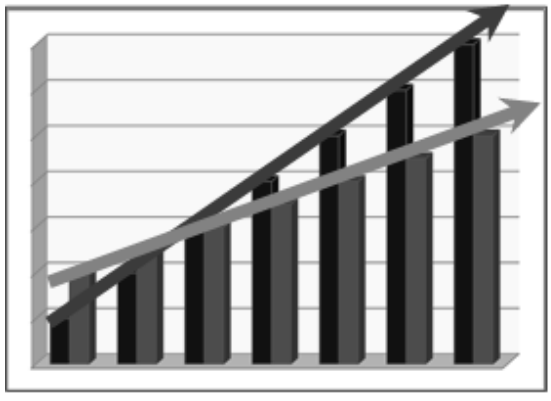
5 SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES IN TWO VARIABLES

I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever asked yourself how businessmen make profits? How can farmers increase their yield or harvest? How parents budget their income on food, education, clothing and other needs? How cellular phone users choose the best payment plan? How students spend their daily allowances or travel from home to school?



MONTHLY BUDGET

Monthly Income	Php42,000.00
House Payment	Php 7,500.00
Food	Php10,000.00
Utilities	Php 2,500.00
Education	Php 8,000.00
Clothing	Php 2,000.00
Transportation	Php 5,000.00
Miscellaneous	Php 3,500.00
Total Expenses	Php38,500.00
Savings	Php 3,500.00

Find out the answers to these questions and determine the vast applications of systems of linear equations and inequalities in two variables through this module.

II. LESSONS AND COVERAGE

In this module, you will examine the above questions when you take the following lessons:

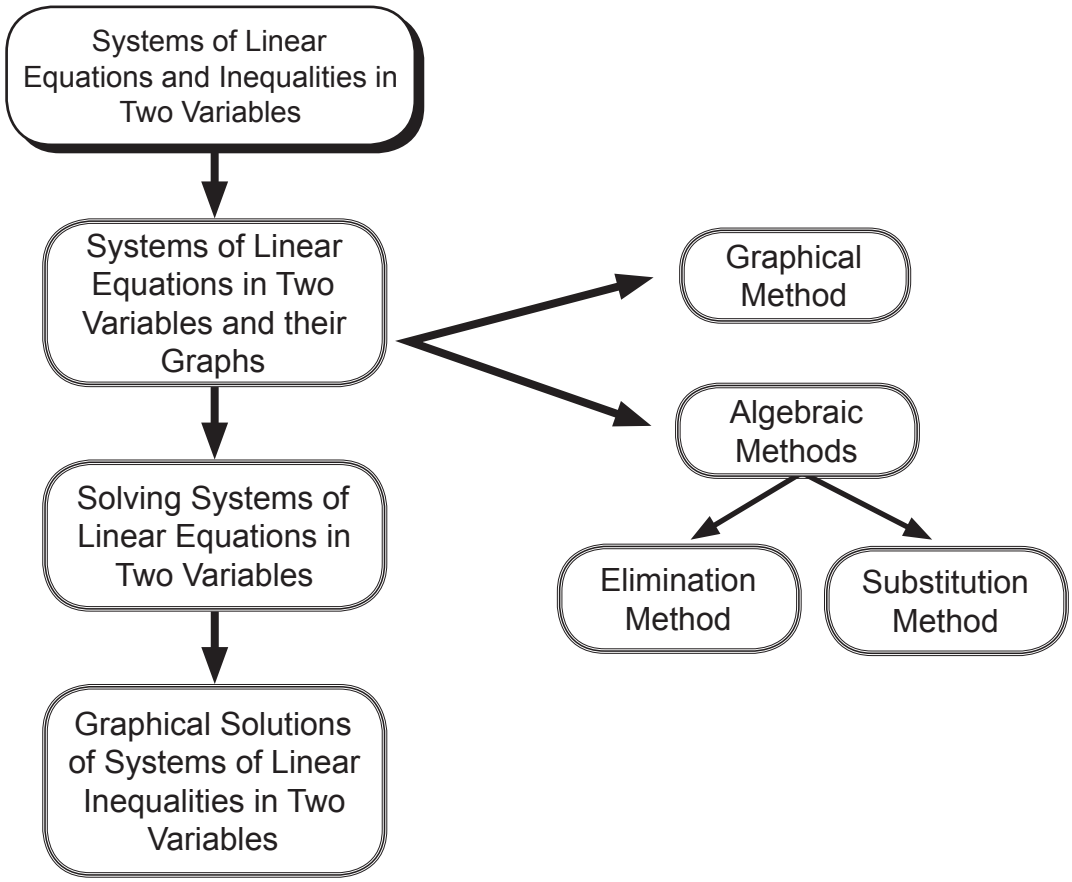
Lesson 1 – Systems of linear equations in two variables and their graphs
Lesson 2 – Solving systems of linear equations in two variables
Lesson 3 – Graphical solutions of systems of linear inequalities in two variables

In these lessons, you will learn to:

Lesson 1	• Describe systems of linear equations and inequalities using practical situations and mathematical expressions. • Identify which systems of linear equations have graphs that are parallel, intersecting, and coinciding. • Graph systems of linear equations in two variables.
Lesson 2	• Solve systems of linear equations by (a) graphing; (b) elimination; (c) substitution. • Solve problems involving systems of linear equations in two variables.
Lesson 3	• Graph system of linear inequalities in two variables. • Solve a system of linear inequalities in two variables by graphing. • Solve problems involving systems of linear inequalities in two variables.

Module Map

Here is a simple map of the lessons that will be covered in this module.



III. PRE - ASSESSMENT

Part I: Find out how much you already know about this module. Choose the letter that you think best answers the question. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. Which of the following is a system of linear equations in two variables?

   a. $2x - 7y = 8$

   b. $3x + 5y = -2$
      $x - 4y = 9$

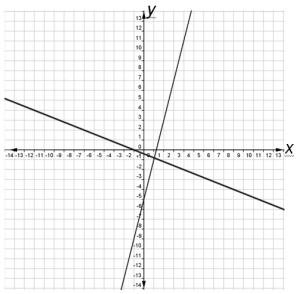
   c. $x + 9y = 2$
      $2x - 3y > 12$

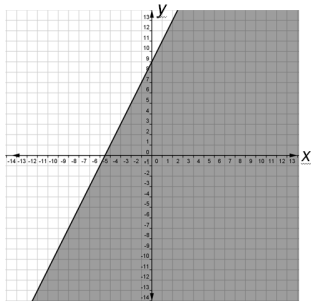
   d. $4x + 1 = 8$
      $3y - 7 = 11$

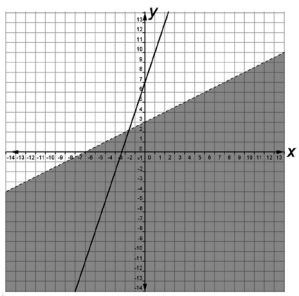
2. What point is the intersection of the graphs of the lines $x + y = 8$ and $2x - y = 1$?

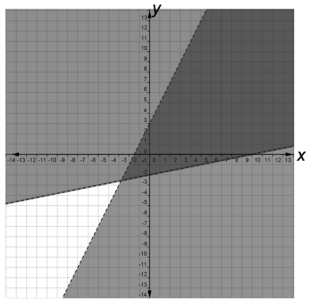
   a. (1, 8) b. (3, 5) c. (5, 3) d. (2, 6)

3. Which of the following is a graph of a system of linear inequalities in two variables?

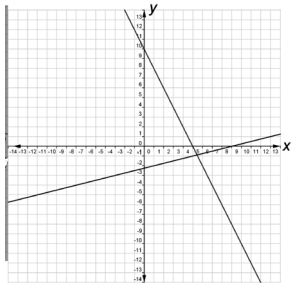
   a. 

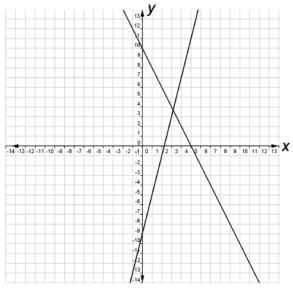
   c. 

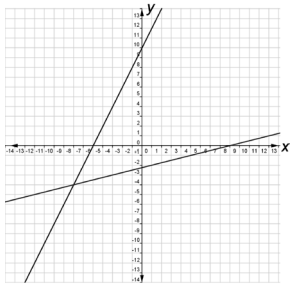
   b. 

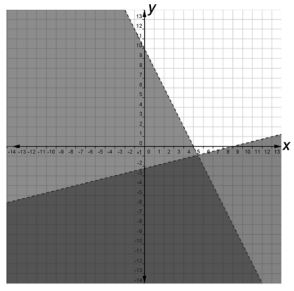
   d. 

4. Which of the following shows the graph of the system $\begin{matrix} 2x + y < 2 \\ x - 4y > 9 \end{matrix}$?

a. 

c. 

b. 

d. 

5. If $2x + y = 9$ and $2x - y = 11$, what is the value of x?

a. 4 b. 5 c. 10 d. 20

6. A car park charges Php 45 for the first 3 hours and Php 5 for every succeeding hour or a fraction thereof. Another car park charges Php 20 for the first 3 hours and Php 10 for every succeeding hour or a fraction thereof. In how many hours would a car owner pay the same parking fee in any of the two car parks?

a. 2 hr b. 3 hr c. 5 hr d. 8 hr

7. How many solutions does a consistent and independent system of linear equations have?

a. 0 b. 1 c. 2 d. Infinite

8. Which of the following ordered pairs satisfy both $2x + 7y > 5$ and $3x - y \leq 2$?

a. (0, 0) b. (10, -1) c. (-4, 6) d. (-2, -8)

9. Mr. Agpalo paid Php 260 for 4 adult's tickets and 6 children's tickets. Suppose the total cost of an adult's ticket and a children's ticket is Php 55. How much does an adult's ticket cost?

a. Php 20 b. Php 35 c. Php 80 d. Php 120

10. Which system of equations has a graph that shows intersecting lines?

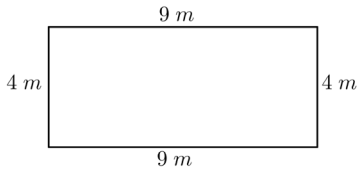
a. $2x + 4y = 14$
$x + 2y = 7$

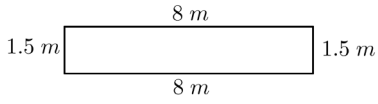
b. $-3x + y = 5$
$6x - 2y = 1$

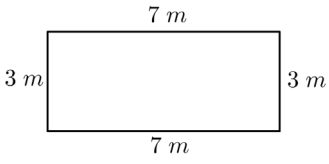
c. $4x + 8y = 7$
$x + 2y = 3$

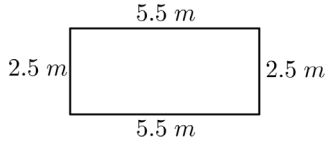
d. $3x + y = 10$
$3x - y = 5$

11. Mr. Bonifacio asked each of his agriculture students to prepare a rectangular garden such that its perimeter is at most 19 m and the difference between its length and its width is at least 5 m. Which of the following could be the sketch of a garden that a student may prepare?

a. 

c. 

b. 

d. 

12. Luisa says that the system $\begin{matrix} 3x + y = 2 \\ 2y = 15 - 6x \end{matrix}$ has no solution. Which of the following reasons would support her statement?

I. The graph of the system of equations shows parallel lines.
II. The graph of the system of equations shows intersecting lines.
III. The two lines as described by the equations in the system have the same slope.

a. I and II b. I and III c. II and III d. I, II, and III

13. Jose paid at most Php 250 for the 4 markers and 3 pencils that he bought. Suppose the marker is more expensive than the pencil and their price's difference is greater than Php 30. Which of the following could be the amount paid by Jose for each item?

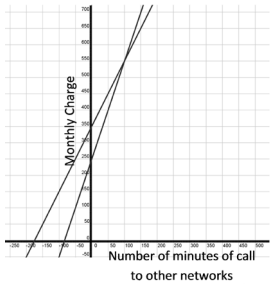
a. Marker: Php 56
Pencil: Php 12

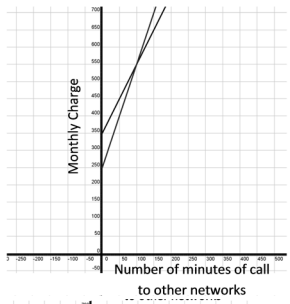
b. Marker: Php 35
Pencil: Php 15

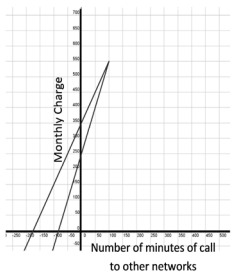
c. Marker: Php 46
Pencil: Php 7

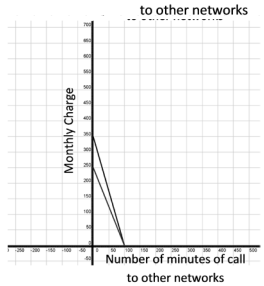
d. Marker: Php 50
Pencil: Php 19

14. Bea wanted to compare the mobile network plans being offered by two telecommunication companies. Suppose Bea's father would like to see the graph showing the comparison of the two mobile network plans. Which of the following graphs should Bea present to his father?

a. 

c. 

