Summer Assignment 2020 – IB Math Analysis & Approaches SL1/Precalculus Intensified

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Purpose of Assignment:

This assignment includes material you learned in previous math courses. Knowledge of this material is essential to your success in the upcoming year. Topics covered in this assignment include functions, function operations, and transformations.

Estimated time to complete Assignment: This assignment should take 3-4 hours.

Due date and method of assessment for Assignment:

The summer assignment will be due during the second week of the 2020-21 school year. Details about the deadline and logistics for submission will be provided by the second day of the school year. You will have a quiz on the summer assignment during the second week of the school year. The quiz will have two parts: on part 1 you will not be allowed to use a calculator, but on part 2 you will be allowed to use a graphing calculator.

Note: You must have your own graphing calculator for this class. A TI-84 (any type) is highly recommended, but TI-83 or non-CAS version of the TI-Nspire calculators are allowed. CAS versions of the TI-Nspire and the TI-89 are not allowed on the IB exam and therefore are not allowed in this class. For this assignment (as opposed to during class) it is also possible to use one of the several online types of graphing calculators, such as www.desmos.com, that are available.

Instructions for Assignment:

This assignment is taken from pages in the textbook that we will be using in class next year. All work should be done neatly on your own paper. Graphs should be done on graph paper.

The pages provided in this summer assignment are just the ones that have the assigned problems on them. You can access a full copy of chapters 2 and 3, which includes example problems, using the following link: https://bit.ly/2Wan8lu

Chapter 2 Exercises:

p.73 2B #1abdef, 2dfh, 5
p.76 2C #1b, 2ac
p.82 2D #1cdkl, 4, 5
p.89 2F #1a, 3
p.90 2G #1, 3
p.96 2I #1ad, 3a, 4a(iii), 4b, 6, 7
p.99 2J #2a, 3

Chapter 3 Exercises:

p.115 3B #2
p.119 3C #3b, 4c
p.123 3E #1b, 3b
p.125 3F #1b, 2b
p.125 3G #1aeg, 3
p.126 3H #1a, 2
p.129 3I #1, 4
p.136 3J #1d, 2abc
p.140 3K #1ace, 3

b What notation could be used to find the cost of the banquet if 125 people attend? Calculate this value.

c What notation could be used to find the number of people who can attend the banquet if Nikita's budget for the banquet is \$1250? Calculate this value.

d What values of n do not make sense for this situation?

a $C(n) = 20n + 320$	
b $C(125) = 20(125) + 320$ $C(125) = 2820$ \$2820	
c $C(n) = 1250$ $20n + 320 = 1250$ $20n = 930$ $n = 46.5$ 46 people	Since rounding up will put the total over \$1250, we round down to 46 people.
d $0 \le n \le a$ where a is the capacity of the room	n cannot be negative and cannot be larger than the number of people the room can hold or the number of people who can be covered by Nikita's budget.

Exercise 2B

1 Calculate the substitutions indicated for each given function.

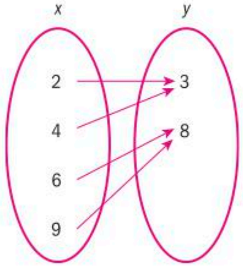
a $g(x) = -x^2 + 2$, $g(-4)$

b $f : x \rightarrow 5x - 1$, $f(-9)$

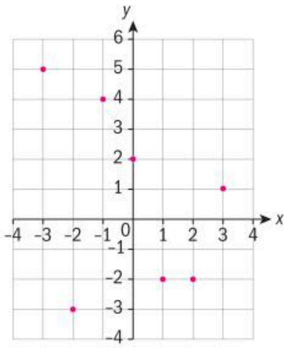
c $C(n) = 20n + 250$, $C(100)$

d $h(x) = -4$, $h(5)$

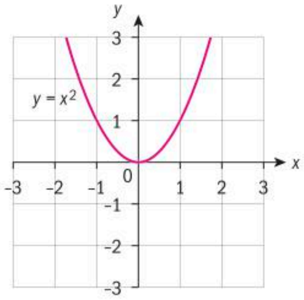
e $f(2)$ for the mapping diagram below



f $f(-3)$ for the graph below



g $f(-1)$ for the graph below



2 If $f(x) = -3x^2 - 1$, $g(x) = -4x + 7$ and $h(x) = 6$, find:

a $f(-3)$ **b** $g(15)$

c $f(1) + g(-1)$ **d** $h(0)$

e $f(x - 2)$ **f** $g(n)$

g $f(1) \times h(1)$ **h** $f(x + 1) \times g(x - 2)$

3 The journey of a car travelling towards Perth, Australia, can be expressed by the relation $d = -75t + 275$, where d is the distance from Perth in km and t is driving time in hours.

a Is this relation a function? Justify your answer.

b Rewrite the equation in functional notation.

c At the start of the journey, how far was the car from Perth?

d What values of t do not make sense for this situation?

4 To convert a temperature from Celsius to Fahrenheit, we use the formula:

$$F(C) = \frac{9}{5}C + 32 .$$

a Is this formula a function? Justify your answer.

b Describe what $F(17)$ is asking and calculate the value.

c Describe what $F(C) = 100$ is asking and calculate the value of C.

d Use the formula to calculate the temperature in degrees Fahrenheit at which water freezes.

e Use the formula to calculate the temperature in degrees Fahrenheit at which water boils.

f The average body temperature of a dog is 38.75°C. Convert this to degrees Fahrenheit.

g Cookies are typically baked at 350°F. Convert this to degrees Celsius.

5 A British telecommunications company offers the following roaming data package to its customers: a flat fee of £25 plus £10 per gigabyte of data

a Express the total cost, C, as a function of the number of gigabytes of data (g).

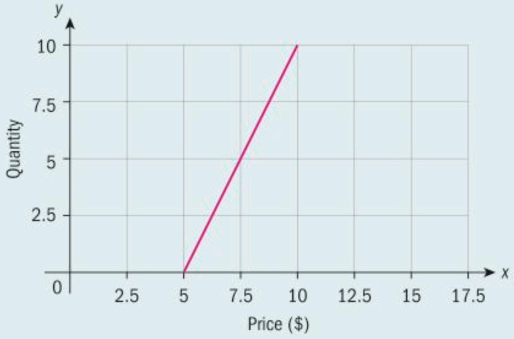
b What values of g do not make sense in this context?

c State the notation that could be used to find the roaming cost for a trip where 14 gb of data is used? Calculate this value.

d State the notation that could be used to find the number of gigabytes of data one can use for a total bill of £100. Calculate this value.

Developing inquiry skills

Let's return to the chapter opener about economics and supply and demand. The equation of the line in the graph on the right is $y = -10 + 2x$. Rewrite this in functional notation and define the variables you choose.



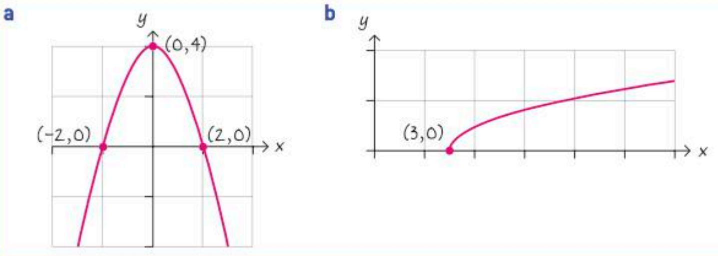
International-mindedness

The development of functions bridged many countries including France (Rene Descartes), Germany (Gottfried Wilhelm Leibnitz) and Switzerland (Leonhard Euler).

Example 8

Using your GDC, sketch the following graphs:

a $f(x) = -x^2 + 4$ **b** $y = \sqrt{x-3}$



When sketching the graph of a function, be sure to include the relevant features of the graph, including the *x*- and *y*-intercepts.

Exercise 2C

1 Use your GDC to sketch the graphs of the following functions:

a $f(x) = -0.01x^2 + 0.5x + 2.56$

b $g : x \to 0.44\sqrt{3.2x - 4.1}$

c $y = 2\sin(x + 3) + 2$

2 Given $f(x) = x^2 + 1$, $g(x) = -3x + 2$ and $h(x) = \sqrt{x}$, use your GDC to sketch the graphs of the following:

a $f(x) + g(x)$ **b** $f(x) - h(x)$

c $f(x) \times g(x)$ **d** $f(x) + g(x) + h(x)$

HINT

For part **1c**, be sure your calculator is in radians, and set the window of your GDC so that the *x*-axis shows from -2π to 2π.

3 Use your GDC to help you draw the graphs of the following functions.

a $y = 0.2x^2 + 1.3x - 4.5$

b $f(x) = 3 \times 2^{0.5x+3} - 1$

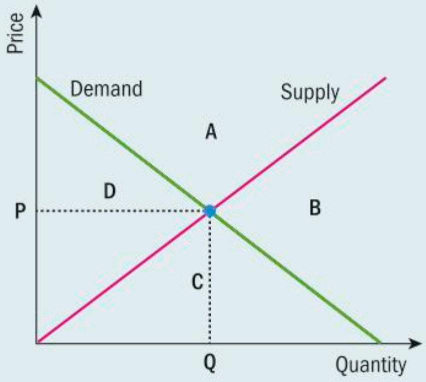
Developing inquiry skills

Let's return to our opening problem about supply and demand.

The equation we generated in section 2.2 for supply was $Q(P) = -10 + 2P$, where P represents price and Q represents the quantity supplies by producers.

If the equation for the quantity of a product demanded by consumers is $D(P) = 30 - 3P$, use your GDC to sketch both graphs on the same set of axes and find the intersection point.

Explain what this intersection point means in terms of supply and demand.



