KENDRIYA VIDYALAYA SANGATHAN

SILCHAR REGION

STUDY MATERIAL

CLASS XI

PHYSICS

2012-2013

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Syllabus

Unit No	Name	Subject Expert
I	Physical World and Measurement	**Mr. S. L. Verma,** KV Aizawl
II	Kinematics	**Mr. S. L. Verma,** KV Aizawl
III	Law of Motion	**Mr. Anurag Jayaswal** KV Silchar
IV	Work, Energy, Power	**Mr. Anurag Jayaswal** KV Silchar
V	Motion of System of Particles and Rigid Body	**Mr. Harendra Kumar** KV Lumding
VI	Gravitation	**Mr. Harendra Kumar** KV Lumding
VII	Properties of Bulk Matter	**Mrs. Laxmi Salam** KV Langjing
VIII	Thermodynamics	**Mrs. Laxmi Salam** KV Langjing
IX	Behaviour of Perfect Gas and Kinetic Theory of gases	**Mr. B. K. Jha,** KV Karimganj
X	Oscillations and Waves	**Mr. B. K. Jha,** KV Karimganj

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MATHEMATICAL TOOLS

Physical constants:-

1. Mass of an electron (M_e) = $9.1x10^{-31}$kg.
2. Mass of a proton (M_p) = 1.6725×10^{-27}kg.
3. Mass of a neutron (M_n) = 1.6746×10^{-27}kg.
4. Charge of an electron (e) = -1.6×10^{-19}c
5. Speed of light in vacuum (c) = 3×10^{8} m/sec.
6. Planck Constant (h) = 6.6256×10^{-34} J x sec .
7. Universal Gravitation constant (G) = 6.67×10^{-11} $Nm^2/kg^{2.}$
8. Avogadro Number (N_A) = 6.023×10^{23} mol^{-1}.
9. Boltzmann constant (K) = 1.38×10^{-23} J/K
10. Stefan Constant (σ) = 5.67×10^{-8} W m^{-2} $K^{-4.}$
11. Wien Displacement Constant (b) = 2.898×10^{-3} m K
12. Solar Constant (S) = 1.388×10^{3} W m^{-2}
13. Mass of the sun (M_S) = 2×10^{30}kg.
14. Mass of the earth (M_E) = 5.98×10^{24}kg
15. Radius of the earth (R_e) = 6400 Km. = 6.4×10^{6}m.
16. Density of earth 5.522×10^{3}kg/m^3.
17. Average angular velocity of the earth = 7.29×10^{-5}rad./sec
18. Average distance between the sun and earth = 1.5×10^{11}m.
19. Average distance between moon and the earth =3.84×10^{8}m.
20. Magnetic Moment of the earth = 6.4×10^{21} Amp. X m^2.

Conversion Coefficients

1. 1Light year = 9.46×10^{15}m.
2. 1 A.U. = 1.496×10^{11}m.
3. 1Å = 10^{-10}m.
4. 1 Pound = 0.4536kg = 453.6 gm
5. 1 Fermi = 10^{-15}m.
6. 1 C.S.L.= 1.4 x Mass of the sun.

7. 1 Shake = 10^{-8}sec
8. 1ev = 1.6 x10^{-19} Joule.
9. 1 Horse Power = 746 Watt.

Quadratic Equation

An equation of second degree is called a quadratic equation. It is of the form :-

$$ax^2 + bx + c = 0$$

The roots of a quadratic equation are

$$X = \frac{-b \pm (b^2 + 4ac)^{1/2}}{2a}$$

Binomial Theorem

If n is any integer, positive or negative or a fraction and x is any real number, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{\underline{|2}} + \ldots$$

If IxI <<1 , then $(1+x)^n = 1 + nx$.

Mensuration :-

1. Area of a circle = $\pi r^2 = \pi D^2/4$
2. Surface area of a sphere = $4\pi r^2 = \pi D^2$
3. Volume of a sphere = $4/3\ \pi r^3$
4. Surface area of a cylinder = $2\pi r(r+l)$
5. Volume of a cylinder = $\pi r^2 l$
6. Curved surface area of a cone = πrl
7. Volume of a cone = $1/3\pi r^2 h$
8. Surface area of a cube = 6x(side)^2
9. Volume of a cube = (side)^3

Fundamental Trigonometric relations

$$\text{Cosec}\theta = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec}\theta = \frac{1}{\text{Cos}\theta}$$

$\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{1}{\text{Tan}\theta}$

$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

$1 + \tan^2\theta = \text{Sec}^2\theta$

$1 + \text{Cot}^2\theta = \text{Cosec}^2\theta$

Sin(A+B) = SinACosB + CosASinB

Cos (A + B) = CosACosB – SinASinB

Sin (A – B) = SinACosB – CosASinB

Cos (A – B) = CosACosB + SinASinB

$$\text{Tan}(\text{A}+\text{B}) = \frac{\text{TanA}+\text{TanB}}{1-\text{TanATanB}}$$

Sin 2A = 2SinACosA

$\text{Cos } 2A = 2\text{Cos}^2A - 1 = 1 - 2\text{Sin}^2A = \text{Cos}^2A - \text{Sin}^2A$

Sin (A+B) + Sin(A-B) = 2 SinACosB

Cos(A+B) + Cos (A-B) = 2CosACosB

$$\text{CosC} + \text{CosD} = 2\text{Cos}\frac{(\text{C}+\text{D})}{2}\text{Cos}\frac{(\text{C}-\text{D})}{2})$$

Logarithms

$\text{Log}_a mn = \text{Log}_a m + \text{Log}_a n$

$\text{Log}_a (\frac{m}{n}) = \text{Log}_a m - \text{Log}_a n$

$\text{Log}_a m = \text{Log}_b m \times \text{Log}_a b$

$\text{Log}_{10}10^3 = \text{Log}_{10}1000 = 3$

$\text{Log}_a 1 = 0$

$\text{Log}_a a = 1$

Average Values

$< \text{Sin}\theta > = 0$, $< \text{Cos}\theta > = 0$

$< \text{Sin}^2\theta > = ½$

$< \text{Cos}^2 \theta > = ½$

Approximate Values

If angle (θ) small then $\theta \longrightarrow 0$

$\text{Sin}\theta \cong \theta$

$\text{Cos}\theta \cong 1$

$\text{Tan}\theta \cong \theta$

Differential Formulae

1. Differentiation of a constant© is zero

$$\frac{dc}{dx} = 0$$

2. $\frac{\mathrm{d(cy)}}{\mathrm{dx}} = \mathrm{c}\frac{\mathrm{dy}}{\mathrm{dx}}$

3. $\frac{d(x^n)}{dx} = nx^{n-1}$

4. $\frac{\mathrm{d[f(x)\pm g(x)]}}{\mathrm{dx}} = \frac{\mathrm{df(x)}}{\mathrm{dx}} \pm \frac{\mathrm{dg(x)}}{\mathrm{dx}}$

5. $\frac{\mathrm{d\{f(x)g(x)\}}}{\mathrm{dx}} = \frac{\mathrm{f(x)dg(x)}}{\mathrm{dx}} + \frac{\mathrm{g(x)df(x)}}{\mathrm{dx}}$

6. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{\{g(x)\}^2}$

7. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

8. $\frac{de^x}{dx} = e^x$

9. $\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx}$

10. $$\frac{dlog_e x}{dx} = \frac{1}{x}$$

11. $$\frac{d(a^x)}{dx} = a^x log_{e^a}$$

12. $$\frac{dlog_a x}{dx} = \frac{1}{x} log_{e^a}$$

13. $$\frac{d(\sin x)}{dx} = \cos x$$

14. $$\frac{d(\cos x)}{dx} = -\sin x$$

15. $$\frac{d(\tan x)}{dx} = sec^2 x$$

16. $$\frac{d(\cot x)}{dx} = -cosec^2 x$$

17. $$\frac{d(cosecx)}{dx} = -\operatorname{cosec} x\ cot\ x$$

18. $$\frac{d(secx)}{dx} = \sec x\ tan\ x$$

Integral Formulae

1. ∫dx =x +c Where c = constant
2. ∫x^{n+1}dx = $\frac{x^{n+1}}{n+1} + C$
3. ∫dx/x = $\log_e$x +c
4. ∫Sinx dx = - Cosx +c
5. ∫Sin ax dx = -$\frac{\text{Cos ax}}{\text{a}}$
6. ∫ Cos x dx = Sin x +c
7. ∫Sec^2x dx = tan x +c
8. ∫ $Cosec^2$x dx = -Cot x +c
9. ∫ Sec x tan x dx = Sec x +c
10. ∫Cosec x Cot x dx = - Cosec x + c
11. ∫ e^x dx = e^x +c

Physical World And Measurement

There are four fundamental forces which govern both macroscopic and microscopic phenomena. There are

(i) Gravitational force (iii) Electromagnetic force

(ii) Nuclear force (iv) Weak force

The relative strengths of these forces are

$$Fg :Fw:Fe:Fs=1:10^{25}:10^{36}:10^{38} .$$

All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.

(a) Fundamental quantities (b) Derived quantities.

The units of the fundamental quantities called fundamental units , and the units of derived quantities called derived units.

System of units:-

(a) MKS (b) CGS (c) FPS (d) SI

- *The dimensions of a physical quantity are the powers to which the fundamental quantities are raised to represent that physical quantity.*

- The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time, is called dimensional equation.
- According to this principle of homogeneity a physical equation will be dimensionally correct if the dimensions of all the terms in the all the terms occurring on both sides of the equation are the same.
- If any equation is dimensionally correct it is not necessary that must be mathematically correct too.
- There are three main uses of the dimensional analysis-

(a) To convert a unit of given physical quantities from one system of units to another system for which we use

$$n_2 = n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$$

(b) To check the correctness of a given physical relation.

(c) To derive a relationship between different physical quantities.

- Significant figures: - The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

For counting of the significant figure rule are as:

(i) All non- zero digits are significant figure.

(ii) All zero between two non-zero digits are significant figure.

(iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant. But such zeros are significant if they come from a measurement.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant.

(v) All zeros to the right of a decimal point are significant.

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. Single zero conventionally placed to the left of the decimal point is not significant.

(vii) The number of significant figures does not depend on the system of units.

- In addition or subtraction, the result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.

- In multiplication or division, the result should be reported to the same number of significant figures as that of the number with minimum of significant figures.
- Accuracy refers to the closeness of a measurement to the true value of the physical quantity and precision refers to the resolution or the limit to which the quantity is measured.
- Difference between measured value and true value of a quantity represents error of measurement.

  It gives an indication of the limits within which the true value may lie.

Mean of n measurements

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \cdots \ldots + a_n}{n}$$

Absolute error (Δa) = a_{mean} - a_i Where a_i = measured value

It may be - positive, negative or zero.

(i) Mean absolute error

(ii) Relative error - it is the ratio of the mean absolute error to the true value.

$$\delta a = |\Delta a| / a_{mean}$$

(iii) The relative error expressed in percent is called percentage error.

The error is communicated in different mathematical operations as detailed below:

(i) For $x = (a \pm b)$, $\Delta x = \pm (\Delta a + \Delta b)$

(ii) For $x = a \times b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

(iii) For $x = a/b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

(iv) For $x = a^n b^m / c^p$ $\Delta x/x = \pm (n\Delta a/a + m \Delta b/b + p\Delta c/c$

Very short answer type questions, (1 mark question)

Q1. State one law that holds good in all natural processes.

Ans. One such laws is the Newton's gravitation law, According to this law everybody in this nature are attracts with other body with a force of attraction which is directly proportional to the product of their masses and inversely proportionally To the square of the distance between them.

Q2: Among which type of elementary particles does the electromagnetic force act?

Ans : Electromagnetic force acts between on all electrically charged particles.

Q3. Name the forces having the longest and shortest range of operation.

Ans : longest range force is gravitational force and nuclear force is shortest range force.

Q4. If 'slap' times speed equals power, what will be the dimensional equation for 'slap'?

Ans . Slap x speed = power

Or slap = power/speed = $[MLT^{-2}]$

Q5. If the units of force and length each are doubled, then how many times the unit of energy would be affected?

Ans : Energy = Work done = Force x length

So when the units are doubled, then the unit of energy will increase four times.

Q6. Can a quantity has dimensions but still has no units?

Ans : No, a quantity having dimension must have some units of its measurement.

Q7. Justify L +L = L and L – L =L.

Ans: When we add or subtract a length from length we get length, So L +L =L AND L – L =L, justify.

Q8. Can there be a physical quantity that has no unit and no dimensions?

Ans : yes, like strain.

Q9. Given relative error in the measurement of length is 0.02, what is the percentage error?

Ans: percentage error = 2 %

Q10. If g is the acceleration due to gravity and λ is wavelength, then which physical quantity does represented by $\sqrt{g\lambda}$.

Ans. Speed or velocity.

Short answer type questions (2 marks)

Q1.If heat dissipated in a resistance can be determined from the relation:

$H = I^2Rt$ joule , If the maximum error in the measurement of current, resistance and time are 2% ,1% , and 1% respectively, What would be the maximum error in the dissipated heat?

Ans: % error in heat dissipated is ±6 %.

Q2. Name any three physical quantities having the same dimensions and also give their dimensions.

Ans : Any group of physical quantities, like work , energy and torque and their dimensions $[ML^2 T^{-2}]$.

Q3. In Van der Wall's equation $(P + a/V^2)(V – b) = RT$, Determine the dimensions of a and b.

Ans : $[a] = [ML^5 T^{-2}]$ and $[b] = [M^0L^3T^0]$.

Q4. Give the limitations of dimensional analysis.

Ans ……………………………

Q5. If $X= a+ bt^2$, where X is in meter and t is in second . find the unit of a and b?

Ans : unit of a is meter and unit of b is $m/sec^{2.}$

Q6. What is meant by significant figures ? State the rules for counting the number of significant figures in a measured quantity?

Ans. …………………………

Q7. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual relative errors.

Ans : For $x = a/b$, $\Delta x/x = \pm (\Delta a/a + \Delta b/b)$

Q8. Deduce the dimensional formulae for the following physical quantities.

A) Gravitational constant.

B) Power

C) coefficient of viscosity

D) Surface tension.

Ans: (A) gravitational constant = $[M^{-1} L^3 T^{-2}]$,

B) Power = $[ML^2T^{-3}]$

C) Coefficient of viscosity = $[ML^{-1} T^{-1}]$

D) Surface tension = $[ML^0T^{-2}]$

Q9. Name the four basic forces in nature. Arrange them in the order of their increasing strengths.

Ans : (i) Gravitational force (ii) Electromagnetic force

(iii) nuclear force (iv) Weak force

The relative strengths of these forces are

$$Fg :Fw:Fe:Fs=1:10^{25}:10^{36}:10^{38} .$$

Q10. Convert 1 Newton force in to Dyne.

Ans : $1N = 10^5$ Dyne.

Short answer type questions (3marks)

Q1. If E,M,J and G respectively denote energy, mass, angular momentum and gravitational constant, Calculate the dimensions of EJ^2/M^5G^2

Q2. The frequency ν of vibration of stretched string depends on its length L its mass per unit length m and the tension T in the string obtain dimensionally an expression for frequency ν.

Q3. What is meant by significant figures .State the rules for counting the number of significant figures in a measured quantity?

Q4. A physical quantity X is given by $X = A^2B^3/C\sqrt{D}$, If the percentage errors of measurement in A,B,C and D are 4%,2%,3% and 1% respectively, then calculate the % error in X.

Q5. If two resistors of resistance $R_1=(4 \pm 0.5)\Omega$ and $R_2=(16 \pm 0.5)\Omega$ are connected (1) In series and (2) Parallel . Find the equivalent resistance in each case with limits of % error.

Q6. The length of a rod measured in an experiment was found to be 2.48m, 2.46, 2.50m and 2.48m and 2.49m, Find the average length , the absolute error in each observation and % error.

Q7. A famous relation in physics relates moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of the light c. A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = m_o / (1 - v^2)^{1/2}$$

Guess where to put the missing c.

Q8. A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 4.2 kgm^2s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the units of time is ϒ sec. show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \Upsilon^2$ in terms of the new units.

Q9. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively, what are the dimensions of Y in MKS system?

Q10. In an experiment, on the measurement of g using a simple pendulum the time period was measured with an accuracy of 0.2 % while the length was measured with accuracy of 0.5%. Calculate the percentage error in the value of g.

Long answer question (5 marks)

Q1. Explain:

(i) Absolute error (iii) Mean absolute error

(ii) Relative error (iv) percentage error

(v) Random error

Q2. Convert:

(i) Gravitational constant (G) = 6.67×10^{-11} $Nm^2\ kg^{-2}$ to $cm^3\ g^{-1}\ s^{-2}$ (ii) The escape velocity v of a body depends on, the acceleration due to gravity 'g' of the planet and the radius R of the planet, Establish dimensionally for relation for the escape velocity.

Q3. Name the four basic forces in nature. Write a brief note of each, hence compare their strengths and ranges.

HOTs

Q1. What are the dimensions of $^1/u_0\epsilon_0$, where symbols have their usual meaning.

Ans : [$M^0\ L^2\ T^{-2}$]

Q2.What is the dimensions of $(^1/_2)\epsilon_0 E^2$, Where E electric field and ϵ_0 permittivity of free space.

Ans : [$M^1\ L^{-1}\ T^{-2}$]

Q3. The pairs of physical quantities that have the same dimensions are:

(a) Reynolds's number and coefficient of friction,

(b) Curie and frequency of a light wave

(c) Latent heat and gravitational potential

(d) Planck's constant and torque.

Ans : (a), (b).

Q4. If L,C,R represent inductance , capacitance and resistance respectively, the combinations having dimensions of frequency are

(a) $^{1}/\sqrt{CL}$ (b) L/C (c) R/L (d) R/C

Ans : (a) and (c).

Q5. If the error in radius is 3%, what is error in volume of sphere?

(a) 3 % (b) 27 % (c) 9 % (d) 6 %

Ans : (c) 9%.

KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

Displacement ≤ Actual distance

*Speed:- It is rate of change of distance covered by the body with respect to time.

Speed = Distance travelled /time taken

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ $sec.^2$ and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,	For deceleration motion
$V = u + at$	$v = u - at$
$S = ut + {}^1/_2\, at^2$	$S = ut - {}^1/_2\, at^2$
$V^2 = u^2 + 2as$	$V^2 = u^2 - 2as$
$Sn = u + {}^a/_2(2n-1)$	$Sn = u - {}^a/_2(2n-1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When V_A and V_B are incident to each other at angle Θ

$$V_{AB} = (V^2{}_A + V^2{}_B - 2V_AV_B\text{Cos }\Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**
  - (i) Only vectors of same nature can be added.
  - (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan\beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

- (iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- (iv) Vector addition is associative,

  $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
- (v) R is maximum if Θ = 0 and minimum if Θ = 180^0.

Subtraction of two vectors :-

- (i) Only vector of same nature can be subtracted.
- (ii) Subtraction of B from A = vector addition of A and (-B),

  $\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$

Where $R = [A^2 + B^2 + 2AB \cos(180 - \Theta)]^{1/2}$ and

$\tan\beta = B \sin(180 - \Theta) / [A + B \cos(180 - \Theta)]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

- (iii) Vector subtraction is not commutative $\overrightarrow{(A} - \vec{B}) \neq (\vec{B} - \vec{A})$
- (iv) Vector subtraction is not associative,

  $(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Here $A_x = A\cos\Theta$ and $A_y = A\sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A}\,.\,\vec{B} = AB\cos\Theta$$

(i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\,\hat{k} = 1$

(ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\,\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad \hat{\jmath} \times \hat{k} = \hat{I}$ $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

$$V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$$

- Horizontal range

$$R = u^2 \sin 2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

- Flight time

$$T = 2u \sin\Theta/g$$

- Height

$$H = u^2 \sin^2\Theta/2g$$

For maximum height $\Theta = 90^0$

$$H_{max.} = u^2/2g$$

Very Short answer type questions (1 marks)

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^0.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : $W = 2\pi/12 = \pi/6$ rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

Short Answer type question (2 marks)

Q1. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n-1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54\ m/sec^2$.

Q3.what is relative velocity in one dimension, if V_A and V_B are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \sin 2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

and Height

$$H = u^2 \sin^2\Theta/2g$$

For $\Theta = 45^0$

$$H = u^2/4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan\beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r\,\omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max}/2$$

$$\text{So} \quad h = R/2$$

$$\text{Or} \quad R = 2h$$

Short answer questions (3 marks)

Q1. If ‘R’ is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0.}$

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{B} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

Long answer question (5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $u\text{Cos}45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and ($90 - \Theta$) ,

$$\text{then} \qquad t_1 = 2u \, Sin\Theta/g$$

$$t_2 = 2u \, Sin(90 - \Theta)/g = 2u \, Cos\Theta/g$$

$$t_1 t_2 = 2R/g \, .$$

NEWTON'S LAWS OF MOTION

Newton' 1st law or Law of Inertia

Every body continues to be in its state of rest or of uniform motion until and unless and until it is compelled by an external force to change its state of rest or of uniform motion.

Inertia

The property by virtue of which a body opposes any change in its state of rest or of uniform motion is known as inertia. Greater the mass of the body greater is the inertia. That is **mass is the measure of the inertia of the body**.

Numerical Application

If, $\vec{F} = 0$; $\vec{u}$ = constant

Physical Application

1. When a moving bus suddenly stops, passenger's head gets jerked in the forward direction.
2. When a stationery bus suddenly starts moving passenger's head gets jerked in the backward direction.
3. On hitting used mattress by a stick, dust particles come out of it.
4. In order to catch a moving bus safely we must run forward in the direction of motion of bus.
5. Whenever it is required to jump off a moving bus, we must always run for a short distance after jumping on road to prevent us from falling in the forward direction.

Key Concept

In the absence of external applied force velocity of body remains unchanged.

Newton' 2nd law

Rate of change of momentum is directly proportional to the applied force and this change always takes place in the direction of the applied force.

$$\frac{d\vec{p}}{dt} \propto \vec{F}$$

or, $\frac{d\vec{p}}{dt} = \vec{F}$ (here proportionality constant is 1)

putting, $\vec{p} = m\vec{v}$

$\vec{F} = \frac{d\vec{p}}{dt}$

or, $\vec{F} = \frac{dm\vec{v}}{dt}$

or, $\boxed{\vec{F} = \frac{md\vec{v}}{dt} + \frac{\vec{v}dm}{dt}}$

or, $\vec{F} = \frac{md\vec{v}}{dt}$ (if m is constant dm/dt = 0)

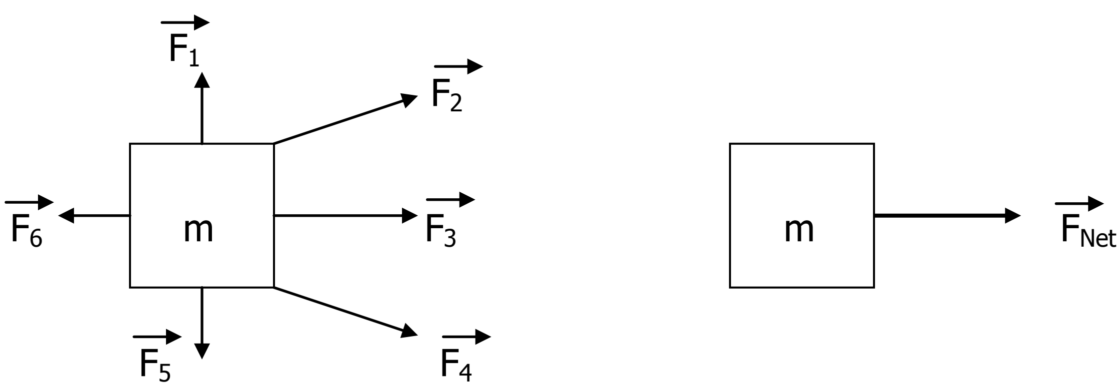
or, $\boxed{\mathbf{\vec{F} = m\vec{a}}}$

Note :- Above result is not Newton's second law rather it is the conditional result obtained from it, under the condition when m = constant.

Numerical Application

$$\boxed{\vec{a} = \frac{\vec{F}_{Net}}{m}}$$

Where $\vec{F}_{Net}$ is the vector resultant of all the forces acting on the body.

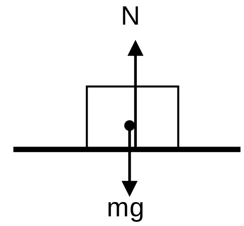


Where, $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6$

Physical Application

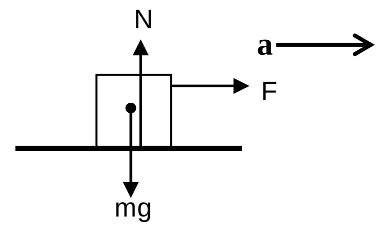
Horizontal Plane

i) Case - 1
Body kept on horizontal plane is at rest.



For vertical direction
N = mg(since body is at rest)

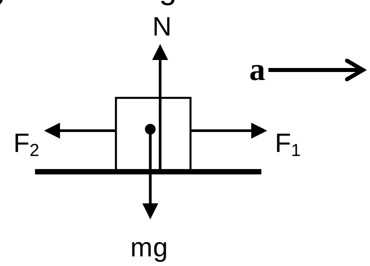
ii) Body kept on horizontal plane is accelerating horizontally under single horizontal force.



For vertical direction
N = mg (since body is at rest)

For horizontal direction
F = ma

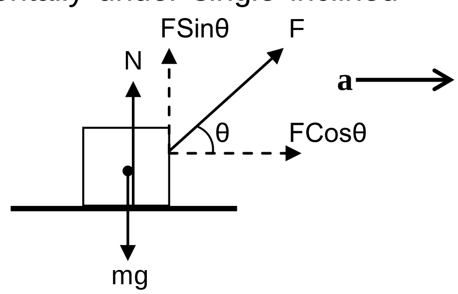
iii) Body kept on horizontal plane is accelerating horizontally towards right under two horizontal forces. ($F_1 > F_2$)



For vertical direction
N = mg (since body is at rest)

For horizontal direction
$F_1 - F_2 = ma$

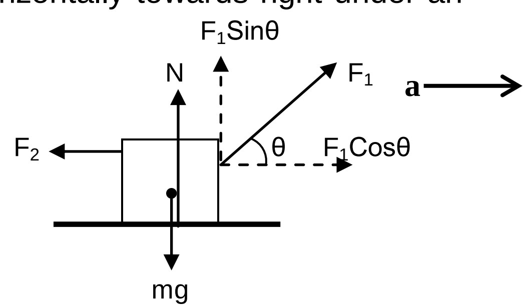
iv) Body kept on horizontal plane is accelerating horizontally under single inclined force



For vertical direction
N + FSinθ = mg (since body is at rest)

For horizontal direction
FCosθ = ma

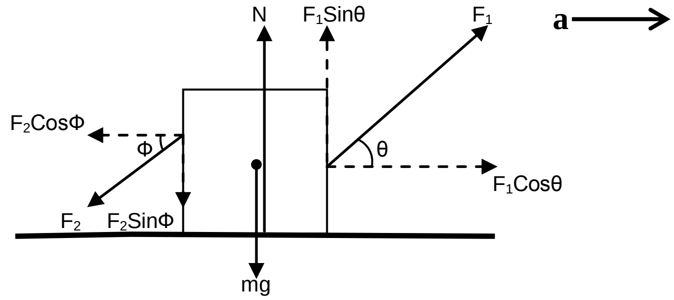
v) Body kept on horizontal plane is accelerating horizontally towards right under an inclined force and a horizontal force.



For vertical direction
$N + F_1 Sin\theta = mg$ (since body is at rest)

For horizontal direction
$F_1 Cos\theta - F_2 = ma$

vi) Body kept on horizontal plane is accelerating horizontally towards right under two inclined forces acting on opposite sides.

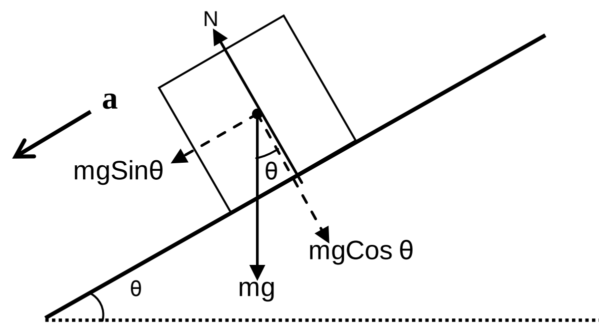


For vertical direction
$N + F_1Sin\theta = mg + F_2\ Sin\Phi$
(since body is at rest)

For horizontal direction
$F_1Cos\theta – F_2Cos\Phi = ma$

Inclined Plane

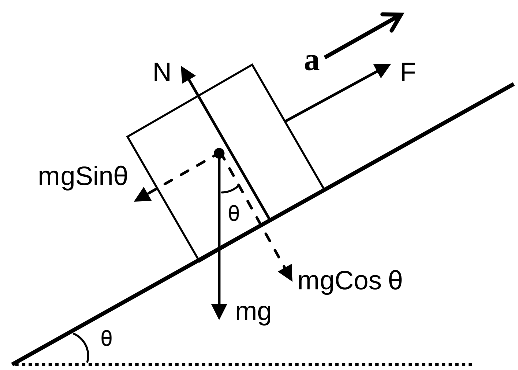
i) Case - 1
Body sliding freely on inclined plane.



Perpendicular to the plane
N = mgCosθ (since body is at rest)

Parallel to the plane
mgSinθ = ma

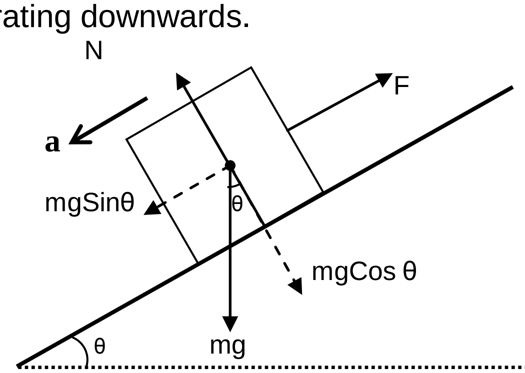
ii) Case - 2
Body pulled parallel to the inclined plane.



Perpendicular to the plane
N = mgCosθ (since body is at rest)

Parallel to the plane
F - mgSinθ = ma

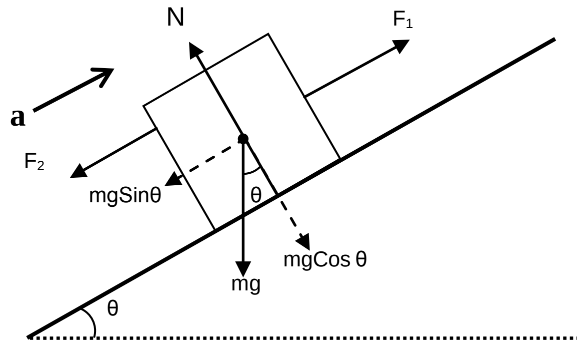
iii) Case - 3
Body pulled parallel to the inclined plane but accelerating downwards.



Perpendicular to the plane
N = mgCosθ (since body is at rest)

Parallel to the plane
mgSinθ - F = ma

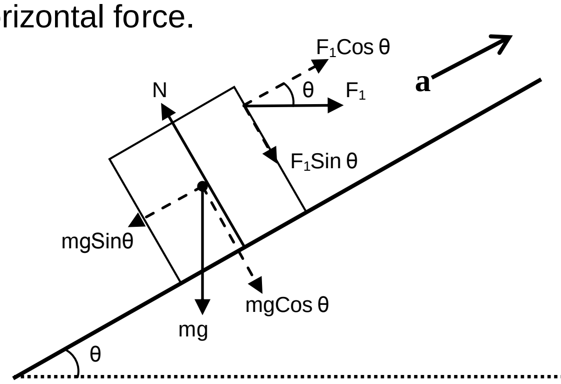
iv) Case - 4
Body accelerating up the incline under the effect of two forces acting parallel to the incline.



Perpendicular to the plane
N = mgCosθ (since body is at rest)

Parallel to the plane
F_1 - F_2 - mgSinθ = ma

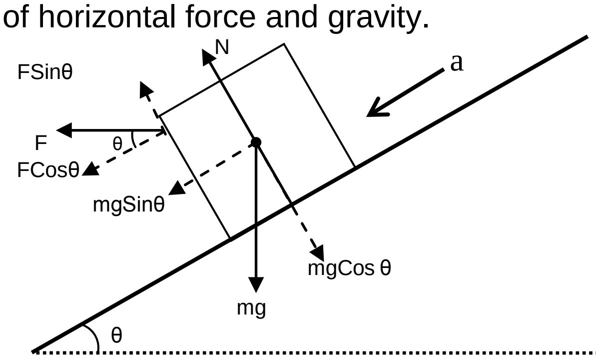
v) Case - 5
Body accelerating up the incline under the effect of horizontal force.



Perpendicular to the plane
N = mgCosθ + F_1Sinθ (since body is at rest)

Parallel to the plane
F_1Cosθ - mgSinθ = ma

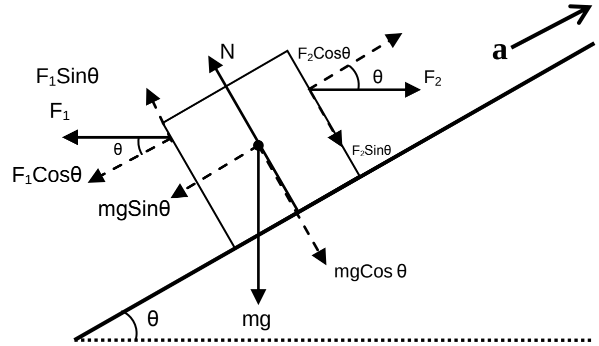
vi) Case - 6
Body accelerating down the incline under the effect of horizontal force and gravity.



Perpendicular to the plane
N + FSinθ = mgCosθ (since body is at rest)

Parallel to the plane
FCosθ + mgSinθ = ma

vii) Case - 7
Body accelerating up the incline under the effect of two horizontal forces acting on opposite sides of a body and gravity.

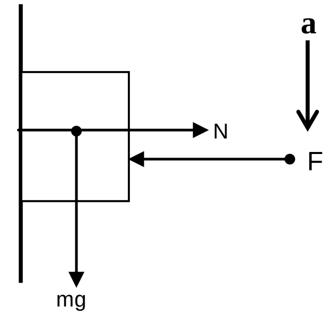


Perpendicular to the plane
N + F_1Sinθ = mgCosθ + F_2Sinθ(since body is at rest)

Parallel to the plane
F_2Cosθ - F_1Cosθ - mgSinθ = ma

Vertical Plane

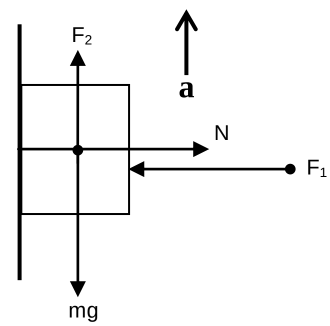
i) Case - 1
Body pushed against the vertical plane by horizontal force and moving vertically downward.



For horizontal direction
mg = ma (since body is at rest)

For vertical direction
F = N

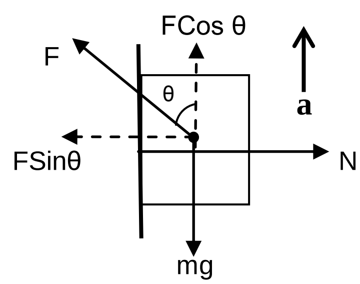
ii) Case - 2
Body pushed against the vertical plane by horizontal force and pulled vertically upward.



For vertical direction
F_2 - mg = ma

For horizontal direction (since body is at rest)
N = F_1

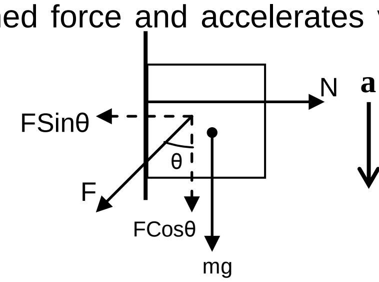
iii) Case - 3
Body pushed against the vertical plane by inclined force and accelerates vertically upward.



For horizontal direction
N = FSinθ (since body is at rest)

For vertical direction
FCosθ – mg = ma

iv) Case - 3
Body pushed against the vertical plane by inclined force and accelerates vertically downward.

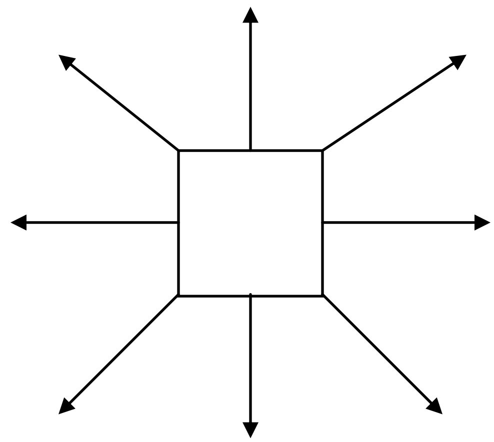


For horizontal direction
N = FSinθ (since body is at rest)

For vertical direction
FCosθ + mg = ma

Tension In A Light String

Force applied by any linear object such as string, rope, chain, rod etc. is known as it's tension. Since string is a highly flexible object so it can only pull the object and can never push. Hence tension of the string always acts away from the body to which it is attached irrespective of the direction.



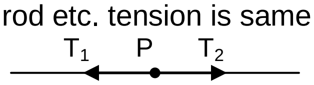
Tension of the string, being of pulling nature, always acts away from the body to which it is attached

Physical Application

i) Flexible wire holding the lamp pulls the lamp in upward direction and pulls the point of suspension in the downward direction.
ii) Rope holding the bucket in the well pulls the bucket in the upward direction and the pulley in the downward direction.
iii) Rope attached between the cattle and the peg pulls the cattle towards the peg and peg towards the cattle.
iv) When a block is pulled by the chain, the chain pulls the block in forward direction and the person holding the chain in reverse direction.

Key Point

In case of light string, rope, chain, rod etc. tension is same all along their lengths.



Consider a point P on a light (massless) string. Let tensions on either side of it be T_1 and T_2 respectively and the string be accelerating towards left under these forces. Then for point P

$$T_1 - T_2 = ma$$

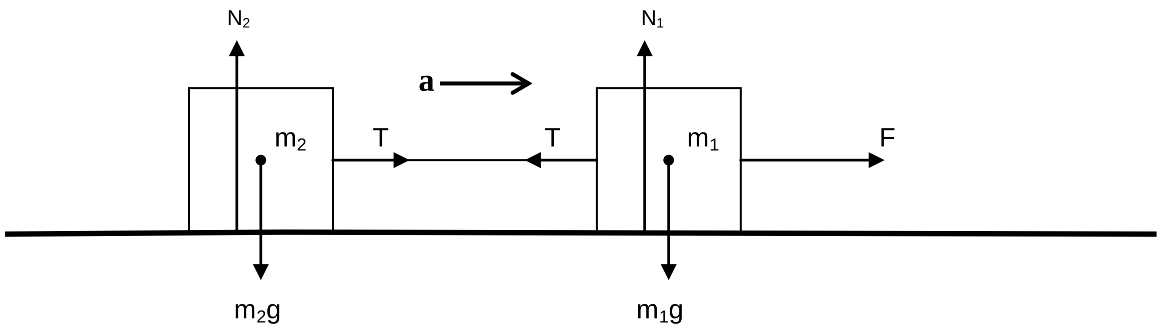
Since string is considered to be light mass m of point P is zero

or, $$T_1 - T_2 = 0$$

or, $$\mathbf{T_1 = T_2}$$

i) Case - 1

Two bodies connected by a string are placed on a smooth horizontal plane and pulled by a horizontal force.



For vertical equilibrium of m_1 and m_2

$N_1 = m_1g$ and $N_2 = m_2g$

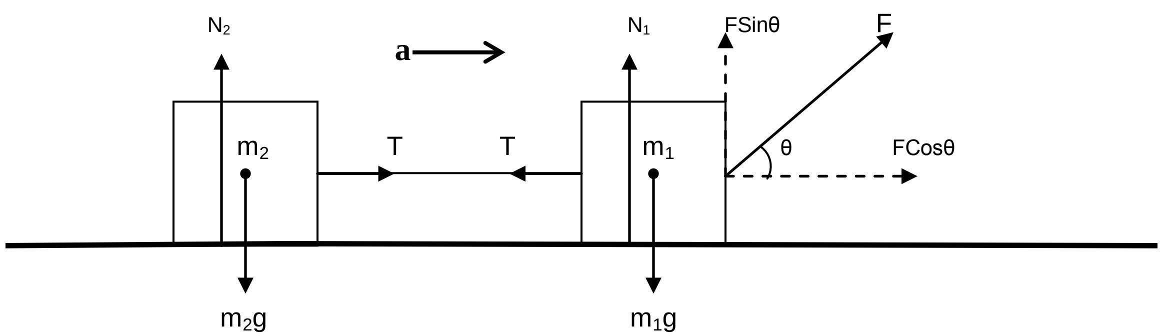
For horizontal acceleration of m_1 and m_2

$F - T = m_1a$ and $T = m_2a$

(Since both the bodies are connected to the same single string they have same acceleration)

ii) Case - 2

Two bodies connected by a horizontal string are placed on a smooth horizontal plane and pulled by a inclined force.



For vertical equilibrium of m_1 and m_2

$N_1 + F\sin\theta = m_1g$ and $N_2 = m_2g$

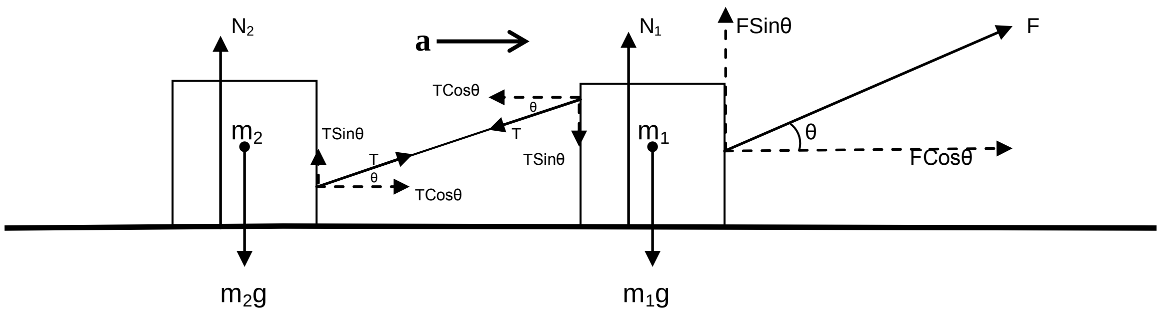
For horizontal acceleration of m_1 and m_2

$F\cos\theta - T = m_1a$ and $T = m_2a$

(since both the bodies are connected to the same single string they have same accelerations)

iii) Case - 3

Two bodies connected by a inclined string are placed on a smooth horizontal plane and pulled by a inclined force.