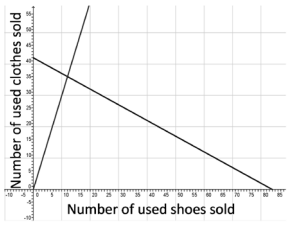
b. 

d. 

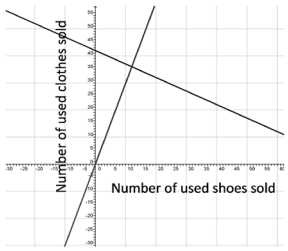
15. Edna and Grace had their meal at a pizza house. They ordered the same kind of pizza and drinks. Edna paid Php 140 for 2 slices of pizza and a drink. Grace paid for Php 225 for 3 slices of pizza and 2 drinks. How much did they pay for the total number of slices of pizza?

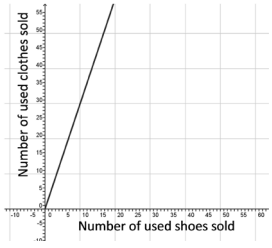
a. Php 55 b. Php 110 c. Php 165 d. Php 275

16. The Senior Citizens Club of a certain municipality is raising funds by selling used clothes and shoes. Mrs. Labrador, a member of the club, was assigned to determine how many used clothes and shoes were sold after knowing the important information needed. She was asked further to present to the club how she came up with the result using graph. Which of the following graphs could Mrs. Labrador present?

a. 

c. 

b. 

d. 

17. The Math Club rented a sound system for their annual Mathematics Camp. They also rented a generator in case of power interruption. After the 3-day camp, the club paid a total amount of Php3,000, three days for the sound system and two days for the generator. If each is rented for one day, the club should have paid a total amount of Php1,100. What was the daily rental cost of the generator?

a. Php 300
b. Php 600
c. Php 800
d. Php 2,400

18. Mrs. Soriano would like to keep track of her family's expenses to have an idea of the maximum or minimum amount of money that she will allot for electric and water consumption, food, clothing, and other needs. Which of the following should Mrs. Soriano prepare?

a. Budget Plan
b. Compilation of Receipts
c. Pricelist of Commodities
d. Bar Graph of Family's Expenses

19. A restaurant owner would like to make a model which he can use as guide in writing a system of equations. He will use the system of equations in determining the number of kilograms of pork and beef that he needs to purchase daily given a certain amount of money (C), the cost (A) of a kilo of pork, the cost (B) of a kilo of beef, and the total weight of meat (D). Which of the following models should he make and follow?

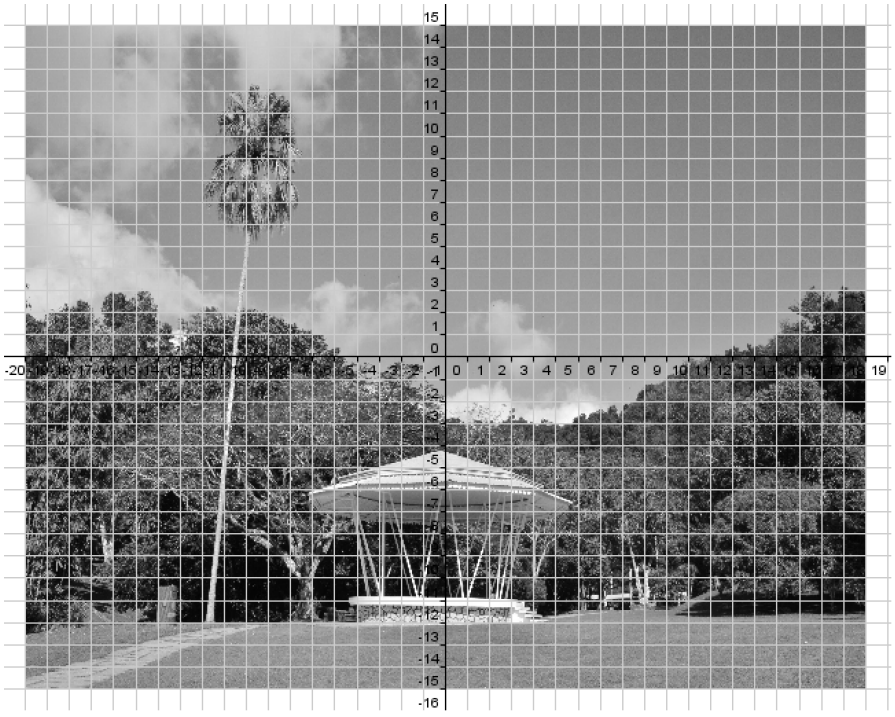
a. $\begin{aligned} Ax - By &= C \\ x + y &= D \end{aligned}$
b. $\begin{aligned} Ax + By &= C \\ x - y &= D \end{aligned}$
c. $\begin{aligned} Ax + By &= C \\ x + y &= D \end{aligned}$
d. $\begin{aligned} Ax - By &= C \\ x - y &= D \end{aligned}$

20. Mrs. Jacinto would like to instill the value of saving and to develop decision-making among her children. Which of the following situations should Mrs. Jacinto present to her children?

a. Buying and selling different items
b. A person putting coins in his piggy bank
c. Buying assorted goods in a department store
d. Making bank deposits in two banks that give different interests

Part II. Illustrate each mathematics concept in the given figure then describe it by completing the statement at the bottom.

Lines	**Slope of a Line**	**y - intercept of a Line**
Points on a Line	**Points**	**Coordinates of Points**
Parallel Lines	**Intersecting Lines**	**Linear Equations**
	Linear Inequality	



My idea of (mathematics concept given) is _____
_____
_____
_____

Part III. Use the situation below to answer the questions that follow.

One Sunday, a Butterfly Exhibit was held at the Quezon Memorial Circle in Quezon City. A number of people, children and adults, went to see the exhibit. Admission was Php 20 each for adults and Php 12 each for children.

Questions:

1. How much did an adult pay for the exhibit? How about a child?

2. Complete the table below for the amount that must be paid by a certain number of adults and children who will watch the exhibit.

Number of Adults	Admission Fee	Number of Children	Admission Fee
2		2	
3		3	
4		4	
5		5	
6		6	

3. How much would 10 adults pay if they watch the exhibit? How about 10 children? Show your solution.

4. If a certain number of adults watched the exhibit, what expression would represent the total admission fee?

   What mathematical statement would represent the total amount that will be collected from a number of children? Explain your answer.

5. Suppose 6 adults and 15 children watch the exhibit. What is the total amount they will pay as admission? Show your solution.

6. If a number of adults and another number of children watch the exhibit, how will you represent the total amount they will pay for the admission? Explain your answer.

7. Suppose the total amount collected was Php 3,000. How many adults and how many children could have watched the exhibit?

8. The given situation illustrates the use of linear equations in two variables. In what other real-life situations are linear equations in two variables applied? Formulate problems out of these situations then solve.

IV. LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate understanding of key concepts of systems of linear equations and inequalities in two variables, formulate real-life problems involving these concepts, and solve these with utmost accuracy using a variety of strategies.

Lesson 1 Systems of Linear Equations in Two Variables and their Graphs

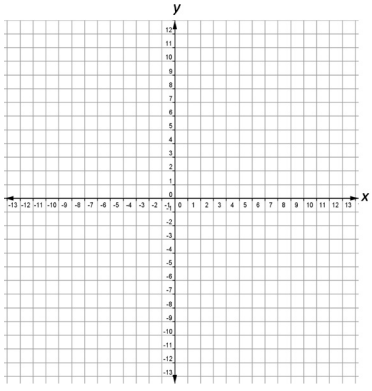
What to Know

Start Lesson 1 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding Systems of Linear Equations in Two Variables and their Graphs. As you go through this lesson, think of the following important question: *"How is the system of linear equations in two variables used in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. To check your work, refer to the answer key provided at the end of this module.

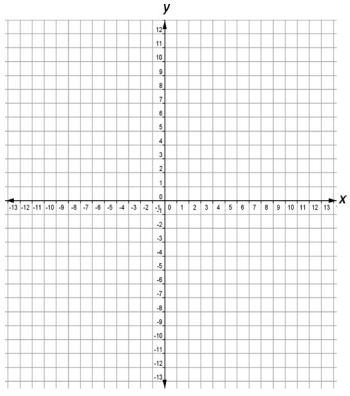
Activity 1 DESCRIBE ME!

Directions: Show the graph of each of the following linear equations in a Cartesian coordinate plane. Answer the questions that follow.

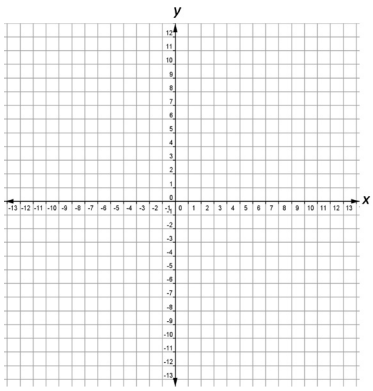
1. $y = 2x + 3$



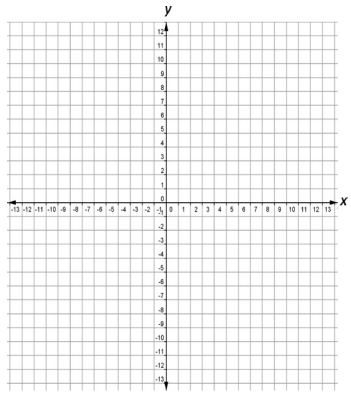
2. $3x - y = 2$



3. $y = 5x - 1$



4. $2x - 3y = 6$



QUESTIONS ?

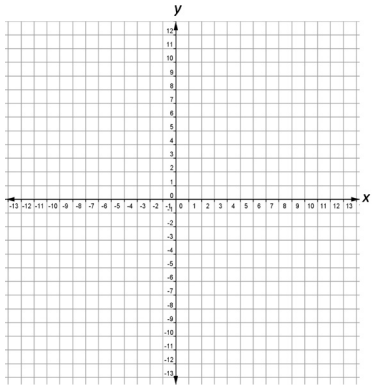
a. How did you graph each linear equation in two variables?
b. How do you describe the graphs of linear equations in two variables?

Were you able to draw and describe the graphs of linear equations in two variables? Suppose you draw the graphs of two linear equations in the same coordinate plane. How would the graphs of these equations look like? You'll find that out when you do the next activity.

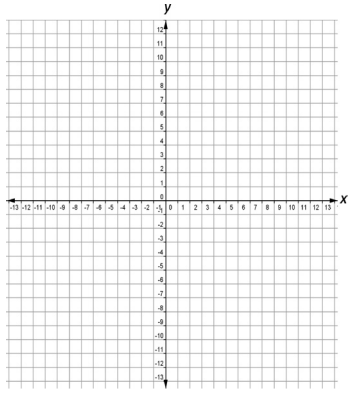
Activity 2 MEET ME AT THIS POINT IF POSSIBLE...

Directions: Show the graph of each pair of linear equations below using the same Cartesian plane then answer the questions that follow.

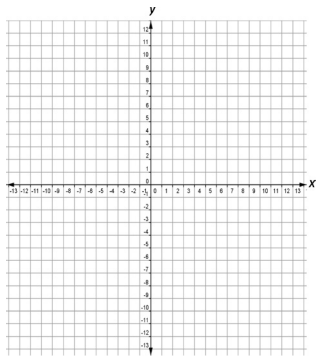
1. $3x + y = 5$ and $2x + y = 9$



2. $3x - y = 4$ and $y = 3x + 2$



3. $x + 3y = 6$ and $2x + 6y = 12$



QUESTIONS ?

a. How did you graph each pair of linear equations?
b. How would you describe the graphs of $3x + y = 5$ and $2x + y = 9$?
How about $3x - y = 4$ and $y = 3x + 2$? $x + 3y = 6$ and $2x + 6y = 12$?
c. Which pair of equations has graphs that are intersecting?
How many points of intersection do the graphs have?
What are the coordinates of their point(s) of intersection?
d. Which pair of equations has graphs that are not intersecting? Why?
How do you describe these equations?
e. Each pair of linear equations forms a system of equations. The point of intersection of the graphs of two linear equations is the solution of the system. How many solutions does each pair of equations have?
e.1) $3x + y = 5$ and $2x + y = 9$
e.2) $3x - y = 4$ and $y = 3x + 2$
e.3) $x + 3y = 6$ and $2x + 6y = 12$
f. What is the slope and the y-intercept of each line in the given pair of equations?

f.1)	$3x + y = 5$;	slope =	y-intercept =
	$2x + y = 9$;	slope =	y-intercept =
f.2)	$3x - y = 4$;	slope =	y-intercept =
	$y = 3x + 2$;	slope =	y-intercept =
f.3)	$x + 3y = 6$;	slope =	y-intercept =
	$2x + 6y = 12$;	slope =	y-intercept =

g. How would you compare the slopes of the lines defined by the linear equations in each system?
How about their y-intercepts?
h. What statements can you make about the solution of the system in relation to the slopes of the lines?
How about the y-intercepts of the lines?
i. How is the system of linear equations in two variables used in solving real-life problems and in making decisions?