Exercise 2D

1 For each relation below, state whether it is a function or not. If yes, state the domain and range.

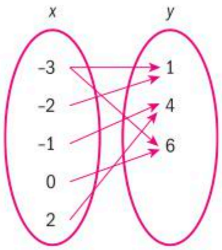
a

x	6	8	8	12
y	1	6	4	−1

b $\{(-5,6),(-2,4),(3,14)\}$

c $\{(-12,7),(-8,-8),(-5,-8)\}$

d

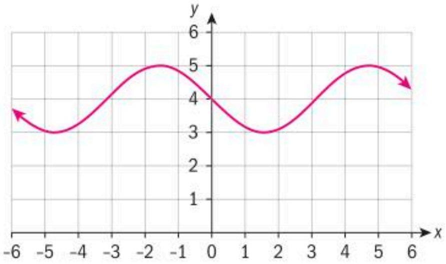


e $f(x)=-2x$

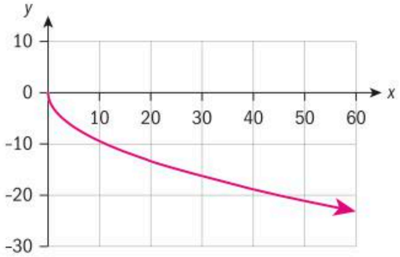
f $g(x)=-4$

g $x=3$

h



i



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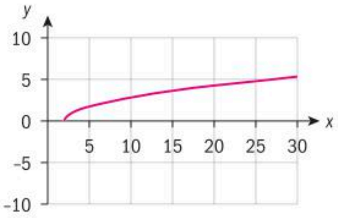

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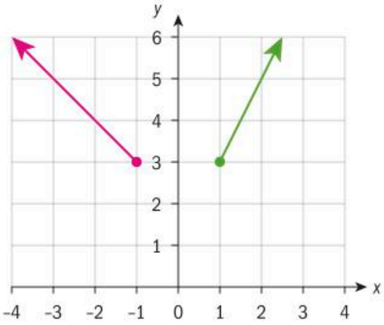

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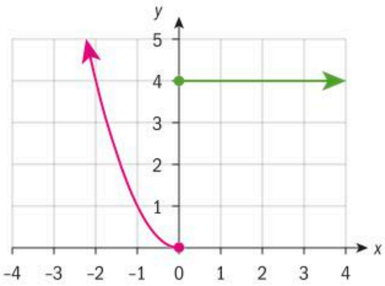


2 Use your GDC to graph the following functions. Sketch the graph and state the domain and range.

a $f(x)=-(x-4)(x+2)(x-1)$

b $g(x)=\dfrac{x+2}{x-1}$

c $f:x\rightarrow -\log(x-1)$

d $g:x\rightarrow 3\cos x+2$

3 Draw the graph of a possible function for each of the following domain and range:

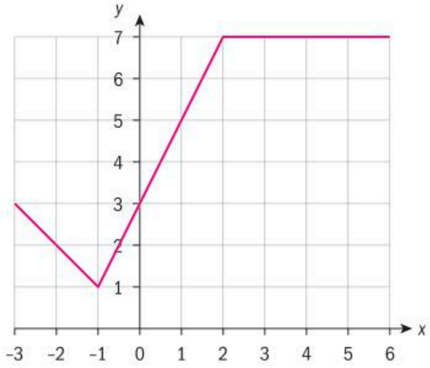
- **a** domain: $x \in \mathbb{R}$
  range: $y \in \mathbb{R}$
- **b** domain: $]-\infty, \infty[$
  range: $[3, \infty[$
- **c** domain: $0 \le x < 6$
  range: $-3 \le y < 2$
- **d** domain: $x > 0$
  range: $y > 0$

4 Consider the piecewise function

$$f(x) = \begin{cases} \frac{1}{3}x + 2, & 0 \le x \le 6 \\ -x + 10, & 6 < x \le 10 \end{cases}$$

- **a** Find each value: **i** $f(6)$ **ii** $f(8)$
- **b** Sketch the graph of f.
- **c** Write down the domain and range of f.

5 Consider the graph of the piecewise function $y = f(x)$, where $-3 \le x \le 6$. Find the equations for the function, including an interval of the domain that applies to each part.

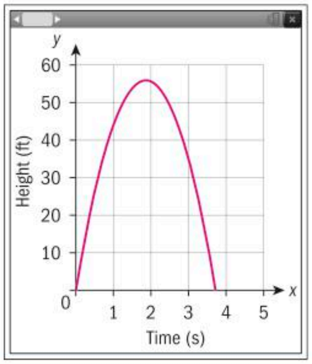


Example 12

The path of a golf ball after being hit by a club can be modelled by the function $h(t) = 60t - 16.1t^2$, where t is the time in seconds and h is the height of the ball in feet.

- **a** Use your GDC to sketch a graph of the ball's path. Label the axes.
- **b** Explain why this graph is a function.
- **c** Use your GDC to find the t-intercepts and maximum value.
- **d** Write down the domain and range for this situation.

a



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Exercise 2F

1 **a** Given the functions

$f(x) = -x^2 + 5x$

$g(x) = 4x - 2$

$h(x) = \sqrt{x} + 1$

find:

i $f(g(x))$ **ii** $f(f(x))$

iii $f(h(x))$ **iv** $(g \circ h)(x)$

v $(f \circ f \circ f)(-1)$ **vi** $g(h(9))$

vii $(g \circ f)(2) + (f \circ g)(2)$

b State the domain of parts **ai–iv.**

2 Create two different functions, $f(x)$ and $g(x)$, such that

a $f(g(x)) = g(f(x))$

b $f(g(x)) \neq g(f(x))$

c $f(2) = g(2)$

3 Given $f(x) = -2x + 5$ and $g(x) = 4x - 1$:

a find an expression for $f(g(x))$

b solve $f(g(x)) = 12$.

4 Given $f(x) = 3x^2 - 6$ and $g(x) = -x + 4$:

a find $f(g(x))$

b using your GDC, sketch the graph of $f(g(x))$

c state the domain and range of $f(g(x))$

5 You purchase a new refrigerator. You have no way to get it home, so you pay the delivery fee of \$25. The sales tax on the delivery fee is 6%.

a Write a function $f(x)$ that represents the cost of the refrigerator and the delivery fee.

b Write a function $g(x)$ that represents the cost of the refrigerator and the tax.

c Find and interpret both $f(g(x))$ and $g(f(x))$.

d If taxes cannot be charged on delivery, which composite function from part **c** should be used to calculate the total amount you must pay to the store?

A composite function can be formed with more than just equations. Let's look at other forms for functions, such as ordered pairs, diagrams and graphs.

Example 15

If $f(x) = \{(-4, -2), (3, 4), (9, 1), (10, -2)\}$ and

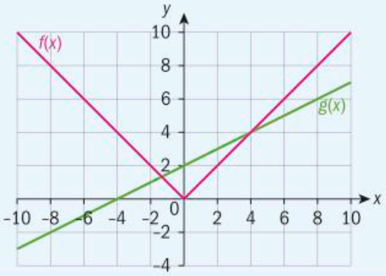
$g(x) = \{(-4, 5), (-2, 3), (0, 6), (7, -4), (11, -4)\}$, find $f(g(-2))$.

$g(-2) = 3$	Start with the inside function, $g(-2)$. Find the point in the function g that has an x-coordinate of -2. The answer to $g(-2)$ will be the y-coordinate of this point.
$f(g(-2)) = f(3) = 4$	Now go to the function f and find the point with an x-coordinate of 3. The y-coordinate is the final answer. This method can also be used with a table of values or mapping diagram.

Example 16

Using the graph of two functions $f(x)$ and $g(x)$ on the right, find:

a $f(g(2))$ **b** $(g \circ f)(-1)$ **c** $g(g(0))$



a $g(2) = 3$	Start with the inside $g(2)$. Look at the graph of the g function and find where $x = 2$. The value will be the y-coordinate of this point.
$f(g(2)) = f(3)$	Now look at the graph of the f function and find where $x = 2$.
$f(3) = 3$	The final answer will be the y-coordinate of this point.
b $f(-1) = 1$ $(g \circ f)(-1) = g(1)$ $g(1) = 2.5$	
c $g(0) = 2$ $g(g(0)) = g(2)$ $g(2) = 3$	

Exercise 2G

1 Given the functions

$f(x) = \{(2, 1), (3, 2), (5, 6), (10, -4)\}$

$g(x) = \{(-2, 3), (3, 11), (6, -3), (11, 0)\}$

a find:

i $f(g(-2))$ **ii** $g(f(5))$

iii $g(g(3))$

b state the domain and range of both functions.

2 Using the table of values below, find:

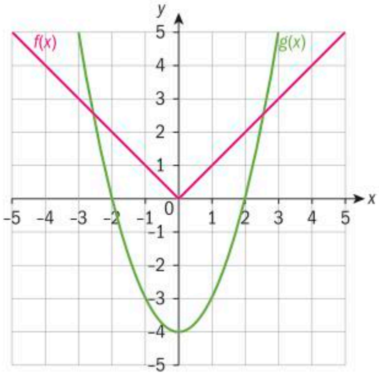
x	-1	3	7	9
$f(x)$	9	6	-2	-1
$g(x)$	-5	7	0	-4

a $(f \circ g)(3)$ **b** $(g \circ f)(-1)$ **c** $(f \circ f)(9)$

3 Using the graph below, find:

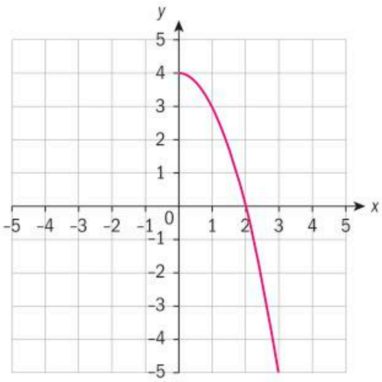
a $g(f(0))$ **b** $g(f(1))$

c $f(g(-2))$ **d** $f(g(0))$



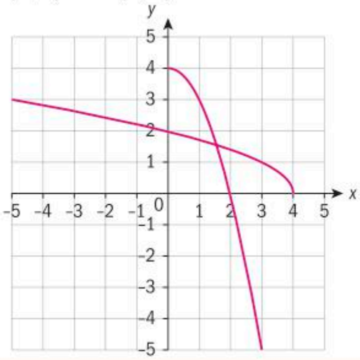
4 Create two functions, $f(x)$ and $g(x)$, each a set of ordered pairs, such that $f(g(-1)) = 2$.

b i This is the graph of $f: x \rightarrow -x^2 + 4$, for $x \geq 0$.



