Subject Name: Antennas & Propagation

Topic Name: Antenna Arrays

(Unit – 2)

Prepared

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Syllabus / Antenna Arrays

1. Broad side arrays
2. End fire arrays
3. Collinear arrays

Example of Antenna Arrays:

Yagi Uda Arrays

Aim and Objective:

- To give insight of basic Knowledge of Antenna Arrays
- To give thorough understanding of the radiation characteristics of types of Antenna Arrays i.e Broad side array, End fire array and Collinear arrays.
- To impart knowledge about applications of Antenna Arrays

Pre – Test MCQ:

1. An antenna is a transitional structure between –
   (a) Free space and guiding element
   (b) Free space and free space
   (c) Guiding element and another guiding element
   (d) All of the above

Answer - a

2. The guiding device used for the antenna system –
   (a) Transmission line
   (b) Co-axial line
   (c) Waveguide
   (d) All of the above

Answer - d

3. If the radiation from an antenna is represented in terms of field strength, it is called –
   (a) Field pattern
   (b) Power pattern
   (c) Radiation pattern
   (d) None of the above

Answer - a

4. A major lobe is defined as the radiation lobe containing –
   (a) Direction of minimum radiation
   (b) Direction of maximum radiation
   (c) Direction of moderate radiation
   (d) None of the above

Answer - b

5. The First-null beam width (FNBW) is defined as the angular measurement between the directions –
   (a) radiating the maximum power
   (b) radiating half of the maximum power

(c) radiating no power
(d) None of the above

Answer - c

6. The half-power beam width (HPBW) is defined as the angular measurement between the directions –
(a) radiating the maximum power
(b) radiating half of the maximum power
(c) radiating no power
(d) None of the above

Answer - b

7. The x-z plane (elevation plane; ϕ = 0) is the principal –
(a) $\boldsymbol{E}$-plane
(b) H-plane
(c) Either E-plane or H-plane
(d) None of the above

Answer - a

8. An Omni directional antenna is a special antenna of type –
(a) Directional
(b) Non-directional
(c) Isotropic
(d) None of the above

Answer - a

9. The ratio of the main beam area to the total beam area is called –
(a) beam efficiency
(b) beam deficiency
(c) stray factor
(d) None of the above

Answer - a

10. Radiation density is defined as power per –
(a) Unit area
(b) Unit angle
(c) Unit field
(d) None of the above

Answer - a

11. Radiation intensity is defines as power per -
    (a) Unit area
    (b) Unit angle
    (c) Unit field
    (d) None of the above

Answer - b

12. The radiation intensity can be obtained by multiplying the radiation density by –
    (a) The distance
    (b) Square of the distance
    (c) Cube of the distance
    (d) None of the above

Answer - b

13. Directivity is defined as the ratio of the maximum radiation intensity to –
    (a) average radiation intensity
    (b) peak radiation intensity
    (c) unity radiation intensity
    (d) None of the above

Answer - a

14. The gain of an antenna is an actual quantity which is less than directivity (D) due to –
    (a) Radiating losses in the antenna
    (b) ohmic losses in the antenna
    (c) inductive losses in the antenna
    (d) All of the above

Answer - b

15. In an antenna system, the maximum power transfer takes place when the antenna is ___________to the source.
    (a) Perfectly matched
    (b) Inductively matched
    (c) Conjugate matched
    (d) None of the above

Answer - c

Pre-requisite

- Basic knowledge of Electromagnetic Fields and Wave guides.
- Basic Knowledge of Antenna Parameters and Radiations

UNIT II: ANTENNA ARRAYS

N element linear array, Pattern multiplication, Broadside and End fire array – Concept of Phased arrays, Adaptive array, Basic principle of antenna Synthesis-Binomial array, Yagi Arrays.

INTRODUCTION – ARRAY ANTENNAS:

- The radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications, it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna.

  Note: Higher directivity is the basic requirement in point-to-point communication, radars and space applications.

- Enlarging the dimensions of single elements often leads to more directive characteristics. Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration*. This new antenna, formed by multielements, is referred to as an array*. Inmost cases, the elements of an array are identical. The individual elements of an array may be of any form (wires, apertures, etc.).
- Thus antenna array can be defined as the system of similar antennas directed to get required high directivity in the desired direction.
- The total field of the array is determined by the vector addition of the fields radiated by the individual elements. The individual element is generally called *element of an antenna array*. This assumes that the current in each element is the same as that of the isolated element (neglecting coupling).
- The antenna array is said to be *linear* if the elements of the antenna array are equally spaced along a straight line. The linear antenna array is said to be *uniform linear* array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line.
- In an array of identical elements, there are at least five controls that can be used to shapethe overall pattern of the antenna. These are:
  - the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
  - the relative displacement between the elements
  - the excitation amplitude of the individual elements
  - the excitation phase of the individual elements
  - the relative pattern of the individual elements

- Practically various forms of the antenna array are used as radiating systems. They are;
  - i. Broadside Array (BSA)
  - ii. End-Fire Array (EFA)
  - iii. Collinear Array
  - iv. Parasitic Array

⇨ Broadside Array (BSA)

- The broadside array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane consisting elements.A typical arrangement of a Broadside array is as shown in Fig. 3-1.
- A broadside array consists number of identical antennas placed parallel to each other along a straight line. This straight line is perpendicular to the axis of individual antenna. It is known as *axis of antenna array.* Thus each element is perpendicular to the axis of antenna array.
- All the individual antennas are spaced equally along the axis of antenna array. The spacing between any two elements is denoted by $'d'$. All the elements are fed with currents with equal magnitude and same phase. As the maximum radiation is directed in broadside direction i.e. perpendicular to the line of axis of array, the radiation pattern for the broadside array is bidirectional.
- Thus broadside array can be defined as the arrangement of antennas in which maximum radiation is in the direction perpendicular to the axis of array and plane containing the elements of array.

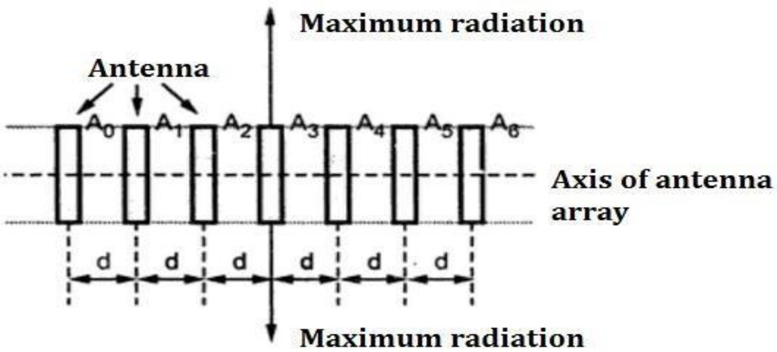


Fig. 3-1 Broadside array

⇨ End-Fire Array (EFA)

- The end fire array is very much similar to the broadside array from the point of view of arrangement. But the main difference is in the direction of maximum radiation. In broadside array, the direction of the maximum radiation is perpendicular to the axis of array; while in the end fire array, the direction of the maximum radiation is along the axis of array.

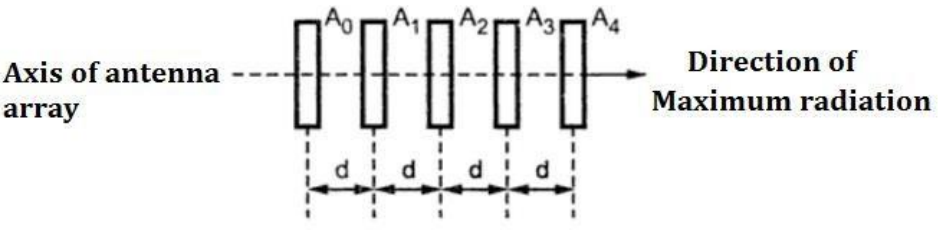


Fig. 3-2 End-fire array

- Thus in the end fire array number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to get entire arrangement unidirectional finally. i.e. maximum radiation along the axis of array as shown in Fig. 3-2.

- Thus end fire array can be defined as an array with direction of maximum radiation coincides with the direction of the axis of array to get unidirectional radiation.

⇨ **Collinear array**

- As the name indicates, in the collinear array, the antennas are arranged co-axially i.e. the antennas are arranged end to end along, a single line as shown in Fig. 3-3 (a) and (b).

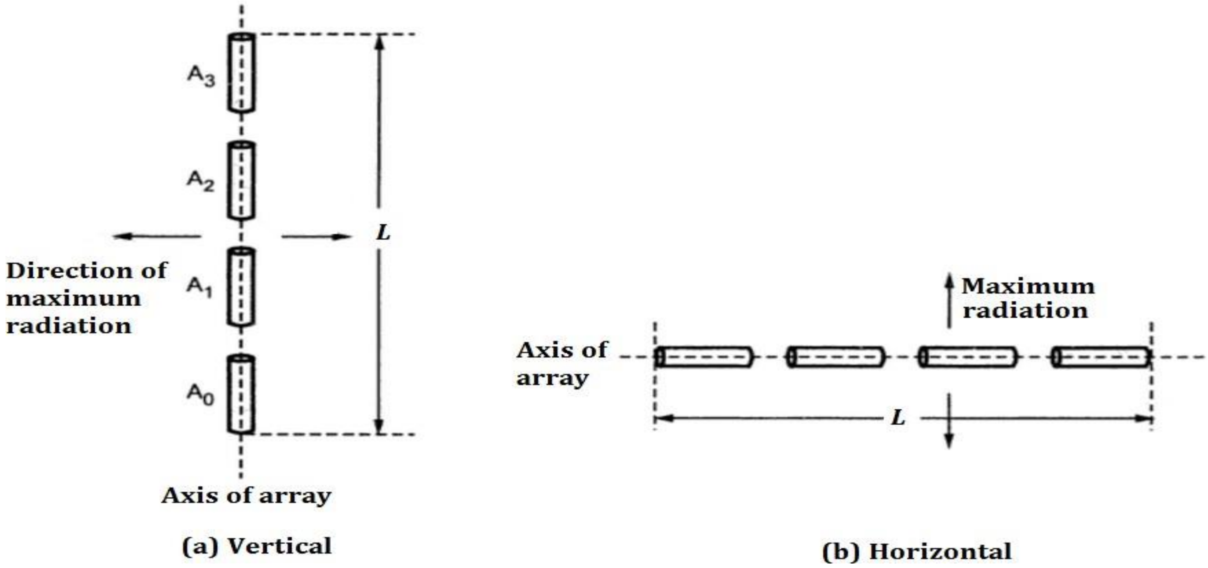


Fig. 3-3 Collinear array

- The individual elements in the collinear array are fed with currents equal in magnitude and phase. This condition is similar to the broadside array. In collinear array the direction of maximum radiation is perpendicular to the axis of array.
- So the radiation pattern of the collinear array and the broadside array is very much similar but the radiation pattern of the collinear array has circular symmetry with main lobe perpendicular everywhere to the principle axis. Thus the collinear array is also called omni directional array or broadcast array.
- The gain of the collinear array is maximum if the spacing between the elements is of the order of $0.3\ \lambda$ to $0.5\ \lambda$.

