SUPPLEMENTAL DOCUMENT SD-6
FOR PART IVC – Quality Assurance/Uncertainty
Measurement Uncertainty for Extrapolations of Net Weight and Unit Count

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Introduction

The following examples demonstrate various approaches for deriving estimates of uncertainty associated with weight and count extrapolations:

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These examples are meant to be illustrative, not exclusive. Laboratories should develop defensible procedures that fit their operational environment and jurisdictional requirements. Notes and calculations are provided to clarify these applications. Calculations are based upon assumptions that populations are normally distributed[1]. Various terms used in this document are defined in the SWGDRUG Recommendations Annex A. The following examples should not be directly applied to methodology used without first considering the specific purpose of the method and its relevant operational environment.

[1] *Where sample weights diverge substantially from a normal distribution, weight extrapolations using small sample sizes (e.g. n = 3) may yield unreliable extrapolations and associated uncertainties.*

A Example 1: Extrapolation of net weight

Scenario:
A laboratory receives an exhibit containing 100 bags of white powder.

Objective:
The analyst needs to determine the total net weight of the powder in the 100 bags. This is done by weighing the powder from a sample of the population and extrapolating to the total population.

Procedure:

A.1 Determine the population size *N*. Only bags which have sufficient similar characteristics are placed in the same population.

In this example, the contents of all 100 bags are visually consistent in substance amount (about 0.5 gram) and physical appearance (i.e. color, texture, etc)[2], hence $N = 100$.

A.2 Select the sample size, *n*, to be weighed[1].

In this example, the analyst chooses a sample size $n = 10$. The 10 units are randomly selected[3] from the total population.
(Results for other *n* values are given later in the section.)

A.3 Measure the weight of the powder in each of the randomly selected units.

The weight (*X*) of the powder in each of the 10 bags is measured by dynamic weighing on a three-place balance (with 0.001 gram readability)[4] as recorded in table 1.1.

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[2] *If the bags or tablets are visually dissimilar, they need to be separated into different groups before continuing with the analysis separately for each group.*

[3] *A "random sample" is defined as "the sample so selected that any portion of the population has an equal (or known) chance of being chosen. Haphazard or arbitrary choice of units is generally insufficient to guarantee randomness" in SWGDRUG Glossary of Terms and Definitions, Annex A.*

[4] *See SWGDRUG Supplemental Document SD-3 for discussion on weighing processes and measurement uncertainty.*

Table 1.1: Individual weights of 10 bags.

Bag	Wt of powder (*X*), gram	Bag	Wt of powder (*X*), gram
1	0.593	6	0.574
2	0.509	7	0.580
3	0.557	8	0.540
4	0.548	9	0.532
5	0.569	10	0.529

A.4 Calculate the average weight per unit, X, the standard deviation, *s*, and the relative standard deviation, RSD.

Average weight per unit, X	= 0.5531 gram
Standard deviation, *s*	= 0.02622 gram
Relative Standard Deviation, RSD[5]	= 4.741%

A.5 Obtain the uncertainty, u_w, associated with the balance used[4].

In this example, the laboratory has determined u_w = 0.00185 gram for a three-place balance.

A.6 Obtain the uncertainty associated with the calculated average weight, $u_{\bar{X}}$. This uncertainty encompasses the standard deviation as well as the number of measurements performed.

$$u_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{0.02622\ g}{\sqrt{10}} = 0.008292$$

[5] *The laboratory's requirement should ensure that the variability of the measurements is small enough that all samples can be considered as belonging to the same population. UNODC and ENFSI Guidelines on Representative Drug Sampling (Reference D.2), page 34 states "In common practice, an acceptance criterion is that the sampling results are taken into consideration if the ratio between the standard deviation s and the average weight X of a drug unit in the sample is less than 0.1 (RSD<10%). Otherwise, an increase of the sample size is required in order to reach the target percentage." In casework, RSDs of sample weights higher than 10% may be encountered (see reference D.6). For such cases, when necessary and feasible, Laboratories may evaluate the RSD acceptance criteria based on weight and type (e.g. pharmaceutical versus illicit) of sample.*

A.7 Calculate the combined uncertainty, u_c, associated with the average weight per unit, by combining the standard uncertainties[6] of the average weight, $u_{\bar{X}}$, and the balance used, u_w,[7] via the root-sum-square (RSS) method.

$$u_c = \sqrt{u_{\bar{X}}^{\ 2} + u_w^{\ 2}} = \sqrt{0.008292^2 + 0.00185^2} = 0.008496 \text{ gram}$$

A.8 Calculate the extrapolated net weight of the 100 bags, *W*, and its associated uncertainty, u_T.

