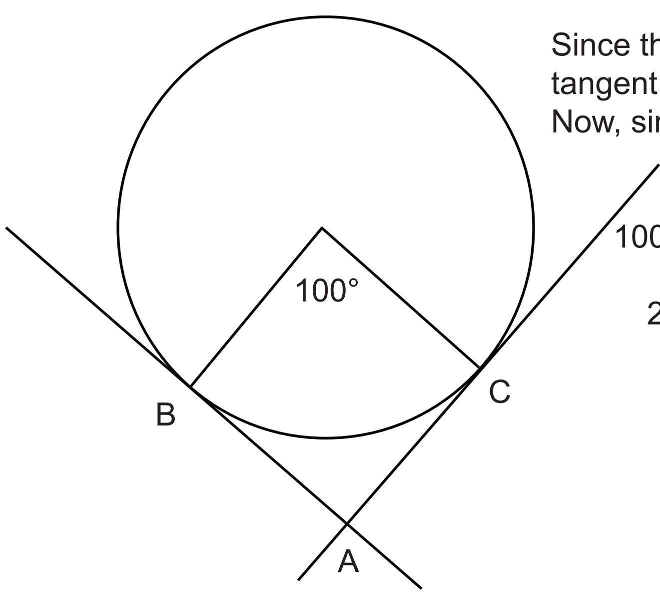
Therefore, the area of the sector is $144\pi/6 = 24\pi$

3. Another useful fact about circles is that a line tangent to a circle is perpendicular to the radius at the point of tangency. (see diagram)



This can be used to solve the following:

Ex.1 Given the diagram below, find the measure of $\measuredangle BAC$.



Solution:

Since the radius is perpendicular to the tangent, m $\measuredangle C$ and m $\measuredangle B$ must be 90°. Now, since the angles of a 4-sided figure must add to 360°, we get

$$100° + 90° + 90° + m\measuredangle BAC = 360°$$

$$280° + m\measuredangle BAC = 360°$$

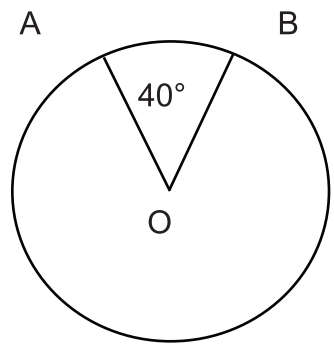
$$m\measuredangle BAC = 80°$$

Try These

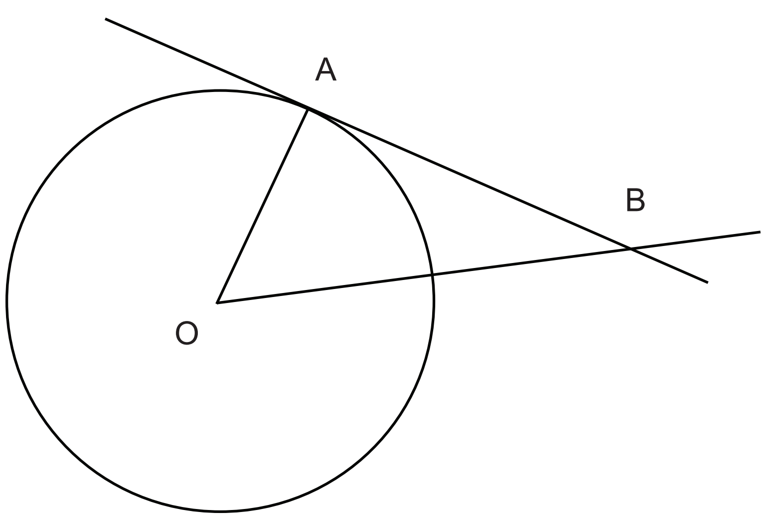
1. Complete the chart by filling in the missing information.

	radius	diameter	circumference	area
a	2			
b		6		
c			8π	
d				25π

2. Given the radius of circle O is 4, find the length of arc AB and the area of sector AOB.



3. Given measure of $\measuredangle ABO = 50°$, find the measure of $\measuredangle AOB$. (Assume $\overline{AB}$ is tangent to circle O at point A.)



F. *Volume and Surface Area*

1. The volume of almost any <u>solid</u> is the area of the base (whatever it is) times the third dimension (height, depth, thickness, etc). It is easier to remember how to find a volume by visualizing and not memorizing.

Ex. 1 Find the volume of a cylinder with a radius of 3 cm and a height of 10 cm.

Solution: Since the base is a circle its area is $\pi r^2 = \pi 3^2 = 9\pi$ cm$^{2.}$ If you multiply by the height, the volume would be 90π cm^3.

2. For <u>cones</u> and <u>pyramids</u>, it's the same, except you must divide by 3. (A pyramid is $\frac{1}{3}$ the volume of a corresponding prism with the same base and height)

Ex. 1 Find the volume of a pyramid with a square base of edge length 5 cm and a heigh of 12 cm.

Solution: The area of the base = $5^2 = 25$ cm^2

The Volume = $\frac{\text{Area of base} \times \text{height}}{3}$

The Volume = $\frac{25 \times 12}{3} = 100$ cm^3

3. To find the surface area of a sphere, use $A = 4\pi r^2$

The volume can be found using $V = \frac{4}{3}\pi r^3$

Ex.1 Find the surface area and volume of a sphere with a radius of 4.

Solution: The surface area (A) = $4\pi r^2 = 4\pi \bullet 4^2 = 64\pi$

The volume (V) = $\frac{4}{3} \bullet \pi \bullet r^3 = \frac{4}{3} \bullet \pi \bullet 4^3 = \frac{256}{3} \bullet \pi$

Note: Most of the time you will want to leave your answer in terms of π.

Try These

1. Find the volume of a prism with a base that is a right triangle with length of 3 cm and 4 cm and a height of 10 cm.

2. Find the volume of a cone which has a radius of 3 and a height of 12.

3. Find the surface area of a sphere with a diameter of 10.
(Hint: Find r.)

4. Find the volume of a sphere with a circumference of 14π.
(Hint: Find r.)

G. *Angles of Polygons*

1. For any polygon, the sum of the interior angles can always be found. The easiest way to do this is to remember the following: The sum of the interior angles of a triangle is 180°. The sum for a 4-sided polygon is 360°. From there, just remember each new side adds another 180°.

2. If it is a regular polygon, the measure of each angle can be found by dividing the sum by the number of sides.

3. The exterior angles always add to 360°.

4. The interior and exterior angle at a vertex add to 180°.

5. The following chart provides a summary:

Number of Sides	Sum of the Interior Angles	Measure of One Angle (if regular)	Sum of the Exterior Angles
3	180	60º	360º
4	360	90º	360º
5	540	108º	360º
6	720	120º	360º
⋮	⋮	⋮	⋮

Ex.1 Find the measure of an interior angle of a regular polygon with 10 sides.

Solution:

Since the figure has 10 sides (7 more sides than a triangle) we must add 7 180°'s to our original 180°. So, the sum of the angles would be 180° + 7(180°) = 1440°. Each angle would therefore be 1440°/10 = 144°.

Alternate solution:

Since the sum of the exterior angles is always 360°, each exterior angle of a 10-sided regular polygon is 36°.
Therefore, each of the interior angles is 180°– 36°= 144°.

Try These

1. Given a regular polygon has 12 sides find:

   (a) The sum of the measures of the interior angles.

   (b) The measure of one interior angle.

   (c) The measure of one exterior angle.

   (d) The sum of the measures of one exterior and one interior angle.

2. Given a regular polygon has an interior angle that measures 135°, how many sides does the polygon have?

H. *Reflections and Size Transformations*

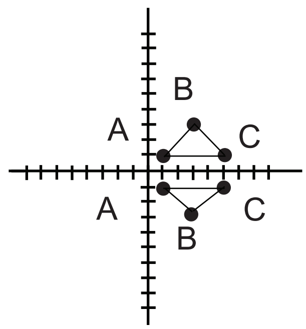
1. *Reflections*

When a 2-dimensional figure is reflected over a line, it becomes the "mirror" image on the other side. Consequently, the new figure is congruent to the original (it has the same area).

> Ex 1: A triangle has vertices at (1,1), (5, 1) and (3, 3). What is the area of this triangle after it has been reflected over the x-axis?

Solution:

Although it is not necessary to do the reflection to answer this question, it does show the process.



The new vertices are (1, -1), (5,-1) and (3, -3).
The base of this triangle is 4, height is 2.

So, the area is $\frac{1}{2}(4)(2) = 4$.

(The same area as the original triangle.)

2. *Size Transformation*

When every side of a figure is increased by the same factor (sometimes called the scale factor (SF) or the magnitude of the transformation), the new figure is the same shape, but may be larger (SF>1), smaller (SF<1), or the same size (SF=1).
Since the figure is increasing (decreasing) in two dimensions, the area of the new figure will be proportioned to the square of the scale factor. The ratio of the perimeters will still be the scale factor, however.

Ex 1: What is the area and perimeter of a 3 by 4 rectangle after each side has doubled in length?

Solution:

The perimeter of the original rectangle is $3 + 4 + 3 + 4 = 14$.
Since every length doubled, the perimeter would also double:
$P = 2(14) = 28$.

The area of the original rectangle is $3 \cdot 4 = 12$.
Since the scale factor is 2, the area would be multiplied by 2^2.
So the final area would be $12 \cdot 4 = 48$.

Try These

1. Find the vertices, area and perimeter of the following figure after it has been reflected over the y-axis (2,1), (2,4), (6,1).

2. If you multiply the circumference of a circle by 3, the area of the new circle will be how many times greater than the area of the original circle?

V. ALGEBRA II

A. *Fractional Exponents*

Fractional exponents are a convenient way to combine roots and exponents. Since $\sqrt{x} \bullet \sqrt{x} = x$ and since $x^{\frac{1}{2}} \bullet x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1$, we can define $\sqrt{x} = x^{\frac{1}{2}}$. Similarly, for any radical, the index (root) becomes the denominator. For example, $\sqrt[4]{x^5} = x^{\frac{5}{4}}$. In general, $\sqrt[n]{x^m} = x^{\frac{m}{n}}$.

This can also be applied to numbers, for example, $8^{\frac{2}{3}} = \sqrt[3]{8^2} = 2^2 = 4$. This is the most common use of fractional exponents on the ACT.

Try These

1. Simplify

a. $25^{\frac{3}{2}}$ b. $27^{\frac{-2}{3}}$

B. *Quadratic Relations*

1. *Circles*

The general equation for a circle is $(x – h)^2 + (y – k)^2 = r^2$, where the center is at (h, k) and r is the radius. If you are given the equation of a circle, you should be able to determine its center, its radius, and, hence, its graph. For example, given $(x – 3)^2 + (y + 4)^2 = 25$, the center would be at (3, -4) and the radius would be $\sqrt{25}$, or 5 . Sometimes, in order to find the center and radius, you will be required to “complete the square.”

