2018/08/21 Seminar at Kochi University of Technology

Superfluidity of Bose-Einstein condensates in ring traps

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References:
MK and Y. Kato, Phys. Rev. A **91**, 053608 (2015).
MK and I. Danshita, arXiv:1712.09403.

Collaborator：Yusuke Kato (University of Tokyo)
Ippei Danshita (Kindai University)

Contents

- Introduction to cold atomic gases and superfluidity

- Part 1. Multiple-swallowtail structures in 2D Bose-Einstein condensate : MK and Y. Kato, Phys. Rev. A **91**, 053608 (2015).

- Part 2. Superflow decay of Bose-Einstein condensate in the ring traps : MK and I. Danshita, arXiv:1712.09403.

Introduction : Cold Atomic Gases

Cold atomic gases

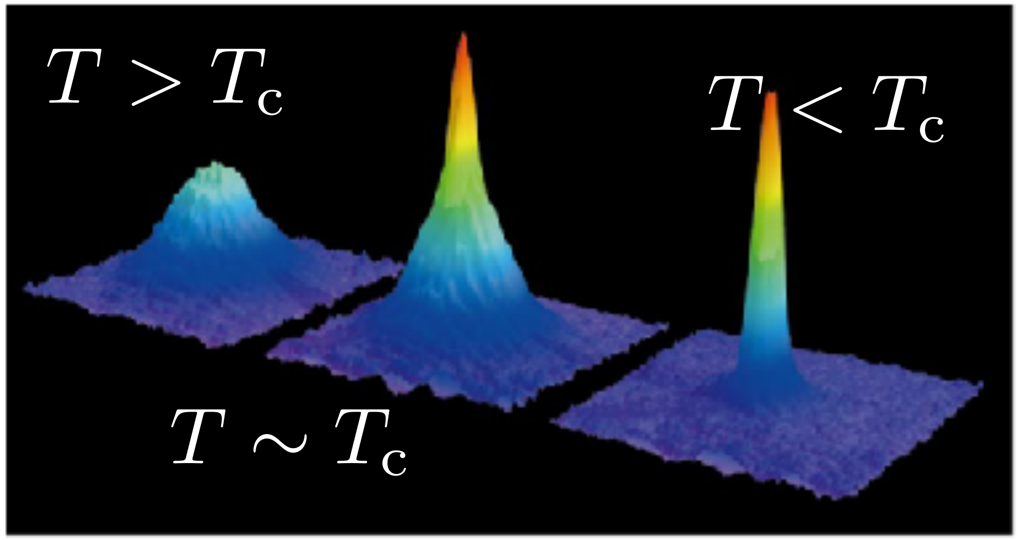
Atoms (typically alkali atoms) are trapped in a vacuum by using a magnetic field or a laser beam.

Bose-Einstein condensation (BEC, 1995) and **Fermi superfluid** (2004) were experimentally realized.

Important features

- Highly controllability
- Very clean systems.

$\Rightarrow$ Ideal quantum many-body systems!



W. Ketterle, Rev. Mod. Phys. **74**, 1131 (2002).

Introduction : Cold Atomic Gases

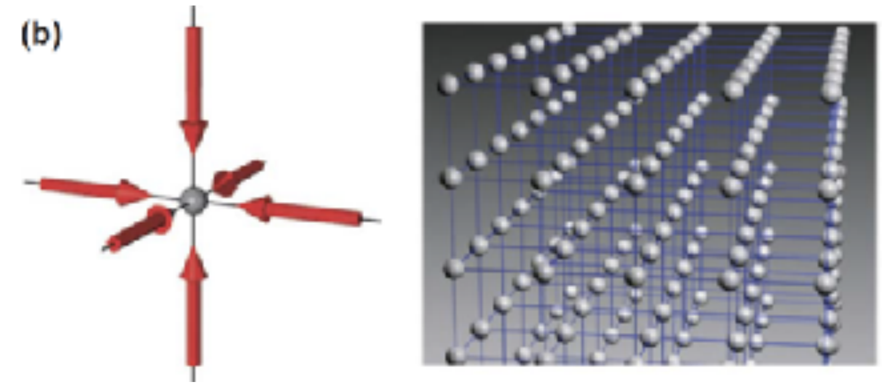
Typical scales of cold atomic gases

Temperature：10nK〜500nK $T_c^{\mathrm{BEC}} \sim O(100\mathrm{nK})$

Density : 10^{-5} times the density of the air

Length：0.1μm〜100μm

Particle number：10^3〜10^7



I. Bloch, Rev. Mod. Phys. **80**, 885 (2008).

What we can do in cold atomic gases

Tune of the inter-particle interaction ⇒ BCS-BEC crossover, supersolid

- **Control of spatial dimension and shape of the trap potential ⇒ Ring trap**
- **Control of disorder** ⇒ Anderson localization, **tuning the defect strength**

Spin-orbit coupling ⇒ Topological physics

Quantum gas microscope ⇒ single site imaging, measurements of entanglement

Methods : Hamiltonian of dilute Bose gases

Hamiltonian for dilute Bose gases

$$\hat{H} = \int d\boldsymbol{r} \left[\frac{\hbar^2}{2m} \boldsymbol{\nabla} \hat{\psi}^\dagger(\boldsymbol{r}) \cdot \boldsymbol{\nabla} \hat{\psi}(\boldsymbol{r}) + U(\boldsymbol{r}) \hat{\psi}^\dagger(\boldsymbol{r}) \hat{\psi}(\boldsymbol{r}) + \frac{g}{2} \hat{\psi}^\dagger(\boldsymbol{r})^2 \hat{\psi}(\boldsymbol{r})^2 \right]$$

$\hat{\psi}(\boldsymbol{r})$: Field operator of bosons $U(\boldsymbol{r})$: external potential

$V(\boldsymbol{r} - \boldsymbol{r}') = g\delta(\boldsymbol{r} - \boldsymbol{r}')$: Contact interaction

$g \equiv \dfrac{4\pi\hbar^2 a}{m}$ a : s-wave scattering length

$a > 0$: repulsive interaction

$a < 0$: attractive interaction

Heisenberg equation

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(\boldsymbol{r}, t) = [\hat{\psi}(\boldsymbol{r}, t), \hat{H}]$$

See, for example, L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Oxford University Press, Oxford, UK, 2003)

Introduction : Gross-Pitaevskii Equation

$\langle\hat{\psi}(\boldsymbol{r},t)\rangle \equiv \Psi(\boldsymbol{r},t)$ Neglecting the fluctuations $\hat{\psi}(\boldsymbol{r},t) \rightarrow \Psi(\boldsymbol{r},t)$

Gross-Pitaevskii (GP) equation (Mean-field theory)

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + U(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) + g|\Psi(\boldsymbol{r},t)|^2\Psi(\boldsymbol{r},t)$$

This approximation is justified under the **low temperature** and the **dilute** conditions.

$n\lambda_{\mathrm{dB}}^3 \gg 1$ Thermal de Brogile wave length >> mean particle distance

$na^3 \ll 1$ s-wave scattering length << mean particle distance

The GP equation can correctly describe the static and dynamical properties of the BEC.

Introduction : Mean-field Approximation

Physical quantities can be obtained by the order parameter.

$\Psi(\boldsymbol{r},t) \equiv \sqrt{n(\boldsymbol{r},t)}e^{i\varphi(\boldsymbol{r},t)}$: Order parameter

$n(\boldsymbol{r},t) \equiv |\Psi(\boldsymbol{r},t)|^2$: local particle density

$\boldsymbol{v}(\boldsymbol{r},t) \equiv \frac{\hbar}{m}\boldsymbol{\nabla}\varphi(\boldsymbol{r},t)$: local velocity field

From the single-valuedness of the order parameter,
the circulation is quantized :

Quantization of the circulation

$$\Gamma \equiv \oint_C d\boldsymbol{r} \cdot \boldsymbol{v}(\boldsymbol{r},t) = \frac{\hbar}{m}\oint_c d\varphi = \frac{2\pi\hbar}{m}W$$

$W \in \mathbb{Z}$: Winding number

Introduction : Superfluidity

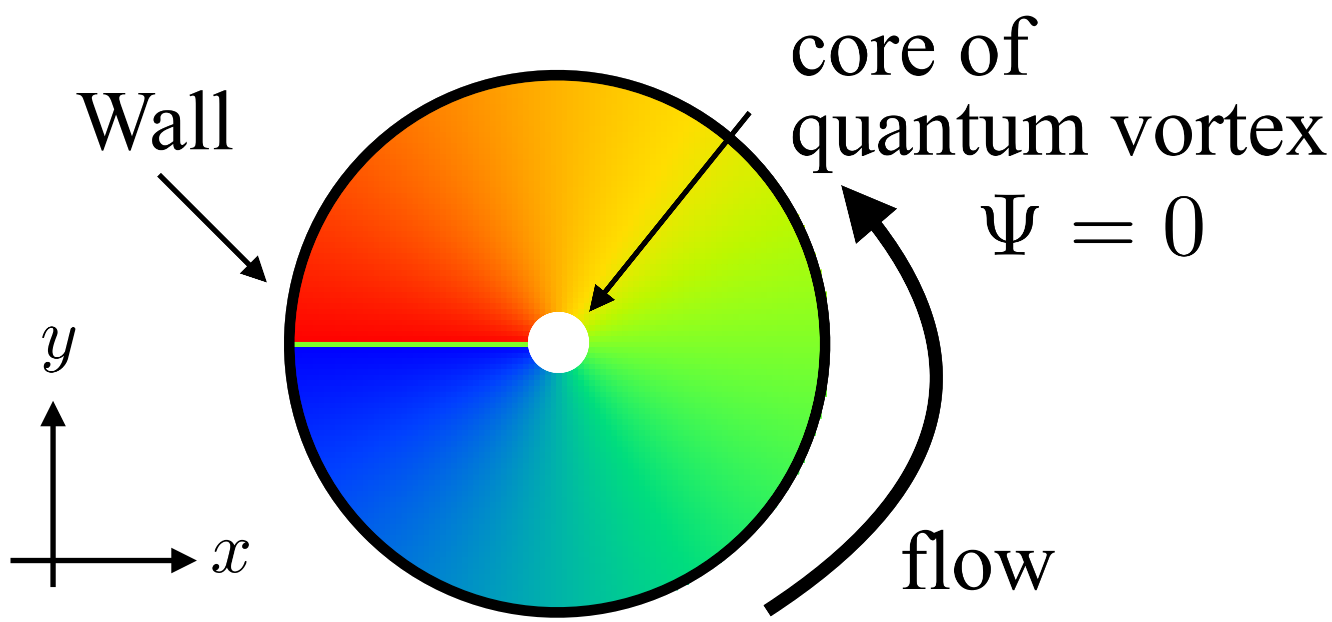
The winding number is a topological number.

⇒ The topological number can not be changed by a perturbation.

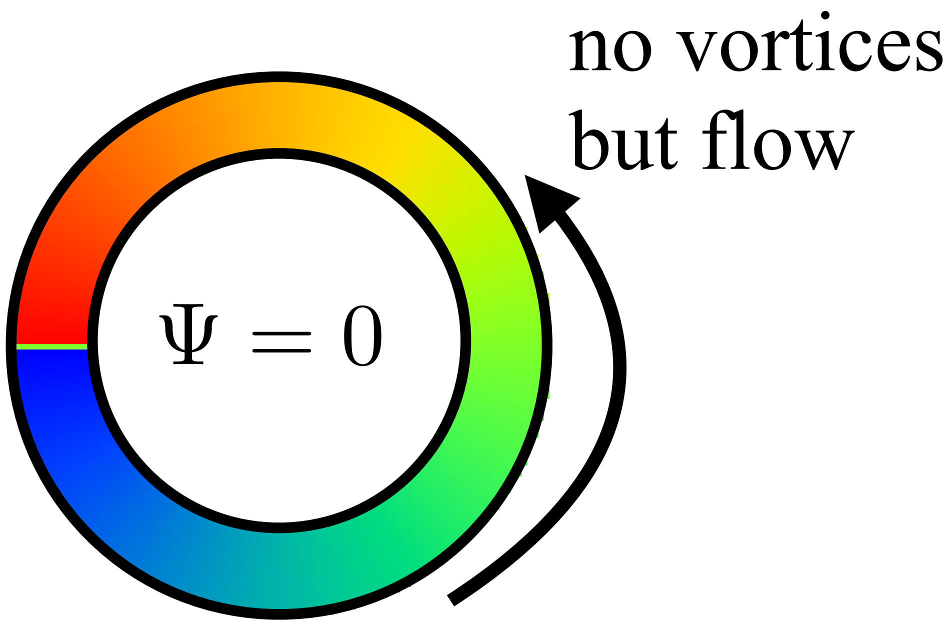
⇒ The finite winding number states have long lifetime.

⇒ Persistent current ! $T_{\text{Lifetime}} \gg T_{\text{experiment}}$

Simply connected systems
(Typical setup of cold atoms)



Multiply connected systems
(Ring-shaped trap)



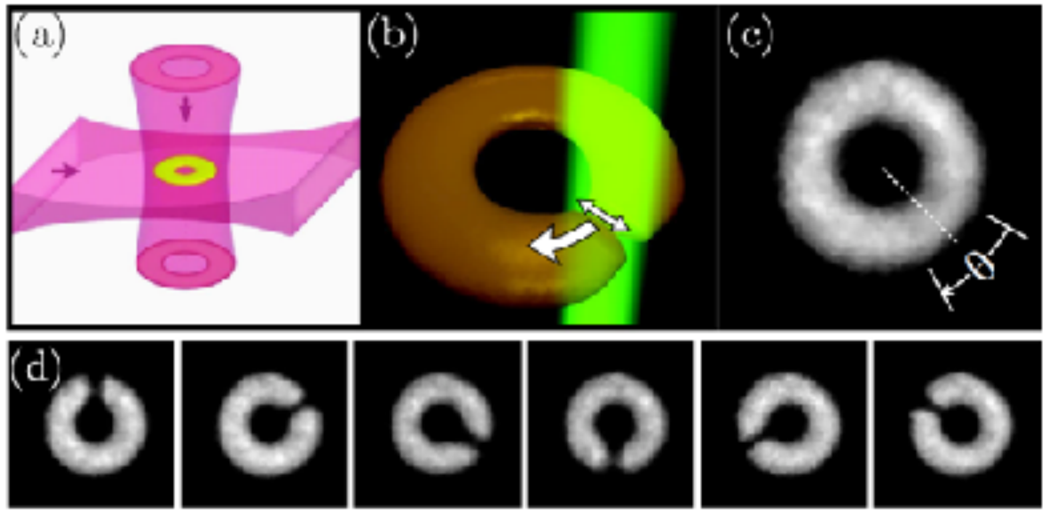
Introduction : BEC in a ring trap

Ring traps are ideal systems for studying superfluidity.

Bose-Einstein condensates confined in a ring trap have been investigated by several experimental groups.

Experimental studies for the ring trap

Ryu *et al*., PRL **99**, 260401 (2007) : Persistent current
Ramanathan *et al*., PRL **106**, 130401 (2011) : Persistent current
Moulder *et al*., PRA **86**, 013629 (2012) : Persistent current
Beattie *et al*., PRL **110**, 025301 (2013) : Two-component BEC
Wright *et al*., PRL **111**, 025302 (2013) : Rotation
Neely *et al*., PRL **111**, 235301 (2013) : Turbulence
Ryu *et al*., New J. Phys. **16**, 013046 (2014) : Rotation
Eckel *et al*., Nature **506**, 200 (2014) : Hysteresis
Eckel *et al*., PRX **4**, 031052 (2014) : Current-phase relations
Corman *et al*., PRL **113**, 135302 (2014) : Kibble-Zurek mechanism
Kumar *et al*., PRA **95**, 021602(R) (2017) : Temperature dependent Decay of persistent current



Wright *et al*., PRL **111**, 025302 (2013).

Red : Persistent current (no stirring)
Blue : stirring the BEC in the ring trap

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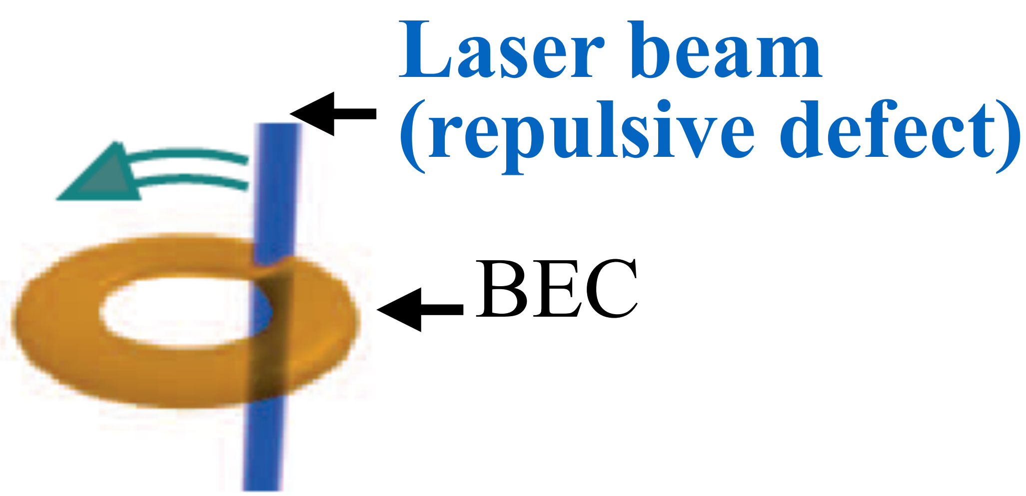
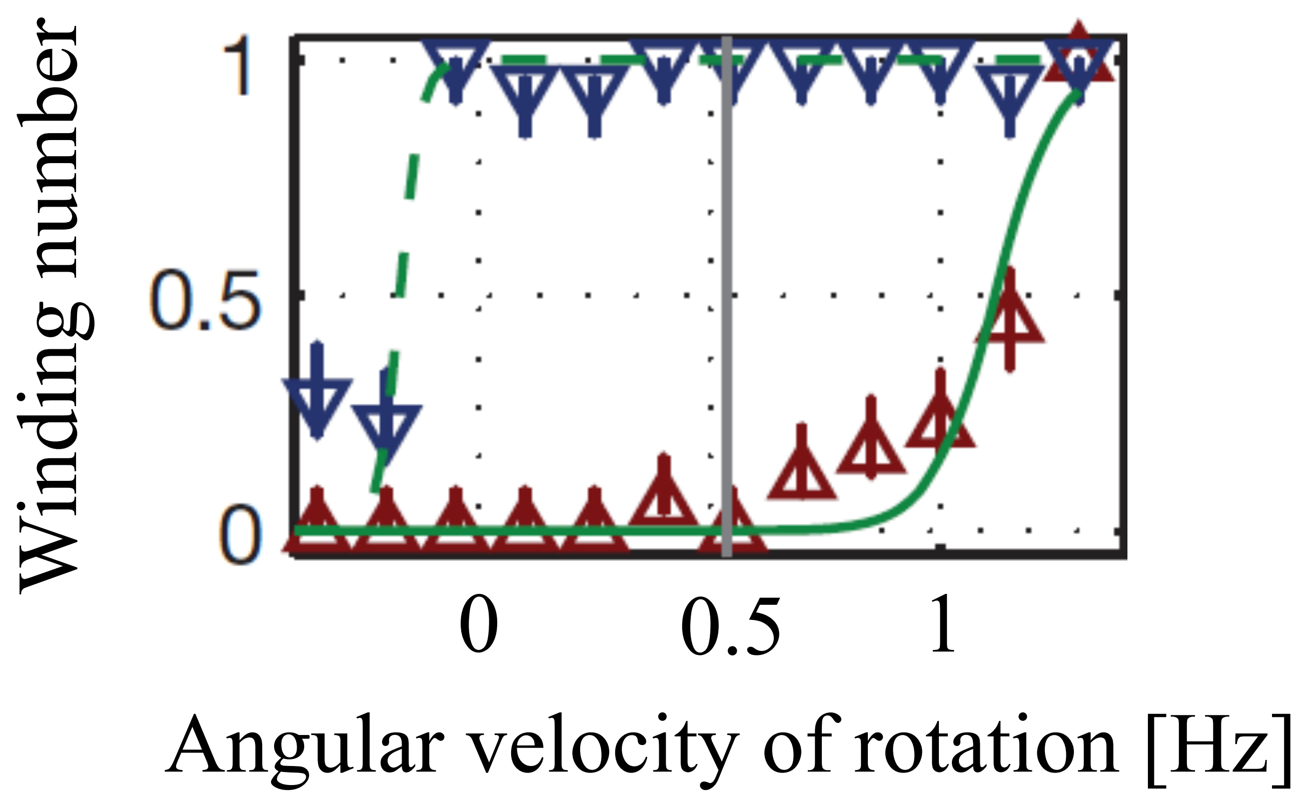
Part 1 : NIST experiment

The NIST group observed hysteresis in stirred BEC.