Extrapolated net weight, $W = N * \bar{X}$ = 100 * 0.5531 g = 55.31 grams
Extrapolated uncertainty, $u_T = N * u_c$ = 100* 0.008496 g = 0.8496 grams

A.9 Obtain the expanded extrapolated uncertainty, U_T, by using the appropriate coverage factor, *k*, (*Student's t* constant for 9 degrees of freedom)[8]. Round up the expanded extrapolated uncertainty, U_T, to two significant figures.

If a 95% level of confidence is used, (coverage factor *k* = 2.26216),

$$U_T = u_T * k = 0.8496 \text{ g} * 2.26216 = 1.922 \text{ grams} \approx 2.0 \text{ grams}$$

If a 99% level of confidence is used (coverage factor *k* = 3.24984),

$$U_T = u_T * k = 0.8496 \text{ g} * 3.24984 = 2.761 \text{ grams} \approx 2.8 \text{ grams}$$

A.10 Report the total extrapolated net weight and its associated uncertainty by matching the extrapolated net weight to the same level of significance (i.e. decimal places) as the rounded expanded uncertainty:

When the 95% level of confidence is used:
The amount of powder in 100 bags is 55.3 grams ± 2.0 grams at a 95% level of confidence, determined by weighing 10 bags and extrapolating to obtain the total net weight.

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[6] *When a sample size of greater than 10% of the population is used, a finite correction factor of* $\sqrt{\left(\frac{N-n}{N}\right)}$ *should be applied to the combined uncertainty (Reference D.2). However, since this correction factor is always less than 1 and decreases as n increases, it reduces the total uncertainty. The finite correction factor was not applied to these examples as omission results in a more conservative estimate of uncertainty.*

[7] *Contributions of uncertainty substantially less than one third of the largest contributor can often be eliminated from consideration (Reference D.3). However, for purposes of this document, the smaller contribution from the balance used* u_w *is included for all calculations.*

[8] *The coverage factor k is obtained from a two-tailed Student's t-distribution with n-1 degrees of freedom. In this function, as n increases with more data points, k decreases.*

When the 99% level of confidence is used:
The amount of powder in 100 bags is 55.3 grams ± 2.8 grams at a 99% level of confidence, determined by weighing 10 bags and extrapolating to obtain the total net weight.

A.11 If the analyst also performs qualitative analysis on each one of the 10 randomly selected bags and all are found to contain cocaine (that is, no negatives found), the following inferences about the population (at the respective 95% or 99% levels of confidence) can be made:

By statistically sampling 10 bags, it is concluded at a 95% level of confidence, that at least 76% of the population contains cocaine.

By statistically sampling 10 bags, it is concluded at a 99% level of confidence, that at least 65% of the population contains cocaine.

The above statistical inferences on the population as well as for other levels of confidence (depending on laboratory's policy and decision), can be calculated using the ENFSI DWG Calculator for Qualitative Sampling of seized drugs (2012). (This calculator can be obtained from the website http://www.enfsi.eu/documents/enfsi-dwg-calculator-qualitative-sampling-seized-drugs-2012).

Appendix 1.1:

Net weights and associated uncertainties extrapolated for other sample sizes are given in Table 1.2. It is noted that as the sample size *n* increases, the expanded extrapolated uncertainty, U_T, decreases. Also, for a given sample size *n*, the expanded uncertainty is larger when a higher level of confidence is used.

Table 1.2: Calculations for sample sizes of *n* = 3, 5, 10, 20 and 30.

Sample size, *n*	**3**	**5**	**10**	**20**	**30**
Avg wt of unit, *X*	0.5530	0.5552	0.5531	0.5514	0.5510
Std deviation, *s*	0.04214	0.03086	0.02622	0.02860	0.02759
% RSD	7.621	5.558	4.741	5.188	5.007
Std uncertainty of avg wt, u_{X}	0.024331	0.013800	0.008292	0.006396	0.005037
Combined std uncertainty, u_c	0.024401	0.013923	0.008496	0.006658	0.005366
Extrapolated uncertainty, u_T	2.4401	1.3923	0.8496	0.6658	0.5366
Extrapolated wt, *W*	55.30	55.52	55.31	55.14	55.10
With 95% Level of Confidence					
Coverage factor, *k*	4.30265	2.77645	2.26216	2.09302	2.04523
Exp extrapolated uncertainty, U_T	10.499	3.866	1.922	1.394	1.097
Lower Wt Limit	44.80	51.65	53.39	53.74	54.00
Upper Wt Limit	65.80	59.39	57.23	56.53	56.20
With 99% Level of Confidence					
Coverage factor, *k*	9.92484	4.60409	3.24984	2.86093	2.75639
Exp extrapolated uncertainty, U_T	24.218	6.410	2.761	1.905	1.479
Lower Wt Limit	31.08	49.11	52.55	53.23	53.62
Upper Wt Limit	79.52	61.93	58.07	57.04	56.58

Raw data of individual sample weights used are given in Table 1.3.

