Common Core State Standards

Mathematics II

Integrated Pathway

Student Workbook
with Scaffolded Practice
Unit 2

WALCH EDUCATION®

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Introduction

The *CCSS Mathematics II Student Workbook with Scaffolded Practice* includes all of the student pages from the Teacher Resource necessary for your day-to-day classroom use. This includes:

- Warm-Ups
- Problem-Based Tasks
- Practice Problems
- Station Activity Worksheets

In addition, it provides Scaffolded Guided Practice examples that parallel the examples in the TRB and SRB. This supports:

- Taking notes during class
- Working problems for preview or additional practice

The workbook includes the first Guided Practice example with step-by-step prompts for solving, and the remaining Guided Practice examples without prompts. Sections for you to take notes are provided at the end of each sub-lesson. Additionally, blank coordinate planes are included at the end of the full unit, should you need to graph.

The workbook is printed on perforated paper so you can submit your assignments and three-hole punched to let you store it in a binder.

Lesson 2.1.1: Graphing Quadratic Functions

Warm-Up 2.1.1

The table below represents the amount of money a car owner spends on repairs as a function of the number of oil changes the owner gets each year. Each oil change costs $20.

Yearly number of oil changes	Amount spent on car repairs ($)
0	500
1	420
2	340
3	260
4	180
5	100

1. Write a linear model of the amount the owner will spend on car repairs as a function of the number of oil changes in 1 year.

2. What is the *y*-intercept? What does it represent?

3. What is the rate of change? What does it mean for the owner?

Scaffolded Practice 2.1.1

Example 1

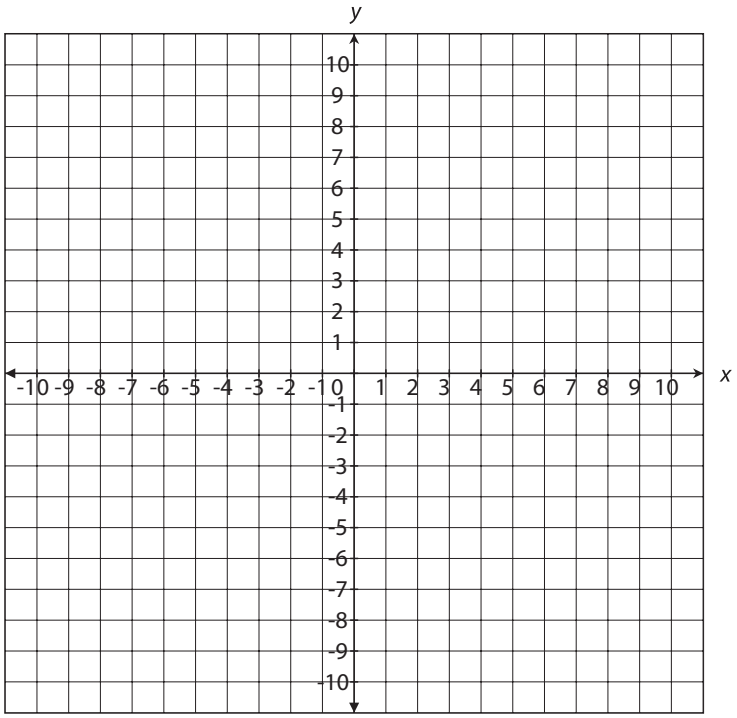
Given the function $f(x) = x^2$, identify the key features of the graph: the extremum, vertex, and y-intercept. Then sketch the graph.

1. Determine the extremum of the graph.

2. Determine the vertex of the graph.

3. Determine the y-intercept of the graph.

4. Graph the function.



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Example 2

Given the function $f(x) = -2x^2 + 16x - 30$, identify the key features of the graph: the extremum, vertex, and y-intercept. Then sketch the graph.

Example 3

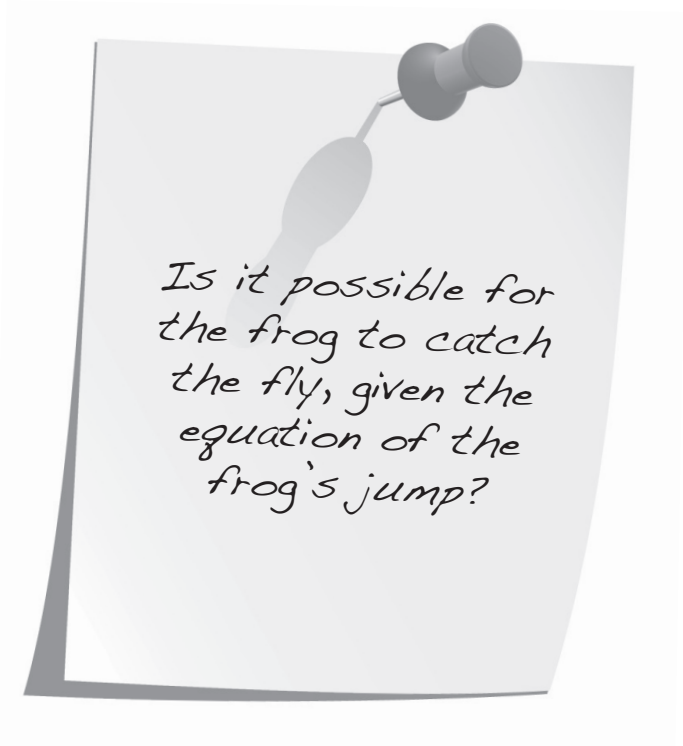
Given the function $f(x) = x^2 + 6x + 9$, identify the key features of its graph: the extremum, vertex, and y-intercept. Then sketch the graph.

Example 4

Given the function $f(x) = -2x^2 - 12x - 10$, identify the key features of its graph: the extremum, vertex, and y-intercept. Then sketch the graph.

Problem-Based Task 2.1.1: How High Can a Frog Jump?

A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation $h(x) = -x^2 + 4x + 1$, where $h(x)$ is the frog's height above the water and x is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?



Practice 2.1.1: Graphing Quadratic Functions

For each function that follows, identify the intercepts, vertex, and maximum or minimum. Then, sketch the graph of the function.

1. $y = x^2 + 6x + 5$

2. $y = x^2 + 2x - 15$

3. $y = -x^2 + 10x - 9$

4. $y = -x^2 - 4x$

5. $y = x^2 - 4x - 12$

6. $y = \frac{1}{2}x^2 + 2x$

7. $y = -x^2 - 4x - 3$

For each problem that follows, determine whether the function has a minimum or maximum, identify the maximum or minimum, and identify the intercepts.

8. Suppose the distance between a boomerang and the person who threw it follows a quadratic relationship in terms of the time t since it was thrown. The equation that models this situation is given by $d(t) = -\frac{1}{2}t^2 + 6t$.

9. The path of a snowboarder performing stunts is given by the equation $y = -16t^2 + 24t + 16$, where t is time in seconds and y is the duration of the stunt.

10. The flight of a boulder launched from a catapult follows the quadratic equation $H(x) = -x^2 + 6x + 16$, where $H(x)$ represents the height of the boulder in feet and x is the horizontal distance in feet the boulder travels after launch.

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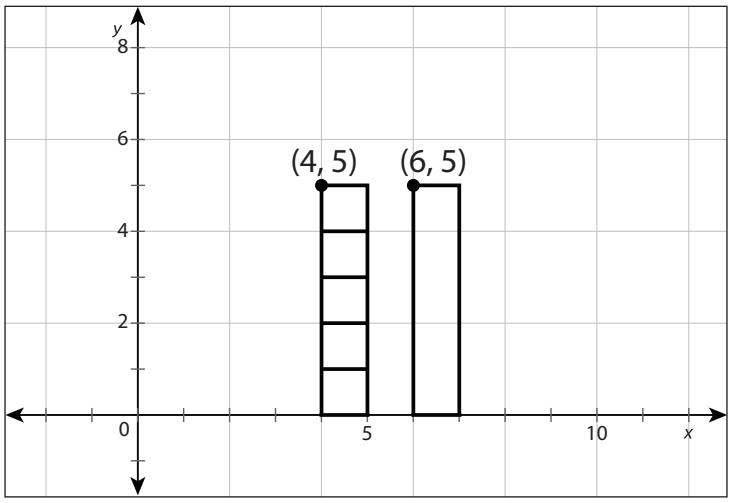
Lesson 2.1.2: Interpreting Various Forms of Quadratic Functions

Warm-Up 2.1.2

A sprinkler is spraying water over a neighbor's fence. The graph shows a stack of moving boxes on the left that the sprinkler must clear and a birdbath on the right that the sprinkler must fill. The water will follow a parabolic path.



1. One possible path the water could travel is given by $y = -\frac{1}{5}x^2 + \frac{8}{5}x + \frac{9}{5}$, where y represents the height in feet and x represents the horizontal distance traveled in feet. What is the vertex of the quadratic equation?

2. Determine the second x-intercept if one x-intercept of the path of the water is (–1, 0).

3. What is the maximum value of the quadratic equation?

4. Sketch the graph of the path of the water.

5. Based on the graph, will the water clear the moving boxes? If it clears the moving boxes, will the water fill the birdbath?

Scaffolded Practice 2.1.2

Example 1

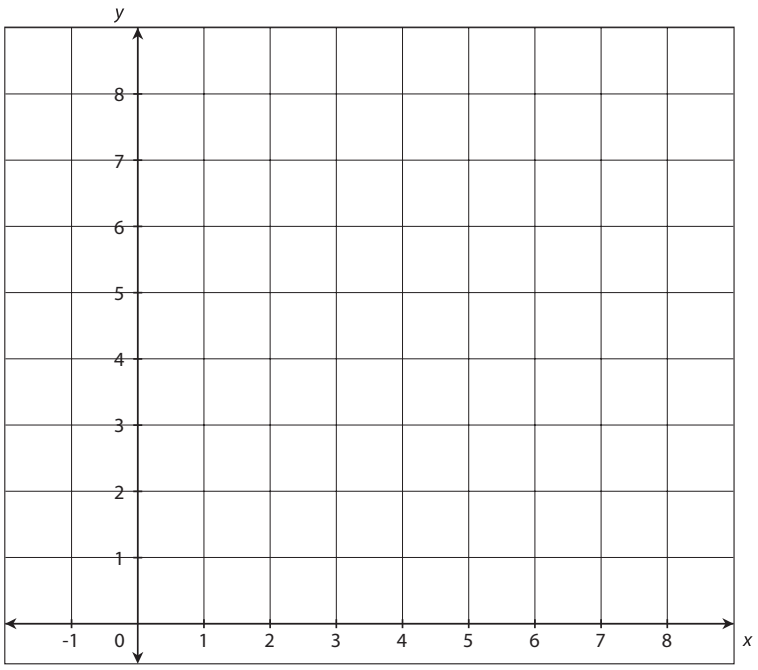
Suppose that the flight of a launched bottle rocket can be modeled by the function $f(x) = -(x - 1)(x - 6)$, where $f(x)$ measures the height above the ground in meters and x represents the horizontal distance in meters from the launching spot at $x = 1$. How far does the bottle rocket travel in the horizontal direction from launch to landing? What is the maximum height the bottle rocket reaches? How far has the bottle rocket traveled horizontally when it reaches its maximum height? Graph the function.

1. Identify the x-intercepts of the function.

2. Determine the maximum height of the bottle rocket.

3. Determine the horizontal distance from the launch point to the maximum height of the bottle rocket.

4. Graph the function.



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Example 2

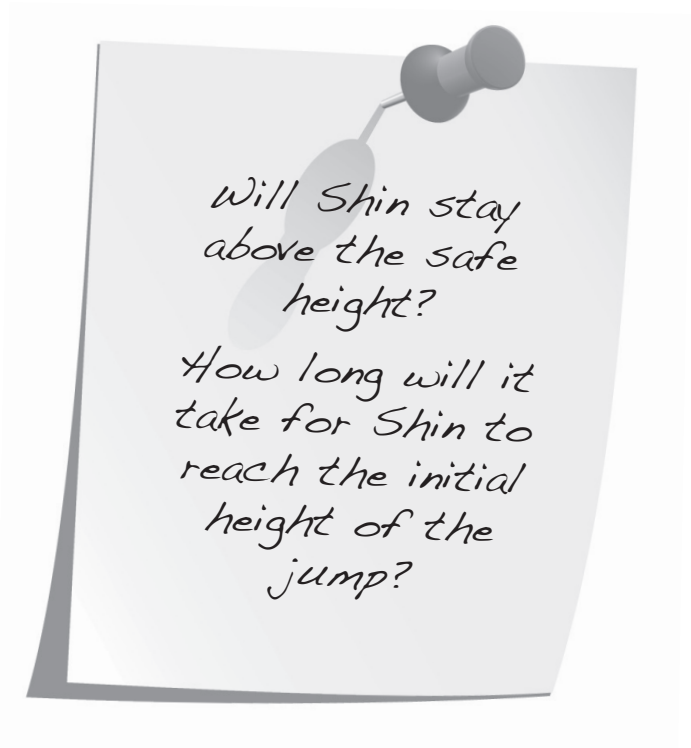
Reducing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 change in price, x, for a particular item is $R(x) = -100(x - 7)^2 + 28{,}900$. What is the maximum value of the function? What does the maximum value mean in the context of the problem? What price increase maximizes the revenue and what does it mean in the context of the problem? Graph the function.

Example 3

A football is kicked and follows a path given by $f(x) = -0.03x^2 + 1.8x$, where $f(x)$ represents the height of the ball in feet and x represents the horizontal distance in feet. What is the maximum height the ball reaches? What horizontal distance maximizes the height? Graph the function.

Problem-Based Task 2.1.2: Is the Glider Safe?

Shin is a beginner hang glider. He's practicing jumping from a certain height, dipping initially, and then rising. Shin should dip to a height no lower than 6 feet above the ground, which is considered a safe height, before changing direction and beginning to rise. The position of Shin's hang glider is given by $y = (x - 4)(x - 6)$, with x representing the time in seconds since Shin starts the initial jump and y representing the distance in feet from the safe height. Will Shin stay above the safe height? How long will it take for Shin to reach the initial height of the jump?



Practice 2.1.2: Interpreting Various Forms of Quadratic Functions

Use the given functions to complete all parts of problems 1–3.

1. $f(x) = x^2 - 6x + 8$
   a. Identify the y-intercept.
   b. Identify the vertex.
   c. Identify whether the function has a maximum or minimum.

2. $f(x) = -0.5(x + 2)(x - 4)$
   a. Identify the x-intercepts.
   b. Determine the y-intercept.
   c. Determine the axis of symmetry.
   d. Determine the vertex.

3. $f(x) = -16(x - 1)^2 + 10$
   a. Identify the vertex.
   b. Identify whether the function has a maximum or minimum.

Use the given information in each scenario that follows to complete the remaining problems.

4. A bird is descending toward a lake to catch a fish. The bird's flight can be modeled by the equation $h(t) = t^2 - 14t + 40$, where $h(t)$ is the bird's height above the water in feet and t is the time in seconds since you saw the bird. Graph the function. What is the vertex? What does the minimum value mean in the context of the problem?

5. A military pilot fires a test missile whose path can be modeled by the equation $f(x) = -(x - 40)(x + 2)$, where $f(x)$ is the height of the missile in miles and x is the number of seconds since the missile was fired. Graph this function. What are the x-intercepts and what do they mean in the context of the problem? After how many seconds is the height of the missile the same as the initial height?

continued

6. The path of a snowboarder performing stunts is given by the equation $f(x) = -16(t - 2)(t + 1)$, where t is time in seconds and y is the duration of the stunt. Graph the function. What are the x-intercepts? Explain the meaning of the x-intercepts in the context of the problem. How long does the stunt last?

7. The flight of a paper airplane follows the quadratic equation $H(x) = -(x - 3)^2 + 25$, where $H(x)$ represents the height of the paper airplane and x is the horizontal distance in feet the airplane travels after it is thrown. Graph the function. What is the vertex? Explain the meaning of the vertex in the context of the problem.

8. The height of a golfer's ball is given by the equation $y = -16x^2 + 32x$, where y represents the height in feet and x represents the time in seconds. Graph the function. What is the vertex and what does it mean in the context of the problem?

9. The revenue, $R(x)$, generated by an increase in price of x dollars for an item is represented by the equation $R(x) = -5(x - 15)(x + 5)$. Graph the function. What are the x-intercepts and what do they represent in the context of the problem? What value of x maximizes the revenue?

10. Decreasing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 decrease in price, x, is $R(x) = -(x - 7)^2$. Graph the function. What is the vertex and what does it represent in the context of the problem?

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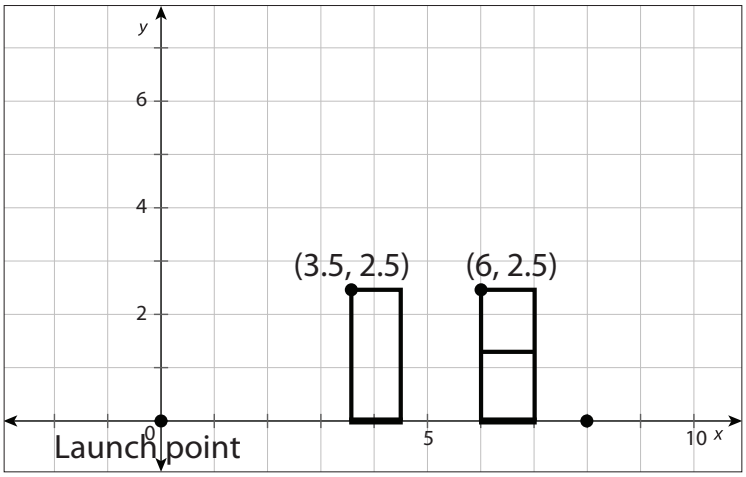
Lesson 2.2.1: Interpreting Key Features of Quadratic Functions

Warm-Up 2.2.1

The object of a popular video game is to launch a boulder to knock over boxes, buildings, and other items. The graph shows an obstacle on the left that the boulder must clear in order to knock over the stack of boxes on the right. The boulder will follow a parabolic path and will launch from (0, 0) and end at (8, 0).



1. What are the *x*-intercepts for the parabola formed by the path of the boulder?

2. What is the axis of symmetry for the parabola formed by the path of the boulder? How do you know?

3. One possible path for the boulder is $y=-\frac{3}{8}x^2+3x$. What is the vertex of the parabola created by this equation?

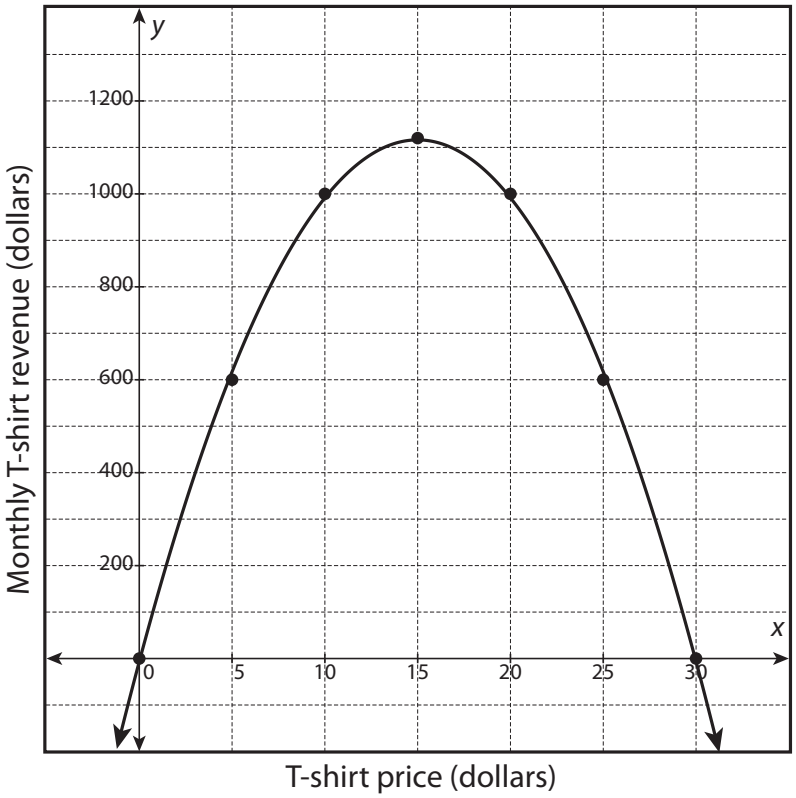
4. Will the boulder clear the obstacle? How do you know?

5. Will the boulder knock down the boxes? How do you know?

Scaffolded Practice 2.2.1

Example 1

A local store's monthly revenue from T-shirt sales is modeled by the function $f(x) = -5x^2 + 150x - 7$. Use the equation and graph to answer the following questions: At what prices is the revenue increasing? Decreasing? What is the maximum revenue? What prices yield no revenue? Is the function even, odd, or neither?



1. Determine when the function is increasing and decreasing.

2. Determine the maximum revenue.

3. Determine the prices that yield no revenue.

4. Determine if the function is even, odd, or neither.

5. Use the graph of the function to verify that the function is neither odd nor even.

continued

Example 2

A function has a minimum value of –5 and x-intercepts of –8 and 4. What is the value of x that minimizes the function? For what values of x is the function increasing? Decreasing?

Example 3

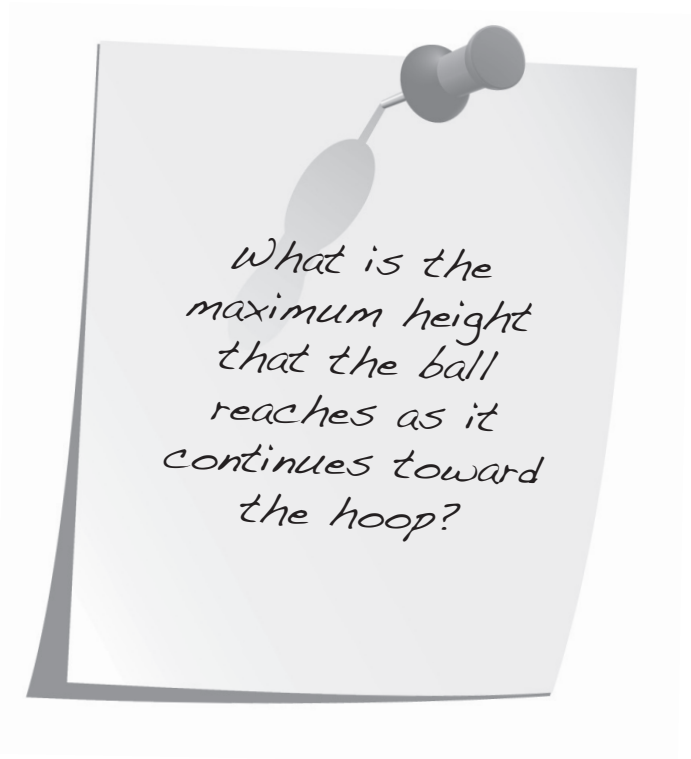
The table below shows the predicted temperatures for a summer day in Woodland, California. At what times is the temperature increasing? Decreasing?

Time	Temperature (°F)
8 A.M.	52
10 A.M.	64
12 P.M.	72
2 P.M.	78
4 P.M.	81
6 P.M.	76

Problem-Based Task 2.2.1: One-on-One Basketball

You and a friend are playing one-on-one basketball at the park. You aim at the hoop and release the ball, which follows a parabolic path. The table below represents the ball's horizontal distance from you versus the height of the ball as it travels toward the center of the hoop, represented by the point (14, 10). Use a quadratic model to determine for what horizontal distances the height of the ball is increasing and decreasing. What is the maximum height that the ball reaches as it continues toward the hoop?

Distance from shooter (feet)	Ball height (feet)
4	10
6	12
12	12
14	10



Practice 2.2.1: Interpreting Key Features of Quadratic Functions

For each of the functions below, use graphing technology to answer the following questions: What are the x-values for which the function is increasing? Decreasing? What is the maximum or minimum value of the function? What are the intercepts? Is the function even, odd, or neither?

1. $f(x) = 3x^2 - 2x - 5$

2. $g(x) = -3x^2 + 10x + 1$

3. $y = 5x^2 + 10x + 11$

4. $h(x) = 2x^2 - 4x - 11$

Given the descriptions of the quadratic functions below, answer the following questions: What is the value of x that minimizes or maximizes the function? For what values of x is the function increasing? Decreasing?

5. A function has a minimum value of –20 and x-intercepts of –1.72 and 0.38.

6. A function has a maximum value of 12.375 and x-intercepts of 0.41 and 1.84.

7. A function has a minimum value of –8.675 and x-intercepts of 1.23 and –0.48.

8. A function has a minimum value of –8.167 and x-intercepts of 1.3 and –1.

continued

Use the tables and scenarios that follow to complete the remaining problems.

9. You and a friend are playing softball. You throw the ball toward your friend's mitt so that the ball follows a parabolic path. The table below represents the ball's distance from you versus the height of the ball as it travels toward the center of the mitt, represented by the point (0, 2). Use a quadratic model to determine for what distances the height of the ball is increasing and decreasing.

Distance from you (feet)	Ball height (feet)
0	2
20	8
30	8
40	6

10. The table below shows the predicted temperatures for an autumn day in Annapolis, Maryland. Use a quadratic model to determine the maximum temperature that Annapolis reaches on this day.

Time	Temperature (°F)
11 A.M.	59
2 P.M.	63
5 P.M.	63
8 P.M.	58
11 P.M.	56

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Lesson 2.2.2: Identifying the Domain of a Quadratic Function

Warm-Up 2.2.2

Gina works in a video game store over her summer break. She earns $8 per hour plus commission on the video games and gaming consoles she sells. This month, her store offered an incentive of an extra day off to the employee who sells the most copies of a certain new game. She makes $3 in commission for each game sold.

1. Write a linear model to represent Gina's take-home pay as a function of the number of games she sells in one 8-hour workday.

2. What is the reasonable domain for this function?

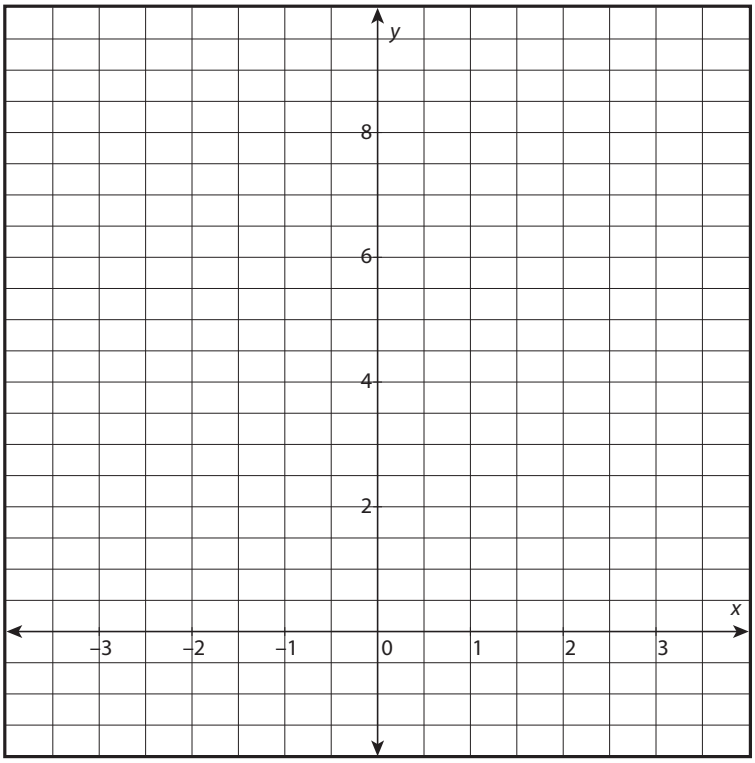
3. What does the domain represent?

Scaffolded Practice 2.2.2

Example 1

Describe the domain of the quadratic function $g(x) = 1.5x^2$.

1. Sketch a graph of the function.



2. Describe what will happen if the function continues.

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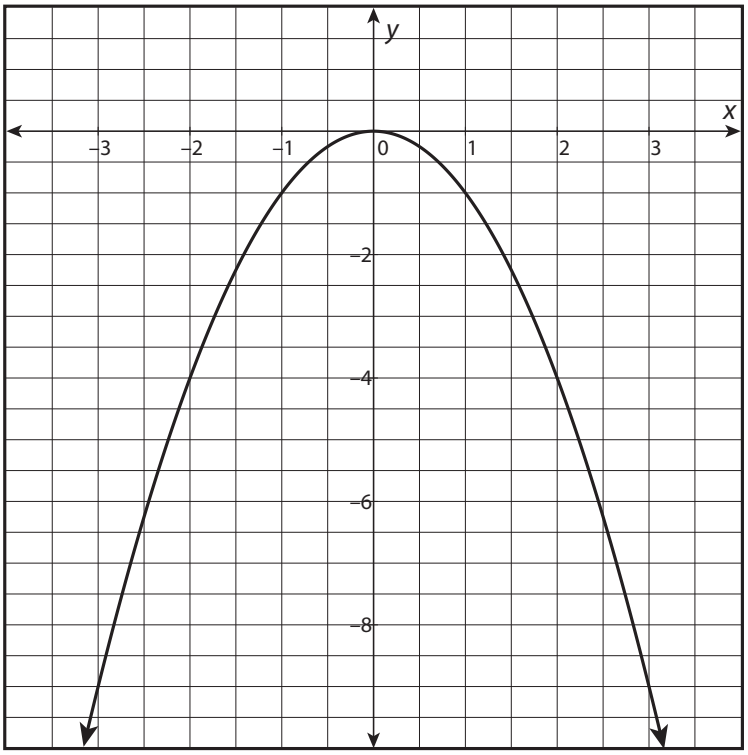
Example 2

Describe the domain of the following function.

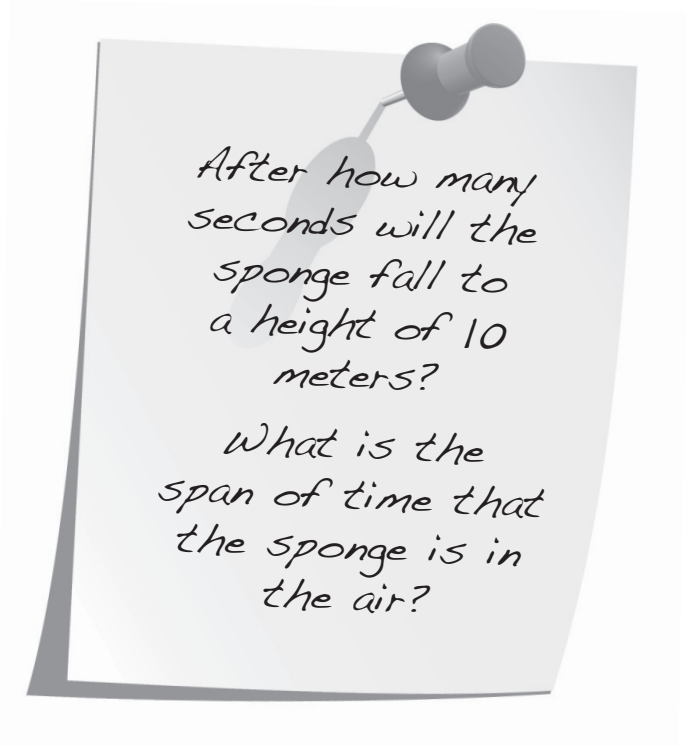


Example 3

Amit is a diver on the swim team. Today he's practicing by jumping off a 14-foot platform into the pool. Amit's height in feet above the water is modeled by $f(x) = -16x^2 + 14$, where x is the time in seconds after he leaves the platform. About how long will it take Amit to reach the water? Describe the domain of this function.

Problem-Based Task 2.2.2: Window Washers

A window washer tosses his wet sponge from a height of 10 meters above ground level to his coworker above him. The sponge reaches its maximum height of 11.25 meters exactly 0.5 second later, but the coworker does not catch the sponge and it falls to ground. After how many seconds will the sponge fall to a height of 10 meters? What is the span of time that the sponge is in the air?



Practice 2.2.2: Identifying the Domain of a Quadratic Function

Use graphing technology to determine the domain of each quadratic function.

1. $y = -x^2 + 7x + 1$

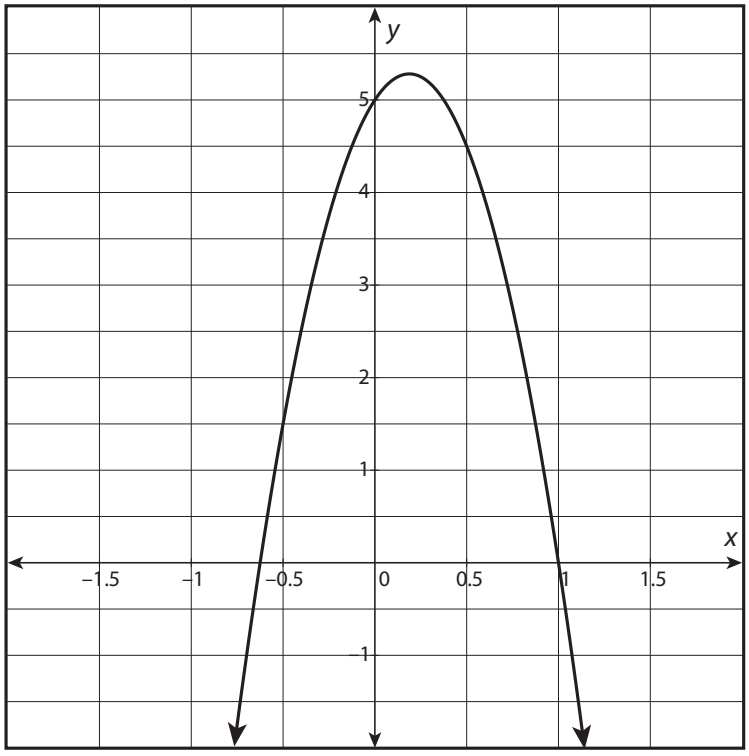
2. $y = -\frac{3}{5}x^2 + 21x - 3$

3. $f(x) = 4x^2 + 5x - 12$

4. $g(x) = x^2 + 12x - 8$

Describe the domain of each of the following functions in words and as an inequality.

5.



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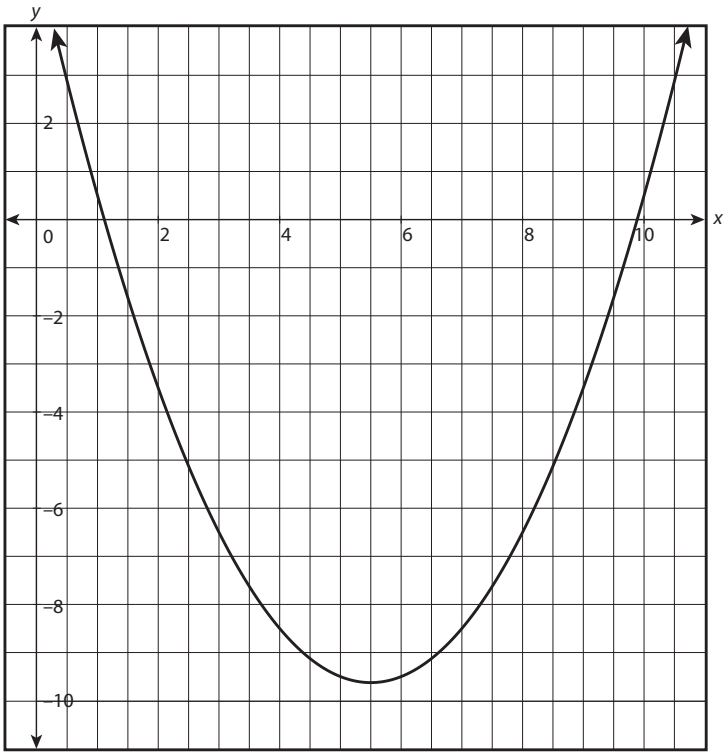

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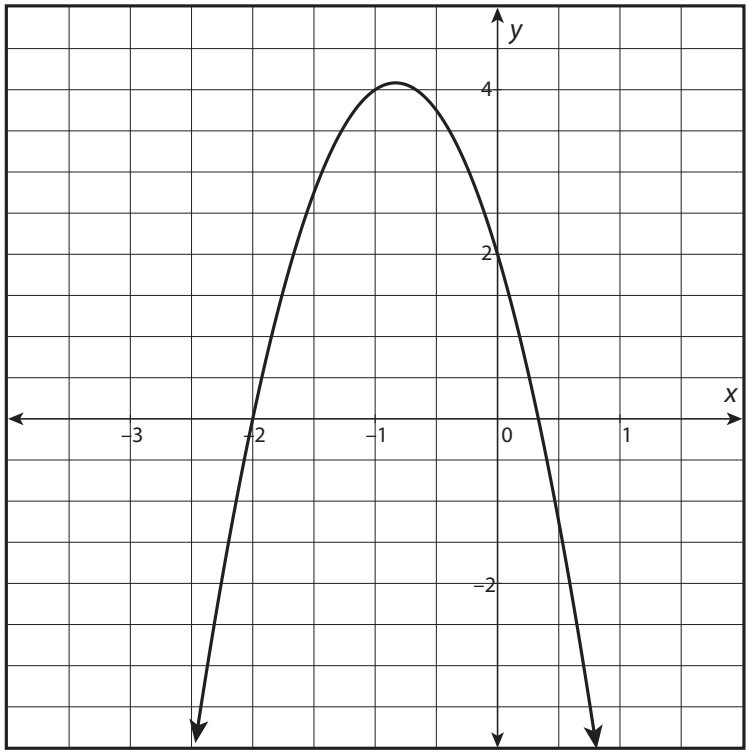

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Use the given information to solve the following problems.

8. A soccer ball is kicked from the ground and travels a parabolic path. The path can be modeled by the function $h(t) = -5t^2 + 19.5t$, where $h(t)$ is the height of the soccer ball in meters above the ground t seconds after being kicked. Assuming the ball lands on level ground, about how long is the ball in the air?

9. A golf ball is shot from the ground using a practice cannon and travels a parabolic path. The path of the ball can be modeled by the function $h(t) = -16t^2 + 150t$, where $h(t)$ is the height of the golf ball in meters above the ground t seconds after being shot. Assuming the ball lands on level ground, about how long does it take the golf ball to hit the ground?

10. The senior class is putting on a talent show to raise money for their senior trip. In the past, the profit from the talent show could be modeled by the function $P(t) = -16x^2 + 600x - 4000$, where x represents the ticket price in dollars. What is the reasonable domain for this function? For what domain value will the profits be maximized?

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Lesson 2.2.3: Identifying the Average Rate of Change

Warm-Up 2.2.3

Data on a certain car shows that its gas mileage can be modeled by a linear function. The car yields 45 miles per gallon (mpg) when driven at 40 miles per hour (mph) and 25 mpg when driven at 80 mph.

1. What is the rate of change of this car's gas mileage?

2. Create the linear model of the car's gas mileage as a function of its speed.

3. Given the model from problem 2, what is always true about this car's gas mileage? Explain.

Scaffolded Practice 2.2.3

Example 1

Calculate the average rate of change for the function $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$.

1. Evaluate the function for $x = 3$.

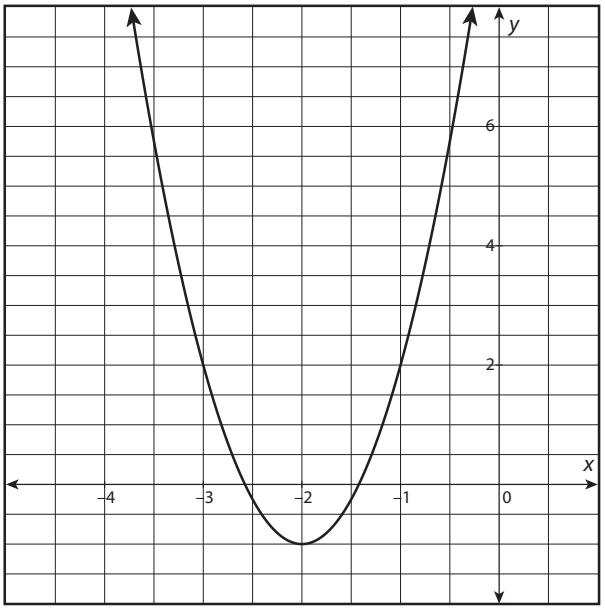
2. Evaluate the function for $x = 1$.

3. Use the average rate of change formula to determine the average rate of change between $x = 1$ and $x = 3$.

continued

Example 2

Use the graph of the function to calculate the average rate of change between $x = -3$ and $x = -2$.



Example 3

For the function $g(x) = (x - 3)^2 - 2$, is the average rate of change greater between $x = -1$ and $x = 0$ or between $x = 1$ and $x = 2$?

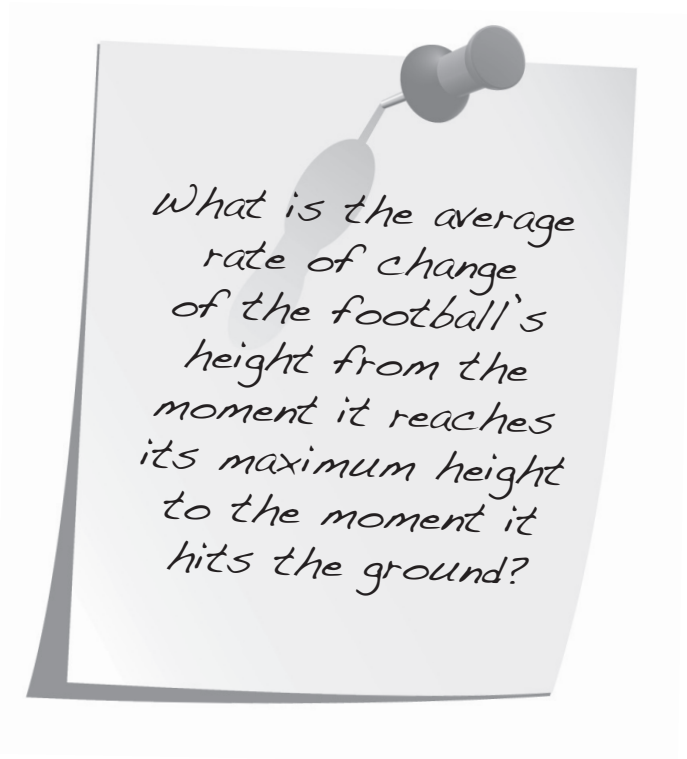
Example 4

Find the average rate of change between $x = -0.75$ and $x = -0.25$ for the following function.

x	*y*
–1	0
–0.75	3.44
–0.5	6.25
–0.25	8.44
0	10
0.25	10.94

Problem-Based Task 2.2.3: Is the Maximum High Enough?

It is Super Bowl season and teams that have made the play-offs have specialists evaluating every aspect of their field game. One particular team received news that their recently injured kicker's field goal kick is modeled by the function $h(t) = -16(x - 1)^2 + 16$, where $h(t)$ is the height of the ball in feet t seconds after it is kicked. If the football needs to clear a 17-foot goalpost, will the ball make it over if this particular team member kicks it? What is the average rate of change of the football's height from the moment it reaches its maximum height to the moment it hits the ground?



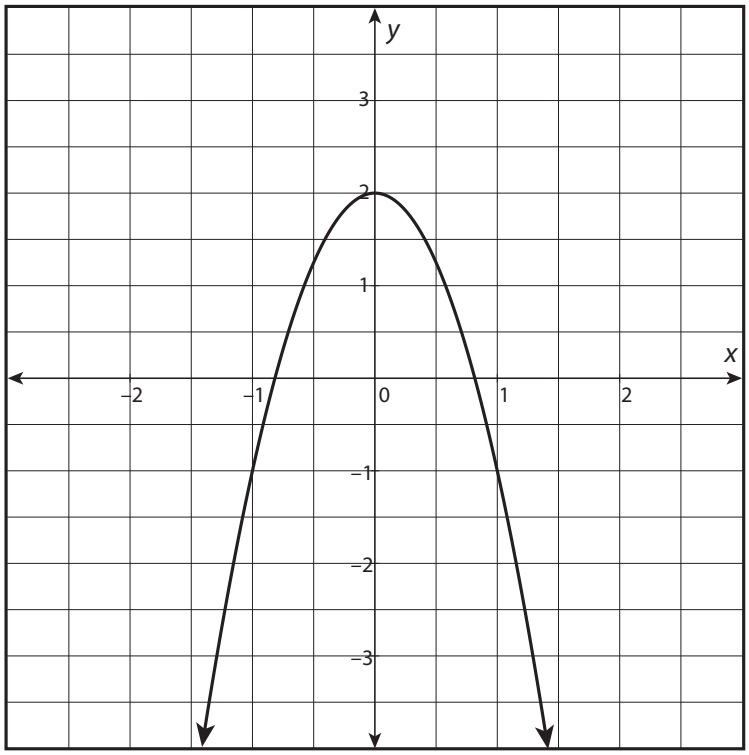
Practice 2.2.3: Identifying the Average Rate of Change

Calculate the average rate of change for each function below between $x = -1$ and $x = 1$.

1. $f(x) = 2(x + 1)^2 - 3$

2. $g(x) = 4 - 3(x - 1)^2$

3. $h(x) = x^2 - 4x + 6$

4. 

5.

x	*y*
–2	–1
–1.5	–1.75
–1	–4
–0.5	–7.75
0	–13
0.5	–19.75
1	28
1.5	–37.75

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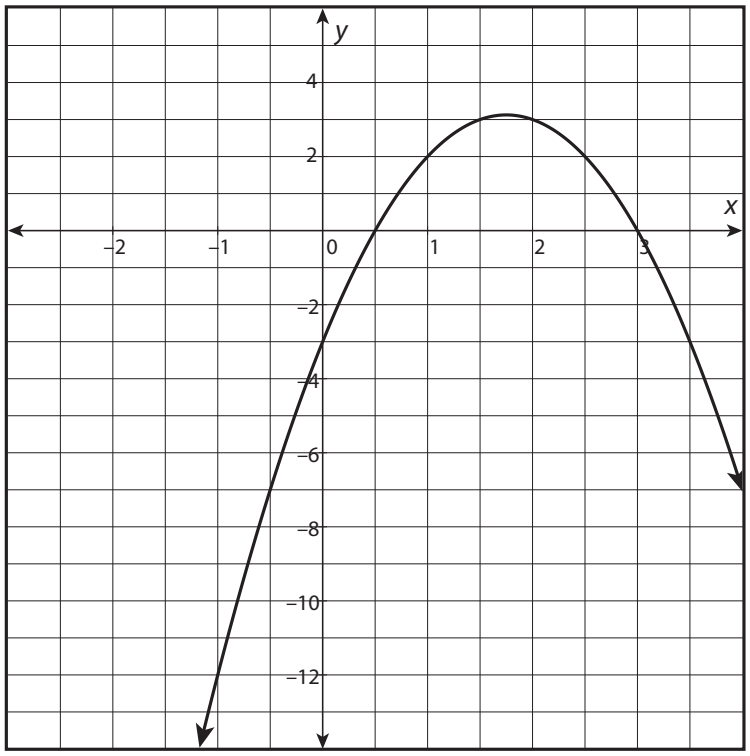

For the following functions, is the average rate of change greater between $x = -2$ and $x = 0$ or between $x = 0$ and $x = 2$?