Ex. 1a Find the center and radius of $(x + 2)^2 + (y - 4)^2 = 16$

Solution: With this example h = -2, k = 4, r = 4
So the center is (-2,4) and the radius is 4

> Ex. 1b Find the center and radius of $x^2 + y^2 - 4x + 6y = 12$

Solution: Step 1: Rearrange

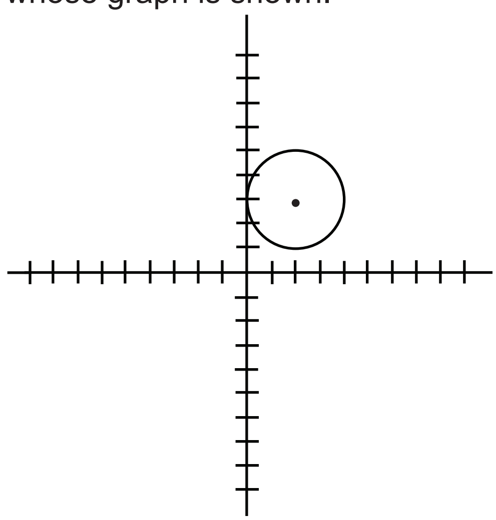
$x^2 - 4x + \_\_\_\_ + y^2 + 6y + \_\_\_\_ = 12$

Step 2: Complete the square by adding the square of half the number in front of x to both sides. (repeat for y).

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$$
$$(x - 2)^2 + (y + 3)^2 = 25$$

Therefore, the center is at (2, -3) and the radius is 5.

Sometimes you will be given the center and radius of a circle (or you are given a graph from which you can determine the center and radius). You should be able to write the equation of the circle. For example, find the equation of the circle whose graph is shown.



Solution:

The center is at (2,3)

and

the radius = 2.

Therefore, the equation of this circle

would be

$(x - 2)^2 + (y - 3)^2 = 4$

2. *Ellipses*

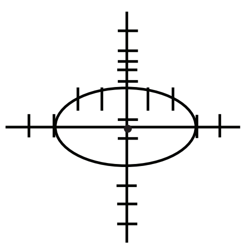
The general equation for an ellipse is usually written as

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, where the center is at (h,k) and the length of the major axis is 2a. Note: if the major axis is vertical, a^2 is under the $(y-k)^2$. As with circles, you should be able to use this information to find the center and the length of the minor and major axis from the equation. For example, given:

$\frac{(x-3)^2}{9}+\frac{y^2}{25}=1,$ the center is at (3,0)

$a^2=25,\ a=5,\ b^2=9,\ b=3.$ So, the length of the major axis is 10, and is vertical, and the length of the minor axis is 6, and is horizontal.

You should also be able to find the equation of an ellipse from its graph. For example, given the graph below, find the equation of the ellipse.



Solution: Since the center is at (0,0) and a = 3 (note, the major axis is horizontal) and b = 2. Therefore, the equation would be

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

3. *Hyperbolas*

If the center of the hyperbola is at the origin (0,0) (which is usually the case on the ACT), then the general equation of a hyperbola is

either $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

In the first case, the vertices are at (a, 0) and (-a, 0). In the second, the vertices are on the y-axis instead (0,a) and (0, -a).

This is probably all you need to know about hyperbolas.

4. *Parabolas*

Parabolas can appear on the ACT in one of two ways.

It might come in the form $y = ax^2 + bx + c$ (for example, $y = x^2 – 4x + 10$). This probably looks like a quadratic equation because — basically — it is.

The vertex of a parabola in this form can be found by recognizing that the x-coordinate of the vertex is $\frac{-b}{2a}$.

Plug this value into the equation for x to find y.

Ex.1 Find the vertex of the parabola $y = x^2 – 4x + 10$.

Solution: a = 1, b = 4, c = 10.

So the x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$.

The y-coordinate of the vertex can be found by substituting 2 for x:

$$(2)^2 - 4(2) + 10 = 6.$$

Therefore, the vertex is at (2, 6).

Also, note that if *a* is positive (as it is in this example) the parabola opens up. If *a* is negative, it opens down.

The second way parabolas may appear would be in the form
$y = a(x – h)^2 + k$ or $x = a(y – k)^2 + h$.

In either case the vertex is at (h, k). The value "*a*" tells you which way the parabola opens. The following chart summarizes this information:

Shape of the Parabola		
	$a > 0$	$a < 0$
$y = a(x-h)^2 + k$	∪	∩
$x = a(y-k)^2 + h$	⊂	⊃

Ex.2 Find the vertex of the parabola $y = \frac{-1}{2}(x - 4)^2 + 1$

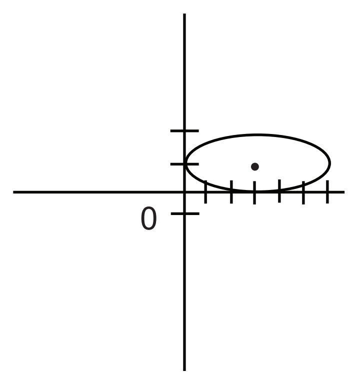
Solution: h = 4 and k = 1 So, the vertex is (4,1)

Try These

1. Find the center and radius of the following circles:

   a. $(x - 3)^2 + (y + 5)^2 = 25$ b. $x^2 + y^2 - 10y = 11$

2. Find the equation for the ellipse.



C. *Absolute Value Equations*

There are several types of absolute value equations that often come up on the ACT. There may be an inequality, such as:

$$|x-a| < b$$

Ex. 1 $|2x+1| < 9$

To solve, we could first eliminate the absolute value bars by expanding:

$\|2x+1\| < 9$	$-9 < 2x+1 < 9$
Then get x by itself	$-9-1 < 2x < 9-1$ $-10 < 2x < 8$
Now we have the solution	$-5 < x < 4$

Another method is to express the inequality conditionally:
To solve these, we again eliminate the absolute value bars by expanding.
These, however, become “or” statements.

Ex. 2 $|3x-2| > 5$

Solution:

This becomes $3x - 2 > 5$ ***or*** $3x - 2 < -5$

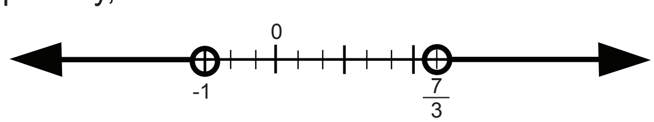
This is because numbers greater than 5 and numbers less than -5 all have absolute values bigger than 5.

Solving all statements for x gives the solution set:

$3x - 2 + 2 > 5 + 2$ or $3x - 2 + 2 < -5 + 2$

$3x > 7$ or $3x < -3$

Graphically, the solution is drawn like this:



Note: Solving absolute value problems with an equal sign works just like method 2 above, except the solution begins using an "or" statement.

Try These

1. $\left| 2x - 1 \right| < 7$

2. $\left| x + 3 \right| > 4$

3. $\left| 2x + 1 \right| = 5$

D. *Matrices*

For the last several years, one matrix question has appeared on every released test. These are usually straight forward applications that are quite easy if you have covered this content (and can remember it).

1. Matrix addition. To add two (or more matrices) they must have the exact same dimensions (e.g., 2 rows by 3 columns).

Ex. 1 $\begin{bmatrix} 2 & 1 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2+3 & 1+5 \\ 4+-1 & -2+4 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$

2. Matrix addition (with multiplication by a number)

Ex. 1 $2\begin{bmatrix} 1 & 2 \\ 4 & -2 \end{bmatrix} + 3\begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & -4 \end{bmatrix} + \begin{bmatrix} -3 & 6 \\ 6 & 12 \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ 14 & 8 \end{bmatrix}$

3. Matrix multiplication (Dimension Analysis). To multiply two matrices together, the “middle numbers” of their dimensions must match: For example, A 2X3 matrix can be multiplied with a 3X4 matrix since the two numbers in the middle (i.e., 3) match. The resulting matrix would be the “outside dimensions”. In this case 2X4. You could not multiply a 2X3 matrix with a 2X3 matrix since the middle numbers (3 and 2) do not match.

4. Matrix Multiplication. To multiply two matrices together, first make sure the middle dimensions match (see part 3). Then create the “shell” of a matrix with the outside dimensions.

Ex. 1 $\underset{2 \times 2}{\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}} \times \underset{2 \times 1}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} = \begin{bmatrix} 2(3) + 1(5) \\ 3(3) + 4(5) \end{bmatrix} = \underset{2 \times 1}{\begin{bmatrix} 11 \\ 29 \end{bmatrix}}$

5. Evaluating Determinants. To evaluate a 2X2 determinant, multiply the “down diagonal” then subtract from this the number obtained by multiplying the “up” diagonal.

Up

Ex. 1 $\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = 3(5) - 1(4) = 15 - 4 = 11$

Down

Try These

Given $A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 4 & 0 \end{bmatrix}$

Find (If Possible)

1. **A + B**

2. **3A + 2B**

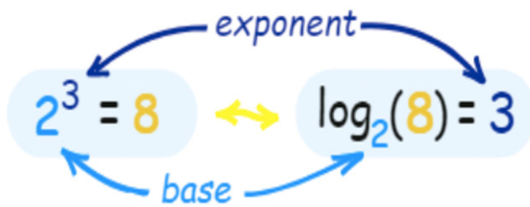
3. **A • B**

4. **C • A**

5. **Evaluate Det [A]**

E. *Logarithms*

The key to solving a logarithm problem on the ACT is often understanding the relationship between the logarithm function and the exponential function.



Understanding this relationship allows one to solve a basic log equation by rewriting it into an exponential equation which most students are more familiar with.

Ex. 1 Solve $\log_2 x = 4$

Solution: Rewrite the problem as exponential equation $2^4 = x$ then solve. The answer is $x = 16$

Ex. 2 Solve $\log_2 8 = x$

Solution: Rewrite the problem as exponential equation $2^x = 8$ then solve. The answer is $x = 3$

Try These

1. Rewrite as an exponential function: $\log_3 9 = 2$

2. Rewrite as a log function: $4^3 = 64$

3. Solve

a. $\log_5 x = 3$

b. $\log_2 32 = x$

VI. TRIGONOMETRY

A. *Basic Relationships*

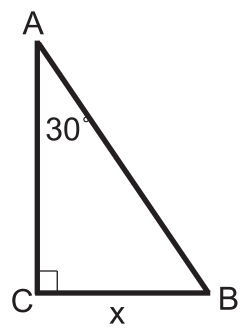
Most trigonometric problems on the ACT can be solved by knowing the three basic ratios:

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos x = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan x = \frac{\text{opposite}}{\text{adjacent}}$$

(If these don't look familiar to you, then don't do this section.)

An example of a problem using these would be the following:

Given $m\sphericalangle A = 30^\circ$, and AC = 10, find BC



Solution:

$$\tan 30^\circ = \frac{x}{10}$$

$$0.577 = \frac{x}{10}$$

$$5.77 = x$$

B. *Identities*

Other trigonometric relationships you should know are the following:

$$\csc x = \frac{1}{\sin x} \qquad \tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\cot x = \frac{1}{\tan x} \qquad \sin^2 x + \cos^2 x = 1$$

Although you might have learned many (many) others, these are the ones you need to know for the ACT. (If a problem requires another identity, it will usually be given.)

