System Partitioning and Improved Bond Graph Model Reduction Using Junction Structure Power Flow

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ABSTRACT

In recent work, the authors have proposed a technique to search quantitatively and systematically for decoupling among dynamic elements of a bond graph model, and to partition models in which decoupling is found. This paper demonstrates how the algorithm increases the scope and robustness of existing physical-domain model reduction techniques, monitors the validity of simplifying assumptions based on decoupling as the system or environment changes, and can improve computation time.

The algorithm is first reviewed, and then applied to a vehicle dynamics problem in which there exists one-way coupling between the longitudinal and pitch dynamics of a medium-sized truck on a smooth road. A subset of the longitudinal dynamic elements drives a partition of predominantly pitch-related elements, with the driven elements not affecting the longitudinal dynamics. After partitioning, each submodel is reduced separately using an existing reduction technique that is based on comparison of element contributions to dominant system dynamics. The resulting model accurately and efficiently predicts both forward speed and pitch angle. Elements are grouped in a more meaningful way for comparison, and greater simplification is possible when the partitioned (as opposed to the fully-coupled) model is reduced. Road roughness is then introduced, and the algorithm quantitatively predicts that assuming one-way coupling between the partitions is no longer valid due to increased pitch motion.

INTRODUCTION AND BACKGROUND

Determining models of appropriate complexity for simulation-based design is as important as ever, despite continued advances in computing power that allow use of increasingly large, high-fidelity models. Determination of the appropriate model should be done in a systematic, quantitative matter, in contrast to *ad hoc* model construction methods based on intuition, experience, or the reassuring notion that greater complexity leads to greater accuracy. The *ad hoc* approach can lead to models that sacrifice accuracy for computation time (or vice-versa) to such an extent that the simulation-based design process becomes inaccurate or time-prohibitive. Wilson and Stein (1995) were thus motivated to define the term "proper modeling" as the systematic formulation of a model of minimum complexity that predicted dynamic system response with sufficient accuracy, and with parameters that retained physical meaning. Subsequent research led to algorithms for deducing a proper model by incrementing model complexity until accuracy criteria were met, for example [Wilson and Stein, 1995; Ferris *et al*., 1998; Walker *et al*., 2000].

While the deduction techniques were frequency-based and therefore effective for linear systems, constructing proper models from the large, generally non-linear models relevant to engineering design required temporal reduction techniques. Previous methods such as [Rosenberg and Zhou, 1988] and [Louca *et al*., 1997] have largely concentrated on finding reduced models whose parameters map to dominant system dynamics. In Louca *et al*. (1997), for example, the Model Order Reduction Algorithm (MORA) calculates "activity" (the time integral of the absolute value of the power flow) of each energy storage and dissipative element of a full model. Elements whose percent contributions to the total system activity fall beneath a user-specified threshold may be eliminated without significantly affecting the dominant system dynamics. This model reduction method only considers the overall system behavior; therefore, user intervention may be required when the analyst is interested in specific outputs.

Ye and Youcef-Toumi (1999) investigated an energy-based sensitivity approach to estimating the contribution of neighboring bond energies to the state of the energy storage element of interest. The goal of the work was to generate reduced models to predict an output of interest regardless of its association with low or high-energy dynamics.

The above techniques aim to reduce model complexity. However, these algorithms do not assess the location or intensity of coupling among elements, and do not use decoupling to guide reduction and simplification. Recent research by the authors focused on reducing the size and complexity of non-linear lumped-parameter models by finding weakly coupled subsystems that can be simulated separately or in parallel. A bond graph-based algorithm is presented in [Rideout *et al.*, 2004] that searches a model for sets of bond graph elements between which one-way coupling exists. Upon finding such subgraphs, the model is then partitioned into "driving" and "driven" subsystems.

The driving subsystem outputs excite the driven, and the driven dynamic responses do not affect those of the driving system.

