*IPMU MS seminar*
*2015 Feb 12th*

# Superconformal Quantum Mechanics from M2-branes

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# Motivations

## M5-branes

| | | |
|---|---|---|
| $R^{1,3} \times \Sigma_g$ | AGT relations | '09 Alday Gaiotto Tachikawa |
| $R^{1,2} \times M_3$ | 3d-3d relations | '11 Dimofte Gaiotto Gukov; Terashima Yamazaki |
| $R^{1,1} \times M_4$ | 2d-4d relations | '13 Gadde Gukov Putrov |

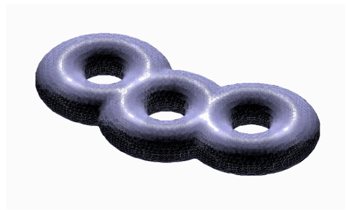
**SCFT on $R^{1,5-d}$ ⟺ Geometry $M_d$**

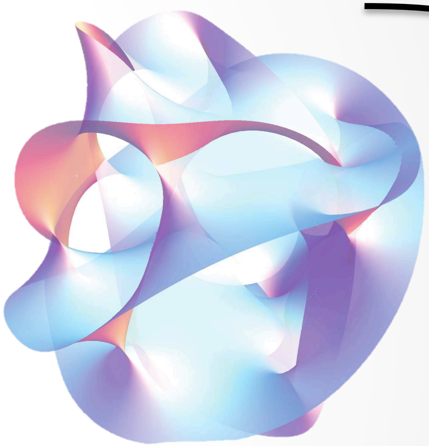
**Q. How about curved M2-branes ?**

# Idea

**M2-branes**

$$\mathbf{R} \times \left[ \Sigma_g \subset \mathbf{CY} \right]$$





**SCQM**

# Outline

**I. SCQM**
(Superconformal Quantum Mechanics)

**II. M2-brane**
(BLG-model & ABJM-model)

Review

**III. SCQM from M2-branes**

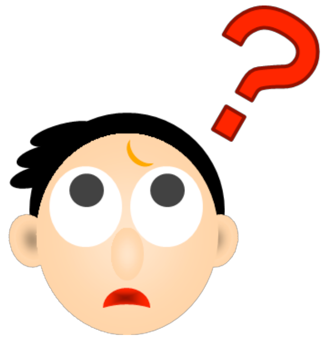
My work !

**IV. Conclusion**

# I. SCQM

# Conf. Transf.

conformal transformation ⇒ transf. which locally preserves the angle between two points



**No angle in 1d・・・**

More precisely

$$\eta_{\rho\sigma} \frac{\partial x'^{\rho}}{\partial x^{\mu}} \frac{\partial x'^{\sigma}}{\partial x^{\nu}} = \Lambda(x)\, \eta_{\mu\nu}$$

generators

- translation $P_\mu$
- dilatation $D$
- ~~rotation~~ $L_{\mu\nu}$
- conformal boost $K_\mu$

**1d conf. transf. => 3 generators H, D, K**

# CQM

Q. How to construct
Confroma Quantum Mechanics (CQM) ?

scale inv. scalar field theory

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - g\phi^{\frac{2d}{d-2}}$$

↓ d=1

$$S = \frac{1}{2}\int dt(\dot{x}^2 - \frac{g}{x^2})$$

**DFF model**

'76 de Alfaro Fubini Furlan

finite conf. transf.

$$t' = \frac{at+b}{ct+d} \qquad x'(t') = \frac{x(t)}{ct+d} \qquad A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \det A = 1$$

1. Translation

$$t' = t - \epsilon_1 \qquad x'(t') = x(t) \qquad A = \begin{pmatrix} 1 & 0 \\ -\epsilon_1 & 1 \end{pmatrix}$$

2. Dilatation

$$t' = e^{-\epsilon_2} t \qquad x'(t') = e^{-\frac{\epsilon_2}{2}} x(t) \qquad A = \begin{pmatrix} e^{-\frac{\epsilon_2}{2}} & 0 \\ 0 & e^{\frac{\epsilon_2}{2}} \end{pmatrix}$$

3. Conformal boost

$$t' = \frac{t}{\epsilon_3 t + 1} \qquad x'(t') = \frac{x(t)}{\epsilon_3 t + 1} \qquad A = \begin{pmatrix} 1 & \epsilon_3 \\ 0 & 1 \end{pmatrix}$$

**Conformal invariant !**

# infinitesimal conf. transf.

$$\delta t = f(t) \qquad \delta x = \frac{1}{2}\dot{f}x \qquad f(t) = a + bt + ct^2$$

$a \to$ H, $b \to$ D, $c \to$ K

⬇ Noether's thm.

Hamiltonian $$H = \frac{1}{2}(p^2 + \frac{g}{x^2})$$

Dilatation generator $$D = tH - \frac{1}{4}(xp + px)$$

Conformal boost generator $$K = t^2H - \frac{1}{2}t(xp + px) + \frac{1}{2}x^2$$

# sl(2,R) conformal alg.

$$[H, D] = iH \qquad [K, D] = -iK \quad [H, K] = 2iD$$

CQM is expected to be solved algebraically !

# But story is not so simple • • •

i. no normalizable ground state !
ii. continuous energy spectrum !

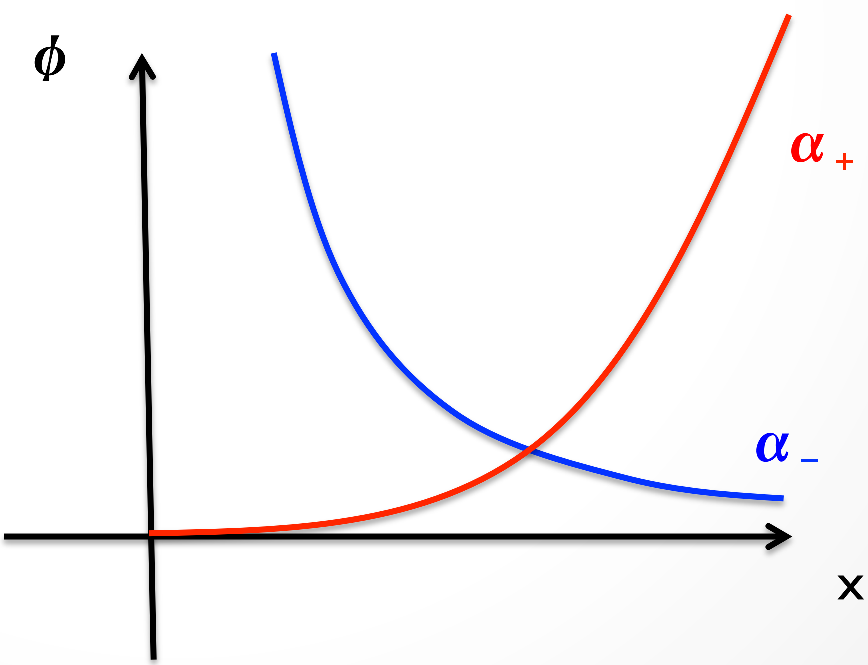
$$\left(-\frac{d^2}{dx^2}+\frac{g}{x^2}\right)\phi(x)=0$$

$$\phi(x)=x^\alpha$$

$$\alpha_+=\frac{1+\sqrt{1+4g}}{2}$$

$$\alpha_-=\frac{1-\sqrt{1+4g}}{2}$$

**Non-normalizable wavefct**



# DFF's proposal

$$G = uH + vD + wK \longrightarrow \frac{\partial G}{\partial t} + i[H, G] = 0.$$