7. $y = \frac{1}{2}(x+2)^2 - 3$

8. $a(x) = -x^2 + 8x + 3$

9. $y = 5x^2 - 6x + 4$

Read the scenario and use the information in it to answer the question.

10. A drop of rain falls from a height of 1,400 feet above the ground. The function $h(t) = -16t^2 + 1400$ is used to model the raindrop's height, $h(t)$, in feet t seconds after it starts to fall. What is the raindrop's average rate of change 2 to 3 seconds after it falls?

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Lesson 2.3.1: Building Functions from Context

Warm-Up 2.3.1

The height of a falling object (that is, its distance from the surface of a planet) can be approximated by the quadratic equation $h(t) = a - \frac{1}{2}gt^2$, where a is the starting height, g is the gravitational constant of the planet, and t is the free-fall time in seconds. Each planet has a different gravitational constant based on its mass. Use the information in the problems below to determine how long it will take an object to fall from a given height to the surface of a particular planet. Round your answers to the nearest hundredth.

1. The gravitational constant of Earth is approximately 32 ft/s^2. If an object is dropped from 1,200 feet above Earth, how many seconds will pass before the object hits the ground?

2. The gravitational constant of Mars is approximately 12.5 ft/s^2. If an object is dropped from 1,200 feet above Mars, how many seconds will pass before the object hits the surface of Mars?

3. The gravitational constant of Jupiter is approximately 84.5 ft/s^2. If an object is dropped from 1,200 feet above Jupiter, how many seconds will pass before the object hits the surface of Jupiter?

Scaffolded Practice 2.3.1

Example 1

A farmer is building a rectangular pen using 100 feet of electric fencing and the side of a barn. In addition to fencing, there will be a 4-foot gate also requiring the electric fencing on either side of the pen. The farmer wants to maximize the area of the pen. How long should he make each side of the fence in order to create the maximum area?

1. Write the expressions that describe the length of each side of the pen.

2. Build the equation that describes the area of the pen.

3. To find the maximum area, use the vertex.

4. Finally, use the *x*-value from the vertex to find the lengths of each side of the pen.

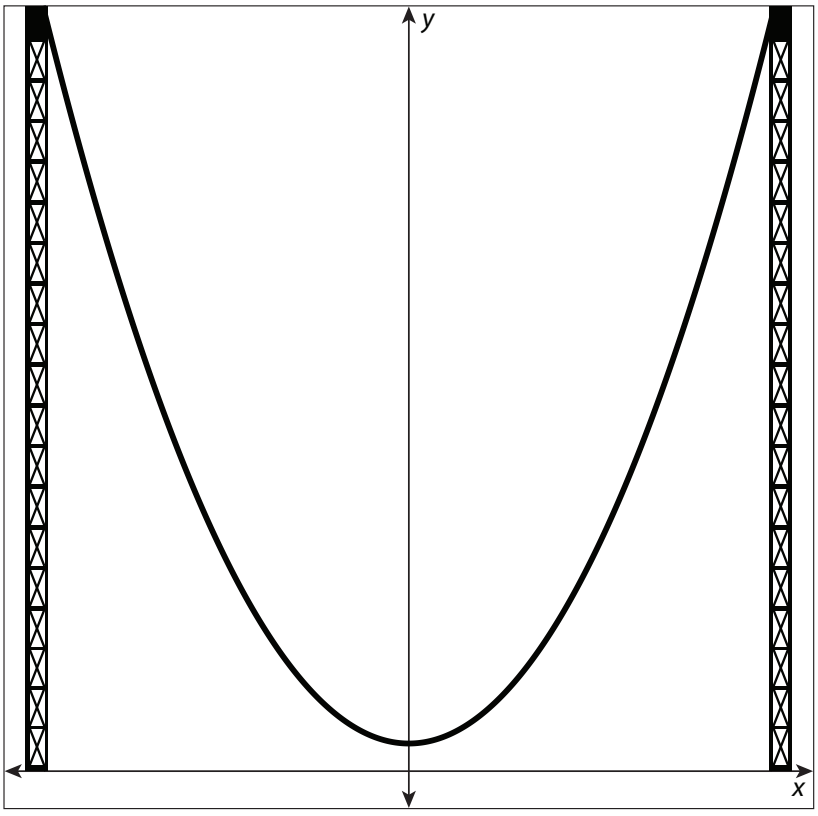
continued

Example 2

An amusement park has commissioned the design of a steel roller coaster with a drop section that is modeled by a parabola. Part of the roller coaster's track will go through an underground tunnel. In this section, the roller coaster will dip 12 feet below ground level. The roller coaster will dip below ground level at a horizontal distance of 24 feet from the peak just before the drop and reemerge to ground level at a horizontal distance of 36 feet from the peak just before the drop. Find an equation of the parabola that describes the drop and the height of the roller coaster at the peak.

Example 3

A suspension bridge has two cables secured at either end of the span by two supporting towers. The cables are attached to the tops of the towers. In the section between the two towers, the cables form a parabolic curve. At their lowest point, the cables are 15 feet from the surface of the bridge. The towers are 400 feet apart, and the vertical distance from the surface of the bridge to the top of each tower is 415 feet. What is a quadratic equation that describes the curve of the cables between the towers?



Problem-Based Task 2.3.1: To Drill or Not to Drill?

Cal owns and operates a small oil field in Texas. The field has 75 oil wells, and each well produces 945 barrels of oil per day. There is enough land in the oil field for Cal to drill more wells, but every additional well will cause oil production to drop by 3 barrels per day for each well. Cal believes that it would be profitable to build more wells in the field, but he is not sure how many to build. Building too few wells won't result in the most possible profit. Building too many wells will not be cost effective and will actually cut his profits. Help Cal create a model that allows him to predict the effect that more wells will have on oil production. Use this model to determine the maximum number of wells Cal should have on his land.

Practice 2.3.1: Building Functions from Context

Use your knowledge of quadratic functions to complete each problem that follows.

1. Expand the linear factors of $f(x)$, where $f(x) = (x - 10)(x + 2)$.

2. Let $g(x) = 3x^2 + 4x - 36$. What is $g(2)$?

3. The product of two consecutive odd integers is 2,915. Build a function that can be used to solve for the integers. What are the two integers?

Use the following scenario to complete problems 4 and 5.

> A suspension bridge has two supporting towers with a cable secured at either end of the span and then draped off the towers. In the section between the two supporting towers, the cable forms a parabolic curve. At the lowest point of the curve, the cable is 20 feet from the surface of the bridge. The supporting towers are 500 feet apart, and rise 420 feet from the surface of the bridge to the top of the tower. Use the height of the left tower as the *y*-intercept.

4. What are the coordinates of the vertex and the *y*-intercept?

5. What is the equation in vertex form of the suspension cable?

continued

Use the following scenario to complete problems 6 and 7.

A museum curator needs to frame a rectangular painting. The painting is 24 inches by 16 inches.

6. If the frame is to be of width x, what is a function for the area of the painting, including the frame?

7. What would be the area of just the frame if the value of x was determined to be 2.5 inches?

Use the following scenario to complete problems 8 and 9.

An amusement park is building a wooden roller coaster with a drop section modeled by a parabola. The roller coaster will travel through an underground tunnel. In this section, the roller coaster will dip 9 feet below ground level. The roller coaster will dip below ground level a horizontal distance of 32 feet from the peak just before the drop and reemerge to ground level a horizontal distance of 80 feet from the peak just before the drop.

8. What is the equation of the parabola that models the drop from the peak to the reemergence at ground level?

9. What is the height of the peak just before this drop?

Use the given information to write a function for problem 10.

10. A family just bought a puppy and wants to fence in part of the yard. The fenced-in area is a rectangular section of the yard that is separate from all other structures on the property. The family has purchased 70 feet of fencing. What function models the area of the pen?

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Lesson 2.3.2: Operating on Functions

Warm-Up 2.3.2

The student council is selling lanyards with the school's name on them. Each lanyard costs $1.50 to purchase from the manufacturer. The student council is selling each lanyard for $5.00.

1. Write a function $f(x)$ that models the cost of buying the lanyards before reselling them.

2. Write a function $g(x)$ that models the income of selling the lanyards.

3. What is the profit from selling one lanyard?

4. Write a function $h(x)$ that models the profit of the sale of the lanyards in terms of $f(x)$ and $g(x)$, and then simplify the function.

Scaffolded Practice 2.3.2

Example 1

Let $f(x) = x^2 - 3x + 4$ and $g(x) = x^2 + 6x - 3$. Build a new function, $h(x)$, for which $h(x) = (f + g)(x)$.

1. Expand the new function, $h(x)$, into a form where substitution can be used.

2. Add the functions.

continued

Example 2

Let $f(x) = 3x + 4$ and $g(x) = 5x - 2$. Build a new function, $h(x)$, for which $h(x) = (f \bullet g)(x)$.

Example 3

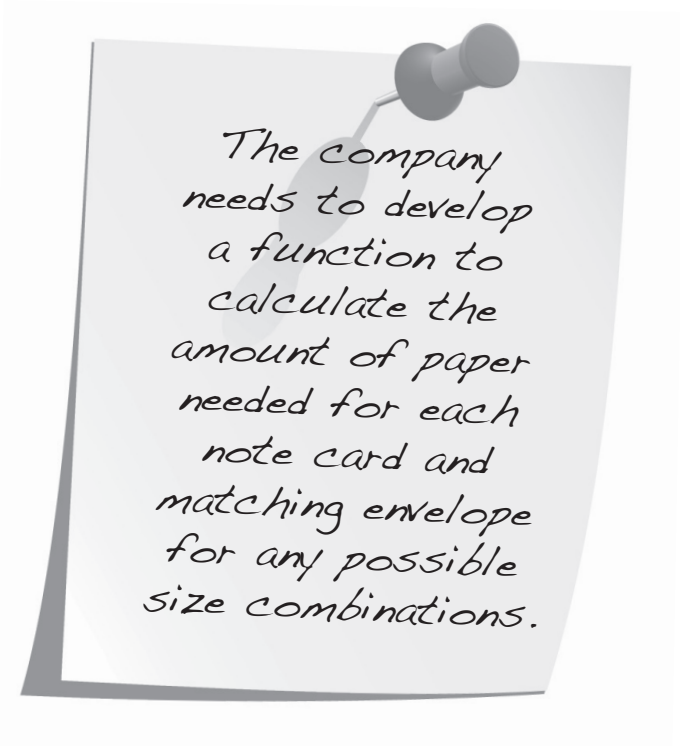
For $f(x) = 3x^2 + 13x - 10$ and $g(x) = x + 5$, find $\left(\frac{f}{g}\right)(x)$. What type of function is the quotient of $\left(\frac{f}{g}\right)(x)$? Are there restrictions on the domain and range of the function $\left(\frac{f}{g}\right)(x)$?

Example 4

Zane is a textiles designer. His latest project is to design a rectangular area rug for a hotel lobby. The dimensions of the lobby are such that one set of walls is twice the length of the other set of walls. The rug must lay centered in the lobby, with each edge of the rug exactly 3 feet from each wall. What is the function in terms of x that describes the area of the lobby? What is the function in terms of x that describes the area of the rug? What is the function that describes the area of the lobby left uncovered by the rug?

Problem-Based Task 2.3.2: Pushing Envelopes

A stationery company makes note cards and matching envelopes in various sizes. Regardless of the size of the note card, the ratio of the height to the width is always 3 : 5. Every envelope is made to be precisely 2 mm wider and 3 mm taller than its matching note card. Finally, the amount of paper required to construct each envelope is equal to 3 times the area of the front of the envelope. The company needs to develop a function to calculate the amount of paper needed for each note card and matching envelope for any possible size combinations.



Practice 2.3.2: Operating on Functions

Use your knowledge of operations on functions to complete the problems that follow.

1. Let $f(x) = x^2 + 7x + 12$ and $g(x) = 5x + 23$. Build a new function, $h(x)$, for which $h(x) = (f + g)(x)$.

2. Let $s(x) = x^2 + 12x + 13$ and $t(x) = 19x + 3$. Build a new function, $u(x)$, for which $u(x) = (s - t)(x)$.

3. Let $j(x) = 2x + 1$ and $k(x) = 11x - 3$. Build a new function, $m(x)$, for which $m(x) = (f \bullet g)(x)$.

4. Let $f(x) = x^2 + 7x + 12$ and $g(x) = x + 3$. Build a new function, $h(x)$, for which $h(x) = \left(\frac{f}{g}\right)x$. State any restrictions on the domain and range.

5. Let $g(x) = -3x^2 - 11x + 15$ and $h(x) = -x^2 + 3$. Build a new function, $k(x)$, for which $k(x) = (h - g)(x)$.

continued

6. Sinopa is designing a rectangular area rug for a room. The dimensions of the room are such that one set of walls is 3 feet longer than the other set of walls. The rug must lay centered in the room with each edge 2 feet from each wall. What is the function in terms of x that describes the area of the room? What is the function in terms of x that describes the area of the rug? What is the function that describes the area of the room left uncovered by the rug?

7. A city parks department increased the lengths of both sides of a park by the same amount. As a result, the park is 120 feet by 200 feet. Build the functions that represent the original side lengths and calculate the area function of the original park using an operation of the functions you built.

8. The altitude of a triangular banner is 4 feet longer than twice its base. Define the functions that describe the altitude and base of the triangle, and then use these functions to build an area function of the banner.

9. The surface area of a cylinder is found by adding the top, bottom, and side surfaces. The top and bottom of a cylinder are circles with an area of πr^2, where r is the radius of the circle. The side surface of a cylinder is given by the equation $2\pi rh$, where r is the radius of the circles on the top and bottom of the cylinder. Define a function $f(r)$ that describes the area of the top and bottom of the cylinder. Define a function $g(r)$ that describes the surface area of the side of the cylinder. Using $f(r)$ and $g(r)$, define a function that describes the surface area of a cylinder.

10. If the cylinder described in the previous problem were cut into thirds that are parallel to the top and bottom, resulting in 3 cylinders of equal height, what would be the new function for the total surface area of the cylinders?

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Lesson 2.4.1: Square Root and Cube Root Functions

Warm-Up 2.4.1

Michael has 24 feet of wooden fencing. He is using all 24 feet of fencing to create one rectangular garden. The perimeter of a rectangle is the sum of its four sides, and the area of a rectangle is its length times its width, or *area* = *length* • *width*. For Michael's garden, if x = the length of the rectangle, then the width = $12 - x$.

1. Write a function that shows all the possible areas of rectangular gardens Michael could make with the 24 feet of fencing.

2. If x is the rectangle's length, what is the domain of x?

3. Create a graph of the function on the determined domain.

4. Which rectangle length and width will result in the largest garden area?

Scaffolded Practice 2.4.1

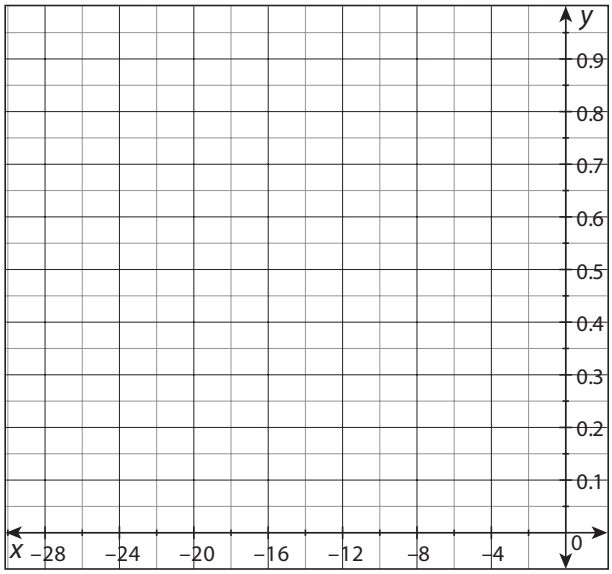
Example 1

The function $y = 0.176\sqrt{x+30}$ has a domain of $-30 \le x \le 0$. Determine the range of the function, then use a graph to estimate the value of y when $x = -10$.

1. Determine the range of the function.

2. Find at least three points on the function, including critical points.

3. Plot the three points and sketch the graph.



4. Use the graph to estimate the function's output at the given input.

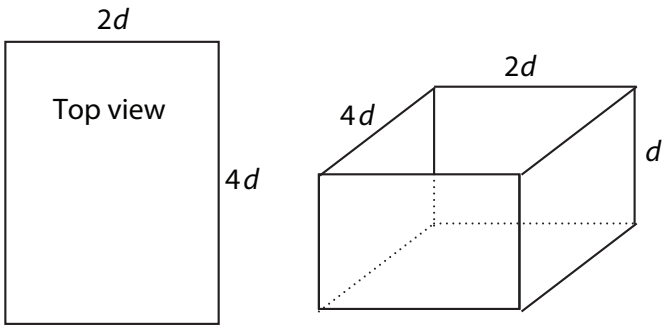
continued

Example 2

Compare the domain, range, graph, and critical values on the graph of $y = -4\sqrt[3]{x} + 2$ to the graph of $y = \sqrt[3]{x}$. How are these differences reflected in the algebraic equations?

Example 3

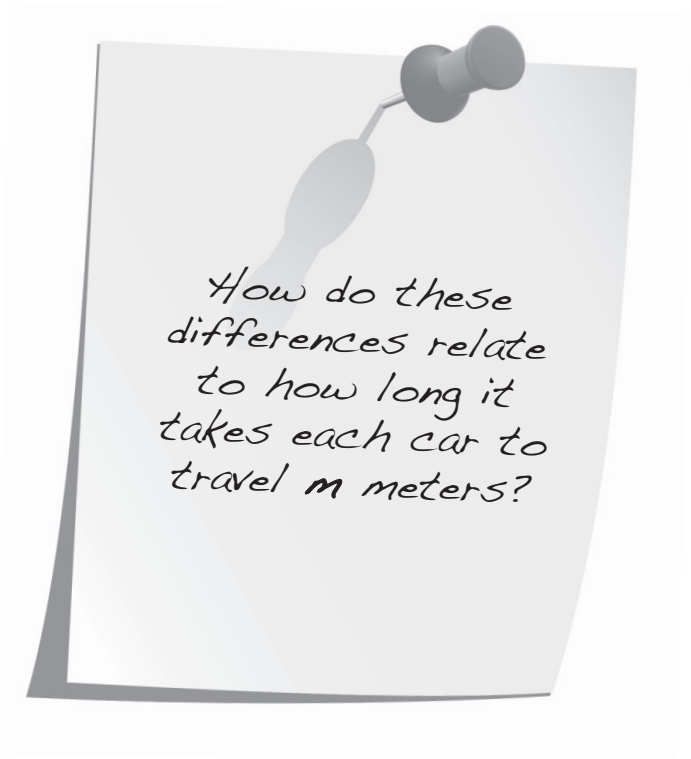
The Stephens family is building a rectangular inground pool. The depth, length, and width will all be related, as shown in the diagrams below.



The depth is d, the length is $4d$, and the width is $2d$. All dimensions are measured in feet. The volume of the pool is the product of the three dimensions: $4d \cdot 2d \cdot d = 8d^3$. The depth, d, can be represented for a pool with any volume using the function $d = \sqrt[3]{\frac{1}{8}V} = \frac{1}{2}\sqrt[3]{V}$. The Stephens family would like a pool with a volume of at least 1,000 ft^3 and no more than 11,000 ft^3. Create a graph to show the Stephens family all the possible values of d for the desired volumes.

Problem-Based Task 2.4.1: Auto Acceleration

Car manufacturers test how quickly a car is capable of accelerating by recording how long it takes the car to accelerate from 0 miles per hour to 60 miles per hour. Sofasta Motors' newest car, the Accelerator, reached 60 miles per hour in 8.1 seconds, which is an acceleration of 3.3 meters per second^2. Vroomy Vehicles' car, the VV1, reached 60 miles per hour in 9.1 seconds, which is an acceleration of 2.9 meters per second^2. The time t_a, in seconds, it takes the Accelerator to travel m meters can be represented using the function $t_a = 0.778\sqrt{m}$. The time t_v, in seconds, it takes the VV1 to travel m meters can be represented using the function $t_v = 0.830\sqrt{m}$. How do the differences in the graphs of the functions relate to the differences in the algebraic representations? How do these differences relate to how long it takes each car to travel m meters?



Practice 2.4.1: Square Root and Cube Root Functions

For problems 1–4, create a graph of each function. Note the domain, range, and any critical points.

1. $y=\sqrt{x+1}$

2. $y=2\sqrt[3]{x}$

3. $y=-\sqrt[3]{x-4}+3$

4. $y=3\sqrt{x}-10$

For problems 5 and 6, create a graph showing each pair of functions. Describe any similarities and differences between the graphs, including domain, range, and critical points. Describe how any similarities and differences are shown in the algebraic functions.

5. $y=\sqrt{x}$ and $y=-\sqrt{x+8}$

6. $y=\sqrt[3]{x}$ and $y=5\sqrt[3]{x}+9$

continued

Use the following information to complete problems 7 and 8.

The radius of a sphere, r, with a given volume, V, is $r = \sqrt[3]{\left(\frac{3}{4\pi}\right)} \bullet \sqrt[3]{V}$.

7. Create a graph to show the radii for spheres with different volumes. Only include a reasonable domain and range in the graph.

8. The volume of an NBA basketball is between 433.5 and 450.3 in^3. Use the graph of $r = \sqrt[3]{\left(\frac{3}{4\pi}\right)} \bullet \sqrt[3]{V}$ to estimate the range of the radii of NBA basketballs.

Read the following scenario and use the information in it to complete problems 9 and 10.

Two soccer players, Aurora and Isla, each kick a ball from the same goal line. The balls each decelerate as they travel down the soccer field. The time in seconds, t, it takes Aurora's ball to travel m meters is $t = 1.061\sqrt{m}$. The time it takes Isla's ball to travel m meters is $t = 0.849\sqrt{m}$.

9. Graph the two functions on the same coordinate plane. Describe any similarities and differences between the graphs.

10. Using the graphs, compare how long it will take Aurora's ball and Isla's ball to travel the 90-meter length of a soccer field.

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Lesson 2.4.2: Absolute Value and Step Functions

Warm-Up 2.4.2

Emery works at a clothing store. He earns extra money based on how much clothing he sells. He earns 15% on the sales he makes, in addition to the flat salary of $400 he makes each week. He can determine how much money he earns in a week, m, by using the following equation: $m = 0.15s + 400$, where s represents his weekly sales, in dollars.

1. If Emery sells $1,400 worth of clothing in a week, how much money will he earn that week?

2. Create a graph that shows Emery's weekly earnings for any sales amount.

3. Use the graph to estimate how much Emery needs to sell in order to earn $750 in a week.

Scaffolded Practice 2.4.2

Example 1

A software program can show a user how far he or she is from a location on a map. Zadie is walking in a straight line down her street. For any time t, in seconds, her distance d from her home, in feet, can be represented by the function $d = 4|t - 20|$. Create a graph to show Zadie's distance from her house. Which point on the graph shows when Zadie has reached her house?

1. Determine a domain for the problem statement.

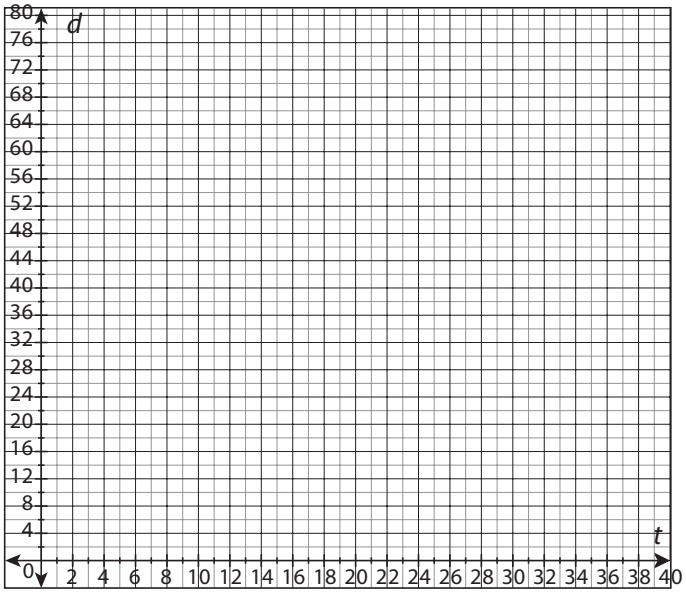
2. Determine the range for the given domain.

3. Find the critical point of the absolute value function.

4. Determine whether the critical point is a minimum or a maximum.

5. Find at least two additional points on the graph. One should have an input less than the critical point, and the other should have an input greater than the critical point.

6. Use the three points to create the graph of the function on the determined domain.



7. Interpret the graph to find when Zadie will arrive at her house.

continued

Example 2

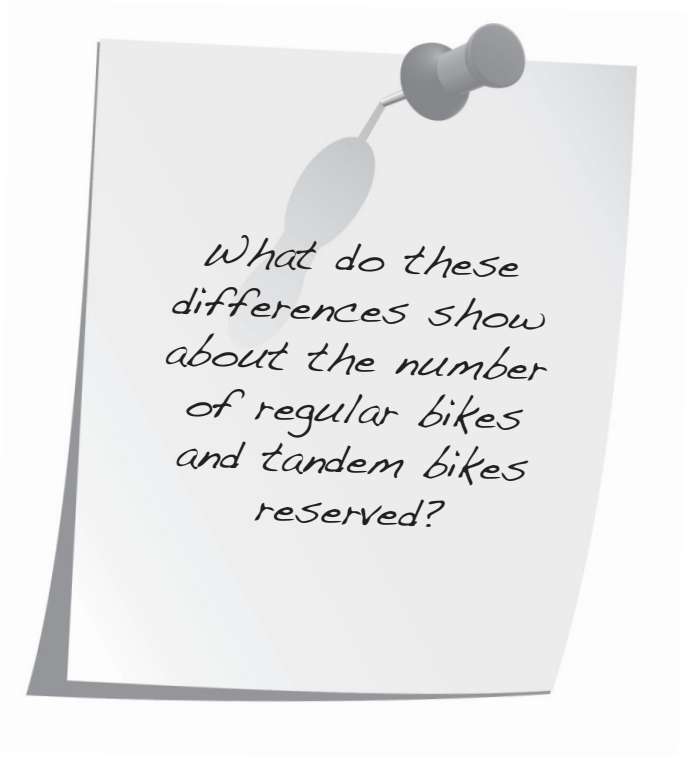
The eleventh grade students are planning a dance. Snacks will be sold at the dance, and the dance committee is trying to determine how many snacks to buy. The dance committee estimates that 60% of the students who buy tickets to the dance will want to buy snacks. Only 200 tickets will be sold. The number of snacks, *s*, to order can be written as a function of the number of tickets, *t*, ordered. For any number of tickets sold, *t*, the committee can order *s* snacks, where $s = \lceil 0.60t \rceil$. Create a graph to show the number of snacks to order for any number of tickets sold.

Example 3

Compare the domain, range, graph, and critical values on the graph of $y = -3|x+1|$ to the graph of $y = |x|$. How are these differences seen in the algebraic equations?

Problem-Based Task 2.4.2: Bike Rentals

A bike store manager reserves a certain number of bikes to be used as rentals. The manager uses the city's information about tourists to estimate how many bikes to reserve each week; however, she does not want to reserve too many bikes. The manager uses the greatest integer function in her estimate. If the city estimates a total of t tourists between 100 and 1,000 in one week, the bike store manager estimates that $r = \lfloor 0.2t \rfloor + 10$ rentals will be needed. The manager also reserves some tandem bicycles which each fit two riders. The number of tandem bikes reserved, n, is estimated using the function $n = \lfloor 0.10t \rfloor$. How do the differences in the graphs of the functions relate to the differences in the algebraic representations? What do these differences show about the number of regular bikes and tandem bikes reserved?



Practice 2.4.2: Absolute Value and Step Functions

For problems 1–4, create a graph of each function. Note the domain, range, and any critical points. Note that $\lfloor\ \rfloor$ represents the greatest integer function, and $\lceil\ \rceil$ represents the least integer function.

1. $y=|x-6|$

2. $y=-\lceil x \rceil+1$

3. $y=10|x|+7$

4. $y=\lfloor 2.5x \rfloor-12$

For problems 5–8, create a graph showing each pair of functions. Describe any similarities and differences between the graphs, including domain, range, and critical points. Describe how any similarities and differences are shown in the algebraic functions.

5. $y=|x|$ and $y=-4|x-9|$

6. $y=|x|$ and $y=|x+8|-15$

continued

7. $y=\lfloor x \rfloor$ and $y=\lfloor 3x+2 \rfloor$

8. $y=\lceil x \rceil$ and $y=2\lceil x \rceil-10$

Read the following scenario and use the information in it to complete problems 9 and 10.

> The number of students in Mrs. Wayland's Math II class depends on the total number of students in the eleventh grade. If there are s students in the eleventh grade, approximately $m_2=\lceil 0.22s \rceil-5$ students are in her Math II class. There are at least 80 students in the eleventh grade, and no more than 200 students in the eleventh grade.

9. Create a graph to show the number of students in Mrs. Wayland's Math II class. Identify the domain and range of the graph.

10. The number of students in Mr. Curt's Math I class, m_1, can be estimated using the function $m_1=\lfloor 0.35s \rfloor$. Compare the graph of the number of students in Mr. Curt's class to the number of students in Mrs. Wayland's class.

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Lesson 2.4.3: Piecewise Functions

Warm-Up 2.4.3

After a hockey player hits a hockey puck, the puck travels at a constant rate of 45 meters per second. The distance in meters, d, traveled by the puck after s seconds can be found using the function $d = 45s$.

1. How long will it take for the puck to travel half the length of a 60-meter hockey rink?

2. Create a graph to show the distance traveled by a puck traveling for any number of seconds. Include a reasonable domain and range in the graph.

3. A second player hits a hockey puck at a constant rate of 50 meters per second, and the distance d_2 can be determined after any time s using the function $d_2 = 50s$. Graph this function on the same graph as the first function.

4. How are the two graphs similar? How are they different? Relate any similarities and differences to the algebraic representations of the functions.

Scaffolded Practice 2.4.3

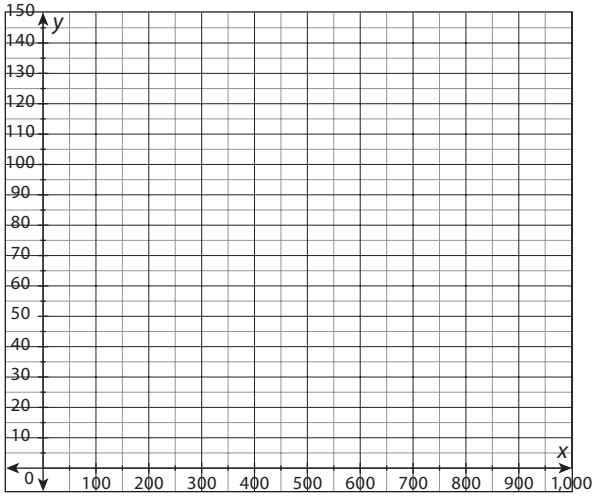
Example 1

A cell-phone plan charges customers a monthly fee, which includes 500 minutes of talk time. After 500 talk minutes, the customer is charged \$0.10 a minute. The total monthly charges, y, for any number of talk minutes, x, can be represented using the piecewise function $y=\begin{cases}50, \text{ if } 0\le x\le 500\\ 50+0.10(x-500), \text{ if } x>500\end{cases}$. Create a graph to show the monthly charges for any number of talk minutes.

1. Determine the domain and range of the first piece of the function.

2. Find at least two points on the graph of the first piece of the function.

3. Use the two points to graph the first piece of the function.



4. Determine the domain and range for the second piece of the function.

5. Find at least two points on the graph of the second piece of the function.

6. Use the two points to graph the second piece of the function.

continued

Example 2

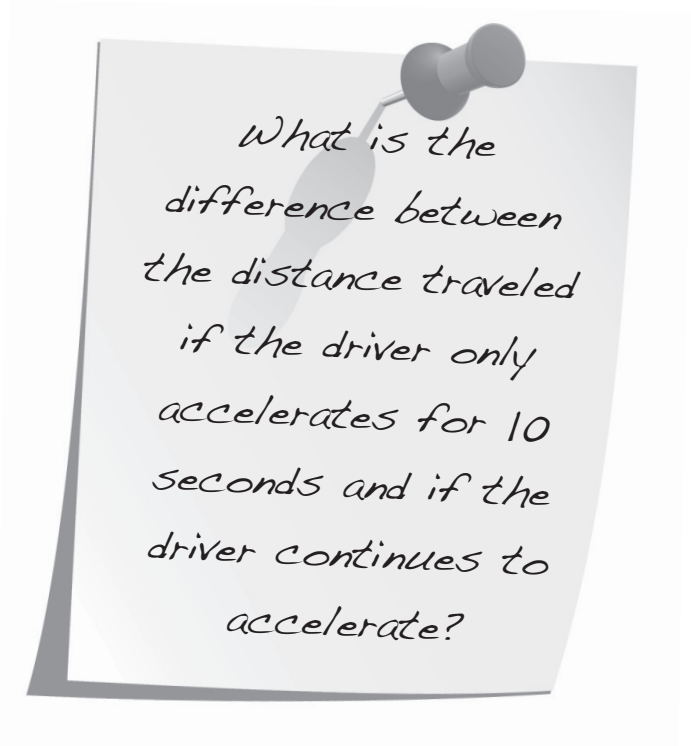
One hockey player passes a puck to a second player. The second player hits the puck instantly. The total meters, m, traveled by the hockey puck after s seconds can be represented using the function $m=\begin{cases}48s, \text{ if } 0\le s\le 1\\ 40s+8, \text{ if } 1<s<2\end{cases}$. Create a graph to show the distance traveled by the puck after s seconds.

Example 3

Create a graph of the function $y=\begin{cases}2, \text{ if } 0\le x<4\\ x^2-14, \text{ if } x\ge 4\end{cases}$.

Problem-Based Task 2.4.3: Stop and Go

A driver is stopped at a red light. When the light turns green, the driver will accelerate, then drive at a steady speed. The total distance traveled by the car, d, in meters, at any time t, in seconds, can be represented using the piecewise function $d_1 = \begin{cases} 1.34t^2, \text{ if } 0 \le t \le 10 \\ 26.8t - 134, \text{ if } t > 10 \end{cases}$. If the driver continues to accelerate, the function $d_2 = 1.34t^2$ would represent the driver's distance traveled. What is the difference between the distance traveled if the driver only accelerates for 10 seconds and if the driver continues to accelerate? Use graphs and algebraic representations to explain your answer.



Practice 2.4.3: Piecewise Functions

For problems 1–6, create a graph for each of the following piecewise functions.

1. $y=\begin{cases}7x, \text{ if } x<3\\ x^2, \text{ if } x\geq 3\end{cases}$

2. $y=\begin{cases}-6, \text{ if } 1\leq x\leq 8\\ x-10, \text{ if } x>8\end{cases}$

3. $y=\begin{cases}-3x+1, \text{ if } x<2\\ 4x-13, \text{ if } x\geq 2\end{cases}$

4. $y=\begin{cases}-x^2+1, \text{ if } -1\leq x<1\\ 10, \text{ if } x\geq 1\end{cases}$

5. $y=\begin{cases}-x+12, \text{ if } x<15\\ x-18, \text{ if } x\geq 15\end{cases}$

6. $y=\begin{cases}2x^2, \text{ if } -2\leq x\leq 2\\ -x^2+12, \text{ if } x>2\end{cases}$

continued

Read the following scenario and use the information in it to complete problems 7 and 8.

Saar is walking to school. He looks at his watch, realizes he is running late, and starts jogging. Saar's distance to school in miles, d, at any time in minutes t, can be represented using the function $d = \begin{cases} -0.1t + 1.5, \text{ if } 0 \le t < 8 \\ -0.2t + 2.3, \text{ if } t \ge 8 \end{cases}$.

7. Create a graph to show Saar's distance from school at any time t.

8. How long does it take Saar to reach school? How can you tell from the graph?

Read the following scenario and use the information in it to complete problems 9 and 10.

Saar compares this walk to school to his walk to school yesterday, when he walked at a steady pace. His distance from school yesterday could be represented by the function $d_2 = -0.12t$.

9. Graph the function for yesterday's walk on the same coordinate plane as the graph of today's walk.

10. Describe any differences in the domain, range, and extreme values between the two graphs. On which day did Saar reach school in less time?

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Lesson 2.5.1: Analyzing Exponential Functions

Warm-Up 2.5.1

If a cube has a side length of s, then the volume of the cube, V, is represented by the equation $V = s^3$. Claire is moving, and is packing her belongings into cube-shaped boxes. A moving company sells cube-shaped boxes with three different side lengths: 15 inches, 18 inches, and 24 inches. The price, p, of each box can be determined using the equation $p = 0.0005 \cdot V$, where p is the price in dollars and V is the box's volume.

1. What is the volume of a cube-shaped box with 18-inch sides?

2. What is the price of a cube-shaped box with 18-inch sides?

3. Claire decides to buy one 15-inch box and two 24-inch boxes. What is the total volume of the three boxes?

4. What is the price of one 15-inch box and two 24-inch boxes?

Scaffolded Practice 2.5.1

Example 1

A school tracks the total number of students enrolled each year. The school uses the change in the total number of students to estimate how many students have been enrolled in the school each year since 2000. If t is the number of years after 2000, the total number of students, $f(t)$, can be estimated using the function $f(t) = 250(0.98)^t$. How is the total number of students changing each year? Is the total number of students growing or decaying?

1. Identify the yearly rate of change in the function.

2. Describe how the rate of change relates to the change of the dependent quantity.

3. Determine whether the dependent quantity is growing or decaying.

continued

Example 2

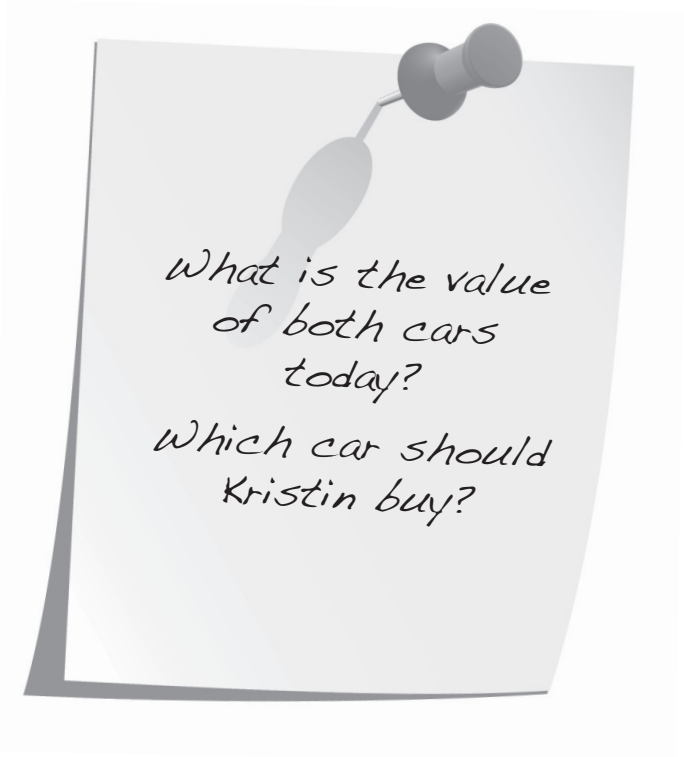
A bank offers a savings account with interest that is compounded monthly. In other words, the interest earned is added to the account every month instead of once a year. Dillon opened a savings account with $500. If t is the time in years the account has been open, the balance in his account, $f(t)$, is $f(t) = 500(1.004)^{12t}$. What is the estimated yearly exponential growth rate? Describe how this rate relates to the yearly change of the balance in Dillon's account.

Example 3

A number of bacteria, $f(t)$, at any time t, in hours, can be estimated using the function $f(t) = 3000(1.24)^t$. What was the initial size of the bacteria colony? Is the bacteria population exponentially decaying or growing?

Problem-Based Task 2.5.1: Which Car Is a Better Buy?

Kristin is buying a new car, and is trying to decide between two particular models. Both cars are the same price today. Kristin knows that the value of each car will change over time. According to her research, the first car, the Roadvana, will have a value of v_1 dollars after t years, modeled by the equation $v_1 = 22{,}000(0.90)^{2t}$. The second car, the Savannah, will have a value of v_2 dollars after t years, modeled by the equation $v_2 = 22{,}000(0.86)^t$. Kristin wants to buy the car that will have a higher value in the future. What is the value of both cars today? Which car should Kristin buy?



Practice 2.5.1: Analyzing Exponential Functions

Find the percent rate of change of $f(t)$ for each unit of t. State whether the function shows exponential growth or decay.

1. $f(t) = 110(0.95)^t$

2. $f(t) = 1.08(1.07)^t$

3. $f(t) = 30(0.90)^{4t}$

4. $f(t) = 63(0.87)^{11t}$

5. $f(t) = 500(1.15)^{2t}$

Use the information below to complete problems 6–8.

> The deer population, p, in a forest preserve t years after 2005 can be estimated using the function $p(t) = 440(0.92)^t$.

6. What was the size of the deer population in 2005?

continued

7. What is the yearly rate of change of the population?

8. The wolf population may be related to the deer population. The wolf population, w, can be estimated t years after 2005 using the function $w(t) = 84(0.98)^{2t}$. Which population is changing faster? Explain your answer.

Use the information below to complete problems 9 and 10.

Neal opens a savings account that earns interest monthly. He can estimate the total dollars in his account, $d(t)$, t years after opening the account by using $d(t) = 4000(1.0008)^{12t}$.

9. How much money did Neal initially put into the account?

10. What is the yearly rate of change of the account? Is it growing or decaying?

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Lesson 2.5.2: Comparing Properties of Functions Given in Different Forms

Warm-Up 2.5.2

Two seagulls dive into the ocean. The given functions represent the height of each seagull above the surface of the ocean as a function of the seagull's horizontal distance from a certain buoy. For each set of functions, determine which bird descends deeper into the ocean.

1. $f(x) = 3(x - 2)^2 - 5$ or $g(x) = \{(-8, 0), (-6, -4), (-4, 0)\}$

2. $f(x) = 3x^2 - 12x + 7$ or $g(x) = \frac{1}{2}(x+2)^2 - 6$

3. $f(x) = 2x^2 - 8x + 11$ or the function outlined in the following table:

x	–3	–1	1	3	5
$g(x)$	11	6	3	2	3

Scaffolded Practice 2.5.2

Example 1

Which function has a greater y-intercept, $f(x) = 8x - 2$ or $g(x) = 2(x - 3)(x + 1)$?

1. Make a general observation.

2. Determine the y-intercept of each function.

3. Compare the y-intercept of each function.

continued

Example 2

Three students are shooting wads of paper with a rubber band, aiming for a trash can in the front of the room. The height of each student's paper wad in feet is given as a function of the time in seconds. Which student's paper wad flies the highest?

- The path of Alejandro's paper wad is modeled by the equation $f(x) = -x^2 + 2x + 7$.
- Melissa's paper wad is estimated to reach the heights shown in the table below.

x	0	2	3	4
y	3	6	7	6

- After 3 seconds, Connor's paper wad achieves a maximum height of 6.5 feet above the floor.

Example 3

Which of the following quadratic functions has a vertex with a larger y-value: $f(x) = 2x^2 - 12x + 25$, or $g(x)$ as presented in the table?

x	–4	–3	–2	0	2
$g(x)$	7	8	7	–1	–17

Example 4

You are considering investing \$5,000 in one of two mutual funds. The first fund will pay \$500 each year. The second fund is predicted to have end-of-year balances as shown in the following table. Which fund should you choose if you want to withdraw your money after 5 years? Which fund should you choose if you want to invest the money for 10 years?

x (year)	0	1	2	3	4	5
$I(x)$ (\$)	5,000	5,200	5,500	5,900	6,400	7,000

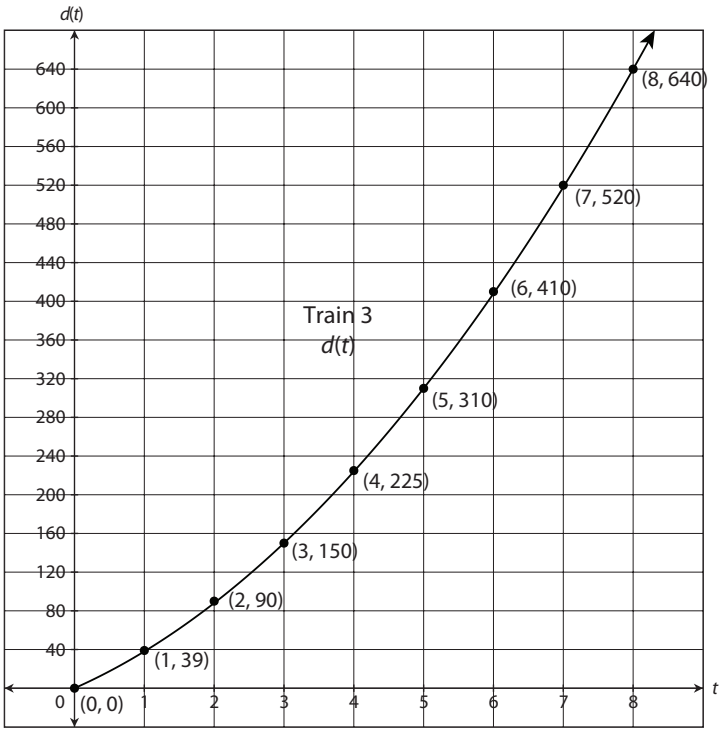
Example 5

Suppose that you have been offered a position at a prestigious company. You may choose how your salary is paid. Option 1 is described by the quadratic equation $S(x) = 2500x^2 + 2500x + 60{,}000$, where x is the number of years you are with the company and $S(x)$ is the yearly salary in dollars. Option 2 has a starting yearly salary of \$35,000, but you will get a 25% raise each year. Make a table of values for each salary and graph both functions on a coordinate plane. If you plan to work for this company for 5 years, which option should you choose? If you plan to work for this company until you retire at age 70, which option should you choose?

Problem-Based Task 2.5.2: On to Washington!

An emergency meeting has been called on Capitol Hill in Washington, D.C. As a newly elected representative for the state of Georgia, you need to find a way to travel to Washington. No flights are available at a time that fits your schedule, but there are three trains that you could take. The distance between Atlanta and Washington, D.C., is 639 miles. Train 1 is a nonstop train that leaves at 10 A.M. and travels at 66 mph. Train 2 also leaves at 10 A.M., but has a few stops that slow it down. The total distance traveled each hour by Train 2 is given in the table below, where t is in hours and $d(t)$ is in miles. Train 3 leaves at noon. The distance traveled for Train 3 can be modeled by the graph below, where t is the time in hours beginning at noon, and d is the distance traveled in miles. Is it possible to determine which train will arrive in Washington first? Which train should you select if you want to minimize time spent on the train? Explain.

t	0	1	2	3	4	5	6	7	8	9	10
$d(t)$	0	48	100	156	216	280	348	420	496	576	660



Is it possible to determine which train will arrive in Washington first? Which train should you select if you want to minimize time spent on the train?