System partitioning is often assumed in engineering design, for example in vehicle dynamics when an experienced engineer assumes that longitudinal vehicle dynamics are not affected by pitch motions, and then uses a point-mass model as opposed to a half-car model. Even for the expert, however, the partitioning assumption should be quantitatively verified. As the road roughness (external inputs) or vehicle loading and suspension stiffness (system parameters) change, the significance of the decoupling between longitudinal and pitch dynamics can gradually increase and render the point mass analysis ineffective.

The current paper has two objectives. First, the partitioning method is reviewed and demonstrated using a truck model in which the longitudinal and pitch dynamics are decoupled. Junction structure and energetic elements can be eliminated, and energetic elements are grouped in a more meaningful way for subsequent reduction using MORA, allowing a greater degree of reduction. Secondly, the ability of the method to quantitatively track the validity of decoupling assumptions is demonstrated by increasing road roughness, thus causing the pitch motion to affect longitudinal dynamics and the partitions to break down.

The method is reviewed in the next section, after which the medium-sized truck case study model is outlined. Simulation results and discussion follow. The paper ends with a summary and conclusions.

MODEL CONDITIONING AND PARTITION SEARCH

Bond graph junction structure represents constraint equations that link the constitutive law variables of dynamic elements, and thus can give insight into the location of weak coupling upon

- computation of a quantitative metric for the contribution of each constraint term
- comparison of their relative contributions
- identification and elimination of negligible terms.

In Rideout *et al*. (2004), the relative activity of bonds at each 0- or 1-junction is proposed as a means of unearthing negligible constraint equation terms.

Partitions are defined as groups of dynamic elements demarcated by negligible constraint equation terms. The insight provided by bond graphs, wherein constraint equations and the relationships among their variables are explicitly represented by 0- and 1-junctions and their bonds, is extremely valuable.

Procedure

Activity [Louca *et al*., 1998] of an element or bond over the time interval t_1 to t_2 is defined as:

$$A = \int_{t_1}^{t_2} |P| \, dt \tag{1}$$

where P is instantaneous power of the element or bond. Activity is always positive, and either constant or monotonically increasing, over a given time interval. The procedure consists of the following general steps, and the reader is referred to [Rideout, 2004] and [Rideout et al., 2004] for greater detail.

Step 0: Construct a bond graph model of the system that can be considered the "full" model inasmuch as its complexity captures the dynamics of interest. Record the outputs of interest from the full model for later comparison with reduced or partitioned models.

Step 1: Calculate the activity of each bond in the graph and compare, at each *n*-port junction, the activity A_i of each of the *n* connected bonds to the junction maximum. Low relative activity of bond *i* at a junction implies that

1) for a 0-junction, the flow f_i can be neglected in the junction flow summation
2) for a 1-junction, the effort e_i can be neglected in the junction effort summation.

Locally inactive bonds at a junction are defined as those with an activity ratio falling below a user-defined threshold:

$$\frac{A_i}{max(A_i)} < \varepsilon \tag{2}$$

Step 2: "Condition" the bond graph by converting the bonds with negligible activity to modulated sources. Table 1 summarizes the conversion implications for bonds depending on the junctions or elements to which they are attached.

The activity of an "internal" junction structure bond (between two junction structure elements) must be compared to the activities of the bonds at two separate bond graph nodes, as shown in Scenario (i) of Table 1. For Scenario (i), Case A, the activity of bond 1 is low compared to the other bonds at the 0-junction, but is on the order of the other bond activities at the 1-junction. The flow input to the 0-junction can be eliminated, and the effort input to the 1-junction can be provided by a modulated effort source. In Case B the bond activity is locally inactive at the 1-junction, but is on the order of the other 0-junction activities. If both Cases (a) and (b) apply, then the bond can be eliminated. If the local activity of an "external" junction structure bond (between the junction structure and an energetic element) is negligible, a trivial driven partition results as shown in Scenarios (ii) and (iii) of Table 1. The generalized impedance *Z* (representing an *I, C*, or *R* element) can then be eliminated from the model. A mechanism thus exists for model reduction; however, decoupling among groups of

elements manifests itself as local inactivity of internal junction structure bonds.