For vertical equilibrium of m_1 and m_2

$\mathbf{N_1 + FSin\theta = m_1g + TSin\theta}$ **and** $\mathbf{N_2 + TSin\theta = m_2g}$

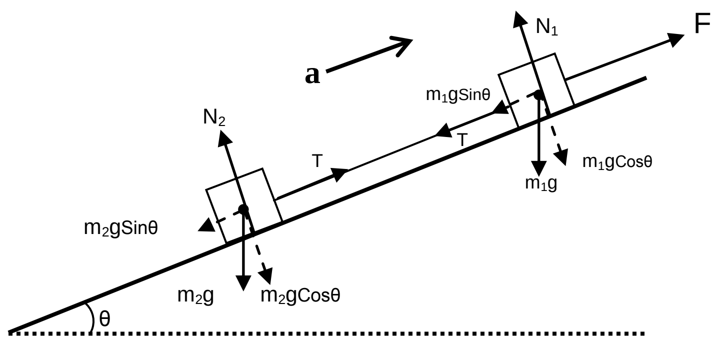
For horizontal acceleration of m_1 and m_2

$\mathbf{FCos\theta - TCos\theta = m_1a}$ **and** $\mathbf{TCos\theta = m_2a}$

(since both the bodies are connected to the same single string they have same accelerations)

iv) Case - 4

Two bodies connected by a string made to accelerate up the incline by applying force parallel to the incline.



For equilibrium of m_1 and m_2 in the direction perpendicular to the plane

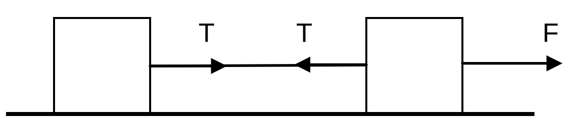
$\mathbf{N_1 = m_1gCos\theta}$ **and** $\mathbf{N_2 = m_2gCos\theta}$

For acceleration of m_1 and m_2 up the incline

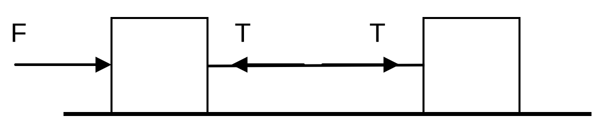
$\mathbf{F - T - m_1gSin\theta = m_1a}$ **and** $\mathbf{T - m_2gSin\theta = m_2a}$

Tension of A light Rigid Rod

Force applied by rod is also known as its tension. Since rod is rigid, it cannot bend like string. Hence rod can pull as well as push. Tension of rod can be of pulling as well as pushing nature but one at a time. Tension of a rod attached to the body may be directed towards as well as away from the body.



Tension of rod is pulling both the blocks



Tension of rod is pushing both the blocks

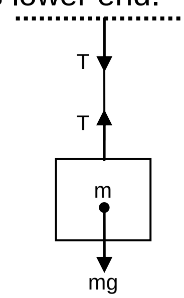
Physical Application

i) Pillars supporting the house **pushes** the house in the upward direction and **pushes** the ground in the downward direction.
ii) Wooden bars used in the chair **pushes** the ground in the downward direction and **pushes** the seating top in the upward direction.
iii) Parallel bars attached to the ice-cream trolley **pushes** the trolley in the forward direction and **pushes** the ice-cream vendor in the backward direction.(when the trolley is being pushed by the vendor)
iv) Rod holding the ceiling fan **pulls** the fan in the upward direction and **pulls** the hook attached to the ceiling in the downward direction.
v) Parallel rods attached between the cart and the bull **pulls** the cart in the forward direction and **pulls** the bull in the backward direction.

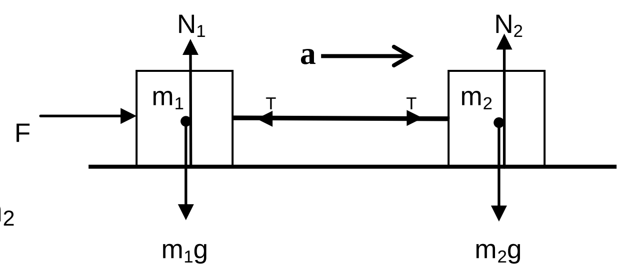
Different Cases of Light Rigid Rod

i) Case - 1
Rod attached from the ceiling and supporting the block attached to its lower end. Since the block is at rest

T = mg



ii) Case - 2
Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pushing force.

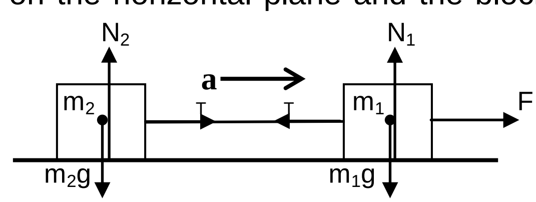


For vertical equilibrium of m_1 and m_2
$N_1 = m_1g$ and $N_2 = m_2g$

For horizontal acceleration of m_1 and m_2
$F - T = m_1a$ and $T = m_2a$

(Since both the bodies connected to the rod will have same acceleration)

iii) Case - 3
Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pulling force.



For vertical equilibrium of m_1 and m_2
$N_1 = m_1g$ and $N_2 = m_2g$

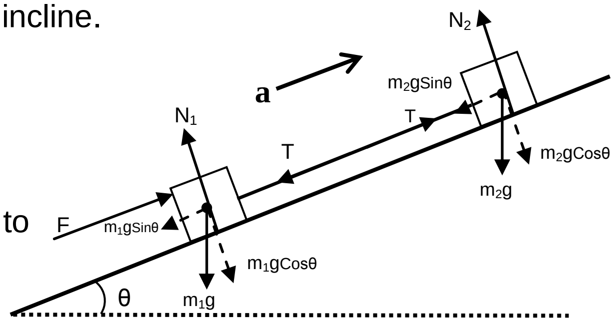
For horizontal acceleration of m_1 and m_2

$F - T = m_1a$ and $T = m_2a$

(Since both the bodies are connected to the same rod they have same acceleration)

iv) Case - 4

Rod is attached between two blocks placed on the incline plane and the blocks are accelerated by pushing parallel to the incline.



For vertical equilibrium of m_1 and m_2

$N_1 = m_1gCos\theta$ and $N_2 = m_2gCos\theta$

For acceleration of m_1 and m_2 parallel to the incline

$F - m_1gSin\theta - T = m_1a$,

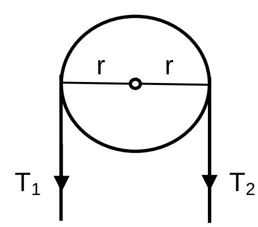
$T - m_2gSin\theta = m_2a$

Fixed Pulley

It is a simple machine in the form of a circular disc or rim supported by spokes having groove at its periphery. It is free to rotate about an axis passing through its center and perpendicular to its plane.

Key Point

In case of light pulley, tension in the rope on both the sides of the pulley is same (to be proved in the rotational mechanics)



Anticlockwise Torque - Clockwise Torque = Moment of Inertia x Angular acceleration

$$T_1 \times r - T_2 \times r = I\alpha$$

Since the pulley is light and hence considered to be massless, it's moment of inertia

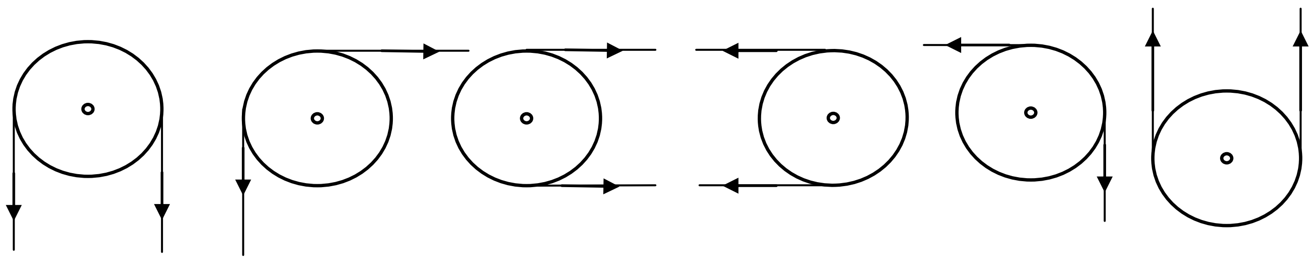
$$I = 0$$

or, $$T_1 \times r - T_2 \times r = 0$$

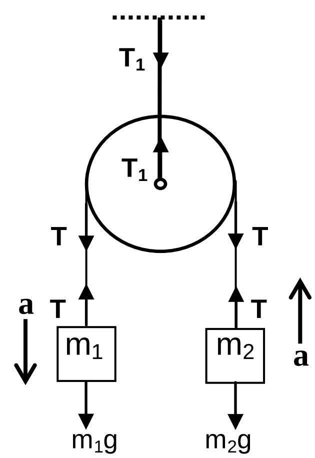
or, $$T_1 \times r = T_2 \times r$$

or, $$\mathbf{T_1 = T_2}$$

Different Cases of Fixed Pulley



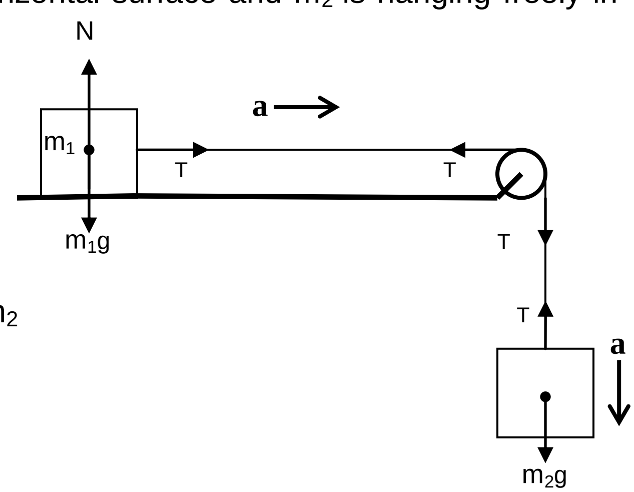
i) Case - 1
Two bodies of different masses ($m_1 > m_2$) are attached at two ends of a light string passing over a smooth light pulley



For vertical equilibrium of pulley
$\mathbf{T_1 = T + T = 2T}$

For vertical acceleration of m_1 and m_2
$\mathbf{m_1g - T = m_1a}$ and $\mathbf{T - m_2g = m_2a}$
m_1 accelerates downwards and m_2 accelerates upwards(m_1>m_2)

ii) Case - 2
Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on a horizontal surface and m_2 is hanging freely in air.

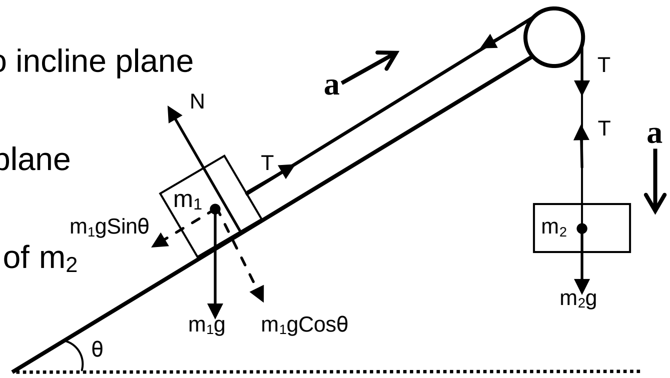


For vertical equilibrium m_1
$\mathbf{N = m_1g}$

For horizontal acceleration of m_1
$\mathbf{T = m_1a}$

For vertically downward acceleration of m_2
$\mathbf{m_2g - T = m_2a}$

iii) Case - 3
Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on an inclined surface and m_2 is hanging freely in air.



For equilibrium of m_1 perpendicular to incline plane
$N = m_1gCos\theta$

For acceleration of m_1 up the incline plane
$T - m_1gSin\theta = m_1a$

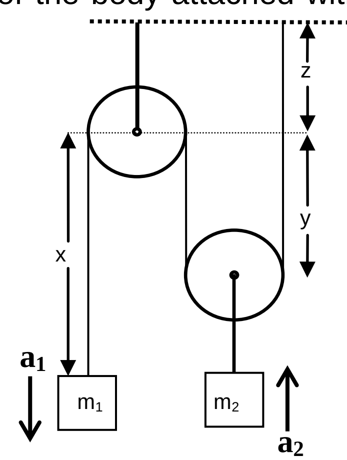
For vertically downward acceleration of m_2
$m_2g - T = m_2a$

Movable Pulley

The pulley which moves in itself is known as movable pulley.

Key Point

In case of light movable pulley, acceleration of a body (pulley) goes on decreasing on increasing the number of strings attached to it. That is the body attached with two ropes moves with half the acceleration of the body attached with single rope.



Length of the string is constant

$x + 2y + z = L$ (Constant)

Differentiating both sides with respect to t (Time)

$$\frac{dx}{dt} + 2\frac{dy}{dt} + \frac{dz}{dt} = \frac{dL}{dt}$$

or, $v_1 + 2v_2 + 0 = 0$ (z and L are constant)

or, $v_1 + 2v_2 = 0$

Again differentiating both sides with respect to t

$$\frac{dv_1}{dt} + 2\frac{dv_2}{dt} = 0$$

or, $a_1 + 2a_2 = 0$

or, $\mathbf{a_1 = - 2a_2}$

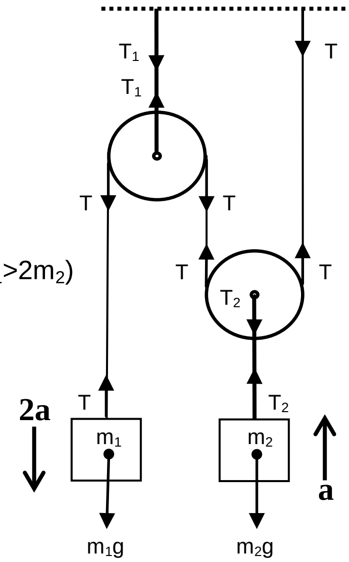
That is acceleration of **m_1** (body attached to a single string) is opposite and twice the acceleration of **m_2** (body attached to a double string)

Different Cases of Light Movable Pulley

i) Case - 1

Mass m_1 is attached at one end of the string and the other end is fixed to a rigid support. Mass m_2 is attached to the light movable pulley.

For vertical acceleration of m_1
$m_1g - T = m_1 2a$ (m_1 is connected to a single string)



For vertical acceleration of m_2
$T_2 – m_2g = m_2a$
(m_1 accelerates downwards and m_2 accelerates upwards since $m_1>2m_2$)

For the clamp holding the first pulley
$T_1 = 2T$

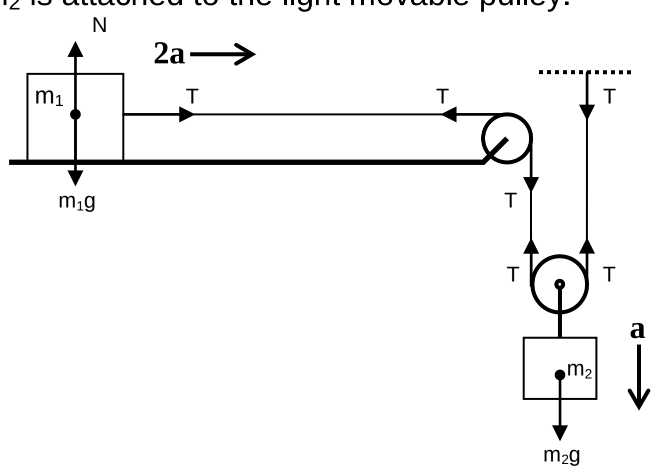
For the clamp holding the movable pulley
$2T - T_2 = m_{pulley}a$
or, $2T - T_2 = 0$ (light pulley)
or, **$2T = T_2$**

ii) Case - 2
Mass m_1 is attached at one end of the string and placed on a smooth horizontal surface and the other end is fixed to a rigid support after passing through a light movable suspended pulley. Mass m_2 is attached to the light movable pulley.

For vertical equilibrium of m_1
$N = m_1g$

For horizontal acceleration of m_1
$T = m_1 2a$

For vertical motion of m_2
$m_2g – 2T = m_2a$

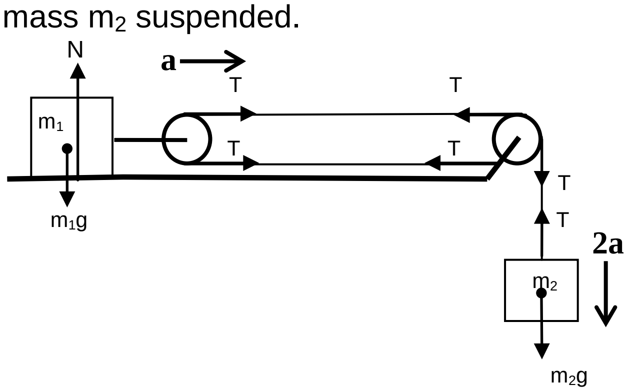


iii) Case - 3
Mass m_1 is attached to the movable pulley and placed on a smooth horizontal surface. One end of the string is attached to the clamp holding the pulley fixed to the horizontal surface and from its other end mass m_2 suspended.

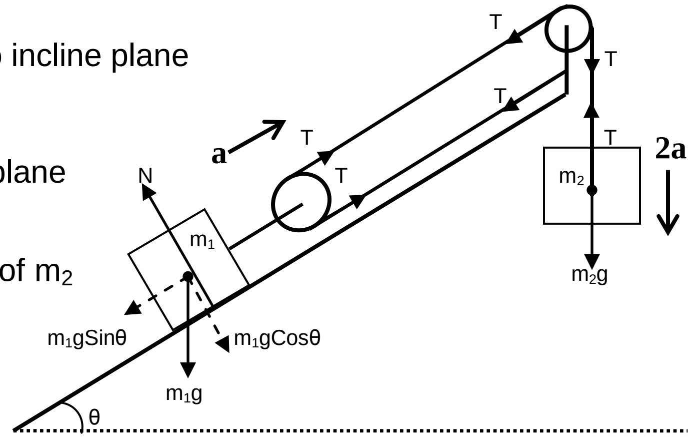
For vertical equilibrium of m_1
$N = m_1g$

For horizontal motion of m_1
$2T = m_1a$

For vertical motion of m_2
$m_2g - T = m_2 2a$



iv) Case - 4
Mass m_1 is attached to a movable pulley and placed on a smooth inclined surface. Mass m_2 is is suspended freely from a fixed light pulley.



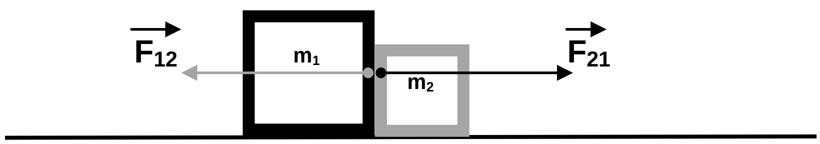
For equilibrium of m_1 perpendicular to incline plane
$N = m_1gCos\theta$

For acceleration of m_1 up the incline plane
$2T - m_1gSin\theta = m_1a$

For vertically downward acceleration of m_2
$m_2g - T = m_22a$

Newton' 3rd law or Law of Action and Reaction

Every action is opposed by an equal and opposite reaction.
or
For every action there is an equal and opposite reaction.



F_{12} is the force on the first body (m_1) due to second body (m_2)
F_{21} is the force on the second body (m_2) due to first body (m_1)

If $\vec{F}_{12}$ is action then $\vec{F}_{21}$ reaction and if $\vec{F}_{21}$ is action then $\vec{F}_{12}$ reaction

Numerical Application

Force on the first body due to second body (F_{12}) is equal and opposite to the force on the second body due to first body (F_{21}).

$$\vec{F}_{21} = -\vec{F}_{12}$$

Physical Application

i) When we push any block in the forward direction then block pushes us in the backward direction with an equal and opposite force.
ii) Horse pulls the rod attached to the cart in the forward direction and the tension of the rod pulls the cart in the backward direction.

iii) Earth pulls the body on its surface in vertically downward direction and the body pulls the earth with the same force in vertically upward direction.
iv) While walking we push the ground in the backward direction using static frictional force and the ground pushes us in the forward direction using static frictional force.
v) When a person sitting on the horse whips the horse and horse suddenly accelerates, the saddle on the back of the horse pushes the person in the forward direction using static frictional force and the person pushes the saddle in the backward direction using static frictional force.
Note – Normal reaction of the horizontal surface on the body is not the reaction of the weight of the body because weight of the body is the force with which earth attracts the body towards its center, hence its reaction must be the force with which body attracts earth towards it.

Linear Momentum

It is defined as the quantity of motion contained in the body. Mathematically it is given by the product of mass and velocity. It is a vector quantity represented by p.

$$\vec{p} = m\vec{v}$$

Principle Of Conservation Of Linear Momentum

It states that in the absence of any external applied force total momentum of a system remains conserved.

Proof-
We know that,

$$\vec{F} = m\vec{a}$$

or,
$$\vec{F} = \frac{m d\vec{v}}{dt}$$

or,
$$\vec{F} = \frac{d m\vec{v}}{dt}$$

or,
$$\vec{F} = \frac{d\vec{p}}{dt}$$

if, $\vec{F} = 0$

$$\frac{d\vec{p}}{dt} = 0$$

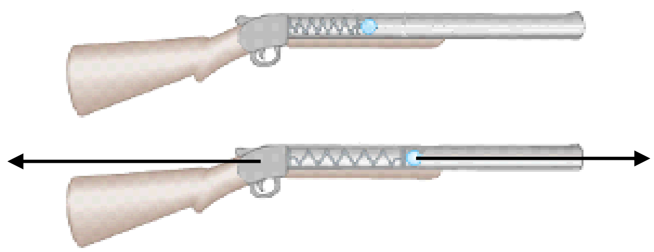
or, $\vec{p}$ **= Constant** (differentiation of constant is zero)

or,
$$\vec{p}_{initial} = \vec{p}_{final}$$

Physical Application

i) Recoil of gun – when bullet is fired in the forward direction gun recoils in the backward direction.
ii) When a person jumps on the boat from the shore of river, boat along with the person on it moves in the forward direction.
iii) When a person on the boat jumps forward on the shore of river, boat starts moving in the backward direction.
iv) In rocket propulsion fuel is ejected out in the downward direction due to which rocket is propelled up in vertically upward direction.

Different Cases of Conservation of Linear Momentum



Recoil of gun

Let mass of gun be m_g and that of bullet be m_b.
Initially both are at rest, hence their initial momentum is zero.

$$p_i = m_g u_g + m_b u_b = 0$$

Finally when bullet rushes out with velocity v_g, gun recoils with velocity v_b, hence their final momentum is

$$p_f = m_g v_g + m_b v_b$$

Since there is no external applied force, from the principal of conservation of linear momentum

$$p_f = p_f$$

or, $$m_g v_g + m_b v_b = 0$$

or, $$m_g v_g = -m_b v_b$$

or, $$\mathbf{v_g = -\frac{m_b v_b}{m_g}}$$

From above expression it must be clear that
1. Gun recoils opposite to the direction of motion of bullet.
2. Greater is the mass of mullet m_b or velocity of bullet v_b greater is the recoil of the gun.
3. Greater is the mass of gun m_g, smaller is the recoil of gun.

Impulse and Impulsive Force

Impulsive Force

The force which acts on a body for very short duration of time but is still capable of changing the position, velocity and direction of motion of the body up to large extent is known as impulsive force.

Example -

1. Force applied by foot on hitting a football.
2. Force applied by boxer on a punching bag.
3. Force applied by bat on a ball in hitting it to the boundary.
4. Force applied by a moving truck on a drum.

Note- Although impulsive force acts on a body for a very short duration of time yet its magnitude varies rapidly during that small duration.

Impulse

Impulse received by the body during an impact is defined as the product of average impulsive force and the short time duration for which it acts.

$$\mathbf{I = F_{avg} \times t}$$

Relation Between Impulse and Linear Momentum

Consider a body being acted upon by an impulsive force, this force changes its magnitude rapidly with the time. At any instant if impulsive force is F then elementary impulse imparted to the body in the elementary time dt is given by

$$dI = F \times dt$$

Hence total impulse imparted to the body from time t_1 to t_2 is

$$I = \int_{t_1}^{t_2} F dt$$

But from Newton's second law we know that

$$F = \frac{dp}{dt}$$

or, $$Fdt = dp$$

Therefore,

$$I = \int_{p_1}^{p_2} dp$$

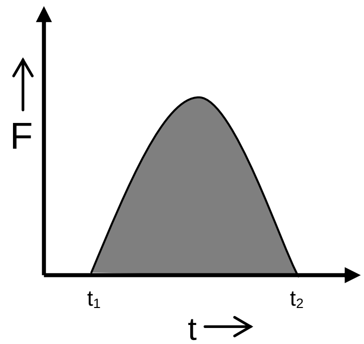
or, $$I = [p]_{p_1}^{p_2}$$

or, $$\mathbf{I = p_2 - p_1}$$

Hence impulse imparted to the body is equal to the change in its momentum.

Graph Between Impulsive Force and Time

With the time on x axis and impulsive force on y axis the graph of the following nature is obtained



Area enclosed under the impulsive force and time graph from t_1 to t_2 gives the impulse imparted to the body from time t_1 to t_2.

Physical Application

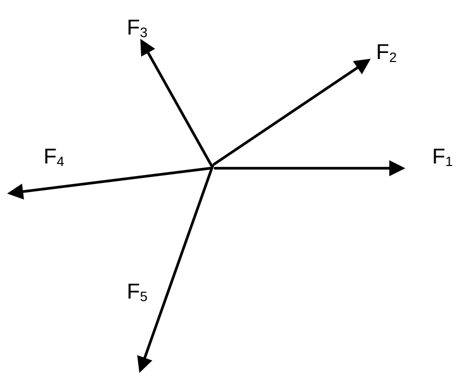
i) While catching a ball a player lowers his hand to save himself from getting hurt.
ii) Vehicles are provided with the shock absorbers to avoid jerks.
iii) Buffers are provided between the bogies of the train to avoid jerks.
iv) A person falling on a cemented floor receive more jerk as compared to that falling on a sandy floor.
v) Glass wares are wrapped in a straw or paper before packing.

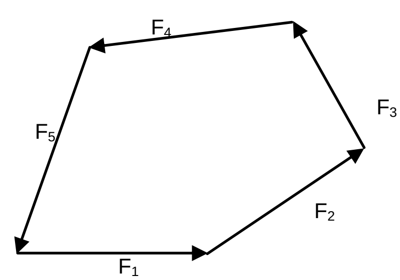
Equilibrium of Concurrent Forces

If the number of forces act at the same point, they are called concurrent forces. The condition or the given body to be in equilibrium under the number of forces acting on the body is that these forces should produce zero resultant.

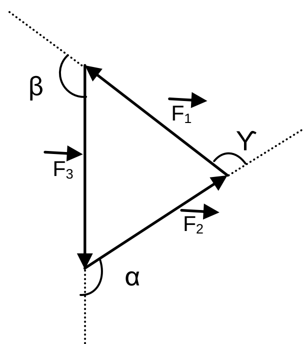
The resultant of the concurrent forces acting on a body will be zero if they can be represented completely by the sides of a closed polygon taken in order.

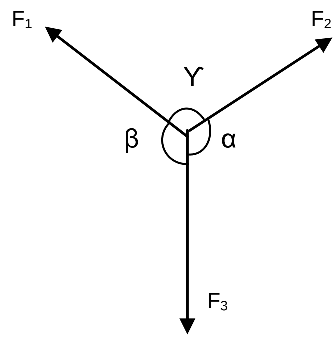
$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = 0$$





Lami's Theorem – It states that the three forces acting at a point are in equilibrium if each force is proportional the sine of the angle between the other two forces.



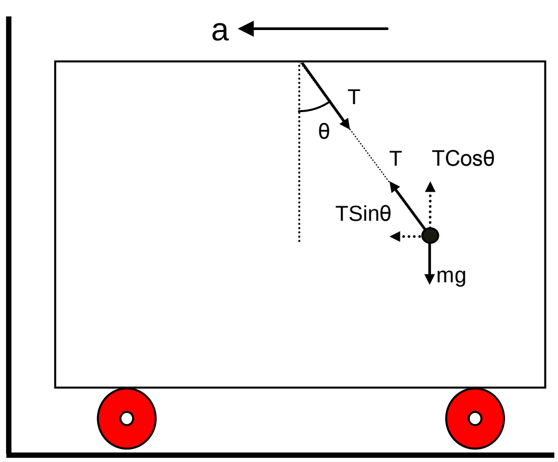


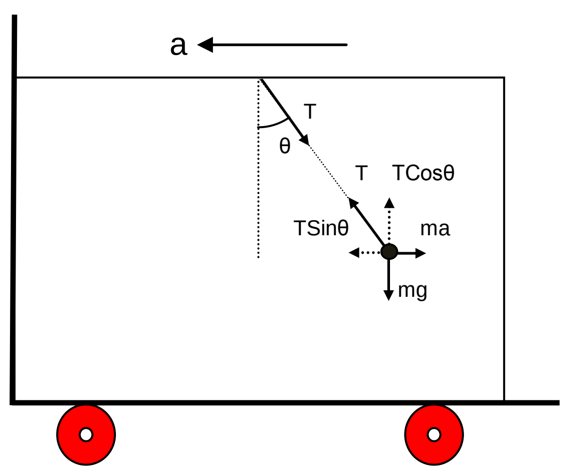
$$\frac{F_1}{\text{Sin}\,\alpha} = \frac{F_2}{\text{Sin}\,\beta} = \frac{F_3}{\text{Sin}\,\gamma}$$

Inertial and Non-inertial Frame of Reference

Frame of reference is any frame with respect to which the body is analyzed. All the frames which are at rest or moving with a constant velocity are said to be inertial frame of reference. In such frame of reference all the three laws of Newton are applicable.

Any accelerated frame of reference is said to be non-inertial frame of reference. In such frames all the three laws of Newton are not applicable as such. In order to apply Newton's laws of motion in a non-inertial frame, along with all other forces a pseudo force F = ma must also be applied on the body opposite to the direction of acceleration of the frame.





Inertial Frame of Reference	Inertial Frame of Reference
(Frame outside the accelerated car) For vertical equilibrium of body $T\cos\theta = mg$ For horizontal acceleration of body, as the body is accelerated along with the car when observed from the external frame $T\sin\theta = ma$ a=0 Therefore, **$\tan\theta = a/g$**	(Frame attached to the accelerated car) For vertical equilibrium of body $T\cos\theta = mg$ For horizontal equilibrium of the body, as the body is at rest when observed from the frame attached to the car $T\sin\theta = ma$ Therefore, **$\tan\theta = a/g$**

Since body is at rest when observed from the non-inertial frame attached to the accelerated car a pseudo force **F = ma** is applied on the body opposite to the acceleration of the car which balance the horizontal component of tension of the string $T\sin\theta$ acting on the body.

Note- From which ever frame we may observe the situation, final result always comes out to be the same.

Reading of Spring Balance

Reading of a spring balance is equal to the tension in the spring of the balance but measured in kilogram.

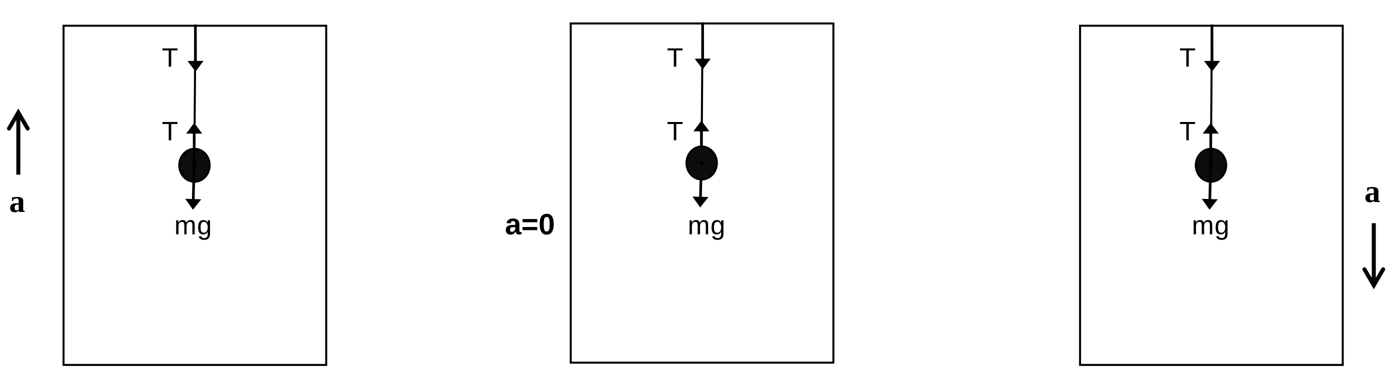
$$\textbf{Reading} = \frac{T}{g}\ \textbf{kgf}$$

Reading of Weighing Machine

Reading of a weighing machine is equal to the normal reaction applied by the machine but measured in kilogram**.**

$$\textbf{Reading} = \frac{N}{g}\ \textbf{kgf}$$

LIFT

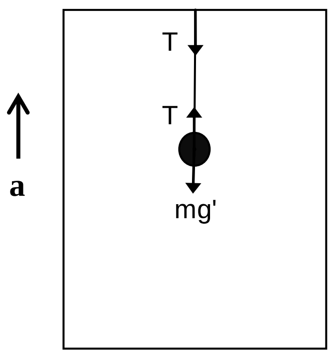


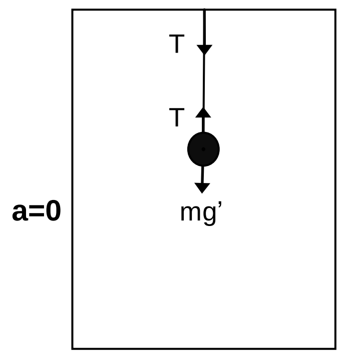
Observer Outside the Lift

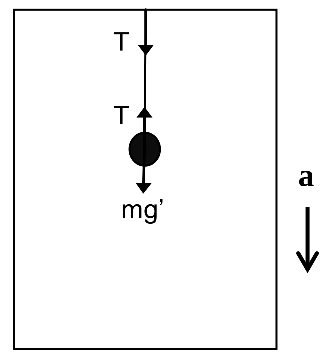
Lift Accelerating Vertically Up
Moving up with increasing velocity.
or
Moving down with decreasing velocity.
For vertical motion of body
$T - mg = ma$
or, $T = mg + ma$
or, $\mathbf{T = m(g + a)}$

Lift Accelerating Vertically Up
Moving up with constant velocity.
or
Moving down with constant velocity.
For vertical motion of body
$a=0$ $\mathbf{T = mg}$

Lift Accelerating Vertically Down
Moving up with decreasing velocity.
or
Moving down with increasing velocity.
For vertical motion of body
$mg - T = ma$
or, $T = mg - ma$
or, $\mathbf{T = m(g - a)}$







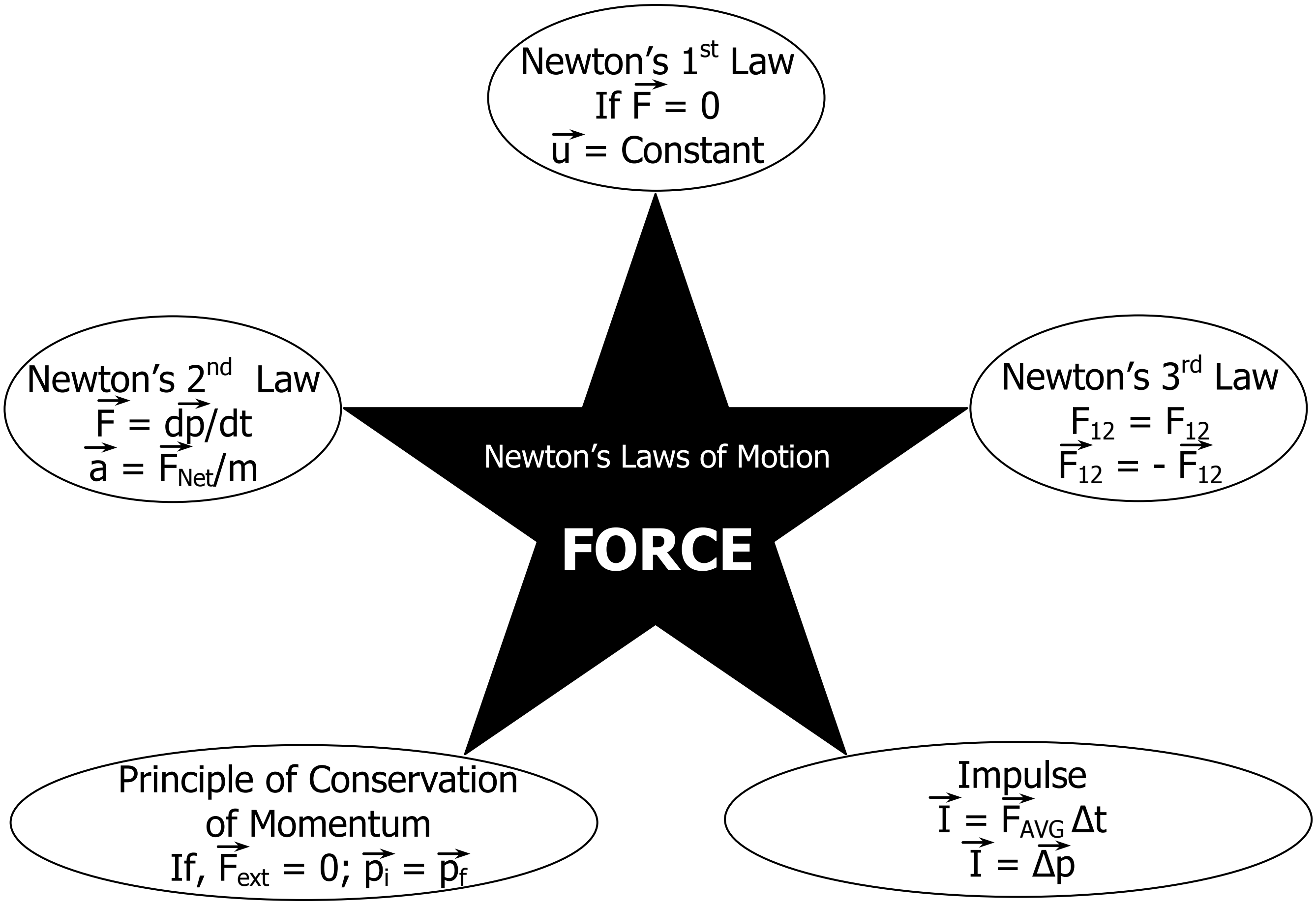
Observer Inside the Lift
(Body is at rest according to the observer inside the lift)

Lift Accelerating Vertically Up
Moving up with increasing velocity.
or
Moving down with decreasing velocity.
Since body is at rest
$\mathbf{T= mg'}$
but, $T = m(g + a)$
therefore, $\mathbf{g' = g + a}$
Where g' is apparent acceleration due to gravity inside the lift.

Lift Accelerating Vertically Up
Moving up with constant velocity.
or
Moving down with constant velocity.
Since body is at rest
$\mathbf{T = mg'}$
but, $T = mg$
therefore, $\mathbf{g' = g}$
Where g' is apparent acceleration due to gravity inside the lift.

Lift Accelerating Vertically Down
Moving up with decreasing velocity.
or
Moving down with increasing velocity.
Since body is at rest
$\mathbf{T = mg'}$
But, $\mathbf{T = m(g - a)}$
therefore, $\mathbf{g' = g - a}$
Where g' is apparent acceleration due to gravity inside the lift.

MEMORY MAP


Newton's 1st Law
If $\vec{F} = 0$
$\vec{u}$ = Constant
Newton's 2nd Law
$\vec{F} = d\vec{p}/dt$
$\vec{a} = \vec{F}_{Net}/m$
Newton's 3rd Law
$F_{12} = F_{12}$
$\vec{F}_{12} = -\vec{F}_{12}$
Newton's Laws of Motion
FORCE
Principle of Conservation of Momentum
If, $\vec{F}_{ext} = 0$; $\vec{p}_i = \vec{p}_f$
Impulse
$\vec{I} = \vec{F}_{AVG}\,\Delta t$
$\vec{I} = \Delta\vec{p}$

FRICTION

Friction - The property by virtue of which the relative motion between two surfaces in contact is opposed is known as friction.

Frictional Forces - Tangential forces developed between the two surfaces in contact, so as to oppose their relative motion are known as frictional forces or commonly friction.
Types of Frictional Forces - Frictional forces are of three types :-
1. Static frictional force
2. Kinetic frictional force
3. Rolling frictional force

Static Frictional Force - Frictional force acting between the two surfaces in contact which are relatively at rest, so as to oppose their relative motion, when they tend to move relatively under the effect of any external force is known as static frictional force. Static frictional force is a self adjusting force and its value lies between its minimum value up to its maximum value.

Minimum value of static frictional force - Minimum value of static frictional force is zero in the condition when the bodies are relatively at rest and no external force is acting to move them relatively.

$$f_{s(min)} = 0$$

Maximum value of static frictional force - Maximum value of static frictional force is $\mu_s N$ (where μ_s is the coefficient of static friction for the given pair of surface and N is the normal reaction acting between the two surfaces in contact) in the condition when the bodies are just about to move relatively under the effect of external applied force.

$$f_{s(max)} = \mu_s N$$

Therefore, $$f_{s(min)} \leq f_s \leq f_{s(max)}$$

or, $$\mathbf{0 \leq f_s \leq \mu_s N}$$

Kinetic Frictional Force - Frictional force acting between the two surfaces in contact which are moving relatively, so as to oppose their relative motion, is known as kinetic frictional force. It's magnitude is almost constant and is equal to $\mu_k N$ where μ_k is the coefficient of kinetic friction for the given pair of surface and N is the normal reaction acting between the two surfaces in contact. It is always less than maximum value of static frictional force.

$$\mathbf{f_k = \mu_k N}$$

Since, $$f_k < f_{s(max)} = \mu_s N$$

Therefore, $$\mu_k N < \mu_s N$$

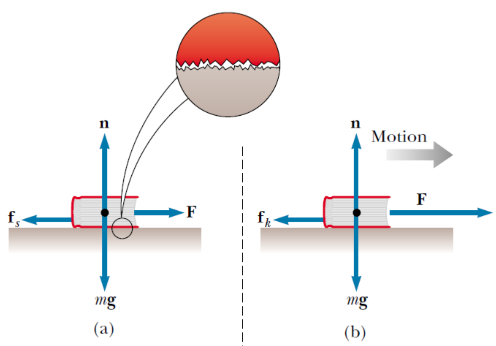
or, $$\mathbf{\mu_k < \mu_s}$$

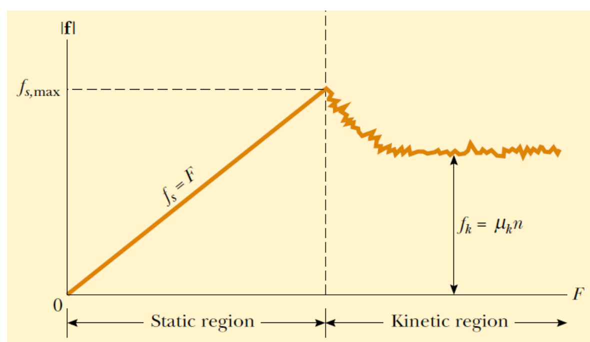
Limiting Frictional Force – The maximum value of static frictional force is the maximum frictional force which can act between the two surfaces in contact and hence it is also known as limiting frictional force.

Laws of Limiting Frictional Force –
1. Static friction depends upon the nature of the surfaces in contact.
2. It comes into action only when any external force is applied to move the two bodies relatively, with their surfaces in contact.
3. Static friction opposes the impending motion.
4. It is a self adjusting force.
5. The limiting frictional force is independent of the area of contact between the two surfaces.