Density profiles of BEC



How do we understand this phenomenon?

⇒ The **swallowtail structure** is a key concept to understand superfluidity.

Eckel *et al*., Nature **506**, 200 (2014)

BEC in the ring trap

For simplicity, we consider a one-dimensional system with periodic boundary condition.

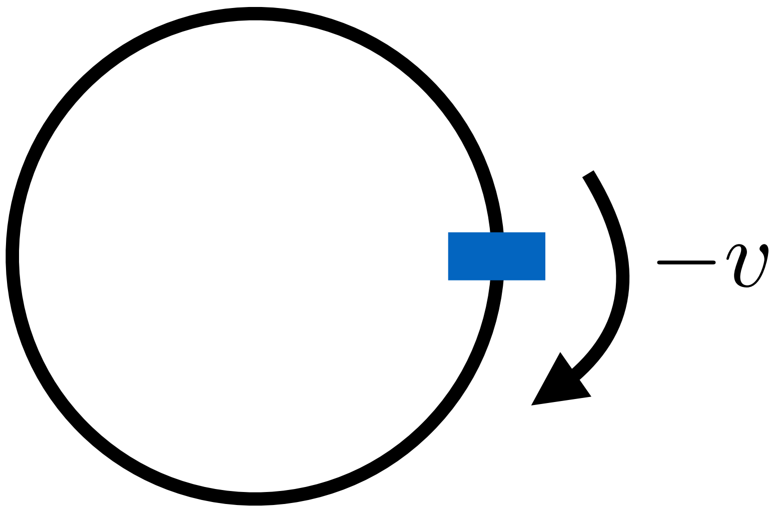
The GP eq. in the laboratory frame

$$i\hbar\frac{\partial}{\partial t'}\Psi'(x',t') = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x'^2} + U'(x',t') + g|\Psi'(x',t')|^2\right]\Psi'(x',t')$$

$$U'(x',t') = U_0 e^{[-(x'+vt')^2/d^2]}$$ Moving defect (Gaussian type)

$$\Psi'(x'+L,t') = \Psi'(x',t')$$ Periodic boundary condition

L : system size



But, this equation is not easy to treat due to the time dependence of the defect.

Lab frame to Moving frame

Convert from the laboratory frame to the moving frame

Unitary transformation $$\Psi(x,t) \equiv \exp\left[\frac{i}{\hbar}\left(\frac{1}{2}mv^2t + mvx'\right)\right]\Psi'(x',t')$$

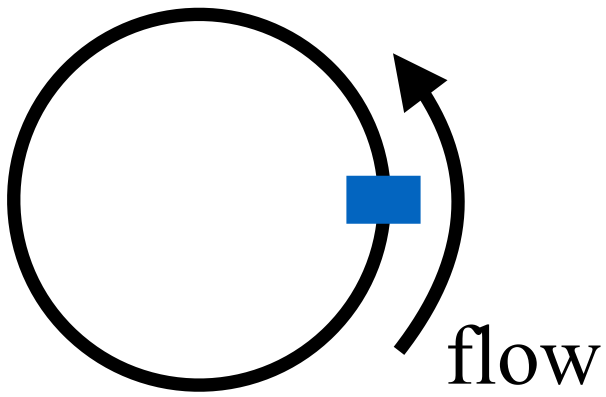
$$x \equiv x' + vt' \qquad t \equiv t'$$

The GP eq. in the moving frame

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x) + g|\Psi(x,t)|^2\right]\Psi(x,t)$$

$U(x) = U_0 e^{-(x/d)^2}$ Static defect

$\Psi(x+L,t) = e^{imvL/\hbar}\Psi(x,t)$ **Twisted Periodic boundary condition**



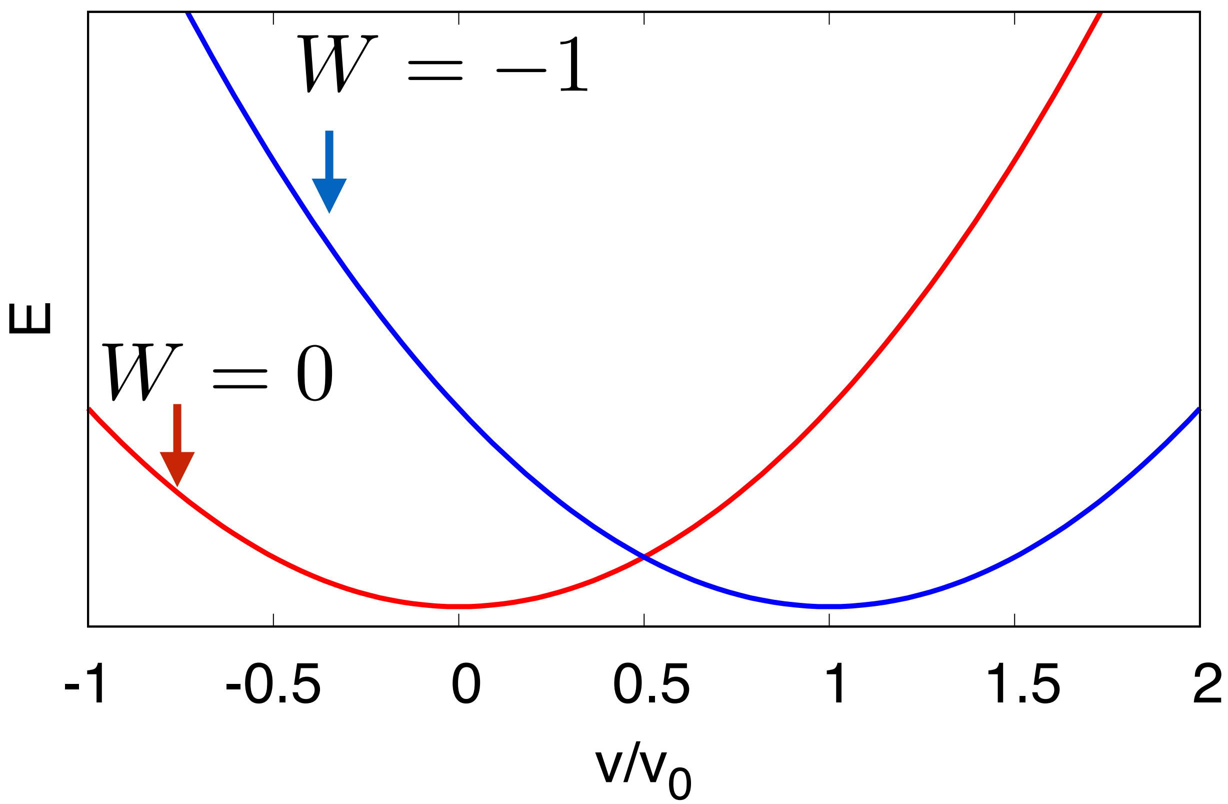
Stationary solution : $\Psi(x,t) = e^{-i\mu t/\hbar}\Psi(x)$

μ : chemical potential

Swallowtail structure in 1D ring

Energy vs rotation velocity (non-interacting case)

$g = 0$ and $U = 0$

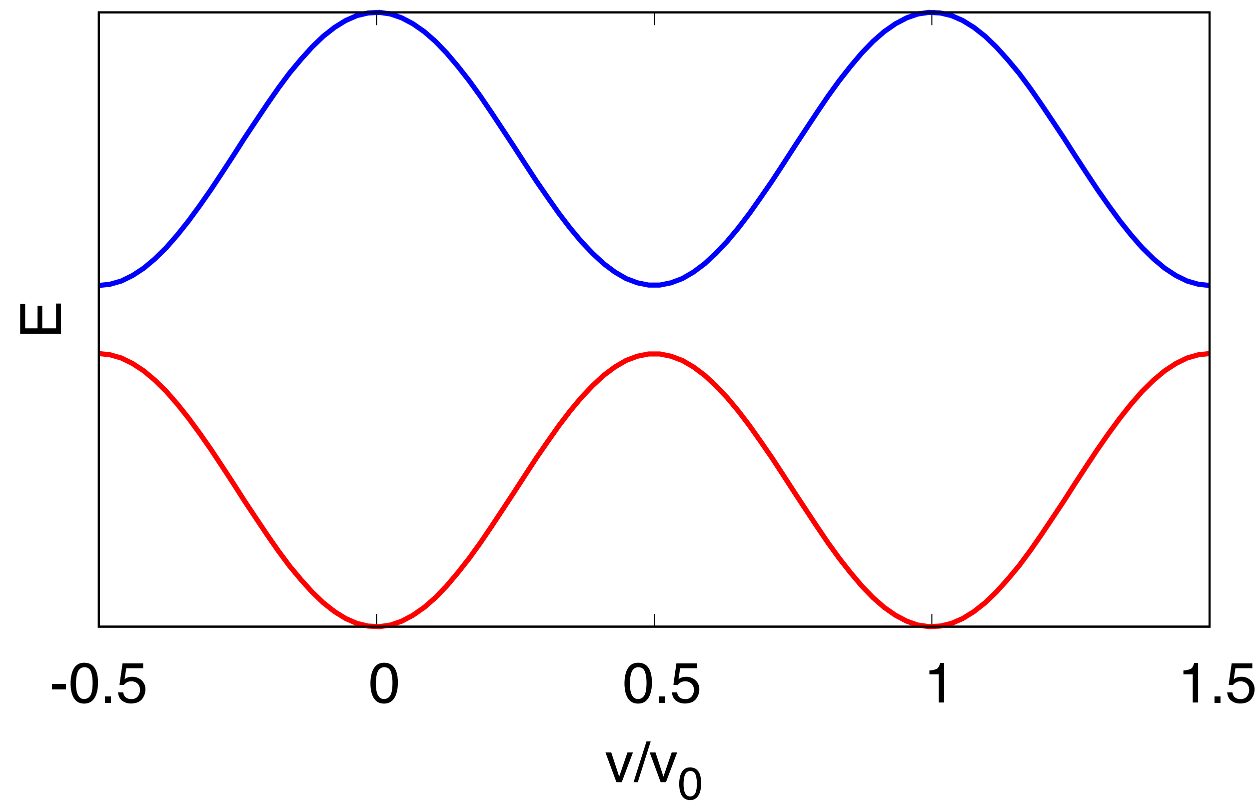


$$\Psi(x) \propto e^{im(v+Wv_0)x/\hbar}$$

$$E = \frac{m}{2}(v + Wv_0)^2$$

$$v_0 \equiv 2\pi\hbar/mL$$

$g = 0$ and $U \neq 0$



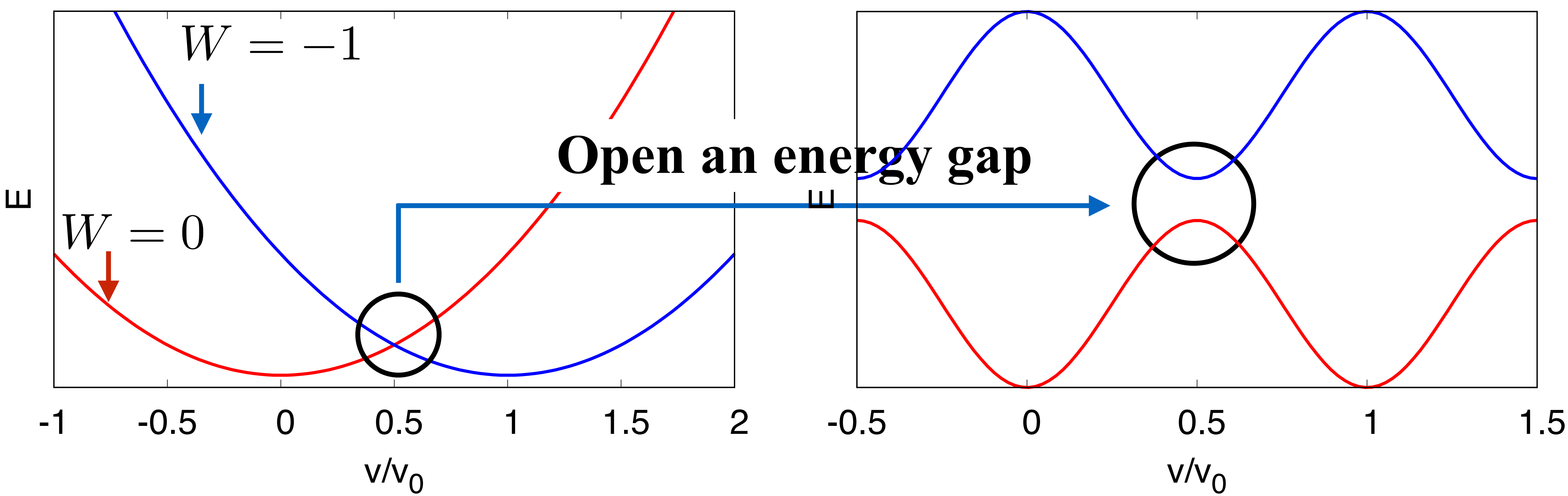
$$\Psi(x+L,t) = e^{imvL/\hbar}\Psi(x,t)$$

is invariant under $v \to v + nv_0$

Swallowtail structure in 1D ring

Energy vs rotation velocity (non-interacting case)

$g = 0$ and $U = 0$ $\qquad$ $g = 0$ and $U \neq 0$



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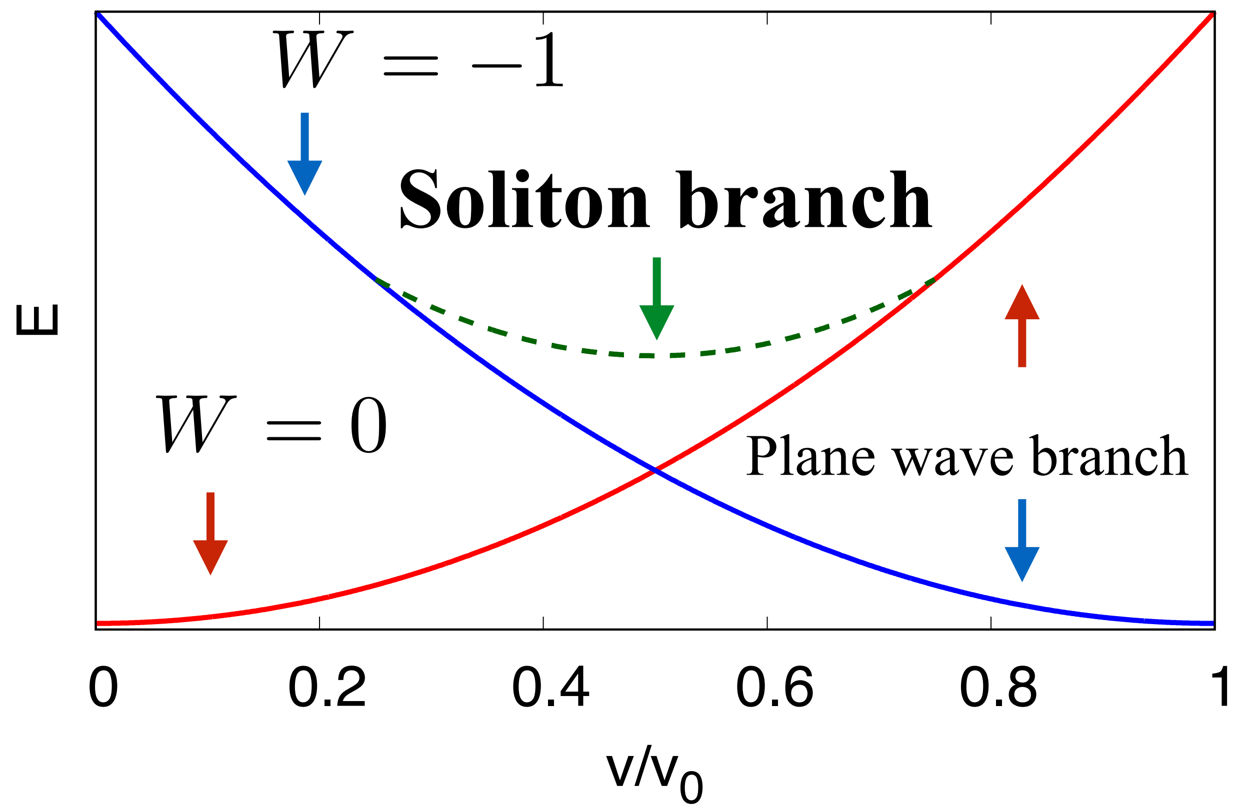
$$\Psi(x+L,t) = e^{imvL/\hbar}\Psi(x,t)$$

is invariant under $v \to v + nv_0$

Swallowtail structure in 1D ring

Energy vs rotation velocity (interacting case)

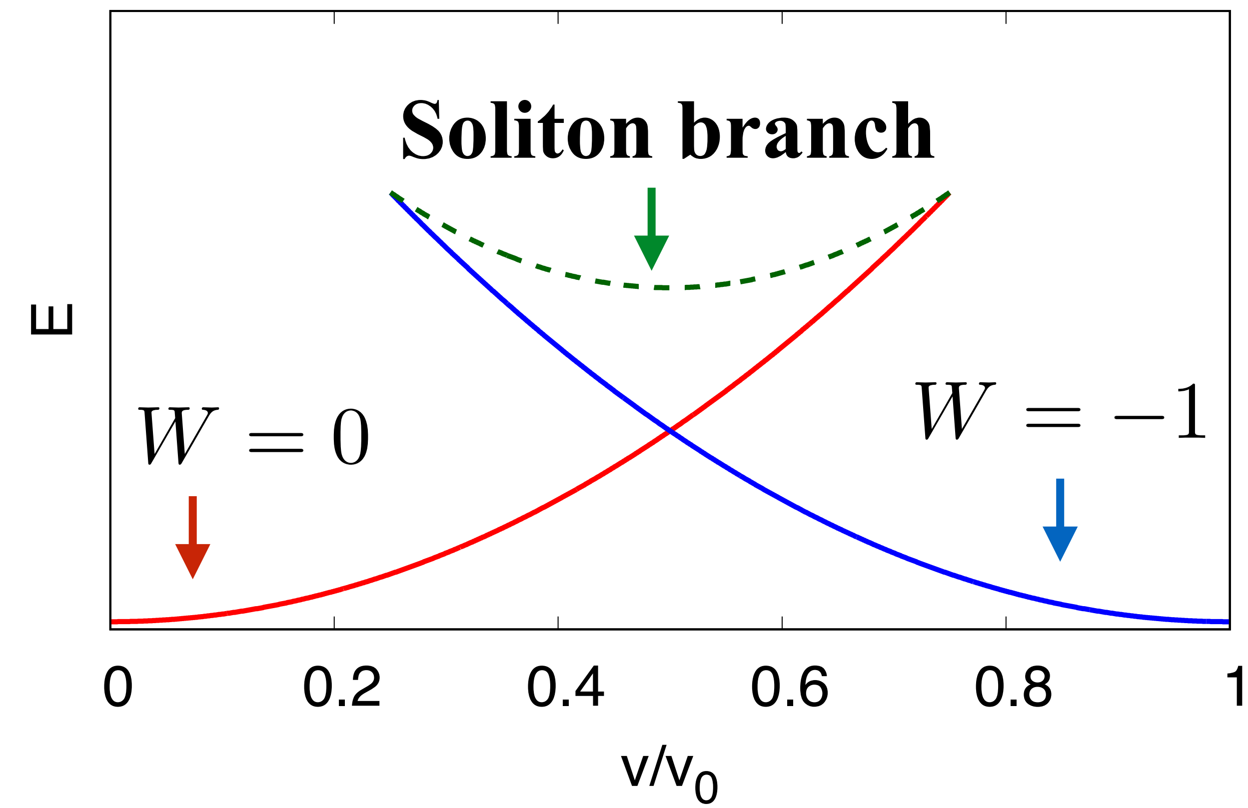
$g > 0$ and $U = 0$



According to the non-linearity, the soliton branches appear.

Solitonic solutions are unstable.

