**Sustainable Fashion Supply Chain Management Under Oligopolistic Competition and Brand Differentiation**

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**Abstract:**

In this paper, we developed a new model of oligopolistic competition for fashion supply chains in the case of differentiated products with the inclusion of environmental concerns. The model assumes that each fashion firm's product is distinct by brand and the firms compete until an equilibrium is achieved. Each fashion firm seeks to maximize its profits as well as to minimize its emissions throughout its supply chain with the latter criterion being weighted in an individual manner by each firm. The competitive supply chain model is network-based and variational inequality theory is utilized for the formulation of the governing Nash equilibrium as well as for the solution of the case study examples. The numerical examples illustrate both the generality of the modeling framework as well as how the model and computational scheme can be used in practice to explore the effects of changes in the demand functions; in the total cost and total emission functions, as well as in the w! eights.

**Keywords:** fashion supply chain management, supply chain networks, product differentiation, brands, oligopolistic competition, sustainable supply chains, alternative transportation modes, Nash equilibrium, game theory, variational inequalities

## 1. Introduction

The fashion and apparel industry faces vast challenges as well as opportunities in the reduction of its environmental impact globally. The demand for apparel that is produced and distributed in a manner that minimizes the use (and discarding) of toxic dyes, raw materials such as cotton grown with pesticides, as well as the generation of waste in terms of textiles and byproducts (including packaging) is coming not only from consumers but, more recently, even from firms such as Levi's, Gap, H&M, and Wal-Mart that wish to enhance or to maintain a positive brand identity (see, e.g., Claudio (2007), Glausiusz (2008), Rosenbloom (2010), and Tucker (2010)).

In addition, organizations such as the Natural Resources Defense Council (NRDC) are now increasingly emphasizing that this industry's reduction of its environmental impacts will require that brands and retailers reexamine their supply chains way back to the inputs into their production processes and take more responsibility even for the fabric utilized (cf. Tucker (2010)).

In order to fix ideas, and to emphasize the scope of the environmental issues associated with the fashion and apparel industry, we now provide some data. According to Claudio (2007), polyester is a man-made fiber whose demand from the fashion industry has doubled in the past 15 years. Its manufacture requires petroleum and releases such emissions as volatile organic compounds and gases such as hydrogen chloride, as well as particulates. Other byproducts associated with its production are emitted in the waste water. However, even natural fibers used in textiles for apparel may also leave a large environmental imprint. For example, the production of cotton, one of the most versatile fibers used in clothing, accounts for a quarter of all the pesticides used in the United States, which is the largest exporter of cotton in the world (see Claudio (2007)). According to the NRDC (see Tucker (2010)), textile manufacturing pollutes as much as 200 tons of water per ton of fabric. In ! China, for example, a textile factory may also burn about 7 tons of carbon emitting coal per ton of fabric produced. In the case of blue jean production, Xintang, located in the northeastern part of the Pearl River Delta in China, is where approximately 200 million pairs of jeans are produced annually for 1,000 different labels. The standard jean dyeing process dispenses into its waste water a mixture of dye, bleach, and detergent and, as a consequence, the production of blue jeans in such a manner is partly to blame for the pollution of the Pearl River (see UPI.com (2010)).

As the production of apparel has become global and competition has intensified (see Gereffi and Memedovic (2003)), with an increased prominence of brands and buyer-driven value chains, new networks are transforming this industry. Interestingly, whereas in 1992 about 49% of all retail apparel sold in the United States was actually made there, by 1999 the proportion had fallen to just 12% (Rabon (2001)). Between 1990 and 2000, the value of apparel imports to the US increased from $25 billion to $64 billion. According to Gereffi and Memedovic (2003), the top exporters of apparel in 2000, with a value of over $1 billion in US dollars, were: China, Hong Kong (now referred to as Hong Kong SAR), the United States, Mexico, and Turkey, whereas in 1980, the major exporters were: Hong Kong SAR, South Korea, Taiwan, China, and the United States. However, as noted in Nagurney and Woolley (2010), with the growing investment and industrialization in developing nations, it is also imp! ortant to evaluate the overall impact at not only the operational level, but also in terms of the environment. For example, between 1988-1995, multinational corporations invested nearly $422 billion worth in new factories, supplies, and equipment in developing countries (World Resources Institute (1998)). Through globalization, firms of industrialized nations may make use of manufacturing plants in developing nations that offer lower production costs; however, more than not, combined with inferior environmental concerns, due, for example, to a looser environmental regulatory system and/or lower environmental impact awareness.

Also, it is imperative to recognize that the accounting of environmental emissions associated with the fashion and apparel industry, especially given its global dimensions in terms of both manufacturing plant locations and demand markets, include emissions generated in the transportation and distribution of the products across oceans and vast tracts of land. For example, H&M, a well-known fast fashion company, is cognizant of the environmental impact of even the fuels used in the transportation of its fashion products as well as the number of shipments needed for distribution. According to the Guardian (2010), H&M has identified that 51% of its carbon imprint in 2009 was due to transportation. In order to reduce the associated emissions, it began more direct shipments that avoided intermediate warehouses, decreased the volumes shipped by ocean and air by 40% and increased the volume of products shipped by rail, resulting in an over 700 ton decrease in the amount of carbo! n dioxide emitted.

In this paper, we develop an oligopoly model for fashion supply chain competition which explicitly considers different brands and different degrees of environmental consciousness and sustainability. The network-based model, which is formulated and studied as a variational inequality problem, captures competition among the firms in manufacturing, transportation/distribution, and storage, and assumes that the firms seek not only to maximize their profits but also care, in an individual way, about the emissions that they generate. The supply chain network oligopoly model that we develop is novel, and fills major research gaps, by contributing to understanding in several dimensions: 1. it handles product differentiation through branding; 2. it explicitly allows for alternative modes of transportation for product distribution as well as the possibility of an option of direct shipment from manufacturing plants, and 3. it enables each fashion/apparel producing firm to individuall! y determine, by use of its individual concern through a weighting factor, its environmental impacts through the emissions that it generates not only in the manufacture of its product but throughout its supply chain, with the ultimate deliveries at the demand markets.