Cause of Friction

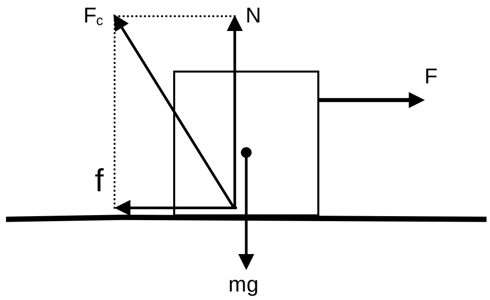
Old View - The surfaces which appear to be smooth as seen through our naked eyes are actually rough at the microscopic level. During contact, the projections of one surface penetrate into the depressions of other and vice versa. Due to which the two surfaces in contact form a saw tooth joint opposing their relative motion. When external force is applied so as to move them relatively this joint opposes their relative motion. As we go on increasing the external applied force the opposition of saw tooth joint also goes on increasing up to the maximum value known as limiting frictional force ($\mu_s N$) after which the joint suddenly breaks and the surfaces start moving relatively. After this the opposition offered by the saw tooth joint slightly decreases and comes to rest at almost constant value ($\mu_k N$)





Modern View – According to modern theory the cause of friction is the atomic and molecular forces of attraction between the two surfaces at their actual point of contact. When any body comes in contact with any other body then due to their roughness at the microscopic level they come in actual contact at several points. At these points the atoms and molecules come very close to each other and intermolecular force of attraction start acting between them which opposes their relative motion.

Contact Force - The forces acting between the two bodies due to the mutual contact of their surfaces are known as contact forces. The resultant of all the contact forces acting between the bodies is known as resultant contact force. Example friction (f) and normal reaction (N) are contact forces and their resultant (F_c) is the resultant is the resultant contact force.



$$\mathbf{F_c = \sqrt{f^2 + N^2}}$$

Since maximum value of frictional force is Limiting frictional force ($\mu_s N$) Therefore maximum value of contact force is

$$F_{c(max)} = \sqrt{(\mu_s N)^2 + N^2}$$

or, $$F_{c(max)} = N\sqrt{\mu_s^{\,2} + 1^2}$$

or, $$\mathbf{F_{c(max)} = N\sqrt{\mu_s^{\,2} + 1}}$$

Angle of Friction – The angle between the resultant contact force (of normal reaction and friction) and the normal reaction is known as the angle of friction.

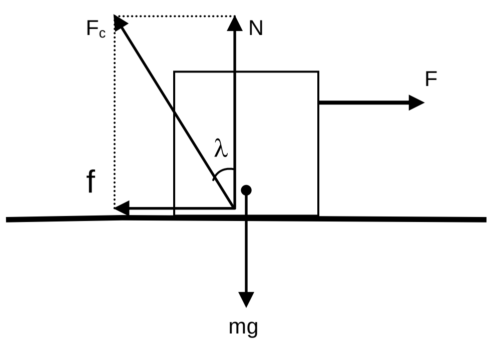
$\text{Tan}\,\lambda = \dfrac{f}{N}$

or, $\boldsymbol{\lambda = \text{Tan}^{-1}\dfrac{f}{N}}$

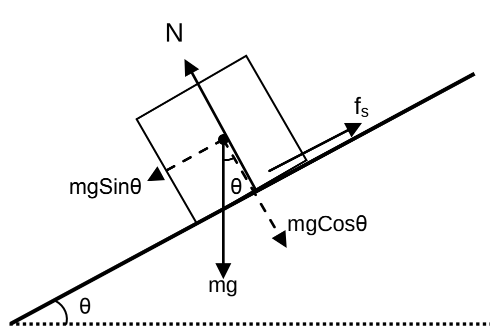
or, $\lambda_{max} = \text{Tan}^{-1}\dfrac{f_{max}}{N}$

or, $\lambda_{max} = \text{Tan}^{-1}\dfrac{\mu_s N}{N}$

or, $\boldsymbol{\lambda_{max} = \text{Tan}^{-1}\,\mu_s}$



Angle of Repose – The angle of the inclined plane at which a body placed on it just begins to slide is known as angle of repose.



Perpendicular to the plane
$\mathbf{N = mgCos\theta}$ (since body is at rest)

Parallel to the plane when body is at rest
$\mathbf{mgSin\theta = f_s}$

When body is just about to slide

$$mgSin\theta = f_{s(max)} = \mu_s N = \mu_s mgCos\theta$$

or, $$Tan\theta = \mu_s$$

or, $$\boldsymbol{\theta = Tan^{-1}\mu_s}$$

Note - Angle of repose is equal to the maximum value of angle of friction

Rolling Frictional Force - Frictional force which opposes the rolling of bodies (like cylinder, sphere, ring etc.) over any surface is called rolling frictional force. Rolling frictional force acting between any rolling body and the surface is almost constant and is given by $\mu_r N$. Where μ_r is coefficient of rolling friction and N is the normal reaction between the rolling body and the surface.

$$\boldsymbol{f_r = \mu_r N}$$

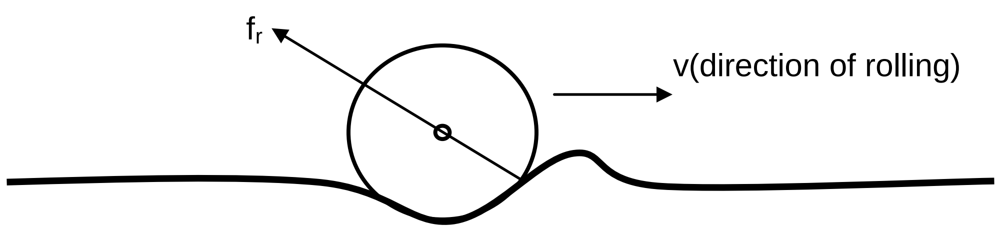
Note – Rolling frictional force is much smaller than maximum value of static and kinetic frictional force.

$$f_r << f_k < f_{s(max)}$$

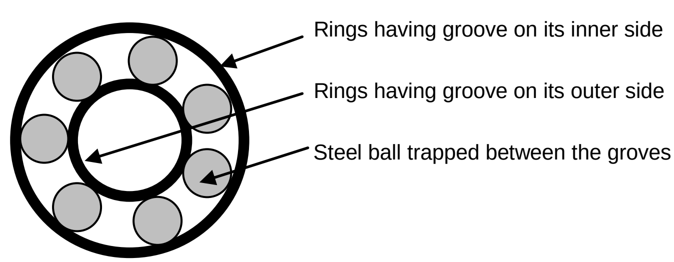
or, $$\mu_r N << \mu_k N < \mu_s N$$

or, $$\boldsymbol{\mu_r << \mu_k < \mu_s}$$

Cause of Rolling Friction – When any body rolls over any surface it causes a little depression and a small hump is created just ahead of it. The hump offers resistance to the motion of the rolling body, this resistance is rolling frictional force. Due to this reason only, hard surfaces like cemented floor offers less resistance as compared to soft sandy floor because hump created on a hard floor
is much smaller as compared to the soft floor.



Need to Convert Kinetic Friction into Rolling Friction – Of all the frictional forces rolling frictional force is minimum. Hence in order to avoid the wear and tear of machinery it is required to convert kinetic frictional force into rolling frictional force and for this reason we make the use of ball-bearings.



Friction: A Necessary Evil – Although frictional force is a non-conservative force and causes lots of wastage of energy in the form of heat yet it is very useful to us in many ways. That is why it is considered as a necessary evil.

Advantages of Friction -
i) Friction is necessary in walking. Without friction it would have been impossible for us to walk.
ii) Friction is necessary for the movement of vehicles on the road. It is the static frictional force which makes the acceleration and retardation of vehicles possible on the road.
iii) Friction is helpful in tying knots in the ropes and strings.
iv) We are able to hold anything with our hands by the help of friction only.

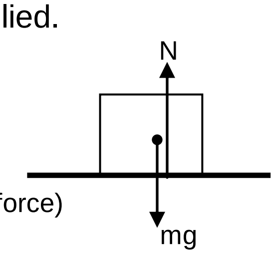
Disadvantages of Friction -
i) Friction causes wear and tear in the machinery parts.
ii) Kinetic friction wastes energy in the form of heat, light and sound.
iii) A part of fuel energy is consumed in overcoming the friction operating within the various parts of machinery.

Methods to Reduce Friction –
i) By polishing – Polishing makes the surface smooth by filling the space between the depressions and projections present in the surface of the bodies at microscopic level and there by reduces friction.
ii) By proper selection of material – Since friction depends upon the nature of material used hence it can be largely reduced by proper selection of materials.
iii) By lubricating – When oil or grease is placed between the two surfaces in contact, it prevents the surface from coming in actual contact with each other. This converts solid friction into liquid friction which is very small.

Physical Application

Horizontal Plane

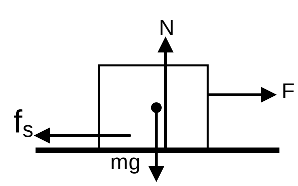
i) Body kept on horizontal plane is at rest and no force is applied.



For vertical equilibrium
N = mg

$\mathbf{f_{friction} = 0}$ (friction is a opposing force and there is no external applied force)

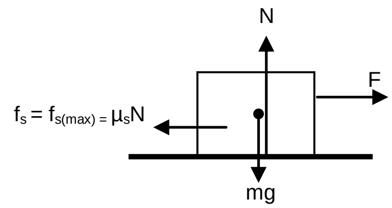
ii) Body kept on horizontal plane is at rest under single horizontal force.



For vertical equilibrium
N = mg (since body is at rest)

For horizontal equilibrium (since body is at rest)
$\mathbf{F = f_s}$

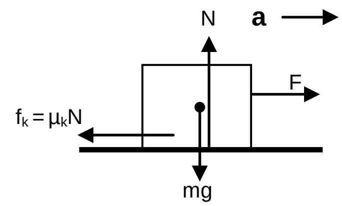
iii) Body kept on horizontal plane is just about to move.



For vertical direction
N = mg (since body is at rest)

For horizontal direction (since body is just about to move)
$\mathbf{F = f_s = f_{s(max)} = \mu_s N}$

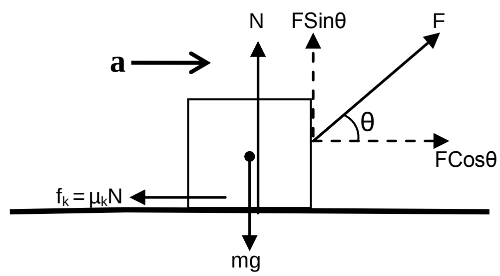
iv) Body kept on horizontal plane is accelerating horizontally.



For vertical direction
N = mg (since body is at rest)

For horizontal direction
$\mathbf{F - f_k = ma}$
or, $\mathbf{F - \mu_k N = ma}$

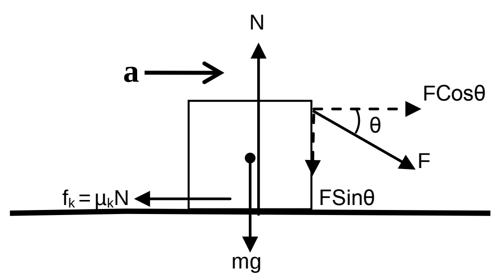
v) Body kept on horizontal plane is accelerating horizontally towards right under single upward inclined force.



For vertical direction
$\mathbf{N + F\sin\theta = mg}$ (since body is at rest)

For horizontal direction
$\mathbf{F\cos\theta - f_k = ma}$
or, $\mathbf{F\cos\theta - \mu_k N = ma}$

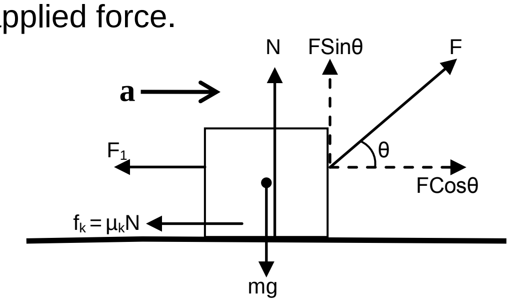
vi) Body kept on horizontal plane is accelerating horizontally towards right under single downward inclined force.



For vertical direction
$\mathbf{N = F\sin\theta + mg}$ (since body is at rest)

For horizontal direction
$\mathbf{F\cos\theta - f_k = ma}$
or, $\mathbf{F\cos\theta - \mu_k N = ma}$

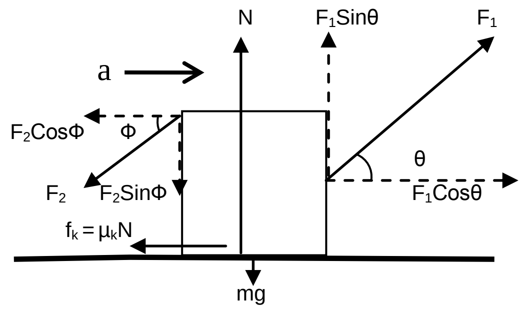
vii) Body kept on horizontal plane is accelerating horizontally towards right under an inclined force and an opposing horizontally applied force.



For vertical direction
$\mathbf{N + F\sin\theta = mg}$ (since body is at rest)

For horizontal direction
$\mathbf{F\cos\theta - F_1 - f_k = ma}$
or, $\mathbf{F\cos\theta - F_1 - \mu_k N = ma}$

vi) Body kept on horizontal plane is accelerating horizontally towards right under two inclined forces acting on opposite sides.

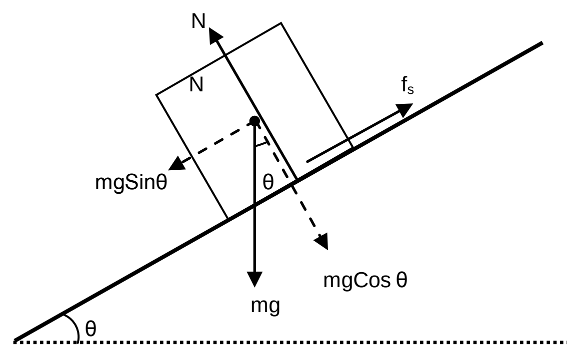


For vertical direction(since body is at rest)
$N + F_1 Sin\theta = mg + F_2\ Sin\Phi$

For horizontal direction
$F_1 Cos\theta - F_2 Cos\Phi - \mu_k N = ma$

Inclined Plane

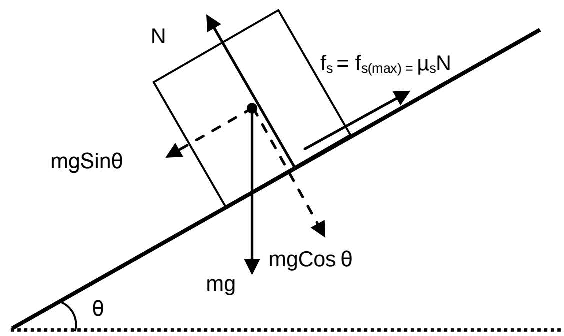
i) Case - 1
Body is at rest on inclined plane.



Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane (since body is at rest)
$mgSin\theta = f_s$

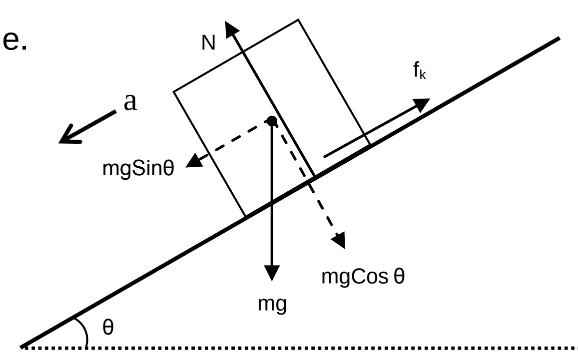
ii) Case - 2
Body is just about to move on inclined plane.



Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane (since body is at rest)
$mgSin\theta = f_s = f_{s(max)} = \mu_s N$

iii) Case - 3
Body is accelerating downwards on inclined plane.

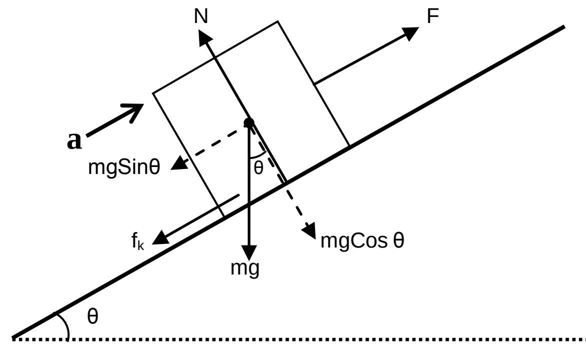


Perpendicular to the plane
$N = mgCos\theta$ (since body is at rest)

Parallel to the plane
$mgSin\theta - f_k = ma$
or, **$mgSin\theta - \mu_k N = ma$**

iv) Case - 4

Body is accelerating up the incline under the effect of force acting parallel to the incline.



Perpendicular to the plane

$N = mgCos\theta$ (since body is at rest)

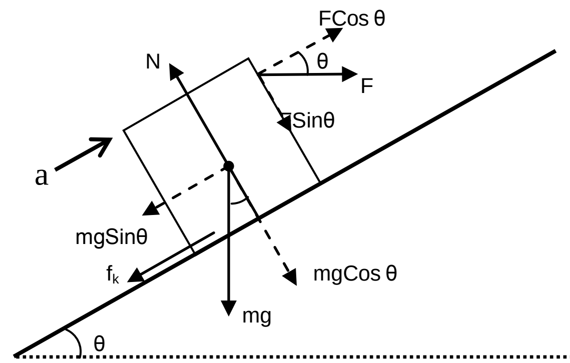
Parallel to the plane

$F - f_k - mgSin\theta = ma$

or, **$F - \mu_k N - mgSin\theta = ma$**

v) Case - 5

Body accelerating up the incline under the effect of horizontal force.



Perpendicular to the plane

$N = mgCos\theta + FSin\theta$ (since body is at rest)

Parallel to the plane

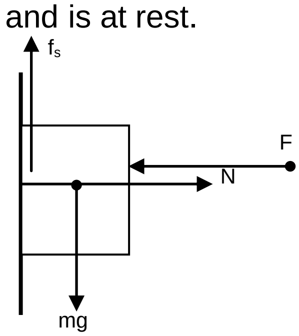
$FCos\theta - mgSin\theta - f_k = ma$

or, **$FCos\theta - mgSin\theta - \mu_k N ma$**

Vertical Plane

i) Case - 1

Body pushed against the vertical plane by horizontal force and is at rest.



For horizontal direction (since body is at rest)

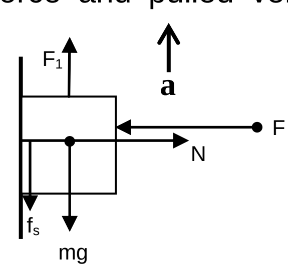
$F = N$

For vertical direction

$mg = f_s$

ii) Case - 2

Body pushed against the vertical plane by horizontal force and pulled vertically upward



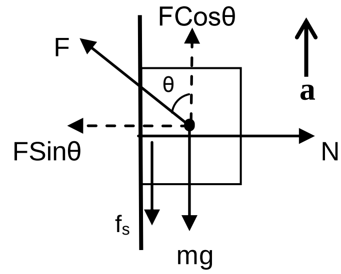
For horizontal direction (since body is at rest)

$F = N$

For vertical direction

$F_1 - mg – f_s = ma$

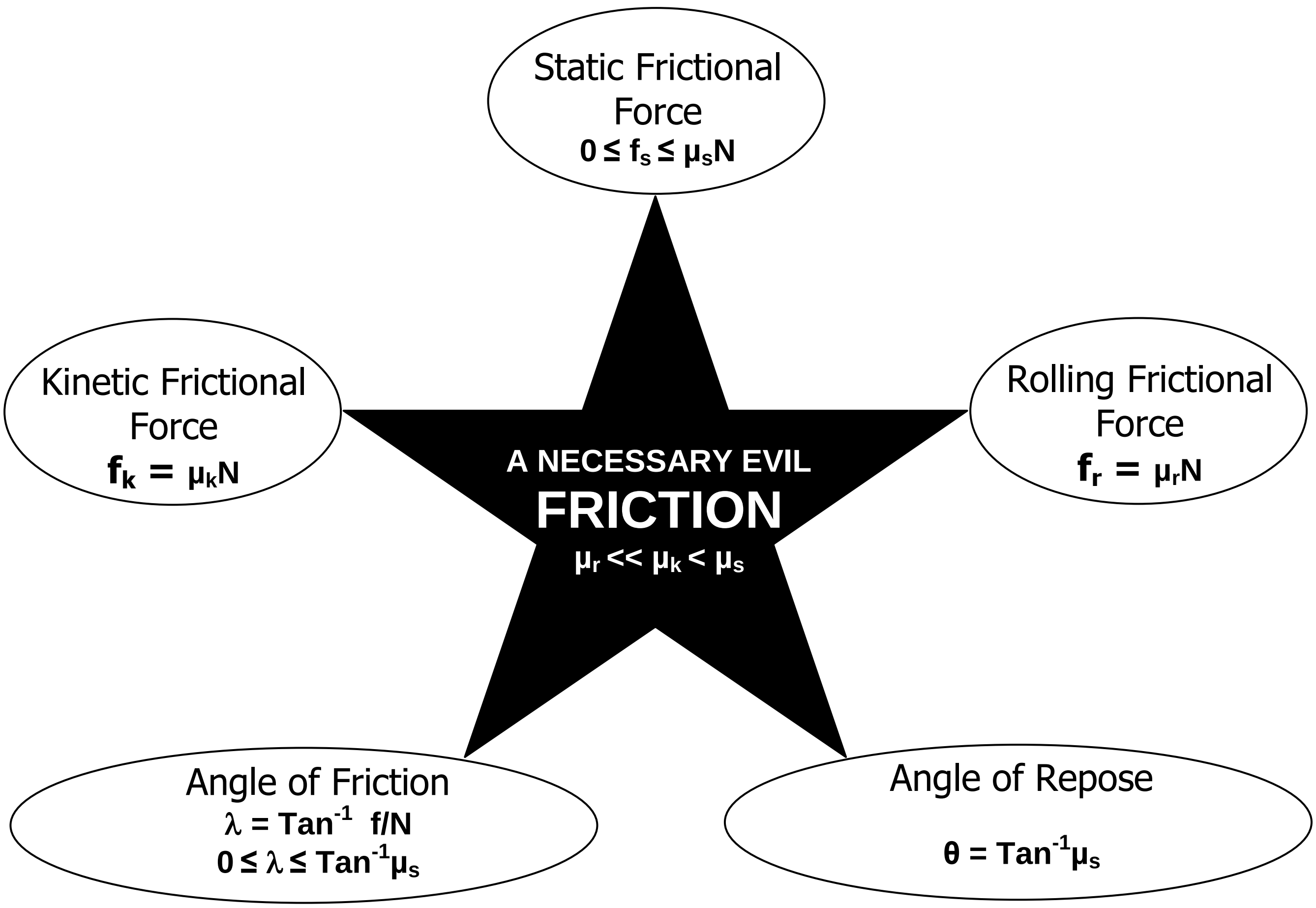
iii) Case - 3
Body pushed against the vertical plane by inclined force and accelerates vertically upward.



For horizontal direction
N = FSinθ (since body is at rest)

For vertical direction
FCosθ - mg - f_s = ma

MEMORY MAP

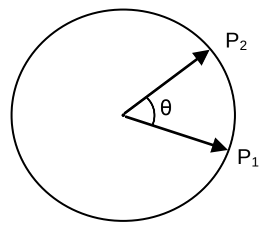

Static Frictional Force
$0 \leq f_s \leq \mu_s N$
Kinetic Frictional Force
$f_k = \mu_k N$
Rolling Frictional Force
$f_r = \mu_r N$
A NECESSARY EVIL
FRICTION
$\mu_r << \mu_k < \mu_s$
Angle of Friction
$\lambda = \text{Tan}^{-1}\ f/N$
$0 \leq \lambda \leq \text{Tan}^{-1}\mu_s$
Angle of Repose
$\theta = \text{Tan}^{-1}\mu_s$

CIRCULAR MOTION

Circular Motion – When a body moves such that it always remains at a fixed distance from a fixed point then its motion is said to be circular motion. The fixed distance is called the radius of the circular path and the fixed point is called the center of the circular path.

Uniform Circular Motion – Circular motion performed with a constant speed is known as uniform circular motion.

Angular Displacement – Angle swept by the radius vector of a particle moving on a circular path is known as angular displacement of the particle. Example :– angular displacement of the particle from P_1 to P_2 is θ.



Relation Between Angular Displacement and Linear Displacement –

Since,

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

$$\text{Anglular Displacement} = \frac{\text{arc } P_1P_2}{\text{radius}}$$

$$\boldsymbol{\theta = \frac{s}{r}}$$

Angular Velocity – Rate of change of angular displacement of a body with respect to time is known as angular displacement. It is represented by ω.

Average Angular Velocity – It is defined as the ratio of total angular displacement to total time taken.

$$\omega_{avg} = \frac{\text{Total Angular Displacement}}{\text{Total Time Taken}}$$

$$\boldsymbol{\omega_{avg} = \frac{\Delta\theta}{\Delta t}}$$

Instantaneous Angular Velocity – Angular velocity of a body at some particular instant of time is known as instantaneous angular velocity.

Or

Average angular velocity evaluated for very short duration of time is known as instantaneous angular velocity.

$$\omega = \lim_{\Delta t \to 0} \omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\boldsymbol{\omega = \frac{d\theta}{dt}}$$

Relation Between Angular Velocity and Linear Velocity

We know that angular velocity

$$\omega = \frac{d\theta}{dt}$$

Putting, $\theta = s/r$

$$\omega = \frac{d\,(s/r)}{dt}$$

or,

$$\omega = \frac{1}{r}\frac{ds}{dt}$$

or,

$$\boldsymbol{\omega = \frac{v}{r}}$$

or,

$$\mathbf{v = r\omega}$$

Time Period of Uniform Circular Motion – Total time taken by the particle performing uniform circular motion to complete one full circular path is known as time period.

In one time period total angle rotated by the particle is 2π and time period is T. Hence angular velocity

$$\omega = \frac{2\pi}{T}$$

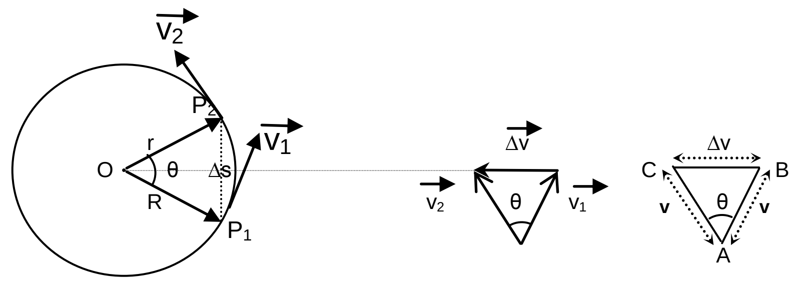
or,

$$\mathbf{T = \frac{2\pi}{\omega}}$$

Frequency - Number of revolutions made by the particle moving on circular path in one second is known as frequency.

$$\mathbf{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

Centripetal Acceleration – When a body performs uniform circular motion its speed remains constant but velocity continuously changes due to change of direction. Hence a body is continuously accelerated and the acceleration experienced by the body is known as centripetal acceleration (that is the acceleration directed towards the center).



Consider a particle performing uniform circular motion with speed v. When the particle changes its position from P_1 to P_2 its velocity changes from $\vec{v_1}$ to $\vec{v_2}$ due to change of direction. The change in velocity from P_1 to P_2 is $\vec{\Delta v}$ which is directed towards the center of the circular path according to triangle law of subtraction of vectors.

From figure ΔOP_1P_2 and ΔABC are similar, hence applying the condition of similarity

$$\frac{BC}{AB} = \frac{P_1 P_1}{O P_1}$$

or, $$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

or, $$\Delta v = \frac{v\Delta s}{r}$$

Dividing both sides by Δt, $$\frac{\Delta v}{\Delta t} = \frac{v\Delta s}{r\Delta t}$$

Taking limit $\Delta t \rightarrow 0$ both sides,

$$\lim_{\Delta t\to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t\to 0} \frac{\Delta\theta}{\Delta t}$$

or, $$\frac{dv}{dt} = \frac{vds}{dt}$$

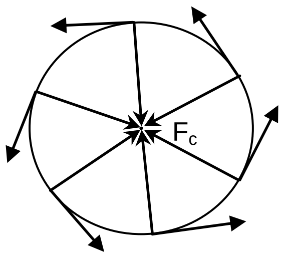
or, $$\boxed{\mathbf{a = \frac{v^2}{r}}}$$

Putting $v = r\omega$, $$\boxed{a = r\boldsymbol{\omega}^2}$$

Since the change of velocity is directed towards the center of the circular path, the acceleration responsible for the change in velocity is also directed towards center of circular path and hence it is known as centripetal acceleration.

Centripetal Force – Force responsible for producing centripetal acceleration is known as centripetal force. Since centripetal acceleration is directed towards the center of the circular path the centripetal force is also directed towards the center of the circular path.

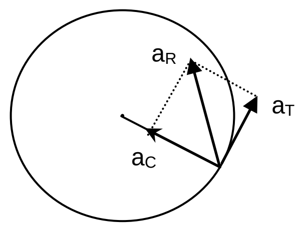
If a body is performing uniform circular motion with speed v and angular velocity ω on a circular path of radius r, then centripetal acceleration is given by



$$\boxed{F_c = \frac{mv^2}{r} = mr\omega^2}$$

Net Acceleration of a Body Performing Non-Uniform Circular Motion

When a body performs non-uniform circular motion its speed i.e. magnitude of instantaneous velocity varies with time due to which it experiences tangential acceleration a_T along with the centripetal acceleration a_C. Since both the accelerations act simultaneously on a body and are mutually perpendicular to each other, the resultant acceleration a_R is given by their vector sum.



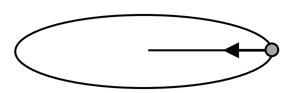
$$\vec{a}_R = \vec{a}_T + \vec{a}_C$$

$$a_R = \sqrt{a_T^2 + a_C^2}$$

Physical Application of Centripetal Force

i) Case - 1

Circular motion of a stone tied to a string.

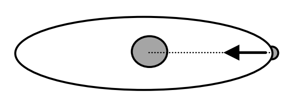


Centripetal force is provided by the tension of the string

$$F_c = \frac{mv^2}{r} = T$$

ii) Case - 2

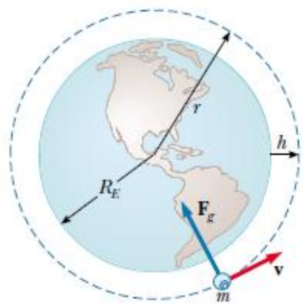
Circular motion of electron around the nucleus.



Centripetal force is provided by the electrostatic force of attraction between the positively charged nucleus and negatively charged electron

$$F_c = \frac{mv^2}{r} = F_E$$

iii) Case - 3
Circular motion of planets around sun or satellites around planet.



Centripetal force is provided by the gravitational force of attraction between the planet and sun

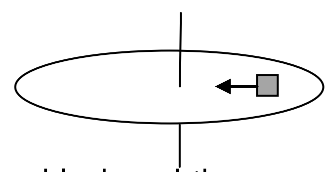
$$F_c = \frac{mv^2}{r} = F_g$$

iv) Case - 4
Circular motion of vehicles on a horizontal road.

Centripetal force is provided by the static frictional force between the road and the tyre of the vehicle.

$$F_c = \frac{mv^2}{r} = f_s$$

v) Case - 5
Circular motion of a block on rotating platform.



Centripetal force is provided by the static frictional force between the block and the platform.

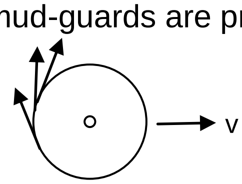
$$F_c = \frac{mv^2}{r} = f_s$$

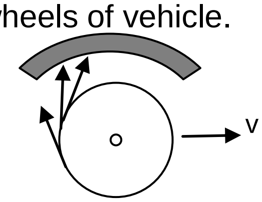
vi) Case - 6
Circular motion of mud particles sticking to the wheels of the vehicle.

Centripetal force is provided by the adhesive force of attraction between the mud particles and the tyres of the vehicle.

$$F_c = \frac{mv^2}{r} = F_{adhesive}$$

At very high speed when adhesive force is unable to provide necessary centripetal force, the mud particles fly off tangentially. In order to prevent the particles from staining our clothes, mud-guards are provided over the wheels of vehicle.

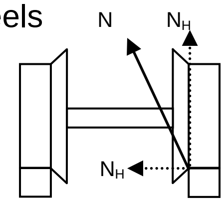




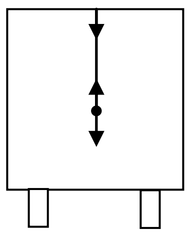
vii) Case - 7
Circular motion of a train on a horizontal track.

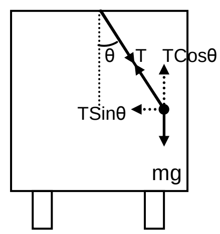
Centripetal force is provided by the horizontal component of the reaction force applied by the outer track on the inner projection of the outer wheels

$$F_c = \frac{mv^2}{r} = N_{Horizontal}$$



viii) Case - 8
Circular motion of a toy hanging from ceiling of vehicle.


Car moving with constant velocity on horizontal road



Car taking a turn with constant velocity on a horizontal road

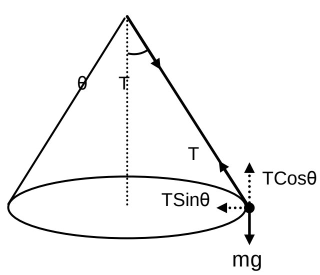
Whenever car takes a turn, string holding the toy gets tilted outward such that the vertical component of the tension of string balances the weight of the body and the horizontal component of tension provides the necessary centripetal force.

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

Therefore, $$\tan\theta = \frac{v^2}{rg}$$

ix) Case - 9
Conical pendulum.



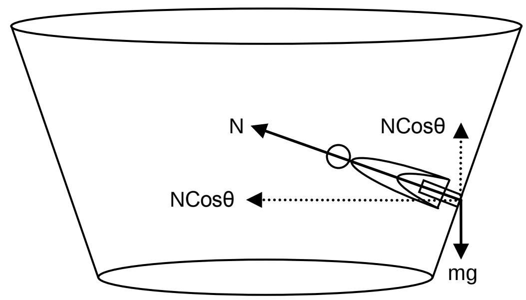
Whenever bob of a pendulum moves on a horizontal circle it's string generates a cone. Such a pendulum is known as conical pendulum. The vertical component of the tension of the string balances the weight of the body and the horizontal component of tension provides the necessary centripetal force.

$$TSin\theta = \frac{mv^2}{r}$$

$$TCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

x) Case - 10
Well of death.



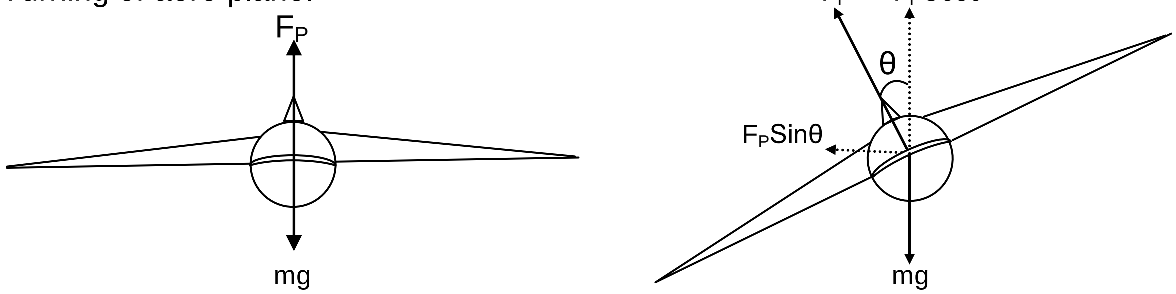
In the well of death, the rider tries to push the wall due to its tangential velocity in the outward direction due to which wall applies normal reaction in the inward direction. The vertical component of the normal reaction balances the weight of the body and its horizontal component provides the necessary centripetal force.

$$NSin\theta = \frac{mv^2}{r}$$

$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

xi) Case - 11
Turning of aero plane.



While taking a turn aero-plane tilts slightly inwards due to which it's pressure force also gets tilted inwards due to which it's pressure force also gets tilted inwards such that it's vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.

$$F_P \sin\theta = \frac{mv^2}{r}$$

$$F_P \cos\theta = mg$$

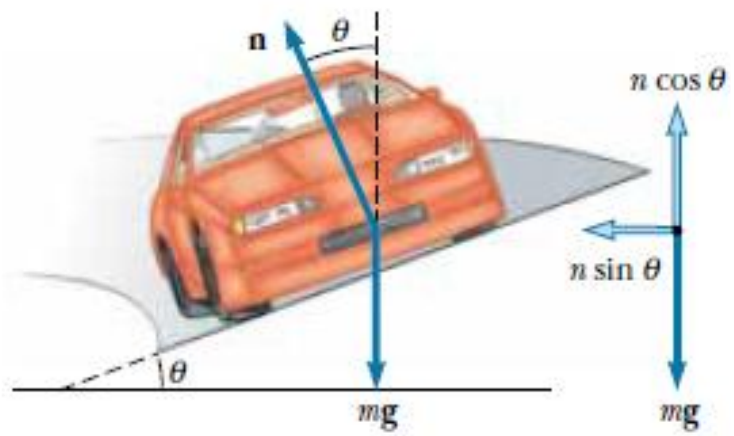
Therefore, $$\mathbf{\tan\theta = \frac{v^2}{rg}}$$

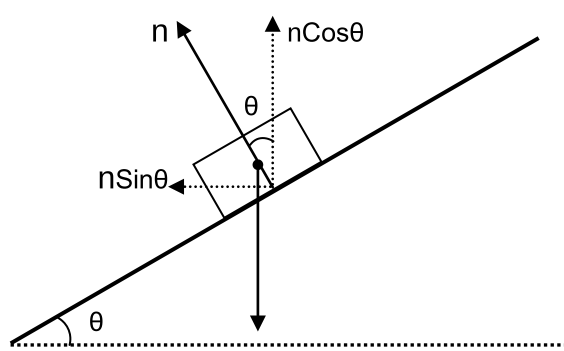
xi) Case - 11

Banking of Roads

In case of horizontal road necessary centripetal force mv^2/r is provided by static frictional force. When heavy vehicles move with high speed on a sharp turn (small radius) then all the factors contribute to huge centripetal force which if provided by the static frictional force may result in the fatal accident.

To prevent this roads are banked by lifting their outer edge. Due to this, normal reaction of road on the vehicle gets tilted inwards such that it's vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.





$$n\sin\theta = \frac{mv^2}{r}$$

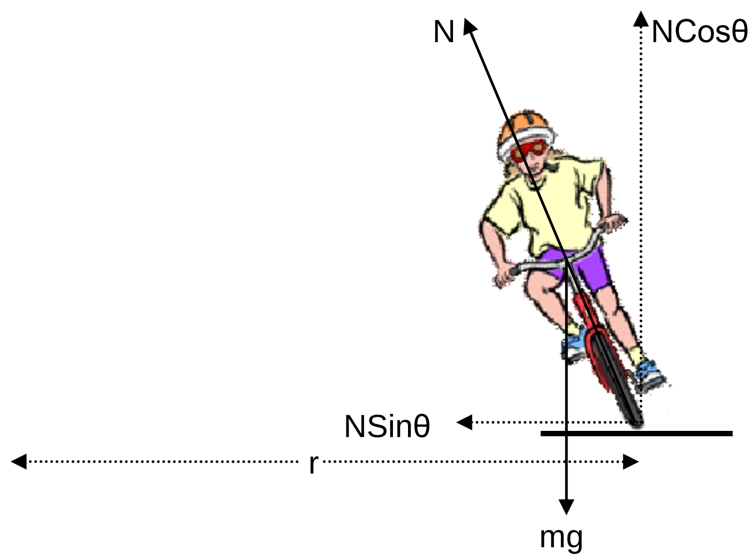
$$n\cos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

xii) Case - 12
Bending of Cyclist

In case of a cyclist moving on a horizontal circular track necessary centripetal force is provided by static frictional force acting parallel along the base. As this frictional force is not passing from the center of mass of the system it tends to rotate the cycle along with the cyclist and make it fall outward of the center of the circular path.

To prevent himself from falling, the cyclist leans the cycle inwards towards the center of the circle due to which the normal reaction of the surface of road on the cycle also leans inward such that that its vertical component balances the weight of the body and the horizontal component provides the necessary centripetal force.

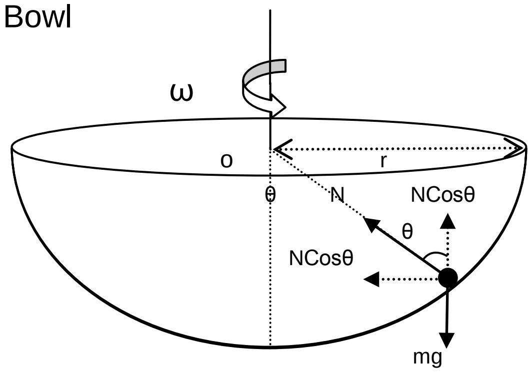


$$NSin\theta = \frac{mv^2}{r}$$

$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

xiii) Case - 13
Motion of a Ball in a Bowl



When the bowl rotates with some angular velocity ω. The vertical component of the normal reaction of the bowl on the ball balances the weight of the body and its horizontal component provides the necessary centripetal force.

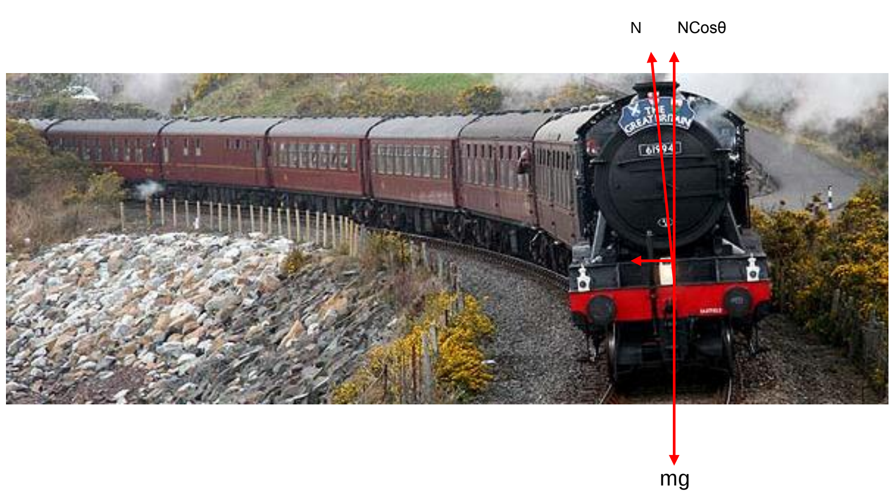
$$NSin\theta = \frac{mv^2}{r}$$

$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

xiv) Case - 14
Motion of a train on the banked tracks.

At the turns tracks are banked by slightly elevating the outer tracks with respect to the inner ones. This slightly tilts the train inwards towards the center of the circular path due to which the normal reaction of the tracks on the train also gets slightly tilted inwards such that the vertical component of the normal reaction balances the weight of the train and it's horizontal component provides the necessary centripetal force.



$$NSin\theta = \frac{mv^2}{r}$$

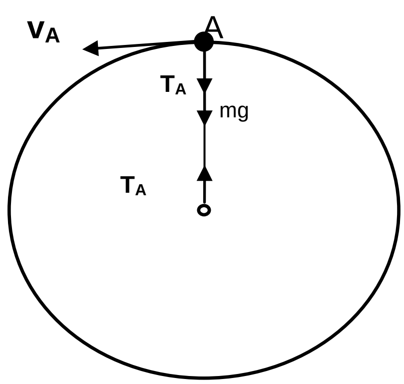
$$NCos\theta = mg$$

Therefore, $$\mathbf{Tan\theta = \frac{v^2}{rg}}$$

Vertical Circular Motion

Whenever the plane of circular path of body is vertical its motion is said to be vertical circular motion.

