7 Systems of Linear Equations

BUILDING ON

- modelling problems using linear relations
- graphing linear functions
- solving linear equations

BIG IDEAS

- A system of two linear equations is solved when the set of ordered pairs that satisfies both equations is determined.
- Multiplying or dividing the equations in a linear system by a non-zero number, or adding or subtracting the equations, produces an equivalent system.
- A system of two linear equations may have one solution, infinite solutions, or no solution.

NEW VOCABULARY

system of linear equations, linear system
solving by substitution
equivalent systems
solving by elimination
infinite
coincident lines

POLAR BEARS In Nunavut, scientists collected data on the numbers of polar bears they encountered, and the bears' reactions. When we know some of these data, we can write, then solve, related problems using linear systems.

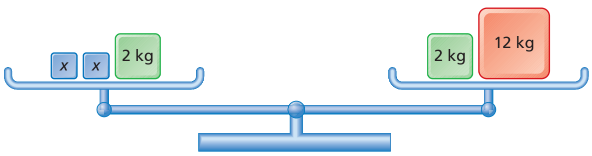
7.1 Developing Systems of Linear Equations

LESSON FOCUS

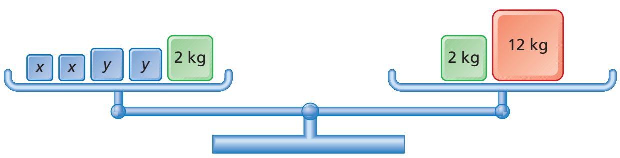
Model a situation using a system of linear equations.

Make Connections

Which linear equation relates the masses on these balance scales?



Which linear equation relates the masses on these balance scales?



How are the two equations the same? How are they different?

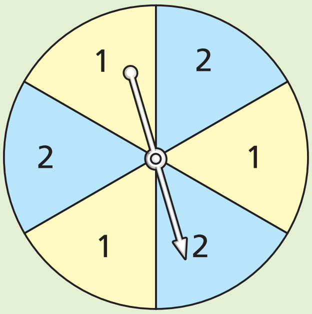
What do you know about the number of solutions for each equation?

Construct Understanding

THINK ABOUT IT

Work with a partner.

A game uses a spinner with the number 1 or 2 written on each sector.



Each player spins the pointer 10 times and records each number.
The sum of the 10 numbers is the player's score.
One player had a score of 17.
How many times did the pointer land on 1 and land on 2?
Write two equations that model this situation.

Why do you need two equations to model this situation?
Is a solution of one equation also a solution of the other equation?
Explain.

A school district has buses that carry 12 passengers and buses that carry 24 passengers. The total passenger capacity is 780. There are 20 more small buses than large buses.

To determine how many of each type of bus there are, we can write two equations to model the situation.
We first identify the unknown quantities.
There are some small buses but we don't know how many.
Let s represent the number of small buses.

There are some large buses but we don't know how many.
Let l represent the number of large buses.
The total passenger capacity is 780.
Each small bus carries 12 people and each large bus carries 24 people.
So, this equation represents the total capacity: $12s + 24l = 780$
There are 20 more small buses than large buses.
So, this equation relates the numbers of buses: $s = l + 20$

Why can we also write the first equation as: $s + 2l = 65$?
Why might it be better to write this equation as: $12s + 24l = 780$?

These two linear equations model the situation:
$12s + 24l = 780$
$s = l + 20$

These two equations form a **system of linear equations** in two variables, s and l.
A system of linear equations is often referred to as a **linear system.**
A *solution* of a linear system is a pair of values of s and l that satisfy both equations.

Suppose you are told that there are 35 small buses and 15 large buses.
To verify that this is the solution, we compare these data with the given situation.
The difference in the numbers of small and large buses is: $35 - 15 = 20$
Calculate the total capacity of 35 small buses and 15 large buses.

$$\begin{aligned}\text{Total capacity} &= 35(12) + 15(24)\\ &= 420 + 360\\ &= 780\end{aligned}$$

The difference in the numbers of small and large buses is 20 and the total passenger capacity is 780. This agrees with the given data, so the solution is correct.

We can also verify the solution by substituting the known values of s and l into the equations.
In each equation, substitute: $s = 35$ and $l = 15$

$12s + 24l = 780$

$$\begin{aligned}\text{L.S.} &= 12s + 24l\\ &= 12(35) + 24(15)\\ &= 420 + 360\\ &= 780\\ &= \text{R.S.}\end{aligned}$$

$s = l + 20$

$$\begin{aligned}\text{L.S.} &= s\\ &= 35\end{aligned}$$

$$\begin{aligned}\text{R. S.} &= l + 20\\ &= 15 + 20\\ &= 35\\ &= \text{L.S.}\end{aligned}$$

For each equation, the left side is equal to the right side. Since $s = 35$ and $l = 15$ satisfy each equation, these numbers are the solution of the linear system.

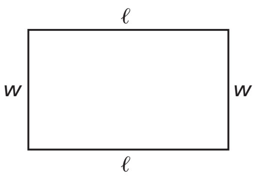
Example 1 Using a Diagram to Model a Situation

a) Create a linear system to model this situation:
The perimeter of a Nunavut flag is 16 ft.
Its length is 2 ft. longer than its width.

b) Denise has determined that the Nunavut flag is 5 ft. long and 3 ft. wide.
Use the linear system from part a to verify that Denise is correct.

SOLUTION

a) Draw a rectangle to represent the flag.
Use the variables l and w to represent the dimensions of the flag in feet.



The perimeter of the flag is 16 ft.
The perimeter is: $l + l + w + w = 2l + 2w$
So, this equation represents the perimeter: $2l + 2w = 16$
The length of the flag is 2 ft. more than the width.
So, this equation relates the dimensions: $l = w + 2$

A linear system that models the situation is:
$2l + 2w = 16$
$l = w + 2$

b) The flag is 5 ft. long and 3 ft. wide.
To verify this solution:
The measurements above confirm that the length is 2 ft. longer than the width.
Calculate the perimeter.

$$\begin{aligned}\text{Perimeter} &= \text{twice the length plus twice the width}\\ &= 2(5\text{ ft.}) + 2(3\text{ ft.})\\ &= 10\text{ ft.} + 6\text{ ft.}\\ &= 16\text{ ft.}\end{aligned}$$

This confirms that the perimeter is 16 ft.
So, the solution is correct.

CHECK YOUR UNDERSTANDING

1. a) Create a linear system to model this situation:
The stage at the Lyle Victor Albert Centre in Bonnyville, Alberta, is rectangular.
Its perimeter is 158 ft.
The width of the stage is 31 ft. less than the length.

b) Sebi has determined that the stage is 55 ft. long and 24 ft. wide. Use the linear system from part a to verify that Sebi is correct.

[Answer: a) $2l + 2w = 158$; $w = l - 31$]

How could you use guess and test to solve the problem?

Example 2 Using a Table to Create a Linear System to Model a Situation

a) Create a linear system to model this situation:
In Calgary, a school raised \$195 by collecting 3000 items for recycling.
The school received 5¢ for each pop can and 20¢ for each large plastic bottle.

b) The school collected 2700 pop cans and 300 plastic bottles.
Use the linear system to verify these numbers.

SOLUTION

a) Choose a variable to represent each unknown number.
Let c represent the number of cans, and let b represent the number of bottles.
Use this information to create a table.

	Refund per Item (\$)	Number of Items	Money Raised (\$)
Can	0.05	c	$0.05c$
Bottle	0.20	b	$0.20b$
Total		3000	195

The third column in the table shows that the total number of items collected can be represented by this equation:
$c + b = 3000$
The fourth column in the table shows that the money raised can be represented by this equation: $0.05c + 0.20b = 195$
So, a linear system that models the situation is:
$c + b = 3000$
$0.05c + 0.20b = 195$

b) To verify the solution:
In each equation, substitute: $c = 2700$ and $b = 300$

$c + b = 3000$

$$\begin{aligned} \text{L.S.} &= c + b \\ &= 2700 + 300 \\ &= 3000 \\ &= \text{R.S.} \end{aligned}$$

$0.05c + 0.20b = 195$

$$\begin{aligned} \text{L.S.} &= 0.05c + 0.20b \\ &= 0.05(2700) + 0.20(300) \\ &= 135 + 60 \\ &= 195 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side.
Since $c = 2700$ and $b = 300$ satisfy each equation, these numbers are the solution of the linear system.

CHECK YOUR UNDERSTANDING

2. a) Create a linear system to model this situation:
A school raised \$140 by collecting 2000 cans and glass bottles for recycling.
The school received 5¢ for a can and 10¢ for a bottle.

b) The school collected 1200 cans and 800 bottles.
Use the linear system to verify these numbers.

[Answer: a) $0.05c + 0.10b = 140$; $c + b = 2000$]

What other linear system could model this situation? Would the solution be different? Explain.

For this situation:

A store display had packages of 8 batteries and packages of 4 batteries.
The total number of batteries was 320.
There were 1.5 times as many packages of 4 batteries as packages of 8 batteries.

Cary wrote this linear system:
$8e + 4f = 320$
$1.5f = e$
where e represents the number of packages of 8 batteries and f represents the number of packages of 4 batteries.

Cary's classmate, Devon, said that the solution of the linear system was:
There are 30 packages of 8 batteries and 20 packages of 4 batteries.

To verify the solution, in each equation Cary substituted: $e = 30$ and $f = 20$

$8e + 4f = 320$

$$\begin{aligned}\text{L.S.} &= 8e + 4f\\ &= 8(30) + 4(20)\\ &= 240 + 80\\ &= 320\\ &= \text{R.S.}\end{aligned}$$

$1.5f = e$

$$\begin{aligned}\text{L.S.} &= 1.5f & \text{R.S.} &= e\\ &= 1.5(20) & &= 30\\ &= 30 & &= \text{L.S.}\end{aligned}$$

Cary said that since the left side is equal to the right side for each equation, the solution is correct.

But when Devon used the problem to verify the solution, she realized that there should be more packages of 4 batteries than packages of 8 batteries, so the solution was wrong.

This illustrates that it is better to consider the given data to verify a solution rather than substitute in the equations. There could be an error in the equations that were written to represent the situation.

What are the correct equations for this situation?

Example 3 Relating a Linear System to a Problem

A store sells wheels for roller skates in packages of 4 and wheels for inline skates in packages of 8.

Create a situation about wheels that can be modelled by the linear system below. Explain the meaning of each variable. Write a related problem.

$8i + 4r = 440$

$i + r = 80$

SOLUTION

$8i + 4r = 440$ ①

$i + r = 80$ ②

We number the equations in a linear system to be able to refer to them easily.

In equation ①:

The variable i is multiplied by 8, which is the number of inline skate wheels in a package.

So, i represents the number of packages of inline skate wheels in the store.

The variable r is multiplied by 4, which is the number of roller skate wheels in a package.

So, r represents the number of packages of roller skate wheels in the store.

Then, equation ① could represent the total number of wheels in all these packages in the store.

And, equation ② could represent the total number of packages of inline skate and roller skate wheels in the store.

A possible problem is:

A store has 80 packages of wheels for inline skates and roller skates.

Inline skate wheels come in packages of 8.

Roller skate wheels come in packages of 4.

The total number of these wheels in all packages is 440.

How many packages of inline skate wheels and how many packages of roller skate wheels are in the store?

CHECK YOUR UNDERSTANDING

3. A bicycle has 2 wheels and a tricycle has 3 wheels.

Create a situation about wheels that can be modelled by the linear system below. Explain the meaning of each variable. Write a related problem.

$2b + 3t = 100$

$b + t = 40$

[Sample Answer: A possible problem is: There are 40 bicycles and tricycles in a department store. The total number of wheels on all bicycles and tricycles in the store is 100. How many bicycles and how many tricycles are in the store?]

Discuss the Ideas

1. When you write a linear system to model a situation, how do you decide which parts of the situation can be represented by variables?

2. What does the *solution* of a linear system mean?

3. How can you verify the solution of a linear system?

Exercises

A

4. Which system of equations is *not* a linear system?

a) $2x + y = 11$
$x = 13 + y$

b) $2x = 11 - y$
$4x - y = 13$

c) $-\frac{1}{2}x - y = \frac{3}{4}$
$\frac{3}{2}x + 2 = -\frac{7}{8}$

d) $-x^2 + y = 10$
$x + y = 5$

5. Which linear systems have the solution $x = -1$ and $y = 2$?

a) $3x + 2y = -1$
$2x - y = 1$

b) $3x - y = -1$
$-x - y = -1$

c) $-3x + 5y = 13$
$4x - 3y = -10$

B

6. Match each situation to a linear system below. Justify your choice. Explain what each variable represents.

a) During a clothing sale, 2 jackets and 2 sweaters cost \$228. A jacket costs \$44 more than a sweater.

b) The perimeter of a standard tennis court for doubles is 228 ft. The width is 42 ft. less than the length.

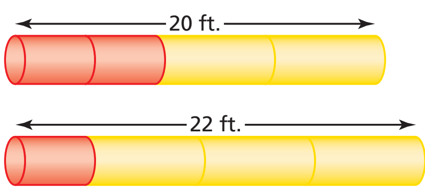
c) At a cultural fair, the Indian booth sold chapatti and naan breads for \$2 each. A total of \$228 was raised. Forty more chapatti breads than naan breads were sold.

i) $2x + 2y = 228$
$x - y = 42$

ii) $2x + 2y = 228$
$x - y = 40$

iii) $2x + 2y = 228$
$x - y = 44$

7. a) Create a linear system to model this situation:
Two different lengths of pipe are joined, as shown in the diagrams.



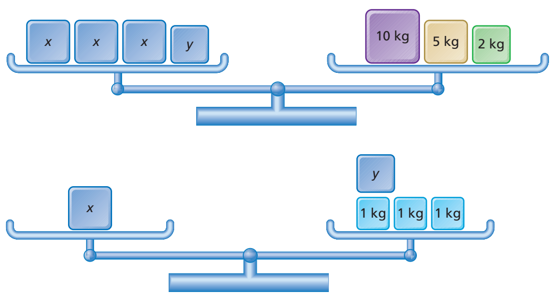
b) Verify that the shorter pipe is 4 ft. long and the longer pipe is 6 ft. long.

8. a) Create a linear system to model this situation:
The perimeter of an isosceles triangle is 24 cm. Each equal side is 6 cm longer than the shorter side.

b) Verify that the side lengths of the triangle are: 10 cm, 10 cm, and 4 cm

9. Teri works in a co-op that sells small and large bags of wild rice harvested in Saskatchewan.

a) These balance scales illustrate the two different sizes of bags of rice: x represents the mass of a large bag and y represents the mass of a small bag. Write a linear system to model the two balance scales.

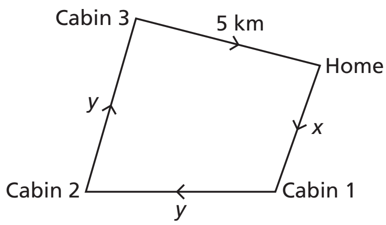


b) Use the diagrams of the balance scales to verify that a small bag of rice has a mass of 2 kg and a large bag has a mass of 5 kg.

c) Use the linear system to verify the masses of the bags in part b.

For questions 10 and 11, write a linear system to model each situation. Then verify which of the given solutions of the related problem is correct.

10. A dogsled team travelled a total distance of 25 km from home to three cabins and then back home. All distances are measured along the trail. The distance from home to cabin 2 is 13 km. What is the distance from home to cabin 1? What is the distance from cabin 1 to cabin 2? (*Solution A*: The distance from home to cabin 1 is 7 km. The distance from cabin 1 to cabin 2 is 6 km. *Solution B*: The distance from home to cabin 1 is 6 km. The distance from cabin 1 to cabin 2 is 7 km.)



11. Padma walked and jogged for 1 h on a treadmill. She walked 10 min more than she jogged. For how long did Padma walk? For how long did she jog? (*Solution A*: Padma walked for 35 min and jogged for 25 min. *Solution B*: Padma jogged for 35 min and walked for 25 min.)

12. Shen used this linear system to represent a situation involving a collection of \$2 and \$1 coins.

$2t + l = 160$
$t + l = 110$

a) What problem might Shen have solved?
b) What does each variable represent?

13. Jacqui wrote a problem about the costs of tickets for a group of adults and children going to a local fair. She modelled the situation with this linear system.

$5a + 2c = 38$
$a - c = 2$

a) What problem might Jacqui have written? Justify your answer.
b) What does each variable represent?

14. Write a situation that can be modelled by this linear system. Explain what each variable means, then write a related problem.

