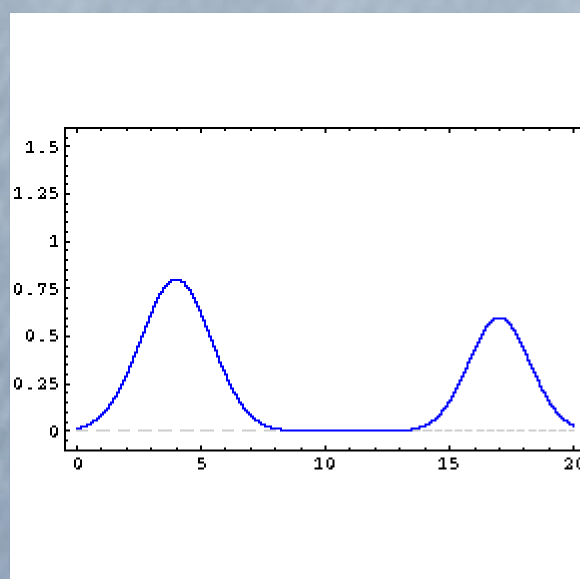
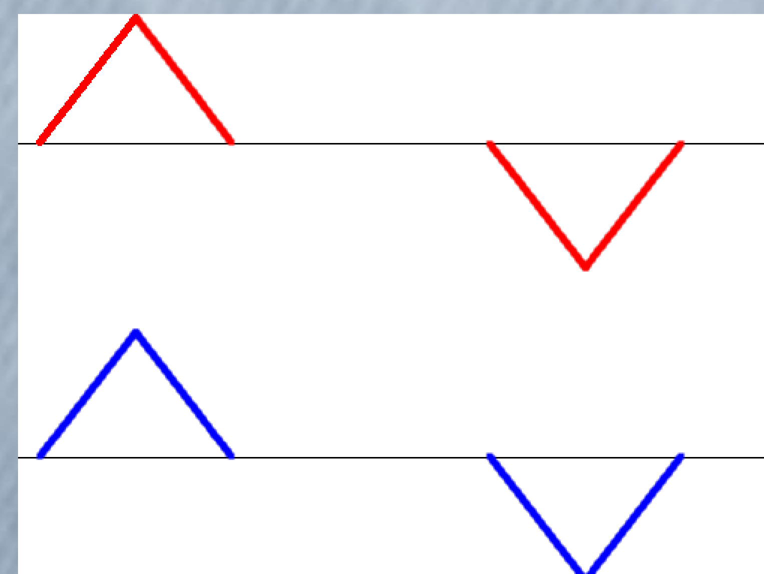
Superposition and Standing Waves

- Superposition
- Constructive and destructive interference
- Standing waves
- Harmonies and tone
- Interference from two sources
- Beats

Principle of Superposition

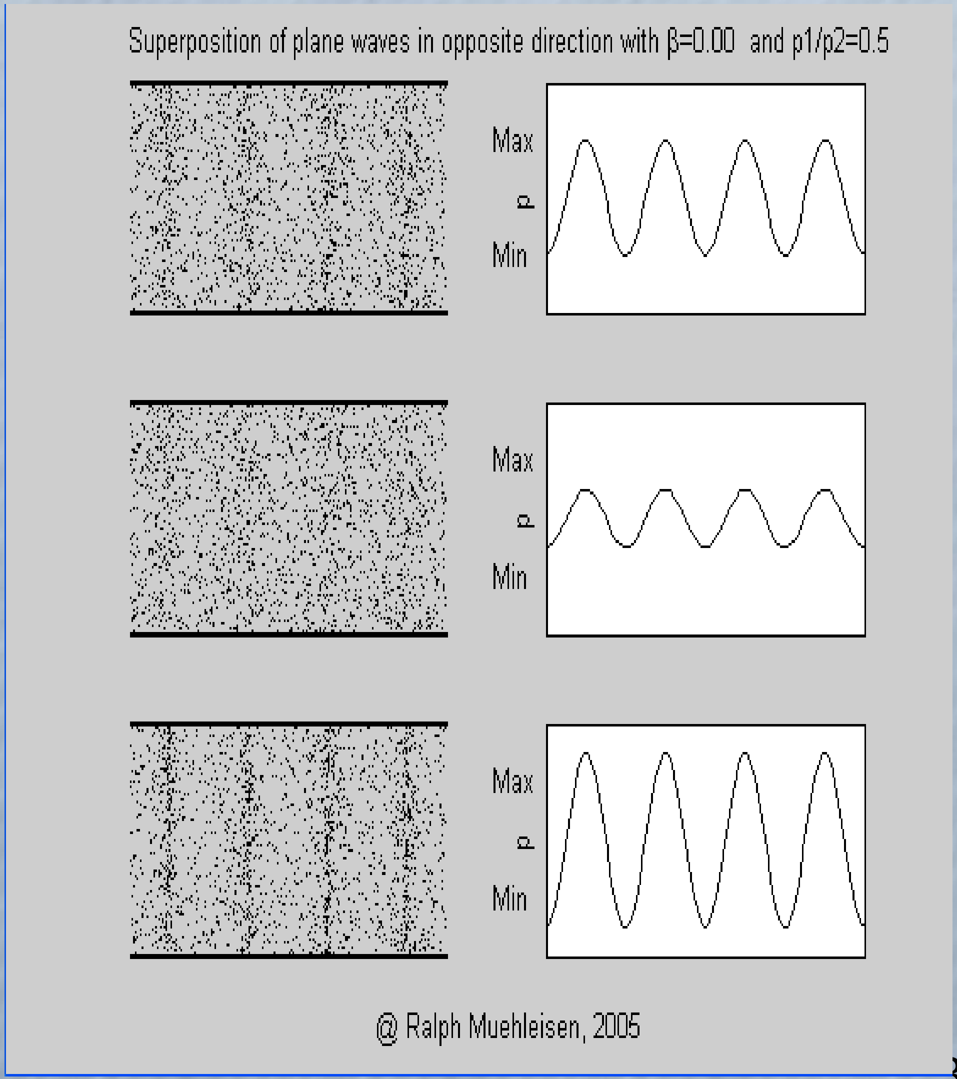
When two or more waves are simultaneously present at a single point in space, the displacement of the medium at that point is the sum of the displacement due to each individual wave.





Constructive and Destructive Interference

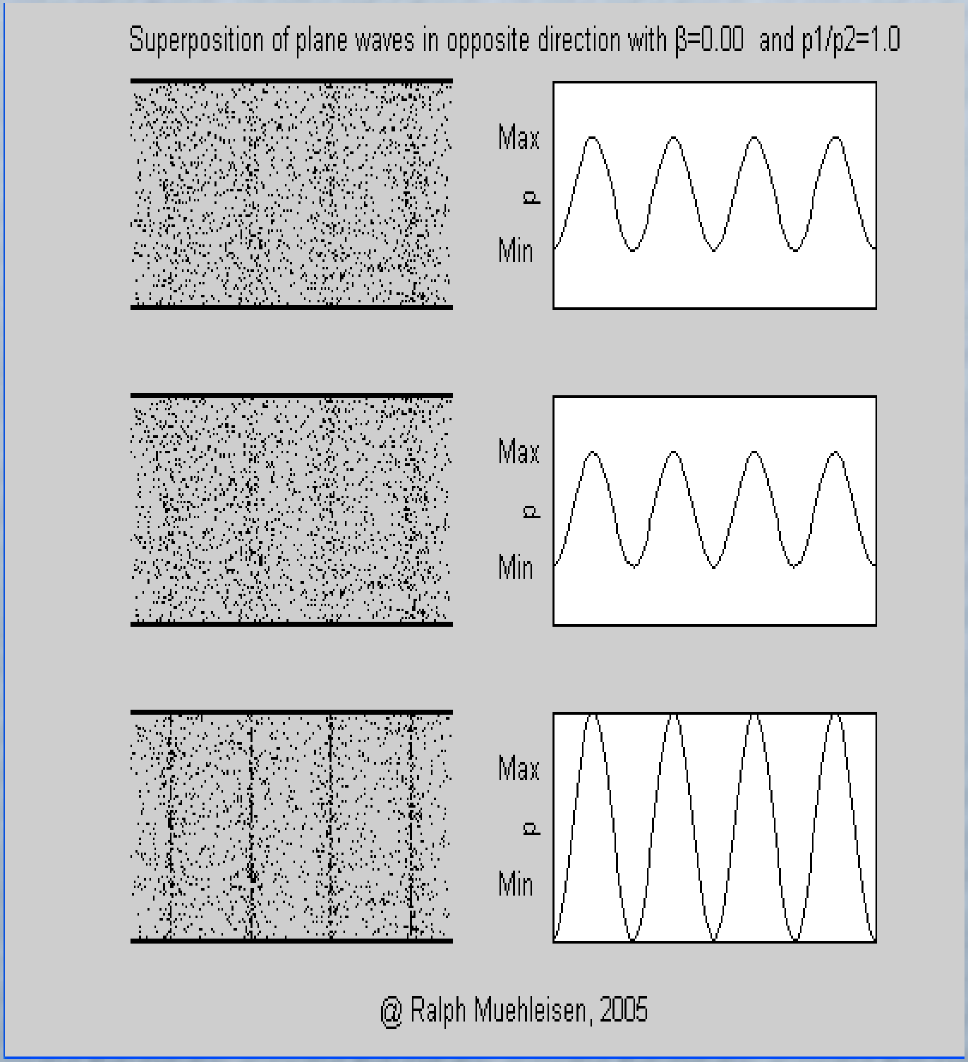
- Constructive – amplitude of the 2 waves is of the same sign
- Destructive – amplitude of the 2 waves is of the opposite sign



