Third
Fractional Calculus, Probability and Non-Local Operators: Applications and Recent Developments

November 18th - 20th 2015

TIMETABLE

	Wednesday, 18-Nov	Thursday, 19-Nov	Friday, 20-Nov
09:30-10:30	Lecture 1 M. Ryznar	Lecture 2 M. Ryznar	Lecture 3 M. Ryznar
10:30-10:45	*Coffee break*	*Coffee break*	*Coffee break*
10:45-11:45	Lecture 1 G. Pagnini	Lecture 2 G. Pagnini	Lecture 3 G. Pagnini
11:45-12:00	*Coffee break*	*Coffee break*	*Coffee break*
12:00-12:45	J. Lörinczi	E. Scalas	M. Bonforte
12:45-13:30	B. Toaldo	F. Polito	C. Cuesta
13:30-15:00	*Lunch*	*Lunch*	*Lunch*
15:00-15:45	N. Georgiou	F. Mainardi	J. Gracia
15:45-16:00	*Coffee break*	*Coffee break*	*Coffee break*
16:00-16:45	M. Trinh	G. Garra	P. Paradisi
16:45-17:30	D. Molina	T. M. Pham	
20:30		*Social Dinner*	

M. RYZNAR – Wroclaw University of Technology, Poland
"Heat kernels and exit time distributions of isotropic Levy processes"

G. PAGNINI – BCAM & Ikerbasque, Spain
"Fractional diffusion with a physical perspective"

J. LÖRINCZI - Loughborough University, UK
"Regime change in the decay rates of stationary states of a class of jump processes"

B. TOALDO - La Sapienza University of Rome, Italy
"Stepped semi-Markov processes and integro-differential equations"

N. GEORGIOU – University of Sussex, UK
"A fractional network and various approximations"

M. TRINH – University of Sussex, UK
"The fractional non-homogeneous Poisson process"

D. MOLINA – BCAM, Spain
"A promising stochastic process to explain the anomalous diffusion of single inside cells"

E. SCALAS – University of Sussex, UK
"On intraday option pricing"

F. POLITO – University of Turin, Italy
"Generalized nonlinear Yule models"

F. MAINARDI – University of Bologna, Italy
"Completely monotone functions in fractional relaxation processes"

G. GARRA – La Sapienza University of Rome, Italy
"Fractional thermoelasticity: Some physical applications and mathematical problems"

T. PHAM BIHN – Belgorod National Research University, Russia
"Physical characterisation of the generalised gray Brownian motion"

M. BONFORTE – Autonoma University of Madrid, Spain
"Fractional nonlinear degenerate diffusion equations on bounded domains"

C. CUESTA – UPV/EHU, Spain
"A non-local KdV-Burgers equation: Numerical study of travelling waves"

J. GRACIA – University of Zaragoza, Spain
"Boundary layers in a two-point boundary value problem with fractional derivatives"

P. PARADISI – ISTI-CNR Pisa, Italy
"Scaling law of diffusivity in a intermittent system with a fractal and a Poisson component"

Scientific and Organizing Committee

Jószef Lörinczi, Loughborough University, UK
Gianni Pagnini, BCAM-Ikerbasque
Enrico Scalas, University of Sussex-UK & BCAM

bcam basque center for applied mathematics

EXCELENCIA SEVERO OCHOA

EUSKO JAURLARITZA GOBIERNO VASCO
HEZKUNTZA, HIZKUNTZA POLITIKA ETA KULTURA SAILA
DEPARTAMENTO DE EDUCACIÓN,

ikerbasque Basque Foundation for Science

HEAT KERNELS AND EXIT TIME DISTRIBUTIONS OF ISOTROPIC LEVY PROCESSES

MICHAŁ RYZNAR

Let $X(t)$ be an isoperimetric Lévy process on $\mathbb{R}^d$. Its transition density (*heat kernel*) $p_t(x)$ is assumed to be unimodal, that it is radial and decreasing function of the space variable, which is equivalent to the fact that its Lévy measure has radial and unimodal density. The most important example is the isoperimetric α - stable process, $0 < \alpha < 2$ with the fractional Laplacian $(\Delta)^{\alpha/2}$ as a generator. Another important class of such processes are subordinate Brownian motions with the generator $\psi(\Delta)$, where ψ is the Laplace exponent of the underlying subordinator.