**G is constant of motion**

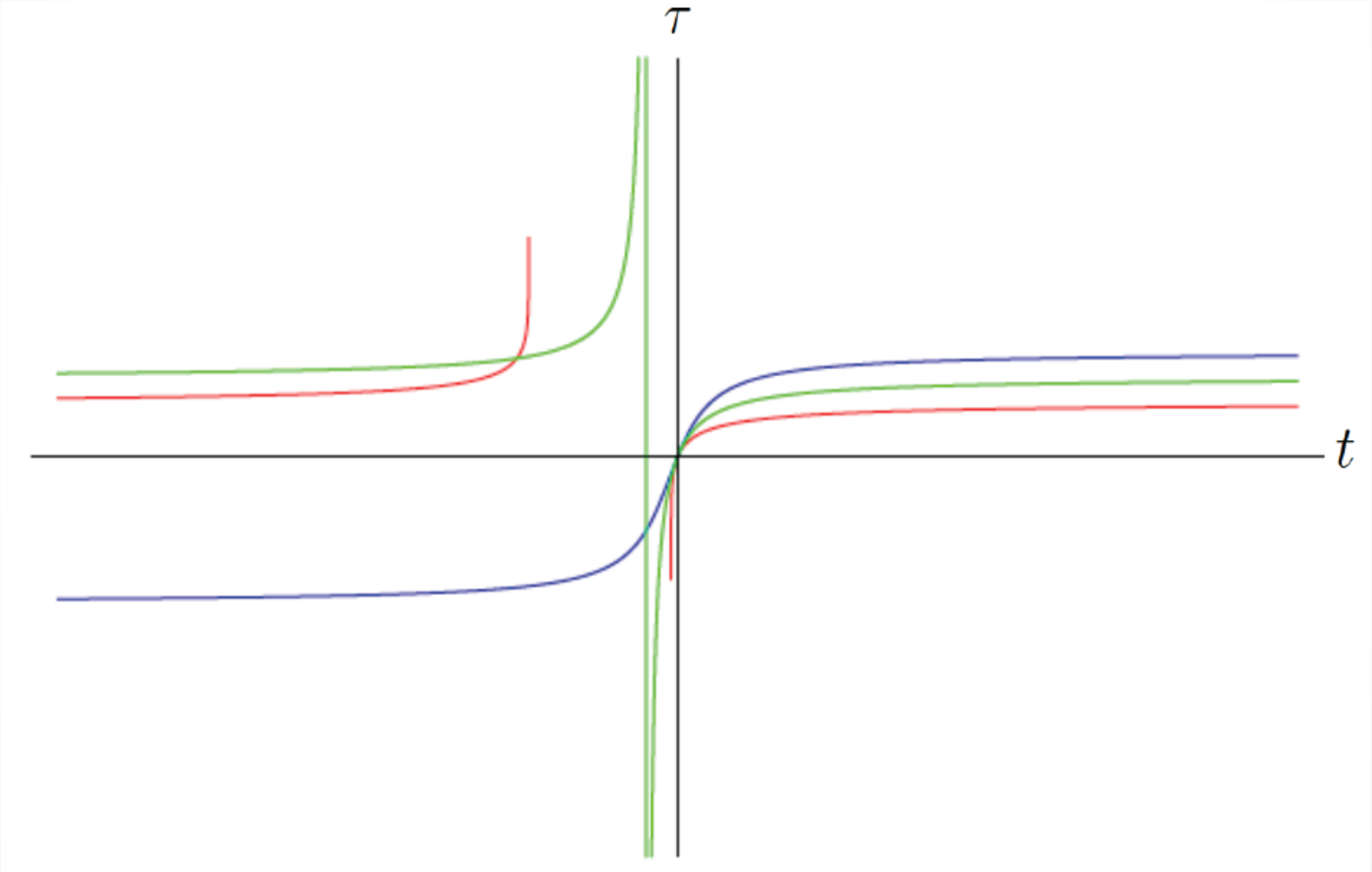
=> G can be used as the new Hamiltonian describing the time evolution