$g > 0$ and $U \neq 0$

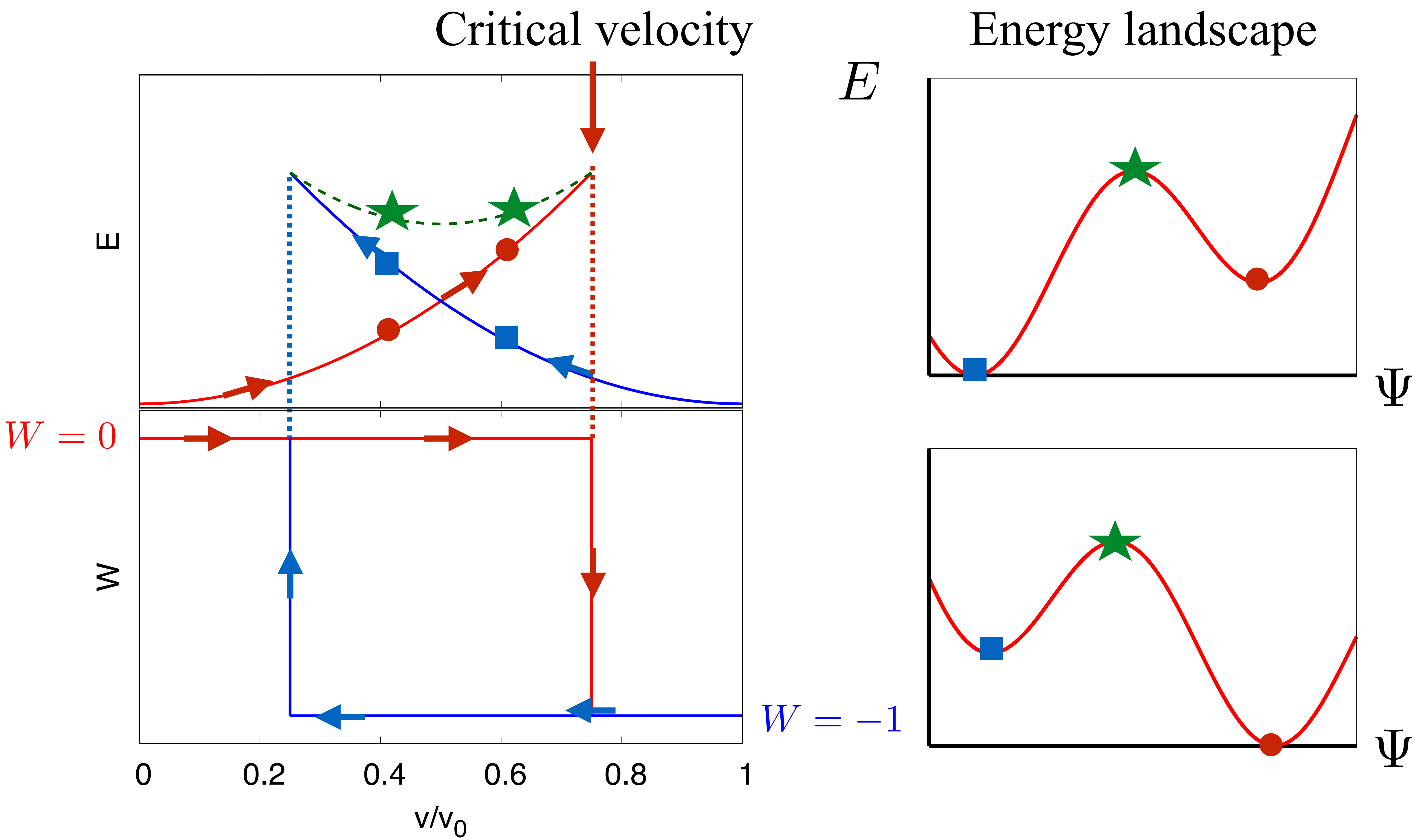


Swallowtail structure

E. J. Mueller, Phys. Rev. A **66**, 063603 (2002).
R. Kanamoto et al., Phys. Rev. A **79**, 063616 (2009).

Physical interpretation of the Swallowtail

The swallowtail structure means the existence of hysteresis.



Physical interpretation of the Swallowtail

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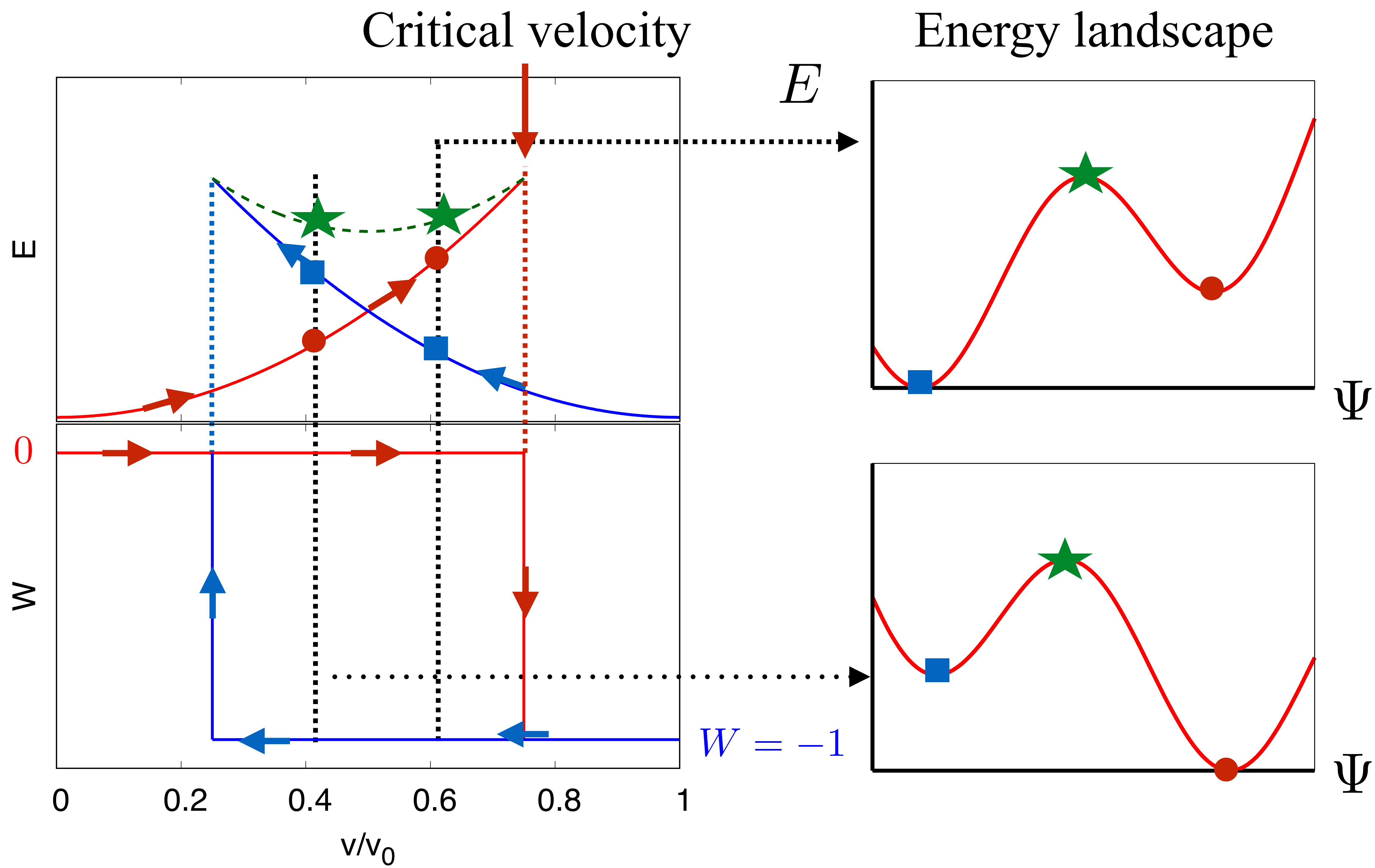
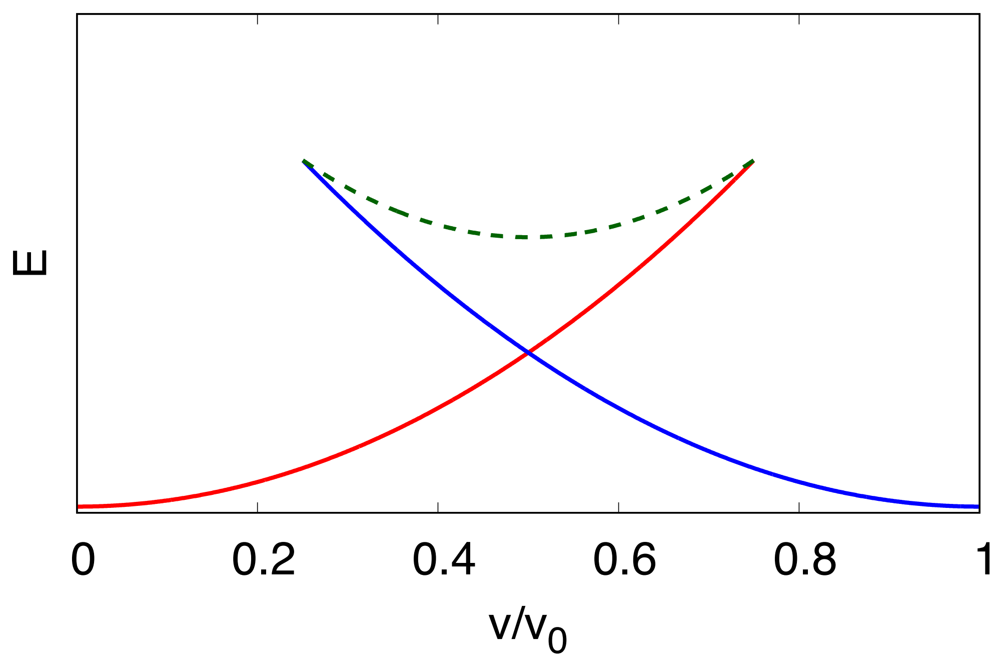


Motivation of this study

In 1D ring systems, the swallow tail structure is important to understand hysteresis phenomena.

Question : What happens in higher dimension?



How about the swallowtail structure in 2D?

Effects of the quantum vortices?

$\Rightarrow$ **We consider 2D torus system with moving defect.**

Model : GP equation in 2D system

The GP eq. in the moving frame (2D)

$$\left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2+U(\boldsymbol{r})+g|\Psi(\boldsymbol{r})|^2\right]\Psi(\boldsymbol{r})=\mu\Psi(\boldsymbol{r})$$

$$U(\boldsymbol{r})=U_0\exp\left[-(\boldsymbol{r}/d)^2\right]$$ Defect

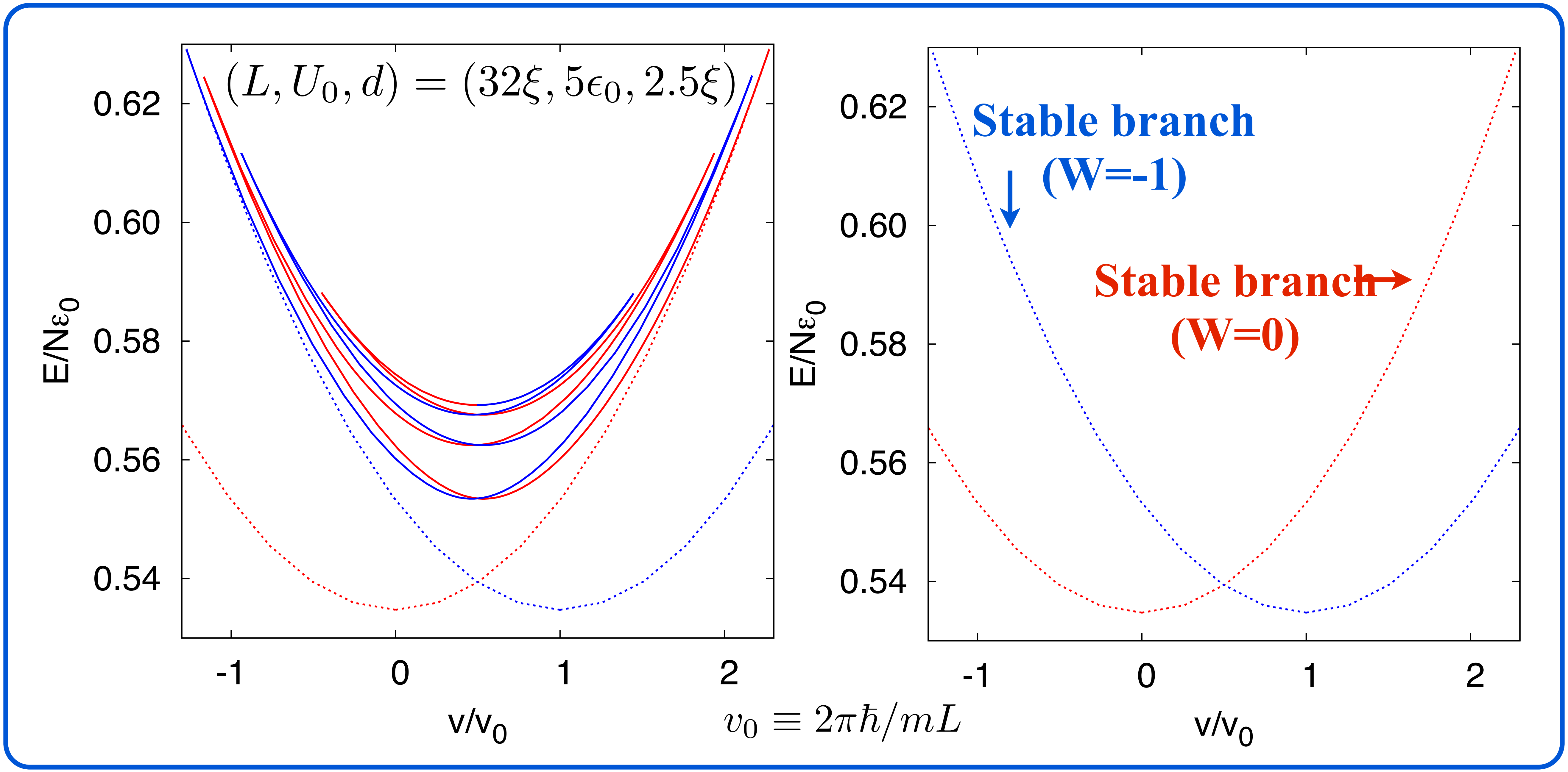
$$\Psi(\boldsymbol{r}+L\boldsymbol{e}_x)=e^{imvL/\hbar}\Psi(\boldsymbol{r})$$
$$\Psi(\boldsymbol{r}+L\boldsymbol{e}_y)=\Psi(\boldsymbol{r})$$
Twisted periodic boundary condition (only x direction)

Numerical method : **pseudo-arclength continuation method**

$\Rightarrow$ easy to obtain unstable solutions

Reference : H. B. Keller, *Lectures on Numerical Methods In Bifurcation Problems (Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1987).*

Results : Multiple-swallowtail structure



$\xi \equiv \hbar/\sqrt{mgn_0}$: healing length

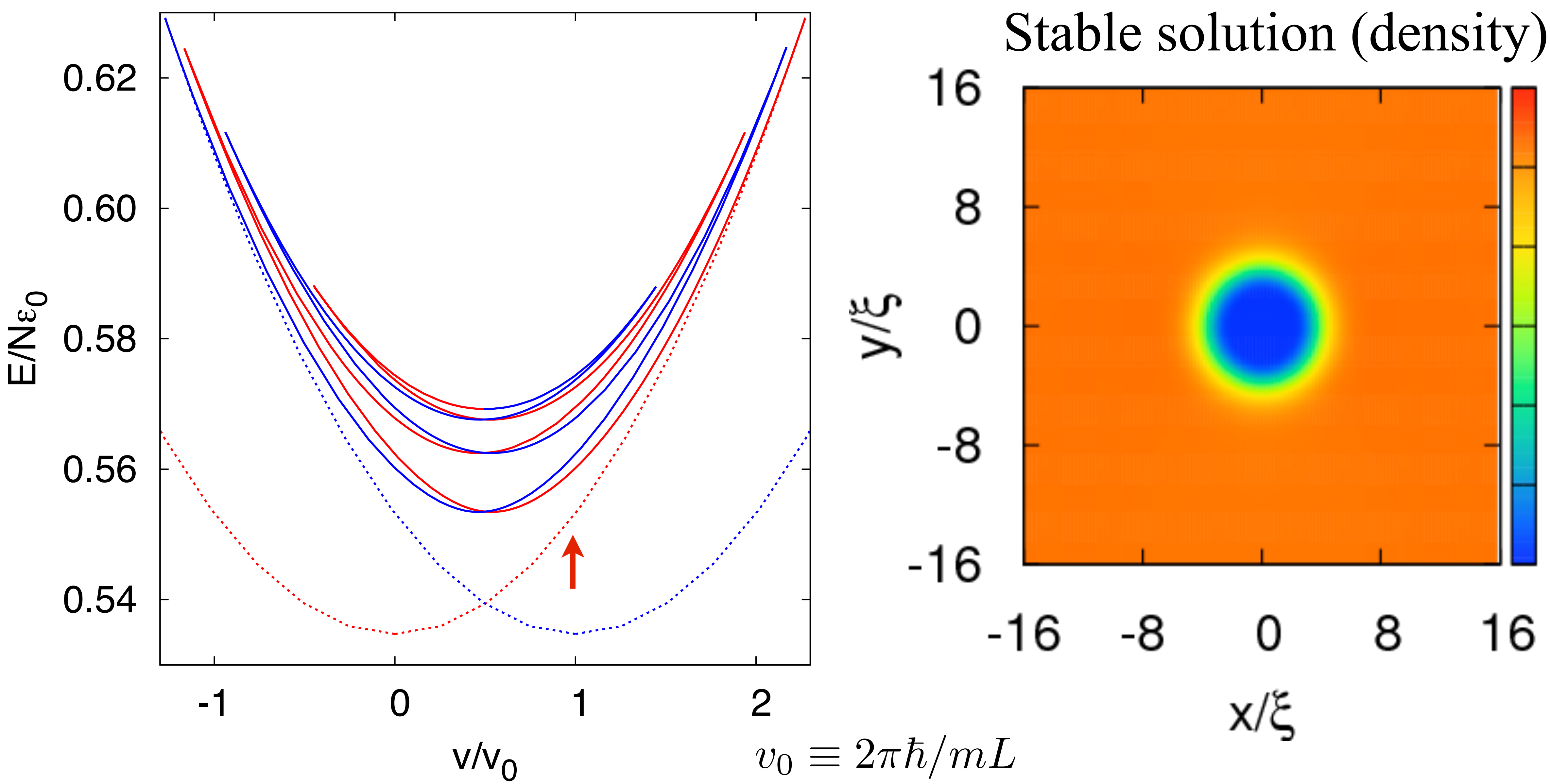
$\epsilon_0 \equiv gn_0$: characteristic energy

n_0 : mean density

Winding number

$$W \equiv \frac{1}{2\pi}\int_{-L/2}^{+L/2} dx\, \frac{\partial \varphi(\boldsymbol{r})}{\partial x}\bigg|_{y=L/2}$$