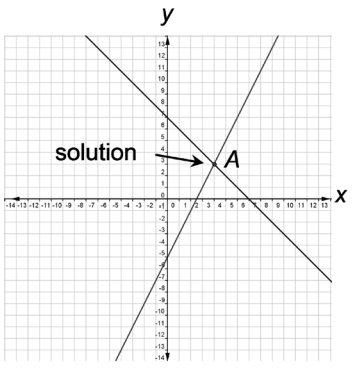
How did you find the preceding activities? Are you ready to learn about systems of linear equations in two variables and their graphs? I'm sure you are. From the activities done, you were able to determine when two lines intersect and when they do not intersect. You were able to relate also the solution of system of linear equations with the slopes and y-intercepts of their graphs. But how are systems of linear equations in two variables used in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on Systems of Linear Equations in Two Variables and their Graphs and the examples presented.

Equations like $x - y = 7$ and $2x + y = 8$ are called **simultaneous linear equations or a system of linear equations** if we want them to be true for the same pair of numbers. The solution of such equations is an ordered pair of numbers that satisfies both equations. The solution set of a system of linear equations in two variables is the set of all ordered pairs of real numbers that makes every equation in the system true.

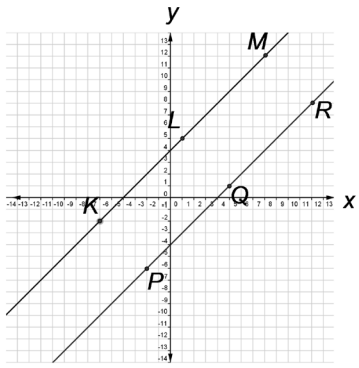
The solution of a system of linear equations can be determined algebraically or graphically. To find the solution graphically, graph both equations on a Cartesian plane then find the point of intersection of the graphs, if it exists. The solution to a system of linear equations corresponds to the coordinates of the points of intersection of the graphs of the equations.

A system of linear equations has:

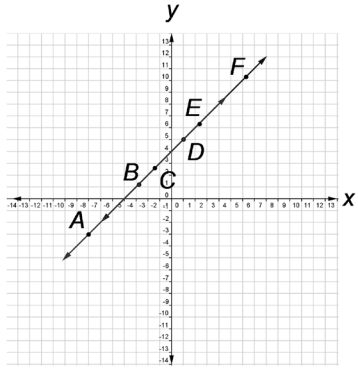
a. only one solution if their graphs intersect.
b. no solution if their graphs do not intersect.
c. infinitely many solutions if their graphs coincide.



Exactly one solution



No solution



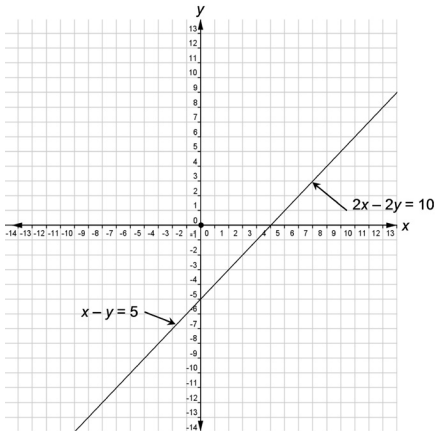
Infinitely Many Solutions

There are three kinds of systems of linear equations in two variables according to the number of solutions. These are:

1. System of consistent and dependent equations

This is a system of linear equations having infinitely many solutions. The slopes of the lines defined by the equations are equal, their y-intercepts are also equal, and their graphs coincide.

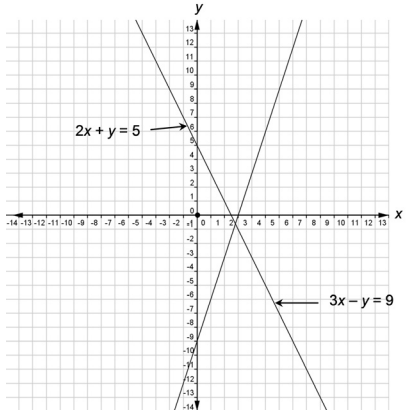
Example: The system of equations $\begin{matrix} x - y = 5 \\ 2x - 2y = 10 \end{matrix}$ is consistent and dependent. The slopes of their lines are equal, their y-intercepts are also equal, and their graphs coincide.



2. System of consistent and independent equations

This is a system of linear equations having exactly one solution. The slopes of the lines defined by the equations are not equal; their y-intercepts could be equal or unequal; and their graphs intersect.

Example: The system of equations $\begin{matrix} 2x + y = 5 \\ 3x - y = 9 \end{matrix}$ is consistent and independent. The slopes of their lines are not equal, their y-intercepts could be equal or unequal, and their graphs intersect.



3. System of inconsistent equations

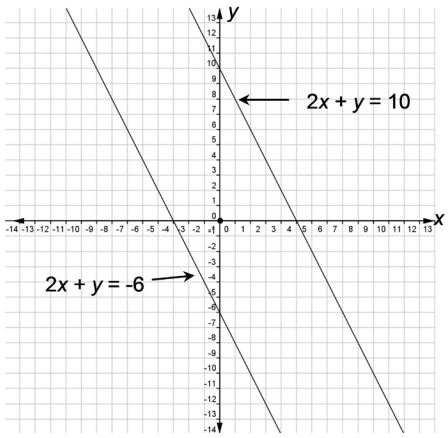
This is a system of linear equations having no solution. The slopes of the lines defined by the equations are equal or have no slopes, their y-intercepts are not equal; and their graphs are parallel.

Example: The system of equations $\begin{matrix} 2x + y = -6 \\ 2x + y = 10 \end{matrix}$ is inconsistent. The slopes of their lines are equal; their y-intercepts are not equal; and their graphs are parallel.

Learn more about Systems of Linear Equations in Two Variables and their Graphs through the WEB. You may open the following links.

1. https://new.edu/resources/solving-linear-systems-by-graphing
2. http://www.mathwarehouse.com/algebra/linear_equation/systems-of-equation/index.php
3. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-228s.html

Systems of linear equations in two variables are illustrated in many real-life situations. A system of linear equations in two variables can be used to represent problems that involve finding values of two quantities such as the number of objects, costs of goods or services, or amount of investments, solutions of which can also be described using graphs. But how are the solutions to problems involving systems of linear equations used in making decisions?



What to Process

Your goal in this section is to apply the key concepts of systems of linear equations in two variables and their graphs. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

Activity 3 CONSISTENT OR INCONSISTENT?

Directions: Determine whether each system of linear equations is consistent and dependent, consistent and independent, or inconsistent. Answer the questions that follow.

1. $2x - y = 7$
   $3x - y = 5$
2. $2x + y = -3$
   $2x + y = 6$
3. $x - 2y = 9$
   $2x - 4y = 18$
4. $8x + 2y = 7$
   $y = -4y + 1$
5. $-3x + y = 10$
   $4x + y = 7$
6. $x - 2y = 9$
   $x + 3y = 14$
7. $6x - 2y = 8$
   $y = 3x - 4$
8. $x + 3y = 8$
   $x - 3y = 8$
9. $2y = 6x - 5$
   $3y = 9x + 1$
10. $3x + 5y = 15$
    $4x - 7y = 10$

QUESTIONS ?

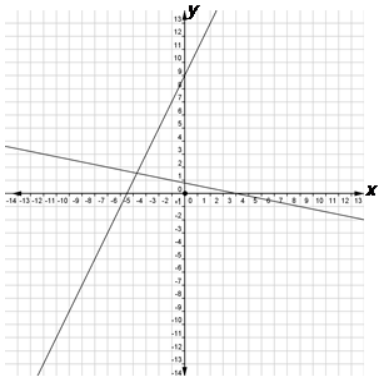
a. How were you able to identify system of equations that are consistent-dependent, consistent-independent and inconsistent?
b. When do you say that a system of linear equations is consistent and dependent? consistent and independent? inconsistent?
c. Give examples of systems of linear equations that are consistent and dependent, consistent and independent, and inconsistent.

Were you able to determine which system of linear equations in two variables is consistent and dependent, consistent and independent, or inconsistent? In the next activity, you will describe the solution set of system of linear equations in two variables through its graph.

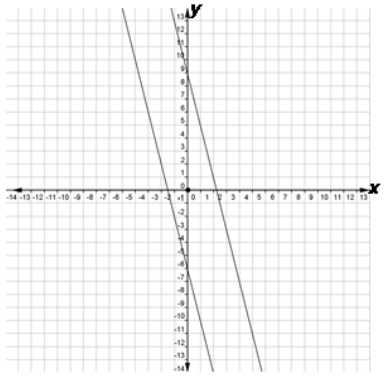
Activity 4 HOW DO I LOOK?

Directions: Describe the solution set of the system of linear equations as shown by the following graphs. Answer the questions that follow.

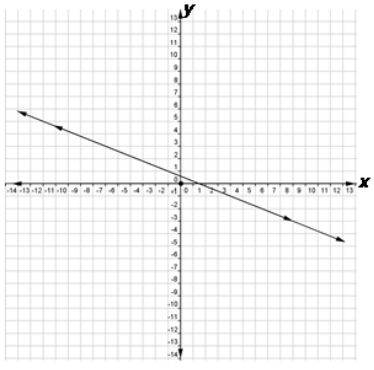
1.



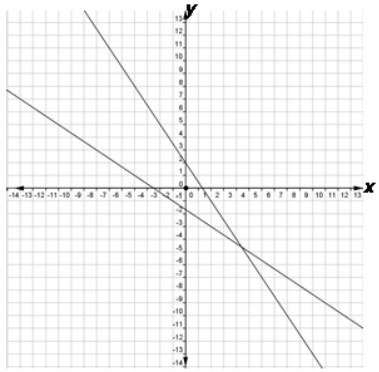
3.



2.



4.



QUESTIONS ?

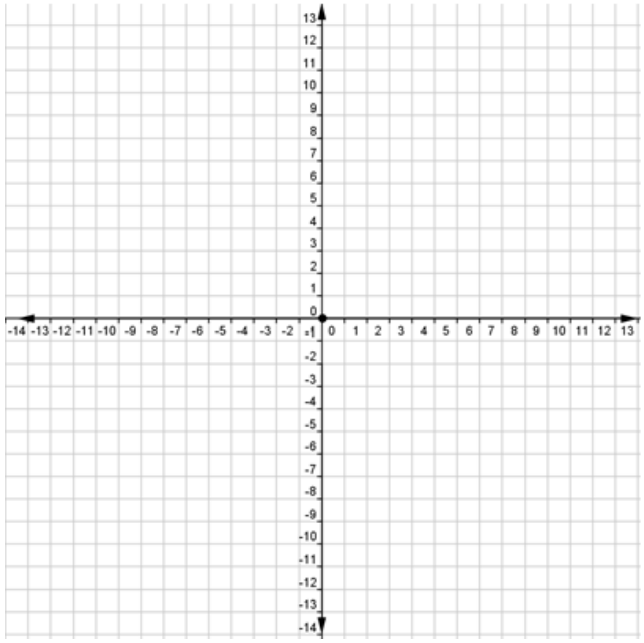
a. How many solution/s does each graph of system of linear equations have?
b. Which graph shows that the system of linear equations is consistent and dependent? consistent and independent? inconsistent? Explain your answer.
c. When do you say that the system of linear equations as described by the graph is consistent and dependent? consistent and independent? inconsistent?
d. Draw graphs of systems of linear equations that are consistent and dependent, consistent and independent, and inconsistent. Describe each graph.

Was it easy for you to describe the solution set of system of linear equations given the graph? In the next activity, you will graph systems of linear equations then describe their solution sets.

Activity 5 DESCRIBE MY SOLUTIONS!

Directions: Graph each of the following systems of linear equations in two variables on the Cartesian coordinate plane. Describe the solution set of each system based on the graph drawn. Answer the questions that follow.

1. $x + y = 8$
   $x + y = -3$
2. $3x - y = 7$
   $x + 3y = -4$
3. $x + 6y = 9$
   $2x + 6y = 18$
4. $x - 2y = 12$
   $6x + 3y = -9$
5. $3x + y = -2$
   $x + 2y = -4$



QUESTIONS ?

a. How did you graph each system of linear equations in two variables?
b. How does the graph of each system look like?
c. Which system of linear equations has only one solution? Why? How about the system of linear equations with no solution? infinite number of solutions? Explain your answer.