In the lecture we describe the behaviour of the transition density $p_t(x)$ under some weak type scaling condition expressed in terms of the Lévy-Khintchine exponent ψ of the process:

$$Ee^{i\langle \xi, X(t)\rangle} = e^{-t\psi(\xi)}, \quad \xi \in \mathbb{R}^d.$$

The scaling conditions are understood as follows. We say that ψ satisfies the weak *lower* scaling condition at infinity (WLSC) if there are numbers $\underline{\alpha} > 0$, $\underline{\theta} \geq 0$ and $\underline{c} \in (0, 1]$, such that

$$\psi(\lambda\theta) \geq \underline{c}\lambda^{\underline{\alpha}}\psi(\theta) \quad \text{for} \quad \lambda \geq 1, \quad \theta > \underline{\theta}.$$

We write $\psi \in \mathrm{WLSC}(\underline{\alpha}, \underline{\theta}, \underline{c})$ or $\psi \in \mathrm{WLSC}$. If $\psi \in \mathrm{WLSC}(\underline{\alpha}, 0, \underline{c})$, then we say that ψ satisfies the global WLSC. The weak *upper* scaling condition at infinity (WUSC) means that there are numbers $\overline{\alpha} < 2$, $\overline{\theta} \geq 0$ and $\overline{C} \in [1, \infty)$ such that

$$\psi(\lambda\theta) \leq \overline{C}\lambda^{\overline{\alpha}}\psi(\theta) \quad \text{for} \quad \lambda \geq 1, \quad \theta > \overline{\theta}.$$

In short, $\psi \in \mathrm{WUSC}(\overline{\alpha}, \overline{\theta}, \overline{C})$ or $\psi \in \mathrm{WUSC}$. Global WUSC means $\mathrm{WUSC}(\overline{\alpha}, 0, \overline{C})$. Here ψ is a radial function, which defines $\psi(\theta) = \psi(\xi)$ for $\xi \in \mathbb{R}^d$, $|\xi| = \theta$.

The main result result regarding the description of the heat kernel will be the following estimate

$$p_t(x) \approx \psi^{-1}(1/t)^d \wedge \frac{t\,\psi(1/|x|)}{|x|^d},$$

which holds locally (small t and x) or globally, contingent on the local or global weak scaling property of the symbol ψ.

Next, we will study the distribution of $\tau_D = \inf\{t > 0 : X_t \notin D\}$ is the time of the first exit of X_t from D. In particular when $D = B_R$ is a centered ball of radius R we provide very precise estimates of the mean exit time depending on the starting point of the process:

$$\mathbb{E}^x \tau_D \approx \sqrt{\frac{1}{\psi(1/\delta_D(x))\psi(1/R)}},$$

where $\delta_D(x) = \mathrm{dist}(x, D^c)$.

Some other results regarding the estimates of the *survival probability* $\mathbb{P}^x(\tau_D > t)$ for more general domains (e.g. exterior domains like complements of balls) will be discussed.

Finally we estimate the *Dirichlet heat kernel* p_D of smooth open sets $D \subset \mathbb{R}^d$ that is the transition density of the process killed on exiting D. The estimates have a form of explicit factorization involving the transition density p of the Lévy process (on the whole of $\mathbb{R}^d$), and the *survival probability* $\mathbb{P}^x(\tau_D > t)$. For instance, if ψ has global lower and upper scalings, and D is a C^2 halfspace-like open set, then

$$p_D(t, x, y) \ \approx\ \mathbb{P}^x(\tau_D > t)\ p(t, x, y)\ \mathbb{P}^y(\tau_D > t), \qquad t > 0,\ x, y \in \mathbb{R}^d,$$

where

$$p_t(x) \approx\ \psi^{-1}(1/t)^d \ \wedge\ \frac{t\,\psi(1/|x|)}{|x|^d},$$

$$\mathbb{P}^x(\tau_D > t)\ \approx\ 1\ \wedge\ \frac{1}{\sqrt{t\,\psi(1/\delta_D(x))}}.$$

Similar estimates will be presented for smooth bounded or exterior domains.

The most of the results disussed in the lectures come from a joint project with Krzysztof Bogdan and Tomasz Grzywny (Wrocław University of Technology).