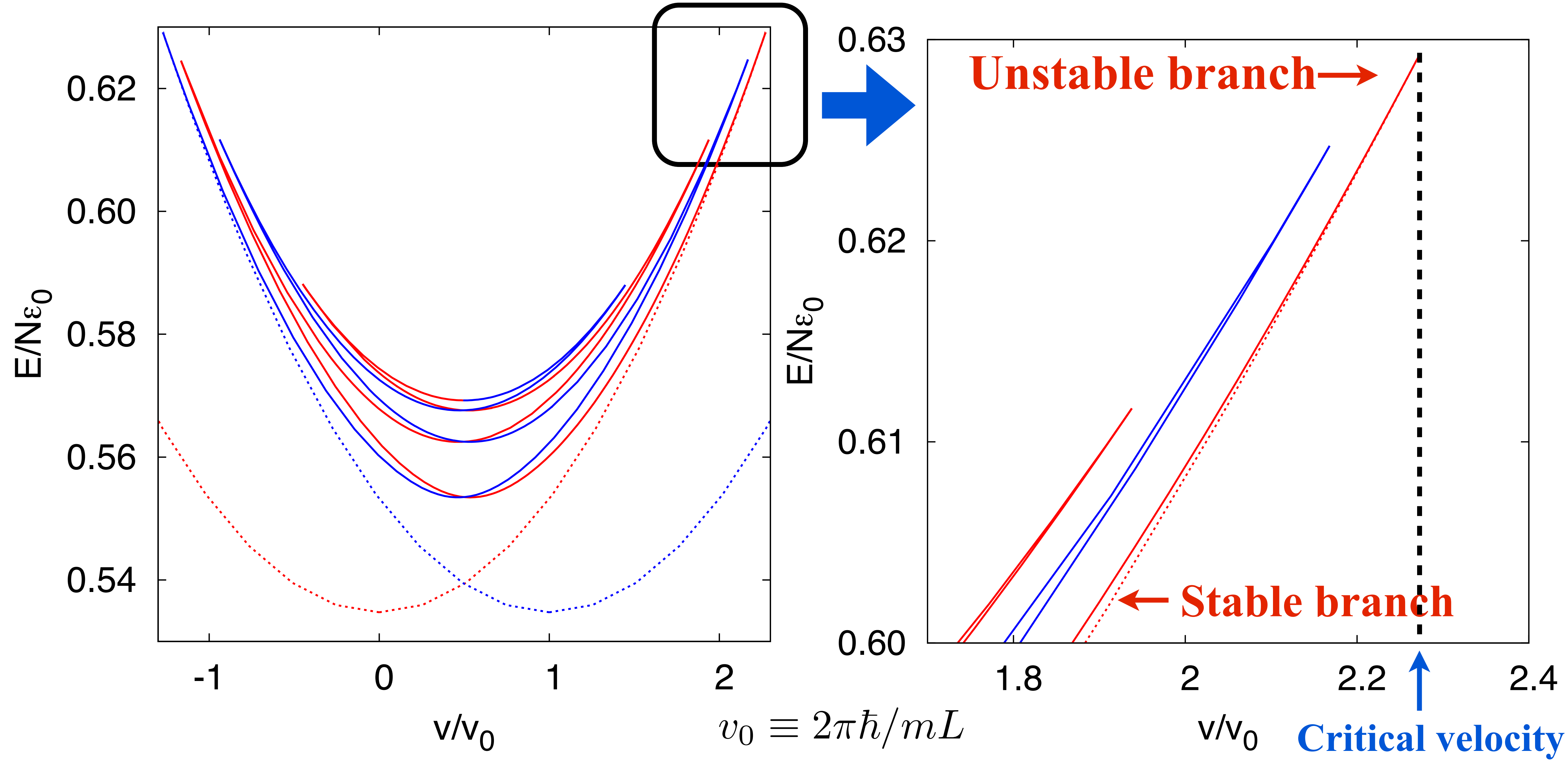
Results : Multiple-swallowtail structure



$v_0 \equiv 2\pi\hbar/mL$

- Stable solution, no quantum vortex

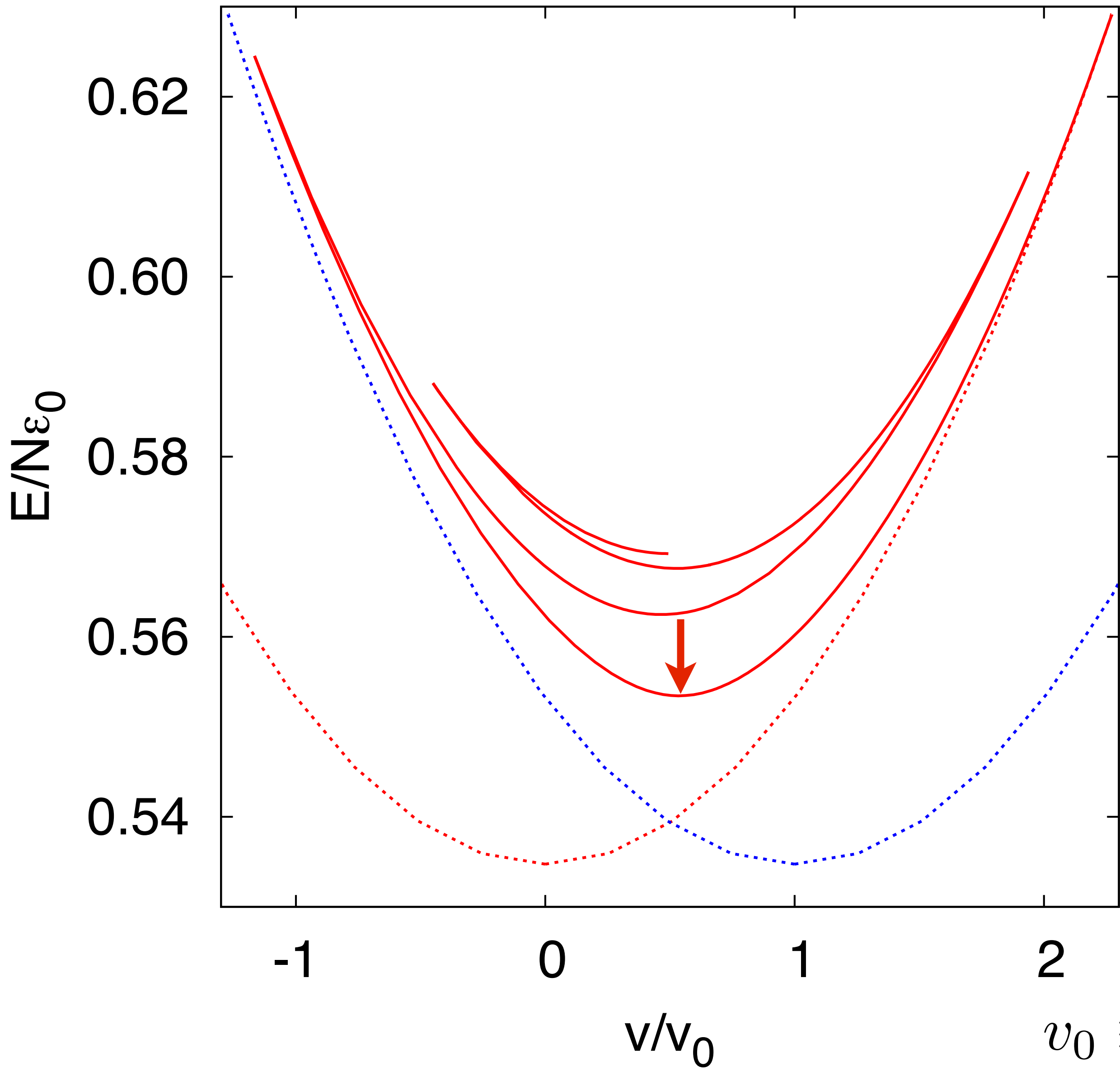
Results : Multiple-swallowtail structure



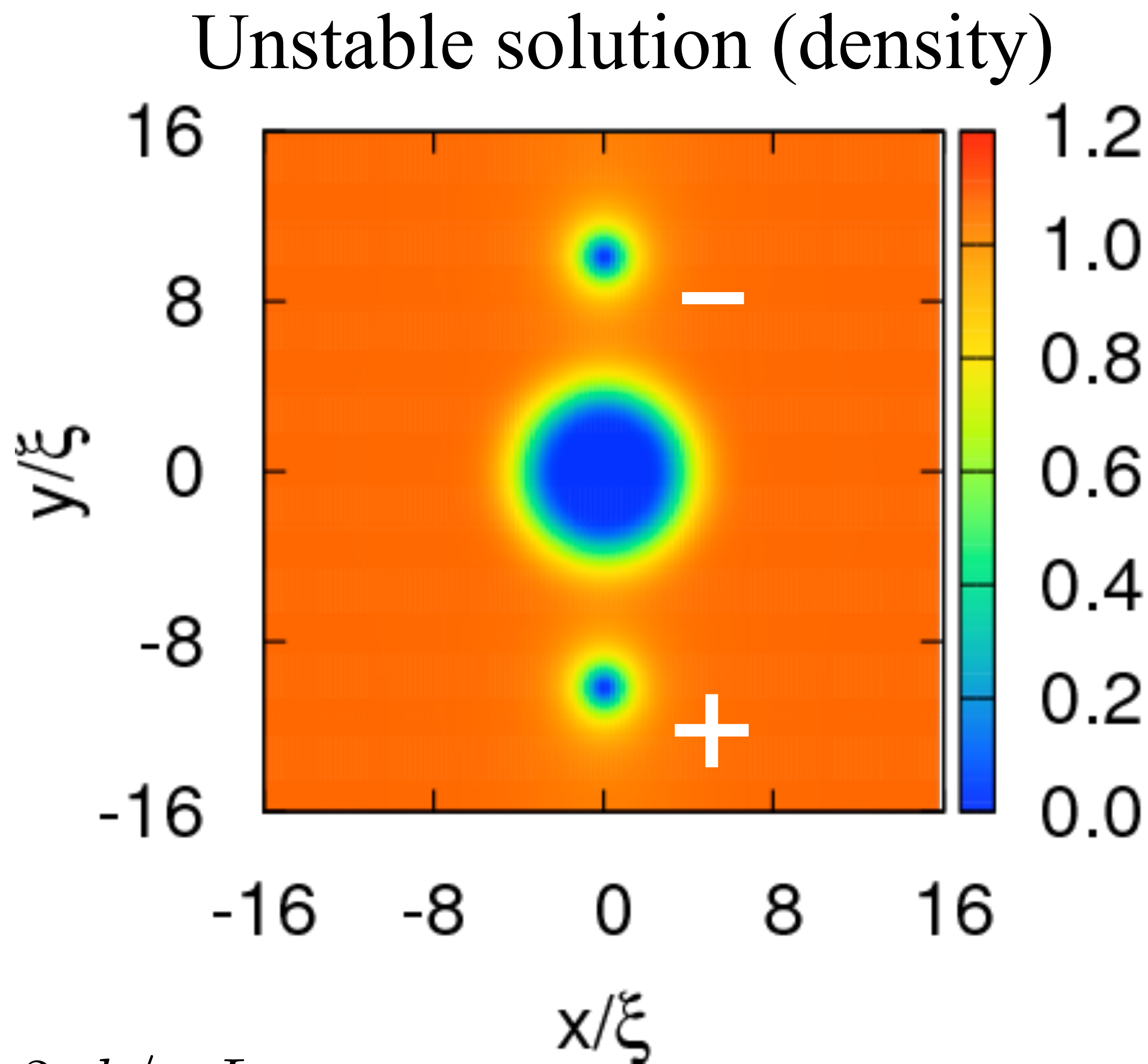
$v_0 \equiv 2\pi\hbar/mL$

- The stable solution and the unstable solution merge at the critical velocity.

Results : Multiple-swallowtail structure

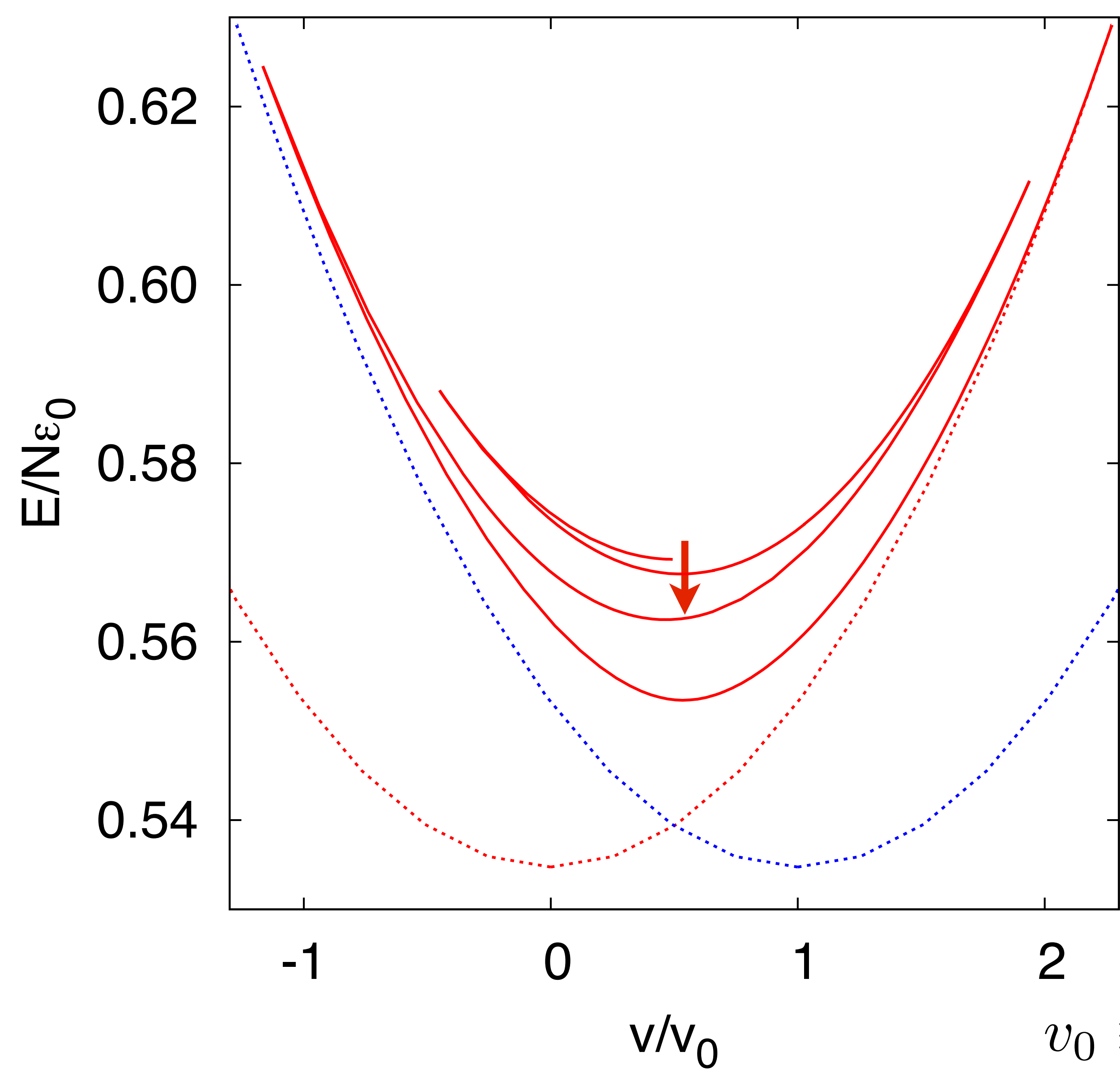


$v_0 \equiv 2\pi\hbar/mL$



- There is one vortex pair. Similar solutions are reported by C. Huepe and M-E. Brachet, Physica D, **140**, 126 (2000).

Results : Multiple-swallowtail structure



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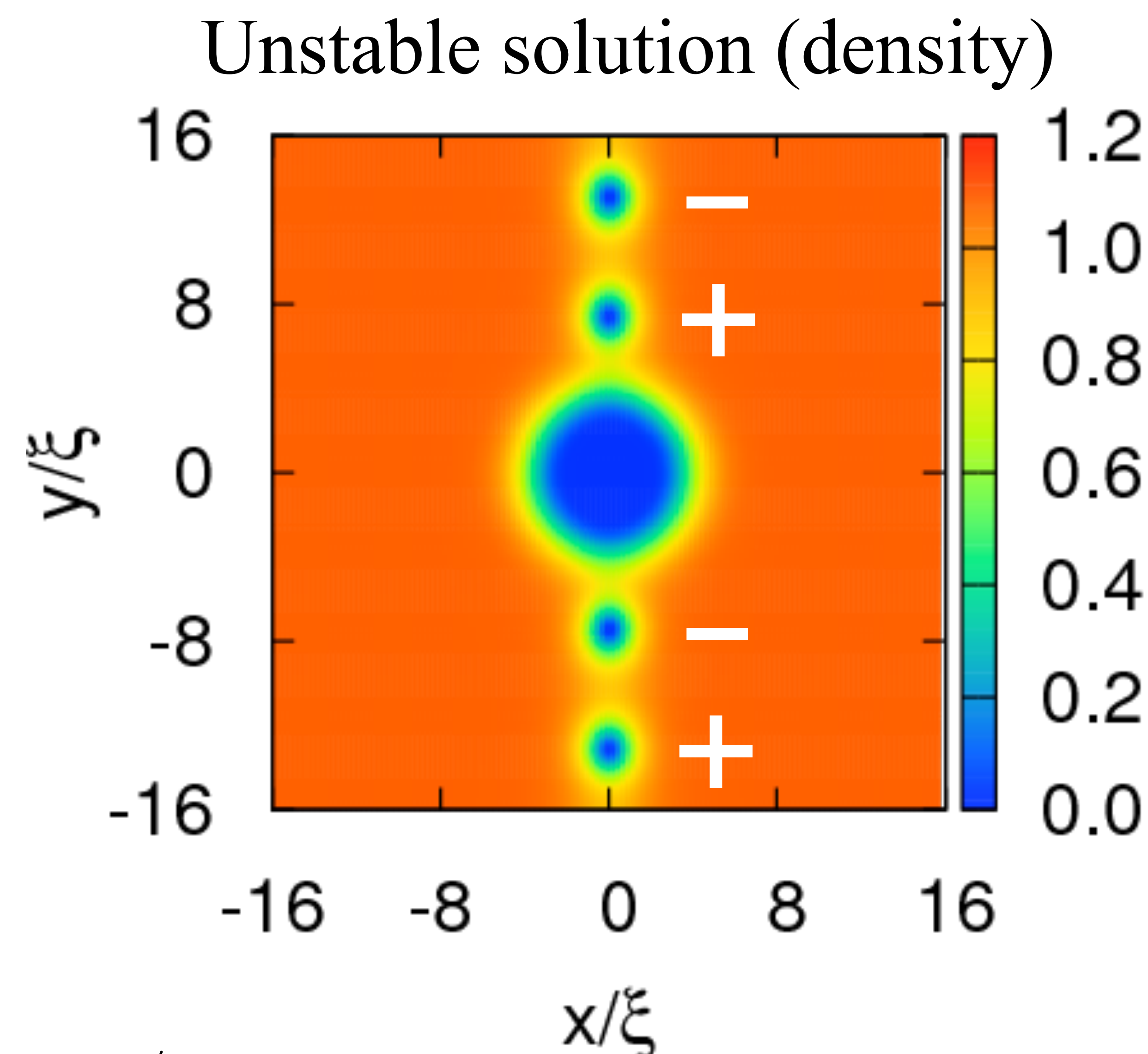
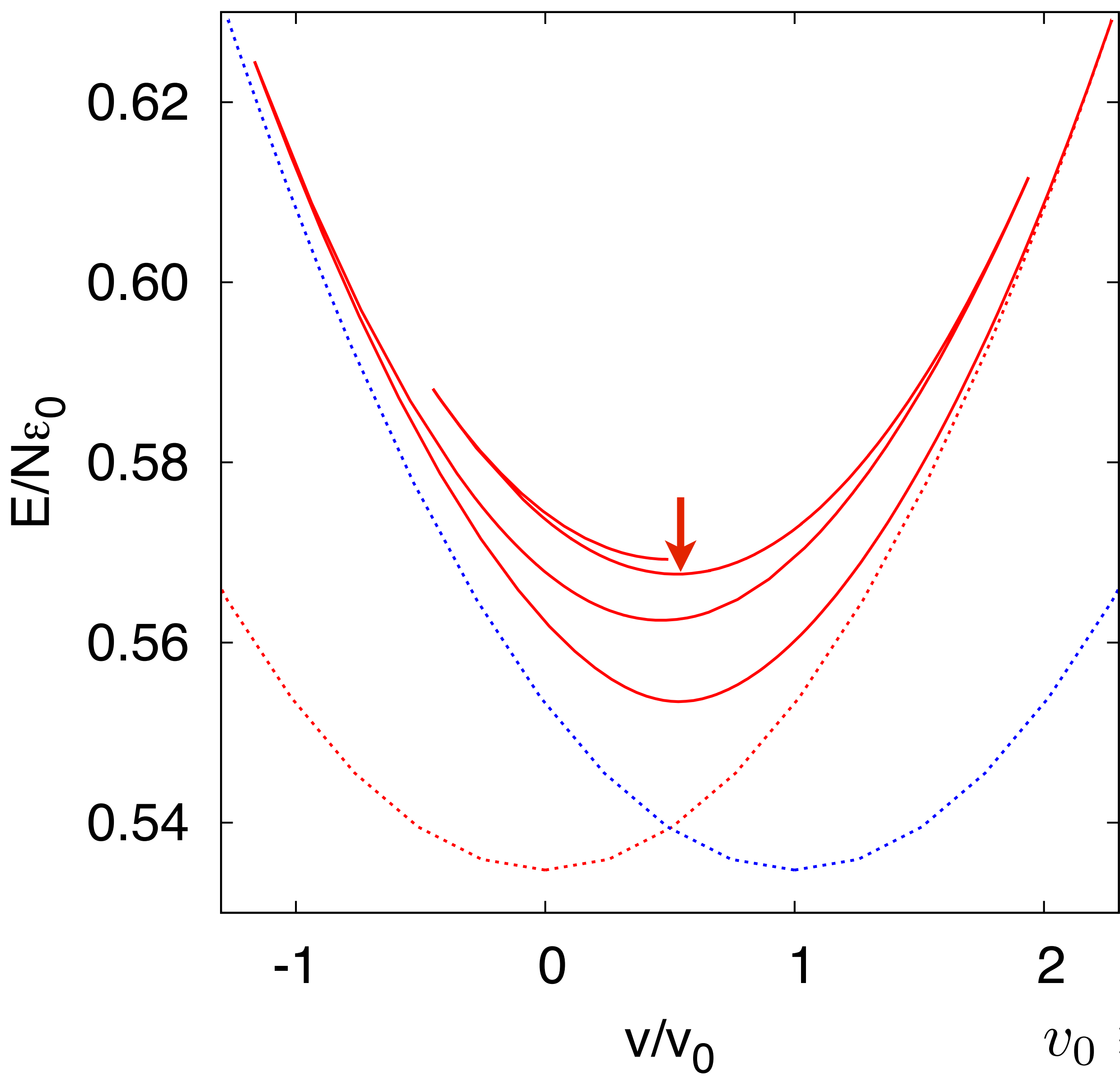
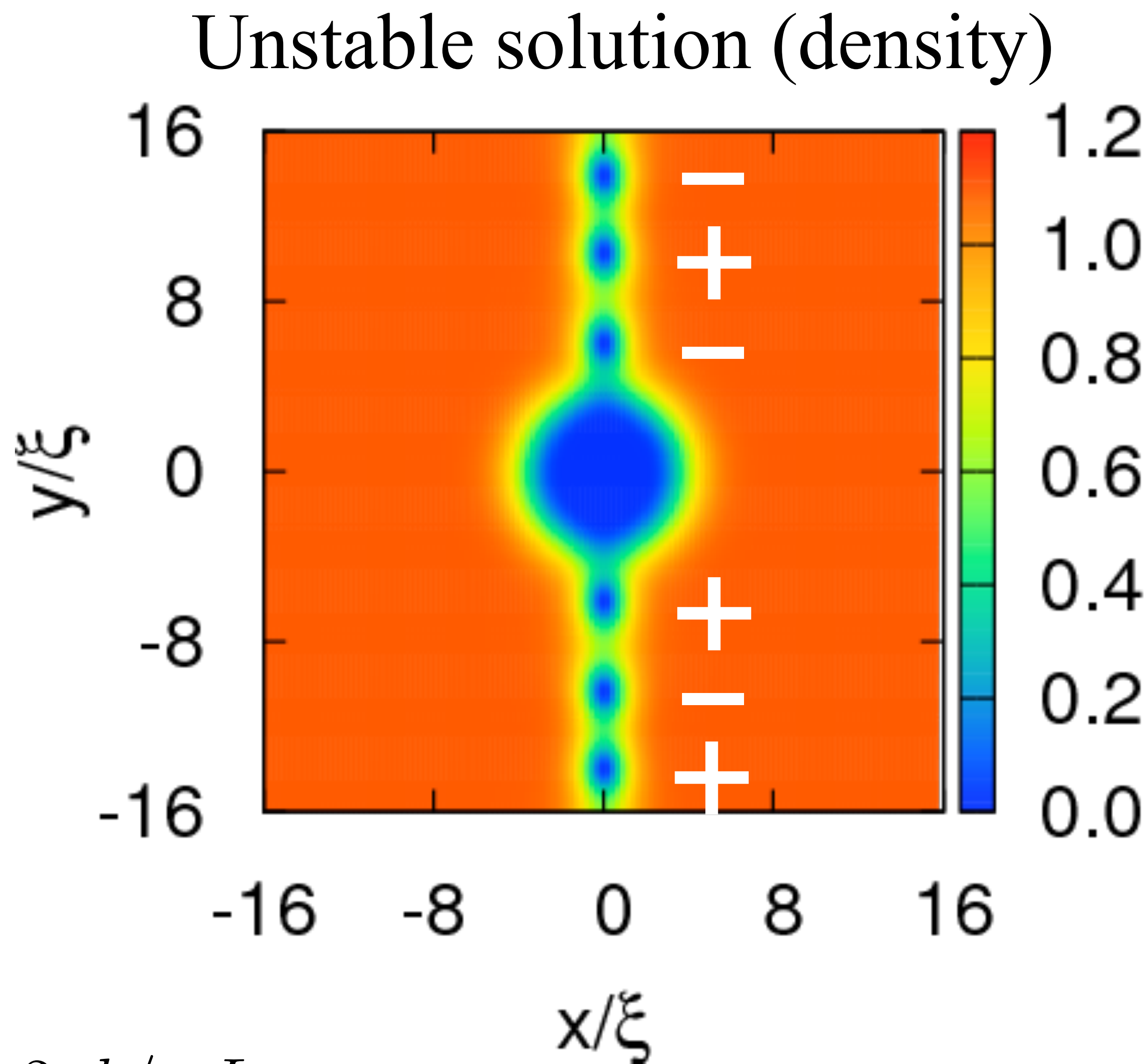