In this section, the discussion was about system of linear equations in two variables and their graphs.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of systems of linear equations in two variables and their graphs. After doing the following activities, you should be able to answer the following question: How is the system of linear equations in two variables used in solving real-life problems and in making decisions?

Activity 6 HOW WELL I UNDERSTOOD...

Directions: Answer the following.

1. How do you describe a system of linear equations in two variables?
2. Give at least two examples of systems of linear equations in two variables.
3. When is a system of linear equations in two variables used?
4. How do you graph systems of linear equations in two variables?
5. How do you describe the graphs of systems of linear equations in two variables?
6. How do you describe systems of linear equations that are consistent and dependent? consistent and independent? inconsistent?
7. Study the situation on the next page:

Jose wanted to construct a rectangular garden such that its perimeter is 28 m and its length is 6 times its width.

a. What system of linear equations represents the given situation?
b. Suppose the system of linear equations is graphed. How would the graphs look like?
c. Is the system consistent and dependent, consistent and independent, or inconsistent? Why?

In this section, the discussion was about your understanding of systems of linear equations in two variables and their graph.

What new realizations do you have about the systems of linear equations in two variables and their graphs? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

Activity 7 HOW MUCH AND WHAT'S THE COST?

Directions: Complete the table below by writing all the school supplies that you use. Indicate the quantity and the cost of each.

School Supply	Quantity	Cost

Formulate linear equations in two variables based from the table. Then use some pairs of these equations to form different systems of equations. Draw the graph of each system of linear equations. Use the rubric provided to rate your work.

Rubric for Real-Life Situations Involving Systems of Linear Equations in Two Variables and their Graphs

4	3	2	1
Systematically listed in the table the data, properly formulated linear equations in two variables that form a system of equations, and accurately drawn the graph of each system of linear equations.	Systematically listed in the table the data, properly formulated linear equations in two variables that form a system of equations but unable to draw the graph accurately.	Systematically listed in the table the data and formulated linear equations in two variables but unable to form systems of equations.	Systematically listed in the table the data.

In this section, your task was to cite three real-life situations where systems of linear equations in two variables are illustrated.

How did you find the performance task? How did the task help you see the real world use of the topic?

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about systems of linear equations in two variables and their graphs. The lesson provided you opportunities to describe systems of linear equations and their solution sets using practical situations, mathematical expressions, and their graphs. You identified and described systems of linear equations whose graphs are parallel, intersecting, or coinciding. Moreover, you were given the chance to draw and describe the graphs of systems of linear equations in two variables and to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the next lesson, Solving Systems of Linear Equations Graphically and Algebraically.

Lesson 2 Solving Systems of Linear Equations in Two Variables

What to Know

Start the lesson by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding Solving Systems of Linear Equations in Two Variables. As you go through this lesson, think of the following important question: H**ow is the system of linear equations in two variables used in solving real-life problems and in making decisions?** To find out the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier.

Activity 1 HOW MUCH IS THE FARE?

Directions: Use the situation below to answer the questions that follow.

Suppose for a given distance, a tricycle driver charges Php 10.00 every passenger while a jeepney driver charges Php 12.00.

1. Complete the table below for the fare collected by the tricycle and jeepney drivers from a certain number of passengers.

Number of Passengers	Amount Collected by the Tricycle Driver	Amount Collected by the Jeepney Driver
1		
2		
3		
4		
5		
10		

15		
20		
25		
30		

2. How did you determine the amount collected by the tricycle and jeepney drivers from their passengers?

3. Suppose in 3 round trips the tricycle and jeepney drivers had had a total of 68 passengers.
   a. How would you find the number of passengers each had?
   b. What mathematical statement will you use to find the number of passengers each had?

      What is the total amount of fare collected from the passengers by the two drivers? Explain how you arrived at your answer.

   c. How would you draw the graph of the mathematical statement obtained in 3b? Draw and describe the graph.

4. Suppose the total fare collected by the tricycle and jeepney drivers is Php 780.
   a. How would you find the number of passengers each carried?
   b. What mathematical statement will you use to find the number of passengers each had?
   c. How would you draw the graph of the mathematical statement obtained in 4b? Draw the graph in the Cartesian coordinate plane where the graph of the mathematical statement in 3b was drawn. Describe the graph.

5. How do you describe the two graphs drawn?

6. What do the graphs tell you?

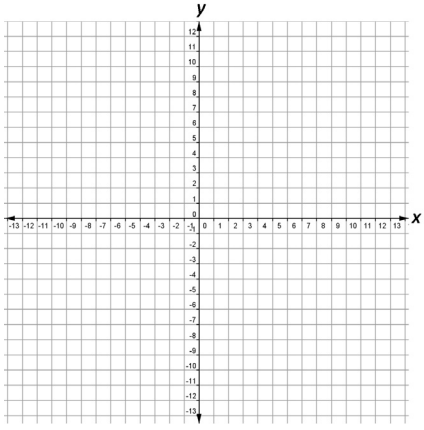
7. How did you determine the number of passengers each driver had?

How did you find the activity? Were you able to use linear equations in two variables to represent a real-life situation? Were you able to find some possible solutions of a linear equation in two variables and draw its graph? In the next activity, you will show the graphs of systems of linear equations in two variables. You need this skill to learn about the graphical solutions of systems of linear equations in two variables.

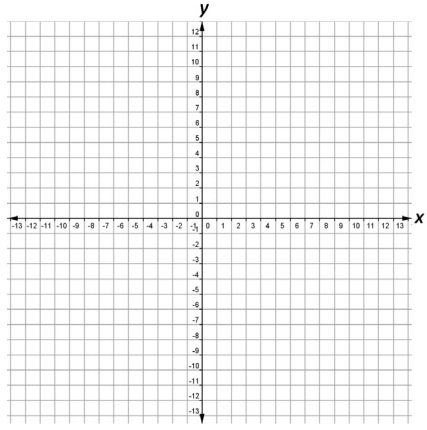
Activity 2 LINES, LINES, LINES...

Directions: Use the situation below to answer the questions that follow.

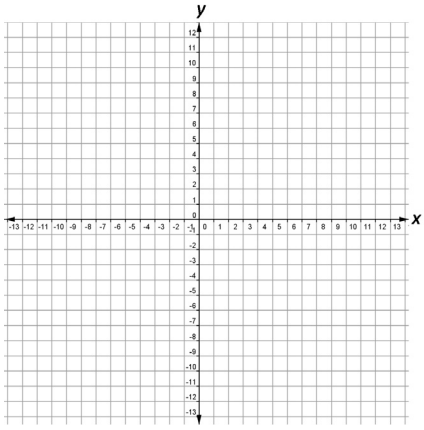
1. $y = x + 7$
   $y = -2x + 1$



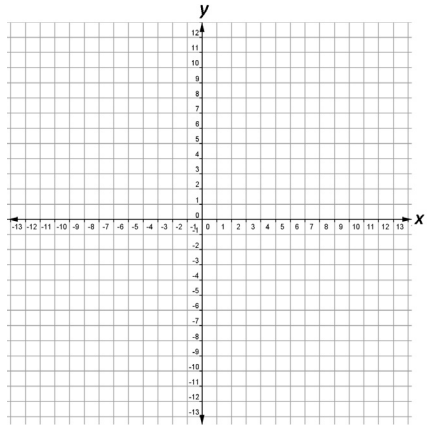
3. $3x + 8y = 12$
   $8x - 5y = 12$



2. $y = 3x - 2$
   $8x + 7y = 15$



4. $x - y = 6$
   $2x + 7y = -6$



QUESTIONS ?

a. How did you show the graph of each system of equations?
b. How do you describe the graph of each system of equations?
c. Are the graphs intersecting lines? If yes, what are the coordinates of the point of intersection of these lines?
d. What do you think do the coordinates of the point of intersection of the lines mean?

Were you able to draw the graph of each system of linear equations in two variables? Were you able to determine and give the meaning of the coordinates of the point of intersection of intersecting lines? As you go through this module, you will learn about this point of intersection of two lines and how the coordinates of this point are determined algebraically. In the next activity, you will solve for the indicated variable in terms of the other variable. You need this skill to learn about solving systems of linear equations in two variables using the substitution method.

Activity 3 IF I WERE YOU...

Directions: Solve for the indicated variable in terms of the other variable. Explain how you arrived at your answer.

1. $4x + y = 11$; $y =$
2. $5x - y = 9$; $y =$
3. $4x + y = 12$; $x =$
4. $-5x - 4y = 16$; $y =$
5. $2x + 3y = 6$; $y =$
6. $-2x + 7y = 18$; $x =$
7. $-3x - 8y = 15$; $x =$
8. $\frac{1}{4}x + 3y = 2$; $x =$
9. $\frac{4}{9}x - \frac{1}{3}y = 7$; $y =$
10. $-\frac{2}{3}x - \frac{1}{2}; y = 8$ $x =$

How did you find the activity? Were you able to solve for the indicated variable in terms of the other variable? In the next activity, you will solve linear equations. You need this skill to learn about solving systems of linear equations in two variables algebraically.

Activity 4 WHAT MAKES IT TRUE?

Directions: Find the value of the variable that would make the equation true. Answer the questions that follow.

1. $5x = 15$
2. $-3x = 21$
3. $9x = -27$
4. $-7x = -12$
5. $\frac{2}{3}x = 8$
6. $x + 7 = 10$
7. $3y - 5 = 4$
8. $2y + 5y = -28$
9. $-3y + 7y = 12$
10. $5x - 2x = -15$

QUESTIONS ?

a. How did you solve each equation?
b. What mathematics concepts or principles did you apply to solve each equation? Explain how you applied these mathematics concepts and principles.
c. Do you think there are other ways of solving each equation? Explain your answer.

Were you able to solve each equation? In solving each equation, were you able to apply the mathematics concepts or principles which you already learned? Solving equations is an important skill that you need to fully develop so you would not find difficulty in solving systems of linear equations in two variables algebraically. But how are systems of linear equations in two variables used in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before you start performing these activities, read and understand first some important notes on solving systems of linear equations and the examples presented.

The solution of a system of linear equations can be determined algebraically or graphically. To find the solution graphically, graph both equations on a Cartesian coordinate plane then find the point of intersection of the graphs, if it exists. You may also use graphing calculator or computer software such as GeoGebra in determining the graphical solutions of systems of linear equations. GeoGebra is a dynamic mathematics software which helps you visualize and understand concepts in algebra, geometry, calculus, and statistics.

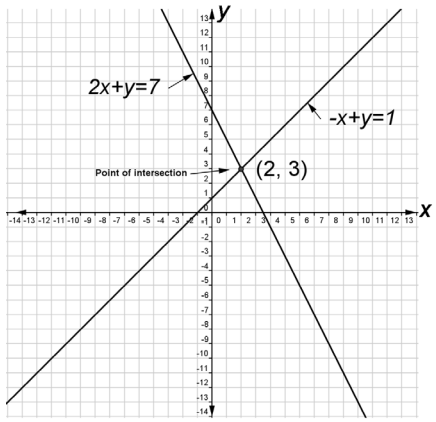
The solution to a system of linear equations corresponds to the coordinates of the points of intersection of the graphs of the equations.

Examples: Find the solutions of the following systems of linear equations graphically.

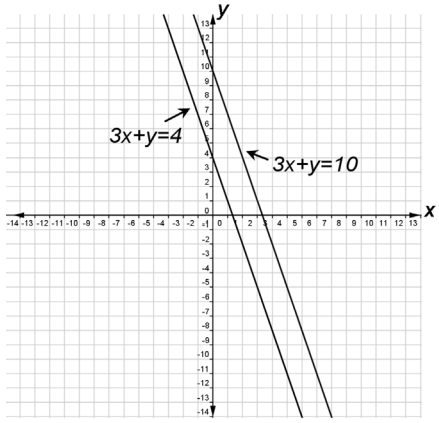
a. $\begin{matrix} 2x + y = 7 \\ -x + y = 1 \end{matrix}$ b. $\begin{matrix} 3x + y = 4 \\ 3x - y = -5 \end{matrix}$ c. $\begin{matrix} x - 2 = -5 \\ 2x - 4y = -10 \end{matrix}$

Answer (a): The graphs of $2x + y = 7$ and $-x + y = 1$ intersect at (2, 3).

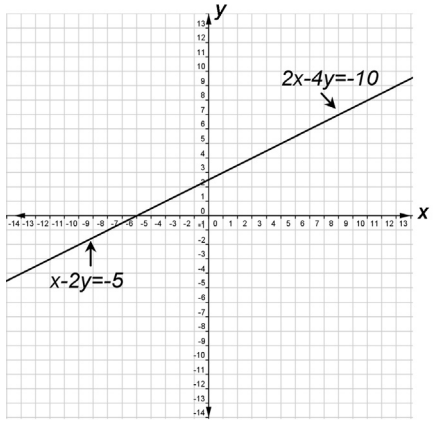
Hence, the solution of the system $\begin{matrix} 2x + y = 7 \\ -x + y = 1 \end{matrix}$ is $x = 2$ and $y = 3$.