Vertical Circular Motion of a Body Tied to a String



Consider a body of mass m tied to a string and performing vertical circular motion on a circular path of radius r. At the topmost point A of the body weight of the body mg and tension T_A both are acting in the vertically downward direction towards the center of the circular path and they together provide centripetal force.

$$T_A + mg = \frac{mv_A^2}{r}$$

<u>Critical velocity at the top most point</u>

As we go on decreasing the v_A , tension T_A also goes on decreasing and in the critical condition when v_A is minimum tension $T_A = 0$. The minimum value of v_A in this case is known as critical velocity $T_{A(Critical)}$ at the point A. From above

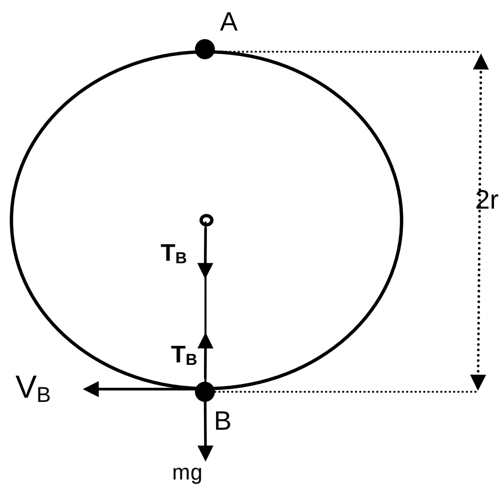
$$0 + mg = \frac{mv_{A(Critical)}^2}{r}$$

or, $$v_{A(Critical)}^2 = rg$$

or, $$\mathbf{v_{A(Critical)} = \sqrt{rg}}$$

If the velocity at point A is less than this critical velocity then the string will slag and the body in spite of moving on a circular path will tend to fall under gravity.

Critical velocity at the lower most point



Taking B as reference level of gravitational potential energy and applying energy conservation

$$E_A = E_B$$

$$P_A + K_A = P_B + K_B$$

$$mg2r + \frac{1}{2}mv_A^2 = mg0 + \frac{1}{2}mv_B^2$$

Putting, $v_A = \sqrt{rg}$

$$mg2r + \frac{1}{2}m(\sqrt{rg})^2 = 0 + \frac{1}{2}mv_B^2$$

or, $\quad 4mgr + mgr = mv_B^2$

or, $\quad 5mgr = mv_B^2$

or, $\quad v_B = \sqrt{5gr}$

This is the minimum possible velocity at the lower most point for vertical circular motion known as critical velocity at point B.

$$\mathbf{v_{B(Critical)} = \sqrt{5gr}}$$

Tension at lowermost point in critical condition

For lowermost point B net force towards the center is centripetal force. Tension T_B acts towards the center of the circular path whereas weight mg acts away from it. Hence,

$$T_B - mg = \frac{mv_B^2}{r}$$

Putting, $v_B = \sqrt{5rg}$

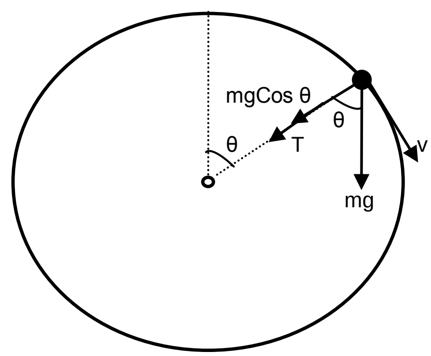
$$T_B - mg = \frac{m5gr}{r}$$

or, $$\mathbf{T_B = 6mg}$$

Hence in critical condition of vertical circular motion of a body tied to a string velocities at topmost and lowermost be $\sqrt{(rg)}$ and $\sqrt{(5rg)}$ respectively and tensions in the strings be 0 and 6mg respectively.

General Condition for Slagging of String in Vertical Circular Motion

For the body performing vertical circular motion tied to a string, slagging of string occurs in the upper half of the vertical circle. If at any instant string makes angle θ with vertical then applying net force towards center is equal to centripetal force, we have



$$T + mgCos\,\theta = \frac{mv^2}{r}$$

For slagging T = 0,

$$0 + mgCos\,\theta = \frac{mv^2}{r}$$

or, $$\mathbf{v = \sqrt{rgCos\,\theta}}$$

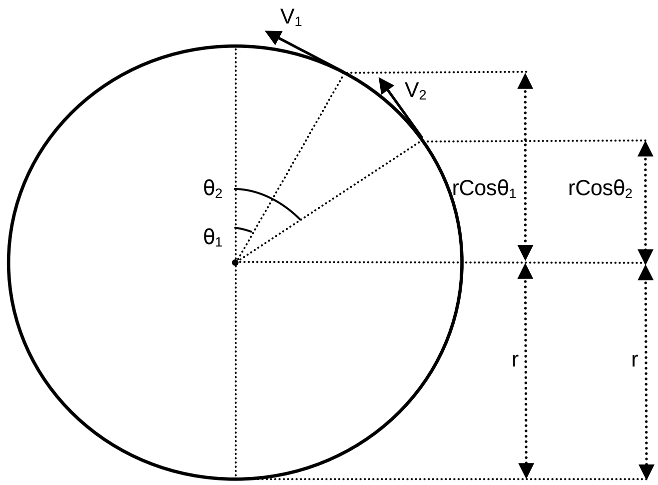
Case-1 At Topmost point θ = 0, therefore $v = \sqrt{rg}$

Case-2 At θ = 60^o, therefore $v = \sqrt{rgCos60} = \sqrt{rg/2}$

Case-3 When string becomes horizontal that is at θ = 90^o, $v = \sqrt{rgCos90} = 0$

Velocity Relationship at different Points of Vertical Circular Motion

Let initial and final velocities of the body performing vertical circular motion be v_1 and v_2 and the angle made by string with the vertical be θ_1 and θ_2. Taking lowermost point of vertical circular path as reference level and applying energy conservation,



$$E_1 = E_2$$

$$P_1 + K_1 = P_2 + K_2$$

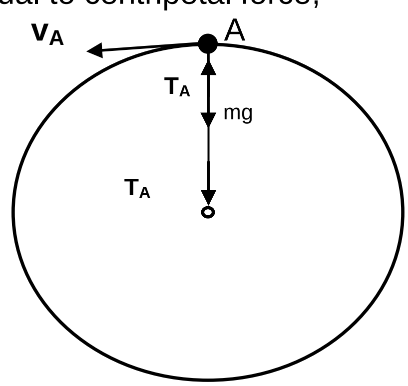
$$mg(r + rCos\theta_1) + \frac{1}{2}mv_1^2 = mg(r + rCos\theta_2) + \frac{1}{2}mv_2^2$$

or, $$mgr(Cos\theta_1 - Cos\theta_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

or, $$\mathbf{(v_2^2 - v_1^2) = 2gr(Cos\theta_1 - Cos\theta_2)}$$

Vertical Circular Motion of a Body Attached to a Rod

Since rod can never slag hence in the critical situation a body attached to the rod may reach the topmost position A of the vertical circular path with almost zero velocity. In this case its weight mg acts in vertically downward direction and tension of rod acts on the body in the vertically upward direction. Applying net force towards center is equal to centripetal force,



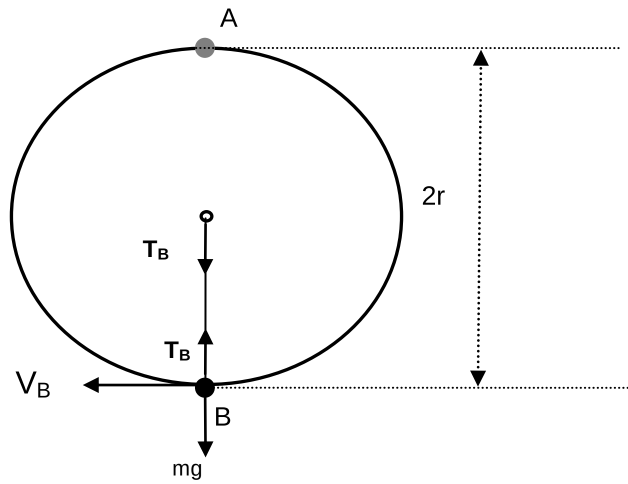
$$mg - T_A = \frac{mv_A^2}{r}$$

Putting $\mathbf{v_A = 0}$ (for critical condition)

$$mg - T_A = 0$$

or, $$\mathbf{T_A = mg}$$

Critical velocity and Tension at the lower most point



Taking B as reference level of gravitational potential energy and applying energy conservation

$$E_A = E_B$$
$$P_A + K_A = P_B + K_B$$
$$mg2r + \frac{1}{2}mv_A^2 = mg0 + \frac{1}{2}mv_B^2$$

Putting, v_A = 0(for critical condition)

$$mg2r + 0 = 0 + \frac{1}{2}mv_B^2$$

or, $$4mgr = mv_B^2$$

or, $$v_B = \sqrt{4rg}$$

This is the minimum possible velocity at the lower most point for vertical circular motion known as critical velocity at point B.

$$\mathbf{v_{B(Critical)} = \sqrt{4rg}}$$

Tension at lowermost point in critical condition

For lowermost point B applying net force towards center is equal to centripetal force. Tension T_B acts towards the center of the circular path whereas weight mg acts away from it in vertically downward direction. Hence,

$$T_B - mg = \frac{mv_B^2}{r}$$

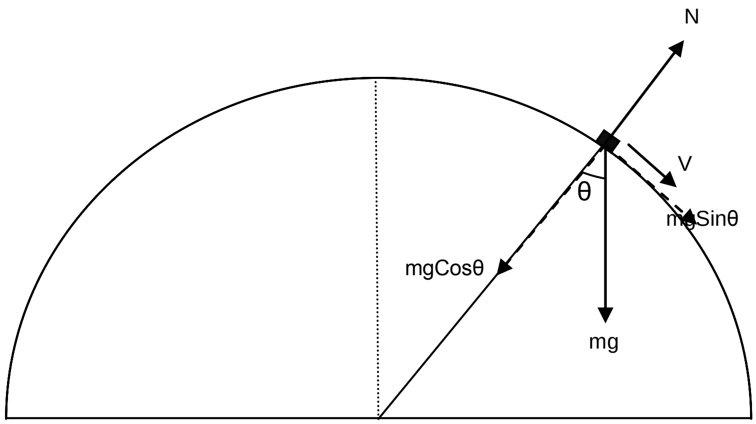
Putting, $v_B = \sqrt{4rg}$

$$T_B - mg = \frac{m4gr}{r}$$

or, $$\mathbf{T_B = 5mg}$$

Hence in critical condition of vertical circular motion of a body attached to the rod velocities at topmost and lowermost be 0 and $\sqrt{4rg}$ respectively and tensions in the rod be mg (pushing nature) and 5mg (pulling nature) respectively.

Motion of A Body Over Spherical Surface



A body of mass m is moving over the surface of the smooth sphere of radius r. At any instant when the radius of sphere passing through the body makes angle θ with the vertical the tangential velocity of the body is v. Since net force towards the center is centripetal force we have

$$mg\cos\theta - N = \frac{mv^2}{r}$$

or, $$\mathbf{N = mg\cos\theta - \frac{mv^2}{r}}$$

if v increases N decreases and when the body just loses contact with the sphere N = 0.

Putting N = 0,

$$0 = mg\cos\theta - \frac{mv^2}{r}$$

or, $$\frac{mv^2}{r} = mg\cos\theta$$

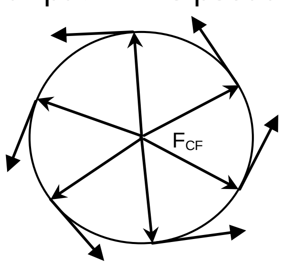
or, $$\mathbf{v = \sqrt{rg\cos\theta}}$$

This is the minimum velocity at which the body loses contact and it is the maximum velocity at which the body remains in contact with the surface.

CENTRIFUGAL FORCE

It is a pseudo force experienced by a body which is a part of the circular motion. It is a non-realistic force and comes into action only when the body is in a circular motion. Once the circular motion of the body stops, this force ceases to act. Its magnitude is exactly same as that of centripetal force but it acts opposite to the direction of the centripetal force that is in the radially outward direction.

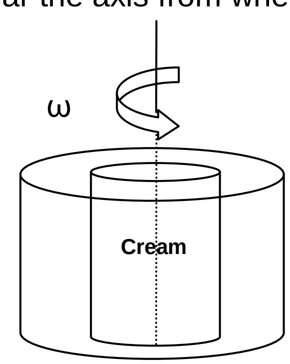
Frame of reference attached to a body moving on a circular path is a non-inertial frame since it an accelerated frame. So when ever any body is observed from this frame a pseudo force $F = ma = mv^2/r = mr\omega^2$ must be applied on the body opposite to the direction of acceleration along with the other forces. Since the acceleration of the frame in circular motion is centripetal acceleration $a = v^2/r$ directed towards the center of the circular path, the pseudo force applied on the bodies observed from this frame is $F = mv^2/r$ directed away from the center of the circular path. This pseudo force is termed as a centrifugal force.



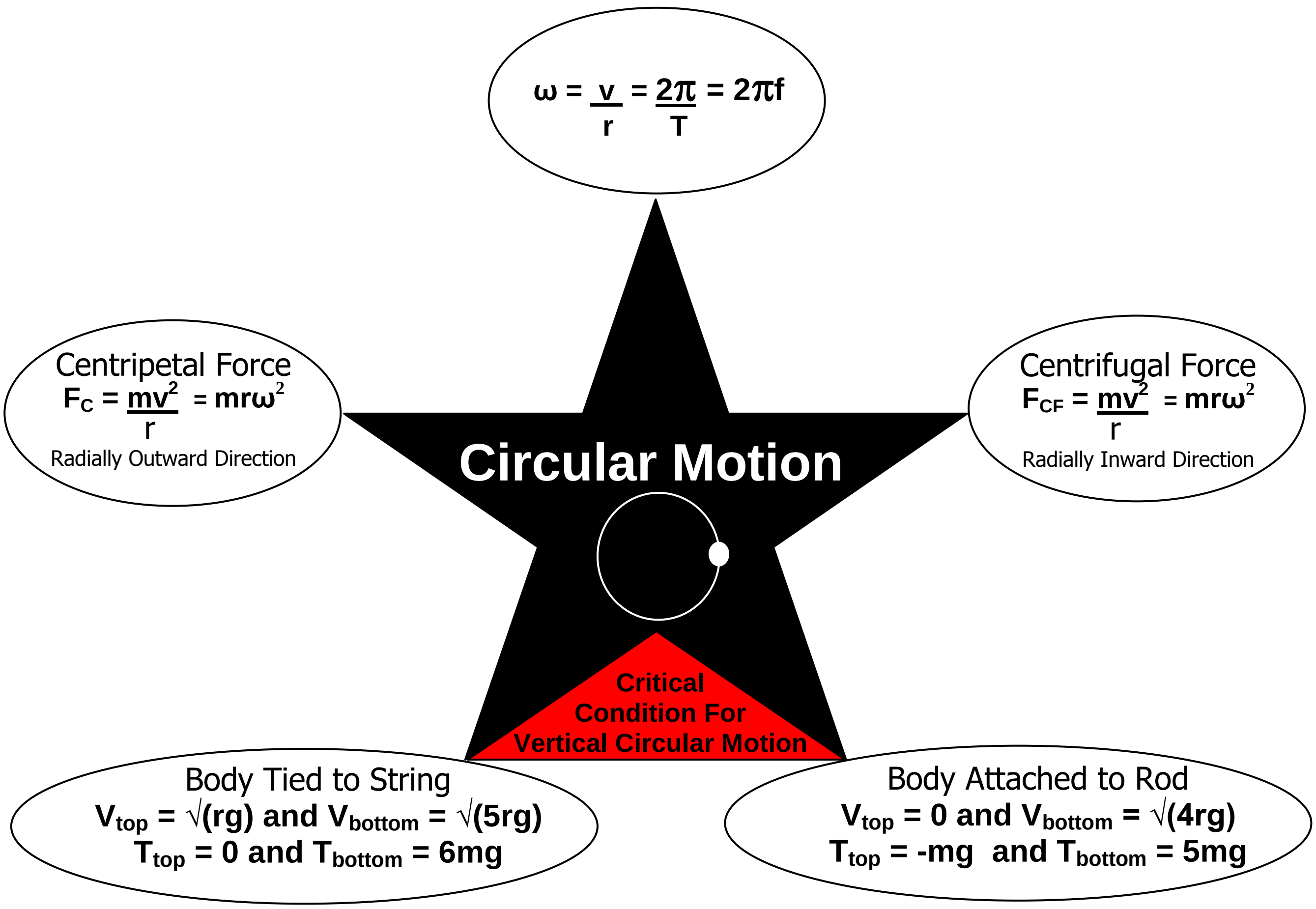
$$\boxed{\mathbf{F_{Centrifugal} = \frac{mv^2}{r} = mr\omega^2}}$$ (Directed in radially outward direction)

CENTRIFUGE

It is an apparatus used to separate cream from milk. It works on the principal of centrifugal force. It is a cylindrical vessel rotating with high angular velocity about its central axis. When this vessel contains milk and rotates with high angular velocity all the particles of milk start moving with the same angular velocity and start experiencing centrifugal force $F_{Centrifugal} = mr\omega^2$ in radially outward direction. Since centrifugal force is directly proportional to the mass of the particles, massive particles of milk on experiencing greater centrifugal force starts depositing on the outer edge of the vessel and lighter cream particles on experiencing smaller centrifugal force are collected near the axis from where they are separated apart.



MEMORY MAP



Critical Condition of Vertical Circular MOtion

Very Short Answer Type 1 Mark Questions

1. Is net force needed to keep a body moving with uniform velocity?
2. Is Newton's 2^{nd} law (F = ma) always valid. Give an example in support of your answer?
3. Action and reaction forces do not balance each other. Why?
4. Can a body remain in state of rest if more than one force is acting upon it?
5. Is the centripetal force acting on a body performing uniform circular motion always constant?
6. The string is holding the maximum possible weight that it could withstand. What will happen to the string if the body suspended by it starts moving on a horizontal circular path and the string starts generating a cone?
7. What is the reaction force of the weight of a book placed on the table?
8. What is the maximum acceleration of a vehicle on the horizontal road? Given that coefficient of static friction between the road and the tyres of the vehicle is μ.
9. Why guns are provided with the shoulder support?
10. While paddling a bicycle what are the types of friction acting on rear wheels and in which direction?

Answer

1. No.
2. It is valid in an inertial frame of reference. In non-inertial frame of reference (such as a car moving along a circular path), Newton's 2^{nd} law doesn't hold apparently.
3. Since they are acting on different bodies.
4. Yes, if all the forces acting on it are in equilibrium.
5. No, only its magnitude remains constant but its direction continuously goes on changing.
6. It will break because tension in the string increases as soon as the body starts moving.
7. The force with which the book attracts the earth towards it.
8. a_{max} = fs(max)/m = μN/m = μmg/m = μg.
9. So that the recoil of gun may be reduced by providing support to the gun by the shoulders.
10. Static friction in forward direction and rolling friction in backward direction.

Short Answer Type 2 Marks Questions

1. Explain why the water doesn't fall even at the top of the circle when the bucket full of water is upside down rotating in a vertical circle?
2. The displacement of a particle of mass 1kg is described by $s = 2t + 3t^2$. Find the force acting on particle? (F = 6N)
3. A particle of mass 0.3 kg is subjected to a force of F = -kx with k = 15 Nm^{-1}. What will be its initial acceleration if it is released from a point 10 cm away from the origin? (a = - 5 ms^{-2})
4. Three forces F_1, F_2 and F_3 are acting on the particle of mass m which is stationary. If F_1 is removed, what will be the acceleration of particle? ($a = F_1/m$)

5. A spring balance is attached to the ceiling of a lift. When the lift is at rest spring balance reads 50 kg of a body hanging on it. What will be the reading of the balance if the lift moves :-
(i) Vertically downward with an acceleration of 5 ms^{-2}
(ii) Vertically upward with an acceleration of 5 ms^{-2}
(iii) Vertically upward with a constant velocity.
Take g = $10 m/s^2$. [(i) 25kgf,(ii) 75kgf, (iii) 50kgf]

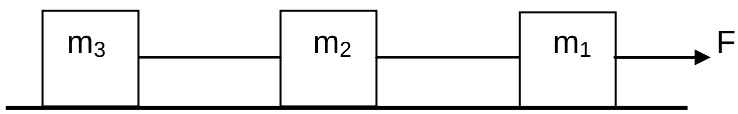
6. Is larger surface area break on a bicycle wheel more effective than small surface area brake? Explain?
7. Calculate the impulse necessary to stop a 1500 kg car moving at a speed of $25 ms^{-1}$? (–37500 N-s)
8.Give the magnitude and directions of the net force acting on a rain drop falling freely with a constant speed of 5 m/s? ($F_{net} = 0$)
9. A block of mass .5kg rests on a smooth horizontal table. What steady force is required to give the block a velocity of 2 m/s in 4 s? (F= .25N)
10. Calculate the force required to move a train of 200 quintal up on an incline plane of 1 in 50 with an acceleration of 2 ms^{-2}. The force of friction per quintal is 0.5 N? (F = 44100N)

Short Answer Type 3 Marks Questions

1. A bullet of mass 0.02 kg is moving with a speed of $10m^{-1}s$. It penetrates 10 cm of a wooden block before coming to rest. If the thickness of the target is reduced to 6 cm only find the KE of the bullet when it comes out? (**Ans :** 0.4 J)
2. A man pulls a lawn roller with a force of F. If he applies the force at some angle with the ground. Find the minimum force required to pull the roller if coefficient of static friction between the ground and the roller is μ?
3. A ball bounces to 80% of its original height. Calculate the change in momentum?
4. A pendulum bob of mass 0.1 kg is suspended by a string of 1 m long. The bob is displaced so that the string becomes horizontal and released. Find its kinetic energy when the string makes an angle of (i) 0°, (ii) 30°, (iii) 60° with the vertical?
5. The velocity of a particle moving along a circle of radius R depends on the distance covered s as F = 2αs where α is constant. Find the force acting on the particle as a function of s?
6. A block is projected horizontally on rough horizontal floor with initial velocity u. The coefficient of kinetic friction between the block and the floor is μ. Find the distance travelled by the body before coming to rest?
7. A locomotive of mass m starts moving so that its velocity v changes according to v = √(α s), where α is constant and s is distance covered. Find the force acting on the body after time t?
8. Derive an expression for the centripetal force?
9. Find the maximum value of angle of friction and prove that it is equal to the angle of repose?
10. State and prove Lami's theorem?

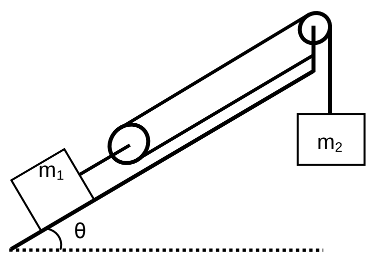
Long Answer Type 5 Marks Questions

1. Find the maximum and minimum velocity of a vehicle of mass m on a banked road of banking angle θ, if coefficient of static friction of the wheels of vehicle with the road is μ?
2. Find the maximum and minimum force applied parallel up the incline on a block of mass m placed on it if angle of inclination is θ and coefficient of static friction with the block is μ so that the block remains at rest?
3. Prove that in case of vertical circular motion circular motion of a body tied to a string velocities at topmost and lowermost point be √(rg) and √(5rg) respectively and tensions in the strings be 0 and 6mg respectively?
4. Find the maximum horizontal velocity that must be imparted to a body placed on the top of a smooth sphere of radius r so that it may not loose contact? If the same body is imparted half the velocity obtained in the first part then find the angular displacement of the body over the smooth sphere when it just loses contact with it?
5. Find the acceleration of the blocks and the tension in the strings?

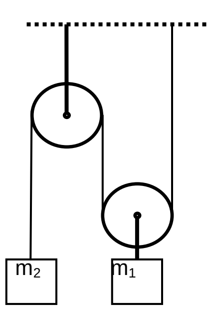


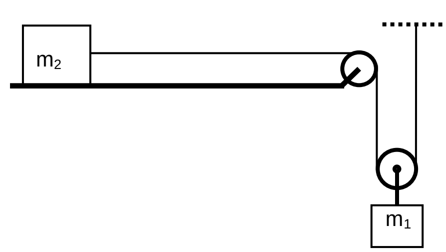
Some Intellectual Stuff

1. Find the acceleration of the blocks m_1 and m_2. All the surfaces are smooth and string and pulley are light? Also find the net force on the clamped pulley?

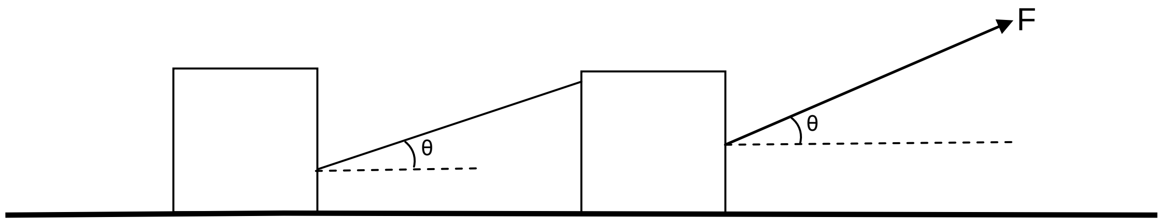


2. A body of mass m explodes into three fragments of with masses in the ratio 2:2:6. If the two similar masses move of perpendicular to each other with the speed of 10m/s each, find the velocity of the third particle and its direction relative to the two other bodies?
3. A mass of 5 kg is suspended by a rope of length 2m from the ceiling. A horizontal force of 50 N is applied at the mid point P of the rope? Calculate the angle that the rope makes with the vertical and the tension in the part of the rope between the point of suspension and point P?. Neglect the mass of the rope. ($g = 10ms^{-2}$)
4. A body moving inside a smooth vertical circular track is imparted a velocity of √(4rg) at the lowermost point. Find its position where it just loses contact with the track?
5.

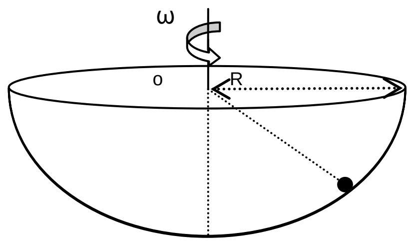




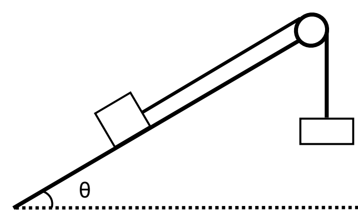
Find in both the cases
(i)Acceleration of the two blocks.(ii)Tension in the clamp holding the fixed pulley?
6. Mass of both the blocks is m and coefficient of kinetic friction with the ground is µ. Find the acceleration of the two blocks and tension in the string attached between the two blocks?



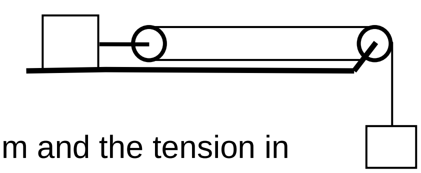
7. A small sphere of mass m is placed in a hemispherical bowl of radius R. Bowl is rotated with angular velocity ω. Find the angle made by the radius of the bowl passing through the sphere with the vertical when the sphere starts rotating with the bowl?



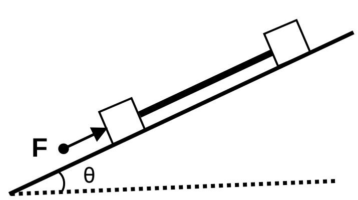
8. Mass of both the blocks is m find net force on the pulley?



9. Mass of both the blocks is m find acceleration of both the blocks and net force on the clamp holding the fixed pulley?



10. Mass of both the blocks is m find acceleration of the system and the tension in the rod?



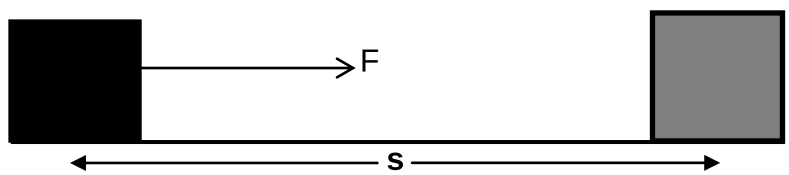
WORK ENERGY AND POWER

WORK

PHYSICAL DEFINITION

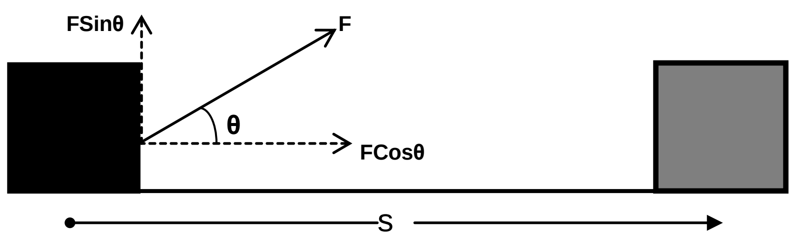
When the point of application of force moves in the direction of the applied force under its effect then work is said to be done.

MATHEMATICAL DEFINITION OF WORK



Work is defined as the product of force and displacement in the direction of force

$$W = F \times s$$

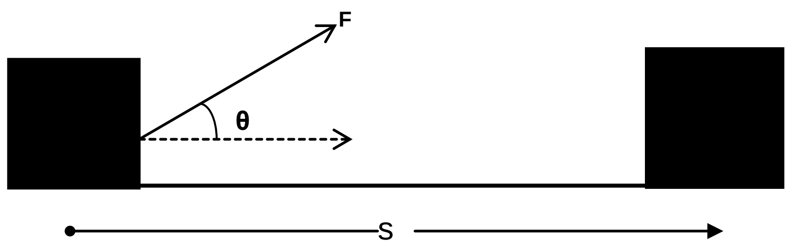


If force and displacement are not parallel to each other rather they are inclined at an angle, then in the evaluation of work component of force (F) in the direction of displacement (s) will be considered.

$$W = (F\cos\theta) \times s$$

or, $$\mathbf{W = Fs\cos\theta}$$

VECTOR DEFINITION OF WORK



Force and displacement both are vector quantities but their product, work is a scalar quantity, hence work must be scalar product or dot product of force and displacement vector.

$$\mathbf{W = \vec{F} \cdot \vec{s}}$$

WORK DONE BY VARIABLE FORCE

Force varying with displacement

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

$$dW = \vec{F} \cdot \vec{ds}$$

$$\mathbf{W = \int_{s_1}^{s_2} \vec{F} \cdot \vec{ds}}$$

Force varying with time

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated.

$$dW = \vec{F} \cdot \vec{ds}$$

Multiplying and dividing by dt,

$$dW = \frac{\vec{F} \cdot \vec{ds}\ dt}{dt}$$

or,

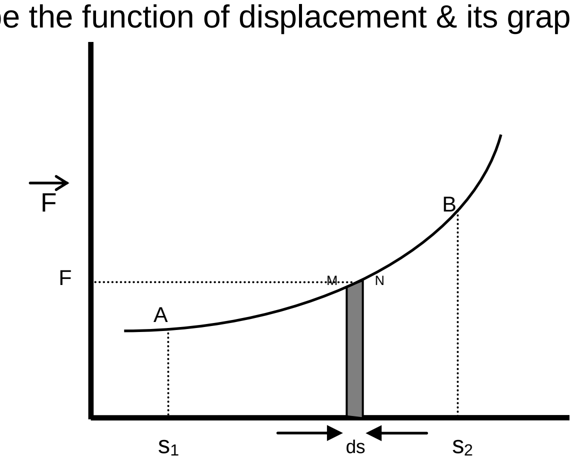
$$dW = \vec{F} \cdot \vec{v}\ dt \quad (v=ds/dt)$$

Total work is obtained by integrating the elementary work from initial to final limits.

$$\mathbf{W = \int_{t_1}^{t_2} \vec{F} \cdot \vec{v}\, dt}$$

WORK DONE BY VARIABLE FORCE FROM GRAPH

Let force be the function of displacement & its graph be as shown.



To find work done from s_1 to s_2 we consider two points M & N very close on the graph such that magnitude of force (F) is almost same at both the points. If elementary displacement from M to N is ds, then elementary work done from M to N is.

dW = F.ds
dW = (length x breadth)of strip MNds
dW = Area of strip MNds

Thus work done in any part of the graph is equal to area under that part. Hence total work done from s_1 to s_2 will be given by the area enclosed under the graph from s_1 to s_2.

W = Area (ABS_2S_1A)

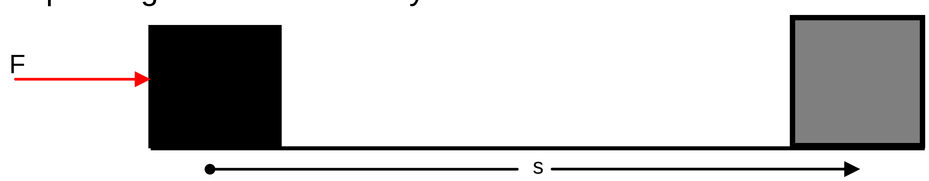
DIFFERENT CASES OF WORK DONE BY CONSTANT FORCE

Case i) Force and displacement are in same direction

$\theta = 0$

Since, $W = Fs \cos\theta$
Therefore $W = Fs \cos 0$
or, **W = Fs**

Ex - Coolie pushing a load horizontally

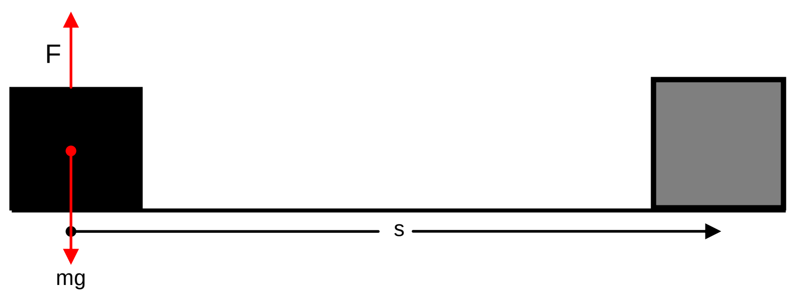


Case ii) Force and displacement are mutually perpendicular to each other

$\theta = 90$

Since, $W = Fs \cos\theta$
Therefore $W = Fs \cos 90$
or, **W = 0**

Ex - coolie carrying a load on his head & moving horizontally with constant velocity. Then he applies force vertically to balance weight of body & its displacement is horizontal.



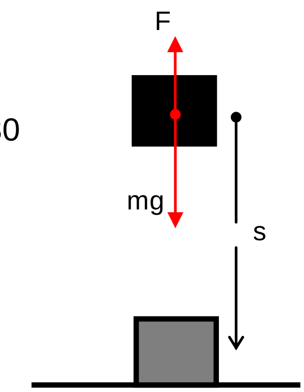
(3) Force & displacement are in opposite direction

$\theta = 180$

Since, $W = Fs \cos\theta$

Therefore $W = Fs \cos 180$

or, $\mathbf{W = -Fs}$



Ex - Coolie carrying a load on his head & moving vertically down with constant velocity. Then he applies force in vertically upward direction to balance the weight of body & its displacement is in vertically downward direction.

ENERGY

Capacity of doing work by a body is known as energy.

Note - Energy possessed by the body by virtue of any cause is equal to the total work done by the body when the cause responsible for energy becomes completely extinct.

TYPES OF ENERGIES

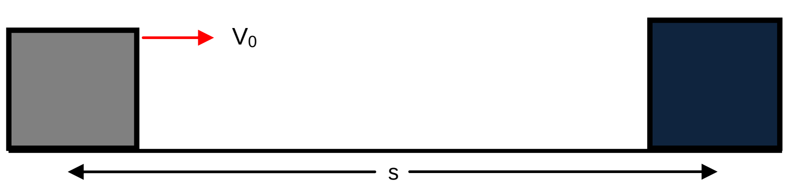
There are many types of energies like mechanical energy, electrical, magnetic, nuclear, solar, chemical etc.

MECHANICAL ENERGY

Energy possessed by the body by virtue of which it performs some mechanical work is known as mechanical energy.
It is of basically two types-
(i) Kinetic energy
(ii) Potential energy

KINETIC ENERGY

Energy possessed by body due to virtue of its motion is known as the kinetic energy of the body. Kinetic energy possessed by moving body is equal to total work done by the body just before coming out to rest.



Consider a body of man (m) moving with velocity (v_o).After travelling through distance (s) it comes to rest.

$$u = v_o$$
$$v = 0$$
$$s = s$$

Applying, $v^2 = u^2 + 2as$

$$0 = v_0^2 + 2as$$

or, $2as = -v_0^2$

or, $a = \frac{-v_o^2}{2s}$

Hence force acting on the body,

$$F = ma$$
$$F_{on\ body} = -\frac{mv_o^2}{2s}$$

But from Newton's third law of action and reaction, force applied by body is equal and opposite to the force applied on body

$$F_{by\ body} = -F_{on\ body}$$
$$= +\frac{mv_o^2}{2s}$$

Therefore work done by body,

$$W = \vec{F}.\vec{s}$$

or, $W = \frac{mv_0^2}{2s}.s.\text{Cos } 0$

or, $W = \frac{1}{2} mv_o^2$

Thus K.E. stored in the body is,

$$\mathbf{K.E. = \frac{1}{2} mv_o^2}$$

KINETIC ENERGY IN TERMS OF MOMENTUM

K.E. of body moving with velocity v is

$$K.E. = \frac{1}{2} mv_o^2$$

Multiplying and dividing by m

$$K = \frac{1}{2}\frac{mv^2 \times m}{m}$$
$$= \frac{1}{2}\frac{m^2v^2}{m}$$

But, mv = p (linear momentum)

Therefore, $\mathbf{K = \frac{p^2}{2m}}$

POTENTIAL ENERGY

Energy possessed by the body by virtue of its position or state is known as potential energy. Example:- gravitational potential energy, elastic potential energy, electrostatic potential energy etc.

GRAVITATIONAL POTENTIAL ENERGY

Energy possessed by a body by virtue of its height above surface of earth is known as gravitational potential energy. It is equal to the work done by the body situated at some height in returning back slowly to the surface of earth.

Consider a body of mass m situated at height h above the surface of earth. Force applied by the body in vertically downward direction is

$$F = mg$$

Displacement of the body in coming back slowly to the surface of earth is

$$s = h$$

Hence work done by the body is

$$W = FsCos\theta$$

or, $$W = FsCos0$$

or, $$W = mgh$$

This work was stored in the body in the form of gravitational potential energy due to its position. Therefore

$$\mathbf{G.P.E = mgh}$$

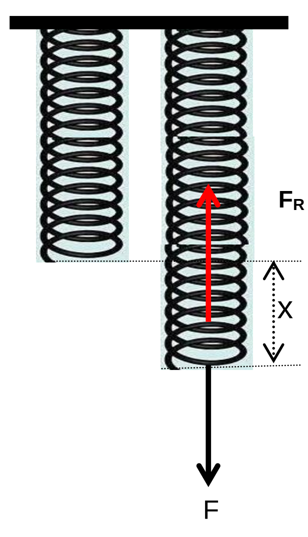
ELASTIC POTENTIAL ENERGY

Energy possessed by the spring by virtue of compression or expansion against elastic force in the spring is known as elastic potential energy.

Spring

It is a coiled structure made up of elastic material & is capable of applying restoring force & restoring torque when disturbed from its original state. When force (F) is applied at one end of the string, parallel to its length, keeping the other end fixed, then the spring expands (or contracts) & develops a restoring force (F_R) which balances the applied force in equilibrium.

On increasing applied force spring further expands in order to increase restoring force for balancing the applied force. Thus restoring force developed within the spring is directed proportional to the extension produced in the spring.



$$F_R \propto x$$

or, $\mathbf{F_R = kx}$ (k is known as spring constant or force constant)

If x = 1, $F_R = k$

Hence force constant of string may be defined as the restoring force developed within spring when its length is changed by unity.

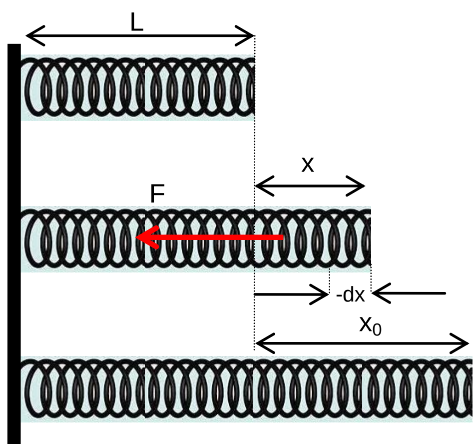
But in equilibrium, restoring force balances applied force.

$$\mathbf{F = F_R = k\,x}$$

If x = 1, F = 1

Hence force constant of string may also be defined as the force required to change its length by unity in equilibrium.

Mathematical Expression for Elastic Potential Energy



Consider a spring of natural length 'L' & spring constant 'k' its length is increased by x_o. Elastic potential energy of stretched spring will be equal to total work done by the spring in regaining its original length.

If in the process of regaining its natural length, at any instant extension in the spring was x then force applied by spring is

$$F = kx$$

If spring normalizes its length by elementary distance dx opposite to x under this force then work done by spring is

$$dW = F. (-dx) . Cos0$$

(force applied by spring F and displacement –dx taken opposite to extension x are in same direction)

$$dW = -kxdx$$

Total work done by the spring in regaining its original length is obtained in integrating dW from x_0 to 0

$$W = \int_{x_0}^{0} -kxdx$$

or, $$W = -k[x^2/2]_{0}^{x_0}$$

or, $$W = - k (0^2/2 - x_0^2/2)$$

o r, $$W = -k (0 - x_0^2/2)$$

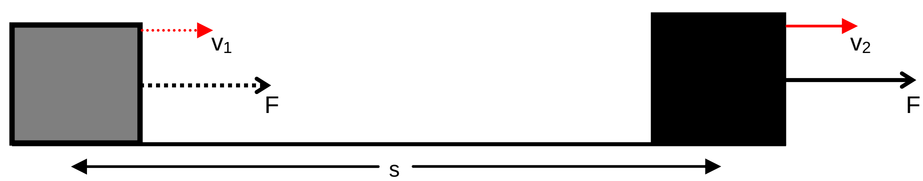
or, $$W = \frac{1}{2} kx_o^2$$

This work was stored in the body in the form of elastic potential energy.

$$\mathbf{E.P.E = \frac{1}{2} kx_o^2}$$

WORK ENERGY THEOREM

It states that total work done on the body is equal to the change in kinetic energy.(Provided body is confined to move horizontally and no dissipating forces are operating).