Wrocław University of Technology
E-mail address: michal.ryznar@pwr.edu.pl

Fractional Diffusion with a Physical Perspective

Gianni Pagnini
gpagnini@bcamath.org
BCAM-Basque Center for Applied Mathematics
Alameda Mazarreo 14, 48009 Bilbao, Basque Country, Spain

November 18, 2015

Abstract

The mini-course is focused on the physical picture that fractional diffusion emerges from motion in random media which show a distribution of temporal or spatial scales. A method is proposed to develop stochastic processes whose one-time one-point probability density function (PDF) is solution of a fractional diffusion equation. This method is related to the so-called generalized grey Brownian motion [1,2].

Such processes can be viewed as Stochastic Solutions of the equation under investigation. Under the physical perspective, the question "Which is the stochastic solution of a certain equation?" is converted into "Which is the stochastic process underlying a certain empirical description?"

The random walkers move in a random medium. Walkers' motion is assumed independent of the medium and, at the same time, medium properties are assumed to be independent of the random walker. Since the walkers are not aware about the environment their motion is assumed to be Gaussian, as known to occur in classical diffusion processes. The characterization of the random environment to have fractional diffusion is discused [3,4].

References

[1] Mura A., Pagnini G., Characterizations and simulations of a class of stochastic processes to model anomalous diffusion. *J. Phys. A: Math. Theor. 41, 285003 (2008)*

[2] Pagnini G Erdélyi–Kober fractional diffusion. *Fract. Calc. Appl. Anal. 15, 117–127 (2012)*

[3] Pagnini G., Short note on the emergence of fractional kinetics. *Physica A 409, 29–34 (2014)*

[4] Pagnini G., Paradisi P., A stochastic solution with Gaussian stationary increments of the symmetric space-time fractional diffusion equation. *Fract. Calc. Appl. Anal. Accepted (2016)*

1

Regime change in the decay rates of stationary states of a class of jump processes

József Lörinczi
J.Lorinczi@lboro.ac.uk
Loughborough Unviersity, UK

November 18, 2015

Abstract

Non-local operators and related jump processes are currently attracting much attention at the interface area across functional analysis, PDE and probability, as well as in applications to mathematical physics and the natural sciences. I will consider a class of Lévy jump processes with the property that the intensity of single large jumps dominates the intensity of all multiple large jumps (jump-paring class). Perturbing such a process by a suitable potential creates a stationary distribution, which can be represented in terms of the ground state of the related non-local Schrödinger operator. First I discuss the spatial decay properties of ground states separately for confining and decaying potentials. Then I will explain that for decaying potentials a sharp qualitative transition occurs in the fall-off patterns as the Lévy intensity is varied from sub-exponential to exponential order and beyond. I will also discuss the mechanism of this transition.

Stepped semi-Markov processes and integro-differential equations

Bruno Toaldo
bruno.toaldo@uniroma1.it
Dipartimento di Scienze Statistiche
Sapienza - Università di Roma
Piazzale Aldo Moro 5, 00195, Rome, Italy

November 18, 2015

Abstract

Stepped semi-Markov processes are right-continuous processes whose paths are stepped functions with a finite number of discontinuities on any finite interval. The waiting times between jumps are in general non exponential r.v.'s. We present the governing equations (Kolmogorov equations) for this type of semi-Markov processes. These equations are integro-differential equations that become fractional when the distribution of the waiting times are i.i.d. r.v.'s J_i satisfying $P(J_i > t) = E_\alpha(-t^\alpha)$, $\alpha \in (0,1)$, where $E_\alpha(\cdot)$ is the Mittag-Leffler function. Furthermore we present different sufficient conditions, on the waiting time distribution, for saying that a stepped semi-Markov process is a time-changed Markov process. This has interesting consequence on the structure of the Kolmogorov equation.

References

[1] M.M. Meerschaert and B. Toaldo. Relaxation patterns and semi-Markov dynamics. *Submitted*, 2015. (available at arXiv:1506.02951)

[2] B. Toaldo. Convolution-type derivatives, hitting-times of subordinators and time-changed C_0-semigroups. *Potential Analysis*, 42(1): 115–140, 2015.

[3] B. Toaldo. Lévy mixing related to distributed order calculus, subordinators and slow diffusions. *Journal of Mathematical Analysis and Applications*, 430(2): 1009 – 1036, 2015.

[4] B. Toaldo. Stepped semi-Markov processes and conjugate subordinators. *In preparation*, 2015.