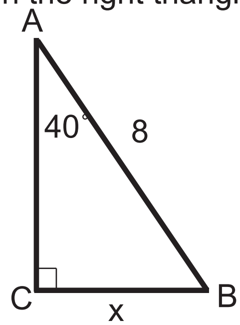
For example: Simplify ***tan x • cot x***

Solution: If you know the basic identities, then this is easy.

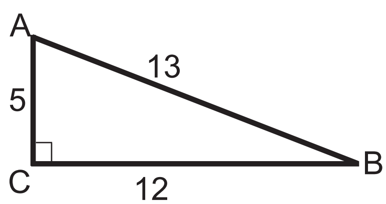
$$\tan x \bullet \cot x = \tan x \bullet \frac{1}{\tan x} = 1$$

Try These

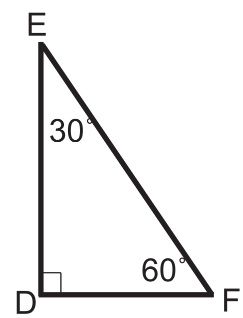
1. Given the right triangle ABC, $m\measuredangle C = 90^\circ$, $m\measuredangle A = 40^\circ$, AB = 8, find BC.



2. Find the cos $\measuredangle$ A.



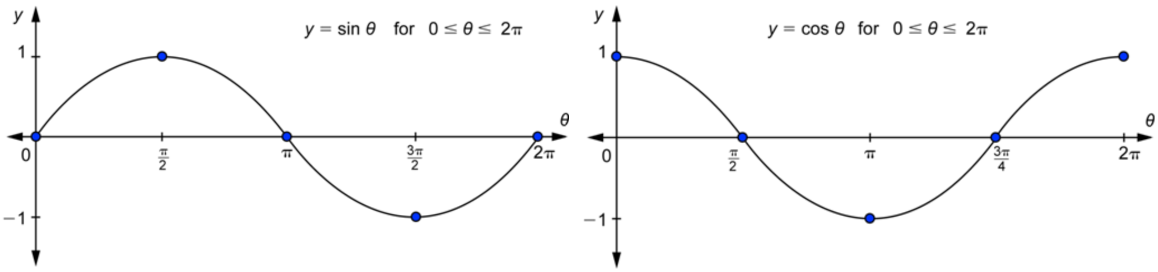
3. Find the sin $\measuredangle$ E.



4. Simplify tan x • sin x + cos x
   (Hint: Change everything to sin, cos; then get a common denominator.)

C. *Sinusoidal Graphs*

The ACT does sometimes require knowledge of Sine (or Cosine) graphs. Usually this involves four concepts: (1) Amplitude (2) Period (3) Horizontal Shift and (4) Vertical Shift.



In General, Sine graphs can be expressed as F(x)=ASinB(X-C) + D, where

A (technically, the absolute value of A) is the amplitude

The period = $\frac{2\pi}{B}$

C = The horizontal shift (Note the minus sign in the equation. So, (X - 3) would represent a horizontal shift of right 3)

D = The vertical shift

For example: Given $F(X) = 3\text{Sin}2\left(X - \frac{\pi}{2}\right) - 5$

Find the amplitude, period, horizontal shift and vertical shift.

Solution: Amplitude = 3, Period = $\frac{2\pi}{2} = \pi$, Horizontal shift = $\frac{\pi}{2}$,
Vertical shift = -5

Try These

Given F(X) = -2Sin4(X+ π) + 3 Find

1. **The amplitude**

2. **The period**

3. **Horizontal shift**

4. **Vertical shift**

D. *The Law of Sines and Cosines*

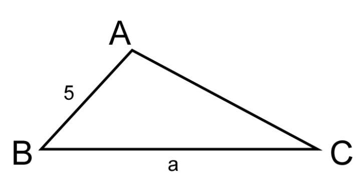
On occasions, the ACT does require knowledge of the Laws of Sines and Cosines. They do NOT, however, require that these formilas be memorized (They will be given as part of the question). Bothe of these are generalized versions of what we normally call "SOHCAHTOA" rules to apply to non-right triangles.

1. The Law of Sines states the ratio of length of each side of a triangle is proportional to the sin of the angle opposite it. Or symbolically,

$$\frac{\text{Sin A}}{a} \quad \frac{\text{Sin B}}{b} \quad \frac{\text{Sin C}}{c}$$

where A, B, and C represent the measures of the 3 angles and a, b, and c the length of the sides. (Note: You could also turn this formula over).

For example: Given the following triangle, find the length of side a



A = 100º
C = 50º

Solution: $\frac{\text{Sin } 100}{a} = \frac{\text{Sin } 50}{5}$ By cross multiplying, 5Sin100 = aSin50

and a= $\frac{5\text{Sin }(100)}{\text{Sin }(50)}$

Note: The ACT will not simpliy. The answer will include the sin ratios

2. The Law of Cosines is really just the Pythagorean Theorem on steroids. For any triangle with sides of length a, b, and c across from angles A B, and Cm respectively, $c^2 = a^2 + b^2 - 2ab(\cos C)$. So, the Pythagorean Theorem is a special case for when C = 90 (The cos90 = 0, so that term drops out). Note, the law can be rewritten to begin with any of the 3 sides and end with the cosine of the angle opposite that side.

> Ex. 1 Given the following: A=40, B=60, C=80, a=6, b=8
> find the length of side c.

Solution: $c^2=6^2 + 8^2 - 2(6)(8)\text{Cos}80$ Note: this would probably be what the answer would look like. Correct answers with the Law of Sines or Cosines are rarely simplified

> Ex. 2 Given a= 4, b=6, c=9, write an expressin using the Law of Cosines for the smallest angle.

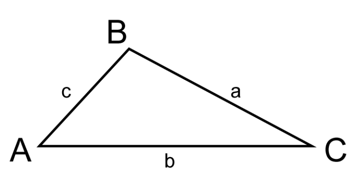
Solution: Since a is the smallest side, A would be the smallest angle.
So, $a^2 = b^2 + c^2 - 2bc\text{Cos}A$.
On the ACT the correct answer would be: $4^2 = 6^2 + 9^2 - 2(6)(9)\text{Cos}A$

Try These

1. Given the following, use the Law of Sines and Cosines to find the missing pieces of the triangle.

A = 40 a = 10
B = 60 b = ?
C = ? c = ?

2. Given a = 10, b = 12, C = 60˚, write an expression using either the Law of Sines or Cosines for the length of side c



a = 10
b = 12
C = 60º

VII. Advanced Topics

A. *Imaginary numbers*

The square root of a negative number is not a real number. To deal with this we define $i = \sqrt{-1}$. This allows us to represent these numbers using "imaginary" numbers. Imaginary numbers and real numbers are both part of a larger set of numbers called complex numbers.

1. Since $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.
   Therefore, $i^5 = i$, $i^6 = -1$, $i^7 = -i$, $i^8 = 1$. This pattern continues forever. Thus, $i^{100} = 1$ (since 100 is a multiple of 4), $i^{101} = i$ (since 101 is "one past" a multiple of 4). Consequently, to simplify i to any power, all you need to do is divide the exponent by 4 and keep the remainder.

2. To simplify other expressions using i, treat i as you would any other variable (like x). When finished, replace all i^2s with -1 and simplify one more time.

Ex. 1 Simplify $\sqrt{-49}$

$$\sqrt{-49} = \sqrt{-1} \bullet \sqrt{49} = i \bullet 7 = 7i$$

Ex. 2 Simplify $(5 - i) \bullet (4 + i)$

Solution: $(5 - i) \bullet (4 + i) = 20 + 5i - 4i - i^2 =$

$$20 + i - i^2 = 20 + i - (-1) = 21 + i$$

Try These

(Note: Some calculators have an "*i*" button.
This would allow most of these questions to be done more easily.)

1. Simplify
   a) i^{50} b) i^{203}

2. Simplify
   a) $(1 - 3i) + (5 + i)$ b) $(5 - i) \bullet (5 + i)$

B. Sequences and Series

1. Sequences of numbers with a "common difference" are called **arithmetic sequences**. For example: 4, 7, 10, 13 (common difference = 3). To find a given term of an arithmetic sequence, recognize that every term after the first is achieved by adding the appropriate number of common differences. Since the first term doesn't get one, the other terms will have one less common difference than their term number. For example: in 4, 7, 10, 13, the 10th term would be found by taking 4 + 9 common differences = 4 + 9(3) = 31.

   The answer could also be computed using the phrase: "1st + (n - 1)(d)" , if you can remember it!

2. **Geometric sequences** are similar to arithmetic sequences; except, in this case, there is a common ratio between terms. That is, each term is found by multiplying the previous term by the same number (called the common ratio). For example: 4, 20, 100, 500 would be a geometric sequence with a common ratio of 5. This answer could also be completed using the phrase: "1st times $(r)^{(n-1)}$, so try to remember this too!!

   To find a term in a geometric sequence, you need to recognize that every term after the first can be found by multiplying by the appropriate number of common ratios. Since the first term again doesn't get one, every other term can be found by multiplying the starting number by the common ratio one less time than the term number. For example: In 4, 20, 100, 500, the 6th term can be found by taking 4 x 5 common ratios = $4 \cdot 5^5$ = 12,500 (easily done with a calculator).

Try These

1. Find the indicated term of the arithmetic sequence 6, 10, 14 . . .

   a) 10th term   b) 100th term

2. Given an arithmetic sequence has a common difference of 5 and the 6th term is 50, what is the first term?

3. Find the indicated term of the geometric sequence 2, 6, 18 . . .

   a) 5th term   b) 10th term

4. Given a geometric sequence has a common ratio of ½ and the fourth term is 12, what is the first term?

Chapter 3

Specific Strategies for the Math Subtest

I. SETTING THE PACE (MATH)

A. *Key Ideas*

1. To get your best score you must be realistic about your goal.
2. In general, the questions toward the beginning are easier than questions/ones toward the end.
3. Skipping problems and going back is a good strategy for your math class where the goal is to score 90% - 100%, but it is NOT a good strategy for the ACT. (An exception will be the 33+ approach where the goal is 95% - 100%.) If you can't do a problem, drop down levels until, as a last resort, you guess. It will not help you to waste time on a problem you don't know by trying twice. (Of course, if you remember how to do a problem later in the test, it is okay to go back.)

B. *Three Approaches*

To get a 19 - 22 Concentrate on 1 - 30 Be selective on 31 - 50 Hunt on 51 - 60

1. The "19 to 22 approach." (This is an average score. If math is a weak area, and you would be happy with a 20, then this is for you!) To get a 20 all you need to know is how to do about 20 questions.* So move slowly on the ones you think you can get. It's okay to spend two minutes on questions you feel you are doing correctly. Discipline yourself to do whatever it takes to get that one right. It's a waste of time to "play with" a question for 2 minutes, waiting for the "bolt of lightning to strike you." With practice you will get to know when you are slowly progressing and when you are just playing.