$x + y = 100$
$x - y = 10$

C

15. Any linear system in two variables can be expressed as:

$Ax + By = C$
$Dx + Ey = F$

a) What do you know about the coefficients B, E, C, and F when the solution is the ordered pair $(0, y)$?
b) What do you know about the coefficients A, D, C, and F when the solution is the ordered pair $(x, 0)$?

16. Show how this system can be written as a linear system.

$$\frac{-3x + 24}{x + y} = -6$$
$$-\frac{x}{5} - \frac{y}{3} = x + y$$

17. **a)** Write a linear system that has the solution $x = 1$ and $y = 1$. Explain what you did.
b) Why is there more than one linear system with the same solution?

18. **a)** Without solving this system, how do you know that $y = 2$ is part of the solution of this linear system?

$x + 2y = 7$
$x + 3y = 9$

b) Solve the system for x. Explain what you did.

Reflect

What do you need to consider when you write a linear system to model a situation? Use one of the exercises to explain.

7.2 Solving a System of Linear Equations Graphically

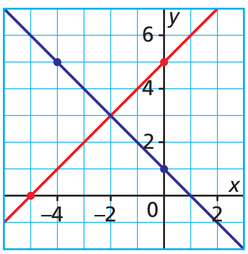
LESSON FOCUS

Use the graphs of the equations of a linear system to estimate its solution.

The town of Kelvington, Saskatchewan, erected 6 large hockey cards by a highway to celebrate the town's famous hockey players. One player is Wendel Clark.

Make Connections

Two equations in a linear system are graphed on the same grid.



What are the equations of the graphs? Explain your reasoning.
What are the coordinates of the point of intersection of the two lines?
Explain why these coordinates are the solution of the linear system.

Construct Understanding

TRY THIS

Work with a partner.
You will need grid paper and a ruler.

Here is a problem about the hockey cards in Kelvington.

The perimeter of each large hockey card is 24 ft.
The difference between the height and width is 4 ft.
What are the dimensions of each card?

A. Create a linear system to model this situation.

B. Graph the equations on the same grid.

C. What are the coordinates of the point of intersection, P, of the two lines?

D. Why must the coordinates of P be a solution of each equation in the linear system?

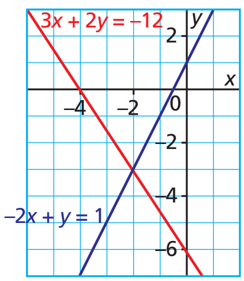
E. What are the side lengths of each large hockey card in Kelvington?

How did you decide on which axis each variable should be graphed?

The solution of a linear system can be estimated by graphing both equations on the same grid. If the two lines intersect, the coordinates (x, y) of the point of intersection are the solution of the linear system.
Each equation of this linear system is graphed on a grid.

$3x + 2y = -12$ ①

$-2x + y = 1$ ②



How could you verify that the graphs represent the equations in the linear system?

We can use the graphs to estimate the solution of the linear system.
The set of points that satisfy equation ① lie on its graph.
The set of points that satisfy equation ② lie on its graph.
The set of points that satisfy both equations lie where the two graphs intersect.
From the graphs, the point of intersection appears to be $(-2, -3)$.
To verify the solution, we check that the coordinates $(-2, -3)$ satisfy both equations.

In each equation, we substitute: $x = -2$ and $y = -3$:

$3x + 2y = -12$

$$\begin{aligned}\text{L.S.} &= 3x + 2y\\ &= 3(-2) + 2(-3)\\ &= -6 - 6\\ &= -12\\ &= \text{R.S.}\end{aligned}$$

$-2x + y = 1$

$$\begin{aligned}\text{L.S.} &= -2x + y\\ &= -2(-2) - 3\\ &= 4 - 3\\ &= 1\\ &= \text{R.S.}\end{aligned}$$

Why must the solution satisfy both equations?

For each equation, the left side is equal to the right side.
Since $x = -2$ and $y = -3$ satisfy each equation, these numbers are the solution of the linear system.

Example 1 Solving a Linear System by Graphing

Solve this linear system.

$x + y = 8$

$3x - 2y = 14$

SOLUTION

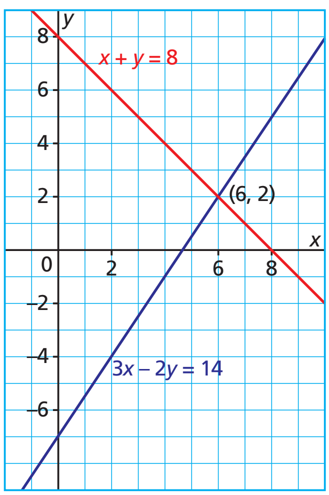
$x + y = 8$ ①

$3x - 2y = 14$ ②

Determine the x-intercept and y-intercept of the graph of equation ①.
Both the x- and y-intercepts are 8.
Write equation ② in slope-intercept form.

$$\begin{aligned}3x - 2y &= 14\\ -2y &= -3x + 14 \quad \text{Divide by } -2 \text{ to solve for } y.\\ y &= \frac{3}{2}x - 7\end{aligned}$$

The slope of the graph of equation ② is $\frac{3}{2}$, and its y-intercept is -7.
Graph each line.
The point of intersection appears to be (6, 2).



Verify the solution. In each equation, substitute: $x = 6$ and $y = 2$

$x + y = 8$

$$\begin{aligned}\text{L.S.} &= x + y\\ &= 6 + 2\\ &= 8\\ &= \text{R.S.}\end{aligned}$$

$3x - 2y = 14$

$$\begin{aligned}\text{L.S.} &= 3x - 2y\\ &= 3(6) - 2(2)\\ &= 18 - 4\\ &= 14\\ &= \text{R.S.}\end{aligned}$$

For each equation, the left side is equal to the right side.
So, $x = 6$ and $y = 2$ is the solution of the linear system.

CHECK YOUR UNDERSTANDING

1. Solve this linear system.
   $2x + 3y = 3$
   $x - y = 4$
   [Answer: (3, –1)]

What other strategy could you use to graph the equations?

Example 2 Solving a Problem by Graphing a Linear System

One plane left Regina at noon to travel 1400 mi. to Ottawa at an average speed of 400 mph. Another plane left Ottawa at the same time to travel to Regina at an average speed of 350 mph. A linear system that models this situation is:

$d = 1400 - 400t$

$d = 350t$

where d is the distance in miles from Ottawa and t is the time in hours since the planes took off

a) Graph the linear system above.

b) Use the graph to solve this problem: When do the planes pass each other and how far are they from Ottawa?

SOLUTION

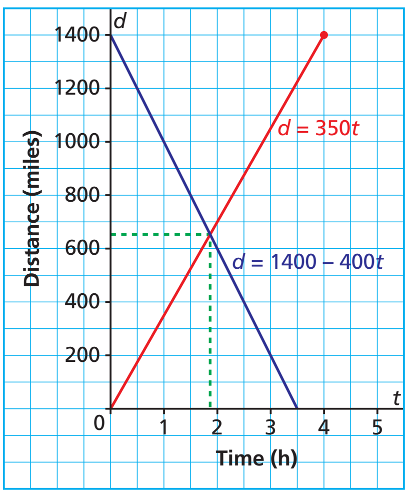
The planes pass each other when they have been travelling for the same time and they are the same distance from Ottawa.

a) Solve the linear system to determine values of d and t that satisfy both equations.

$d = 1400 - 400t$ ①

$d = 350t$ ②

Each equation is in slope-intercept form.
For the graph of equation ①, the slope is -400 and the vertical intercept is 1400.
For the graph of equation ②, the slope is 350 and the vertical intercept is 0.
Graph the equations.



CHECK YOUR UNDERSTANDING

2. Jaden left her cabin on Waskesiu Lake, in Saskatchewan, and paddled her kayak toward her friend Tyrell's cabin at an average speed of 4 km/h. Tyrell started at his cabin at the same time and paddled at an average speed of 2.4 km/h toward Jaden's cabin. The cabins are 6 km apart. A linear system that models this situation is:

$d = 6 - 4t$

$d = 2.4t$

where d is the distance in kilometres from Tyrell's cabin and t is the time in hours since both people began their journey

a) Graph the linear system above.

b) Use the graph to solve this problem: When do Jaden and Tyrell meet and how far are they from Tyrell's cabin?

[Answer: b) after travelling for approximately 54 min and at approximately 2.3 km from Tyrell's cabin]

How would the equations in the linear system change if you wanted to determine when the planes meet and how far they are from Regina? Explain.

b) The graphs appear to intersect at (1.9, 650); that is, the planes appear to pass each other after travelling for 1.9 h and at a distance of 650 mi. from Ottawa.
To verify the solution, use the given information.
The plane travelling from Regina to Ottawa travels at 400 mph.
So, in 1.9 h, it will travel: $400(1.9)$ mi. $= 760$ mi.
So, it will be: $(1400 - 760)$ mi., or 640 mi. from Ottawa.
The plane travelling from Ottawa to Regina travels at 350 mph.
So, in 1.9 h, its distance from Ottawa will be:
$350(1.9)$ mi. $= 665$ mi.
The time and distance are approximate because these measures cannot be read accurately from the graph.
0.9 h is $60(0.9)$ min $= 54$ min
The planes pass each other after travelling for approximately 1 h 54 min and when they are approximately 650 mi. from Ottawa.

How could you solve the problem without using a linear system?

Example 3 Solving a Problem by Writing then Graphing a Linear System

a) Write a linear system to model this situation:
To visit the Head-Smashed-In Buffalo Jump interpretive centre near Fort Macleod, Alberta, the admission fee is \$5 for a student and \$9 for an adult. In one hour, 32 people entered the centre and a total of \$180 in admission fees was collected.

b) Graph the linear system then solve this problem: How many students and how many adults visited the centre during this time?

SOLUTION

a) Use a table to help develop the equations.

Given:	Creating a Linear System
There are students and adults.	Let s represent the number of students. Let a represent the number of adults.
There are 32 people.	One equation is: $s + a = 32$
Cost per student is \$5.	$5s$ dollars represents the total cost for the students.
Cost per adult is \$9.	$9a$ dollars represents the total cost for the adults.
Cost for all the people is \$180.	Another equation is: $5s + 9a = 180$

(Solution continues.)

CHECK YOUR UNDERSTANDING

3. **a)** Write a linear system to model this situation:
Wayne received and sent 60 text messages on his cell phone in one weekend.
He sent 10 more messages than he received.

b) Graph the linear system then solve this problem: How many text messages did Wayne send and how many did he receive?

[Answers: a) $s + r = 60$; $s - r = 10$
b) 35 sent and 25 received]

The linear system is:

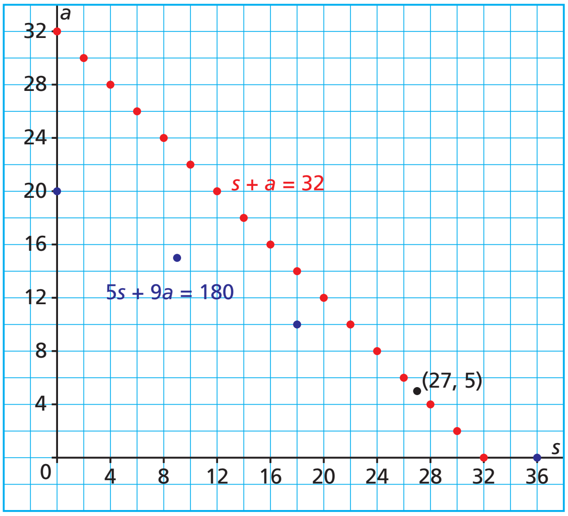
$s + a = 32$ ①

$5s + 9a = 180$ ②

b) Use intercepts to graph each line.

Equation	*a*-intercept	*s*-intercept
$s + a = 32$	32	32
$5s + 9a = 180$	20	36

Since the data are discrete, place a straightedge through the intercepts of each line and plot more points on each line.



The point of intersection appears to be (27, 5).
Verify this solution.
Determine the cost for 27 students at \$5 each and 5 adults at \$9 each:

27 students at \$5 each	=	\$135
+ 5 adults at \$9 each	=	\$45
32 people for		\$180

The total number of people is 32 and the total cost is \$180, so the solution is correct.

Twenty-seven students and 5 adults visited the centre.

Suppose the equations had been graphed with *s* on the vertical axis and *a* on the horizontal axis. Would the graphs have been different? Would the solution have been different? Explain.

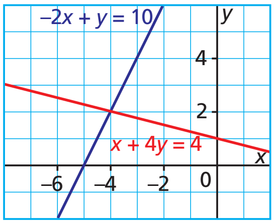
Discuss the Ideas

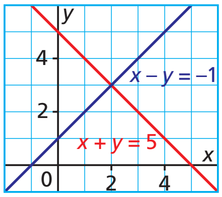
1. What steps do you follow to solve a system of linear equations by graphing?
2. What are some limitations to solving a linear system by graphing?

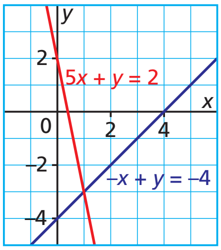
Exercises

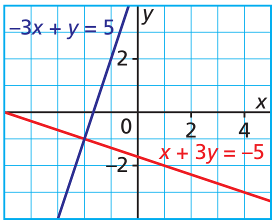
A

3. Determine the solution of each linear system.

a) 

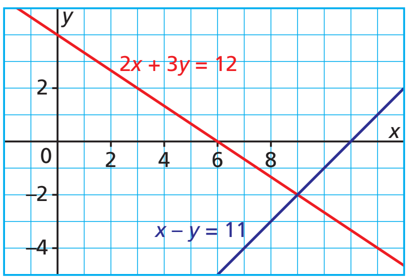
b) 

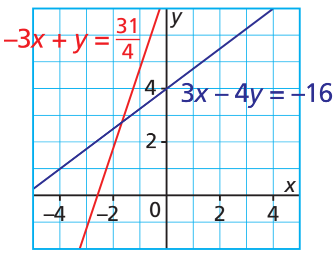
c) 

d) 

B

4. For each linear system, use the graphs to determine the solution. Explain how you know whether the solution is exact or approximate.

a) 

b) 

5. a) Solve each linear system.

i) $x + y = 7$
$3x + 4y = 24$

ii) $x - y = -1$
$3x + 2y = 12$

iii) $5x + 4y = 10$
$5x + 6y = 0$

iv) $x + 2y = -1$
$2x + y = -5$

b) Choose one linear system from part a. Explain the meaning of the point of intersection of the graphs of a system of linear equations.

6. Emil's solution to this linear system was (500, 300). Is his solution exact or approximate? Explain.

$3x - y = 1149$
$-x + 2y = 142$

7. Solve each linear system.

a) $2x + 4y = -1$
$3x - y = 9$

b) $5x + 5y = 17$
$x - y = -1$

c) $x + y = \frac{23}{4}$
$x - y = \frac{3}{4}$

d) $3x + y = 6$
$x + y = -\frac{4}{3}$

8. Two companies charge these rates for printing a brochure:
Company A charges \$175 for set-up, and \$0.10 per brochure.
Company B charges \$250 for set-up, and \$0.07 per brochure.
A linear system that models this situation is:
$C = 175 + 0.10n$
$C = 250 + 0.07n$
where C is the total cost in dollars and n is the number of brochures printed

a) Graph the linear system above.

b) Use the graph to solve these problems:

i) How many brochures must be printed for the cost to be the same at both companies?

ii) When is it cheaper to use Company A to print brochures? Explain.

9. Part-time sales clerks at a computer store are offered two methods of payment:
Plan A: \$700 a month plus 3% commission on total sales
Plan B: \$1000 a month plus 2% commission on total sales.
A linear system that models this situation is:
$P = 700 + 0.03s$
$P = 1000 + 0.02s$
where P is the clerk's monthly salary in dollars and s is the clerk's monthly sales in dollars

a) Graph the linear system above.

b) Use the graph to solve these problems:

i) What must the monthly sales be for a clerk to receive the same salary with both plans?

ii) When would it be better for a clerk to choose Plan B? Explain.

For questions 10 to 13, write a linear system to model each situation. Solve the related problem. Indicate whether your solution is exact or approximate.

10. The area of Stanley Park in Vancouver is 391 hectares. The forested area is 141 hectares more than the rest of the park. What is the area of each part of the park?

11. In the American Hockey League, a team gets 2 points for a win and 1 point for an overtime loss. In the 2008–2009 regular season, the Manitoba Moose had 107 points. They had 43 more wins than overtime losses. How many wins and how many overtime losses did the team have?

12. Annika's class raised \$800 by selling \$5 and \$10 movie gift cards. The class sold a total of 115 gift cards. How many of each type of card did the class sell?