We now discuss some of the related literature. Nagurney and Woolley (2010) also developed multicriteria supply chain network models for sustainability but focused on cost-minimization and system-optimization, as opposed to profit maximization, and oligopolistic competition, as we do in this paper. They identified a synergy measure to evaluate potential cost and environmental synergies associated with firms that are involved in mergers and/or acquisitions. More recently, Nagurney and Nagurney (2010) proposed a multicriteria network design model for the sustainable engineering of supply chains that also focused on optimization and not on oligopolistic competition with profit maximization. Nagurney (2010a), on the other hand, developed a supply chain network design model in the case of oligopolistic competition but did not include environmental issues. Moreover, in the latter model it was assumed that the product that was being produced by multiple competing firms was homogene! ous. In the fashion/apparel industry, on the other hand, brands are distinct and consumers who purchase apparel and fashion products may be brand-sensitive due to status appeal, reputation, environmental awareness, etc. Hence, the model in this paper considers the fashion or apparel product manufactured by a given firm to be differentiated by brand from similar products produced by the other firms that it competes with. Our supply chain network oligopoly model is new from this perspective and also due to the inclusion of environmental emissions within a multicriteria, competitive decision-making framework.

Nagurney and Yu (2011) focused on multicriteria decision-making for fashion supply chain management with the minimization of cost (see also Nagurney (2010b)) and the minimization of time as relevant criteria and assumed that the demand for the fashion product was known at each demand market. The model in this paper, in contrast, assumes that the demand for the particular brand is elastic and not fixed and considers multiple, competing firms rather than a single firm. For an edited volume on fashion supply chain management, a relatively new area of application of rigorous tools, see Choi (2011), the focused journal special issue edited by Choi and Chen (2008), and the papers by Sen (2008) and Brun et al. (2008).

Hence, this paper builds on the existing literature in sustainable supply chain management, with a focus on system-wide issues and in an industry in which competition and brands are the reality. Indeed, as early as Beamon (1999), Sarkis (2003), Corbett and Kleindorfer (2003), Nagurney and Toyasaki (2003, 2005), Sheu, Chou, and Hu (2005), Kleindorfer, Singhal, and van Wassenhove (2005), Nagurney, Liu, and Woolley (2007), and Linton, Klassen, and Jayaraman (2007) it has been argued that sustainable supply chains are critical for the examination of operations and the environment, with sustainable fashion being a more recent topic in both research and practice (see e.g., de Brito, Carbone, and Meunier Blanquart (2008) and the references therein). Sustainable supply chains have arisen as a focus for special issues (see Piplani, Pujawan, and Ray (2008)) and have advanced to a degree that even policies to reduce emissions have been explored in rigorous frameworks (see Wu et al. (2! 006), Nagurney, Liu, and Woolley (2006), and Chaabane, Ramudhin, and Paquet (2010)). For a thorough survey of sustainable supply chain management until 2008, see Seuring and Muller (2008). Nevertheless, a general, rigorous modeling and computational framework that captures oligopolistic competition, brand differentiation, and environmental concerns, in a supply chain network setting has not, heretofore, been constructed.

This paper is organized as follows. In Section 2, we develop the new sustainable fashion supply chain network oligopoly model with brand differentiation and provide some qualitative properties. In Section 3, we present the computational procedure which we then apply in Section 4 to compute solutions to a spectrum of numerical examples that comprise our case study. The case study illustrates both the generality of our framework and its applicability. We also provide managerial insights based on our computational case study. In Section 5, we summarize our findings, discuss the many directions that future research can take, and present our conclusions.

## 2. The Sustainable Fashion Supply Chain Network Oligopoly Model

We consider a finite number of $I$ fashion firms, with a typical firm denoted by $i$, who are involved in the production, storage, and distribution of a fashion product and who compete noncooperatively in an oligopolistic manner. Each firm corresponds to an individual brand representing the product that it produces.

Each fashion firm is represented as a network of its economic activities (cf. Figure 1). Each fashion firm seeks to determine its optimal product quantities by using Figure 1 as a schematic. Each fashion firm $i$; $i = 1, \ldots, I$, hence, is considering $n^i_M$ manufacturing facilities/plants; $n^i_D$ distribution centers, and serves the same $n_R$ demand markets. Let $L^i$ denote the set of directed links representing the economic activities associated with firm $i$; $i = 1, \ldots, I$. Let $G = [N, L]$ denote the graph consisting of the set of nodes $N$ and the set of links $L$ in Figure 1, where $L \equiv \cup_{i=1,\ldots,I} L^i$.

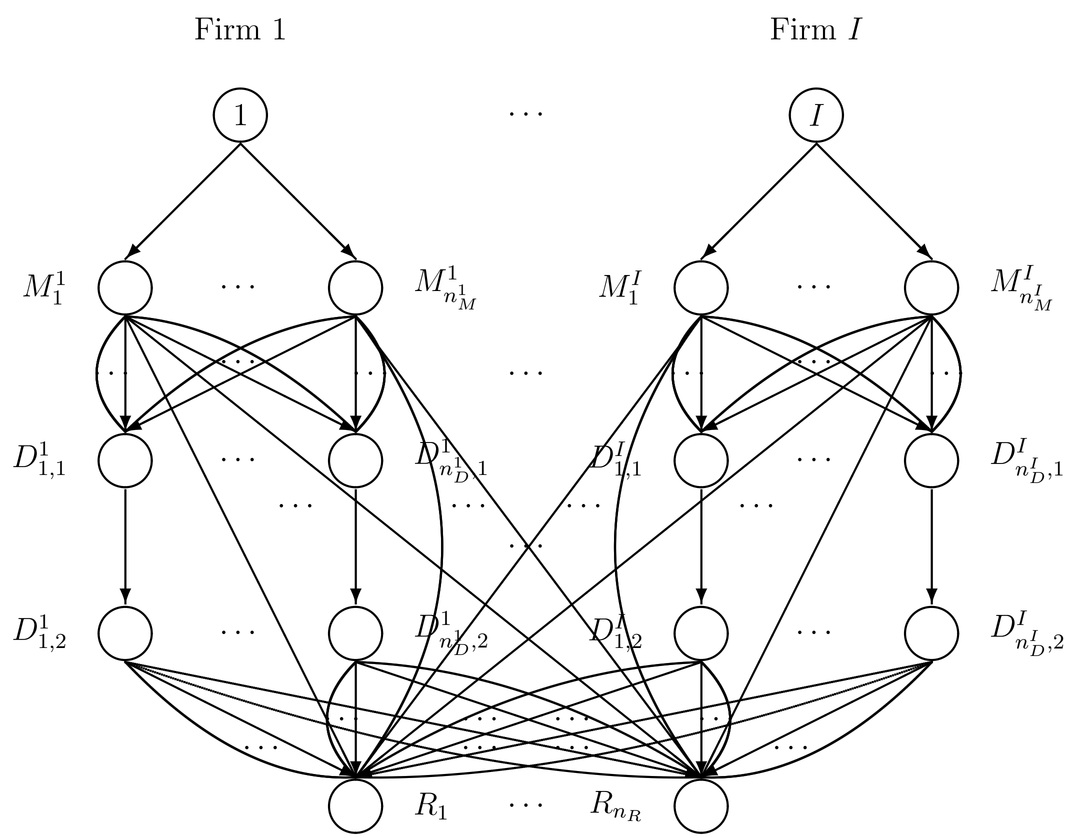


Figure 1: The Fashion Supply Chain Network Topology of the Oligopoly

The links from the top-tiered nodes $i$; $i = 1, \ldots, I$, representing the respective fashion firm, in Figure 1 are connected to the manufacturing nodes of the respective firm $i$, which are denoted, respectively, by: $M^i_1, \ldots, M^i_{n^i_M}$, and these links represent the manufacturing links.