Notice that the x-intercept is $(2, 0)$ and the y-intercept is $(0, 4)$.

ii For the inverse, the axes intercepts will be $(0, 2)$ and $(4, 0)$.



To sketch the graph of an inverse, graph the original function and pick some points (like the x-intercepts, y-intercepts, etc) and switch the coordinates. This will result in a reflection over the line $y = x$.

$$(x, y) \rightarrow (y, x)$$

Exercise 2I

1 Determine algebraically whether the following pairs of functions are inverses.

a $f(x) = -2x + 2$ and $g(x) = \frac{1}{2}x + 2$

b $f: x \rightarrow 2x^3 + 4$ and $g: x \rightarrow \sqrt[3]{\frac{x-3}{2}}$

c $f(x) = \sqrt{3x-2}$ and $g(x) = \frac{x^2}{3} + \frac{2}{3}$

d $g(x) = -\frac{3}{4}x + 5$ and $h(x) = -\frac{4x-20}{3}$

2 Find the x-intercept and y-intercepts of the line $y = -4x + 2$. Explain how these points can help you to graph the inverse.

3 i For each function below, using your GDC, sketch its graph.

ii On the same set of axes, sketch the graph of its inverse.

iii Determine the equation of the inverse.

a $f(x) = -4x^2 + 4$

b $g(x) = -2\sqrt{x} + 5$ **c** $g: x \rightarrow \frac{1}{2}x + 6$

4 **a** For each function below, state the domain and range of its inverse. Use your GDC if necessary.

- **i** $f(x) = 2x^2 - 5x + 6$ or $x \geq 1.25$
- **ii** $f: x \to -3x + 1$ **iii** $g: x \to 2^x + 3$
- **iv** $g(x) = -\sqrt{-x+2} + 1$

b Explain how you can use the domain and range of a function to find the domain and range of its inverse.

5 Create a one-to-one function. State the equation of the function and of its inverse and explain why the inverse is also a one-to-one function.

6 If $f(x) = 2x - 5$:

- **a** solve $f(x) = 11$
- **b** find $f^{-1}(x)$
- **c** find $f^{-1}(11)$.
- **d** What do you notice about your answers to parts **a** and **c**?
- **e** Create a general rule to explain your answer for part **d**.

7 Given $f(x) = -2x - 1$ and $g(x) = -3x^2$, show that $g \circ f^{-1}(-1) = 0$.

Investigation 10

1 Draw the graph of the following functions:

a $f(x) = x$ **b** $f: x \to \frac{1}{x}$ **c** $g(x) = 3 - x$

2 For each function above, draw its inverse. What do you notice?

3 All of these functions are called **self-inverse functions.**

- **a** Write a definition of a self-inverse function.
- **b** Write an algebraic definition similar to the algebraic definition for regular inverses: $f(g(x)) = g(f(x)) = x$.

4 **Conceptual** How does a graph help you identify a self-inverse function?

5 The self inverse function $f(x) = x$ is also called the **identity function**. Write a definition of the identity function.

6 What does the identity function map? Think of inputs and outputs.

7 **Factual** What do you think the identity line is?

TOK

Do you think that mathematics is just the manipulation of symbols under a set of rules?

Example 21

Show that the function $f(x) = \frac{x}{x-1}$ is a self-inverse function.

$f(f(x)) = x$ $\dfrac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}\right)-1} = x$ $\dfrac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}\right)-\left(\frac{x-1}{x-1}\right)} = x$	We must show that $f(f(x)) = x$.

Continued on next page

Exercise 2J

1 Show graphically that $f(x) = -x$ is a self-inverse function.

2 **a** Show that $y = 3 - x$ is a self-inverse function.

  b Show that $y = -2 - x$ is a self-inverse function.

  c Show that $y = \frac{1}{2} - x$ is a self-inverse function.

  d Write a generalization from your answers in parts **a–c.**

3 Show that $f(x) = \frac{-x-2}{5x+1}$ is a self-inverse function.

4 Find the value of m such that $g(x) = \frac{2x-4}{x+m}$ is a self-inverse function.

International-mindedness

Which do you think is superior; the Bourbaki group analytical approach or the Mandelbrot visual approach to mathematics?

Developing your toolkit

Now do the Modelling and investigation activity on page 106.

Chapter summary

- A mathematical **relation** is a relationship between any set of ordered pairs.
- A **function** is a special type of relation. A relation is a function if each input relates to only one output. In other words, one value of the independent variable does not give more than one value of the dependent variable. A function cannot have a repeating x-value with different y-values.
- When writing an expression for a function, there are several different notations. For example, $f(x)$, $g(x)$, $f: \longrightarrow$. These are all called **functional notation**.

  $f(x)$, said as "f of x", represents a function f with independent variable x.
- Functional notation is a quick way of expressing substitution. Rather than saying "Given $y = 3x - 5$, find the value of y if $x = 2$", we can simply state: "Given $f(x) = 3x - 5$, find $f(2)$".
- **Draw** and **sketch** are two of the IB command terms you need to know.

  Draw: Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.

  Sketch: Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
- The **domain** of a function is all possible values of the independent variable, or x.

  The **range** of a function is all possible values of the dependent variable, or y.
- There are two different types of data we may encounter: discrete data and continuous data.

  Discrete data is graphed as separate, distinct points.

  Continuous data is graphed as lines or curves.

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Lines 1 and 4 are perpendicular, since $m_1 \times m_4 = \left(\frac{2}{3}\right)\times\left(-\frac{3}{2}\right) = -1$.	Since two lines L_1 and L_2 are perpendicular if $m_1 \times m_2 = -1$
Lines 2 and 3 are parallel, since $m_2 = m_3 = -\frac{2}{3}$.	Since two lines L_1 and L_2 are parallel if $m_1 = m_2$

Exercise 3B

1 Determine whether lines 1 and 2 which pass through the given points are parallel, perpendicular or neither.

a Line 1: (3, 6) and (6, 11)
Line 2: (4, −1) and (9, 2)

b Line 1: (5, −1) and (3, 7)
Line 2: (−1, 4) and (0, 0)

2 A line passes through the points (3, 2) and $(x, 5)$ and is perpendicular to a line with gradient $\frac{4}{3}$. Find the value of x.

3 Liam works up to 60 hours each week. His weekly pay, in dollars, depends on the number of hours he works, as shown in the graph.

a Find the gradient for each line segment in the graph.

b Explain the meaning of each gradient in the context of Liam's work.

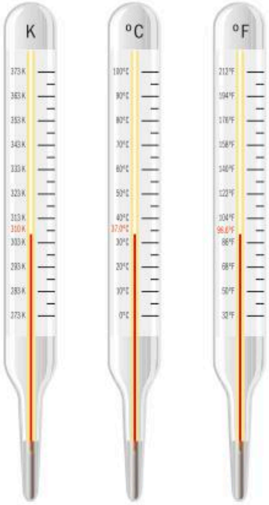
3.2 Linear functions

Why do we have more than one scale for measuring temperature?

What represents a greater increase in temperature: an increase of one-degree Celsius or an increase of one-degree Fahrenheit?

At what temperature is degrees Celsius equal to degrees Fahrenheit?

In section 3.1 you looked at the rate of change, or gradient, between two connected variables.



Exercise 3C

1 Write down the gradient and y-intercept for the following lines:

- **a** line 1: $y = 3x - 7$
- **b** line 2: $y = -\frac{2}{3}x + 4$
- **c** line 3: $y = -2$

2 Write down the equation of the line with gradient $\frac{1}{5}$ passing through $(0, 1)$.

3 Find the equation, in gradient-intercept form, of the following lines:

- **a** the line that passes through the point $(0, -1)$ and is parallel to the line $y = 4x - 3$
- **b** the line that passes though the points $(-3, -2)$ and $(1, 10)$

4 Write down the following:

- **a** the equation of the vertical line that passes through $(8, -1)$
- **b** the equation of the horizontal line that passes through the point $(-3, -10)$
- **c** the equation of the line perpendicular to $y = 1$ that passes through $(9, 5)$
- **d** the point of intersection of the lines $x = -2$ and $y = 7$

Different forms for the equation of a straight line

The equation of a straight line written in the form $y = mx + c$ is said to be written in **gradient-intercept form**.

Reflect Can you explain why this form of the equation of a straight line is called the gradient-intercept form?

There are, however, other ways in which you can write the equation of a straight line.

Consider a line that passes through the point (x_1, y_1) and let (x, y) be any other point on the line. Then the gradient of the line is given by $m = \frac{y - y_1}{x - x_1}$. This can be rewritten as $y - y_1 = m(x - x_1)$.

The **point-gradient form** of the equation of a line is $y - y_1 = m(x - x_1)$. In this equation the parameters tell you the gradient m and a point (x_1, y_1) that lies on the line.

TOK

Descartes showed that geometric problems could be solved algebraically and vice versa.

What does this tell us about mathematical representation and mathematical knowledge?

Example 7

Find the equation, in point-gradient form, of the following lines:

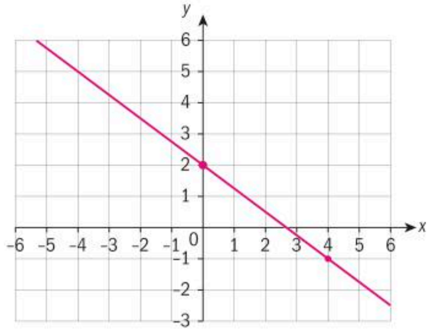
- **a** the line that passes through $(-4, 5)$ and is parallel to the line with equation $y = -\frac{1}{2}x - 3$
- **b** the line that passes through the points $(-1, 2)$ and $(3, -4)$.