- There are two vortex pair.

Results : Multiple-swallowtail structure

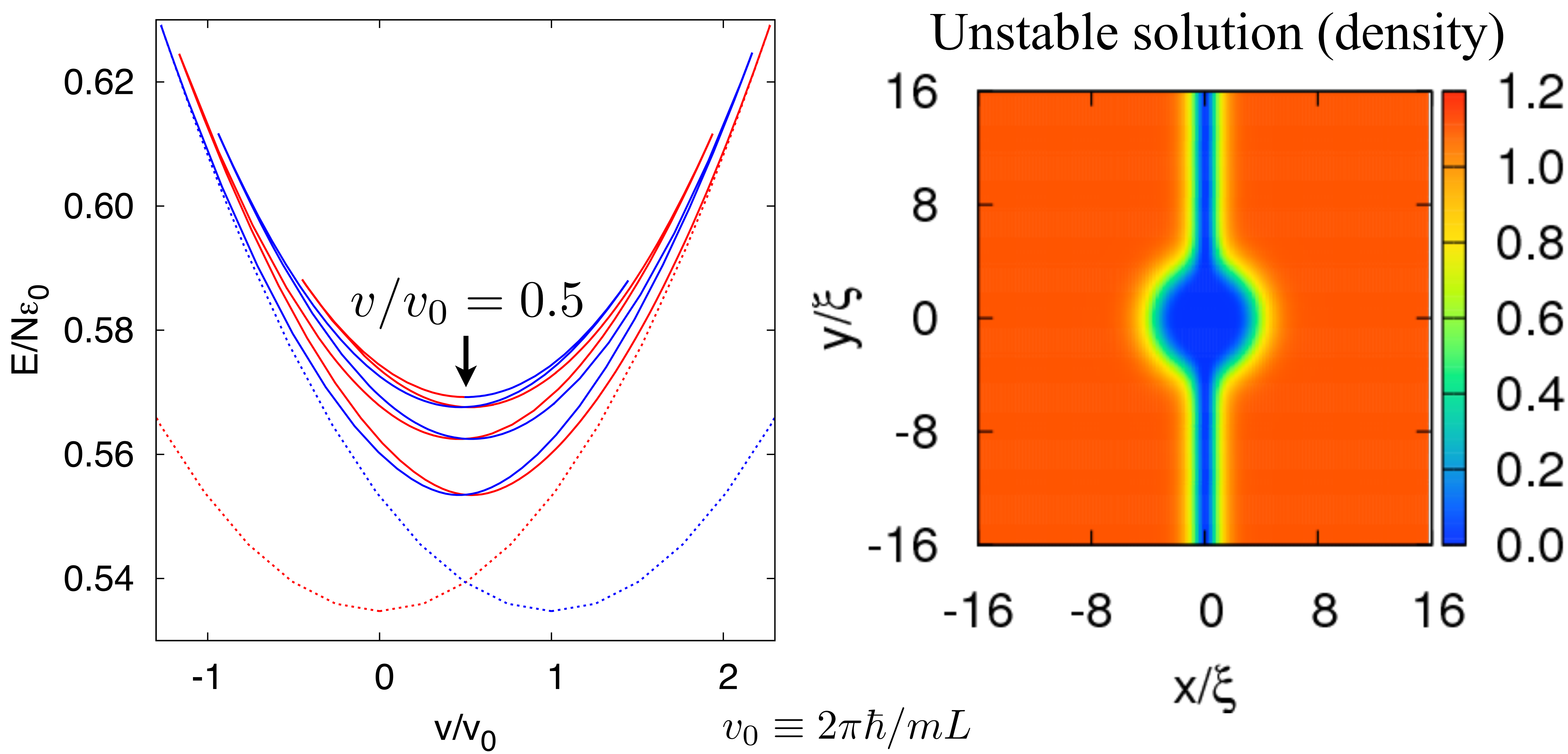




$v_0 \equiv 2\pi\hbar/mL$

- There are three vortex pair.

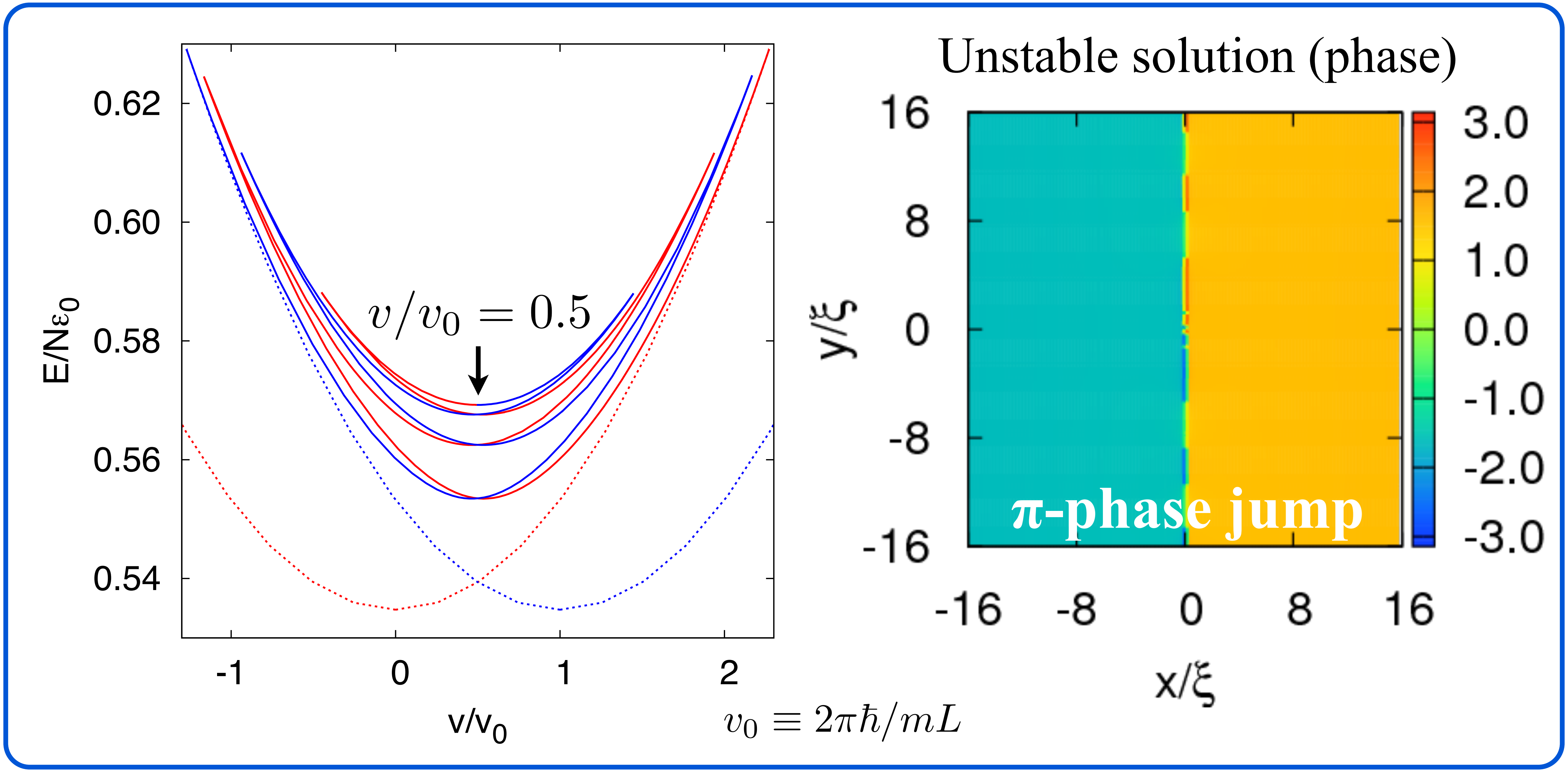
Results : Multiple-swallowtail structure



- The winding number changes by forming dark soliton (self-induced phase slip)

R. Kanamoto, *et al.*, Phys. Rev. A **79**, 063616 (2009).

Results : Multiple-swallowtail structure



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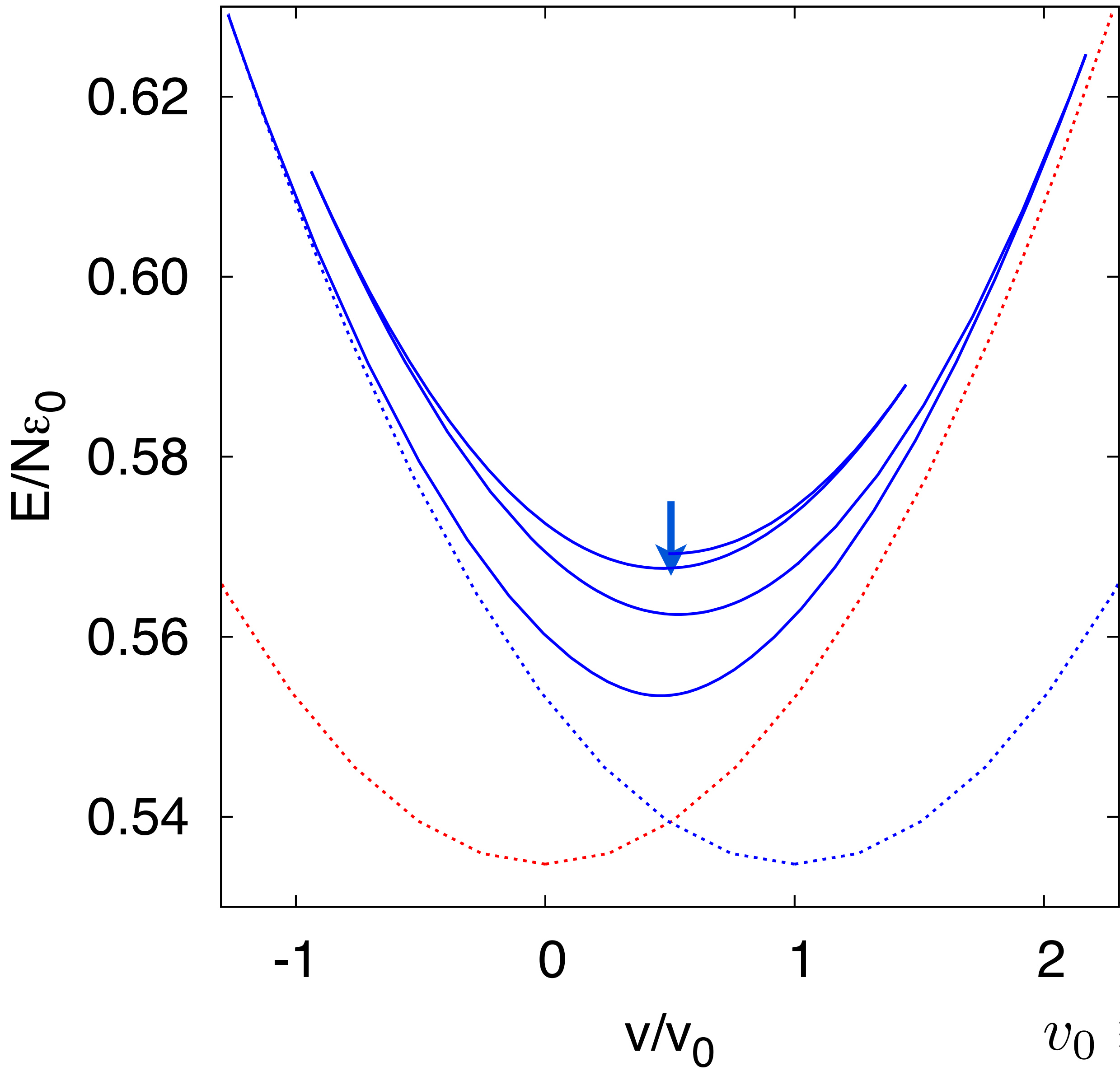
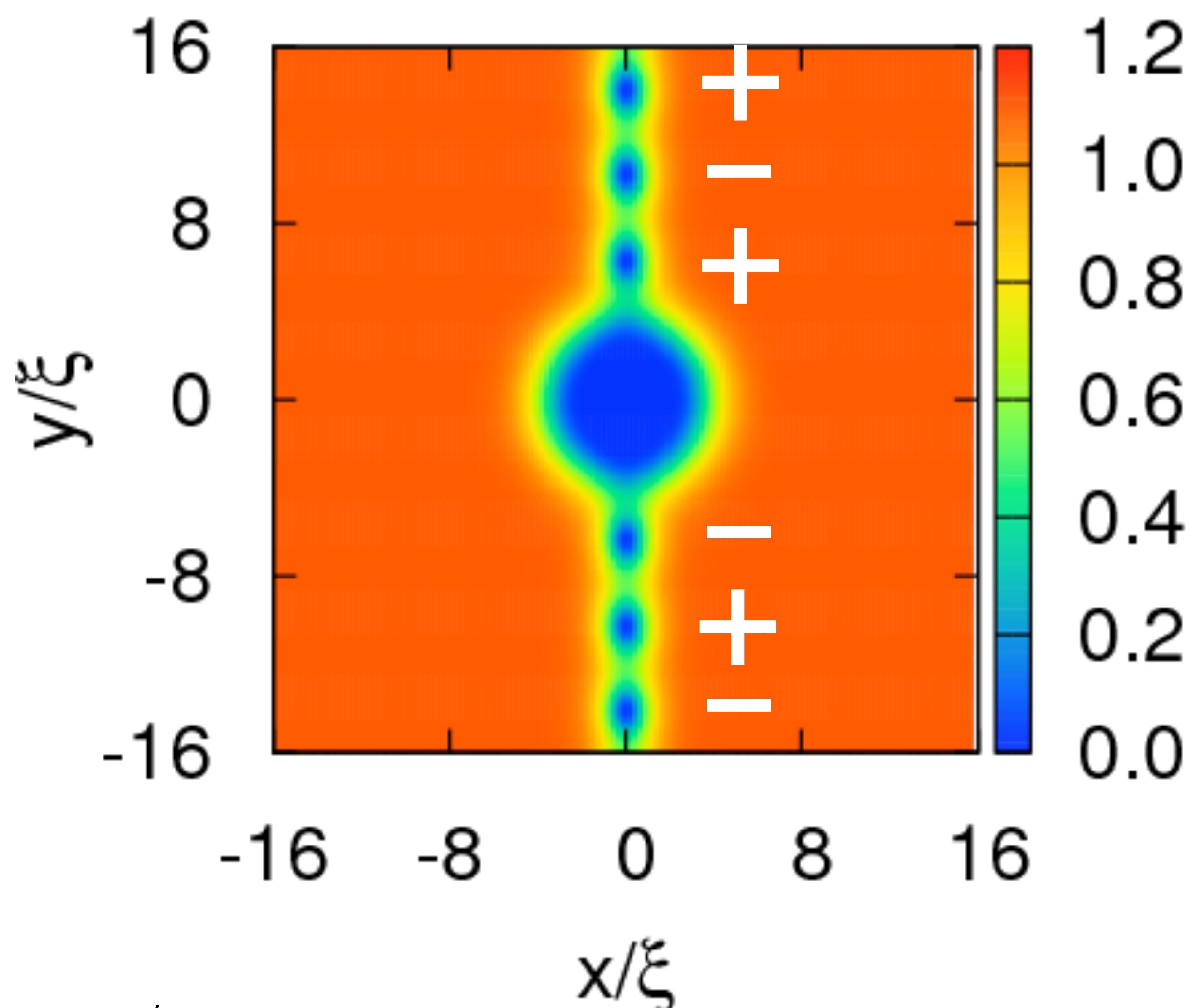