The links from the manufacturing nodes, in turn, are connected to the distribution center nodes of each fashion firm $i$; $i = 1, \ldots, I$, which are denoted by $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$. These links correspond to the shipment links between the manufacturing plants and the distribution centers where the product is stored. Observe that there are alternative shipment links to denote different possible modes of transportation (which would also have associated with them different levels of emissions). Different modes of transportation may include: rail, air, truck, sea, as appropriate.

The links joining nodes $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$ with nodes $D^i_{1,2}, \ldots, D^i_{n^i_D,2}$ for $i = 1, \ldots, I$ correspond to the storage links. Finally, there are possible shipment links joining the nodes $D^i_{1,2}, \ldots, D^i_{n^i_D,2}$ for $i = 1, \ldots, I$ with the demand market nodes: $R_1, \ldots, R_{n_R}$. Here we also allow for multiple modes of transportation, as depicted using multiple arcs in Figure 1.

In addition, in order to represent another possible option, as was noted for H&M in the Introduction, we allow for the possibility that a firm may wish to have the product transported directly from a manufacturing plant to a demand market, and avail itself of one or more transportation shipment modes.

We emphasize that the network topology in Figure 1 is only representative, for definiteness. In fact, the model can handle any prospective supply chain network topology provided that there is a top-tiered node to represent each firm and bottom-tiered nodes to represent the demand markets with a sequence of directed links, corresponding to at least one path, joining each top-tiered node with each bottom-tiered node. Hence, different supply chain network topologies to that depicted in Figure 1 correspond to distinct fashion supply chain network problems.

Let $d_{ik}$ denote the demand for fashion firm $i$'s product; $i = 1, \ldots, I$, at demand market $k$; $k = 1, \ldots, n_R$. Note that in our model, we consider the general situation where the products of all these fashion firms are not homogeneous but are differentiated by *brand*. For more details on the theory of product differentiation, see Beath and Katsoulacos (1991), Shy (1996), and Carlton and Perloff (2004).

Let $x_p$ denote the nonnegative flow on path $p$ joining (origin) node $i$; $i = 1, \ldots, I$ with a (destination) demand market node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^i_k} x_p = d_{ik}, \quad k = 1, \ldots, n_R;\ i = 1, \ldots, I, \tag{1}$$

where $P^i_k$ denotes the set of all paths joining the origin node $i$; $i = 1, \ldots, I$ with destination node $R_k$, and $P \equiv \cup_{i=1,I} \cup_{k=1,n_R} P^i_k$, denotes the set of all paths in Figure 1. According to (1), the demand for fashion firm $i$'s product at demand point $k$ must be equal to the sum of the product flows from firm $i$ to that demand market. We group the demands $d_{ik}$; $k = 1, \ldots, n_R$; $i = 1, \ldots, I$ into the $n_R \times I$-dimensional vector $d$.

Here, for the sake of generality, we refer to the bottom-tiered nodes in Figure 1 as demand markets. Of course, they may correspond to retailers.

We assume that there is a demand price function (sometimes also referred to as the inverse demand function) associated with each fashion firm's product at each demand market. We denote the demand price of fashion firm $i$'s product at demand market $R_k$ by $\rho_{ik}$ and we assume, as given, the demand price functions:

$$\rho_{ik} = \rho_{ik}(d), \quad k = 1, \ldots, n_R; \, i = 1, \ldots, I, \tag{2}$$

that is, the price for fashion firm $i$'s product at a particular demand market may depend upon not only the demands for this fashion product at the other demand markets, but also on the demands for the other substitutable fashion products at all the demand points. Hence, (2) captures competition on the demand side of the competitive fashion supply chain network. Such demand price functions are of the form utilized in the study of differentiated oligopolies (cf. Singh and Vives (1984), Häckner (2000), Shy (1996), and Carlton and Perloff (2004)) but are not limited to being linear, as is commonly assumed in economics. Also, we do not limit ourselves to a single demand market since the firms compete in multiple demand markets Recall that (2) reflects the price that consumers at demand market $k$ are willing to pay for the brand produced by firm $i$. We assume that the demand price functions are continuous, continuously differentiable and monotone decreasing.

In addition, let $f_a$ denote the flow on link $a$. We must have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{3}$$

where $\delta_{ap} = 1$ if link $a$ is contained in path $p$ and $\delta_{ap} = 0$, otherwise. In other words, the flow on a link is equal to the sum of flows on paths that contain that link.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{4}$$

Observe that, since the firms share no links, we do not need to distinguish with superscripts the individual firm path and link flows. We group the path flows into the vector $x \in R_+^{n_P}$. We assume that all vectors are column vectors.

The total operational cost on a link, be it a manufacturing/production link, a shipment/distribution link, or a storage link is assumed, in general, to be a function of the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{5}$$

where $f$ is the vector of all the link flows. The above total cost expressions capture competition among the firms for resources used in manufacturing, transportation, and storage of their fashion products. We assume that the total cost on each link is convex and is continuously differentiable.

Let $X_i$ denote the vector of strategy variables associated with firm $i$; $i = 1, \ldots, I$, where $X_i$ is the vector of path flows associated with firm $i$, that is, $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, where $P^i \equiv \cup_{k=1,\ldots,n_R} P_k^i$, and $n_{P^i}$ denotes the number of paths from firm $i$ to the demand markets. $X$ is then the vector of all the firms' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

The profit function $\pi_i$ of firm $i$; $i = 1, \ldots, I$, is the difference between the firm's revenue and its total costs, that is,

$$\pi_i = \sum_{k=1}^{n_R} \rho_{ik}(d) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \hat{c}_a(f). \tag{6}$$

In addition, all the fashion firms are concerned with their environmental impacts along their supply chains, but, possibly, to different degrees. The emission-generation function associated with link $a$, denoted by $\hat{e}_a$, is assumed to be a function of the product flow on that link, that is,

$$\hat{e}_a = \hat{e}_a(f_a), \quad \forall a \in L. \tag{7}$$

These functions are assumed to be convex and continuously differentiable.