Answer (b): The graphs of $3x + y = 4$ and $3x + y = 10$ are parallel. Hence, the system $\begin{matrix} 3x + y = 4 \\ 3x - y = -5 \end{matrix}$ has no solution.



Answer (c): The graphs of $x - 2y = -5$ and $2x - 4y = -10$ coincide. Hence, the system $\begin{matrix} x - 2 = -5 \\ 2x - 4y = -10 \end{matrix}$ has infinite number of solutions.



A system of linear equations can be solved algebraically by **substitution** or **elimination methods.**

To solve a system of linear equations by substitution method, the following procedures could be followed:

a. Solve for one variable in terms of the other variable in one of the equations. If one of the equations already gives the value of one variable, you may proceed to the next step.
b. Substitute to the second equation the value of the variable found in the first step. Simplify then solve the resulting equation.
c. Substitute the value obtained in (b) to any of the original equations to find the value of the other variable.
d. Check the values of the variables obtained against the linear equations in the system.

Example: Solve the system $\begin{matrix} 2x + y = 5 \\ -x + 2y = 5 \end{matrix}$ by substitution method.

Solution: Use $2x + y = 5$ to solve for y in terms of x.

Subtract $-2x$ from both sides of the equation.

$2x + y - 2x = 5 - 2x \rightarrow y = 5 - 2x$

Substitute $5 - 2x$ in the equation $-x + 2y = 5$.

$-x + 2(5 - 2x) = 5$

Simplify.

$-x + 2(5) + 2(-2x) = 5 \rightarrow -x + 10 - 4x = 5$

$-5x = 5 - 10 \rightarrow -5x = -5$

Solve for x by dividing both sides of the equation by -5.

$\frac{-5x}{-5} = \frac{-5}{-5} \rightarrow x = 1$

Substitute 1, value of x, to any of the original equations to solve for y.

$-x + 2y = 5 \rightarrow -1 + 2y = 5$

Simplify.

$-1 + 2y = 5 \rightarrow 2y = 5 + 1 \rightarrow 2y = 6$

Solve for y by dividing both sides of the equation by 2.

$\frac{2y}{2} = \frac{6}{2} \rightarrow y = 3$

Check the values of the variables obtained against the linear equations in the system.

1. $2x + y = 5$; $x = 1$ and $y = 3$
   $2(1) + 3 = 5 \rightarrow 2 + 3 = 5 \rightarrow 5 = 5$
   Hence, $x = 1$ and $y = 3$ are true to $2x + y = 5$.

2. $-x + 2y = 5$; $x = 1$ and $y = 3$
   $-1 + 2(3) = 5 \rightarrow -1 + 6 = 5 \rightarrow 5 = 5$
   Hence, $x = 1$ and $y = 3$ are true to -x + 2y = 5

Therefore, the solution to the system $\begin{matrix} 2x + y = 5 \\ -x + 2y = 5 \end{matrix}$ is the ordered pair (1, 3).

To solve a system of linear equations in two variables by the elimination method, the following procedures could be followed:

a. Whenever necessary, rewrite both equations in standard form $Ax + By = C$.
b. Whenever necessary, multiply either equation or both equations by a nonzero number so that the coefficients of x or y will have a sum of 0. (Note: The coefficients of x and y are additive inverses.)
c. Add the resulting equations. This leads to an equation in one variable. Simplify then solve the resulting equation.
d. Substitute the value obtained to any of the original equations to find the value of the other variable.
e. Check the values of the variables obtained against the linear equations in the system.

Example: Solve the system $\begin{matrix} 3x + y = 7 \\ 2x - 5y = 16 \end{matrix}$ by elimination method.

Solution: Think of eliminating y first.
Multiply 5 to both sides of the equation $3x + y = 7$.

$5(3x + y = 7) \rightarrow 15x + 5y = 35$

Add the resulting equations.

$$\begin{array}{r} 15x + 5y = 35 \\ 2x - 5y = 16 \\ \hline 17x \quad = 51 \end{array}$$

Solve for x by dividing both sides of the equation by 17.

$17x = 51 \rightarrow \frac{17x}{17} = \frac{51}{17} \rightarrow x = 3$

Substitute 3, value of x, to any of the original equations to solve for y.

$2x - 5y = 16 \rightarrow 2(3) - 5y = 16$

Simplify.

$6 - 5y = 16 \rightarrow -5y = 16 - 6 \rightarrow -5y = 10$

Solve for y by dividing both sides of the equation by -5.

$$-5y = 10 \quad \rightarrow \quad \frac{-5y}{-5} = \frac{10}{-5} \quad \rightarrow \quad y = -2$$

Check the values of the variables obtained against the linear equations in the system.

1. $3x + y = 7$; $x = 3$ and $y = -2$
   $3(3) + (-2) = 7 \rightarrow 9 - 2 = 7 \rightarrow 7 = 7$
   Hence, $x = 3$ and $y = -2$ are true to $3x + y = 7$.

2. $2x - 5y = 16$; $x = 3$ and $y = -2$
   $2(3) - 5(-2) = 16 \rightarrow 6 + 10 = 16 \rightarrow 16 = 16$
   Hence, $x = 3$ and y = -2 are true to $2x - 5y = 16$.

Therefore, the solution to the system $\begin{matrix} 3x + y = 7 \\ 2x - 5y = 16 \end{matrix}$ is the ordered pair (3, -2).

Systems of linear equations in two variables are applied in many real-life situations. They are used to represent situations and solve problems related to uniform motion, mixture, investment, work, and many others. Consider the situation below.

A computer shop hires 12 technicians and 3 supervisors for total daily wages of Php 7,020. If one of the technicians is promoted to a supervisor, the total daily wages become Php 7,110.

In the given situation, what do you think is the daily wage for each technician and supervisor? This problem can be solved using system of linear equations.

Let x = daily wage of a technician and y = daily wage of a supervisor. Represent the total daily wages before one of the technicians is promoted to a supervisor.

$$12x + 3y = 7{,}020$$

Represent the total daily wages after one of the technicians is promoted to a supervisor.

$$11x + 4y = 7{,}110$$

Use the two equations to find the daily wages for a technician and a supervisor.

$$12x + 3y = 7{,}020$$
$$11x + 4y = 7{,}110$$

Solve the system graphically or by using any algebraic method.

Let's solve the system using Elimination Method. Multiply both sides of the first equation by 4 and the second equation by 3 to eliminate y.

$$\begin{matrix} 12x + 3y = 7{,}020 \\ 11x + 4y = 7{,}110 \end{matrix} \rightarrow \begin{matrix} 4(12x + 3y = 7{,}020) \\ 3(11x + 4y = 7{,}110) \end{matrix} \rightarrow \begin{matrix} 48x + 12y = 28{,}080 \\ 33x + 12y = 21{,}330 \end{matrix}$$

The resulting system of linear equations is $\begin{matrix} 48x + 12y = 28{,}080 \\ 33x + 12y = 21{,}330 \end{matrix}$

.

Subtract the terms on both sides of the resulting equations.

$$\begin{array}{l} 48x + 12y = 28{,}080 \\ \underline{33x + 12y = 21{,}330} \\ 15x \qquad = 6{,}750 \end{array}$$

Using the equation $15x = 6{,}750$, solve for x by dividing both sides of the equation by 15.

$$15x = 6{,}750 \quad \rightarrow \quad \frac{15x}{15} = \frac{6{,}750}{15} \quad \rightarrow \quad x = 450$$

The daily wage of a technician is **Php 450.**

Learn more about Systems of Linear Equations in Two Variables and their Graphs through the WEB. You may open the following links.

1. http://www.mathguide.com/lessons/Systems.html
2. http://www.mathwarehouse.com/algebra/linear_equation/systems-of-equation/index.php
3. http://edhelper.com/LinearEquations.htm
4. http://www.purplemath.com/modules/systlin1.htm
5. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-229s.html
6. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-232s.html
7. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-233s.html
8. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-234s.html
9. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-235s.html
10. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-236s.html

Find the daily wage of a supervisor by substituting 450 to x in any of the original equations. Then, solve the resulting equation.

$$\begin{array}{lll} 12x + 3y = 7{,}020; & x = 450 & \\ 12(450) + 3y = 7{,}020 & \rightarrow \quad 5{,}400 + 3y = 7{,}020 & \\ & \rightarrow \quad 3y = 7{,}020 - 5{,}400 & \\ & \rightarrow \quad 3y = 1{,}620 & \rightarrow \frac{3y}{3} = \frac{1{,}620}{3} \\ & \rightarrow \quad y = 540 & \end{array}$$

The daily wage of a supervisor is **Php 540.**

Answer: The daily wages for a technician and a supervisor are **Php 450 and Php 540**, respectively.

You have seen how system of linear equations is used to solve real-life problem. In what other real-life situations are systems of linear equations in two variables illustrated or applied? How is the system of linear equations in two variables used in solving real-life problems and in making decisions?

Now that you learned about solving systems of linear equations in two variables graphically and algebraically, you may now try the activities in the next section.

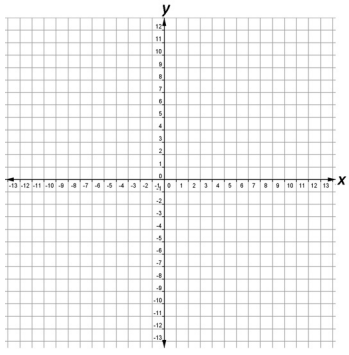
What to Process

Your goal in this section is to learn and understand solving systems of linear equations graphically and algebraically. Use the mathematical ideas and the examples presented in the preceding section in answering the activities provided.

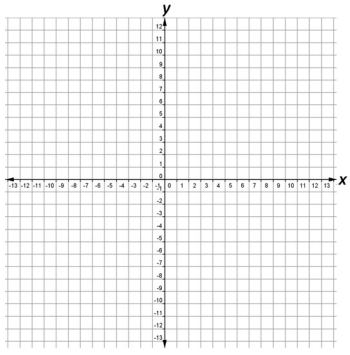
Activity 5 WHAT SATISFIES BOTH?

Directions: Solve each of the following systems of linear equations graphically then check. You may also use GeoGebra to verify your answer. If the system of linear equations has no solution, explain why.

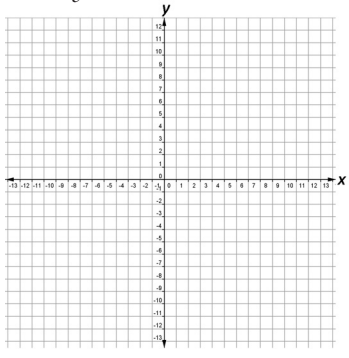
1. $x + y = -7$
   $y = x + 1$



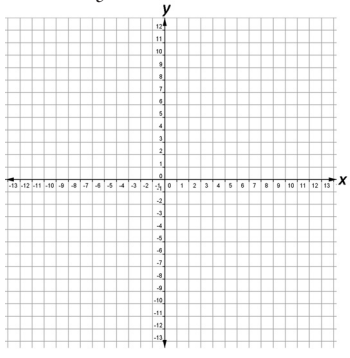
3. $3x + y = 2$
   $2y = 4 - 6x$



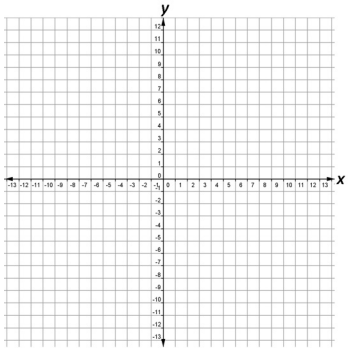
2. $x - y = 5$
   $x + 5y = -7$



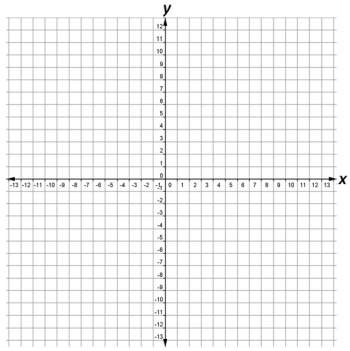
4. $x + y = 4$
   $2x - 3y = 3$



5. $y = 5x - 2$
$5x - 3y = -14$



6. $2x - 3y = 5$
$3y = 10 + 2x$



Were you able to determine the solution of each system of linear equations in two variables graphically? In the next activity, you will determine the resulting equation when the value of one variable is substituted to a given equation.

Activity 6 TAKE MY PLACE!

Directions: Determine the resulting equation by substituting the given value of one variable to each of the following equations. Then solve for the other variable using the resulting equation. Answer the questions that follow.