Continued on next page

Example 10

a Draw the graph of $3x + 4y - 8 = 0$. **b** Sketch the graph of $3x - 2y + 9 = 0$.

a

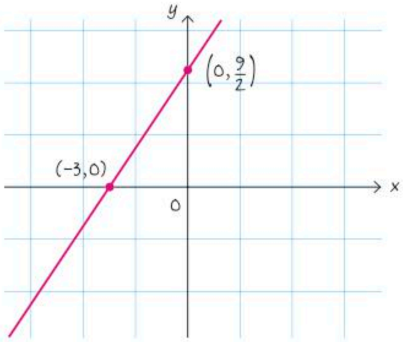


One method of graphing a line given in general form is to rewrite the equation in gradient-intercept form.

$$3x + 4y - 8 = 0$$
$$4y = -3x + 8$$
$$y = -\frac{3}{4}x + 2$$

You can now see that the y-intercept is $(0, 2)$ and gradient is $-\frac{3}{4}$. Use this information to draw the graph of the line.

b



Another method you can use to graph a line given in general form is to find the intercepts.

To find the x-intercept, substitute 0 for y.

$$3x - 2(0) + 9 = 0$$
$$3x = -9$$
$$x = -3$$

To find the y-intercept, substitute 0 for x.

$$3(0) - 2y + 9 = 0$$
$$-2y = -9$$
$$y = \frac{9}{2} = 4\frac{1}{2}$$

Notice that you were directed to **sketch,** rather than **draw** the graph. When sketching, you should show the key features of the graph, such as the axial intercepts of the line and the correct general shape of the graph.

Exercise 3E

1 Write the equation of each of these lines in the general form $ax + by + d = 0$ where a, b and d are integers.

a $y = \frac{1}{6}x - 3$

b The line with gradient $-\frac{2}{3}$ and y-intercept $(0, 4)$.

c The line with gradient -1 that passes through $(-3, 2)$.

2 Change the general form of the equation of each line to gradient-intercept form, $y = mx + c$.

a $3x + y - 5 = 0$

b $2x - 4y + 8 = 0$

c $5x + 2y + 7 = 0$

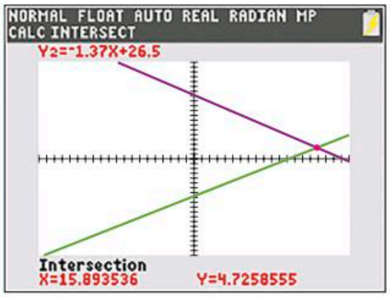
3 Sketch the graph of the line and label the coordinates of the axial intercepts.

a $x + 2y + 6 = 0$

b $2x - 6y + 8 = 0$

Example 12

Solve $1.26x - 15.3 = -1.37x + 26.5$ by using your GDC to find a point of intersection.



Intersection
X=15.893536 Y=4.7258555

$x = 15.9$

Graph the left-hand side and the right-hand side of the equation: $y = 1.26x - 15.3$ and $y = -1.37x + 26.5$.

You may need to adjust your viewing window to see the point of intersection.

Find the point of intersection. The x-coordinate of the point of intersection is the solution to the equation.

The answer is shown correct to 3 s.f.

Exercise 3F

1 Use your GDC to find the point of intersection for each pair of lines:

a $y = 2x - 1$ and $y = 3x + 1$

b $y = 2x + 1$ and $4x + 2y = 8$

c $y = 4.3x + 7.2$ and $y = 0.5x - 6.4$

d $2x - 3y = -1$ and $y = -\frac{3}{4}x + 2$

2 Solve each equation by using your GDC to find a point of intersection:

a $3x - 4 = 0.5x - 1.75$

b $6.28x + 15.3 = 2.29x - 4.85$

3 The following equations give the weekly salary an employee can earn at two different sales jobs, where x is the amount of sales in euros and y is the weekly salary in euros. Find the amount of sales for which the weekly salaries would be equal.

$y = 0.16x + 200$

$y = 0.10x + 300$

Linear functions

A function whose graph is a line is called a **linear function**. You can write a linear function as an equation, such as $y = 2x + 3$, or using function notation, $f(x) = 2x + 3$. You can think of $f(x)$ as another representation for y.

Exercise 3G

1 Consider the functions $f(x) = -x + 5$, $g(x) = 2x + 3$ and $h(x) = \frac{1}{3}x - 4$. Find the following:

a $f(3)$ **b** $g(0)$

c $h(6) - g(1)$ **d** $f(2) + g(-1)$

e $(f \circ g)(4)$ **f** $(h \circ f)(-7)$

g $(f \circ g)(x)$ **h** $(h \circ f)(x)$

2 Give the domain and range of each function:

a $f(x) = 3x + 8$ **b** $h(x) = x - 6$

3 Sketch a graph of the following:

- **a** a linear function with range {6}
- **b** a line that is not a function.

4 Find $f^{-1}(x)$ for each of the following linear functions. Give your answers in the form $f^{-1}(x) = mx + c$.

a $f(x)=\frac{1}{2}x+4$ **b** $f(x)=-3x+9$

Reflect Why are the gradients of a linear function and its inverse reciprocals?

Example 13

A video streaming service charges a monthly service fee of £6.99 and an additional £0.99 for each 24-hour download rental of a premium movie. The total monthly cost of this service, in GBP, is $f(x) = 0.99x + 6.99$, where x is the number of premium movie rentals that month.

- **a** Find the total monthly cost for a month in which there are five premium movie rentals.
- **b** Find $f^{-1}(x)$ and tell what x and $f^{-1}(x)$ represents in this function.
- **c** Find the number of premium move rentals downloaded in a month where the total monthly cost was £18.87.

a $f(5) = 0.99(5) + 6.99 = 11.94$ The total monthly cost is £11.94.	Substitute 5 for x and evaluate.
b $f(x) = 0.99x + 6.99$	
$y = 0.99x + 6.99$	Write the function in terms of x and y.
$x = 0.99y + 6.99$	Interchange x and y.
$x - 6.99 = 0.99y$ $y=\frac{x-6.99}{0.99}$	Solve for y.
$f^{-1}(x)=\frac{x-6.99}{0.99}$, where £$x$ is the total monthly cost and $f^{-1}(x)$ is the number of premium move rentals downloaded during the month	Express with the notation $f^{-1}(x)$.
c $f^{-1}(18.87)=\frac{18.87-6.99}{0.99}$ $= 12$ premium movie rentals	Find $f^{-1}(18.87)$.

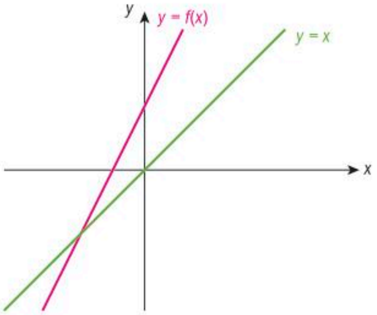
Exercise 3H

1 Find $f^{-1}(x)$ for each of the following linear functions. Give your answers in the form $f^{-1}(x) = mx + c$.

a $f(x) = 4x - 5$ **b** $f(x)=-\frac{1}{6}x+3$

c $f(x) = 0.25x + 1.75$

2 The graph of a linear function $y = f(x)$ and the line $y = x$ is shown below. Copy the graphs and then add a sketch of the graph of $y = f^{-1}(x)$.



3 A t-shirt company imprints logos on t-shirts. The company charges a one-time set-up fee of \$65 and \$10 per shirt. The total cost of x shirts, in CAD, is given by $f(x) = 10x + 65$.

a Find the total cost for 55 t-shirts.

b Find $f^{-1}(x)$ and tell what x and $f^{-1}(x)$ represent in this function.

c Find the number of t-shirts in an order with a total cost of \$5065.

Linear models

A linear function which describes the relationship between two variables which are connected in real-life is called a **linear model**.

A linear model is used to analyse and predict how the dependent variable will change in response to the independent variable changing.

International mindedness

The term "function" was introduced by the German mathematician Gottfried Wilhelm Leibniz in the 17th century, and the notation was coined by Swiss Leonard Euler in the 18th century.

Example 14

The table shows the relationship between the number of days a car is rented and the cost of the rental.

Number of days (x)	Cost in euros ($f(x)$)
1	30.25
2	60.50
3	90.75
4	121.00
5	151.25

a Without plotting a graph, show algebraically that the relationship between the number of days and the cost is a linear relationship.

b Find a linear model for this relationship. Give your answer as a function in gradient-intercept form.

c Find the cost of renting a car for seven days.

a $m = \dfrac{60.50 - 30.25}{2 - 1} = 30.25$

$m = \dfrac{90.75 - 60.50}{3 - 2} = 30.25$

You need to find the gradient of each line segment joining consecutive points in the table.

Continued on next page

c Use your model to explain whether an increase of one-degree Fahrenheit or an increase of one-degree Celsius is a greater change in temperature.

d Find the value at which the temperature in degrees Celsius is equal to the temperature in degrees Fahrenheit.

a $m = \frac{59-32}{15-0} = \frac{9}{5} = 1.8$	Find the gradient. You could use any two values given in the table.
$F(C) = 1.8C + 32$	The y-intercept is 32, so $b = 32$.
b $F(27) = 1.8(27) + 32 = 80.6$ The equivalent measure is 80.6°F.	Substitute 27 for C.
c For every increase of 1 degree Celsius, there is an increase of 1.8 degrees Fahrenheit. This indicates that an increase of one-degree Celsius is a greater change in temperature than an increase of one-degree Fahrenheit.	
d $x = 1.8x + 32$	Let C and F both equal x.
$-0.8x = 32$ $x = -40$ −40°C is the same temperature as −40°F.	Solve for x.

Exercise 3I

1 The force applied to a spring and the extension of the spring are connected by a linear relationship.

When a spring holds no mass, its extension is zero. When a force of 160 newtons (N) is applied, the extension of the spring is 5 cm.

a Find a linear model for the extension of the spring in terms of the force applied. Make sure you state clearly the variables you use to represent each quantity.

b Find the extension of the spring when a force of 370 N is applied.

2 Frank is a salesman. He is paid a basic weekly salary, and he also earns a percentage of commission on every sale he makes.

In a certain week, Frank makes sales totalling £1500. His total pay for that week is £600.

In another week, Frank makes sales totalling £2000. His total pay for that week is £680.

a Find, in gradient-intercept form, an equation that relates Frank's total weekly pay, £y, with his total sales revenue, £x.

b Explain the meaning of both the gradient and y-intercept in your model.

c Find Frank's total weekly pay when his weekly sales total £900.