Table 1.3: Individual sample weights of 30 bags used in examples.

Bag	Wt of powder (*X*), gram	Bag	Wt of powder (*X*), gram	Bag	Wt of powder (*X*), gram
1	0.593	11	0.583	21	0.593
2	0.509	12	0.510	22	0.530
3	0.557	13	0.540	23	0.548
4	0.548	14	0.582	24	0.581
5	0.569	15	0.552	25	0.539
6	0.574	16	0.530	26	0.579
7	0.580	17	0.509	27	0.530
8	0.540	18	0.580	28	0.532
9	0.532	19	0.520	29	0.511
10	0.529	20	0.590	30	0.560

Step A.7 shows that the combined uncertainty, u_c, is contributed by two terms: the standard uncertainties of the average weight, $u_{\bar{X}}$, and the weighing process, u_w. If a balance of a different uncertainty is used, the combined uncertainty will change. Similarly, the distribution of the individual weights of the population will affect the combined uncertainty. To illustrate the impact of the weight distribution of the population on the extrapolation of the total net weight, another 30 bags from a different population (one that has been tested to be normally distributed) are individually weighed on the same balance. The individual weights of these 30 bags are given in Table 1.4 below and the associated calculations given in Table 1.5. It is noted that the RSD values listed in Table 1.5 are all much smaller than those for Table 1.2 (above). This consequentially gives rise to smaller expanded extrapolated uncertainty, U_T, for all sample sizes in Table 1.5 as compared to Table 1.2.

Table 1.4: Individual sample weights of 30 bags from a normally distribution population.

Bag	Wt of powder (*X*), gram	Bag	Wt of powder (*X*), gram	Bag	Wt of powder (*X*), gram
1	0.553	11	0.557	21	0.552
2	0.549	12	0.557	22	0.554
3	0.557	13	0.552	23	0.555
4	0.554	14	0.555	24	0.557
5	0.550	15	0.555	25	0.551
6	0.553	16	0.556	26	0.557
7	0.556	17	0.557	27	0.557
8	0.557	18	0.547	28	0.556
9	0.555	19	0.554	29	0.551
10	0.556	20	0.556	30	0.552

Table 1.5: Calculations for sample sizes of *n* = 3, 5, 10, 20 and 30.

Sample size, *n*	**3**	**5**	**10**	**20**	**30**
Avg wt of unit, *X*	0.5530	0.5526	0.5540	0.5543	0.5543
Std deviation, *s*	0.004000	0.003209	0.002789	0.002886	0.002728
% RSD	0.7233	0.5808	0.5034	0.5206	0.4922
Std uncertainty of avg wt, u_{X}	0.0023094	0.0014353	0.0008819	0.0006452	0.0004981
Combined std uncertainty, u_c	0.002959	0.002341	0.002049	0.001959	0.001916
Extrapolated uncertainty, u_T	0.2959	0.2341	0.2049	0.1959	0.1916
Extrapolated wt, *W*	55.30	55.26	55.40	55.43	55.43
With 95% Level of Confidence					
Coverage factor, *k*	4.30265	2.77645	2.26216	2.09302	2.04523
Exp extrapolated uncertainty, U_T	1.273	0.650	0.464	0.410	0.392
Lower Wt Limit	54.03	54.61	54.94	55.02	55.03
Upper Wt Limit	56.57	55.91	55.86	55.84	55.82
With 99% Level of Confidence					
Coverage factor, *k*	9.92484	4.60409	3.24984	2.86093	2.75639
Exp extrapolated uncertainty, U_T	2.937	1.078	0.666	0.561	0.528
Lower Wt Limit	52.36	54.18	54.73	54.87	54.90
Upper Wt Limit	58.24	56.34	56.07	55.99	55.95

B Example 2: Extrapolation of net weight in conjunction with a hypergeometric sampling plan

Scenario:
The scenario is the same as Example 1, where the laboratory receives an exhibit containing 100 bags of white powder. Sentencing penalty in this jurisdiction increases if the amount of substance containing cocaine exceeds 25 grams.

Objective:
The analyst will use statistical sampling to determine, to a 99% level of confidence, if the jurisdictional weight threshold is exceeded or not. This example does not take purity of the powder into account because it is not jurisdictionally relevant.

Procedure:

B.1 The analyst needs to determine how many bags must be sampled to determine if the 25-gram threshold weight is exceeded.