Table 1 shows an internal bond connected to a 1- and a 0-junction. Other possible internal bond connections are to *(M)TF* or *(M)GY* elements. If both bonds are locally inactive compared to their respective junctions, then the transformer or gyrator can be eliminated. If one bond is locally inactive, a modulated source in sequence with an *(M)TF* or *(M)GY* results. The transformer or gyrator can be incorporated into the source, resulting in the equivalent conditioned junction structure shown in Table 2.

Note that the bonds with negligible activity at a junction must be causally weak at that junction in order to be conditioned. In other words, a bond with negligible activity that imposes flow on a 1-junction (effort on a 0-junction) cannot be converted to a modulated flow (effort) source. The input to the modulated source, i.e., the shared power variable of the junction, must be causally imposed on the junction by another bond with significant activity.

Step 3: Identify bond subgraphs (collections of bond graph elements) that are connected only by modulating signals. If removing the modulating signals due to bond conversion results in two or more separate bond graphs, then subgraphs have resulted and the most important pre-requisite for partitioning has been met.

Step 4: Identify driving and driven partitions – subgraphs between which all modulating signals carry information in one direction.

Step 5: Reduce the model as follows. If the only outputs of interest are in driving partitions, eliminate the driven partitions. If an output of interest is associated with a driven partition element, replace its driving partition(s) with the necessary inputs (time histories) to excite the driven partition.

Some examples of subgraphs S (indicated as polygons), and driving and driven partitions S_G and S_N are shown in Figure 1. Note that a partition can be simultaneously both driving and driven.

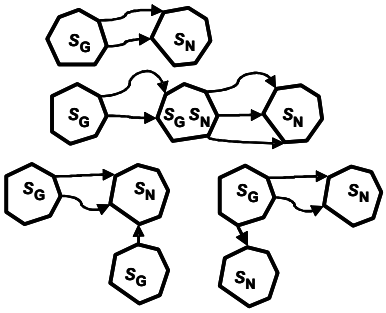


Figure 1. Subgraphs and partitions

Table 1. Interpretation of locally inactive bonds

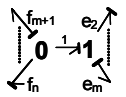
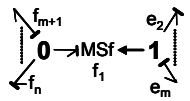
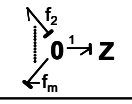
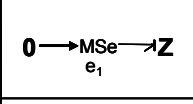
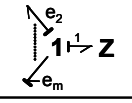
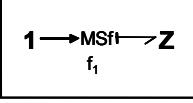
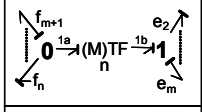
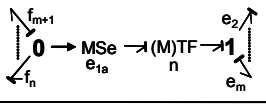
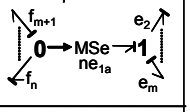
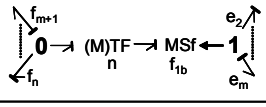
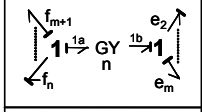
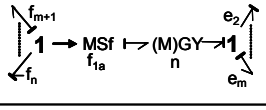
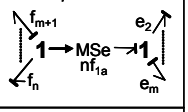
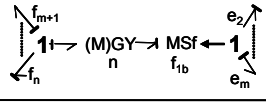
Scenario (i)	Case A. $A_1 << A_{m+1,\ldots n}$	Case B. $A_1 << A_{2,\ldots m}$
		