⇨ **Parasitic array**

- In order to overcome feeding problems of the antenna, sometimes, the elements of the array are fed through the radiation from the nearby element. The array of antennas in which the parasitic elements get the power through electromagnetic coupling with driven element which is in proximity with the parasitic element is known as parasitic array.
- The simplest form of the parasitic array consists one driven element and one parasitic element. In multielement parasitic array, there may be one or more driving elements and also one or more parasitic elements. So in general the multielement parasitic array is the array with at least one driven element and one or more parasitic elements.
- The common example of the parasitic array with linear half wave dipoles as elements of array is Yagi-Uda array or simply Yagi antenna.
- The amplitude and the phase of the current induced in the parasitic element depends on the spacing between the driven element and parasitic element. To make the radiation pattern

unidirectional, the relative phases of the currents are changed by adjusting the spacing between the elements. This is called tuning of array. For a spacing between the driven and parasitic element equal to $\lambda/4$ and phase difference of $\pi/2$ radian, unidirectional radiation pattern is obtained.

ARRAY OF POINT SOURCES

- The array of point sources is nothing but the array of an isotropic radiators occupying zero volume. For the greater number of point source in the array, the analysis of antenna array becomes complicated and time consuming. Also the simplest condition of number of point sources in the array is two. Then conveniently analysis is done by considering first two point sources, which are separated by distance d and having same polarization. The results obtained for only two point sources can be further extended for $'n'$ number of point sources in the array.
- Let us consider the array of two isotropic point sources, with a distance of separation $'d'$ between them. The polarization of two isotropic point sources is assumed to be the same. To derive different expressions following conditions can be applied to the antenna array ;
    1. Two point sources with currents of equal magnitudes and with same phase.
    2. Two point sources with currents of equal magnitude but with opposite phase.
    3. Two point sources with currents of unequal magnitudes and with any phase.

⇨ **Two Point Sources with Currents Equal in Magnitude and Phase:**

- Consider two point-sources 1 and 2 separated by distance $'d'$ and both the point sources are supplied with currents equal in magnitude and phase as shown in Fig. 3-4.
- Let point P far away from the array and the distance between point P and point sources 1 and 2 be r_1 and r_2 respectively.

  Assuming far-field observations ,

$$r_1 \approx r_2 \approx r \qquad \text{-------- (3.1)}$$

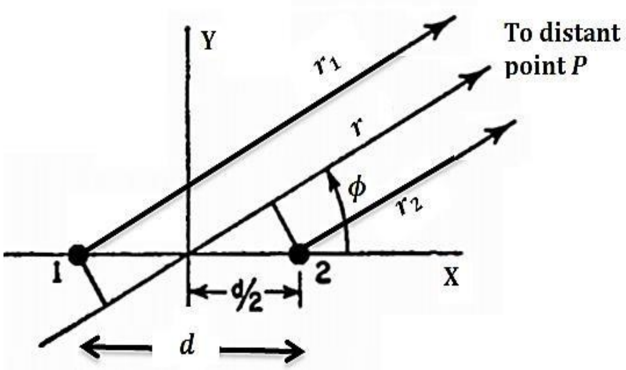


Fig. 3-4 Two element array

- The radiation from the point source 2 will reach earlier at point P than that from point source 1 because of the path difference. The extra distance is travelled by the radiated wave from point source 1 than that by the wave radiated from point source 2. Hence path difference is given by,

$$Path\ difference = \frac{d}{2}\cos\phi + \frac{d}{2}\cos\phi = d\cos\phi \quad \text{-------- (3.2)}$$

- The path difference can be expressed in terms of wavelength as ;

$$Path\ difference = \frac{d}{\lambda}\cos\phi$$

Hence the phase difference $'\psi'$ is given by ;

$$Phase\ difference\ \psi = 2\pi \times Path\ difference$$

$$\psi = 2\pi \times \frac{d}{\lambda}\cos\phi = \frac{2\pi}{\lambda} d\cos\phi$$

$$\psi = kd\cos\phi \quad \text{-------- (3.3)}$$

$k = \frac{2\pi}{\lambda}$

- Let $E_1 = E_0.e^{-j\frac{\psi}{2}}$ is field component due topoint source 1. Similarly, let $E_2 = E_0.e^{j\frac{\psi}{2}}$ is field component due to point source 2. Therefore, the total far-field at a distant point P is ;

$$E_T = E_1 + E_2 = E_0.e^{-j\frac{\psi}{2}} + E_0.e^{j\frac{\psi}{2}}$$

$$E_T = E_0\left(e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right) = 2E_0\cos\frac{\psi}{2} \quad \text{-------- (3.4)}$$

Note that the amplitude of both the field components is E_0 as currents are same and the point sources are identical.

Substituting value of ψ from Eqn. (3.3), we get,

$$E_T = 2E_0\cos\left[\frac{kd\cos\phi}{2}\right] \quad \text{-------- (3.5)}$$

- Above equation represents total field intensity at point P, due to two point sources having currents of same amplitude and phase. The total amplitude of the field at point P is $2E_0$ while the phase shift is $(kd\cos\phi)/2$. By putting $2E_0 = 1$, then the pattern is said to be normalized.

⇨ **<u>Maxima direction:</u>**

- From Eqn. (3.5), the total field is maximum when $\cos\left[\frac{kd\cos\phi}{2}\right]$ is maximum. Maximum value of cosine function is ±1. Hence the condition for maxima is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = \pm 1 \quad \text{-------- (3.6)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$k = \frac{2\pi}{\lambda}\ ;\ d = \frac{\lambda}{2}$

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = \pm 1$$

$$i.e,\quad \frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi,\ where,\ n = 0,1,2,....$$

If $n = 0$, then ; $\frac{\pi}{2}\cos\phi_{max} = 0$

$$\cos\phi_{max} = 0 \quad ; \quad \phi_{max} = 90°\ \text{or}\ 270° \text{--------------- (3.7)}$$

⇨ **<u>Minima direction:</u>**

- From Eqn. (3.5), the total field is minimum when $\cos\left[\frac{kd\cos\phi}{2}\right]$ is minimum. Minimum value of cosine function is 0. Hence the condition for maxima is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = 0 \qquad \text{-------- (3.8)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = 0$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{mi} = \cos^{-1}(0) = \pm(2n+1)\ \frac{\pi}{2}, where, n = 0,1,2, \ldots.$$

If $n = 0$, then ; $\quad \frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$

$$\cos\phi_{min} = \pm 1 \quad ; \quad \phi_{min} = 0° \text{ or } 180° \qquad \text{-------- (3.9)}$$

⇨ **<u>Half power point directions:</u>**

- When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

$$\cos\left[\frac{kd\cos\phi}{2}\right] = \pm\frac{1}{\sqrt{2}} \qquad \text{-------- (3.10)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\cos\left[\frac{\pi}{2}\cos\phi\right] = \pm\frac{1}{\sqrt{2}}$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm\frac{\pi}{4}, where, n = 0,1,2, \ldots.$$

If $n = 0$, then ; $\quad \frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$

$$\cos\phi_{HPPD} = \pm\frac{1}{2} \quad ; \quad \phi_{HPPD} = \pm 60° \text{ or } \pm 120° \qquad \text{-------- (3.11)}$$

- The field pattern drawn with E_T against ϕ for $d = \lambda/2$, then the pattern is bidirectional as shown in Fig. 3-5. The field pattern obtained is bidirectional and it is a figure of eight (8). If this patterns is rotated by 360° about axis, it will represent three dimensional doughnut shaped space pattern.

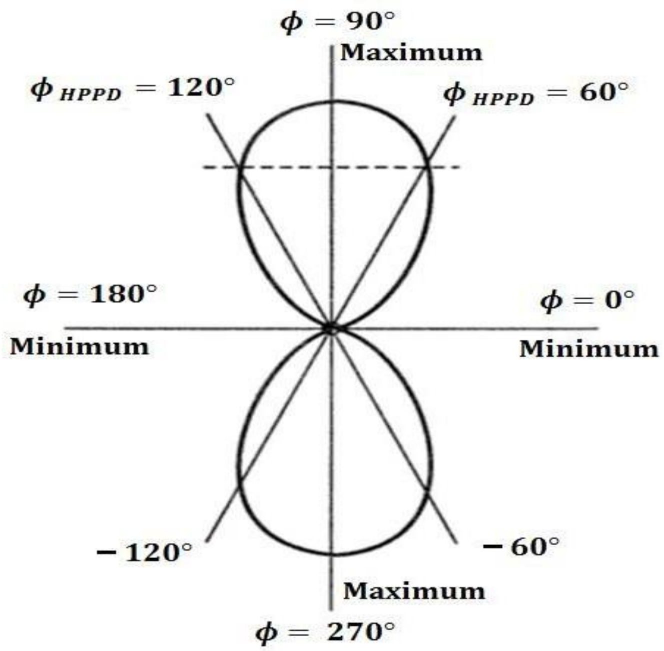


Fig. 3-5 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and phase.