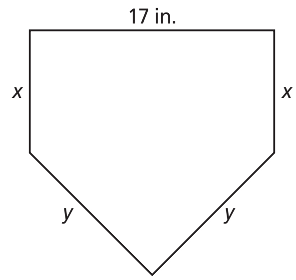
13. A group of adults and students went on a field trip to the Royal Tyrell Museum, near Drumheller, Alberta. The total admission fee was \$152. There were 13 more students than adults. How many adults and how many students went on the field trip?

14. a) Write a linear system to model this situation: A box of 36 golf balls has a mass of 1806 g. When 12 balls are removed, the mass is 1254 g.

b) Use a graph to solve this problem: What is the mass of the box and the mass of one golf ball?

c) Why was it difficult to determine a solution?

15. The home plate in a baseball diamond is a pentagon with perimeter 58 in. Each shorter side, x, is $3\frac{1}{2}$ in. less than each longer side, y. What are the values of x and y?



16. a) Solve this linear system by graphing.

$2x + 7y = 3$
$4x + 3y = 7$

b) Why is the solution approximate?

C

17. Emma solved a linear system by graphing. She first determined the intercepts of each line.

Equation	x-intercept	y-intercept
1	5	5
2	4	6

a) Write a linear system that Emma could have solved. Explain your work.

b) Draw the graphs to determine the solution.

18. One equation of a linear system is $y = 2x + 1$. The solution of the linear system is in the third quadrant. What might the second equation be? Explain how you determined the equation.

19. a) Suppose you want to solve this linear system by graphing. How do you know that the lines are perpendicular?

$2x + 3y = -5$

$\frac{x}{2} - \frac{y}{3} = 2$

b) Create another linear system where the lines are perpendicular. Explain what you did.

Reflect When you solve a linear system graphically, how can you determine whether the solution is approximate or exact?

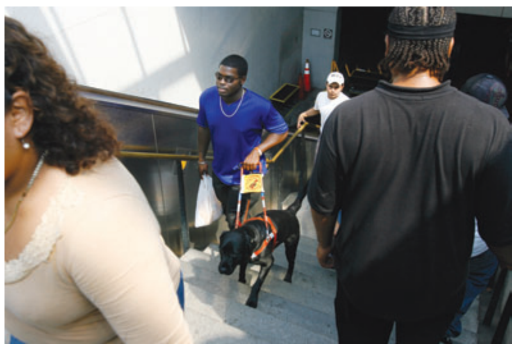
MATH LAB 7.3 Using Graphing Technology to Solve a System of Linear Equations

LESSON FOCUS

Determine and verify the solution of a linear system using graphing technology.

Make Connections

In 2006, the population of Canada was 31 612 897. The population of the eastern provinces was 12 369 487 more than the population of the territories and western provinces.



What linear system models this situation?
How could you determine the population of the territories and western provinces?
How could you determine the population of the eastern provinces?
Why can't you determine an exact solution by graphing on grid paper?

Construct Understanding

TRY THIS

Work with a partner.
You will need:

- a graphing calculator or computer with graphing software

Léa's school had a carnival to celebrate *Festival du Voyageur*.
The school raised $1518.75 by charging an adult $3.75 and a student $2.50.
The total attendance was 520.
How many adults and how many students attended?

Before you graph, how can you tell in which quadrant the point of intersection lies?

What table settings would you use? Why did you choose these?

A. Write a linear system to model this situation.

B. Express each equation in slope-intercept form. Graph each line.

C. Determine the coordinates of the point of intersection of the lines. Are these coordinates exact or approximate? Explain.