Scenario (ii)	**Case A. $A_1 << A_{2,\ldots m}$**	
		
Scenario (iii)	**Case A. $A_1 << A_{2,\ldots m}$**	
		

Table 2. Transformer and gyrator conditioning

Transformer	Case A. $A_{1a} << A_{m+1,\ldots n}$	Equivalent
		
	Case B. $A_{1b} << A_{2,\ldots m}$	**Equivalent**
		
Gyrator	**Case A. $A_{1a} << A_{m+1,\ldots n}$**	**Equivalent**
		
	Case B. $A_{1b} << A_{2,\ldots m}$	**Equivalent**
		

CASE STUDY: DECOUPLING OF CLASS VI TRUCK VEHICLE DYNAMICS

In Louca *et al.* (2001), a model of a Class VI International 4700-series delivery truck that incorporated engine, powertrain, and vehicle dynamics was reduced using MORA. The resulting proper model accurately and efficiently predicted vehicle speed – an output associated with the dominant longitudinal dynamics – during vehicle acceleration and deceleration on a smooth, flat road. The vertical/pitch elements such as the suspension compliance and sprung mass rotational inertia were eliminated. Therefore, any pitch related variable, e.g., pitch angle, could no longer be accurately predicted.

Rideout (2004) demonstrated that MORA could be used to generate a proper model to predict both forward speed V and pitch angle θ for the same vehicle accelerating with full throttle from a standstill, and encountering a 1:10

slope after 1200 feet of travel. Only the vehicle dynamics were studied, and the model had 24 energy storage and dissipative elements in contrast to the 55 elements in the integrated engine-drivetrain-vehicle model of [Louca *et al*., 2001]. To capture longitudinal dynamics, activities were calculated over a time window that spanned the entire maneuver duration. To rank pitch elements, activities were calculated over a second window of length approximately equal to one pitch oscillation at the onset of the most severe pitch transient. By eliminating the intersection of the two resulting sets of inactive elements using a 97% MORA activity threshold, a proper model was generated that eliminated 5 of the 24 energetic elements:

- front unsprung mass vertical inertia
- front wheel inertia
- front tire damping
- rear tire damping
- front tire longitudinal slip resistance

Despite the attempt to select a second time window during which pitch dynamic effects would be maximized, 45% of the total activity during that window was contributed by the sprung mass longitudinal degree of freedom and the aerodynamic drag.

Successful partitioning of the model into longitudinal and pitch dynamics would make the activity hierarchy insensitive to the exact choice of time window. Elements in one partition would not affect the activity indices of elements in the others. In the present case study, individual partitions could be reduced using MORA without the infiltration of longitudinal elements into the pitch activity hierarchy.

Model conditioning and partition search

Bond activities were locally compared at 0- and 1-junctions, with external junction structure bonds included in the comparison but not subject to conditioning. Energetic element elimination was not done by local comparison as in Scenarios (ii) and (iii) of Table 1, but by application of MORA post-partitioning. A user-defined threshold of 95% was set to determine which bonds were locally inactive.

Figure 2 shows the bond graph, with inactive bonds indicated by a block arrow originating from the junction at which the bond's activity is negligible. The reader is referred to Table 3 for a description of the elements. The element at the origin of each arrow drives a modulated source upon conditioning. Four *MTF* elements can be entirely eliminated. The block arrows indicate a driving subgraph with longitudinal elements, and a driven subgraph (for which bonds and elements are shown in grey), containing predominantly vertical and rotational elements.

While the conditioned bond graph has subgraphs between which all new modulating signals travel in one direction, a "subgraph loop" results from the pre-existing modulating signals from the rear tire normal force 0-junction to the rear tire rolling and slip resistance elements as shown in Figure 2. For the truck a nominal signal value of -45000 Newtons can be identified as the static tire load. The tire resistances can be replaced with non-modulated elements using this constant value, allowing partitioning to proceed. The change did not affect simulation results, as shown in Figure 3. The figure shows two nearly identical plots (one with and one without modulation) for pitch angle, and two nearly identical plots for forward speed, during the most severe pitch transient.