Each fashion firm aims to minimize the total amount of emissions generated in the manufacture, storage, and shipment of its product. Hence, the other objective of firm $i$; $i = 1, \ldots, I$, is given by:

$$\text{Minimize} \quad \sum_{a \in L^i} \hat{e}_a(f_a). \tag{8}$$

We can now construct a weighted function, which we refer to as the utility function (cf. Fishburn (1970), Chankong and Haimes (1983), Yu (1985), Keeney and Raiffa (1992), Nagurney and Dong (2002)), associated with the two criteria faced by each firm. The term $\omega_i$ is assumed to be the price that firm $i$ would be willing to pay for each unit of emission on each of its links. This term, hence, represents the environmental concern of firm $i$, with a higher $\omega_i$ denoting a greater concern for the environment. Consequently, the multicriteria decision-making problem faced by fashion firm $i$; $i = 1, \ldots, I$, is:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \hat{c}_a(f) - \omega_i \sum_{a \in L^i} \hat{e}_a(f_a). \tag{9}$$

In view of (1)-(9), we may write:

$$U = U(X), \tag{10}$$

where $U$ is the $I$-dimensional vector of all the firms' utilities.

We now consider the usual oligopolistic market mechanism in which the $I$ firms select their product path flows (which correspond to quantity decision variables in the Cournot oligopoly framework) in a noncooperative manner, each one trying to maximize its own utility. We seek to determine a path flow pattern $X^*$ for which the $I$ firms will be in a state of equilibrium as defined below.

**Definition 1: Supply Chain Network Cournot-Nash Equilibrium**

*A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm $i$; $i = 1, \ldots, I$:*

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{11}$$

*where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.*

Note that, according to (11), an equilibrium is established if no firm can individually improve its utility, by changing its production path flows, given the production path flow decisions of the other firms.

The variational inequality formulations of the Cournot-Nash (Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980)) sustainable fashion supply chain network problem satisfying Definition 1, in both path flows and link flows, respectively, are given in the following theorem.

**Theorem 1**

*Assume that for each fashion firm $i$; $i = 1, \ldots, I$, the utility function $U_i(X)$ is concave with respect to the variables in $X_i$, and is continuously differentiable. Then $X^* \in K$ is a sustainable fashion supply chain network Cournot-Nash equilibrium according to Definition*

*1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{I}\langle \nabla_{X_i}U_i(X^*)^T, X_i - X_i^*\rangle \geq 0, \quad \forall X \in K, \tag{12}$$

*where $\langle \cdot,\cdot\rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i}U_i(X)$ denotes the gradient of $U_i(X)$ with respect to $X_i$. The solution of variational inequality (12) is equivalent to the solution of the variational inequality: determine $x^* \in K^1$ satisfying:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \omega_i\frac{\partial \hat{E}_p(x^*)}{\partial x_p} - \rho_{ik}(x^*) - \sum_{l=1}^{n_R}\frac{\partial \rho_{il}(x^*)}{\partial d_{ik}}\sum_{p\in P_l^i} x_p^*\right]\times[x_p - x_p^*] \geq 0,\ \forall x \in K^1, \tag{13}$$

*where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, $\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b\in L^i}\sum_{a\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}$ and $\frac{\partial \hat{E}_p(x)}{\partial x_p} \equiv \sum_{a\in L^i}\frac{\partial \hat{e}_a(f_a)}{\partial f_a}\delta_{ap}$.*

*In addition, (13) can be re-expressed in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, such that:*

$$\sum_{i=1}^{I}\sum_{a\in L^i}\left[\sum_{b\in L^i}\frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \omega_i\frac{\partial \hat{e}_a(f_a^*)}{\partial f_a}\right]\times[f_a - f_a^*]$$
$$+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d^*)}{\partial d_{ik}}d_{il}^*\right]\times[d_{ik} - d_{ik}^*] \geq 0, \quad \forall (f,d) \in K^2, \tag{14}$$

*where $K^2 \equiv \{(f,d) | \exists x \geq 0,\ and\,(1),(3),\ and\,(4)\,hold\}$.*

**Proof:** Variational inequality (12) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). Observe now that

$$\nabla_{X_i}U_i(X) = \left[\frac{\partial U_i}{\partial x_p}; p \in P_k^i; k = 1,\ldots,n_R\right], \tag{15}$$

where for each path $p$; $p \in P_k^i$,

$$\frac{\partial U_i}{\partial x_p} = \frac{\partial\left[\sum_{l=1}^{n_R}\rho_{il}(d)\sum_{p\in P_l^i}x_p - \sum_{b\in L^i}\hat{c}_b(f) - \omega_i\sum_{b\in L^i}\hat{e}_b(f_b)\right]}{\partial x_p}$$

$$= \sum_{l=1}^{n_R}\frac{\partial\left[\rho_{il}(d)\sum_{p\in P_l^i}x_p\right]}{\partial x_p} - \frac{\partial\left[\sum_{b\in L^i}\hat{c}_b(f)\right]}{\partial x_p} - \omega_i\frac{\partial\left[\sum_{b\in L^i}\hat{e}_b(f_b)\right]}{\partial x_p}$$

$$= \rho_{ik}(d) + \sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d)}{\partial d_{ik}}\frac{\partial d_{ik}}{x_p}\sum_{p\in P_l^i}x_p - \sum_{a\in L^i}\frac{\partial\left[\sum_{b\in L^i}\hat{c}_b(f)\right]}{\partial f_a}\frac{\partial f_a}{\partial x_p} - \omega_i\sum_{a\in L^i}\frac{\partial\left[\sum_{b\in L^i}\hat{e}_b(f_b)\right]}{\partial f_a}\frac{\partial f_a}{\partial x_p}$$

$$= \rho_{ik}(d) + \sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d)}{\partial d_{ik}}\sum_{p\in P_l^i}x_p - \sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap} - \omega_i\sum_{a\in L^i}\frac{\partial \hat{e}_a(f_a)}{\partial f_a}\delta_{ap}. \tag{16}$$

The demand price functions (2) can be re-expressed in light of (1) as functions of path flows. By making use then of the definitions of $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ and $\frac{\partial \hat{E}_p(x)}{\partial x_p}$ above, variational inequality (13) is immediate. In addition, the equivalence between variational inequalities (13) and (14) can be proved with (1) and (3).□

Variational inequalities (13) and (14) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $n$-dimensional Euclidean space. Indeed, if we define the column vectors: $X \equiv x$ and

$$F(X) \equiv \Big[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \omega_i \frac{\partial \hat{E}_p(x)}{\partial x_p} - \rho_{ik}(x) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x)}{\partial d_{ik}} \sum_{p \in P_l^i} x_p;$$
$$p \in P_k^i;\ k = 1, \ldots, n_R;\ i = 1, \ldots, I\Big], \tag{18}$$

and $\mathcal{K} \equiv K^1$ then (13) can be re-expressed as (17). If we define the column vectors: $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$, such that:

$$F_1(X) = \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} + \omega_i \frac{\partial \hat{e}_a(f_a)}{\partial f_a};\ a \in L^i;\ i = 1, \ldots, I\right],$$
$$F_2(X) = \left[-\rho_{ik}(d) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d)}{\partial d_{ik}} d_{il};\ k = 1, \ldots, n_R;\ i = 1, \ldots, I\right], \tag{19}$$

and $\mathcal{K} \equiv K^2$ then (14) can be re-expressed as (17).