Equation	Value of Variable	Equation	Value of Variable
1. $4x + y = 7$;	y: $x + 3$	4. $5x + 2y = 8$;	x: $3y + 1$
2. $x + 3y = 12$;	x: $4 - y$	5. $4x - 7y = -10$;	y: $x - 4$
3. $2x - 3y = 9$;	y: $x - 2$	6. $-5x = y - 4$;	y: $3x + 5$

QUESTIONS ?

a. How did you determine each resulting equation?
b. What resulting equations did you arrive at?
c. How did you solve each resulting equation?
d. What mathematics concepts or principles did you apply to solve each resulting equation?
e. How will you check if the value you got is a solution of the equation?

How did you find the activity? Do you think it would help you perform the next activity? Find out when you solve systems of linear equations using the substitution method.

Activity 7 SUBSTITUTE THEN SOLVE!

Directions: Determine the resulting equation by substituting the given value of one variable to each of the following equations. Then solve for the other variable using the resulting equation. Answer the questions that follow.

1. $x + y = 8$
   $y = x + 6$
2. $x = -y + 7$
   $x - y = -9$
3. $y = 2x$
   $4x + 3y = 20$
4. $y = 2x + 5$
   $3x - 2y = -5$
5. $2x + 5y = 9$
   $-x + y = 2$
6. $3x + y = 2$
   $9x + 2y = 7$
7. $x - y = -3$
   $3x + y = 19$
8. $4x + y = 6$
   $x - 2y = 15$
9. $2x + y = 10$
   $4x + 2y = 5$
10. $-x + 3y = -2$
    $-3x + 9y = -6$

a. How did you use substitution method in finding the solution set of each system of linear equations?
b. How did you check the solution set you got?
c. Which system of equations is difficult to solve? Why?
d. Which system of equations has no solution? Why?
e. Which system of equations has infinite number of solutions? Explain your answer.

Were you able to find the solution set of each system of linear equations? Do you think this is the most convenient way of solving a system of equations? In the next activity, you will be determining the number(s) that must be multiplied to the terms of one or both equations in a system of equations. This will lead you in finding the solution set of a system of linear equations in two variables using the elimination method.

Activity 8 ELIMINATE ME!

Directions: Determine the number(s) that must be multiplied to one or both equations in each system to eliminate one of the variables. Justify your answer.

1. $5x - 2y = 12$
   $2x + y = 7$
2. $x + 3y = 5$
   $4x + 2y = 7$

To eliminate x	To eliminate y

3. $x - 4y = 12$
   $5x + y = -5$

4. $-3x + 2y = 7$
   $5x - 4y = -2$

5. $-2x - 5y = 10$
   $3x - 2y = 6$

6. $9x - 5y = 8$
   $3x + 7y = 12$

To eliminate x	To eliminate y

How did you find the activity? Do you think it would help you perform the next activity? Find out when you solve systems of linear equations using the elimination method.

Activity 9 ELIMINATE THEN SOLVE!

Directions: Solve each system of linear equations by elimination method then check. Answer the questions that follow.

1. $3x + 2y = -4$
   $2x - y = -12$

2. $7x - 2y = 4$
   $5x + y = 15$

3. $5x + 2y = 6$
   $-2x + y = -6$

4. $2x + 3y = 7$
   $3x - 5y = 1$

5. $x - 4y = 9$
   $3x - 2y = 7$

6. $3x + 7y = 12$
   $5x - 4y = 20$

7. $2x + y = 9$
   $x - 2y = 6$

8. $5x + 2y = 10$
   $3x - 7y = -4$

9. $2x + 7y = -5$
   $3x - 8y = -5$

10. $-3x + 4y = -12$
    $2x - 5y = 6$

QUESTIONS ?

a. How did you use the elimination method in solving each system of linear equations?
b. How did you check the solution set you got?
c. Which system of equations is difficult to solve? Why?
d. When is the elimination method convenient to use?
e. Among the three methods of solving systems of linear equations in two variables, which do you think is the most convenient to use? Which do you think is not? Explain your answer.

In this section, the discussion was about solving systems of linear equations in two variables using the graphical and algebraic methods.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about solving systems of linear equations in two variables, let's go deeper by moving on to the next section.

What to Understand

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of the different methods of solving systems of linear equations in two variables. After doing the following activities, you should be able to answer the following question: **How is the system of linear equations in two variables used in solving real-life problems and in making decisions?**

Activity 10 LOOKING CAREFULLY AT THE GRAPHS...

Directions: Answer the following questions:

1. How do you determine the solution set of a system of linear equations from its graph?
2. Do you think it is easy to determine the solution set of a system of linear equations by graphing? Explain your answer.
3. When are the graphical solutions of systems of linear equations difficult to determine?
4. How would you check if the solution set you found from the graphs of linear equations in a system are the solutions?
5. What do you think are the advantages and the disadvantages of the graphical method of solving systems of linear equations? Explain your answer.

Were you able to answer all the questions in the activity? Do you have better understanding now of the graphical method of solving systems of linear equations? In the next activity, you will be given the opportunity to deepen your understanding of solving systems of linear equations using the substitution method.

Activity 11 HOW SUBSTITUTION WORK...

Directions: Use the system of linear equations $\begin{matrix} 5x - 2y = 3 \\ 2x + y = 12 \end{matrix}$ to answer the following:

1. How do you describe each equation in the system?
2. How will you solve the given system of equations?
3. Do you think the substitution method is more convenient to use in finding the solution set of the system? Explain your answer.
4. What is the solution set of the given system of equations? Explain how you arrived at your answer.
5. When is the substitution method convenient to use in solving systems of linear equations?
6. Give two examples of systems of linear equations in two variables then solve using the substitution method.

How did you find the activity? Were you able to have better understanding of the substitution method of solving systems of linear equations? In the next activity, you will be given the opportunity to deepen your understanding of solving systems of linear equations using the elimination method.

Activity 12 ELIMINATE ONE TO FIND THE OTHER ONE

Directions: Use the system of linear equations $\begin{matrix} 3x - 5y = 8 \\ 2x + 7y = 6 \end{matrix}$ to answer the following questions:

1. How do you describe each equation in the system?
2. How will you solve the given system of equations?
3. Which algebraic method of solving system of linear equations do you think is more convenient to use in finding its solution set? Why?
4. What is the solution set of the given system of equations? Explain how you arrived at your answer.
5. When is the elimination method convenient to use in solving systems of linear equations?
6. Give two examples of systems of linear equations in two variables then solve using the elimination method.

The activity provided you opportunities to deepen your understanding of solving systems of linear equations in two variables using the elimination method. You were able to find out also systems of linear equations that can be solved conveniently by using substitution or elimination method. In the next activity, you will extend your understanding of systems of linear equations in two variables as to how they are used in solving real-life problems.

Activity 13 SOLVE THEN DECIDE!

Directions: Answer each of the following questions. Show your complete solutions and explanations/justifications.

1. Which of the following is more economical when renting a vehicle? Justify your answer.

   LG's Rent a Car: Php 1,500 per day plus Php 35 per kilometer traveled
   Rent and Drive: Php 2,000 per day plus Php 25 per kilometer traveled

2. Luisa sells two brands of Tablet PC. She receives a commission of 12% on one brand and 8% on the other brand. If she is able to sell 2 Tablet PC's, one for each brand, the total cost is Php 42,000 and the amount that she will receive as commission is Php 4,400.

   a. What is the cost of each brand of Tablet PCs?
   b. How much commission does she receive on one brand? How about on the other brand?
   c. Suppose you are Luisa and you wish to earn more. Which Tablet PC will you ask the customers or clients to buy? Why?

3. Which of the following mobile networks has a better offer? Justify your answer.

   World Celcom: Php 500 monthly charge
   Free calls and texts to World Celcom subscribers
   Php 6.50 per minute of call to other networks

   Smartlink: Php 650 monthly charge
   Free calls and texts to Smartlink subscribers
   Php 5 per minute of call to other networks

4. Mr. Salonga has two investments. His total investment is Php 400,000. He receives 3% interest on one investment and 7% interest on the other. The total interest that Mr. Salonga receives in a year is Php 16,000.

   a. How much money does Mr. Salonga have in each investment?
   b. Suppose you were Mr. Salonga. In which investment will you place more money? Why?

5. The school canteen sells chicken and egg sandwiches. It generates an income of Php 2 for every chicken sandwich sold and Php 1.25 for every egg sandwich. Yesterday, they were able to sell all 420 sandwiches prepared and generated an income of Php 615. The teacher in charge of the canteen realized that the canteen could have earned more if additional sandwiches are prepared.

   a. How many chicken sandwiches was the canteen able to sell on that day? How about egg sandwiches?
   b. If you were the teacher in charge of the canteen, which kind of sandwich would you prepare more? Why?

What new insights do you have about solving systems of linear equations? What new connections have you made for yourself?

Let's extend your understanding. This time, apply what you have learned in real life by doing the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task in which you will demonstrate your understanding of solving systems of linear equations in two variables.

Activity 14 PLAY THE ROLE OF ...

Cite situations in real life where systems of linear equations in two variables are applied. Form a group of 5 members and role play each situation. With your groupmates, formulate problems out of these situations then solve in as many ways as you can.

Activity 15 SELECT THE BEST POSTPAID PLAN

1. Make a list of all postpaid plans being offered by different mobile network companies.
2. Use the postpaid plans to formulate problems involving systems of linear equations in two variables. Solve the problems formulated. Use the rubric provided to rate your work.
3. Determine the best postpaid plan that each company offers. Explain your answer.
4. Determine the mobile network company that you will recommend to your parents, older brothers or sisters, or relatives if ever they apply for a postpaid plan. Justify your choice.

Rubric on Problems Formulated and Solved

Score	Descriptors
6	Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate.
5	Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
4	Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
3	Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details.
2	Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension.
1	Poses a problem but demonstrates minor comprehension, not being able to develop an approach.

Source: D.O. #73 s. 2012

In this section, your tasks were to cite real-life situations and formulate and solve problems involving systems of linear equations in two variables.

How did you find the performance task? How did the task help you see the real world application of systems of linear equations in two variables?

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about solving systems of linear equations in two variables using the graphical and algebraic methods namely: substitution and elimination methods. In this lesson, you were able to find different ways of finding the solutions of systems of linear equations and given the opportunity to determine the advantages and disadvantages of using each method and which is more convenient to use. Using the different methods of solving systems of linear equations, you were able to find out which system has no solution, one solution, and infinite number of solutions. More importantly, you were given the chance to formulate and solve real-life problems, make decisions based on the problems, and demonstrate your understanding of the lesson by doing some practical tasks. Your understanding of this lesson will be extended in the next lesson, Graphical Solutions of Systems of Linear Inequalities in Two Variables. The mathematical skills you acquired in finding the graphical solutions of systems of linear equations can also be applied in the next lesson.

Lesson 3 Graphical Solutions of Systems of Linear Inequalities in Two Variables

What to Know

Start Lesson 3 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding Graphical Solutions of Systems of Linear Inequalities in Two Variables. As you go through this lesson, think of the following important question: **How is the system of linear inequalities in two variables used in solving real-life problems and in making decisions?** To find out the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier.

Activity 1 SUMMER JOB

Directions: Use the situation below to answer the questions that follow.

Nimfa lives near a beach resort. During summer vacation, she sells souvenir items such as bracelets and necklaces which are made of local shells. Each bracelet costs Php 85 while each piece of necklace is Php 110. She needs to sell at least Php 15,000 worth of bracelets and necklaces.

QUESTIONS ?

1. Complete the table below.

Number of bracelets sold	Cost	Number of necklaces sold	Cost	Total Cost
1		1		
2		2		
3		3		

4		4		
5		5		
10		10		
15		15		
20		20		
25		25		
30		30		
40		40		
50		50		
60		60		
80		80		
100		100		

2. How much would Nimfa's total sale if she sells 5 pieces of bracelets and 5 pieces of necklaces?

   How about if she sells 10 pieces of bracelets and 20 pieces of necklaces?

3. What mathematical statement would represent the total sale of bracelets and necklaces? Describe the mathematical statement then graph.

4. Nimfa wants to have a total sale of at least Php 15,000. What mathematical statement would represent this? Describe the mathematical statement then graph.

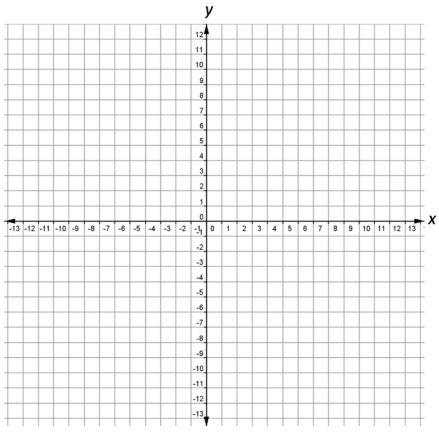
5. How many bracelets and necklaces should Nimfa sell to have a total sale of at least Php 15,000? Give as many answers as possible then justify.