Standing Wave

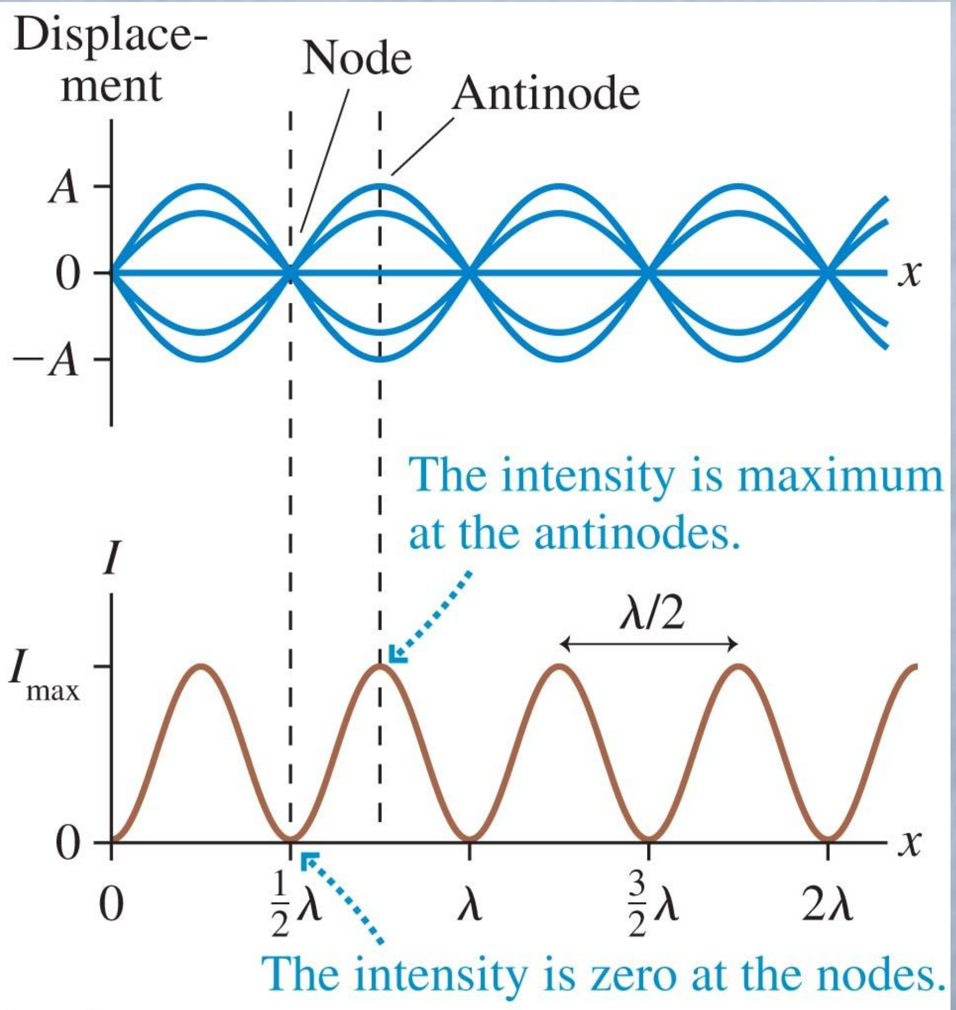
Two waves traveling in the opposite directions with the same amplitude

The two waves interfere and create a standing wave



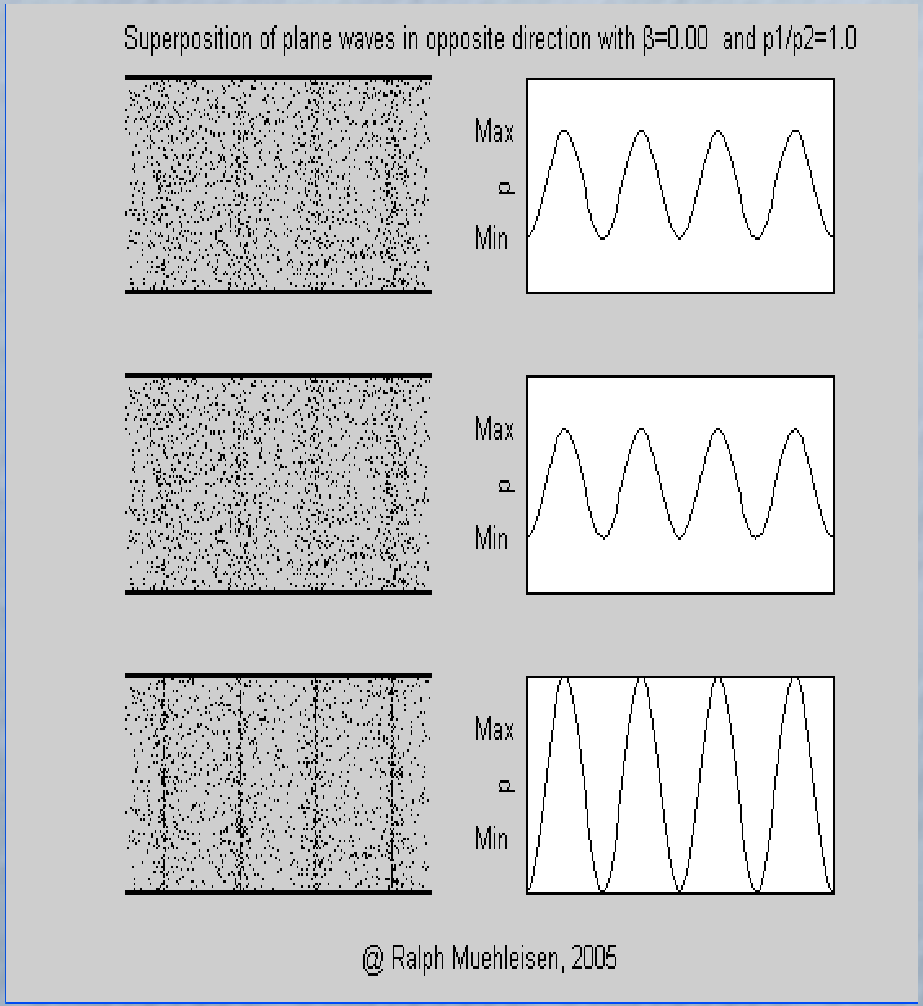
Nodes and Antinodes

- Nodes – displacement does not change
- Antinodes – displacement changes with maximum amplitude



Nodes and Antinodes – longitudinal waves

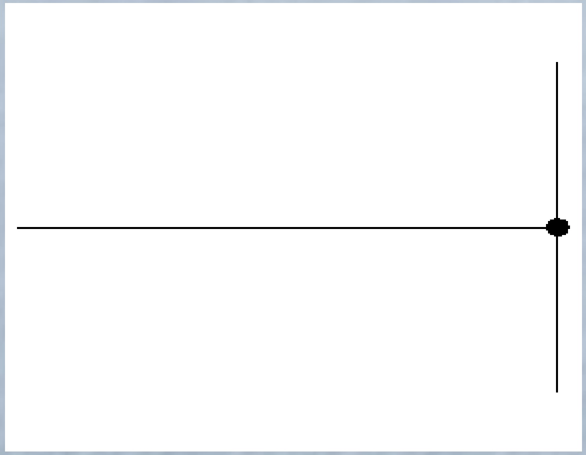
Nodes and antinodes can be defined as pressure or velocity. Text book defines as pressure – other sources define as velocity of particles in the medium

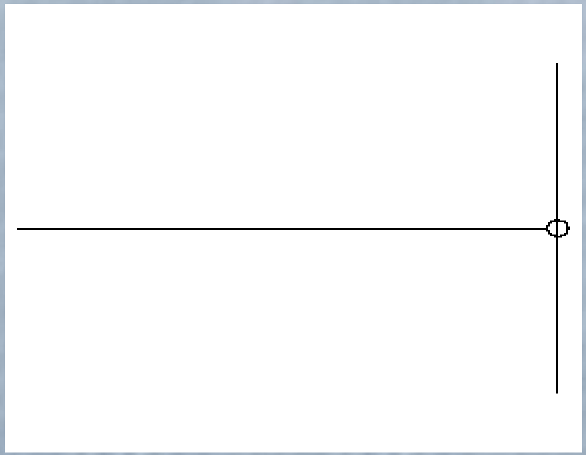


Reflections

When a wave meets a boundary it is reflected.