⇨ **<u>Two Point Sources with Currents Equal in Magnitude and opposite phase:</u>**

- Consider two point sources separated by distance $'d'$ and supplied with currents equalin magnitude but opposite phase. Consider Fig.3-4, all theconditions are exactly same except the phase of the currents is opposite i.e. 180°. Withthis condition, the total field at far point P is given by,

$$E_T = (-E_1) + E_2 \qquad \text{-------- (3.12)}$$

- Assuming equal magnitudes of currents, the fields at point P due to the point sources 1 and 2 can be written as ; $E_1 = E_0 . e^{-j\frac{\psi}{2}}$

$$E_2 = E_0 . e^{j\frac{\psi}{2}}$$

- Therefore, the total far-field at a distant point P is ;

$$E_T = (-E_1) + E_2 = -E_0 . e^{-j\frac{\psi}{2}} + E_0 . e^{j\frac{\psi}{2}}$$

$$E_T = E_0 \left(e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}\right) = j(2E_0) \sin\frac{\psi}{2} \qquad \text{-------- (3.13)}$$

- Now as the condition for two point sources with currents in phase and out of phaseis exactly same, the phase angle can be written as ;

$$\psi = kd \cos\phi \qquad \text{-------- (3.14)}$$

Substituting value of ψ in Eqn. (2.93), we get,

$$E_T = j(2E_0) \sin\left[\frac{kd\cos\phi}{2}\right] \qquad \text{-------- (3.15)}$$

By putting $(2E_0) = 1$, then the pattern is said to be normalized.

Maxima direction:

- From Eqn. (3.15), the total field is maximum when $\sin[\frac{kd\cos\phi}{2}]$ is maximum. Maximum value of sine function is ±1. Hence the condition for maxima is given by,

$$\sin[\frac{kd\cos\phi}{2}] = \pm 1 \qquad \text{-------- (3.16)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$k = \frac{2\pi}{\lambda} \; ; d = \frac{\lambda}{2}$$

$$\sin[\frac{\pi}{2}\cos\phi] = \pm 1$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{max} = \sin^{-1}(\pm 1) = \pm(2n+1)\frac{\pi}{2}, where, n = 0,1,2, ...$$

$$\text{If } n = 0, \text{ then ;} \quad \frac{\pi}{2}\cos\phi_{mi} = \pm\frac{\pi}{2}$$

$$\cos\phi_{max} = \pm 1 \quad ; \quad \phi_{ma} = 0° \text{or } 180° \text{--------------- (3.17)}$$

Minima direction:

- From Eqn. (3.15), the total field is minimum when $\sin[\frac{kd\cos\phi}{2}]$ is minimum. Minimum value of sine function is 0.Hence the condition for maxima is given by,

$$\sin[\frac{kd\cos\phi}{2}] = 0 \qquad \text{-------- (3.18)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\sin[\frac{\pi}{2}\cos\phi] = 0$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{min} = \sin^{-1}(0) = \pm n\pi, where, n = 0,1,2, ...$$

$$\text{If } n = 0, \text{ then ;} \quad \frac{\pi}{2}\cos\phi_{min} = 0$$

$$\cos\phi_{min} = 0 \quad ; \quad \phi_{min} = 90° \text{ or } 270° \quad \text{-------- (3.19)}$$

Half power point directions:

- When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

$$\sin[\frac{kd\cos\phi}{2}] = \pm\frac{1}{\sqrt{2}} \qquad \text{-------- (3.20)}$$

- Let spacing between the two point sources be $\lambda/2$, then ;

$$\sin[\frac{\pi}{2}\cos\phi] = \pm\frac{1}{\sqrt{2}}$$

$$i.e, \quad \frac{\pi}{2}\cos\phi_{HPPD} = \sin^{-1}(\pm\frac{1}{\sqrt{2}}) = \pm\frac{\pi}{4}, where, n = 0,1,2, ...$$

If $n = 0$, then ; $\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}; \qquad \phi_{HPPD} = \pm 60° \text{ or } \pm 120° \qquad \text{-------- } (3.21)$$

- Thus from the conditions of maxima, minima and half power points, the field pattern can be drawn with E_T against ϕ for $d = \lambda/2$ as shown in Fig. 3-6.

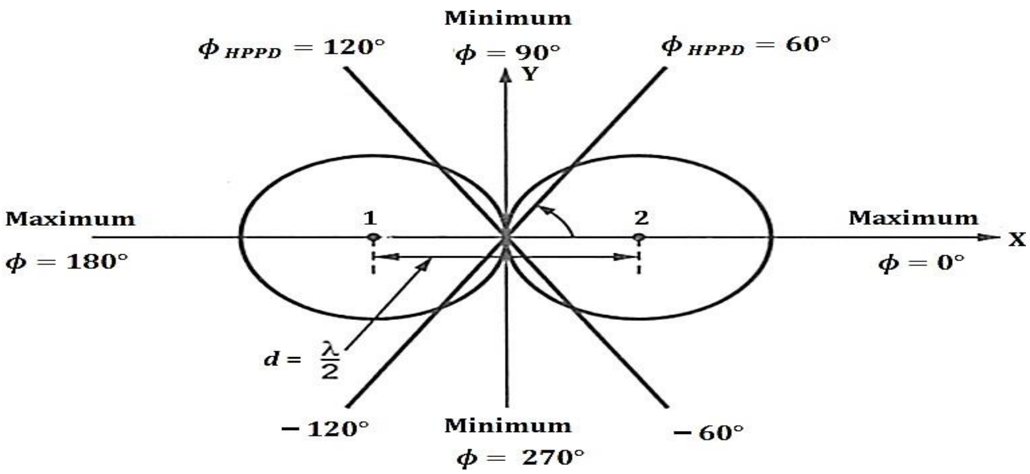


Fig. 3-6 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and out of phase.

⇨ **<u>Two Point Sources with Currents Unequal in Magnitudeand with any Phase:</u>**

- Let us consider, two point sources are separated by distance d and supplied with currents which are different in magnitudes and with any phase difference say α, as shown in Fig. 3-7 (a).
- Assume that source 1 is taken as reference for phase. The amplitude of the fields due to source 1 and source 2 at the distant point P is E_1 and E_2 respectively, in which E_1 is greater than E_2, as shown in the vector diagram in Fig. 3-7 (b).

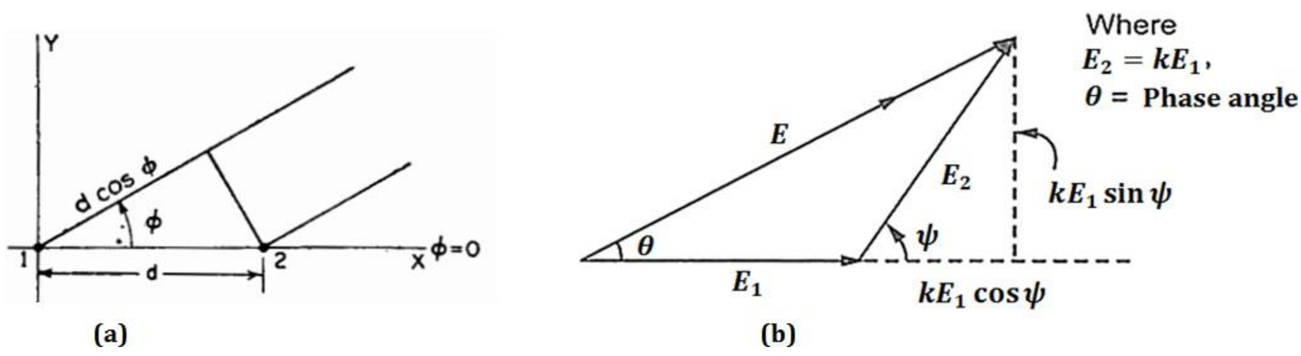


Fig. 3-7 (a) Two point sources with currents unequal in magnitude and with any phase (b) Vector diagram

- Now the total phase difference between the radiations by the two point sources at any far point P is given by,

$$\psi = kd \cos \phi + \alpha \qquad \text{-------- (3.22)}$$

where α is the phase angle with which current I_2 leads current I_1.

Then the resultant field at point P is given by,

$$E_T = E_1 e^{j0} + E_2 e^{j\psi} \qquad \text{-------- (3.23)}$$

(Source 1 is assumed to be reference hence phase angle is 0)

$$E_T = E_1 + E_2 e^{j\psi} = E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

Let $\frac{E_2}{E_1} = k$ $\qquad E_T = E_1[1 + k(\cos \psi) + j \sin \psi]$ -------- (3.24)

Note that $E_1 > E_2$, the value of k is less than unity and varies from $0 \leq k \leq 1$.

- The magnitude and phase angle of the resultant field at point P is given by,

$$|E_T| = E_1\sqrt{(1 + k \cos \psi)^2 + j(k \sin \psi)^2} \qquad \text{-------- (3.25)}$$

$$\theta = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi} \qquad \text{-------- (3.26)}$$

N- ELEMENT UNIFORM LINEAR ARRAY:

- An array of N elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line.
- Consider uniform linear array of $\boldsymbol{N}$ isotropic point sources with all the individual elements spaced equally at distance $'d'$ from each other and all elements are fed with currents equal in magnitude and uniform progressive phase shift along line as shown in Fig. 3-8.