D. How could you verify the solution in Step C by using tables of values?

E. Verify your solution by using the data in the given problem.

F. How can you use your results to determine the number of adults and the number of students who attended the carnival?

Assess Your Understanding

1. To solve this linear system:
$x + 2y = 8$
$3x + 4y = 20$
Gerard entered each equation into his graphing calculator in this form:

```
Plot1 Plot2 Plot3
\Y1 = -X/2+4
\Y2 = -3X/4+5
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

Gerard created a table of values for these equations.

X	Y1	Y2
0	4	5
1	3.5	4.25
2	3	3.5
3	2.5	2.75
4	2	2
5	1.5	1.25
6	1	.5

X=0

a) How could Gerard use the table to determine the solution of the linear system?

b) Describe a different strategy Gerard could use to solve this linear system.

2. **a)** Explain how you would use graphing technology to determine the solution of this linear system.

$$3x - 6y = 14$$
$$x + y = \frac{7}{6}$$

b) What is the solution?

3. A community group purchased 72 cedar tree and spruce tree seedlings to plant for an Earth Day project. The number of cedar trees was twice the number of spruce trees. How many of each type of tree seedling did the group purchase?

4. **a)** Each linear system below contains the equation $x + 2y = 3$. Solve each system by graphing.

i) $x + 2y = 3$, $2x - y = 1$ **ii)** $x + 2y = 3$, $2x - y = 6$ **iii)** $x + 2y = 3$, $2x - y = 11$ **iv)** $x + 2y = 3$, $2x - y = 16$

b) What patterns are there in the linear systems and their solutions? Use the patterns to predict another linear system that would extend the pattern. Explain your prediction.

c) Solve the linear system to check your prediction.

5. When you graph to solve a linear system that contains fractional coefficients, will you always get an approximate solution? Use the linear systems below to explain.

System A

$$\frac{1}{2}x + y = 3$$
$$x + \frac{1}{2}y = 3$$

System B

$$2x + y = \frac{23}{6}$$
$$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$$

CHECKPOINT 1

Connections

Given this Situation and Related Problem

A store has boxes containing 1500 golf balls.

There are 5 more boxes containing 12 balls than boxes containing 24 balls.

How many of each size of box does the store have?

Identify the Variables

Let x represent the number of boxes with 12 balls.

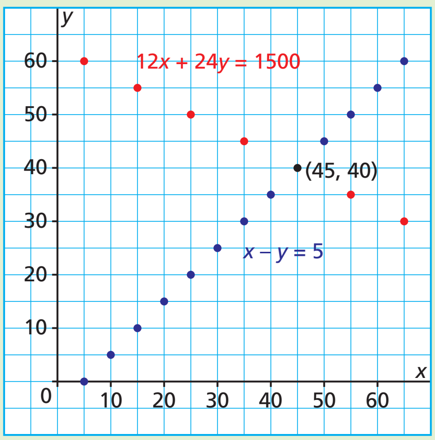
Let y represent the number of boxes with 24 balls.

Model the Situation with a Linear System

$12x + 24y = 1500$

$x - y = 5$

Graph to Locate the Point of Intersection



Identify the Solution

$x = 45$

$y = 40$

Use the equations to verify the solution.

Use the problem to verify the solution.

Concept Development

- **In Lesson 7.1**
  – You defined a linear system, wrote linear systems to model problems, and related linear systems to problems.
- **In Lesson 7.2**
  – You solved a linear system by graphing the linear equations on grid paper and determining the coordinates of their point of intersection.
  – You identified whether a solution to a linear system was exact or approximate.
- **In Lesson 7.3**
  – You solved linear systems using graphing technology by first expressing each equation in the form $y = f(x)$.
  – You used technology to determine whether the solution of a linear system was exact or approximate.
  – You verified the solution of a linear system by examining tables of values.

Assess Your Understanding

7.1

1. a) Create a linear system to model this situation:
The Commonwealth stadium in Edmonton, Alberta, has the largest video screen JumboTron in the world. Its perimeter is 128 ft. Its width is 16 ft. less than its length.

b) The length of the screen is 40 ft. and its width is 24 ft.
  i) Use the equations to verify these dimensions.
  ii) Use the problem to verify these dimensions.

2. Create a situation that can be modelled by this linear system, then write a related problem.
$10x + 5y = 850$
$x - y = 10$

7.2

3. Solve this linear system by graphing on grid paper. Describe your strategy.
$2x + y = 1$
$x + 2y = -1$

4. A fitness club offers two payment plans:
Plan A: an initiation fee of $75 plus a user fee of $5 per visit
Plan B: a user fee of $10 per visit
A linear system that models this situation is:
$F = 75 + 5v$
$F = 10v$
where F is the total fee in dollars and v is the number of visits
a) Graph this linear system.
b) Under what conditions is Plan A cheaper? Justify your answer.

5. a) Write a linear system to model this situation:
A group of students and adults went to the Vancouver Aquarium in B.C. The admission fee was $21 for a student and $27 for an adult. The total cost for 18 people was $396.

b) Use a graph to solve this problem:
How many students and how many adults went to the aquarium?

7.3

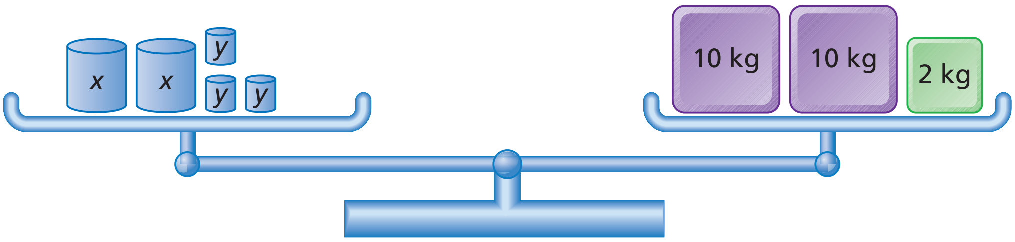
6. a) Write a linear system to model this situation:
A large tree removes 1.4 kg of pollution from the air each year. A small tree removes 0.02 kg each year. An urban forest has 15 000 large and small trees. Together, these trees remove 7200 kg of pollution each year.

b) Use graphing technology to solve this problem: How many of each size of tree are in the forest?

7.4 Using a Substitution Strategy to Solve a System of Linear Equations

LESSON FOCUS

Use the substitution of one variable to solve a linear system.



Make Connections

Look at the picture above.
Is there enough information to determine the masses of the container labelled x and the container labelled y? Explain.
How would your answer change if you knew that each container labelled x had a mass of 5 kg? Explain.

Construct Understanding

THINK ABOUT IT

Work with a partner.

Solve each linear system without graphing.

- $3x + 5y = 6$
  $x = -4$
- $2x + y = 5$
  $y = -x + 3$

What strategies did you use?
How can you check that each solution is correct?

In Lessons 7.2 and 7.3, you solved linear systems by graphing. This strategy is time-consuming even when you use graphing technology, and you can only approximate the solution. We use algebra to determine an exact solution. One algebraic strategy is called **solving by substitution.** By using substitution, we transform a system of two linear equations into a single equation in one variable, then we use what we know about solving linear equations to determine the value of that variable.

Consider this linear system.

$3x + 4y = -4$ ①

$x + 2y = 2$ ②

In equation ②, the variable x has coefficient 1. So, solve equation ② for x.

$x + 2y = 2$ ②

$x = -2y + 2$

> Why did we solve for x instead of for y?

Since the solution of the linear system is the point of intersection of the graphs of the two lines, the x-coordinate must satisfy both equations. We substitute the expression for x in the other equation.

Substitute $x = -2y + 2$ in equation ①.

$3x + 4y = -4$ ①

$3(-2y + 2) + 4y = -4$ Use the distributive property to simplify.

$-6y + 6 + 4y = -4$ Collect like terms.

$-2y = -10$ Solve for y.

$y = 5$

When we know the value of one variable, we substitute for that variable in one of the original equations then solve that equation for the other variable.

> Would we get the same solution if we solved equation ② for y instead of for x? Explain.

Substitute $y = 5$ in equation ②.

$x + 2y = 2$ ②

$x + 2(5) = 2$

$x + 10 = 2$ Solve for x.

$x = -8$

To verify the solution is correct, we substitute for both variables in the original equations.

In each equation, substitute: $x = -8$ and $y = 5$

$3x + 4y = -4$ ①

L.S. $= 3x + 4y$

$= 3(-8) + 4(5)$

$= -24 + 20$

$= -4$

$=$ R.S.

$x + 2y = 2$ ②

L.S. $= x + 2y$

$= -8 + 2(5)$

$= -8 + 10$

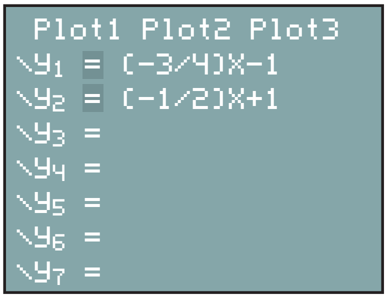
$= 2$

$=$ R.S.

> Would we get the same solution if we substituted $y = 5$ in equation ① instead of equation ②? Explain.

For each equation, the left side is equal to the right side, so the solution is: $x = -8$ and $y = 5$

Here is the graphical solution for the linear system on page 417:



Example 1 Solving a Linear System by Substitution

Solve this linear system.

$2x - 4y = 7$

$4x + y = 5$

SOLUTIONS

Method 1

$2x - 4y = 7$ ①

$4x + y = 5$ ②

Solve equation ② for y.

$4x + y = 5$ ②

$y = 5 - 4x$

Substitute $y = 5 - 4x$ in equation ①.

$2x - 4y = 7$ ①

$2x - 4(5 - 4x) = 7$ Simplify, then solve for x.

$2x - 20 + 16x = 7$

$18x = 27$

$x = 1.5$

Substitute $x = 1.5$ in equation ②.

$4x + y = 5$ ②

$4(1.5) + y = 5$ ②

$6 + y = 5$

$y = -1$

CHECK YOUR UNDERSTANDING

1. Solve this linear system.

$5x - 3y = 18$

$4x - 6y = 18$

[Answer: $x = 3$ and $y = -1$]

Method 2

When no equation in a linear system has a variable with coefficient 1, it is helpful if there are two like terms where one term is a multiple of the other term.

$2x - 4y = 7$ ①
$4x + y = 5$ ②

In equation ②, the term $4x$ can be written as $2(2x)$:

$2(2x) + y = 5$ ③

When we change the form of an equation, we label it with a different number.

Solve equation ① for $2x$.

$2x - 4y = 7$ ①
$2x = 7 + 4y$

Substitute $2x = 7 + 4y$ in equation ③.

$$\begin{aligned} 2(2x) + y &= 5 \quad ③ \\ 2(7 + 4y) + y &= 5 \\ 14 + 8y + y &= 5 \\ 9y &= -9 \\ y &= -1 \end{aligned}$$

Simplify, then solve for y.

Substitute $y = -1$ into equation ①.

$$\begin{aligned} 2x - 4y &= 7 \\ 2x - 4(-1) &= 7 \\ 2x + 4 &= 7 \\ 2x &= 3 \\ x &= 1.5 \end{aligned}$$

Solve for x.

Verify the solution.
In each equation, substitute: $x = 1.5$ and $y = -1$

$2x - 4y = 7$ ①

$$\begin{aligned} \text{L.S.} &= 2x - 4y \\ &= 2(1.5) - 4(-1) \\ &= 3 + 4 \\ &= 7 \\ &= \text{R.S.} \end{aligned}$$

$4x + y = 5$ ②

$$\begin{aligned} \text{L.S.} &= 4x + y \\ &= 4(1.5) - 1 \\ &= 6 - 1 \\ &= 5 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is: $x = 1.5$ and $y = -1$

Why did we write $4x = 2(2x)$ and not $4y = 2(2y)$?

Verify that we get the same solution when we substitute $y = -1$ into equation ②.

Example 2 Using a Linear System to Solve a Problem

a) Create a linear system to model this situation:
Nuri invested \$2000, part at an annual interest rate of 8% and the rest at an annual interest rate of 10%. After one year, the total interest was \$190.

b) Solve this problem: How much money did Nuri invest at each rate?

SOLUTIONS

a) Use a table to help develop the equations.

Given:	Creating a Linear System
There are two investments.	Let x dollars represent the amount invested at 8%. Let y dollars represent the amount invested at 10%.
The total investment was \$2000.	One equation is: $x + y = 2000$
x dollars at 8%	The interest is 8% of x dollars: $0.08x$
y dollars at 10%	The interest is 10% of y dollars: $0.10y$
The total interest is \$190.	Another equation is: $0.08x + 0.10y = 190$

The linear system is:

$x + y = 2000$ ①

$0.08x + 0.10y = 190$ ②

b) Method 1

Because the coefficients of x and y in equation ① are 1, use substitution to solve the linear system.

Solve for y in equation ①.

$x + y = 2000$ ①

$y = -x + 2000$

Substitute $y = -x + 2000$ in equation ②.

$0.08x + 0.10y = 190$ ②

$0.08x + 0.10(-x + 2000) = 190$ Use the distributive property.

$0.08x - 0.10x + 200 = 190$ Collect like terms.

$-0.02x + 200 = 190$

$-0.02x = -10$ Divide each side by -0.02 to solve for x.

$x = 500$

Substitute $x = 500$ in equation ①.

$x + y = 2000$

$500 + y = 2000$ Solve for y.

$y = 1500$

Nuri invested \$500 at 8% and \$1500 at 10%.

CHECK YOUR UNDERSTANDING

2. a) Create a linear system to model this situation:
Alexia invested \$1800, part at an annual interest rate of 3.5% and the rest at an annual interest rate of 4.5%. After one year, the total interest was \$73.

b) Solve this problem: How much money did Alexia invest at each rate?

[Answers: a) $x + y = 1800$; $0.035x + 0.045y = 73$
b) Alexia invested \$800 at 3.5% and \$1000 at 4.5%.]

Why might you want to multiply equation ② by 100?

Method 2

Use graphing technology to solve the system and the problem.
Express each equation in slope-intercept form.

$x + y = 2000$ $\qquad\qquad$ $0.08x + 0.10y = 190$

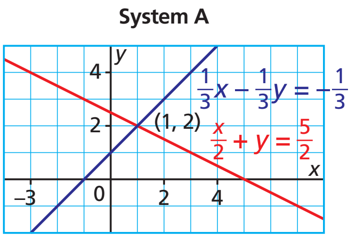
$y = -x + 2000$ $\qquad\qquad$ $0.10y = -0.08x + 190$

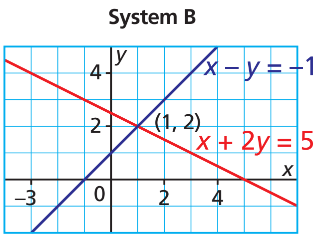
$y = -0.8x + 1900$

Graph each equation.

The lines intersect at: (500, 1500)
The solution is: $x = 500$ and $y = 1500$
So, Nuri invested \$500 at 8% and \$1500 at 10%.
To verify the solution, use the data in the given problem.
Add the amounts: \$500 + \$1500 = \$2000, which is the total amount invested
8% of 500 is \$40 and 10% of \$1500 is \$150. Add the interest.
The total interest is: \$40 + \$150 = \$190
These numbers match those given in the problem, so the solution is correct.

These two linear systems have the same graphs and the same solution $x = 1$ and $y = 2$.





In System A, we can multiply the first equation by 2, and the second equation by 3 to write equivalent equations with integer coefficients. The result is the equations in System B.

System A			**System B**	
$\frac{x}{2} + y = \frac{5}{2}$	①	Multiply each term by 2.	$x + 2y = 5$	③
$\frac{1}{3}x - \frac{1}{3}y = -\frac{1}{3}$	②	Multiply each term by 3.	$x - y = -1$	④

The two linear systems have the same solution $x = 1$ and $y = 2$ because the corresponding equations are equivalent; that is, equation ① is equivalent to equation ③, and equation ② is equivalent to equation ④.
Multiplying or dividing the equations in a linear system by a non-zero number does not change the graphs. So, their point of intersection, and hence, the solution of the linear system is unchanged.

A system of equivalent equations is called an **equivalent linear system** and has the same solution as the original system.

When an equation in a linear system has coefficients or a constant term that are fractions, we can multiply by a common denominator to write an equivalent equation with integer coefficients.

Example 3 Solving a Linear System with Fractional Coefficients

Solve this linear system by substitution.

$$\frac{1}{2}x + \frac{2}{3}y = -1$$

$$y = \frac{1}{4}x - \frac{5}{3}$$

SOLUTIONS

Method 1

$$\frac{1}{2}x + \frac{2}{3}y = -1 \quad ①$$

$$y = \frac{1}{4}x - \frac{5}{3} \quad ②$$

Write an equivalent system with integer coefficients.
For equation ①:

$$\frac{1}{2}x + \frac{2}{3}y = -1$$

Multiply each term by 6.

$$6\left(\frac{1}{2}x\right) + 6\left(\frac{2}{3}y\right) = 6(-1)$$

Simplify.

$$3x + 4y = -6 \quad ③$$

For equation ②:

$$y = \frac{1}{4}x - \frac{5}{3}$$

Multiply each term by 12.

$$12(y) = 12\left(\frac{1}{4}x\right) - 12\left(\frac{5}{3}\right)$$

Simplify.

$$12y = 3x - 20 \quad ④$$

CHECK YOUR UNDERSTANDING

3. Solve this linear system by substitution.

$$\frac{1}{2}x - \frac{4}{5}y = -2$$

$$y = \frac{1}{4}x - \frac{3}{8}$$

[Answer: $x = -\frac{23}{3}$ and $y = -\frac{55}{24}$]

Why did we multiply equation ① by 6 and equation ② by 12?

Solve equation ③ for $3x$.

$$3x + 4y = -6 \qquad ③$$
$$3x = -4y - 6$$

Equations ③ and ④ form a linear system that is equivalent to the linear system formed by equations ① and ②.

Substitute for $3x$ in equation ④.

$$12y = 3x - 20 \qquad ④$$
$$12y = (-4y - 6) - 20$$
$$12y = -4y - 26 \qquad \text{Solve for } y.$$
$$16y = -26$$
$$y = -\frac{26}{16}, \text{ or } -\frac{13}{8}$$

Substitute $y = -\frac{13}{8}$ into equation ④.

$$12y = 3x - 20 \qquad ④$$
$$12\left(-\frac{13}{8}\right) = 3x - 20 \qquad \text{Solve for } x.$$
$$-\frac{39}{2} = 3x - 20$$
$$20 - \frac{39}{2} = 3x$$
$$\frac{40}{2} - \frac{39}{2} = 3x$$
$$\frac{1}{2} = 3x \qquad \text{Divide by 3.}$$
$$x = \frac{1}{6}$$

Method 2

$$\frac{1}{2}x + \frac{2}{3}y = -1 \qquad ①$$
$$y = \frac{1}{4}x - \frac{5}{3} \qquad ②$$

Since equation ② is solved for y, substitute for y in equation ①.

$$\frac{1}{2}x + \frac{2}{3}y = -1 \qquad ①$$
$$\frac{1}{2}x + \frac{2}{3}\left(\frac{1}{4}x - \frac{5}{3}\right) = -1 \qquad \text{Use the distributive property.}$$
$$\frac{1}{2}x + \frac{1}{6}x - \frac{10}{9} = -1 \qquad \text{Collect like terms.}$$
$$\frac{2}{3}x - \frac{10}{9} = -1 \qquad \text{Solve for } x.$$
$$\frac{2}{3}x = -1 + \frac{10}{9}$$
$$\frac{2}{3}x = \frac{1}{9} \qquad \text{Divide by } \frac{2}{3}.$$
$$x = \frac{1}{9}\left(\frac{3}{2}\right)$$
$$x = \frac{1}{6}$$

(Solution continues.)

Substitute $x = \frac{1}{6}$ in equation ②.

$y = \frac{1}{4}x - \frac{5}{3}$ ②

$y = \frac{1}{4}\left(\frac{1}{6}\right) - \frac{5}{3}$

$y = \frac{1}{24} - \frac{5}{3}$

$y = \frac{1}{24} - \frac{40}{24}$

$y = -\frac{39}{24}$, or $-\frac{13}{8}$

Verify the solution.

In each equation, substitute: $x = \frac{1}{6}$ and $y = -\frac{13}{8}$

$\frac{1}{2}x + \frac{2}{3}y = -1$ ① $y = \frac{1}{4}x - \frac{5}{3}$ ②

$$\begin{aligned}
\text{L.S.} &= \frac{1}{2}\left(\frac{1}{6}\right) + \frac{2}{3}\left(-\frac{13}{8}\right) \\
&= \frac{1}{12} - \frac{13}{12} \\
&= -1 \\
&= \text{R.S.}
\end{aligned}$$

$$\text{L.S.} = -\frac{13}{8}$$

$$\begin{aligned}