3 A new fitness gym is offering two membership plans.

Plan A: A one-off enrollment fee of \$79.99, and a further monthly fee of \$9.99 per month

Plan B: no enrollment fee, and monthly fees of \$20.00 per month

a For each plan, find an expression for the total cost of a person's membership with the number of months that person has been a member. State clearly the variables you use.

After a certain number of months, Plan A becomes more cost-effective than Plan B.

b Use the models from part **a** to determine how many months a person needs to be a member before Plan A becomes more cost-effective than Plan B.

4 Liam works up to 60 hours each week. His weekly pay, £p, depends on the number of hours, h, he works. This information is presented in the graph shown.

a Find the equations for the piecewise function.

b Find the amount Liam is paid in a week that he works:

i 22 hours

ii 47 hours

5 Office Resource has 3000 printers available to sell in a certain month. On average, sales drop by 6.5 printers for each €1 increase in price. This is modelled by the demand function, $q = -6.5p + 3000$, where €p is the sales price and q is the number of printers sold during the month.

a Find the number of printers the model predicts Office Resource sells in a month, if the selling price is €200.

b Explain how raising the sales price by €20 affects the sales.

The manufacturer supplying the printers to Office Resource controls supply according to the function $q = 48p - 1600$, where q is the number of printers supplied during the month and €p is the price Office Resource must pay the manufacturer for each printer they supply. This function is known as the supply function.

c Find the price per printer Office Resource must pay to be supplied with 2000 printers a month.

d Graph the supply and demand functions on your GDC and then sketch the graphs on your paper.

e When the quantity supplied equals the quantity demanded the market is said to be in equilibrium. Find the equilibrium price and the equilibrium demand.

Reflect What type of real-life situations can be modelled by linear functions?

How can you use linear models in real-life situations?

Developing inquiry skills

In the opening scenario for this chapter you looked at how crates of emergency supplies were dropped from a plane.

The function $g(t) = -5t + 670$ gives the height of the crate when the parachute is open as a function of the number of seconds after the crate leaves the plane.

You were asked to explain what -5, the coefficient of t, represents. Do you agree with the answer you gave? If not, what is your answer now?

a The graph of $y = g(x)$ can be obtained from the graph of $y = f(x)$ by translating left 4 units and a reflection in the x-axis. $g(x) = -(x+4)^2$	Translation 4 units left $\Rightarrow h = -4$. Reflection in x-axis $\Rightarrow a = -1$.
b The graph of $y = g(x)$ can be obtained from the graph of $y = f(x)$ by translating right 3 units, a vertical compression with scale factor $\frac{1}{2}$, and translating up 2 units. $g(x) = \frac{1}{2}(x-3)^2 + 2$	Translation right 3 units $\Rightarrow h = 3$. Vertical compression with scale factor $\frac{1}{2}$ $\Rightarrow a = \frac{1}{2}$. Translation up 2 units $\Rightarrow h = 2$.

Exercise 3J

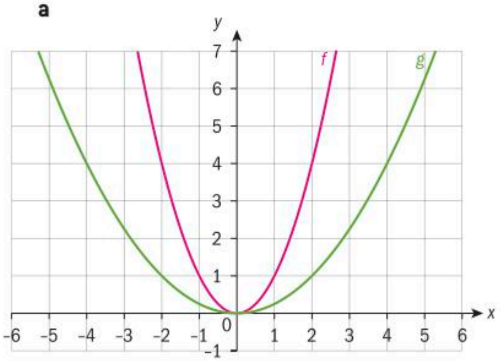
1 Sketch the parent quadratic, $y = x^2$, and the graph of $y = g(x)$ on the same axes. Then write down the coordinates of the vertex and the equation of the axis of symmetry for the graph of g.

a $g(x) = (x+3)^2$ **b** $g(x) = -x^2 + 4$

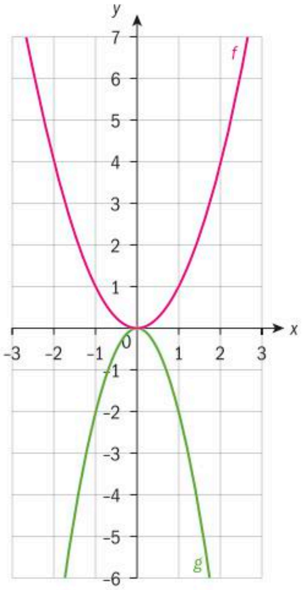
c $g(x) = \frac{1}{4}x^2$ **d** $g(x) = 2(x-4)^2 - 3$

2 Describe the transformations of the graph of $f(x) = x^2$ that lead to the graph of g. Then write an equation for $g(x)$.

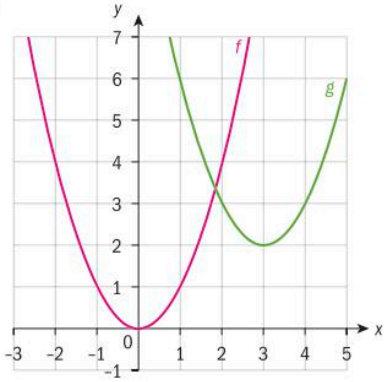
a



b

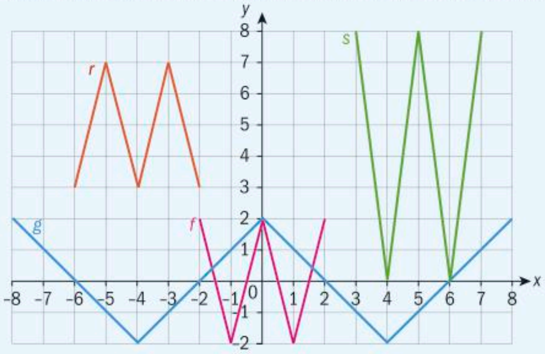


c



Example 19

Functions g, r and s are transformation of the graph of f. Find the functions g, r and s in terms of f.



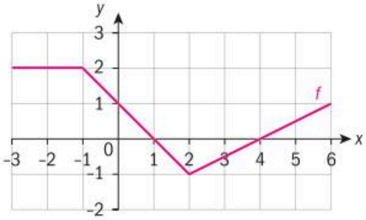
$g(x) = f\left(\frac{1}{4}x\right)$	The graph of g is a horizontal stretch of the graph of f with scale factor 4. $\frac{1}{q} = 4 \Rightarrow q = \frac{1}{4}$
$r(x) = -f(x+4) + 5$	The graph of r can be obtained by translating the graph of f left 4 units, reflecting in the x-axis, and then translating up 5 units. $h = -4$ and $k = 5$
$s(x) = 2f(x-5) + 4$	The graph of g can be obtained by translating the graph of f right 5 units, vertical stretching with scale factor 2, and translating up 4 units. $a = 2 \quad h = 5 \quad k = 4$

Reflect What is the relationship between the graphs of $y = f(x)$ and $y = f(-x)$?

The graph of $y = f(qx)$ is a horizontal stretch or compression of the graph of $y = f(x)$. For which values of q is the transformation a stretch rather than a compression?

Exercise 3K

1 The graph of $y = f(x)$, where $-3 \le x \le 6$, is shown. Copy the graph of f and draw these functions on the same axes.



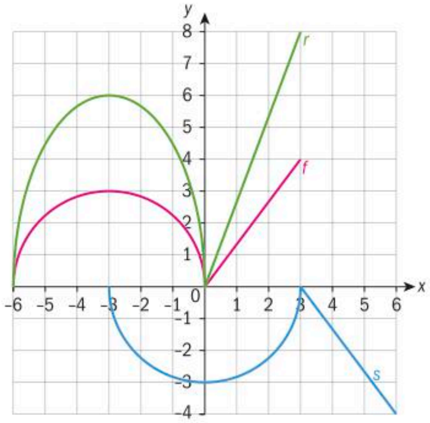
a $g(x) = f(-x)$ **b** $g(x) = -f(x)$

c $g(x) = f(2x)$ **d** $g(x) = 3f(x)$

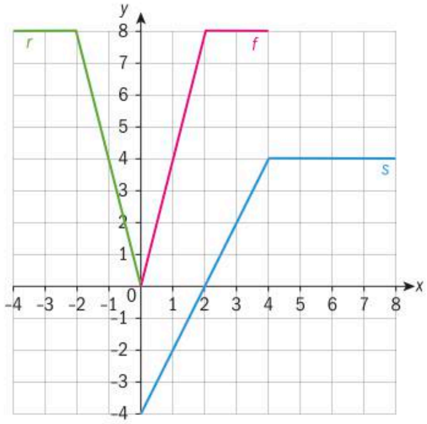
e $g(x) = f(x+6)$ **f** $g(x) = f(x) - 3$

2 The graphs of functions r and s are transformation of the graph of f. Find the functions r and s in terms of f.

a



b



3 The diagram shows the graph of $y = f(x)$, for $2 \le x \le 8$.

a Write down the range of f.

Let $g(x) = f(-x)$.

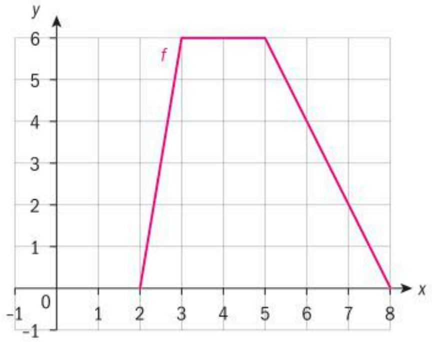
b Sketch the graph of g.

c Write down the domain of g.

The graph of h can be obtained by a vertical translation of the graph of g. The range of h is $-4 \le y \le 2$.

d Find the equation for h in terms of g.

e Find the equation for h in terms of f.



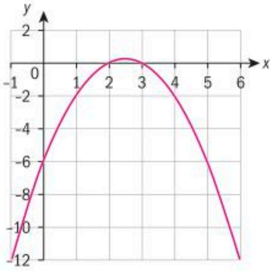
Developing inquiry skills

In the opening scenario for this chapter you looked at how crates of emergency supplies were dropped from a plane.