Practice 2.5.2: Comparing Properties of Functions Given in Different Forms

Use the following information to solve problems 1–3.

Natalie is considering which method of travel—car, train, or plane—would be best to travel the flight distance of 747 miles from Atlanta to New York City. Use this distance for each problem. One rule of thumb is to estimate traveling by car at 60 mph. The train can be modeled by the equation $T(x) = -2.4x^2 + 90.8x + 1.59$, where x represents the number of hours and $T(x)$ represents the number of miles traveled. The table below represents the time and distance traveled during the plane trip.

Hours	0	0.5	1	1.5	2	2.5
Miles	0	149	300	455	612	747

1. If the car and the train leave Atlanta at the same time, which one arrives in New York City first? Is this mode of travel faster for the entire trip?

2. Estimate the vertex for the train. Is this vertex reasonable within the context of the problem? Why or why not?

3. If the car and the train both leave Atlanta at 7 A.M. and the plane leaves Atlanta at 4:30 P.M., determine which would arrive in New York City first.

Use the following information to solve problems 4–7.

Three turtles are running a race. They are free to roam in any direction. The location of the first turtle, Elmer, can be given by the equation $E(t) = t^2 - 4t + 4$, where $E(t)$ is the distance in feet from the starting line and t is the number of seconds since the race started. The location of the second turtle, Fred, is given by the equation $F(t) = 3(t - 2)^2 - 18$. The location of the third turtle, George, is given in the following table.

t	1	2	3	4	5
$G(t)$	–18	–20	–18	–12	–2

4. Which turtle is winning the race at $t = 2$?

continued

5. Which turtle is winning the race at $t = 6$?

6. Is there a point at which any of the turtles are tied? If so, at what time(s)?

7. If the finish line were at 40 feet, which turtle would you predict to win? Why?

Use the given information to answer questions 8–10.

8. Which of the following parabolas has the vertex with the smallest y-value: a parabola with two x-intercepts with $a > 0$, or a parabola with two x-intercepts with $a < 0$?

9. Which function achieves a lower maximum value: a parabola with no x-intercepts and $a < 0$, or a parabola with two x-intercepts and $a < 0$?

10. You've been offered jobs at two different companies, and each company offers a different form of payment. At one company, each day's take-home pay will be the sum of that day's wages plus the previous day's wages, starting with \$1 on the first day and continuing as shown in the table below. The other company will pay you \$0.01 on the first day, then double that amount every subsequent day, and is modeled by the equation $y = 0.01(2)^{(x-1)}$. On what day does the exponential function surpass the quadratic model?

Day	1	2	3	4	5
Take-home pay (\$)	1	3	6	10	15

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Lesson 2.6.1: Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$

Warm-Up 2.6.1

As a fund-raiser for the senior prom, the student council has decided to charge a small fee for parking in the school parking lot. Students can purchase yearly parking passes for $5 each. The school spent a total of $10 to buy a year's supply of the tags the students hang in their windows to verify that they paid for the parking spot.

1. Build a function that models the revenue the school will make selling parking passes.

2. Graph the function.

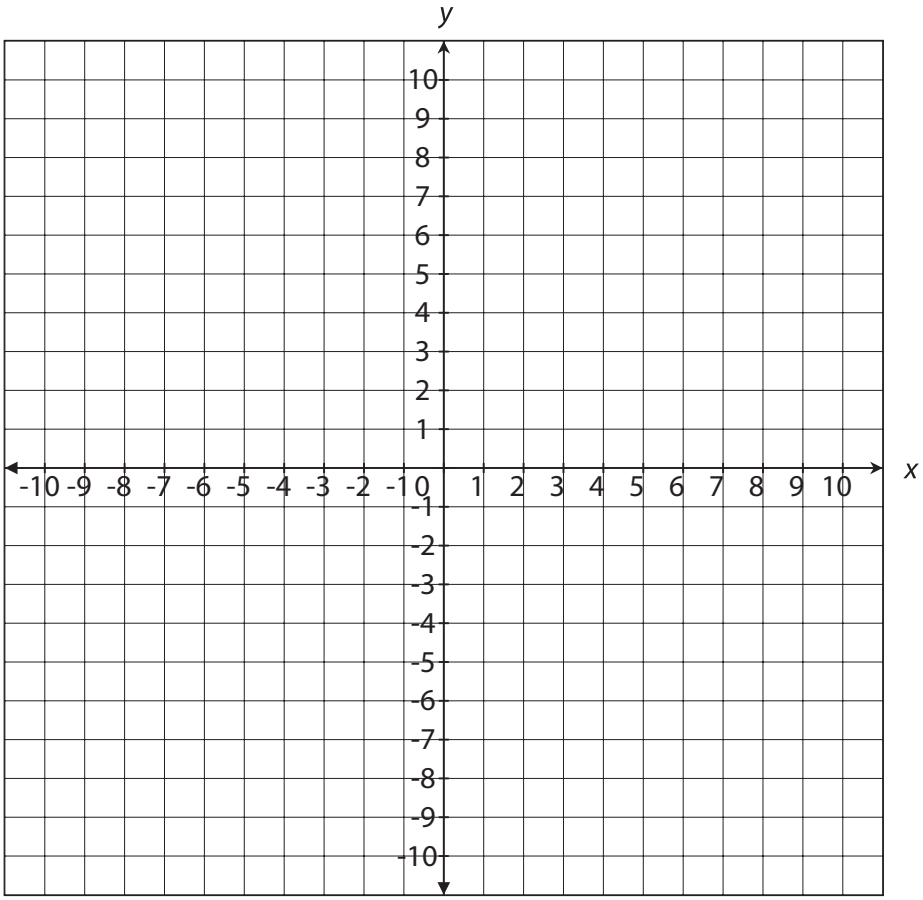
3. What would be the effect on the graph if the school were able to find a company that only charges $8 for the tags?

Scaffolded Practice 2.6.1

Example 1

Consider the function $f(x) = x^2$ and the constant $k = 2$. What is $f(x) + k$? How are the graphs of $f(x)$ and $f(x) + k$ different?

1. Substitute the value of k into the function.
2. Use a table of values to graph the functions on the same coordinate plane.



3. Compare the graphs of the functions.

continued

Example 2

Consider the function $f(x) = x^2$ and the constant $k = -3$. What is $f(x) + k$? How are the graphs of $f(x)$ and $f(x) + k$ different?

Example 3

Consider the function $f(x) = x^2$, its graph, and the constant $k = 4$. What is $f(x + k)$? How are the graphs of $f(x)$ and $f(x + k)$ different?

Example 4

Consider the function $f(x) = x^2$ and the constant $k = -1$. What is $f(x + k)$? How are the graphs of $f(x)$ and $f(x + k)$ different?

Example 5

The revenue function for a model helicopter company is modeled by the curve $f(x) = -5x^2 + 400x$, where x is the number of helicopters built per month and $f(x)$ is the revenue. The owner wants to include rent in the revenue equation to determine the company's profit per month. The company pays \$2,250 per month to rent its warehouse. In terms of $f(x)$, what equation now describes the company's profit per month? Compare the vertices of the original function and the transformed function.

Problem-Based Task 2.6.1: The Catch

On the last play of a football game, the offense is on the opposing team's 35-yard line. The offense is losing by 4 points, but can win by making a touchdown. The quarterback backs away 5 yards from behind the line of scrimmage and throws the ball to his receiver, who makes the catch at the goal line for the touchdown and the win. The quarterback's release point is 6 feet above the ground, the same height at which the receiver caught the ball. Also, the ball was thrown such that its maximum height was 15 feet above the ground.

Given the above information, build and graph the equation of the football's path, with the x-axis representing the distance from the line of scrimmage and the y-axis as the height of the football above the ground.

Practice 2.6.1: Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$

For problems 1–3, let $f(x) = x^2$. Write a function that translates f as described.

1. 2 units to the left

2. 3 units up

3. 5 units to the right and 2 units down

For problems 4–6, let $f(x) = x^2$. Graph $g(x)$ by translating the graph of f. State the vertex of the translated function.

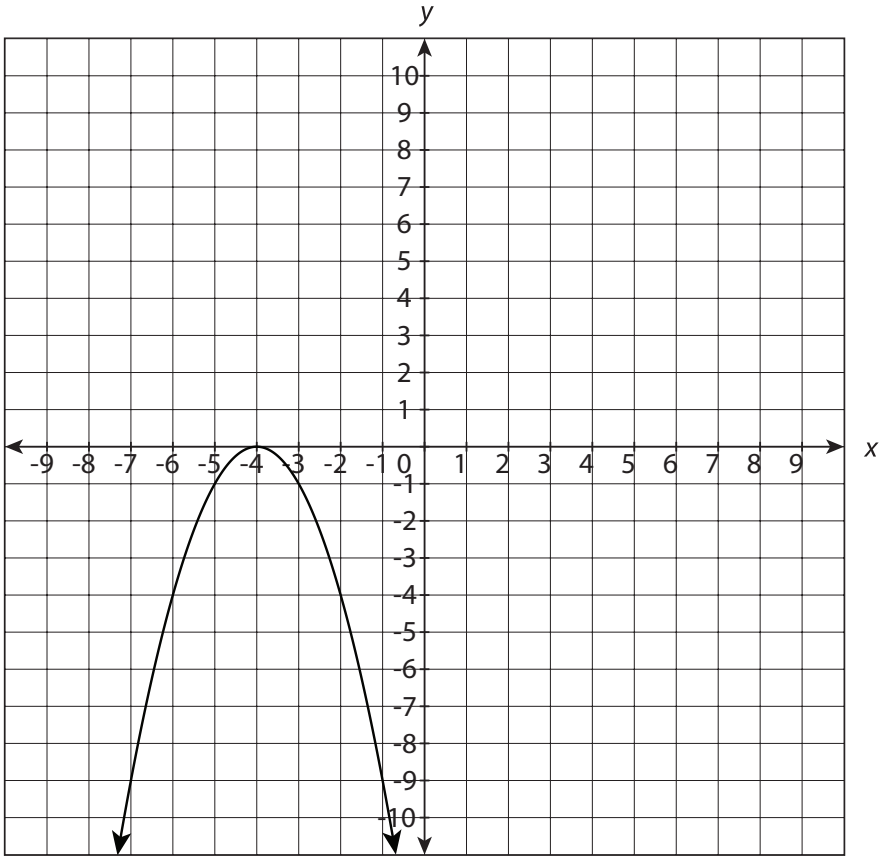
4. $g(x) = (x - 2)^2$

5. $g(x) = x^2 - 4$

6. $g(x) = (x + 1)^2 + 3$

Use what you know about translations of functions to solve each problem.

7. The graph shown below is a translation of $f(x) = -x^2$. Write an equation for the graph and state the value of k that was used to transform the function.



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8. A mother and her daughter went golfing. The mother hit first. Her ball followed the path modeled by the equation $f(x) = -0.0009x^2 + 0.2088x$ in the direction of the hole, and landed 18 yards short of the hole. The daughter teed off 18 yards closer to the hole because she is a beginner. She realized that if she could hit the ball on the same trajectory as her mother, her ball would land right by the hole. What is the equation that describes the path that the daughter's ball should follow?

9. A basketball is thrown from a height of 4 feet so that its path is modeled by the function $f(x) = -0.03x^2 + 1.3x + 4$. If the exact same shot is taken from a balcony that is 12 feet above where the original shooter was standing, how far away will the ball hit the ground? What is the equation that models this shot?

10. Simon has a toy that launches hollow plastic balls. The launched balls always follow a path modeled by the function $f(x) = -\frac{1}{8}(x-8)^2 + 8$ when the launcher is at the "origin." If the launcher is lifted up 2 feet and moved forward 5 feet, will a launched ball land in a basket that is on a 4-foot high stool 20 feet from the origin? What is the function that models this new launcher position?

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Lesson 2.6.2: Replacing $f(x)$ with $k \bullet f(x)$ and $f(k \bullet x)$

Warm-Up 2.6.2

An architectural firm is designing a one-story, square office building that is required to have a 4-foot space in every exterior wall for an exit. The area of the office building is given by the equation $f(x) = (x + 4)(x + 4) = (x + 4)^2$.

1. Given $f(x) = (x + 4)^2$, what would be the area of the building if $x = 20$ feet?

2. If the firm used the same blueprint with the area found in problem 1 to design a building with 3 identical floors, what would be the total area of the new building's office space?

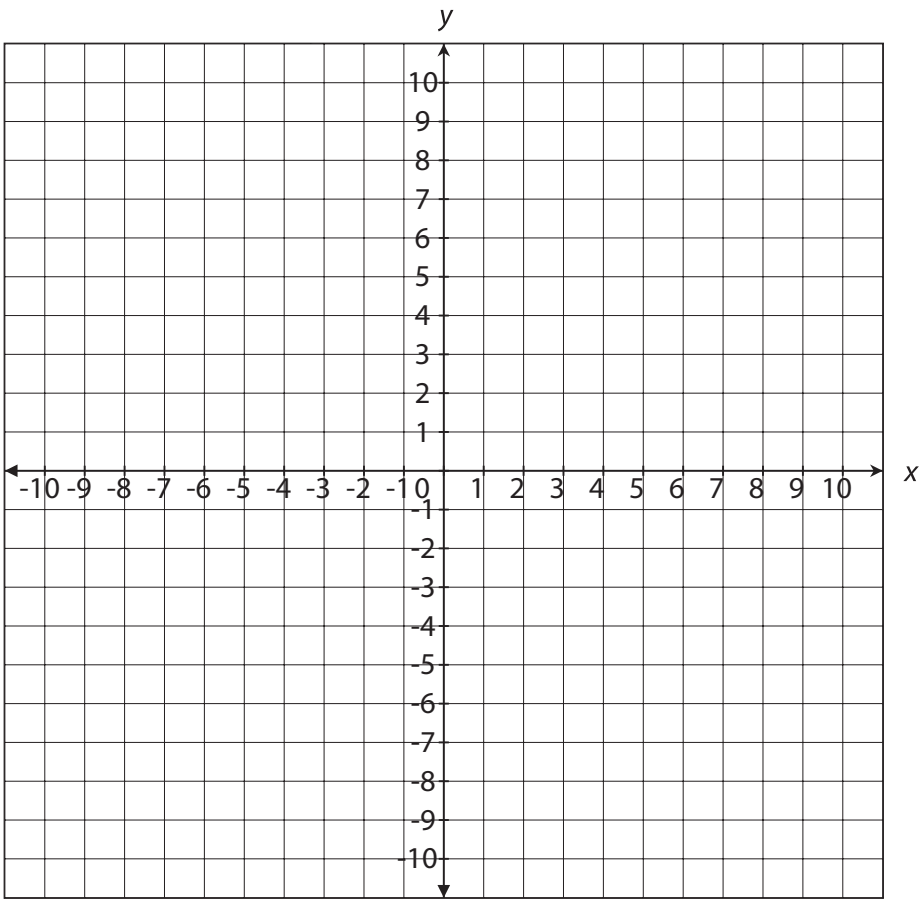
3. If the firm were to double the length of each wall in the original design, what would be the new area of the one-story building?

Scaffolded Practice 2.6.2

Example 1

Consider the function $f(x) = x^2$, its graph, and the constant $k = 2$. What is $k \cdot f(x)$? How are the graphs of $f(x)$ and $k \cdot f(x)$ different? How are they the same?

1. Substitute the value of k into the function.
2. Use a table of values to graph the functions.



3. Compare the graphs.

continued

Example 2

Consider the function $f(x) = x^2 - 81$, its graph, and the constant $k = 3$. What is $f(k \cdot x)$? How do the vertices and the x-intercepts of $f(x)$ and $f(k \cdot x)$ compare?

Example 3

Consider the function $f(x) = x^2 - 6x + 8$, its graph, and the constant $k = -1$. What is $k \cdot f(x)$? How do the graphs of $f(x)$ and $k \cdot f(x)$ compare?

Example 4

Consider the function $f(x) = x^2 - 6x + 8$, its graph, and the constant $k = -1$. What is $f(k \cdot x)$? How do the graphs of $f(x)$ and $f(k \cdot x)$ compare?

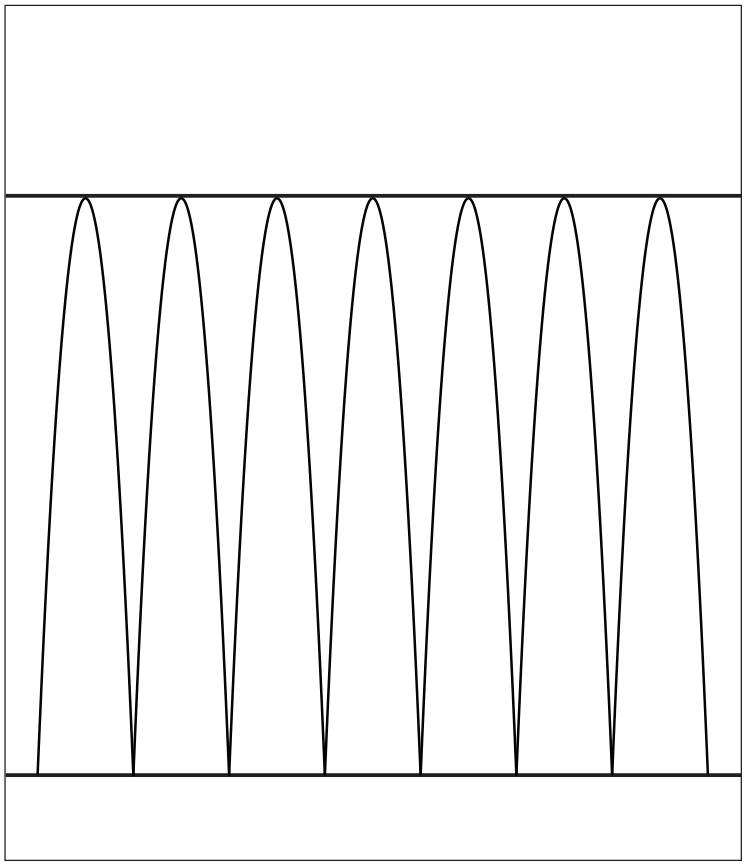
Example 5

The dimensions of a rectangular garden edged with wood are such that the longer sides are 3 times the length of the shorter sides. Keeping the same ratio of side lengths, which would result in having a larger garden area: making the existing garden 5 times larger, or building 4 more gardens identical in size to the first?

Problem-Based Task 2.6.2: Fewer Parabolas, Please

A city hired a civil engineering firm to draw up plans for a 420-foot bridge that would use 7 downward-facing parabolic arches to support the span. The resulting plans called for the arches to be 75 feet high, with a distance of 60 feet between the bases of each arch. A diagram of this plan is shown below.

After seeing the drawings, city councilors asked the civil engineering firm to make a second set of drawings using only 5 evenly spaced parabolic curves but covering the same 420-foot span. What are the equations of the 5 parabolas in the new plan?



What are the equations of the 5 parabolas in the new plan?

Practice 2.6.2: Replacing $f(x)$ with $k \bullet f(x)$ and $f(k \bullet x)$

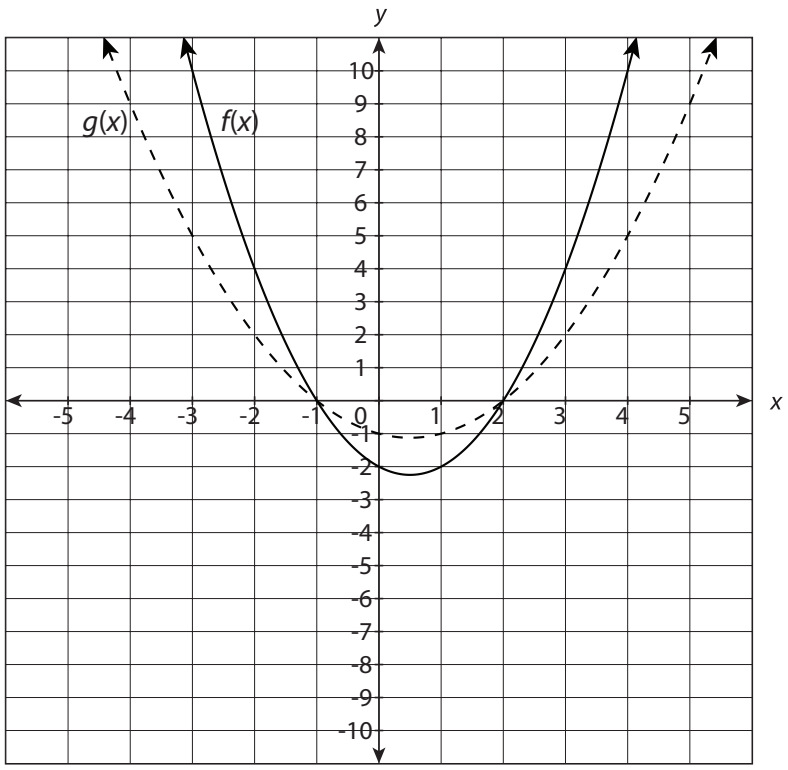
Use what you have learned about transformations of functions to solve problems 1 and 2.

1. For the function $f(x) = x^2 + x - 6$, find $2 \bullet f(x)$, and describe the changes that occur to the graph of f as a result of multiplying the function by 2. Check your answers by comparing the two functions on your graphing calculator.

2. For the function $f(x) = x^2 + x$, find $f(3x)$, and describe the changes that occur to the graph of f as a result of multiplying the variable x by 3. Check your answers by comparing the two functions on your graphing calculator.

Use the graphs and the given information to complete problems 3 and 4.

3. Consider the graphs of the functions $f(x)$ and $g(x)$ shown below. The equation for $f(x)$ is $f(x) = x^2 - x - 2$. What could be the equation for $g(x)$?



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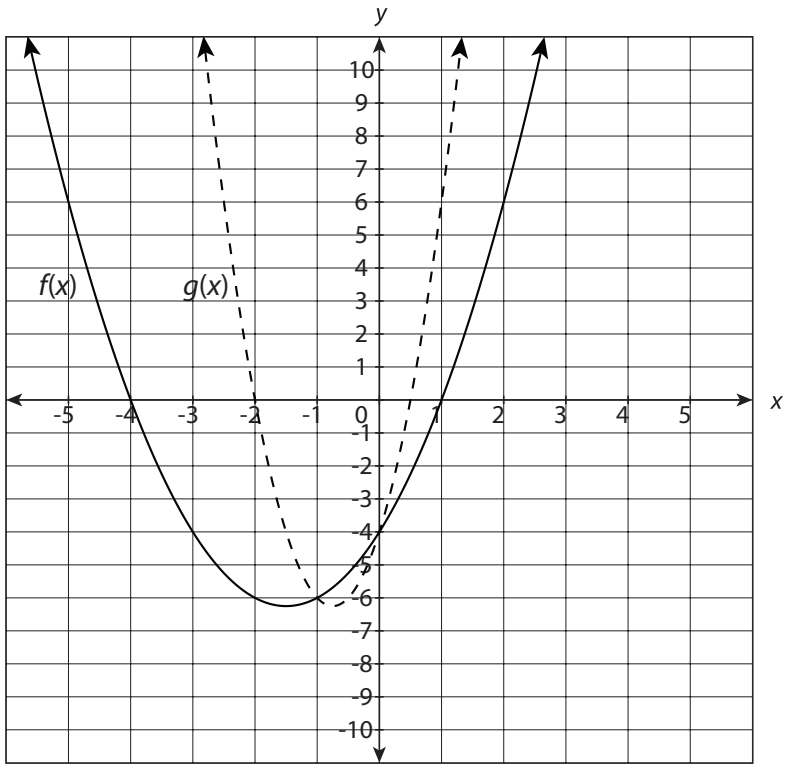
4. Consider the graphs of the functions $f(x)$ and $g(x)$ shown below. The equation for $f(x)$ is $f(x) = x^2 + 3x - 4$. What could be the equation for $g(x)$?



Complete each of the following tasks for the functions in problems 5–7.

- Graph $f(x)$ and $g(x)$ on your graphing calculator.
- Determine the scale factor and the transformation(s): horizontal stretch, horizontal compression, vertical stretch, vertical compression, reflection over the x-axis, or reflection over the y-axis.
- Describe the similarities and differences of the graphs.

5. $f(x) = x^2 - x - 2$; $g(x) = -2f(x)$

6. $f(x) = x^2 - x - 2$; $g(x) = f(-2x)$

7. $f(x) = x^2 - 1$; $g(x) = -\frac{1}{2} \cdot f(x)$

continued

Read each scenario and use the given information to solve problems 8–10.

8. A farmer has a rectangular goat pen such that one side is 2 times as long as the other side. He would like to have more space for his goats, and he is deciding between two options. He could either double the lengths of the sides of the existing pen, or he could build a second pen of the same size as the first. Which option would give him the most area for his goats? Explain your answer in terms of $k \bullet f(x)$ and $f(k \bullet x)$.

9. A company that produces skateboards knows the equation that models profit per month is $f(x) = 3x^2 + 300x$, where x is the price charged per skateboard. If the company plans to expand with the hopes of doubling its profits, should the new model for the company's profit be $f(2x)$, $f\left(\frac{1}{2}x\right)$, $2 \bullet f(x)$, or $\frac{1}{2} \bullet f(x)$? Explain.

10. Jada and Jayla are twins on the same softball team. They can each hit the ball so that it follows a path modeled by the equation $f(x) = -0.01x^2 + 0.98x + 2$. Jada says that the ball would go farther if it followed the path $g(x) = f(2x)$. Jayla says the ball would go farther if it followed the path $g(x) = 2 \bullet f(x)$. Who is correct? Which equation for $g(x)$ would allow the ball to achieve the same height as the ball in the original equation?

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Lesson 2.7.1: Finding Inverse Functions

Warm-Up 2.7.1

Ari received $100 as a birthday gift and has decided to save this money as well as add $20 of his own money each month. The total money Ari has saved can be estimated using the equation $y = 20x + 100$, where x is the number of months and y is the total amount of money saved.

1. How much money will Ari have saved after 6 months?

2. How long will it take Ari to save $400?

3. Suppose Ari decided to save $50 each month. The amount of money in his savings account after x months can now be estimated using the equation $y = 50x + 100$. How long will it take Ari to save $400 with his new savings plan?

Scaffolded Practice 2.7.1

Example 1

Lana is driving home from her friend's house. She is driving at a steady speed, and her distance from her home, in miles, can be represented by the function $f(x) = -40x + 15$, where x is her driving time in hours. Find the inverse function $f^{-1}(x)$ to show when, in hours, Lana will be x miles from home.

1. Determine if the function is one-to-one.

2. Rewrite the function $f(x)$ in the form "$y =$."

3. Switch x and y in the original equation of the function.

4. Solve the new equation for y by using inverse operations.

5. Replace y with $f^{-1}(x)$ to show that the equation is the inverse of $f(x)$.

continued

Example 2

A high school is selling tickets to a school play. The school is using the money earned from selling tickets to pay for the play. Any extra money after expenses will be profit. The profit, in dollars, can be represented using the equation $p(x) = 12x - 600$, where x is the number of tickets sold. Find the inverse of $p(x)$ to show the number of tickets that need to be sold in order to earn a given profit.

Example 3

Find the inverse function of $f(x) = 4x^2$. Use a restricted domain so the inverse is a function.

Problem-Based Task 2.7.1: Falling Keys

Piper drops her keys from a third-story window to a friend standing on the sidewalk. The distance between the keys and the ground, in feet, can be represented by the function $f(x) = -16x^2 + 32$, where x is the time in seconds. Piper would like to be able to determine the time, in seconds, at which the keys were any given distance from the ground. What is the function with height in feet from the ground as the independent variable? After about how many seconds were the keys 4 feet above the ground?

Practice 2.7.1: Finding Inverse Functions

Find the inverse of each function for problems 1–6. State the domain and range of both the function and its inverse. Restrict the domain of the function if needed.

1. $f(x) = -x^2$

2. $f(x) = 5x - 1$

3. $f(x) = -x + 3$

4. $f(x) = x^2 + 7$

5. $f(x) = \frac{1}{4}x - 4$

6. $f(x) = -3x + 8$

continued

For problems 7–10, state a reasonable domain for the problem statement, find the inverse function $f^{-1}(x)$, and identify the independent and dependent quantities of the inverse.

7. Gary is jogging. The total distance he has traveled, in miles, can be estimated using the function $f(x) = 9x$, where x is his jogging time in hours.

8. Camryn sells computers. She earns commission for her computer sales plus a fixed wage for each day she works. Her daily earnings, in dollars, can be estimated using the function $f(x) = 0.15x + 50$, where x represents her computer sales in dollars.

9. The distance a motorboat travels, in meters, can be estimated using the equation $f(x) = 1.9x^2$, where x is the time in seconds, for $0 \leq x \leq 10$.

10. A rock band is selling tickets to a concert at a theater. The band earns money for each ticket sold, but has to pay some of the earnings to the theater. The total money earned by the band can be estimated using the function $f(x) = 7.5x - 300$, where x is the number of tickets sold.

Name: Date:

Notes

Name: Date:

Notes

Station 1

At this station, you will find graph paper and a ruler. Work together to graph the following quadratic equation:

$$y = x^2 + 6x + 9$$

1. Write this quadratic equation as a quadratic function.

2. What are the values of a, b, and c in the quadratic function?

   a = _____

   b = _____

   c = _____

To graph the function, you need the vertex, x-intercept, and y-intercept.

3. If the x-value of the vertex is found by $x = \frac{-6}{2(1)} = -3$, then write this x calculation using the general terms a, b, and/or c.

4. If the y-value of the vertex is found by $y = f\left(\frac{-6}{2(1)}\right) = f(-3) = 0$, then write this y calculation using the general terms a, b, and/or c.

5. Based on problems 3 and 4, how can you find the vertex of the graph for $f(x) = ax^2 + bx + c$?

   What is the vertex of the quadratic function $x^2 + 6x + 9 = 0$?

continued

6. How do you find the *x*-intercept of a function? (*Hint*: $y = f(x)$)

7. How do you find the *y*-intercept of a function?

8. What are the intercepts for $y = x^2 + 6x + 9$?

9. On your graph paper, graph the function using the vertex, *x*-intercept, and *y*-intercept.

10. What shape is the graph? Why do you think the graph has this shape?

Station 2

At this station, you will find a graphing calculator. As a group, follow the steps according to your calculator model to graph $y = x^2 + 4$ and $y = x^2 - 4$.

On a TI-83/84:

Step 1: Press [Y=]. At Y_1, type [X,T,θ,n][x^2] [+][4].

Step 2: Press [GRAPH].

On a TI-Nspire:

Step 1: Arrow over to the graphing icon and press [enter]. At *f1*(*x*), enter [*x*], hit the [x^2] key, then type [+][4].

Step 2: Press [enter].

1. What shape is the graph?

2. Does the graph open upward or downward?

3. Which term do you think makes the graph open upward or downward? Explain your reasoning.

On a TI-83/84:

Step 3: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 3: Press [ctrl], then [T].

4. What information does your calculator show?

5. How can you use this information to find the vertex of the graph?

What is the vertex of the graph?

continued

On a TI-83/84:

Step 4: Press [Y=]. At Y_2, type [X,T,θ,n][x^2] [–][4].

Step 5: Press [GRAPH].

On a TI-Nspire:

Step 4: Press [ctrl][tab] to go back to the graphing window. Use the touch pad to select ">>" on the bottom left of the screen. At *f2*(*x*), enter [*x*], hit the [x^2] key, then type [–][4].

Step 5: Press [enter].

6. What shape is the graph?

7. Does the graph open upward or downward?

8. Which term do you think makes the graph open upward or downward? Explain your reasoning.

On a TI-83/84:

Step 6: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 6: Press [ctrl], then [T]. Press [ctrl], then [T] a second time to refresh the screen.

9. What information does your calculator show?

10. How can you use this information to find the vertex of the graph of $y = x^2 - 4$?

    What is the vertex of $y = x^2 - 4$?

11. Why do the graphs for $y = x^2 + 4$ and $y = x^2 - 4$ have different vertices?

Station 3

At this station, you will find a graphing calculator. As a group, follow the steps according to your calculator model to graph $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$.

On a TI-83/84:

Step 1: Press [Y=]. At Y_1, type [X,T,θ,n][x^2]. At Y_2, type [3][X,T,θ,n][x^2].

Step 2: Press [GRAPH].

On a TI-Nspire:

Step 1: Arrow over to the graphing icon and press [enter]. At *f1*(*x*), enter [*x*], then hit the [x^2] key. Arrow down. At *f2*(*x*), enter [3][*x*], then hit the [x^2] key.

Step 2: Press [enter].

1. Why do both graphs have the same vertex?

2. Which graph is wider, $y = x^2$ or $y = 3x^2$?

   Why is one graph wider than the other?

On a TI-83/84:

Step 3: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 3: Press [ctrl], then [T].

3. What information does your calculator show?

4. What is the relationship between Y1 and Y2 in the table?

   How does this relationship relate to $y = x^2$ and $y = 3x^2$?

continued

On a TI-83/84:

Step 4: Press [Y=]. At Y_3, type [0][.][5] [X,T,θ,n][x^2].

Step 5: Press [GRAPH].

On a TI-Nspire:

Step 4: Press [ctrl][tab] to go back to the graphing window. Use the touch pad to select ">>" on the bottom left of the screen. At *f3*(*x*), enter [0][.][5][*x*], then hit the [x^2] key.

Step 5: Press [enter].

5. Why is the graph of $y = 0.5x^2$ wider than $y = x^2$ and $y = 3x^2$?

On a TI-83/84:

Step 6: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 6: Press [ctrl], then [T]. Press [ctrl], then [T] a second time to refresh the screen.

6. What information does your calculator show?

7. What is the relationship between Y1 and Y3 in the table?

   How does this relationship relate to $y = x^2$ and $y = 0.5x^2$?

8. What is the relationship between Y2 and Y3 in the table?

   How does this relationship relate to $y = 3x^2$ and $y = 0.5x^2$?

Station 4

At this station, you will find graph paper and a ruler. Work together to graph the following quadratic equations:

$f(x) = x^2 - x - 6$ and $f(x) = -x^2 + x - 6$

1. What are the values of a, b, and c in each quadratic function?

$f(x) = x^2 - x - 6$	$f(x) = -x^2 + x - 6$
$a =$ _____	$a =$ _____
$b =$ _____	$b =$ _____
$c =$ _____	$c =$ _____

2. Use the information in problem 1 to find the vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a} \right) \right)$ for each function. Show your work.

3. Find the x-intercepts of $f(x) = x^2 - x - 6$ using factoring. Show your work.

4. On your graph paper, graph $f(x) = x^2 - x - 6$ using its vertex and x-intercepts.

5. Does the parabola open upward or downward? Explain your answer.

continued

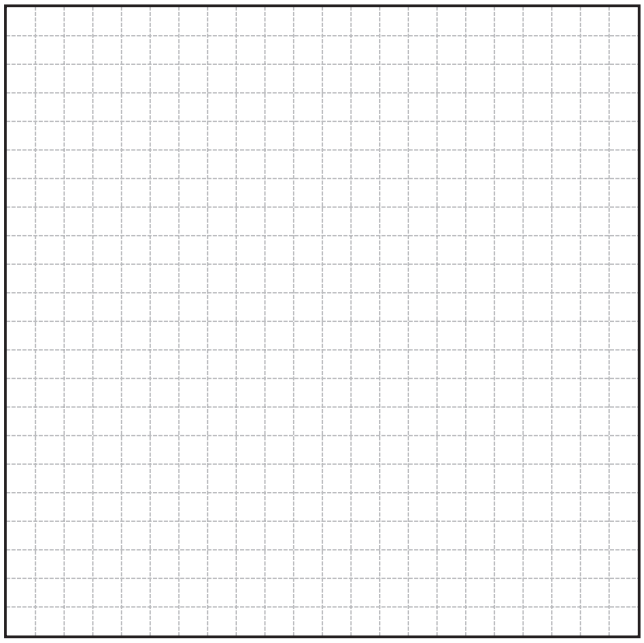
6. Fill out the table below to help you graph $f(x) = -x^2 + x - 6$.

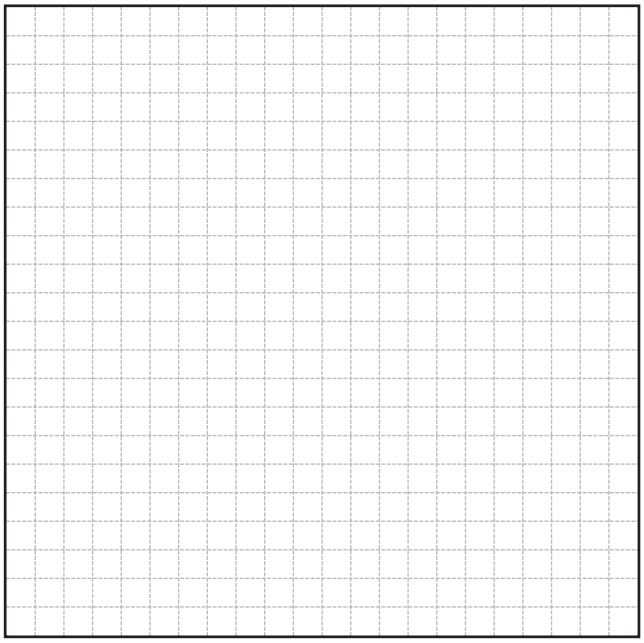
x	$y = f(x)$
–4	
0	
4	

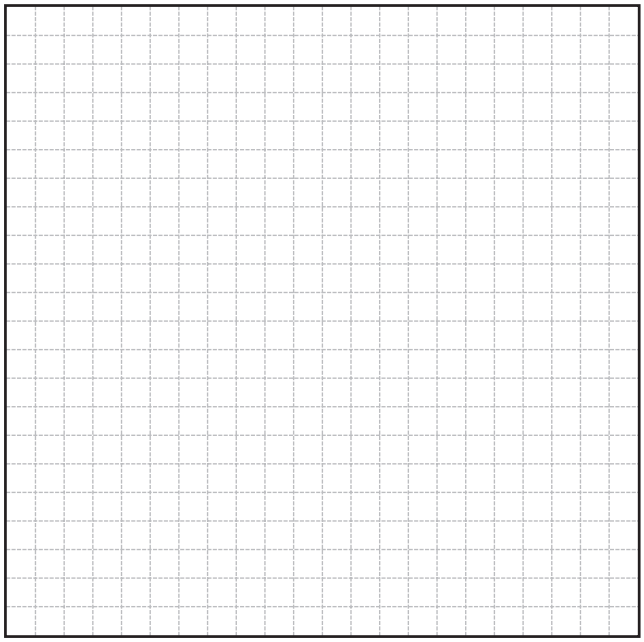
Graph $f(x) = -x^2 + x - 6$ on your graph paper.

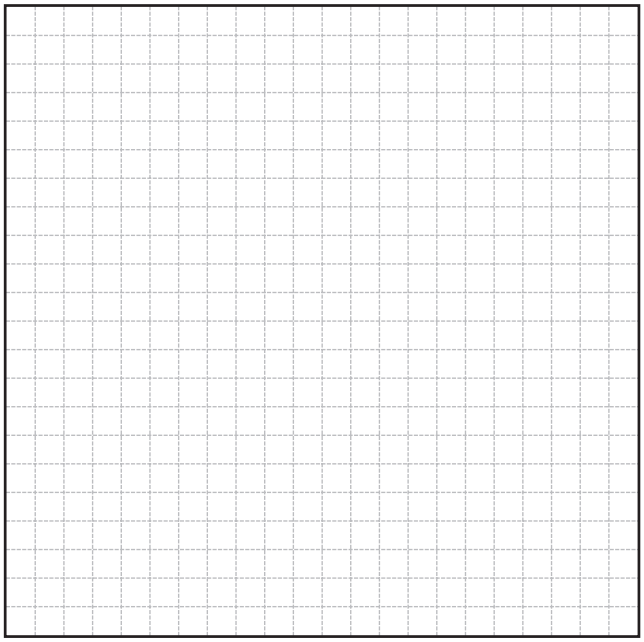
7. Does the graph open upward or downward? Explain your answer.

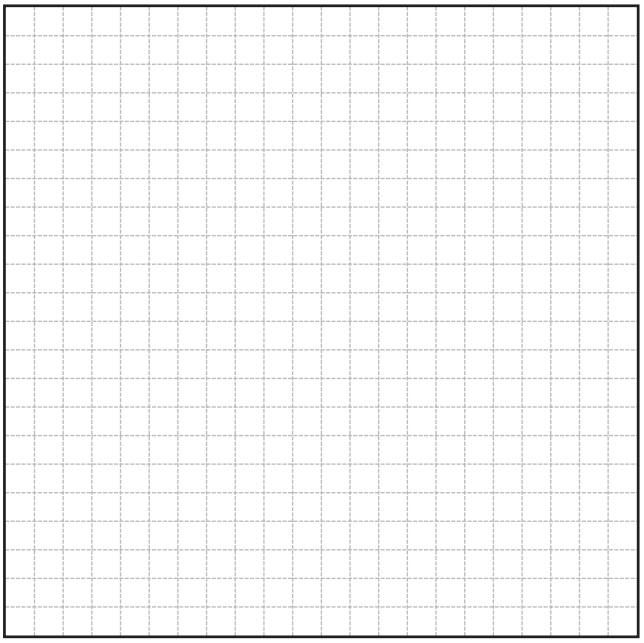
8. Will the graph of $f(x) = -x^2 + x - 6$ have x-intercepts? Why or why not?