   **Note: 20 correct + 1/5 of the other 40 correct by guessing gives a total of 28. This is about a 20 on most ACTs.*

To get a 23 - 26 Concentrate on 1 - 40 Be selective on 41 - 50 Hunt on 51 - 60

2. The "23 to 26 approach." (If you scored between 19 and 22 on a previous ACT, or if you are a good math student, this one's for you.) To get a 25 on the math section, you need to know how to do about 30 – 35 questions correctly.* As before, you do not need to rush the ones you think you are getting. To get a 25, you should count on doing about 35 – 40 problems using math, using strategies on another 10 – 15, and guessing on about 5 – 10.

**Note: 35 correct + 1/5 of the other 25 correct by guessing gives a total of 40. This is about a 25 on most ACTs.*

To get a 27 - 32
Concentrate on 1 - 50
Be selective on 51 - 60

3. The "27+ approach." (If you scored 25+ on a previous ACT and you are an exceptional math student, you want to aim for this score.)

To get a 33 - 36
Do 1 - 50 in 40 minutes
Spend the rest of the time on 51 - 60 and any problem(s) you skipped

To get a 31 or above you must be able to do most of the problems.* In addition, you must be able to do the easier problems faster than students who are seeking 20 or 25: you will need the additional time to do the hard questions. To get 30 or above, you must be able to do about 45 – 50 questions using math, about 5 – 10 using strategies, and guess on no more than about 5.

**Note: 50 correct + 1/5 of the other 10 by guessing gives a total of 52. This is about a 30 on most ACT's.*

C. *Pacing Markers*

1. If your goal is to score 20 to 24 on the math test, you should go slower than 1 question per minute.

2. If your goal is 25 you could average close to a minute/question.

3. If your goal score is over 25, you will need to get ahead of a minute per question. (5 to 10 max)

II. INTRODUCTION: THREE WAYS TO DO MATH QUESTIONS ON THE ACT

The Algebraic Door

Note: All examples are in the booklet "Preparing for the ACT Assessment."

There is usually more than one door that "opens" ACT questions.

A. (Level 1) Using Mathematics [Just like in your math class.]

1. One mathematical way
Ex. problem # 47

A more Intuitive Door

2. Another way
Ex. # 47

B. **(Level 2) Using Multiple Choice Strategies** [These are the ways your math teacher probably didn't show you.]

1. *Estimation*: This is by far the most important strategy for doing questions you cannot do using the standard mathematical procedure.
   Ex. # 19

Note: Many times estimation will only allow you to eliminate 2 or 3 choices. Pick the most reasonable answer from all viable alternatives. (That is, don't sweat it, just pick the one that looks the closest.)

2. Work backwards from the answers.
   Ex # 3

3. Plug in numbers.
   Ex. # 18

C. **(Level 3) Guessing**

You can't beat random guessing. Pick a letter and stay with it. This is the most efficient way to guess and you reduce your risk of not bubbling a question in by mistake.

III. GENERAL STRATEGIES

A. Write in the test booklet.

B. Use "good mechanics" on every question:

1. Read slowly (in phrases). Reading one time slowly is better than twice quickly. Pay special attention to a series of questions, all associated with the same given information. This passage approach will enable you to pace yourself more effectively during the test.

2. Make sure you understand the question before you start solving.

3. Make sure you've answered the question.

4. Check each problem before you move on.
(This is more efficient than going back to check.)

C. Know the directions (They're the same as in the practice test.)

D. Answer every question.

IV. SPECIFIC LEVEL I STRATEGIES

NOTE: Several problems do not have examples in the free test. Use ACT test 1165D, as indicated.

A. Introduction: Chapter 2 presented a comprehensive listing of the math skills required on this test. Here are specific ACT questions. As before, real ACT questions from "Preparing for the ACT Assessment" are referenced.

B. Strategies for Pre-algebra/Algebra

1. Know how to go from an average to a total.
Ex. # 28

2. Use your calculator for computation.
Ex. # 15

3. Know the linear equation: $y = mx + b$.
Ex. # 17

4. Know how to solve a linear equation.
Ex. # 22

5. Think proportionately.
Ex. # 1

6. Know basic probability.
Ex. 1) # 2 Ex. 2) #5 Ex. 3) #37

7. Remember the rules of exponents.
Ex. #16 Ex. #25

8. Know how to translate an expression into symbols.
   Ex. # 16 (1165D)

9. Know how to apply the fundamental counting principle to a ***series of choices*** question.
   Ex. # 6 (1165D)

10. Know how to do fractions on your calculator
    Ex. #30

11. Know how to do scientific notation on your calculator
    Ex. #25

C. Strategies for Geometry

1. Label all information: Start with what you are given, work toward what you want.
   Ex. # 35

2. Draw a picture.
   Ex. # 21

3. Draw the coordinate axis.
   Ex. # 9

4. Know how to apply scale factors.
   Ex. # 47

5. Use the Pythagorean Theorem to find lengths or distances.
   Ex. 1) #19 Ex. 2) #26

6. Divide figures into rectangles, triangles and circles.
   Ex. # 49

7. Know the “180° rules”.
   Ex. # 7

8. Know how to apply special right triangles
   (30°– 60°– 90° , 45°– 45°– 90°, and 3 - 4 - 5).
   Ex. #19

9. Remember that you can calculate the midpoint of a line segment by taking the average value of the x - coordinates, followed by finding the average of the y - coordinates.
   Ex. # 9

D. Strategies for Advanced Algebra

1. Know how to apply “FOIL.”
   Ex. # 10 (1165D)

2. Know the equation of a circle $(x-h)^2 + (y-k)^2 = r^2$.
   Ex. # 42 (1165D)

3. Know how to factor. (Note: Factoring cubes is unusual for the ACT math test.)
   Ex. # 27

E. Strategies for Trigonometry

1. Know how to apply ***SOH CAH TOA***
   (*sin x = opp/hyp, cos x = adj/hyp, tan x = opp/adj*).
   Ex. # 39

2. Know how to use the law of sines and cosines.

3. Know the identity $\text{Sin}^2 + \text{Cos}^2 = 1$
   Ex. #54

V. SPECIFIC LEVEL 2 STRATEGIES
(If you can't do the problem using pure math, try these.)

A. *Estimation.*

1. Eliminate unreasonable choices.
   Ex. # 32

2. Use your "eyeball protractor."
   Ex. # 7

3. Estimate distances using your "ruler."
   Ex. 1) # 19   Ex. 2) # 33

4. Draw your own diagram and estimate from it.
   Ex. 1) # 9

5. Estimate by using "computational landmarks."
   (e.g. .20 = 1/5, .25 = 1/4, .33 = 1/3, .5 = 1/2, .67 = 2/3, π = 3)
   Ex. # 23

B. Check the choices
(start in the middle or with the easiest choice).
Ex. 1) # 3   Ex. 2) # 12
Ex. 3) # 22   Ex. 4) # 31
Ex. 5) # 38

C. Check the Choices. (Use your calculator.)
Ex. 1) # 3 Ex. 2) # 38

D. Make up and Substitute Numbers; use numbers to help your thinking.
Ex. # 18

E. Do Part of the Problem Completely and Correctly. (*It's better to do half a problem correctly and eliminate than to fake the whole thing.*)
Ex. # 25

VI. LEVEL 3 GUESSING STRATEGIES
(*This is what you should do if you have absolutely no idea how to work the problem. Remember, estimation is not guessing.*)

A. Random Guess. Pick your favorite strategy. On average you will get 1 out of 5 (20%) correct. Do this quickly.

Remember: ANSWER EVERY QUESTION!

VII. STRATEGIES FOR HOW TO GET THE MOST OUT OF YOUR CALCULATOR ON THE ACT.

A. General Strategies

1. Use a calculator that you are familiar with.

2. A calculator is only appropriate for some questions.
If you do not know how to work a question, don't waste time playing with it on your calculator.

3. Enter numbers carefully – always mentally estimate the answer to make sure that the calculator's answer is reasonable.

4. Make sure that you have good (preferably new) batteries on test day.

B. Specific Level I Strategies

1. Use your calculator to do the 4 basic math operations.
   Ex. 1) # 8 Ex. 2) # 23

2. Use your calculator to evaluate expressions and functions.
   Ex. 1) # 4 Ex. 2) # 8

Note: With most graphing calculators you can program in algebraic expressions, such as in problem 30. You can store the values of variables and have the calculator evaluate the expression. With practice, this can be both quicker and more reliable.

VIII. SPECIAL STRATEGIES FOR GRAPHING CALCULATORS.

Note: In general these strategies will help you only if you are very familiar with your calculator. In addition, for these strategies to be beneficial, you must practice them with real ACT test problems.

A. Have the following programs stored for execution:

1. **The Pythagorean Theorem.** I would have 2 programs: one for when you have two legs (to find the hypotenuse), and one for when you have one leg and the hypotenuse (to find the other leg). Please note that this can be used to find most distances on the ACT.
   Ex. # 19

2a. **The Quadratic Formula.** Ideally, this program would express the answer in radical form (if irrational) and in complex form if the solutions are not real. If it doesn't, then make sure that you know how to find the correct answer by approximating the five choices. *Note: You can use the quadratic formula as an aid in factoring. Since this formula gives you the roots, you can then work backwards one step to the factors.*

2b. **The Discriminant.** This is the part of the Quadratic Formula that is under the square root $b^2 - 4ac$. The Discriminant (D) tells you about the solutions to a quadratic equation. If:

$D < 0$, there are 2 complex zeros (roots)
$D = 0$, there is 1 real zero (root)
$D > 0$, there are 2 real zeros (roots)

Ex. # 52

3. Have a program available to solve a system of equations (for at least two equations and two variables).

   Ex. # 19 (1165D)

B. Consider computing the following using the graphing capabilities of your calculator.

   1. Solve equations by entering the left side as your first equation(Y1) and the right side as your second equation (Y2). To find the solution, graph the two equations and locate the point of intersection. Use the answer choices as an aid with setting your window. Note: Any variable can be your "X". If your calculator has a table function, put your table into "ask" mode and plug in the 5 choices. The choice that makes Y1 = Y2 is the correct choice.

   2. Find the zeros (roots) of a function by graphing. (*This may also help with factoring.*)

   3. Solve inequalities by entering the whole inequality. Use the choices to set the x-window; set the y-window to go from – 1 to 1. Graphing will produce the solution.

C. Have the formulas listed on the following pages saved as programs for reference.

   Ex. 1) # 27 Ex. 2) # 11

Note: You can save any formula by putting it in as a program, then reference this formula as needed by editing.

Add to this list anything else you want to have with you.

Delete any formula that you have no problem remembering.