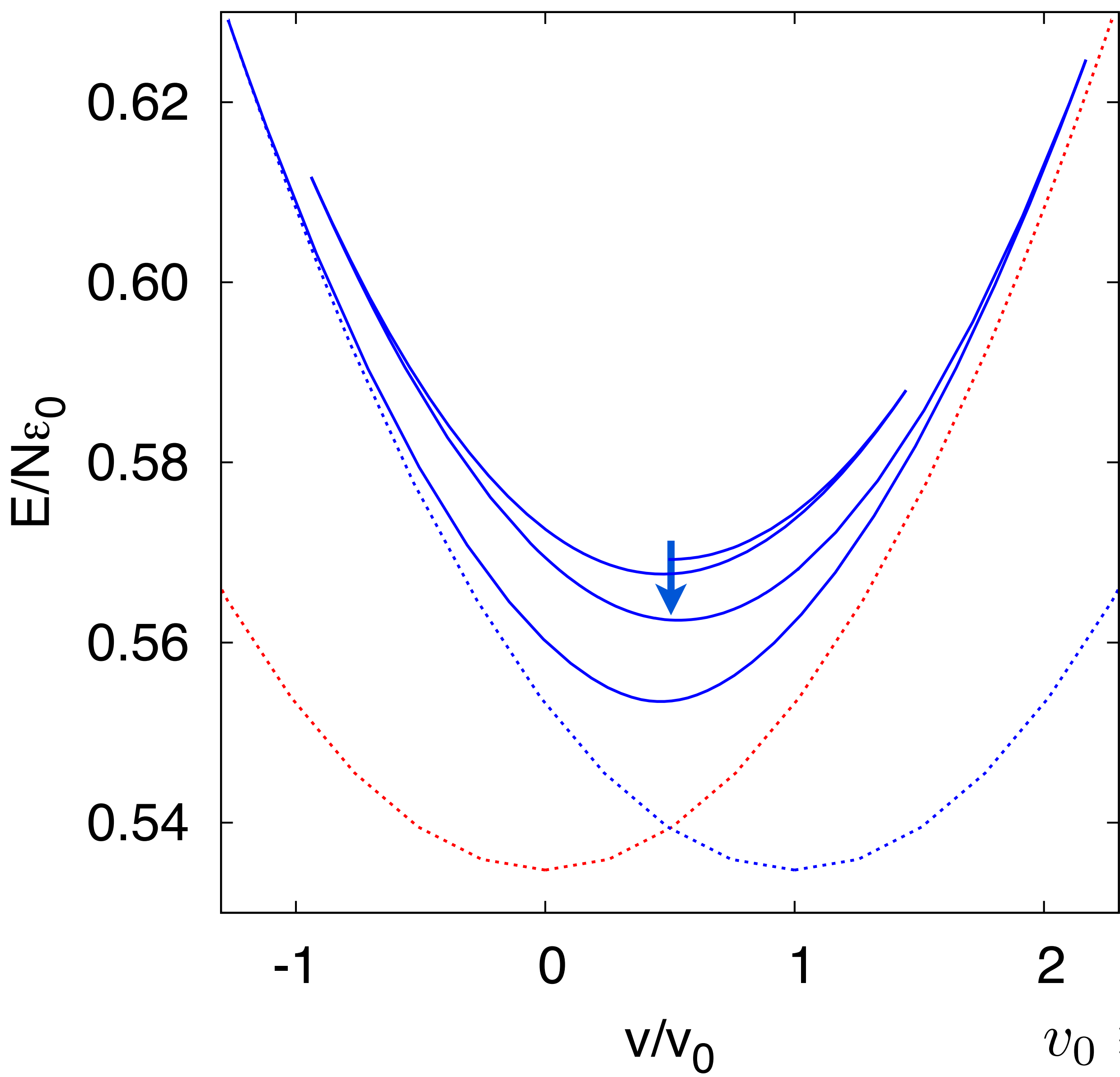
$v_0 \equiv 2\pi\hbar/mL$

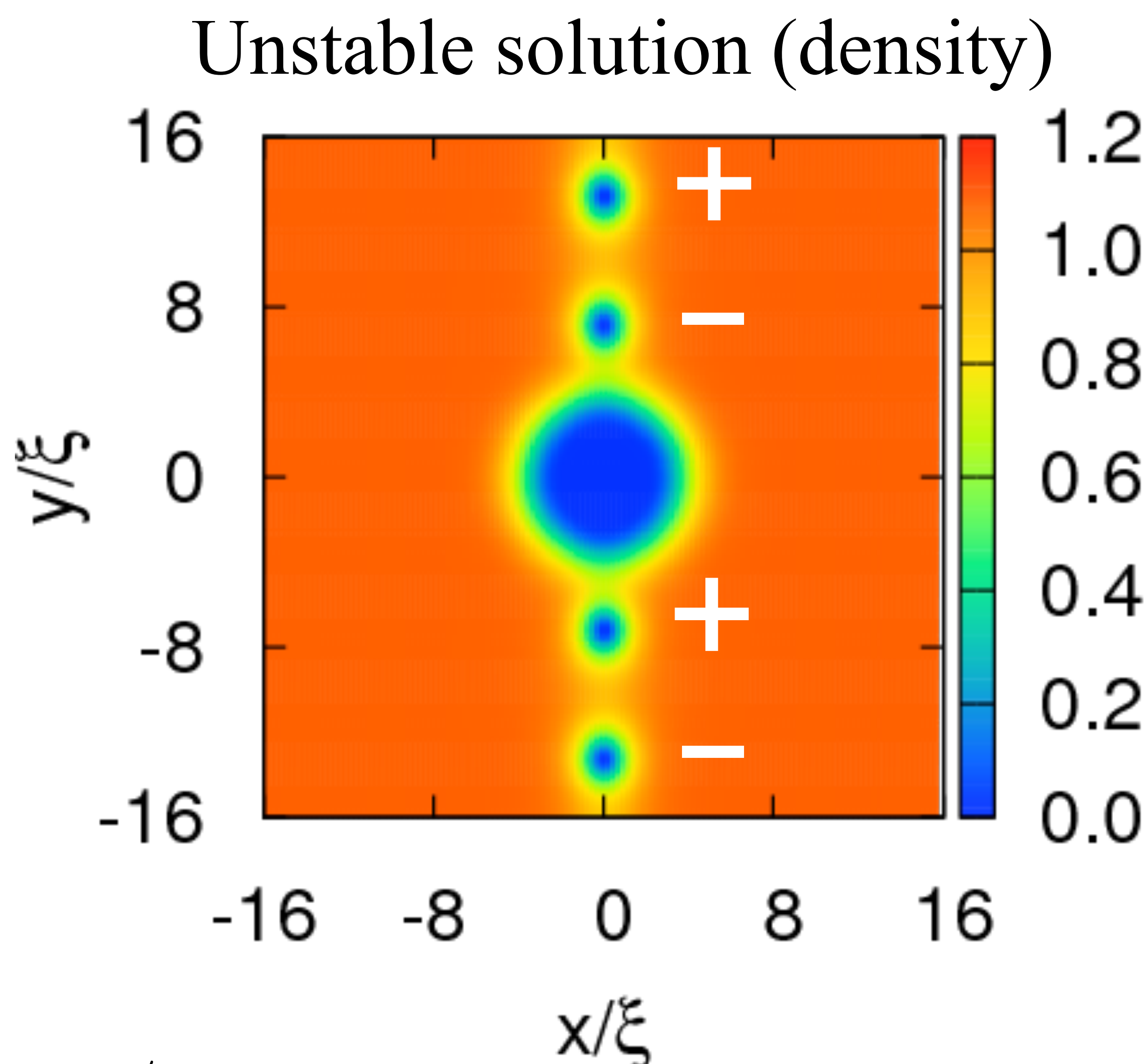
Unstable solution (density)



- There are three vortex pair.

Results : Multiple-swallowtail structure

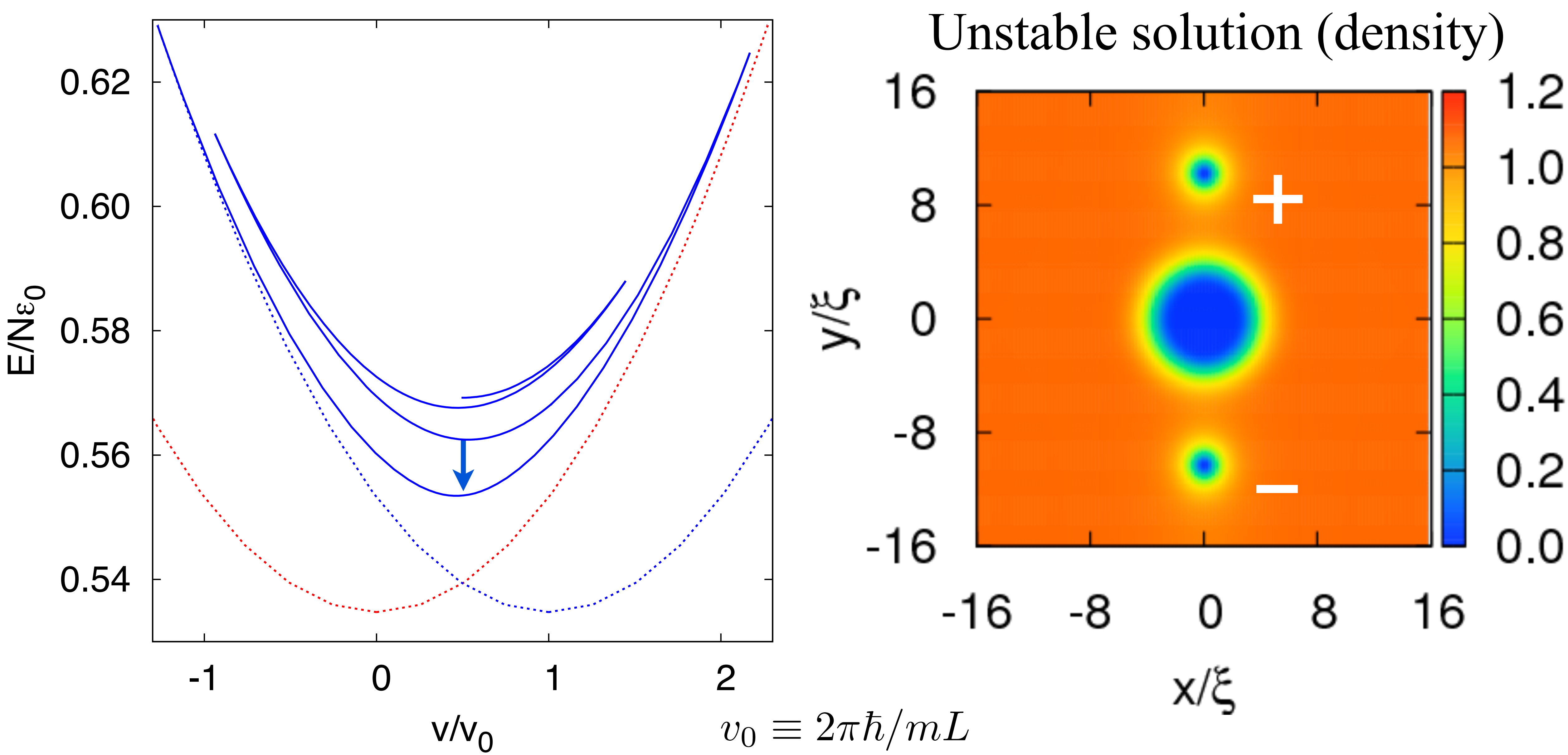




$v_0 \equiv 2\pi\hbar/mL$

- There are two vortex pair.

Results : Multiple-swallowtail structure

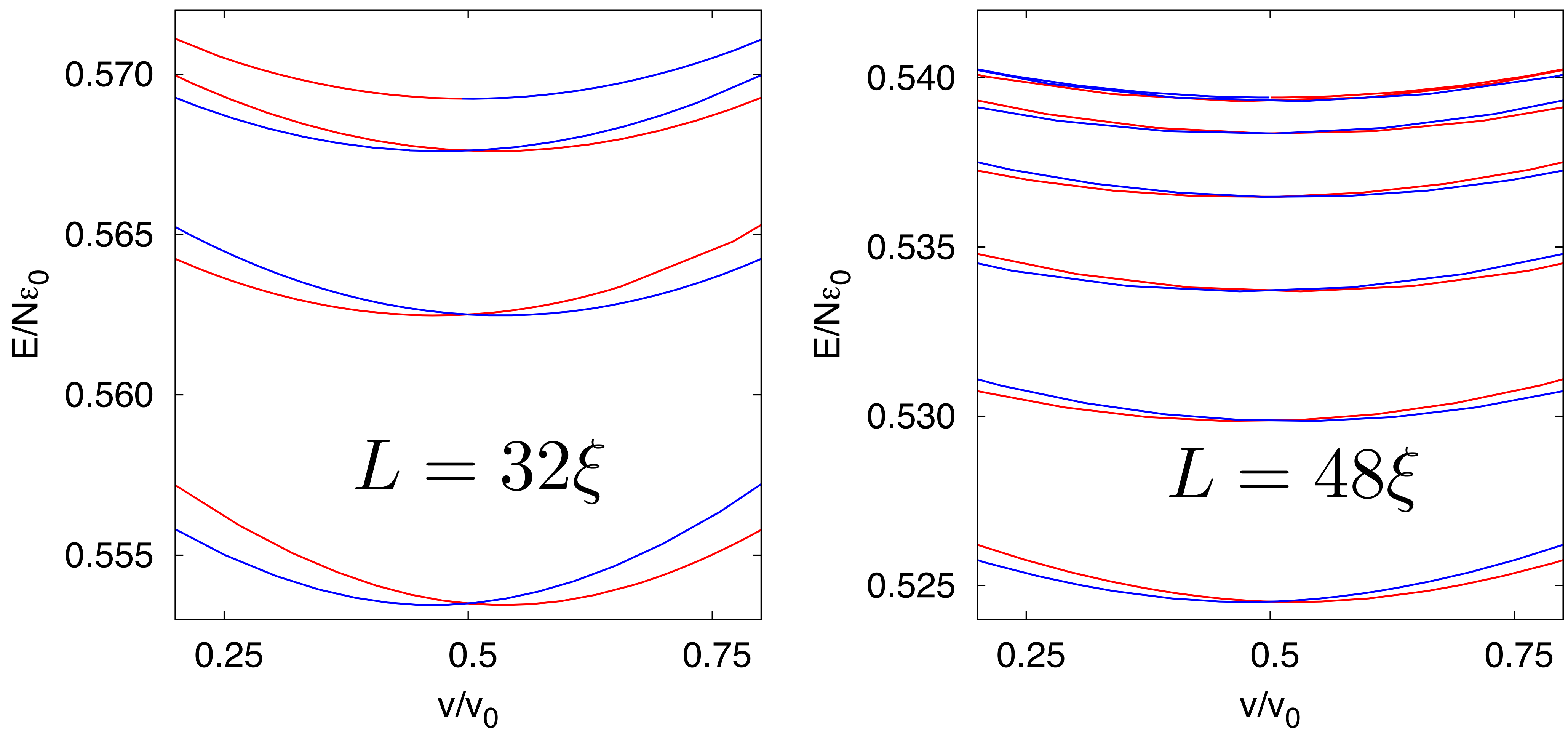


$v_0 \equiv 2\pi\hbar/mL$

- There is one vortex pair.

Results : Relation between swallow tail in 1D

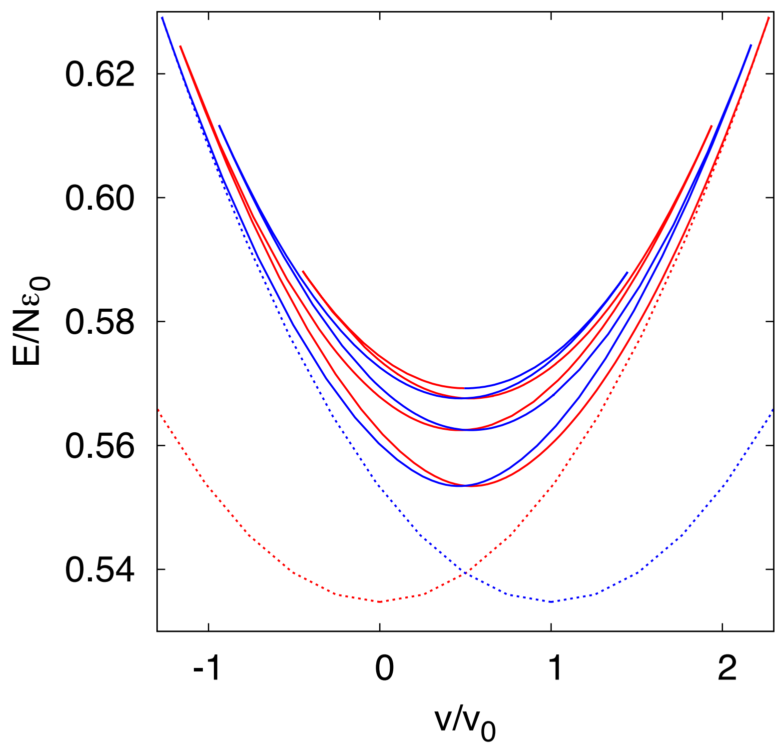
The system size dependence of the number of branches.

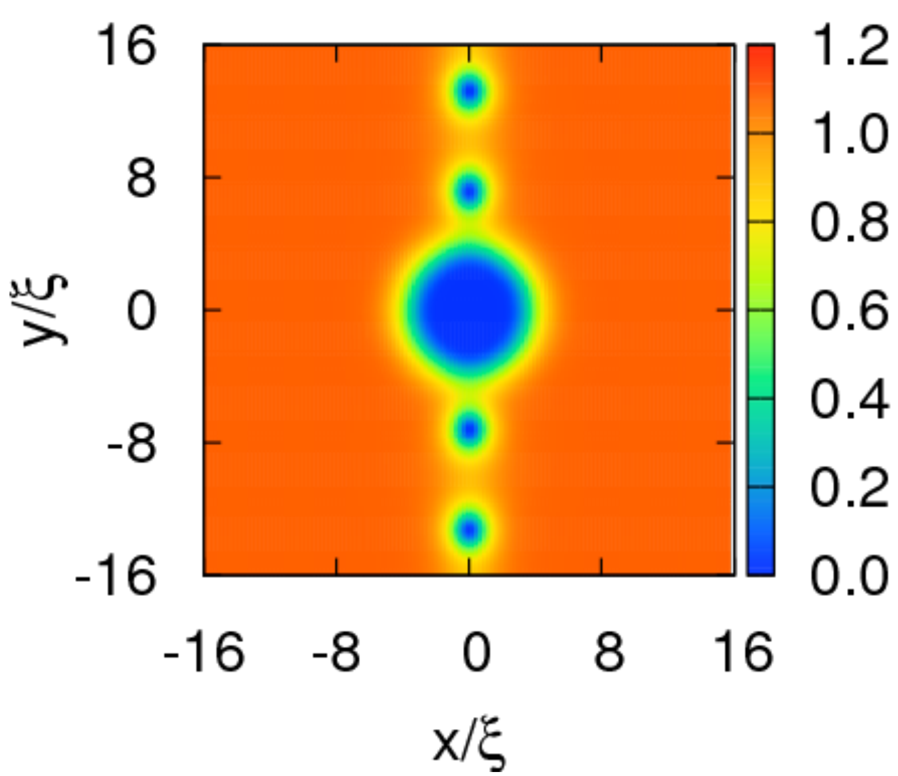


This result suggests that in the small size limit, the multiple-swallowtail structure reduces to normal swallowtail structure

Summary (Part1)

- We studied two-dimensional BEC with moving defect.
- In contrast to 1D, the multiple-swallowtail structures exist in 2D.
- In the small size limit, the multiple-swallowtail structures reduce to the normal swallowtail.