Consider a body of man m moving with initial velocity v_1. After travelling through displacement s its final velocity becomes v_2 under the effect of force F.

$$u = v_1$$
$$v = v_2$$
$$s = s$$

Applying, $$v^2 = u^2 + 2as$$
$$v_2^2 = v_1^2 + 2as$$

or, $$2as = v_2^2 - v_1^2$$

or, $$a = \frac{v_2^2 - v_1^2}{2s}$$

Hence external force acting on the body is

$$F = ma$$
$$F = m \frac{v_2^2 - v_1^2}{2s}$$

Therefore work done on body by external force

$$W = \vec{F} . \vec{s}$$

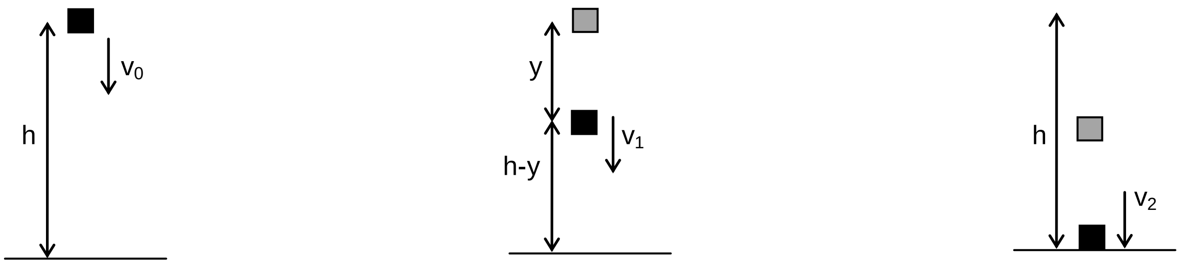
or, $$W = m \frac{v_2^2 - v_1^2}{2s} . s .Cos 0$$

(since force and displacement are in same direction) or, $W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

or, $\mathbf{W = K_2 - K_1}$

or, $\mathbf{W = \Delta K}$

PRINCIPLE OF CONSERVATION OF ENERGY



It states that energy can neither be creased neither be destroyed. It can only be converted from one form to another.

Consider a body of man m situated at height h & moving with velocity v_o. Its energy will be.

$$E_1 = P_1 + K_1$$

or, $\mathbf{E_1 = mgh + \frac{1}{2} m v_o^2}$

If the body falls under gravity through distance y, then it acquires velocity v_1 and its height becomes (h-y)

$u = v_o$
$s = y$
$a = g$
$v = v_1$

From $v^2 = u^2 + 2as$

$$v_1^2 = v_o^2 + 2gy$$

Energy of body in second situation

$$E_2 = P_2 + K_2$$

or, $E_2 = mg(h-y) + \frac{1}{2} mv^2$

or, $E_2 = mg(h-y) + \frac{1}{2} m(v_o^2 + 2gy)$

or, $E_2 = mgh - mgy + \frac{1}{2} mv_o^2 + mgy$

or, $\mathbf{E_2 = mgh + \frac{1}{2} m v_o^2}$

Now we consider the situation when body reaches ground with velocity v_2

$u = v_o$
$s = h$
$a = g$
$v = v_2$

From $$v^2 = u^2 + 2as$$
$${}_2{}^2 = v_o^2 + 2gh$$

Energy of body in third situation

$$E_3 = P_3 + K_3$$

or, $$E_3 = mg0 + ½\ mv_2{}^2$$

or, $$E_3 = 0 + ½\ m\ (v_o{}^2 + 2gh)$$

or, $$\mathbf{E_3 = ½\ mv_o{}^2 + mgh}$$

From above it must be clear that $\mathbf{E_1 = E_2 = E_3}$. This proves the law of conservation of energy.

CONSERVATIVE FORCE

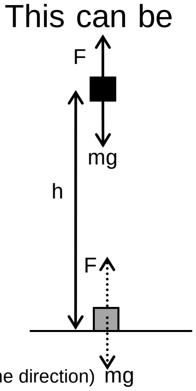
Forces are said to be conservative in nature if work done against the forces gets conversed in the body in form of potential energy. Example:- gravitational forces, elastic forces & all the central forces.

PROPERTIES OF CONSERVATIVE FORCES

1. Work done against these forces is conserved & gets stored in the body in the form of P.E.
2. Work done against these forces is never dissipated by being converted into non-usable forms of energy like heat, light, sound etc.
3. Work done against conservative forces is a state function & not path function i.e. Work done against it, depends only upon initial & final states of body & is independent of the path through which process has been carried out.
4. Work done against conservative forces is zero in a complete cycle.

TO PROVE WORK DONE AGAINST CONSERVATIVE FORCES IS A STATE FUNCTION

Consider a body of man m which is required to be lifted up to height h. This can be done in 2 ways.
(i) By directly lifting the body against gravity
(ii) By pushing the body up a smooth inclined plane.



Min force required to lift the body of mass m vertically is

$$F = mg$$

And displacement of body in lifting is

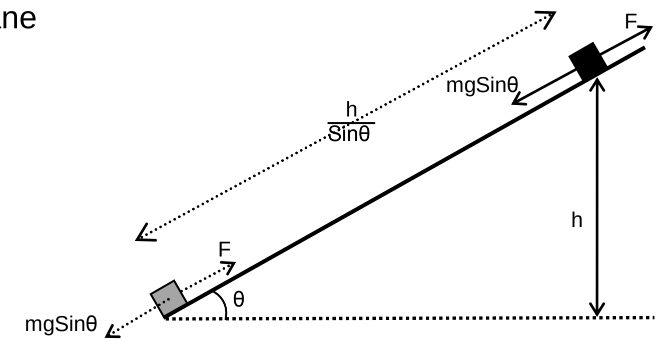
$$s = h$$

Hence work done in lifting is

$$W_1 = FsCos0^o$$ (since force and displacement are in same direction)

$$\mathbf{W_1 = mgh}$$

Now we consider the same body lifted through height h by pushing it up a smooth inclined plane



Min force required to push the body is

$$F = mgSin\theta$$

And displacement of body in lifting is

$$s = \frac{h}{Sin\theta}$$

Hence work done in pushing is

$$W_2 = FsCos0$$

or, $$W_2 = mgSin\theta \cdot \frac{h}{Sin\theta} \cdot 1$$

or, $$\mathbf{W_2 = mgh}$$

From above $W_1 = W_2$ we can say that in both the cases work done in lifting the body through height 'h' is same.

To Prove That Work Done Against Conservative Forces Is Zero In A Complete Cycle

Consider a body of man m which is lifted slowly through height h & then allowed to come back to the ground slowly through height h.

For work done is slowly lifting the body up,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical up displacement of the body is

$$s = h$$

Hence work done is

$$W = FsCos\theta$$

or, $W_I = FsCos0$ (since force and displacement are in same direction)

or, $\mathbf{W_I = mgh}$ (since force and displacement are in same direction)

For work done is slowly bringing the body down,
Minimum force required in vertically upward direction is

$$F = mg$$

Vertical down displacement of the body is

$$s = h$$

Hence work done is

or, $W_2 = FsCos180$(since force and displacement are in opposite direction)

or, $\mathbf{W_2 = - mgh}$

Hence total work done against conservative forces in a complete cycle is

$$W = W_1 + W_2$$

or, $W = (mgh) + (-mgh)$

or, $\mathbf{W = 0}$

NON-CONSERVATIVE FORCES

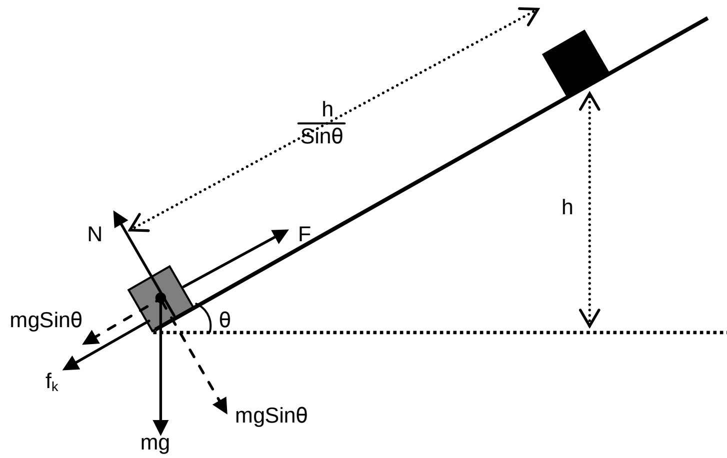
Non conservative forces are the forces, work done against which does not get conserved in the body in the form of potential energy.

PROPERTIES OF NON-CONSERVATIVE FORCES

1. Work done against these forces does not get conserved in the body in the form of P.E.
2. Work done against these forces is always dissipated by being converted into non usable forms of energy like heat, light, sound etc.
3. Work done against non-conservative force is a path function and not a state function.
4. Work done against non-conservative force in a complete cycle is not zero.

PROVE THAT WORK DONE AGAINST NON–CONSERVATIVE FORCES IS A PATH FUNCTION

Consider a body of mass (m) which is required to be lifted to height ‘h’ by pushing it up the rough incline of inclination.



Minimum force required to slide the body up the rough inclined plane having coefficient of kinetic friction μ with the body is

$$F = mgSin\theta + f_k$$

or, $$F = mgSin\theta + \mu N$$

or, $$F = mgSin\theta + \mu mgCos\theta$$

Displacement of the body over the incline in moving through height h is

$$s = \frac{h}{Sin\theta}$$

Hence work done in moving the body up the incline is

$$W = F.s.Cos0$$ (since force and displacement are in opposite direction)

or, $$W = (mgSin\theta + \mu mgCos\theta).\frac{h}{Sin\theta}.1$$

or, $$\mathbf{W = mgh + \frac{\mu mgh}{Tan\theta}}$$

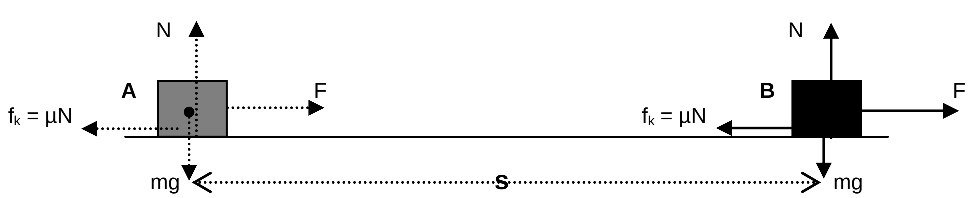
Similarly if we change the angle of inclination from θ to θ_1, then work done will be

$$\mathbf{W_1 = mgh + \frac{\mu mgh}{Tan\theta_1}}$$

This clearly shows that work done in both the cases is different & hence work done against non-conservative force in a path function and not a state function i.e. it not only depends upon initial & final states of body but also depends upon the path through which process has been carried out.

To Prove That Work Done Against Non-conservative Forces In A Complete Cycle Is Not Zero

Consider a body displaced slowly on a rough horizontal plane through displacement s from A to B.



Minimum force required to move the body is

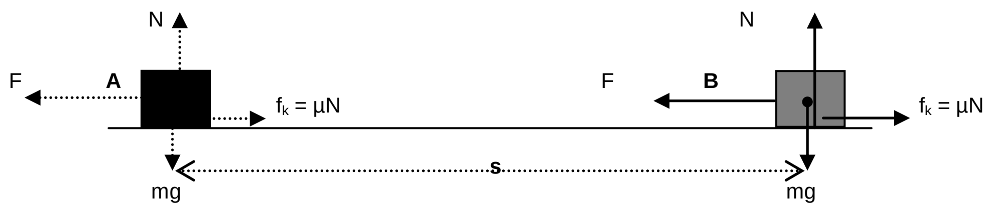
$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0 \text{(since force and displacement are in same direction)}$$

or, $$\mathbf{W = \mu mgs}$$

Now if the same body is returned back from B to A



Minimum force required to move the body is

$$F = f_k = \mu N = \mu mg$$

Work done by the body in displacement s is

$$W = F.s.Cos0 \text{(since force and displacement are in same direction)}$$

or, $$\mathbf{W = \mu mgs}$$

Hence total work done in the complete process

$$\mathbf{W = W_1 + W_2 = 2\mu mgs}$$

Note - When body is returned from B to A friction reverse its direction.

POWER

Rate of doing work by a body with respect to time is known as power.

Average Power

It is defined as the ratio of total work done by the body to total time taken.

$$\mathbf{P_{avg} = \frac{Total\ work\ done}{Total\ time\ taken} = \frac{\Delta W}{\Delta t}}$$

Instantaneous Power

Power developed within the body at any particular instant of time is known as instantaneous power.

Or

Average power evaluated for very short duration of time is known as instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} P_{avg}$$

or, $$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$P_{inst} = \frac{dW}{dt}$$

or, $$P_{inst} = \frac{d\vec{F}\cdot\vec{s}}{dt}$$

or, $$P_{inst} = \vec{F}\cdot\frac{d\vec{s}}{dt}$$

or, $$\mathbf{P_{inst} = \vec{F}\cdot\vec{v}}$$

EFFICIENCY

It is defined as the ratio of power output to power input.

Or

It is defined as the ratio of energy output to energy input.

Or

I It is defined as the ratio of work output to work input.

$$\eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}}$$

PERCENTAGE EFFICIENCY

Percentage Efficiency = Efficiency x 100

$$\textbf{Percentage Efficiency} = \eta = \frac{P_{Output}}{P_{Input}} = \frac{E_{Output}}{E_{Input}} = \frac{W_{Output}}{W_{Input}} \times 100$$

COLLISION

Collision between the two bodies is defined as mutual interaction of the bodies for a short interval of time due to which the energy and the momentum of the interacting bodies change.

Types of Collision

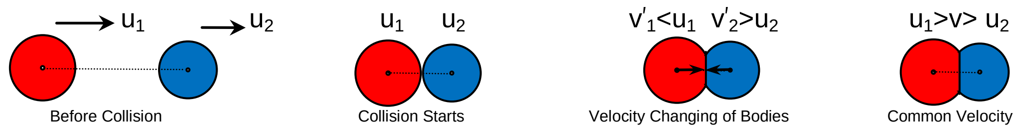
There are basically three types of collisions-

i) Elastic Collision – That is the collision between perfectly elastic bodies. In this type of collision, since only conservative forces are operating between the interacting bodies, both kinetic energy and momentum of the system remains constant.

ii) Inelastic Collision – That is the collision between perfectly inelastic or plastic bodies. After collision bodies stick together and move with some common velocity. In this type of collision only momentum is conserved. Kinetic energy is not conserved due to the presence of non-conservative forces between the interacting bodies.

iii) Partially Elastic or Partially Inelastic Collision – That is the collision between the partially elastic bodies. In this type of collision bodies do separate from each other after collision but due to the involvement of non-conservative inelastic forces kinetic energy of the system is not conserved and only momentum is conserved.

Collision In One Dimension – Analytical Treatment



Consider two bodies of masses m_1 and m_2 with their center of masses moving along the same straight line in same direction with initial velocities u_1 and u_2 with m_1 after m_2. Condition necessary for the collision is $u_1 > u_2$ due to which bodies start approaching towards each other with the velocity of approach $u_1 - u_2$.
Collision starts as soon as the bodies come in contact. Due to its greater velocity and inertia m_1 continues to push m_2 in the forward direction whereas m_2 due to its small velocity and inertia pushes m_1 in the backward direction. Due to this pushing force involved between the two colliding bodies they get deformed at the point of contact and a part of their kinetic energy gets consumed in the deformation of the bodies. Also m_1 being pushed opposite to the direction of the motion goes on decreasing its velocity and m_2 being pushed in the direction of motion continues increasing its velocity. This process continues until both the bodies acquire the same common velocity v. Up to this stage there is maximum deformation in the bodies maximum part of their kinetic energy gets consumed in their deformation.

Elastic collision

In case of elastic collision bodies are perfectly elastic. Hence after their maximum deformation they have tendency to regain their original shapes, due to which they start pushing each other. Since m_2 is being pushed in the direction of motion its velocity goes on increasing and m_1 being pushed opposite to the direction of motion its velocity goes on decreasing. Thus condition necessary for separation i.e. $\mathbf{v_2>v_1}$ is attained and the bodies get separated with velocity of separation $\mathbf{v_2 - v_1}$.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is again returned back to the system when the bodies regain their original shapes. Hence in such collision energy conservation can also be applied along with the momentum conservation.

Applying energy conservation

$$E_i = E_f$$
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$
$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ ....(i)}$$

Applying momentum conservation

$$p_i = p_f$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ....(ii)}$$

Dividing equation (i) by (ii)

$$u_1 + v_1 = v_2 + u_2$$

or, $$\mathbf{v_2 - v_1 = u_1 - u_2}$$

or, **Velocity of separation = Velocity of approach**

or, $$v_2 = v_1 + u_1 - u_2$$

Putting this in equation (i)

$$\mathbf{v_1 = \frac{(m_1-m_2)u_1}{(m_1+m_2)} + \frac{2m_2\,u_2}{(m_1+m_2)}}$$

Similarly we can prove

$$\mathbf{v_2 = \frac{(m_2-m_1)u_2}{(m_1+m_2)} + \frac{2m_1\,u_1}{(m_1+m_2)}}$$

Case 1- If the bodies are of same mass,

$$m_1 = m_2 = m$$

$$\mathbf{v_1 = u_2}$$

$$\mathbf{v_2 = u_1}$$

Hence in perfectly elastic collision between two bodies of same mass, the velocities interchange.ie. If a moving body elastically collides with a similar body at rest. Then the moving body comes at rest and the body at rest starts moving with the velocity of the moving body.

Case 2- If a huge body elastically collides with the small body,

$m_1 >> m_2$

m_2 will be neglected in comparison to m_1

$$v_1 = \frac{(m_1-0)u_1}{(m_1+0)} + \frac{2.0.\,u_2}{(m_1+0)}$$

$$\mathbf{v_1 = u_1}$$

and

$$v_2 = \frac{(0-m_1)u_2}{(m_1+0)} + \frac{2m_1\,u_1}{(m_1+0)}$$

$$\mathbf{v_2 = -u_2 + 2u_1}$$

If, $u_2 = 0$

$$\mathbf{v_2 = 2u_1}$$

Hence if a huge body elastically collides with a small body then there is almost no change in the velocity of the huge body but if the small body is initially at rest it gets thrown away with twice the velocity of the huge moving body.eg. collision of truck with a drum.

Case 3- If a small body elastically collides with a huge body,

$m_2 >> m_1$

m_1 will be neglected in comparison to m_2

$$v_1 = \frac{(0-m_2)u_1}{(0+m_2)} + \frac{2m_2\,u_2}{(0+m_2)}$$

or, $\mathbf{v_1 = -u_1 + 2u_2}$

If $u_2 = 0$

$$\mathbf{v_1 = -u_1}$$

and

$$v_2 = \frac{(m_2-0)u_2}{(0+m_2)} + \frac{2.0.u_1}{(0+m_2)}$$

$$\mathbf{v_2 = u_2}$$

Hence if a small body elastically collides with a huge body at rest then there is almost no change in the velocity of the huge body but if the huge body is initially at rest small body rebounds back with the same speed.eg. collision of a ball with a wall.

Inelastic collision

In case of inelastic collision bodies are perfectly inelastic. Hence after their maximum deformation they have no tendency to regain their original shapes, due to which they continue moving with the same common velocity.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is permanently consumed in the deformation of the bodies against non-conservative inelastic forces. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

Applying momentum conservation

$$p_i = p_f$$

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

or, $m_1u_1 + m_2u_2 = (m_1+m_2)v$

or, $$\mathbf{v = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)}}$$

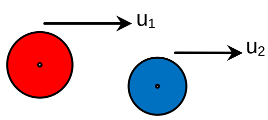
Partially Elastic or Partially Inelastic Collision

In this case bodies are partially elastic. Hence after their maximum deformation they have tendency to regain their original shapes but not as much as perfectly elastic bodies. Hence they do separate but their velocity of separation is not as much as in the case of perfectly elastic bodies i.e. their velocity of separation is less than the velocity of approach.

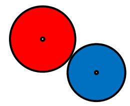
In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is only slightly returned back to the system. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

$$\mathbf{(v_2 - v_1) < (u_1 - u_2)}$$

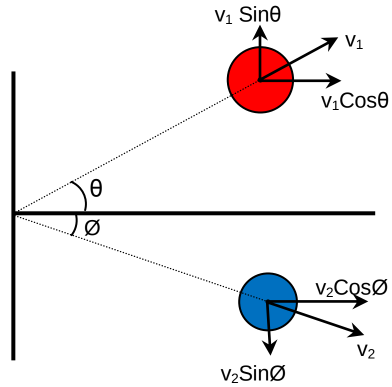
Collision In Two Dimension – Oblique Collision



Before Collision



Collision Starts



After Collision

When the centers of mass of two bodies are not along the same straight line, the collision is said to be oblique. In such condition after collision bodies are deflected at some angle with the initial direction. In this type of collision momentum conservation is applied separately along x-axis and y-axis. If the collision is perfectly elastic energy conservation is also applied.

Let initial velocities of the masses m_1 and m_2 be u_1 and u_2 respectively along x-axis. After collision they are deflected at angles θ and Ø respectively from x-axis, on its either side of the x axis.

Applying momentum conservation along x-axis

$$p_f = p_i$$

$$\mathbf{m_1v_1\ Cos\theta + m_2v_2\ Cos\,Ø = m_1u_1 + m_2u_2}$$

Applying momentum conservation along y-axis

$$p_f = p_i$$

$$m_1v_1\ Sin\theta - m_2v_2\ Sin\,Ø = m_10 + m_20$$

or, $$m_1v_1\ Sin\theta - m_2v_2\ Sin\,Ø = 0$$

or, $$\mathbf{m_1v_1\ Sin\theta = m_2v_2\ Sin\,Ø}$$

In case of elastic collision applying energy conservation can also be applied

$$K_f = K_i$$

$$\mathbf{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2}$$

Coefficient Of Restitution

It is defined as the ratio of velocity of separation to the velocity of approach.

$$\mathbf{e = \frac{Velocity\ of\ separation}{Velocity\ of\ approach}}$$

or,
$$\mathbf{e = \frac{(v_2 - v_1)}{(u_1 - u_2)}}$$

Case-1 For perfectly elastic collision, velocity of separation is equal to velocity of approach, therefore

$$\mathbf{e = 1}$$

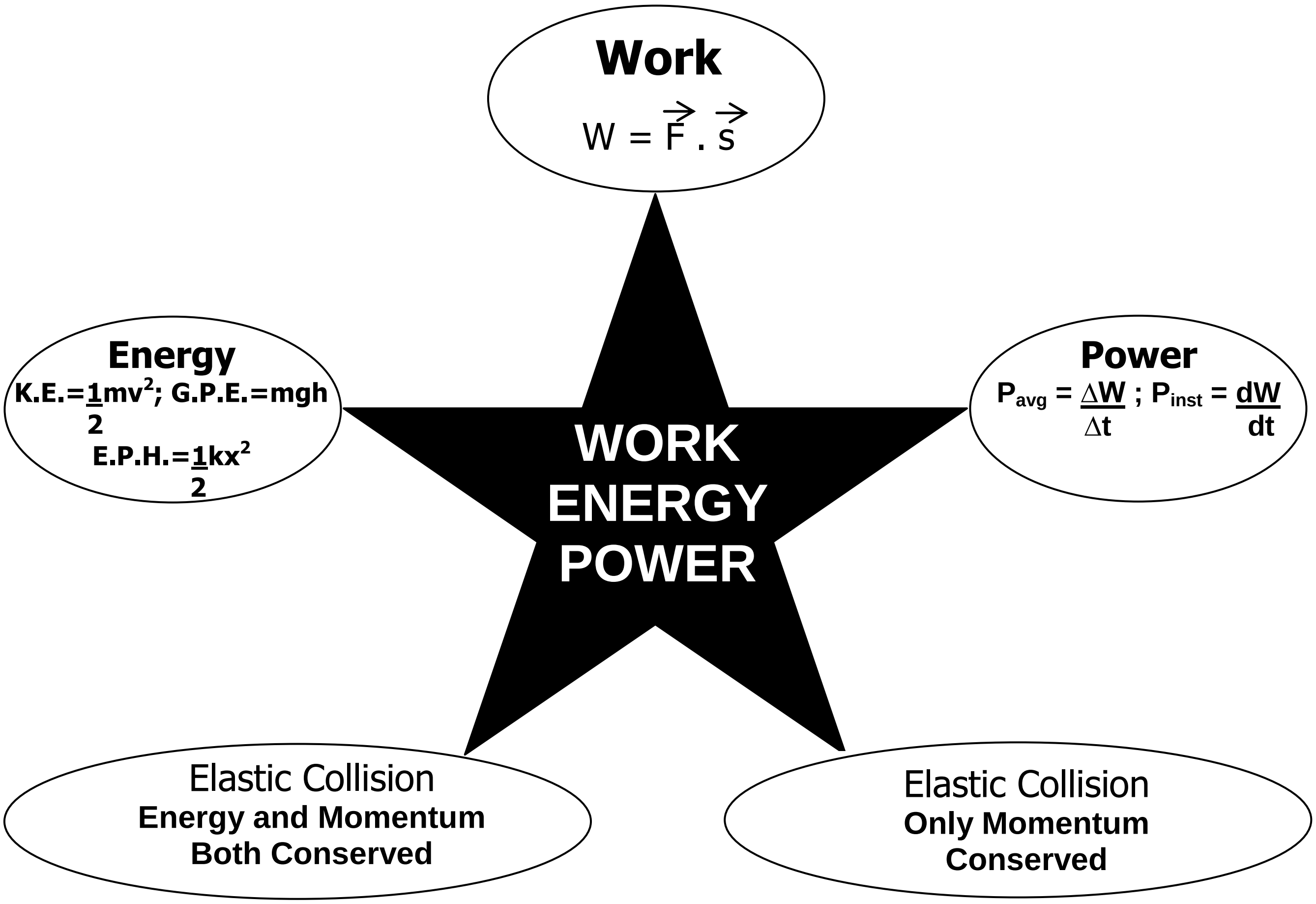
Case-2 For perfectly inelastic collision, velocity of separation is zero, therefore

$$\mathbf{e = 0}$$

Case-3 For partially elastic or partially inelastic collision, velocity of separation is less than velocity of approach, therefore

$$\mathbf{e < 1}$$

MEMORY MAP


Work
$W = \vec{F} \cdot \vec{s}$
Energy
K.E.$=\frac{1}{2}mv^2$; G.P.E.$=mgh$
E.P.H.$=\frac{1}{2}kx^2$
Power
$P_{avg} = \frac{\Delta W}{\Delta t}$; $P_{inst} = \frac{dW}{dt}$
WORK
ENERGY
POWER
Elastic Collision
Energy and Momentum
Both Conserved
Elastic Collision
Only Momentum
Conserved

Very Short Answer Type 1 Mark Questions

1. Define the conservative and non-conservative forces? Give example of each?
2. A light body and a heavy body have same linear momentum. Which one has greater K.E? (Ans: Lighter body has more K.E.)
3.If the momentum of the body is doubled by what percentage does its K.E changes? (300%)
4. A truck and a car are moving with the same K.E on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance? (Truck)
5. What happens to the P.E of a bubble when it rises up in water? (decrease)
6. Define spring constant of a spring?
7. What happens when a sphere collides head on elastically with a sphere of same mass initially at rest?
8. Derive an expression for K.E of a body of mass m moving with a velocity v by calculus method.
9. After bullet is fired, gun recoils. Compare the K.E. of bullet and the gun. (K.E. of bullet > K.E. of gun)
10. In which type of collision there is maximum loss of energy?

Very Short Answer Type 2 Marks Questions

1. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10% what is the speed with which the bob arrives the lowest point? (**Ans :** 6m/s)
2. Find the work done if a particle moves from position $\vec{r_1}$ = (4i + 3j + 6k)m to a position $\vec{r}_2$ = (14i = 13j = 16k) under the effect of force, $\vec{F}$ = (4i + 4j - 4k)N? (**Ans :** 40J)
3. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is stretched further through 0.1m calculate work done? (**Ans :** 4000 Nm–1, 60 J)
4. A pump on the ground floor of a building can pump up water to fill a tank of volume $30m^3$ in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump? The efficiency of the pump is 30%. (**Ans :** 43.556 kW)
5. Spring of a weighing machine is compressed by 1cm when a sand bag of mass 0.1 kg is dropped on it from a height 0.25m. From what height should the sand bag be dropped to cause a compression of 4cm? (**Ans :** 4m)
6. Show that in an elastic one dimensional collision the velocity of approach before collision is equal to velocity of separation after collision?
7. A spring is stretched by distance x by applying a force F. What will be the new force required to stretch the spring by 3x? Calculate the work done in increasing the extension?
8. Write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by length. Draw the graphs to show the variation of potential energy and force with elongation?
9. How does a perfectly inelastic collision differ from perfectly elastic collision? Two particles of mass m_1 and m_2 having velocities u_1 and u_2 respectively make a head on collision. Derive the relation for their final velocities?

10. In lifting a 10 kg weight to a height of 2m, 250 Joule of energy is spent. Calculate the acceleration with which it was raised?(g=10m/s^2) (**Ans :** 2.5m/s^2)

Short Answer Type 3 Marks Questions

1. An electrical water pump of 80% efficiency is used to lift water up to a height of 10m.Find mass of water which it could lift in 1hrour if the marked power was 500 watt?
2. A cycle is moving up the incline rising 1 in 100 with a const. velocity of 5m/sec. Find the instantaneous power developed by the cycle?
3. Find % change in K.E of body when its momentum is increased by 50%.
4. A light string passing over a light frictionless pulley is holding masses m and 2m at its either end. Find the velocity attained by the masses after 2 seconds.
5. Derive an expression for the centripetal force experienced by a body performing uniform circular motion.
6. Find the elevation of the outer tracks with respect to inner. So that the train could safely pass through the turn of radius 1km with a speed of 36km/hr. Separation between the tracks is 1.5m?
7. A block of mass m is placed over a smooth wedge of inclination θ. With what horizontal acceleration the wedge should be moved so that the block must remain stationery over it?
8. Involving friction prove that pulling is easier than pushing if both are done at the same angle.
9. In vertical circular motion if velocity at the lowermost point is $\sqrt{(6rg)}$ where find the tension in the string where speed is minimum. Given that mass of the block attached to it is m?
10. A bullet of mass m moving with velocity u penetrates a wooden block of mass M suspended through a string from rigid support and comes to rest inside it. If length of the string is L find the angular deflection of the string.

Long Answer Type 5 Marks Questions

1. What is conservative force? Show that work done against conservative forces is a state function and not a path function. Also show that work done against it in a complete cycle is zero?
2. A body of man 10 kg moving with the velocity of 10m/s impinges the horizontal spring of spring constant 100 Nm^{-1} fixed at one end. Find the maximum compression of the spring? Which type of mechanical energy conversion has occurred? How does the answer in the first part changes when the body is moving on a rough surface?
3. Two blocks of different masses are attached to the two ends of a light string passing over the frictionless and light pully. Prove that the potential energy of the bodies lost during the motion of the blocks is equal to the gain in their kinetic energies?

4. A locomotive of mass m starts moving so that its velocity v is changing according to the law v $\sqrt{(as)}$, where a is constant and s is distance covered. Find the total work done by all the forces acting the locomotive during the first t seconds after the beginning of motion?
5. Derive an expression for the elastic potential energy of the stretched spring of spring constant k. Find the % change in the elastic potential energy of spring if its length is increased by 10%?

Some Intellectual Stuff

1. A body of mass m is placed on a rough horizontal surface having coefficient of static friction μ with the body. Find the minimum force that must be applied on the body so that it may start moving? Find the work done by this force in the horizontal displacement s of the body?
2. Two blocks of same mass m are placed on a smooth horizontal surface with a spring of constant k attached between them. If one of the block is imparted a horizontal velocity v by an impulsive force, find the maximum compression of the spring?
3. A block of mass M is supported against a vertical wall by a spring of constant k. A bullet of mass m moving with horizontal velocity v_0 gets embedded in the block and pushes it against the wall. Find the maximum compression of the spring?
4. Prove that in case of oblique elastic collision of a moving body with a similar body at rest, the two bodies move off perpendicularly after collision?
5. A chain of length L and mass M rests over a sphere of radius R ($L < R$) with its one end fixed at the top of the sphere. Find the gravitational potential energy of the chain considering the center of the sphere as the zero level of the gravitational potential energy?

MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

CONCEPTS.

.Centre of mass of a body is a point where the entire mass of the body can be supposed to be concentrated

For a system of *n*-particles, the centre of mass is given by

$$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + ... + m_n\vec{r_n}}{m_1 + m_2 + m_3 + ... + m_n} = \frac{\sum_{i=1}^{i=n} m_i\vec{r_1}}{M}$$

.Torque τ *The turning effect of a force with respect to some axis, is called moment of force or torque due to the force.* Torque is measured as the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the axis of rotation.

$$\vec{\tau}=\vec{r} \times \vec{F}$$

.Angular momentum ($\vec{L}$). It is the rotational analogue of linear momentum and is measured as the product of linear momentum and the perpendicular distance of its line of axis of rotation.

Mathematically: If $\vec{P}$ is linear momentum of the particle and $\vec{r}$ its position vector, then angular momentum of the particle, $\vec{L} = \vec{r} \times \vec{P}$

(a)In Cartesian coordinates : $L_Z = xp_y - yp_x$

(b)In polar coordinates : $L = r\ p\ sin\emptyset$,

Where $\emptyset$ is angle between the linear momentum vector $\vec{P}$ and the position of vector $\vec{r}$.

S.I unit of angular momentum is kg m^2s^{-1}.

Geometrically, angular momentum of a particle is equal to twice the product of mass of the particle and areal velocity of its radius vector about the given axis.

.Relation between torque and angular momentum:

(i)$\vec{\tau} = \frac{d\vec{L}}{dt}$ (ii) If the system consists of n-particles, then $\vec{\tau} = \frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt} + \frac{d\vec{L}_3}{dt} + \cdots + \frac{d\vec{L}_n}{dt}$.

.Law of conservation of angular momentum. If no external torque acts on a system, then the total angular momentum of the system always remain conserved.

Mathematically: $\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n = \vec{L}_{total} = a\ constant$

.**Moment of inertia(I).**the moment of inertia of a rigid body about a given axis of rotation is the sum of the products of masses of the various particles and squares of their respective perpendicular distances from the axis of rotation.

Mathematically: I=$m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$= $\sum_{i=1}^{i=n} m_i\, r_i^2$

SI unit of moment of inertia is kg m^2.

MI corresponding to mass of the body. However, it depends on shape & size of the body and also on position and configuration of the axis of rotation.

Radius of gyration (K).it is defined as the distance of a point from the axis of rotation at which, if whole mass of the body were concentrated, the moment of inertia of the body would be same as with the actual distribution of mass of the body.

Mathematically :K=$\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}$= rms distance of particles from the axis of rotation.

SI unit of gyration is m. Note that the moment of inertia of a body about a given axis is equal to the product of mass of the body and squares of its radius of gyration about that axis i.e. I=Mk^2.

.**Theorem of perpendicular axes.** It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moment of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through the lamina.

Mathematically: $I_z = I_x + I_{y'}$

Where x & y-axes lie in the plane of the Lamina and z-axis is perpendicular to its plane and passes through the point of intersecting of x and y axes.

.Theorem of parallel axes. It states that the moment of inertia of a rigid body about any axis is equal to moment of inertia of the body about a parallel axis through its center of mass plus the product of mass of the body and the square of the perpendicular distance between the axes.

Mathematically: $I = I_c + M\,h^2$, where I_c is moment of inertia of the body about an axis through its centre of mass and h is the perpendicular distance between the two axes.

.**Moment of inertia** of a few bodies of regular shapes:

i. M.I. of a rod about an axis through its c.m. and perpendicular to rod, $I = \frac{1}{12}\,ML^2$
ii. M.I. of a circular ring about an axis through its centre and perpendicular to its plane, $I = MR^2$
iii. M.I. of a circular disc about an axis through its centre and perpendicular to its plane, $I = \frac{1}{2}\,MR^2$
iv. M.I. of a right circular solid cylinder about its symmetry axis, $I = \frac{1}{2}\,MR^2$
v. M.I. of a right circular hollow cylinder about its axis = MR^2
vi. M.I. of a solid sphere about its diameter, $I = \frac{2}{5}\,MR^2$
vii. M.I. of spherical shell about its diameter, $I = \frac{2}{3}\,MR^2$

.**Moment of inertia and angular momentum.** The moment of inertia of a rigid body about an axis is numerically equal to the angular momentum of the rigid body, when rotating with unit angular velocity about that axis.

Mathematically: $K.E\ of\ rotation = \frac{1}{2} I\ \omega^2$

.**Moment of inertia and kinetic energy of rotation.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the kinetic energy of rotation of the body, when rotation with unit angular velocity about that axis.

Mathematically:$K.E.of\ rotation = \frac{1}{2}\ I\omega^2$

.**Moment of inertia and torque.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce a unit angular acceleration in the body BOUT THE GIVEN AXIS.

MATHEMATICALLY: $\tau = Ia$

.**Law of conservation of angular momentum.** If no external torque acts on a system, the total angular momentum of the system remains unchanged.

Mathematically:
$I\omega = constant\ vector, i.e., in\ magnitude, I_1\omega_1 = I_2\omega_2,$
$provides\ no\ external\ torque\ acts\ on\ the\ system.$

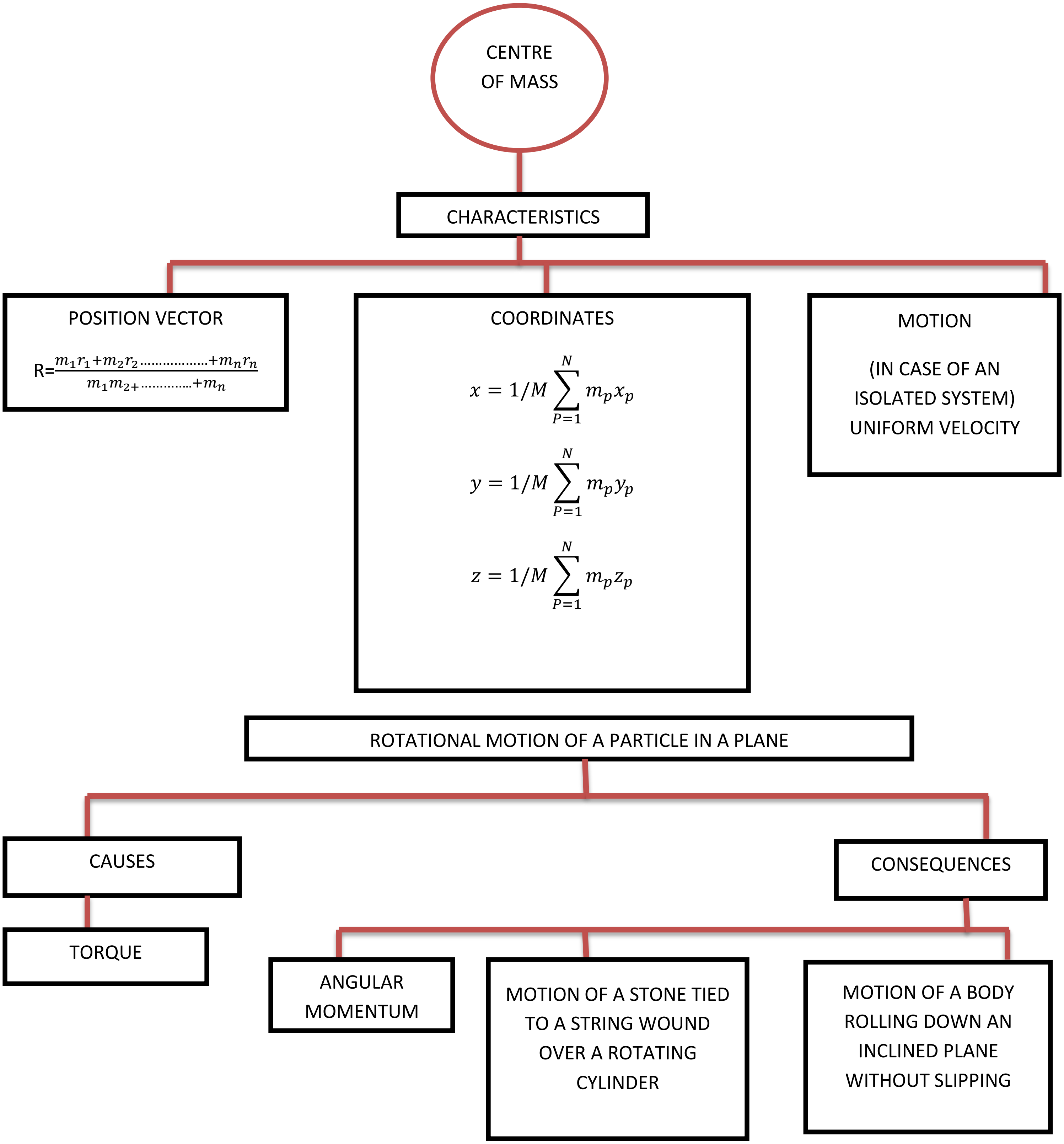
For translational equilibrium of a rigid body, $\vec{F}$=$\sum_i F_i$=0

For rotational equilibrium of a rigid body, $\vec{\tau} = \sum_i \vec{\tau}_i = 0$

1.The following table gives a summary of the **analogy** between various quantities describing linear motion and rotational motion.

s.no.	Linear motion	s.no.	Rotational motion
1.	**Distance/displacement (s)**	**1.**	**Angle or angular displacement (θ)**
2.	**Linear velocity,** $\vartheta = \frac{ds}{dt}$	**2.**	**Angular velocity,** $\omega = \frac{d\theta}{dt}$
3.	**Linear acceleration,** $\alpha = \frac{dv}{dt} = \frac{d^2r}{dr^2}$	**3.**	**Angular acceleration=**$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dr^2}$
4.	**Mass (m)**	**4.**	**Moment of inertia (**I**)**
5.	**Linear momentum,** $p = m\ v$	**5.**	**Angular momentum,** $L = I\omega$
6.	**Force,** $F = m\ a$	**6.**	**Torque,** $\tau = Ia$
7.	**Also, force** $F = \frac{dp}{dt}$	**7.**	**Also, torque,** $\tau = \frac{dL}{dt}$
8.	**Translational KE,** $K_T = \frac{1}{2}mv^2$	**8.**	**Rotational KE,** $K_R = \frac{1}{2}I\omega^2$
9.	**Work done,** $W = Fs$	**9.**	**Work done,** $W = \tau\theta$
	Power, $P = F\ v$		**Power,** $P = \tau\omega$

10.		**10.**	
11.	**Linear momentum of a system is conserved when no external force acts on the system.**	**11.**	**Angular momentum of a system is conserved when no external torque acts on the system**
12.	**Equation of translator motion** **i.** $v = u + at$ **ii.** $s = ut + \frac{1}{2}at^2$ **iii.** $v^2 - u^2 = 2as$, *where the symbols have their usual meaning.*	**12.**	**Equations of rotational motion** **i.** $\omega_2 = \omega_1 + at$ **ii.** $\theta = \omega_1 t + \frac{1}{2}at^2$ **iii.** $\omega_2^2 - \omega_1^2 = 2a\theta$, *where the symbols have their usual meaning.*


CENTRE OF MASS
CHARACTERISTICS
POSITION VECTOR
$R=\frac{m_1r_1+m_2r_2 \ldots\ldots\ldots\ldots + m_nr_n}{m_1m_{2+}\ldots\ldots\ldots + m_n}$
COORDINATES
$x = 1/M \sum_{P=1}^{N} m_p x_p$
$y = 1/M \sum_{P=1}^{N} m_p y_p$
$z = 1/M \sum_{P=1}^{N} m_p z_p$
MOTION
(IN CASE OF AN ISOLATED SYSTEM) UNIFORM VELOCITY
ROTATIONAL MOTION OF A PARTICLE IN A PLANE
CAUSES
TORQUE
CONSEQUENCES
ANGULAR MOMENTUM
MOTION OF A STONE TIED TO A STRING WOUND OVER A ROTATING CYLINDER
MOTION OF A BODY ROLLING DOWN AN INCLINED PLANE WITHOUT SLIPPING

1 Marks Questions

1. If one of the particles is heavier than the other, to which will their centre of mass shift?

Answer:- The centre of mass will shift closer to the heavier particle.