ACT Math formula list:

1. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
   $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

2. The Pythagoream Theorem: $c^2 = a^2 + b^2$

3a. For a 30˚ - 60˚ - 90˚ triangle: short leg = 1, long leg = $\sqrt{3}$ and hypotenuse = 2

3b. For a 45˚ - 45˚ - 90˚ triangle: legs = 1, hypotenuse = $\sqrt{2}$

4. Area: Triangle: $A = \frac{1}{2}bh$
   Rectangle: $A=bh$
   Trapezoid: $A = h\frac{(b1 + b2)}{2}$
   Circle: $A = \pi r^2$

5. Volume: Prism and cylinders: $V = Bh$ (B = area of the base)
   Cones and pyramids: $V = \frac{1}{3}Bh$ (B= area of the base)
   Sphere: $V = \frac{4}{3}\pi r^3$

6. Linear Equation: $y = mx + b$, slope $= \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$
   y-intercept = b

7. Circle Equation: $(x-h)^2 + (y-k)^2 = r^2$, center: (h,k)

8. Ellipse Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, center: (h,k)

9. Parabola Equation: $y = a(x-h)^2 + k$, vertex: (h,k)

10. $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$, $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$, $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

ACT Math formula list [continued]:

11. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\csc\theta = \dfrac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\sin^2\theta = 2\sin\theta\cos\theta$

12. $f(x) = A\sin B(x - C) + D$, where: $|A| = \textit{amplitude}$,

period = $\dfrac{2\pi}{\text{B}}$, C = horizontal shift, D = vertical shift

IX. DIAGNOSING MISTAKES

Most of the problems missed on a standardized test can be attributed to one of the following causes.

A. Mistake Categories

1. You made a careless error in reading.
2. You made a careless error in computation.
3. The problem required you to use mathematics you used to understand but have forgotten.
4. The problem required you to use mathematics you never understood.
5. You knew the math, but the problem required you to use what you knew in a way that did not occur to you.
6. You ran out of time.

B. Correcting Your Approach

If the reason you missed questions falls into one of the above categories,

1. Spend more time at the beginning of each problem. Don't begin to compute until you have fully conceptualized the problem.
2. Slow down on your computations and check your answers by estimation.
3. Note the content you didn't remember and review it before you take the test. Put that concept on your list of *"Things to Remember."*
4. Try more level 2 approaches next time, i.e., use more multiple choice strategies.
5. Not much you can do about these except to keep practicing.
6. If you feel as if you could have done the problems you did not get to, then try to go a little faster. If you feel as if you could not have gotten them anyway, then your approach was correct, i.e., you ran out of time at the appropriate place.

Chapter 4
Introduction to the Science Subtest

I. DESCRIPTION OF THE TEST

About 10 years ago ACT renamed the Science Reasoning Test as the "Science Test". Although this might not seem like a big deal, it did signal an evolution of the test that is continuing. While the focus of the test remains on identifying and applying research principles, the amount of factual information from high school science classes required by this test is increasing every year. As a result, if you take rigorous science classes you will benefit for at least three reasons:

You will

1. Review facts from elementary school science you must know.
2. Learn 5 core research principles that are the focus of the test.
3. Develop a "science sense" that will help you make educated guesses when you can't find the answer in the passage: if the question is about "greenhouse gasses," choose "global warming."

Also, remember that this test is difficult to complete in the allotted time (40 questions in 35 minutes). Consequently, good pacing is critical.

II. THE THREE TYPES OF PASSAGES ON THE SCIENCE SUBTEST All ACT Science tests now have six passages, with each passage having either six or seven questions associated with it. The three types of passages you will encounter are summarized below:

A. Data Representations:(2-3) These passages will present a diagram, figure or a table representing data. You will be asked questions that require an interpretation of the diagram/table.
(See passage IV, pg. 46-47)

B. Research Summaries:(2-3) These passages will present the results of a series of experiments. You will be asked questions about the results of the research as well as the method of the research.
(See passage I, pgs. 40-41)

C. Conflicting Viewpoints:(1) This passage (there is one on each ACT) will present two or more viewpoints on the same scientific issue. You will be asked questions which will require you to compare and contrast these viewpoints.
(See passage II, pgs. 42-43)

The material used in the Science section is drawn from biology, chemistry, physics, and the physical sciences (geology, astronomy, and Earth science). Because of the length of the test (40 questions in 35 minutes), most of the items require only 30 seconds to answer; so, you're not really forced to do any serious "reasoning." You will need to read fast. The techniques provided are designed to help you take the test efficiently and to get your best score.

III. THE ABCs OF PREPARING FOR THE SCIENCE SUBTEST

A. Don't do a science review. Although this may help a little, reviewing your old biology, chemistry, and physics notes is not an efficient way to study for the ACT Science test.

B. Learn specific strategies for this test.

C. Connect what you know to the test by doing practice tests.

IV. WHAT THIS COURSE WILL PROVIDE

A. Facts about elementary school science that you will need to remember.

B. Five research principles that form the core of the test.

C. Pacing strategies appropriate for various goal scores.

D. General strategies that apply to all the passages.

E. Specific strategies for each of the three kinds of passages.

Chapter 5

Setting the pace on the Current ACT Science Test (June 2018)
Score Range: 18-20

1. Spend most of your time (about 6 minutes per passage) on the first two questions. These questions might make you work hard or look hard, but not think hard.
2. After, you are sure the first two questions are correct, spend whatever time you have left looking at the third and/or fourth questions. Answer either of these only if you are sure you have the correct answer.
3. Guess the “letter of the day” on all unanswered questions.

Score Range: 21-24

1. Make sure you get the first two questions on all 6 passages. These questions might make you work hard or look hard, but not think hard.
2. After you are sure the first two questions are correct, keep going with passages you think you understand. Guess the “letter of the day” on questions you’re not sure of and on the last half of passages you don’t understand.
3. Plan to guess on the last one or two questions of most passages. Spend 5-6 minutes (or more) with most passages. (7 minutes with the conflicting viewpoints passage if you choose to do it).

Score Range: 25-28

1. Make sure you get the first three or four questions on all 6 passages. The first two questions might make you work hard or look hard, but not think hard. The next two questions will be application questions that require you to understand the charts and tables, but not do any serious analytical thinking.
2. After you are sure the three or four are correct. Finish the passages you feel comfortable with, guess at the rest of the questions on any passage you are not understanding.
3. To make up time, guess on the last question of any passage. Spend 5-6 minutes with most passages. (7 minutes with the conflicting viewpoints passage if you choose to do it).

Score Range: 29-32

1. Make sure you get every question except the last question on all 6 passages. The first two questions might make you work hard or look hard, but not think hard. The next two questions will be application questions that require you to understand the charts and tables, but not do any serious analytical thinking. The last two or three questions will be the hardest and require more time.
2. Plan to guess on the last question of several passages. Spend 5-6 minutes with most passages. (7 minutes with the conflicting viewpoints passage).

Score Range: 33-36

1. Make sure you get every question except the last question on all 6 passages. The first two questions might make you work hard or look hard, but not think hard. The next two questions will be application questions that require you to understand the charts and tables, but not do any serious analytical thinking. The last two or three questions will be the hardest and require more time.
2. Plan to spend extra time on the last question of several passages. Spend 5-6 minutes with most passages. (7 minutes with the conflicting viewpoints passage).

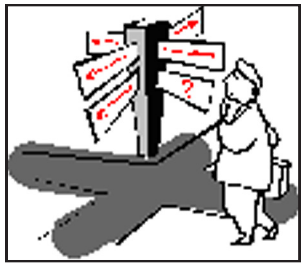
Final comments on pacing.

1. Always remember to make up time:

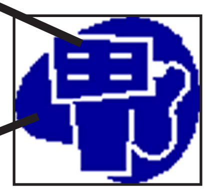
Don't go faster, give-up quicker!

II. To read or not to read…that is the question.
Answer…Read to find the "road map".

The ACT Science Test



Bell City Diner
Specialties: Burgers
Swordfish
Description:
Opened in 1899, The Diner has served over 1,000,000 guests from 50 states and 6 countries.
Our Swordfish has received a 5 star rating from The Mobile Guide.



Freeport Gas Station

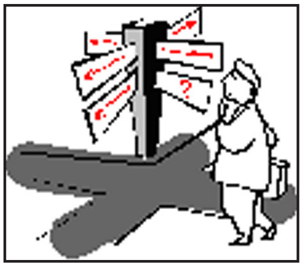
Gas Prices: Regular Unleaded $3.39
Special Unleaded $3.49
Premium Unleaded $3.59

Description:
Our grocery section has all the essentials: bread, milk and soda

Hours:
M-F 7 a.m. to 11 p.m.
Saturday/Sunday 8 a.m. to Midnight

South Town
The Dollar Store
Specialty: Everything's a dollar

Description:
Since 1939, The Dollar Store has sold more than 10 million products to over 5 million customers. The store is located at 5th and Main, 2 blocks west of Broadway.



Questions

1. Where is the Diner?

   A. Freeport
   B. South Town
   C. Bell City
   D. North City

2. What Business is located in Freeport?
   A. Diner
   B. Gas Station
   C. Quick Copy
   D. Dollar Store

3. What is the speciality at the Diner?
   A. Shrimp
   B. Swordfish
   C. Bass
   D. Tuna

4. How much does Regular Unleaded gas sell for at the gas station?
   A. $3.39
   B. $3.49
   C. $3.59
   D. $3.89

5. Who has served the most customers?
   A. Diner
   B. Gas Station
   C. Dollar Store
   D. Can’t be determined

6. Where is the Dollar Store?

   A. East of Broadway
   B. West of Broadway
   C. North of Main
   D. South of Main

Discussion:

1. Was this an easy passage?
2. Looking back, did you have to read all the detail before you turned the page?
3. Did the **"Landmarks"** help you find the answer to each question?
4. Did you use ***all*** the information provided?

<ins>Summary</ins>: Learning how to "road map" a passage will require you to be able to recognize the major "landmarks". For each type of passage, these landmarks will be identified for you.

III. GENERAL STRATEGIES

Read to find the "road map"

A. **Read to find the "Roadmap": *Never*** read the introductory paragraph for the data representations. Read the experiments only when it becomes necessary. ***Always*** read the conflicting viewpoints passage (or guess on it).

B. Try to eliminate two choices. The two choices that are left might be very close. Make sure you identify how remaining choices are different.

C. If you're stuck (you can't eliminate even two choices) guess quickly with your "science sense". Don't use the passage. Look at the question, look at the choices and pick the most reasonable.

D. Always work the "Where's Waldo?" questions. These are questions that make you look hard, but not think hard. Waldo might be a fact or a word, find it just like you found Waldo.

IV. HOW TO APPROACH THE DATA REPRESENTATION PASSAGES

A. *General Principles*

1. Read to find the landmarks and get to the questions (about 20 to 30 seconds, or less). ACT science tests tend to ask "easy" questions about "hard" diagrams. You do not need to fully understand the diagrams, tables, or charts to answer the questions.

2. Make notes on the diagrams and tables. This will save you the time it takes to rethink what you've written.

3. Make conclusions based upon what is provided in the diagram. (That is, don't go beyond what is given.) If a choice makes you make an assumption or a conclusion using information not given, it is probably wrong.