MK and Y. Kato, Phys. Rev. A **91**, 053608 (2015).

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NIST experiments (Kumar *et al.*, 2017)

The NIST group observed decay of superflow in a ring trap.

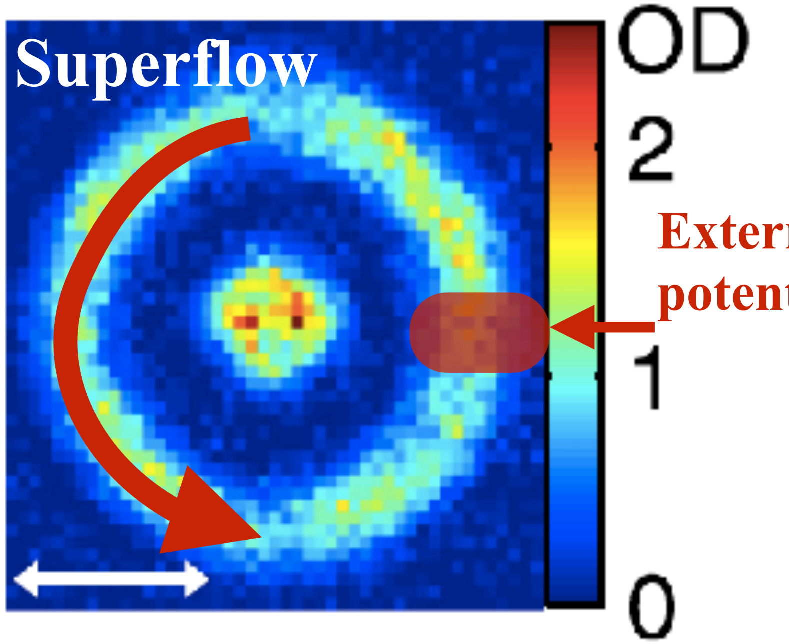
Initial state is winding number = 1.

$$W = \frac{1}{2\pi}\oint_C d\boldsymbol{r}\cdot\boldsymbol{\nabla}\varphi(\boldsymbol{r},t)$$

Applying an external potential to cause decay of superflow (break the rotational symmetry).

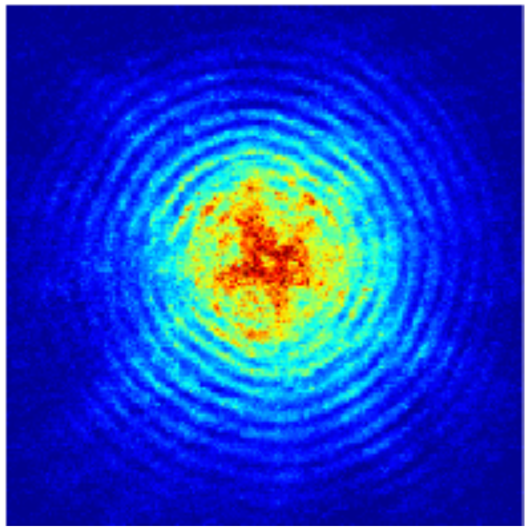
They measured the winding number by **interference experiment**.

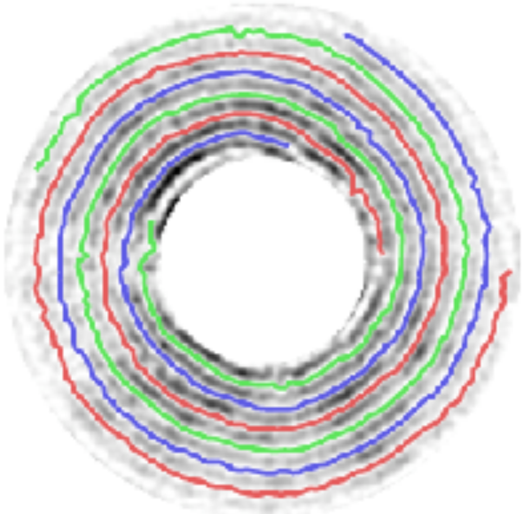
$$|\Psi_{\mathrm{Ring}}(\boldsymbol{r})+\Psi_{\mathrm{Center}}(\boldsymbol{r})|^2 \sim \cos(W\theta + Kr)$$



Kumar *et al.*, PRA **95**, 021602(R) (2017).

Interference pattern (TOF image)



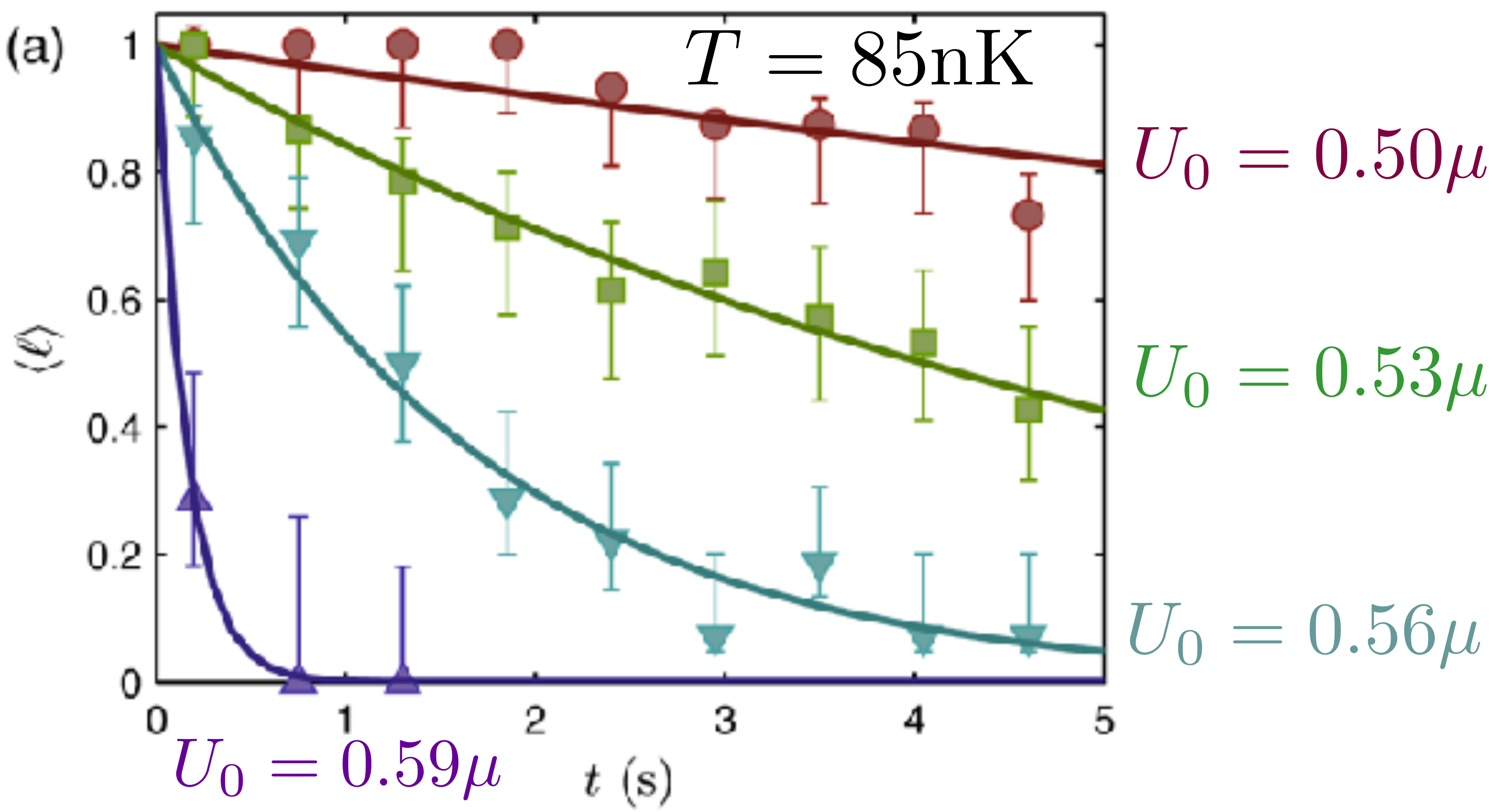


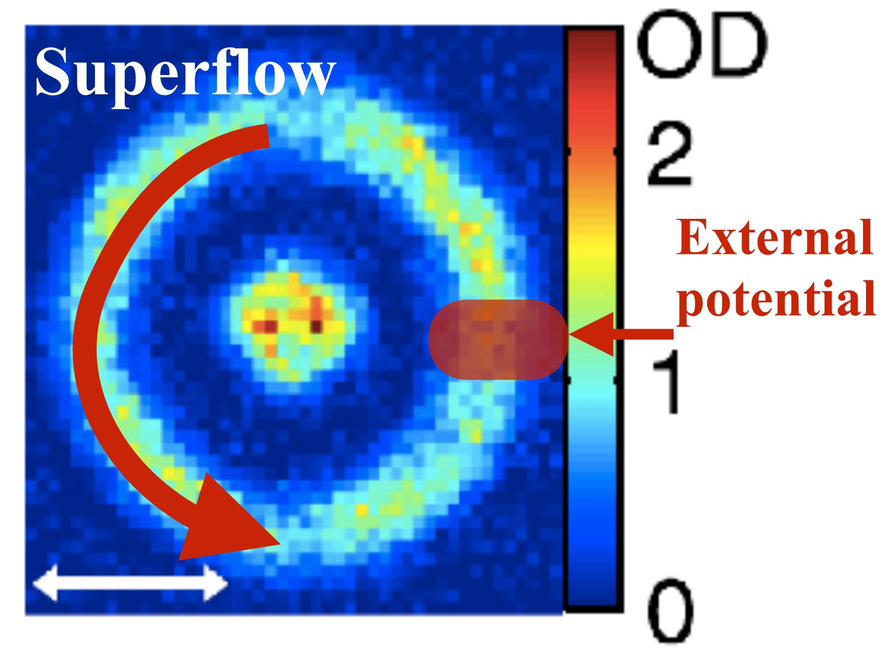
$W = 3$

Eckel *et al.*, PRX **4**, 031052 (2014)

NIST experiments (Kumar *et al.*, 2017)

Angular momentum vs time





U_0 : strength of the external potential

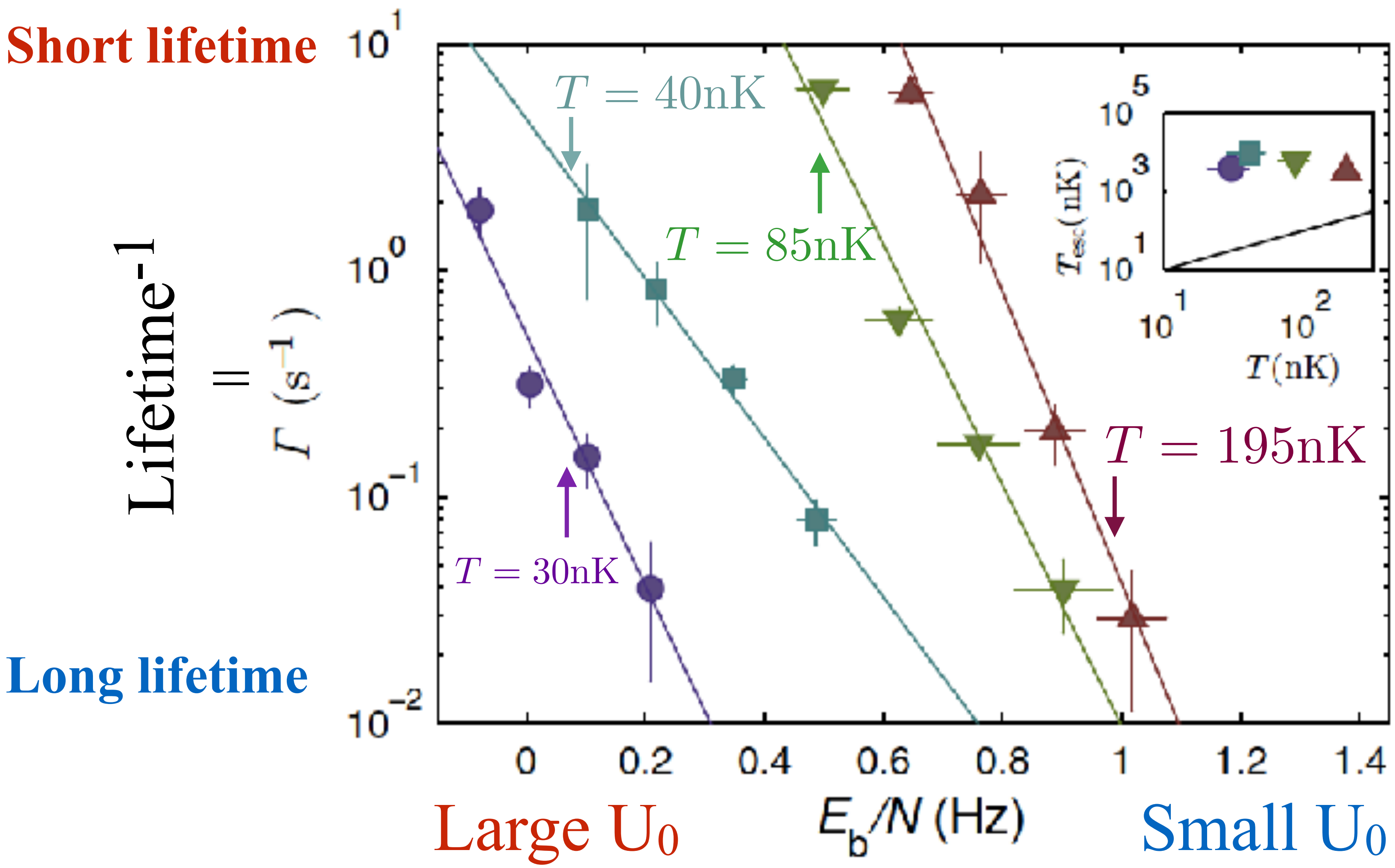
Kumar *et al.*, PRA **95**, 021602(R) (2017).

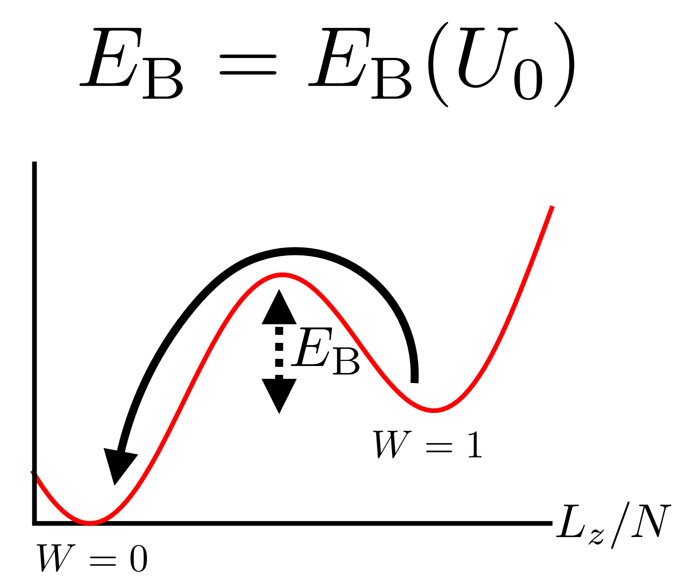
As the potential becomes strong, the superflow decay rapidly.

The lifetime of the superflow is about **1 second**.

NIST experiment : Decay rate of superflow

Decay rate vs energy barrier per particle





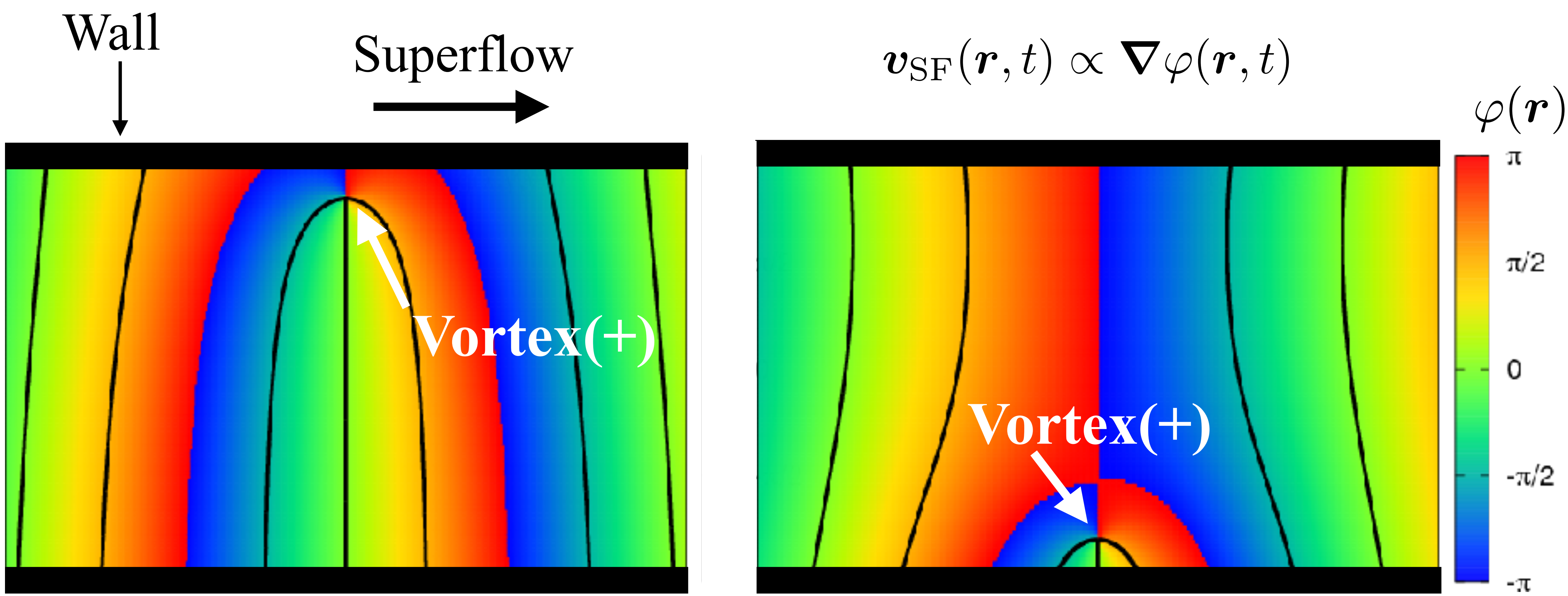
Kumar *et al.*, PRA **95**, 021602(R) (2017).

The decay rate clearly depends on the **temperature**.

⇒ **Thermal phase slip** is a possible decay mechanism.

What is Phase slip?

When a vortex moves perpendicular to the superflow in the container, the winding number changes.



$$\Psi(\boldsymbol{r}) \sim e^{2\pi i W x/L} \qquad \Psi(\boldsymbol{r}) \sim e^{2\pi i (W-1) x/L}$$

This dynamical process is induced by thermal fluctuations.

Thermal phase slips?

If the thermal phase slip occurs, the decay rate should be the **Arrhenius law**.