A hard boundary will invert the reflection, a soft boundary will keep the original sense

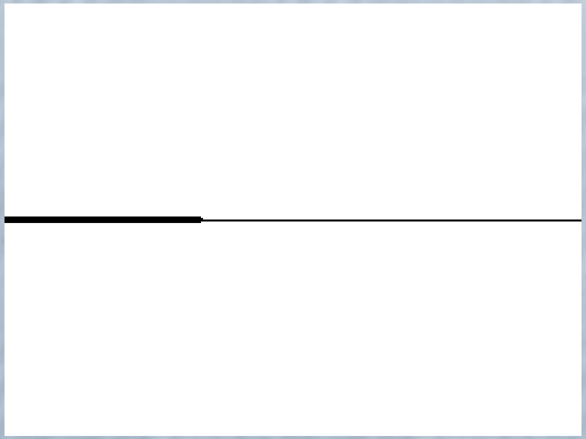


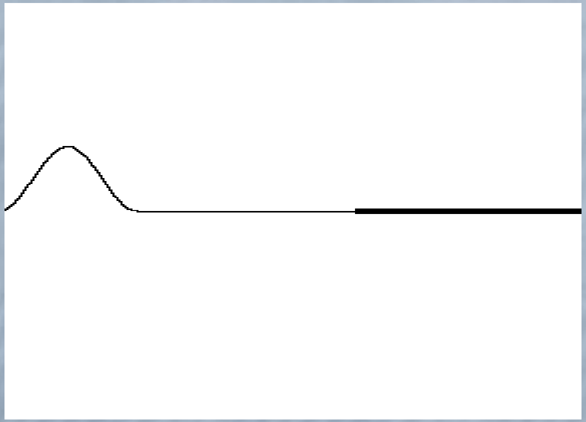


Animation courtesy of Dr. Dan Russell, Kettering University
http://paws.kettering.edu/~drussell

Reflection at a discontinuity

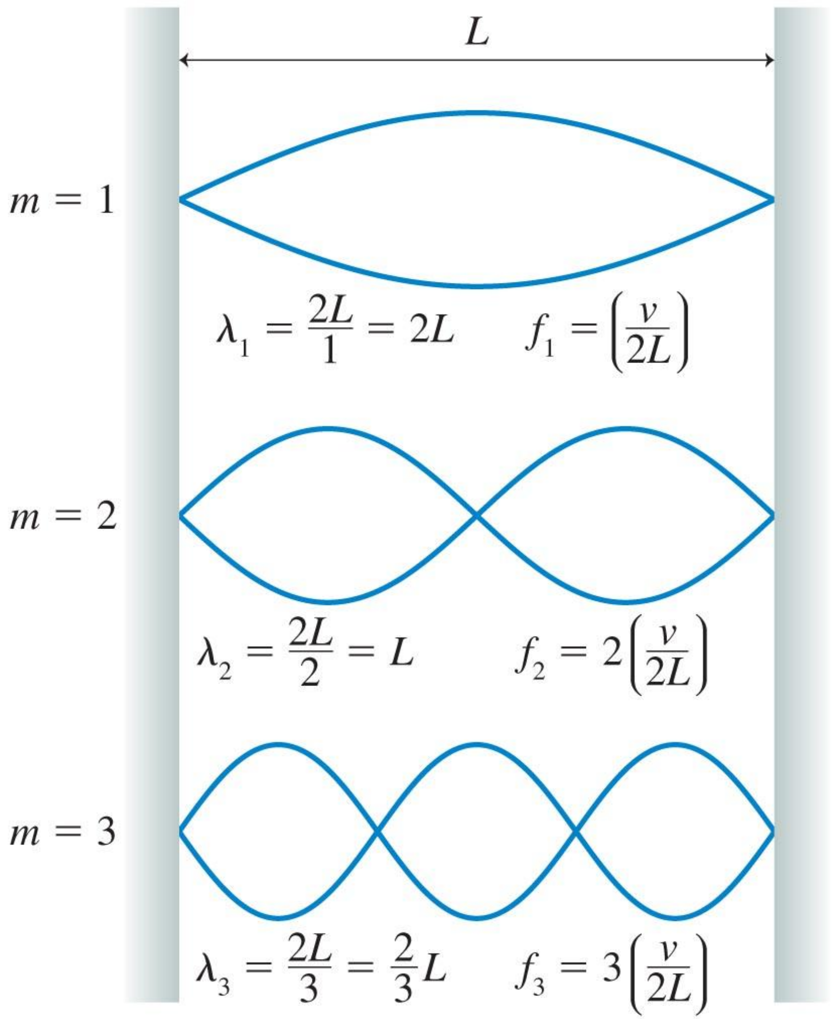
- At a discontinuity in the medium – e.g. passing from higher to lower density, we get partial transmission and partial reflection.
- From low to high density we also get an inversion at the reflection



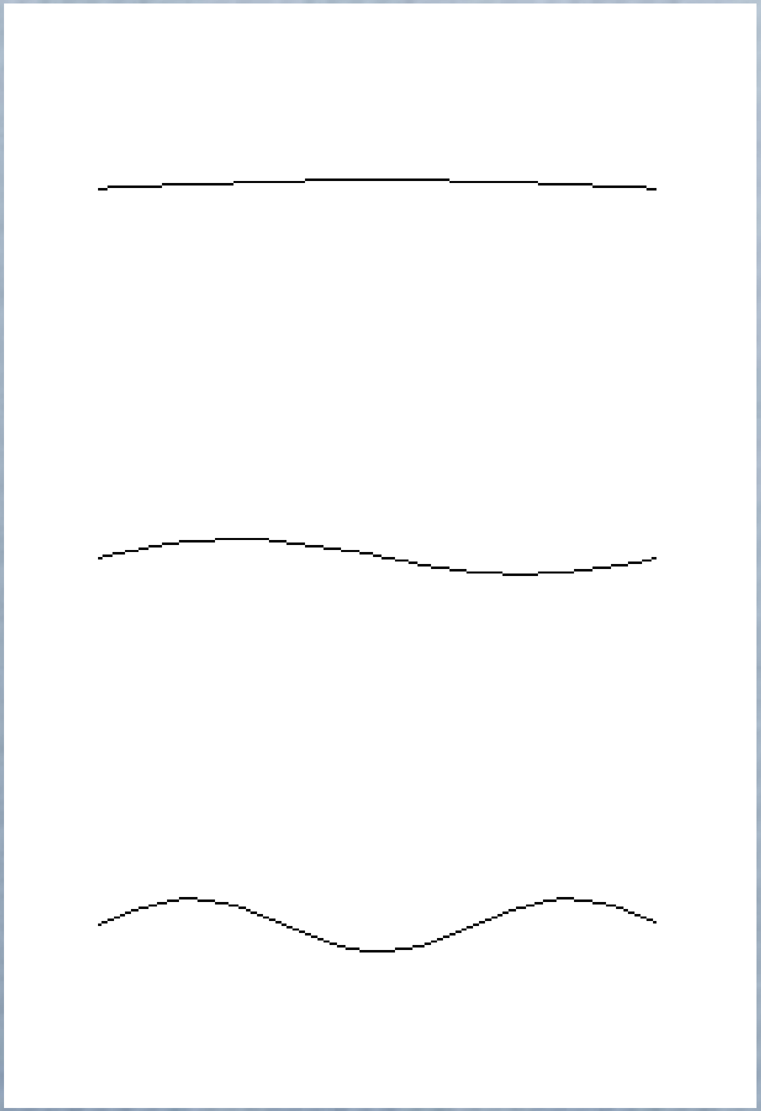


Modes

- Certain wavelengths will fit on a fixed length of medium.
- These are called **modes**
- The number of antinodes gives us the **mode number**