To obtain an estimation of the number of bags that must be sampled, the specified statutory threshold weight (25 grams) is divided by the average net weight ($\bar{X}$) per unit (obtained from Example 1).

Estimated number of bags = $\frac{\text{statutory threshold weight}}{\bar{X}} = \frac{25}{0.5531}$ = 45.1 (46 bags)

Therefore, a minimum of 46 bags must be sampled to provide strong evidence that the threshold weight is exceeded. To compensate for the uncertainty associated with the weighing process, the analyst decides to increase the number of bags to 50.

B.2 Determine the sample size *n* that needs to be qualitatively tested to demonstrate that at least 50 of the 100 bags contain cocaine at a 99% level of confidence.

Method 1: 99% level of confidence corresponds to an α of 0.01 (level of confidence = 0.99 = 1-α). Proceed to use a hypergeometric sampling calculator to determine the sample size needed. (See http://www.enfsi.eu/documents/enfsi-dwg-calculator-qualitative-sampling-seized-drugs-2012)

Using the hypergeometric sampling calculator and the appropriate parameters (N = 100, α = 0.01, proportion of positives = 0.5, with no negatives expected), the sample size is determined to be 7.

<u>or</u>

Method 2: Manually determine the number of bags, *n*, to test by multiplying the resulting conditional probabilities for the 50 bags needed. The number of bags to be sampled will be indicated by the first instance resulting in a probability value (*p*-value) below the established significance level of 0.01 (corresponding to a 99% level of confidence).

$$P_n = P_{(bag\ 1)} * P_{(bag\ 2)} * P_{(bag\ 3)} * P_{(bag\ 4)} * \ldots * P_{(bag\ n)}$$

$$= \frac{50-1}{100} * \frac{50-2}{100-1} * \frac{50-3}{100-2} * \frac{50-4}{100-3} * \ldots * \frac{50-n}{100-(n-1)}$$

$$= \frac{{}^{50-1}C_n}{{}^{100}C_n}$$

= *P* (all *n* bags in the sample contain cocaine)

The following calculations show the *p*-values (and resulting levels of confidence, LoC) obtained for each successive sample tested (with no negatives found) until a value below 0.01 is obtained (which is sample 7):

$P_1 = \frac{49}{100} =$ 0.4900 (51.00% LoC)

$P_2 = \frac{49}{100} * \frac{48}{99} =$ 0.2376 (76.24% LoC)

$P_3 = \frac{49}{100} * \frac{48}{99} * \frac{47}{98} =$0.1139 (88.61% LoC)

$P_4 = \frac{49}{100} * \frac{48}{99} * \frac{47}{98} * \frac{46}{97} =$0.0540 (94.60% LoC)

$P_5 = \frac{49}{100} * \frac{48}{99} * \frac{47}{98} * \frac{46}{97} * \frac{45}{96} =$ 0.0253 (97.47% LoC)

$P_6 = \frac{49}{100} * \frac{48}{99} * \frac{47}{98} * \frac{46}{97} * \frac{45}{96} * \frac{44}{95} =$ 0.0117 (98.83% LoC)

$P_7 = \frac{49}{100} * \frac{48}{99} * \frac{47}{98} * \frac{46}{97} * \frac{45}{96} * \frac{44}{95} * \frac{43}{94} =$ 0.0054 (99.46% LoC)

Therefore, the number of bags *n* needed for testing is 7.

B.3 A total of 7 bags are randomly selected for chemical analysis[9] and confirmed to contain cocaine.

Since all 7 bags are found to contain cocaine, it can be stated, to a 99.46% level of confidence, that at least 50 of the 100 bags contain cocaine.

The total net weight of 50 bags, W_{50}, can be extrapolated from the average net weight per unit (obtained from Example 1):

$$W_{50} = 50 * \bar{X} = 50 * 0.5531 \text{ g} = 27.655 \text{ grams}$$

B.4 The combined standard uncertainty, u_c, associated with the average weight per unit as calculated from Example 1 is:

$$u_c = 0.008496 \text{ gram}$$

The extrapolated uncertainty for 50 bags, u_{T50}, is calculated as

$$u_{T50} = u_c * 50 = 0.008496 \text{ g} * 50 = 0.4248 \text{ gram}$$

[9] *Sample size determination may be made prior to weight determination. For example, a laboratory may mandate inferences to be made for 90% of all populations at a 95% level of confidence, irrespective of statutory weight thresholds. In this instance, the extrapolated net weight may be based on the weights of the individual items weighed and the appropriate coverage factor would be selected to calculate the expanded uncertainty. As an example, for N=100, to achieve a 95% level of confidence, the sample size n = 23. The average weight per tablet $\bar{X}$ and standard deviation s are determined and entered using the ENFSI DWG Calculator for Qualitative Sampling of seized drugs (2012), see http://www.enfsi.eu/documents/enfsi-dwg-calculator-qualitative-sampling-seized-drugs-2012.*