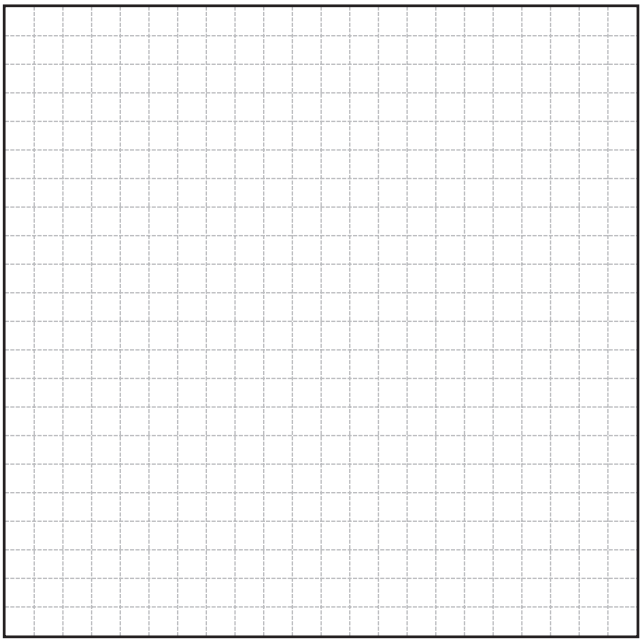


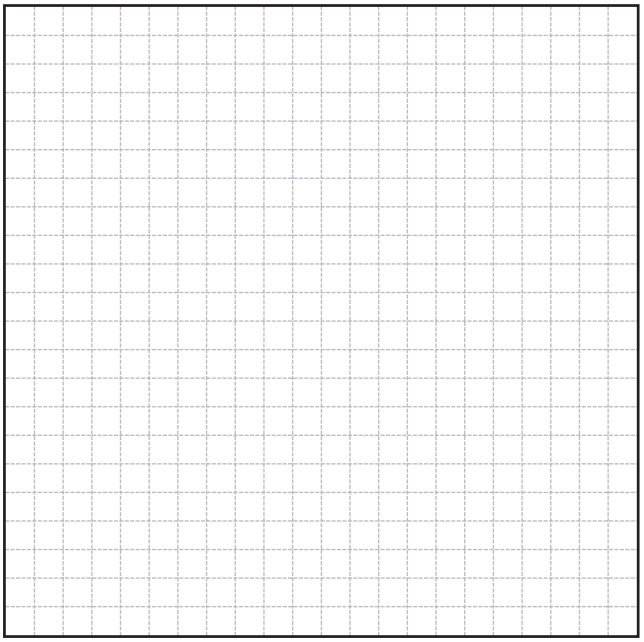


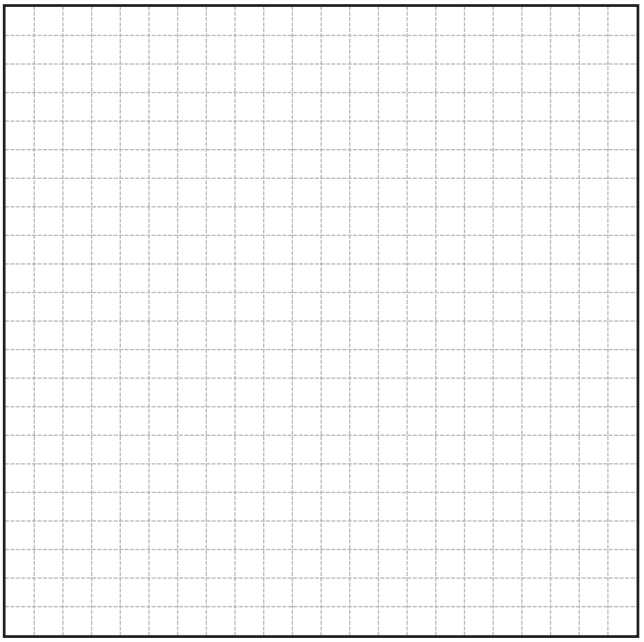


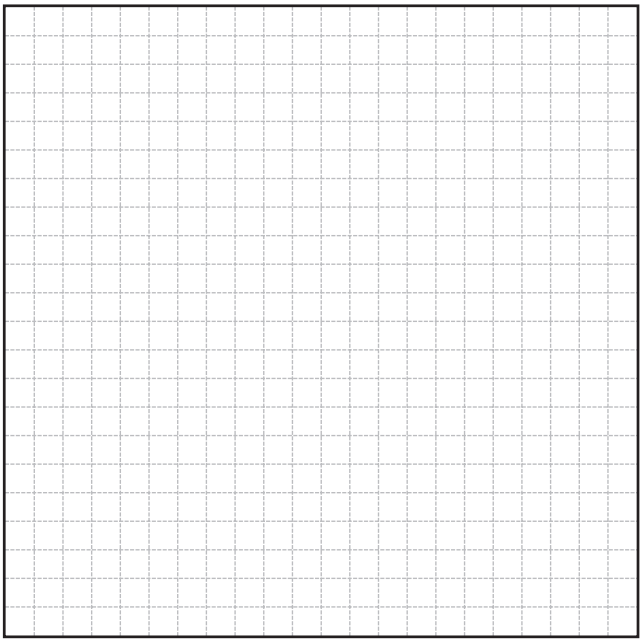


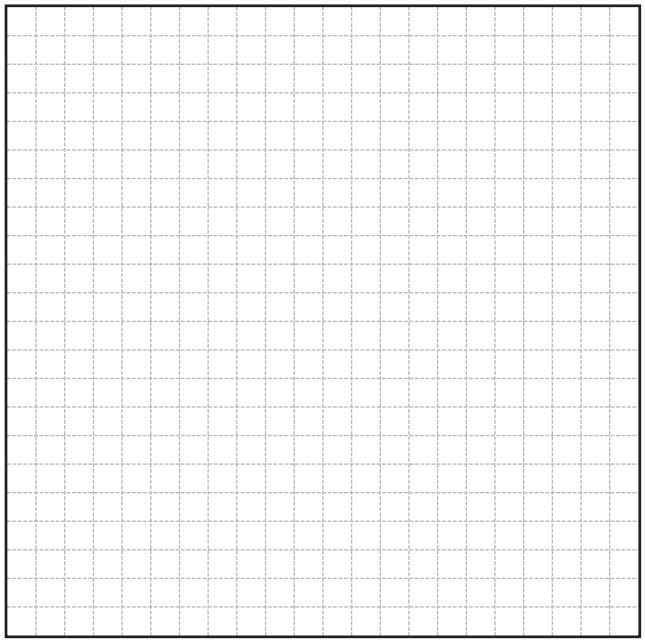


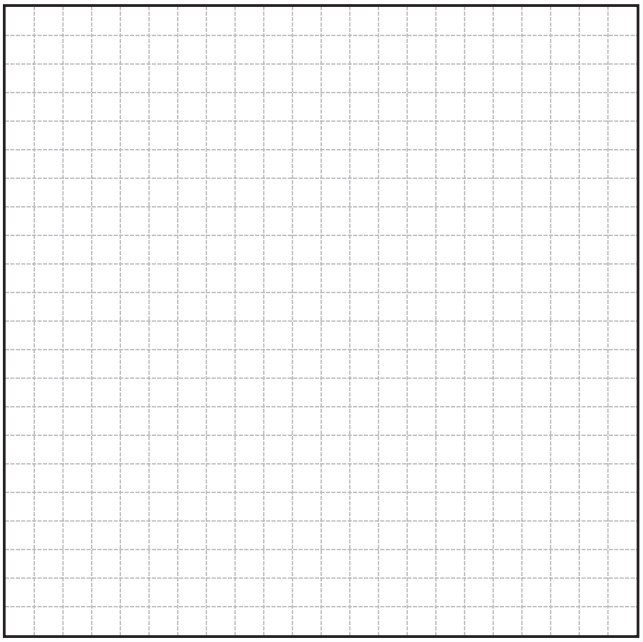


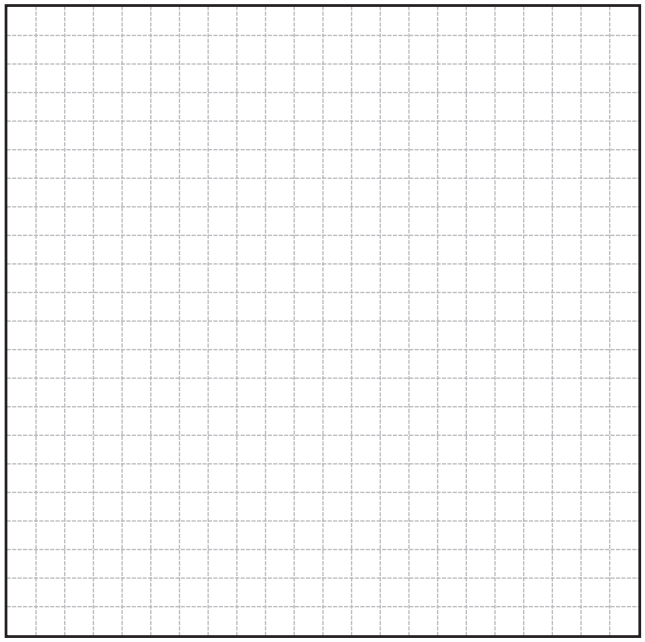


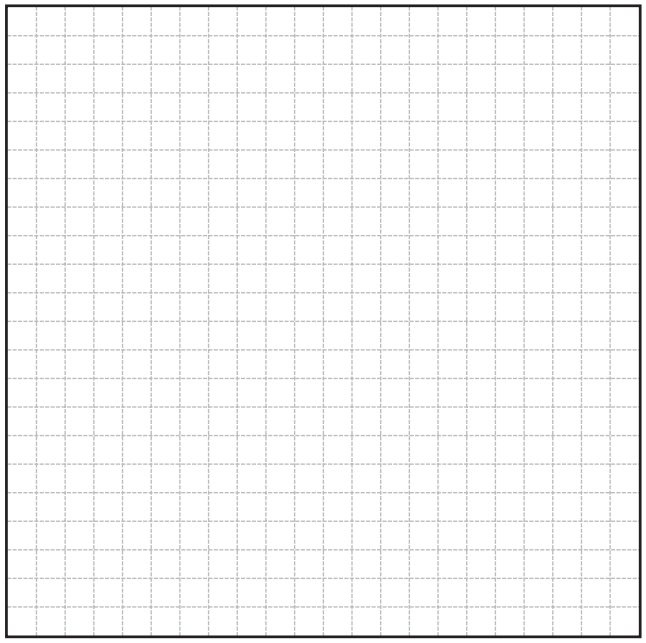


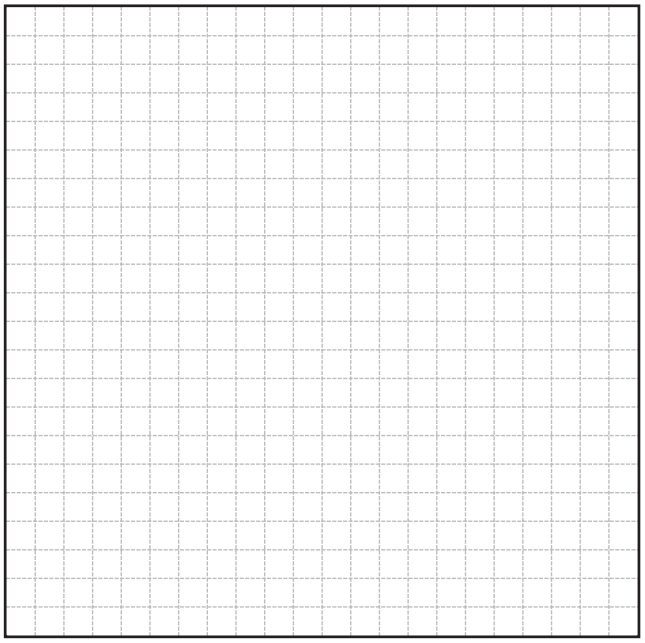


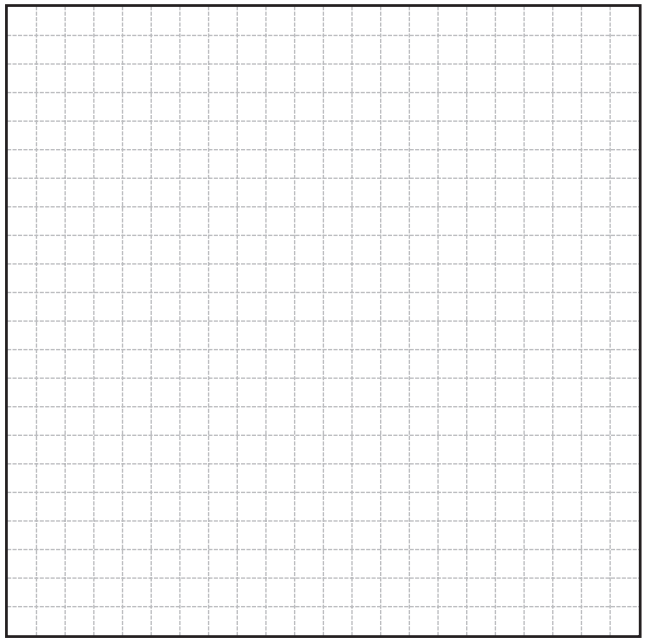


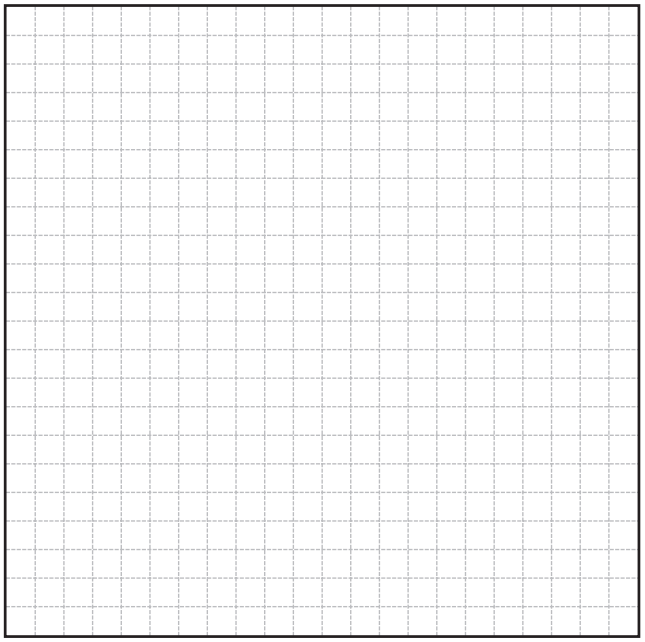


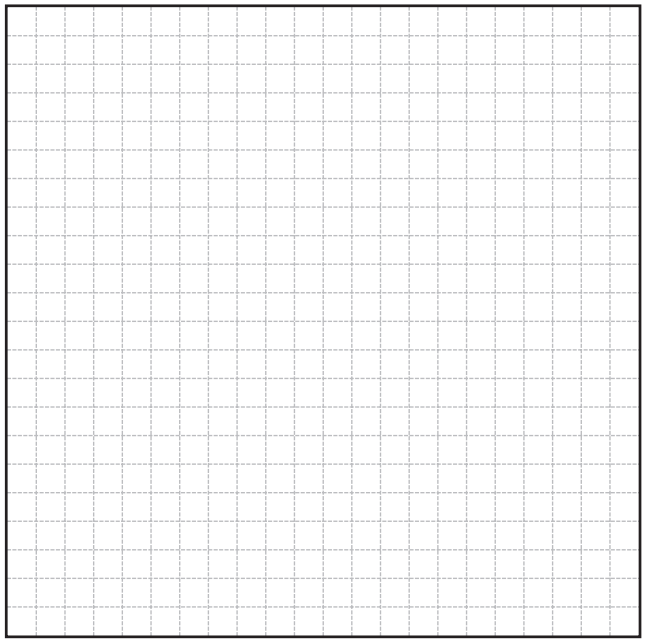


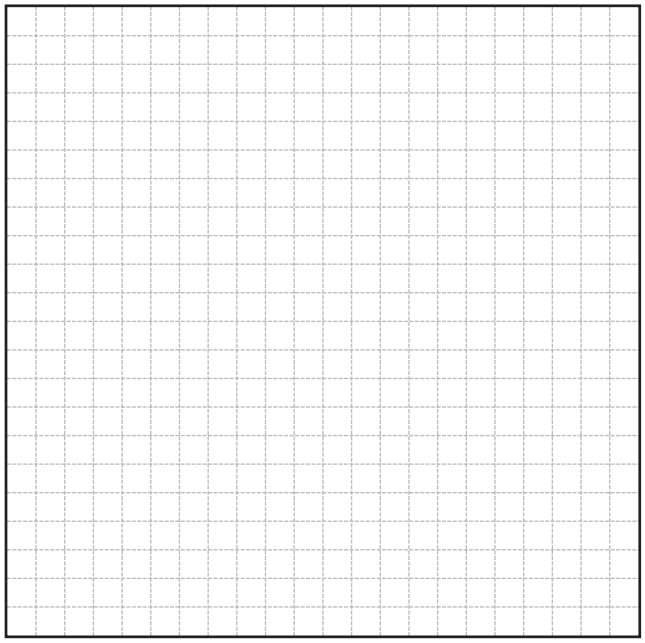


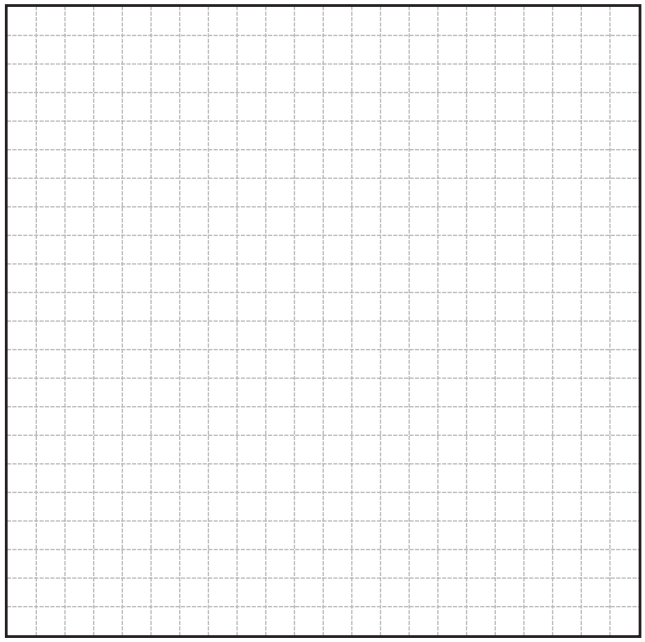


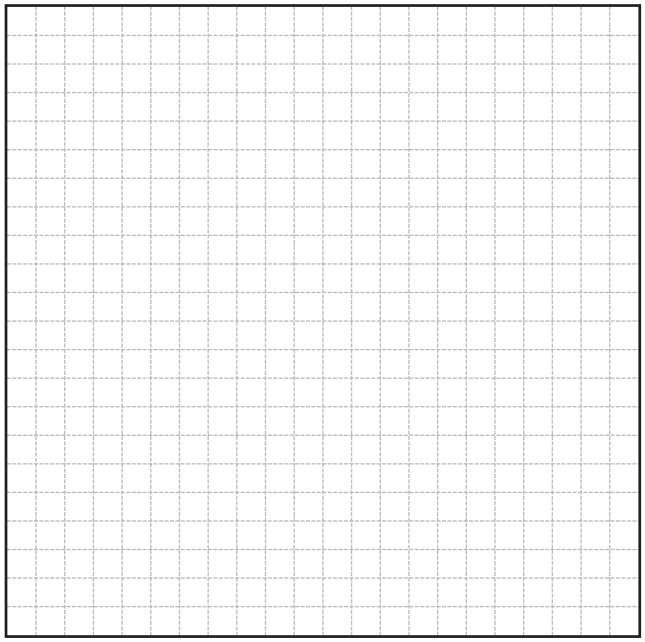


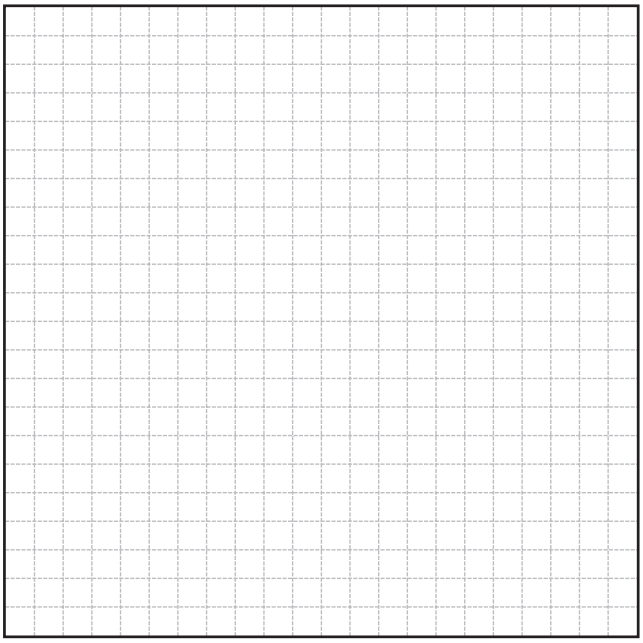


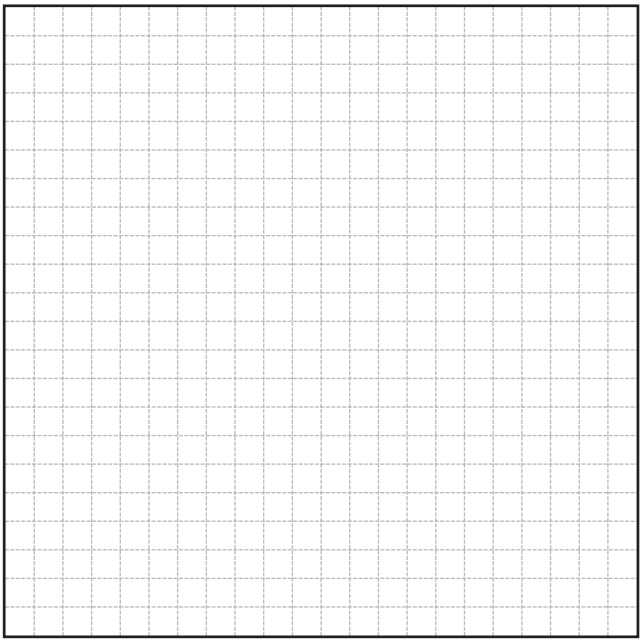


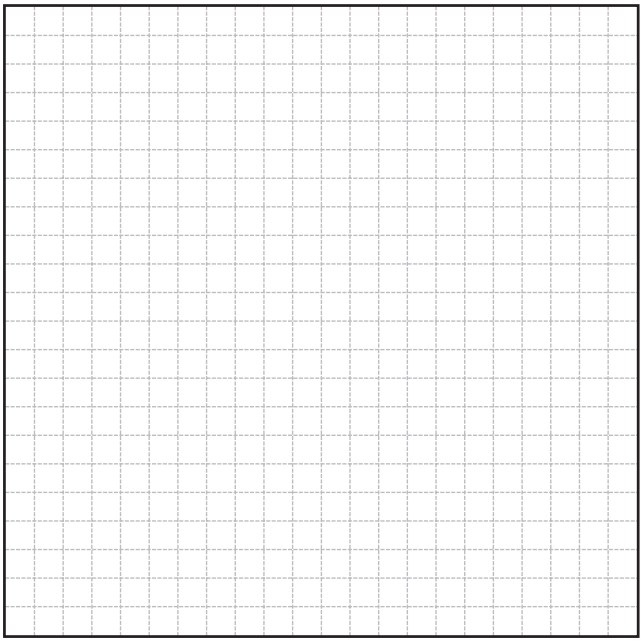


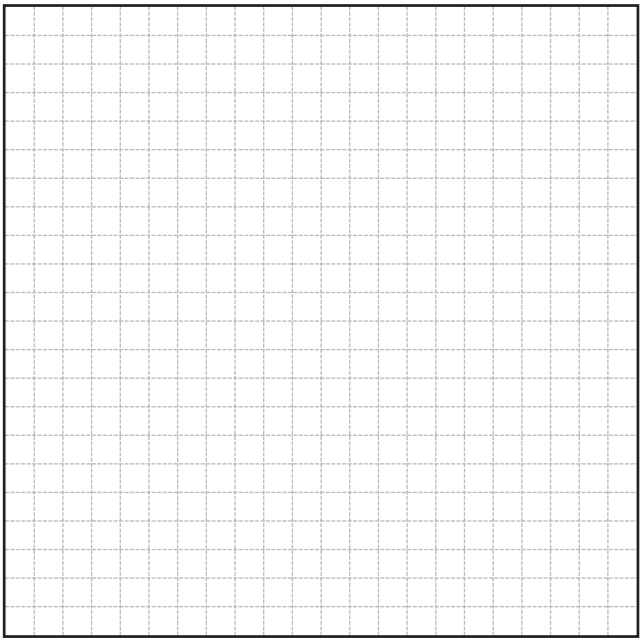


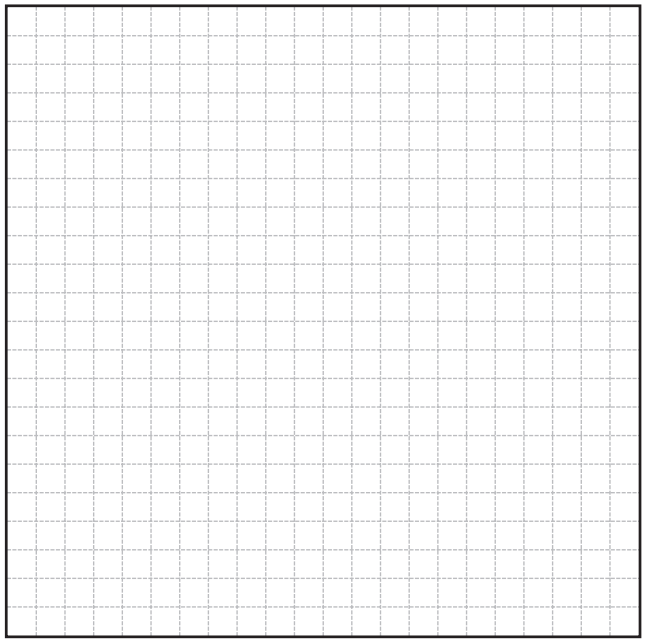


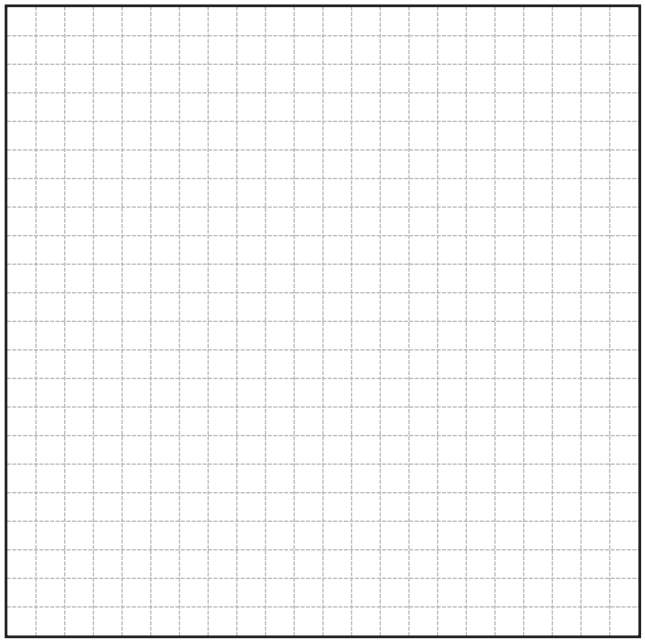


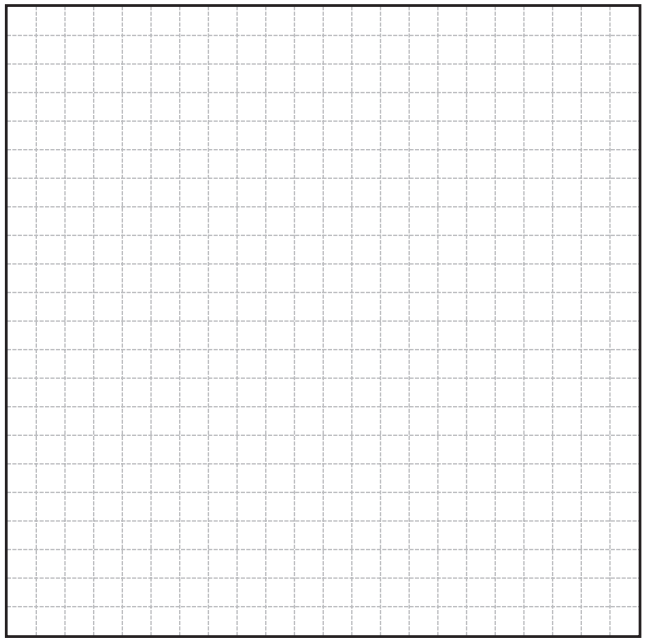


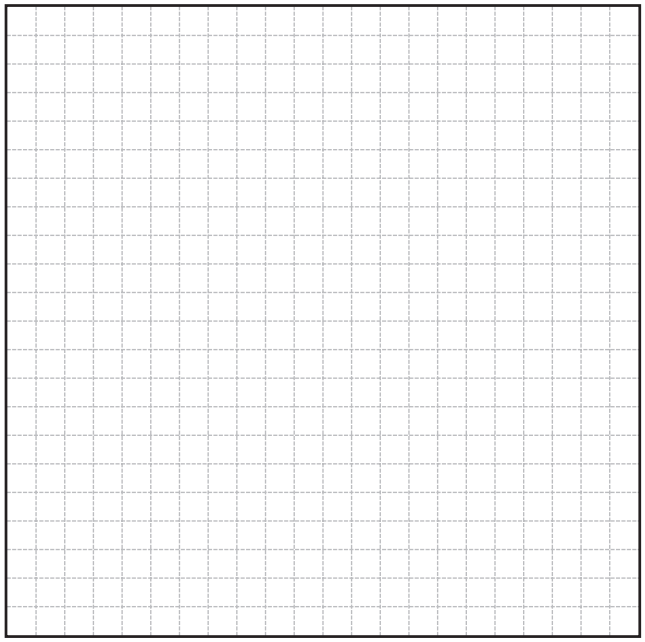


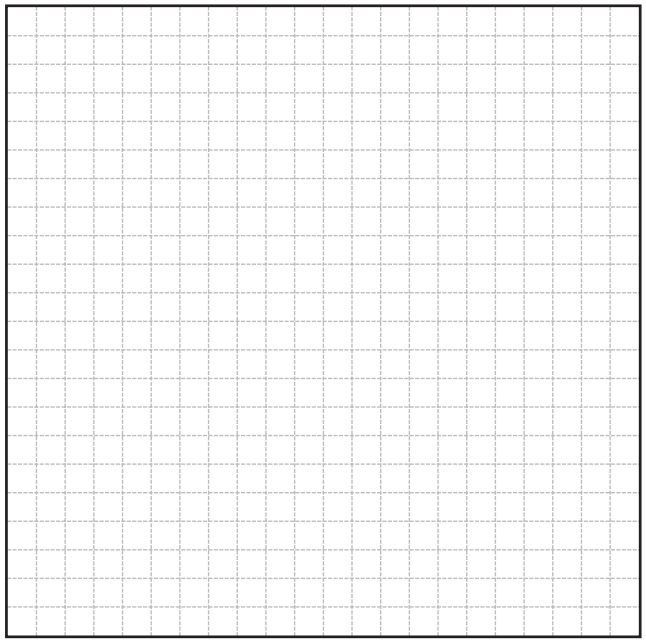


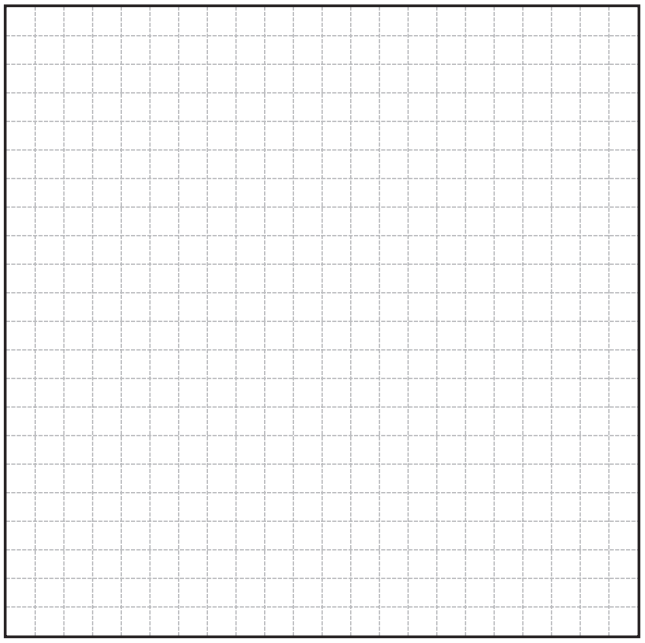


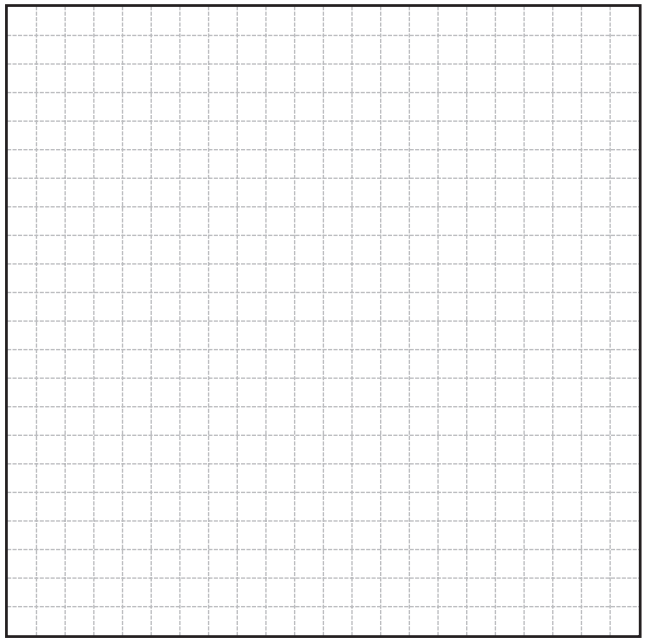


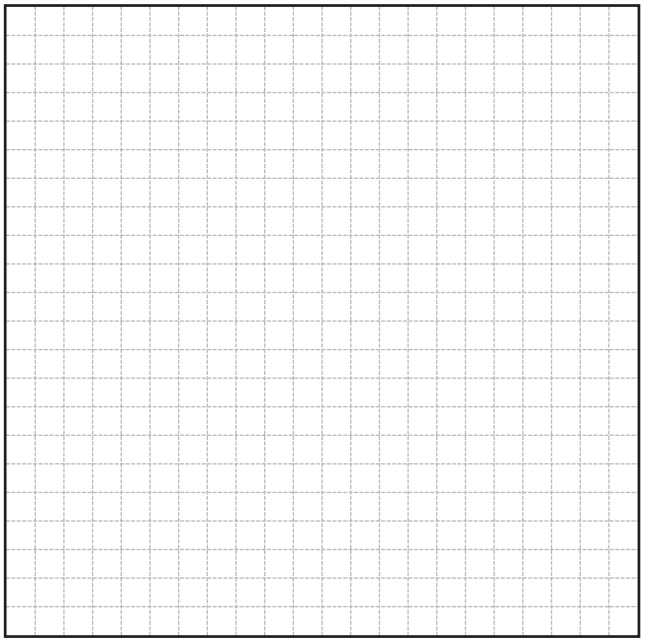


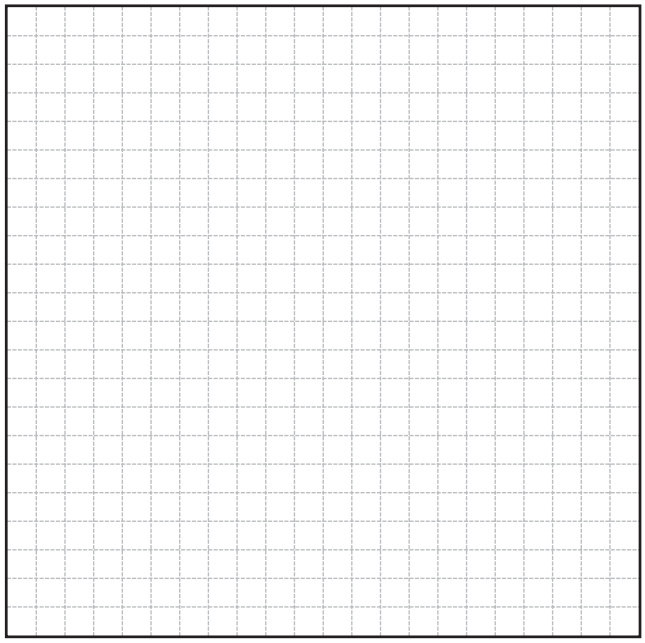


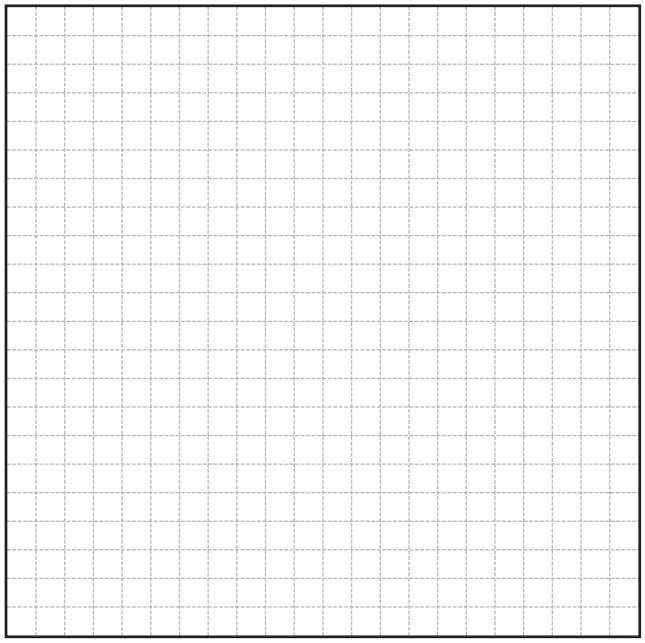


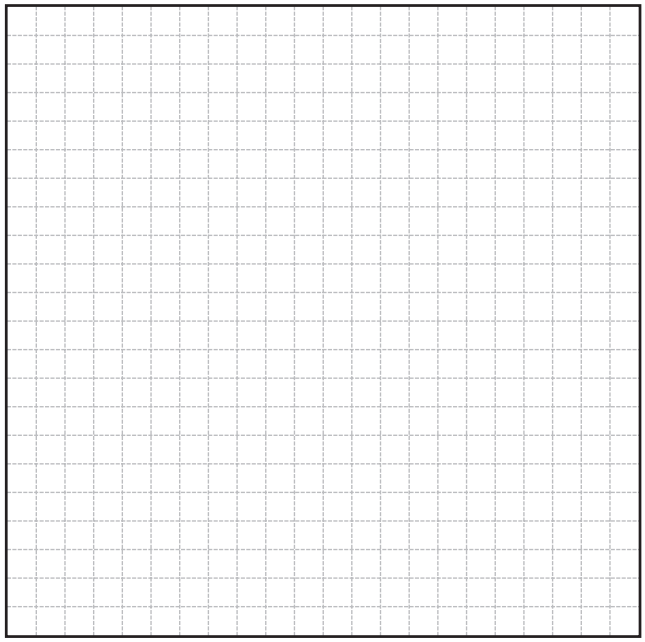


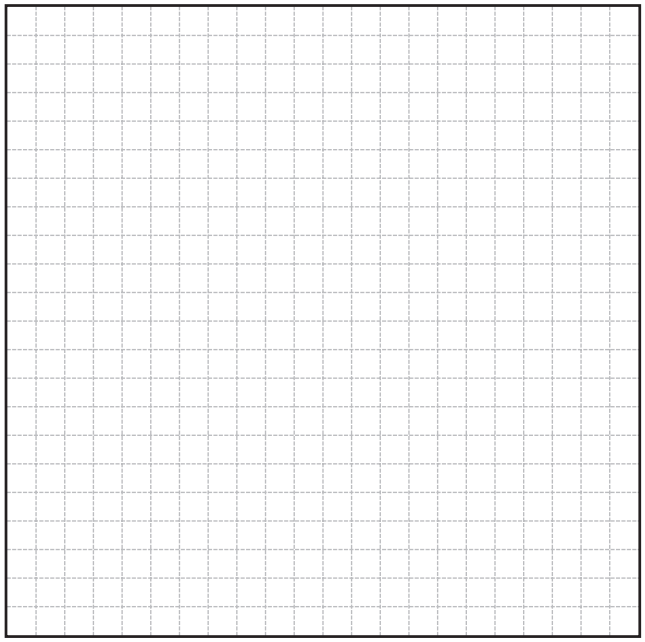


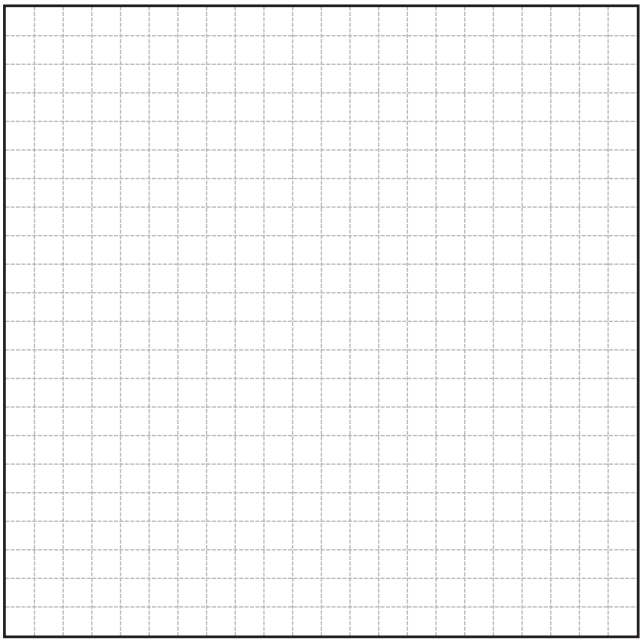


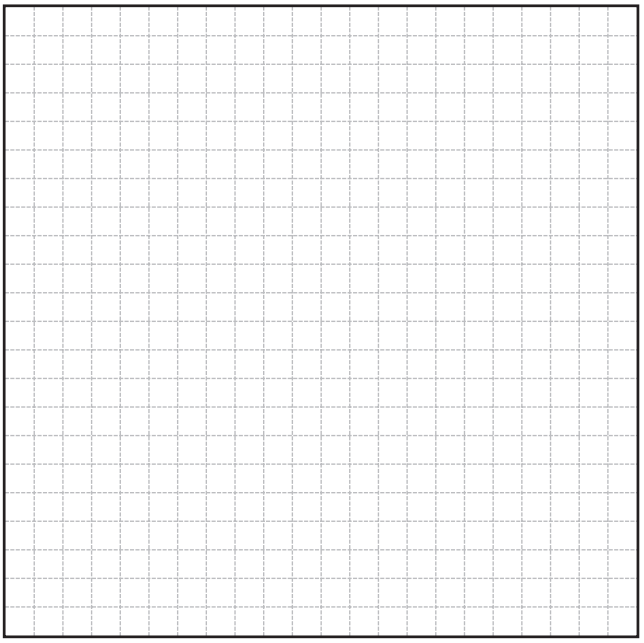


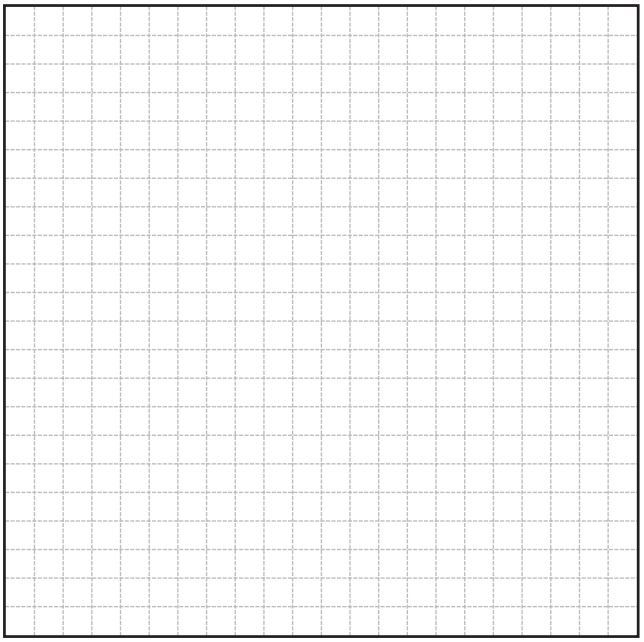


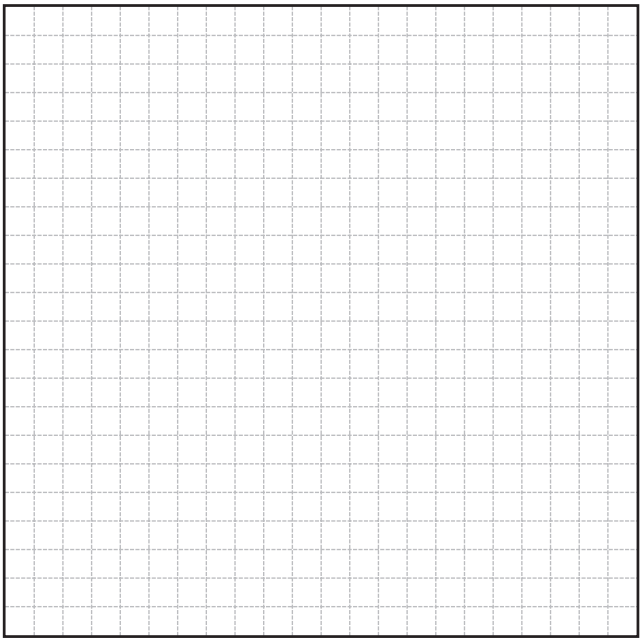


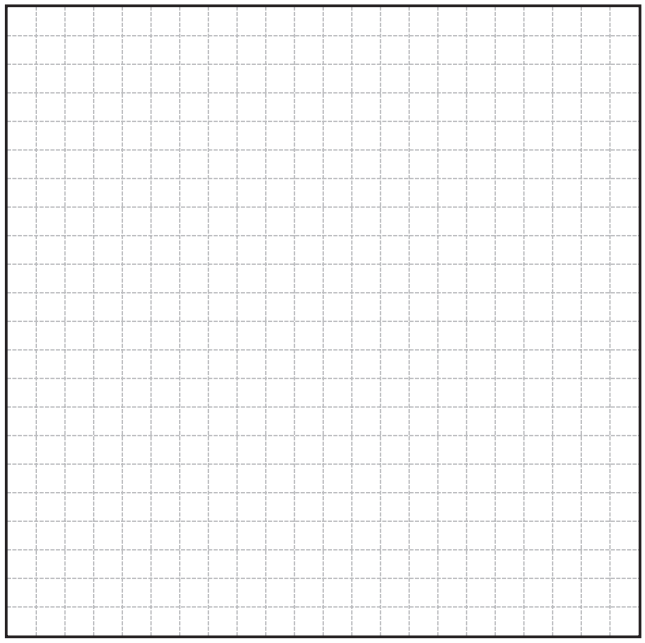


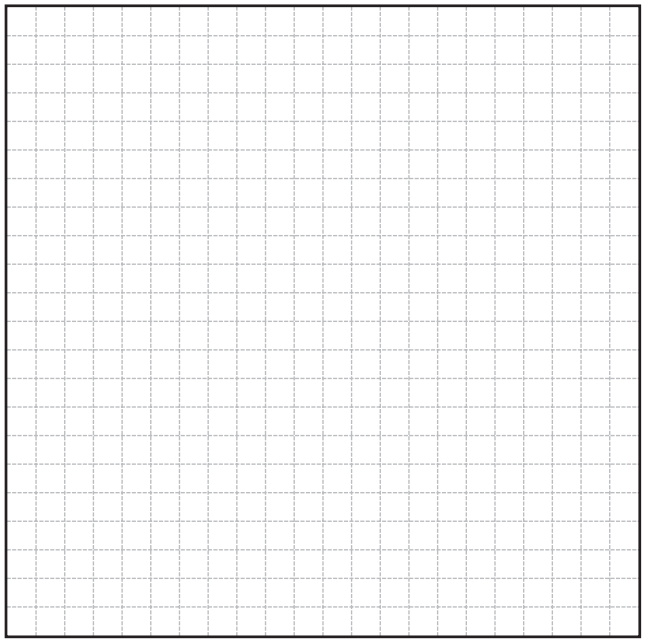


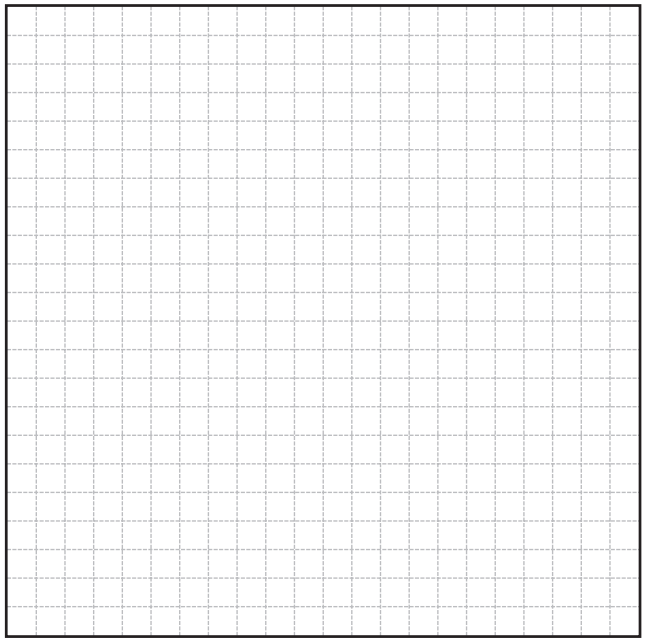


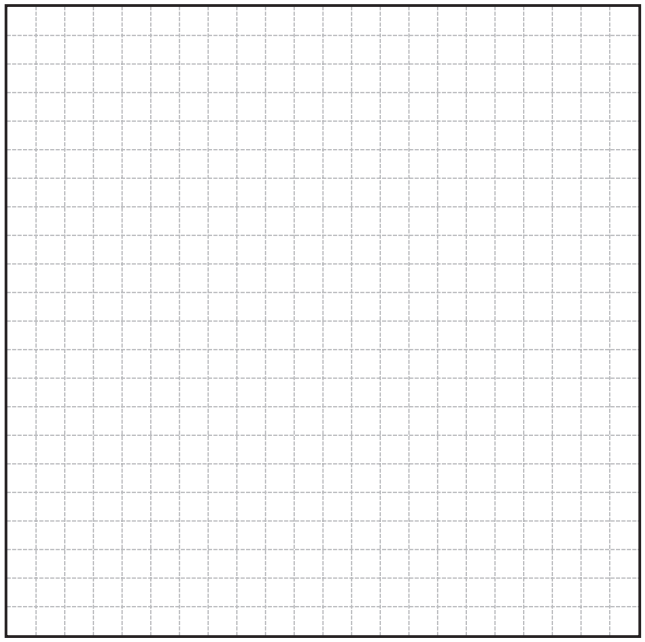


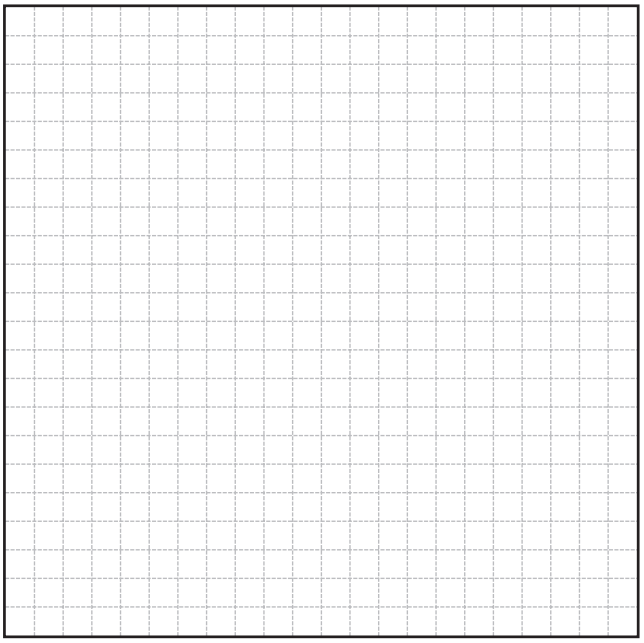


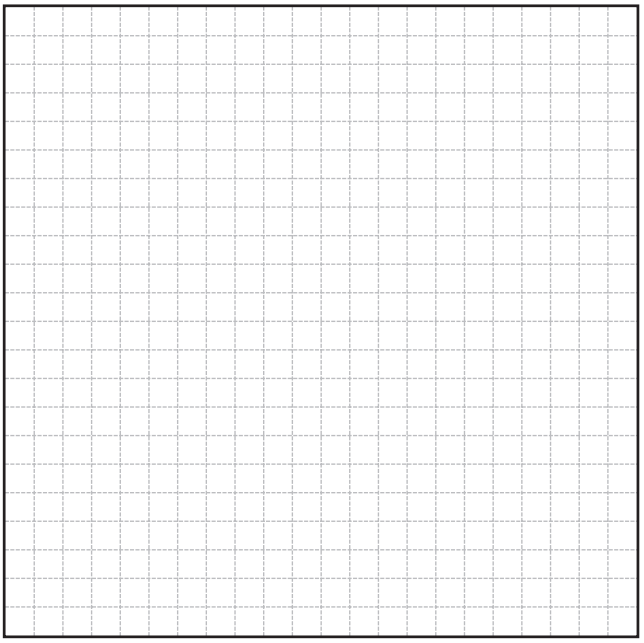


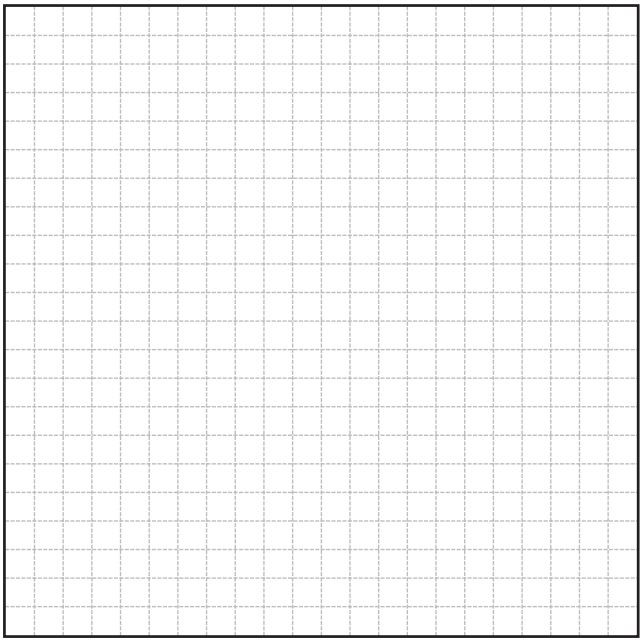


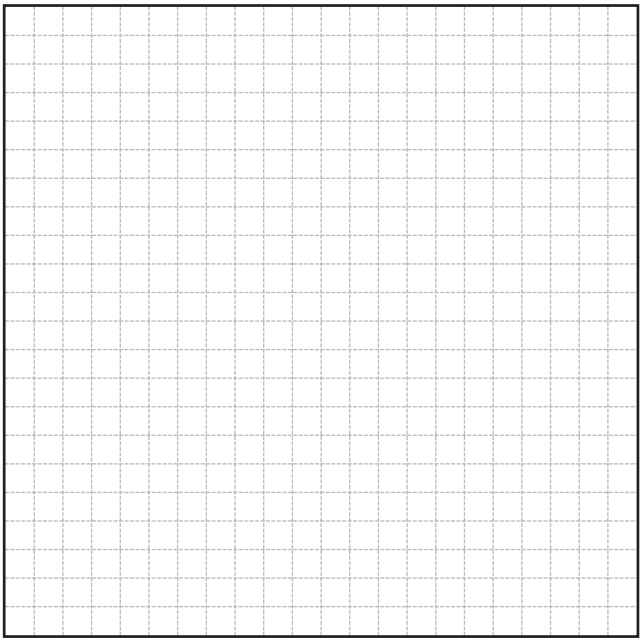


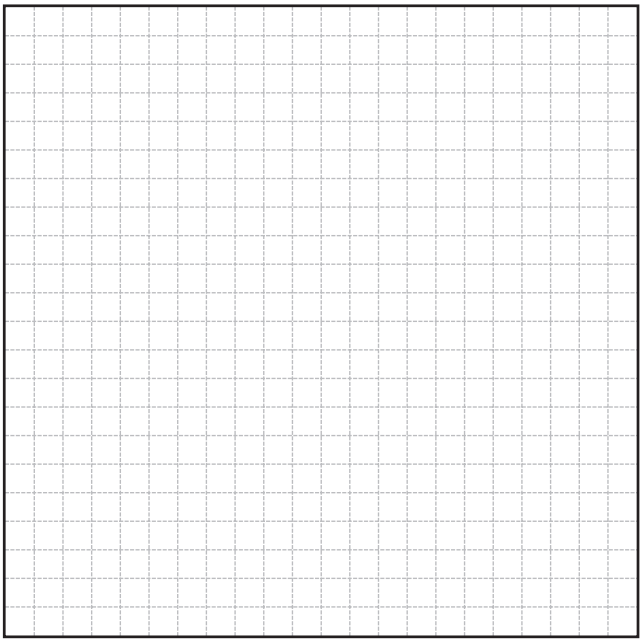