A fractional network and various approximations

Nicos Georgiou
N.Georgiou@sussex.ac.uk
Pevensey III, 5C15-A
University of Sussex, UK

November 18, 2015

Abstract

We study a temporally evolving network with Mittag-Leffler inter-event times. We prove fractional Kolmogorov equations and then use them as a tool to approximate observables for other power-law inter-event times. Finally, we discuss the speed of convergence to equilibrium. This is joint work with Enrico Scalas and Istvan Kiss.

The fractional non-homogeneous Poisson process

Mailan Trinh
M.Trinh@sussex.ac.uk
University of Sussex, UK

November 18, 2015

Abstract

In this talk I discuss possible generalizations of the fractional homogeneous Poisson process. In particular, we consider the non-homogeneous Poisson process subordinated by an inverse alpha -stable subordinator and derive a fractional differential-integral-difference equation related to its marginal probabilities. Moreover, I would like to show how the FNPP and its governing equation can be reduced to the non-fractional and or the homogeneous case.

A promising stochastic process to explain the anomalous diffusion of single particle inside cells

Daniel Molina-García[1], *Tuan Pham Minh*[1,2] *and Gianni Pagnini*[1,3]
[1] BCAM - Basque Center for Applied Mathematics
Mazarredo 14, 48009 Bilbao, Basque Country - Spain
dmolina@bcamath.org
[2] Department of Theoretical Physics, Belgorod National Research University
14 Studencheskaya, 308015 Belgorod - Russia
[3] Ikerbasque - Basque Foundation for Science
Calle de María Díaz de Haro 3, 48013 Bilbao, Basque Country - Spain

Abstract

Internal particles of cells often reveal stochastic trajectories $X(t)$ that do not follow the normal diffusion rule, i.e., the time-average mean square displacement $\overline{\delta^2(\Delta)} = \int_0^T [X(t'+\Delta) - X(t')]^2 dt'/(T-\Delta)$ is not proportional to the lag time Δ. Instead, they often show a sub-diffusive motion where $\overline{\delta^2(\Delta)} \propto \Delta^\alpha, \alpha < 1$. Even if several models are able to reproduce this so-called anomalous diffusion, it is not common that they can explain other important observables at the same time. For example, the motion of mRNA molecules inside E. coli cells is non-ergodic since $\overline{\delta^2(\Delta)}$ behaves as a random variable depending on the particle even for large measurement times T [1, 3]. While this characteristic is well-explained by the continuous time random walk (CTRW), on the other side, this model is unable to explain the experimental data for the observable known as p-variation [2]. Instead, a model like the fractional Brownian motion (fBm) explain quite well the last one but, on the contrary, it is ergodic. In this talk, we will show that the stochastic process known as generalised gray Brownian motion, which one-time one-point probability density function evolves according to an equation with fractional derivatives in the Erdélyi–Kober sense [4, 5], can explain both behaviours at the same time. In addition, we will show some results for a more general model which is able to reproduce also ageing, the fact that the measured observables depend on the starting time of the measurement.

References

[1] I. Golding and E. C. Cox. Physical nature of bacterial cytoplasm. *Phys. Rev. Lett.*, 96(9):098102, 2006.

[2] M. Magdziarz and J. Klafter. Detecting origins of subdiffusion: p-variation test for confined systems. *Phys. Rev. E*, 82(1):011129, 2010.

[3] R. Metzler, J.-H. Jeon, A. G. Cherstvy, and E. Barkai. Anomalous diffusion models and their properties: non-stationarity, non-ergodicity, and ageing at the centenary of single particle tracking. *Phys. Chem. Chem. Phys.*, 16(44):24128–24164, 2014.

[4] A. Mura and G. Pagnini. Characterizations and simulations of a class of stochastic processes to model anomalous diffusion. *J. Phys. A: Math. Theor.*, 41(28):285003, 2008.

[5] G. Pagnini. Erdélyi-Kober fractional diffusion. *Fractional Calculus and Applied Analysis*, 15(1):117–127, 2012.

On intraday option pricing

Enrico Scalas
E.Scalas@sussex.ac.uk
University of Sussex and BCAM
UK and Spain

November 18, 2015

Abstract

A stochastic model for pure-jump diffusion (the compound renewal process) can be used as a zero-order approximation and as a phenomenological description of tick-by-tick price fluctuations. This leads to an exact and explicit general formula for the martingale price of a European call option. A complete derivation of this result is presented by means of elementary probabilistic tools. Emphasis is given to inter-trade durations following the Mittag-Leffler distribution.