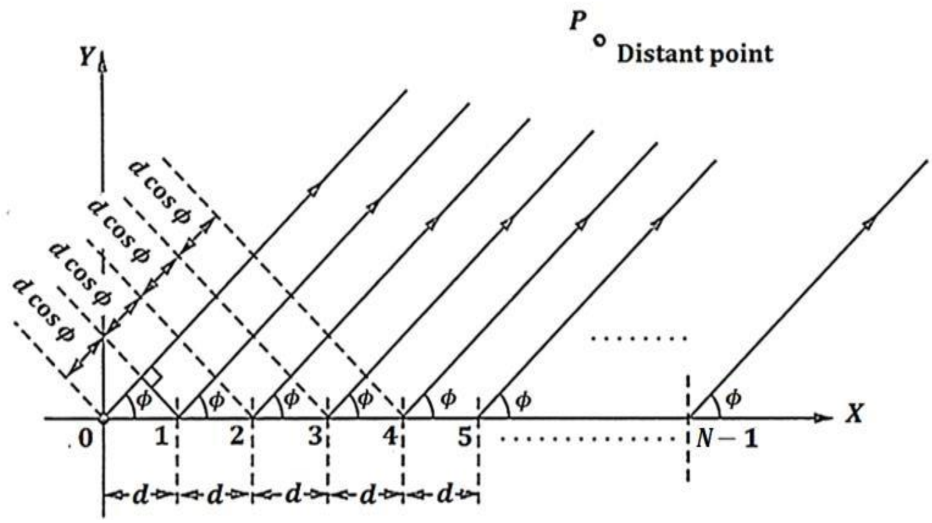


Fig. 3-8 Uniform linear array of N elements

- The total resultant field at the distant point P is obtained by adding the fields due to N individual sources vectorically. Hence,

$$E_T = E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots\cdots\cdots + E_0 e^{(N-1)j\psi}$$

$$E_T = E_0(1 + e^{j\psi} + e^{2j\psi} + \cdots\cdots\cdots + e^{(N-1)j\psi}) \quad \text{-------- (3.27)}$$

- Note that $\psi = kd\cos\phi + \alpha$ indicates the total phase difference of the fields from adjacent sources calculated at point . Simlarly α is the progressive phase shift betweentwo adjacentpoint sources. The value of α may lie between 0°and 180°. If $\alpha = 0°$ we get N element uniform linear broadside array. If $\alpha = 180°$, we get N element uniformlinear end firearray.

  Multiply by $e^{j\psi}$ on both sides ;

$$E_T e^{j\psi} = E_0(e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots\cdots\cdots + e^{Nj\psi}) \quad \text{-------- (3.28)}$$

- Subtract Eqn. (3.27) and (3.28) ;

$$E_T(1 - e^{j\psi}) = E_0(1 - e^{Nj\psi})$$

$$\frac{E_T}{E_0} = \frac{(1 - e^{Nj\psi})}{(1 - e^{j\psi}} = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right] \quad \text{-------- (3.29)}$$

- If the reference point is the physical center of the array, then Eqn. (2.94)reduces to ;

$$\frac{E_T}{E_0} = AF = \left[\frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}\right]$$

  which is the ***antenna array factor.***

- If $\psi = 0$, the maximum value of E_T is determined using L'Hospital's rule ;

$$E_{T\,ma} = E_0\, N$$

- Thus the maximum value of E_T is N times the field from a single source. To normalize the field pattern , so that the maximum value of each is equal to unity. The normalized field pattern is given by ;

$$(E_T)_N = \frac{E_T}{E_{T\,max}} = \left[\frac{\sin(\frac{N}{2}\psi)}{N\,\sin(\frac{1}{2}\psi)}\right] \quad \text{-------- (3.30)}$$

- Eqn. (3.30) is the normalized array factor ;

$$(AF)_N = \left[\frac{\sin(\frac{N}{2}\psi)}{N\sin(\frac{1}{2}\psi)}\right] \quad \text{-------- (3.31)}$$

BROADSIDE ARRAY (BSA)

- An array is said to be broadside array, if maximum radiation occurs in direction perpendicular to array axis.
- In broadside array, individual elements are equally spaced along a line and each element is fed with current of equal magnitude and same phase.
- The total phase difference of the fields at point P from adjacent sources is given by,

$$\psi = kd\cos\phi + \alpha \qquad \text{-------- (3.32)}$$

- The normalized array factor for $'N'$ elements ;

$$(AF)_N = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{1}{2}\psi\right)}\right] \qquad \text{-------- (3.33)}$$

Major lobe

- In broadside array sources should be in phase i.e., $\alpha = 0°$ and $\psi = 0$ for maximum must be satisfied.

$$\psi = kd\cos\phi + \alpha = 0$$

$$kd\cos\phi + \alpha = 0 \qquad \because \alpha = 0$$

$$\cos\phi = 0 \quad ; \qquad \phi_m = 90° \text{ or } 270°$$

Nulls

- To find the nulls of the array Eqn. (3.33) is set to zero ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 0 \quad ⇨ \quad \frac{N}{2}\psi\Big|_{\phi=\phi_n} = \pm n\pi$$

For BSA $\alpha = 0°$

$$\frac{N}{2}(kd\cos\phi_n + \alpha) = \pm n\pi \quad ⇨ \quad \phi_n = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right) \qquad \text{-------- (3.34)}$$

where ; $n = 1, 2, 3 \ldots$

Maxima of minor lobes (secondary maxima)

- The maximum value of Eqn. (3.33) occur when ;

$$k = \frac{2\pi}{\lambda}$$

$$\sin\left(\frac{N}{2}\psi\right) = 1 \quad ⇨ \quad \frac{N}{2}\psi\Big|_{\phi=\phi_s} = \pm(2s+1)\frac{\pi}{2}$$

$$\frac{N}{2}(kd\cos\phi_s + \alpha) = \pm(2s+1)\frac{\pi}{2}$$

$$\phi_s = \cos^{-1}\left\{\frac{1}{kd}\left[\pm\frac{(2s+1)\pi}{N} - \alpha\right]\right\}$$

For BSA $\alpha = 0°$

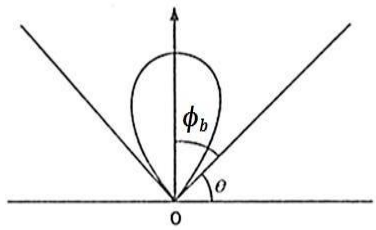
$$\phi_s = \cos^{-1}\left\{\frac{1}{kd}\left[\pm\frac{(2s+1)\pi}{N}\right]\right\} \qquad s = 1,2,3,\ldots$$

$$\phi_s = \cos^{-1}\left[\pm\frac{(2s+1)\lambda}{2Nd}\right] \qquad \text{-------- (3.35)}$$

Beamwidth of major lobe

- Beamwidth is defined as angle between first null and maximum of major lobe (or) Beamwidth is the angle equal to twice the angle between first null and the major lobe maximum. $BWFN = 2 \times \phi_b = 2 \times (90 - \phi_n)$

$$90 - \phi_n = \phi_b \quad ⇨ \quad 90 - \phi_b = \phi_n \qquad \text{-------- (3.36)}$$



$$90 - \phi_b = \cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right)$$

Take cosine on both sides ;

$$\cos(90 - \phi_b) = \cos\left(\cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right)\right)$$

$$\sin \phi_b = \pm \frac{n\lambda}{Nd}$$

For first null $n = 1$

$$\sin \phi_b = + \frac{\lambda}{Nd}$$

Nd indicates the total length of the array L

$$BWFN = 2 \times \phi_b = + \frac{2\lambda}{Nd} \qquad \text{-------- (3.37)}$$

$$BWFN = \frac{2\lambda}{L} = \frac{2}{(L/\lambda)} \quad rad$$

$$BWFN = \frac{114.6°}{(L/\lambda)} \quad deg \qquad \text{-------- (3.38)}$$

Half power beamwidth (HPBW)

$$HPBW = \frac{BWFN}{2} = \frac{1}{(L/\lambda)} \quad rad$$

$$HPBW = \frac{57.3°}{(L/\lambda)} \quad deg \qquad \text{-------- (3.39)}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 2(L/\lambda) \qquad \text{-------- (3.40)}$$

> **Example:** A broadside array of identical antennas consists of isotropic radiators separated by a distance $\boldsymbol{d = \lambda/2}$. Obtain positions of maxima and minima of the radiation pattern.

Solution:

Length of the array: $Nd = 4\left(\frac{\lambda}{2}\right) = \boldsymbol{2\lambda}$

Major lobe: $\phi_m = 90°$ or $270°$

Maxima of minor lobes (Secondary maxima):

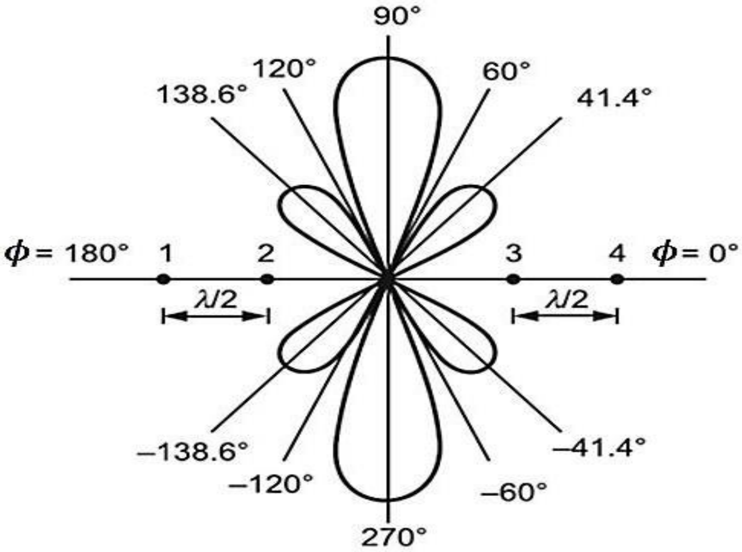
$$\phi_s = \cos^{-1}\left[\pm \frac{(2s+1)\lambda}{2Nd}\right] \qquad s = 1,2,3, \ldots \qquad s = 1\,;\ \pm 41.4°\,,\ \pm 138.6°$$

∴ These are the 4 minor lobe maxima of the array of 4 isotropic radiators fed in phase and spaced $\boldsymbol{\lambda/2}$ apart. No other maxima exist for $s \geq 2$, because for $s = 2$, $\cos^{-1}\left(\pm \frac{5}{4} \gg 1\right)$ whereas cosine value is always << 1.