2. Can centre of mass of a body coincide with geometrical centre of the body?

Answer:- Yes, when the body has a uniform mass density.

3.Which physical quantity is represented by a product of the moment of inertia and the angular velocity?

Answer:- Product of I and ω represents angular momentum(L=I ω).

4.What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ denote the adjacent sides of a parallelogram drawn from a point and the area of parallelogram is $\frac{1}{2}$AB.

Answer:- Area of parallelogram=$|\vec{A} \times \vec{B}|$ = AB$\sin\theta$ = $\frac{1}{2}$AB. (Given)

$$\sin\theta = \frac{1}{2} = \sin 30^0 \text{ or } \theta = 30^0$$

5. Which component of linear momentum does not contribute to angular momentum?

Answer:- The radial component of linear momentum makes no contribution to angular momentum.

6.A disc of metal is melted and recast in the form of solid sphere. What will happen to the moment of inertia about a vertical axis passing through the centre ?

Answer:- Moment of inertia will decrease, because $I_d = \frac{1}{2}\, m\, r^2$ and $I_s = \frac{2}{5}\, m\, r^2$, the radius of sphere formed on recasting the disc will also decrease.

7. What is rotational analogue of mass of body?

Answer:- Rotational analogue of mass of a body is moment of inertia of the body.

8. What are factors on which moment of inertia depend upon?

Answer:- Moment of inertia of a body depends on position and orientation of the axis of rotation. It also depends on shape, size of the body and also on the distribution of mass of the body about the given axis.

9. Is radius of gyration of a body constant quantity?

Answer:- No, radius of gyration of a body depends on axis of rotation and also on distribution of mass of the body about the axis.

10. **Is the angular momentum of a system always conserved? If no, under what condition is it conserved?**

Answer:- No, angular momentum of a system is not always conserved. It is conserved only when no external torque acts on the system.

2 Marks Questions

1. Why is the handle of a screw made wide?

Answerwer:- Turning moment of a force= force × distance(r) from the axis of rotation. To produce a given turning moment, force required is smaller, when r is large. That's what happens when handle of a screw is made wide.

2. Can a body in translatory motion have angular momentum? Explain.

Answer:- Yes, a body in translatory motion shall have angular momentum, the fixed point about which angular momentum is taken lies on the line of motion of the body. This follows from $|L|$= r p $\sin \Theta$.

L=0, only when Θ =0^0or Θ=180^0.

3. A person is sitting in the compartment of a train moving with uniform velocity on a smooth track. How will the velocity of centre of mass of compartment change if the person begins to run in the compartment?

Answer:- We know that velocity of centre of mass of a system changes only when an external force acts on it. The person and the compartment form one system on which no external force is applied when the person begins to run. Therefore, there will be no change in velocity of centre of mass of the compartment.

4. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its K.E is halved, what happens to the angular momentum?

Answer:- $\mathrm{L = m\,v\,r}$ and $\mathrm{v = r\,\omega = r(2\,\pi\,n)}$

$$r = \frac{v}{2\,\pi\,n} \qquad \therefore \qquad L = mv\left(\frac{v}{2\,\pi\,n}\right) = \frac{m\,v^2}{2\,\pi\,n}$$

As,

K.E= $\frac{1}{2}\,mv^2$, therefore, $L = \frac{K.E}{\pi n}$

When K.E. is halved and frequency (n) is doubled, $L = \frac{K.E'}{\pi n'} = \frac{K.E/2}{\pi(2n)} = \frac{K.E}{4\pi n} = \frac{L}{4}$

i.e. angular momentum becomes one fourth.

5. An isolated particle of mass m is moving in a horizontal plane(x-y), along the x-axis at a certain height above the ground. It explodes suddenly into two fragments of masses m/4 and 3 m/4. An instant later, the smaller fragments is at y= +15 cm. What is the position of larger fragment at this instant?

Answer:- As isolated particle is moving along x-axis at a certain height above the ground, there is no motion along y-axis. Further, the explosion is under internal forces only. Therefore, centre of mass remains stationary along y-axis after collision. Let the co-ordinates of centre of mass be (x_{cm} , 0).

Now, $y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2} = 0 \qquad \therefore \qquad m_1y_1 + m_2y_2 = 0$

Or $y_2 = \frac{-m_1y_1}{m_2} = \frac{-m/4}{3m/4} \times 15 = -5\ cm$

So, larger fragment will be at y= -5 ; along x-axis.

6. Why there are two propellers in a helicopter?

Answerwer:- If there were only one propeller in a helicopter then, due to conservation of angular momentum, the helicopter itself would have turned in the opposite direction.

7. A solid wooden sphere rolls down two different inclined planes of the same height but of different inclinations. (a) Will it reach the bottom with same

speed in each case ? (b) Will it take longer to roll down one inclined plane than other ? Explain.

Answer:- (a) Yes, because at the bottom depends only on height and not on slope.

(b) Yes, greater the inclination(θ), smaller will be time of decent, as $t \propto 1/\sin\theta$.

8. There is a stick half of which is wooden and half is of steel. It is pivoted at the wooden end and a force is applied at the steel end at right angles to its length. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is angular acceleration more and why?

Answer:- We know that torque, τ= Force $\times$ Distance = $I\,\alpha$ = constant

$$\therefore\ \alpha = \frac{\tau}{I} \quad \text{i.e} \quad \alpha \propto \frac{1}{I}$$

Angular acc. (α) will be more, when I is small, for which lighter material(wood) should at larger distance from the axis of rotation I.e. when stick is pivoted at the steel end.

9. Using expressions for power in rotational motion, derive the relation $= \mathbf{I}\,\boldsymbol{\alpha}$, where letters have their usual meaning.

Answer:- We know that power in rotational motion, $P = \tau\,\omega$ ……….(i)

and K.E. of motion, E= $\frac{1}{2} I\,\omega^2$

……….(ii)

As power= time rate of doing work in rotational motion, and work is stored in the body in the form of K.E.

$$\therefore \qquad P = \frac{d}{dt}\text{ (K.E. of rotation)}$$

$$= \frac{d}{dt}\left(\frac{1}{2}\,I\,\omega\right) = \frac{1}{2} I \times 2\omega\left(\frac{d\omega}{dt}\right)$$

$$P = I\,\omega\,\alpha$$

Using (i), $P = \tau\,\omega = I\,\omega\,\alpha$ or $\tau = I\,\alpha$, which is the required relation.

10. Calculate radius of gyration of a cylindrical rod of mass m and length L about an axis of rotation perpendicular to its length and passing through the centre.

Answer:- K=? , mass= m , length=L

Moment of inertia of the rod about an axis perpendicular to its length and passing through the centre is

$$\mathrm{I} = \frac{\mathrm{mL}^2}{12}$$

Also, $\mathrm{I} = \mathrm{mK}^2$ $\quad \therefore \ \mathrm{mK}^2 = \frac{\mathrm{mL}^2}{12}$ $\quad$ or $\quad \mathrm{K} = \frac{\mathrm{L}}{\sqrt{12}} = \frac{\mathrm{L}}{2\sqrt{3}}$.

3 Marks Questions

1. Explain that torque is only due to transverse component of force. Radial component has nothing to do with torque.

2. Show that centre of mass of an isolated system moves with a uniform velocity along a straight line path.

3. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved ? Explain.

Ans:- Here, $\mathrm{L} = \mathrm{I}\,\omega$ = constant

K.E. of rotation, $\mathrm{K} = \frac{1}{2}\,\mathrm{I}\,\omega^2$

$$\mathrm{K} = \frac{1}{21}\,\mathrm{I}^2\,\omega^2 = \frac{\mathrm{L}^2}{21}$$

As L is constant, $\therefore \mathrm{K} \propto 1/\mathrm{I}$

When moment of inertia(I) decreases, K.E. of rotation(K) increases. Thus K.E. of rotation is not conserved.

4. How will you distinguish between a hard boiled egg and a raw egg by spinning each on a table top?

Ans:- To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate shall be raw. This is because in a raw egg, liquid matter inside tries to get away from its axis of rotation. Therefore, its moment of inertia I increases. As $\tau = \mathrm{I}\,\alpha$ = constant, therefore, α decreases i.e. raw egg will spin with smaller angular acceleration. The reverse is true for a hard boiled egg which will rotate more or less like a rigid body.

5.Equal torques are applied on a cylindrical and a hollow sphere. Both have same mass and radius. The cylinder rotates about its axis and the sphere rotates about one of its diameters. Which will acquire greater speed? Explain.

6.Locate the centre of mass of uniform triangular lamina and a uniform cone.

7. **A thin wheel can stay upright on its rim for a considerable length when rolled with a considerable velocity, while it falls from its upright position at the slightest disturbance when stationary. Give reason.**
Answer:- When the wheel is rolling upright, it has angular momentum in the horizontal direction i.e., along the axis of the wheel. Because the angular momentum is to remain conserved, the wheel does not fall from its upright position because that would change the direction of angular momentum. The wheel falls only when it loses its angular velocity due to friction.

8. Why is the speed of whirl wind in a tornado so high?
Answer:- In a whirl wind, the air from nearby region gets concentrated in a small space thereby decreasing the value of moment of inertia considerably. Since, I ω= constant, due to decrease in moment of inertia, the angular speed becomes quite high.

9. Explain the physical significance of moment of inertia and radius of gyration.

10. Obtain expression for K.E. of rolling motion.

5 Marks Questions

1. Define centre of mass. Obtain an expression for perpendicular of centre of mass of two particle system and generalise it for particle system.

2. Find expression for linear acceleration of a cylinder rolling down on a inclined plane.
A ring, a disc and a sphere all of them have same radius and same mass roll down

on inclined plane from the same heights. Which of these reaches the bottom (i) earliest (ii) latest ?

3. (i) Name the physical quantity corresponding to inertia in rotational motion. How is it calculated? Give its units.
(ii)Find expression for kinetic energy of a body.

4. State and prove the law of conservation of angular momentum. Give one illustration to explain it.

5. State parallel and perpendicular axis theorem.
Define an expression for moment of inertia of a disc R, mass M about an axis along its diameter.

TYPICAL PROBLEMS

1. A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system.

Ans:-

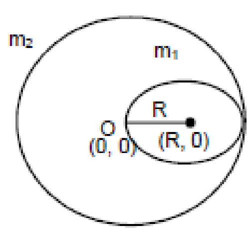
Let the centre of the bigger disc be the origin.

2R = Radius of bigger disc

R = Radius of smaller disc

$$m_1 = \pi R^2 \times T \times \rho$$

$m_2 = \pi(2R)^2 \times T \times \rho$, where T = Thickness of the two discs



ρ =Density of the two discs

∴ The position of the centre of mass

$=\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$

$x_1 = R \qquad y_1 = 0$

$x_2 = 0 \qquad y_2 = 0$

$$\left(\frac{\pi R^2 T\rho R + 0}{\pi R^2 T\rho + \pi(2R)^2 T\rho}, \frac{0}{m^1 + m^2}\right) = \left(\frac{\pi R^2 T\rho R}{5\pi R^2 T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

2. Two blocks of masses 10 kg and 20 kg are placed on the x-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of centre of mass unchanged ?

<u>*Ans:-*</u> Two masses m_1 and m_2 are placed on the X-axis

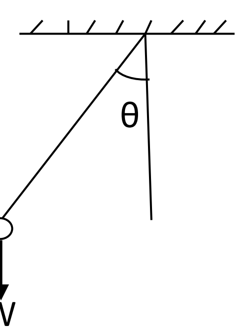
m_1 = 10 kg , m_2 = 20kg

The first mass is displaced by a distance of 2 cm

$$\therefore \overline{X_{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$
$$\Rightarrow 0 = \frac{20 + 20x_2}{30}$$
$$\Rightarrow 20 + 20x_2 = 0$$
$$\Rightarrow 20 = -20x_2$$
$$\Rightarrow x_2 = -1cm$$

∴ The 2nd mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

3. A simple of length l is pulled aside to make an angle θ with the vertical. Find the magnitude of the torque of the weight w of the bob about the point of suspension. When is the torque zero ?



Ans:- A simple of pendulum of length l is suspended from a rigid support.

A bob of weight W is hanging on the other point.

When the bob is at an angle θ with the vertical,

then total torque acting on the point of suspension = i = F × r

⇒ W r sin θ = W l sin θ

At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

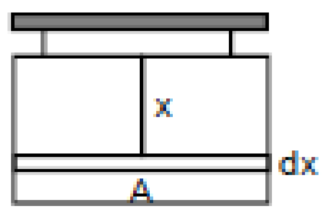
4. A square plate of mass 120 g and edge 5.0 cm rotates about one of edges. If it has a uniform angular acceleration of 0.2 rad/s^2, what torque acts on the plate ?

Ans:- A square plate of mass 120 gm and edge 5 cm rotates about one of the edge. Let take a small area of the square of width dx and length a which is at a distance x from the axis of

rotation.

Therefore mass of that small area

m/a^2 × a dx(m=mass of the square ; a= side of the plate)



$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)]_0^a$$
$$= ma^2/3$$

Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

= {(120 × 10^{-3} × 5^2 × 10^{-4})/3} 0.2

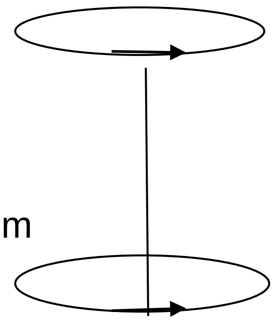
= 0.2 × 10^{-4} = 2 × 10^{-5} N-m.

5. A wheel of moment of inertia 0.10 kg-m^2 is rotating about a shaft at an angular speed of 160 rev/minute. A second wheel is set into rotation at 300 rev/minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200 rev/minute. Find the moment of

inertia of the second wheel.

Ans:- Wheel (1) has

I_1= 0.10 kg-m^2, ω_1 = 160 rev/min



Wheel (2) has
I_2= ? ; ω_2 = 300 rev/min
Given that after they are coupled, ω = 200 rev/min
Therefore if we take the two wheels to bean isolated system
Total external torque = 0
Therefore, $I_1\omega_1 + I_1\omega_2 = (I_1 + I_1)\omega$
⇒ 0.10 × 160 + I_2 × 300 = (0.10 + I_2) × 200
⇒ 5I_2 = 1 – 0.8
⇒ I_2 = 0.04 kg-m^2.

GRAVITATION

CONCEPTS

- **Kepler's law of planetry motion**

(a) Kepler's first law (law of orbit): Every planet revolves around the sun in an elliptical orbit with the sun is situated at one focus of the ellipse.

(b) Kepler's second law (law of area): The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time , i.e., the areal velocity of the planet around the sun is constant.

(c) Kepler's third law (law of period): The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semimajor axis of the elliptical orbit of the planet around the sun.

- Gravitation is the name given to the force of attraction acting between any two bodies of the universe.

- Newton's law of gravitation: It states that gravitational force of attraction acting between two point mass bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, i.e., $F=Gm_1m_2/r^2$, where G is the universal gravitational constant.

- Gravitational constant (G): It is equal to the force of attraction acting between two bodies each of unit mass, whose centres are placed unit distance apart. Value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula $G = [M^{-1}L^3T^{-2}]$. In SI unit, the value of $G = 6.67X10^{-11} Nm^2kg^{-2}$.

- Gravity: It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth. Gravity is the measure of weight of the body. The weight of a body of mass m=mass X acceleration due to gravity=mg. The unit of weight of a body will be the same as those of force.

- **Acceleration due to gravity (g):** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is vector quantity. The value of g changes with height, depth, rotation of earth the value of g is zero at the centre of the earth. The value of g on the surface of earth is 9.8 ms^{-2}. The acceleration due to gravity (g) is related with gravitational constant (G) by the relaion, $g=GM/R^2$ where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity**:

(a) Effect of altitude, $g'=Gr^2/(R+h)^2$ and $g'=g(1-2h/R)$

The first is valid when h is comparable with R and the second relation is valid when h<<R.

The value of g decreases with increase in h.

(b) Effect of depth $g'=g(1-d/R)$

The acceleration due to gravity decreases with increase in depth d and becomes zero at the center of earth.

(c) Effect of rotation of earth: $g'=g-R\ \omega^2 cos^2\lambda$

The acceleration due to gravity on equator decreases on account of rotation of earth and increase with the increase in latitude of a place.

- Gravitational field: It is the space around a material body in which its gravitational pull can be experienced by other bodies. The strength of gravitational field at a point is the measure of gravitational intensity at that point. The intensity of gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field. The intensity of gravitational field at a point distance r from the center of the body of mass M is given by

$E=GM/r^2$=acceleration due to gravity.

- **Gravitational potential:** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point, V=work done(W)/test mass(m_0)= -GM/r. $V=\frac{W}{m_0}= -\frac{GM}{r}$

  Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, E= -dV/dr

  - **Gravitational potential energy of a body,** at a point in the gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point without acceleration.

    Gravitational potential energy U=gravitational potential X mass of body $=-\frac{GM}{r}$ X m.

  - **Inertial mass of a body** is defined as the force required to produce unit acceleration in the body.

    Gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity.

    Inertial mass of a body is identical to the gravitational mass of that body. The main difference is that the gravitational mass of a body is affected by the presence of other bodies near it. Whereas the inertial mass of a body remains unaffected by the presence of other bodies near it.

  - **Satellite:** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

(a) Orbital speed of satellite is the speed required to put the satellite into given orbit around earth.

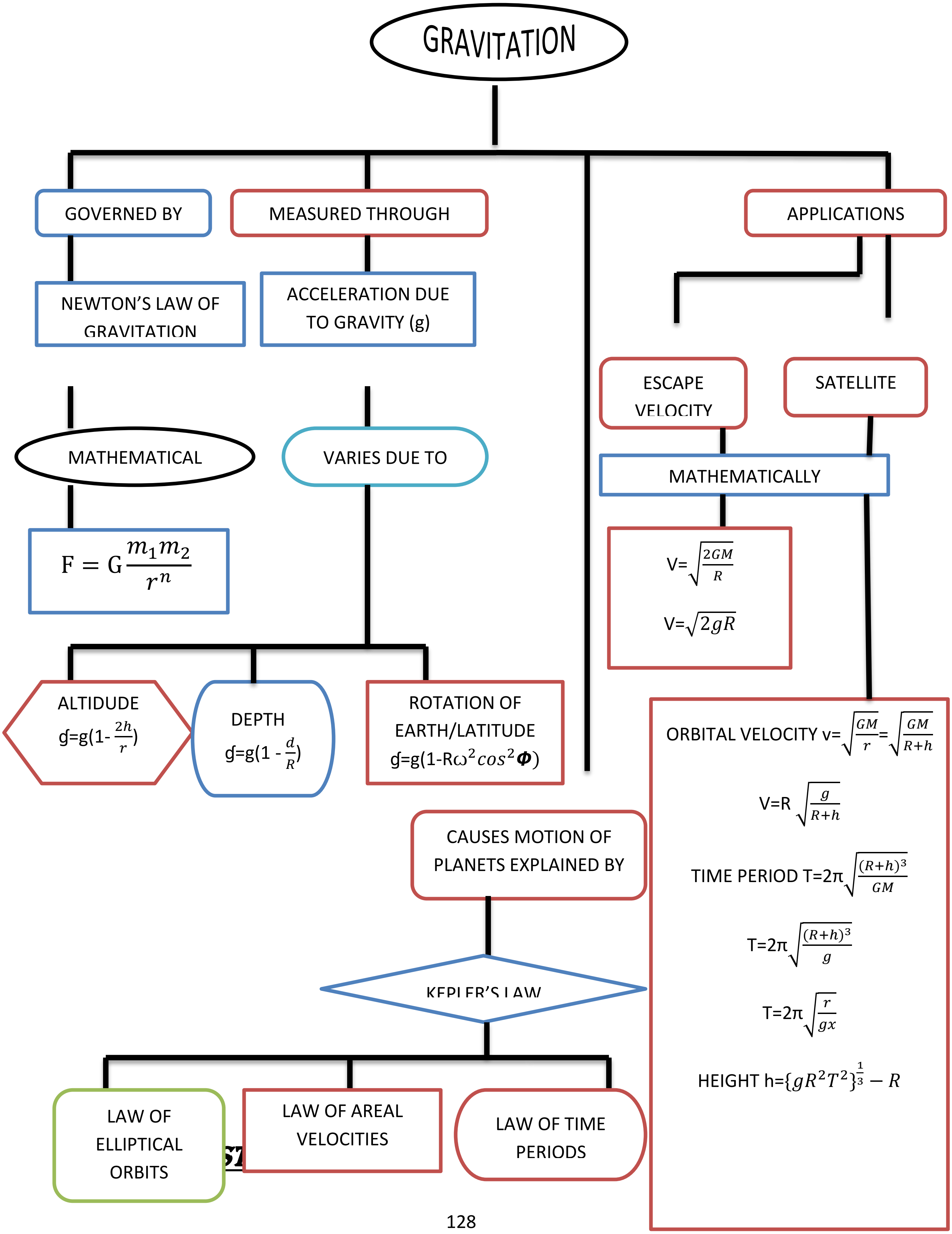
- Time period of satellite(T): It is the time taken by satellite to complete one revolution around the earth. T=$\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$
- Height of satellite above the earth surface:

$$h=\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3}-R$$

- Total energy of satellite, E=P.E +K.E=$\frac{-GMm}{2(R+h)}$

Blinding energy of satellite = -E = GM m/(R+h)

- Geostationary satellite: A satellite which revolves around the earth with the same angular speed in the same direction as is done by the earth around its axis is called geostationary or geosynchronous satellite. The height of geostationary satellite is = 36000 km and its orbital velocity = 3.1 km s^{-1}.
- Polar satellite: It is that satellite which revolves in polar orbit around earth ,i.e. , polar satellite passes through geographical north and south poles of earth once per orbit.
- Escape speed: The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth(or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape velocity v_e is given by, v_e =$\sqrt{\frac{2GM}{R}}$=$\sqrt{2gR}$. For earth, the value of escape speed is 11.2kms^{-1}.
- For a point close to the earth's surface , the escape speed and orbital speed are related as $v_e=\sqrt{2}\,v_o$
- Weightlessness: It is a situation in which the effective weight of the body becomes zero.


GRAVITATION
GOVERNED BY
MEASURED THROUGH
APPLICATIONS
NEWTON'S LAW OF GRAVITATION
ACCELERATION DUE TO GRAVITY (g)
ESCAPE VELOCITY
SATELLITE
MATHEMATICAL
VARIES DUE TO
MATHEMATICALLY
$F = G\frac{m_1 m_2}{r^n}$
V=$\sqrt{\frac{2GM}{R}}$
V=$\sqrt{2gR}$
ALTIDUDE
ɡ=g(1-$\frac{2h}{r}$)
DEPTH
ɡ=g(1 - $\frac{d}{R}$)
ROTATION OF EARTH/LATITUDE
ɡ=g(1-R$\omega^2 cos^2\boldsymbol{\phi}$)
ORBITAL VELOCITY v=$\sqrt{\frac{GM}{r}}$=$\sqrt{\frac{GM}{R+h}}$
V=R$\sqrt{\frac{g}{R+h}}$
TIME PERIOD T=2π$\sqrt{\frac{(R+h)^3}{GM}}$
T=2π$\sqrt{\frac{(R+h)^3}{g}}$
T=2π$\sqrt{\frac{r}{gx}}$
HEIGHT h={$gR^2T^2\}^{\frac{1}{3}} - R$
CAUSES MOTION OF PLANETS EXPLAINED BY
KEPLER'S LAW
LAW OF ELLIPTICAL ORBITS
LAW OF AREAL VELOCITIES
LAW OF TIME PERIODS

Q1.When a stone of mass m is falling on the earth of mass M; find the acceleration of earth if any?

Ans. Force exerted by falling stone on earth, F=mg

Acceleration of earth=$\frac{F}{M}$=$\frac{mg}{M}$

Q2.Why G is called a universal constant?

Ans. It is so because the value of G is same for all the pairs of the bodies (big or small) situated anywhere in the universe.

Q3.According to Kepler's second law the radius vector to a planet from the sun sweeps out equal area in equal interval of time. The law is a consequence of which conservation law.

Ans. Law of Conservation of angular momentum.

Q4.What are the factors which determine ; Why some bodies in solar system have atmosphere and others don't have?

Ans. The ability of a body (planet) to hold the atmosphere depends on acceleration due to gravity.

Q5.What is the maximum value of gravitational potential energy and where?

Ans. The value of gravitational potential energy is negative and it increases as we move away from the earth and becomes maximum (zero) at infinity.

Q6.The gravitational potential energy of a body at a distance r from the center of earth is U. What is the weight of the body at that point?

Ans. U=$\frac{GMm}{r}$=$(\frac{GM}{r^2})$ r m=g r m= (mg) r

Q7.A satellite revolving around earth loses height. How will its time period be changed?

Ans. Time period of satellite is given by; T=$2\pi\sqrt{\frac{(R+h)^3}{GM}}$. Therefore ,T will decrease, when h decreases.

Q8.Should the speed of two artificial satellites of the earth having different masses but the same orbital radius, be the same?

Ans.Yes it is so because the orbital speed of a satellite is independent of the mass of a satellite. Therefore the speeds of the artificial satellite will be of different masses but of the same orbital radius will be the same.

Q9.Can a pendulum vibrate in an artificial satellite?

Ans. No, this is because inside the satellite, there is no gravity ,i.e., g=0.

As t = 2π$\sqrt{l/g}$, hence, for g=0 , t = ∞ . Thus, the pendulum will not vibrate.

Q10.Why do different planets have different escape speed?

Ans. As, escape speed =$\sqrt{2GM/R}$, therefore its value are different for different planets which are of different masses and different sizes.

2 MARKS QUESTIONS

Q1.Show that weight of all body is zero at Centre of earth?

Ans. The value of acceleration due to gravity at a depth d below the surface of earth of radius R is given by ɠ=g(1-d/R).At the center of earth, (dept)d=R; so, ɠ =0.The weight of a body of mass m at the centre of earth =mg'=m x 0=0.

Q2.If a person goes to a height equal to radius of the earth from its surface. What would be his weight relative to that on the earth.

Ans. At the surface of the earth, weight W=mg=GM m/R^2.

At height h =R , weight W'=mg'=$\frac{GM\,m}{(R+h)^2}$=$\frac{GM\,m}{(R+R)^2}$ $\frac{W'}{W}$=$\frac{R^2}{(2R)^2}$ =$\frac{1}{4}$ W'=$\frac{W}{4}$

It means the weight would reduce to one-fourth of the weight on the surface of earth.

Q3.What will be the effect on the time period of a simple pendulum on taking to a mountain?

Ans. The time period of a pendulum, T=2π$\sqrt{l/g}$, i.e., T=∝1/$\sqrt{g}$.As the value of g is less at mountain than at plane, hence time period of simple pendulum will be more at mountain than at plane though the change will be very small.

Q4.A satellite is revolving around the earth, close to the surface of earth with a kinetic energy E. How much kinetic energy should be given to it so that it escapes from the surface of earth?

Ans. Let $v_{0,}v_e$ be the orbital and escape speeds of the satellite, then v_e=$\sqrt{2v_{0,}}$.

Energy in the given orbit,$E_1 = \frac{1}{2}mv_0^2 = E$

Energy for the escape speed,$E_2 = \frac{1}{2}m\, v_e^2 = \frac{1}{2}m\left(\sqrt{2}v_0^2\right) = 2E$

Energy required to be supplied =$E_2 - E_1 = E$.

Q5.A tennis ball and a cricket ball are to be projected out of gravitational field of the earth. Do we need different velocities to achieve so?

Ans. We require the same velocity for the two balls, while projecting them out of the gravitational field. It is so because, the value of escape velocity does not depend upon the mass of the body to be projected [i.e. , v_e=$\sqrt{2gR}$].

Q6.Suppose the gravitational force varies inversely as the nth power of the distance. Show that the time period of a planet in circular orbit of radius R around the sun will be proportional to $R^{(n+1)/2}$.

Ans. $\frac{GM\,m}{R^n} = mR(\frac{2\pi}{T})^2$

$$T^2 = \frac{R\, \times\, 4\pi^2 \times\, R^n}{GM} = \frac{4\pi^2 R^{(n+1)}}{GM}$$

$T = \frac{2\pi}{\sqrt{GM}} \cdot R^{(n+1)/2}$

$T \propto R^{(n+1)/2}$

Q7.Draw graphs showing the variation of acceleration due to gravity with (a)height above the earth's surface, (b)depth below the Earth's surface.

Ans.(a)The variation of g with height h is related by relation g∝1/r^2 where r=R+h. Thus, the variation of g and r is a parabolic curve.

(b)The variation of g with depth is released by equation g'=g(1-d/R) i.e. g'∝ $(R-d)$ .Thus, the variation of g and d is a straight line.

Q8.Why does moon have no atmosphere?

Ans. Moon has no atmosphere because the value of acceleration due to gravity 'g' on surface of moon is small. Therefore, the value of escape speed on the surface of moon is small. The molecules of atmospheric gases on the surface of the moon have thermal speeds greater than the escape speed. That is why all the molecules of gases have escaped and there is no atmosphere on moon.

Q9.A rocket is fired with a speed v=2$\sqrt{gR}\, near$ the earth's surface and directed upwards. Find its speed in interstellar space.

Ans. Let v be the speed of rocket instellar space.

Using law of conservation of energy, we have$\frac{1}{2}m(2\sqrt{gR})^2$=$\frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$

=$\frac{1}{2}m(\sqrt{2gR})^2 + \frac{1}{2}mv^2$

$$v^2 = 4gR - 2gR$$

$$v = \sqrt{2gR}$$

3 marks questions

Q1.Explain how knowledge of g helps us to find (i) mass of earth and (ii)mean density of earth?

Q2. Obtain the expression for orbital velocity, time period, and altitude of a satellite.

Q3. What do you understand by 'Escape velocity'? Derive an expression for it in terms of parameters of given planet.

Q4. What do you understand by gravitational field, Intensity of gravitational field . Prove that gravitational intensity at a point is equal to the acceleration due to gravity at that point.

Q5.A mass M is broken into two parts of masses $m_1 and\, m_2$. How are $m_1\ and\, m_2$ related so that force of gravitational attraction between the two parts is maximum.

Ans. Let $m_1 = m,$ then $m_2 = M - m$. Gravitational force of attraction between them when placed distance r apart will be $= \frac{Gm(M-m)}{r^2}$.

Differentiating it w.r.t. m, we get

$$\frac{dF}{dm} = \frac{G}{r^2}\left[m\frac{d}{dm}(M-m) + (M-m)\frac{dm}{dm}\right] = \frac{G}{r^2}[m(-1) + M - m] = \frac{G}{r^2}(M-2m)$$

If F is maximum, then $\frac{dF}{dm} = 0$;

Then $\frac{G}{r^2}(M-2m) = 0$ or M=2m or m=$\frac{M}{2}$

Q6.Two particles of equal mass move in a circle of radius r under the action of their mutual gravitational attraction. Find the speed of each particle if its mass is m.

Ans. The two particles will move on a circular path if they always remain dramatically opposite so that the gravitation force on one particle due to other is directed along the radius. Taking into consideration the circulation of one particle we have

$$\frac{mv^2}{r} = \frac{Gmm}{(2r)^2} \text{ or } v = \sqrt{\frac{Gm}{4r}}$$

Q7.The magnitude of gravitational field at distances $r_1\ and\ r_2$ from the centre of a uniform sphere of radius R and mass M are $I_1 and\ I_2$ respectively. Find the ratio of (I_1/I_2) if $r_1 > R\ and\ r_2 < R$.

Ans. When $r_1 > R$, the point lies outside the sphere . Then sphere can be considered to be a point mass body whose whole mass can be supposed to be concentrated at its Centre. Then gravitational intensity at a point distance $r_1$from the Centre of the sphere will be,$I_1 = GM/r_1^2$

When $r_2 < R$, the point P lies inside the sphere. The unit mass body placed at P, will experience gravitational pull due to sphere of radius r_2, whose mass is M'=$\frac{M\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi R^3} = \frac{Mr_2^3}{R^3}$.

Therefore, the gravitational intensity at P will be ,

$$I_2 = \frac{GMr_2^3}{R^3} \cdot \frac{1}{r_2^2} = \frac{GMr_2}{R^3}$$

$$\frac{I_1}{I_2} = \frac{GM}{r_1^2} \cdot \frac{R^3}{GMr_2} = \frac{R^3}{r_1^2 r_2}$$

Q8.Two bodies of masses $m_1 and\ m_2$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Find their relative velocity of approach at a separation distance r between them.

Ans. Let v_r be the relative velocity of approach of two bodies at a distance r apart. The reduced mass of the system of two particles is , $\mu = \frac{m_1 m_2}{m_1+m_2}$.

According to law of conservation of mechanical energy.

Decrease in potential energy = increase in K.E.

$0 - \left(-\frac{Gm_1m_2}{r}\right) = \frac{1}{2}\mu v_r^2$ or $\frac{Gm_1m_2}{r} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2$ or $v_r = \sqrt{\frac{2G(m_1+m_2)}{r}}$

Q9.Since the moon is gravitationally attracted to the earth, why does it not simply crash on earth?

Ans. The moon is orbiting around the earth in a certain orbit with a certain period . The centripetal force required for the orbital motion is provided to the gravitational pull of earth. The moon can crash into the earth if its tangential velocity is reduced to zero. AS moon has tangential velocity while orbiting around earth, it simply falls around the earth rather than into it and hence cannot crash into the earth.

Q10.What are the conditions under which a rocket fired from earth, launches an artificial satellite of earth?

Ans. Following are the basic conditions: (i) The rocket must take the satellite to a suitable height above the surface of earth for ease of propulsion.

(ii)From the desired height, the satellite must be projected with a suitable speed, called orbital speed.

(iii)In the orbital path of satellite, the air resistance should be negligible so that its speed does not decrease and it does not burn due to the heat produced.

5 MARKS QUESTIONS

Q1.State Kepler's laws of planetary motion. Prove second Kepler's law using concept of conservation of angular motion.

Q2.State universal law of gravitation. What is the significance of this law. Find the expression for acceleration due to gravity.

Q3.Explain the variation of acceleration due to gravity with (I) altitude (ii) depth

Q4. Define gravitational potential energy. Derive the expression for gravitational potential energy. What is the maximum value of gravitational potential energy?

Q5.What is escape speed? Derive the expressions for it. Calculate escape speed for the Earth.

TYPICAL PROBLEMS

Q1.Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Ans. The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Consider the motion of one of the particles. The force on the particle is $F = \frac{Gm^2}{4R^2}$. If the speed is v, its acceleration is v^2/R.

Thus by Newton's Law,

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$V = \sqrt{\frac{Gm}{4R}}$$

Q2.A particle is fired vertically upward with a speed of 3.8km/s. Find the maximum height attained by the particle. Radius of earth=6400km and g at the surface=9.8m/s. Consider only earth's gravitation.

Ans. At the surface of the earth, the potential energy of the earth-particle system is $\frac{GMm}{R}$ with usual symbol. The kinetic energy is 1/2 mv^2 where $v_0 = 9.8km/s$. At the maximum height the kinetic energy is zero. If the maximum height reached is H, the potential energy of the earth-particle system at this instant is $-\frac{GMm}{R+H}$. Using conservation of energy , $-\frac{GMm}{R} + \frac{1mv^2}{2} = -\frac{GMm}{R+H}$

Writing GM=g$R^2 and$ dividing by m,

$$-gR + \frac{v_0^2}{2} = \frac{-gR^2}{R+H}$$

$$\frac{R^2}{R+H} = R - \frac{v_0^2}{2g}$$

$$R + H = \frac{R^2}{R - \frac{v_0^2}{2g}}$$

Putting the value of R,$v_0 and$ g on right side,

$$R + H = \frac{(6400\ km)^2}{6400 - \frac{(9.8km/s)^2}{2\ X\ 9.8s^{-2}}}$$

$$= 27300km$$

H = (27300 - 6400)km =20900km

3. Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the center.

Ans. A small section of rod is considered at ‘x’ distance mass of the element = (M/L). dx = dm

$dE_1 = \frac{G(dm)X1}{(d^2+x^2)}$ $= 2.\frac{G(dm)}{(d^2+x^2)}.\frac{d}{\sqrt{(d^2+x^2}} = \frac{2GMd\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$

Total gravitational field

E=$\int_0^{L/2} \frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$

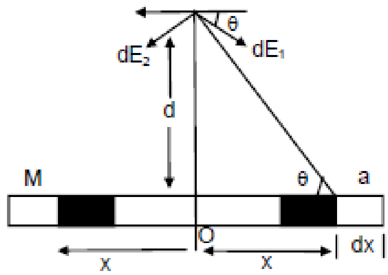
Integrating the above equation it can be found that,

$$E = \frac{2GM}{d\sqrt{L^2+4d^2}}$$

Resultant dE = 2 dE_1 sin θ

$$= 2 \times \frac{G(dm)}{(d^2+x^2)} \times \frac{d}{\sqrt{(d^2+x^2}} = \frac{2 \times GM \times d\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$$

Total gravitational field



$$E=\int_0^{L/2} \frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$$

Integrating the above equation it can be found that,

$$E = \frac{2GM}{d\sqrt{L^2 + 4d^2}}$$

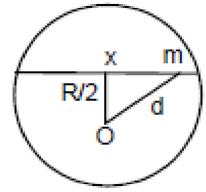
Q4.A tunnel is dug along a diameter of the earth. Find the force on a particle of mass m placed in the tunnel at a distance x from the centre.

Ans. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (\frac{1}{2})\sqrt{4x^2 + R^2}$$

M be the mass of the earth, M' the mass of the sphere of radius d/2.

Then M = (4/3) π$R^3 \rho$



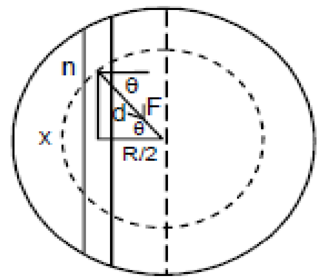
M' = (4/3)π$d^3 \tau$

Or $\frac{M\prime}{M} = \frac{d^3}{R^3}$

 Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

So, Normal force exerted by the wall = F cosθ



$$\frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2}$$

Therefore I think normal force does not depend on x.

Q5. (a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation.

(b)If the satellite is directly above the north pole at some instant , find the time it takes to come over equatorial plane. Mass of the earth=$6X10^{24}kg$

Ans.(a) Angular speed f earth & the satellite will be same

$$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$$

Or

$$\frac{1}{24\ X\ 3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

Or $12|3600 = 3.14\sqrt{\frac{(R+h)^3}{gR^2}}$

Or $\frac{(R+h)^2}{gR^2} = \frac{(12\ X\ 3600)^2}{(3.14)^2}$

Or $\frac{(6400+h)^3 X\, 10^9}{9.8X(6400)^2 X10^6} = \frac{(12X3600)^2}{(3.14)^2}$

Or $\frac{(6400+h)^3 X10^9}{6272X10^9} = 432 \times 10^4$

Or $(6400 + h)^3 = 6272 \times 432 \times 10^4$

Or $6400 + h = (6272 \times 432 \times 10^4)^{1/3}$

Or $h = (6272 \times 432 \times 10^4)^{\frac{1}{3}} - 6400$

$= 42300\ m.$

(b)Time taken from north pole to equator = (1/2) t

$$= \left(\frac{1}{2}\right) X6.28 \sqrt{\frac{(43200 + 6400)^3}{10\ X\ (6400)^2\ X\ 10^6}} = 3.14 \sqrt{\frac{(497)^3 X\ 10^6}{(64)^2\ X\ 10^{11}}}$$

$$= 3.14 \sqrt{\frac{497\ X\ 497\ X\ 497}{64\ X\ 64\ X\ 10^5}} = 6\ hour.$$

MECHANICS OF SOLID AND FLUID

- **Deforming force**:- A force acting on a body which produces change in its shape of body instead of its state of rest or uniform motion of the body.
- **Elasticity:**-The property of matter by virtue which it regains its original shape and size, when the deforming forces are removed is called elasticity.
- **Plasticity**:- The inability of a body to return to its original shape and size, when the deforming forces are removed is called plasticity.
- **Hooke's law**:- when a wire is loaded within elastic limit, the extension produced in wire is directly proportional to the load applied.

  OR

  Within elastic limit stress α strain

  $$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

- **Stress** :- Restoring force set up per unit area when deforming force acts on the body

  $$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

  S.I Unit of stress = N/m^2 or Pascal (Pa)

  Dimensional formula = $M^a L^b T^c$

Types of stress:-

Stress →
- → Normal stress →
  - → Tensile stress(When there is an increase in dimension of the body along the direction of force)
  - → Compression stress(when there is decrease in dimension)
- → Tangential stress (When deforming force acts tangential to the surface of body)

Strain:- The ratio of change in dimension to the original dimension is called strain

It has no unit

Types of strain:-

→ Longitudinal strain= $\frac{\text{Change in Length}}{\text{Original Length}}$

→ Volumetric strain = $\frac{\text{Change in Volume}}{\text{Original Volume}}$

→ Sharing Strain = φ =$\frac{\Delta L}{L}$, Where ΔL =displacement of the face on which force is applied and L is the height of the face

Hooke's L aw:- Within elastic limit, stress α strain

$$\frac{\text{Strss}}{\text{Strain}} = \text{Constant (Modulus of Elasticity)}$$

Modulus of elasticity are of 3 types.

(1) **Young's Modulus (Y)** = $\frac{Normal\ stress}{Longitudinal\ Strain}$

(2) **Bulk Modulus (K)** = $\frac{Normal\ stress}{Volumetric\ Strain}$

(3) **Modulus of rigidity modulus (η)** =$\frac{Tangential\ stress}{Shearing\ \ Strain}$

- **Compressibility** : the reciprocal of bulk modulus of a material is called its compressibility

  Compressibility = 1/K

  Stress – Strain- diagram
- **Proportionality limit(P)** – The stress at the limit of proportionality point P is known as proportionality limit
- **Elastic limit** - the maximum stress which can be applied to a wire so that on unloading it return to its original length is called the elastic limit
- **Yield point(Y)-** The stress, beyond which the length of the wire increase virtually for no increase in the stress
- **Plastic region**- the region of stress- strain graph between the elastic limit and the breaking point is called the plastic region.

- **Fracture point or Breaking point(B)-** the value of stress corresponding to which the wire breaks is called breaking point
- **Work done in stretching a wire per unit volume/energy sored per unit volume of specimen**

  = ½ x stress x strain
- **Elastic after effect:-** The delay in regaining the original state by a body after the removal of the deforming force is called elastic after effect.
- **Elastic fatigue**:- the loss in strength of a material caused due to repeated alternating strains to which the material is subjected.
- **Poisson's ratio(ϭ)** :- The ratio of lateral strain to longitudinal strain is called

$$\text{Poisons ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

- Relation between Y,K,¶, ϭ
  1. Y=3K(1-2 ϭ)
  2. Y=2¶(1+ ϭ)
  3. $ϭ = \frac{3k-2¶}{2¶+6k}$
  4. $\frac{9}{\gamma}$ = 1/K +3/¶
- Applications of elasticity
  1. Metallic part of machinery is never subjected to a stress beyond the elastic limit of material.
  2. Metallic rope used in cranes to lift heavy weight are decided on the elastic limit of material
  3. In designing beam to support load (in construction of roofs and bridges)
  4. Preference of hollow shaft than solid shaft
  5. Calculating the maximum height of a mountain

MECHANICS OF FLUID

- **Pressure** :The force/threat acting per unit area is called pressure

  S.I Unit of pressure is N/M^2 or pascal (Pa)

Dimensional formula ($ML^{-1}T^{-2}$)

- **Pascal's law:-** Pressure applied to an enclosed fluid is transmitted to all part of the fluid and to the wall of the container.
- **Application of Pascal's law:-**

  (1) Hydraulic lift, presses etc.