The function $h(t) = -4.9t^2 + 720$ gives the height of the crate during free fall.

How could you transform the parent graph $h(t) = t^2$ to give the function $h(t) = -4.9t^2 + 720$? What do these transformations tell you about the motion of the crate in this context?

d



Exercise 2A

1 **a** Functon

b Function

c Not a function as there are tickets with different costs (adults vs child), so the same number of tickets could a different total cost.

d Function

e Not a function

f Function

g Function

h Function

i Not a function

j Function

k Function

l Function

m Not a function

2 Answers will vary

3 “All functions are relations, but not all relations are functions.”

Exercise 2B

1 **a** $g(-4) = -14$

b $f: \rightarrow -9 = -46$

c $C(100) = 2250$

d $h(5) = -4$

e 3 **f** 5

g 1

2 **a** $f(-3) = -28$

b $g(15) = -53$

c $f(1) + g(-1) = 7$

d 6

e $f(x-2) = -3x^2 + 12x - 12$

f $g(n) = -4n + 7$

g −24

h $f(x+1) = -3x^2 - 6x - 4$

$g(x-2) = -4x + 15$

$f(x+1) \times g(x-2)$
$= 12x^3 - 21x^2 - 74x - 60$

3 **a** Yes, it is a function. Every value of t will yield only one value of d.

b $d(t) = -75t + 275$

c $d(0) = 275$ km

d $0 < t < n$, where n is the amount of time it takes to drive to Perth.

4 **a** Yes, it is a function. Every temperature in Celsius will only yield one temperature in Fahrenheit.

b $F(17)$ is asking what temperature in °F is equivalent to 17 °C.

$F(17) = 62.6$°F.

c $F(C) = 100$ is asking what temperature in °C is equivalent to 100°F.

$C = 37.7 \approx 37.8$°C

d $F(0) = 32$°F

e $F(100) = 212$°F

f $F(38.75) = 101.75$ °F

g $C = 176.\bar{6} \approx 177$°C

5 **a** $C(g) = 10g + 25$

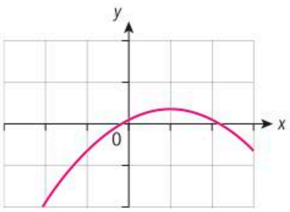
b ~~$g > 0$~~ $g < 0$

c $C(14) = 165$

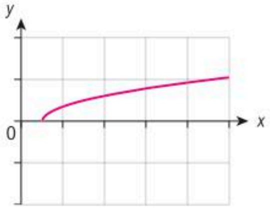
d $g = 7.5$ gigs

Exercise 2C

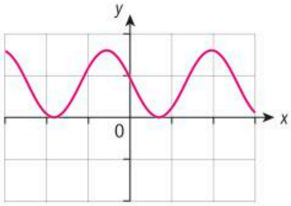
1 **a**



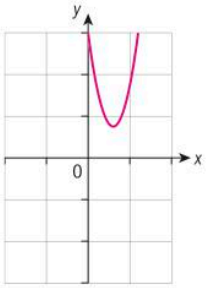
b



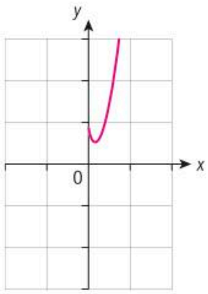
c



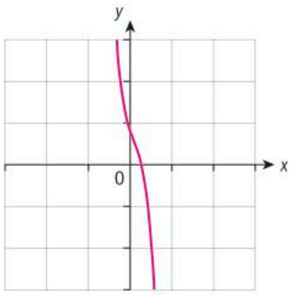
2 **a**



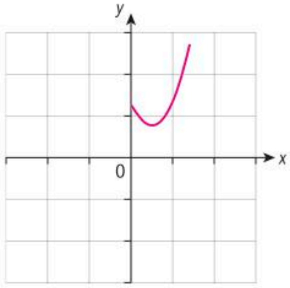
b



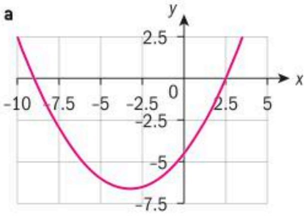
c



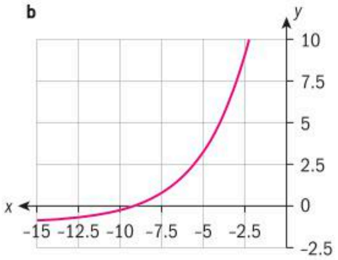
d



3 **a**



b



Exercise 2D

1 **a** Not a function

b Domain: {−5, −1, 3}

Range: {4, 6, 14}

c Domain: {−12, −8, −5}

Range: {−8, 7}

d Not a function

e Domain: $x \in \mathbb{R}$,] −∞, ∞[or (−∞, ∞)

Range: $y \in \mathbb{R}$,] −∞, ∞[or(−∞, ∞)

f Domain: $x \in \mathbb{R}$,] −∞, ∞[or (−∞, ∞)

Range: {4}

g Not a function

h Domain: $x \in \mathbb{R}$,] −∞, ∞[or (−∞, ∞)

Range: $3 \le y \le 5$, [3, 5]

i Domain: $x \ge 0$, [0, ∞[or [0, ∞)

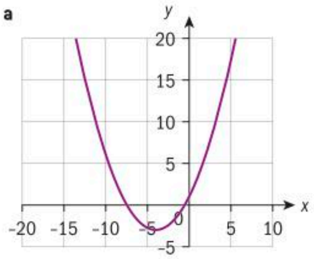
Range: $y \le 0$,] −∞, 0] or (−∞, 0]

i Not a function

k. domain: x≤-1,x≥1
range: y≥3

l. not a function.

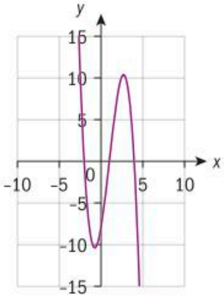
2 **a**



Domain: $x \in \mathbb{R}$,] −∞,∞[or(−∞,∞)

Range: $y \in \mathbb{R}$,] −∞,∞[or (−∞,∞)

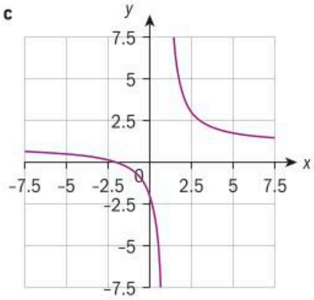
b



Domain: $x \in \mathbb{R}, x \ne 1$,] −∞, 1 [∪]1, ∞[or (−∞, 1)∪(1, ∞)

Range: $y \in \mathbb{R}, y \ne 0$,] −∞, 0[∪]0, ∞[or (−∞, 0)∪(0, ∞)

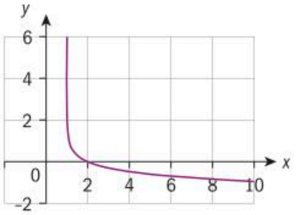
c



Domain: $x > 1$,]1, ∞[or (1, ∞)

Range: $y > -2$,] −2, ∞[or (−2, ∞)

d

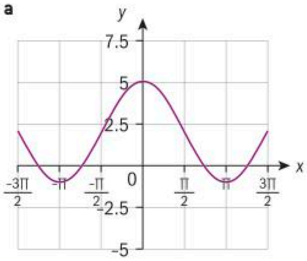


Domain: $x \in \mathbb{R}$,] −∞,∞ [or(−∞, ∞)

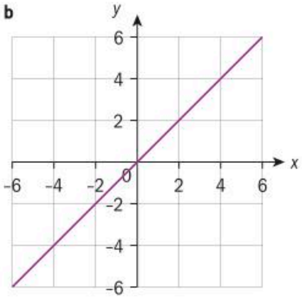
Range: $-1 \le y \le 5$, [−1, 5]

3 Answers will vary.

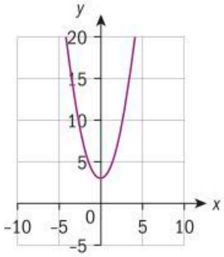
a



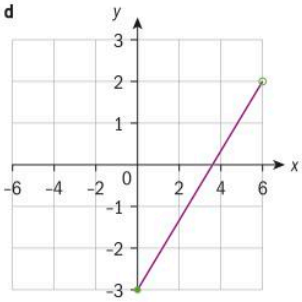
b



c



d



4 **a** **i** 4 **ii** 2

b

c $0 \le x \le 10; 0 \le y \le 4$

5 $f(x) = \begin{cases} -x, & -3 \le x \le -1 \\ 2x+3, & -1 < x \le 2 \\ 7, & 2 < x \le 6 \end{cases}$

It is also possible to include −1 in the second interval rather than the first and 2 in the third interval rather than the second.

Exercise 2E

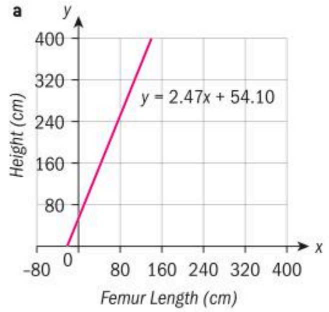
1 **a** $C(n) = 40 + 21n$, where C is the cost and n is the number of hours.

b Domain: $x \ge 0$, $[0, \infty[$ or $[0, \infty)$

Range: $y \ge 0$, $[0, \infty[$ or $[0, \infty)$

c $C(4) = \$14$

2 **a**



b Domain eg: $10 \le f \le 80$

Range eg: $75.8 \le h \le 228$

c $h(51) \sim 180$ cm

d $f \approx 43.3$ cm

3 **a** $I(t) = 10000\left(1 + \frac{0.025}{12}\right)^{12t}$

b The equation satisfies the vertical line test.

c Domain: $t \in \mathbb{R}, t \ge 0$. Range: $[10000, \infty)$.

d Javier needs 27 years and 10 months to double his money.