\text{R.S.} &= \frac{1}{4}\left(\frac{1}{6}\right) - \frac{5}{3} \\
&= \frac{1}{24} - \frac{5}{3} \\
&= \frac{1}{24} - \frac{40}{24} \\
&= -\frac{39}{24}, \text{ or } -\frac{13}{8} \\
&= \text{L.S.}
\end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$x = \frac{1}{6}$ and $y = -\frac{13}{8}$

Discuss the Ideas

1. When you have a system of linear equations, how can you form an equivalent linear system?
2. When you want to write an equivalent equation with integer coefficients, how do you decide which number to multiply by? Use an example to explain.
3. What are some advantages of solving a linear system using the substitution strategy rather than graphing?

Exercises

A

4. Use substitution to solve each linear system.

a) $y = 9 - x$
$2x + 3y = 11$

b) $x = y - 1$
$3x - y = 11$

c) $x = 7 + y$
$2x + y = -10$

d) $3x + y = 7$
$y = x + 3$

5. Solve each linear system.

a) $2x + 3y = 11$
$4x - y = -13$

b) $4x + y = -5$
$2x + 3y = 5$

c) $x + 2y = 13$
$2x - 3y = -9$

d) $3x + y = 7$
$5x + 2y = 13$

B

6. a) In each linear system, identify two like terms and say how they are related.

i) $2x - 3y = 2$
$4x - 4y = 2$

ii) $40x + 10y = 10$
$3x + 5y = 5$

iii) $-3x + 6y = 9$
$5x - 2y = -7$

iv) $-3x + 4y = 6$
$9x + 3y = 27$

b) Solve each linear system in part a.

7. a) Suppose you wanted to solve a linear system in the fewest steps. Which of these systems would you choose? Why?

i) $x - y = -5$
$x = -1$

ii) $x - y = -5$
$-x - y = 3$

iii) $2x - 3y = 7$
$x - 2y = 3$

b) Solve each linear system in part a. Explain what you did.

8. a) For each equation, identify a number you could multiply each term by to ensure that the equation has only integer coefficients and constants. Explain why you chose that number. Create an equivalent linear system.

$\frac{x}{3} - \frac{y}{2} = 2$

$\frac{5x}{6} + \frac{3y}{4} = 1$

b) Verify that both linear systems in part a have the same solution.

9. a) For each equation, choose a divisor. Create an equivalent linear system by dividing each term in the equation by that divisor.

$2x + 2y = -4$
$-12x + 4y = -24$

b) Show that both linear systems in part a have the same solution.

For questions 10 to 18, write a linear system to model each situation. Solve the linear system to solve the related problem.

10. A study recorded the reactions of 186 polar bears as they were approached by a tundra buggy. Some bears did not appear to respond, while others responded by sitting, standing, walking away, or running away. There were 94 more bears that did not respond than did respond. How many bears responded and how many bears did not respond?

11. Louise purchased a Métis flag whose length was 90 cm longer than its width. The perimeter of the flag was 540 cm. What are the dimensions of the flag?

12. Forty-five high school students and adults were surveyed about their use of the internet. Thirty-one people reported a heavy use of the internet. This was 80% of the high school students and 60% of the adults. How many students and how many adults were in the study?

13. Many researchers, such as those at the Canadian Fossil Discovery Centre at Morden, Manitoba, involve students to help unearth fossil remains of 80 million-year-old reptiles. Forty-seven students are searching for fossils in 11 groups of 4 or 5. How many groups of 4 and how many groups of 5 are searching?

14. An art gallery has a collection of 85 Northwest Coast masks of people and animals. Sixty percent of the people masks and 40% of the animal masks are made of yellow cedar. The total number of yellow cedar masks is 38. How many people masks and how many animal masks are there?

15. Sam scored 80% on part A of a math test and 92% on part B of the math test. His total mark for the test was 63. The total mark possible for the test was 75. How many marks is each part worth?

16. Five thousand dollars was invested in two savings bonds for one year. One bond earned interest at an annual rate of 2.5%. The other bond earned 3.75% per year. The total interest earned was $162.50. How much money was invested in each bond?

17. Tess has a part-time job at an ice-cream store. On Saturday, she sold 76 single-scoop cones and 49 double-scoop cones for a total revenue of $474.25. On Sunday, Tess sold 54 single-scoop cones and 37 double-scoop cones for a total revenue of $346.25. What is the cost of each cone?

18. Joel has a part-time job that pays him $40 per weekend. Sue has a part-time job that paid a starting bonus of $150, then $30 per weekend. For how many weekends would Joel have to work before he earns the same amount as Sue? Justify your answer.

19. Solve each linear system.

a) $\frac{1}{2}x + \frac{2}{3}y = 1$
$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2}$

b) $\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$
$x - y = -\frac{4}{3}$

c) $\frac{1}{3}x - \frac{3}{8}y = 1$
$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2}$

d) $\frac{7}{4}x + \frac{4}{3}y = 3$
$\frac{1}{2}x - \frac{5}{6}y = 2$

20. This linear system was used to solve a problem about the cost of buying reams of printer paper and ink cartridges for a school computer lab:
$7.50r + 45c = 375$
$r - c = 15$
a) Create a situation that can be modelled using the linear system. Write a related problem.
b) Solve the system and the problem.

21. Create a situation that can be modelled by this linear system. Write a related problem. Solve the system and the problem.
$2x + 4y = 98$
$x + y = 27$

22. a) Write a linear system that is equivalent to this system. Explain what you did.
$2x - y = -4$
$3x + 2y = 1$
b) Solve each linear system. How do your solutions show that the systems are equivalent?

C

23. One weekend, members of a cycling club rode on the KVR trail (Kettle Valley Railway) uphill from Penticton in Okanagan, B.C. The uphill climb reduced the cyclists' usual average speed by 6 km/h and they took 4 h to get to Chute Lake. On the return trip, the downhill ride increased the cyclists' usual average speed by 4 km/h. The return trip took 2 h.
a) What is the usual average speed?
b) What is the distance from Penticton to Chute Lake?

24. Researchers at the Nk'Mip Desert and Heritage Centre in Osoyoos, B.C., measured the masses of 45 female rattlesnakes and 100 male rattlesnakes as part of a study. The mean mass of all the snakes was 194 g. The mean mass of the males was 37.7 g greater than the mean mass of the females. What was the mean mass of the males and the mean mass of the females? Show your work.

25. After an airplane reached its cruising altitude, it climbed for 10 min. Then it descended for 15 min. Its current altitude was then 1000 m below its cruising altitude. The difference between the rate of climb and the rate of descent was 400 m/min. What were the rate of climb and the rate of descent?

26. Explain why x will always equal y in the solution of this linear system for any non-zero values of A, B, and C; $A \neq B$.

$Ax + By = C$

$Bx + Ay = C$

27. The solution of this linear system is (–2, 3). What are the values of A and B? Show your work.

$Ax + By = -17$

$Bx + Ay = 18$

Reflect

When you use the substitution strategy to solve a linear system, which do you choose first: the equation you start with or the variable you solve for? Use an example to justify your answer.

THE WORLD OF MATH

Careers: Electronics Technician

An electronics technician assembles, installs, troubleshoots, and repairs electronic equipment used by consumers, business, and industry. A technician can use a solution of a linear system to determine the current flowing through an electrical circuit and the voltage across the circuit.

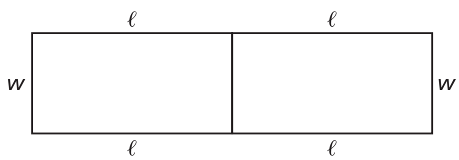
7.5 Using an Elimination Strategy to Solve a System of Linear Equations

LESSON FOCUS

Use the elimination of one variable to solve a linear system.

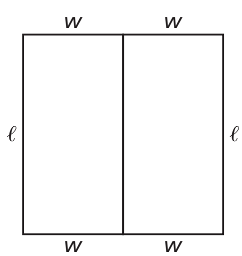
Make Connections

A carpenter placed two identical plywood sheets end to end and measured their perimeter.



Perimeter: 40 ft.

The carpenter placed the sheets side to side and measured their perimeter.



Perimeter: 32 ft.

Suppose you wanted to determine the dimensions of a piece of this plywood.
Which linear system would model the situation?
How would you solve the linear system?

Construct Understanding

TRY THIS

Work with a partner.
Use graphing technology if it is available.

Use this linear system:
$x + 2y = 10$ ①
$x + y = 7$ ②

A. Create equation ③ by adding like terms in equation ① and equation ②. Graph all three equations on the same screen or grid. What do you notice?

B. Create equation ④ by subtracting equation ② from equation ①. Graph all four equations on the same screen or grid. What do you notice?

C. How would subtracting equation ② from equation ① help you solve the system? Would adding the two equations help you solve the system? Explain.

D. Create and solve your own linear system where each equation has the same y-coefficient. Add and subtract the equations to create two other linear equations. Graph all four equations. What do you notice?

E. How does adding or subtracting equations help you solve a linear system?

Why does adding or subtracting the like terms in two linear equations always result in another linear equation?

In Lesson 7.4, you learned that multiplying or dividing each equation in a linear system by a non-zero number does not change the solution because the linear systems that are formed are equivalent.

Similarly, adding or subtracting the two equations in a linear system produces equivalent linear systems. We use this property to solve a linear system by first eliminating one variable by adding or subtracting the two equations. This is called **solving by elimination.**

Why did we put brackets around equation ② when we subtracted it?

Consider this linear system.

$2x + y = -7$ ①

$x + y = -4$ ②

Because the coefficients of y are equal, we can eliminate y by subtracting the equations.

Subtract equation ② from equation ① to determine the value of x.

$$\begin{aligned} 2x + y &= -7 && ① \\ -(x + y &= -4) && ② \\ \hline 2x + y - x - y &= -7 - (-4) && \text{Collect like terms.} \\ x &= -3 \end{aligned}$$

Substitute $x = -3$ into equation ① to determine the value of y.

$$\begin{aligned} 2x + y &= -7 && ① \\ 2(-3) + y &= -7 && \text{Solve for } y. \\ -6 + y &= -7 \\ y &= -1 \end{aligned}$$

Verify the solution. In each equation, substitute: $x = -3$ and $y = -1$

$2x + y = -7$ ①

$$\begin{aligned} \text{L.S.} &= 2x + y \\ &= 2(-3) + (-1) \\ &= -6 - 1 \\ &= -7 \\ &= \text{R.S.} \end{aligned}$$

$x + y = -4$ ②

$$\begin{aligned} \text{L.S.} &= x + y \\ &= -3 + (-1) \\ &= -4 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is: $x = -3$ and $y = -1$

We may need to multiply one or both equations by constants before we can eliminate a variable by adding or subtracting.

Example 1 Solving a Linear System by Subtracting to Eliminate a Variable

Solve this linear system by elimination.

$3x - 4y = 7$

$5x - 6y = 8$

SOLUTION

$3x - 4y = 7$ ①

$5x - 6y = 8$ ②

No like terms have equal coefficients.

CHECK YOUR UNDERSTANDING

1. Solve this linear system by elimination.

   $2x + 7y = 24$

   $3x - 2y = -4$

   [Answer: $x = 0.8$ and $y = 3.2$]

Consider the x-terms.
Their least common multiple is 15.
To make these terms equal, multiply equation ① by 5 and multiply equation ② by 3.

5 × equation ①: $5(3x - 4y = 7) \rightarrow 15x - 20y = 35$ ③
3 × equation ②: $3(5x - 6y = 8) \rightarrow 15x - 18y = 24$ ④

These equations form an equivalent linear system.

To solve this linear system, subtract equation ④ from equation ③ to eliminate x.

$$\begin{aligned} 15x - 20y &= 35 \quad ③ \\ -(15x - 18y &= 24) \quad ④ \\ \hline -20y - (-18y) &= 35 - 24 \\ -2y &= 11 \\ y &= -5.5 \end{aligned}$$

What number would you multiply each equation by to be able to eliminate y by subtracting?

Substitute $y = -5.5$ in equation ①.

$$\begin{aligned} 3x - 4y &= 7 \quad ① \\ 3x - 4(-5.5) &= 7 \quad \text{Solve for } x. \\ 3x + 22 &= 7 \\ 3x &= -15 \\ x &= -5 \end{aligned}$$

Verify the solution.
In each equation, substitute: $x = -5$ and $y = -5.5$

$3x - 4y = 7$ ①

$$\begin{aligned} \text{L.S.} &= 3x - 4y \\ &= 3(-5) - 4(-5.5) \\ &= -15 + 22 \\ &= 7 \\ &= \text{R.S.} \end{aligned}$$

$5x - 6y = 8$ ②

$$\begin{aligned} \text{L.S.} &= 5x - 6y \\ &= 5(-5) - 6(-5.5) \\ &= -25 + 33 \\ &= 8 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = -5$ and $y = -5.5$

In a linear system, we may have to write equivalent equations with integer coefficients before we apply the elimination strategy.

Example 2 Solving a Linear System by Adding to Eliminate a Variable

Use an elimination strategy to solve this linear system.

$\frac{2}{3}x - \frac{1}{2}y = 4$

$\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2}$

SOLUTION

$\frac{2}{3}x - \frac{1}{2}y = 4$ ①

$\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2}$ ②

Multiply each equation by a common denominator.

6 × equation ①: $6\left(\frac{2}{3}x - \frac{1}{2}y = 4\right)$

$$6\left(\frac{2}{3}x\right) - 6\left(\frac{1}{2}y\right) = 6(4)$$

$$4x - 3y = 24 \quad ③$$

4 × equation ②: $4\left(\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2}\right)$

$$4\left(\frac{1}{2}x\right) + 4\left(\frac{1}{4}y\right) = 4\left(\frac{5}{2}\right)$$

$$2x + y = 10 \quad ④$$

An equivalent linear system is:

$4x - 3y = 24$ ③

$2x + y = 10$ ④

To eliminate y, multiply equation ④ by 3 so both y-terms have the numerical coefficient, 3.

3 × equation ④: $3(2x + y = 10) \rightarrow 6x + 3y = 30$ ⑤

An equivalent linear system is:

$6x + 3y = 30$ ⑤

$4x - 3y = 24$ ③

CHECK YOUR UNDERSTANDING

2. Use an elimination strategy to solve this linear system.

$\frac{3}{4}x - y = 2$

$\frac{1}{8}x + \frac{1}{4}y = 2$

[Answer: $x = 8$ and $y = 4$]

Suppose you multiplied equation ① by $\frac{1}{2}$. How would that help you eliminate one variable?

Add:

$$\begin{array}{rl} 6x + 3y = 30 & ⑤ \\ + \ (4x - 3y = 24) & ③ \\ \hline 10x \quad = 54 & \\ x \quad = 5.4 & \end{array}$$

Solve for x.

Substitute $x = 5.4$ in equation ①.

$$\frac{2}{3}x - \frac{1}{2}y = 4 \qquad ①$$

$$\frac{2}{3}(5.4) - \frac{1}{2}y = 4$$

$$3.6 - \frac{1}{2}y = 4$$

Collect like terms.

$$-\frac{1}{2}y = 4 - 3.6$$

$$-\frac{1}{2}y = 0.4$$

Multiply each side by -2 to solve for y.

$$y = -2(0.4)$$

$$y = -0.8$$

Verify the solution.
In each equation, substitute: $x = 5.4$ and $y = -0.8$

$$\frac{2}{3}x - \frac{1}{2}y = 4 \qquad ①$$

$$\begin{aligned} \text{L.S.} &= \frac{2}{3}x - \frac{1}{2}y \\ &= \frac{2}{3}(5.4) - \frac{1}{2}(-0.8) \\ &= 3.6 + 0.4 \\ &= 4 \\ &= \text{R.S.} \end{aligned}$$

$$\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2} \qquad ②$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{2}x + \frac{1}{4}y \\ &= \frac{1}{2}(5.4) + \frac{1}{4}(-0.8) \\ &= 2.7 - 0.2 \\ &= 2.5 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
$x = 5.4$ and $y = -0.8$

What other strategy could you use to solve this linear system?

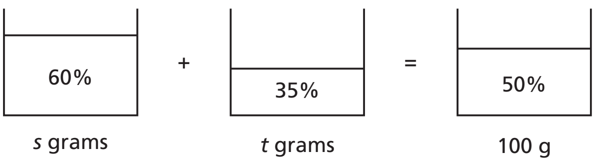
Example 3 Using a Linear System to Solve a Problem

a) Write a linear system to model this situation:
An alloy is a mixture of metals. An artist was commissioned to make a 100-g bracelet with a 50% silver alloy. He has a 60% silver alloy and a 35% silver alloy.

b) Solve this problem:
What is the mass of each alloy needed to produce the desired alloy?

SOLUTION

a) Let s grams represent the mass of 60% alloy and let t grams represent the mass of 35% alloy.
Sketch a diagram.



From the diagram:
The 100-g mass of the bracelet is the sum of mass s grams of 60% alloy and mass t grams of 35% alloy.
So, the first equation is: $s + t = 100$

The mass of silver in the bracelet is 50% of 100 g:
$0.50(100 \text{ g}) = 50 \text{ g}$
This 50 g of silver is made up of 60% of s plus 35% of t.
So, the second equation is: $0.60s + 0.35t = 50$

The linear system is:
$s + t = 100$ ①
$0.60s + 0.35t = 50$ ②

CHECK YOUR UNDERSTANDING

3. a) Write a linear system to model this situation:
An artist was commissioned to make a 625-g statue of a raven with a 40% silver alloy. She has a 50% silver alloy and a 25% silver alloy.

b) Solve this problem: What is the mass of each alloy needed to produce the desired alloy?

[Answers: a) $f + t = 625$; $0.50f + 0.25t = 250$
b) 375 g of the 50% alloy; 250 g of the 25% alloy]

b) Multiply equation ① by 0.60, then subtract to eliminate s.

$0.60 \times$ equation ①: $0.60(s + t = 100)$

$0.60s + 0.60t = 60$ ③

Use the distributive property.

Subtract equation ③ from equation ②.

$$\begin{array}{rl} 0.60s + 0.35t = 50 & ② \\ -(0.60s + 0.60t = 60) & ③ \\ \hline -0.25t = -10 & \\ t = 40 & \end{array}$$

Divide each side by -0.25 to solve for t.

Substitute $t = 40$ in equation ①.

$$\begin{aligned} s + t &= 100 \quad ① \\ s + 40 &= 100 \\ s &= 60 \end{aligned}$$