Landmarks

- **Variables**
- **Scales**
- **Units**

4. Identify the variables. These are your landmarks! First note **what** is charted or graphed. Come back for details and specific pieces of data. If there are charts, note the variables and any changes that occur between two charts. If there are coordinate axes, note the variables on each axis and how each variable is being scaled (how it is marked). Once you have identified the variables, specific relationships that are needed to answer questions can be determined. **The Landmarks in the passage become your clues in the questions.**

5. Recommended times: Seven questions 6 minutes each. Six questions pacing 5 minutes each.

Now try passage IV, in the "*Preparing for the ACT Assessment*" booklet.

B. *When you are stuck on a data representation question, these strategies will usually work.*

1. Eliminate choices that use variables not in the question.

2. Pick a choice that seems to come from the same place in the passage.

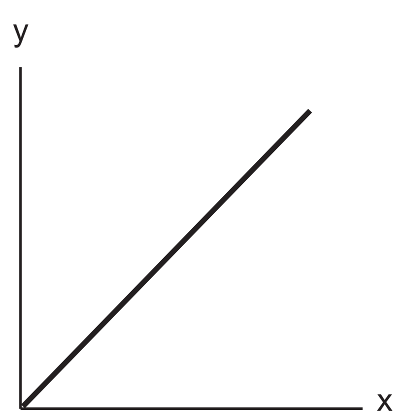
3. Pick what seems logical from the table or chart, not the reading.

4. After eliminating bad choices, pick the choice that is most closely associated with the main idea of the passage.

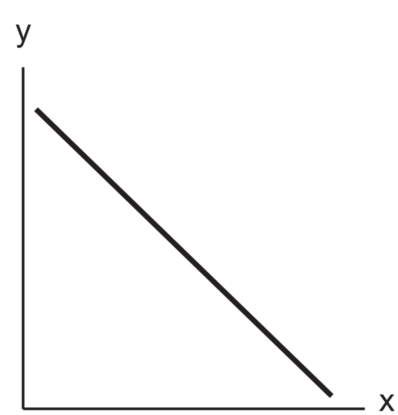
C. *Specific Techniques*

1. Ask yourself these key questions when viewing graphs, tables, and charts:
   - *How is the table/chart/graph labeled? What is it saying?*
   - *What are the variables?*
   - *How are the variables measured? What units are used? (e.g. grams, cubic centimeters, hectares,) and are they being expressed as percentages or ratios?*

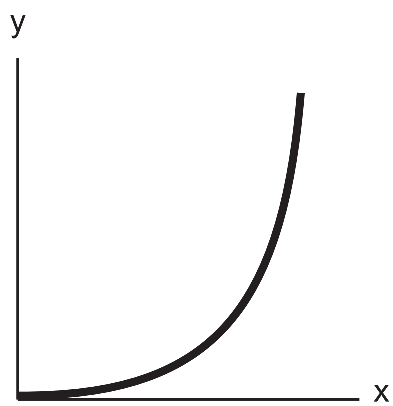
2. The movement of data on a coordinate graph on ACT tests may be linear (straight) or curvilinear (curved). They may demonstrate a positive relationship between the two variables (i.e., an increase in one causes an increase in the other), or they may demonstrate a negative relationship (i.e., an increase in one causes a decrease in the other). The following examples show these relationships.



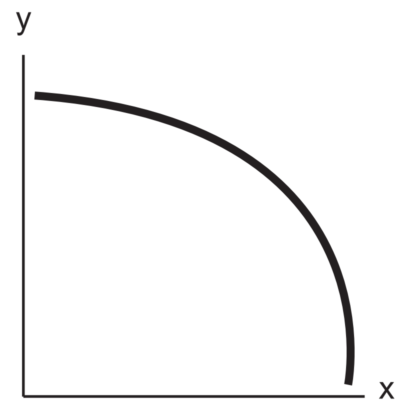
Linear / Positive



Linear / Negative



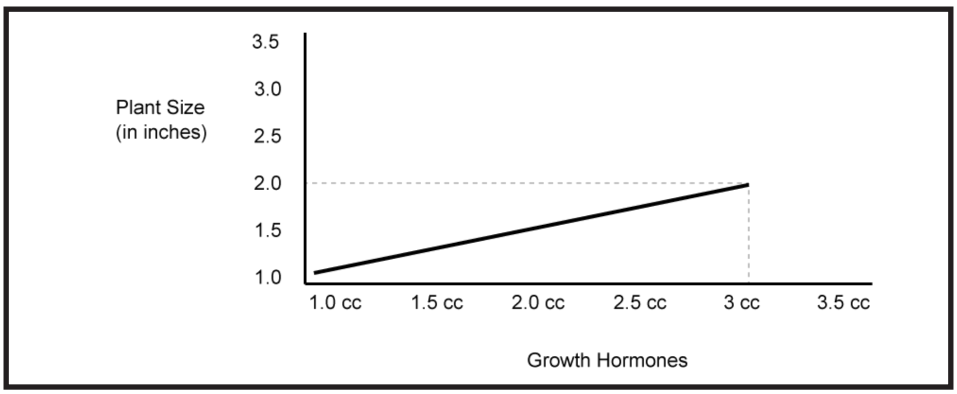
Curvilinear / Positive



Curvilinear / Negative

3. *Interpolation*

One of the most effective ways to deal with coordinate graphs (and even tables) on the ACT Science subtest is to use a technique called interpolation ("reading between the lines"). Interpolation is a method used to determine specific data points that may not be directly presented to you on the graph.



Referring to the graph above, consider this common question that requires the use of interpolation: "What would be the corresponding increase in plant size if a plant is given a 2.7 cc dose of growth hormone?"

a. 1.7 inches
b. 1.8 feet
c. 3.0 inches
d. 4.0 inches

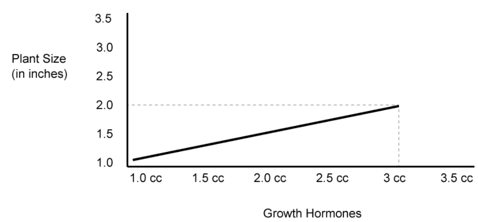
To use interpolation to find the correct answer, place your pencil on the horizontal axis at a point close to where 2.7 would be (between 2.5 and 3.0). Once you are confident that you are close, move your pencil straight upwards until you reach the line of the graph, and then move your pencil directly to the left until you reach the vertical axis. The value where your pencil is now is the corresponding plant size. The correct answer in this case is a.

Remember: Always pay close attention to the unit of measurement. Some students may have rushed to select 2.0 feet because it was a round number, and close enough to the right number. They would have been wrong, because 2.0 feet is 24 inches, and this is nowhere near correct.

Interpolation is a strong tool, but use your pencil carefully, watch the units of measurement, and use common sense.

4. *Extrapolation*

Extrapolation is another scientific method that the ACT will test you on. It is a means of "extending" a graph beyond the data given in order to predict data points not shown at this scale.



Consider the same graph as we did the interpolation. You might be asked a question like this: " A group of scientists decided to double the amount of growth hormone from 3.5 cc to 7.0 cc. What would be the corresponding increase in size?"

a. 2.0 inches
b. 2.7 inches
c. something less than 2.7 inches
d. something more than 2.7 inches

The correct answer is d. Although we cannot determine the exact answer, we can infer (or extrapolate) from the figure that it must be greater than 2.7 inches.

<u>Note</u>: Extrapolation on the ACT will always require you to go just beyond the data provided.

V. HOW TO APPROACH THE RESEARCH SUMMARY PASSAGES

These are the three experimental reasoning information sets on the ACT Science subtest. You will be presented with a research problem, and the objective of the research will be detailed in a one-paragraph passage. This passage is generally followed by three or four experiments. Some of this information will come in the form of graphs or tables. If so, you can use the skills discussed earlier to decipher these as needed. On this part of the test, the ACT is testing your ability to understand the scientific method. The ACT test-makers have their own way of testing your abilities, so it is to your benefit to try a few sections in your practice booklet.

A. *General Principles*

1. It's okay to spend some time with the passage (30 sec. to 1 min.). Look for the general idea (major landmarks) and then go back for details (i.e., see the forest first; come back for the trees). You can come back and read the parts that you need to.

2. Make notes on the passage.

3. You can usually ignore big words and chemical equations. The passages will either tell you what it means or you don't need to know it.

4. If applicable, note what changes between experiments.

5. Identify the variables. The experiment is investigating the effects of some of these variables. Usually these variables are under the control of the experimenter. Similarly, you will want to note the variables the experiment is investigating the effects on. Also note the factors that are being held constant. These are the landmarks! You will usually find them by looking at the tables and figures. **The Landmarks in the passage become the clues in the questions.**

6. Recommended times: Seven questions pacing, 6 minutes each. Six questions pacing, 5 minutes each.

Try passage I.

B. *When you are stuck on a research summary, these strategies will usually work:*

1. Eliminate choices that use variables not in the passage.

2. Look for directional words in the question.
   Pick a choice in the same direction.

3. Pick what seems to be the most reasonable choice.
   (See question 10.)

C. *Specific Techniques*

1. When reviewing the passage, you can easily identify the research objective by underlining the first and last sentence. You must know the object of the study.

2. When looking at each experiment, you must identify

   a. The variables. How are they measured?
      Write all of this down.
   b. The procedure used to measure these variables.
      Write it down.
   c. The results of the experiment.
      Write these down as well (or underline them).

Note: If there are graphs or tables, you can find most of this information in these.

3. Pay attention to any controls that are used in the experiment. Sometimes they are stated explicitly and sometimes they are presented in the table as a group that is not receiving the experimental variable.

4. When comparing more than one experiment, look for trends in the data. Again, if you are given the luxury of graphs or tables, all of the above will be made simpler—just use the skills you gained in the data representation section.

VI. HOW TO HANDLE THE CONFLICTING VIEWPOINTS PASSAGES

Although there may appear to be two “conflicting viewpoints” passages in some ACT assessments, I assure you there will be only one per test. Each test will provide several paragraphs that depict some scientific theory or event that is open for debate. In fact, one should treat these passages as a debate. Each debater will acknowledge the theory or event. The debaters will then state an opinion, a hypothesis, and an assumption. It doesn’t matter who is right, so do not get into the mindset of trying to determine who sounds more scientific. Your goal is to understand the differences between the two (or more) points of view.

A. *General Principles*

1. Approach these the same way as you did the passages in the reading subtest.

2. Read the passage carefully enough to understand the “heart” of the conflict. There will usually be two viewpoints, and they will always be opposing.

3. Read the opening paragraph carefully. It contains information before the viewpoints, which provides known facts that will be important.

4. Make notes on the passage. Consider making a chart to compare the viewpoints. For each model, list areas of agreement and areas of disagreement.

5. Try to think of several descriptors that distinguish the viewpoints. Such as, “slow vs. fast” or “soft vs. hard.”

6. If the viewpoints have names, the names will provide important information.

7. Neither viewpoint will be “correct.”

8. Recommended reading times: Seven minutes.

   Try passage II.