Step 1: Enter Confidence level (1-α)	0.95	α=	0.05
Step 2: Enter Population (*N*)	90	*t*=	2.074
Step 3: Enter Sample Size (*n*)	23	*n/N*=	0.256
Step 4: Enter Mean Weight (x-bar)	0.55209	Q=	0.863
Step 5: Enter Sample Standard Deviation (*s*)	0.0283	rsd(%) =	5.1
Step 6: Enter Number of Negative Results in Sample (r)	0	P=	1
Confidence Interval =	**0.55209**	**±**	0.011

Figure above shows the results obtained from the ENFSI DWG Calculator for Qualitative Sampling of seized drugs.

The total expanded uncertainty (U_{T50}), at 99% level of confidence, and rounded up to two significant figures (coverage factor *k* = 3.7074 using Student's *t* value for 6 degrees of freedom) is

$$U_{T50} = u_{T50} * k = 0.4248 \text{ g} * 3.7074 = 1.574 \text{ g} \approx 1.6 \text{ gram}$$

B.5 The analyst compares the calculated extrapolated weight of the 50 bags, W_{50}, minus the expanded uncertainty, U_{T50} , (truncated to the same level of significance) against the statutory threshold of 25 grams.

The weight of 50 bags is 27.6 grams ± 1.6 grams at a 99% level of confidence. The lower end of the weight range is = 27.6 – 1.6 grams = 26.0 grams (which is above 25-grams statutory threshold).

B.6 If the lower end of the weight range is below the statutory threshold, additional bag(s) may need to be sampled in order to demonstrate that the statutory threshold has been exceeded.

B.7 The results of the analysis can be reported in either of the following ways:

1) A total of 100 indistinguishable bags were received. By using statistical sampling of 7 bags, it is concluded at a 99% level of confidence that at least 50% of the population contains cocaine. The extrapolated net weight of 50 bags is 27.6 grams ± 1.6 grams at a 99% level of confidence.

2) A total of 100 indistinguishable bags were received. Using statistical (hypergeometric) sampling and by testing 7 bags, cocaine is present in at least 26.0 grams of powder at a level of confidence of at least 98%.

   Explanation on deriving the overall level of confidence (i.e. at least 98%): The second report option gives an overall level of confidence of at least 98% for the weight and identity of the powder. Each of these parameters is individually tested at a 99% level of confidence. Since these two statements are not independent of each other, the Bonferroni correction (Reference D.1, p 155-156) is used in the calculation of the overall confidence level. This is obtained by determining the value of (1 – 0.01 – 0.01)*100%. If the two statements are independent, the multiplication rule of probability can be used instead, giving an overall level of confidence of 99%*99% = 98.01%.

Appendix 2.1:

To contrast the practicality of using hypergeometric sampling to identify a proportion of a population, the following example is given:

<u>To determine the content of all 100 bags</u>

If a sampling size of 7 is used to determine the content of all 100 bags, the probability of failure = P_7

$$= P_{(bag\ 1)} * P_{(bag\ 2)} * P_{(bag\ 3)} * P_{(bag\ 4)} * P_{(bag\ 5)} * P_{(bag\ 6)} * P_{(bag\ 7)}$$

$$= \frac{{}^{100-1}C_7}{{}^{100}C_7}$$

$$= \frac{99}{100} * \frac{98}{99} * \frac{97}{98} * \frac{96}{97} * \frac{95}{96} * \frac{94}{95} * \frac{93}{94} = 0.93 \text{ (7\% confident)}$$

As illustrated in this case, if only 7 bags are sampled, the analyst is only 7% confident that all 100 bags contain a substance containing cocaine.

If a 95% level of confidence is needed for the reporting of content of all 100 bags, the sampling size needs to be increased as shown below:

$$\frac{99}{100} * \frac{98}{99} * \frac{97}{98} * \frac{96}{97} * \frac{95}{96} * \dots * \frac{5}{6} = 0.05 \text{ (95\% confident)}$$

giving a sample size of 95.

Therefore, it is often practical to report that a certain proportion of the population is positive instead of reporting on the entire population. This can be achieved by using statistical sampling. Using the same example of a total population of 100 bags, if the laboratory only needs to report on the content of 90 bags, the sampling size would reduce to 23:

$$\frac{89}{100} * \frac{88}{99} * \frac{87}{98} * \frac{86}{97} * \frac{85}{96} * \dots * \frac{67}{78} = 0.047 \text{ (95.3\% confident)}$$

As seen from this example, if the laboratory needs to report on the content of all 100 bags at a confidence level of 95%, a total of 95 bags need to be tested. In contrast, if the laboratory only needs to report on the content of 90 bags at the same confidence level, the number of bags to be tested is reduced to 23 (a reduction of 75%).