$$d\tau = \frac{dt}{u + vt + wt^2}$$

$$q(\tau) = \frac{x(t)}{\sqrt{u + vt + wt^2}}$$

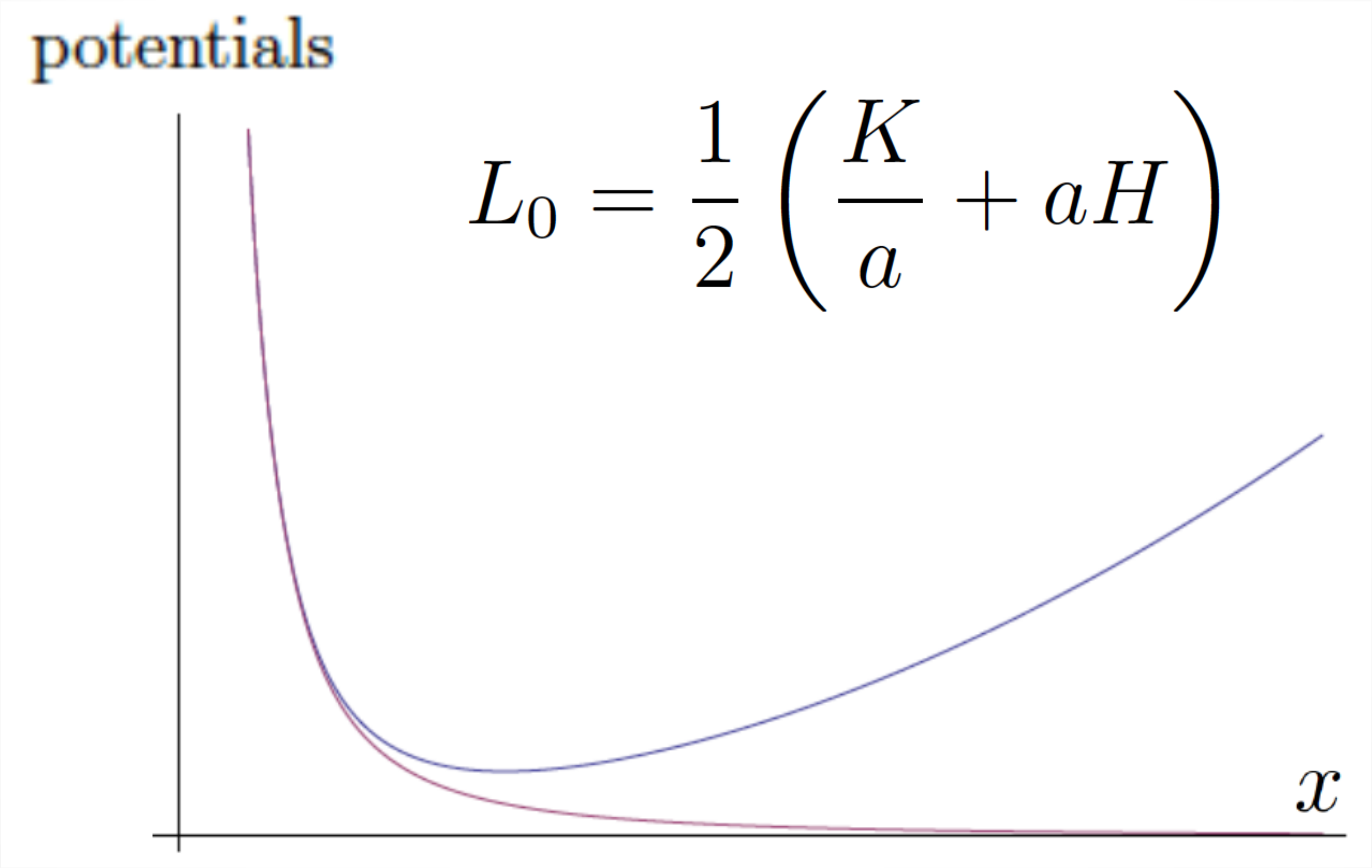
**G can be non-compact !**

$$\Delta = v^2 - 4uw$$



compactness condition for G

$$v^2 - 4uw < 0.$$



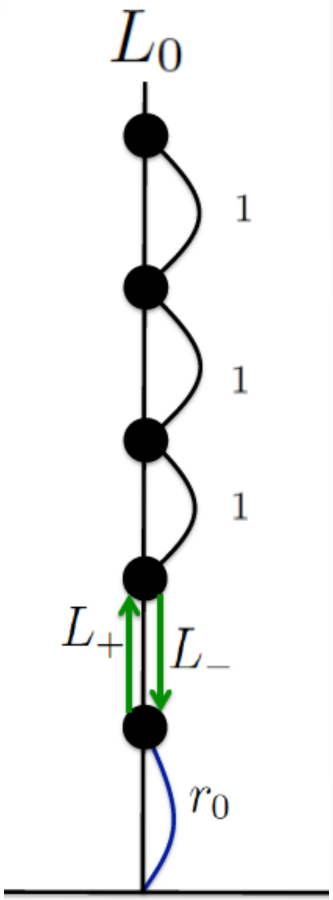
eigenvalue

$$L_0|n\rangle = r_n|n\rangle$$

$$r_n = r_0 + n, \quad n = 0, 1, 2, \cdots$$

$$\mathcal{C}_2 = r_0(r_0 - 1)$$

$$r_0 = \frac{1}{2}\left(1 + \sqrt{g^2 + \frac{1}{4}}\right)$$

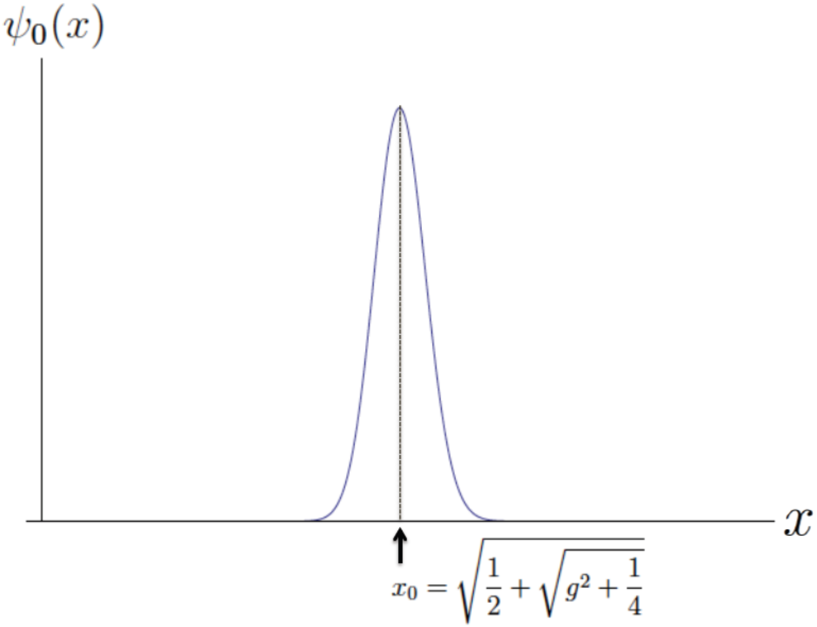



Physical quantities can be computed from the eigenstate of $L_0$ !

# wavefunction

'76 de Alfaro et al.

$$\psi_0(x) = \sqrt{\frac{2}{\Gamma(2r_0)}} e^{-\frac{x^2}{2}} x^{\frac{1}{2}+\sqrt{g^2+\frac{1}{4}}}$$




$$\psi_n(x) = \sqrt{\frac{\Gamma(n+1)}{2\Gamma(n+2r_0)}} x^{-\frac{1}{2}} \left(\frac{x^2}{a}\right)^{r_0} e^{-\frac{x^2}{2a}} L_n^{2r_0-1}\left(\frac{x^2}{a}\right)$$

# correlation function

'76 de Alfaro et al. '12 Jackiw et al.

$$F_2(t_1,t_2)=\langle t_1|t_2\rangle=\frac{\Gamma(2r_0)a^{2r_0}}{[2i(t_1-t_2)]^{2r_0}}$$

$$F_3(t;t_2,t_1)=\langle t_2|B(t)|t_1\rangle$$

$$=\langle 0|B(0)|0\rangle\left(\frac{i}{2}\right)^{2r_0+\Delta}\frac{\Gamma(2r_0)a^{2r_0}}{(t-t_1)^{\Delta}(t_2-t)^{\Delta}(t_1-t_2)^{-\Delta+2r_0}}$$

$$F_4(t_1,t_2,t_3,t_4)=\langle 0|B(0)|0\rangle\langle 0|\tilde{B}(0)|0\rangle\frac{\Gamma(2r_0)}{2^{\Delta+\tilde{\Delta}+2r_0}}$$

$$\times\frac{1}{(t_{13})^{\Delta-r_0}(t_{24})^{\tilde{\Delta}-r_0}(t_{12})^{\tilde{\Delta}+r_0}(t_{34})^{\Delta+r_0}(t_{14})^{2r_0-\Delta-\tilde{\Delta}}}x^{r_0}\ {}_2F_1(\Delta,\tilde{\Delta};2r_0;x)$$

# Gauged Quantum Mechanics

$$L = \frac{1}{2} D_0 z D_0 \overline{z} + c A_0 \qquad Dz := \dot{z} + i A_0 z$$

integrate out auxiliary gauge field

$$L = \frac{1}{2}\left(\dot{x}^2 - \frac{c^2}{x^2}\right)$$

DFF-model !