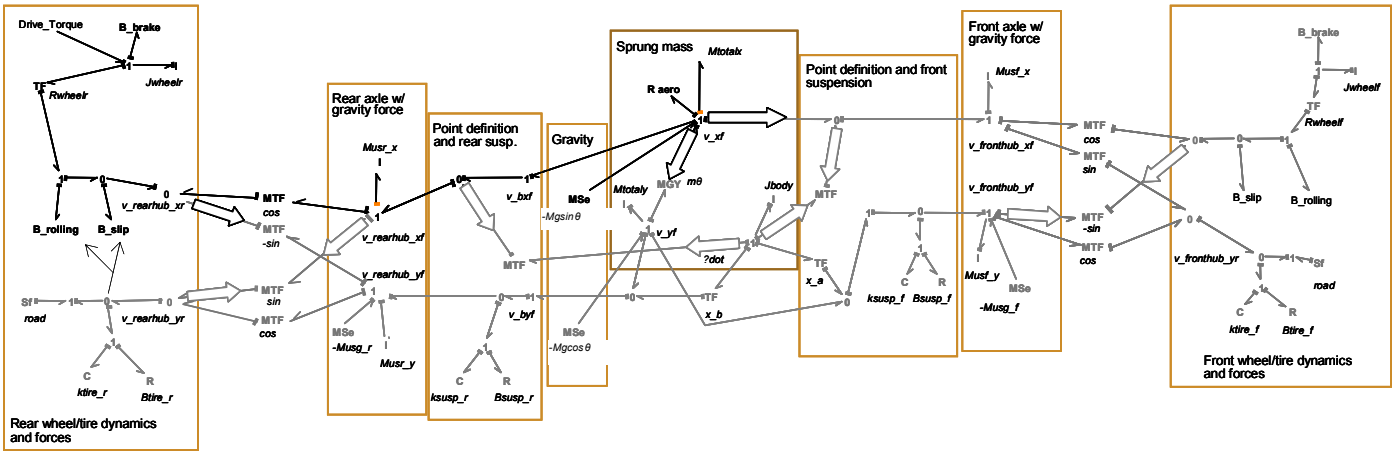


Figure 2. Class VI truck bond graph showing locally inactive bonds

Table 3. Half-car bond graph elements

Bond Graph Segment	Energy Element	Description
Sprung mass	Mtotalx Mtotaly Jbody R_aero	sprung mass – x d.o.f. sprung mass – y d.o.f. sprung mass rotational inertia aerodynamic drag
Point definition and front suspension	ksusp_f Bsusp_f	front suspension stiffness front suspension damping rate
Front axle	Musf_x Musf_y	unsprung mass – x d.o.f. unsprung mass – y d.o.f.
Front wheel/tire dynamics	ktire_f Btire_f B_slip B_rolling Jwheelf B_brake	front tire stiffness front tire damping rate longitudinal slip resistance rolling resistance front wheel rotational inertia brake
Point definition and rear suspension	ksusp_r Bsusp_r	rear suspension stiffness rear suspension damping rate
Rear axle	Musr_x Musr_y	unsprung mass – x d.o.f. unsprung mass – y d.o.f.
Rear wheel/tire dynamics	ktire_r Btire_r B_slip B_rolling Jwheelr B_brake Drive_Torque	rear tire stiffness rear tire damping rate longitudinal slip resistance rolling resistance rear wheel rotational inertia brake drive torque from input file