Sixty grams of 60% silver alloy and 40 g of 35% silver alloy are needed. To verify the solution, use the given data.
The bracelet has a mass of 100 g; and 60 g + 40 g = 100 g.
60 g of 60% alloy was used in the bracelet.
So, the mass of silver in the bracelet from this alloy is:
60% of 60 g = 36 g
40 g of 35% alloy was used in the bracelet.
So, the mass of silver in the bracelet from this alloy is:
35% of 40 g = 14 g
The mass of silver in the bracelet is 36 g + 14 g = 50 g, which is 50% of the mass of the 100-g bracelet.
So, 60 g of 60% silver alloy and 40 g of 35% silver alloy are needed.

Why couldn't you determine a solution if the bracelet was made with a 25% silver alloy?

THE WORLD OF MATH

Math Fact: A Linear System with Three Variables

We can use both the substitution and elimination strategies to solve a linear system with three equations in three variables:

$$\begin{array}{rl} x + 4y + 3z = 5 & ① \\ x + 3y + 2z = 4 & ② \\ x + y - z = -1 & ③ \end{array}$$

We can begin by eliminating the variable x. Subtract equation ② from equation ①, then subtract equation ③ from equation ②.

$$\begin{array}{rl} x + 4y + 3z = 5 & ① \\ -(x + 3y + 2z = 4) & ② \\ \hline y + z = 1 & ④ \end{array} \qquad \begin{array}{rl} x + 3y + 2z = 4 & ② \\ -(x + y - z = -1) & ③ \\ \hline 2y + 3z = 5 & ⑤ \end{array}$$

The result is equations ④ and ⑤, which form a linear system in two variables.
Solve the linear system for y and z. Then solve for x.
Verify your solution.

Example 4 Solving by Determining the Value of Each Variable Independently

Solve this linear system.

$2x + 3y = 8$

$5x - 4y = -6$

SOLUTION

$2x + 3y = 8$ ①

$5x - 4y = -6$ ②

Eliminate x first. The least common multiple of the coefficients of x is 10.

Multiply equation ① by 5 and equation ② by 2, then subtract.

$$\begin{array}{ll} 5 \times \text{equation ①:} & 10x + 15y = 40 \\ 2 \times \text{equation ②:} & \underline{-(10x - 8y = -12)} \\ & 23y = 52 \\ & y = \dfrac{52}{23} \end{array}$$

Eliminate y next. The least common multiple of the coefficients of y is 12.

Multiply equation ① by 4 and equation ② by 3, then add.

$$\begin{array}{ll} 4 \times \text{equation ①:} & 8x + 12y = 32 \\ 3 \times \text{equation ②:} & \underline{+(15x - 12y = -18)} \\ & 23x = 14 \\ & x = \dfrac{14}{23} \end{array}$$

Verify the solution.

In each equation, substitute: $x = \frac{14}{23}$ and $y = \frac{52}{23}$

$2x + 3y = 8$ ①

$$\begin{aligned} \text{L.S.} &= 2\left(\frac{14}{23}\right) + 3\left(\frac{52}{23}\right) \\ &= \frac{28}{23} + \frac{156}{23} \\ &= \frac{184}{23} \\ &= 8 \\ &= \text{R.S.} \end{aligned}$$

$5x - 4y = -6$ ②

$$\begin{aligned} \text{L.S.} &= 5\left(\frac{14}{23}\right) - 4\left(\frac{52}{23}\right) \\ &= \frac{70}{23} - \frac{208}{23} \\ &= -\frac{138}{23} \\ &= -6 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is: $x = \frac{14}{23}$ and $y = \frac{52}{23}$

CHECK YOUR UNDERSTANDING

4. Solve this linear system.

$3x + 9y = 5$

$9x - 6y = -7$

[Answer: $x = -\frac{1}{3}$ and $y = \frac{2}{3}$]

What is an advantage of not substituting $y = \frac{52}{23}$ into one of the equations to determine x?

Discuss the Ideas

1. When you use the elimination strategy to solve a linear system, how can you tell whether you will add equations or subtract them?

2. When you use the elimination strategy to solve a linear system, how do you know when you need to begin by multiplying?

Exercises

A

3. Use an elimination strategy to solve each linear system.

   a) $x - 4y = 1$
   $x - 2y = -1$

   b) $3a + b = 5$
   $9a - b = 15$

   c) $3x - 4y = 1$
   $3x - 2y = -1$

   d) $3x - 4y = 0$
   $5x - 4y = 8$

4. For each linear system, write an equivalent linear system where both equations have:

   i) the same x-coefficients

   ii) the same y-coefficients

   a) $x - 2y = -6$
   $3x - y = 2$

   b) $15x - 2y = 9$
   $5x + 4y = 17$

   c) $7x + 3y = 9$
   $5x + 2y = 7$

   d) $14x + 15y = 16$
   $21x + 10y = -1$

5. Solve each linear system in question 4.

B

6. Use an elimination strategy to solve each linear system.

   a) $2x + y = -5$
   $3x + 5y = 3$

   b) $3m - 6n = 0$
   $9m + 3n = -7$

   c) $2s + 3t = 6$
   $5s + 10t = 20$

   d) $3a + 2b = 5$
   $2a + 3b = 0$

7. Solve each linear system. Explain what you did for part d.

   a) $8x - 3y = 38$
   $3x - 2y = -1$

   b) $2a - 5b = 29$
   $7a - 3b = 0$

   c) $18a - 15b = 4$
   $10a + 3b = 6$

   d) $6x - 2y = 21$
   $4x + 3y = 1$

For questions 8 to 11, model each situation with a linear system then solve the problem.

8. The mean attendance at the Winnipeg Folk Festival for 2006 and 2008 was 45 265. The attendance in 2008 was 120 more than the attendance in 2006. What was the attendance in each year?

9. Talise folded 545 metal lids to make cones for jingle dresses for herself and her younger sister. Her dress had 185 more cones than her sister's dress. How many cones are on each dress?

10. Years ago, people bought goods with beaver pelts instead of cash. Two fur traders purchased some knives and blankets from the Hudson's Bay Company store at Fort Langley, B.C. The items and the cost in beaver pelts for each fur trader are shown below:
    10 knives + 20 blankets = 200 beaver pelts
    15 knives + 25 blankets = 270 beaver pelts
    What is the cost, in beaver pelts, of one knife and of one blanket?

11. Bernard used an electronic metronome to help him keep time to a guitar piece he was learning to play. He played at a moderate tempo for 4.5 min and at a fast tempo for 30 s. Bernard played a total of 620 beats on the metronome. The rate for the moderate tempo was 40 beats/min less than the rate for the fast tempo. What is the rate in beats per minute for each tempo?

12. Solve each linear system. Explain what you did for part a.

a) $\frac{a}{2} + \frac{b}{3} = 1$

$\frac{a}{4} - \frac{2b}{3} = -1$

b) $\frac{x}{2} + \frac{y}{2} = 7$

$3x + 2y = 48$

c) $0.03x + 0.15y = 0.027$

$-0.5x - 0.5y = 0.05$

d) $-1.5x + 2.5y = 0.5$

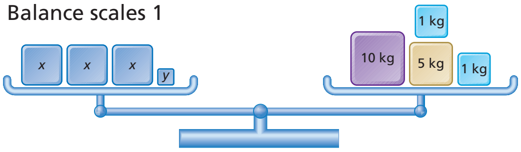
$2x + y = 1.5$

13. The 2008–09 Edmonton Oilers had 25 players, 17 of whom were over 6 ft. tall. Seven-ninths of the Canadian players were over 6 ft. tall. Three-sevenths of the foreign players were over 6 ft. tall. How many players were Canadian and how many were foreign?

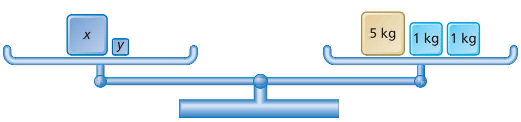
14. Melody surveyed the 76 grade 10 students in her school to find out who played games online. One-quarter of the girls and $\frac{3}{4}$ of the boys said they played online games with someone over the weekend. Thirty-nine students played online games that weekend. How many girls and how many boys did Melody survey?

15. a) Which linear system is modelled by these two balance scales?

Balance scales 1



Balance scales 2



b) From Balance scales 1, suppose you remove mass x and mass y from the left side and 7 kg from the right side. How do you know that the scales will still be balanced?

c) How does this process help you determine the value of x and the value of y?

d) How is this process related to the elimination strategy for solving a linear system?

16. To visit the Manitoba Children's Museum in Winnipeg:

- One adult and 3 children pay \$27.75.
- Two adults and 2 children pay \$27.50.

Which ticket is more expensive? Justify your answer.

17. A co-op that sells organic food made 25 kg of soup mix by combining green peas that cost \$5/kg with red lentils that cost \$6.50/kg. This mixture costs \$140. What was the mass of peas and the mass of lentils in the mixture?

18. This linear system models a problem about a pentagon.

$3x + 2y = 21$

$x - y = 2$

What might the problem be? Solve the problem you suggest.

19. a) Write a problem that can be modelled by this linear system. Explain how you created the problem.

$3x + y = 17$

$x + y = 7$

b) Solve the problem you created.

20. Suppose you want to eliminate one variable in the linear system below by adding.

a) What are two different ways to eliminate a variable?

$3x + 4y = 29$

$2x - 5y = -19$

b) Solve the system using the two ways you described in part a.

21. This table shows the numbers of males and females in a study of colour blindness.

	Female	Male	Total
Colour blind	2	12	14
Not colour blind	98	88	186
Total	100	100	200

a) Use the data in the table to create a situation that can be modelled by a linear system.

b) Pose and solve a related problem.

C

22. Cam invested in a stock and a bond for one year. At the end of the year, the stock had lost 10.5% and the bond had gained 3.5%. The total loss for both investments was $84. If Cam had invested the bond amount into the stock and the stock amount into the bond, he would have lost only $14. How much money did Cam invest in the stock and in the bond?

23. In the equation $2x + 5y = 8$, the difference in consecutive coefficients and constant term is 3.

a) Write another equation whose coefficients and constant differ by 3. Solve the linear system formed by these equations.

b) Write, then solve two different systems of linear equations for which the coefficients and constant term in each equation differ by 3.

c) Compare your solutions in parts a and b.

d) Use algebra to verify that when the coefficients and constant term in the linear equations differ by a constant in this way, then the solution of the linear system will always be the same.

24. A farmer in Saskatchewan harvested 1 section (which is 640 acres) of wheat and 2 sections of barley. The total yield of grain for both areas was 99 840 bushels. The wheat sold for $6.35/bushel and the barley sold for $2.70/bushel. The farmer received $363 008 for both crops.

a) What was the yield of each section in bushels/acre?

b) Some farmers use hectares instead of acres or sections to measure area. One acre is 0.4047 ha. Would you have to write and solve a different linear system to determine the yield in bushels/hectare? Explain.

Reflect

You have used graphing, substitution, and elimination to solve a linear system. For each strategy, give an example of a linear system that you think would be best solved using that strategy. Justify your choices.

CHECKPOINT 2

Connections

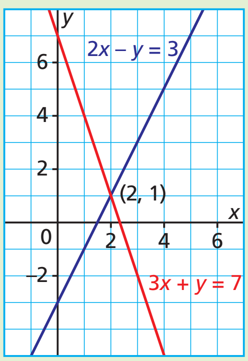
Linear System

$3x + y = 7$

$2x - y = 3$

Solve by graphing

Use graphing software, a graphing calculator, or grid paper.



Solve by substitution

$3x + y = 7$ ①

$2x - y = 3$ ②

Solve equation ① for y.

$y = -3x + 7$

Substitute for y in equation ②.

$2x - (-3x + 7) = 3$

$5x - 7 = 3$

$5x = 10$

$x = 2$

Substitute for x in equation ①.

$3(2) + y = 7$

$6 + y = 7$

$y = 1$

Solution: $x = 2$ and $y = 1$

Solve by elimination

$3x + y = 7$ ①

$2x - y = 3$ ②

Add the equations to eliminate y.

$5x = 10$

$x = 2$

Substitute for x in equation ①.

$3(2) + y = 7$

$6 + y = 7$

$y = 1$

Solution: $x = 2$ and $y = 1$

Verify the solution.
Substitute for x and y in each equation to check that the values satisfy the equations.

$3x + y = 7$ ①	$2x - y = 3$ ②
L.S. $= 3x + y$	L.S. $= 2x - y$
$= 3(2) + 1$	$= 2(2) - 1$
$= 7$	$= 3$
= R.S.	= R.S.

Since the left side is equal to the right side for each equation, the solution is: $x = 2$ and $y = 1$

Concept Development

- **In Lesson 7.4**
  - You solved linear systems by using the substitution strategy.
  - You created equivalent equations by multiplying or dividing each term in an equation by a non-zero number.
- **In Lesson 7.5**
  - You solved linear systems by using the elimination strategy.
  - You created an equivalent linear system by adding or subtracting the two equations in a linear system.

Assess Your Understanding

7.4

1. Solve each linear system by substitution.

a) $5x + y = 4$
$x + y = 2$

b) $3x - y = 1$
$2x + y = -1$

c) $\frac{x}{3} + \frac{y}{4} = -\frac{9}{4}$
$\frac{5x}{6} - \frac{3y}{4} = -\frac{17}{4}$

2. a) Write a linear system to model this situation:
A store sold Inukshuk replicas made with either 6 or 7 stones.
The total number of stones in all Inukshuit sold was 494. The store sold 13 more Inukshuit made with 6 stones than those made with 7 stones.

b) Solve this problem: How many of each type of replicas were sold?

3. One thousand dollars was invested in two savings bonds for one year. One bond earned interest at an annual rate of 5.5%. The other bond earned 4.5% per year. The total interest earned was \$50. How much money was invested in each bond?

7.5

4. Solve each linear system by elimination.

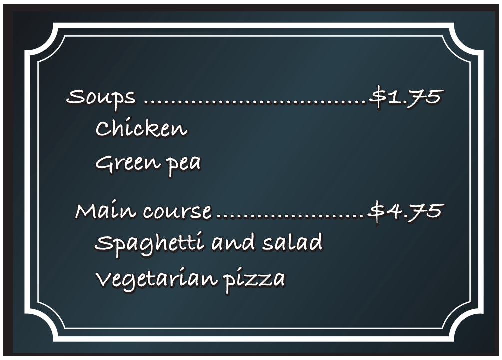
a) $3x - y = -11$
$-x + y = -1$

b) $\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3}$
$\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8}$

c) $0.5x - 0.3y = 0.15$
$-0.3x + 0.5y = -0.65$

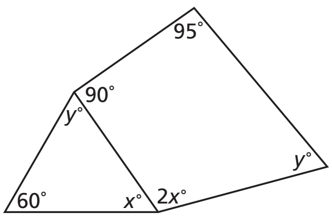
d) $x + 2y = -2$
$-2x + y = 6$

5. Each time Trisha went to the school cafeteria, she bought either a bowl of soup for \$1.75 or a main course for \$4.75. During the school year, she spent \$490 and bought 160 food items. How many times did Trisha buy soup? How many times did she buy a main course? Justify your answers.



6. The volumes of two acid solutions differed by 1000 mL. The larger volume contained 5.5% acid. The smaller volume contained 4.5% acid. The total volume of acid in both solutions was 100 mL. What was the volume of each solution?

7. Determine the values of x and y in this diagram. How can you check that your answers are correct?



7.6 Properties of Systems of Linear Equations

LESSON FOCUS

Determine the numbers of solutions of different types of linear systems.

Make Connections

Phil was teased by his grandparents to determine their ages given these clues.

The sum of our ages is 151.
Add our two ages. Double this sum is 302.
What are our ages?

Can Phil determine his grandparents' ages given these clues? Why or why not?

Construct Understanding

THINK ABOUT IT

Work in a group of 3.
Use graphing technology if it is available.

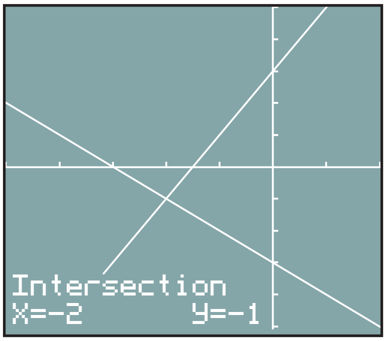
Each linear system below contains the equation: $-2x + y = 2$
Solve each linear system by graphing.

System 1	**System 2**	**System 3**
$-2x + y = 2$	$-2x + y = 2$	$-2x + y = 2$
$2x + y = 2$	$-2x + y = 4$	$-4x + 2y = 4$

Share your results with your group.
How many solutions does each linear system have?

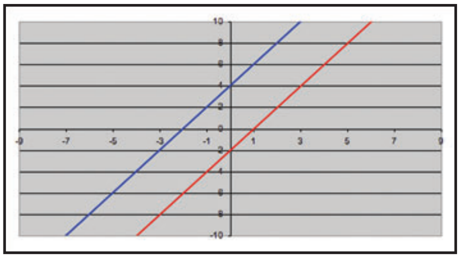
All the linear systems you studied in earlier lessons have had exactly one solution.
We can graph a linear system to determine how many solutions it has.

- Here are the graphs of the linear system:
  $x + y = -3$
  $-2x + y = 3$
  The graphs intersect at $(-2, -1)$.
  So, there is only one solution: $x = -2$ and $y = -1$
  Because the slopes of the lines are different, the lines intersect at exactly one point.



- Here are the graphs of the linear system:
  $-4x + 2y = 8$
  $-2x + y = -2$

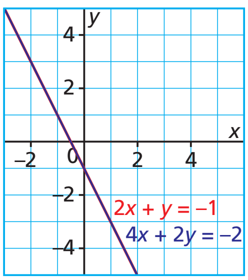
How can you tell from the equations that the lines are parallel?



The graphs do not intersect.
So, there is no solution.
Because the slopes of the lines are equal, the lines are parallel.

- Here are the graphs of the linear system:
  $2x + y = -1$
  $4x + 2y = -2$

How can you tell from the equations that they represent the same line?



Since the graphs coincide, every point on one line is also a point on the other line; so all the points on the line are solutions.
So, there are **infinite** solutions.
Because the lines have equal slopes and the same y-intercept, they are **coincident lines.**

Infinite means unlimited or without bound.

When lines coincide, they are **coincident.**

Example 1 Determining the Number of Solutions of a Linear System

Determine the number of solutions of each linear system.

a) $x + y = -2$
$-2x - 2y = 4$

b) $4x + 6y = -10$
$-2x - y = -1$

c) $3x + y = -1$
$-6x - 2y = 12$

SOLUTION

Write each equation in slope-intercept form to identify the slope and y-intercept of each line.

a) $x + y = -2$ ①
$-2x - 2y = 4$ ②

For equation ①:

$x + y = -2$ — Subtract x from each side.

$y = -x - 2$ ③

The slope is -1 and the y-intercept is -2.

For equation ②:

$-2x - 2y = 4$ — Add $2x$ to each side.

$-2y = 2x + 4$ — Divide by -2 to solve for y.

$$\frac{-2y}{-2} = \frac{2x}{-2} + \frac{4}{-2}$$

$y = -x - 2$ ④

Equation ④ is the same as equation ③, so the slope is -1 and the y-intercept is -2.
The slope-intercept forms of both equations are the same, so the lines are coincident and the linear system has infinite solutions.

b) $4x + 6y = -10$ ①
$-2x - y = -1$ ②

For equation ①:

$4x + 6y = -10$ — Subtract $4x$ from each side.

$6y = -4x - 10$ — Divide by 6 to solve for y.

$$\frac{6y}{6} = \frac{-4x}{6} - \frac{10}{6}$$

$$y = -\frac{2}{3}x - \frac{5}{3} \quad ③$$

The slope is $-\frac{2}{3}$ and the y-intercept is $-\frac{5}{3}$.

CHECK YOUR UNDERSTANDING

1. Determine the number of solutions of each linear system.