For completeness, we provide some qualitative properties of the solution to variational inequalities (13) and (14). In particular, we derive existence and uniqueness results.

Since the feasible set $K^1$ is not compact we cannot obtain existence of a solution simply from the assumption of the continuity of $F$. However, we can impose a rather weak condition to guarantee existence of a solution pattern. Considering that the demand prices should be nonnegative, the demand $d_{ik}$ for fashion firm $i$'s product; $i = 1, \ldots, I$, at every demand market $k$; $k = 1, \ldots, n_R$, is actually bounded. Consequently, in light of (1), we have that:

$$\mathcal{K}_b \equiv \{x | 0 \leq x \leq b\}, \tag{20}$$

where $b > 0$ and $x \leq b$ means that $x_p \leq b$ for all $p \in P_k^i$; $k = 1, \ldots, n_R$ and $i = 1, \ldots, I$. Then $\mathcal{K}_b$ is a bounded, closed, and convex subset of $R_+^{n_P}$. Thus, the following variational equality

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \tag{21}$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and $F$ is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we have the following theorem:

**Theorem 2: Existence**

*There exists at least one Nash Equilibrium, equivalently, at least one solution to variational inequality (13) (equivalently, (14)), since in the light of the demand price functions (2), there exists a $b > 0$, such that variational inequality (21) admits a solution in $\mathcal{K}_b$ with*

$$x^b \leq b. \tag{22}$$

In addition, we now provide a uniqueness result.

**Theorem 3: Uniqueness**

*With Theorem 2, variational inequality (21) and, hence, variational inequality (14) admits at least one solution. Moreover, if the function $F(X)$ of variational inequality (14), as defined in (19), is strictly monotone on $\mathcal{K} \equiv K^2$, that is,*

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K},\ X^1 \neq X^2. \tag{23}$$

*then the solution to variational inequality (14) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.*

Theorem 2 provides a condition, which is reasonable in practice, that will guarantee that, for each firm, the quantities manufactured at each plant, stored at each distribution center, and shipped, via each mode of transportation, will be unique. For other applications to supply chain problems in which strict monotonicity is assumed, see Nagurney (2006) and the references therein.

## 3. The Algorithm

In this Section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of sustainable fashion supply chain network oligopoly models governed by variational inequality (13) yields subproblems that can be solved explicitly and in closed form.

Specifically, recall that at an iteration $\tau$ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{24}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and $F$ is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{25}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in $n$-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an $n$-dimensional function from $\mathcal{K}$ to $R^n$, with $F(X)$ being continuous (see also (17)).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein. Applications to the solution of network oligopolies can be found in Nagurney, Dupuis and Zhang (1994) and Nagurney (2010a).

### Explicit Formulae for the Euler Method Applied to the Sustainable Fashion Supply Chain Network Oligopoly Variational Inequality (13)

The elegance of this procedure for the computation of solutions to the sustainable fashion supply chain network oligopoly problem modeled in Section 2 can be seen in the following explicit formulae. In particular, (24) for the sustainable fashion supply chain network oligopoly model governed by variational inequality problem (13) yields the following closed form expressions for the fashion product path flows:

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\rho_{ik}(x^{\tau}) + \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x^{\tau})}{\partial d_{ik}} \sum_{p \in P_l^i} x_p^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p} - \omega_i \frac{\partial \hat{E}_p(x^{\tau})}{\partial x_p})\}, \; \forall p \in P_k^i, \forall k, \forall i. \tag{26}$$

In the next Section, we solve sustainable fashion supply chain network oligopoly problems using the above algorithmic scheme.

## 4. Case Study with Managerial Insights

In this Section, we present a case study in which we solve sustainable fashion supply chain management problems under oligopolistic competition and brand differentiation numerically. In our case study there are two fashion firms, Firm 1 and Firm 2, each of which is involved in the production, storage, and distribution of a single fashion product, which is differentiated by its brand. Each firm has, at its disposal, two manufacturing plants, two distribution centers, and serves a single demand market. Hence, the topology is as depicted in Figure 2. The manufacturing plants $M_1^1$ and $M_1^2$ are located in the United States, whereas the manufacturing plants $M_2^1$ and $M_2^2$ are located off-shore with lower operational costs. However, the demand market is in the United States as are the distribution centers.

The case study consists of three problem sets of examples.

For the computation of solutions to the numerical examples, we implemented the Euler method, as discussed in Section 3, using Matlab. The convergence tolerance was $\epsilon = 10^{-6}$, and the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We considered the algorithm to have converged (cf. (26)) when the absolute value of the difference between successive path flows differed by no more than the above $\epsilon$. We initialized the algorithm by setting the demand of each fashion firm's product at 10 and equally distributed the demand among all the paths for each firm.

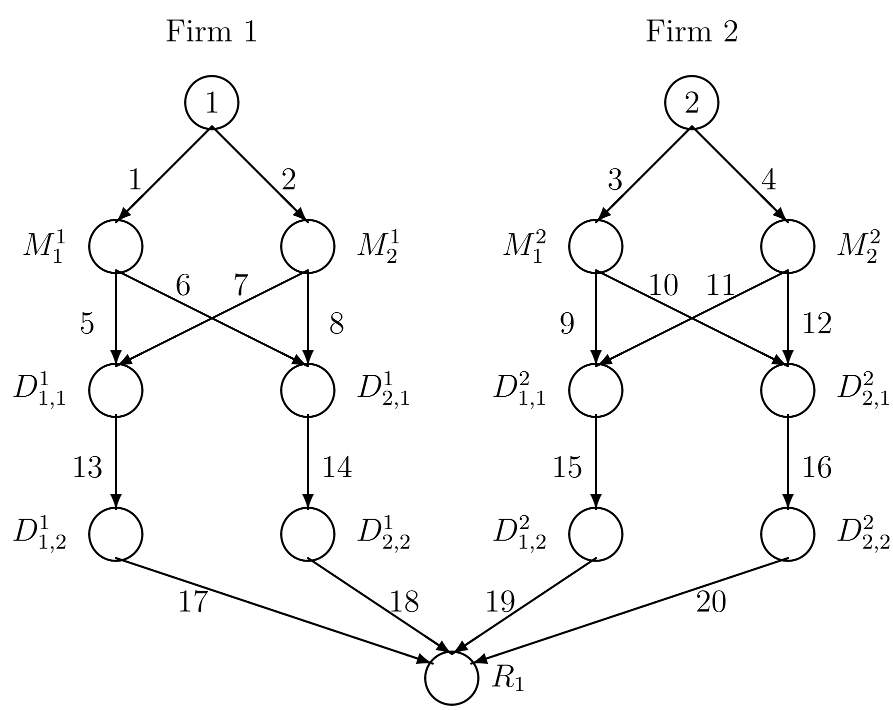


Figure 2: The Fashion Supply Chain Network Topology for the Case Study

Table 1: Total Cost and Total Emission Functions with Link Flow Solution for Example 1