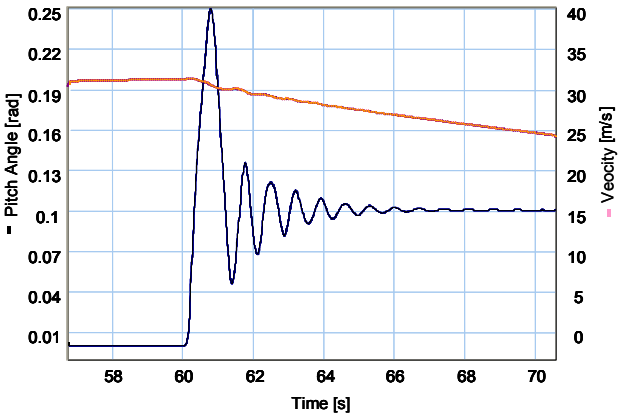


Figure 3. Fully-coupled model with and without rear tire modulating signals

Simulation Results and Model Reduction

Figures 4 and 5 show the forward velocity and pitch angle predictions for the full and partitioned models. There is a slight degradation of prediction of the small pitch transients when the brakes are released, but the significant peaks and decay rate remain accurate. Applying MORA to the separate partitions with a 97% activity threshold, and eliminating the elements that comprise the union of the two resulting sets of inactive elements, suggests two further eliminations in addition to the original five. The original five elements along with:

- rear wheel inertia *Jwheelr*
- front unsprung mass *Musf_x*

can be eliminated without changing the predictions from those of Figures 4 and 5.

Computation time comparison

The computational times of the various models are compared in Table 3. MORA was applied to the full model and to each partition. The table summarizes the numbers of equations, variables, independent and dependent states, and algebraic loops in the models. The absolute and relative tolerances for all experiments were *1e-4* and *1e-3* respectively.

The Baseline, Traditional MORA, and Conditioned Model computation times are based on running the model with the junction structure of both partitions present. The Driving Partition time is the time required to predict the longitudinal velocity and generate the input files necessary to excite the driven partition. Neither the junction structure nor the energetic elements of the driven partition are present. The Driven Partition time is that required to predict the pitch angle using input file excitation only, without the driving partition junction structure and energetic elements.

Conditioning two-way power bonds can be seen to reduce computation time, even though the partitions are not separated. Conditioning can also break algebraic loops. Significant savings result when simulation can be restricted to individual partitions. When running the driving and driven partitions sequentially to predict the pitch angle in the absence of previously stored driving inputs, the computation time is on the order of that of the fully coupled model. However, case studies of larger systems [Rideout, 2004] have shown sequential partition simulation to be significantly faster than simulation of the fully coupled or conditioned models.

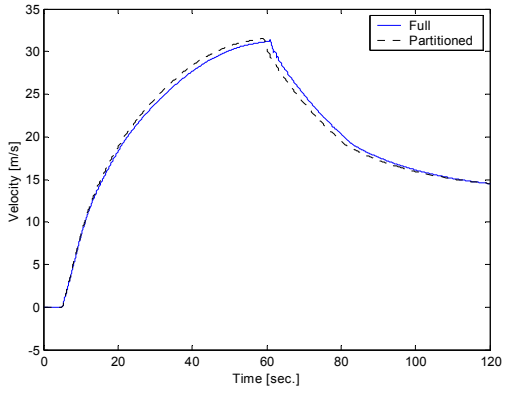


Figure 4. Full and partitioned model velocity predictions

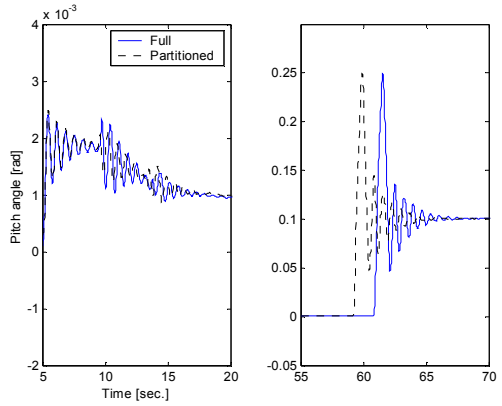


Figure 5. Full and partitioned model pitch angle predictions

Table 4. Computation time comparison

Model	Ind. States	Dep. States	Alg. Loops	Comp. Time (s)	No. of Steps
Baseline	44	2	0	3.688	46345
Traditional MORA	37	2	2	2.272	36125
Conditioned Model	44	2	0	3.085	43165
Driving Partition	11	1	0	1.958	10349
Driven Partition	26	0	0	1.890	18669