Nulls:

$$\phi_n = \cos^{-1}\left(\pm \frac{n\lambda}{Nd}\right) \qquad n = 1,2,3,\ldots \qquad n = 1\,;\ \pm 60°\,,\ \pm 120° \qquad n = 2\,;\ \pm 0°\,,\ \pm 180°$$

∴ 0°, 60°, 120°, 180°, −60°, −120° are six minor lobe minima of the array of 4 isotropic radiators spaced $\boldsymbol{\lambda/2}$ apart. No other minima (nulls) exist for which cosine functions becomes more than one.



END-FIRE ARRAY (EFA)

- An array is said to be end-fire array, if the direction of maximum radiation coincides with the array axis.
- In end-fire array, individual elements are equally spaced along a line and each element is fed with current of equal magnitude and opposite phase.
- The total phase difference of the fields at point P from adjacent sources is given by,

$$\psi = kd\cos\phi + \alpha \qquad \text{-------- (3.41)}$$

- The normalized array factor for $'N'$ elements ;

$$(AF)_N = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{1}{2}\psi\right)}\right] \qquad \text{-------- (3.42)}$$

Major lobe

- In end-fire array $\psi = 0$ and $\phi = 0°$ or $180°$

$$\psi = kd\cos\phi + \alpha = 0$$

$\psi = 0$ and $\phi = 0°$ ⇨ $\alpha = -kd$ -------- (3.43)

$\psi = 0$ and $\phi = 180°$ ⇨ $\alpha = kd$ -------- (3.44)

$$\phi_m = 0° \text{ or } 180°$$

Nulls

- To find the nulls of the array Eqn. (3.42) is set to zero ;

$$\sin\left(\frac{N}{2}\psi\right) = 0 \quad \Rightarrow \quad \frac{N}{2}\psi\Big|_{\phi=\phi_n} = \pm n\pi$$

$$k = \frac{2\pi}{\lambda}$$

For EFA $\alpha = -kd$

$$\frac{N}{2}(kd\cos\phi_n + \alpha) = \pm n\pi$$

$$\frac{N}{2}(kd\cos\phi_n - kd) = \pm n\pi$$

where ; $n = 1, 2, 3 \ldots$

$$\frac{Nd}{\lambda}(\cos\phi_n - 1) = \pm n \quad \text{-------- (3.45)}$$

$$2\sin^2\frac{\phi_n}{2} = \pm\frac{n\lambda}{Nd}$$

$$\phi_n = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right) \quad \text{-------- (3.46)}$$

Further simplication : From Eqn. (3.45) , note that the value of $(\cos\phi_n - 1)$ is always less than 1, Hence it is negative. So consider negative values of R.H.S ;

From Eqn. (3.45):

$$\frac{Nd}{\lambda}(\cos\phi_n - 1) = -n$$

$$\phi_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right) \quad \text{-------- (3.47)}$$

Maxima of minor lobes (secondary maxima)

- The maximum value of Eqn. (3.42) occur when ;

$$\sin\left(\frac{N}{2}\psi\right) = 1 \quad \Rightarrow \quad \frac{N}{2}\psi\Big|_{\phi=\phi_s} = \pm(2s+1)\frac{\pi}{2}$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{N}{2}(kd\cos\phi_s + \alpha) = \pm(2s+1)\frac{\pi}{2}$$

For EFA $\alpha = -kd$

$$(\cos\phi_s - 1) = \pm(2s+1)\frac{\lambda}{2Nd} \quad s = 1,2,3,\ldots \quad \text{-------- (3.48)}$$

$$\phi_s = \cos^{-1}\left[\pm(2s+1)\frac{\lambda}{2Nd} + 1\right] \quad \text{-------- (3.49)}$$

Further simplication : From Eqn. (3.48) , note that the value of $(\cos\phi_s - 1)$ is always less than 1, Hence it is negative. So consider negative values of R.H.S ;

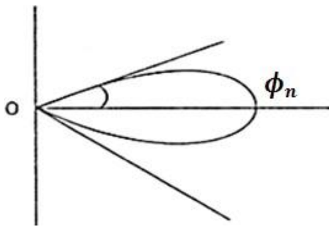
From Eqn. (3.48):

$$(\cos\phi_s - 1) = -(2s+1)\frac{\lambda}{2Nd}$$

$$\phi_s = \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right] \quad \text{-------- (3.50)}$$

Beamwidth of major lobe

- Beamwidth is defined as angle between first null and maximum of major lobe (or) Beamwidth is the angle equal to twice the angle between first null and the major lobe maximum.
$BWFN = 2 \times \phi_n$



$$\phi_n = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$\sin\frac{\phi_n}{2} = \pm\sqrt{\frac{n\lambda}{2Nd}}$$

For small angles ; $\sin\phi_n \approx \phi_n$

$$\frac{\phi_n}{2} \approx \pm\sqrt{\frac{n\lambda}{2Nd}}$$

$$\phi_n = \pm\sqrt{\frac{2n\lambda}{Nd}} \qquad \text{-------- (3.51)}$$

Nd indicates the total length of the array L

$$\phi_n = \pm\sqrt{\frac{2n\lambda}{Nd}}$$

For first null $n = 1$

$$\phi_n = \pm\sqrt{\frac{2\lambda}{L}}$$

$$BWFN = 2 \times \phi_n = \pm 2\sqrt{\frac{2\lambda}{L}} \qquad \text{-------- (3.52)}$$

$$BWFN = \pm 2\sqrt{\frac{2}{(L/\lambda)}}\ rad$$

$$BWFN = 114.6^\circ\sqrt{\frac{2}{(L/\lambda)}}\ deg \qquad \text{-------- (3.53)}$$

Half power beamwidth (HPBW)

$$HPBW = \frac{BWFN}{2} = \pm\sqrt{\frac{2}{(L/\lambda)}}$$

$$HPBW = 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}}\ deg \qquad \text{-------- (3.54)}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 4(L/\lambda) \qquad \text{-------- (3.55)}$$

Example: A end-fire array of identical antennas consists of isotropic radiators separated by a distance $\boldsymbol{d = \lambda/2}$. Obtain positions of maxima and minima of the radiation pattern.

Solution:

Length of the array: $Nd = \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$

Major lobe: $\phi_m = 0°$ or $180°$

Maxima of minor lobes (Secondary maxima):

$$\phi_s = \cos^{-1}\left[1 - (2s+1)\frac{\lambda}{2Nd}\right] \qquad s = 1,2,3,\dots$$

$s = 1\,;\ \pm 75.5°$

$s = 2\,;\ \pm 104.5°$

$s = 3\,;\ \pm 138.6°$

∴ These are the 6 minor lobe maxima of the array of 4 isotropic radiators fed spaced $\boldsymbol{\lambda/2}$ apart. No other maxima exist for $s \geq 4$, because for $s = 4$, $\cos^{-1}\left(\pm\frac{5}{4}\right) \gg 1$, whereas cosine value is always << 1.

Nulls:

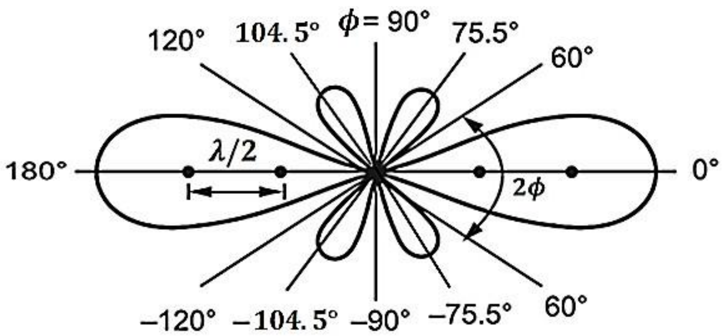
$$\phi_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right) \qquad n = 1,2,3,\dots$$

$n = 1\,;\ \pm 60°$

$n = 2\,;\ \pm 90°$

$n = 3\,;\ \pm 120°$

∴ $\pm 60°, \pm 90°, \pm 120°$ are six minor lobe minima of the array of 4 isotropic radiators spaced $\boldsymbol{\lambda/2}$ apart. No other minima (nulls) exist for which cosine functions becomes more than one.



HANSEN WOODYARD END-FIRE ARRAY

- In end-fire array, the maximum radiation can be obtained along the axis of the array, if the progressive phase shift α between the elements is given by ;

$$= \pm kd \qquad ; \alpha = -kd \text{ for maximum } \phi = 0° \text{ direction}$$
$$\alpha = kd \text{ for maximum } \phi = 180° \text{direction}$$

- It is found that the field produced in the direction $\phi = 0°$is maximum, but the directivity is not maximum. In many applications it is necessary to have the maximum possible directivity of the linear array.