Exercise 2F

1 **a** **i** $f(g(x)) = -16x^2 + 36x - 6$

ii $f(f(x)) = -x^4 + 10x^3 - 30x^2 + 25x$

iii $f(h(x)) = -x + 3\sqrt{x} + 4$

iv $g \circ h(x) = 4\sqrt{x} + 2$

v $f(-1) = -6$

$f \circ f(-1) = -66$

$f \circ f \circ f(-1) = 4686$

vi $g(h(9)) = 14$

vii $g \circ f(2) = 22$

$f \circ g(2) = -6$

$$g \circ f(2) + f \circ g(2) = 22 - 6 = 16$$

b **i** $x \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

ii $x \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

iii $x \ge 0$ or $[0, \infty[$ or $[0, \infty)$

iv $x \ge 0$ or $[0, \infty[$ or $[0, \infty)$

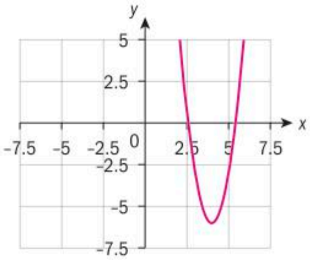
2 Answers will vary.

3 **a** $f(g(x)) = -8x + 7$

b $x = -\frac{5}{8}$

4 **a** $f(g(x)) = 3x^2 - 24x + 42$

b



c Domain: $x \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

Range: $y \ge -6$ *or* $[-6, \infty[$ *or* $[-6, \infty)$

5 **a** $f(x) = x + 25$

b $g(x) = 1.06x$

c $f(g(x)) = 1.06x + 25$; this represented only paying tax on the price of the fridge.

$g(f(x)) = 1.06(x + 25)$; this represents paying tax on both the price of the fridge and the delivery fee.

d $f(g(x))$

Exercise 2G

1 **a** **i** $g(-2) = 3$

$f(g(-2)) = f(3) = 2$

ii $f(5) = 6$

$g(f(5)) = g(6) = -3$

iii $g(3) = 11$

$g(g(3)) = g(11) = 0$

b For $f(x)$:

Domain: $\{2, 3, 5, 10\}$

Range: $\{-4, 1, 2, 6\}$

For $g(x)$:

Domain: $\{-2, 3, 6, 11\}$

Range: $\{-3, 0, 2, 11\}$

2 **a** $g(3) = 7$

$f \circ g(3) = f(7) = -2$

b $f(-1) = 9$

$g \circ f(-1) = g(9) = -4$

c $f(9) = -1$

$f \circ f(9) = f(-1) = 9$

3 **a** $f(0) = 0$

$g(f(0)) = g(0) = -4$

b $f(1) = 1$

$g(f(1)) = g(1) = -3$

c $g(-2) = 0$

$f(g(-2)) = f(0) = 0$

d $g(-0) = -4$

$f(g(0)) = f(-4) = 4$

4 Answers will vary; $g(x)$ contains a point with an x-coordinate of −1; $f(x)$ must have a point with a y-coordinate of 2.

Exercise 2H

1 **a** $f(n) = x - 100$

$g(n) = 2.20n$

b $f(n)$ represents that you receive commission on every new person who signs up after the first 100 people.

$g(n)$ represents that you receive 2.20 GBP for each person (after the first 100) who sign up.

c **i** $f(224) = 124$

ii $g(n) = 272.80$ GBP

d **i** $S(276) = 387.20$ GBP

ii $x = 152$ people

2 **a** $f(x) = x - \$25$; this could represent \$25 off the price of the TV.

$g(x) = 1.10x$; this could represent a tax of 10%.

b **i** You paid $699.99 - 25$ $= \$674.99$

ii After tax, the TV cost 1.10×674.99 $= 742.489 \approx \$742.49$

c **i** $P(x) = \$1.10(x - \$25) + 49.99$

ii $P(525.99) \approx \$601.08$

3 Answer will vary.

Exercise 2I

1 **a** $f(g(x)) = -x - 2$

Since $f(g(x)) \neq x$, these are not inverses.

b $f(g(x)) = x$

$g(f(x)) = x$

Since $f(g(x)) = g(f(x)) = x$, these are inverses.

c $f(g(x)) = x$

$g(f(x)) = x$

Since $f(g(x)) = g(f(x)) = x$, these are inverses.

d $g(h(x)) = x$

$h(g(x)) = x$

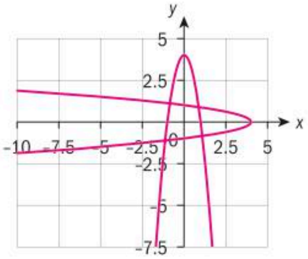
Since $f(g(x)) = g(f(x)) = x$, these are inverses.

2 x-intercept: $\left(\frac{1}{2}, 0\right)$

y-intercept: $(0, 2)$

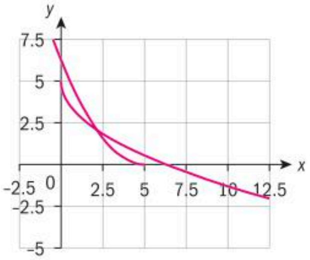
Since you only need two points to graph a line, you can switch the coordinates to find two point that the inverse passes through: $\left(0, \frac{1}{2}\right)$ and $(2, 0)$.

3 **a** **i** & **ii**



iii $f^{-1}(x) = \pm\sqrt{\frac{-x}{4} + 1}$

b **i** & **ii**



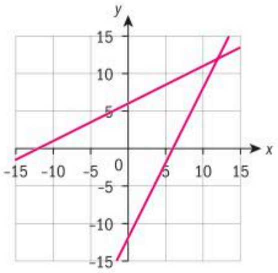
iii $g^{-1}(x) = \left(\frac{-x+5}{2}\right)^2, x \geq 0$

Note 1: The domain restriction is needed since the original function $g(x) = -2\sqrt{x} + 5$ would have the same restriction.

Note 2: The inverse, $g^{-1}(x)$, can be simplified further if desired:

$y = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{25}{4}, x \geq 0$

c **i** & **ii**



iii $g : x \rightarrow \frac{1}{2}x + 6$

$y = \frac{1}{2}x + 6$

$x = \frac{1}{2}y + 6$

$\frac{1}{2}y = x - 6$

$y = 2x - 12$

$g^{-1}(x) = 2x - 12$

4 **a** **i** Domain: $x \geq X$ or $[X, \infty[$ or $[X, \infty)$

Range: $y \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

ii Domain: $x \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

Range: $y \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

iii Domain: $x \in \mathbb{R}$ or $]-\infty, \infty[$ or $(-\infty, \infty)$

Range: $y > 3$ or $]3, \infty[$ or $(3, \infty)$

iv Domain: $x \leq 1$ or $[-\infty, 1]$ or $(-\infty, 1]$

Range: $y \leq 2$ or $[-\infty, 2]$ or $(-\infty, 2]$

b The domain of the function becomes the range of its inverse, and the range of the function becomes the domain of its inverse.

5 Answers will vary. In order for the function to be a one-to-one function, the inverse must be a function.

6 **a** $x = 8$

b $2y = x + 5$

$$y = \frac{x+5}{2}$$

$$f^{-1}(x) = \frac{x+5}{2}$$

c $f^{-1}(11) = \frac{11+5}{2}$

$$f^{-1}(11) = \frac{16}{2}$$

$$f^{-1}(11) = 8$$

d $f(x) = 11$ gives the same answer as $f^{-1}(11)$.

e $f(x) = n = f^{-1}(n)$

7 $f(x) = -2x - 1$

$y = -2x - 1$

$x = -2y - 1$

$2y = -x - 1$

$$y = \frac{-x-1}{2}$$

$$f^{-1}(x) = \frac{-x-1}{2}$$

$$g \circ f^{-1}(x) = -3\left(\frac{-x-1}{2}\right)^2$$

$$g \circ f^{-1}(x) = -3\frac{(-x-1)^2}{4}$$

$$g \circ f^{-1}(x) = -3\frac{(x^2+2x+1)}{4}$$

$$g \circ f^{-1}(x) = \frac{-3x^2-6x-3}{4}$$

$$g \circ f^{-1}(-1) = \frac{-3(-1)^2-6(-1)-3}{4}$$

$$g \circ f^{-1}(-1) = \frac{-3(1)-6(-1)-3}{4}$$

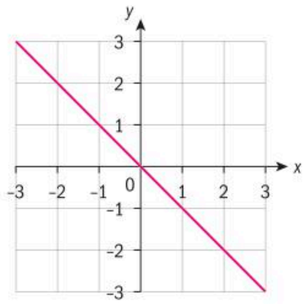
$$g \circ f^{-1}(-1) = \frac{-3+6-3}{4}$$

$$g \circ f^{-1}(-1) = \frac{0}{4}$$

$$g \circ f^{-1}(-1) = 0$$

Exercise 2J

1



2 **a** $f(f(x)) = x$

$3 - (3 - x) = x$

$3 - 3 + x = x$

$x = x$

b $f(f(x)) = x$

$-2 - (-2 - x) = x$

$-2 + 2 + x = x$

$x = x$

c $f(f(x)) = x$

$$\frac{1}{2} - \left(\frac{1}{2} - x\right) = x$$

$$\frac{1}{2} - \frac{1}{2} - x = x$$

$x = x$

d $f(x) = n - x$, $n \in \mathbb{R}$ is a self-inverse.