B. *Specific Techniques*

1. Identify the factual content of the point of conflict. The theory or event of interest is stated in the first sentence and in the concluding sentence. Sometimes you will find words that are bracketed, italicized, or parenthesized. The purpose of these designated words is to lead you to recognize some type of process or sequence of events. Once you have used these clues to identify the facts, ask yourself, what is the object of this debate?

2. Next, identify the disagreement between the two points of view. You will usually be presented with either two scientists, two models, or two theories that represent conflicting positions on a subject. These positions will include information about the following:
   a. General agreement on the underlying event or theory
   b. The opinions of the point of concern
   c. The hypotheses being advanced
   d. The assumptions used to support these hypotheses

3. Remember, no one is right in this debate. Do not take sides because you feel that one of the two sides seems more correct or more scientific. Act like an ambivalent reporter—just consider the facts.

4. Consider the following things when you are answering conflicting viewpoint questions:
   a. These questions will try to determine how well you can compare the two viewpoints. Such questions will usually ask you to find a point of common ground on which both sides would agree.* Finding this common ground is a good way to determine the theory or event that is the subject of the debate.

   b. You will often be asked to understand the assumptions that are being made by each of the viewpoints. Remember that an assumption is an unwritten belief that, if false, would make all of the conclusions based on this assumption invalid. A typical question asks you to identify which assumptions must be made in order for a particular viewpoint to be accepted. This requires you to find an assumption that would bolster a given viewpoint. Do not look outside of the particular viewpoint text in question to find this assumption.

*The opening paragraph contains known information agreed to by all viewpoints.

c. Another common type of question asks you to understand what types of statements might weaken or strengthen a particular argument. It may ask, “In order to weaken view 1, which of the following must be true?” Four statements will follow. Look for the one that either contradicts view 1 or supports view 2. When looking for a response that strengthens a particular argument, identify the one that either tends to support the argument in question or weakens the alternative argument.

d. Each question will offer you clues as to where you should look to find the answer. For instance, “according to view 1” means look closely at the statement of view 1.

V. Obtaining feedback from a practice test.

If you ran out of time, or were close to running out of time with a science practice test, then you will want to work on taking the test more efficiently. One key is the ability to identify questions you know you are going to miss. Try the following steps after taking a science practice test:

First, select your second choice for each question you missed.
Then re-grade.

If your second choice is correct, you may have benefited by slowing down on that question.

If your second choice is incorrect, this is a question you should have given up on quicker.

The next time you take a practice test, try to identify questions you know you are probably going to miss. Giving up quickly on these questions will allow you to go slower on the others.

Appendix A

Appendix A

Summary of the Content on the Math Subtest of the ACT
(Pre-algebra and Statistics)

Fractions, Decimals and Percents:
- To change from a fraction to a decimal, divide bottom into top.
- To change from a decimal to a fraction, read as a fraction, write as a fraction, then reduce.
- To change from a decimal to a percent, (or vice versa) move the decimal 2 places. (Note: 1. = 100%)

Percent Problems:
Is means =, ***of*** means multiply.

What number, what is, and what percent refers to the variable.

$$\%\text{ increase or decrease} = \frac{\text{Change}}{\text{Original}}$$

Mult/Div Fractions:
Get rid of mixed numbers, cancel, then multiply straight across.

Add/Sub Fractions:
Get a common denominator first.

Order of Operation:
1. grouping symbols (work inside-out)
2. exponents
3. div. & mult. (work left to right)
4. subt. & add. (work left to right)

Evaluating Expressions:
Replace each letter by its value and simplify.

Simplify Expressions:
Distribute parentheses and then combine similar terms.

Scientific Notation:
Put a number in scientific notation by moving the decimal point until the number is between 1 and 10, and then multiply by the appropriate power.

Proportions:
Solve by cross-multiplying: $\frac{a}{b} = \frac{c}{d}$

$ad = bc$

Statistics:
- mean (average): Find by adding all the scores and dividing by number of scores. *(Note: If you know the average of a set of scores, you can find the total by multiplying the average by the number of scores.)*
- median: Find by arranging the scores from high to low. The median is the score "in the middle." (Note: If there is an even number of scores, average the two scores in the middle.)
- mode: The most frequent score. *(Note: If more than one score is the most frequent, each of these scores is the mode.)*

Probability:
The probability that something will occur =

$$\frac{\textit{The number of ways it can occur}}{\textit{The total number of possible occurrences}}$$

Example: 2 blue and 3 white balls are in a hat; the probability of drawing a blue ball =

$$\frac{\textit{number of blue balls}}{\textit{total number of balls}} = \frac{2}{5}$$

For any experiment, the sum of the probabilities of the outcomes must equal 1.

If two events are independent (unrelated) then the probability of both occurring is the product. (Multiply)

Fundamental Counting Principle:
With a series of choices, the total number of ways to choose is the product of the number of ways for each choice.

Summary of the Content on the Math Subtest of the ACT (Algebra)

Even & Odd Numbers: Any multiple of 2 is even.

even + even = even	even • even = even
odd + odd = even	even • odd = even
even + odd = odd	odd • odd = odd

Real Number Properties:

Natural Numbers: {1, 2, 3, …}
Whole Numbers: {0, 1, 2, 3, …}
Integers: {…-2, -1, 0, 1, 2…}
Rational Number: any number that ends or repeats when written as a decimal

Unit: {1}	commutative	(a + b) = (b + a)
Prime: {2, 3, 5, 7…}	associative	(a + b) + c = a + (b + c)
Composite: {4, 6, 8, 9…}	distributive	a(b + c) = ab + ac

Multiplying Polynomials:

$(x + a)(x + b) = x^2 + bx + ax + ab$
With more terms, multiply each member of the first group by each member of the second.

Rational Equations: (variables in the denominator):

Solve as you would any equation, except you must check:
Any value which makes the denominator = 0, makes the fraction undefined and, therefore, cannot be used as a solution.

Factoring:

1. Always take out the GCF (greatest common factor).

2. (2 terms) Look for the difference between the squares.

   $a^2 – b^2 = (a – b)(a + b)$

   Look for the difference or sum of cubes

   $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

   $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

3. (3 terms) General factoring (trial and error).

4. (4 terms) Factor by grouping.

<u>Systems of Equations:</u>

1. Solve by addition method: Multiply one or both equations by a number which, upon adding the equations, makes a variable drop out.

2. Solve using substitution: Solve for one variable in one equation. Substitute this expression for the variable in the other equation.

<u>Solving Equations:</u>

1. Remove fractions
2. Distribute (   )
3. Combine similar terms
4. Get the variable on 1 side
5. Add & Subtract
6. Multiply & Divide

<u>Solving Inequalities:</u>

Solve using same procedure as equations. (Reverse inequality sign when multiplying or dividing by a negative.)

Open circle ***<u>does not</u>*** include endpoint, Closed circle does.

<u>Exponents:</u>

$x^a \bullet x^b = x^{a+b}$; $\quad \dfrac{x^a}{x^b} = x^{a-b}$; $\quad (x^a)^b = x^{a \cdot b}$

$x^{-n} = \dfrac{1}{x^n}$; $\quad \dfrac{1}{x^{-n}} = x^n$;

$x^0 = 1$; $\quad -x^0 = -(1) = -1$

<u>Square Roots:</u>

<u>Simplify</u>: pull out as much as possible

<u>Multiplication and Division</u>:

Multiply Outsides and multiply insides.

Note: You can cancel outside pieces with outsides, insides with insides.

<u>Addition and Subtraction</u>:

Simplify first. Then add similar roots. (Insides must be the same.)

<u>Approximate by Estimation</u>:

$\sqrt{2} \approx 1.4$	$\sqrt{6} \approx 2.4$
$\sqrt{3} \approx 1.7$	$\sqrt{7} \approx 2.6$
$\sqrt{4} = 2$	$\sqrt{8} \approx 2.8$
$\sqrt{5} \approx 2.2$	$\sqrt{9} = 3$

Summary of the Content on the Math Subtest of the ACT (Geometry)

Triangle Theorems and Definitions:

The ∢'s of a triangle add up to 180°.
Acute ∢: less than 90°.
Right ∢: 90°.
Obtuse ∢: greater than 90°.
In any triangle, the biggest side is opposite the biggest ∢, etc.

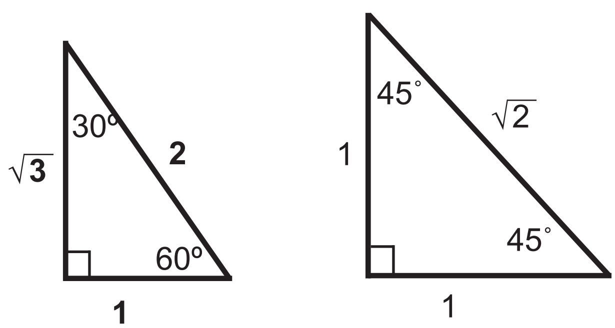
If 2 sides of a triangle are equal, then so are the angles opposite them, and vice versa. (Isosceles Triangle Theorem)

Any 2 sides of a triangle must, together, be longer than the third.

rt **Δ** : $leg^2 + leg^2 = hyp^2$

3, 4, 5; 6, 8, 10 etc.
5, 12, 13; 10, 24, 26 etc.

Pythagorean triples



Special Triangles

Formulas:

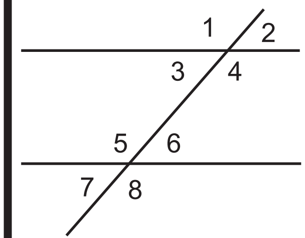
Area: Parallelogram: $A = b \cdot h$
(*Note: includes square and rectangle*)
Triangle: A = ½ *bh* Circle: A = πr^2
Distance: (use Pythagorean Theorem)
Midpoint: (avg. of x's, avg. of y's)
Perimeter: Circle: C = $2\pi r = \pi D$
Volume: Box: V = *lwh* = *Bh*
(B = area of base)
Pyramid:

$$V = \frac{1}{3}Bh$$

Sphere:

$$V = \frac{4}{3}\pi r^3$$

Parallel Lines:



m ∢1 = m ∢5
m ∢2 = m ∢6 corresponding
m ∢3 = m ∢7 angles
m ∢4 = m ∢8

m∢3=m∢6 alternate interior
m∢5=m∢4

m∢3 + m∢5 = 180° same side interior
m∢4 + m∢6 = 180°

Vertical ∢s (∢1, ∢4) are always equal.
Two angles which make a line (∢5, ∢6) add up to 180°. (m∢5 + m∢6 = 180°)

Coordinate Geometry:

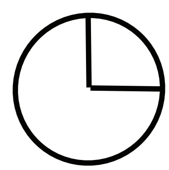
To find the slope or y-int. of a line, solve for y.
Y = mx + b m = slope b = y-int.

$$Note: \quad m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

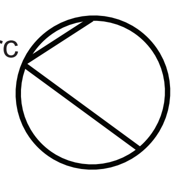
Parallel lines have equal slopes,
Perpendicular lines have negative reciprocal slopes.