- In 1938, Hansen and Woodyard proposed certain conditions for the end-fire case which are helpful in enhancing the directivity without altering other characteistics of the end-fire array. These conditions are known as Hansen –Woodyard conditions for end-fire radiation.
- According to Hansen –Woodyard conditions, the phase shift between closely spaced radiators of a very long array should be ;

$$\alpha = -(kd + 2.94/n) \approx -(kd + \pi/n) \text{ for maximum } \phi = 0° \text{ direction}$$

$$\alpha = +(kd + 2.94/n) \approx +(kd + \pi/n) \text{ for maximum } \phi = 180° \text{ direction}$$

Directivity

- Directivity can be expressed in terms of the total length of the array ;

$$D_{max} = 1.789[4(L/\lambda)] \quad \text{-------- (3.56)}$$

Parameter :	Broadside array	End-fire array	Hansen –Woodyard End-fire array
Directivity	$D_{max} = 2(L/\lambda)$	$D_{max} = 4(L/\lambda)$	$D_{max} = 1.789[4(L/\lambda)]$
	Where ; $L = Nd$		

PRINCIPLE OF PATTERN MULTIPLICATION

- *The field pattern of an array of non-isotropic but similar sources is the product of the pattern of the individual sources and the pattern of isotropic point sources having the same locations, relative amplitudes, and phase as the non-isotropic point sources.* This is referred to as pattern multiplication for arrays of identical elements.

$$\text{Total field } (E) = \underbrace{\{E_i(\theta,\phi) \times E_a(\theta,\phi)\}}_{\text{Multiplication of Field pattern}} \times \underbrace{\{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}}_{\text{Addition of Phase Pattern}} \quad \text{-------- (3.57)}$$

where ; $E_i(\theta,\phi)$ = Field pattern of individual source

$E_a(\theta,\phi)$ = Field pattern of array of isotropic source

$E_{pi}(\theta,\phi)$ = Phase pattern of individual source

$E_{pa}(\theta,\phi)$ = Phase pattern of array of isotropic source

⇨ ***RADIATION PATTERN OF 4-ISOTROPIC ELEMENTS FED IN PHASE & SPACED $\lambda/2$ APART***

- Consider a 4-element array of antennas as shown in Fig. 3-8, in which the spacing between the elements is $\boldsymbol{\lambda/2}$ and the currents are in-phase ($\alpha = 0$). The pattern can be obtained directly by adding the four electric fields due to the 4 antennas. However the same radiation pattern can be obtained by pattern multiplication in the following manner.

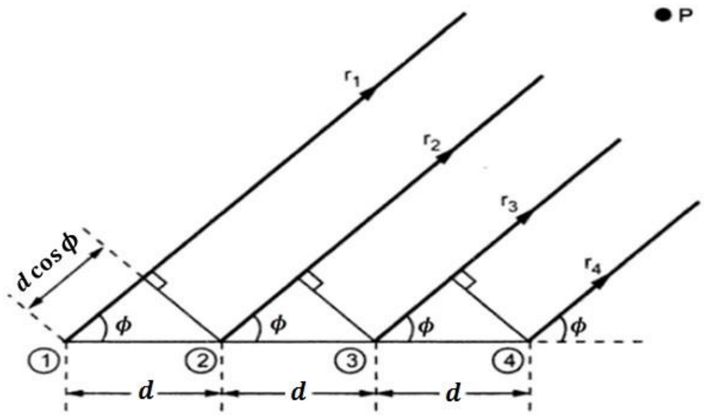
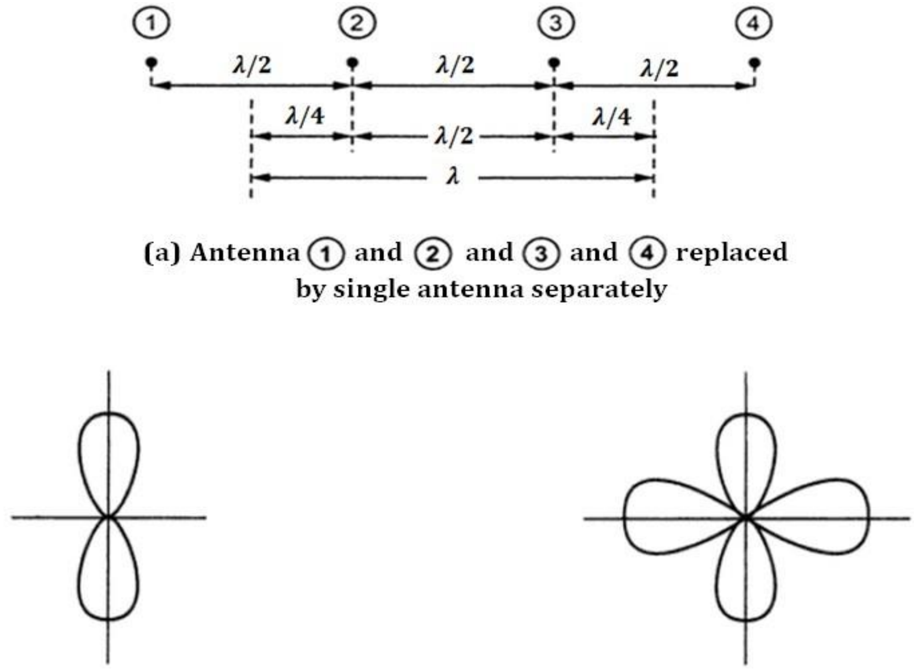


Fig. 3-8 Linear array of 4 isotropic elements spaced $\lambda/2$ apart , fed in-phase

- Two isotropic point sources spaced $\lambda/2$ apart fed in-phase provides a bidirectional pattern as in Fig. 3-9 (b). Now the elements 1 and 2 are considered as one unit and this new unit is considered to be placed between the midway of elements 1, 2 and similarly the elements 3,4 as shown in Fig. 3-9 (a).



(a) Antenna ① and ② and ③ and ④ replaced by single antenna separately

(b) Radiation pattern of 2 antennas spaced at distance $\lambda/2$ and fed with equal currents in phase

(c) Radiation pattern of 2 antennas spaced at distance λ and fed with equal currents in phase

Fig. 3-9

- 4 elements spaced $\lambda/2$ have been replaced by 2 units spaced λ and therefore the problem of determining radiation of 4 elements has been reduced to find out the radiation pattern of 2 antennas spaced λ apart as in Fig. 3-9 (a).

$$\left\{\begin{matrix} Resultant\ radiation \\ pattern\ of\ 4\ elements \end{matrix}\right\} = \left\{\begin{matrix} Resultant\ radiation \\ pattern\ of\ individual\ elements \end{matrix}\right\} \times \left\{\begin{matrix} rray\ of\ 2\ units \\ spaced\ \lambda \end{matrix}\right\}$$

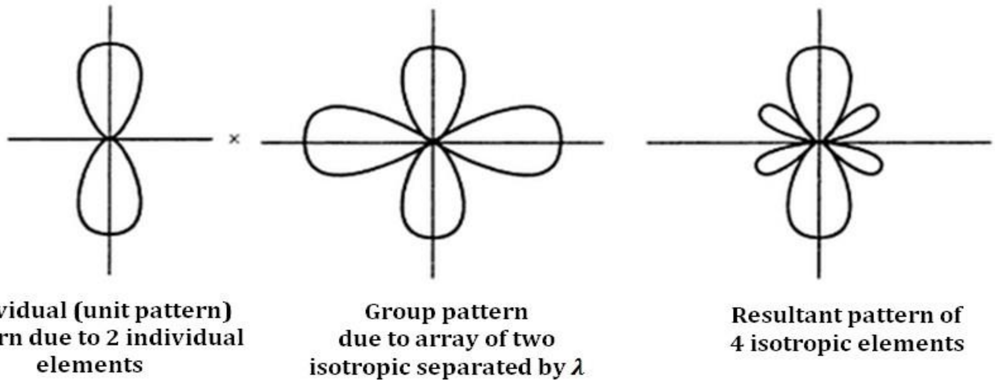


Fig. 3-10 Resultant radiation pattern of 4 isotropic elements by pattern multiplication

- Here the width of the principal lobe is the same as the width of the corresponding lobe of the group pattern. The number of secondary lobes can be determined from the nulls in the resultant pattern, which is sum of the nulls in the unit and group patterns.

CONCEPT OF PHASED ARRAYS, ADAPTIVE ARRAY

- In case of the broadside array and the end fire array, the maximum radiation can be obtained by adjusting the phase excitation beween elements in the direction normal and along the axis of array respectively.
- That means in other words elements of antenna array can be phased in particular way. So we can obtain an array which gives maximum raidation in any direction by controlling phase excitation in each element. Such an array is commonly called phased array.
- The array in which the phase and the amplitude of most of the elements is variable, provided that the direction of maximum radiation (beam direction) and pattern shape along with the side lobes is controlled, is called as phased array.
- Suppose the array gives maximum radiation in direction $\phi = \phi_0$ where $0 \leq \phi_0 \leq 180°$, then the phase shift that must be controlled can be obtained as follows.

$$\psi = kd\cos\phi + \alpha|_{\phi=\phi_0} = 0 \qquad \text{-------- (3.58)}$$

- Thus from Eqn. (3.58), it is clear that the maximum radiation can be achieved in any direction if the progressive phase difference between the elements is controlled. The electronic phased array operates on the same principle.
- Consider a three element array as shown in the Fig. 3-11. The element of array is considered as $\lambda/2$ dipole. All the cables used are of same length. All the three cables are brought together at common feed point. Here mechanical switches are used. Such switch is installed one at each antenna and one at a common feed point.
- All the switches are ganged together. Thus by operating switch, the beam can be shifted to any phase shift.
- To make operation reliable and simple, the ganged mechanical switch is replaced by PIN dipole which acts as electronic switch. But for precision in results, the number of cables should be minimised.

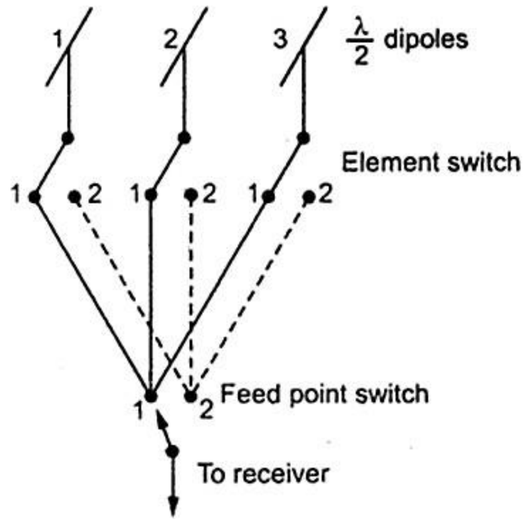


Fig. 3-11 Phased array with mechanical switches at elements and feed point

- To make operation reliable and simple, the ganged mechanical switch is replaced by PIN dipole which acts as electronic switch. But for precision in results, the number of cables should be minimised.
- In many applications phase shifter is used instead of controlling phase by switching cables. It can be achieved by using ferrite device. The conducting wires are wrapped around the phase shifter. The current flowing through these wires controls the magnetic field within ferrite and then the magnetic field in the ferrite controls the phase shift.
- The phased array for specialized functional utility are recognized by different names such as frequency scanning array, retroarray and adoptive array.
- The array in which the phase change is controlled by varying the frequency is called frequency scanning array. This is found to be the simplest phased array as at each element separate phase control is not necessary. A simple transmission line fed frequency scanning array as shown in the Fig. 3-12.