C Example 3: Extrapolation of unit count

Scenario: The laboratory receives a large container with numerous illicitly pressed tablets.

Objective:
The analyst needs to determine the total number of tablets present in the container and its associated uncertainty by direct weighing of a sample of individual tablets and extrapolating to obtain the total count.

Procedure:

C.1 Determine whether all the tablets in the container can be treated as one population.

Since all the tablets in the container are visually similar, they will be treated as one population.

C.2 Measure the net weight of all the tablets.

The total weight, *TW,* of the total population of tablets is determined to be 701.5 grams based on dynamic weighing on a balance with 0.1 gram readability.

C.3 Choose the number of individual tablets to weigh.

In this example, the analyst randomly samples and weighs 10 tablets (n = 10). (Results for other n values are given later in the section.)

The weight of each tablet X is determined by dynamic weighing on a balance with 0.0001 gram readability as in Table 3.1.

Table 3.1: Individual weights of 10 tablets.

Tablet	Wt of tablet (X), gram	Tablet	Wt of tablet (X), gram
1	0.3084	6	0.3437
2	0.3225	7	0.2918
3	0.3349	8	0.3116
4	0.2981	9	0.3077
5	0.3293	10	0.3426

C.4 Calculate the average weight per tablet, $\bar{X}$, the standard deviation of the tablet weight, *s*, and the relative standard deviation, RSD.

Average weight per tablet, $\bar{X}$	= 0.31906 gram
Standard deviation, *s*	= 0.018287 gram
Relative standard deviation, RSD	= 5.7314 %

C.5 The number of tablets in the container is estimated by dividing the total weight of all the tablets, *TW*, by the average weight per tablet, $\bar{X}$.

Estimated number of tablets in container = $\frac{TW}{\bar{X}} = \frac{701.5}{0.31906} = 2198.6$

C.6 Obtain the uncertainty associated with the two balances used[4]:

Uncertainty for one-place balance (0.1 g readability), u_{w1} = 0.35810 gram
Uncertainty for four-place balance (0.0001 g readability), u_{w2} = 0.0004840 gram

C.7 Calculate the relative uncertainties of both weighing processes.

Relative uncertainty of the total weight of tablets, u'_{TW}:

$$u'_{TW} = \frac{u_{TW}}{TW} = \frac{u_{w1}}{TW} = \frac{0.35810\ g}{701.5\ g} = 0.00051048$$

Relative uncertainty of average weight per tablet, $\bar{X}$:

$$u'_{\bar{X}} = \frac{u_{\bar{X}}}{\bar{X}} = \frac{\sqrt{\left(\frac{s}{\sqrt{n}}\right)^2 + (u_{w2})^2}}{\bar{X}} = \frac{\sqrt{\left(\frac{0.018287\ g}{\sqrt{10}}\right)^2 + (0.0004840\ g)^2}}{0.31906\ g} = 0.018188$$

C.8 Combine the two relative standard uncertainties (u'_{TW} and $u'_{\bar{X}}$) to obtain the combined relative standard uncertainty, u'_c, associated with the extrapolated tablet count.

$$u'_c = \sqrt{{u'_{TW}}^2 + {u'_{\bar{X}}}^2} = \sqrt{(0.00051048)^2 + (0.018188)^2} = 0.018195$$

C.9 Determine the absolute combined uncertainty, u_c, for the tablet count by multiplying the combined relative standard uncertainty, u'_c, by the estimated number of tablets.

$u_c = u'_c *$ number of tablets= 0.018195 * 2198.6 = 40.004

C.10 Expand the combined uncertainty, u_c, using the appropriate coverage factor *k*.

At a 95% level of confidence for *n* = 10, the coverage factor *k* = 2.26216.
Expanded uncertainty, $U_c = u_c$ * *k* = 40.004 * 2.26216 = 90.496 tablets.

If a 99% level of confidence is used, the coverage factor *k* = 3.24984.
Expanded uncertainty, $U_c = u_c$ * *k* = 40.004 * 3.24984 = 130.007 tablets.

C.11 Report the total extrapolated tablet number, and its associated uncertainty, truncating or rounding to the nearest whole number as per laboratory's policy. In this example, the number of tablets is truncated while the associated uncertainty is rounded up for a conservative approach.