**Hamiltonian reduction (Routh reduction)**

$$\tilde{\mathcal{M}}_c = \mu^{-1}(c)$$

# Gauged Matrix Model

$$S = \int dt \left[ \mathrm{Tr}(DXDX) + \frac{i}{2}\left(\overline{Z}DZ - D\overline{Z}Z\right) + c\mathrm{Tr}A \right]$$

$$DX := \dot{X} + i[A, X]$$

↓ integrate out auxiliary gauge field

$$S = \frac{1}{2}\int dt \left[ \sum_a \dot{x}_a^2 - \sum_{a\neq b} \frac{c^2}{(x_a - x_b)^2} \right]$$

Calogero model !

**Hamiltonian reduction**
**(Routh reduction)**

(Super)conformal Quantum Mechanics
can be constructed by "**gauging**"!

'91 Polychronakos; '08 Fedoruk et al.

# Supersymmetry

Supersymmetric quantum mechanics (SQM)
was originally introduced as the simple model of SUSY QFT

'81 Witten

But SUSY in QM is
much more **fruitful** and **exotic** !

i. #(component in supermultiplet) $\geqq$ #(SUSY)

ii. #(physical boson) $\neq$ #(fermion)

# Superspace and Superfield

Witten's construction is restricted to specific $\mathcal{N}$=2 SQM

↓

Superspace & superfield formalism is useful for the construction of SQM

↓

But only $\mathcal{N} \leqq 8$ SQM is available

'00 Pashnev et al. ; '02 Gates et al.

| $\mathcal{N}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_{\mathcal{N}}$ | 1 | 2 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 32 | 64 | 64 | 128 | 128 | 128 | 128 |

**#(components in supermultiplet) ≧ #(SUSY)**

# AD map

#(physical boson) $\neq$ #(fermion)

Why does this happen in 1d ?

Hodge dual $\quad * : d\Omega_p \longrightarrow d\Omega_{d-p-2}$

0-form $\leftrightarrow$ (-1)-form

physical boson $\leftrightarrow$ auxiliary boson

**Automorphic Duaity Map** **‘96 Gates et al.**

From the above facts

i) Only $\mathcal{N}$=1,2,4,8 SQM have been constructed via superspace & superfield formulation.

ii) Supermultiplet is denoted by

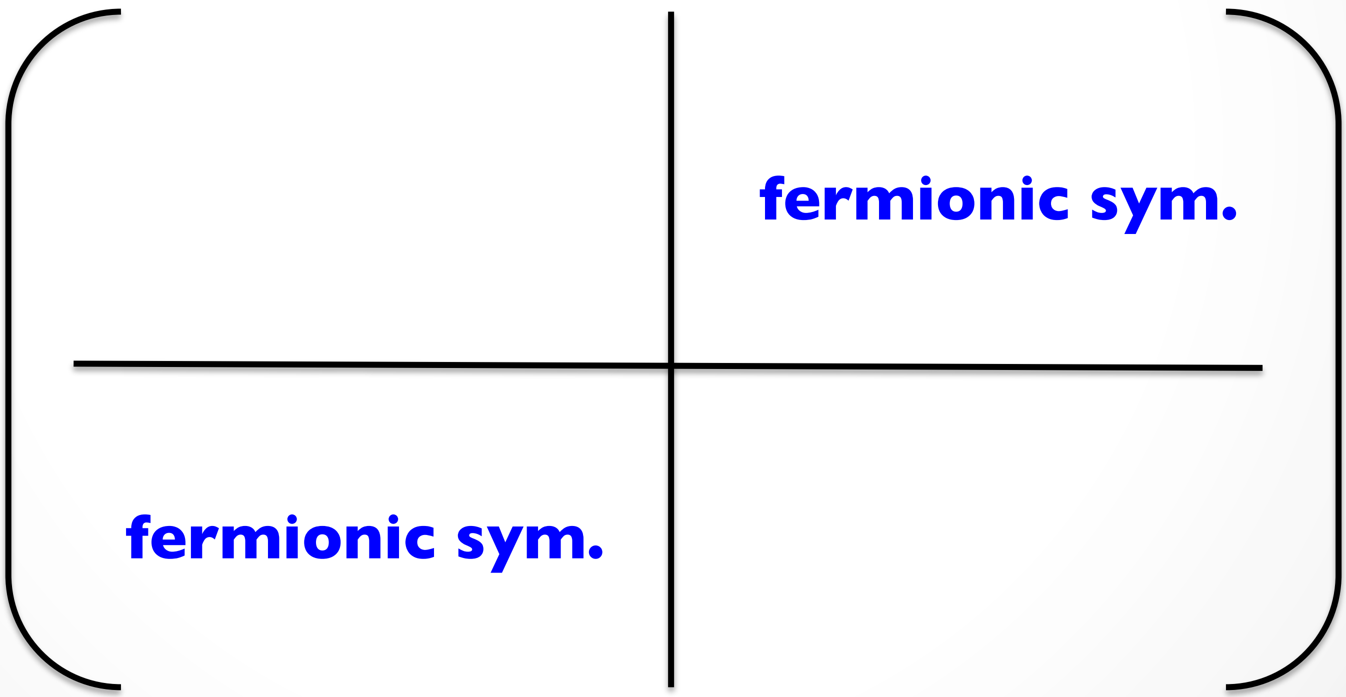
**(#(boson), $\mathcal{N}$, $\mathcal{N}$-#(boson))**

# Superconformal symmetry

conf. sym. + SUSY = superconf. sym.