a) $x + y = 3$
$-2x - y = -2$

b) $4x + 6y = -10$
$-2x - 3y = 5$

c) $2x - 4y = -1$
$3x - 6y = 2$

[Answers: a) one solution
b) infinite solutions
c) no solution]

For equation ②:

$-2x - y = -1$ Add $2x$ to each side.

$-y = 2x - 1$ Multiply each side by -1 to solve for y.

$y = -2x + 1$ ④

The slope is -2 and the y-intercept is 1.
Because the slopes are different, the lines intersect at exactly one point and the linear system has one solution.

> Why don't you need to identify the y-intercepts to show there is only one solution?
> What is the solution?

c) $3x + y = -1$ ①

$-6x - 2y = 12$ ②

For equation ①:

$3x + y = -1$ Subtract $3x$ from each side.

$y = -3x - 1$ ③

The slope is -3 and the y-intercept is -1.

For equation ②:

$-6x - 2y = 12$ Add $6x$ to each side.

$-2y = 6x + 12$ Divide each side by -2 to solve for y.

$$\frac{-2y}{-2} = \frac{6x}{-2} + \frac{12}{-2}$$

$y = -3x - 6$ ④

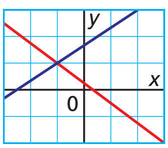
The slope is -3 and the y-intercept is -6.
Because the slopes are equal and the y-intercepts are different, the lines are parallel and the linear system has no solution.

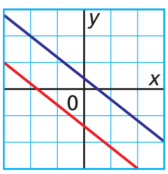
> Why is it important to identify the y-intercepts when the equations in a linear system have the same slope?

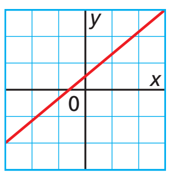
When you attempt to solve a linear system of two equations in two variables, there are only three possibilities. You can determine the number of solutions using different methods.

Possible Solutions for a Linear System

Intersecting Lines	**Parallel Lines**	**Coincident Lines**
One solution	No solution	Infinite solutions







Example 2 Creating a Linear System with 0, 1, or Infinite Solutions

Given the equation $-2x + y = 4$, write another linear equation that will form a linear system with:

a) exactly one solution

b) no solution

c) infinite solutions

SOLUTION

a) For a linear system with one solution, the lines must have different slopes so they can intersect.
Write the given equation in slope-intercept form to identify its slope.

$-2x + y = 4$ Add $2x$ to each side.

$y = 2x + 4$

The slope of the line is 2.
Let the second line have slope 3 and any y-intercept, such as 2.
Then its equation is:

$y = 3x + 2$

A possible linear system with exactly one solution is:

$y = 2x + 4$
$y = 3x + 2$

b) For a linear system with no solution, the lines must be parallel but not coincident so there is no point of intersection.
The lines must have equal slopes but different y-intercepts.
From part a, the given equation is equivalent to $y = 2x + 4$, with slope 2.
The second line must also have slope 2.
Let its y-intercept be 5.
Then its equation is:

$y = 2x + 5$

A possible linear system with no solution is:

$y = 2x + 4$
$y = 2x + 5$

CHECK YOUR UNDERSTANDING

2. Given the equation $-6x + y = 3$, write another linear equation that will form a linear system with:

a) exactly one solution

b) no solution

c) infinite solutions

[Sample Answers: a) $y = 2x + 4$ b) $y = 6x - 3$ c) $-12x + 2y = 6$]

c) For a linear system with infinite solutions, the lines must be coincident so they intersect at every possible point.
The equations must be equivalent.
The given equation is: $-2x + y = 4$
To determine an equivalent equation, multiply each term by a non-zero number, such as -3.
$(-3)(-2x) + (-3)(y) = (-3)(4)$ Simplify.
$6x - 3y = -12$
A possible linear system with infinite solutions is:
$6x - 3y = -12$
$-2x + y = 4$

How is a linear system with infinite solutions like a linear system with no solution?
How are the systems different?

Discuss the Ideas

1. Without solving a linear system, what strategies could you use to determine how many solutions it has? Use an example to explain.
2. Why do you get the equation of a coincident line when you multiply or divide each term of the equation of a linear function by a non-zero number?
3. How can you use the slopes of the lines in a linear system to determine the number of possible solutions? How can you use the intercepts?

THE WORLD OF MATH

Historical Moment: Linear Systems and the Babylonians

Babylon was a city-state, the remains of which are in Iraq. The Babylonians had a superior knowledge of mathematics including the ability to solve problems that can be modelled by linear systems. Some of these problems are preserved on clay tablets. For example, a tablet dating from around 300 BCE contains a problem similar to this:

Two fields have a total area of 1800 square units. One field produces grain at the rate of $\frac{2}{3}$ of a bushel per square unit while the other field produces grain at the rate of $\frac{1}{2}$ of a bushel per square unit.

The total yield is 1100 bushels. What is the area of each field?

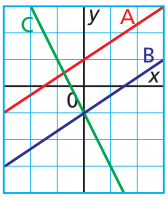
How would you solve this problem?

Exercises

A

4. **a)** Without graphing, determine the slope of the graph of each equation.
  i) $-x + y = 5$
  ii) $-x - y = 10$
  iii) $-2x + 2y = 10$
  iv) $x + y = 5$
b) Which lines in part a are parallel?
c) Which lines in part a intersect?

5. The graphs of three lines are shown below.



a) Identify two lines that form a linear system with exactly one solution.
b) Identify two lines that form a linear system with no solution.

6. Use these 6 equations:

$4x + 2y = 20$ $\quad$ $x - 3y = 12$
$5x - 15y = -60$ $\quad$ $2x + y = 10$
$6x + 3y = 5$ $\quad$ $2x - 6y = 24$

Write a linear system that has:
a) no solution
b) exactly one solution
c) infinite solutions

B

7. Determine the number of solutions of each linear system.

a) $x + 2y = 6$
$x + y = -2$

b) $3x + 5y = 9$
$6x + 10y = 18$

c) $2x - 5y = 30$
$4x - 10y = 15$

d) $\frac{x}{2} + \frac{y}{3} = \frac{1}{2}$
$\frac{x}{2} + \frac{y}{3} = \frac{1}{4}$

8. The first equation of a linear system is given. Write a second equation to form a linear system that satisfies each condition. Explain your reasoning.
a) The second line intersects the line $-2x + y = 1$ in the first quadrant.
b) The second line does not intersect the line $-2x + y = 1$.
c) The second line coincides with the line $-2x + y = 1$.

9. The table below shows some properties of the graphs of 3 linear equations. For the linear system formed by each pair of equations, how many solutions are there? Explain your reasoning.
a) A and B
b) A and C
c) B and C

Equation	Slope	*y*-intercept
A	−0.5	4
B	−0.5	2
C	0.5	4

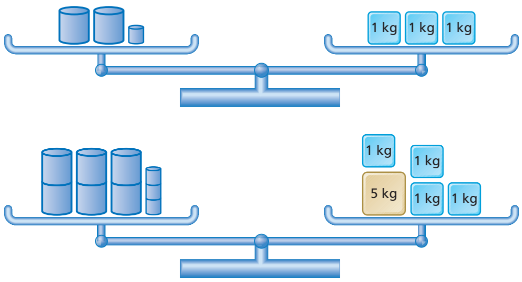
10. Marc wrote the two equations in a linear system in slope-intercept form. He noticed that the signs of the two slopes were different. How many solutions will this linear system have? Explain.

11. Two lines in a linear system have the same slope. What information do you need to determine whether the linear system has no solution or infinite solutions?

12. Use the equation $3x - 4y = 12$ as an equation in three different linear systems. Write a second equation so that each system has a different number of solutions. Explain what you did for each system.

For questions 13 to 17, tell whether the linear system that models each problem has one solution, no solution, or infinite solutions. Justify your answer. You do not have to solve the problems.

13. What are the masses of a large container and a small container?



14. Nadine has a cup of nickels and a cup of dimes. The total number of coins is 300 and their value is \$23.25. How many coins are in each cup?

15. Prana has a savings account and a chequing account with a total balance of \$85. His parents doubled the amount in each account and the new total balance is \$170. How much money does Prana have in each account?

16. The total attendance at a weekend Pow Wow was 568. There were 44 more people on Sunday than on Saturday. What was the attendance for each day?

17. Tickets for a guided tour of La Maison Gabrielle-Roy in Saint-Boniface, Manitoba, cost \$5 for an adult and \$3 for a student. Seventy-five tickets were sold for \$275. How many adults and how many students visited La Maison Gabrielle-Roy?

18. Use the terms "slope" and "y-intercept" to describe the conditions for two lines to have 0, 1, or infinite points of intersection.

19. **a)** Write a linear system that has infinite solutions.

b) Explain what happens when you try to solve the system using elimination.

20. **a)** Write a linear system that has no solution.

b) Explain what happens when you try to solve the system using elimination.

21. Given the graph of a linear system, is it always possible to determine the number of solutions of the linear system? Use examples to explain your reasoning.

C

22. **a)** Determine the number of solutions of each system.

i) $2x + 3y = 4$
$4x + 6y = 8$

ii) $2x + 3y = 4$
$4x + 6y = 7$

iii) $2x + 3y = 4$
$4x + 5y = 8$

b) How can you compare corresponding coefficients to help you determine whether a system has no solution, one solution, or infinite solutions?

23. In the linear system below, $AE - DB = 0$; show that the system does not have exactly one solution.

$Ax + By = C$
$Dx + Ey = F$

24. **a)** For what values of k does the linear system below have:

i) exactly one solution?

ii) infinite solutions?

$$\frac{1}{2}x + \frac{5}{3}y = 2$$
$$kx + \frac{5}{2}y = 3$$

b) Explain why there is no value of k for which the linear system has no solution.

Reflect

Use examples to explain why the only possible solutions of a linear system are: no solution, one solution, or infinite solutions.

STUDY GUIDE

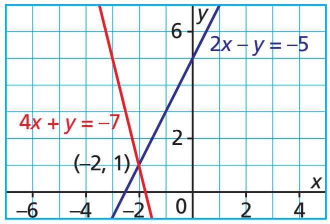
CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ A system of two linear equations is solved when the set of ordered pairs that satisfies both equations is determined.	This means that: ■ Substituting (x, y) into the two equations of a linear system determines whether the ordered pair is a solution. You can determine this solution using graphs, or algebraic strategies.
■ Multiplying or dividing the equations in a linear system by a non-zero number, or adding or subtracting the equations produces an equivalent system.	■ You can create an equivalent system by multiplying or dividing each term in a linear equation by a constant. The solution of the new system is the same as the original solution. You can create an equivalent system by adding or subtracting the like terms in the equations of a linear system. The solution of the new system is the same as the original system.
■ A system of two linear equations may have one solution, infinite solutions, or no solution.	■ You can determine the number of solutions by graphing or by comparing the slopes and y-intercepts of the graphs of the linear equations.

Reflect on the Chapter

- How do you determine the number of solutions of a linear system?
- What are some different strategies you can use to solve a linear system?
- Why does multiplying or dividing each term in each equation or adding or subtracting the equations not change the solution of a linear system?

SKILLS SUMMARY

Skill	Description	Example
Solve a linear system by graphing. [7.2, 7.3]	To solve a linear system by graphing: **1.** Draw the graphs after determining their intercepts, or their slopes and y-intercepts. **2.** The coordinates of the point of intersection are the solution of the linear system. **3.** Verify the solution by substituting the coordinates into the equations.	For this linear system: $2x - y = -5$ $4x + y = -7$  The solution is: $x = -2$ and $y = 1$
Solve a linear system algebraically. [7.4, 7.5]	To solve a linear system algebraically: **1.** Use substitution or elimination. **2.** Verify the solution by substituting for x and y in both equations to check that the coordinates of the point of intersection satisfy both equations.	For this linear system: $2x - y = -5$ ① $4x + y = -7$ ② Use elimination. Add the equations. $6x = -12$ $x = -2$ Substitute for x in equation ①. $2(-2) - y = -5$ $y = 1$ The solution is: $x = -2$ and $y = 1$
Determine the number of solutions of a linear system. [7.6]	To determine the number of solutions in a linear system, use Step 1 or Step 2: **1.** Compare the graphs of the equations. **2.** Compare the slopes and y-intercepts of the lines.	The graphs of the linear system above have different slopes, so there is exactly one solution. The graphs of the linear system below have the same slope and the same y-intercept, so there are infinite solutions: $2x + 4y = 6$ $4x + 8y = 12$ The graphs of the linear system below have the same slope and different y-intercepts, so there is no solution: $2x + 4y = 6$ $4x + 8y = 10$

REVIEW

7.1

For questions 1 and 2, create a linear system to model the situation, then identify which is the correct solution for the related problems. Justify your choice.

1. **a)** The situation is:
In 2009, the Bedford Road Invitational Tournament (BRIT) in Saskatoon, Saskatchewan, held its 41st annual basketball tournament. Teams from outside Saskatchewan have won the tournament 17 more times than teams from Saskatchewan.

b) The related problems are:
How many times have teams from Saskatchewan won the BRIT? How many times have teams from outside Saskatchewan won the BRIT? (*Solution A*: teams from Saskatchewan have won 29 times and teams from outside Saskatchewan have won 12 times. *Solution B*: teams from Saskatchewan have won 12 times and teams from outside Saskatchewan have won 29 times.)

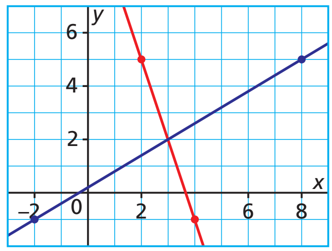
2. **a)** The situation is:
Yvette operates a snow-blowing business. She charges \$15 for a small driveway and \$25 for a large driveway. One weekend, Yvette made \$475 by clearing snow from 25 driveways.

b) The related problems are:
How many small driveways did Yvette clear? How many large driveways did she clear? (*Solution A*: Yvette cleared 10 small driveways and 15 large driveways.
Solution B: Yvette cleared 15 small driveways and 10 large driveways.)

3. Kyle wrote this linear system to model a problem he created about the cost of tickets and popcorn for a group of people to go to a movie theatre. What problem might he have written?
$9.95t + 5.50p = 76.20$
$t - p = 3$

7.2

4. **a)** Which linear system is modelled by this graph? Explain how you know.



b) What is the solution of the linear system? Is it exact or approximate? How do you know?

5. To solve the linear system below by graphing, George and Sunita started with different steps:
$-x + 4y = 10$ ①
$4x - y = -10$ ②
George's Method
Equation ①: plot (0, 2.5) and (–10, 0)
Equation ②: plot (0, 10) and (–2.5, 0)

Sunita's Method
Equation ①: graph $y = \frac{1}{4}x + 2.5$
Equation ②: graph $y = 4x + 10$

a) Explain what each student will probably do next.

b) Choose either method. Solve the linear system by graphing.

6. Explain how you would solve this linear system by graphing on grid paper. You do not have to draw the graphs.
$x - y = 15$
$2x + y = 6$

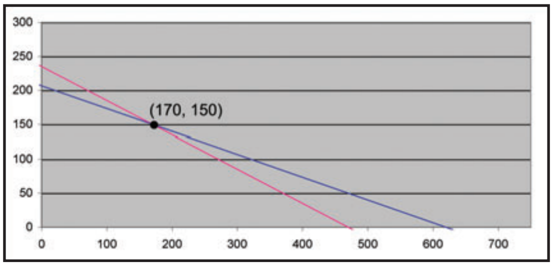
7. **a)** Graph to solve this linear system.
$4x - 2y = 1$
$3x - 4y = 16$

b) Tell whether the solution is exact or approximate, and how you know.

7.3

8. **a)** Write a linear system to model this situation: Table salt contains 40% sodium, and health experts recommend that people limit their sodium intake. For breakfast, Owen ate 2 bowls of cereal and 4 slices of bacon that contained a total of 940 mg of sodium. Natalie ate 1 bowl of cereal and 3 slices of bacon that contained a total of 620 mg of sodium.

b) This graph represents a linear system for the situation in part a. What does each line in the graph represent?



c) Solve this related problem: How much sodium is in 1 bowl of cereal and in 1 slice of bacon? Is the solution exact or approximate? How could you find out?

9. Use graphing technology to solve each linear system.

a) $2x + 3y = 13$
$5x - 2y = 1$

b) $y = \frac{1}{6}x - 2$
$y = -\frac{1}{6}x + 2$

c) $4x - 5y = 20$
$8x + 5y = 19$

d) $\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16}$
$-2x + 4y = 20$

7.4

10. Solve each linear system by substitution.

a) $x + y = -5$
$x + 3y = -15$

b) $7x + y = 10$
$3x - 2y = -3$

c) $\frac{1}{2}x + 3y = \frac{5}{6}$
$\frac{1}{3}x - 5y = \frac{16}{9}$

d) $0.6x - 0.2y = -0.2$
$-0.03x - 0.07y = 0.17$

11. **a)** Why did Laura multiply equation ① by 4 and equation ② by 6 before she solved this linear system?

$-\frac{3}{2}x - \frac{1}{4}y = -\frac{1}{2}$ ①

$\frac{1}{3}x + \frac{5}{6}y = \frac{19}{3}$ ②

b) Why will the new linear system have the same solution as the original system?

c) Solve the linear system.

12. **a)** Write a linear system to model this situation: Paul made bannock to celebrate National Aboriginal Day. He measured $5\frac{3}{4}$ cups of flour using a $\frac{1}{4}$ cup measure and a $\frac{2}{3}$ cup measure. Paul used 1 more $\frac{1}{4}$ cup measure than $\frac{2}{3}$ cup measure.

b) Solve this related problem: How many measures of each size did Paul use?