Modes

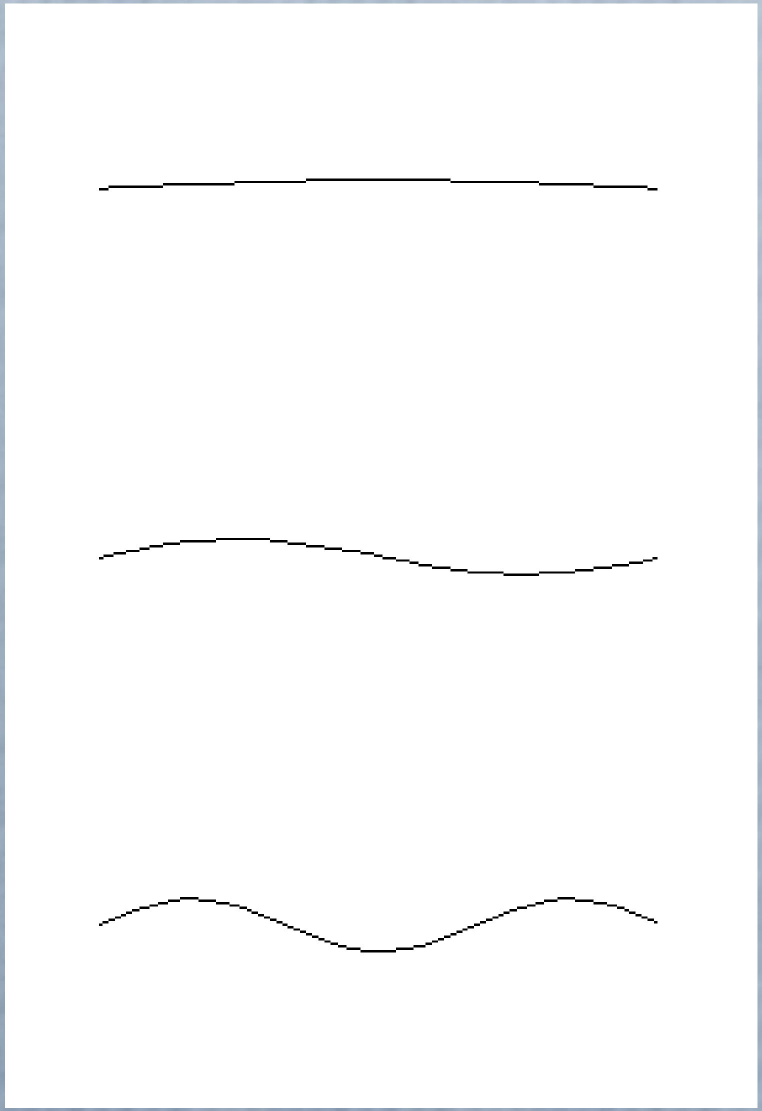


The wavelengths of the modes for a medium length L, can be described by

$$\lambda_m = \frac{2L}{m}$$

$$m = 1,2,3,4,...$$

Modes

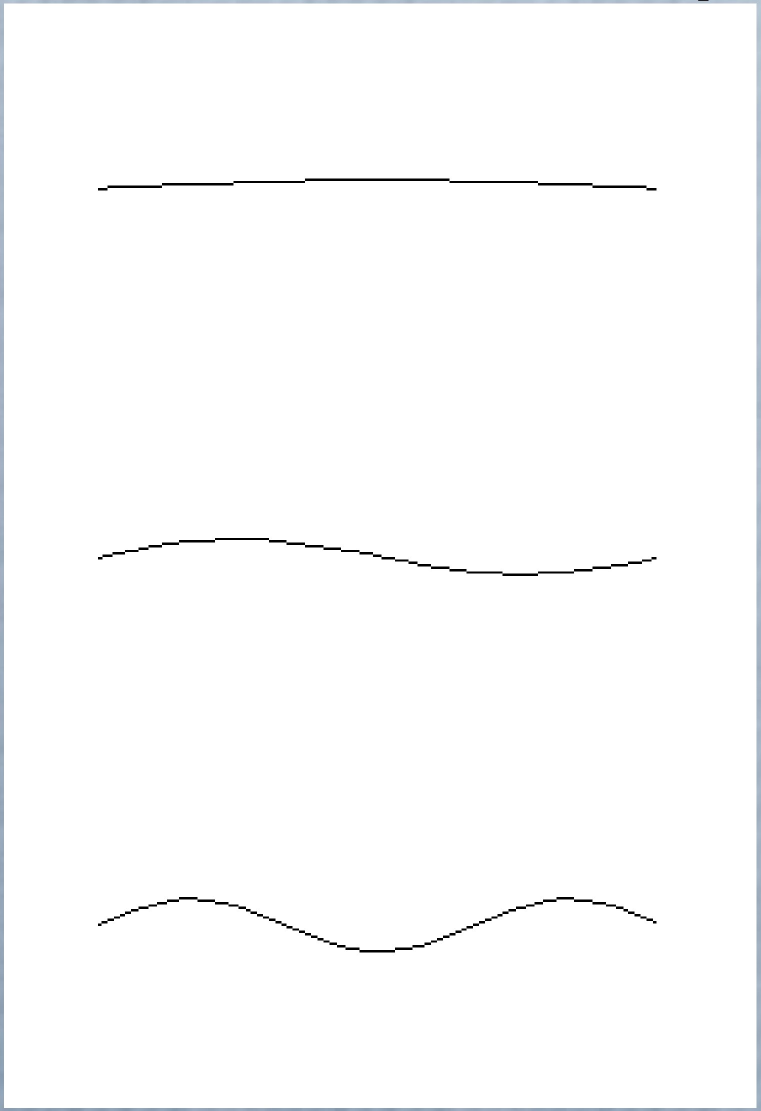


The frequencies of the modes for a medium length L, can be described by

$$f_m = \frac{v}{\lambda_m} = \frac{mv}{2L}$$

$$m = 1,2,3,4,\ldots$$

Special Modes



When m=1 we get the lowest frequency, called the **fundamental frequency**

$$f_1 = \frac{v}{2L}$$

$$f_m = m f_1$$

$$m = 1,2,3,4\ldots$$

Applications

- Stringed instruments – we know the velocity of the wave in the string is:

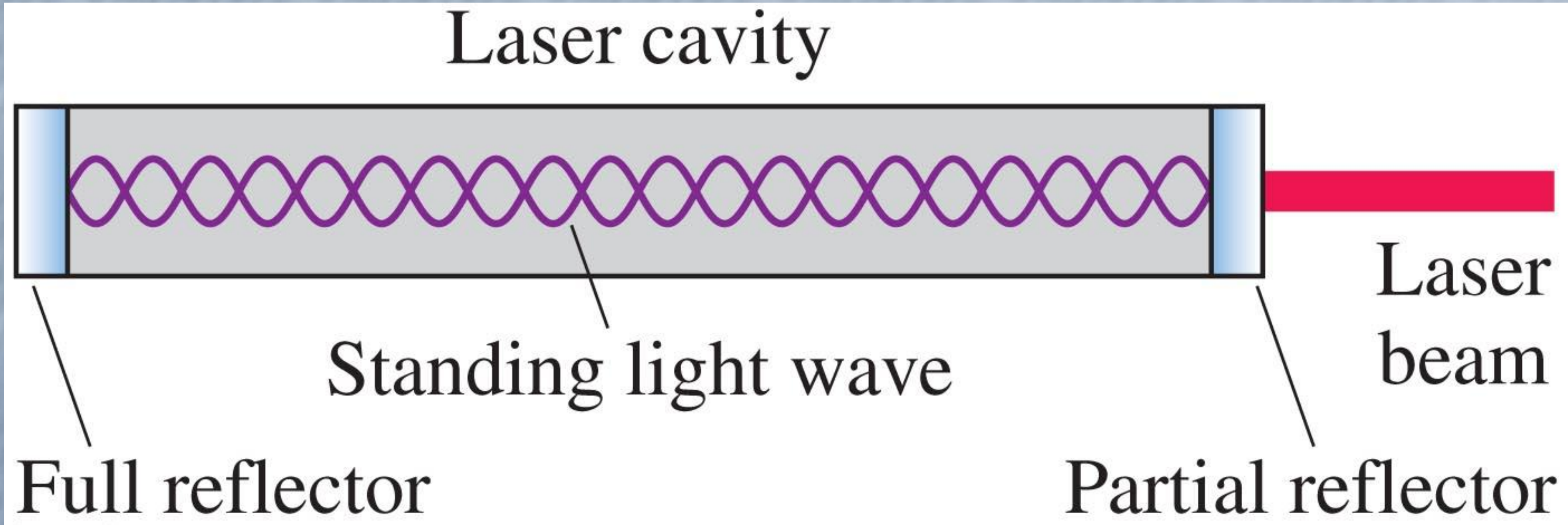
$$v = \sqrt{\frac{T_s}{\mu}}$$

- To keep the tension on stringed instruments the same, the strings **linear density, μ** is changed

$$f_1 = \frac{1}{2L}\sqrt{\frac{T_s}{\mu}}$$

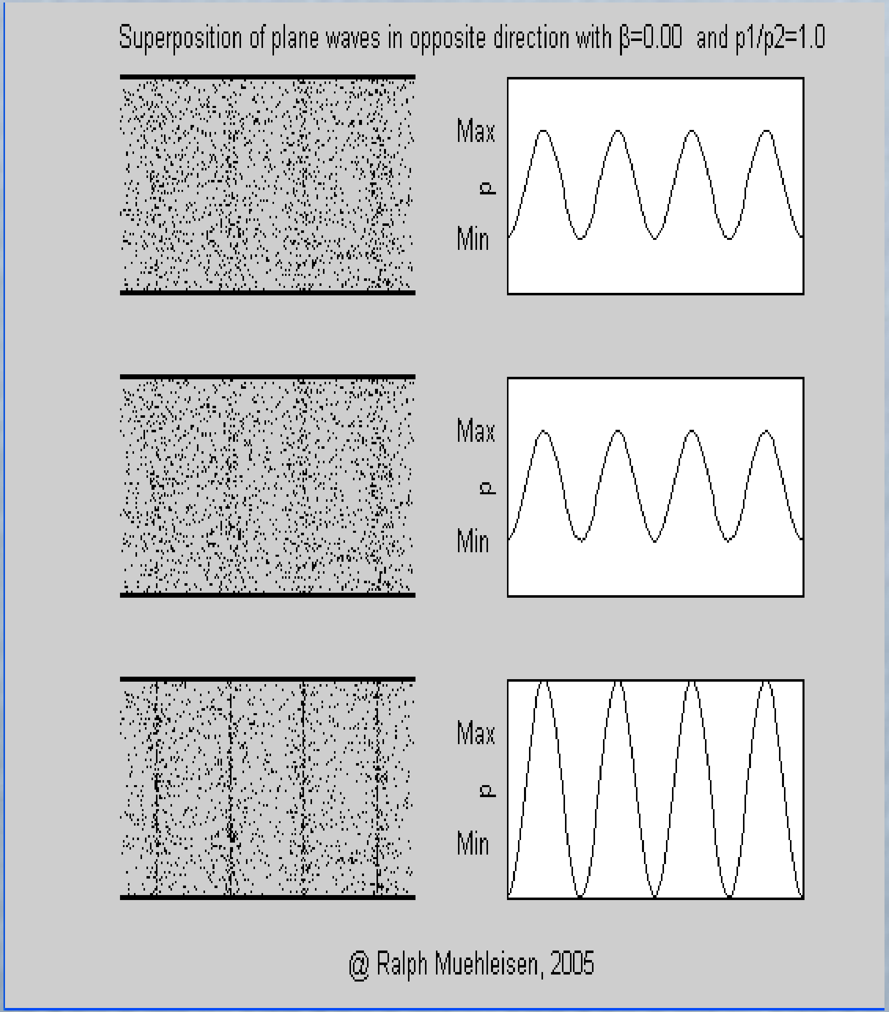
Lasers

The laser has a full reflector and partial reflector. The light produced in the cavity is leaked at one end by a mirror that is only 99% efficient.