  (2) Hydraulic brakes
- **Pressure exerted by liquid column**:- $P = h\rho g$, where h= depth of liquid, ρ=density , g=acc$_n$. due to gravity.
- **Variation of pressure with depth**: $P = P_a + h\rho g$, where P_a =atmospheric pressure
- **Atmospheric pressure**:- The pressure exerted by atmosphere is called atmospheric pressure.

  At sea level, atmospheric pressure= 0.76m of Hg column

  Mathematically 1 atm = 1.013×10^5 Nm^{-2}
- **Archimedes' principle**:- It states that when a body is immersed completely or partly in a fluid, it loses in weight equal to the weight of the fluid displaced by it.

  Mathematically: Apparent weight = True weight – $V\rho g$

  Where V is volume of fluid displaced, ρ is its density.
- **Viscosity**:- It is the property of liquid (or gases) due to which a backward dragging force acts tangentially between two layers of liquid when there is relative motion between them.
- **Newton's formula for Viscous force**:- the viscous force between two liquid layer each of area A and having a velocity gradient dv/dx is

  $\mathbf{F = \eta A (dv/dx)}$, where **η** is coefficient of viscosity
- **Coefficient of viscosity**:- It is define as the tangential viscous force which maintains a unit velocity gradient between two parallel layers each of unit area

  S.I unit of coefficient of viscosity is poiseuille or pascal-second

- **Poiseuille's equation**:- when a liquid of coefficient of viscosity flows through a tube of length 'l' and radius r, then the volume of liquid following out per second is given

  $$V = \pi P r^4 / 8\eta l \ ,$$

  Where P is the difference of pressure between the two ends of the tube.
- **Stoke's law**: The backward dragging force acing on a small sphere of radius r falling with uniform velocity *v* through a medium of coefficient of viscosity is given by

  $$F = 6\pi \eta r v$$

- **Terminal velocity**:- It is the maximum constant velocity acquired by the body while falling freely in a viscous medium

  The terminal velocity v of a spherical body of radius r and density ϭ while falling freely in a viscous medium of viscosity ¶ , density is given by

  $$V = \frac{2}{q} \frac{r2}{\eta} (\sigma - \rho) g$$

- **Stream line**:- It is the path, straight or curved, the tangent at any point to which given the direction of the flow of liquid at that point
- **Tube of flow**:- A tube of flow is a bundle of stream lines having the same velocity of fluid elements over any cross section perpendicular to the direction of flow
- **Stream line flow**:- the flow of the liquid in which each molecule of the liquid passing through a point travels along the same path and with the some velocity as the preceeding molecule passing through the same point
- **Laminar flow**:- the flow of liquid, in which velocity of the layer varies from maximum at the axis to minimum for the layer in contact with the wall of the tube is called laminar flow.
- **Turbulent flow**:- It is the flow of liquid in which a liquid moves with a velocity greater than its critical velocity. The motion of the particles of liquid becomes disorderly or irregular.

- **Critical velocity**:- It is that velocity of liquid flow, upto which the flow of liquid is streamlined and above which its flow becomes turbulent. Critical velocity of a liquid (V_c) flowing through a tube is given by

  $$V_c = K\eta / \rho r$$

  Where **ρ** is the density of liquid following through a tube of radius r and **η** the coefficient of viscosity of liquid
- **Reynold's umber**:- It is a pure number which determines the nature of flow of liquid through a pipe

  Quantitatiively Renold's number N = $\rho D V_c / \eta$

  Where **η** is coefficient of viscosity of liquid , **ρ** is density of liquid D is the diameter of the tube, V_c is critical velocity

  For stream line flow, Reynold's number <2000

  For turbulent flow, Reynold's number > 3000

  For uncertain flow, 2000<Reynold's number<3000

- **Theorem of continuity** : If there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant

  Mathematically $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$

  It is called the equation of continuity

  For in compressible liquid $\rho_1 = \rho_2$ Therefore the equation continuity becomes

  $$a_1 v_1 = a_2 v_2$$

  Bernoulli's theorem:- It states that for an in compressible non-viscous liquid in steady flow, the total energy i.e. pressure energy, potential energy and kinetic energy remains constant its flow.

  Mathematically $\frac{P}{\rho} + gh + ½ v^2$ = Constant

  $$\frac{P}{\rho g} + h + \frac{v2}{2g} = \text{Constant}$$

The term $\frac{P}{\rho g}$, h and $\frac{v2}{2g}$ are called pressure head, gravitational head and velocity head respectively.

- **Application of Bernoull's theorem**
  - (i) Working of Bunsen burner
  - (ii) Lift of an air foil
  - (iii) Spinning of ball (Magnus effect)
  - (iv) Sprayer
  - (v) Ping pong ball in air jet.

- **Toricelli's theorem/speed of efflux**:-It states that the velocity of efflux i.e. the velocity with which the liquid flows out of an orifice (i.e. a narrow hole) is equal to that which is freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

  Quantitatively velocity of efflus

  $$V = \sqrt{2gh}$$

  Venturimeter:- It is a device use to measure the rate of flow of liquid. Venturimeter consists of a wide tube having a constriction in the middle. If a_1 and a_2 are the areas of cross section of the wide end and the threat, p_1 and p_2 are the pressure of liquid, then velocity of the liquid entering at the wide end is given by $V_1=a_2 \quad \sqrt{2(P_1-P_2)\ \rho(a_1^2-a_2^2)}$

- **Surface tension (T)** :- It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contract so as to occupy a minimum surface area

  Mathematically $\quad T = F/l$

  S.I Unit is : Nm^{-1} $\quad$ Dimensional formula : ML^0T^{-2}

  Surface Energy : The potential energy per unit area of the surface film is called the surface energy.

  $$\text{Surface energy} = \frac{\textit{Work done in increasing the surface area}}{\textit{Increase in area}}$$

Surface tension is numerally equal to surface energy

- **Excess of pressure inside a drop and double**:- There is excess of pressure on concave side of a curved surface
  1. Excess of pressure inside a liquid drop = 2T/R
  2. Excess of pressure inside a liquid bubble = 4T/R
  3. Excess of pressure inside an air bubble = 2T/R, Where T is the surface tension , R = radius of liquid drop
- **Angle of contact**:- The angle which the tangent to the free surface of the liquid at the point of contact makes with the wall of the containing vessel, is called the angle of contact

  For liquid having convex meniscus, the angle of contact is obtuse and for having concave meniscus, the angle of contact is acute.
- **Capillary tube**:- A tube of very fine bore is called capillary tube
- **Capillarity**:-The rise or fall of liquid inside a capillary tube when it is dipped in it is called capillarity
- **Ascent formula**:- when a capillary tube of radius 'r' is dipped in a liquid of density *s* and surface tension T, the liquid rises or depresses through a height,

  $$H= 2T\cos\theta \ /r\,\rho\ g$$

  There will be rise a liquid when angle of contact θ is acute. There will be fall in liquid when angle of contact θ is obtuse.

<u>Thermal expansion and calorimetry</u>

- Heat- it is a form of energy, which produce in us the sensation of warmth
- Temperature:- The degree of hotness or coldness of a body is called temperature
- Thermometer- It is a device used to measure the temperature of a body
- Scales of temperature:- there are four scales of temperature. Given below is scales of temp with lower and upper fixed point

  <u>Temperature scales</u> <u>Lower fixed point (Melting point office)</u> <u>Upper fixed point (Boiling point of water)</u>

1. Celsius	0^0C	100^0C
2. Fahrenheit	32^0F	212^0F
3. Reamur	0^0R	80^oR
4. Kelvin	273K	373K

- **Relation between the various temperature scales**

  If C,F, R and K are temperature of a body on Celsius, Fahrenheit , Reumer and Kelvin scale, then

  $$\mathbf{C/5 = F-32/9 = R/4 = K-273/5}$$

- **Thermal expansion**:- all solid expands on heating. There are three types of expansion.

  (1) Liner expansion- When a solid rod of initial length 'l' is heated through a temperature ΔT then its new length $\mathbf{l' = l(1+\alpha\Delta T)}$, where α is called coefficient of liner expansion

  (2) Superficial expansion- when a solid of initial surface area A is heated through temperature then its new Area is $\mathbf{A' = A(1 + \beta\Delta T)}$, where β is coefficient of superficial expansion

  (3) Cubical expansion- when a solid of initial volume V is heated through a temperature ΔT then its new volume is $\mathbf{V' = V(1 + \Upsilon\Delta T)}$,where ϒ is the coefficient of cubical expansion.

- Relation between α ,β and ϒ

  $$\mathbf{\alpha = \beta/2 = \Upsilon/3}$$

- In case of liquid $\mathbf{\Upsilon_r = \Upsilon_a + \Upsilon_g}$

  Where Υ_r=Coefficient of real expansion of a liquid

  Υ_a= Coefficient of apparent expansion of liquid

  Υ_g= Coefficient of cubical expansion of the vessel

  Thermal capacity = It is the amount of heat required to raise its temperature through one degree

- **Water equivalent** :- It is the mass of water which absorbs or emits the same amount of heat as is done by the body for the same rise or fall in temperature. It is represented by W = mc

- **Specific heat** :- It is the amount of heat required to raise the temperature of unit mass of substance through unit degree Celsius

$$C = \Delta Q/m\Delta T$$

- **Latent heat** :- It is define as the quantity of heat required to change the unit mass of the substance from its one state completely to another state at constant temperature

  Mathematically Q = ML
- *Types of Latent heat* →
  - Latent heat of fusion (L_f)
  - Latent heat of vaporization (L_v)

- **Calorimeter** :- Device used for measuring heat
- **Principle of calorimetry** :- Heat loss by hot body = Heat gain by cold body
- **Transfer of heat** :- there are three modes by which heat transfer takes place

  (1) **Conduction**:- It is the process by which heat is transmitted from one point to another through a substance in the direction of fall of temperature without the actual motion of the particles of the substance. When two opposite faces of a slab, each of cross section A and separated by a distance d are maintained at temperature T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t

  $Q = K A (T_1 - T_2)t/d$ Where K is coefficient of thermal conductivity of the mater

- **Coefficient of thermal conductivity**:- It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept at one degree difference of temperature

  S.I unit of coefficient of thermal conductivity : $J\,S^{-1}m^{-1}K^{-1}$ or $W\,m^{-1}K^{-1}$

  (2) **Convection**:- It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance.

  (3) **Radiation**:- It is the process by which heat is transmitted from one place to another without heating the intervening medium

- **Newton's laws of cooling**:- It states that the rate of loss of heat or rate of cooling of a body is directly proportional to the temperature difference between the body and the surrounding, provided the temperature difference is small

  Mathematically $-dQ/dt = K(T-T_0)$
- **Perfect black body**:- It is a body which absorbs heat radiations of all the wavelengths, which fall on it and emits the full radiation spectrum on being heated.
- **Stefan's law:-** It states that the total amount of heat energy radiated per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the substance of the body

  Mathematically $E \; \alpha \; T^4$

  $E = \Sigma T^4$ Where is called Stefan's constant

  It's value is $5.67 \times 10^{-8}\ JS^{-1}m^{-2}k^{-4}$

Wein's displacement law:- According to this law, the wavelength λ_m of maximum intensity of emission of black body radiation is inversel y proportional to absolute temperature (T) of black body.

$$\boldsymbol{\lambda_m \; \alpha \; \frac{1}{T}}$$

$\boldsymbol{\lambda_m T = b}$ where b is wien's constant

Questions with **(mark) are HOTs Question

<u>1 MARK QUESTIONS</u>

Q.1 A wire is stretched by a force such that its length becomes double. How will the Young's modulus of the wire be affected?

Ans. Young's modulus remains the same.

Q.2 How does the Young's modulus change with rise in temperature?

Ans. Young's modulus of a material decreases with rise in temperature.

Q.3 Which of the three modulus of elasticity – Y, K and η is possible in all the three states of matter (solid, liquid and gas)?

Ans. Bulk modulus (K)

Q.4 The Young's modulus of steel is much more than that for rubber. For the same longitudinal strain, which one will have greater stress?

Ans. Stress= Y X longitudinal strain. So steel will have greater stress.

Q.5 Which of the two forces – deforming or restoring is responsible for elastic behavior of substance?

Ans. Restoring force.

Q.6. Which mode of transfer of heat is the quickest?

Ans. Radiation.

** Q. 7 A boat carrying a number of large stones is floating in a water tank. What will happen to the level of water if the stones are unloaded into the water?

Ans. The level of water will fall because the volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

Q.8. A rain drop of radius r falls in air with a terminal velocity v. What is the terminal velocity of a rain drop of radius 3r ?

Ans. $v = \frac{2r^2(\varsigma-\sigma)g}{9\eta}$ $\qquad$ v α r^2

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2 \qquad \rightarrow \qquad v_2=\left(\frac{3r}{r}\right)v_1^2=9v_1$$

**Q. 9 When air is blown in between two balls suspended close to each other , they are attracted towards each other. Why?

Ans. On blowing air between the two balls, the air velocity increases, decreasing pressure. The pressure on the outer side of the ball being more will exert forces on the balls, so they move towards each other.

Q.10. Why does air bubble in water goes up?

Ans. The terminal velocity , $v = \frac{2r^2(\varsigma-\sigma)g}{9\eta}$ As the density of air ς is less than density of water σ, the terminal velocity is negative. For this reason air bubbles moves upward.

2 MARKS QUESTIONS

Q.11 Steel is more elastic than rubber. Explain.

Ans. Consider two wire, one of steel and another of rubber having equal length L and cross sectional area A . When subjected to same deforming force F, the extension produce in steel is l_S and in rubber is l_R such that $l_R > l_S$.

Then $Y_s = \frac{FL}{Al_s}$ and $Y_r = \frac{FL}{Al_r}$

$\frac{Y_s}{Y_r} = \frac{l_r}{l_s}$

As $l_s < l_r \rightarrow Y_s > Y_r$

Hence steel is more elastic.

Q.12. A wire stretches by a certain amount under a load. If the load and radius are both increased to four times, find the stretch caused in the wire.

Ans. For a wire of radius r stretched under a force F,

$Y = \frac{FL}{\pi r^2 L}$ or $l = \frac{FL}{\pi r^2 Y}$

Let l' be the extension when both the load and the radius are increased to four times, Then, l' = $\frac{4F\ X\ L}{\pi(4r)^2 L}$ = $\frac{FL}{4\pi r^2 Y}$ = $\frac{l}{4}$

Q. 13. Calculate the percentage increase in the length of a wire of diameter 2mm stretched by a force of 1kg F. Young's modulus of the material of wire is 15 X 10^{10}Nm^{-2}.

Ans. F = 1 Kg F = 9.8N Y = 15 X 10^{10}Nm^{-2} r = $\frac{2}{2}$ = 1mm = 10^{-3}m

Cross section of wire , $\pi r^2 = \pi$ X $(10^{-3})^2 = \pi$ X 10^{-6}m^2

Now Y = $\frac{FL}{al}$

$\frac{l}{L} = \frac{F}{aY} = \frac{9.8}{\pi\ X\ 10^{-6}\ X\ 15\ X\ 10^{10}}$ = 2.1 X 10^{-5}

Percentage increase = 2.1 X 10^{-5} X 100 = 0.0021%

Q. 14. The pressure of a medium is changed from 1.01 X 10^5 pa to 1.165 X 10^5 pa and changed in volume is 10% keeping temperature constant. Find the bulk modulus of the medium.

Ans. Here Δp = 1.165 X 10^5 – 1.01 X 10^5 = 0.155 X 10^5pa

$\frac{\Delta V}{V}$ = 10 % = 0.1

Now K = $\frac{\Delta P}{\frac{\Delta V}{V}} = \frac{0.155\ X\ 10^5}{0.1}$ = 1.55 X 10^5 pa

Q.15. 27 identical drops of water are falling down vertically in air each with a terminal velocity of 0.15m/s. If they combine to form a single bigger drop, what will be its terminal velocity?

Ans. Let r = radius of each drop, v = 0.15 m/s

Now $v = \frac{2r2(\varsigma - \sigma)g}{9\eta}$ -------------------------------------(1)

Let R be the radius of the big drop.

Volume of big drop = Volume of 27 small drops

$$\frac{4}{3}\pi R^3 = 27 \text{ X } \frac{4}{3}\pi r^3$$

R = 3r

Let v_1 be the terminal velocity of bigger drop

$$v_1 = \frac{2R^2(\varsigma-\sigma)g}{9\eta} \text{ -----------------------------------(2)}$$

$$\frac{v_1}{v} = \frac{R^2}{r^2} = 9$$

v_1 = 9v = 9 X 0.15 = 1.35 m/s

Q.16. Water flows through a horizontal pipe line of varying cross section at the rate of $0.2m^3s^{-1}$. Calculate the velocity of water at a point where the area of cross section of the pipe is $0.02m^2$.

Ans. Rate of flow = av

$$v = \frac{Rate\ of\ flow}{a}$$

Rate of flow = $0.2m^3s^{-1}$ a = $0.02m^2$

$$v = \frac{0.2m^s s^{-1}}{0.02\text{m}^2} = 10 \text{ ms}^{-1}$$

Q. 17. A cylinder of height 20m is completely filled with water. Find the efflux water (in m s-1) through a small hole on the side wall of the cylinder near its bottom. Given g = 10m/s.

Ans Here h = 20m , g = 10 m/s

Velocity of efflux , v = $\sqrt{2gh}$ = $\sqrt{2 \ X\ 10\ X\ 20}$ = 20m/s

**Q.18. At what common temperature would a block of wood and a block of metal appear equally cold or equally hot when touched?

Ans. When touched an object appear cold if heat flows from our hand to the object. On the other hand it appears hot, if heat flows from the object towards our hand. Therefore a block of wood and block of metal will appear equally cold or equally hot if there is no exchange of heat between hand and the block. So the two blocks will appear equally cold or equally hot if they are at the same temperature as that of our hands i.e. the temperature of our body.

Q.19. A piece of chalk immersed into water emits bubbles in all directions. Why?

Ans. A piece of chalk has extremely narrow capillaries. As it is immersed in water, water rises due to capillary action. The air present in the capillaries in the chalk is forced out by the rising water. As a result bubbles are emitted from the chalk in all the directions.

3 MARKS

Q. 20. Water at a pressure of 4×10^4 Nm-2 flows at $2ms^{-1}$ through a pipe of $0.02m^2$ cross sectional area which reduces to $0.01m^2$. What is the pressure in the smaller cross section of the pipe?

Ans. $a_1v_1 = a_2v_2$

$$v_2 = \frac{a_1v_1}{v_2} = \frac{0.02 \, X \, 2}{0.01} = 4\text{m/s}$$

Again $$\frac{P_1}{\varsigma} + \frac{1}{2}v_1^2 = \frac{P_2}{\varsigma} + \frac{1}{2}v_2^2$$

$$P_1 = P_2 - \frac{1}{2}\varsigma\,(v_1^2 - v_2^2)$$

$P_1 = 3.4 \times 10^4 Nm^{-2}$

Q.21. What is surface tension and surface energy? Derive the relation between surface tension and surface energy.

Q.22. Derive equation of continuity for steady and irrotational flow of a perfectly mobile and incompressible fluid. What conclusion is drawn from it?

Q.23 What is Stoke's law? Derive the relation by the method of dimension.

Q.24. A piece of iron of mass 0.1 kg is kept inside a furnace, till it attains the temperature of the furnace. The hot piece of iron is dropped into a calorimeter containing 0.24 Kg of water at 20℃. The mixture attains an equilibrium temperature of 60℃. Find the temperature of the furnace. Given water equivalent of calorimeter = 0.01 kg and specific heat of iron = 470 J Kg^{-1} $K^{-1.}$

Ans. Let θ_1 be the temperature of the furnace i.e of the piece of iron.

Heat lost by the piece of iron Q = M_1C_1 $(\theta_1 - \theta)$

Here M_1 = 0.1 Kg C_1 = 470 J Kg^{-1} K^{-1} $\theta = 60℃$

Q = 0.1 X 470 $(\theta_1 - 60)$ = 47 $(\theta_1 - 60)$ ------------------------- (1)

Heat gain by water and the calorimeter , Q = $(M_2 + w)$ C_2 $(\theta - \theta_2)$

M_2 = 0.24 Kg w = 0.01 Kg θ_2 = 20℃ C_2 = Specific heat of water = 4200 J Kg^{-1} K^{-1}

Q = (0.24 + 0.01) x 4200 X (60 – 20) = 42000 -------------------------(2)

From (1) and (2) 47 $(\theta_1 - 60)$ = 42000

θ_1 = 953.62 ℃

**Q. 25. Calculate the energy spent in spraying a drop of mercury of 1 cm radius into 10^6 droplets all of same size. Surface tension of mercury is 35 x 10^{-3} Nm^{-1}.

Ans. T = 35 x 10^{-3} Nm^{-1} R = 1 cm

Let r be the radius of each small drop, when the original drop is spitted into 10^6 small drops.

Then $10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$$r = 10^{-2} R$$

$$r = 10^{-2} \times 1 = 10^{-2} \text{ cm}$$

Initial surface area of the original drop $= 4\pi R^2 = 4\pi \times 1^2 = 4\pi \text{cm}^2$

Final surface area of the 10^6 small drops $= 10^6 \times 4\pi r^2 = 10^6 \times 4\pi \times (10^{-2})^2 = 400\,\pi \text{ cm}^2$

Therefore increase in surface area $= 400\,\pi - 4\pi = 396\,\pi \text{cm}^2 = 396\pi \times 10^{-4} \text{m}^2$

Therefore energy spent $=$ T X increase in surface area $= 35 \times 10^{-3} \times 396\pi \times 10^{-4} = 4.354 \times 10^{-3}$N

Q.26. A liquid takes 10 minutes to cool from 70℃ to 50℃. How much time will it take to cool from 60℃ to 40℃ ? The temperature of the surrounding is 20℃.

Ans. 1st case $\theta_1 = 70^0$C $\theta_2 = 50^0$C $\theta_0 = 20$℃ t = 10 minutes

Using $\frac{\theta_1 - \theta_2}{t} = k\left(\frac{\theta_{1+}\theta_2}{2} - \theta_0\right)$, we get

$$\frac{20}{10} = k\,(60 - 20) = 40\,k$$

$$K = \frac{1}{20}$$

For 2nd case $\theta_1 = 60^0$C $\theta_2 = 40^0$C $\theta_0 = 20$℃ t = ?

Using $\frac{\theta_1 - \theta_2}{t} = k\left(\frac{\theta_{1+}\theta_2}{2} - \theta_0\right)$, we get

$$\frac{20}{t} = \frac{1}{20}\,(50 - 20) = \frac{3}{2}$$

$$t = \frac{40}{3} = 13.33 \text{ minutes}$$

**Q. 28. A slab of stone of area $0.36m^2$ and thickness of 0.1m is exposed to the lower surface of steam at 100℃. A block of ice at 0℃ rest on the upper surface of the slab. In one hour 4.8 Kg of ice is melted. Calculate the thermal conductivity of stone.

Ans. Here $A = 0.36m^2$, d = 0.1m , $T_1 – T_2 = 100 – 0 = 100$℃ t = 1hr = 3600 sec

Mass of ice melted M = 4.8 Kg

We know Latent heat of ice L = 336×10^3 J Kg^{-1}

Heat required to melt the ice Q = ML = $4.8 \times 336 \times 10^3 = 1.613 \times 10^6$ K

Now $Q = \frac{KA(T_1 - T_2)t}{d}$

$$1.613 \times 10^6 = \frac{K \, X \, 0.36 \, X \, 100 X \, 3600}{0.1}$$

$K = 1.245 \, wm^{-10}C^{-1}$

<u>5 MARKS</u>

Q. 28. Define capillarity and angle of contact. Derive an expression for the ascent of liquid inside a capillary tube where it is dipped in a liquid.

Q. 29. Show that there is always excess of pressure on the concave side of the meniscus of a liquid. Obtain the expression for the excess of pressure inside (i) a liquid drop (ii) liquid bubble.

Q. 30. State and prove the Bernoulli’s principle. Give two practical application of it.

Q.31. Define terminal velocity. Show that the terminal velocity v of a sphere of radius r, density ς falling vertically through a viscous fluid of density **σ** and coefficient of viscosity η is given by

$$v = \frac{2(\varsigma - \sigma) r^2 g}{\eta}$$

Q. 32. State and explain Hooke's law. A wire is fixed at one end and is subjected to increasing load at the other end. Draw a curve between stress and strain. With the help of the curve, explain the term elastic limit, yield point, breaking point and permanent set. How this curve does may be used to distinguish between ductile and brittle substances.

THERMODYNAMICS

- **Thermal Equilibrium**:- Two systems are said to be in thermal equilibrium with each other if they have the same temperature.
- **Thermo dynamical system**:- An assembly of large numbers of particles having same temperature, pressure etc is called thermo dynamical system.
- **Thermodynamic variables** :- The variables which determine the thermodynamic behavior of a system are called thermodynamic variables
- **Zeroth law of Thermodynamics** :- IT states that if two system A and B are in thermal equilibrium with a third system C , then the two system A and B are also in thermal equilibrium with each other.
- **Thermodynamic Process** :- A thermodynamic process is said to be taking place , if the thermodynamic variable of the system change with time.
- **Types of thermodynamic Process**:-

  (1) *Isothermal process* – process taking place at constant temperature.

  (2) *Adiabatic process* – process where there is no exchange of heat.

  (3) *Isochoric process* – process taking place at constant volume

  (4) *Isobaric process* –Process taking place at constant Pressure.

  (5) *Cyclic process*:- Process where the system returns to its original state.
- **Equation of state** : A relation between pressure, volume and temperature for a system is called its equation of state .
- **Indicator diagram (P-V diagram**) :- The graphical representation of the state of a system with the help of two thermodynamical variables is called indicator diagram of the system.
- **Internal energy of a gas** :- It is the sum of kinetic energy and the intermolecular potential energy of the molecules of the gas. Internal energy is a function of temperature.
- **First law of Thermodynamics** :- It states that if an amount of heat dQ I added to a system , a part of heat is used in increasing its internal energy while the remaining part of heat may be used up as the external work done dW by the system.

Mathematically $dQ = dU + dW$

$dQ = dU + PdV$.

- **Work done during expansion / compression of gas**:- When the volume of gas changes from V_1 to V_2 , the work done is given by $W = \oint_{V1}^{V2} PdV$ = Area under the P -V diagram.

- **Thermodynamical operations are**

(1) **Isothermal process** : A thermodynamic process that takes place at constant temperature is called an isothermal process.

- Equation of state for isothermal process : PV = constant.
- Work done during an isothermal process

$$W_{iso} = RT \log_e \frac{V2}{V1} = 2.303\ RT \log_e \frac{V2}{V1}$$

(2) **Adiabatic process** : A thermodynamic process that takes place in such a manner that no heat enters or leaves the system is called adiabatic process

➔ Equation of state for adiabatic process

(i) PV^{γ} = constant (ii) $TV^{\gamma-1}$ =constant (iii) $\frac{P^{\gamma-1}}{T^{\gamma}}$ =constant

➔ Work done during adiabatic change

$$W_{adia} = \frac{R(T1-T2)}{(\gamma-1)}$$

- **Reversible process** :- It is a process in which the system can be retraced to its original state by reversing the condiditions.
- **Irreversible process**:- It is a process in which the system cannot be retraced to its original state by reversing the conditions.

- **Second law of thermodynamics:**

- **Kelvin's statement of second law** – It is impossible to derive a continous supply of work by cooling a body to a temperature lower than that of the coldest of its surrounding.
- **Clausius statement of second law** – It is impossible for a self –acting machine unaided by any external agency to transfer heat from a body to another body at higher temperature.

- **Heat Engine** – a heat engine is a device for converting heat energy continuously into a mechanical work.
  - Component of heat engine- (i) source of heat (ii) Sink (iii) Working substance
- **Efficiency of heat Engine** :-It is defined as the ratio of the external work obtained to the amount of heat energy absorbed from the heat source.

  Mathematically

$$\eta = \frac{W}{Q1} = \frac{Q1 - Q2}{Q1} = 1 - \frac{Q2}{Q1}$$

- **Carnot's heat Engine** :- it is an ideal heat Engine which is based on carnot's reversible cycle.

  Efficiency of carnot's heat Engine

$$\eta = 1 - \frac{Q2}{Q1} = 1 - \frac{T2}{T1}$$

- **Refrigerator or Heat pump**:- it is heat engine working backward.
- **Co-efficient of performance** : It is the ratio of heat absorbed from cold body to the work done by the refrigerator.

Mathematically $$\beta = \frac{Q2}{W} = \frac{Q2}{Q1-Q2} = \frac{T2}{T1-T2}$$

All Questions with **(mark) are HOTs Question

Q1 Which Thermodynamical variable is defined by the first law of thermodynamics? 1

Ans: Internal energy.

Q2 What is the amount of work done in the Cyclic process? 1

Ans:It is numerically equal to the area of the cyclic process.

Q3 Out of the parameters- temperature, pressure,work and volume, which parameter does not

Characterize the thermodynamics state of matter? 1

Ans: Work

Q4 What is the nature of P-V diagram for isobaric and isochoric process? 1

Ans: The P-V diagram for an isobaric process is a straight line parrel to the volume axis while that

For an isochoric process is a straight line parallel to pressure axis.

Q5 On what factors does the efficiency of Carnot engine depends? 1

Ans: Temperature of the source of heat and sink.

** Q6 Can we increase the temperature of gas without supplying heat to it? 1

Ans: Yes, the temperature of gas can be by compressing the gas under Adiabatic condition.

Q7 Why does the gas get heated on compression? 1

Ans: Because the work done in compressing the gas increases the internal energy of the gas.

Q8 Which thermodynamic variable is defined by Zeroth law of thermodynamics? 1

Ans: Temperature

Q9 Can the whole of work be converted into heat? 1

Ans: Yes ,Through friction.

Q10 In a Carnot engine, temperature of the sink is increased. What will happen to its efficiency? 1

Ans: We know $\eta = 1 - \frac{T2}{T1}$

On increasing the temperature of the sink (T_2), the efficiency of the Carnot engine will decrease

**Q11 If hot air rises , why is it cooler at the top of mountain than near the sea level ? 2

Ans: Since atmospheric pressure decreases with height, pressure at the top of the mountain is lesser. When the hot air rises up,it suffer adiabatic expansion at the top of the mountain.For an adiabatic change,first law of thermodynamics may be express as

$$dU + dW = 0 \qquad (dQ = 0)$$

$$dW = -dU$$

Therefore work done by the air in rising up (dW =+ve) result in decrease in the internal

Energy of the air (dU = -ve) and hence a fall in the temperature.

Q12 What happen to the internal energy of a gas during (i) isothermal expansion (ii) adiabatic Expansion? 2

Ans: In isothermal expansion ,temperature remains constant.Therefore internal energy which is a function of temperature will remain constant.

(ii)for adiabatic change dQ = 0 and hence first law of thermodynamics becomes

$$0 = dU + dW$$

$$dW = - dU$$

During expansion, work is done by the gas i.e. dW is positive. Hence ,dU must be negative.

Thus ,in an adiabatic expansion , the internal energy of the system will decrease.

Q13.Air pressure in a car increases during driving. Explain Why? 2

Ans: During driving as a result of the friction between the tyre and road ,the temperature of

The tyre and the air inside it increases. Since volume of the tyre does not change, due to increase in temperature ,pressure of the increases (due to pressure law).

Q14 The efficiency of a heat engine cannot be 100%. Explain why ? 2

Ans: The efficiency of heat engine $\eta = 1 - \frac{T2}{T1}$

The efficiency will be 100% or 1, if $T_2 = 0$ K.

Since the temperature of 0 K cannot be reached, a heat engine cannot have 100% efficiency.

Q15 In an effort to cool a kitchen during summer, the refrigerator door is left open and the kitchen door and windows are closed. Will it make the room cooler ?

Ans: The refrigerator draws some heat from the air in front of it. The compressor has to do some

Mechanical work to draw heat from the air at lower temperature. The heat drawn from the air together with the work done by the compressor in drawing it, is rejected by the refrigerator with the help of the radiator provided at the back to the air. IT follows that in each cycle, the amount of heat rejected to the air at the back of the refrigerator will be greater than that is drawn from the air in front of it. Therefore temperature of the room will increase and make hotter.

Q16 Why cannot the Carnot's engine be realised in practice?

Ans: Because of the following reasons

(i) The main difficulty is that the cylinder should come in contact with the source,sink and stand again and again over a complete cycle which is very difficult to achieve in practice.

(ii) The working substance should be an ideal gas however no gas fulfils the ideal gas behaviour.

(iii) A cylinder with a perfectly frictionless piston cannot be realised

Q17 A slab of ice at 273K and at atmospheric pressure melt.(a) What is the nature of work done on

The ice water system by the atmosphere?(b)What happen to the internal energy of the ice- Water system?

Ans: (a) The volume of the ice decreases on melting. Hence the work done by the atmosphere on The ice – water system is positive in nature.

(b) Since heat is absorbed by the ice during melting, the internal energy of the ice-water system increases.

Q18 Why is the conversion of heat into work not possible without a sink at lower temperature? 2

Ans:For converting heat energy into work continuosly, a part of the heat energy absorbed from the source has to be rejected.The heat energy can be rejected only if there is a body whose

Temperature is less than that of the source. This body at lower temperature is called sink.

** Q19 Can water be boiled without heating ? 2

Ans:Yes, water can be boil without heating. This is done by increasing the pressure on the surface of water inside a closed insulated vessel. By doing so, the boiling point of the water decreases to the room temperature and hence starts boiling.

Q20 What are the limitations of the first law of thermodynamics ? 2

Ans: The limitations are --- (i) It does not tells us the directions of heat transfer

(ii) it does not tell us how much of the heat is converted into work.

(iii)it does not tell us under what conditions heat is converted into work.

**Q21 Calculate the fall in temperature when a gas initially at 72^0C is expanded suddenly to eight times its original volume. Given γ= 5/3. 3

Ans: Let $V_1 = x$ cc $V_2 = 8x$ cc

$T_1 = 273+72 = 345$ K ¥ = 5/3 , $T_2 = ?$

Using the relation $T_1 V_1^{¥-1} = T_2 V_2^{¥-1}$

Therefore $T_2 = T_1 (V_1/V_2)^{¥-1}$

$= 345x(1/8)^{2/3}$

Taking log of both sides, we get

Log T_2 = log 345 – 2/3 log 8

=2.5378 – 2/3(0.9031)

= 2.5378 -0.6020 = 1.9358

Or T_2 =86.26 K

Therefore the fall in temperature = 345 -86.26 258.74 K

Q22 A Carnot engine whose source temperature is at 400K takes 100 Kcal of heat at this temperature in each cycle and gives 70 Kcal to the sink. Calculate (i) the temperature of the sink

(ii) the efficiency of the engine.

Ans: Here T_1 =400K , Q_1 = 100 Kcal , Q_2 = 70 Kcal

T_2 = ? ,¶ = ?

(i) $Q_1/Q_2 = T_1/T_2$

Or $T_2 = (Q_2/Q_1)T_1$

Or T_2 =70/100 x400

Or T_2 = 280 K

(ii) $\eta = 1- T_2/T_1$

= 1 – 280/400

= 1- 0.7 = 0.3

Or % of η= 0.3 x100 =30 %

Q23 If at 50^oC and 75 cm of mercury pressure, a definite mass of gas is compressed (i)slowly

(iii) suddenly, then what will be the final pressure and temperature of the gas in each case, if the final volume is one fourth of the initial volume? Given γ = 1.5

Ans:(I) When the gas is compressed slowly, the change is isothermal.

Therefore $P_2 V_2 = P_1 V_1$

$P_2 = P_1V_1/V_2$

$=(75x\ V_1/V_1)x4$ = 300 cm of mercury

Temperature remains constant at 50^oC

(ii)When the gas is compressed suddenly, the change is adiabatic

As per $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

$P_2 = P_1(V_1/V_2)^{\gamma}$

$= 75 \times (4)^{1.5}$ =600 cm of Hg

Also $T_2 V_2^{¥-1} = T_1 V_1^{¥-1}$

$T_2 = T_1 (V_1/V_2)^{¥-1}$ = $323x(4)^{(1.5-1)}$ = 646K

=646-273 =373 ^oC

Q24 Two engines A and B have their sources at 400K and 350 K and sink at350K and 300K

Respectively. Which engine is more efficient and by how much? 3

Ans: For engine A T_1 =400K , T_2 = 350 K

Efficiency $\eta_A = 1 - T_2/T_1$

=1-350/400 =1/8

% of η_A = 1/8 X100 = 12.5%

For Engine B $T_1 = 350$ K , $T_2 = 300$ K

Efficiency $\eta_B = 1 – T_2/T_1$

$= 1- 300/350 =1/7$

% of η_B =1/7 x100 =14.3%

Since $\eta_B > \eta_A$ so engine A is much more efficient than engine B by (14.3% - 12.5%) = 1.8%

** Q25 Assuming a domestic refrigerator as a reversible heat engine working between melting point

Of ice and the room temperature at 27^oC, calculate the energy in joule that must be supplied to freeze 1Kg of water at 0^oC. 3

Ans: Here $T_1 = 27+273 =300K$, $T_2 =0 +273 = 273$

Mass of water to be freezed , M = 1 Kg = 1000g

Amount of heat that should be removed to freeze the water

Q_2 = ML =1000X 80 cal

= 1000X80 X4.2 =3.36×10^5 J

Now $Q_1 = (T_1/T_2)X Q_2 = (300/273)X3.36x10^5 = 3.692 x10^5$ J

Therefore energy supplied to freeze the water

$W =Q_1 – Q_2 = 3.693x10^5 - 3.36 x10^5$

$= 3.32 x10^5$ J

** Q26 A refrigerator freezes 5Kg of water at 0^oC into ice at 0^oc in a time interval of 20 minutes. Assume that the room temperature is 20^oC, calculate the minimum power needed to accomplish it.

Ans : Amount of heat required to convert water into ice at 0^oC,

Q_2 =mL =(5Kg)x(80)Kcal/Kg

= 400 Kcal

Now T_1 =20^oC =273+20 = 293K

T_2 = 0^oc 0+273 =273 K

We know that $Q_2/W = T_2/(T_1 – T_2)$

Or $W = Q_2 \times (T_1 – T_2)/T_2$

= 400x(293- 273)/273

= 29.3 Kcal = $29.3 \times 4.2 \times 10^3$J

=123×10^3 J

Time t = 20 min =20x60 =1200s

Power needed P = W/t =$123 \times 10^3/1200$

=102.5 W

**Q27 The temperature T_1 and T_2 of two heat reserviour in an ideal carnot engine are 1500^oC and

500^oC. Which of this increasing the temperature T_1 by 100^oC or decreasing T_2 by 100^oC would result in greater improvement of the efficiency of the engine?

Ans: Using ¶ =$1 – T_2/T_1$ =$(T_1- T_2)/T_1$

(1)increasing T_1 by 100^oC ¶$_1$ = (1600 -500)/(1600 + 273)

= 1100/1873 = 59 %

(ii) Decreasing T_2 by 100^0C ¶$_2$ = 1500 –(500-100)/(1500 +273)

=1100/1773 = 67%

Therefore decreasing T_2 by 100^oC results in greater improvement of efficiency.

Q28 State the first law of thermodynamics and discussed the application of this law to the boiling process. 3

Q29 What is thermodynamic system ? Prove that work done by thermodynamic system is equal to the area under P-V diagram. 3

30 Prove that $C_p - C_v = R$, for an ideal gas . 3

Q31 What is isothermal process / State two essential conditions for such a process to takes place. Show analytically that the work by one mole of an ideal gas during volume expansion from V_1 V_2 at temperature T is given by
$W = RT \log_e V_2/V_1$ 5

Q32 Define an adiabatic process. State two essential conditions for such a process to takes place.Derive an expression for adiabatic process to takes place. 5

Q33 Discuss the four steps of Carnot's cycle and show that the efficiency is given by ¶ = $1 - T_2/ T_1$, Where T_1 and T_2 are the temperature of the source and sink respectively. 5

Q34 Describe the working of refrigerator as heat pump. Derive the expression of its coefficient of performance. If the door of a refrigerator is kept open for a long time ,will it make the room warm or cool ? 5

Q35 What is the need of introducing the second law of thermodynamics ? State the Kelvin –Planck and Claussius statement of second law of thermodynamics and show that both the statement are equivalent. 5

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KINETIC THEORY OF GASES

Boyle's Law: *At constant temperature volume of given mass of gas is inversely proportional to its pressure.*

$$V \, \alpha \, \frac{1}{P} \text{ or } PV = \text{constant}$$

Charle's Law: *At constant pressure volume of a given mass of gas is directly proportional to its absolute temperature.*

$$V \, \alpha \, T \text{ or } \frac{V}{T} = \text{constant}$$

*For 1° rise in temp.

$$V_t = Vo(1 + \frac{t}{273.15})$$

Gay Lussac's Law: At constant volume, pressure of a given mass of gas is directly proportional to its absolute temp.

$$\frac{P}{T} = \text{constant.}$$

For 1^0C rise in temperature $P_t = P_o \, (1 + \frac{t}{273.15})$

Ideal Gas Equation: for n mole of gas

PV=nRT,

for 1 mole , PV=RT

Universal gas constant: R = 8.31 J $mol^{-1}K^{-1}$

Boltzmann constant: $k_B = \frac{R}{N_A}$**where k_B = Boltzmann constant, N_A = Avogadro's no.**

Ideal gas: A gas which obeys gas law strictly is an ideal or perfect gas. The molecules of such a gas are of point size and there is no force of attraction between them.

Assumptions of Kinetic Theory of Gases

1. All gases consist of molecules which are rigid, elastic spheres identical in all respect for a given gas.

2. The size of a molecule is negligible as compared with the average distance between two molecules.

3. During the random motion, the molecules collide with one another and with the wall of the vessel.The collisions are almost instantaneous.

4. The molecular density remains uniform throughout the gas.

5. The collisions are perfectly elastic in nature and there are no forces of attraction or repulsion between them.

Pressure exerted by gas:

$$P = \frac{1}{3}.\frac{M}{V}\overline{v^2} = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}mn\overline{v^2}$$

Where: n=no. of molecules per unit volume.

m=mass of each molecule.

$\overline{v^2}$ =mean of square speed.

V = Volume

M = mass of gas

Average Kinetic energy of a gas: If M is molecular mass and V is molecular volume and m is mass of each molecule. Then

1. Mean K.E per mole of a gas,

$$E = \frac{1}{2}M\overline{v^2} = \frac{3}{2}PV = \frac{3}{2}RT = \frac{3}{2}K_B N_A T$$

2. Mean K.E per molecule of a gas,

$$\vec{E} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}k_B T$$

3. K.E of 1gram of gas,

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{RT}{M_0}$$ M_0 gram molecular weight

Avogadro Law: Equal volume of all gases under similar condition of temp. and pressure contain equal number of molecules.