| Link $a$ | $\hat{c}_a(f)$ | $\hat{e}_a(f_a)$ | $f_a^*$ |
|---|---|---|---|
| 1 | $10f_1^2 + 10f_1$ | $.05f_1^2 + .5f_1$ | 6.09 |
| 2 | $f_2^2 + 7f_2$ | $.1f_2^2 + .8f_2$ | 19.94 |
| 3 | $10f_3^2 + 7f_3$ | $.1f_3^2 + f_3$ | 4.83 |
| 4 | $f_4^2 + 5f_4$ | $.15f_4^2 + 1.2f_4$ | 19.93 |
| 5 | $f_5^2 + 4f_5$ | $.08f_5^2 + f_5$ | 2.85 |
| 6 | $f_6^2 + 6f_6$ | $.1f_6^2 + f_6$ | 3.24 |
| 7 | $2f_7^2 + 30f_7$ | $.15f_7^2 + 1.2f_7$ | 8.63 |
| 8 | $2f_8^2 + 20f_8$ | $.15f_8^2 + f_8$ | 11.31 |
| 9 | $f_9^2 + 3f_9$ | $.25f_9^2 + f_9$ | 3.67 |
| 10 | $f_{10}^2 + 4f_{10}$ | $.25f_{10}^2 + 2f_{10}$ | 1.17 |
| 11 | $1.5f_{11}^2 + 30f_{11}$ | $.4f_{11}^2 + 1.5f_{11}$ | 9.28 |
| 12 | $1.5f_{12}^2 + 20f_{12}$ | $.45f_{12}^2 + f_{12}$ | 10.64 |
| 13 | $f_{13}^2 + 3f_{13}$ | $.01f_{13}^2 + .1f_{13}$ | 11.48 |
| 14 | $f_{14}^2 + 2f_{14}$ | $.01f_{14}^2 + .15f_{14}$ | 14.55 |
| 15 | $f_{15}^2 + 1.8f_{15}$ | $.05f_{15}^2 + .3f_{15}$ | 12.95 |
| 16 | $f_{16}^2 + 1.5f_{16}$ | $.08f_{16}^2 + .5f_{16}$ | 11.81 |
| 17 | $2f_{17}^2 + f_{17}$ | $.08f_{17}^2 + f_{17}$ | 11.48 |
| 18 | $f_{18}^2 + 4f_{18}$ | $.1f_{18}^2 + .8f_{18}$ | 14.55 |
| 19 | $f_{19}^2 + 5f_{19}$ | $.3f_{19}^2 + 1.2f_{19}$ | 12.95 |
| 20 | $1.5f_{20}^2 + f_{20}$ | $.35f_{20}^2 + 1.2f_{20}$ | 11.81 |

**Problem Set 1**

In the first set of examples, fashion Firm 1 cares about the emissions that it generates much more than Firm 2 does, which is indicated by the respective values of $\omega_1$ and $\omega_2$, where $\omega_1 = 5$ and $\omega_2 = 1$. In addition, Firm 1 utilizes more advanced technologies in its supply chain activities in order to lower the emissions that it generates, but at relatively higher costs. The total cost and the total emission functions for all the links are given in Table 1.

**Example 1**

At the demand market $R_1$, the consumers reveal their preferences for the product of Firm 1, through the demand functions, with the demand price functions for the two fashion products being given by:

$$\rho_{11}(d) = -d_{11} - .2d_{21} + 300, \quad \rho_{21}(d) = -2d_{21} - .5d_{11} + 300.$$

The computed equilibrium link flows are reported in Table 1. For completeness, we also provide the computed equilibrium path flows. There were four paths for each firm and we label the paths as follows (please refer to Figure 2): for Firm 1:

$$p_1 = (1, 5, 13, 17), \quad p_2 = (1, 6, 14, 18), \quad p_3 = (2, 7, 13, 17), \quad p_4 = (2, 8, 14, 18);$$

and for Firm 2:

$$p_5 = (3, 9, 15, 19), \quad p_6 = (3, 10, 16, 20), \quad p_7 = (4, 11, 15, 19), \quad p_8 = (4, 12, 16, 20).$$

The computed equilibrium path flow pattern was:

$$x^*_{p_1} = 2.85, \quad x^*_{p_2} = 3.24, \quad x^*_{p_3} = 8.63, \quad x^*_{p_4} = 11.31,$$

$$x^*_{p_5} = 3.67, \quad x^*_{p_6} = 1.17, \quad x^*_{p_7} = 9.28, \quad x^*_{p_8} = 10.64.$$

Therefore, the demand for the Firm 1's product was 26.03 and the price was 269.02, while the demand for Firm 2's product was 24.76 and the price was 237.47.

The total cost for Firm 1 was: $2,860.11$; the total emissions that it generated: 182.03, and its revenue was: $7,002.35$ yielding a profit of $4,142.25$. The total cost for Firm 2 was: $2,386.61$; its total emissions: 368.50, and its revenue: $5,879.36$ yielding a profit of $3,492.75$. The utilities (cf. (9)) for Firm 1 and for Firm 2 were: $3,232.11$ and $3,124.25$, respectively.