Number of tablets: 2198 ± 91
The number of tablets is an extrapolated estimated value based on the individual weights of 10 tablets and the uncertainty value represents an expanded uncertainty at a 95% level of confidence.

Number of tablets: 2198 ± 131
The number of tablets is an extrapolated estimated value based on the individual weights of 10 tablets and the uncertainty value represents an expanded uncertainty at a 99% level of confidence.

Appendix 3.1:

Examples of other sample sizes *n* = 3, 5, 30 and 50 taken from the same population are given in Table 3.2, together with data from *n* = 10 for comparison. Raw data of tablet weights used for Table 3.2 are given in Table 3.3. It is noted that the extrapolated combined uncertainty, u_c, is smaller as the sample size gets bigger. Also, for a given sample size *n*, the expanded uncertainty, U_c, is larger when a higher level of confidence is used.

It should be the laboratory's decision and policy to determine the sample size *n* needed for the extrapolation of number of units. Smaller *n* is more time efficient but results in a much larger expanded uncertainty, U_c. Larger *n* takes more time to complete the analysis but has the benefit of a smaller expanded uncertainty

Table 3.2: Calculations for sample sizes of *n* =3, 5, 10, 30 and 50.

Sample size, *n*	**3**	**5**	**10**	**30**	**50**
Avg wt per tablet, $\bar{X}$	0.32193	0.31864	0.31906	0.32337	0.32510
Std deviation, *s*	0.013259	0.015163	0.018287	0.017731	0.019186
% RSD	4.1186	4.7587	5.7314	5.4833	5.9016
Extrapolated tablet count, $\frac{TW}{\bar{X}}$	2179.0	2201.5	2198.6	2169.3	2157.8
Std uncert of avg wt, $u_{\bar{X}}$	0.0076551	0.0067811	0.0057828	0.0032373	0.0027133
Rel. uncert of net wt, u'_{TW}	0.00051048	0.00051048	0.00051048	0.00051048	0.00051048
Rel. uncert of avg wt, $u'_{\bar{X}}$	0.023826	0.021336	0.018188	0.010122	0.008478
Combined rel uncert, u'_c	0.023832	0.021342	0.018195	0.010135	0.008493
Extrapolated combined uncert, u_c	51.930	46.985	40.004	21.987	18.327
With 95% Level of Confidence					
Coverage factor, *k*	4.30265	2.77645	2.26216	2.04523	2.00958
Expanded uncert, U_c	223.435	130.450	90.496	44.968	36.828
With 99% Level of Confidence					
Coverage factor, *k*	9.92484	4.60409	3.24984	2.75639	2.67995
Expanded uncert, U_c	515.393	216.322	130.007	60.604	49.114

Table 3.3: Individual weight of tablets for Table 3.2.

Tablet	Wt of tablet (X), gram	Tablet	Wt of tablet (X), gram	Tablet	Wt of tablet (X), gram
1	0.3084	21	0.3152	41	0.3580
2	0.3225	22	0.2763	42	0.3090
3	0.3349	23	0.3058	43	0.3251
4	0.2981	24	0.3014	44	0.3459
5	0.3293	25	0.3376	45	0.3054
6	0.3437	26	0.3313	46	0.3195
7	0.2918	27	0.3388	47	0.2802
8	0.3116	28	0.3192	48	0.3463
9	0.3077	29	0.3323	49	0.2802
10	0.3426	30	0.3348	50	0.3356
11	0.3476	31	0.3462		
12	0.3450	32	0.3317		
13	0.3196	33	0.3322		
14	0.3171	34	0.3272		
15	0.3321	35	0.3305		
16	0.3441	36	0.3383		
17	0.3435	37	0.3456		
18	0.3240	38	0.3456		
19	0.3293	39	0.3106		
20	0.3155	40	0.3408		

Appendix 3.2:

To illustrate the impact of the weight distribution on the extrapolation of the unit count, three distinct populations of tablets were evaluated. Group 1 contains 50 Ecstasy tablets, Group 2 contains 50 illegitimate pharmaceutical tablets, and Group 3 contains 50 legitimate pharmaceutical tablets.