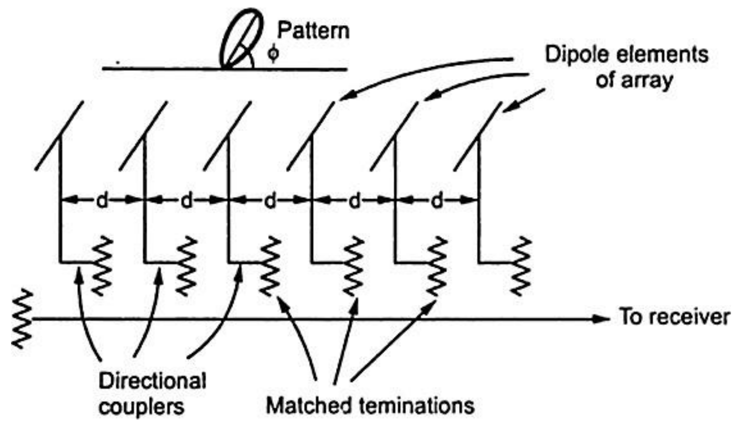


Fig. 3-12 Frequency scanning line fed phased array.

- Each element of the scanning array is fed by a transmission line via directional coupler. Note that the directional couplers are fixed in position, while the beam scanning is done with a frequency change. To avoid reflections and to obtain pure form of the travelling wave, the transmission line is properly terminated of the load.

- The main advantage of the frequency scaning array is that there are no moving parts and no switches and phase shifters are required.
- The array which automatically reflects an incoming signal back to the source is called retroarray. It acts as a retroreflector similar to the passive square corner reflector. That means the wave incident on the array is received and transmitted back in the same direction.
- In other words, each element of the retroarray reradiates signal which is actually the conjugate of the received one. Simplest form of the retroarray is the Van Atta array as X shown in the Fig. 3-13 in which 8 identical dipole elements are used, with pairs formed between elements l and 8, 2 and 7, 3 and 6, 4 and 5 using cables of equal length. If the wave arrives at angle say , then it gets transmitted in the same direction.

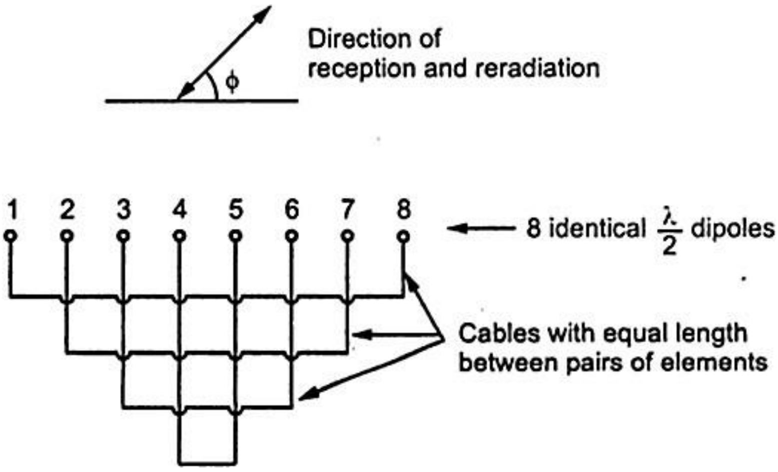


Fig. 3.15 Van Atta retorarray

- An array which automatically turn the maximum beam in the desired direction while turn the null in the undesired direction is called adoptive array. The adpotive array adjust itself in the desired direction with awareness of its enviomment.
- In modem adoptive arrays, the output of each element in the array is sampled, digitized and then processed using computers. Such arrays are commonly called smart antennas.

BASIC PRINCIPLE OF ANTENNA SYNTHESIS - (BINOMIAL ARRAY)

- In case of uniform linear array, to increase the directivity, the array length has to be increased. But when the array length increases, the secondary or side lobes appear in the pattern. In some of the special applications, it is desired to have single main lobe with no minor lobes.
- That means the minor lobes should be eliminated completely or reduced to minimum level as compared to main lobe. To achieve such pattern, the array is arranged in such away that the broad side array radiate more strongly at the centre than that from edges.
- To reduce the side lobe level, John Stone proposed that sources have amplitudes proportional to the coefficients of a binomial series of the form ;

$$(1+x)^{m-1} = 1 + (m-1)x + \frac{(m-1)(m-2)}{2!}x^2 + \frac{(m-1)(m-2)(m-3)}{3!}x^3 + \dots \quad \text{-------- (3.59)}$$

where m is the number of radiating sources in the array.

Binomial array is an array whose elements are excited according to the current levels determined by the binomial coefficient.

- The positive cooefficients of the series expansion for different values of m are ;

$$\begin{array}{lccccccccccccccccccc}
m=1 & & & & & & & & & & 1 & & & & & & & & & \\
m=2 & & & & & & & & & 1 & & 1 & & & & & & & & \\
m=3 & & & & & & & & 1 & & 2 & & 1 & & & & & & & \\
m=4 & & & & & & & 1 & & 3 & & 3 & & 1 & & & & & & \\
m=5 & & & & & & 1 & & 4 & & 6 & & 4 & & 1 & & & & & \\
m=6 & & & & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & & & \\
m=7 & & & & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & & & \\
m=8 & & & 1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 & & \\
m=9 & & 1 & & 8 & & 28 & & 56 & & 70 & & 56 & & 28 & & 8 & & 1 & \\
m=10 & 1 & & 9 & & 36 & & 84 & & 126 & & 126 & & 84 & & 36 & & 9 & & 1
\end{array}$$

-------- (3.60)

- The above represents Pascal's triangle. If the values of m are used to represent the number of elements of the array, then the coefficients of the expansion represent the relative amplitudes of the elements. Since the coefficients are determined from a binomial series expansion, the array is known as binomial array.

<u>Nonuniform amplitude arrays of even and odd number of elements:</u>

- Assuming that the amplitude excitation is symmetrical about the origin, the array factor for a nonuniform amplitude array can be written as ;

$$(AF)_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos[(2n-1)u]$$

$$(AF)_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u] \quad \text{-------- (3.61)}$$

where

$$u = \frac{\pi d}{\lambda}\cos\phi$$

where a_n's are the excitation coefficients of the array elements.

- From Eqn's (3.60) and (3.61) , the amplitude coefficients for the following arrays are:

1. Two elements ($2M = 2$)

   $a_1 = 1$

2. Three elements ($2M + 1 = 3$)

   $2a_1 = 2 \Rightarrow a_1 = 1$

   $a_2 = 1$

3. Four elements ($2M = 4$)

   $a_1 = 3$

   $a_2 = 1$

4. Five elements ($2M + 1 = 5$)

   $2a_1 = 6 \Rightarrow a_1 = 3$

   $a_2 = 4$

   $a_3 = 1$

- Binomial array's do not exhibit any minor lobes provided the spacing between the elements is equal or less than one-half of a wavelength.
- The design using a $\lambda/2$ spacing leads to a pattern with no minor lobes, the half-power beamwidth and maximum directivity for $d = \lambda/2$ spacing in terms of the numbers of elements or the length of the array are given by ;

$$HPBW\ (d = \lambda/2) \approx \frac{0.75}{\sqrt{L/\lambda}} \quad \text{-------- (3.62)}$$

$$D_{max} = 1.77\sqrt{1 + 2L/\lambda} \quad \text{-------- (3.63)}$$

- The advantages of binomial array is that there are no side lobes in the resultant pattern. The disadvantages are i. Beam width of the main lobe is large which is undesirable ii. Directivity is small and high excitation levels are required for the center elements of large arrays.

Yagi Arrays:

Yagi Uda Antenna:

Yagi-Uda arrays or Yagi-Uda antennas are high gain antennas. The antenna was first invented by a Japanese Prof. S. Uda in early 1940's and described in English by Prof. H. Yagi. Hence the antenna name Yagi-Uda antenna was given after Prof. S. Uda and Prof. H. Yagi. A basic Yagi-Uda antenna consists a driven element, one reflector and one or more directors. Basically it is an array of one driven element and one of more parasitic elements. The driven element is a folded dipole made of a metallic rod which is excited.

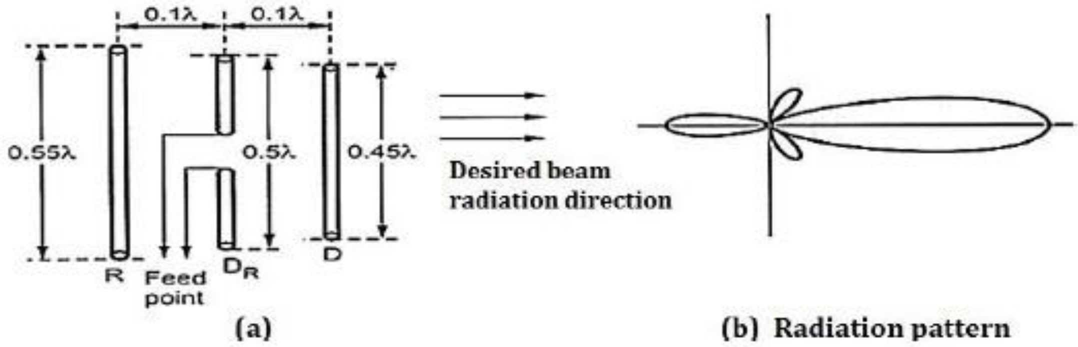
A Yagi-Uda antenna uses both the reflector (R) and the director (D) elements in same antenna.The element at the back side of the driven element is the reflector. It is of the larger length compared with remaining elements. The element in front of the driven element is the director which is of lowest length. Directors and reflector are called parasitic elements. All the elements are placed parallel and close to each other as shown in Fig. 1. The length of the folded dipole is about $\lambda/2$ and it is at resonance. Length of the director is less than $\lambda/2$ and length of the reflector is greater than $\lambda/2$.