3 $f(f(x)) = x$

$$\frac{-\left(\frac{-x-2}{5x+1}\right)-2}{5\left(\frac{-x-2}{5x+1}\right)+1} = x$$

$$\frac{\frac{(x+2)}{(5x+1)} - \frac{2(5x+1)}{(5x+1)}}{\frac{(-5x-10)}{(5x+1)} + \frac{(5x+1)}{(5x+1)}} = x$$

$$\frac{\frac{x+2-10x-2}{5x+1}}{\frac{-9}{5x+1}} = x$$

$$\frac{-9x}{-9} = x$$

$x = x$

4 $f(f(x)) = x$

$$\frac{2\left(\frac{2x-4}{x+m}\right)-4}{\left(\frac{2x-4}{x+m}\right)+m} = x$$

$$\frac{\left(\frac{4x-8}{x+m}\right)-4\left(\frac{x+m}{x+m}\right)}{\left(\frac{2x-4}{x+m}\right)+m\left(\frac{x+m}{x+m}\right)} = x$$

$$\frac{\frac{4x-8-4x-4m}{x+m}}{\frac{2x-4+xm+m^2}{x+m}} = x$$

$$\frac{-4m-8}{2x-4+xm+m^2} = x$$

$-4m - 8 = x(2x - 4 + xm + m^2)$

$-4m - 8 = 2x^2 - 4x + x^2m + m^2$

$-4m - 8 = (2 + m)x^2 - 4x + m^2$

$0 = 2 + m$

$m = -2$

Chapter review

1 **a** Yes **b** No **c** Yes

d No **e** Yes **f** Yes

g Yes **h** No **i** Yes

j Yes **k** Yes **l** No

m No

2 **a** Domain: {−5, −1, 0, 1, 4, 9}

Range: {−8, −1, 0, 1, 6, 9}

b Domain: {0, 2, 3, 4}

Range: {−2, 2, 3}

c Domain: {−8, −5, 0, 1}

Range: {−2, 2, 3}

d Domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$ or $]-\infty, \infty[$

Range: $y \in \mathbb{R}$ or $(-\infty, \infty)$ or $]-\infty, \infty[$

e Domain: $-3 < x \leq 3$ or $(-3, 3]$ or $]-3, 3]$

Range: $-3 < x \leq -1$ or $(-3, -1]$ or $]-3, -1]$

f Domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$ or $]-\infty, \infty[$

Range: $x \geq -12.25$ or $[12.25, \infty)$ or $[12.25, \infty[$

Exercise 3B

1 **a** $m_1 = \frac{5}{3}$ $m_2 = \frac{3}{5}$ neither

b $m_1 = -4$ $m_2 = -4$ parallel

2 -1

3 **a** 8, 12

b For the first 40 hours Liam works in a week he is paid \$8 per hour. For each hour over 40 hours that Liam works, he earns \$12 per hour.

Exercise 3C

1 **a** $m = 3$; $(0, -7)$

b $m = -\frac{2}{3}$; $(0, 4)$

c $m = 0$; $(0, -2)$

2 $y = \frac{1}{5}x + 1$

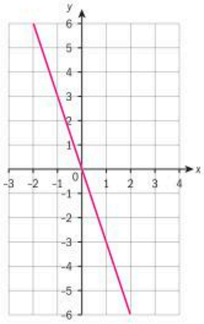
3 **a** $y = 4x - 1$ **b** $y = 3x + 7$

4 **a** $x = 8$ **b** $y = -10$

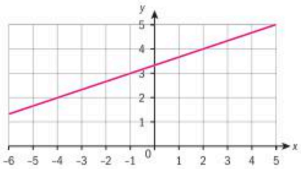
c $x = 9$ **d** $(-2, 7)$

Exercise 3D

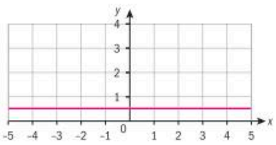
1 **a**



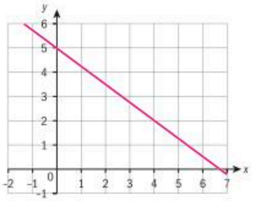
b



c



d



2 $y - 6 = -3(x - 2)$

3 **a** -3

b $y + 4 = -3(x + 3)$ or $y - 2 = -3(x + 5)$

c Change the equations to slope-gradient form: $y = -3x - 13$

Exercise 3E

1 **a** $x - 6y - 18 = 0$

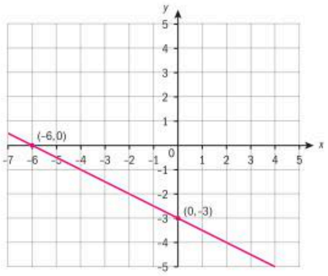
b $2x + 3y - 12 = 0$

c $x + y + 1 = 0$

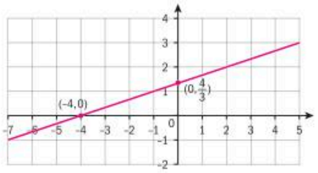
2 **a** $y = -3x + 5$ **b** $y = \frac{1}{2}x + 2$

c $y = -\frac{5}{2}x - \frac{7}{2}$

3 **a**



b



Exercise 3F

1 **a** $(-2, -5)$ **b** $(0.75, 2.5)$

c $(-3.58, -8.19)$

d $(1.18, 1.12)$

2 **a** 0.9

b -5.05

3 \$1666.67

Exercise 3G

1 **a** 2 **b** 3 **c** -7

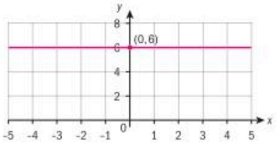
d 4 **e** -6 **f** 0

g $-2x + 2$ **h** $-\frac{1}{9}x - \frac{7}{3}$

2 **a** **Domain:** all real numbers
Range: all real numbers

b **Domain:** all real numbers
Range: all real numbers

3 **a**



b No vertical line is a function as the y corresponding to the x-coordinate of the x-intercept is not unique (in fact, any y corresponds to it).

4 **a** $f^{-1}(x) = 2x - 8$

b $f^{-1}(x) = -\frac{1}{3}x + 3$

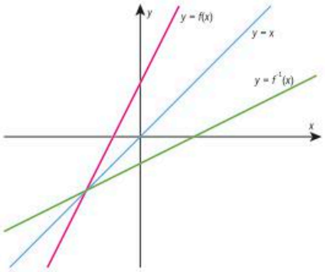
Exercise 3H

1 **a** $f^{-1}(x) = \frac{1}{4}x + \frac{5}{4}$

b $f^{-1}(x) = -6x + 18$

c $f^{-1}(x) = 4x - 7$

2



3 **a** \$615

b $f^{-1}(x) = \frac{1}{10}x - 6.5$, where x is the total cost of a t-shirt order in dollars and $f^{-1}(x)$ is the number of t-shirts in the order

c 500 t-shirts

Exercise 3I

1 **a** $d = \frac{1}{32}F$ or $F = 32d$, where d = distance in cm and F = force in newtons

b 11.5625 cm (exact) or 11.6 cm to 3 s.f.

2 **a** $y = 0.16x + 360$

b The y-intercept represents Frank's weekly salary of £360. The gradient shows that Frank's commission is 16% of his sales.

c £504

3 **a** Plan A: $c(m) = 9.99m + 79.99$ and Plan B: $c(m) = 20m$, where c is the cost for m months at the gym

b Month 8

4 **a**

$$p(h) = \begin{cases} 8h, & 0 \le h \le 40 \\ 12h - 160, & 40 < h \le 60 \end{cases}$$

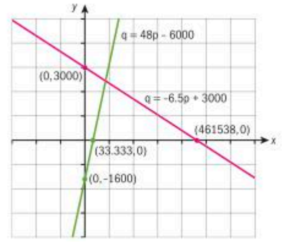
b **i** £176 **ii** £404

5 **a** 1700

b The model predicts that raising the price €20 will result in 130 fewer printers sold.

c 75 Euro

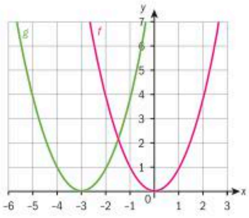
d



e €75

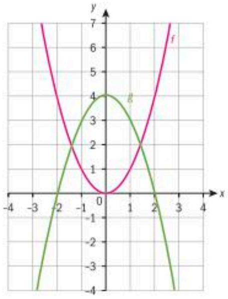
Exercise 3J

1 **a**



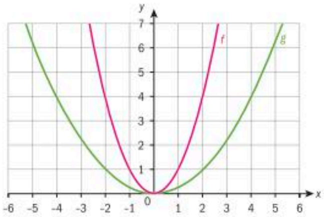
Axis: $x = -3$ vertex: $(-3, 0)$

b



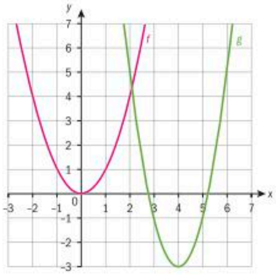
Axis: $x = 0$ vertex: $(0, 4)$

c



Axis: $x = 0$ vertex: $(0, 0)$

d



Axis: $x = 4$ vertex: $(4, -3)$

2 **a** Vertical compression with scale factor $\frac{1}{4}$; $g(x) = \frac{1}{4}x^2$

b Reflection in ***x***-axis, vertical stretch with scale factor 2; $g(x) = -2x^2$

c Horizontal translation right 3, vertical translation up 2; $g(x) = (x-3)^2 + 2$

d Horizontal translation left 3, vertical stretch with scale factor $\frac{3}{2}$, vertical translation down 5; $g(x) = \frac{3}{2}(x+3)^2 - 5$

Exercise 3K

1 **a** The graph is reflected about the y-axis.

b The graph is reflected about the x-axis.

c The graph is compressed horizontally with scale factor $\frac{1}{2}$.

d The graph is stretched vertically with scale factor 3.

e The graph is translated to the left by 6 units.

f The graph is translated downwards by 3 units.

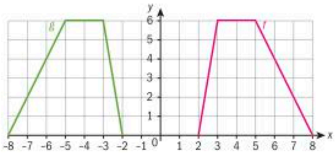
2 **a** $r(x) = 2f(x)$; $s(x) = -f(x-3)$

b $r(x) = f(-x)$;

$s(x) = f\left(\frac{1}{2}x\right) - 4$

3 **a** $0 \le y \le 6$

b



c $-8 \le x \le -2$

d $h(x) = g(x) - 4$

e $h(x) = f(-x) - 4$

Exercise 3L

1 x-intercepts: $(-2.81, 0)$, $(0.475, 0)$; y-intercept: $(0, -4)$; vertex: $(-1.17, -8.08)$

2 x-intercepts: none; y-intercept: $(0, -3)$; vertex: $(0.726, -0.785)$

3 **Domain:** $x \in \mathbb{R}$
Range: $f(x) \le 9.125$