SL(2)=SU(1,1)=Sp(2)   R-sym

**Lie supergroup**



# 1d superconformal group

| supersymmetry | supergroup | R-symmetry |
|---|---|---|
| $\mathcal{N}=1$ | $OSp(1\|2)$ | 1 |
| $\mathcal{N}=2$ | $SU(1,1\|1)$ | $U(1)$ |
| $\mathcal{N}=3$ | $OSp(3\|2)$ | $SU(2)$ |
| $\mathcal{N}=4$ | $SU(1,1\|2)$ | $SU(2)$ |
| | $D(2,1;\alpha), \alpha \neq -1, 0,$ | $SU(2) \times SU(2)$ |
| $\mathcal{N}=5$ | $OSp(5\|2)$ | $SO(5)$ |
| $\mathcal{N}=6$ | $SU(1,1\|3)$ | $SU(3) \times U(1)$ |
| | $OSp(6\|2)$ | $SO(6)$ |
| $\mathcal{N}=7$ | $OSp(7\|2)$ | $SO(7)$ |
| | $G(3)$ | $G_2$ |
| $\mathcal{N}=8$ | $OSp(8\|2)$ | $SO(8)$ |
| | $SU(1,1\|4)$ | $SU(4) \times U(1)$ |
| | $OSp(4^*\|4)$ | $SU(2) \times SO(5)$ |
| | $F(4)$ | $SO(7)$ |
| $\mathcal{N}>8$ | $OSp(\mathcal{N}\|2)$ | $SO(\mathcal{N})$ |
| | $SU(1,1\|\frac{\mathcal{N}}{2})$ | $SU(\frac{\mathcal{N}}{2}) \times U(1)$ |
| | $OSp(4^*\|\frac{\mathcal{N}}{2})$ | $SU(2) \times Sp(\frac{\mathcal{N}}{2})$ |

constructed via superfield

conjectured via superfield

**my work**

# Conjectured $\mathcal{N} \geqq 8$ SCQM

$\mathcal{N}$>8 SCQM is not available from superfield & superspace.

However, **SU(1,1|$\mathcal{N}$/2) SCQM** action
is conjectured from $\mathcal{N}$=4 **SU(1,1|2) SCQM**.

$$S = \int dt \left[ \dot{x}^2 + i \left( \overline{\psi}_i \dot{\psi}^i - \dot{\overline{\psi}}_i \psi^i \right) - \frac{\left(c + \overline{\psi}_i \psi^i\right)^2}{x^2} \right]$$

'88 Ivanov et al.

# II. M2-brane

# M2-brane

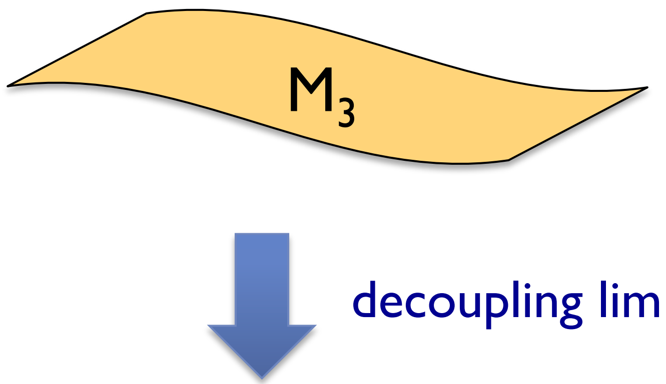
11d SUGRA    '78 Cremmer et al.

| | | |
|---|---|---|
| metric | $g_{MN}$ | 44 |
| gravitino | $\psi_{M\alpha}$ | 128 |
| 3-form gauge field | $C_{MNP}$ | 84 |

(1+2) dimensional object in 11d SUGRA background

**M2-brane (electric)**

(1+2) dimensional extended object in 11d SUGRA bckd.




(1+2) dimensional world-volume theory on the branes

# World-volume theory of M2-branes

Required information

- $X^I$ (position of M2-brane)
- $\Psi$ (SUSY)
- $A_\mu$ (internal d.o.f.)
- conformal (IR theory of D2-brane (3d SYM))

$M_3 = \mathbf{R}^{1,2} \longrightarrow$ **'07 BLG-model**
**'08 ABJM-model**

# BLG-model

'08 Bagger Lambert; Gustavsson

- 3d $\mathcal{N} = 8$ superconformal CS-matter w/ OSp(8|4)

Conf(R$^{1,2}$)

$$\begin{pmatrix} Sp(4) & \text{fermionic} \\ \text{fermionic} & SO(8) \end{pmatrix}$$

R-sym

- Lie 3-alg.

$$\begin{cases} [T^a, T^b, T^c] = f^{abc}{}_d T^d & \text{structure constant} \\ h^{ab} = \mathrm{Tr}(T^a, T^b) & \text{metric} \end{cases}$$

field content

| | |
|---|---|
| $X^I_a$ | $\mathbf{8}_v$ |
| $\Psi_a$ | $\mathbf{8}_c$ |
| $A_{\mu ab}$ | $\mathbf{1}$ |

SUSY parameter

| | |
|---|---|
| $\epsilon$ | $\mathbf{8}_s$ |

Lagrangian

$$\mathcal{L}_{\rm BLG} = -\frac{1}{2}D^\mu X^{Ia} D_\mu X^I_a + \frac{i}{2}\overline{\Psi}^a_{\dot{A}}\Gamma^\mu_{\dot{A}\dot{B}} D_\mu \Psi_{\dot{B}a} + \frac{i}{4}\overline{\Psi}_{\dot{A}b}\Gamma^{IJ}_{\dot{A}\dot{B}} X^I_c X^J_d \Psi_{\dot{B}a} f^{abcd} - V(X) + \mathcal{L}_{TCS}$$

$$V(X) = \frac{1}{12} f^{abcd} f^{efg}{}_d X^I_a X^J_b X^K_c X^I_e X^J_f X^K_g$$

$$\mathcal{L}_{TCS} = \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}\right)$$

# ABJM-model

'08 Aharony et al.

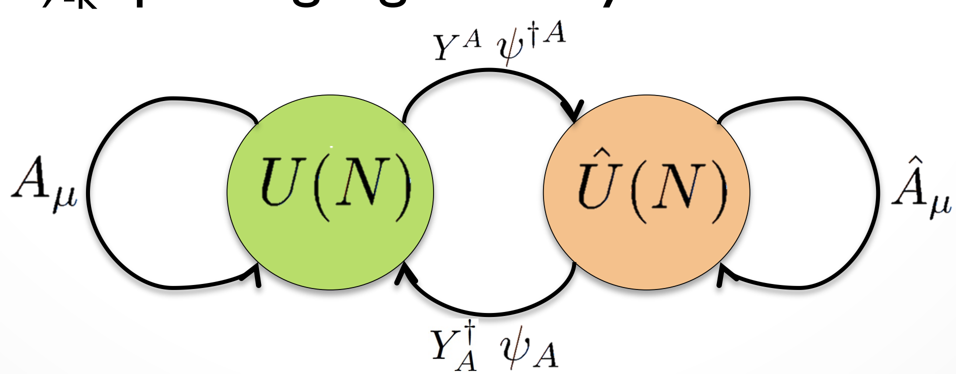
- 3d $\mathcal{N} = 6$ superconfomal CS-matter w/ Osp(6|4)

Conf($R^{1,2}$)

$$\begin{pmatrix} Sp(4) & \text{fermionic} \\ \text{fermionic} & SU(4) \cong SO(6) \end{pmatrix}$$

R-sym

- $U(N)_k \times U(N)_{-k}$ quiver gauge theory



# Moduli space & brane interpretation

$$\mathcal{M}_{N,k} = \frac{(\mathbb{C}^4/\mathbb{Z}_k)^N}{S_N} = \mathrm{Sym}^N(\mathbb{C}^4/\mathbb{Z}_k)$$

'08 Aharony et al.

| | | |
|---|---|---|
| ABJM-model | N M2-branes propagating in | $\mathbb{C}^4/\mathbb{Z}_k$ |
| SO(4) BLG-model k=1 | 2 M2-branes propagating in | $\mathbb{R}^8$ |
| Spin(4) BLG-model k=2 | 2 M2-branes propagating in | $\mathbb{C}^4/\mathbb{Z}_2$ |

# III. SCQM from M2-branes

# World-volume theories of M2-branes

$M_3 = \mathbf{R}^{1,2}$ $\longrightarrow$ **'07 BLG**

**'08 ABJM**

$M_3 = \mathbf{R} \times \Sigma$ $\longrightarrow$ **My work**

energy scale characterized by volume of $\Sigma$ >> enegy

***Further IR limit can be taken !***

# IR QM emerging

Namely

**IR QM with energy scale << vol($\Sigma$)$^{-1}$**

## *How to derive?*

Step1. BPS eq. → low-energy conf.