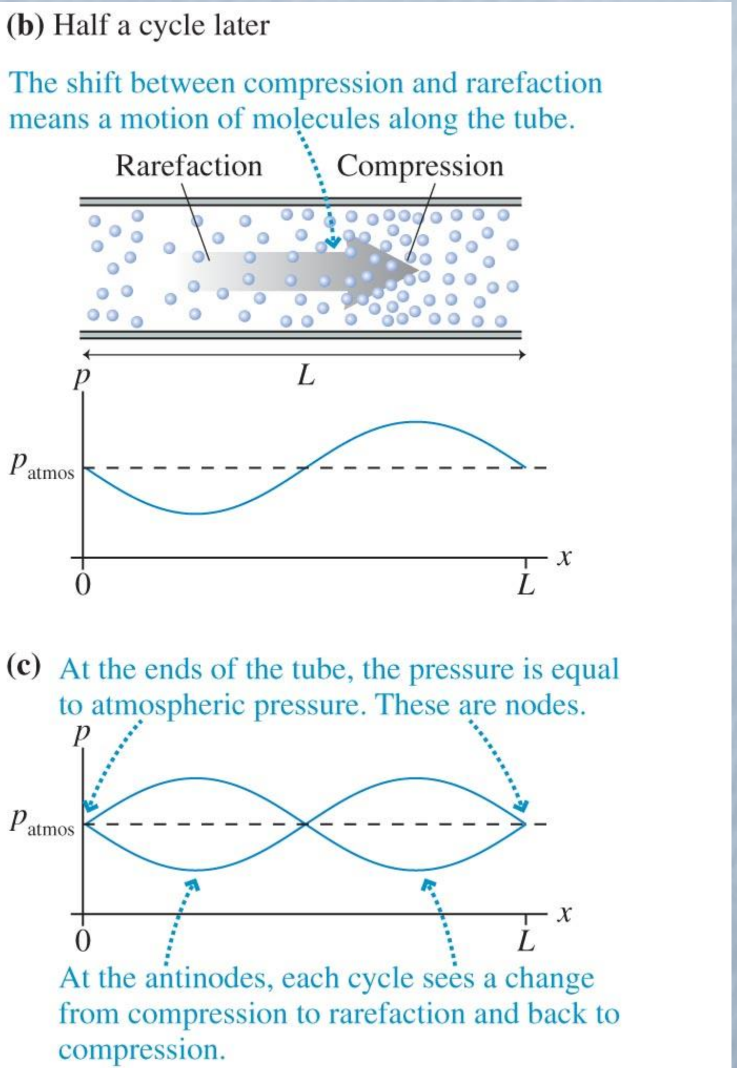
Standing sound waves in pipes

- A closed end pipe will reflect the wave
- An open end pipe will partially transmit and partially reflect the sound wave – it is a discontinuity in the medium



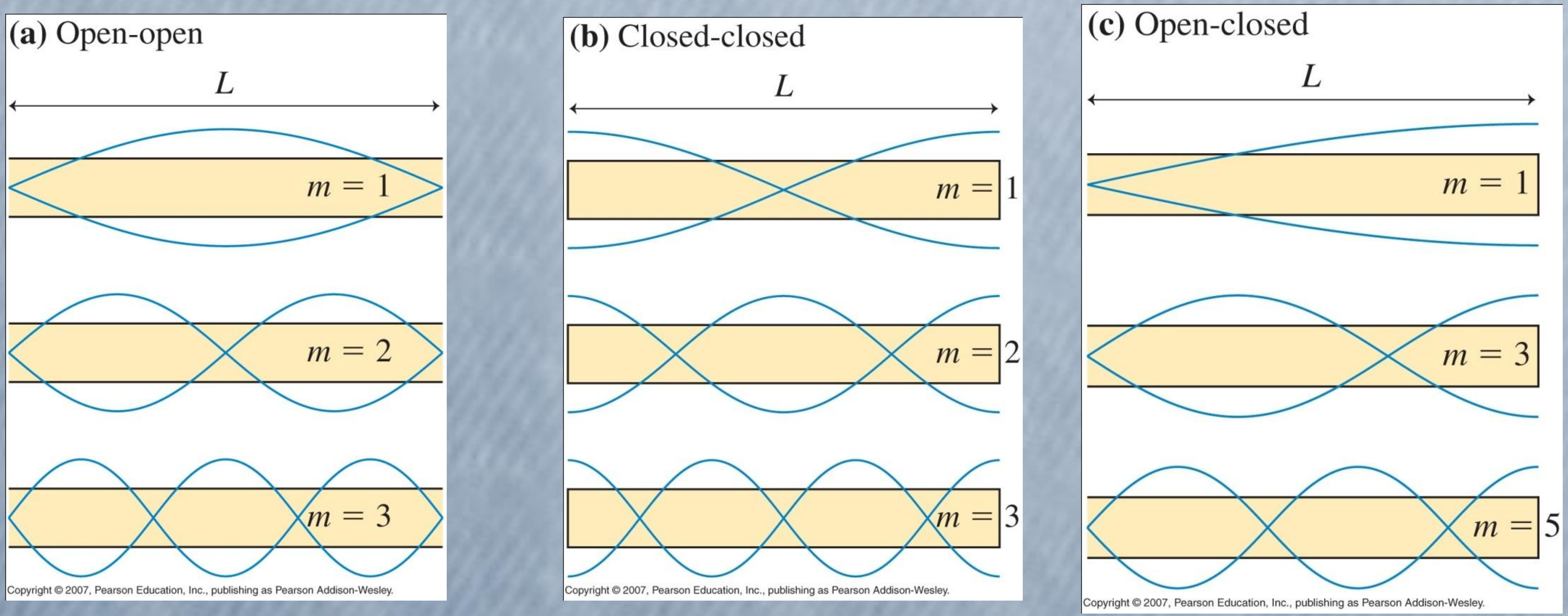
Sound waves in a pipe

- The open end of a pipe will be a pressure node – the pressure will constant
- A closed end of the pipe will be a pressure antinode – the pressure fluctuates from minimum to maximum value



Sound wave modes in a pipe

Representation of longitudinal waves in open-open, closed-closed and open-closed pipes



Standing waves in an open-closed pipe

We can get one quarter wavelengths in an open-closed pipe:

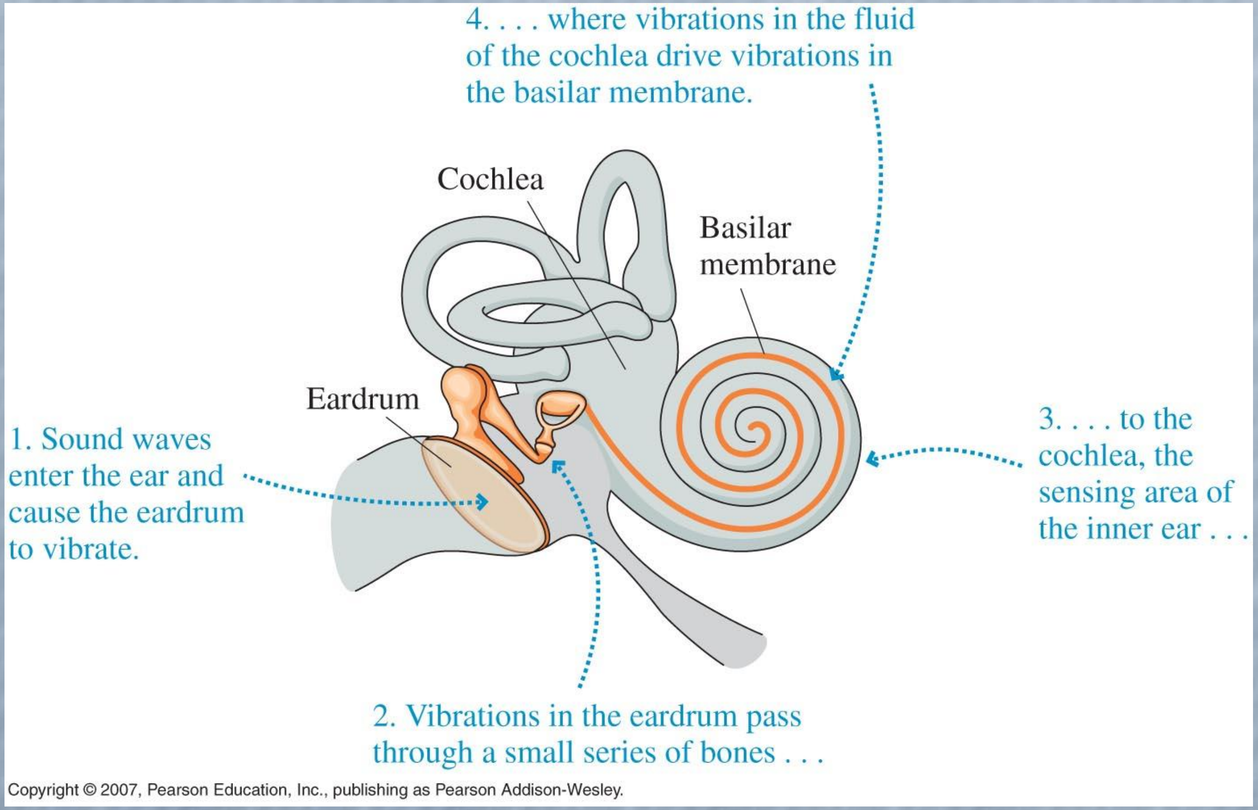
$$\lambda_m = \frac{4L}{m}$$

$$f_m = \frac{mv}{4L}$$

$$m = 1,3,5,7\ldots$$

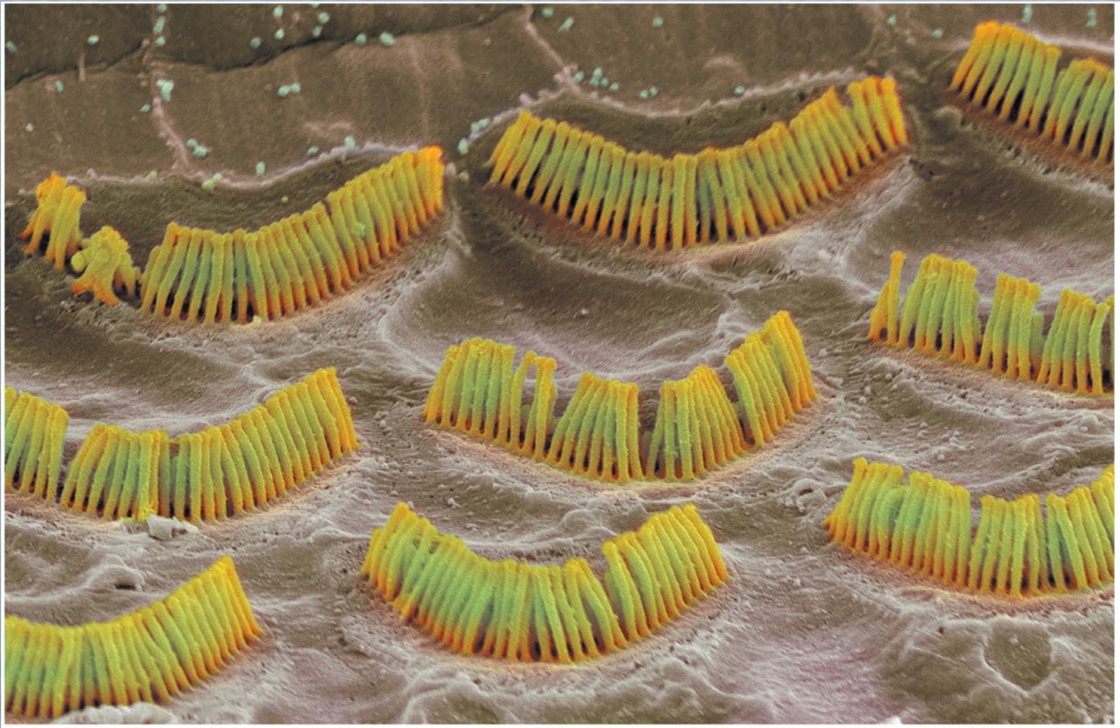
Physics of the human ear

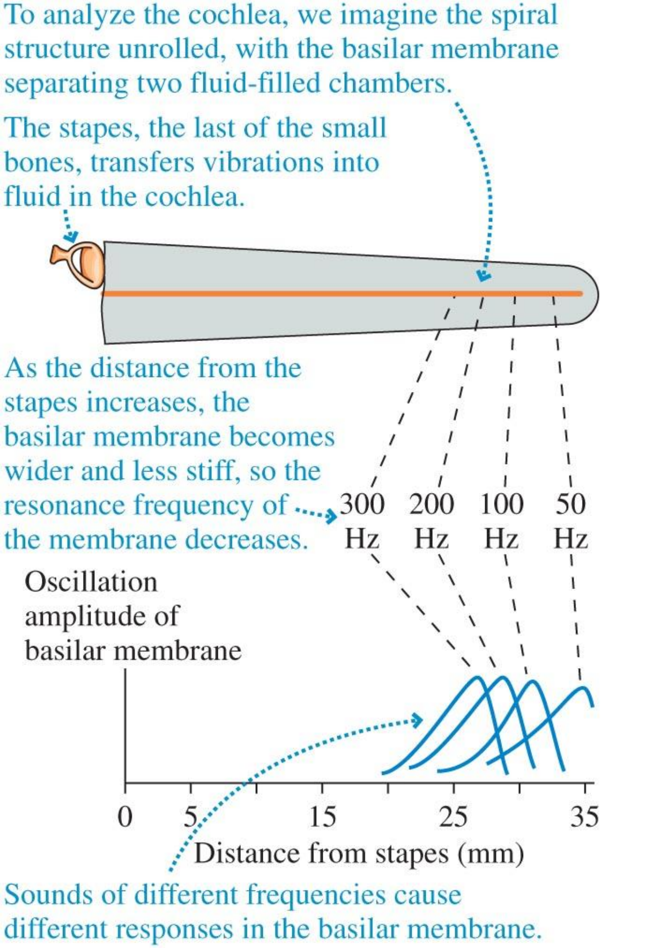
Sound travels into the ear, vibrates the ear drum, which amplifies the sound, and sends it down the cochlea



Physics of the human ear

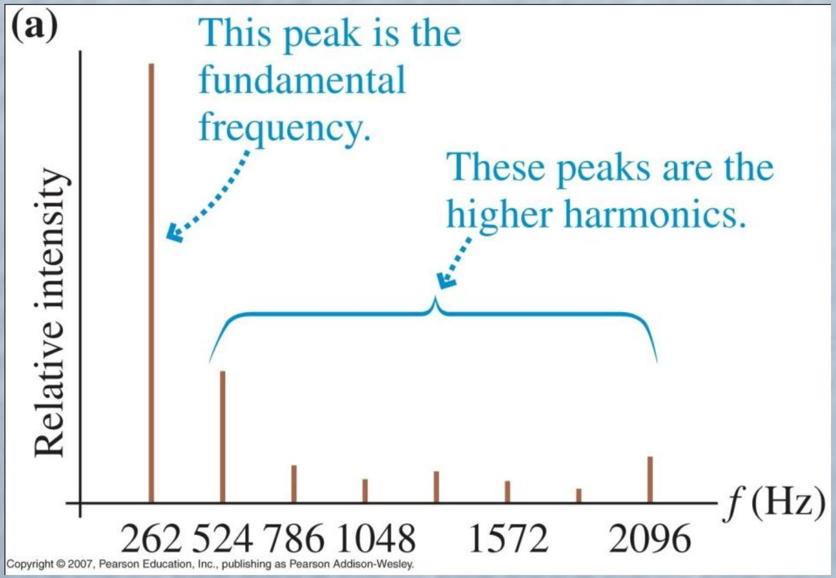
The sound resonates hair cells in the cochlea (0.5nm) to fire neurons

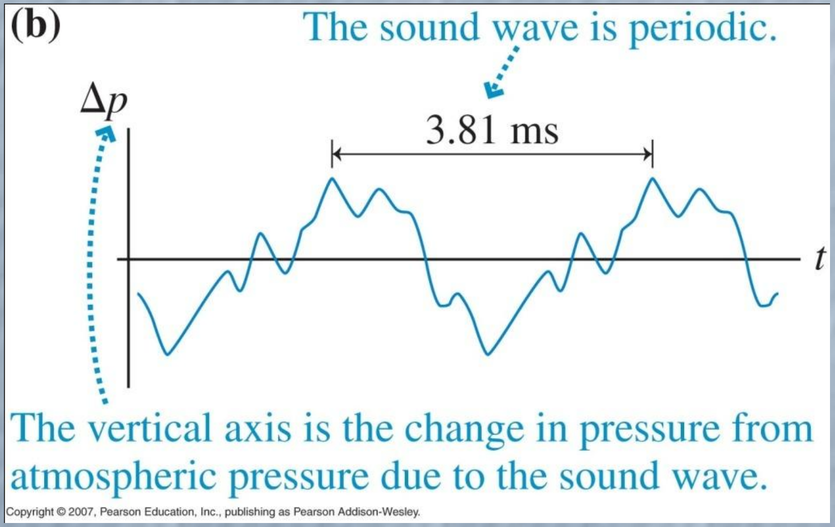


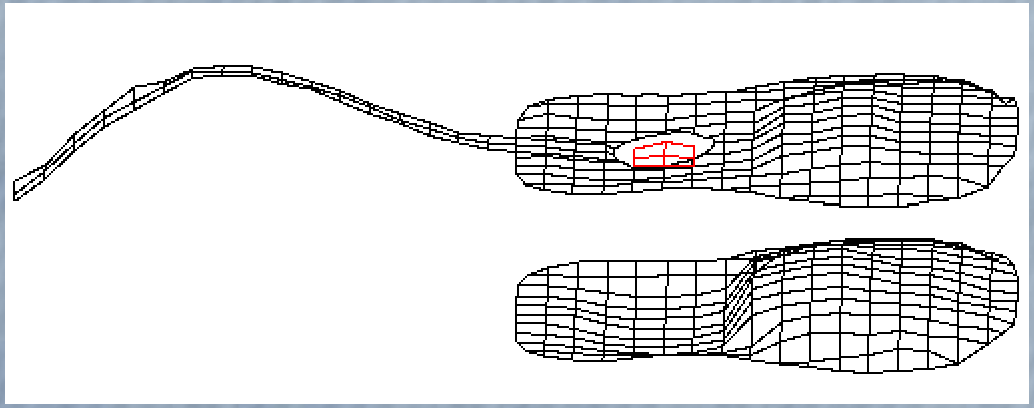


Shape of sound

A guitar string will have many higher frequencies, or harmonics. They add to the tone quality, or timbre.