How did you find the activity? Were you able to use linear inequalities in two variables to represent a real-life situation? Were you able to find some possible solutions of a linear inequality in two variables and draw its graph? In the next activity, you will show the graphs of systems of linear equations and inequalities in two variables. You need this skill to learn about the graphical solutions of systems of linear inequalities in two variables.

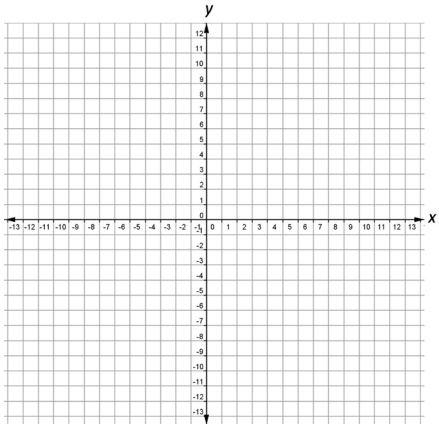
Activity 2 A LINE OR HALF OF A PLANE?

Directions: Draw the graphs of the following linear equations and inequalities in two variables. Answer the questions that follow.

1. $3x + y = 10$
2. $5x - y = 12$
3. $2x + 3y = 15$
4. $3x - 4y = 8$
5. $4x + 7y = -8$



6. $3x + y < 10$
7. $5x - y > 12$
8. $2x + 3y \leq 15$
9. $3x - 4y \geq 8$
10. $4x + 7y < -8$



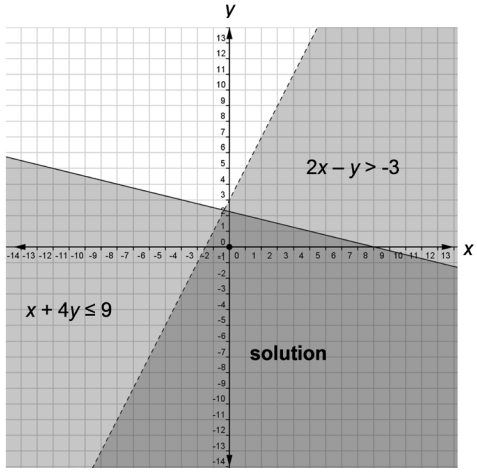
a. How did you graph each mathematical statement?
b. Compare the graphs of $3x + y = 10$ and $3x + y < 10$. What statements can you make?
How about $5x - y = 12$ *and* $5x - y > 12$*?* $2x + 3y = 15$ *and* $2x + 3y \leq 15$*?*
c. How do you differentiate the graphs of linear equations and inequalities in two variables?
d. How many solutions does a linear equation in two variables have? How about linear inequalities in two variables?
e. Suppose you draw the graphs of $3x + y < 10$ and $5x - y > 12$ on another Cartesian coordinate plane. How would you describe their graphs? What ordered pairs satisfy both inequalities?

Were you able to draw the graph of each mathematical statement? Were you able to compare the graphs of linear equations and inequalities in two variables? Were you able to determine the ordered pairs that satisfy two linear inequalities? Finding solutions of linear inequalities leads you in understanding the graphical solutions of systems of linear inequalities in two variables. But how are systems of linear inequalities in two variables used in solving real-life problems and in making decisions? You will find these out in the activities in the next section. To prepare yourself in performing these activities, read and understand first some important notes on the Graphical Solutions of Systems of Linear Inequalities in Two Variables and the examples presented.

To solve a system of inequalities in two variables by graphing, draw the graph of each inequality on the same rectangular coordinate plane. Each time, shade the solution set of each inequality. The solution set of the system is the region where the shadings overlap.

Example: To solve the system $\begin{matrix} 2x - y > -3 \\ x + 4y \leq 9 \end{matrix}$ graphically, graph $2x - y > -3$ and $x + 4y \leq 9$ on the same Cartesian coordinate plane. The region where the shadings overlap is the graph of the solution to the system.

Like systems of linear equations in two variables, systems of linear inequalities are also applied in many real-life situations. They are used to represent situations and solve problems related to uniform motion, mixture, investment, work, and many others.

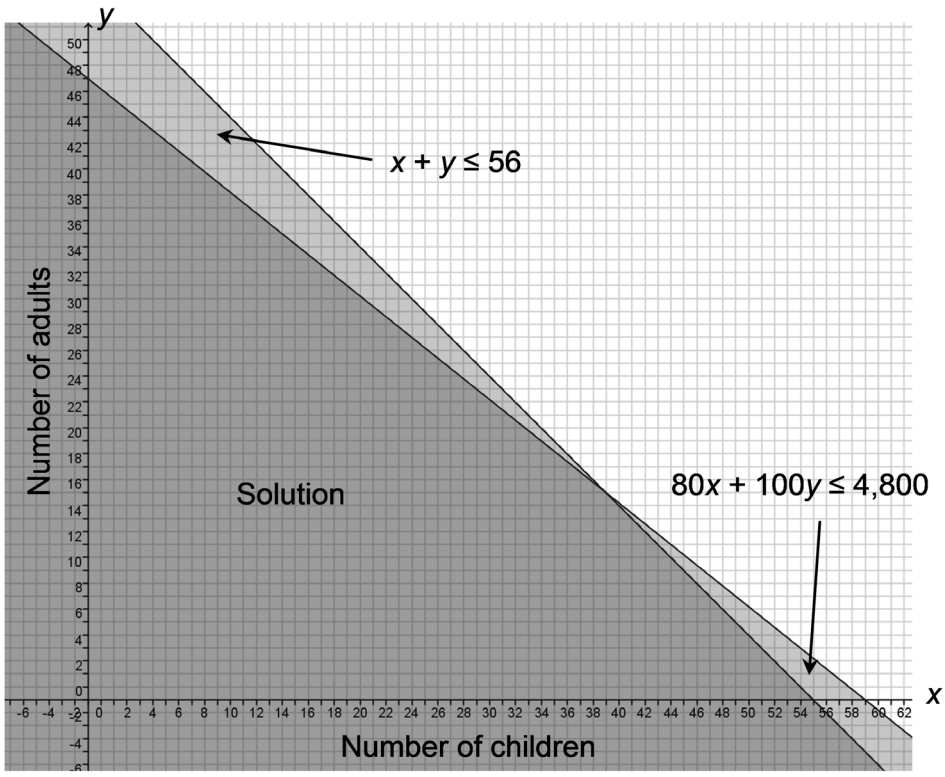


Example: There are at most 56 people composed of children and adults who are in a bus. Each child and adult paid Php 80 and Php 100, respectively. If the total amount collected was not more than Php 4,800, how many children and adults are in the bus?

Solution: Let x = number of children in the bus
y = number of adults in the bus

Represent the number of people in the bus as $x + y \leq 56$.
Represent the amount collected as $80x + 100y \leq 4{,}800$.

Use the two inequalities to find the number of children and adults who are in the bus. Write these as a system of linear inequalities then solve graphically.



The region where the shadings overlap is the graph of the solution to the system. Consider any point in this shaded region then substitute its coordinates in the system to check.

Consider the point whose coordinates are (20, 30). Check this against the inequalities $x + y \leq 56$ and $80x + 100y \leq 4{,}800$.

If $x = 20$ and $y = 30$, then $20 + 30 \leq 56$. The first inequality is satisfied.

If $x = 20$ and $y = 30$, then $80(20) + 100(30) \leq 4{,}800$ or $1{,}600 + 3{,}000 \leq 4{,}800$ or $4{,}600 \leq 4{,}800$.

The second inequality is also satisfied. This means that one possible number of children in the bus is 20 and the number of children is 30.

However, not all points in the region where the shadings overlap are solutions to the given situation. Only those values of x greater than or equal to zero ($x \geq 0$) and those values of y greater than or equal to zero ($x \geq 0$) can only be considered. Can you think of the reason? Definitely, the number of children and adults can never be negative.

Learn more about Systems of Linear Equations in Two Variables and their Graphs through the WEB. You may open the following links.

1. http://www.purplemath.com/modules/syslneq.htm
2. https://new.edu/resources/solving-systems-of-linear-inequalities-two-variables
3. http://www.netplaces.com/algebra-guide/graphing-linear-relationships/graphing-linear-inequalities-in-two-variables.htm
4. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-238s.html
5. http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-240s.html

You have seen how system of linear inequalities in two variables is used to solve real-life problem. In what other real-life situations are systems of linear inequalities in two variables illustrated or applied? How is the system of linear inequalities in two variables used in solving real-life problems and in making decisions?

Now that you learned about the graphical solutions of systems of linear inequalities in two variables, you may now try the activities in the next section.

What to Process

Your goal in this section is to learn and understand solving systems of linear inequalities in two variables graphically. Use the mathematical ideas and the examples presented in answering the activities provided.

Activity 3 DO I SATISFY YOU?

Directions: Determine if each ordered pair is a solution to the system of linear inequality $2x + 5y < 10$ / $3x - 4y \geq -8$. Then, answer the questions that follow.

1. (3, 5)
2. (-2, -10)
3. (5, -12)
4. (-6, -8)
5. (0, 0)
6. (2, 15)
7. (-6, 10)
8. (-12, 1)
9. (0, 2)
10. (5, 0)

QUESTIONS ?

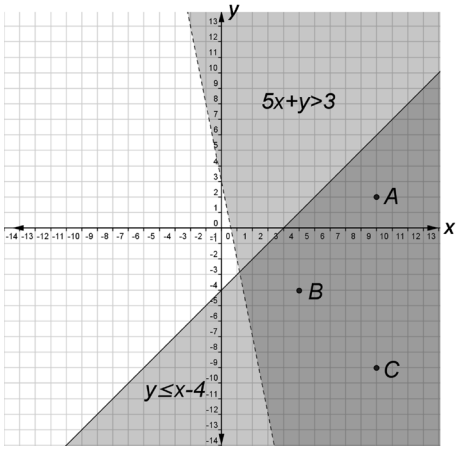
a. How did you determine if the given ordered pair is a solution of the system?
b. How did you know if the given ordered pair is not a solution of the system?
c. How many solutions do you think the system of inequalities has?

Were you able to find out if each ordered pair is a solution of the given system of linear inequalities in two variables? In the next activity, you will determine the graphical solutions of systems of linear inequalities in two variables.

Activity 4
AM I IN THAT REGION?

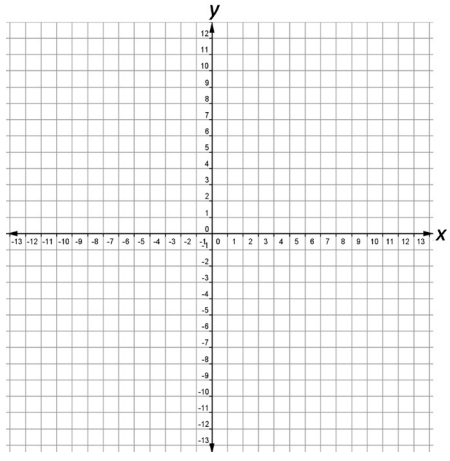
Directions: Solve the following systems of inequalities graphically then give three ordered pairs satisfying the inequalities. Show that the ordered pairs satisfy the inequalities. Answer the questions that follow. The first one is done for you.

1. $5x + y > 3$
   $y \leq x - 4$

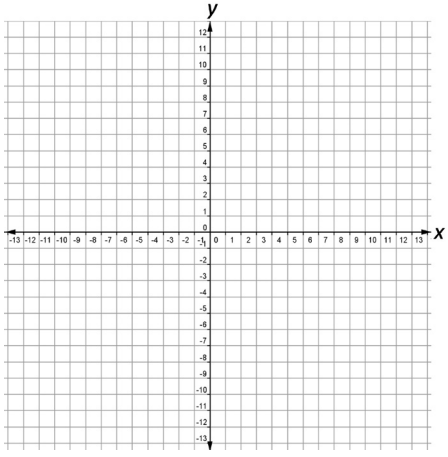


Some ordered pairs satisfying the system of inequalities are (10, 2), (5, -4), and (10, -9).