**Example 2**

Example 2 had the identical data to that of Example 1 except that the consumers were more price-sensitive with respect to fashion Firm 2's product. The demand price function associated with Firm 2's product was now:

$$\rho_{21}(d) = -3d_{21} - .5d_{11} + 300.$$

**Example 3**

Example 3 had the same data as Example 1 but now the consumers were even more price-sensitive with respect to fashion Firm 2's product, with the demand price function for Firm 2's product now given by:

$$\rho_{21}(d) = -4d_{21} - .5d_{11} + 300.$$

**Example 4**

Example 4 had the identical data as Example 1 except that the demand price function associated with fashion Firm 2's product was now:

$$\rho_{21}(d) = -5d_{21} - .5d_{11} + 300.$$

Table 2: Computed Equilibrium Demands, Prices, Profits, Total Emissions, and Utilities for Examples 1, 2, 3, and 4

| | Example 1 | Example 2 | Example 3 | Example 4 |
|---|---|---|---|---|
| The demand for Firm 1's product | 26.03 | 26.12 | 26.18 | 26.22 |
| The demand for Firm 2's product | 24.76 | 20.68 | 17.75 | 15.55 |
| The price of Firm 1's product | 269.02 | 269.75 | 270.27 | 270.67 |
| The price of Firm 2's product | 237.47 | 224.91 | 215.91 | 209.14 |
| The profit of Firm 1 | 4,142.25 | 4,168.39 | 4,187.19 | 4,201.36 |
| The profit of Firm 2 | 3,492.75 | 2,881.09 | 2,451.92 | 2,134.32 |
| The emissions of Firm 1 | 182.03 | 183.00 | 183.70 | 184.23 |
| The emissions of Firm 2 | 368.50 | 269.75 | 208.22 | 167.04 |
| The utility of Firm 1 | 3,232.11 | 3,253.39 | 3,268.70 | 3,280.23 |
| The utility of Firm 2 | 3,124.25 | 2,611.34 | 2,243.70 | 1,967.27 |

The computed equilibrium demands, prices, profits, emissions, and utilities for Examples 1, 2, 3, and 4 are reported in Table 2.

We now provide some managerial insights from these examples. Note that the changes in the demand price function for fashion Firm 2's product (cf. Example 1 through Example 4) indicate that the consumers are becoming more price-sensitive with respect to fashion Firm 2's product. With the consumers' increasing environmental concerns, the demand for fashion Firm 2's product decreases significantly, since fashion Firm 2 does not have as good of a reputation in terms of environmental sustainability as Firm 1 does. In addition, the profit of fashion Firm 2 drops dramatically. The total emissions of Firm 1 increase slightly, whereas those of Firm 2 decrease substantially from Example 1 through Example 4.

**Problem Set 2**

Problem Set 2 also consisted of 4 examples. In this set of examples, we assumed that Firm 2 was now more environmentally conscious and raised $\omega_2$ from 1 to 5. Hence, in this set of examples, Firm 1 and Firm 2 both had their $\omega$ weights equal to 5. Examples 5 through 8 had their data identical to the data in Examples 1 through 4, respectively, except for the larger value of $\omega_2$.

The computed equilibrium demands, prices, profits, emissions, and utilities for Examples 5, 6, 7, and 8 are reported in Table 3.

Interestingly, the weights, the $\omega_i$s, may also be interpreted as *taxes* in that a governmental authority may impose a tax associated with carbon emissions, for example, that each firm

Table 3: Computed Equilibrium Demands, Prices, Profits, Total Emissions, and Utilities for Examples 5, 6, 7, and 8

| | Example 5 | Example 6 | Example 7 | Example 8 |
|---|---|---|---|---|
| The demand for Firm 1's product | 26.18 | 26.23 | 26.27 | 26.30 |
| The demand for Firm 2's product | 17.35 | 15.13 | 13.41 | 12.04 |
| The price of Firm 1's product | 270.34 | 270.74 | 271.05 | 271.30 |
| The price of Firm 2's product | 252.20 | 241.50 | 233.23 | 226.65 |
| The profit of Firm 1 | $4,189.75$ | $4,204.08$ | $4,215.16$ | $4,224.00$ |
| The profit of Firm 2 | $3,019.58$ | $2,563.82$ | $2,225.83$ | $1,965.67$ |
| The emissions of Firm 1 | 183.79 | 184.33 | 184.74 | 185.07 |
| The emissions of Firm 2 | 191.27 | 152.71 | 125.73 | 106.03 |
| The utility of Firm 1 | $3,270.78$ | $3,282.44$ | $3,291.47$ | $3,298.66$ |
| The utility of Firm 2 | $2,063.20$ | $1,800.29$ | $1,597.17$ | $1,435.52$ |

must pay. Hence, this set of examples in which the $\omega_i$ terms are equal for both firms with a value of 5 reflects also this scenario (see, e.g., Dhanda, Nagurney, and Ramanujam (1999)).

Comparing the results of Examples 5, 6, 7, and 8 with the results for Examples 1, 2, 3, and 4, respectively, we observe that, as expected, Firm 2 now emits a significantly lower amount of emissions for each demand price function that it is faced with whereas Firm 1 now emits, in each example in this problem set, a slightly higher amount than it emitted in the corresponding example in the first problem set.

Nevertheless, an increase in environmental concerns is not sufficient for fashion Firm 2 to attract more demand and to increase its profits, since it has not modified its pollution-abatement technologies and, indeed, its total cost functions and total emission functions remain as in Problem Set 1. Such information is clearly useful to managers and our theoretical and computational framework allows managers to conduct sensitivity analysis to investigate the effects on profits and emissions of changes in the data.

**Problem Set 3**

In our third and final problem set we varied both the total cost functions and the total emission functions of Firm 2, and also investigated the situation that the firms faced identical functions throughout the supply chain.

**Example 9**

Example 9 had the identical data to that in Example 5 except that fashion Firm 2 now

Table 4: Total Cost and Total Emission Functions for Example 10

| Link $a$ | $\hat{c}_a(f)$ | $\hat{e}_a(f_a)$ |
|---|---|---|
| 1 | $10f_1^2 + 10f_1$ | $.05f_1^2 + .5f_1$ |
| 2 | $f_2^2 + 7f_2$ | $.1f_2^2 + .8f_2$ |
| 3 | $10f_3^2 + 10f_3$ | $.05f_3^2 + .5f_3$ |
| 4 | $f_4^2 + 7f_4$ | $.1f_4^2 + .8f_4$ |
| 5 | $f_5^2 + 4f_5$ | $.08f_5^2 + f_5$ |
| 6 | $f_6^2 + 6f_6$ | $.1f_6^2 + f_6$ |
| 7 | $2f_7^2 + 30f_7$ | $.15f_7^2 + 1.2f_7$ |
| 8 | $2f_8^2 + 20f_8$ | $.15f_8^2 + f_8$ |
| 9 | $f_9^2 + 4f_9$ | $.08f_9^2 + f_9$ |
| 10 | $f_{10}^2 + 6f_{10}$ | $.1f_{10}^2 + f_{10}$ |
| 11 | $2f_{11}^2 + 30f_{11}$ | $.15f_{11}^2 + 1.2f_{11}$ |
| 12 | $2f_{12}^2 + 20f_{12}$ | $.15f_{12}^2 + f_{12}$ |
| 13 | $f_{13}^2 + 3f_{13}$ | $.01f_{13}^2 + .1f_{13}$ |
| 14 | $f_{14}^2 + 2f_{14}$ | $.01f_{14}^2 + .15f_{14}$ |
| 15 | $f_{15}^2 + 3f_{15}$ | $.01f_{15}^2 + .1f_{15}$ |
| 16 | $f_{16}^2 + 2f_{16}$ | $.01f_{16}^2 + .15f_{16}$ |
| 17 | $2f_{17}^2 + f_{17}$ | $.08f_{17}^2 + f_{17}$ |
| 18 | $f_{18}^2 + 4f_{18}$ | $.1f_{18}^2 + .8f_{18}$ |
| 19 | $2f_{19}^2 + f_{19}$ | $.08f_{19}^2 + f_{19}$ |
| 20 | $f_{20}^2 + 4f_{20}$ | $.1f_{20}^2 + .8f_{20}$ |