Avogadro Number:

$$N_A = 6.0225 \times 10^{23} \text{ mol}^{-1}$$

Graham's Law of diffusion:

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

r = rate of diffusion

ρ = density

Delton's law of partial pressure: Total pressure exerted by a mixture of non-reacting gases occupying a given volume is equal to the sum of partial pressures which gas would exert if it alone occupied the same volume at given temp.

$$P = P_1 + P_2 + P_3 + \ldots \ldots \ldots$$

Average Speed :- $\overline{V} = \frac{v_1+v_2+v_3+\ldots+v_n}{n}$

$$\bar{v} = \sqrt{\frac{8k_bT}{\pi m}} = \sqrt{\frac{8RT}{\pi M_0}}$$

Root mean square:

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

Most probable speed:

$$V_{mp} = \sqrt{\frac{2K_BT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

Relation between :$\bar{v}$, $\overline{v_{rms}}$ & v_{mp}

$$\bar{v} = 0.92v_{rms}, v_{mp} = 0.816v_{rms}$$

$$v_{rms} : \bar{v} : v_{mp} = 1.73 : 1.6 : 1.41$$

Therefore: $v_{rms} > \bar{v} > v_{mp}$

Degree of freedom:

f = 3N-k

where , f = no. of degree of freedom.

N = no. of of atoms in a molecule. k = no. of independent relation between the atoms.

1. Monoatomic gas – 2 degree of freedom.
2. Diatomic gas – 5 degree of freedom.

Law of equipartion of energy: For any thermodynamical system in thermal equilibrium, the energy of the system is equally divided amongst its various degree of freedom and energy associated with each degree of freedom corresponding to each molecule is $\frac{1}{2} K_B T$, where K_B is the Boltzmann's constant and T is absolute temperature.

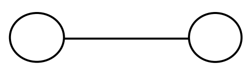
- The law of equipartition of energy holds good for all degrees of freedom whether translational , rotational or vibrational.
- A monoatomic gas molecule has only translational kinetic energy

$$E_t = 1/2mV_x^2 + 1/2mV_y^2 + 1/2mV_z^2 = 3/2K_BT$$

So a monoatomic gas molecule has only three (translational) degrees of freedom.

- In addition to translational kinetic energy, a diatomic molecule has two rotational
  Kinetic energies

$$E_t + E_r = 1/2mV_x^2 + 1/2mV_y^2 + 1/2mV_z^2 + 1/2I_YW_y^2 + 1/2I_zW_z^2$$



Here the line joining the two atoms has been taken as x-axis about which there is no rotation. So, the degree of freedom of a diatomic molecule is 5, it does not vibrate.

At very high temperature, vibration is also activated due to which two extra degree of freedom emerge from vibrational energy. Hence at very high temperature degree of freedom of diatomic molecule is seven.

***(**Each translational and rotational degree of freedom corresponds to one mole of absorption of energy and has energy $1/2k_BT$).

Internal Energies & specific heats of monoatomic, diatomic& polyatomic gases:

1. If 'f' is degree of freedom then for a gas of polyatomic molecules energy associated with 1 mole of gas,

$$U = \frac{f}{2}RT \quad , \quad C_v = \frac{f}{2}R$$

$$C_p = \left(1 + \frac{f}{2}\right) R, \ \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

2. For a monoatomic gas f=3,

$$U = \frac{3}{2}RT \quad , \quad C_v = \frac{3}{2}R$$

$$C_p = \frac{5}{2} \ R, \qquad \gamma = 1.66$$

3. For a diatomic gas with no vibrational mode f=5, so

$$U = \frac{5}{2}RT \quad , \quad C_v = \frac{5}{2}R$$

$$C_p = \frac{7}{2} \ R, \qquad \gamma = 1.4$$

4. For a diatomic gas with vibrational mode f=7, so

$$U = \frac{7}{2}RT \quad , \quad C_v = \frac{7}{2}R$$

$$Cp = \frac{9}{2} \ R \ , \gamma = 1.28$$

Meanfree path: It is the average distance covered by a molecule between two successive collisions. It is given by,

$$\overline{\lambda} = \frac{1}{\sqrt{2}(n\pi d^2)}$$

Where,n is no. density and 'd' is diameter of the molecule.

Brownian Equation :-The zig-zag motion of gas molecules is Brownian motion which occurs due to random collision of molecules.

Memory Map

<u>Kinetic Theory of gases</u>

1. $V_{rms} = \sqrt{\frac{3p}{\rho}}$

2. $E = \frac{3}{2}\ \mathrm{RT}$

3. $V_{rms} = \sqrt{\frac{3\mathrm{RT}}{\mathrm{M}}} = \sqrt{\frac{3\mathrm{PV}}{\mathrm{M}}}$

4. $V\ \alpha\ \sqrt{T}$

Law of Equipartion of Energy

$\frac{1}{2}\, m v_x^{\,2} = \frac{1}{2}\, m v_y^{\,2}$

$= \frac{1}{2}\, m v_{2=}^{\ \ 2} = \frac{1}{2}\, k_B T$

$P = \frac{1}{3}\ P\overrightarrow{C^2}$

Mean free Path

$\bar{\lambda} = \frac{1}{\sqrt{2}\, n\pi d^2}$

Specific Heats

$r = 1 + \frac{2}{f}$ where $r = \frac{C_p}{C_v}$

and f=degree of freedom

(1 Marks Question)

1. What type of motion is associated with the molecules of a gas?

Ans:- Brownian motion.

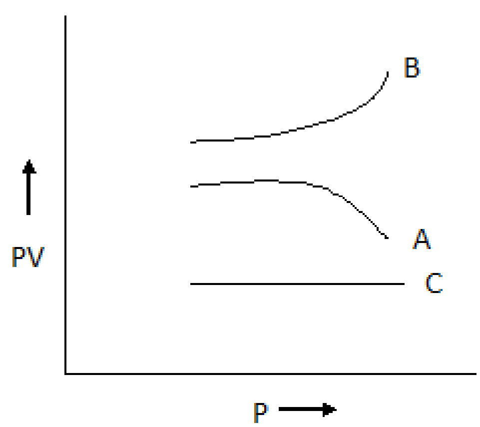
2. On which factors does the average kinetic energy of gas molecules depend?

Ans:- The average K.E. of a gas molecule depends only on the absolute temperature of the gas and is directly proportional to it.

3. Why do the gases at low temperature and high pressure, show large deviations from ideal behaviour?

Ans:- At low temperature and high pressure , the intermolecular attractions become appreciable. So, the volume occupied by the gas molecules cannot be neglected in comparison to the volume of the gas. Hence the real gases show large from ideal gas behaviour.

4. Following fig. shows the variation of the product PV with respect to the pressure (P) of given masses of three gases, A,B,C. The temperature is kept constant. State with proper arguments which of these gases is ideal.



Ans:- Gas ‘C’ is ideal because PV is constant for it. That is gas ‘C’ obeys Boyle’s law at all pressures.

5. When a gas is heated, its temperature increases. Explain it on the basis of kinetic theory of gases.

Ans:- When a gas is heated, the root mean square velocity of its molecules increases. As $V_{rms} \alpha \sqrt{T}$ so temperature of the gas increases.

6. The ratio of vapour densities of two gases at the same temperature is 8:9. Compare the rms. velocity of their molecules?

Ans :- $\frac{(Vrms)1}{(Vrms)2} = \sqrt{\frac{M2}{M1}} = \sqrt{\frac{\rho 2}{\rho 1}} = \sqrt{\frac{9}{8}} = 3:2\sqrt{2}$

7. Cooking gas containers are kept in a lorry moving with uniform speed. What will be the effect on temperature of the gas molecules?

Ans:- As the lorry is moving with a uniform speed, there will be no change in the translational motion or K.E. of the gas molecules. Hence the temperature of the gas will remain same.

8. What is the mean translational kinetic energy of a perfect gas molecule at temperature T?

Ans:- A perfect gas molecule has only translational K.E.

$$E = 3/2\ k_B T$$

9. Name two factors on which the degrees of freedom of a gas depend?

Ans:- (i) Atomicity of the gas molecule.

(ii) Shape of the molecule.

(iii) Temperature of gas.

10. Define absolute zero, according to kinetic interpretation of temperature?

Ans:- Absolute zero is the temperature at which all molecular motion ceases.

(2 Marks question)

1. Write the relation between the pressure and kinetic energy per unit volume of a gas. Water solidifies into ice at 273 K. What happens to the K.E. of water molecules?

Ans:- P = 2/3 E. The K.E. of water molecules gas partly converted into the binding energy of the ice.

2. The absolute temperature of a gas is increased 4 times its original value. What will be the change in r.m.s. velocity of its molecules?

Ans:- $V_{rms} \alpha \sqrt{T}$

$V'_{rms} \alpha \sqrt{4T}$

$V'_{rms} / V_{rms} = 2$

$V'_{rms} = 2V_{rms}$

Change in rms velocity of molecules = $V'_{rms} - V_{rms}$

$= V_{rms}$

3.What will be the ratio of the root mean square speeds of the molecules of an ideal gas at 270K and 30K?

Ans :- $V_{rms} / V'_{rms} = \sqrt{\frac{T}{T'}} = \sqrt{\frac{270}{30}} = 3 : 1$

4.A mixture of Helium and Hydrogen gas is filled in a vessel at 30 degree Celsius. Compare the root mean square velocities of the molecules of these gases at this temperature.

(atomic weight of Hydrogen is 4)

Ans :- $(V_{rms})He/(V_{rms})H_2 = \{(M_{H2})/(M_{He})\}^{1/2} = \sqrt{\frac{2}{4}} = 1 : 2\sqrt{2}$

5.The velocities of three molecules are 3V,4V and 5V.Determine the root mean square velocity.

Ans:- V_{rms} = $\sqrt{\frac{50}{3}V} = 4.08V$

6.Write the equation of state for 16g of O_2**.**

Ans :- No. of moles in 32g of O_2 = 1

No. of moles in 16g of O_2 = 1/9 x 16 = ½

As pv = nRT and n=1/2

So, PV= $\frac{1}{2}$ RT

7.Should the specific heat of monoatomic gas be less than, equal to or greater than that of a diatomic gas at room temperature? Justify your answer.

Ans :- Specific heat of a gas at constant volume is equal to f/2R.

For monoatomic gases f = 3 so C_v = 3/2 R.

For diatomic gases f = 5 so C_v = 5/2 R.

Hence the specific heat for monoatomic gas is less than that for a diatomic gas.

8. A gas in a closed vessel is at the pressure P_o. If the masses of all the molecules be made half and their speeds be made double, then find the resultant pressure?

Ans:- $P_o = \frac{1}{3}\ \frac{mN}{V}\ \overline{V^2}$ $\quad = \frac{1}{3}\ \frac{mN}{2V}(\ 2V)^2$ = $2P_0$

9. A box contains equal number of molecules of hydrogen and oxygen. If there is a fine hole in the box, which gas will leak rapidly? Why?

Ans :- $V_{rms} \propto \frac{1}{\sqrt{M_0}}$

Hence hydrogen gas will leak more rapidly because of its smaller molecular mass.

10. When a gas filled in a closed vessel is heated through 1^0C, its pressure increases by 0.4 %. What is the initial temperature of the gas?

Ans:- P\` = P = 0.4/100. P,  T\` = T + 1

By Gay Lussac's law P/T = (P + 0.4/100.P)/T + 1,

$$\frac{P}{T} = \left(P + \frac{.4}{100}\ P\right) \div \ (T+1)$$

$$\frac{(\mathrm{P+.004P})}{\mathrm{T+1}} = \frac{\mathrm{P(1.004)}}{\mathrm{T+1}}$$

T+1= (1.004)T

1=.004T

T=250K

(3 Marks Questions)

1. Show that rms velocity of O_2 is $\sqrt{2}$ times that of SO_2. Atomic wt. of Sulphur is 32 and that of oxygen is 16.

<u>Ans</u>. $V \propto \frac{1}{\sqrt{M}}$. $\frac{V_{O_2}}{V_{SO_2}} = \sqrt{\frac{64}{32}} = \sqrt{2}$

Or $v_{O_2} = \sqrt{2}\ SO_2$.

2. Calculate the temperature at which rms velocity of SO_2 is the same as that of Oxygen at 27℃.

<u>Ans</u>. For O_2, $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R \times 300}{32}}$

For SO_2, $V_{rms} = \sqrt{\frac{3R\acute{T}}{M_0}} = \sqrt{\frac{3R \times \acute{T}}{64}}$

As $V_0 = V$ $\therefore \sqrt{\frac{3R\acute{T}}{64}} = \sqrt{\frac{3R \times 300}{32}}$

$\acute{T}$ = 600t = 600 – 273 = 327℃.

3. Calculate the total no. of degrees of freedom possessed by the molecules in 1cm^3 of H_2 gas at NTP

<u>Ans</u>. No. of H_2 Molecules in 22.4 liters or 22400 cm^3 at NTP $= 6.02 \times 10^{23}$.

$\therefore$ No. of H_2 Molecules in 1 cm^3 at NTP $= \frac{6.02 \times 10^{23}}{22400} = 2.6875 \times 10^{19}$.

No. of degrees of freedom associated with each H_2 (a diatomic) molecule = 5

∴ Total no. of degree of freedom associated with 1cm^3 gas

$= 2.6875 \times 10^{19} \times 5 = 1.3475 \times 10^{20}$.

4. Derive Boyle's law on the basis of Kinetic Theory of Gases.

5. Derive Charles's law on the basis of Kinetic Theory of Gases.

6. State Dalton's law of partial pressures. Deduce it from Kinetic Theory of Gases.

7. Using the expression for pressure exerted by a gas, deduce Avogadro's law and Graham's law of diffusion.

8. State the number of degree of freedom possessed by a monoatomic molecule in space. Also give the expression for total energy possessed by it at a given temperature. Hence give the total energy of the atom at 300 K.

9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of helium gas atom at -20℃? Atomic mass of argon = 39.9 u and that of helium = 4.0 u.

Ans. Root mean square speed for argon at temperature T

$$V = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

Root mean square speed for helium at temp. 20℃ is

$$\acute{V} = \sqrt{\frac{3R \times 253}{4}}$$

As V = $\acute{V}$ so we have $\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$

$$= \frac{T}{39.9} = \frac{253}{4} \quad \text{or } T = \frac{253 \times 39.9}{4}$$

T = 2523.7 K

10. From a certain apparatus the diffusion rate of Hydrogen has an average value of 28.7 cm^3 s^{-1}; the diffusion of another gas under the same conditions is measured to have an average rate of 7.2cm^{3}s^{-1}. Identify the gas.

Ans. From Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_1}{M_2}}$$

$$M_2 = \left(\frac{r_1}{r_2}\right)^2 M_1 = \left(\frac{28.7}{7.2}\right)^2 \times 2$$
$$= 31.78 \approx 32$$

Thus the unknown gas is Oxygen.

(Long Questions)

11. Prove that the pressure exerted by a gas is P = $\frac{\mathbf{1}}{\mathbf{3}}\rho \overline{c^2}$ where ρ is the density and c is the root mean square velocity.

12.What are the basic assumptions of Kinetic Theory of Gases? On their basis derive an expression for the pressure exerted by an ideal gas.

Oscillations and Waves

- **Periodic Motion**: A motion which repeats itself over and over again after a regular interval of time.
- **Oscillatory Motion**: A motion in which a body moves back and forth repeatedly about a fixed point.
- **Periodic function**: A function that repeats its value at regular intervals of its argument is called periodic function. The following sine and cosine functions are periodic with period T.

  $f(t) = \sin\frac{2\pi t}{T}$ and $g(t) = \cos\frac{2\pi t}{T}$

  These are called Harmonic Functions.

  Note :- All Harmonic functions are periodic but all periodic functions are not harmonic.

One of the simplest periodic functions is given by

$f(t) = A\cos\omega t \quad [\omega = 2\pi/T]$

If the argument of this function *ωt* is increased by an integral multiple of 2π radians, the value of the function remains the same. The function f(t) is then periodic and its period, T is given by

$T = \frac{2\pi}{\omega}$

Thus the function f(t) is periodic with period T

$f(t) = f(t + T)$

Linear combination of sine and cosine functions

$f(t) = A\sin\omega t + B\cos\omega t$

A periodic function with same period T is given as

$A = D\cos\phi$ and $B = D\sin\phi$

∴*f(t) = D sin (ωt + ø)*

∴ $D = \sqrt{A^2 + B^2}$ *and* $ø = \tan^{-1}\frac{x}{a}$

- **Simple Harmonic Motion (SHM):** A particle is said to execute SHM if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from mean position and is always directed towards mean position.

  Restoring Force ∝ Displacement

  $$F \propto x$$

  $$F = -kx$$

  Where 'k' is force constant.
- **Amplitude**: Maximum displacement of oscillating particle from its mean position.

  $x_{Max} = \pm A$
- **Time Period:** Time taken to complete one oscillation.
- **Frequency:** $= \frac{1}{T}$. Unit of frequency is Hertz (Hz).

  $$1 \text{ Hz} = 1\ s^{-1}$$
- **Angular Frequency:** $\omega = \frac{2\pi}{T} = 2\pi\nu$

  S.I unit ω = rad s^{-1}
- **Phase:**
  1. The Phase of Vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.

     It is denoted by ø.
  2. **Initial phase or epoch**: The phase of particle corresponding to time t = 0.

     It is denoted by ø.
- **Displacement in SHM :**

  $$x = A\cos(\omega t + ø_0)$$

  Where, x = Displacement,

  A = Amplitude

  ωt = Angular Frequency

  $ø_0$ = Initial Phase.

Case 1: When Particle is at mean position x = 0

$$v = -\omega\sqrt{A^2 - 0^2} = -\omega A$$

$$v_{max} = \omega A = \frac{2\pi}{T} A$$

Case 2: When Particle is at extreme position x = $\pm A$

$$v = -\omega\sqrt{A^2 - A^2} = 0$$

<u>Acceleration</u>

Case 3: When particle is at mean position x = 0,

acceleration = $-\omega^2(0)$ = 0.

Case 4: When particle is at extreme position then

$x = A$ acceleration = $-\omega^2 A$

➢ **Formula Used** :

1. $x = A\cos(\omega t + \varnothing_0)$

2. v = $\frac{dx}{dt} = -\omega\sqrt{A^2 - x^2}$, $v_{max} = \omega A$.

3. $a = \frac{dv}{dt} = \omega^2 A\cos(\omega t + \varnothing_0)$

$$= -\omega^2 x$$

$a_{max} = \omega^2 A$

4. *Restoring force* F = $-kx = -m\omega^2 x$

Where k = force constant & $\omega^2 = \frac{k}{m}$

5. *Angular freq.* $\omega = 2\pi\nu = 2\pi/T$

6. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Displacement}}{\textbf{Acceleration}}} = 2\pi\sqrt{\frac{x}{a}}$

7. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Inertia Factor}}{\textbf{Spring Factor}}} = 2\pi\sqrt{\frac{m}{k}}$

8. P.E at displacement 'y' from mean position

$E_P = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \sin^2\omega t$

9. K.E. at displacement 'y' from the mean position

$E_K = \frac{1}{2}k(A^2 - y^2) = \frac{1}{2}m\omega^2(A^2 - y^2)$

$= \frac{1}{2}m\omega^2 A^2 \cos^2\omega t$

10. Total Energy at any point

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 mA^2 v^2$

11. Spring Factor $K = F/y$

12. Period Of oscillation of a mass 'm' suspended from a massless spring of force constant '*k*'

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For two springs of spring factors k_1 and k_2 connected in parallel effective spring factor

$$k = k_1 + k_2 \qquad \therefore T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

13. For two springs connected in series, effective spring factor 'k' is given as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad Or \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Note:- When length of a spring is made 'n' times its spring factor becomes $\frac{1}{n}$ times and hence time period increases $\sqrt{n}$ times.

14. When spring is cut into 'n' equal pieces, spring factor of each part becomes '*nk*'.

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

15. Oscillation of simple pendulum

$$T = 2\pi\sqrt{l/g}$$

$$\nu = \frac{1}{2\pi}\sqrt{g/l}$$

16. For a liquid of density ρ contained in a U-tube up to height 'h'

$$T = 2\pi\sqrt{h/g}$$

17. For a body dropped in a tunnel along the diameter of earth

$T = 2\pi\sqrt{R/g}$, where **R** = Radius of earth

18. **Resonance**: If the frequency of driving force is equal to the natural frequency of the oscillator itself, the amplitude of oscillation is very large then such oscillations are called resonant oscillations and phenomenon is called resonance.

Waves

Angular wave number: It is phase change per unit distance.

i.e. $k = \frac{2\pi}{\lambda}$, S.I unit of k is radian per meter.

Relation between velocity, frequency and wavelength is given as :- $V = \upsilon\lambda$

Velocity of Transverse wave:-

(i)In solid molecules having modulus of rigidity 'η' and density 'ρ' is

$$V = \sqrt{\frac{\eta}{\rho}}$$

(ii)In string for mass per unit length 'm' and tension 'T' is $V = \sqrt{\frac{T}{m}}$

Velocity of longitudinal wave:-

(i) in solid $V = \sqrt{\frac{Y}{\rho}}$, Y= young's modulus

(ii) in liquid $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

(iii) in gases $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

According to Newton's formula: When sound travels in gas then changes take place in the medium are isothermal in nature. $V = \sqrt{\frac{P}{\rho}}$

According to Laplace: When sound travels in gas then changes take place in the medium are adiabatic in nature.

$V = \sqrt{\frac{P\gamma}{\rho}}$ 'Where $\gamma = \frac{Cp}{Cv}$

Factors effecting velocity of sound :-

(i) Pressure – No effect

(ii) Density – $V \alpha \frac{1}{\sqrt{\rho}}$ or $\frac{V1}{V2} = \sqrt{\frac{\rho 1}{\rho 2}}$

(iii) Temp- $V \alpha \sqrt{T}$ or $\frac{V1}{V2} = \sqrt{\frac{T1}{T2}}$

Effect of humidity :– sound travels faster in moist air

(iv) Effect of wind –velocity of sound increasing along the direction of wind.

Wave equation:– if wave is travelling along + x-axis

(i) Y=A sin ($\omega t - kx$), Where, $k = \frac{2\pi}{\lambda}$

(ii) Y=A sin $2\pi\ (\frac{t}{T} - \frac{x}{\lambda})$

(iii) Y=A sin $\frac{2\pi}{T}$ (vt-x)

If wave is travelling along –ve x- axis

(iv) Y=A sin ($\omega t + kx$) ,Where ,$k = \frac{2\pi}{\lambda}$

(v) Y=A sin $2\pi\ (\frac{t}{T} + \frac{x}{\lambda})$

(vi) Y=A sin $\frac{2\pi}{T}$ (vt+x)

Phase and phase difference

Phase is the argument of the sine or cosine function representing the wave.

$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Relation between phase difference ($\Delta\phi$) and time interval (Δt) is $\boldsymbol{\Delta\phi = -\frac{2\pi}{T}\Delta t}$

Relation between phase difference (Δp) and path difference (Δx) is $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

Equation of stationary wave:-

(1) Y_1 =a sin $2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (incident wave)

Y_1 = $\pm$ a sin $2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$ (reflected wave)

Stationary wave formed

Y = Y_1+Y_2 = $\pm$ 2a cos $\frac{2\pi x}{\lambda}$ sin $\frac{2\pi t}{T}$

(2) For (+ve) sign antinodes are at x= 0, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

And nodes at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

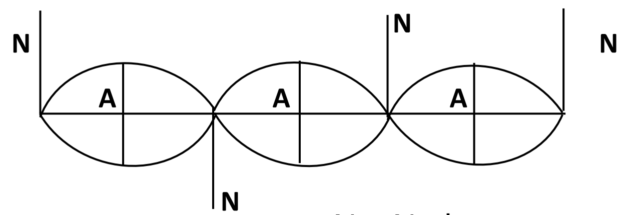
(3) For (-ve) sign antinodes are at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

Nodes at x= 0, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

(4)Distance between two successive nodes or antinodes are $\frac{\lambda}{2}$ and that between nodes and nearest antinodes is $\frac{\lambda}{4}$.

(5) Nodes-point of zero displacement-

Antinodes- point of maximum displacement-



N = Nodes

A = Antinodes

Mode of vibration of strings:-

a) $\upsilon = \frac{p}{2L}\sqrt{\frac{T}{m}}$ Where ,T= Tension

M= mass per unit length

υ = frequency, V=velocity of second, p = 1, 2, 3, …

b) When stretched string vibrates in P loops υ p= $\frac{p}{2L}\sqrt{\frac{T}{m}} = p\upsilon$

c) For string of diameter D and density ρ

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

d) Law of length $\nu \propto \frac{1}{L}, \nu L = constant$

ORGANPIPES

1. In an organ pipe closed at one end only odd harmonics are present

$\upsilon_1 = \frac{V}{4L}$ (fundamental)

$\upsilon_2 = 3\upsilon$ (third harmonic or first overtone)

$\upsilon_3 = 5\upsilon$

$\upsilon_n = (2n-1)\ \upsilon$

2. In an open organ pipe at both ends both odd and even harmonics are present.

$\upsilon'_1 = \frac{V}{2L} = \upsilon'$ (first harmonic)

$\upsilon'_2 = 2\upsilon'$ (second harmonic or first overtone)

$\upsilon'3 = 3\upsilon'$

$\upsilon'n = (2n-1)\ \upsilon'$

3. Resonance tube: If L_1 and L_2 are the first and second resonance length with a tuning fork of frequency 'υ'then the speed of sound. v = 4v$(L_1 + 0.3D)$

Where ,D=internal diameter of resonance tube

$$v = 2v(L_2 - L_2)$$

End correction=0.3D= $\dfrac{L2 - L1}{2}$

Beats formation

1. Beat frequency = No. of beats per second = Difference in frequency of two sources.

$$b = \nu_1 - \nu_2$$

2. $\nu_2 = \nu_1 \pm b$
3. If the prong of tuning fork is filed, its frequency increases. If the prong of a tuning fork is loaded with a little way, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

Doppler effect in sound

1. If V, Vo, Vs, and Vm are the velocity of sound , observes, source and medium respectively, then the apparent frequency

$$\upsilon 1 = \frac{V + Vm - Vo}{V + Vm - Vs} \times \upsilon$$

2. If the medium is at rest (v_m= o),then

$$\upsilon' = \frac{V - Vo}{V - Vs} \times \upsilon$$

3. All the velocity are taken positive with source to observer (S$\rightarrow$O) direction and negative in the opposite (O$\rightarrow$S) direction

(Questions)

(1 marks questions)

1. Which of the following relationships between the acceleration 'a' and the displacement 'x' of a particle involve simple harmonic motion?

(a) a=0.7x (b) $a=-200x^2$ (c) a = -10x (d) $a=100x^3$

Ans: - (c) reprent SHM.

2. Can a motion be periodic and not oscillatory?

Ans: - Yes, for example, uniform circular motion is periodic but not oscillatory.

3. Can a motion be periodic and not simple harmonic? If your answer is yes, give an example and if not, explain why?

Ans:- Yes, when a ball is doped from a height on a perfectly elastic surface ,the motion is oscillatory but not simple harmonic as restoring force F=mg=constant and not $F\alpha$ -x, which is an essential condition for S.H.M.

4. A girl is swinging in the sitting position. How will the period of the swing change if she stands up?

Ans:-The girl and the swing together constitute a pendulum of time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the girl stands up her centre of gravity is raised. The distance between the point of suspension and the centre of gravity decreases i.e. length 'l' decreases .Hence the time period 'T' decreases.

5. The maximum velocity of a particle, executing S.H.M with amplitude of 7mm is 4.4 m/s. What is the period of oscillation?

Ans: - $V_{max}=\omega A = \frac{2\pi}{T}A$, $T = \frac{2\pi A}{\text{Vmax}} = \frac{2 \times 22 \times .007}{7 \times 4.4} = 0.01s$

6. Why the longitudinal wave are also called pressure waves?

Ans: - Longitudinal wave travel in a medium as series of alternate compressions and rare fractions i.e. they travel as variations in pressure and hence are called pressure waves.

7. How does the frequency of a tuning fork change, when the temperature is increased?

Ans: -As the temperature is increased, the length of the prong of a tuning fork increased .This increased the wavelength of a stationary waves set up in the tuning fork. As frequency,

$\upsilon = \frac{1}{\lambda}$, So the frequency of tuning fork decreases.

8. An organ pipe emits a fundamental node of a frequency 128Hz. On blowing into it more strongly it produces the first overtone of the frequency 384Hz. What is the type of pipe –Closed or Open?

Ans: - The organ pipe must be closed organ pipe, because the frequency the first overtone is three times the fundamental frequency.

9. All harmonic are overtones but all overtones are not harmonic. How?

Ans: -The overtones with frequencies which are integral multiple of the fundamental frequency are called harmonics. Hence all harmonic are overtones. But overtones which are non-integrals multiples of the fundamental frequency are not harmonics.

10. What is the factor on which pitch of a sound depends?

Ans: - The pitch of a sound depends on its frequency.

(2 Marks questions)

1. At what points is the energy entirely kinetic and potential in S.H.M? What is the total distance travelled by a body executing S.H.M in a time equal to its time period, if its amplitude is A?

Ans. The energy is entirely kinetic at mean position i.e. at y=0. The energy is entirely potential at extreme positions i.e.

$$y = \pm A$$

Total distance travelled in time period T = 2A + 2A = 4A.

2. A simple pendulum consisting of an inextensible length 'l' and mass 'm' is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2. Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards?

Ans. When the lift is stationary, T = $2\pi\sqrt{\frac{l}{g}}$

When the lift accelerates upwards with an acceleration of 4.5 m/s^2

$T' = 2\pi\sqrt{\frac{l}{g+45}}$

Therefore, the time period decreases when the lift accelerates upwards.

3. Does the function y = $\sin^2\omega t$ represent a periodic or a S.H.M? What is period of motion?

Ans. Displacement y = $\sin^2\omega t$

Velocity v = $\frac{dy}{dt}$ = 2sin ωt × cosωt × ω

v = ω sin 2ωt

Acceleration a = $\frac{dv}{dt}$ = ω×cos 2ωt × 2ω

a = 2 ω^2cos 2ωt.

As the acceleration is not proportional to displacement y, the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω.

∴ Time Period T = $\frac{2\pi}{Angular\ freq.} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

4. All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?

Ans. All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So these functions cannot be used to represent bounded motion like SHM.

5. A simple Harmonic Motion is represented by $\frac{d^2x}{dt^2}$+αx = 0. What is its time period?

Ans. $\frac{d^2x}{dt^2} = -\alpha x$ Or a = – αx

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{x}{\alpha x}} = \frac{2\pi}{\sqrt{\alpha}}$$

$$\mathbf{T = \frac{2\pi}{\sqrt{\alpha}}}$$

6. The Length of a simple pendulum executing SHM is increased by 2.1%. What is the percentage increase in the time period of the pendulum of increased length?

Ans. Time Period, T = $2\pi\sqrt{\frac{l}{g}}$ i.e. T $\propto\sqrt{l}$.

The percentage increase in time period is given by

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100 \text{ (for small variation)}$$

$= \frac{1}{2} \times 2.1\%$

= 1.05%

7. A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from x = A to x = A/2.

Ans. Displacement from mean position = A – $A/2$ = $A/2$.

When the motion starts from the positive extreme position, y = A cosωt.

$$\therefore \frac{A}{2} = A\cos\frac{2\pi}{T}t.$$

$\cos\frac{2\pi}{T}t$ = ½ = $\cos\frac{\pi}{3}$

or $\frac{2\pi}{T}t = \frac{\pi}{3}$

$\therefore$ t = $T/6$

8. An open organ pipe produces a note of frequency 5/2 Hz at 15^0C, calculate the length of pipe. Velocity of sound at 0^0C is 335 m/s.

Ans. Velocity of sound at 15^0C

$V=V_0+0.61xt$ =335+0.61x15 =344.15 m/s. (Thermal coefficient of velocity of sound wave is .61/^{0}C)

Fundamental frequency of an organ pipe

$$\upsilon = \frac{V}{4L}, \quad \therefore L = \frac{V}{4\upsilon} = \frac{344.15}{4\times 512} = 0.336m$$

9. An incident wave is represented by Y(x, t)=20sin(2x-4t).Write the expression for reflected wave

(i) From a rigid boundary

(ii) From an open boundary.

Ans.(i) The wave reflected from a rigid boundary is

$$Y (x, t) = -20\sin (2x+4t)$$

(i)The wave reflected from an open boundary is

$$Y (x, t) = 20\sin (2x+4t)$$

Explain why

(i) in a sound wave a displacement node is a pressure antinode and vice- versa

(ii) The shape of pulse gets- distorted during propagation in a dispersive medium.

Ans. (i) At a displacement node the variations of pressure is maximum. Hence displacement node is the a pressure antinode and vice-versa.

(ii)When a pulse passes through a dispersive medium the wavelength of wave changes.

So, the shape of pulse changes i.e. it gets distorted.

(3 Marks Questions)

1. The speed of longitudinal wave `V` in a given medium of density ρ is given by the formula, use this formula to explain why the speed of sound in air.

   (a) is independent at pressure

   (b) increases with temperature and

   (c) increases with humidity

2. Write any three characteristics of stationary waves.

Ans. (i) in stationary waves, the disturbance does not advance forward. The conditions of crest and trough merely appear and disappear in fixed position to be followed by opposite condition after every half time period. (ii) The distance between two successive nodes or antinodes is equal to half the wavelength. (iii) The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

3. Show that the speed of sound in air increased by .61m/s for every 1^0 C rise of temperature.

Ans. $V \alpha \sqrt{T}$

$$\frac{Vt}{Vo} = \sqrt{\frac{t+273}{0+273}}$$

$V_t = V_0 \, (1 + \frac{t}{273})^{1/2} = V_0 \, (1 + \frac{1}{2} . \frac{t}{273})$

$V_t = V_0 + \frac{V \circ \times t}{546}$

At, 0^0C speed of sound in air is 332 m/s.

$\therefore V_t - V_0 = \frac{332 \times t}{546}$

When t= 1^0C , $V_t - V_0$ = 0.61m/s.

4. Find the ratio of velocity of sound in hydrogen gas $\Upsilon = \frac{7}{5}$ to that in helium gas $\Upsilon = \frac{5}{3}$ at the same temperature. Given that molecular weight of hydrogen and helium are 2 and 4 respectively.

Ans. $V = \sqrt{\frac{\gamma RT}{M}}$

At constant temperature,

$\frac{V_H}{V_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \frac{M_H}{M_{He}}} = \sqrt{\frac{715}{5/3}} . \frac{4}{2}$ = 1.68.

5. The equation of a plane progressive wave is, $y = 10Sin2\pi(t - 0.005x)$ where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, y = $10Sin2\pi(t - 0.005x)$………….. (1)

Standard equation for harmonic wave is, $y = A \, Sin2\pi(\frac{t}{T} - \frac{x}{\lambda})$…………… (2)

Comparing eqn (1) & (2), $A = 10, \ \frac{1}{t} = 1, \ \frac{1}{\lambda} = 0.005$

(i) Amplitude A = 10cm

(ii) Frequency $\nu = \frac{1}{T} = 1Hz$

(iii) Wavelength $\lambda = \frac{1}{0.005} = 200cm$

(iv) Velocity v = $\nu\,\lambda$ = 1 x 200 = 200cm/s

6. Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. Y = A Sin (2π$\nu t + \phi_o)$

= 0.01 Sin $(1200\pi t + \frac{\pi}{6})$

7. The amplitude of oscillations of two similar pendulums similar in all respect are 2cm & 5cm respectively. Find the ratio of their energies of oscillations.

Ans. $\frac{E_1}{E_2} = (\frac{A_1}{A_2})^2 = \left(\frac{2}{3}\right)^2 = 4:25$

8. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

Ans. A periodic motion repeats after a definite time interval T. So,

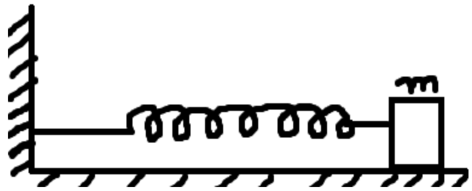
$$y(t) = y(t+T) = y(t+2T)\ etc.$$

9. A spring of force constant 1200N/m is mounted horizontal table. A mass of 3Kg is attached to the free end of the spring, pulled sideways to a distance of 2.0cm and released.

(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?



Ans. Here k = 1200N/m, m = 3Kg, A = 2cm = 2×0.01m

(i) $\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2} \times \frac{1}{3.14}\sqrt{\frac{1200}{3}} = 3.2s^{-1}$

(ii) $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1200}{3}} = 20s^{-1}$

Maximum acceleration = $\omega^2 A = (20)^2 \times 2 \times 10^{-2} = 8m/s^2$

(iii) Maximum speed = $\omega A = 20 \times 2 \times 10^{-2} = 0.40m/s$

10. Which of the following function of time represent, (a) simple harmonic (b) periodic but not SHM and (c) non periodic ?

(i) Sinωt- Cosωt (ii) $Sin^3\omega t$ (iii) $3Cos(\frac{\pi}{2} - 2\omega t)(iv)\exp(-\omega^2 t^2)$

Ans. (i) $x(t) = Sin\omega t - Cos\omega t = \sqrt{2Sin(\omega t - \frac{\pi}{2})}$, so the function is in SHM.

(ii) $x(t) = Sin^3\omega t = \frac{1}{4}(3Sin\omega t - Sin3\omega t)$, represent two separate SHM motion but their combination does not represent SHM.

(iii) $x(t) = 3Cos\left(\frac{\pi}{4} - 2\omega t\right) = 3Cos(2\omega t - \frac{\pi}{4})$, represent SHM.

(iv) $\exp(-\omega^2 t^2) = non\ periodic.$

(5 Marks Questions)

1. (a) A light wave is reflected from a mirror. The incident & reflected wave superimpose to form stationary waves. But no nodes & antinodes are seen, why?

(b) A standing wave is represented by y=2ASinKxCoswt.If one of the component wave is $y_1 = ASin(\omega t - Kx)$, what is the equation of the second component wave?

Ans. (a) As is known, the distance between two successive nodes or two successive antinodes is $\frac{\lambda}{2}$. The wavelength of visible light is of the order of $10^{-7}m$. As such as a small distance cannot be detected by the eye or by a ordinary optical instrument. Therefore, nodes and antinodes are not seen.

(b) As, $2SinACosB = Sin(A+B) + Sin(A-B)$

$$y = 2ASinKxCos\omega t$$

$$= ASin(Kx+\omega t) + ASin(Kx-\omega t)$$

According to superposition principle,

$$y = y_1 + y_2$$

$$and\ y_1 = ASin(\omega t - Kx) = -ASin(Kx - \omega t)$$

$$y_2 = y - y_1 = 2ASinKxCos\omega t + ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) + 2ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) - 2ASin(\omega t - Kx)$$

2. Discuss Newton's formula for velocity of sound in air. What correction was made to it by Laplace and why?

Ans. According to Newton the change in pressure & volume in air is an isothermal process. Therefore he calculated, $v = \sqrt{\frac{p}{\rho}}$ on substituting the require value he found, the velocity of sound was not in close agreement with the observation value. Then Laplace pointed out the error in Newton's formula. According to Laplace the change in pressure and volume is an adiabatic process. So he calculated the value of sound as, $v = \sqrt{\frac{\Upsilon r}{\rho}}$ on putting require value he found velocity of sound as 332m/s very closed to observed theory.

3. (a) What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing wave.

(b) Draw fundamental nodes of vibration of stationary wave in (i) closed pipe, (ii) in an open pipe.

4. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4.

5. Explain Doppler's effect of sound. Derive an expression for the apparent frequency where the source and observer are moving in the same direction with velocity Vs and Vo respectively, with source following the observer.

[Ans. = $\nu^{|} = \frac{v-v_o}{v-v_s} * \nu$]

6. For a travelling harmonic wave, $y = 2Cos(10t - 0.008x + 0.35)$ where x & y are in cm and t in second. What is the phase difference between oscillatory motions at two points separated by a distance of (i) 4cm (ii) 0.5m (iii)$\frac{\lambda}{2}$ $(iv)\frac{3\lambda}{4}$?

Ans.$y = 2Cos(10t - 0.008x + 0.35) \dots\dots\dots (i)$

We know, $y = ACos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right) \dots\dots\dots (ii)$

From (i) & (ii), $\frac{2\pi}{\lambda} = 0.008, \lambda = \frac{2\pi}{0,008} cm = \frac{2\pi}{0.80} m.$

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda} * path\ difference = \frac{2\pi}{\lambda} * \Delta x.$

(i) When $\Delta x = 4cm$, $\Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 4 = 3.2rad$.

(ii) When$\Delta x = 0.5m$, $\Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 0.5 = 0.40rad.$

(iii) When $\Delta x = \frac{\lambda}{2}$, $\Delta\phi = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi rad.$

(iv) When$\Delta x = \frac{3\lambda}{4}$, $\Delta\phi = \frac{2\pi}{\lambda} * \frac{3\lambda}{4} = \frac{3\pi}{2} rad.$

7. **(i)** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly exited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).

Ans. (i) For the fundamental mode,

λ = 2 L = 2 x 100 = 200 cm = 2m.

Frequency v = 2.53 kHz = 2530 Hz

Speed of sound, v = vλ = 2530 x 2 = 5060 m/s

=5.06 km/s

(ii) Length of pipe L= 20 cm = 0.2 m

Speed of sound v= 340 m/s

Fundamental frequency of closed organ pipe

$v = v/4L = \frac{340}{4\times 0.2} = 425$ Hz sw can be excited

Fundamental frequency of open organ pipe

$v' = \frac{v}{2L} = \frac{340}{2\times 0.2} = 850$ Hz

Hence source of frequency 430 Hz will not be in resonance with open organ pipe.

8. A train stands at a platform blowing a whistle of frequency 400 Hz in still air.

(i) What is the frequency of the whistle heard by a man running

(a)Towards the engine 10 m/s.

(b) Away from the engine at 10 m/s?

(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound received by the running man in each case?

Take speed of sound in still air = 340 m/s.

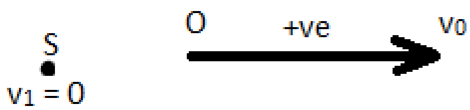
Ans.(i) (a) When the man runs towards the engine

V_0 = -10 m/s, v_1= 0

$$v' = \frac{v - v_0}{v - v_s} \times v = \frac{340+10}{340-0} \times 400 = \frac{350}{340} \times 400 = 411.8$$

(b) When the man runs away from the engine

V_0 = + 10 m/s, $v_s = 0$



$$v'' = \frac{v - v_0}{v - v_s} \times v = \frac{340-10}{340-0} \times 400 = \frac{330}{340} \times 400 = 388.2 \text{ Hz}$$

(ii) (a) When the man runs towards the engine , relative velocity of sound

$v' = v + v_0 = 340 + 10 = 350$ m/s

(b) When the man runs away from the engine, relative velocity of sound

$v' = v - v_0 = 340 - 10 = 330$ m/s .

(iii) The wavelength of sound is not affected by the motion of the listener. Its value is

$$\lambda = \frac{v}{v} = 340/400 = 0.85\text{m}$$

9. What is a spring factor? Derive the expression for resultant spring constant when two springs having constants k_1 and $k_2$are connected in (i) parallel and (ii) in series.

10. Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation is equal to the average potential energy over the same period. At what distance from the mean position is the kinetic energy in simple harmonic oscillator equal potential energy?