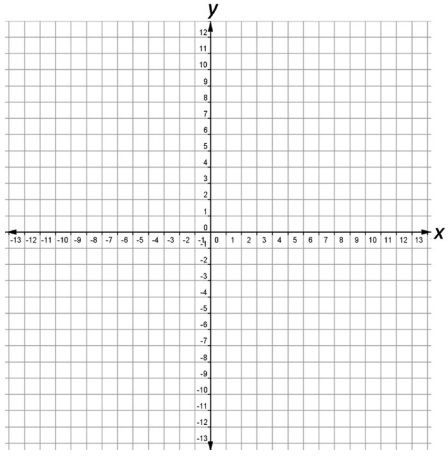
3. $2x - y \geq -2$
   $y < x + 4$



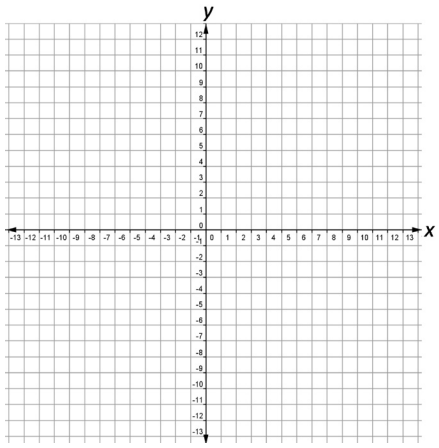
2. $x + y \geq 7$
   $3x - y \leq 10$



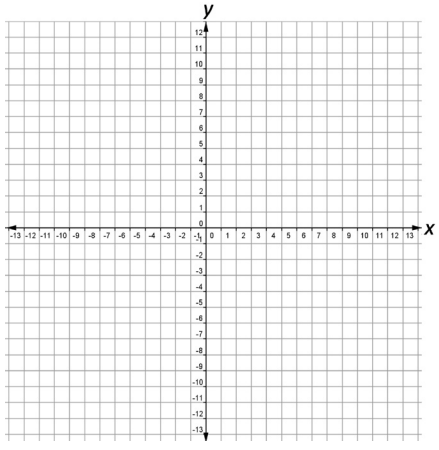
4. $y > 2x - 9$
   $y < 4x + 1$



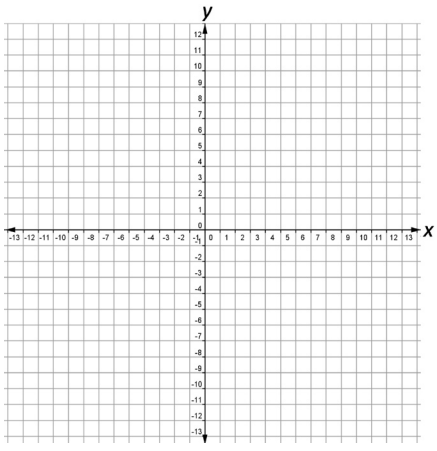
5. $x + y < 12$
$y < -3x + 5$



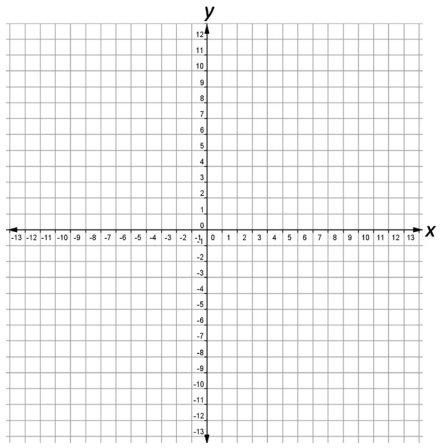
6. $y > 2x + 7$
$2x - y < 12$



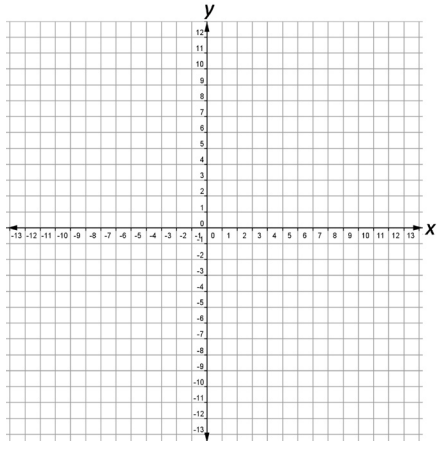
7. $x + 3y > 9$
$x - 3y \leq 9$



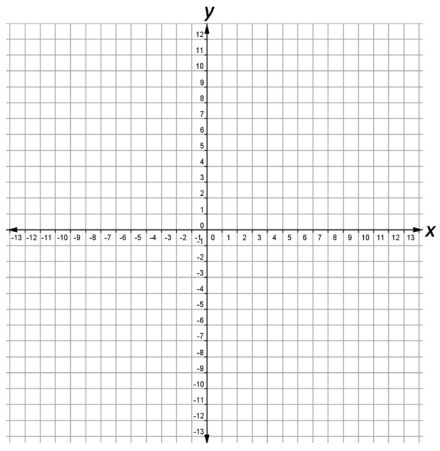
8. $2x - y \geq 10$
$2y \geq 5x + 1$



9. $2x - y < 11$
$3x + 5y \geq 8$



10. $6x + 2y \geq 9$
$3x + y \leq -6$



QUESTIONS ?

a. How did you determine the graphical solutions of each system of linear inequalities in two variables?
b. How did you know that the ordered pairs you listed are solutions of the system of inequalities?
c. Which system of linear inequalities has no solution? Why?
d. When do you say that a system of linear inequalities has solutions? has no solutions?
e. Give 2 examples of system of linear inequalities in two variables having no solutions. Justify your answer.

In this section, the discussion was about solving systems of linear inequalities in two variables graphically.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about the graphical solutions of systems of linear inequalities in two variables, let's go deeper by moving on to the next section.

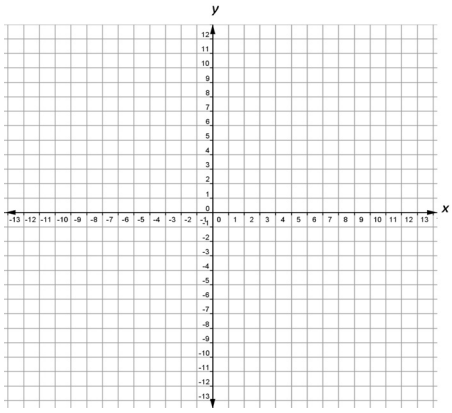
What to Understand

Your goal in this section is to take a closer look at some aspects of the graphical solutions of systems of linear inequalities in two variables. After doing the following activities, you should be able to answer the following question: **"How is the system of linear inequalities in two variables used in solving real-life problems and in making decisions?"**

Activity 5
REGION IN A PLANE

Directions: Answer the following questions.

1. Show the graph of the solution of the system $\begin{matrix} 2x + 5y < 15 \\ 3x - y \geq 8 \end{matrix}$. Use the Cartesian coordinate plane on the next page.



2. How would you describe the graphs of $2x + 5y < 15$ and $3x - y \geq 8$?
3. How would you describe the region where the graphs of $2x + 5y < 15$ and $3x - y \geq 8$ meet?
4. Select any three points in the region where the graphs of $2x + 5y < 15$ and $3x - y \geq 8$ meet. What statements can you make about the coordinates of these points?
5. How would you describe the graphical solutions of the system
   $2x + 5y < 15$?
   $3x - y \geq 8$
6. How is the graphical solution of a system of linear inequalities determined?

   How is it similar or different from the graphical solution of system of linear equations?

Were you able to answer all the questions in the activity? Do you have better understanding now of the graphical solutions of system of linear inequalities in two variables? In the next activity, you will be given the opportunity to deepen further your understanding of solving systems of linear inequalities in two variables graphically.

Activity 6 LOOKING CAREFULLY AT THE REGION...

Directions: Answer the following questions.

1. How do you determine the solution set of a system of linear inequalities in two variables from its graph?
2. Do you think it is easy to determine the solution set of a system of linear inequalities by graphing? Explain your answer.
3. In what instance will you find it difficult to determine the solution set of a system of linear inequalities from its graph?

4. How would you know if the solutions you found from the graphs of linear inequalities in a system are true?
5. What do you think are the advantages and the disadvantages of finding the solution set of a system of linear inequalities graphically? Explain your answer.
6. Is it possible to find the solution set of a system of linear inequalities in two variables algebraically? Give examples if there are any.

The activity provided you opportunities to deepen your understanding of solving systems of linear inequalities in two variables graphically. In the next activity, you will extend your understanding of systems of linear inequalities in two variables as to how they are used in solving real-life problems and in making decisions.

Activity 7 SOLVE THEN DECIDE!

Directions: Answer each of the following. Show your complete solutions and explanations.

1. Tickets in a play cost Php 250 for adults and Php 200 for children. The sponsor of the show collected a total amount of not more than Php 44,000 from more than 150 adults and children who watched the play.

    a. What mathematical statements represent the given situation?
    b. Draw and describe the graphs of the mathematical statements.
    c. How will you find the number of children and adults who watched the play?
    d. Give 4 possible numbers of adults and children who watched the play. Justify your answer.
    e. The sponsor of the show realized that if the prices of the tickets were reduced, more people would have watched the play. If you were the sponsor of the play, would you reduce the prices of the tickets? Why?

2. Mr. Agoncillo has savings account in two banks. The combined amount of these savings is at least Php 150,000. One bank gives an interest of 4% while the other bank gives 6%. In a year, Mr. Agoncillo receives at most Php 12,000.

    a. What mathematical statements represent the given situation?
    b. Draw and describe the graphs of the mathematical statements.
    c. How will you determine the amount of savings in each bank account?

d. Give 4 possible amounts of savings in both accounts. Justify your answer.
e. If you were Mr. Agoncillo, in what bank account would you place greater amount of money? Why?

3. Mrs. Burgos wants to buy at least 30 kilos of pork and beef for her restaurant business but has to spend no more than Php 12,000. A kilo of pork costs Php 180 and a kilo of beef costs Php 220.

a. What mathematical statements represent the given situation?
b. Draw and describe the graphs of the mathematical statements.
c. How will you determine the amount of pork and beef that Mrs. Burgos needs to buy?
d. Give 4 possible amounts of pork and beef that Mrs. Burgos needs to buy. Justify your answer.
e. Mrs. Burgos observed that every week, the number of people coming to her restaurant is increasing. She decided to buy more pork and beef to meet the demands of her customers. If you were Mrs. Burgos, would you do the same? Why?

What new insights do you have about the graphical solutions of systems of linear inequalities in two variables? What new connections have you made for yourself?

Let's extend your understanding. This time, apply in real life what you have learned by doing the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of the graphical solutions of systems of linear inequalities in two variables.

Activity 8 PLAY THE ROLE OF ...

Cite situations in real life where systems of linear inequalities in two variables are applied. Form a group of 5 members and role play each situation. With your groupmates, formulate problems out of these situations then solve in as many ways as you can.

Activity 9 JOIN THE CAMP!

Directions: Perform the following activity. Refer to the situation below.

You are one of the members of the Boys Scouts of the Philippines in your school who will be joining the National Jamboree next month. Your scout master assigned you together with your troop members to take charge of all the camping materials needed such as tents, ropes, bamboos, cooking utensils, fire woods, and other necessary materials. He also asked you to prepare the food menu for the duration of the jamboree including the ingredients.

1. Make a list of all camping materials needed including the quantity of each.
2. Use the camping materials and their quantities to formulate problems involving systems of linear inequalities in two variables. Solve the problems formulated. Use the rubric provided to rate your work.
3. Determine if the camping materials needed are enough for the number of boys scouts who will join the jamboree. Explain your answer.

Rubric on Problems Formulated and Solved

Score	Descriptors
6	Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate.
5	Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
4	Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
3	Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details.
2	Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension.
1	Poses a problem but demonstrates minor comprehension, not being able to develop an approach.

Source: D.O. #73 s. 2012

How did you find the performance task? How did the task help you see the real-world applications of systems of inequalities in two variables? What important things have you learned in the activity? What values are being practiced?

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about the graphical solutions of systems of linear inequalities in two variables. In this lesson, you were able to use the graphical method of finding the solutions of systems of linear inequalities and given the opportunity to determine the advantages and disadvantages of using such method. Using this method of solving systems of linear inequalities, you were able to find out which system has no solution and infinite number of solutions. More importantly, you were given the chance to formulate and solve real-life problems, make decisions based on the problems, and demonstrate your understanding of the lesson by doing some practical tasks.

Glossary of Terms:

1. Elimination Method – This is an algebraic method of solving systems of linear equations. In this method, the value of one variable is determined by eliminating the other variable. To eliminate the variable, some mathematical operations are followed.

2. GeoGebra – This is a dynamic mathematics software that can be used to visualize and understand concepts in algebra, geometry, calculus, and statistics.

3. Graphical Method – This is a method of finding the solution(s) of a system of linear equations by graphing.

4. Simultaneous linear equations or system of linear equations – a set or collection of equations that one solves all together at once.

5. Simultaneous linear inequalities or system of linear inequalities – a set or collection of inequalities that one solves all together at once.

6. Solution to a system of linear equations - This corresponds to the coordinates of the points of intersection of the graphs of the equations.

7. Substitution Method – This is an algebraic method of solving systems of linear equations. In this method, the expression equivalent to one variable in one equation is substituted to the other equation to solve for the other variable.

8. System of consistent and dependent equations – This is a system of linear equations having infinitely many solutions. The slopes of the lines defined by the equations are equal, their y-intercepts are also equal, and their graphs coincide.

9. System of consistent and independent equations – This is a system of linear equations having exactly one solution. The slopes of the lines defined by the equations are not equal, their y-intercepts could be equal or unequal, and their graphs intersect at exactly one point.

10. System of inconsistent equations – This is a system of linear equations having no solution. The slopes of the lines defined by the equations are equal or have no slopes, their y-intercepts are not equal, and their graphs are parallel.

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