acquired more expensive advanced emission-reducing manufacturing technologies, resulting in new total cost and emission functions associated with the manufacturing links, as below:

$$\hat{c}_3(f) = 10f_3^2 + 10f_3, \quad \hat{c}_4(f) = f_4^2 + 7f_4,$$

$$\hat{e}_3(f_3) = .05f_3^2 + .5f_3, \quad \hat{e}_4(f_4) = .1f_4^2 + .8f_4.$$

**Example 10**

Example 10 had the same data as Example 9 but now fashion Firm 2 made even a greater effort to lower its emissions, not only focusing on its manufacturing processes, but also on all other supply chain activities. The total cost and the total emission functions for all the links are provided in Table 4.

**Example 11**

Example 11 had the identical data as in Example 10 except that the effort made by fashion Firm 2 to protect the environment was now also disseminated to the consumers, leading to

Table 5: Computed Equilibrium Demands, Prices, Profits, Total Emissions, and Utilities for Examples 1, 5, 9, 10, and 11

| | Example 1 | Example 5 | Example 9 | Example 10 | Example 11 |
|---|---|---|---|---|---|
| The demand for Firm 1's product | 26.03 | 26.18 | 26.18 | 26.11 | 26.00 |
| The demand for Firm 2's product | 24.76 | 17.35 | 17.75 | 20.80 | 26.00 |
| The price of Firm 1's product | 269.02 | 270.34 | 270.27 | 269.73 | 268.80 |
| The price of Firm 2's product | 237.47 | 252.20 | 251.40 | 245.34 | 268.80 |
| The profit of Firm 1 | 4,142.25 | 4,189.75 | 4,187.18 | 4,167.60 | 4,134.29 |
| The profit of Firm 2 | 3,492.75 | 3,019.58 | 3,020.61 | 3,137.78 | 4,134.29 |
| The emissions of Firm 1 | 182.03 | 183.79 | 183.70 | 182.97 | 181.73 |
| The emissions of Firm 2 | 368.50 | 191.27 | 182.55 | 127.73 | 181.73 |
| The utility of Firm 1 | 3,232.11 | 3,270.78 | 3,268.68 | 3,252.75 | 3,225.64 |
| The utility of Firm 2 | 3,124.25 | 2,063.20 | 2,107.85 | 2,500.92 | 3,225.64 |

the change in the demand for Firm 2's product, with the new demand price function faced by Firm 2 given by:

$$\rho_{21}(d) = -d_{21} - .2d_{11} + 300.$$

Hence, in Example 11, Firm 1 and 2 were *identical*.

The computed equilibrium demands, prices, profits, emissions, and utilities for Examples 1, 5, 9, 10, and 11 are given in Table 5.

Based on the results for Examples 5, 9, and 10, the advanced manufacturing technologies utilized by fashion Firm 2 did improve its performance, but not significantly, while Firm 2's environmental efforts throughout its supply chain notably enhanced its profit and utility. Furthermore, and this is relevant also to managers, the change in consumers' attitudes towards Firm 2 can assist Firm 2 in obtaining as much profit as that of Firm 1. Observe that the profit of Firm 1 in Example 11, however, was not as high as what it achieved in Example 1, which means that if Firm 1 wishes to maintain its competitive advantage, it must pay continuing attention to its emissions. A comparison of the results in Example 10 and Example 11, in turn, suggests that the development of a positive image for a firm in terms of its environmental consciousness and concern may also be an effective marketing strategy for fashion firms.

The above case study demonstrates that consumers' environmental consciousness can be a valuable incentive to spur fashion companies to reexamine their supply chains so as to reduce their environmental pollution, which can, in turn, help such companies to obtain competitive advantages and increased profits.

## 5. Summary and Conclusions and Suggestions for Future Research

In this paper, we focused on the fashion and apparel industry, which presents unique challenges and opportunities in terms of environmental sustainability. We developed a competitive supply chain network model, using variational inequality theory, that captures oligopolistic competition with fashion product brand differentiation. The variational inequality model assumes that each firm seeks to maximize its profits and to minimize the emissions that it generates throughout its supply chain as it engages in its activities of manufacturing, storage, and distribution, with a weight associated with the latter criterion. The model allows for alternative modes of transportation from manufacturing sites to distribution centers and from distribution centers to the demand markets, since different modes of transportation are known to emit different amounts of emissions.

The variational inequality-based competitive supply chain network model advances the state-of-the-art of supply chain modeling in several ways: 1. it captures competition through brand differentiation, which is an important feature of the fashion industry; 2. it allows for each firm to individually weight its concern for the environment in its decision-making, and 3. alternatives such as multiple modes of transportation can be investigated.

In order to demonstrate the generality of the model and the proposed computational scheme, we presented a case study, in which, through a series of numerical examples, we demonstrated the effects of changes on the demand price functions; the total cost and total emission functions, as well as the weights associated with the environmental criterion on the equilibrium product demands, the product prices, profits, and utilities. We noted that the environmental weights could also be interpreted as taxes and, thus, in exploring different values an authority such as the government could assess a priori the effects on the firms' emissions and profits.

The case study also demonstrated that consumers can have a major impact, through their environmental consciousness, on the level of profits of firms in their favoring of firms that adopt environmental pollution-abatement technologies for their supply chain activities. The numerical examples in the case studies were selected for their transparency and for reproducibility purposes.

Future research may take several directions, including: the empirical application of our framework to a large-scale problem; the inclusion of multiple products produced by each firm, with the retainment of brand differentiation, and the incorporation of multiple pollutants.

We hope that the ideas and results in this paper can be used to enhance sustainable fashion supply chain management in both theory and practice.

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