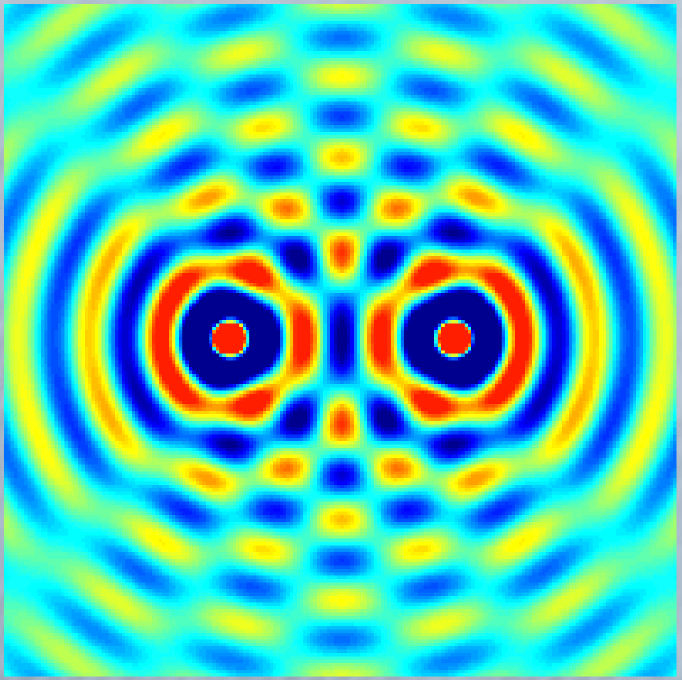






Interference

Two wave sources operating at the same frequency will add (constructively and destructively) and lead to interference patterns.

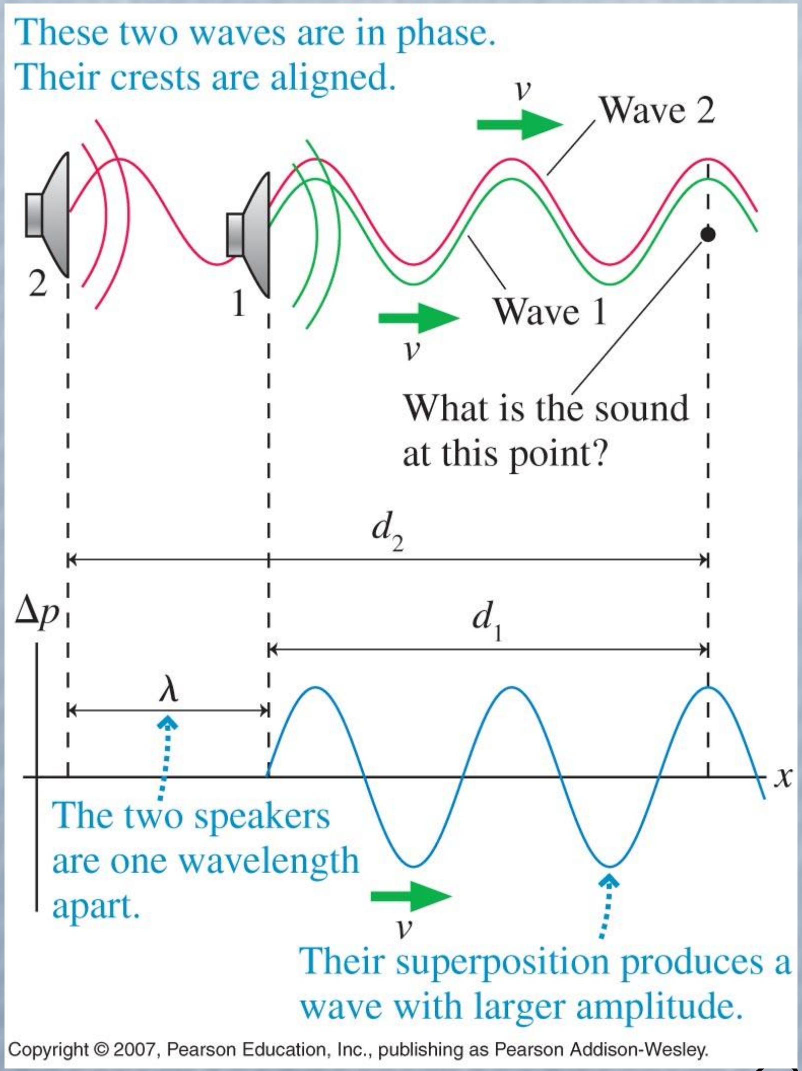


Constructive interference

- Amplitudes will add when the waves are in phase
- This happens when the path length difference is a whole number of wavelengths.

$$\Delta d = m\lambda$$

$$m = 0,1,2,3,\ldots$$

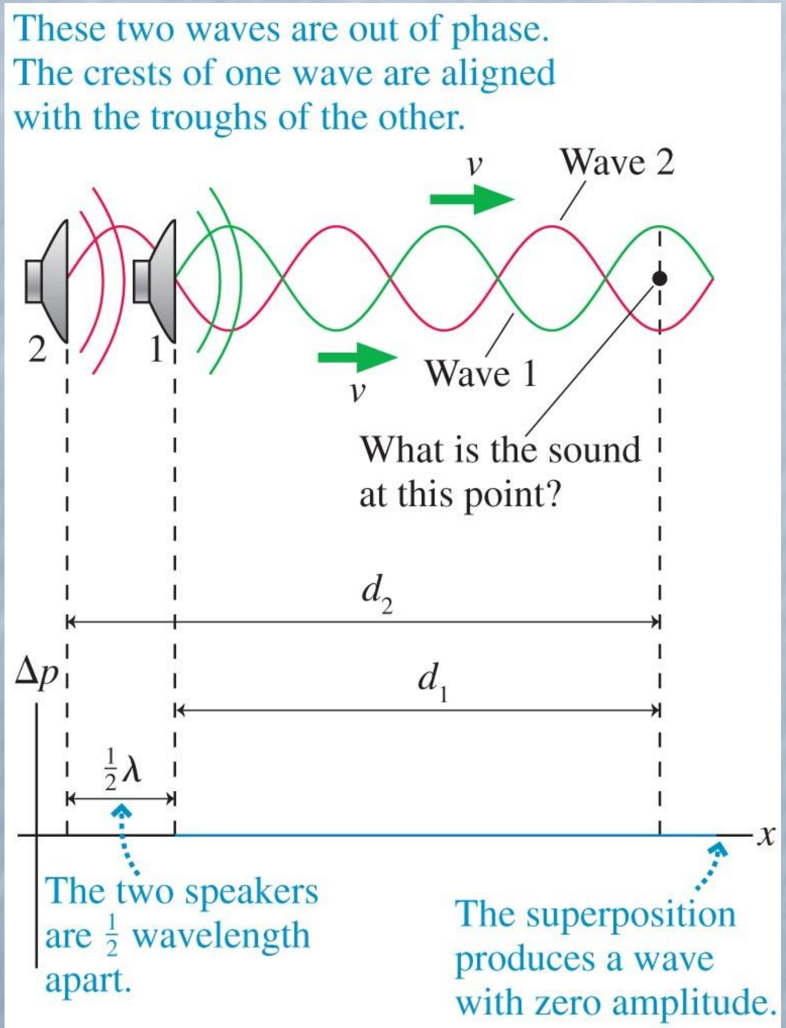


Destructive interference

- Amplitudes will cancel when the waves are out of phase
- This happens when the path length difference is a half wavelength off.