Circles:

Central ∢ = arc



Inscribed ∢ = ½ arc



Circle = 360º

A radius is ⊥ to a tangent

Sum of the angles of a Polygon:

Sides	Sum
3	180º
4	360º
5	540º

(For each new side add 180º)
For regular polygons, all sides and angles are equal.

Scale Factor: For any 2 similar figures (same shape) the ratio of any corresponding lengths is called the scale factor.
Note: Ratio lengths =SF, ratio areas = $(SF)^2$
In 3 dimensions: ratio volumes = $(SF)^3$

Similar Triangles:
Set up a proportion, solve by cross-multiplying.

Summary of the Content on the Math Subtest of the ACT (Advanced Alg. & Trig.)

Quadratic Equations: $ax^2 + bx + c = 0$

To solve: 1) Factor
2) Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To find the vertex, minimum or maximum,
Step 1: let $x = \frac{-b}{2a}$
Step 2: plug this value in for x to find y.

Quadratic Relations:

Circle $(x-h)^2 + (y-k)^2 = r^2$
(h, k) = center r = radius

Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
(h,k) = center
Note: If the major axis is vertical, the larger number will be under the $(y-k)^2$

Parabola $y = a(x-h)^2 + k$
(h,k) = Vertex
$\pm a$ (opens up/down)
$x = a(y-k)^2 + h$
(h,k) = Vertex
$\pm a$ Opens right/left

Hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

(h,k) = center

Laws of Logarithms:

Log A + Log B = Log AB

Log A – Log B = Log $\frac{A}{B}$

Log A^B = B Log A

Absolute Value Equations:

$|x-a| < b$; $-b < x - a < b$

$|x-a| > b$; $x - a > b$ or $x - a < -b$

$|x-a| = b$; $x - a = b$ or $x - a = -b$

Roots and Radicals:

$$\sqrt[m]{x^n} = x^{\frac{n}{m}}$$

Discriminant:

$D = b^2 - 4ac$
D < 0: no real solutions (zeros)
D = 0: 1 real solution (zero)
D > 0: 2 real solutions (zeros)

Trig Identities:

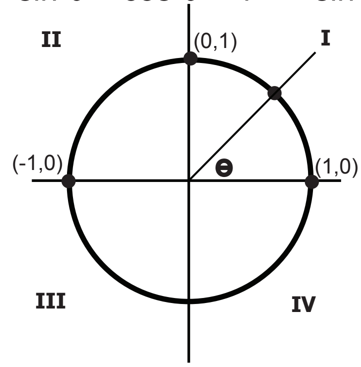
soh | cah | toa

$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = 2\sin\theta\cos\theta$



If (x,y) is the point where the terminal side hits the unit circle then:
$\sin\theta = y$
$\cos\theta = x$
$\tan\theta = y/x$

f(x) = A sin B(x – C) + D
A = amplitude period = $\frac{2\pi}{B}$
C = horizontal shift D = vertical shift

Appendix B

Answers to the "**Try These**"

Page 8

1. .25, 25%
2. 7/20, 35%
3. 3/2, 1.5
4. .375, 37.5%
5. 1/250, .4%
6. 13/200, .065

Page 10

1. 6.25%
2. $255
3. $20,000

Page 12

1. 5/2 or 2.5
2. 5/3
3. 15/8
4. 15 1/8

Page 15

1. 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
2. $2 \cdot 2 \cdot 5 \cdot 5$
3. a. 96
   b. 600
   c. $x^2 y^3 z$
   d. $30a^2 b^2 c^3 d$

Page 19

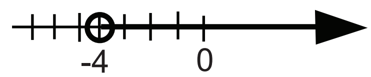
1. a. 0
   b. 5/9
   c. 8/9
2. mean = 11, median = 10, mode = 8 and 10
3. 77.6
4. 100

Page 20

1. $-ab^2 + a^2b$
2. $-7x - 26$
3. $3x^2 + 13x - 10$
4. $2x^3 - x^2 - 35x - 50$

Page 23

1. $y = 6$
2. $x > -4$,



Page 25

1. 3, -4
2. The roots are $x = 5$ and $x = -2$ (5 is extraneous.)

Page 28

1. a. -14
   b. $-98/3$
2. 6
3. 19
4. $h = \dfrac{A - 2\pi r^2}{2\pi r}$
5. $4(2r + 10) = 8r + 40$

Page 32

1. $x(2x - 5)(2x + 5)$
2. $(x + 4)(x - 1)$
3. $(2x + 1)(x - 3)$
4. $(3x - y)(x - 2y)$

Page 34

1. 1, -7
2. 4, 2
3. $-5 \pm \sqrt{22}$

Page 36

1. $12x^3y$
2. $4x^8y^7$
3. $20x^2y^2$
4. $2x^5y^5$

Page 40

1. $6\sqrt{2}$
2. $\sqrt{10}$
3. 5
4. $72\sqrt{2}$

Page 43

1. (4, – 1)
2. (2, 1)
3. Identity (Infinite # of solutions)
4. No solution

Page(s) 48 – 49

1. x = 30°, y = 150°, z = 60°
2. a. Yes, right triangle
   b. No
   c. Triangle, not a right triangle
3. a. 8
   b. $6\sqrt{3}$, 12
   c. 8, 16
   d. 4, 4

Page 51

1. x = 120°, y = 60°
2. angles 4, 5, 8, 9, 12
3. m∢A = 120°, m∢B = 60°,
   m∢D = 120°

Page 55

1. a. area = 40, perimeter = 32
   b. area = 24, perimeter = 20
2. a. 18
   b. 56

Page 61

1. slope = 3/5 , y – int = – 3
2. y = 2/3 x - 3
3. y = -4/3 x + 4 or 4x + 3y = 12
4. y = -2x + 8

Page 64

1.

	r	**d**	**C**	**A**
a	2	4	4π	4π
b	3	6	6π	9π
c	4	8	8π	16π
d	5	10	10π	25π

2. $8\pi/9$, $16\pi/9$
3. 40°

Page 66

1. $60cm^3$
2. 36π
3. 100π
4. $1{,}372\pi/3$

Page 68

1. a. 1800°
   b. 150°
   c. 30°
   d. 180°
2. 8 sides

Page 70

1. (-2, 1)(-2, 4)(-6 ,1)
   area = 6, perimeter = 12
2. 9

Page 71

1. a. 125
   b. 1/9

Page 75

1. a. center (3, -5); radius = 5
   b. center (0, 5); radius = 6
2. $\frac{(x-3)^2}{9}+\frac{(y-1)^2}{1}=1$

Page 77

1. -1 < x < 3
2. x < –7 or x > 1
3. x = 4 or x = –6

Page 79

1. $\begin{bmatrix} 3 & 9 \\ -1 & 3 \end{bmatrix}$
2. $\begin{bmatrix} 7 & 22 \\ -4 & 9 \end{bmatrix}$
3. $\begin{bmatrix} 6 & 5 \\ -1 & -10 \end{bmatrix}$
4. not possible
5. 11

Page 80

1. $3^2 = 9$
2. $\log_4 64 = 3$
3a. 125
3b 5

Page 82

1. 5.144
2. 5/13
3. 1/2
4. sec x

Page 84

1. 2
2. $\pi/2$
3. left π
4. 3

Page 85

1. b = 13.5, c = 15.3, C = 80˚
2. $c = \sqrt{10^2 + 12^2 - 2(10)(12)\cos 60^\circ}$

Page 86

1. a. –1
   b. - i
2. a. 6 - 2i
   b. 26

Page 87

1. a. 42
   b. 402
2. 25
3. a. 162
   b. 39,366
4. 96

Appendix C

Content Guide to the ACT Math Test Sub-section

Subjects found on Every ACT: ***ACT test #***

<u>**Basic Algebra**</u>	**1165D**	**1163E**	**1460E**	**1467F**
Evaluating Expressions	1	23, 57	1, 42	4, 7, 30
Percents	7, 35, 46	27	2, 15, 42	22, 47
Linear Equations (Solve)	4	21	14	
Linear Equations (Translate)	51	32, 36	6	14
Linear Equations (Both)	14	33, 34	27	2
Factoring	13	31, 35	25, 35	17, 24, 60
Proportions	3, 14, 23	13, 17, 20	5, 55	3, 25, 59
Simplifying Expressions	54, 58	15	12	27, 29
Basic Probability	45	9, 13, 17	4	9

<u>Plane Geometry and Coordinate Geometry</u>

Angles of a Triangle	9, 43, 55	30	7	11
Slope	18, 24, 50	49	28, 33	16, 20
Area of a Parallelogram (usually a rectangle)	27, 39	11, 39	8, 16	32
Parallel Line Theorems	25, 49	58	23	31, 54
Special Triangles	5	30, 52	57	
Pythagorean Theorem	5, 32	40, 45	26, 36, 37, 52	33, 39
Circle formulas (2r, r^2)	31	14, 37	44	50

<u>Advanced Algebra and Trigonometry</u>

Basic ratios (SOHCAHTOA)	21, 38	42, 59	32, 38, 52	38, 57
Advanced Trig (Identities, Half/Double-angles)	54, 59	56	32, 58	48

<u>Other</u>

Basic Skills (directional reading)	2, 11, 20	7, 24, 26, 29	9, 13, 21, 32, 41	1, 2, 14, 15, 19, 28, 34, 43
Inductive Reasoning (LOGIC)	26	56	10, 46	6

Most ACT Math Test (50% or higher as noted)

Basic Algebra	1165D	1163E	1460E	1467F
Linear Inequalities (80%)	8	3, 47	20	46
Fundamental Counting Principle (90%)	6	51	19	
Rules of Exponents (90%)	22	4, 44	22, 45	8, 26
Least Common Multiples (90%)			40	10
Evaluating Functions (90%)	12, 47	23, 32	18	
Solving Formulas (90%)	3	21, 34	31, 54	21
Absolute Value (90%)	29, 53, 56	54		18
Averages (90%)	37, 51	1	48	
Systems of Equations (90%)	19	38, 47	17	13
FOIL (80%)	10	19	3	36
Distance = rateXtime (50%)	17	14, 46		

Plane Geometry and Coordinate Geometry				
Scale Factor (80%)	23	12	30, 36, 38 ,47, 51	
Perimeter of a polygon (90%)	39	2, 37		40
Equation of a line (60%)	50			9
Midpoint (80%)	30	50	34, 52	4
Reflections/Rotations and Translations (60%)		60	11	45, 49, 53
Area of a Triangle (90%)	25, 39	22		
Triangle Inequality (50%)				
Angles of a Polygon (50%)		55		

Advanced Algebra and Trigonometry				
Quadratic Equations (80%)	34, 52		25	17, 35
Equation of a Circle (80%)	42		60	41
Ellipses and Parabolas (60%)	54, 59		53	5
Geometric Sequences (60%)		18	29	

Other				
Logic (50%)		5, 16, 24, 43	(All logic problems are from the 1163E)	