However, the fit to the Arrhenius law fails.

$$\Gamma = \Omega_{\rm a} e^{-E_{\rm B}/(k_{\rm B}T_{\rm esc})} \quad \Rightarrow$$

$(\Omega_{\rm a}, T_{\rm esc})$: fitting parameters.

$$T \neq T_{\rm esc}$$

$30 \sim 195\text{nK} \quad 3 \sim 9\mu\text{K}$

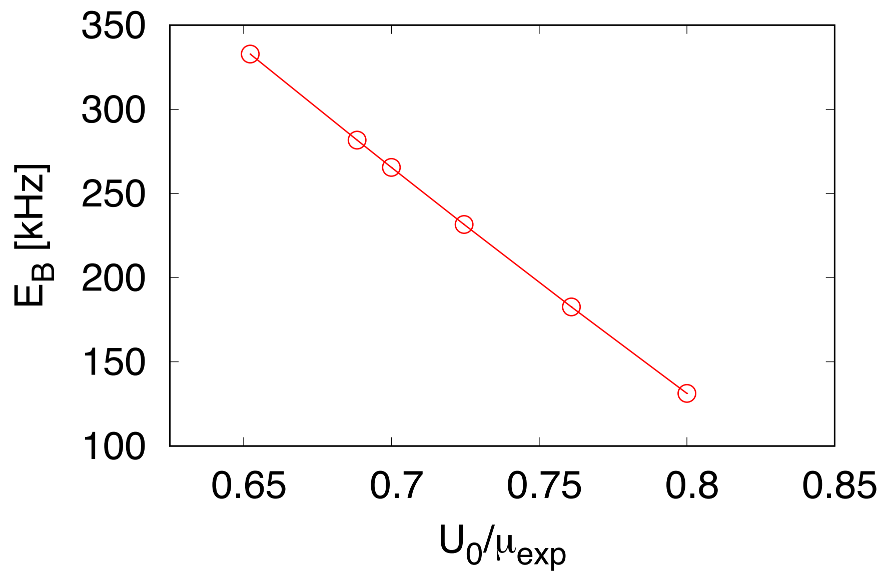
Case	T (nK)	T_c (nK)	$\omega_z/2\pi$ (Hz)	$N/10^5$	μ/h (kHz)	T_{esc} (nK)	Ω_a (s^{-1})
I	30(10)	470(30)	974(7)	4.46(26)	2.91(12)	$3.9(6)\times10^3$	$5(2)\times10^{-1}$
II	40(12)	370(40)	518(4)	6.71(39)	2.93(11)	$9.2(8)\times10^3$	$4.8(9)\times10^0$
III	85(20)	370(40)	520(10)	6.48(46)	2.68(11)	$5.9(8)\times10^3$	$1.9(4)\times10^3$
IV	195(30)	470(30)	985(4)	4.22(26)	2.66(08)	$3.2(4)\times10^3$	$1.2(2)\times10^5$

$\Rightarrow$ **The thermal phase slip is irrelevant in the experiment.**

Kumar *et al.*, PRA **95**, 021602(R) (2017).

Motivation of our work

Actually, our calculation of the energy barrier shows that the thermally activated phase slip is irrelevant.



Our calculation about the energy barrier

$$E_{\mathrm{B}} \sim O(100\mathrm{kHz}) \quad k_{\mathrm{B}}T \sim h \times 0.6\mathrm{kHz}$$
$$(T = 30\mathrm{nK})$$

$$\mathrm{Lifetime} \sim \Gamma^{-1} \sim e^{E_{\mathrm{B}}/(k_{\mathrm{B}}T)} \sim e^{100}$$

$\Rightarrow$ The lifetime is much longer than the experimental timescale.

Aim of our study

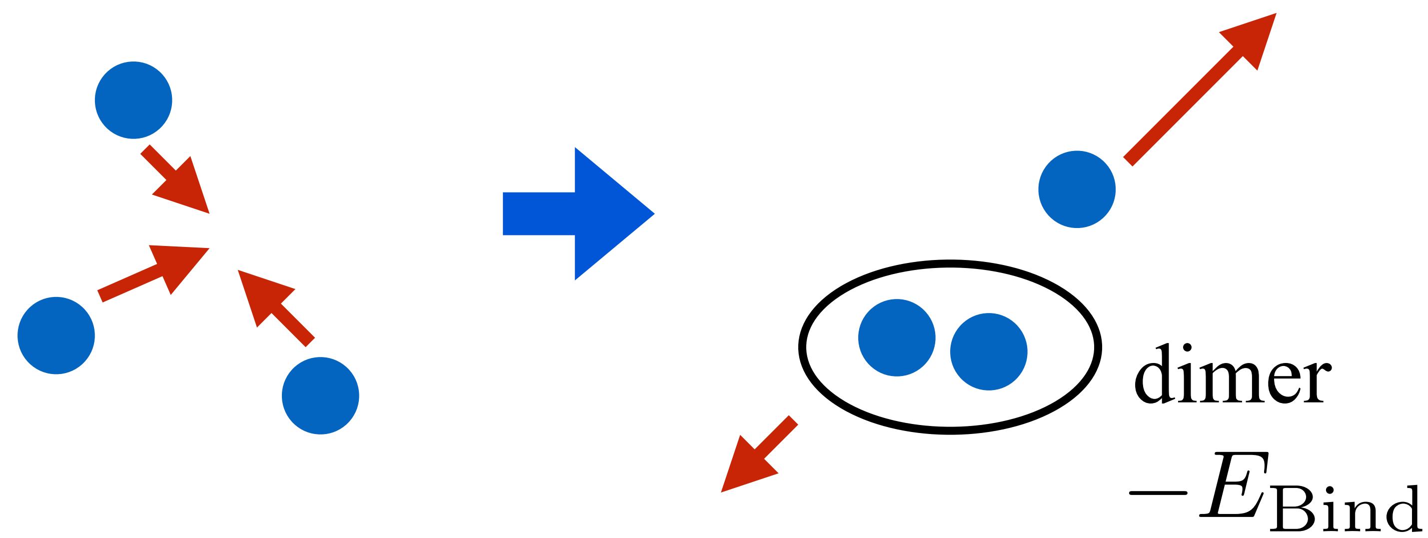
Reveal the origin of the decay observed in the NIST experiment.

Our proposal : hybrid effects of the finite temperature effects and the **three-body loss induced decay**

What is Three-body loss?

Three-body loss : inelastic collision between three atoms

The atoms get the large kinetic energy and escape from the trap.



The energy barrier is a function of the particle number.

$\Rightarrow$ Decreasing the particle number, the energy barrier may vanish.

$\Rightarrow$ The three-body loss induced decay may be relevant!

Numerical simulation with three-body loss term

Simulation : Dissipative quasi-2D GP eq.

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r}) + g_{2\mathrm{D}}|\Psi(\boldsymbol{r},t)|^2\right]\Psi(\boldsymbol{r},t) - \frac{i\hbar}{2}L_3'|\Psi(\boldsymbol{r},t)|^4\Psi(\boldsymbol{r},t)$$

$L_3' = \frac{L_3}{\sqrt{3}\pi a_z^2}$ $\quad L_3 = 1.1\times 10^{-30}\ \mathrm{cm}^6/\mathrm{s}$: three-body loss rate

Stamper-Kurn *et. al*., PRL. **80**, 2027 (1998).
Kagan *et al*., PRL **81**, 933 (1998).

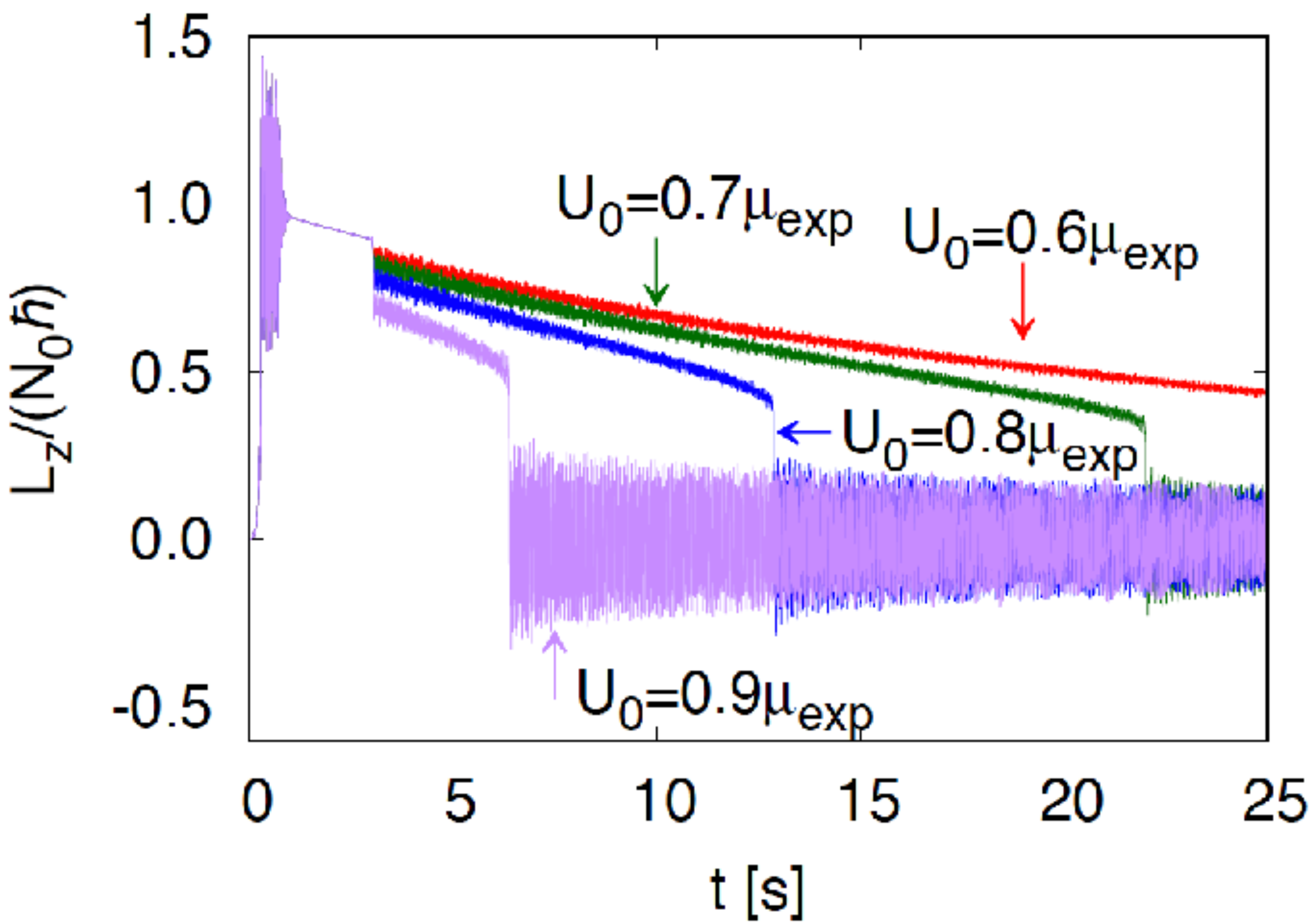
Equation of continuity :

$$\frac{\partial}{\partial t}n(\boldsymbol{r},t) + \boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{r},t) = -L_3 n(\boldsymbol{r},t)^3$$

Breaks the particle number conservation

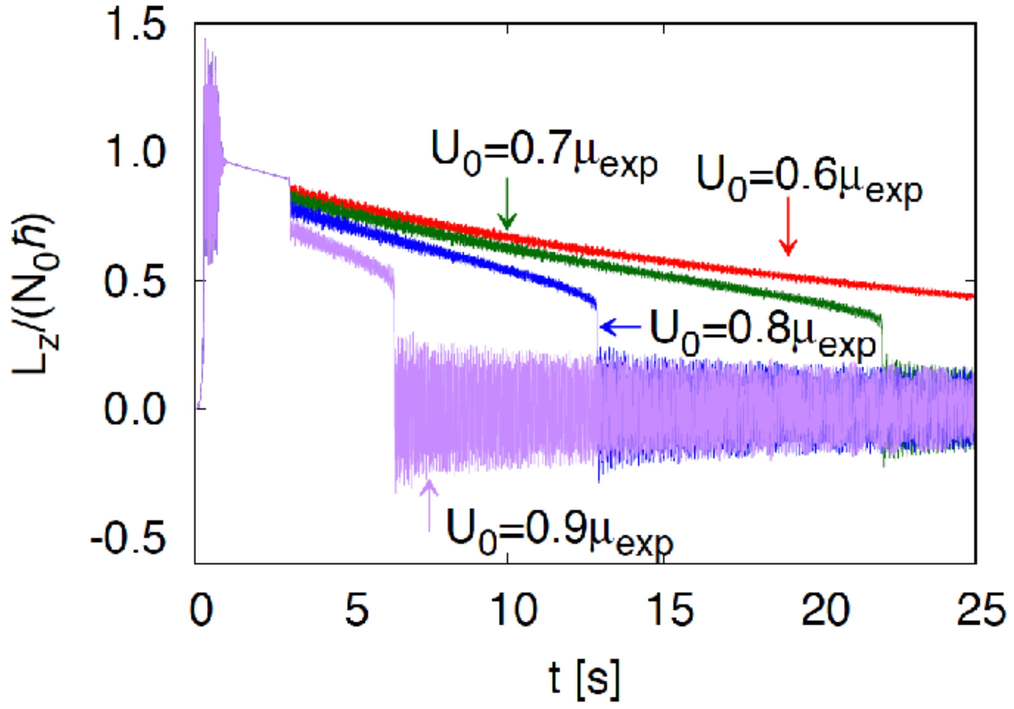
Results : three-body loss induced decay

Numerical simulation at T=0 (GP eq. with three-body loss term)



Our results show that the three-body loss induced decay exists!

Results : Comparison with experiments



Theoretical results $U_0 = 0.6884\mu_{\rm exp}$

$$T_{\rm decay} \sim 23.3{\rm s}$$

Experimental results

$$T_{\rm decay} \sim 6.7{\rm s}$$

Our results are about 3.5 times longer than the experimental lifetime.

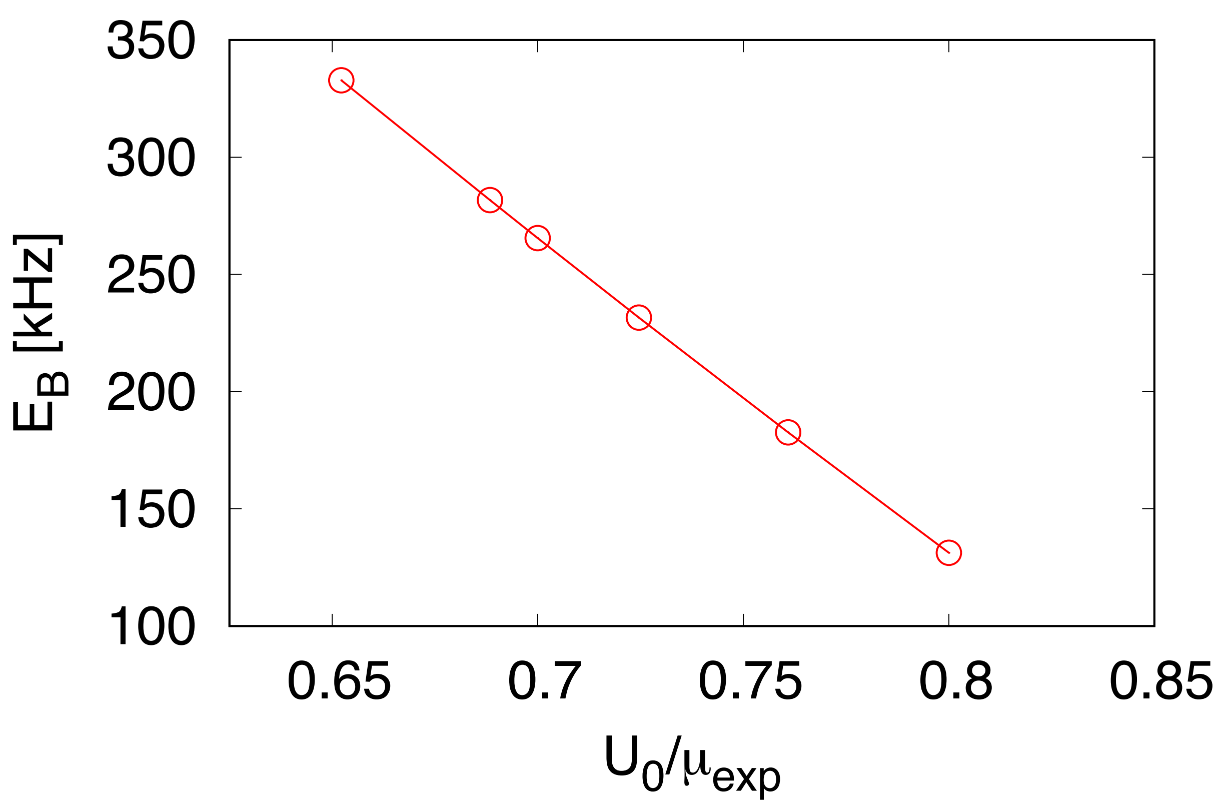
However, our results do not contain the finite temperature effects.

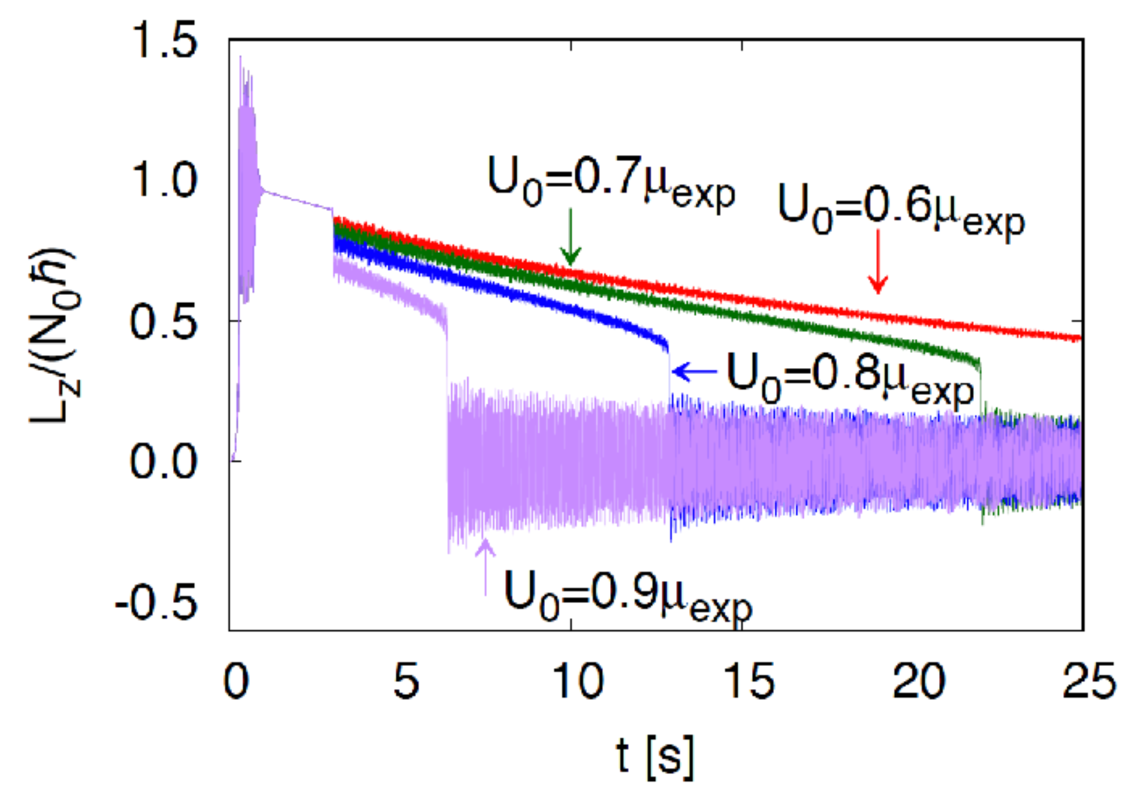
Generally, the finite temperature effects accelerate the decay.

Our proposal : The hybrid effects of the finite temperature and the three-body loss is the origin of the decay in the NIST experiment.

Summary (Part2)

- We calculated the energy barrier in the ring trapped BEC.
- The thermally activated phase slip is irrelevant in the NIST experiment.
- We proposed that the hybrid effects of the finite temperature and the three-body loss induced decay is possible decay mechanism.





MK and I. Danshita, arXiv:1712.09403.

Method : Pseudo-Arclength Continuation Method

Non-linear equations (λ is a parameter) : $G_i(\boldsymbol{u}; \lambda) = 0$

Instead of solving this equation, we regard λ as a variable and add an equation :

Pseudo-arclength continuation method

$$G_i(\boldsymbol{u}, \lambda) = 0, \quad N(\boldsymbol{u}, \lambda) = 0$$

H. B. Keller, *Lectures on Numerical Methods In Bifurcation Problems* (Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1987).

$$N(\boldsymbol{u}, \lambda) \equiv \dot{\boldsymbol{u}}_0 \cdot (\boldsymbol{u} - \boldsymbol{u}_0) + \dot{\lambda}_0(\lambda - \lambda_0) - \Delta s$$

represents a plane that is perpendicular to the tangent (u_0, λ_0) and is separated by Δs from (u_0, λ_0).

We can obtain unstable solutions by this method when the saddle-node bifurcation occurs.