13. When 30 identical rectangular tables are placed end to end, their perimeter is 306 ft. When the same tables are placed side by side, their perimeter is 190 ft.

a) Draw a diagram of the first 3 tables to illustrate each arrangement.

b) Write a linear system to model the situation.

c) Solve the linear system to solve this related problem: What is the width and length of each table?

14. Sofia sketched a design for a blanket. She made the design with 150 shapes that were equilateral triangles and squares. Eighty-three of the shapes were blue. Forty percent of the triangles and 60% of the squares were blue. How many triangles and how many squares were in the design?

7.5

15. Solve each system by elimination.

a) $-3x - y = 5$
$2x + y = -5$

b) $2x - 4y = 13$
$4x - 5y = 8$

16. a) In the linear system below, which number would you multiply one equation by to help you eliminate y in the next step? Explain.

$3x - 4y = 8.5$ ①
$4x + 2y = 9.5$ ②

b) What would be your next step in solving the linear system?

c) Solve the linear system.

17. The key in one type of basketball court has the shape of a rectangle and a semicircle, with perimeter approximately $68\frac{5}{6}$ ft. The length of the rectangular part of the key is 7 ft. longer than its width.

a) Write a linear system to model the situation above.

b) Solve this related problem: To the nearest foot, what are the length and the width of the rectangular part?

7.6

18. a) Write two linear systems where one system has infinite solutions and the other system has no solution.

b) How can you use graphs to show the number of solutions of each linear system?

c) How can comparing the slope-intercept forms for the equations in the linear system help you determine the number of solutions?

19. Grace and Olivia have 2-digit numbers on their hockey jerseys. They wrote three sets of clues to help some friends identify these numbers.
Clue 1: The difference between the two numbers is 33. When you triple each player's number then subtract, the difference is 99.
Clue 2: The sum of the two numbers is 57. When you divide each number by 3 then add the quotients, the sum is 20.
Clue 3: The sum of the two numbers is 57. Their difference is 33.

a) Which clues do not provide sufficient information to identify the two numbers? Explain.

b) Identify the numbers using the clues that are sufficient. Verify that you are correct.

20. Determine the number of solutions for each linear system. Describe the strategies you used.

a) $-x + 5y = 8$
$2x - 10y = 7$

b) $-\frac{3}{2}x + \frac{1}{4}y = -\frac{1}{4}$
$\frac{3}{4}x - \frac{y}{8} = \frac{1}{8}$

c) $0.5x + y = 0.3$
$-x + 2y = 0.6$

d) $2x - y = -5$
$6x - 3y = 15$

21. a) Explain how calculating the slopes of the graphs of the equations of a linear system helps you determine whether the system has only 1 solution. Use an example to explain.

b) Are the slopes of the graphs sufficient information to help you distinguish between a system that has no solution and a system that has infinite solutions?
Use an example to explain.

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. Which statement below is true for this linear system?

$3x - 2y = 4.5$ ①

$-x + \frac{y}{2} = -1.25$ ②

A. If you multiply equation ① by 3, then add the new equation to equation ②, you can eliminate x.

B. There is one solution because the slopes of the lines are different.

C. If you replace equation ② with $4x - 2y = -5$, the new system will have the same solution as the original system.

D. The solution of the linear system is: $x = 1$ and $y = -0.75$

2. Which system has exactly one solution?

A. $y = 3x - 2$
$y = -4x + 5$

B. $4x - 2y = -0.2$
$-x + 0.5y = 0.05$

C. $y = 3x - 2$
$y = 3x + 2$

D. $\frac{1}{3}x + \frac{1}{2}y = \frac{1}{6}$
$\frac{1}{6}x + \frac{1}{4}y = \frac{1}{6}$

3. Use the linear systems in question 2 as examples to help you explain why a linear system may have no solution, exactly one solution, or infinite solutions.

4. Liam created a problem about a group of students and adults planning to travel on the Light Rail Transit (LRT) in Calgary, Alberta. He wrote this linear system to model the situation.

$1.75s + 2.50a = 15.50$

$s - a = 4$

a) Write a problem that can be modelled by the linear system above.

b) Solve your problem. Show how you know your solution is correct.

5. a) Solve each linear system.

i) $-3x - 4y = -2$
$x + 2y = 3$

ii) $-0.5x + 0.2y = -1$
$0.3x - 0.6y = -1.8$

iii) $x - \frac{1}{3}y = \frac{4}{3}$
$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2}$

b) Use one linear system from part a to explain the meaning of the point of intersection of the graphs of the equations of a linear system.

6. a) Write a linear system to model this situation:
A stained glass design was made of yellow squares, each with area 25 cm^2; and red right triangles, each with area 12.5 cm^2. The design used 90 shapes and covered an area of 1500 cm^2.

b) Solve this related problem: How many squares and how many triangles were used?

Exercise Mind and Body

Cross training involves varying the types of exercises you do in each workout, to use different muscles and different amounts of energy. For example, you might run and lift weights in one workout, then swim in the next workout.

Stationary bike: energy used – 420 Calories per hour

Swimming: energy used – 480 Calories per hour

PART A: INVESTIGATING RELATIONS

This table shows the approximate rate at which energy is used, in Calories per hour, for three physical activities. The rate at which energy is used is related to the mass of the person doing the activity.

Activity	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)	Mass (kg)	Rate of Energy Use (Cal/h)
Stationary bike	50	350	60	420	70	490	80	560
Swimming	50	400	60	480	70	560	80	640
Walking uphill	50	300	60	360	70	420	80	480

- Choose one physical activity from the table. Write four ordered pairs for this activity. Graph the ordered pairs. Use graphing technology if it is available. Is there a linear relationship between mass and the rate at which energy is used? How do you know?

- Write an equation to represent the relation between mass and the rate at which energy is used. Use the slope-intercept form of the equation. Explain what you did.
- Write an equation to represent the relation between mass and the rate at which energy is used for each of the other activities in the table.
- Use your equations to create a table of values for a mass that is not listed in the table above.
- Alex has a mass of 60 kg. He used 345 Calories in 45 min by riding a stationary bike followed by a swim.

  Create a linear system to represent this situation.

  Use the linear system to solve this problem:
  How much time did Alex spend on each activity?

PART B: INVESTIGATING OTHER LINEAR RELATIONS

- Do some research about other physical activities that are not listed in the table in Part A. Determine the rate at which energy is used in Calories per unit time.
- Conduct investigations, create situations, then pose problems that involve: linear relations or functions; linear systems; graphing; points of intersection; and so on.

PROJECT PRESENTATION

Your completed project can be presented in a written or an oral format but should include:

- A list of investigations you conducted, the situations you created, and the problems you posed, along with explanations of your strategies for solving the problems
- A display of the graphs that you made, including an explanation of how and why you created them and how you interpreted them

EXTENSION

Because exercise and nutrition play an important role in the health of people, much research has been conducted in these areas. Many linear relations have been discovered during these studies.

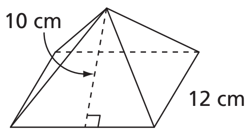
- Investigate other linear relations related to exercise, nutrition, and physical activities.
- Identify different types of energy and how they are measured. You might use an internet search using key words such as: joule, kilojoule, calorie, and kilocalorie.
- Write a brief report on the linear relations you discovered and how you know they are linear.

CUMULATIVE REVIEW Chapters 1–7

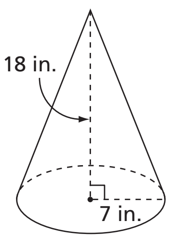
1

1. Convert each measurement.
   a) 290 cm to feet and the nearest inch
   b) 5 yd. to the nearest centimetre
   c) 8 km to miles and the nearest yard
   d) 6500 in. to the nearest metre
   e) 82 000 mm to feet and the nearest inch
   f) 16 mi. to the nearest hundredth of a kilometre

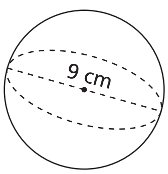
2. For each object, calculate the surface area to the nearest square unit, and the volume to the nearest cubic unit.
   a) right square pyramid



   b) right cone



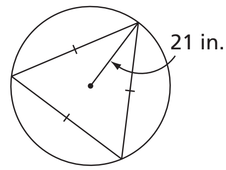
   c) sphere



2

3. A tree is supported by a set of four guy wires. All the guy wires are anchored to the ground at points 3 m from the base of the tree. The guy wires are attached to the tree at a height of 4.5 m above the ground. To the nearest tenth of a degree, calculate the measure of the angle of inclination of a guy wire.

4. An equilateral triangle is inscribed in a circle. The radius of the circle is 21 in. Determine the side length of the triangle to the nearest tenth of an inch.



3

5. Expand and simplify.
   a) $(9 + s)(9 + s)$
   b) $(3a - 5)(2a - 3)$
   c) $(2n + 3p)(5n - 4p)$
   d) $(8s - t)(8s + t)$
   e) $(w + 4)(-2w^2 + 7w - 8)$
   f) $(4 + 3x - 2x^2)(-2 + 2x + 3x^2)$

6. Find and correct the errors in each factorization.
   a) $14a^3b^2 - 28b^3c^2 + 21a^2c^3 = 7b^2(2a^3 - 4bc^2 + 3a^2c^3)$
   b) $n^2 - n - 12 = (n + 4)(n - 3)$
   c) $36r^2 - 64s^2 = (6r + 8s)(6r - 8s)$
   d) $6m^2 + 23m - 18 = (6m - 9)(m + 2)$
   e) $w^2 - 22wx + 121x^2 = (w + 11x)^2$
   f) $30c^2 + 11cd - 30d^2 = (5c - 5d)(6c + 6d)$

4

7. a) Write each radical in simplest form.
   i) $\sqrt{45}$ ii) $\sqrt[3]{128}$ iii) $\sqrt[4]{932}$ iv) $\sqrt{539}$
   b) Write each mixed radical as an entire radical.
   i) $12\sqrt{3}$ ii) $3\sqrt[3]{7}$ iii) $2\sqrt[5]{15}$ iv) $5\sqrt{17}$

8. Simplify. Show your work.
   a) $(a^{-1}b^{-2})^2(a^4b^{-1})$
   b) $\left(\dfrac{c^{-2}d^{11}}{c^4d^{-4}}\right)^{-\frac{1}{3}}$
   c) $\dfrac{13x^3yz^{-2}}{-52x^{-2}y^4z^2}$
   d) $\dfrac{-18a^{-\frac{1}{2}}b^{-5}}{3a^{\frac{5}{2}}b^{-3}}$

9. Evaluate. Show your work.

a) $\left(\frac{3}{4}\right)^{\frac{5}{3}}\left(\frac{3}{4}\right)^{\frac{1}{3}}$

b) $\dfrac{(-3.5)^{-\frac{7}{2}}}{(-3.5)^{-\frac{11}{2}}}$

c) $\left[\left(-\frac{5}{6}\right)^{8}\right]^{\frac{1}{4}}$

d) $\dfrac{(0.064)^{\frac{5}{3}}(0.064)^{\frac{8}{3}}}{(0.064)(0.064)^{\frac{11}{3}}}$

5

10. This table represents a relation.

Name	Sport
Perdita Felicien	track
Donovan Bailey	track
Nancy Greene	skiing
Annamay Pierse	swimming
Justin Morneau	baseball
Steve Nash	basketball

a) Describe the relation in words.

b) Represent this relation as:

i) a set of ordered pairs

ii) an arrow diagram

11. a) Explain why the table of values below represents a function.

Volume of Gasoline (L), v	Cost, C ($)
1	1.09
2	2.18
3	3.27
4	4.36

b) Identify the independent variable and the dependent variable. Justify your choices.

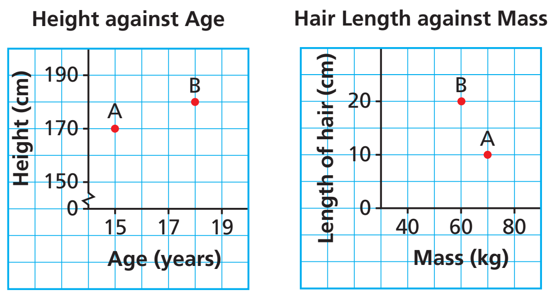
c) Write the domain and range.

d) Write an equation for this function in function notation.

e) Determine the value of $C(25)$. What does this number represent?

f) Determine the value of v when $C(v) = 50$. What does this number represent?

12. These graphs give information about two students.



Which statements are true? Justify your answers.

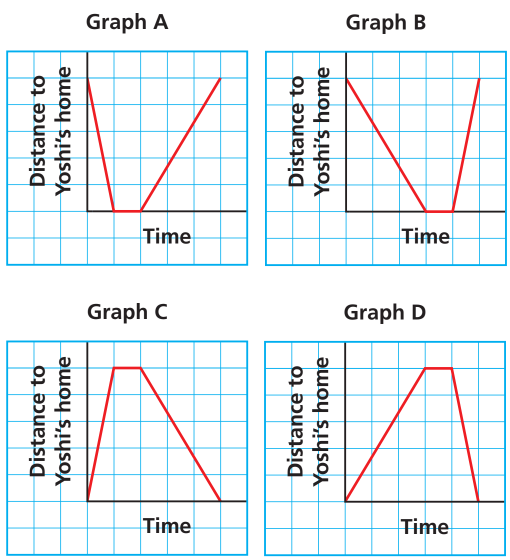
a) The older student is shorter.

b) The student with the lesser mass has longer hair.

c) The older student has longer hair.

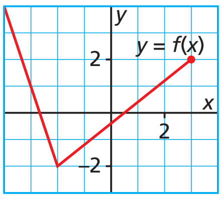
d) The shorter student has the lesser mass.

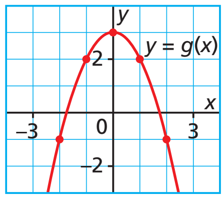
13. a) After playing basketball in the park, Yoshi walks home. He has a snack, then goes back to the park on his skateboard. Which graph best matches this situation? Explain your choice.



b) Choose one of the graphs in part a that did not describe Yoshi's journey. Describe a possible situation for the graph.

14. Determine the domain and range of the graph of each function.

a) 

b) 

15. a) For each equation, create a table of values when necessary, then graph the relation.

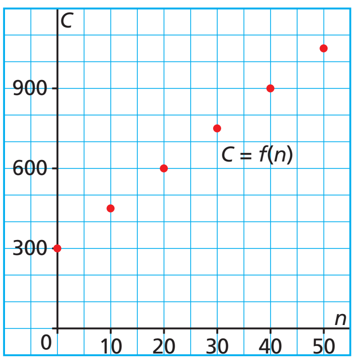
i) $y = 4$ ii) $y = -2x + 1$

iii) $y = x^2 - 2$ iv) $x = 2$

v) $x + 2y = 6$ vi) $x - y^2 = 1$

b) Which equations in part a represent linear relations? How do you know?

16. This graph shows the cost for a banquet room rented for a birthday party. The cost, C dollars, is a function of the number of people, n, who attend.



a) Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axis. Describe what this point represents.

b) Determine the rate of change. What does it represent?

c) Identify the domain and range. Are there any restrictions? Explain.

d) What is the cost when 50 people attend the birthday party?

e) How many people can attend the party for a cost of $675?

6

17. Draw the line through P(−1, 3) with each given slope. Write the coordinates of 3 other points on each line. How did you determine these points?

a) 2 b) −3 c) $\frac{3}{4}$ d) $-\frac{2}{5}$

18. The coordinates of two points on two different lines are given. Are the two lines parallel, perpendicular, or neither? Justify your choice.

a) W(−3, 5), X(8, 3) and C(6, 6), D(1, 8)

b) J(3, −4), K(9, 2) and P(5, −4), Q(2, −1)

c) R(−3, 2), S(1, −6) and E(−2, 1), F(−5, 7)

d) G(2, −5), H(2, 3) and M(3, −3), N(0, −3)

19. a) Graph these equations on the same grid without using a table of values.

i) $y = x - 5$

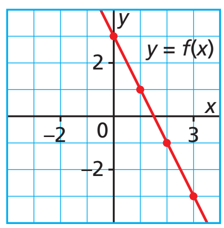
ii) $y = \frac{1}{3}x - 5$

iii) $y = -\frac{3}{2}x - 5$

iv) $y = -4x - 5$

b) Each equation in part a is written in the form $y = mx - 5$. When you change the value of m, how does it affect the graph?

20. A student said that the equation of this line is $y = \frac{1}{2}x - 3$.

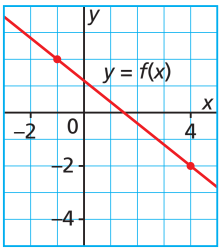


a) What mistakes did the student make?

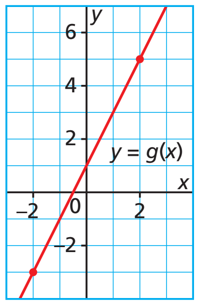
b) What is the correct equation of the line?

21. a) For each line, write an equation in slope-point form.

i)



ii)



b) Write each equation in part a in slope-intercept form, then determine the x- and y-intercepts of each graph.

22. Loretta works as a cook in northern Saskatchewan. She is paid \$14 an hour plus \$200 a week for working in a remote location. Let t represent the time in hours Loretta works each week and let d represent her weekly pay in dollars.

a) Write an equation that relates d and t.

b) What will Loretta's weekly earnings be when she works 35 h?

c) For how many hours did Loretta work when her weekly pay was \$718?

d) Can Loretta earn exactly \$600 in one week? Explain.

23. a) Use intercepts to graph each equation.

i) $7x + y + 14 = 0$ ii) $2x - 5y - 20 = 0$

b) What is the slope of each line in part a? What strategy did you use?

24. a) Write each equation in general form.

i) $y = \frac{5}{4}x - \frac{3}{5}$ ii) $y + 4 = \frac{2}{3}(x - 1)$

b) For each of the 3 forms of the equation of a linear relation, describe when each form may be the best representation.

7

25. A store sells a 1-L can of paint for \$9.60 and a 4-L can of paint for \$20.80. The store has 140 cans of paint, for a total selling price of \$2206.40. Write a linear system that models this situation.

26. Tickets for the school play cost \$8 for an adult and \$5 for a student. The total revenue for one performance was \$1122, with 32 more students than adults in the audience.

a) Write a linear system that models this situation.

b) Solve the linear system in part a by graphing.

27. Use graphing technology to solve this linear system.

$\frac{5}{2}x + \frac{4}{3}y = 7\frac{4}{9}$

$1.5x - 4.8y = 1.2$

28. Arne scored 87.5% on part A of a math exam and 75% on part B of this exam. His total mark for the exam was 87. The total mark possible for the exam was 108. How many marks was each part worth?

29. Use an elimination strategy to solve each linear system.

a) $4x - 3y = 10$
$6x + 2y = 11$

b) $3x + 4y = 1$
$2x - 6y = -21$

30. Use $5x + 3y = 15$ as an equation in 3 different linear systems. Write the second equation so that each system has a different number of solutions. Explain what you did for each system.

31. Explain why Problem 1 has no solution and Problem 2 has infinite solutions.

Problem 1
The difference between two numbers is 75.
Triple each number. Subtract the products.
The difference is 200.
What are the two numbers?

Problem 2
The sum of two numbers is 40.
Divide each number by 2. Add the quotients.
The sum is 20.
What are the two numbers?