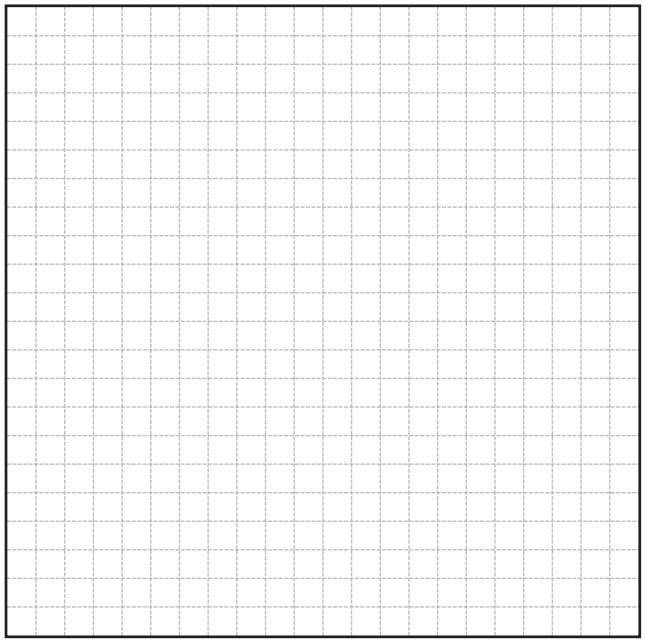


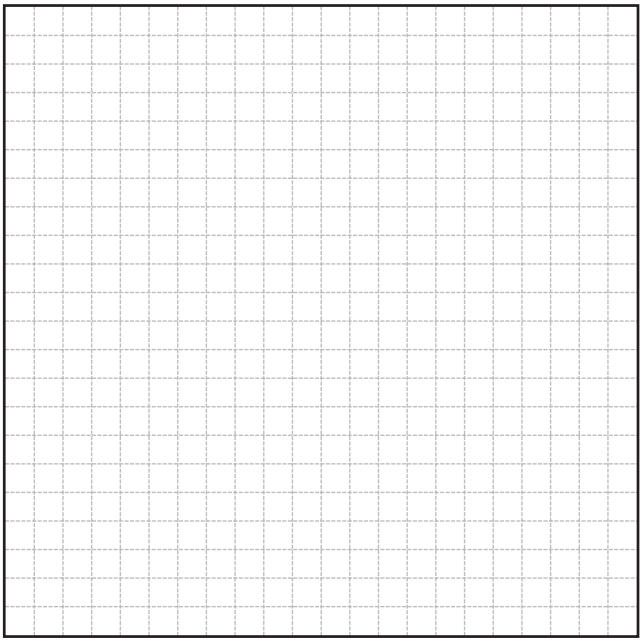


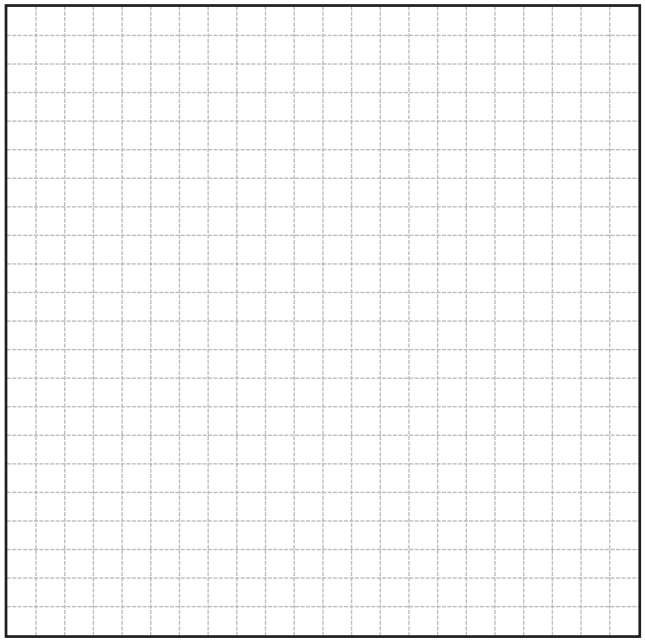


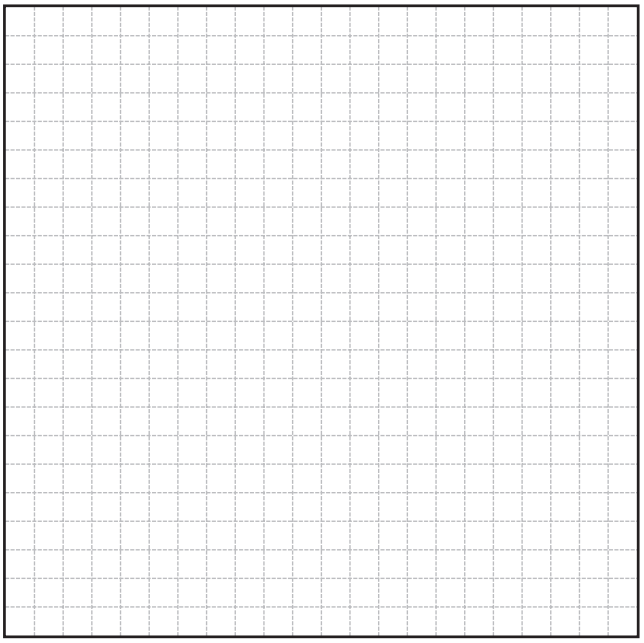


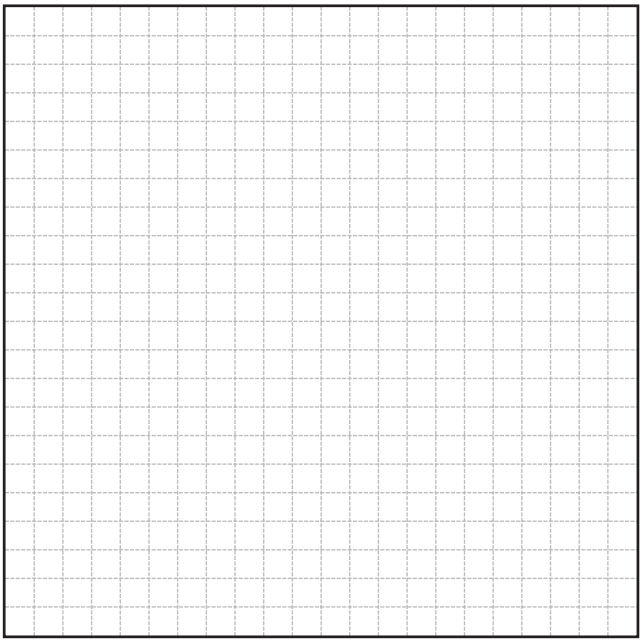


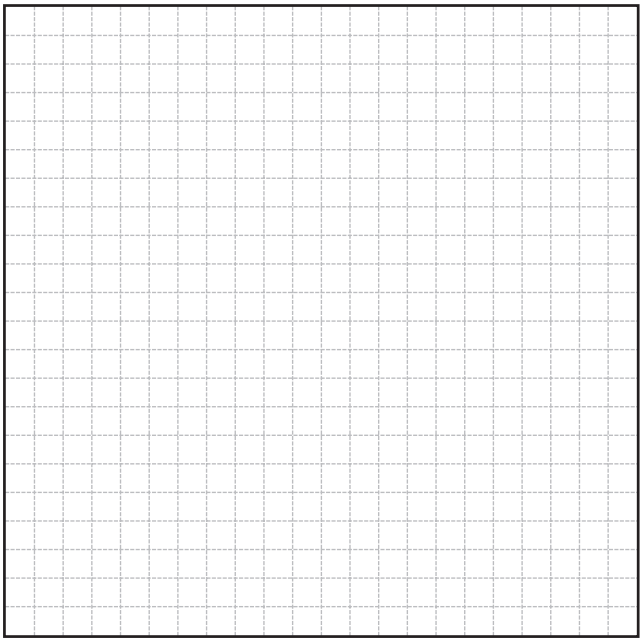


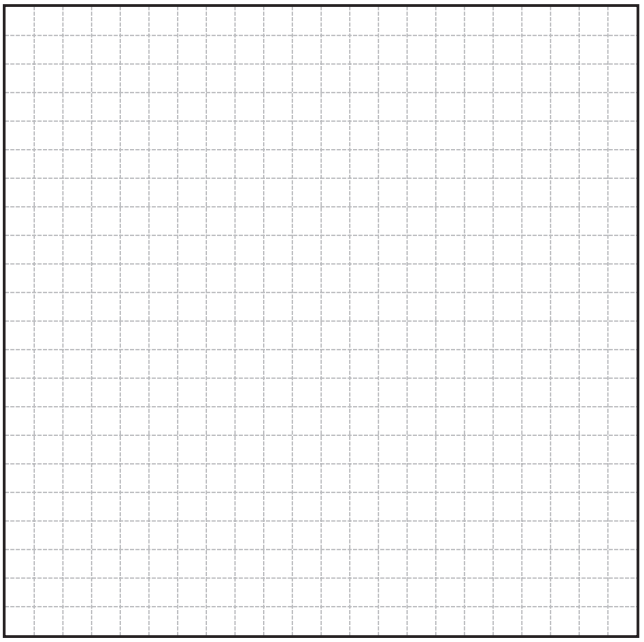


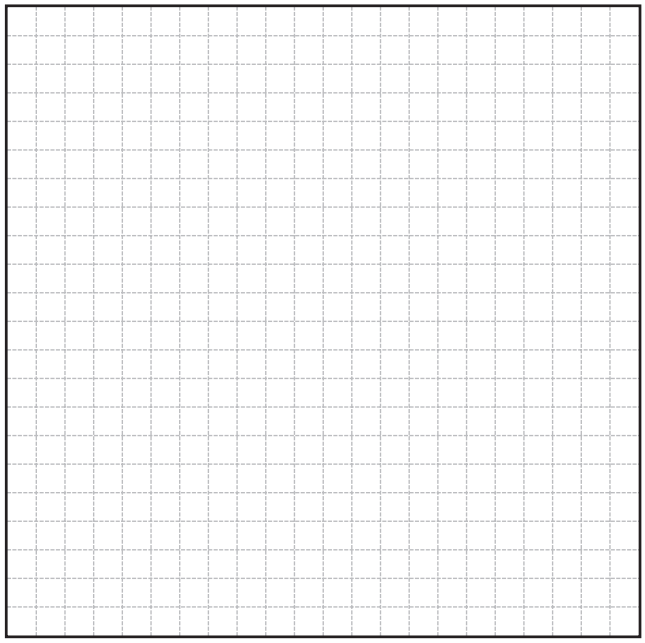


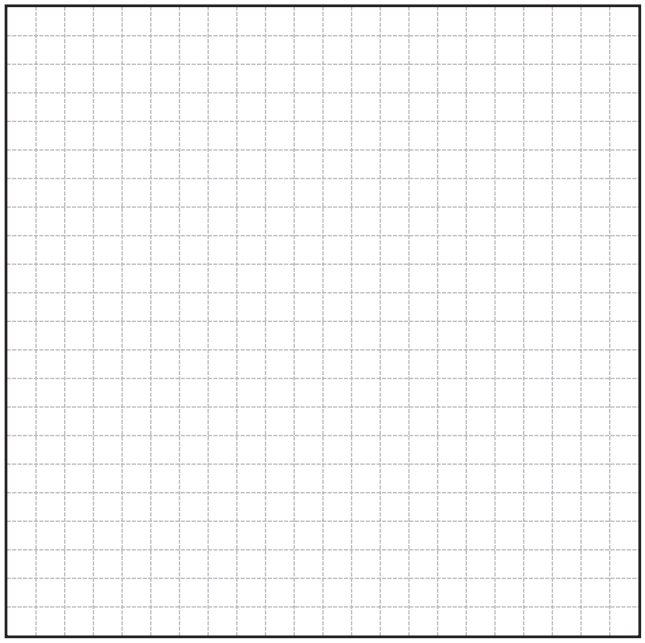


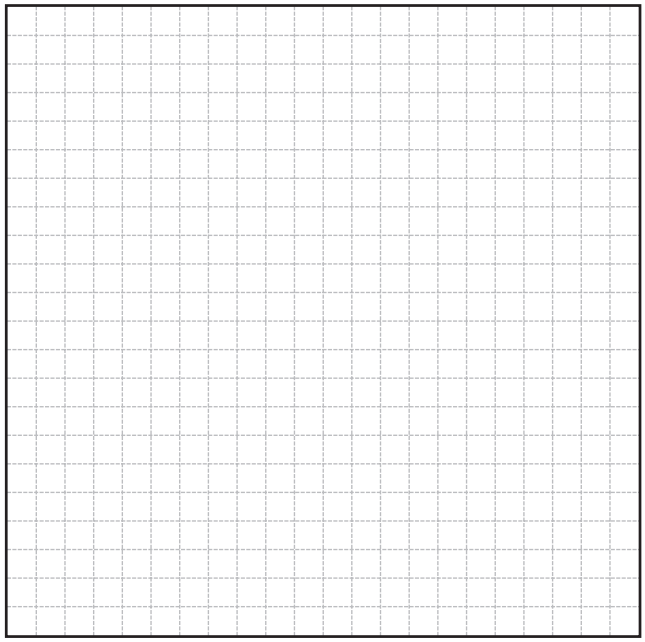


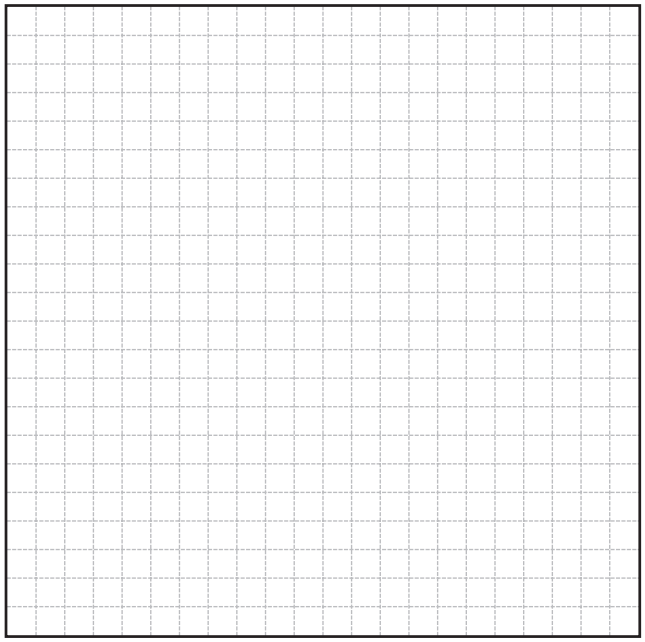


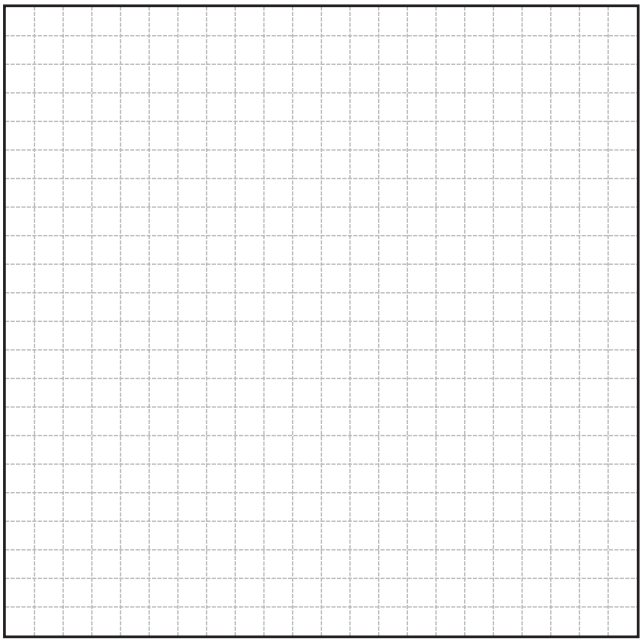


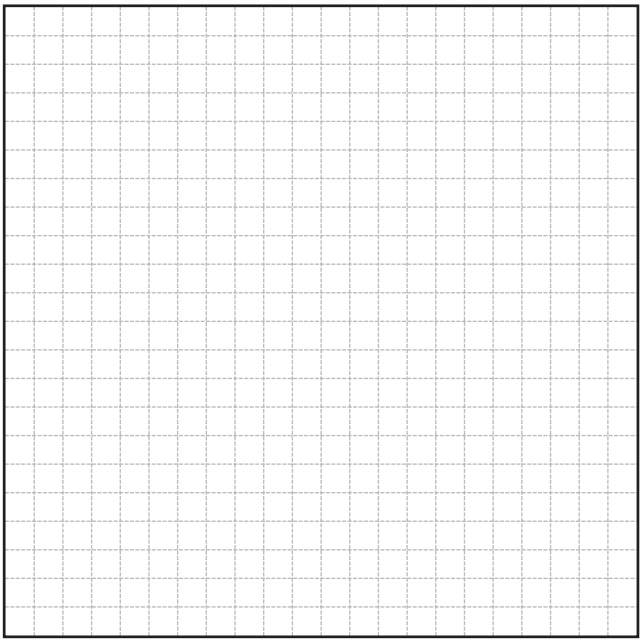


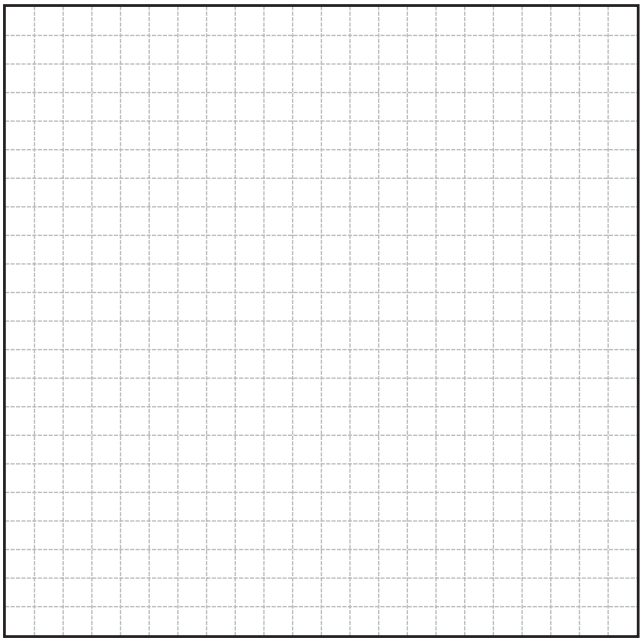


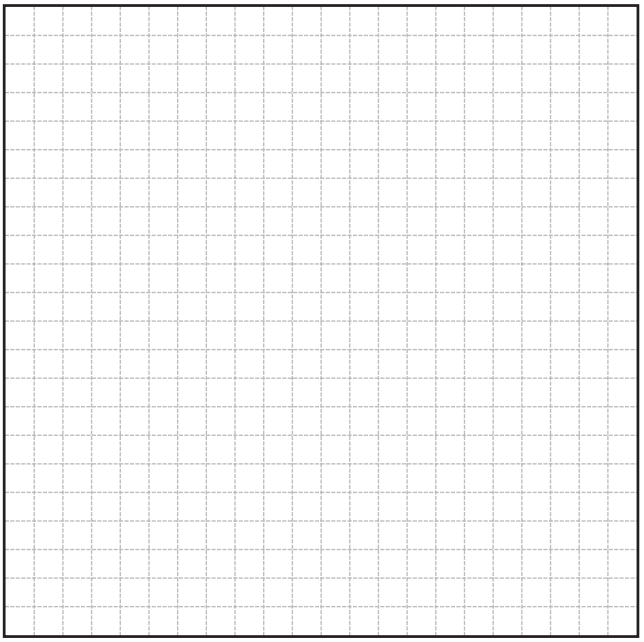


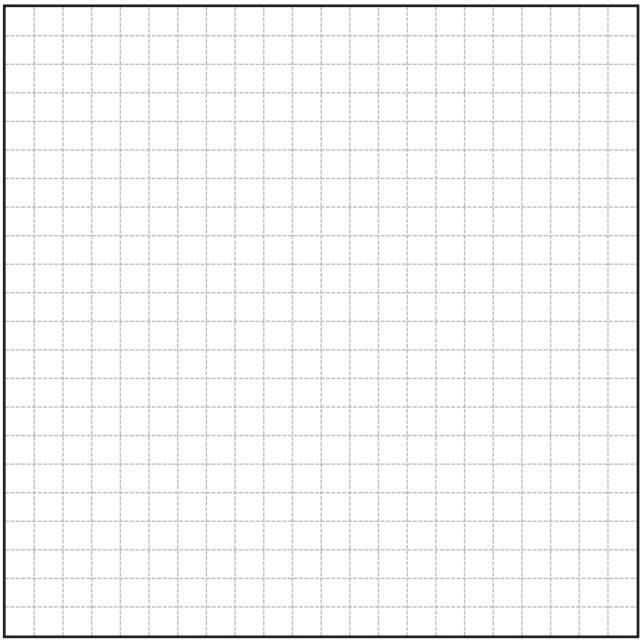


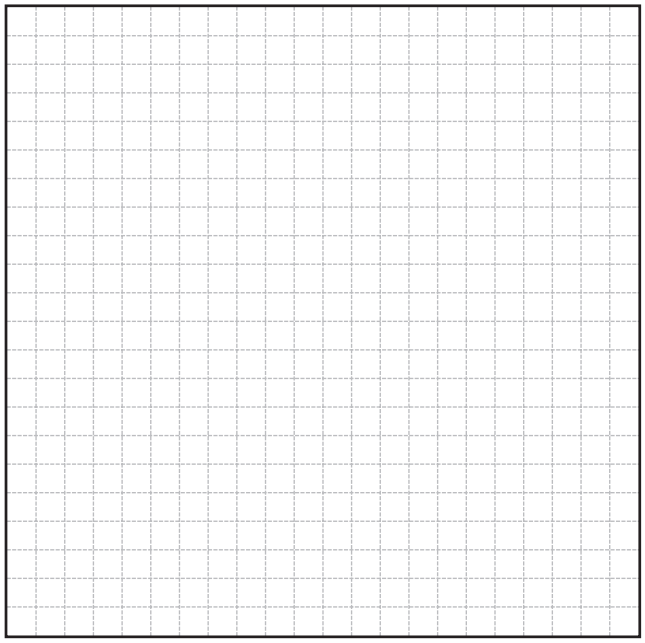


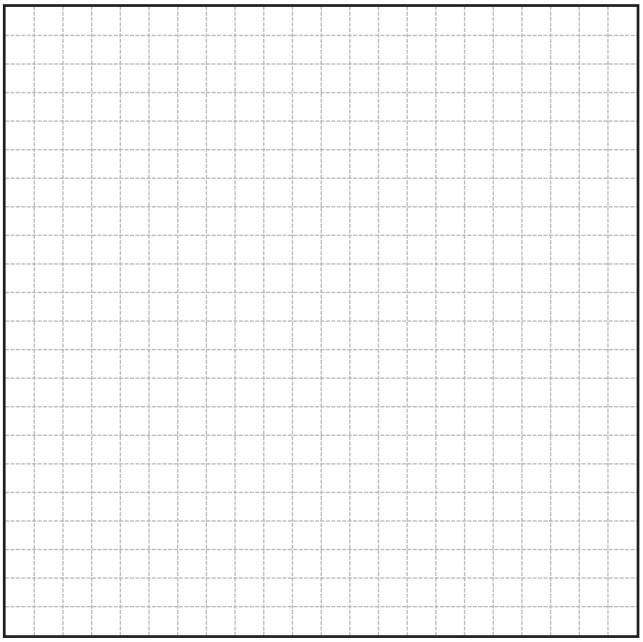


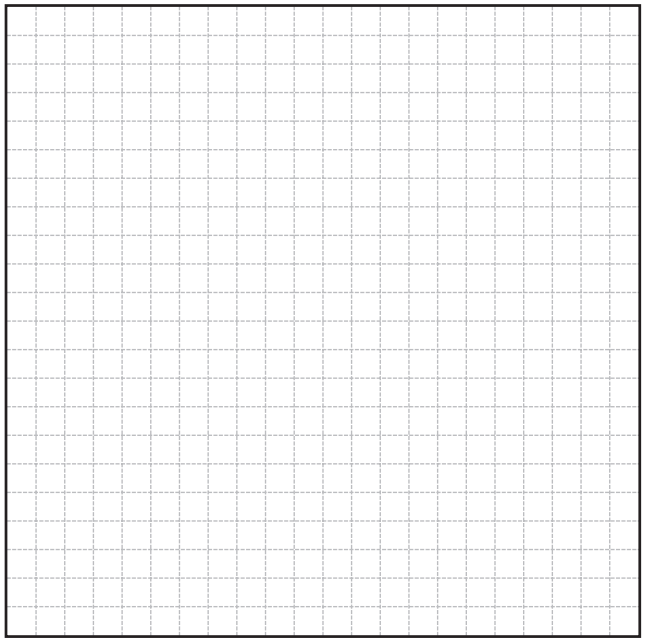


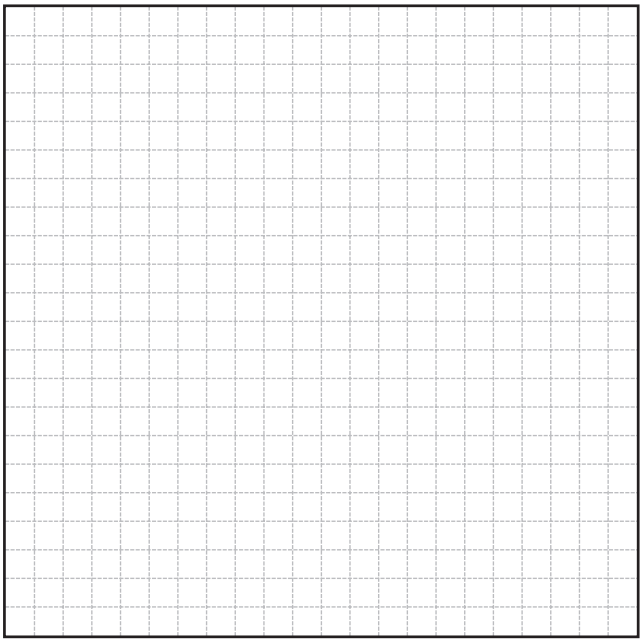


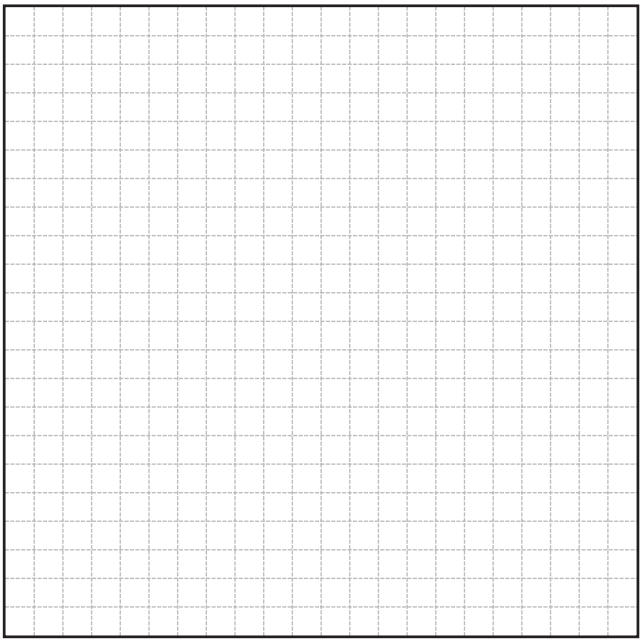


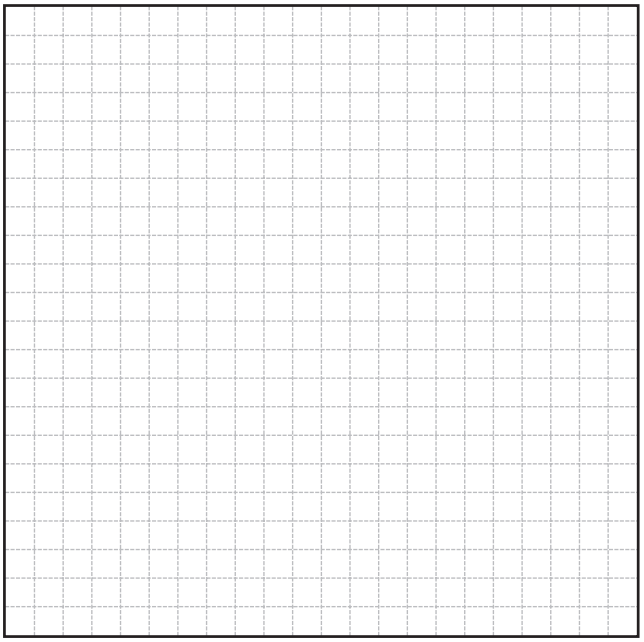


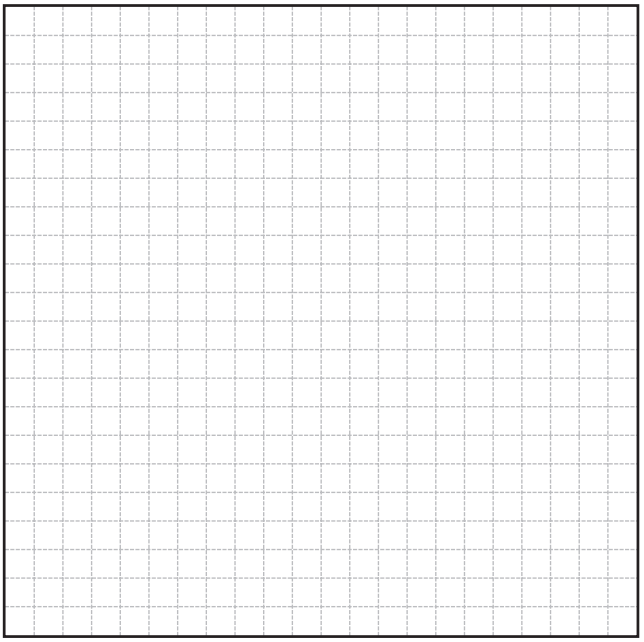


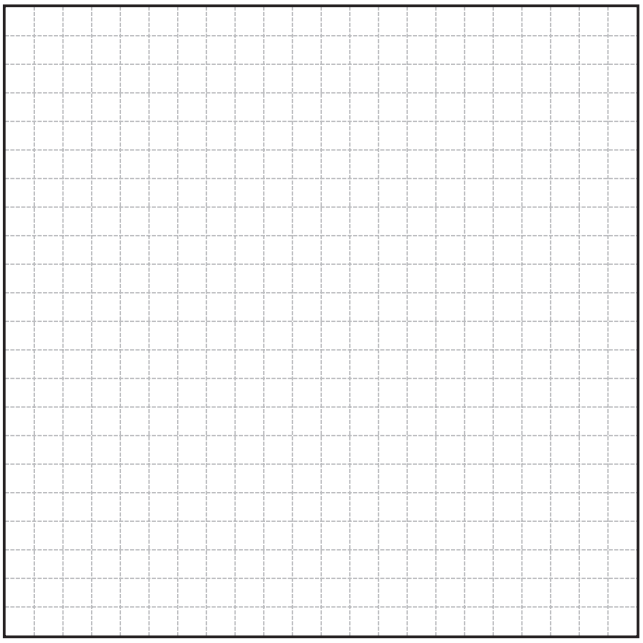


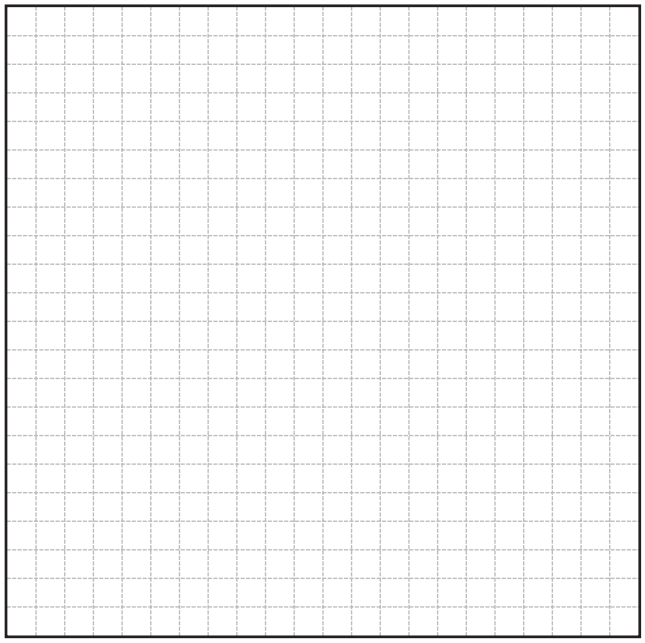


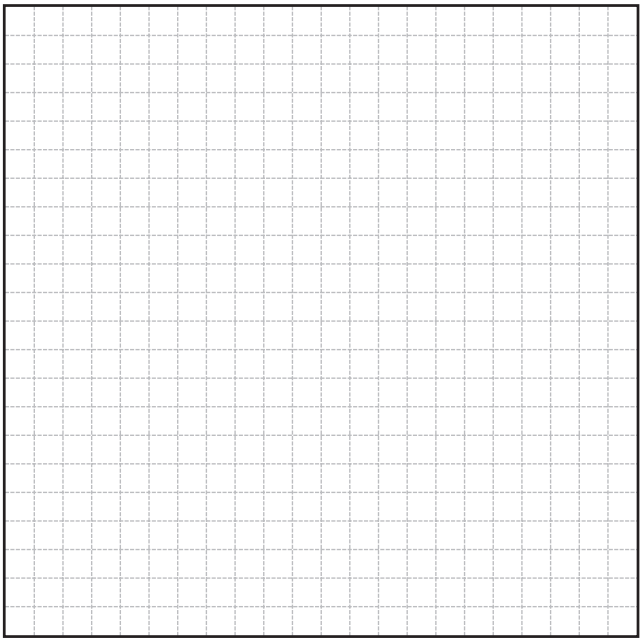


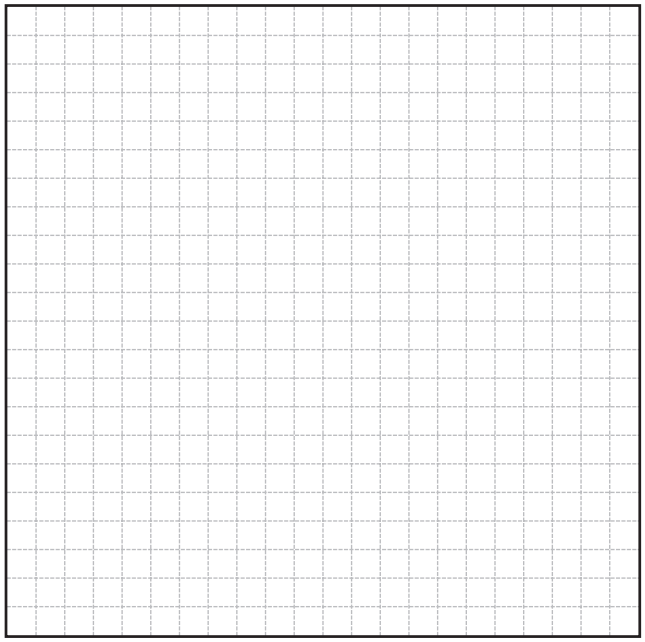


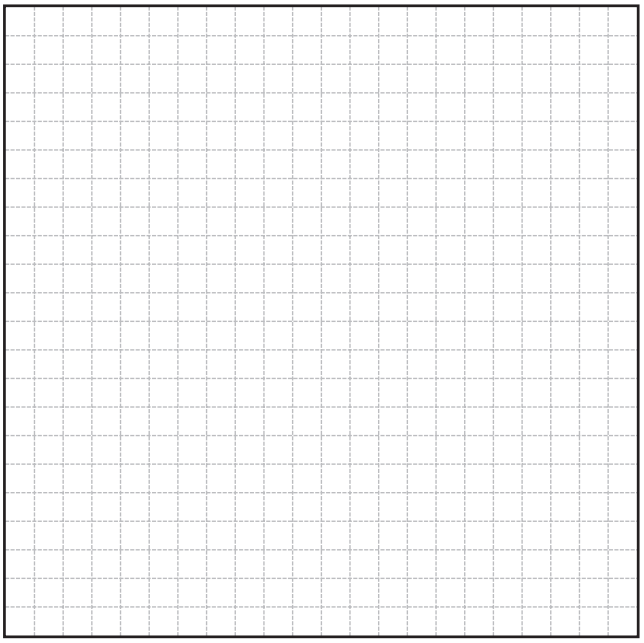


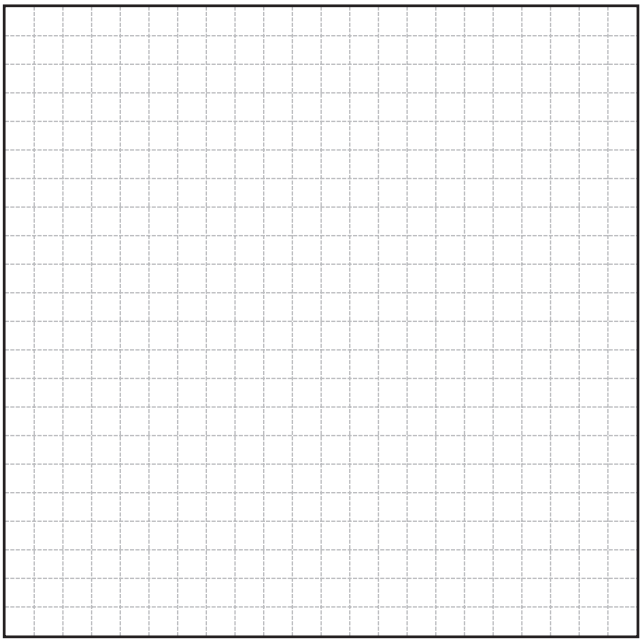


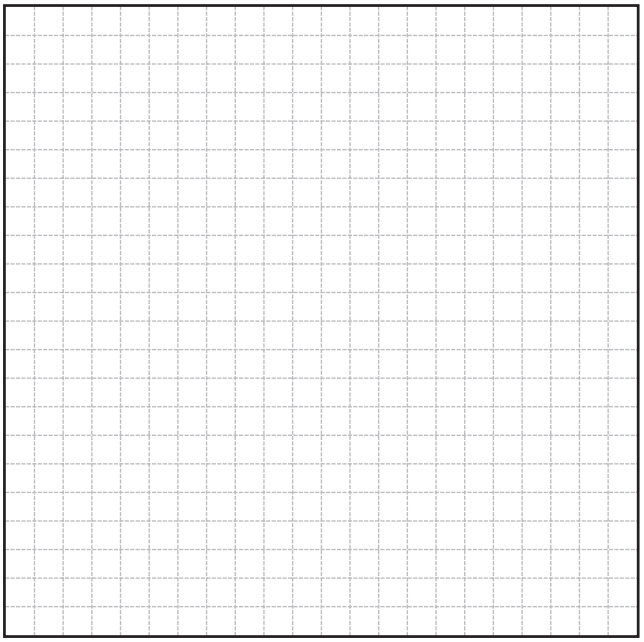


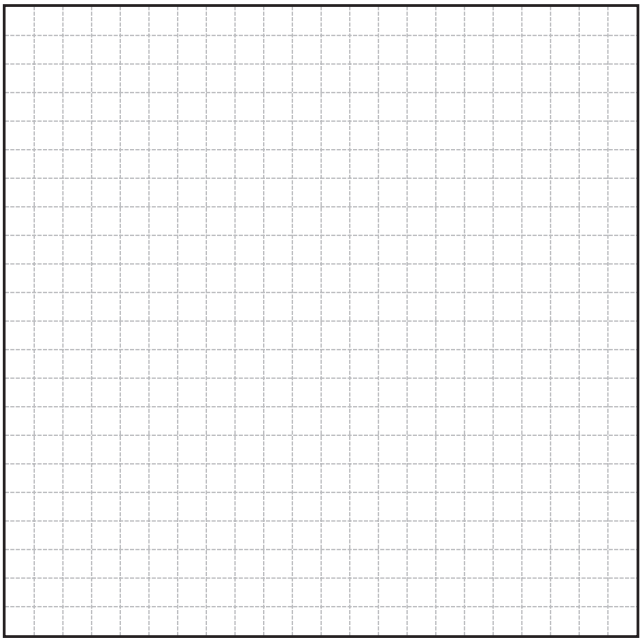


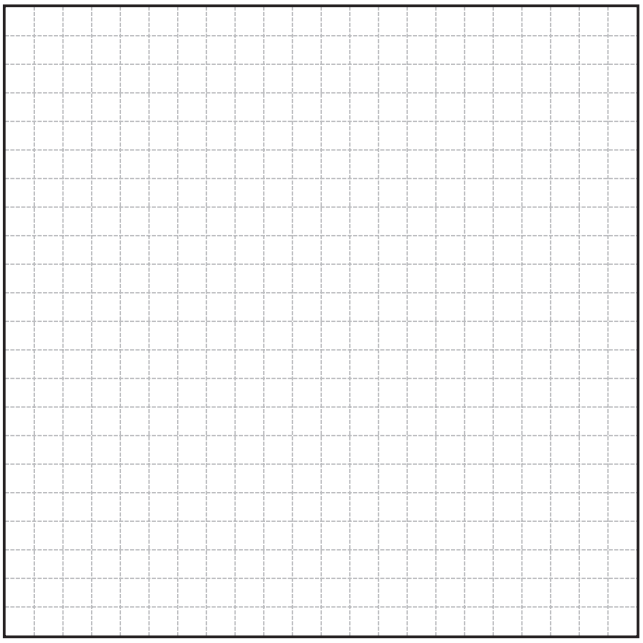


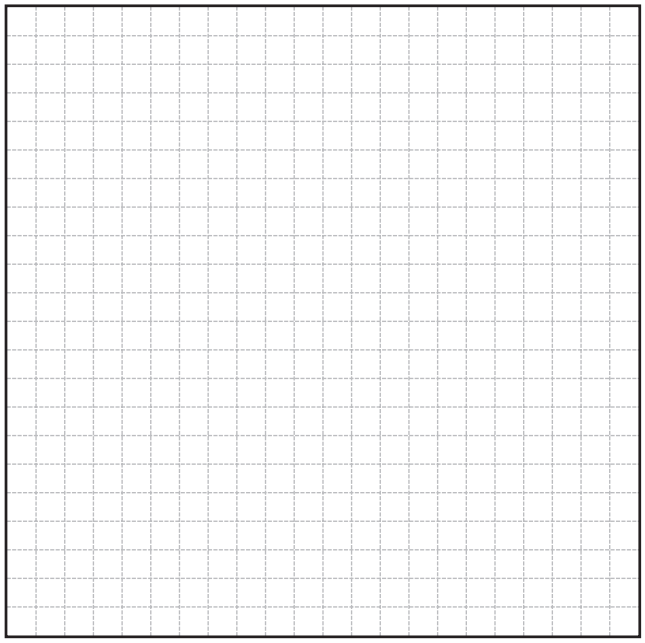


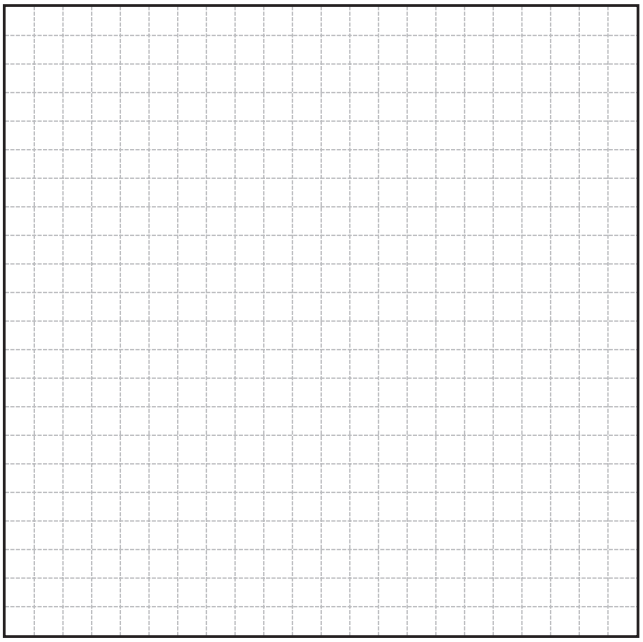


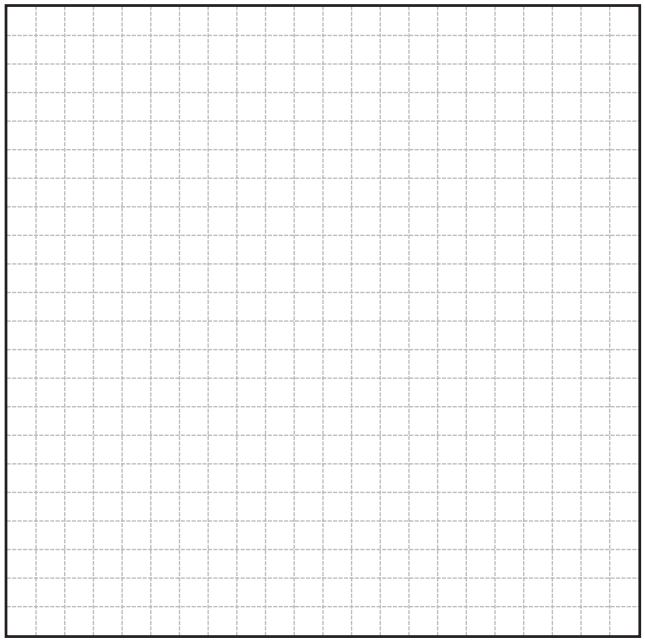


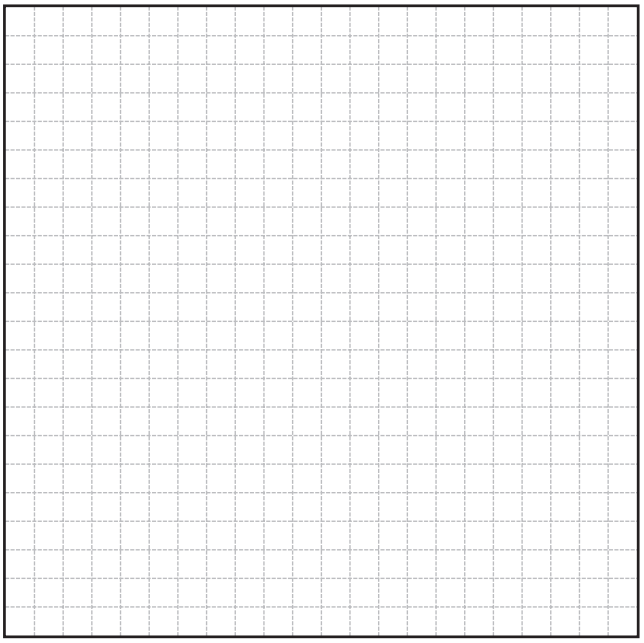


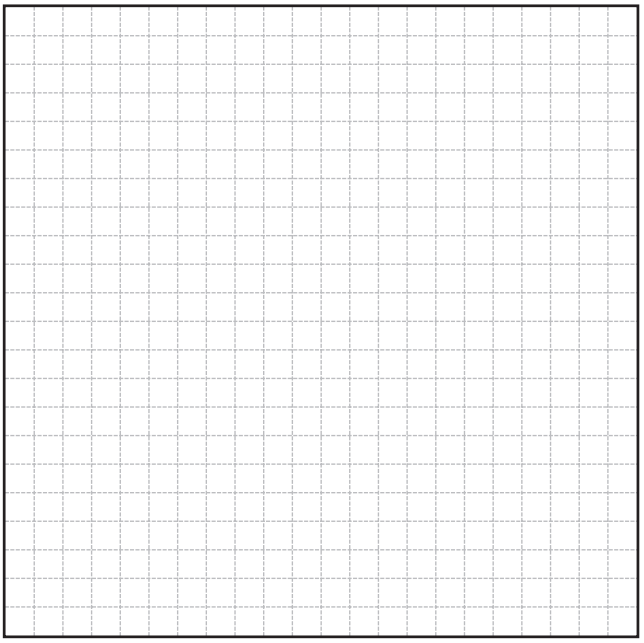


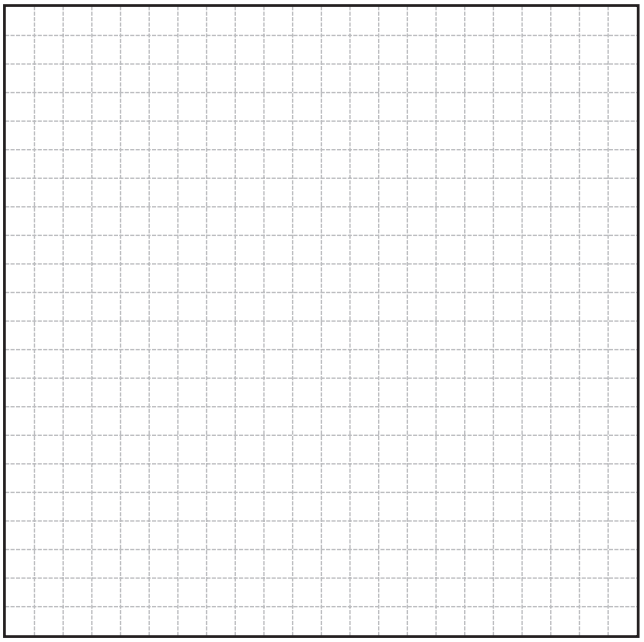


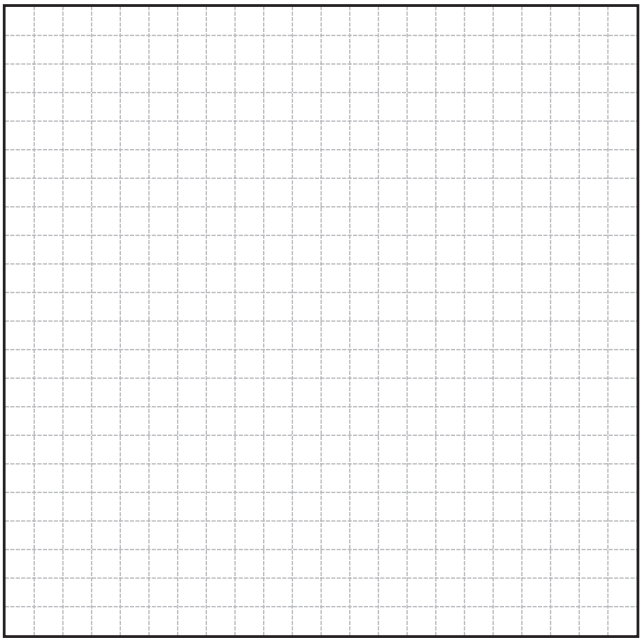


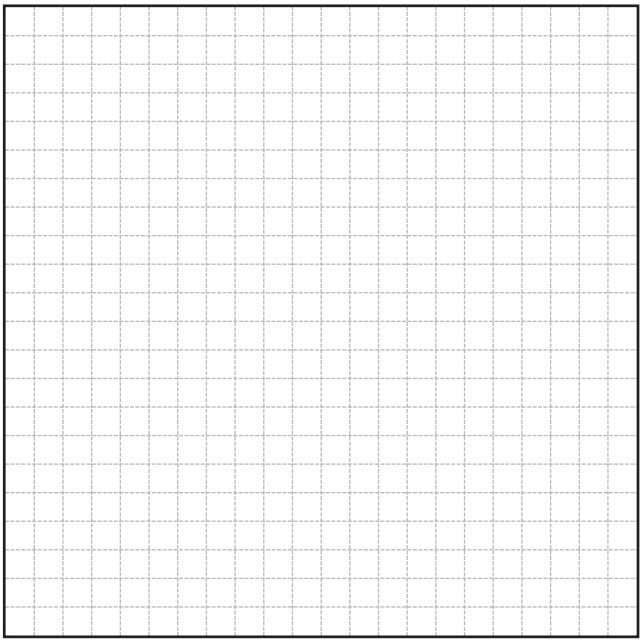


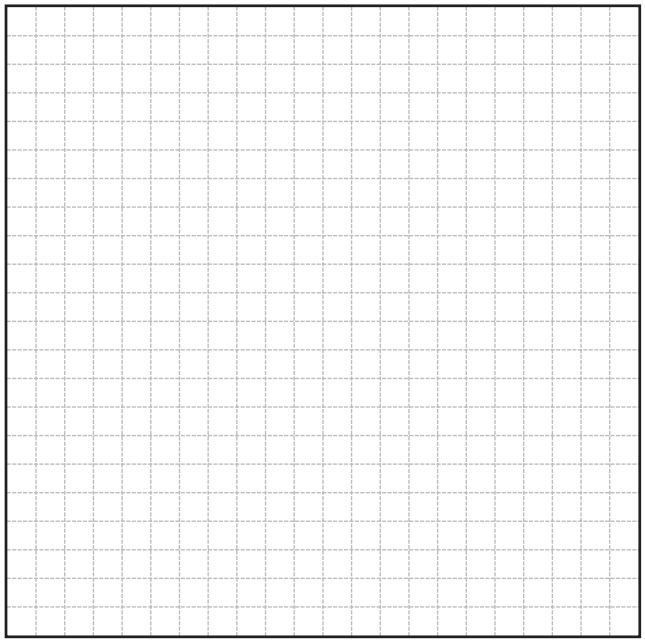


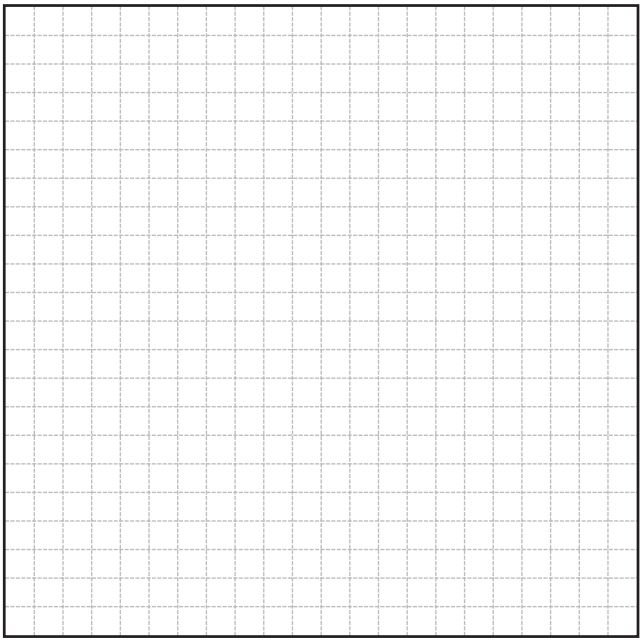


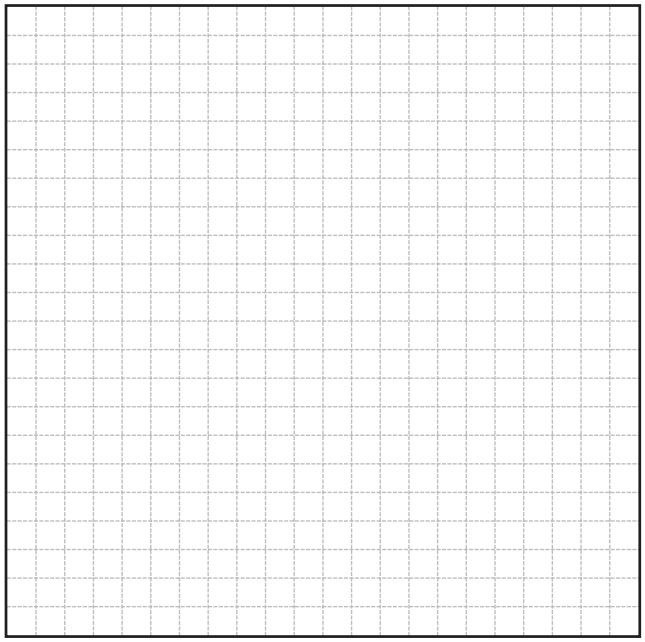