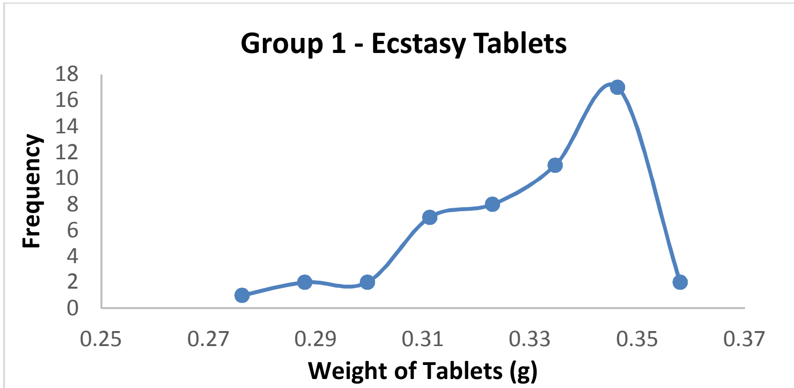
Tablets from each group look visually similar. The total weight of each group of 50 tablets is weighed using a one-place balance (with uncertainty of 0.3581 gram). A sample size of 10 tablets from each group is randomly sampled for individual weighing using a four-place balance (with uncertainty of 0.000484 gram). The calculations for the extrapolation of tablet count for the three groups are shown in Table 3.4 below.

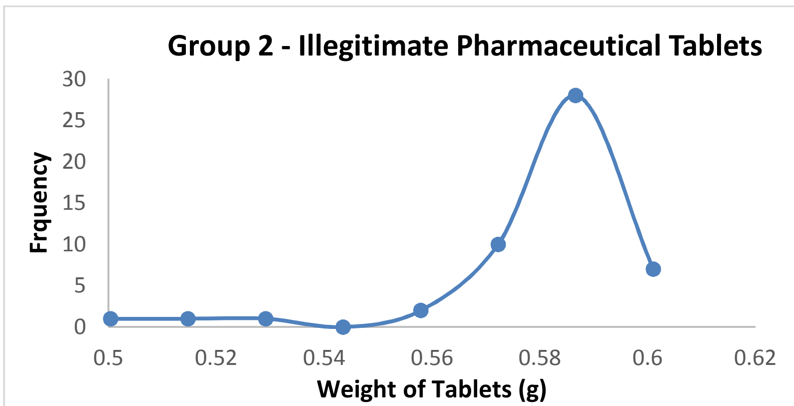
The RSD of the sample, and hence the expanded uncertainty of the extrapolation, depends on the distribution curve. A normally distributed population with a small standard deviation will give smaller expanded uncertainty.

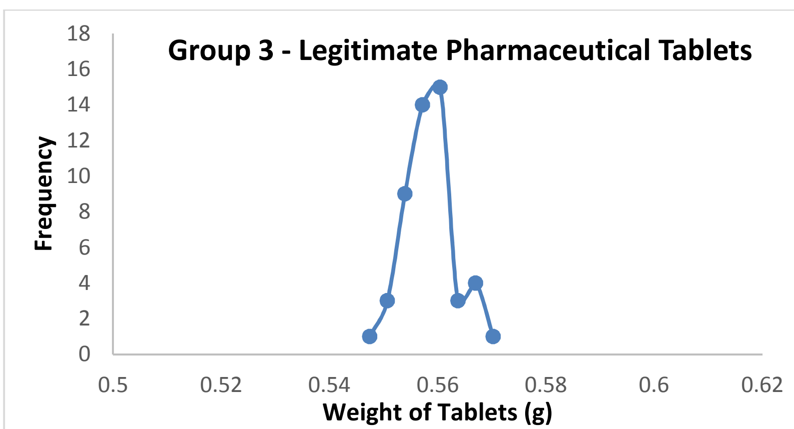
Table 3.4: Calculations for 3 groups of tablets each with sample sizes of 10.

	Group 1	**Group 2**	**Group 3**
Sample Description	Ecstasy Tablets	Illegitimate pharmaceutical tablets	Legitimate pharmaceutical tablets
Total Weight of 50 tablets, *TW*	16.3	28.7	27.9
Avg wt per tablet, X	0.31906	0.58253	0.55591
Std deviation, *s*	0.018287	0.011608	0.0052800
% RSD	5.73142	1.9926	0.94980
Extrapolated tablet count, $\frac{TW}{\overline{X}}$	51.088	49.268	50.188
Std uncert of avg wt, $u_{\overline{X}}$	0.0181877	0.0036706	0.0016697
Rel. uncert of total wt, u'_{TW}	0.021969	0.012477	0.012835
Rel. uncert of avg wt, $u'_{\bar{X}}$	0.0181877	0.0063557	0.0031272
Combined rel uncert, u'_c	0.028521	0.014003	0.013211
Extrapolated combined uncert, u_c	1.45707	0.68989	0.66301
With Level of Confidence = 95% (*k* = 2.26216)			
Expanded uncert, U_c	3.296	1.561	1.500
With Level of Confidence = 99% (*k* = 3.24984)			
Expanded uncert, U_c	4.735	2.242	2.155

Figure 1: Charts showing the distribution of the tablet weights of the three groups of tablets.







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