Generalized nonlinear Yule models

Federico Polito
federico.polito@unito.it
Department of Mathematics
University of Torino
Via Carlo Alberto, 10, 10123 Torino, Italy

November 18, 2015

Abstract

We propose a fractional nonlinear modification of the classical Yule model often studied in the context of macrovolution. The model is analyzed and interpreted in the framework of the development of networks such as the World-Wide Web. Nonlinearity is introduced by replacing the linear birth process governing the growth of the in-links of each specific webpage with a fractional nonlinear birth process with completely general birth rates. Furthermore the fractionality added by the presence of fractional operators furnishes the model with a persistent memory. Among the main results we derive the explicit distribution of the number of in-links of a webpage chosen uniformly at random taking separated the contribution to the asymptotics and the finite time correction. The mean value of the latter distribution is also calculated explicitly in the most general case. Furthermore, in order to show the usefulness of our results, we particularize them in the case of specific birth rates giving rise to a saturating behaviour, a property that is often observed in nature.

Completely monotone functions in fractional relaxation processes

Francesco Mainardi
francesco.mainardi@bo.infn.it
University of Bologna, Italy

November 18, 2015

Abstract

In this talk the condition of complete monotonicity (CM) is discussed for the response functions characterizing relaxation processes modelled by constitutive equations of fractional order. Our models concern both mechanical and dielectric relaxation. We point out that CM is an essential property for the physical acceptability and realizability of the models since it ensures, for instance, that in isolated systems the energy decays monotonically as expected from physical considerations. Studying the conditions under which the response function of a system is CM is therefore of fundamental importance. The purpose of this talk is to summarize the results obtained by the author with some collaborators for visco-elastic and dielectric media where the response functions are shown to be of the Mittag-Leffler type.

FRACTIONAL THERMOELASTICITY: SOME PHYSICAL APPLICATIONS AND MATHEMATICAL PROBLEMS

ROBERTO GARRA,
DIPARTIMENTO DI SCIENZE STATISTICHE,
SAPIENZA UNIVERSITÁ DI ROMA,
ROBERTO.GARRA@UNIROMA1.IT

Abstract. In the framework of the general theory of Gurtin and Pipkin [3] of heat conduction with finite velocity, fractional models of heat propagation have recently gained a discrete development, giving rise to the so-called fractional thermoelasticity field of research [5]. The object of this talk is the discussion of the application of fractional thermoelasticity in two completely different physical fields of research. We first consider the problem of second sound propagation in nonlinear rigid conductors within the theory of heat conduction with memory. In this context we generalize the Hardy-Hardy-Maurer model [4] and we show that the obtained governing equations are nonlinear time-fractional equations linearizable by change of variable.

The second application regards the propagation of nonlinear thermoelastic waves in fluid-saturated porous media, from an early model of Bonafede [1]. In this case we investigate the application of the generalized heat balance equation on the derivation of the model equations and we show a particular solution by means of the invariant subspace method [2]. The main aim is then to suggest some new mathematical problems, in the framework of fractional thermoelasticity, where memory effects and nonlinear terms are both not negligible.

References

[1] M. Bonafede, Hot fluid migration: an efficient source of ground deformation: application to the 1982–1985 crisis at Campi Flegrei-Italy, *Journal of Volcanology and Geothermal Research*, 48: 187–198, (1991)

[2] V. Galaktionov and S. Svirshchevskii, *Exact solutions and invariant subspaces of nonlinear partial differential equations in mechanics and physics.* Chapman and Hall/CRC applied mathematics and nonlinear science series, (2007)

[3] M.E. Gurtin, A.C. Pipkin, A general theory of heat conduction with finite wave speeds, *Archive for Rational Mechanics and Analysis*, 31: 113126, (1968)

[4] Jordan, P. M., Second-sound propagation in rigid, nonlinear conductors, *Mechanics Research Communications*, 68, 52-59, (2015)

[5] Y. Povstenko, *Fractional Thermoelasticity*, Solid Mechanics and Its Applications, Vol.219, Springer, (2015)

Date: November 3, 2015.