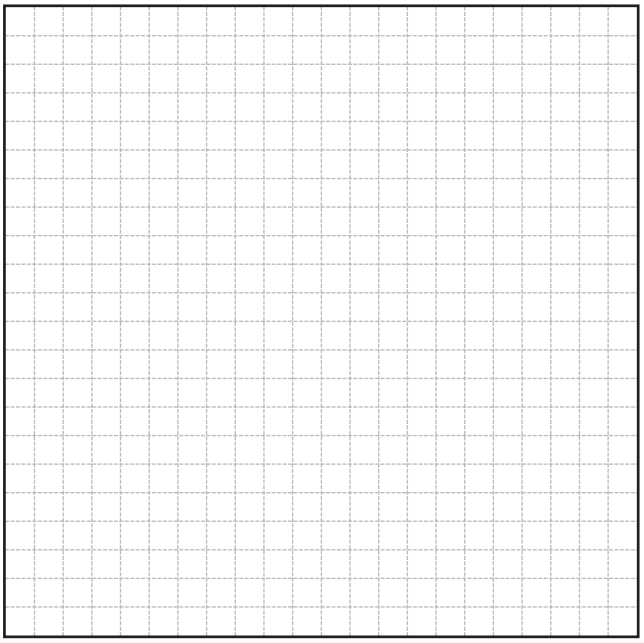


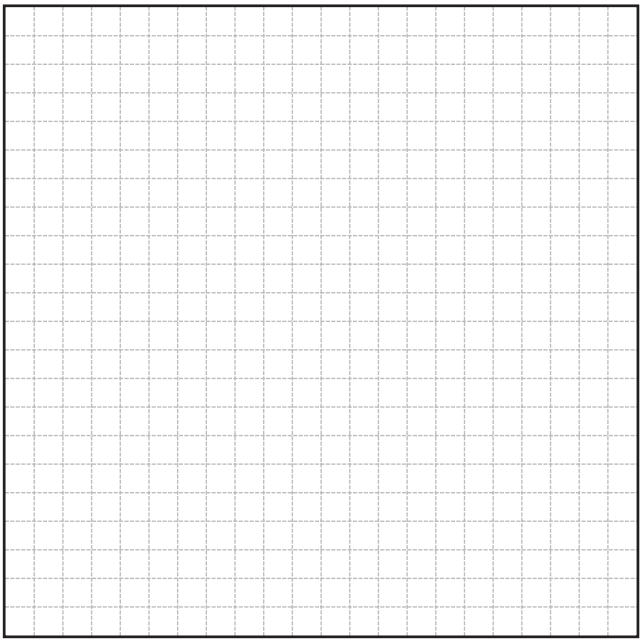


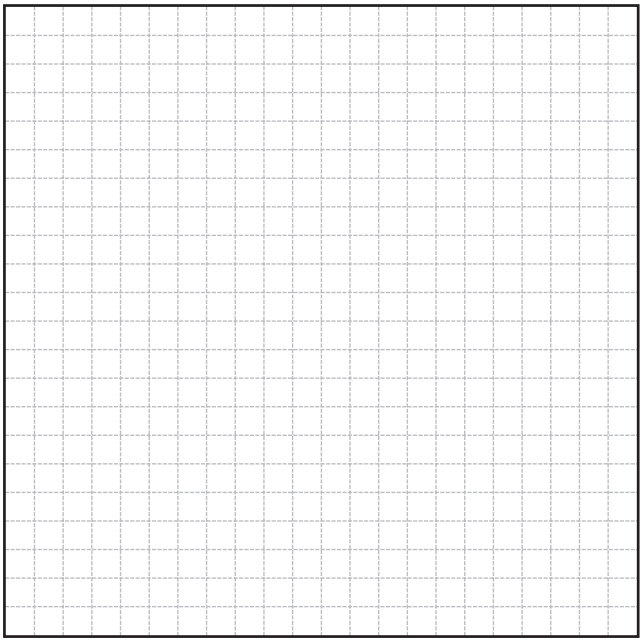


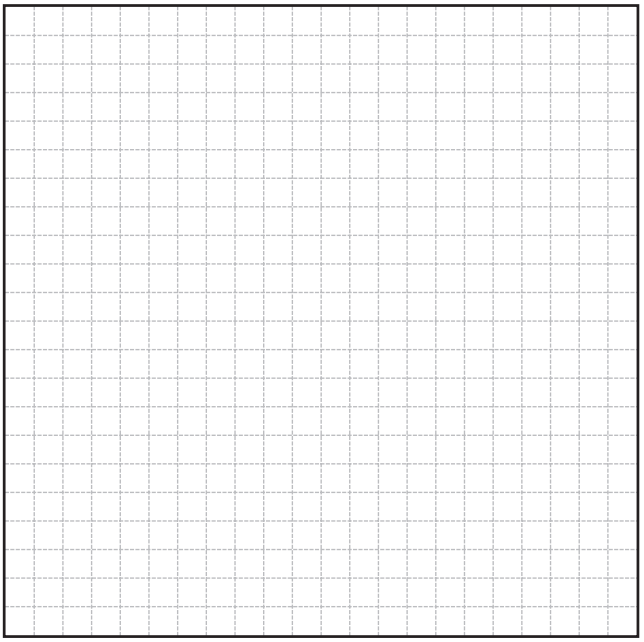


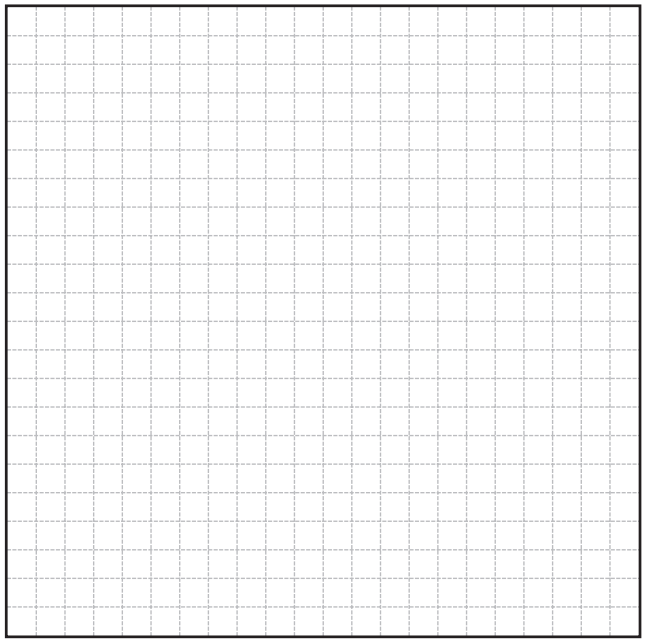


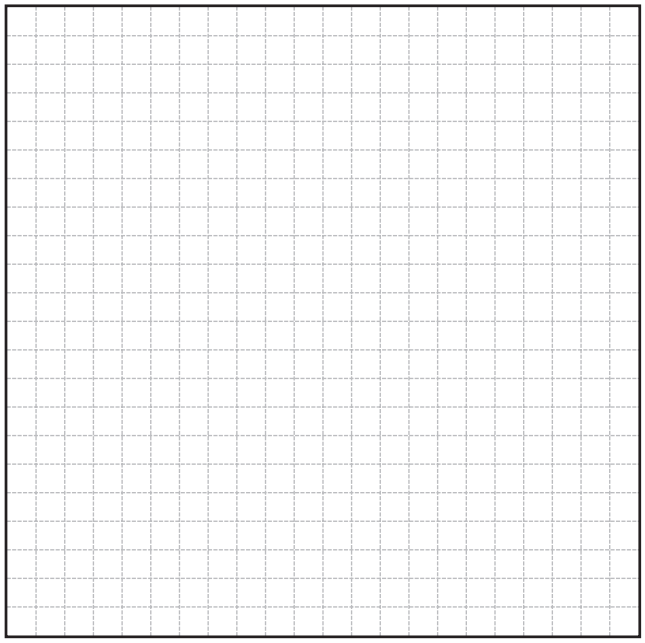


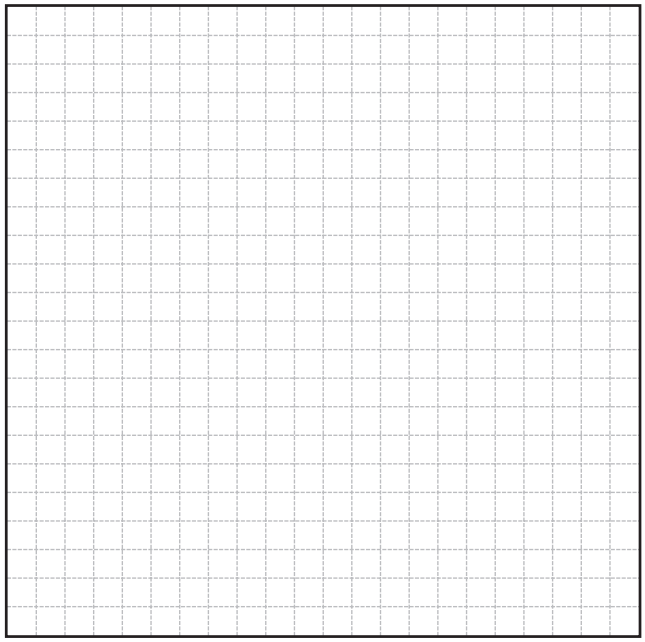


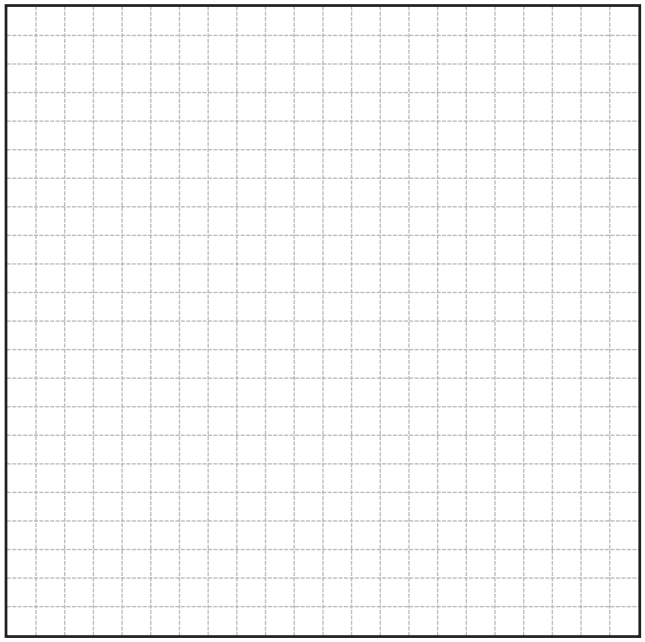


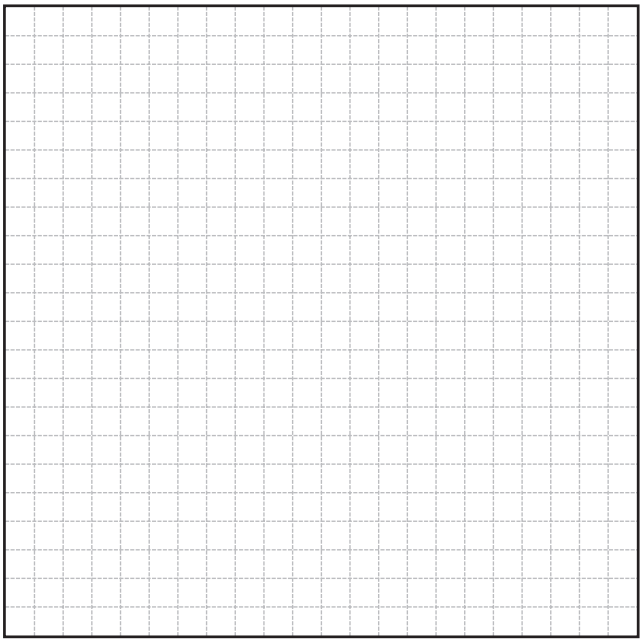


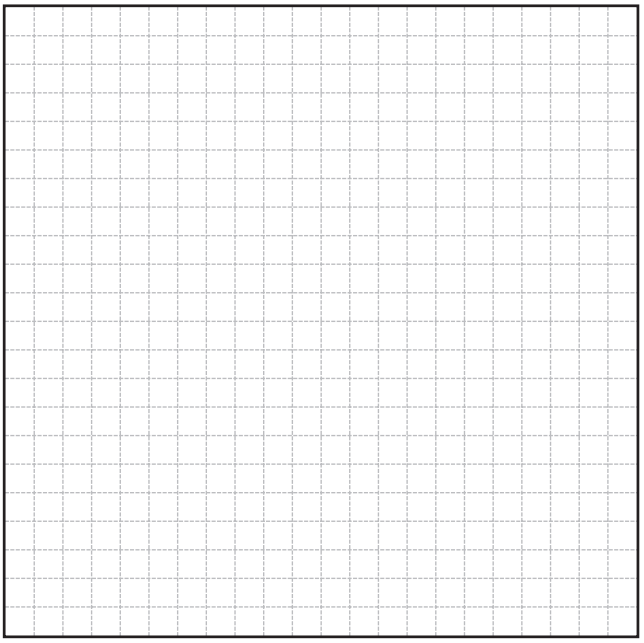


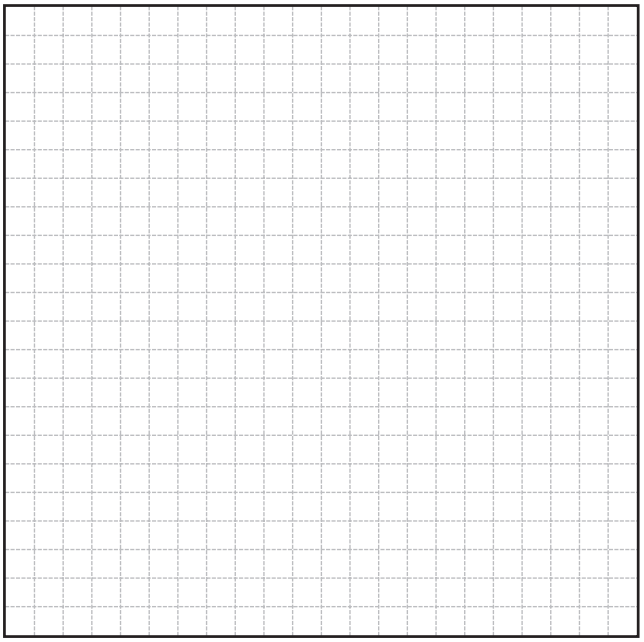


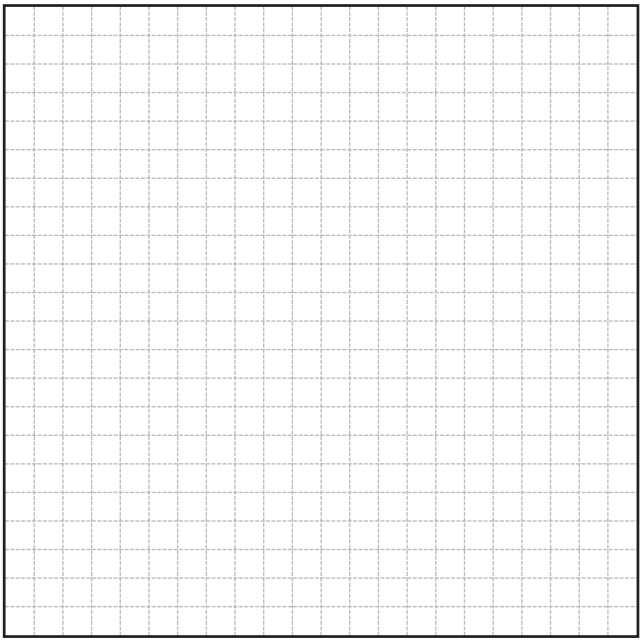


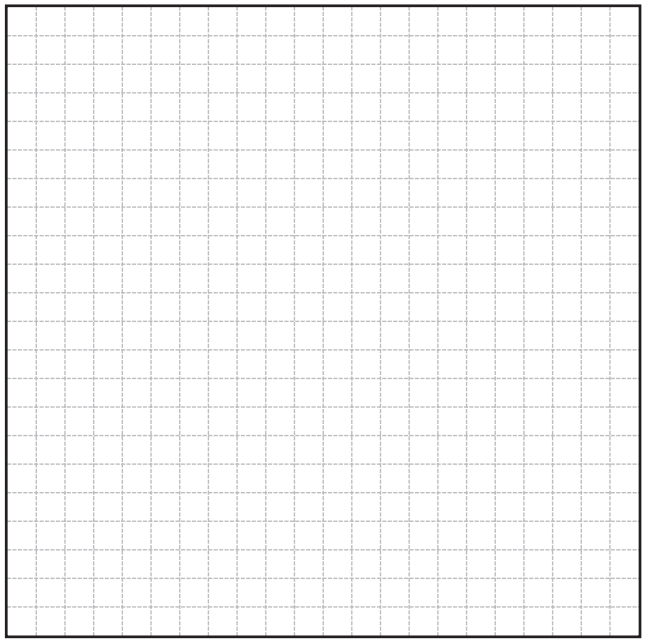


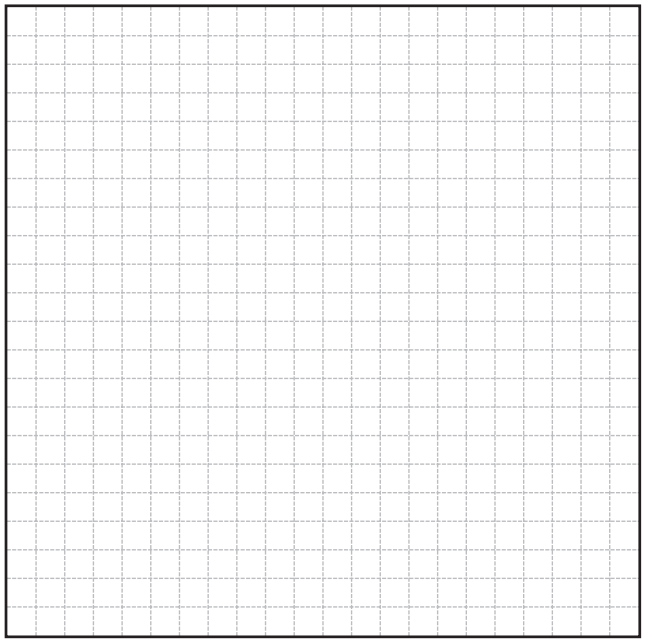


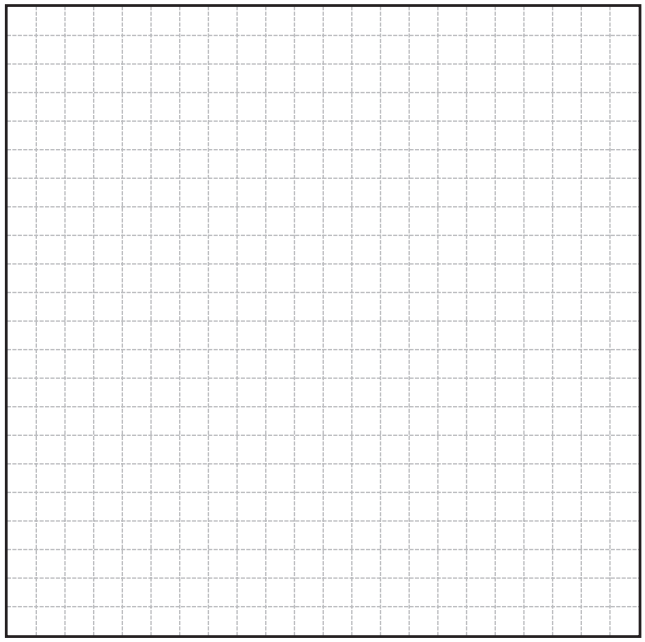


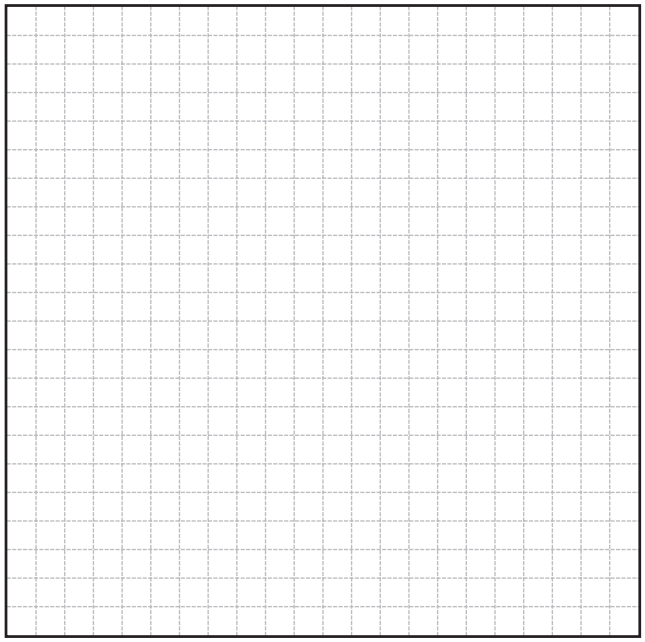


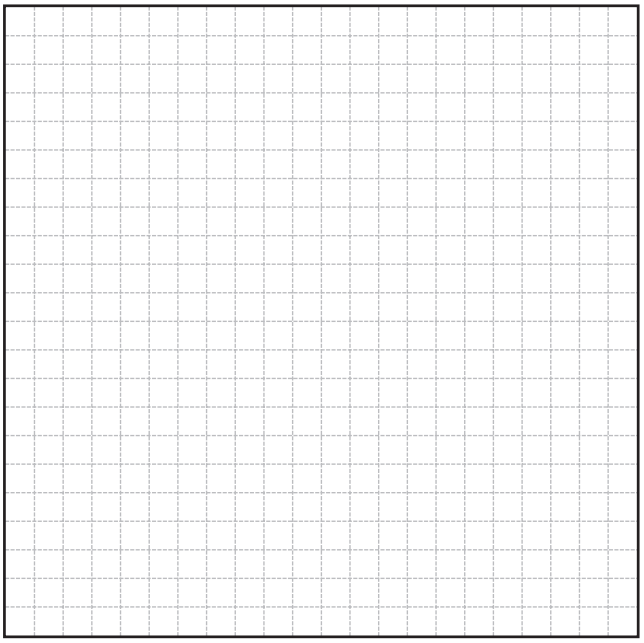


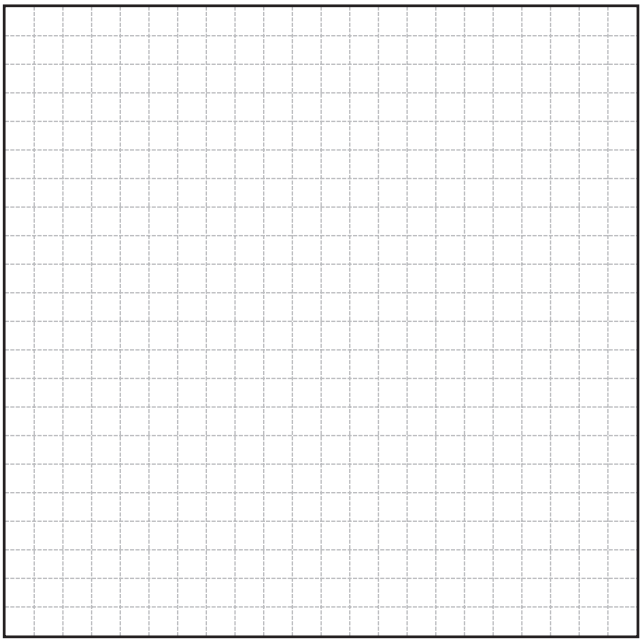


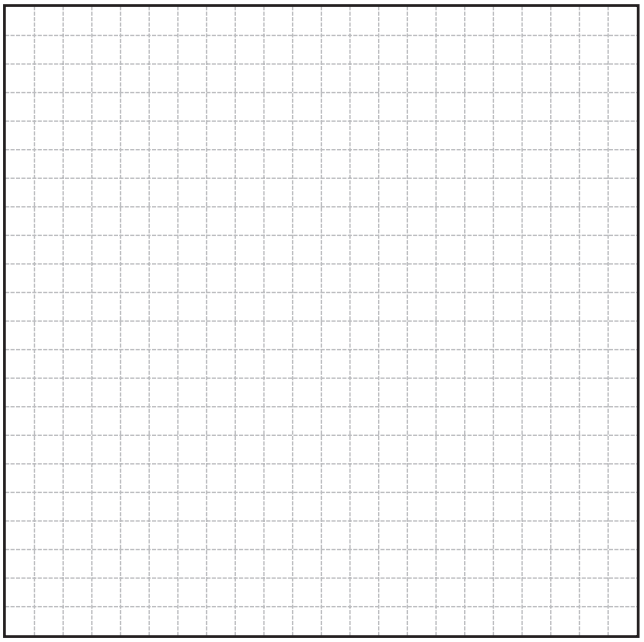


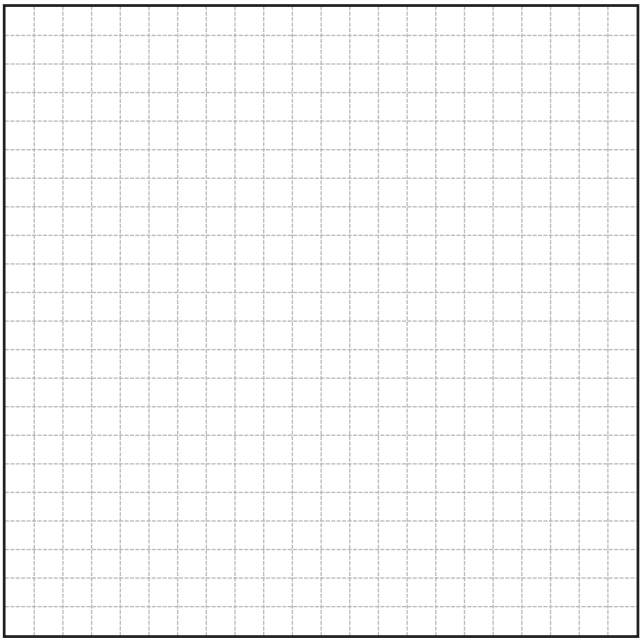


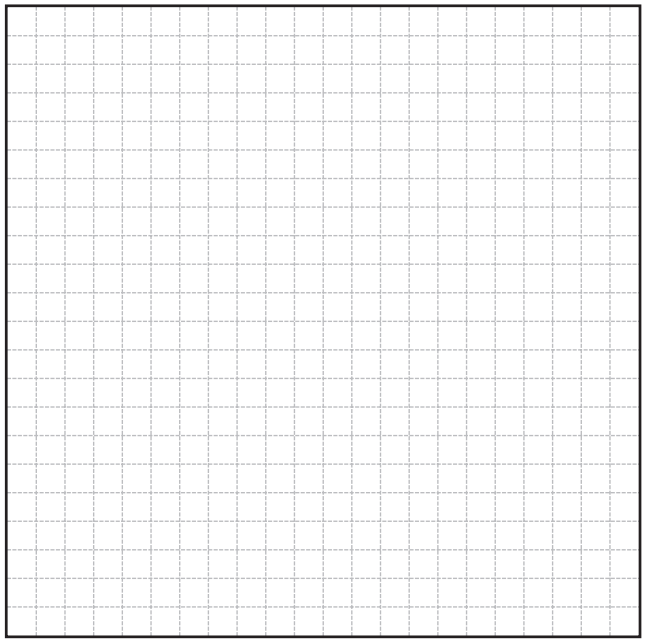


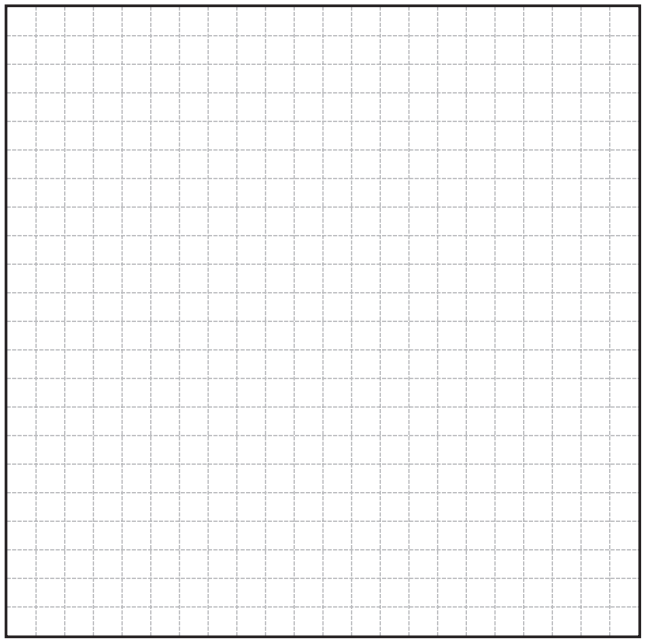


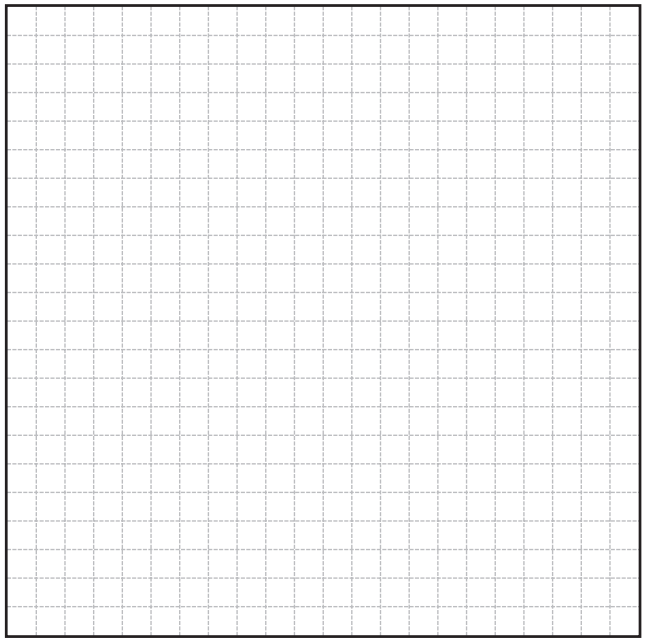


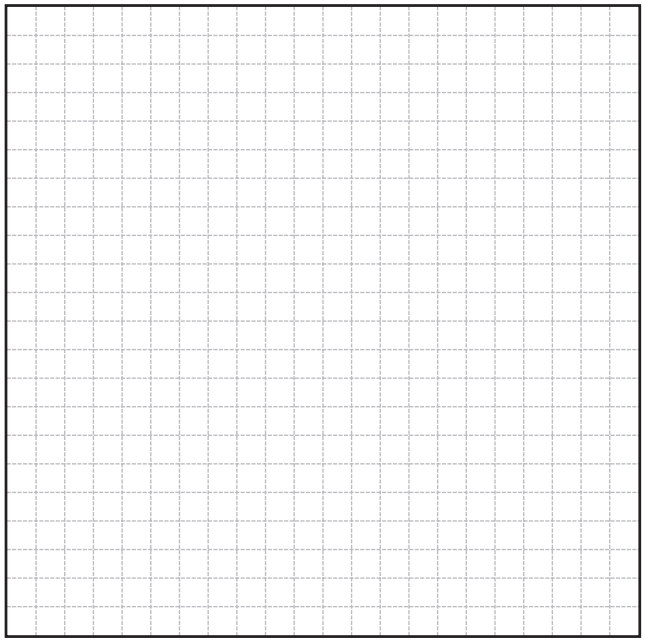


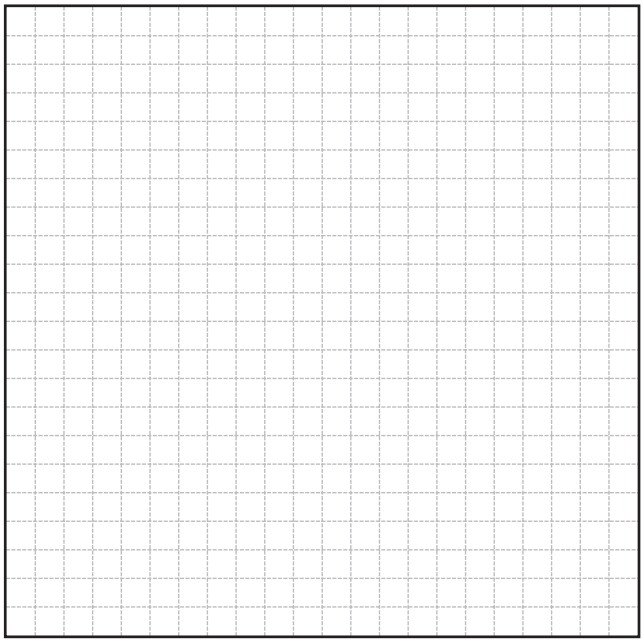


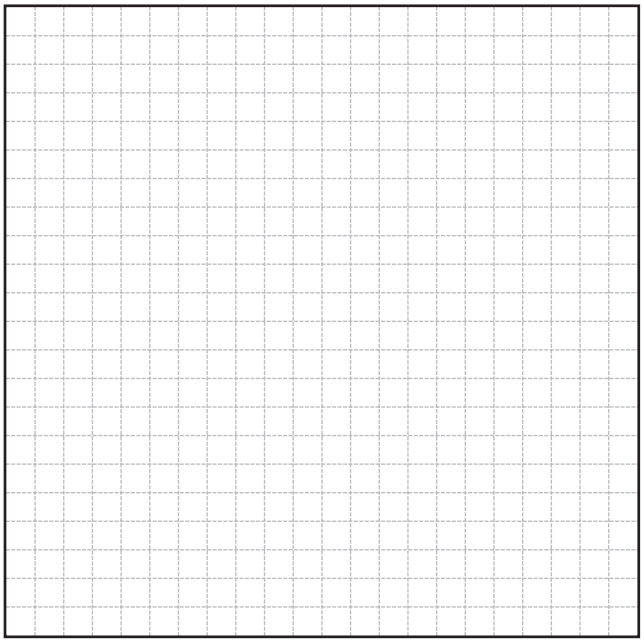


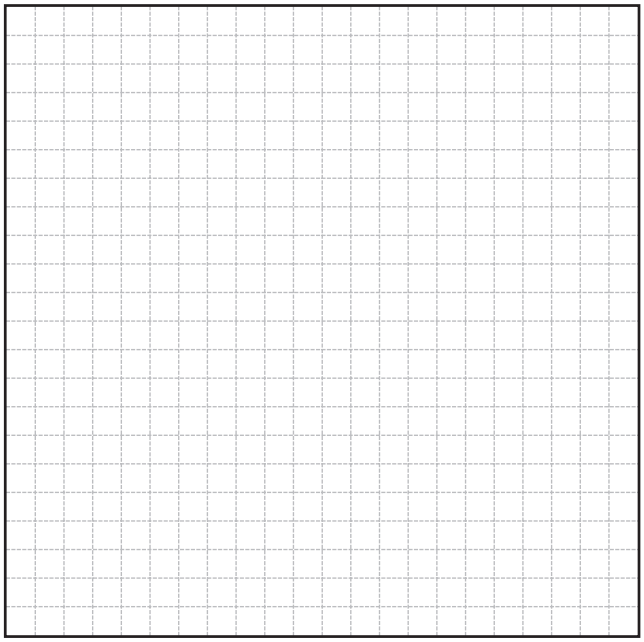


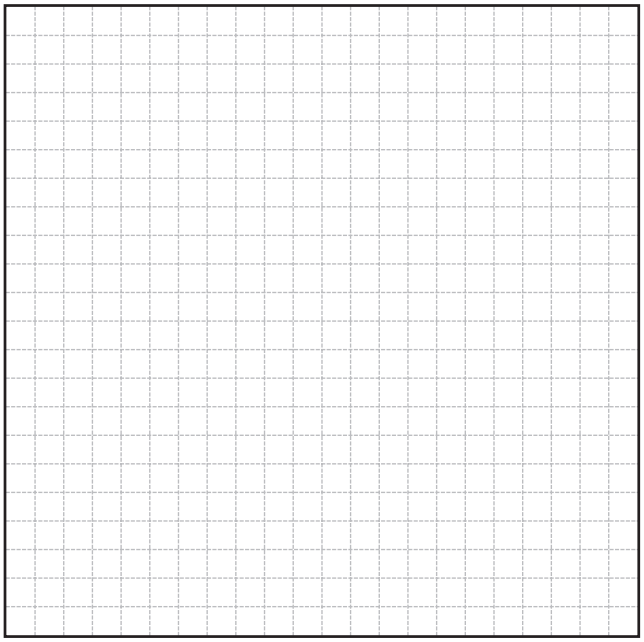












ALGEBRA

Functions	
$f(x)$	Function notation, "f of x"
$f^{-1}(x)$	Inverse function notation
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f+g)(x) = f(x) + g(x)$	Addition
$(f-g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division
$\frac{f(b)-f(a)}{b-a}$	Average rate of change
$f(-x) = -f(x)$	Odd function
$f(-x) = f(x)$	Even function
$f(x) = \lfloor x \rfloor$	Floor/greatest integer function
$f(x) = \lceil x \rceil$	Ceiling/least integer function
$f(x) = a\sqrt[3]{(x-h)} + k$	Cube root function
$f(x) = \sqrt[n]{(x-h)} + k$	Radical function
$f(x) = a\lvert x-h \rvert + k$	Absolute value function
$f(x) = \frac{p(x)}{q(x)};\ q(x) \neq 0$	Rational function