The parasitic element receive excitation through the induced e.m.f. as current flows in the driven element.The phase and amplitude of the currents through the parasitic elements mainly depends on the length of the elements and spacing between the elements. To vary reactance of any element, the dimensions of the elements are readjusted. Generally the spacing between the driven and the parasitic elements is kept nearly 0.1 λ to 0.15 λ.

$$Reflector\ length = \frac{152}{f_{MHz}}\ meter$$

$$Driver\ element\ length = \frac{143}{f_{MHz}}\ meter$$

$$Director\ length = \frac{137}{f_{MHz}}\ meter$$



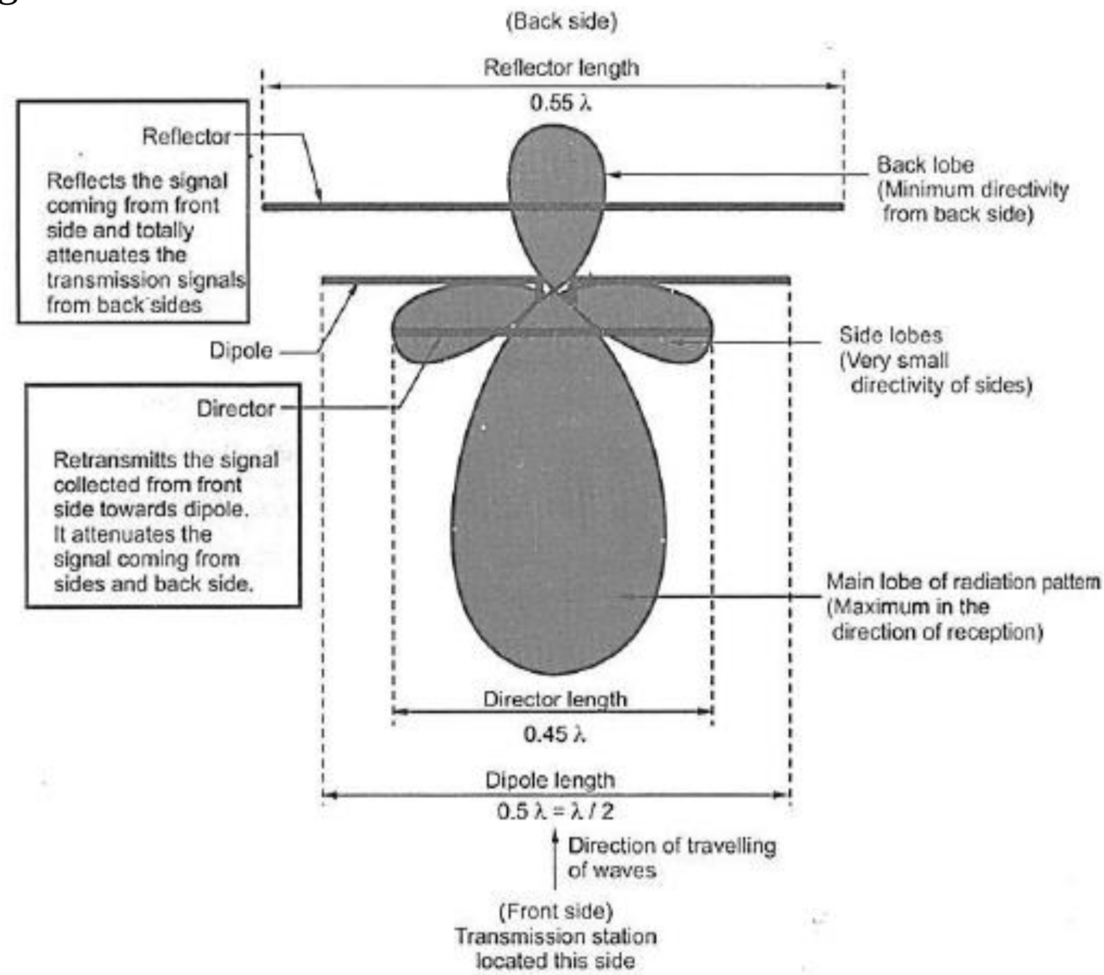
Working of Yagi Antenna:

The parasitic element is used either to direct or to reflect, the radiated energy forming compact directional antenna. If the parasitic element is greater than length $\lambda/2$, (i.e. reflector) then it is inductive in nature. Hence the phase of the current in such element lags the induced voltage.If the parasitic element is less than resonant length $\lambda/2$ (i.e. director), then it is capacitive in nature.

Hence the current in director leads the induced voltage.The directors adds the fields of the driven element in the direction away from the driven element. If more than one directors are used, then each director will excite the next.To increase the gain of the Yagi-Uda antenna, the number of directors is increased in the beam direction.

To get good excitation, the elements are closely spaced.The driven element radiates from front to rear (i.e., from reflector to director). Part of this radiation induces currents in the parasitic elements which reradiate almost all radiations. With the proper lengths of the parasitic elements and the spacing between the elements, the backward radiation is cancelled and the radiated energy is added in front.

Yagi Antenna Radiation Pattern:



Applications of Yagi Antenna:

- Yagi-Uda array is the most popular antenna for the reception of terrestrial television signals in the VHF band (30 MHz-300 MHz).
- The array for this application is constructed using aluminium pipes.
- The driven element is usually a folded dipole, which gives four times the impedance of a standard dipole.
- Thus, a two-wire balanced transmission line having a characteristic impedance of 300 Ω can be directlyconnected to the input terminals of the Yagi-Uda array.
- Yagi-Uda arrays have been used in the HF, VHF, UHF, and microwave frequency bands.

Post – MCQ:

1. The antenna array is defined as the system of similar antennas directed to get required ----------------- in the desired direction.
   (a) High gain
   (b) High directivity
   (c) High bandwidth
   (d) All of the above

Answer - d

2. Find the odd element out –
   (a) Broadside array
   (b) End fire array
   (c) Co-linear array
   (d) Yagi array

Answer - d

3. The Broadside array is defined as an array having maximum radiation-------------------- the axis array
   a) Perpendicular to
   b) Along
   c) Parallel
   d) None of the above

Answer - a

4. The collinear array is also called as –
   (a) Broadcast array
   (b) Broad fire array
   (c) Omni directional array
   (d) All of the above

Answer - a

5. The End fire array is defined as an array having maximum radiation-------------------- the axis array

a) Perpendicular to
b) Along
c) Parallel
d) None of the above

Answer - b

6. An array is said to be uniform array if the array elements are fed with –
   (a) Equal amplitudes and any phase shift
   (b) Equal amplitudes and uniform progressive phase shift
   (c) Unequal amplitudes and any phase shift
   (d) None of the above

Answer - b

7. In a broadside array, the direction of maximum radiations indicated by –
   (a) 0 and 180 degrees
   (b) 90 and 270 degrees
   (c) 90 and 180 degrees
   (d) None of the above

Answer -b

8. In a End fire array, the direction of minimum radiations indicated by –
   (a) 0 and 180 degrees
   (b) 90 and 270 degrees
   (c) 90 and 180 degrees
   (d) None of the above

Answer - b

9. The relation between directivity and the array factor length is given by –
   (a) $D = 2 (L/\lambda)$
   (b) $D = 3(L/\lambda)$
   (c) $D = 4(l/\lambda)$
   (d) None of the above

Answer - c

10. In a phase array, the maximum radiation in any direction can be obtained by controlling ------------------ excitation in each element.
    (a) Angle
    (b) Phase
    (c) Amplitude
    (d) None of the above

Answer - b

11. The Adaptive array is an array which turns the ---------------beam in the desired direction and ----------in the undesired direction.
    (a) Minimum and maximum
    (b) Maximum and zero
    (c) Maximum and minimum
    (d) None of the above

Answer - b

12. The frequency scanning array is an array in which phase change can be controlled by Varying the –

    (a) Phase
    (b) Frequency
    (c) amplitude
    (d) Any of the above

Answer -b

13. The array in which the incoming signal received and sent back in the same direction is called as-
    (a) End fire array
    (b) Frequency scanning array
    (c) Van Atta array
    (d) None of the above

Answer - c

14. Binomial array is an array whose elements are excited according to the current levels determined by the –
    (a) binomial coefficients
    (b) binary coefficients
    (c) integer coefficients
    (d) All of the above

Answer - a

15. The important characteristic of a binomial array is –
    (a) Small beam width
    (b) High directivity
    (c) No side lobes
    (d) All of the above

Answer - c

Conclusion:

At the end of the topic, students will be able –

1) To understand the basic concept of Antenna Arrays and their radiation characteristics.

2) To understand the principle of types of Antenna arrays

3) To get exposure to different types like Broadside array and End fire arrays etc

4) To know the applications of Antenna arrays – example Yagi uda array

References:

1. John D Kraus," Antennas for all Applications", 4th Edition, McGraw Hill, 2010.
2. R.E. Collin," Antennas and Radio wave Propagation", McGraw Hill 1985.
3. Constantine.A. Balanis "Antenna Theory Analysis and Design", Wiley Student Edition, 4th Edition 2016.
4. Rajeswari Chatterjee, "Antenna Theory and Practice" Revised Second Edition New Age International Publishers, 2006.

Assignments:

1. Explain the working principle of Broadside antenna array and derive an expression for the resultant electric field.
2. Explain the working principle of End fire antenna array and derive an expression for the resultant electric field
3. Derive an expression for electric field intensity of an array of N - isotropic sources of (i) Equal amplitude and same phase (ii) Equal amplitude and opposite phase.
4. Write a short note on antenna arrays.
5. Differentiate between BSA and EFA.