Key words and phrases. Nonlinear thermoelastic waves, heat conduction with memory, fractional differential equations.

Physical characterisation of the generalised gray Brownian motion

Tuan Pham Minh
Belgorod National Research University and BCAM
14 Studencheskaya, 308015 Belgorod - Russia
physicsidea@gmail.com

Daniel Molina-García and Gianni Pagnini
BCAM - Basque Center for Applied Mathematics
Mazarredo 14, 48009 Bilbao, Basque Country - Spain
dmolina@bcamath.org
gpagnini@bcamath.org

Abstract

The generalised gray Brownian motion (ggBm) is a parametric class of stochastic processes that models both fast and slow anomalous diffusion [1, 2]. It can be represented as the product of a random variable and a fractional Brownian motion (fBm). In this talk, we will present a wider class of processes of the form $X_{\alpha,\beta,H}(t) = \sqrt{t^\alpha \Lambda_\beta}\, X_H(t)$, where $\alpha \in [-1, \infty)$, Λ_β is a random variable distributed according to the one-side M-Wright/Mainardi function with $\beta \in (0, 1]$ and $X_H(t)$ is the fBm with Hurst index $H \in (0, 1)$. We will see that the ensamble-average mean square displacement (EAMSD) of the processes depends on H but it is perturbed by α, while the ensamble-average temporal-average mean square displacement (EATAMSD) behaves as a power-law of the measurement time T with exponent α, when T is large enough and $\alpha \neq -1$. In the limit case $\alpha = -1$, the dependence is $\log T/T$. This dependence on T is an important feature of the process because it reproduces ageing, a characteristic of many biological processes [3]. Furthermore, by finding the analytical expression for the Ergodicity Breaking parameter (EB), we show that this parameter depends solely on β. This result confirms the fact that β is the only parameter controlling the ergodicity of the processes, being ergodic only in the limit case $\beta = 1$.

References

[1] A. Mura and G. Pagnini. Characterizations and simulations of a class of stochastic processes to model anomalous diffusion. *J. Phys. A: Math. Theor.*, 41(28):285003, 2008.

[2] G. Pagnini. Erdélyi-kober fractional diffusion. *Fractional Calculus and Applied Analysis*, 15(1):117–127, 2012.

[3] A. V. Weigel, B. Simon, M. M. Tamkun, and D. Krapf. Ergodic and nonergodic processes coexist in the plasma membrane as observed by single-molecule tracking. *Proc. Natl. Acad. Sci. USA*, 108(16):6438–6443, 2011.

Fractional nonlinear degenerate diffusion equations on bounded domains

Matteo Bonforte
matteo.bonforte@uam.es
Universidad Autnoma de Madrid, Spain

November 18, 2015

Abstract

We investigate quantitative properties of nonnegative solutions $u(t,x) \geq 0$ to the nonlinear fractional diffusion equation, $\partial_t u + \mathcal{L}F(u) = 0$ posed in a bounded domain, $x \in \Omega \subset \mathbb{R}^N$, with appropriate homogeneous Dirichlet boundary conditions. As $\mathcal{L}$ we can use a quite general class of linear operators that includes the two most common versions of the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, in a bounded domain with zero Dirichlet boundary conditions, but it also includes many other examples since our theory only needs some basic properties that are typical of "linear heat semigroups". The nonlinearity F is assumed to be increasing and is allowed to be degenerate, the prototype is the power case $F(u) = |u|^{m-1}u$, with $m > 1$.

In this talk we propose a suitable class of solutions of the equation, called weak dual solutions, and cover the basic theory: existence, uniqueness of such solutions, and we establish upper bounds of two forms (absolute bounds and smoothing effects), as well as weighted-L^1 estimates. The class of weak dual solutions is very well suited for that work. The standard Laplacian case $s = 1$ is included and the linear case $m = 1$ can be recovered in the limit.

We will also present more advanced estimates: sharp upper and lower boundary behaviour, various forms of Harnack inequalities and higher regularity estimates. When the nonlinearity is of the form $F(u) = |u|^{m-1}u$, with $m > 1$, such global Harnack estimates are the key tool to understand the sharp asymptotic behaviour of the solutions.