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a
∞	Infinity
[	Inclusive on the lower bound
]	Inclusive on the upper bound
(	Non-inclusive on the lower bound
)	Non-inclusive on the upper bound

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Exponential Equations	
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Quadratic Functions and Equations	
$x=\frac{-b}{2a}$	Axis of symmetry
$x=\frac{p+q}{2}$	Axis of symmetry using the midpoint of the x-intercepts
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	Vertex
$f(x) = ax^2 + bx + c$	General form
$f(x) = a(x - h)^2 + k$	Vertex form
$f(x) = a(x - p)(x - q)$	Factored/intercept form
$b^2 - 4ac$	Discriminant
$x^2+bx+\left(\frac{b}{2}\right)^2$	Perfect square trinomial
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula
$(ax)^2-b^2=(ax+b)(ax-b)$	Difference of squares
$(x - h)^2 = 4p(y - k)$	Standard form for a parabola that opens up or down
$(y - k)^2 = 4p(x - h)$	Standard form for a parabola that opens right or left
$F(h, k + p)$	Focus for a parabola that opens up or down
$F(h + p, k)$	Focus for a parabola that opens right or left
$y = k - p$	Directrix for a parabola that opens up or down
$x = h - p$	Directrix for a parabola that opens right or left

Exponential Functions	
$1 + r$	Growth factor
$1 - r$	Decay factor
$f(t) = a(1+r)^t$	Exponential growth function
$f(t) = a(1-r)^t$	Exponential decay function
$f(x) = ab^x$	Exponential function in general form

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Equations of Circles	
$(x-h)^2 + (y-k)^2 = r^2$	Standard form
$x^2 + y^2 = r^2$	Center at (0, 0)
$Ax^2 + By^2 + Cx + Dy + E = 0$	General form

Properties of Radicals
$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Imaginary Numbers
$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Radicals to Rational Exponents
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Multiplication of Complex Conjugates
$(a + bi)(a - bi) = a^2 + b^2$

Properties of Exponents	
Property	General rule
Zero Exponent	$a^0 = 1$
Negative Exponent	$b^{-\frac{m}{n}} = \frac{1}{b^{\frac{m}{n}}}$
Product of Powers	$a^m \bullet a^n = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$\left(b^m\right)^n = b^{mn}$
Power of a Product	$(bc)^n = b^n c^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

DATA ANALYSIS

Rules and Equations	
$P(E)=\dfrac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in sample space}}$	Probability of event E
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	Addition rule
$P(\overline{A})=1-P(A)$	Complement rule
$P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$	Conditional probability
$P(A\cap B)=P(A)\bullet P(B\mid A)$	Multiplication rule
$P(A\cap B)=P(A)\bullet P(B)$	Multiplication rule if A and B are independent
${}_nC_r=\dfrac{n!}{(n-r)!r!}$	Combination
${}_nP_r=\dfrac{n!}{(n-r)!}$	Permutation
$n!=n\bullet(n-1)\bullet(n-2)\bullet\ldots\bullet 1$	Factorial

Symbols	
$\varnothing$	Empty/null set
$\cap$	Intersection, “and”
$\cup$	Union, “or”
$\subset$	Subset
$\overline{A}$	Complement of Set A
!	Factorial
${}_nC_r$	Combination
${}_nP_r$	Permutation

GEOMETRY

Symbols	
$\overset{\frown}{ABC}$	Major arc length
$\overset{\frown}{AB}$	Minor arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
$\square$	Parallelogram
A'	Prime
$^\circ$	Degrees
θ	Theta
ϕ	Phi
π	Pi

Area	
$A = lw$	Rectangle
$A = \frac{1}{2}bh$	Triangle
$A = \pi r^2$	Circle
$A = \frac{1}{2}(b_1 + b_2)h$	Trapezoid

Trigonometric Ratios		
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Identities
$\sin\theta = \cos(90^\circ - \theta)$
$\cos\theta = \sin(90^\circ - \theta)$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sin^2\theta + \cos^2\theta = 1$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Volume	
$V = lwh$	Rectangular prism
$V = Bh$	Prism
$V = \frac{1}{3}\pi r^2$	Cone
$V = \frac{1}{3}Bh$	Pyramid
$V = \pi r^2 h$	Cylinder
$V = \frac{4}{3}\pi r^3$	Sphere

Distance Formula
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Dilation
$D_k(x, y) = (kx, ky)$

Pi Defined
$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{circumference}}{2 \bullet \text{radius}}$

Circumference of a Circle	
$C = 2\pi r$	Circumference given the radius
$C = \pi d$	Circumference given the diameter

Converting Between Degrees and Radians
$\frac{\text{radian measure}}{\pi} = \frac{\text{degree measure}}{180}$

Midpoint Formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Inverse Trigonometric Functions
Arcsin $\theta = \sin^{-1}\theta$
Arccos $\theta = \cos^{-1}\theta$
Arctan $\theta = \tan^{-1}\theta$

Arc Length	
$s = \theta r$	Arc length (θ in radians)

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English		Español
	A	
absolute value a number's distance from 0 on a number line; the positive value of a quantity	U2-153	**valor absoluto** distancia de un número a partir del 0 en una recta numérica; valor positivo de una cantidad
absolute value function a function with a variable inside an absolute value	U2-153	**función de valor absoluto** función con una variable dentro de un valor absoluto
acute triangle a triangle in which all of the angles are acute (less than 90°)	U5-294	**triángulo agudo** triángulo en el que todos los ángulos son agudos (menos de 90°)
Addition Rule If A and B are any two events, then the probability of A or B, denoted $P(A \text{ or } B)$, is given by: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	U4-3	**Regla de la suma** Si A y B son dos eventos cualquiera, entonces la probabilidad de A o B, que se indica con $P(A \text{ o } B)$, está dada por: $P(A \text{ o } B) = P(A) + P(B) - P(A \text{ y } B)$. Con el uso de notación de conjuntos, la regla es $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
adjacent angles angles that lie in the same plane and share a vertex and a common side. They have no common interior points.	U5-223	**ángulos adyacentes** ángulos en el mismo plano que comparten un vértice y un lado común. No tienen puntos interiores comunes.
adjacent side the leg next to an acute angle in a right triangle that is not the hypotenuse	U5-493	**lado adyacente** el cateto junto a un ángulo agudo en un triángulo rectángulo que no es la hipotenusa
alternate exterior angles angles that are on opposite sides of the transversal and lie on the exterior of the two lines that the transversal intersects	U5-223	**ángulos exteriores alternos** ángulos en lados opuestos de la transversal que se sitúan en el exterior de las dos líneas que corta la transversal
alternate interior angles angles that are on opposite sides of the transversal and lie within the interior of the two lines that the transversal intersects	U5-223	**ángulos interiores alternos** ángulos que están en los lados opuestos de la transversal y se ubican en el interior de las dos líneas que corta la transversal
altitude the perpendicular line from a vertex of a figure to its opposite side; height	U5-130 U5-547	**altitud** línea perpendicular desde el vértice de una figura hasta su lado opuesto; altura

English		Español
Angle-Angle (AA) Similarity Statement If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	U5-80	**Criterio de semejanza ángulo-ángulo (AA)** Si dos ángulos de un triángulo son congruentes con dos ángulos de otro triángulo, entonces los triángulos son similares.
angle bisector a ray that divides an angle into two congruent angles	U5-130 U6-69	**bisectriz del ángulo** semirrecta que divide un ángulo en dos ángulos congruentes
angle of depression the angle created by a horizontal line and a downward line of sight to an object that is below the observer	U5-547	**ángulo de depresión** ángulo creado por una línea horizontal y una línea de mira descendente en relación a un objeto que se encuentra por debajo del observador
angle of elevation the angle created by a horizontal line and an upward line of sight to an object that is above the observer	U5-547	**ángulo de elevación** ángulo creado por una línea horizontal y una línea de mira ascendente en relación a un objeto que se encuentra por encima del observador
arc part of a circle's circumference	U6-3	**arco** parte de la circunferencia de un círculo
arc length the distance between the endpoints of an arc; written as $m\overset{\frown}{AB}$	U6-167	**longitud de arco** distancia entre los extremos de un arco; se expresa como $m\overset{\frown}{AB}$
arccosine the inverse of the cosine function, written $\cos^{-1}\theta$ or $\arccos\theta$	U5-547	**arcocoseno** inversa de la función coseno; se expresa $\cos^{-1}\theta$ o $\arccos\theta$
Archimedes a Greek mathematician, physician, engineer, and inventor who lived from 287–212 B.C.; considered to be one of the greatest mathematicians of all time	U6-197	**Arquímedes** fue un matemático, físico, ingeniero e inventor griego que vivió entre 287 y 212 A.C.; se lo considera uno de los matemáticos más importantes de todos los tiempos
arcsine the inverse of the sine function, written $\sin^{-1}\theta$ or $\arcsin\theta$	U5-547	**arcoseno** inversa de la función seno; se expresa $\text{sen}^{-1}\theta$ o $\arcsen\theta$
arctangent the inverse of the tangent function, written $\tan^{-1}\theta$ or $\arctan\theta$	U5-547	**arcotangente** inversa de la función tangente; se expresa $\tan^{-1}\theta$ o $\arctan\theta$
asymptote a line that a function gets closer and closer to, but never crosses or touches	U3-243	**asíntota** línea a la que se acerca cada vez más una función sin cruzarla ni tocarla

English		Español
average rate of change the ratio of the difference of output values to the difference of the corresponding input values: $\frac{f(b)-f(a)}{b-a}$; a measure of how a quantity changes over some interval	U2-53	**tasa de cambio promedio** proporción de la diferencia de valores de salida a la diferencia de valores correspondientes de entrada: $\frac{f(b)-f(a)}{b-a}$; medida de cuánto cambia una cantidad en cierto intervalo
axis of symmetry of a parabola the line through the vertex of a parabola about which the parabola is symmetric. The equation of the axis of symmetry is $x=\frac{-b}{2a}$.	U2-2 U3-108 U6-310	**eje de simetría de una parábola** línea que atraviesa el vértice de una parábola sobre la que la parábola es simétrica. La ecuación del eje de simetría es $x=\frac{-b}{2a}$.
B		
base the quantity that is being raised to a power in an exponential expression; in a^x, a is the base. Also, the side that is opposite the vertex angle of an isosceles triangle.	U1-2 U5-294	**base** cantidad elevada a una potencia en una expresión exponencial; en a^x, a es la base. También, el lado que es opuesto al ángulo vértice de un triángulo isósceles.
base angle an angle formed by the base and one congruent side of an isosceles triangle	U5-294	**ángulo base** ángulo formado por la base y un lado congruente de un triángulo isósceles
binomial a polynomial with two terms	U3-2	**binomio** polinomio con dos términos
bisect to cut in half	U6-197	**bisecar** cortar por la mitad
C		
Cavalieri's Principle The volumes of two objects are equal if the areas of their corresponding cross sections are in all cases equal.	U6-197	**Principio de Cavalieri** Los volúmenes de dos objetos son iguales si las superficies de sus correspondientes secciones transversales son en todos los casos iguales.

English		Español
ceiling function also known as the least integer function; a function represented as $y=\lceil x \rceil$. For any input x, the output is the smallest integer greater than or equal to x; for example, $\lceil -3 \rceil = -3$, $\lceil 2.1 \rceil = 3$, and $\lceil -2.1 \rceil = -2$.	U2-153	**función techo** también conocida como función del mínimo entero; función representada como $y=\lceil x \rceil$. Para cualquier entrada x, la salida es el entero más pequeño mayor que o igual a x; por ejemplo, $\lceil -3 \rceil = -3$, $\lceil 2.1 \rceil = 3$, y $\lceil -2.1 \rceil = -2$.
center of a circle the point in the plane of the circle from which all points on the circle are equidistant. The center is not part of the circle; it is in the interior of the circle.	U6-249	**centro de un círculo** punto en el plano del círculo desde el cual son equidistantes todos los puntos del círculo. El centro no es parte del círculo: se encuentra en el interior del círculo.
center of dilation a point through which a dilation takes place; all the points of a dilated figure are stretched or compressed through this point	U5-31	**centro de dilatación** punto a través del cual se produce una dilatación; todos los puntos de una figura dilatada se alargan o comprimen a través de este punto
central angle an angle with its vertex at the center of a circle	U6-3 U6-167	**ángulo central** ángulo con su vértice en el centro de un círculo
centroid the intersection of the medians of a triangle	U5-294	**centroide** intersección de las medianas de un triángulo
chord a segment whose endpoints lie on the circumference of the circle	U6-3	**cuerda** segmento cuyos extremos se ubican en la circunferencia del círculo
circle the set of all points in a plane that are equidistant from a reference point in that plane, called the center. The set of points forms a two-dimensional curve that measures 360º.	U3-380 U6-3 U6-249 U6-310	**círculo** conjunto de todos los puntos de un plano equidistantes desde un punto de referencia en ese plano, denominado centro. El conjunto de puntos forma una curva bidimensional que mide 360º.
circumcenter the intersection of the perpendicular bisectors of a triangle	U5-294 U6-69	**circuncentro** intersección de las bisectrices perpendiculares de un triángulo
circumference the distance around a circle; $C = 2\pi r$ or $C = \pi d$, for which C represents circumference, r represents the circle's radius, and d represents the circle's diameter.	U6-3 U6-167	**circunferencia** distancia alrededor de un círculo; $C = 2\pi r$ o $C = \pi d$, en donde C representa la circunferencia, r representa el radio del círculo y d, su diámetro.

English		Español
circumscribed angle the angle formed by two tangent lines whose vertex is outside of the circle	U6-3	**ángulo circunscrito** ángulo formado por dos líneas tangentes cuyo vértice está fuera del círculo
circumscribed circle a circle that contains all vertices of a polygon	U5-294 U6-69	**círculo circunscrito** círculo que contiene todos los vértices de un polígono
circumscribed triangle triangle whose sides are tangent to an interior circle	U6-69	**triángulo circunscrito** triángulo cuyos lados son tangentes a un círculo interior
closed interval an interval that includes its endpoints	U3-243	**intervalo cerrado** intervalo que incluye sus extremos
closure a system is closed, or shows closure, under an operation if the result of the operation is within the system	U1-34	**cierre** un sistema es cerrado, o tiene cierre, en una operación si el resultado de la misma está dentro del sistema
coefficient the number multiplied by a variable in an algebraic expression	U3-2	**coeficiente** número multiplicado por una variable en una expresión algebraica
cofunction a trigonometric function whose ratios have the same values when applied to the two acute angles in the same right triangle. The sine of one acute angle is the cofunction of the cosine of the other acute angle.	U5-493	**cofunción** función trigonométrica cuyas proporciones tienen los mismos valores cuando se aplican a los dos ángulos agudos en el mismo triángulo rectángulo. El seno de un ángulo agudo es la cofunción del coseno del otro ángulo agudo.
collinear points points that lie on the same line	U5-31	**puntos colineales** puntos que se ubican en la misma línea
combination a subset of a group of objects taken from a larger group of objects; the order of the objects does not matter, and objects may be repeated. A combination of size r from a group of n objects can be represented using the notation ${}_nC_r$, where ${}_nC_r = \frac{n!}{(n-r)!r!}$.	U4-153	**combinación** subconjunto de un grupo de objetos tomado de un grupo de objetos más grande; el orden de los objetos no importa y los objetos pueden repetirse. Una combinación de tamaño r de un grupo de n objetos puede representarse con la notación ${}_nC_r$, donde ${}_nC_r = \frac{n!}{(n-r)!r!}$.

English		Español
common external tangent a tangent that is common to two circles and does not intersect the segment joining the radii of the circles	U6-134	**tangente común externa** tangente común a dos círculos que no corta el segmento que une los radios de los círculos
common internal tangent a tangent that is common to two circles and intersects the segment joining the radii of the circles	U6-134	**tangente común interna** tangente común a dos círculos que corta el segmento que une los radios de los círculos
common tangent a line tangent to two circles	U6-134	**tangente común** recta tangente a dos círculos
complement a set whose elements are not in another set, but are in some universal set being considered. The complement of set A, denoted by $\overline{A}$, is the set of elements that are in the universal set, but not in A. The event does not occur. The probability of an event not occurring is 1 minus the probability of the event occurring, $P\left(\overline{A}\right)=1-P(A)$.	U4-3	**complemento** conjunto cuyos elementos no se encuentran en otro conjunto, pero están en algún conjunto universal que se considera. El complemento del conjunto A, que se indica con $\overline{A}$, es el conjunto de elementos que se encuentran en el conjunto universal, pero no en A. El evento no se produce. La probabilidad de que un evento no se produzca es 1 menos la probabilidad de que se produzca, $P\left(\overline{A}\right)=1-P(A)$.
complementary angles two angles whose sum is 90°	U5-223 U5-493	**ángulos complementarios** dos ángulos cuya suma es 90°
complex conjugate the complex number that when multiplied by another complex number produces a value that is wholly real; the complex conjugate of $a + bi$ is $a - bi$	U1-65	**conjugado de número complejo** número complejo que cuando se multiplica por otro número complejo produce un valor totalmente real; el conjugado complejo de $a + bi$ es $a - bi$
complex conjugates two complex numbers of the form $a + bi$ and $a - bi$	U3-188	**conjugados de números complejos** dos números complejos de la forma $a + bi$ y $a - bi$
complex number a number in the form $a + bi$, where a and b are real numbers, and i is the imaginary unit	U1-65 U3-188	**número complejo** número en la forma $a + bi$, donde a y b son números reales e i es la unidad imaginaria
complex number system all numbers of the form $a + bi$, where a and b are real numbers, including complex numbers (neither a nor b equal 0), real numbers ($b = 0$), and imaginary numbers ($a = 0$)	U1-65	**sistema de números complejos** todos los números de la forma $a + bi$, donde a y b son números reales, incluidos los números complejos (ni a ni b son iguales a 0), reales ($b = 0$) e imaginarios ($a = 0$)

English		Español
compound event the combination of two or more simple events	U4-77	**evento compuesto** combinación de dos o más eventos simples
compound interest interest earned on both the initial amount and on previously earned interest	U3-349	**interés compuesto** interés devengado tanto de la cantidad inicial como del interés previamente devengado
compound probability the probability of compound events	U4-77	**probabilidad compuesta** probabilidad de eventos compuestos
compression a transformation in which a figure becomes smaller; compressions may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	U5-31	**compresión** transformación en la que una figura se hace más pequeña; las compresiones pueden ser horizontales (cuando afectan sólo la longitud horizontal), verticales (cuando afectan sólo la longitud vertical), o en ambos sentidos
concave down a graph of a curve that is bent downward, such as a quadratic function with a maximum value	U2-53	**cóncavo hacia abajo** gráfico de una curva que se inclina hacia abajo, tal como una función cuadrática con un valor máximo
concave polygon a polygon with at least one interior angle greater than 180° and at least one diagonal that does not lie entirely inside the polygon	U5-424	**polígono cóncavo** polígono con al menos un ángulo interior de más de 180° y con al menos una diagonal que no se ubica por completo dentro de él
concave up a graph of a curve that is bent upward, such as a quadratic function with a minimum value	U2-53	**cóncavo hacia arriba** gráfico de una curva que se inclina hacia arriba, tal como una función cuadrática con un valor mínimo
concavity with respect to a curve, the property of being arched upward or downward. A quadratic with positive concavity will increase on either side of the vertex, meaning that the vertex is the minimum or lowest point of the curve. A quadratic with negative concavity will decrease on either side of the vertex, meaning that the vertex is the maximum or highest point of the curve.	U2-54 U2-112	**concavidad** con respecto a una curva, la propiedad de ser arqueado hacia arriba o hacia abajo. Una función cuadrática con concavidad positiva se incrementará en ambos lados del vértice, lo que significa que el vértice es el punto mínimo o más bajo de la curva. Una función cuadrática con concavidad negativa disminuirá a cada lado del vértice, lo que significa que el vértice es el punto máximo o más alto de la curva.

English		Español
concentric circles coplanar circles that have the same center	U6-3	**círculos concéntricos** círculos coplanares que tienen el mismo centro
concurrent lines lines that intersect at one point	U5-294	**rectas concurrentes** rectas con intersección en un punto
conditional probability of *B* given *A* the probability that event *B* occurs, given that event *A* has already occurred. If *A* and *B* are two events from a sample space with $P(A) \neq 0$, then the conditional probability of *B* given *A*, denoted $P(B\|A)$, has two equivalent expressions: $P(B\|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\text{number of outcomes in } (A \text{ and } B)}{\text{number of outcomes in } A}$.	U4-77	**probabilidad condicional de *B* dado *A*** la probabilidad de que el evento *B* se produzca, dado que el evento *A* ya se ha producido. Si *A* y *B* son dos eventos de un espacio muestral con $P(A) \neq 0$, entonces la probabilidad condicional de *B* dado *A*, indicado $P(B\|A)$ tiene dos expresiones equivalentes: $P(B\|A) = \frac{P(A \text{ y } B)}{P(A)} = \frac{\text{numero de resultados en } (A \text{ y } B)}{\text{numero de resultados en } A}$.
cone a solid or hollow object that tapers from a circular or oval base to a point	U6-197	**cono** objeto sólido o hueco que se estrecha desde una base circular u ovalada hasta un punto
congruency transformation a transformation in which a geometric figure moves but keeps the same size and shape; a dilation where the scale factor is equal to 1	U5-31	**transformación de congruencia** transformación en la cual una figura geométrica se mueve pero mantiene el mismo tamaño y la misma forma; dilatación en la que el factor de escala es igual a 1
congruent arcs two arcs that have the same measure and are either of the same circle or of congruent circles	U6-3	**arcos congruentes** dos arcos que tienen la misma medida y son parte del mismo círculo o de círculos congruentes
consecutive angles angles that lie on the same side of a figure	U5-424	**ángulos consecutivos** ángulos ubicados en el mismo lado de una figura
constant term a term whose value does not change	U3-2	**término constante** término cuyo valor no cambia

English		Español
converse of the Pythagorean Theorem If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.	U5-130	**conversa del teorema de Pitágoras** Si la suma de los cuadrados de las medidas de dos lados de un triángulo equivale al cuadrado de la medida del lado más largo, entonces el triángulo es rectángulo.
convex polygon a polygon with no interior angle greater than 180°; all diagonals lie inside the polygon	U5-424	**polígono convexo** polígono sin ángulo interior de más de 180°; todas las diagonales están dentro del polígono
coordinate proof a proof that involves calculations and makes reference to the coordinate plane	U5-294	**prueba de coordenadas** prueba que involucra cálculos y hace referencia al plano de coordenadas
corollary a theorem that accompanies another theorem and is usually easily deduced from the other theorem	U3-188	**corolario** teorema que acompaña a otro teorema y por lo general se deduce con facilidad del primero
Corollary to the Fundamental Theorem of Algebra If $P(x)$ is a polynomial function of degree $n \geq 1$ with complex coefficients, then the related equation $P(x) = 0$ has exactly n complex solutions (roots), if a double solution is counted as two separate solutions.	U3-188	**Corolario del teorema fundamental del álgebra** Si $P(x)$ es una función polinómica de grado $n \geq 1$ con coeficientes complejos, entonces la ecuación relacionada $P(x) = 0$ tiene exactamente n soluciones complejas (raíces), si una solución doble se cuenta como dos soluciones individuales.
corresponding angles angles in the same relative position with respect to the transversal and the intersecting lines	U5-223	**ángulos correspondientes** ángulos en la misma posición relativa con respecto a las líneas transversal y de intersección
corresponding sides sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.	U5-31	**lados correspondientes** lados de dos figuras que están en la misma posición relativa a la figura. En las transformaciones, los lados correspondientes son los de preimagen e imagen, entonces $\overline{AB}$ y $\overline{A'B'}$ son los lados correspondientes, etc.

English		Español
cosecant the reciprocal of the sine ratio, $\csc\theta = \frac{1}{\sin\theta}$; the cosecant of $\theta = \csc\theta = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$	U5-493 U5-548	**cosecante** razón inversa del seno, $\csc\theta = \frac{1}{\text{sen}\,\theta}$; la cosecante de $\theta = \csc\theta = \frac{\text{longitud de la hipotenusa}}{\text{longitud del lado opuesto}}$
cosine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the side adjacent to the length of the hypotenuse; the cosine of $\theta = \cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$	U5-493	**coseno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud de lado adyacente a la longitud de la hipotenusa; el coseno de $\theta = \cos\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud de la hipotenusa}}$
cotangent the reciprocal of tangent, $\cot\theta = \frac{1}{\tan\theta}$; the cotangent of $\theta = \cot\theta = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$	U5-494 U5-548	**cotangente** recíproco de la tangente, $\cot\theta = \frac{1}{\tan\theta}$; la cotangente de $\theta = \cot\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud del lado opuesto}}$
critical number of a polynomial inequality an x-value that makes $f(x) = 0$, where $f(x)$ is a polynomial function and the inequality is written in any of these forms: $f(x) < 0$, $f(x) \le 0$, $f(x) > 0$, or $f(x) \ge 0$	U3-243	**número crítico de una desigualdad polinómica** valor de x que hace $f(x) = 0$, donde $f(x)$ es una función polinómica y la desigualdad se expresa en cualquiera de estas formas: $f(x) < 0$, $f(x) \le 0$, $f(x) > 0$, o $f(x) \ge 0$
critical number of a rational inequality an x-value that makes $f(x) = 0$ or makes $f(x)$ undefined, where $f(x)$ is a rational function and the inequality is written in any of these forms: $f(x) < 0$, $f(x) \le 0$, $f(x) > 0$, or $f(x) \ge 0$	U3-243	**número crítico de una desigualdad racional** valor de x que hace $f(x) = 0$ o $f(x)$ indefinido, donde $f(x)$ es una función racional y la desigualdad se expresa en cualquiera de estas formas: $f(x) < 0$, $f(x) \le 0$, $f(x) > 0$, o $f(x) \ge 0$

English		Español
cube root For any real numbers a and b, if $a^3 = b$, then a is a cube root of b. The cube root of b is written using a radical: $\sqrt[3]{b}$.	U2-153	**raíz cúbica** para cualquiera de los números reales a y b, si $a^3 = b$, entonces a es la raíz cúbica de b. La raíz cúbica de b se escribe con un radical: $\sqrt[3]{b}$.
cube root function a function that contains the cube root of a variable. The general form is $y = a\sqrt[3]{(x-h)} + k$, where a, h, and k are real numbers.	U2-153	**función raíz cúbica** función que contiene la raíz cúbica de una variable. La forma general es $y = a\sqrt[3]{(x-h)} + k$, donde a, h, y k son números reales.
curve the graphical representation of the solution set for $y = f(x)$. In the special case of a linear equation, the curve will be a line.	U2-112	**curva** representación gráfica del conjunto de soluciones para $y = f(x)$. En el caso especial de una ecuación lineal, la curva será una recta.
cylinder a solid or hollow object that has two parallel bases connected by a curved surface; the bases are usually circular	U6-197	**cilindro** objeto sólido o hueco que tiene dos bases paralelas conectadas por medio de una superficie curva; las bases por lo general son circulares
	D	
decay factor $1 - r$ in the exponential decay model $f(t) = a(1 - r)^t$, or b in the exponential function $f(t) = ab^t$ if $0 < b < 1$; the multiple by which a quantity decreases over time. The general form of an exponential function modeling decay is $f(t) = a(1 - r)^t$.	U2-252 U3-349	**factor de decaimiento** $1 - r$ en el modelo de decaimiento exponencial $f(t) = a(1 - r)^t$, o b en la función exponencial $f(t) = ab^t$ si $0 < b < 1$; el múltiplo por el que una cantidad disminuye con el tiempo. La forma general de una función exponencial que determina decaimiento es $f(t) = a(1 - r)^t$.
decay rate r in the exponential decay model $f(t) = a(1 - r)^t$	U2-252 U3-349	**tasa de decaimiento** r en el modelo de decaimiento exponencial $f(t) = a(1 - r)^t$
decreasing the interval of a function for which the output values are becoming smaller as the input values are becoming larger	U2-54	**decreciente** intervalo de una función por el que los valores de salida se hacen más pequeños a medida que los valores de entrada se hacen más grandes
decreasing function a function such that as the independent values increase, the dependent values decrease	U2-153	**función decreciente** función en la que a medida que aumentan los valores independientes, disminuyen los dependientes

English		Español
degree of a one-variable polynomial the greatest exponent attached to the variable in the polynomial	U3-188	**grado de un polinomio de una variable** el mayor exponente anexado a la variable en el polinomio
dependent events events that are not independent. The outcome of one event affects the probability of the outcome of another event.	U4-3 U4-77	**eventos dependientes** eventos que no son independientes. El resultado de un evento afecta la probabilidad del resultado de otro.
dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function	U3-243	**variable dependiente** designada en el eje de *y*; cantidad que se basa en los valores de entrada de la variable independiente; variable de salida de una función
diagonal a line that connects nonconsecutive vertices	U5-424	**diagonal** línea que conecta vértices no consecutivos
diameter a straight line passing through the center of a circle connecting two points on the circle; equal to twice the radius	U6-3	**diámetro** línea recta que atraviesa el centro de un círculo y conecta dos puntos en él; equivale a dos veces del radio
dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point	U5-31	**dilatación** transformación en la que una figura se amplía o se reduce por un factor de escala en relación con un punto central
directrix of a parabola a line that is perpendicular to the axis of symmetry of a parabola and that is in the same plane as both the parabola and the focus of the parabola; the fixed line referenced in the definition of a parabola	U6-249 U6-310	**directriz de una parábola** línea perpendicular al eje de simetría de una parábola que está en el mismo plano tanto de la parábola como de su foco; línea fija mencionada en la definición de parábola
discriminant an expression whose solved value indicates the number and types of solutions for a quadratic. For a quadratic equation in standard form ($ax^2 + bx + c = 0$), the discriminant is $b^2 - 4ac$.	U3-33	**discriminante** expresión cuyo valor resuelto indica la cantidad y los tipos de soluciones para una ecuación cuadrática. En una ecuación cuadrática en forma estándar ($ax^2 + bx + c = 0$), el discriminante es $b^2 - 4ac$.

English		Español
disjoint events events that have no outcomes in common. If A and B are disjoint events, then they cannot both occur. Disjoint events are also called mutually exclusive events.	U4-3	**eventos disjuntos** eventos que no tienen resultados en común. Si A y B son eventos disjuntos, entonces no pueden producirse ambos. También se denominan eventos mutuamente excluyentes.
dissection breaking a figure down into its components	U6-198	**disección** desglose de una figura en sus componentes
distance formula a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	U5-2 U6-249 U6-310	**fórmula de distancia** fórmula que señala la distancia entre puntos (x_1, y_1) y (x_2, y_2) es igual a $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
dodecagon a 12-sided polygon	U6-198	**dodecágono** polígono de 12 lados
domain the set of all input values (x-values) that satisfy the given function without restriction	U2-54 U2-153 U3-243	**dominio** conjunto de todos los valores de entrada (valores de x) que satisfacen la función dada sin restricciones
double root two roots that are equal	U3-188	**raíz doble** dos raíces que son iguales
double solution two solutions that are equal	U3-188	**solución doble** dos soluciones que son iguales
E		
element an item in a set; also called a member	U4-4	**elemento** ítem en un conjunto; también se denomina miembro
empty set a set that has no elements, denoted by $\varnothing$. The empty set is also called the null set.	U4-4	**conjunto vacío** conjunto que no contiene elementos, indicado con $\varnothing$. También se denomina conjunto nulo.
end behavior the behavior of the graph as x approaches positive infinity and as x approaches negative infinity	U2-54 U3-243	**comportamiento final** el comportamiento de la gráfica al aproximarse x a infinito positivo o a infinito negativo
enlargement a dilation of a figure where the scale factor is greater than 1	U5-32	**ampliación** dilatación de una figura en la que el factor de escala es mayor que 1
equal sets sets with all the same elements	U4-4	**conjuntos iguales** conjuntos con todos los mismos elementos
equiangular having equal angles	U5-294	**equiangular** que tiene ángulos iguales

English		Español
equidistant a point or points that lie the same distance away from a given object	U5-223 U6-69	**equidistante** punto o puntos que están a la misma distancia de un determinado objeto
equilateral triangle a triangle with all three sides equal in length	U5-295	**triángulo equilátero** triángulo que tiene los tres lados de la misma longitud
even function a function that, when evaluated for $-x$, results in a function that is the same as the original function; $f(-x) = f(x)$	U2-54	**función par** función que, cuando se la evalúa para $-x$, tiene como resultado una función que es igual a la original; $f(-x) = f(x)$
event an outcome or set of outcomes of an experiment. An event is a subset of the sample space.	U4-4	**evento** resultado o conjunto de resultados de un experimento. Un evento es un subconjunto del espacio de muestral.
expected value an estimate of value that is determined by finding the product of a total value and a probability of a given event	U4-196	**valor esperado** estimación de valor que se determina al encontrar el producto de un valor total y una probabilidad de un evento determinado
experiment a process or action that has observable results. The results are called outcomes.	U4-4	**experimento** proceso o acción con consecuencias observables. Las consecuencias se denominan resultados.
exponent the quantity that shows the number of times the base is being multiplied by itself in an exponential expression; also known as the power. In a^x, x is the power/exponent.	U1-2	**exponente** cantidad que muestra el número de veces que la base se multiplica por sí misma en una expresión exponencial; también se denomina potencia. En a^x, x es la potencia o exponente.
exponential decay an exponential equation with a base, b, that is between 0 and 1 ($0 < b < 1$); can be represented by the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the decay rate, t is time, and y is the final value	U2-252 U3-349	**decaimiento exponencial** ecuación exponencial con una base, b, que está entre 0 y 1 ($0 < b < 1$); puede representarse con la fórmula $y = a(1 - r)^t$, en la que a es el valor inicial, $(1 - r)$ es la tasa de decaimiento, t es el tiempo y y es el valor final
exponential decay model an exponential function, $f(t) = a(1 - r)^t$, where $f(t)$ is the final output value at the end of t time periods, a is the initial value, r is the percent decrease per time period (expressed as a decimal), and t is the number of time periods	U2-253 U3-349	**modelo de decaimiento exponencial** función exponencial, $f(t) = a(1 - r)^t$, en la que $f(t)$ es el valor de salida final despues de t períodos de tiempo, a es el valor inicial, r es el porcentaje de disminución por período (expresado como decimal), y t es la cantidad de períodos

English		Español
exponential equation an equation of the form $y = ab^x$, where x is the independent variable, y is the dependent variable, and a and b are real numbers	U1-2	**ecuación exponencial** ecuación de la forma $y = ab^x$, en la que x es la variable independiente, y es la variable dependiente, y a y b son números reales
exponential expression an expression that contains a base and a power/exponent	U1-2 U3-349	**expresión exponencial** expresión que incluye una base y una potencia o exponente
exponential function a function with the general form $f(t) = ab^t$, where a is the initial value, b is the growth or decay factor, t is the time, and $f(t)$ is the final output value	U2-253 U3-349	**función exponencial** función con la forma general $f(t) = ab^t$, en la que a es el valor inicial, b es el factor de crecimiento o decaimiento, t es el tiempo, y $f(t)$ es el valor de salida final
exponential growth an exponential function with a base, b, greater than 1 ($b > 1$); can be represented by the formula $f(t) = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the growth rate, t is time, and $f(t)$ is the final value	U2-253 U3-350	**crecimiento exponencial** función exponencial con una base, b, mayor que 1 ($b > 1$); puede representarse la fórmula $f(t) = a(1 + r)^t$, en la que a es el valor inicial, $(1 + r)$ es la tasa de crecimiento, t es el tiempo, y $f(t)$ es el valor final
exponential growth model an exponential function, $f(t) = a(1 + r)^t$, where $f(t)$ is the final output value at the end of t time periods, a is the initial value, r is the percent increase per time period (expressed as a whole number or decimal), and t is the number of time periods	U2-253 U3-350	**modelo de crecimiento exponencial** función exponencial, $f(t) = a(1 - r)^t$, en la que $f(t)$ es el valor de salida final despues de t períodos de tiempo, a es el valor inicial, r es el porcentaje de aumento por período (expresado como entero o decimal), y t es la cantidad de períodos
exterior angle of a polygon an angle formed by one side of a polygon and the extension of another side	U5-295	**ángulo exterior de un polígono** ángulo formado por un lado de un polígono y la extensión de otro lado
exterior angles angles that lie outside a pair of parallel lines	U5-223	**ángulos exteriores** ángulos que están fuera de un par de líneas paralelas
extraneous solution (extraneous root) of an equation a solution of an equation that arises during the solving process, but which is not a solution of the original equation	U3-244	**solución extraña (raíz extraña) de una ecuación** solución de una ecuación que surge durante el proceso de resolución pero que no es una solución de la ecuación original

English		Español
extrema the minima or maxima of a function	U2-2 U2-54 U2-154	**extremos** los mínimos o máximos de una función
	F	
factor (noun) one of two or more numbers or expressions that when multiplied produce a given product	U3-2	**factor** uno de dos o más números o expresiones que al multiplicarse dan un producto determinado
factor (verb) to write an expression as the product of its factors	U3-33	**factorizar** escribir una expresión como el producto de sus factores
factored form of a quadratic function the intercept form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function; also known as intercept form of a quadratic function	U2-2	**forma factorizada de una función cuadrática** forma de intercepto de una ecuación cuadrática, se expresa como $f(x) = a(x - p)(x - q)$, en la que p y q son los interceptos de x de la función; también se conoce como forma de intercepto de una función cuadrática
factorial the product of an integer and all preceding positive integers, represented using a ! symbol; $n! = n \bullet (n - 1) \bullet (n - 2) \bullet \ldots \bullet 1$. For example, $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$. By definition, $0! = 1$.	U4-153	**factorial** producto de un entero y todos los enteros positivos anteriores, que se representa con el símbolo !; $n! = n \bullet (n - 1) \bullet (n - 2) \bullet \ldots \bullet 1$. Por ejemplo, $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$. Por definición, $0! = 1$.
family of functions a set of functions whose graphs have the same general shape as their parent function. The parent function is the function with a simple algebraic rule that represents the family of functions.	U3-244	**familia de funciones** conjunto de funciones cuyos gráficos tienen la misma forma general que su función principal. La función principal es la función con una regla algebraica simple que representa la familia de funciones.
first difference in a set of data, the change in the y-value when the x-value is increased by 1	U2-253	**primera diferencia** en un conjunto de datos, el cambio en el valor y cuando el valor x aumenta por 1

English		Español
floor function also known as the greatest integer function; a function represented as $y = \lfloor x \rfloor$. For any input x, the output is the largest integer less than or equal to x; for example, $\lfloor -3 \rfloor = -3$, $\lfloor 2.1 \rfloor = 2$, and $\lfloor -2.1 \rfloor = -3$.	U2-154	**función piso** también conocida como la función del mayor entero; función representada como $y = \lfloor x \rfloor$. Para cualquier entrada x, la salida es el entero más grande que es menor que o igual a x; por ejemplo, $\lfloor -3 \rfloor = -3$, $\lfloor 2.1 \rfloor = 2$, y $\lfloor -2.1 \rfloor = -3$.
flow proof a graphical method of presenting the logical steps used to show an argument. In a flow proof, the logical statements are written in boxes and the reason for each statement is written below the box.	U5-130	**prueba de flujo** método gráfico para presentar los pasos lógicos utilizados para mostrar un argumento. En una prueba de flujo, las declaraciones lógicas se expresan en casillas y la razón de cada declaración se escribe debajo de la casilla.
focus of a parabola a fixed point on the interior of a parabola that is not on the directrix of the parabola but is on the same plane as both the parabola and the directrix; the fixed point referenced in the definition of a parabola	U6-249 U6-311	**foco de una parábola** punto fijo en el interior de una parábola que no está en la directriz de la parábola sino en el mismo plano que la parábola y la directriz; punto fijo mencionado en la definición de parábola
function a relation in which every element of the domain is paired with exactly one element of the range; that is, for every value of x, there is exactly one value of y.	U2-112 U2-346	**función** relación en la que cada elemento del dominio se empareja con un único elemento del rango; es decir, para cada valor de x, existe exactamente un valor de y.
function notation the use of $f(x)$, which means "function of x," instead of y or another dependent variable in an equation of a function; $f(x) = 2x + 1$ and $y = 2x + 1$ are equivalent functions	U2-346	**notación de funciones** el uso de $f(x)$, que significa "función de x", en lugar de y u otra variable dependiente en la ecuación de una función; $f(x) = 2x + 1$ e $y = 2x + 1$ son funciones equivalentes
Fundamental Theorem of Algebra If $P(x)$ is a polynomial function of degree $n \geq 1$ with complex coefficients, then the related equation $P(x) = 0$ has at least one complex solution (root).	U3-189	**Teorema fundamental del álgebra** Si $P(x)$ es una función polinómica de grado $n \geq 1$ con coeficientes complejos, entonces la ecuación relacionada $P(x) = 0$ tiene al menos una solución compleja (raíz).

English		Español
	G	
general form of an equation of a circle $Ax^2 + By^2 + Cx + Dy + E = 0$, where $A = B$, $A \neq 0$, and $B \neq 0$	U6-249	**forma general de ecuación de un círculo** $Ax^2 + By^2 + Cx + Dy + E = 0$, en la que $A = B$, $A \neq 0$, y $B \neq 0$
greatest common factor (GCF) the largest factor that two or more terms share	U3-34	**máximo común divisor (GCF)** el factor más grande que comparten dos o más términos
greatest integer function also known as the floor function; a function represented as $y = \lfloor x \rfloor$. For any input x, the output is the largest integer less than or equal to x; for example, $\lfloor -3 \rfloor = -3$, $\lfloor 2.1 \rfloor = 2$, and $\lfloor -2.1 \rfloor = -3$.	U2-154	**función del mayor entero** también conocida como función piso; función que se representa como $y = \lfloor x \rfloor$. Para cualquier entrada x, la salida es el entero más grande que es menor que o igual a x; por ejemplo, $\lfloor -3 \rfloor = -3$, $\lfloor 2.1 \rfloor = 2$, y $\lfloor -2.1 \rfloor = -3$.
growth factor the multiple by which a quantity increases over time	U2-253 U3-350	**factor de crecimiento** múltiplo por el que una cantidad aumenta con el tiempo
growth rate the rate of increase in size per unit of time; r in the exponential growth model $f(t) = a(1 + r)^t$	U2-253 U3-350	**tasa de crecimiento** tasa de aumento de tamaño por unidad de tiempo; r en el modelo de crecimiento exponencial $f(t) = a(1 + r)^t$
	H	
half-closed interval an interval that includes one endpoint but not the other; also called a half-open interval	U3-244	**intervalo medio cerrado** intervalo que incluye un punto final pero no el otro; también denominado intervalo medio abierto
half-open interval an interval that includes one endpoint but not the other; also called a half-closed interval	U3-244	**intervalo medio abierto** intervalo que incluye un punto final pero no el otro; también denominado intervalo medio cerrado
horizontal asymptote a line defined as follows: The line $y = b$ is a horizontal asymptote of the graph of a function f if $f(x)$ gets closer to b as x either increases or decreases without bound.	U3-244	**asíntota horizontal** línea recta que se define de la siguiente manera: La línea $y = b$ es una asíntota horizontal del gráfico de una función f si $f(x)$ se acerca a b a medida que x aumenta o disminuye sin límites.