Effect of road roughness on partitionability

Adding road roughness weakens the decoupling, and the extent to which pitch motion affects forward speed can be estimated with a simple recalculation of the bond activities. A washboard road elevation of $0.04\sin(\pi/4)x$ meters was applied, with x being the distance traversed by the vehicle, between 200 and 600 meters of travel. The time interval over which the truck encountered the roughness was 21.4 to 45 seconds. The vehicle responses for the smooth and rough flat roads are compared in Figure 6. The pitch response is much more severe, as expected, and the longitudinal velocity is visibly affected. The forward acceleration of the truck is reduced, and a high-frequency component due to sprung mass rotation appears in the velocity time series.

The net activity was calculated over the time interval, and the increases in relative bond activities of the inactive bonds in Figure 2 are summarized in Table 5. Figure 7 expands the portions of the bond graph containing the partition boundaries, and the boundary bonds are numbered to assist the reader in interpreting Table 5. The partition breaks down at the rear coordinate transformation from sprung mass body-fixed to road-aligned coordinates, as forces tangential and normal to the road have significant components in both orthogonal sprung mass directions. The most severe blurring of the partition results from the increased importance of vertical and longitudinal front suspension and tire forces on the forward motion of the sprung mass.

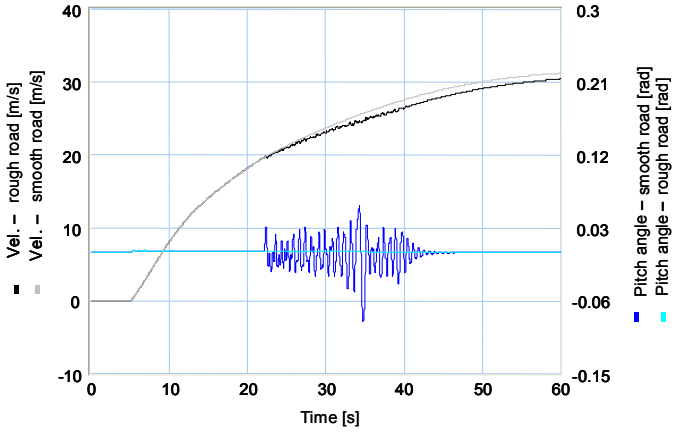


Figure 6. Vehicle responses on rough road

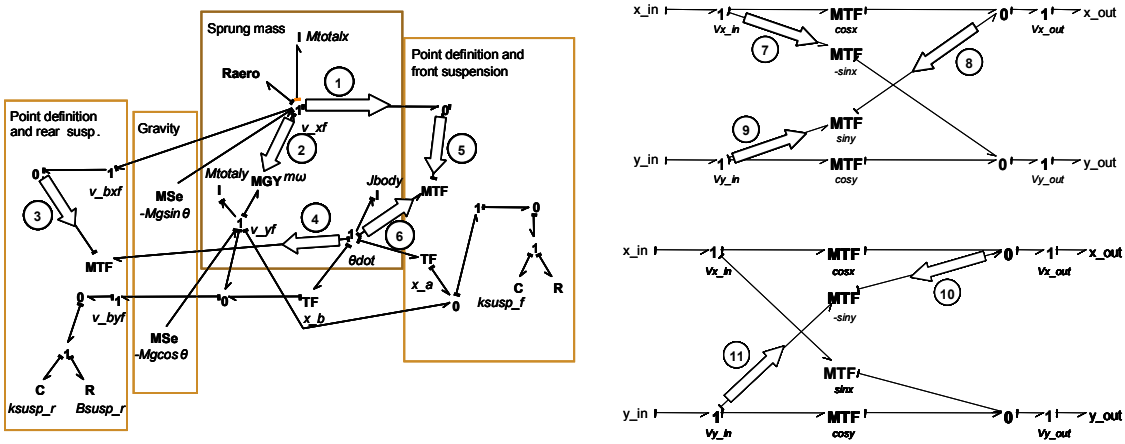


Figure 7. Partition boundary bonds

Table 5. Locations of two-way coupling

BOND	DESCRIPTION	ACTIVITY %MAX	
		SMOOTH ROAD	ROUGH ROAD
1	long. force on sprung mass from front tires / suspension	4.3	**13.8**
2	gyrational longitudinal force on sprung mass	1.9	4.7
3	long. vel. component of rear hub due to rotation	0.02	0.16
4	moment about c.g. from long. rear tire / suspension force	3.8	3.8
5	long. velocity component of front hub due to rotation	0.13	0.26
6	moment about c.g. from long. front tire / suspension force	1.1	0.87
7	body-fixed y rear susp. force component tangent to road	2.4	**8.6**
8	rear tire vel. normal to road, body-fixed long. component	~0	0.03
9	body-fixed long. rear susp. force component normal to road	0.7	0.64
10	front tire vel. normal to road, body-fixed long. component	0.01	0.05
11	body-fixed long. front susp. force component normal to road	0.02	0.03

DISCUSSION

Quantitative partitioning is a powerful tool for maximizing insight into system coupling and minimizing modeling errors based on intuition and assumptions. The user is granted physical insight into the changes in system coupling as the environment or design variables change, and is not restricted to either a full model or an assumed form of the decoupled model. As the system parameters or environment change, models of intermediate complexity can result that are still partitionable even if certain bonds can no longer be conditioned. For large models, multiple partitioning options may exist. The preferred choice may depend on the ability to assign preferred causality to the partitions, or on the creation or breaking of algebraic loops.