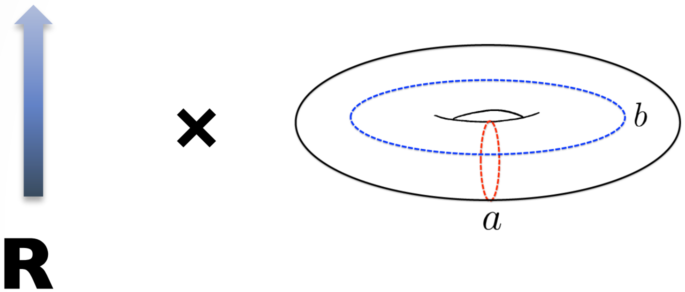
Step2. Integration over the Riemann surface

‘95 Bershadsky Sadov Vafa; ‘06 Kapustin Witten

# $\mathcal{A}_4$ BLG-model/T²

## Step1. BPS eq. → low-energy conf.

$$[X^I, X^J, X^K] = 0 \qquad D_z X^I = 0 \qquad D_{\bar{z}} X^I = 0$$




$$\int_a \omega = 1, \qquad \int_b \omega = \tau$$

**BPS conf.**

$$X^{I+2} = \begin{pmatrix} \cos\theta^I \\ \sin\theta^I \\ 0 \\ 0 \end{pmatrix} r^I \qquad \tilde{A}_z = \begin{pmatrix} 0 & -2\pi\frac{\Theta}{\tau-\bar{\tau}}\omega_z & 0 & 0 \\ 2\pi\frac{\Theta}{\tau-\bar{\tau}}\omega_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{A}^3_{z4}(z,\bar{z}) \\ 0 & 0 & -\tilde{A}^3_{z4}(z,\bar{z}) & 0 \end{pmatrix}$$

## Step2. Integration over the Riemann surface

$$S=\int_{\mathbb{R}} dt\left[\frac{1}{2}D_0X^{Ia}D_0X^I_a-\frac{i}{2}\overline{\Psi}^{\alpha a}D_0\Psi_{\alpha a}-kC_1(E)\tilde{A}^1_{02}\right]$$

**$\mathcal{N}$=16 Superconformal gauged QM**

↓ integrate out auxiliary gauge field

$$S=\frac{1}{2}\int_{\mathbb{R}} dt\left[\dot{q}^2+\sum_{I\neq K}(\dot{r}^I)^2-i\overline{\Psi}^{\alpha a}\dot{\Psi}_{\alpha a}\right.$$
$$\left.-\frac{\left[kC_1(E)+\sum_{I\neq K}h^I+i\overline{\Psi}^{\alpha}_{A}\Psi_{\alpha B}\right]^2}{q^2}-\sum_{I\neq K}\frac{(h^I)^2}{(r^I)^2}\right]$$

**Hamiltonian reduction**
**(Routh reduction)**

decoupled motion associated with local charge

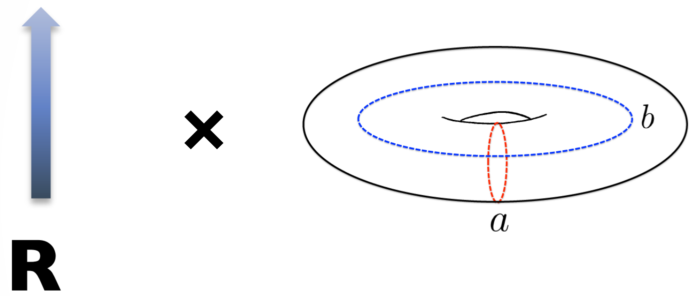
$$S = \frac{1}{2} \int_{\mathbb{R}} dt \left[ \dot{q}^2 - i\overline{\Psi}^{\alpha a} \dot{\Psi}_{\alpha a} - \frac{\left(kC_1(E) + i\overline{\Psi}^{\alpha}_{A} \Psi_{\alpha B}\right)^2}{q^2} \right]$$

# OSp(16|2) SCQM

# ABJM-model/T$^2$

Step1. BPS eq. → low-energy conf.

$$D_z Y^A = 0, \quad D_{\bar{z}} Y^A = 0 \quad Y^C Y_C^\dagger Y^B - Y^B Y_C^\dagger Y^C = 0 \quad Y^C Y_A^\dagger Y^D = 0$$




$$\int_a \omega = 1, \qquad \int_b \omega = \tau$$

**BPS conf.**

$$Y^A = \mathrm{diag}(y_1^A, \cdots, y_N^A)$$

$$A_z = \mathrm{diag}\left(A_{z11}, \cdots, A_{zNN}\right)$$

$$\hat{A}_z = A_z + \partial_z \varphi = \mathrm{diag}\left(A_{z11} + \partial_z \varphi_1, \cdots, A_{zNN} + \partial_z \varphi_N\right)$$

# Step2. Integration over the Riemann surface

$$\int_{\mathbb{R}} dt \left[ D_0 \overline{y}_A^a D_0 y_a^A - i\psi^{\dagger \alpha A a} D_0 \psi_{\alpha A a} + k C_1(E_a) \mathcal{A}_{0a}^{-} \right]$$

**$\mathcal{N}$=12 superconformal gauged QM**

integrate out gauge field

$$\begin{aligned} S = \int_{\mathbb{R}} dt \sum_{a=1}^{N} \Bigg[ & \dot{x}_a^2 - \frac{i}{2} \sum_{A \neq B} \left( \psi^{\dagger \alpha A a} \dot{\psi}_{\alpha A a} - \dot{\psi}^{\dagger A a} \psi_{\alpha A a} \right) \\ & + \sum_{A \neq B} (\dot{r}_a^A)^2 - \frac{i}{2} \left( \lambda^{\dagger \alpha a} \dot{\lambda}_{\alpha a} - \dot{\lambda}^{\dagger \alpha a} \lambda_{\alpha a} \right) \\ & - \frac{\left[ k C_1(E_a) + \sum_{A \neq B} h_a^A + \sum_{A \neq B} \psi^{\dagger \alpha A a} \psi_{\alpha A a} + \lambda^{\dagger \alpha a} \lambda_{\alpha a} \right]^2}{4 x_a^2} - \sum_{A \neq B} \frac{(h_a^A)^2}{4 (r_a^A)^2} \Bigg] \end{aligned}$$