English		Español
horizontal compression squeezing of the parabola toward the *y*-axis	U2-294	**compresión horizontal** contracción de la parábola hacia el eje *y*
horizontal stretch pulling of the parabola and stretching it away from the *y*-axis	U2-294	**estiramiento horizontal** jalar de la parábola y estirarla lejos del eje *y*
hypotenuse the side opposite the vertex of the 90° angle in a right triangle	U5-494	**hipotenusa** lado opuesto al vértice del ángulo de 90° en un triángulo rectángulo
	I	
identity an equation that is true regardless of what values are chosen for the variables	U3-189 U5-494 U5-548	**identidad** ecuación verdadera independientemente de los valores elegidos para las variables
imaginary number any number of the form *bi*, where *b* is a real number, $i = \sqrt{-1}$, and $b \neq 0$	U1-65 U3-189	**número imaginario** cualquier número de la forma *bi*, en el que *b* es un número real, $i = \sqrt{-1}$, y $b \neq 0$
imaginary unit, *i* the letter *i*, used to represent the non-real value, $i = \sqrt{-1}$	U1-65 U3-189	**unidad imaginaria, *i*** la letra *i*, utilizada para representar el valor no real $i = \sqrt{-1}$
incenter the intersection of the angle bisectors of a triangle	U5-295 U6-69	**incentro** intersección de las bisectrices del ángulo de un triángulo
increasing the interval of a function for which the output values are becoming larger as the input values are becoming larger	U2-54	**creciente** intervalo de una función para el que los valores de salida se hacen más grandes a medida que los valores de entrada también se vuelven más grandes
increasing function a function such that as the independent values increase, the dependent values also increase	U2-154	**función creciente** función en la que a medida que aumentan los valores independientes, también aumentan los valores dependientes
independent events events such that the outcome of one event does not affect the probability of the outcome of another event	U4-4 U4-77	**eventos independientes** eventos en los que el resultado de un evento no afecta la probabilidad del resultado de otro evento
independent variable labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function	U3-244	**variable independiente** designada en el eje *x*; cantidad que cambia según los valores seleccionados; variable de entrada de una función
infinity going on without bound; represented by the symbol ∞	U3-244	**infinito** continuación sin límites; se representa con el símbolo ∞

English		Español
inflection point a point on a curve at which the sign of the curvature (i.e., the concavity) changes	U2-54	**punto de inflexión** punto en una curva en el que cambia el signo de la curvatura (es decir, la concavidad)
inscribed angle an angle formed by two chords whose vertex is on the circle	U6-4	**ángulo inscrito** ángulo formado por dos cuerdas cuyo vértice está en el círculo
inscribed circle a circle whose tangents form a triangle	U5-295 U6-69	**círculo inscrito** círculo cuyos tangentes forman un triángulo
inscribed quadrilateral a quadrilateral whose vertices are on a circle	U6-69	**cuadrilátero inscrito** cuadrilátero cuyos vértices están en un círculo
inscribed triangle a triangle whose vertices are on a circle	U6-69	**triángulo inscrito** triangulo cuyos vértices están en un círculo
integer a number that is not a fraction or a decimal	U1-2	**entero** un número que no es una fracción ni un decimal
intercept the point at which a line intercepts the *x*- or *y*-axis	U2-2	**intercepto** punto en el que una línea intercepta el eje *x* o *y*
intercept form the factored form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function	U2-2 U3-108	**forma de intercepto** forma factorizada de una ecuación cuadrática, expresada como $f(x) = a(x - p)(x - q)$, donde p y q son los interceptos de x de la función
intercepted arc an arc whose endpoints intersect the sides of an inscribed angle and whose other points are in the interior of the angle	U6-4	**arco interceptado** arco cuyos extremos intersecan los lados de un ángulo inscrito y cuyos otros puntos se sitúan en el interior del ángulo
interior angle of a polygon an angle formed by two sides of a polygon	U5-295	**ángulo interior de un polígono** ángulo formado por dos lados de un polígono
interior angles angles that lie between a pair of parallel lines	U5-223	**ángulos interiores** ángulos ubicados entre un par de líneas paralelas
intersection a set whose elements are each in both of two other sets. The intersection of sets A and B, denoted by $A \cap B$, is the set of elements that are in both A and B.	U4-4	**intersección** conjunto cuyos elementos están todos en otros dos conjuntos. La intersección de los conjuntos A y B, indicada por $A \cap B$, es el conjunto de elementos que se encuentran tanto en A como en B.

English		Español
interval the set of all real numbers between two given numbers. The two numbers on the ends are the endpoints. The endpoints might or might not be included in the interval depending on whether the interval is open, closed, or half-open/half-closed.	U2-253 U3-34 U3-244	**intervalo** conjunto de todos los números reales entre dos números dados. Los dos números en los finales son los extremos. Los extremos podrían o no estar incluidos en el intervalo, según si el intervalo está abierto, cerrado, o medio abierto o medio cerrado.
interval notation a way of representing an interval using a pair of parentheses, a pair of brackets, or a parenthesis and a bracket	U3-244	**notación de intervalos** modo de representar un intervalo con un par de paréntesis, un par de corchetes, o un paréntesis y un corchete
inverse function the function that results from switching the x- and y-variables in a given function; the inverse of $f(x)$ is written as $f^{-1}(x)$	U2-346	**función inversa** función que se produce como resultado de cambiar las variables x y y en una función determinada; la inversa de $f(x)$ se expresa como $f^{-1}(x)$
inverse operation the operation that reverses the effect of another operation	U2-346	**operación inversa** operación que revierte el efecto de otra
irrational number numbers that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; any number that cannot be written as a decimal that ends or repeats	U1-3 U3-34 U6-198	**números irracionales** números que no pueden expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que no puede expresarse como decimal finito o periódico
isosceles trapezoid a trapezoid with one pair of opposite parallel lines and congruent legs	U5-424	**trapezoide isósceles** trapezoide con un par de líneas paralelas opuestas y catetos congruentes
isosceles triangle a triangle with at least two congruent sides	U5-295	**triángulo isósceles** triángulo con al menos dos lados congruentes
	K	
key features of a quadratic the x-intercepts, y-intercept, where the function is increasing and decreasing, where the function is positive and negative, relative minimums and maximums, symmetries, and end behavior of the function used to describe, draw, and compare quadratic functions	U2-54 U3-109	**características clave de una función cuadrática** interceptos de x, intercepto de y, donde la función aumenta y disminuye, donde la función es positiva y negativa, máximos y mínimos relativos, simetrías y comportamiento final de la función utilizado para describir, dibujar y comparar las funciones cuadráticas

English		Español
kite a quadrilateral with two distinct pairs of congruent sides that are adjacent	U5-424	**cometa** cuadrilátero con dos pares distintos de lados congruentes que son adyacentes
	L	
leading coefficient the coefficient of the term with the highest power. For a quadratic equation in standard form ($y = ax^2 + bx + c$), the leading coefficient is a.	U2-112 U3-34	**coeficiente líder** coeficiente del término con la mayor potencia. En una ecuación cuadrática en forma estándar ($y = ax^2 + bx + c$), el coeficiente líder es a.
least common denominator (LCD) of fractions the least common multiple of the denominators of the fractions	U3-244	**mínimo común denominador (LCD) de fracciones** múltiplo mínimo común de los denominadores de las fracciones
least common multiple (LCM) of polynomials with two or more polynomials, the common multiple of the polynomials that has the least degree and the least positive constant factor	U3-244	**mínimo común múltiplo (LCM) de polinomios** con dos o más polinomios, el múltiplo común de los polinomios que tiene el menor grado y el menor factor constante positivo
least integer function also known as the ceiling function; a function represented as $y = \lceil x \rceil$. For any input x, the output is the smallest integer greater than or equal to x; for example, $\lceil -3 \rceil = -3$, $\lceil 2.1 \rceil = 3$, and $\lceil -2.1 \rceil = -2$.	U2-154	**función de mínimo entero** también conocida como función techo; función representada como $y = \lceil x \rceil$. Para cualquier entrada x, la salida es el entero más pequeño mayor que o igual a x; por ejemplo, $\lceil -3 \rceil = -3$, $\lceil 2.1 \rceil = 3$, y $\lceil -2.1 \rceil = -2$.
legs congruent sides of an isosceles triangle	U5-295	**catetos** lados congruentes de un triángulo isósceles
like terms terms that contain the same variables raised to the same power	U1-34 U3-2	**términos semejantes** términos que contienen las mismas variables elevadas a la misma potencia
limit the value that a sequence approaches as a calculation becomes more and more accurate	U6-198	**límite** valor al que se aproxima una secuencia cuando un cálculo se vuelve cada vez más exacto
line segment a part of a line that is noted by two endpoints, (x_1, y_1) and (x_2, y_2)	U5-2	**segmento de recta** parte de una línea comprendida entre dos extremos, (x_1, y_1) y (x_2, y_2)

English		Español
linear function a function that can be written in the form $f(x) = mx + b$, in which m is the slope, b is the y-intercept, and the graph is a straight line	U2-253 U2-346	**función lineal** función que puede expresarse en la forma $f(x) = mx + b$, en la que m es la pendiente, b es el intercepto de y, y la gráfica es una línea recta
linear pair a pair of adjacent angles whose non-shared sides form a straight angle	U5-223	**par lineal** par de ángulos adyacentes cuyos lados no compartidos forman un ángulo recto
literal equation an equation that involves two or more variables	U3-109	**ecuación literal** ecuación que incluye dos o más variables
	M	
major arc part of a circle's circumference that is larger than its semicircle	U6-4	**arco mayor** parte de la circunferencia de un círculo que es mayor que su semicírculo
maximum the largest y-value of a quadratic equation	U2-2 U3-109	**máximo** el mayor valor de y de una ecuación cuadrática
median of a triangle the segment joining the vertex to the midpoint of the opposite side	U5-295	**mediana de un triángulo** segmento que une el vértice con el punto medio del lado opuesto
member an item in a set; also called an element	U4-4	**miembro** ítem en un conjunto; también se denomina elemento
midpoint a point on a line segment that divides the segment into two equal parts	U5-2 U5-295	**punto medio** punto en un segmento de recta que lo divide en dos partes iguales
midpoint formula formula that states the midpoint of a segment created by connecting (x_1, y_1) and (x_2, y_2) is given by the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	U5-2 U5-295 U6-311	**fórmula de punto medio** fórmula que establece el punto medio de un segmento creado al conectar (x_1, y_1) con (x_2, y_2) está dado por la fórmula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
midsegment a line segment joining the midpoints of two sides of a figure	U5-295	**segmento medio** segmento de recta que une los puntos medios de dos lados de una figura
midsegment triangle the triangle formed when all three of the midsegments of a triangle are connected	U5-295	**segmento medio de un triángulo** triángulo que se forma cuando los tres segmentos medios de un triángulo están conectados

English		Español
minimum the smallest *y*-value of a quadratic equation	U2-3 U3-109	**mínimo** el menor valor de *y* en una ecuación cuadrática
minor arc part of a circle's circumference that is smaller than its semicircle	U6-4	**arco menor** parte de la circunferencia de un círculo que es menor que su semicírculo
monomial an expression with one term, consisting of a number, a variable, or the product of a number and variable(s)	U1-34 U3-2	**monomio** expresión con un solo término, que consiste en un número, una variable, o el producto de un número y una o más variables
Multiplication Rule the probability of two events, A and B, is $P(A \text{ and } B) = P(A) \bullet P(B \mid A) = P(B) \bullet P(A \mid B)$; for independent events A and B, the rule is $P(A \text{ and } B) = P(A) \bullet P(B)$.	U4-77	**Regla de multiplicación** probabilidad de que dos eventos, A y B, sea $P(A \text{ y } B) = P(A) \bullet P(B \mid A) = P(B) \bullet P(A \mid B)$; para eventos independientes A y B, la regla es $P(A \text{ y } B) = P(A) \bullet P(B)$.
mutually exclusive events events that have no outcomes in common. If A and B are mutually exclusive events, then they cannot both occur. Mutually exclusive events are also called disjoint events.	U4-4	**eventos mutuamente excluyentes** eventos que no tienen resultados en común. Si A y B son eventos mutuamente excluyentes, entonces no pueden producirse ambos. También se denominan eventos disjuntos.
N		
neither describes a function that, when evaluated for $-x$, does not result in the opposite of the original function (odd) or the original function (even)	U2-54	**ni** describe una función que, cuando se evalúa para $-x$, no tiene como resultado lo opuesto de la función original (impar) ni la función original (par)
non-rigid motion a transformation done to a figure that changes the figure's shape and/or size	U5-32	**movimiento no rígido** transformación hecha a una figura que cambia su forma o tamaño
nonadjacent angles angles that have no common vertex or common side, or have shared interior points	U5-224	**ángulos no adyacentes** ángulos que no tienen vértices ni lados comunes, o que tienen puntos interiores compartidos
null set a set that has no elements, denoted by $\varnothing$. The null set is also called the empty set.	U4-4	**conjunto nulo** conjunto que no tiene elementos, indicado con $\varnothing$. También se denomina conjunto vacío.

English		Español
	O	
obtuse triangle a triangle with one angle that is obtuse (greater than 90°)	U5-295	**triángulo obtuso** triángulo con un ángulo que es obtuso (de más de 90°)
odd function a function that, when evaluated for $-x$, results in a function that is the opposite of the original function; $f(-x) = -f(x)$	U2-54	**función impar** función que, cuando se evalúa para $-x$, tiene como resultado una función que es lo opuesto a la función original; $f(-x) = -f(x)$
one-to-one a relationship wherein each point in a set of points is mapped to exactly one other point	U2-346	**unívoca** relación en la que cada punto de un conjunto de puntos se corresponde con otro con exactitud
open interval an interval that does not include its endpoints	U3-244	**intervalo abierto** intervalo que no incluye sus extremos
opposite side the side across from an angle	U5-494	**lado opuesto** lado al otro lado de un ángulo
orthocenter the intersection of the altitudes of a triangle	U5-295	**ortocentro** intersección de las alturas de un triángulo
outcome a result of an experiment	U4-4	**resultado** consecuencia de un experimento
	P	
parabola the U-shaped graph of a quadratic equation; the set of all points that are equidistant from a fixed line, called the directrix, and a fixed point not on that line, called the focus. The parabola, directrix, and focus are all in the same plane. The vertex of the parabola is the point on the parabola that is closest to the directrix.	U2-3 U3-109 U6-250 U6-311	**parábola** gráfico de una ecuación cuadrática en forma de U; conjunto de todos los puntos equidistantes de una línea fija denominada directriz y un punto fijo que no está en esa línea, llamado foco. La parábola, la directriz y el foco están todos en el mismo plano. El vértice de la parábola es el punto más cercano a la directriz.
paragraph proof statements written out in complete sentences in a logical order to show an argument	U5-130	**prueba de párrafo** declaraciones redactadas en oraciones completas en orden lógico para demostrar un argumento
parallel lines lines in a plane that either do not share any points and never intersect, or share all points; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$	U5-130	**líneas paralelas** líneas en un plano que no comparten ningún punto y nunca se cortan, o que comparten todos los puntos; se expresan como $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$

English		Español
parallelogram a special type of quadrilateral with two pairs of opposite sides that are parallel; denoted by the symbol $\square$	U5-424	**paralelogramo** un tipo especial de cuadrilátero con dos pares de lados opuestos paralelos; se expresa con el símbolo $\square$
parent function a function with a simple algebraic rule that represents a family of functions. The graphs of the functions in the family have the same general shape as the parent function.	U3-244	**función principal** función con una regla algebraica simple que representa una familia de funciones. Los gráficos de las funciones en la familia tienen la misma forma general que la función principal.
percent of change $\frac{\text{amount of change}}{\text{original amount}}$, written as a percent	U3-350	**porcentaje de cambio** se expresa como porcentaje $\frac{\text{porcentaje de cambio}}{\text{cantidad original}}$
perfect square trinomial a trinomial of the form $x^2+bx+\left(\frac{b}{2}\right)^2$ that can be written as the square of a binomial	U3-34 U6-250 U6-311	**trinomio cuadrado perfecto** trinomio de la forma $x^2+bx+\left(\frac{b}{2}\right)^2$ que puede expresarse como el cuadrado de un binomio
permutation a selection of objects where the order matters and is found either using n^r, if repetitions are allowed, or by using ${}_nP_r=\frac{n!}{(n-r)!}$, where n is the number of objects to select from and r is the number of objects being selected and ordered.	U4-153	**permutación** selección de objetos en la que el orden importa y se encuentra con el uso de n^r, si se permiten las repeticiones, o con ${}_nP_r=\frac{n!}{(n-r)!}$, donde n es la cantidad de objetos de donde seleccionar y r es la cantidad de objetos seleccionados y ordenados.
perpendicular bisector a line that intersects a segment at its midpoint at a right angle	U5-224 U6-69	**bisectriz perpendicular** línea que corta un segmento en su punto medio en ángulo recto
perpendicular lines two lines that intersect at a right angle (90°). The lines form four adjacent and congruent right angles.	U5-224	**líneas perpendiculares** dos líneas que se cortan en un ángulo recto (90°). Las líneas forman cuatro ángulos rectos adyacentes y congruentes.

English		Español
phi **(*ϕ*)** a Greek letter sometimes used to refer to an unknown angle measure	U5-494	***fi (ϕ)*** letra del alfabeto griego que se utiliza a veces para referirse a la medida desconocida de un ángulo
***pi* (*π*)** the ratio of circumference of a circle to the diameter; equal to approximately 3.14	U6-4	***pi (π)*** proporción de la circunferencia de un círculo al diámetro; equivale aproximadamente a 3.14
piecewise function a function that is defined by two or more expressions on separate portions of the domain	U2-154	**función por partes** función definida por dos o más expresiones en porciones separadas del dominio
plane a flat, two-dimensional figure without depth that has at least three non-collinear points and extends infinitely in all directions	U5-224	**plano** figura plana, bidimensional, sin profundidad, que tiene al menos tres puntos no colineales y se extiende infinitamente en todas direcciones
point of concurrency a single point of intersection of three or more lines	U5-295 U6-69	**punto de concurrencia** punto único de intersección de tres o más líneas
point of tangency the only point at which a line and a circle intersect	U6-134	**punto de tangencia** punto único de intersección entre una línea y un círculo
point(s) of intersection the ordered pair(s) where graphed functions intersect on a coordinate plane; these are also the solutions to systems of equations	U3-380	**puntos de intersección** pares ordenados en los que se intersecan funciones representadas en gráficos en un plano de coordenadas; son también las soluciones a sistemas de ecuaciones
polyhedron a three-dimensional object that has faces made of polygons	U6-198	**poliedro** objeto tridimensional que tiene caras compuestas por polígonos
polynomial a monomial or the sum of monomials	U1-34 U3-2	**polinomio** monomio o suma de monomios
polynomial function a function whose rule is a one-variable polynomial; $P(x)$ is a polynomial function if $P(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$, where n is a nonnegative integer and $a_n \neq 0$	U3-189	**función polinómica** función cuya regla es un polinomio de una variable; $P(x)$ es una función polinómica si $P(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$, donde n es un entero no negativo y $a_n \neq 0$
postulate a true statement that does not require a proof	U5-224	**postulado** declaración verdadera que no requiere prueba

English		Español
power the quantity that shows the number of times the base is being multiplied by itself in an exponential expression; also known as the exponent. In a^x, x is the power/exponent.	U1-3	**potencia** cantidad que muestra el número de veces que la base se multiplica por sí misma en una expresión exponencial; también se denomina exponente. En a^x, x es la potencia o exponente.
prime an expression that cannot be factored	U3-34	**número primo** expresión que no puede ser factorizada
probability a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur	U4-4	**probabilidad** número de 0 a 1 inclusivo o porcentaje de 0% a 100% inclusivo que indica cuán probable es que se produzca un evento
probability model a mathematical model for observable facts or occurrences that are assumed to be random; a representation of a random phenomenon	U4-4	**modelo de probabilidad** modelo matemático para hechos o sucesos observables que se presumen aleatorios; representación de un fenómeno aleatorio
probability of an event *E* denoted $P(E)$, and is given by $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$ in a uniform probability model	U4-4	**probabilidad de un evento *E*** se expresa como $P(E)$, y está dado por $P(E) = \frac{\text{número de resultados en } E}{\text{número de resultados en el espacio de muestreo}}$ en un modelo de probabilidad uniforme
proof a set of justified statements organized to form a convincing argument that a given statement is true	U5-130 U5-224	**prueba** conjunto de declaraciones justificadas y organizadas para formar un argumento convincente de que determinada declaraciónes verdadera
proportional having a constant ratio to another quantity	U5-80	**proporcional** que tiene una proporción constante con otra cantidad
pyramid a solid or hollow polyhedron object that has three or more triangular faces that converge at a single vertex at the top; the base may be any polygon	U6-198	**pirámide** objeto poliedro sólido o hueco con tres o más caras triangulares que convergen en un único vértice en la parte superior; la base puede ser cualquier polígono
Pythagorean identity a trigonometric identity that is derived from the Pythagorean Theorem. The primary Pythagorean identity is $\sin^2\theta + \cos^2\theta = 1$.	U5-548	**identidad Pitagórica** identidad trigonométrica que deriva del teorema de Pitágoras. La identidad Pitagórica principal es $\text{sen}^2\theta + \cos^2\theta = 1$.

English		Español
Pythagorean Theorem a theorem that relates the length of the hypotenuse of a right triangle (c) to the lengths of its legs (a and b). The theorem states that $a^2 + b^2 = c^2$.	U5-548 U6-250	**Teorema de Pitágoras** teorema que relaciona la longitud de la hipotenusa de un triángulo rectángulo (c) con las longitudes de sus catetos (a y b). El teorema establece que $a^2 + b^2 = c^2$.
	Q	
quadratic equation an equation that can be written in the form $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$	U3-2 U3-34	**ecuación cuadrática** ecuación que se puede expresar en la forma $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic expression an algebraic expression that can be written in the form $ax^2 + bx + c$, where x is the variable, a, b, and c are constants, and $a \neq 0$	U3-3	**expresión cuadrática** expresión algebraica que se puede expresar en la forma $ax^2 + bx + c$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic formula a formula that states the solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. A quadratic equation in this form can have no real solutions, one real solution, or two real solutions.	U3-34 U3-245 U3-380	**fórmula cuadrática** fórmula que establece que las soluciones de una ecuación cuadrática de la forma $ax^2 + bx + c = 0$ están dadas por $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Una ecuación cuadrática en esta forma tener ningún solución real, o tener una solución real, o dos soluciones reales.
quadratic function a function that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of any quadratic function is a parabola.	U2-3 U2-253 U2-346 U3-109 U6-250 U6-311	**función cuadrática** función que puede expresarse en la forma $f(x) = ax^2 + bx + c$, donde $a \neq 0$. El gráfico de cualquier función cuadrática es una parábola.

English		Español
quadratic inequality an inequality that can be written in the form $ax^2 + bx + c < 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c > 0$, or $ax^2 + bx + c \geq 0$	U3-34	**desigualdad cuadrática** desigualdad que puede expresarse en la forma $ax^2 + bx + c < 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c > 0$, o $ax^2 + bx + c \geq 0$
quadratic-linear system a system of equations in which one equation is quadratic and one is linear	U3-380	**sistema lineal cuadrático** sistema de ecuaciones en el que una ecuación es cuadrática y una es lineal
quadratic polynomial in one variable a one-variable polynomial of degree 2; it can be written in the form $ax^2 + bx + c$, where $a \neq 0$	U3-189	**polinomio cuadrático en una variable** polinomio de una variable de grado 2; se puede expresar en la forma $ax^2 + bx + c$, donde $a \neq 0$
quadrilateral a polygon with four sides	U5-424	**cuadrilátero** polígono con cuatro lados
	R	
radian the measure of the central angle that intercepts an arc equal in length to the radius of the circle; π radians = 180°	U6-167	**radián** medida del ángulo central que intercepta un arco de longitud igual al radio del círculo; π radianes = 180°
radian measure the ratio of the arc intercepted by the central angle to the radius of the circle	U6-167	**medida de radián** proporción del arco interceptado por el ángulo central al radio del círculo
radical expression an expression containing a root, such as $\sqrt[5]{9}$	U1-3	**expresión radical** expresión que contiene una raíz, tal como $\sqrt[5]{9}$
radical function a function with the independent variable under a root. The general form is $y = a\sqrt[n]{(x-h)} + k$, where n is a positive integer root and a, h, and k are real numbers.	U2-154	**función radical** función con la variable independiente bajo una raíz. La forma general es $y = a\sqrt[n]{(x-h)} + k$, donde n es una raíz de entero positivo y a, h, y k son números reales.
radius the distance from the center to a point on the circle; equal to one-half the diameter	U6-4 U6-250	**radio** distancia desde el centro a un punto en el círculo; equivale a la mitad del diámetro
random number generator a tool to select a number without following a pattern, where the probability of any number in the set being generated is equal	U4-196	**generador de números aleatorios** herramienta para seleccionar un número sin seguir un patrón, por lo que la probabilidad de generar cualquier número del conjunto es igual

English		Español
range the set of all outputs of a function; the set of *y*-values that are valid for the function	U2-154 U3-245	**rango** conjunto de todas las salidas de una función; conjunto de valores de *y* que son válidos para la función
rate a ratio that compares measurements with different kinds of units	U3-245	**tasa** proporción que compara medidas con distintos tipos de unidades
ratio the relation between two quantities; can be expressed in words, fractions, decimals, or as a percentage	U3-245 U5-494	**proporción** relación entre dos cantidades; puede expresarse en palabras, fracciones, decimales o como porcentaje
ratio identities identities comprised of other trigonometric identities; the following two identities are ratio identities: $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\cot\theta=\frac{\cos\theta}{\sin\theta}$	U5-548	**identidades de proporciones** identidades que constan de otras identidades trigonométricas; las dos identidades siguientes son identidades de proporciones: $\tan\theta=\frac{\text{sen}\,\theta}{\cos\theta}$ y $\cot\theta=\frac{\cos\theta}{\text{sen}\,\theta}$
ratio of similitude a ratio of corresponding sides; also known as the scale factor	U5-80	**proporción de similitud** proporción de lados correspondientes; se conoce también como factor de escala
rational equation an equation that includes the ratio of two rational expressions, in which a variable appears in the denominator of at least one rational expression	U3-245	**ecuación racional** ecuación que incluye la proporción de dos expresiones racionales, en la que aparece una variable en el denominador de al menos una expresión racional
rational exponent an exponent of the form $\frac{m}{n}$, where *m* and *n* are integers. If *m* and *n* are positive integers and *a* is a real number, then $a^{\frac{m}{n}}=\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}$.	U3-350	**exponente racional** exponente de la forma $\frac{m}{n}$, donde *m* y *n* son enteros. Si *m* y *n* son enteros positivos y *a* es un número real, entonces $a^{\frac{m}{n}}=\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}$.
rational expression an expression made of the ratio of two polynomials, in which a variable appears in the denominator of a polynomial	U3-245	**expresión racional** expresión formada por la proporción de dos polinomios, en la que aparece una variable en el denominador de un polinomio

English		Español
rational function a function that can be written in the form $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$	U3-245	**función racional** función que puede expresarse en la forma $f(x)=\frac{p(x)}{q(x)}$, donde $p(x)$ y $q(x)$ son polinomios y $q(x) \neq 0$
rational inequality an inequality that includes the ratio of two rational expressions, in which a variable appears in the denominator of at least one rational expression	U3-245	**desigualdad racional** desigualdad que incluye la proporción de dos expresiones racionales, en la que aparece una variable en el denominador de al menos una expresión racional
rational number any number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; any number that can be written as a decimal that ends or repeats	U1-3 U3-34	**números racionales** números que pueden expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
real numbers the set of all rational and irrational numbers	U1-3 U1-65 U3-35	**números reales** conjunto de todos los números racionales e irracionales
reciprocal a number that, when multiplied by the original number, has a product of 1	U5-494	**recíproco** número que multiplicado por el número original tiene producto 1
reciprocal identities trigonometric identities that define cosecant, secant, and cotangent in terms of sine, cosine, and tangent: $\csc\theta=\frac{1}{\sin\theta};\ \sec\theta=\frac{1}{\cos\theta};\ \cot\theta=\frac{1}{\tan\theta}$	U5-548	**identidades recíprocas** identidades trigonométricas que definen cosecante, secante y cotangente en términos de seno, coseno y tangente: $\csc\theta=\frac{1}{\text{sen}\theta};\ \sec\theta=\frac{1}{\cos\theta};\ \cot\theta=\frac{1}{\tan\theta}$
rectangle a special parallelogram with four right angles	U5-424	**rectángulo** paralelogramo especial con cuatro ángulos rectos
reduction a dilation where the scale factor is between 0 and 1	U5-32	**reducción** dilatación en la que el factor de escala está entre 0 y 1
Reflexive Property of Congruent Segments a segment is congruent to itself; $\overline{AB} \cong \overline{AB}$	U5-131	**Propiedad reflexiva de congruencia de segmentos** un segmento es congruente con él mismo; $\overline{AB} \cong \overline{AB}$

English		Español
relative frequency (of an event) the number of times an event occurs divided by the number of times an experiment is performed	U4-4	**frecuencia relativa (de un evento)** cantidad de veces que un evento se produce dividido por la cantidad de veces que se realiza el experimento
remote interior angles interior angles that are not adjacent to the exterior angle	U5-295	**ángulos interiores remotos** ángulos interiores que no son adyacentes al ángulo exterior
restricted domain a subset of a function's defined domain	U2-154	**dominio restringido** subconjunto del dominio definido de una función
restricted range a subset of a function's defined range	U2-154	**rango restringido** subconjunto del rango definido de una función
rhombus a special parallelogram with all four sides congruent	U5-425	**rombo** paralelogramo especial con sus cuatro lados congruentes
right angle an angle measuring 90º	U5-224	**ángulo recto** ángulo que mide 90º
right triangle a triangle with one angle that measures 90º	U5-295 U5-494	**triángulo rectángulo** triángulo con un ángulo que mide 90º
rigid motion a transformation done to a figure that maintains the figure's shape and size or its segment lengths and angle measures	U5-32	**movimiento rígido** transformación que se realiza a una figura que mantiene su forma y tamaño o las longitudes de sus segmentos y las medidas de ángulos
root the inverse of a power/exponent; the root of a number x is a number that, when multiplied by itself a given number of times, equals x	U1-3	**raíz** inversa de una potencia o exponente; la raíz de un número x es un número que, multiplicado por sí mismo una cantidad determinada de veces, equivale a x
root(s) solution(s) of a quadratic equation	U3-35	**raíces** soluciones de una ecuación cuadrática
S		
same-side exterior angles angles that lie on the same side of the transversal and are outside the lines that the transversal intersects; sometimes called consecutive exterior angles	U5-224	**ángulos exteriores del mismo lado** ángulos que se ubican en el mismo lado de la transversal y están fuera de las líneas que corta la transversal; a veces se denominan ángulos exteriores consecutivos

English		Español
same-side interior angles angles that lie on the same side of the transversal and are in between the lines that the transversal intersects; sometimes called consecutive interior angles	U5-224	**ángulos interiores del mismo lado** ángulos que se ubican en el mismo lado de la transversal y están en medio de las líneas que corta la transversal; a veces se los denomina ángulos interiores consecutivos
sample space the set of all possible outcomes of an experiment	U4-4	**espacio de muestreo** conjunto de todos los resultados posibles de un experimento
scale factor a multiple of the lengths of the sides from one figure to the transformed figure. If the scale factor is larger than 1, then the figure is enlarged. If the scale factor is between 0 and 1, then the figure is reduced.	U5-32 U5-494	**factor de escala** múltiplo de las longitudes de los lados de una figura a la figura transformada. Si el factor de escala es mayor que 1, entonces la figura se agranda. Si el factor de escala se encuentra entre 0 y 1, entonces la figura se reduce.
scalene triangle a triangle with no congruent sides	U5-295	**triángulo escaleno** triángulo sin lados congruentes
secant the reciprocal of cosine, $\sec\theta = \frac{1}{\cos\theta}$; the secant of θ = $\sec\theta = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$	U5-494 U5-548	**secante** recíproco del coseno, $\sec\theta = \frac{1}{\cos\theta}$; secante de θ = $\sec\theta = \frac{\text{longitud de la hipotenusa}}{\text{longitud del lado adyacente}}$
secant line a line that intersects a circle at two points	U6-4	**línea secante** recta que corta un círculo en dos puntos
second difference in a set of data, the change in successive first differences	U2-253	**segunda diferencia** en un conjunto de datos, el cambio en sucesivas primeras diferencias
sector a portion of a circle bounded by two radii and their intercepted arc	U6-167	**sector** porción de un círculo limitado por dos radios y el arco que cortan
Segment Addition Postulate If B is between A and C, then $AB + BC = AC$. Conversely, if $AB + BC = AC$, then B is between A and C.	U5-131	**Postulado de la suma de segmentos** Si B está entre A y C, entonces $AB + BC = AC$. A la inversa, si $AB + BC = AC$, entonces B se encuentra entre A y C.
semicircle an arc that is half of a circle	U6-4	**semicírculo** arco que es la mitad de un círculo

English		Español
set a collection or list of items	U4-4	**conjunto** colección o lista de elementos
Side-Angle-Side (SAS) Similarity Statement If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.	U5-131	**Criterio de semejanza lado-ángulo-lado (SAS)** Si las medidas de dos lados de un triángulo son proporcionales a las medidas de dos lados correspondientes de otro triángulo y los ángulos incluidos son congruentes, entonces los triángulos son similares.
Side-Side-Side (SSS) Similarity Statement If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.	U5-131	**Criterio de semejanza lado-lado-lado (SSS)** Si las medidas de los lados correspondientes de dos triángulos son proporcionales, entonces los triángulos son similares.
similar two figures that are the same shape but not necessarily the same size; the symbol for representing similarity between figures is $\sim$	U5-80 U5-494	**similar** dos figuras que tienen la misma forma pero no necesariamente el mismo tamaño; el símbolo para representar similitud entre figuras es $\sim$
similarity transformation a rigid motion followed by a dilation; a transformation that results in the position and size of a figure changing, but not the shape	U5-80	**transformación de similitud** movimiento rígido seguido por una dilatación; transformación que tiene como resultado el cambio de posición y tamaño, pero no la forma, de una figura
simple event an event that has only one outcome; sometimes called a single event	U4-77	**evento simple** evento que sólo tiene un resultado; a veces se denomina evento único
sine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the hypotenuse; the sine of θ = $\sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$	U5-494	**seno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud de la hipotenusa; sen de θ = $\text{sen}\,\theta = \frac{\text{longitud del lado opuesto}}{\text{longitud de la hipotenusa}}$

English		Español
slope the measure of the rate of change of one variable with respect to another variable; slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m.	U2-54	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m.
slope formula a formula that states the slope of the line through (or the line segment connecting) $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$	U6-311	**fórmula de pendiente** fórmula que determina la pendiente de la línea que atraviesa (o el segmento de recta que conecta) $A\ (x_1, y_1)$ y $B\ (x_2, y_2)$ es $\frac{y_2 - y_1}{x_2 - x_1}$
sphere a three-dimensional surface that has all its points the same distance from its center	U6-198	**esfera** superficie tridimensional que tiene todos sus puntos a la misma distancia de su centro
square a special parallelogram with four congruent sides and four right angles	U5-425	**cuadrado** paralelogramo especial con cuatro lados congruentes y cuatro ángulos rectos
square root For any real numbers a and b, if $a^2 = b$, then a is a square root of b. The square root of b is written using a radical: $\sqrt{b}$.	U2-154	**raíz cuadrada** para cualquier número real a y b, si $a^2 = b$, entonces a es la raíz cuadrada de b. La raíz cuadrada de b se expresa con un radical: $\sqrt{b}$.
square root function a function that contains a square root of a variable	U2-154	**función raíz cuadrada** función que contiene una raíz cuadrada de una variable
square root of a negative number a number defined such that for any positive real number a, $\sqrt{-a} = i\sqrt{a}$.	U3-189	**raíz cuadrada de un número negativo** número definido de forma tal que para cualquier número real positivo a, $\sqrt{-a} = i\sqrt{a}$.
standard form of a quadratic function a quadratic function written as $f(x) = ax^2 + bx + c$, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term	U2-3 U3-109	**forma estándar de función cuadrática** función cuadrática expresada como $f(x) = ax^2 + bx + c$, donde a es el coeficiente del término cuadrático, b es el coeficiente del término lineal, y c es el término constante
standard form of an equation of a circle $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius	U3-380 U6-250 U6-311	**forma estándar de ecuación de un círculo** $(x - h)^2 + (y - k)^2 = r^2$, donde (h, k) es el centro y r es el radio

English		Español
standard form of an equation of a parabola $(x-h)^2 = 4p(y-k)$ for parabolas that open up or down; $(y-k)^2 = 4p(x-h)$ for parabolas that open right or left. For all parabolas, $p \neq 0$ and the vertex is (h, k).	U6-250 U6-311	**forma estándar de ecuación de una parábola** $(x-h)^2 = 4p(y-k)$ para parábolas que abren hacia arriba o hacia abajo; $(y-k)^2 = 4p(x-h)$ para parábolas que abren a la derecha o a la izquierda. Para todas las parábolas, $p \neq 0$ y el vértice es (h, k).
step function a function that is a series of disconnected constant functions	U2-154	**función escalonada** función que es una serie de funciones constantes desconectadas
straight angle an angle with rays in opposite directions; i.e., a straight line	U5-224	**ángulo recto** ángulo con semirrectas en direcciones opuestas; es decir, línea recta
stretch a transformation in which a figure becomes larger; stretches may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	U5-32	**ampliación** transformación en la que una figura se hace más grande; las ampliaciones pueden ser horizontales (cuando afectan sólo las longitudes horizontales), verticales (cuando afectan sólo las longitudes verticales), o en ambos sentidos
subset a set whose elements are in another set. Set A is a subset of set B, denoted by $A \subset B$, if all the elements of A are also in B.	U4-5	**subconjunto** conjunto cuyos elementos están en otro conjunto. El conjunto A es un subconjunto del conjunto B, indicado por $A \subset B$, si todos los elementos de A se encuentran también en B.
substitution the replacement of a term of an equation by another term that is known to have the same value	U3-380	**sustitución** reemplazo de un término de una ecuación por otro que se sabe que tiene el mismo valor
supplementary angles two angles whose sum is 180°	U5-224 U5-295	**ángulos suplementarios** dos ángulos cuya suma es 180°
Symmetric Property of Congruent Segments If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.	U5-131	**Propiedad simétrica de congruencia de segmentos** Si $\overline{AB} \cong \overline{CD}$, entonces $\overline{CD} \cong \overline{AB}$.
system of equations a set of equations with the same unknowns	U3-380	**sistema de ecuaciones** conjunto de ecuaciones con las mismas incógnitas

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T		
tangent a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the adjacent side; the tangent of θ = $\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$	U5-495	**tangente** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud del lado adyacente; tangente de θ = $\tan\theta = \frac{\text{longitud del lado opuesto}}{\text{longitud del lado adyacente}}$
tangent line a line that intersects a circle at exactly one point and is perpendicular to the radius of the circle	U6-4 U6-134	**recta tangente** línea que corta un círculo en exactamente un punto y es perpendicular al radio del círculo
term a number, a variable, or the product of a number and variable(s)	U1-34 U3-3 U3-189	**término** número, variable, o producto de un número y una o más variables
test interval for a polynomial or rational inequality in *x*, an interval on the *x*-axis formed by one or more critical numbers. The sign of the function on the test interval is the same as the sign of the function value at any *x*-value in the interval.	U3-245	**intervalo de prueba** para una desigualdad polinómica o racional en *x*, intervalo en el eje *x* formado por uno o más números críticos. El signo de la función del intervalo de prueba es el mismo que el del valor de la función en cualquier valor de *x* en el intervalo.
theorem a statement that is shown to be true	U5-131 U6-311	**teorema** declaración que se demuestra que es verdadera
theta (θ) a Greek letter commonly used to refer to unknown angle measures	U5-495	***teta* (θ)** letra griega que se utiliza por lo general para referirse a medidas de ángulos desconocidas
transformation adding or multiplying a constant to a function that changes the function's position and/or shape	U2-294	**transformación** suma o multiplicación de una constante con una función que cambia la posición y/o forma de la función
Transitive Property of Congruent Segments If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.	U5-131	**Propiedad transitiva de congruencia de segmentos** Si $\overline{AB} \cong \overline{CD}$, y $\overline{CD} \cong \overline{EF}$, entonces $\overline{AB} \cong \overline{EF}$.

English		Español
translation transforming a function where the shape and size of the function remain the same but the function moves horizontally and/or vertically; adding a constant to the independent or dependent variable	U2-294	**traslación** transformación de una función en la que la forma y el tamaño de la función permanecen iguales pero la función se traslada en sentido horizontal y/o vertical; suma de una constante a la variable independiente o dependiente
transversal a line that intersects a system of two or more lines	U5-224	**transversal** línea que corta un sistema de dos o más líneas
trapezoid a quadrilateral with exactly one pair of opposite parallel lines	U5-425	**trapezoide** cuadrilátero con exactamente un par de líneas paralelas opuestas
trigonometry the study of triangles and the relationships between their sides and the angles between these sides	U5-495	**trigonometría** estudio de los triángulos y las relaciones entre sus lados y los ángulos entre ellos
trinomial a polynomial with three terms	U3-3	**trinomio** polinomio con tres términos
two-column proof numbered statements and corresponding reasons that show the argument in a logical order	U5-131	**prueba de dos columnas** declaraciones numeradas y las razones correspondientes que muestran el argumento en orden lógico
two-way frequency table a frequency table that shows two categories of characteristics, one in rows and the other in columns. Each cell value is a frequency that shows how many times two different characteristics appear together, or how often characteristics are associated with a person, object, or type of item that is being studied.	U4-77	**tabla de frecuencia de dos vías** tabla de frecuencia que muestra dos categorías de características, una en filas y la otra en columnas. Cada valor de celda es una frecuencia que demuestra cuántas veces dos características diferentes aparecen juntas, o con qué frecuencia las características se asocian con una persona, objeto, o tipo de elemento que se está analizando.
U		
uniform probability model a probability model in which all the outcomes of an experiment are assumed to be equally likely	U4-5	**modelo de probabilidad uniforme** modelo de probabilidad en el que se presume que todos los resultados de un experimento son igualmente probables

English		Español
union a set whose elements are in at least one of two other sets. The union of sets A and B, denoted by $A \cup B$, is the set of elements that are in either A or B or both A and B.	U4-5	**unión** conjunto cuyos elementos están al menos en uno de otros dos conjuntos. La unión de los conjuntos A y B, indicada por $A \cup B$, es el conjunto de elementos que están en A o en B, o a la vez en A y B.
universal set a set of all elements that are being considered in a particular situation. In a probability experiment, the universal set is the sample space.	U4-5	**conjunto universal** conjunto de todos los elementos que se consideran en una situación particular. En un experimento de probabilidad, el conjunto universal es el espacio de muestreo.
	V	
variable a letter used to represent a value or unknown quantity that can change or vary	U3-3	**variable** letra que se utiliza para representar un valor o cantidad desconocida que puede cambiar o variar
Venn diagram a diagram that shows how two or more sets in a universal set are related	U4-5	**diagrama de Venn** diagrama que muestra cómo se relacionan dos o más conjuntos en un conjunto universal
vertex angle angle formed by the legs of an isosceles triangle	U5-295	**ángulo vértice** ángulo formado por los catetos de un triángulo isósceles
vertex form a quadratic function written as $f(x) = a(x - h)^2 + k$, where the vertex of the parabola is the point (h, k); the form of a quadratic equation where the vertex can be read directly from the equation	U2-3 U3-109	**fórmula de vértice** función cuadrática que se expresa como $f(x) = a(x - h)^2 + k$, donde el vértice de la parábola es el punto (h, k); forma de una ecuación cuadrática en la que el vértice se puede leer directamente de la ecuación
vertex of a parabola the point on a parabola that is closest to the directrix and lies on the axis of symmetry; the point at which the curve changes direction; the maximum or minimum	U2-3 U2-112 U3-109 U6-250 U6-311	**vértice de una parábola** punto en una parábola que está más cercano a la directriz y se ubica sobre el eje de simetría; punto en el que la curva cambia de dirección; el máximo o mínimo
vertical angles nonadjacent angles formed by two pairs of opposite rays	U5-224	**ángulos verticales** ángulos no adyacentes formados por dos pares de semirrectas opuestas

English		Español
vertical asymptote a line defined as follows: The line $x = a$ is a vertical asymptote of the graph of a function f if $f(x)$ either increases or decreases without bound as x gets closer to a.	U3-245	**asíntota vertical** recta definida de la siguiente manera: La línea $x = a$ es una asíntota vertical del gráfico de una función f si $f(x)$ aumenta o disminuye sin límites a medida que x se acerca a a.
vertical compression squeezing of the parabola toward the x-axis	U2-294	**compresión vertical** contracción de la parábola hacia el eje x
vertical stretch pulling of the parabola and stretching it away from the x-axis	U2-294	**estiramiento vertical** jalar y estirar la parábola lejos del eje x
	W	
wholly imaginary a complex number that has a real part equal to 0; written in the form $a + bi$, where a and b are real numbers, i is the imaginary unit, $a = 0$, and $b \neq 0$: $0 + bi$	U1-65	**totalmente imaginario** número complejo que tiene una parte real igual a 0; se expresa en la forma $a + bi$, donde a y b son números reales, i es la unidad imaginaria, $a = 0$, y $b \neq 0$: $0 + bi$
wholly real a complex number that has an imaginary part equal to 0; written in the form $a + bi$, where a and b are real numbers, i is the imaginary unit, $b = 0$, and $a \neq 0$: $a + 0i$	U1-65	**totalmente real** número complejo que tiene una parte imaginaria igual a 0; se expresa en la forma $a + bi$, donde a y b son números reales, i es la unidad imaginaria, $b = 0$, y $a \neq 0$: $a + 0i$
	X	
***x*-intercept** the point at which the graph crosses the x-axis; written as $(x, 0)$	U2-3 U3-109	**intercepto de *x*** punto en el que el gráfico cruza el eje x; se expresa como $(x, 0)$
	Y	
***y*-intercept** the point at which the graph crosses the y-axis; written as $(0, y)$	U2-3 U3-109	**intercepto de *y*** punto en el que el gráfico cruza el eje y; se expresa como $(0, y)$
	Z	
Zero Product Property If the product of two factors is 0, then at least one of the factors is 0.	U3-35	**Propiedad de producto cero** Si el producto de dos factores es 0, entonces al menos uno de los factores es 0.
zeros the x-values of a function for which the function value is 0	U3-189	**ceros** valores de x de una función para la que el valor de la función es 0