In the Class VI truck case study, the 120 second time window was used to calculate activities for MORA after partitioning, despite the presence of varying degrees of transient and steady state response of both longitudinal and pitch elements. Activity windowing that crosses several "regimes" of the response is useful for assessing the cumulative effect of one set of dynamics on another. If one were interested in estimating fuel economy for a driving cycle on real roads with known grades and occasional pitch-inducing events such as potholes, then the time window over the entire cycle, in concert with the partitioning algorithm, determines the predictive ability of a point mass model with no vertical and rotational degrees of freedom.

The selection of the threshold remains somewhat arbitrary, as nonlinear systems in general will not allow an analytical mapping of the activity ratio at a partition boundary to the resulting discrepancy of the predicted system outputs before and after partitioning. Recent research on model quality assessment [Sendur *et al.*, 2002] shows the potential to automate the selection of the activity threshold based on practical engineering design criteria. Quantitative model quality assessment also suggests a future method for determining the range of inputs and parameters over which one-way coupling assumptions are valid. Currently, the partitions are defined for a particular parameter vector and set of inputs, and the size of the region in the design and environment space in which the partitions are valid has not been formally researched.

SUMMARY AND CONCLUSIONS:

An algorithm has been presented in which an arbitrary bond graph model can be searched for subgraphs between which one-way coupling occurs. If such decoupling is found, the model can be partitioned into driving and driven submodels that can be simulated independently, generally with significantly less computational expense as both energetic elements and junction structure are eliminated. The algorithm expands the scope of physical-domain non-linear model reduction beyond the elimination of energy storage and dissipative elements that do not affect dominant dynamics. Partitioning creates groups of energetic elements to which reduction methods such as MORA can be applied separately, without "driving" elements dominating the "driven" element activity hierarchy. The retention of parameters from the full model maximizes physical insight into the nature of intra-system coupling.

A vehicle dynamics case study demonstrated the partitioning algorithm and the resultant computational savings, and showed how the strength of one-way coupling can be monitored quantitatively as the environment or system parameters change during the design cycle.

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