**Hamiltonian reduction**
**(Routh reduction)**

decoupled motion associated with local charge

$$S=\int_{\mathbb{R}} dt \sum_{a=1}^{N}\left[\dot{x}_a^2 - i\psi^{\dagger\alpha Aa}\dot{\psi}_{\alpha Aa} - \frac{\left(kC_1(E_a)+\psi^{\dagger\alpha Aa}\psi_{\alpha Aa}\right)^2}{4x_a^2}\right]$$

# SU(1,1|6) SCQM

**$\mathcal{N}$ > 8 SCQM conjectured Lagrangian**

$$S_{\mathcal{N}\geq 4}=\int dt\left[\dot{x}\dot{x}+i\left(\bar{\psi}_k\dot{\psi}^k-\dot{\bar{\psi}}_k\psi^k\right)-\frac{\left(m+\bar{\psi}_k\psi^k\right)^2}{x^2}\right]$$

'88 Ivanov et al.

**exactly same !**

# flat branes

flat BPS branes

↓ decoupling limit

| world-volume theory of flat BPS branes | = | SUSY gauge theory |
|---|---|---|

Dp-branes → (1+p) dimensional SYM

M2-branes → BLG-model、ABJM-model

# Curved Branes

Curved BPS branes

$$\cup$$

Calibrated submfd (SUSY cycle) $C$ $\subset$ ambient space $X$

decoupling limit

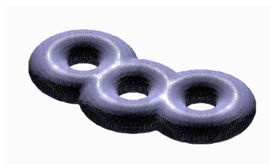
world-volume theory of curved BPS branes = **Topologically twisted theory**

'95 Bershadsky Sadov Vafa

# Curved M2-branes

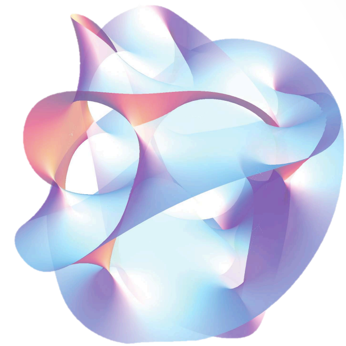
wrapped M2-branes
around Riemann surface $\Sigma_g$

$\cup$



holomorphic curve
(SUSY 2-cycle) $\boldsymbol{\Sigma_g}$

$\subset$



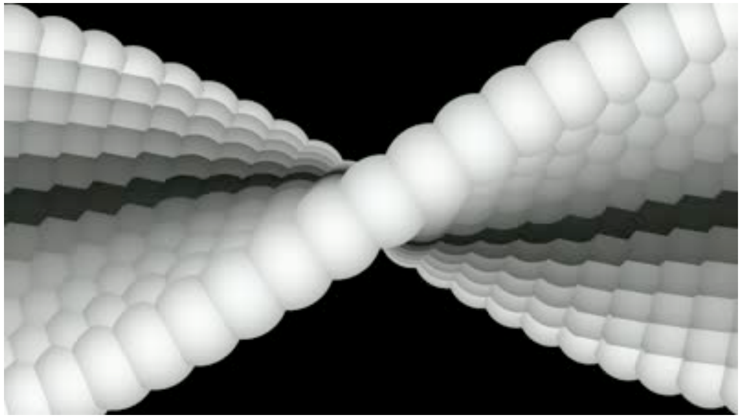
CY mfd $X$

decoupling limit

**Q. What is the world-volume theory**

Now consider

**topological twisting !**



# Topological twisting

'88 Witten

Topological twisting = Modifying **the Euclidean rotational sym. group** with **the R-sym. group**

Brane perspective • • •

rotational sym. group on **the SUSY-cycle**

rotational sym. group of **the transverse space**

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M2-brane perspective • • •

rotational sym. group on **the SUSY-cycle**

$SO(2)_E$

rotational sym. group of **the transverse space**

$SO(8)_R$

Consider

$$X = \mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \mathcal{L}_3 \oplus \mathcal{L}_4 \to \Sigma_g$$

$SO(8)_R \to SO(2)_1 \times SO(2)_2 \times SO(2)_3 \times SO(2)_4$

$SO(2)_E$ — Cartan generator of $SO(8)_R$ — Twisted $SO(2)'_E$

$$s' := s - \sum_{i=1}^{n} a_i T_i$$

$$\deg(\mathcal{L}_i) = \begin{cases} 2|g-1|a_i & (g \neq 1) \\ a_i & (g = 1) \end{cases}$$

$$\sum_{i=1}^{n} a_i = \begin{cases} -1 & (g = 0) \\ 0 & (g = 1) \\ 1 & (g > 1) \end{cases}$$

$n=1 \to CY_2$ $\mathcal{N}=8$

$n=2 \to CY_3$ $\mathcal{N}=4$

$n=3 \to CY_4$ $\mathcal{N}=2$

$n=4 \to CY_5$ $\mathcal{N}=2$

# BLG theory probing K3

$$SO(2)_E \times SO(8)_R \longrightarrow SO(2)'_E \times SO(6)_R$$

| | | |
|---|---|---|
| $X^I$ | $\mathbf{8}_v$ | $\mathbf{6}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_{-2}$ |
| | | $\phi^I$   $\Phi_z$   $\Phi_{\overline{z}}$ |
| | | flat space   $N_\Sigma$ |
| $\Psi$ | $\mathbf{8}_{c+} \oplus \mathbf{8}_{c-}$ | $\mathbf{4}_2 \oplus \overline{\mathbf{4}}_0 \oplus \mathbf{4}_0 \oplus \overline{\mathbf{4}}_{-2}$ |
| | | $\Psi_z$   $\tilde{\lambda}$   $\psi$   $\tilde{\Psi}_{\overline{z}}$ |
| $\epsilon$ | $\mathbf{8}_{s+} \oplus \mathbf{8}_{s-}$ | $\mathbf{4}_0 \oplus \overline{\mathbf{4}}_2 \oplus \mathbf{4}_{-2} \oplus \overline{\mathbf{4}}_0$ |
| | | $\epsilon$   $\tilde{\epsilon}_z$   $\epsilon_{\overline{z}}$   $\tilde{\epsilon}$ |