A non-local KdV-Burgers equation: Numerical study of travelling waves

Carlota M. Cuesta
carlotamaria.cuesta@ehu.es
UPV/EHU, Basque Country Spain

November 18, 2015

Abstract

We present numerical simulations that support our previous study of travelling wave solutions of a Korteweg-de Vries-Burgers equation with a non-local diffusion term. This model equation arises in the analysis of a shallow water flow by performing formal asymptotic expansions associated to the triple-deck regularisation (which is an extension of classical boundary layer theory). The resulting non-local operator is of the fractional derivative type with order between 1 and 2. Travelling wave solutions are typically analysed in relation to shock formation in the full shallow water problem. In this talk we give numerical evidence of stability of non-monotone travelling wave. We also confirm the existence of travelling waves that are everywhere monotone except over a bounded interval where they exhibit oscillations. We shall discussed in some detail the recent numerical method developed for this purpose.

Joint work with F. de la Hoz.

Third Fractional Calculus, Probability and Non-local Operators: Applications and Recent Developments
Bilbao – Basque Country – Spain
18–20 November 2015

Boundary layers in a two-point boundary value problem with fractional derivatives

José Luis Gracia
IUMA and Department of Applied Mathematics, University of Zaragoza
Pedro Cerbuna 12, 50009 - Zaragoza, Spain
jlgracia@unizar.es

Martin Stynes
Applied Mathematics Division, Beijing Computational Science Research Center
Haidian District, Beijing 100084, China
m.stynes@csrc.ac.cn, m.stynes@ucc.ie

In this talk a two-point boundary value problem is considered on the interval $[0, 1]$ where the leading term in the differential operator is either a Riemann-Liouville or a Caputo fractional derivative of order δ with $1 < \delta < 2$. In addition to the singular behaviour that the solution can have at $x = 0$, the solution may exhibit a boundary layer at $x = 1$ when δ is near 1. The conditions on the data of the problem under which this layer appears are investigated. Numerical examples will illustrate our results.

Scaling law of diffusivity in a intermittent system with a fractal and a Poisson component

Paolo Paradisi
paolo.paradisi@isti.cnr.it
ISTI CNR Pisa, Italy

November 18, 2015

Abstract

Complex systems are typically associated with a birth-death process of cooperation, defined by a renewal point process. The renewal events represent the rapid transitions among two self-organized, metastable states or from a self-organized to a non-organized condition and vice versa.

The complexity of a cooperative system is affected by the presence of noise, i.e., random fluctuations with short-term memory.

In this talk we will introduce and discuss a stochastic point process given as a superposition of two contributions: a renewal process with fractal statistics of the inter-event times (inverse power-law distribution) and a (renewal) Poisson process, the last one being a prototype of a noisy component.

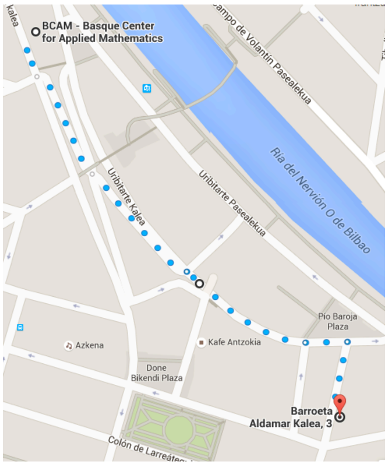
We show that, when this point process drives a particular diffusion process generated by the associated telegraph signal, then a counter-intuitive result is found: despite the presence of a fractal intermittent signal generating fast diffusion, also denoted as super-diffusion, a normal diffusion scaling emerges in the long-time limit, thus allowing to define a long-time diffusivity parameter. Interestingly, the diffusivity parameter obeys a scaling law. In particular, it depends on a power of the Poisson rate, which defines the "noise intensity", through an exponent that is a linear function of the fractal power index.

Third

Fractional Calculus, Probability and Non-Local Operators: Applications and Recent Developments

November 18th - 20th 2015

LUNCH FOR SPEAKERS		
Wednesday, 18 November 2015	Thursday, 19 November 2015	Friday, 20 November 2015
Restaurante EL MUELLE Barroeta Aldamar 3– Bilbao	**Restaurante LA ROCA** C/ Ercilla 1 – Bilbao	**Restaurante BIKANDI ETXEA** Campo Volantín 4 - Bilbao



DINNER FOR SPEAKERS Thursday, 19 November 2015
Restaurante LARRUZZ Calle Uribitarte 24 – Bilbao **At 20:30**