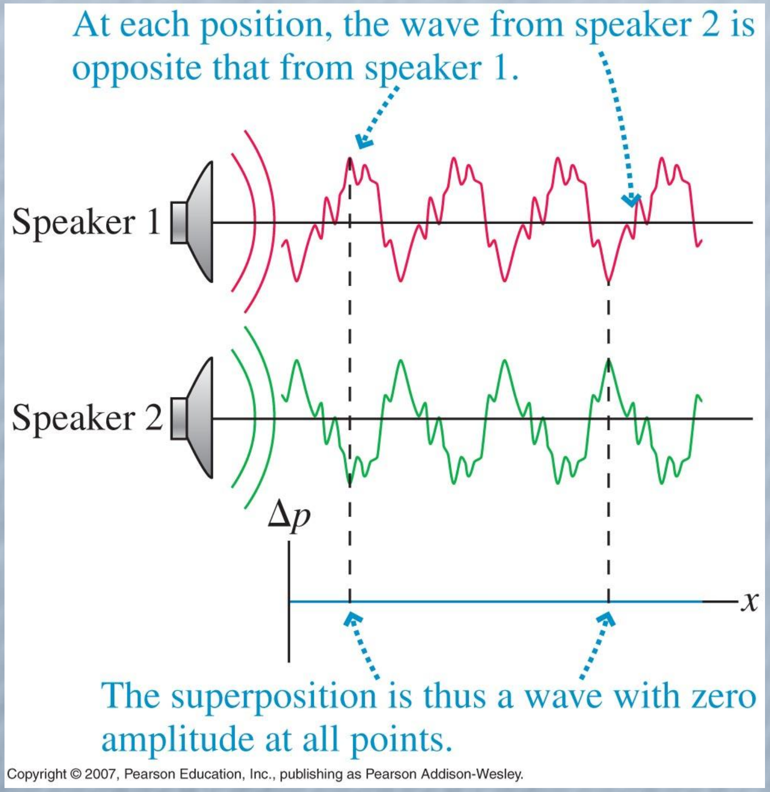
$$\Delta d = \left(m + \frac{1}{2} \right) \lambda$$

$$m = 0,1,2,3,\ldots$$



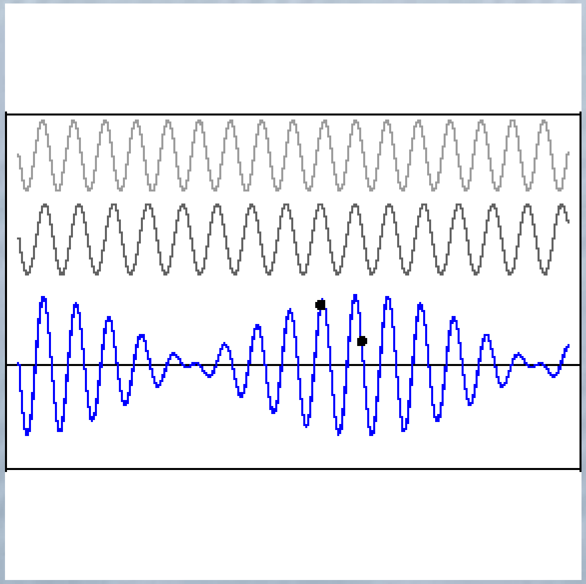
Destructive interference in head phones

- Active noise reduction is when the incoming sound is inverted and rebroadcast
- Commonly used on air flights.
- Selective frequency response.



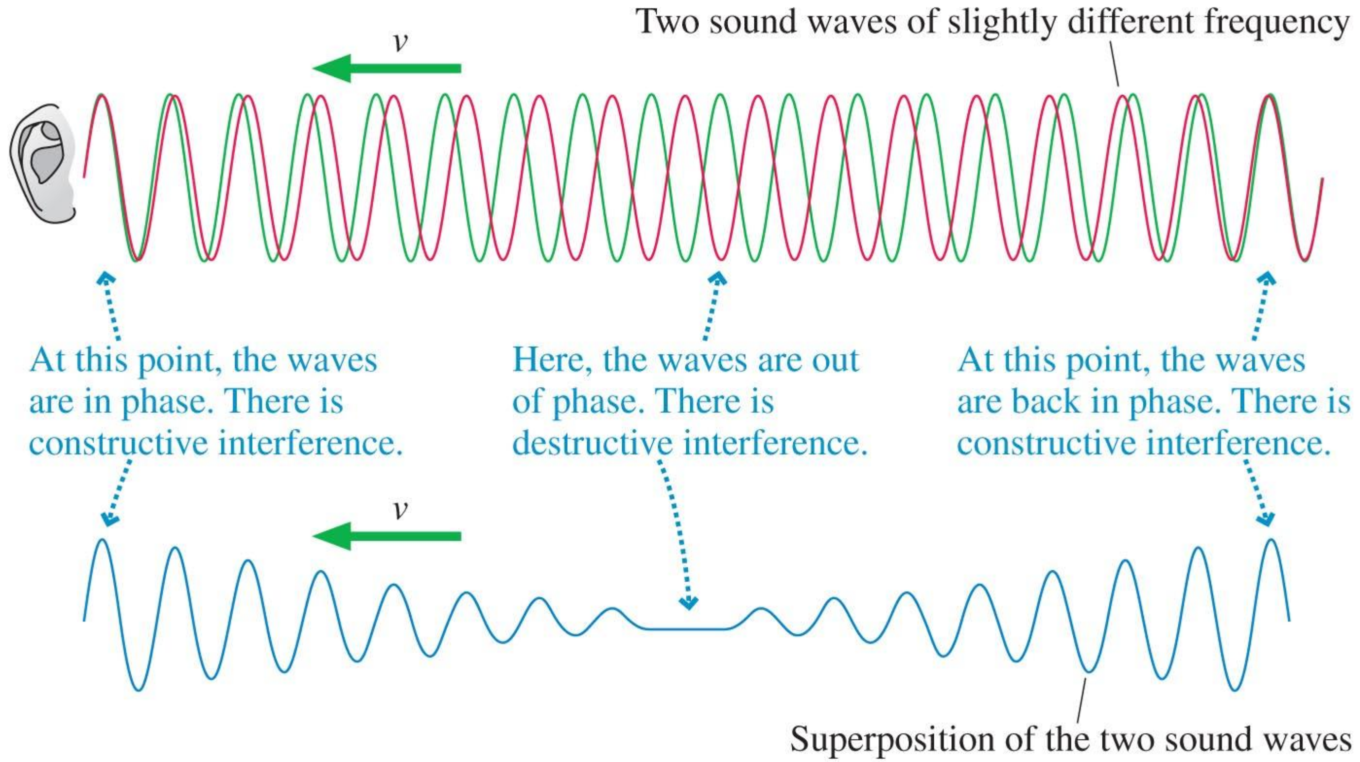
Beats

- Consider two waves of slightly different frequency
- The amplitudes add and cancel and give rise to beats.



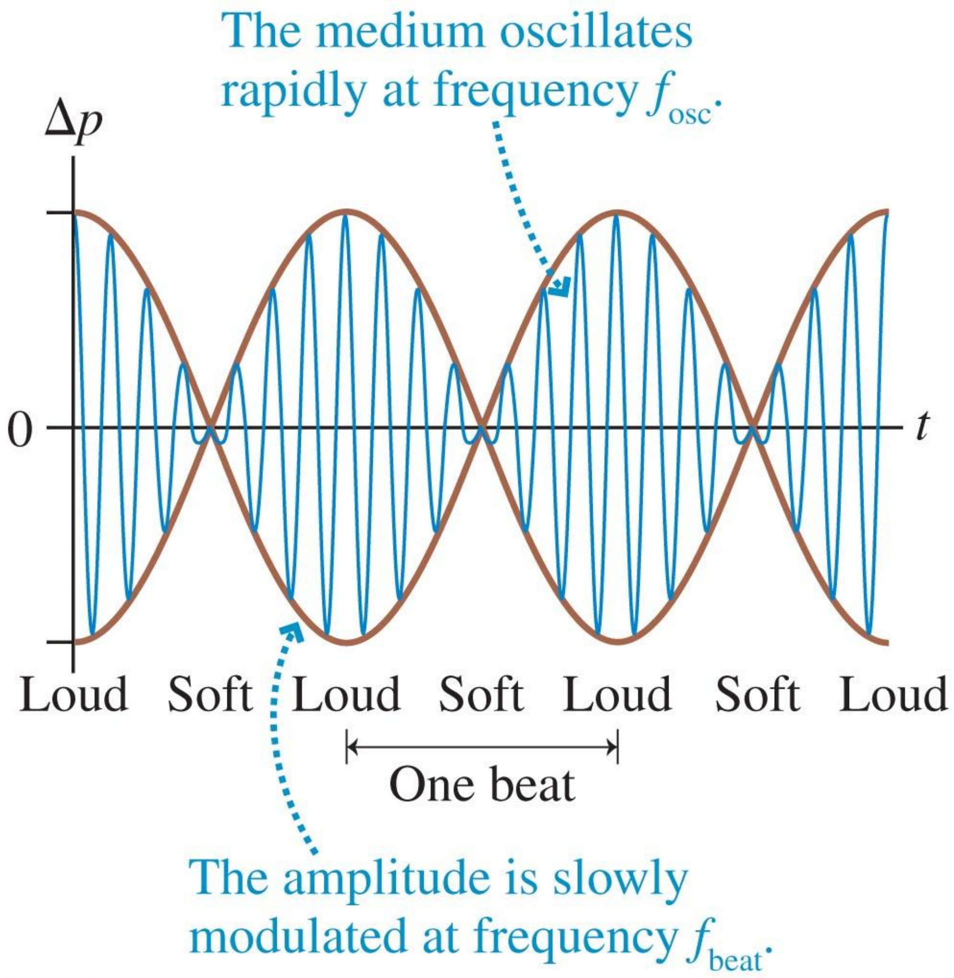
Beats

The time between the beats is dependent on the difference between the two frequencies



Beats

- There are 2 new frequencies, the frequency of the oscillation, f_{osc}, and the beat frequency, f_{beat}



$$f_{osc} = \frac{1}{2}\left(f_1 + f_2\right)$$

$$f_{beat} = \left| f_1 - f_2 \right|$$

Summary

- Superposition
- Constructive and destructive interference
- Standing waves
- Harmonies and tone
- Interference from two sources
- Beats

Homework problems

Chapter 16 Problems

41, 54, 56, 61, 62, 67