$\mathcal{N} = 8$ SUSY

Twisted K3 BLG Lagrangian

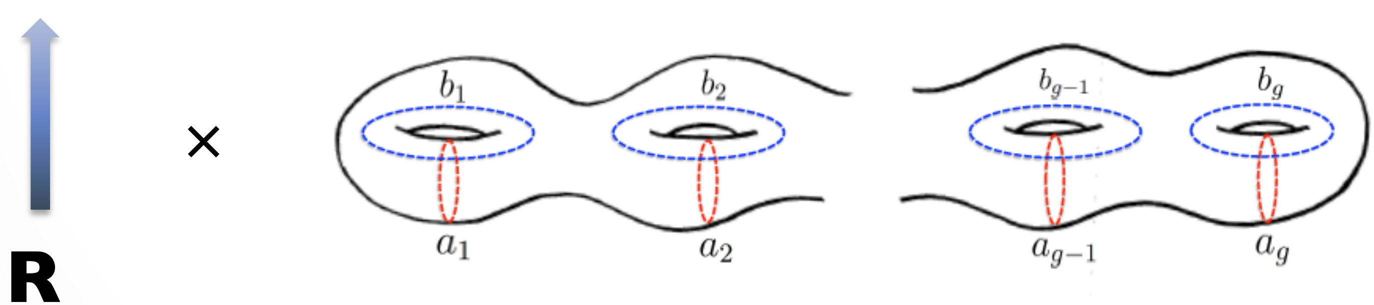
$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}(D_0\phi^I, D_0\phi^I) - (D_z\phi^I, D_{\overline{z}}\phi^I) + (D_0\Phi_z, D_0\Phi_{\overline{z}}) - 2(D_z\Phi_{\overline{w}}, D_{\overline{z}}\Phi_w) \\
& + (\bar{\tilde{\lambda}}, D_0\psi) + (\overline{\Psi}_z, D_0\tilde{\Psi}_{\overline{z}}) - (\bar{\tilde{\Psi}}_{\overline{z}}, D_0\Psi_z) - 2i(\bar{\tilde{\Psi}}_{\overline{z}}, D_z\psi) + 2i(\bar{\tilde{\lambda}}, D_{\overline{z}}\Psi_z) \\
& + \frac{i}{2}(\bar{\tilde{\lambda}}\hat{\Gamma}^{IJ}, [\phi^I, \phi^J, \psi]) - i(\bar{\tilde{\Psi}}_{\overline{z}}\hat{\Gamma}^{IJ}, [\phi^I, \phi^J, \Psi_z]) \\
& + 2i(\overline{\psi}\hat{\Gamma}^I, [\Phi_{\overline{z}}, \phi^I, \Psi_z]) - 2i(\bar{\tilde{\lambda}}\hat{\Gamma}^I, [\Phi_z, \phi^I, \tilde{\Psi}_{\overline{z}}]) \\
& + i(\bar{\tilde{\lambda}}, [\Phi_z, \Phi_{\overline{z}}, \psi]) - 2i(\bar{\tilde{\Psi}}_{\overline{w}}, [\Phi_z, \Phi_{\overline{z}}, \Psi_w]) \\
& - \frac{1}{12}([\phi^I, \phi^J, \phi^K], [\phi^I, \phi^J, \phi^K]) - \frac{1}{2}([\Phi_z, \phi^I, \phi^J], [\Phi_{\overline{z}}, \phi^I, \phi^J]) \\
& - \frac{1}{2}([\Phi_z, \Phi_w, \phi^I], [\Phi_{\overline{z}}, \Phi_{\overline{w}}, \phi^I]) - \frac{1}{2}([\Phi_z, \Phi_{\overline{w}}, \phi^I], [\Phi_{\overline{z}}, \Phi_w, \phi^I]) \\
& + \frac{1}{6}([\Phi_z, \Phi_w, \Phi_v], [\Phi_{\overline{z}}, \Phi_{\overline{w}}, \Phi_{\overline{v}}]) + \frac{1}{2}([\Phi_z, \Phi_w, \Phi_{\overline{v}}], [\Phi_{\overline{z}}, \Phi_{\overline{w}}, \Phi_v]) + \mathcal{L}_{\mathrm{TCS}}
\end{aligned}
$$

# Step1. BPS eq. → low-energy conf.

$$D_z\phi^I = 0, \quad D_{\overline{z}}\phi^I = 0$$
$$D_z\Phi_{\overline{z}} = 0, \quad D_{\overline{z}}\Phi_z = 0$$

$$[\phi^I, \phi^J, \phi^K] = 0$$

$$[\Phi_z, \Phi_{\overline{z}}, \phi^I] = 0, \quad [\Phi_z, \phi^I, \phi^J] = 0, \quad [\Phi_{\overline{z}}, \phi^I, \phi^J] = 0$$
$$[\Phi_w, \Phi_{\overline{w}}, \Phi_z] = 0, \quad [\Phi_{\overline{w}}, \Phi_w, \Phi_{\overline{z}}] = 0.$$




$$\int_{a_i} \omega_j = \delta_{ij}, \quad \int_{b_i} \omega_j = \Omega_{ij}$$

## BPS conf.

$$\phi^I = 0 \qquad \Phi_z = \sum_{i=1}^{g} \begin{pmatrix} \frac{1}{2}\left(e^{-i\varphi} x_A^i + e^{i\varphi} x_B^i\right) \\ \frac{i}{2}\left(e^{-i\varphi} x_A^i - e^{i\varphi} x_B^i\right) \\ 0 \\ 0 \end{pmatrix} \omega_i$$

$$\tilde{A}_z = \begin{pmatrix} 0 & \partial_z \varphi(z,\overline{z}) & 0 & 0 \\ -\partial_z \varphi(z,\overline{z}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{A}^3_{z4}(z,\overline{z}) \\ 0 & 0 & -\tilde{A}^3_{z4}(z,\overline{z}) & 0 \end{pmatrix}$$

## Step2. Integration over the Riemann surface

$$S = \int_{\mathbb{R}} dt \left[ \sum_{i,j} (\mathrm{Im}\ \Omega)_{ij} \left( D_0 x^{ia} D_0 \overline{x}^j_a + \overline{\Psi}^{ia} D_0 \tilde{\Psi}^j_a - \overline{\tilde{\Psi}}^{ia} D_0 \Psi^j_a \right) - k C_1(E) \tilde{A}^1_{02} \right]$$

Information of the curved Riemann surface

Effective action possesses
- **SL(2,R) conformal symmetry**
- **$\mathcal{N}$ = 8 SUSY**

**$\mathcal{N}$=8 superconformal gauged QM**

may describe **curved M2-branes** !

Now testing…
(reduced Gromov-Witten inv. etc.)

# IV. Conclusion

# Conclusion

We found $\mathcal{N} \geqq 8$ **SCQM** models
which are not available
via superfield & superspace formalism.

They may describe
**the M2-branes**
**wrapping a Riemann surface**.

## Future work

- Other Calabi-Yau case
- ABJM with $g \neq 1$ case
- Analysis of the M2-branes through SCQM

## Application

- $AdS_2/CFT_1$ (Black hole physics)
- 3=1+2 (AGT like relation arising from M2)
- reduced Gromov-Witten inv.
- construction of new SCQM