Tab 5: Geometry
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Tab 5: Geometry
Master Materials List

Three-dimensional cube model
Grid paper
Snap cubes
Rulers
Calculator
Markers

Picture This!: Transparencies and handouts
Texas "T" Activity: Transparencies and handouts

The following materials are not in the notebook. They can be accessed on the CD through the links below.

Isometric dot paper (1 per participant) and transparencies (1 per group)

Activity: Picture This!

TEKS: This activity supports teacher content knowledge needed for:

(G.6) **Dimensionality and the geometry of location**. The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.

The student is expected to:

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

Overview: This activity encourages participants to explore and draw orthographic and isometric views. It explores some non-technical aspects of orthographic drawings and the relationship between isometric and orthographic drawings.

Materials:
Isometric dot paper
Grid paper
Snap cubes
Rulers
Transparencies: 1-8 (pages 5-11 – 5-18)
Isometric dot paper (1 per participant) and transparencies (1 per group)
Three-dimensional cube model
Handout 1 (pages 5-4 – 5-5)
Handout 2 (pages 5-6 – 5-8)
Handout 3 (pages 5-9 – 5-10)

Vocabulary:
axonometric
isometric
diametric
trimetric
orthographic
perspective
vanishing point

Grouping: Each participant should complete these activities. Participants may work in groups of 3-4 so that they can help one another.

Time: 1 to 1 ½ hours

Lesson:

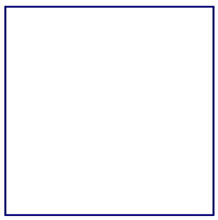
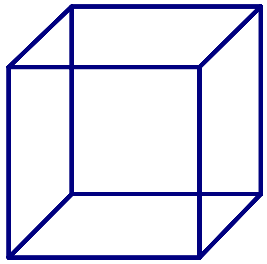
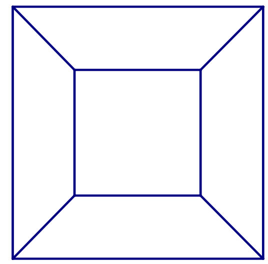
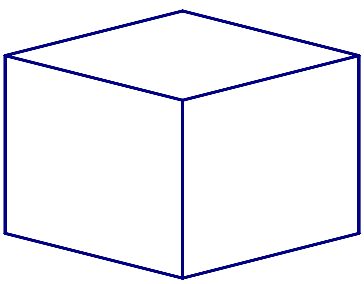
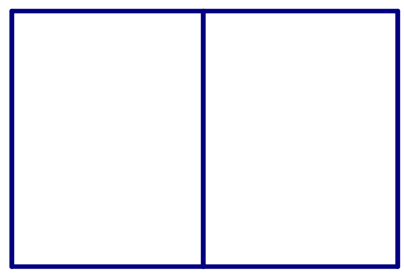
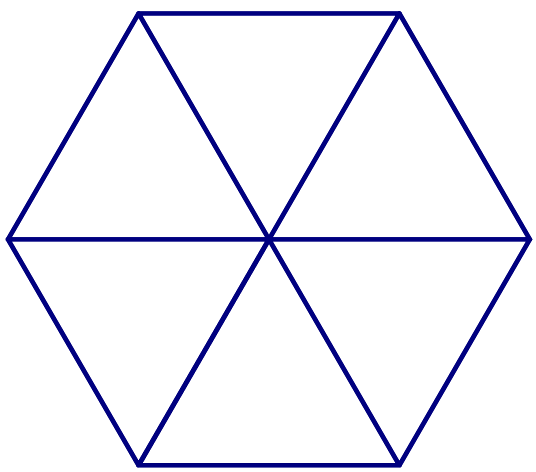
Procedures	Notes
Part I: 1. Distribute Handout 1, **Part 1 Diagrams,** (pages 5-4 – 5-5) to participants and at least one model of a cube to each group. Have participants determine which of the diagrams could represent a cube.	Participants should be in groups for discussion; allow time for discovery that all of these diagrams represent a cube. Ask the participants why the diagrams of the cube in Part I look so different. *The diagrams show the cube from different angles and perspectives.*
2. Provide participants with Handout 2, **Drawings in Two-Dimensions,** (pages 5-6 – 5-8) terms with definitions. Discuss and define perspective views, vanishing point, axonometric views and orthographic views. Have participants decide in their groups which type of drawing each diagram from Handout 1 represents. Groups should be prepared to share their choices with the whole group.	(See discussion of vocabulary below) Transparencies 1-6 (pages 5-11 – 5-16) for each diagram are provided.
Part II:	
3. Distribute two pre-made snap cube examples, a transparency of isometric dot paper (see Materials List for a link to this), and transparency pens to each group; also provide a sheet of isometric dot paper for each participant. Have each participant sketch the isometric view of the given example, and then have the group sketch the view onto their transparency. Each group will share its result.	Be sure to bring the participants back together for a whole class discussion. The trainer can make two of the given sample shapes, or make two simple shapes of his/her own. Optional: To save prep time, have the participants build the models shown on Transparency 7 (page 5-17).
4. Give participants a sheet of grid paper and have them sketch the three orthographic views (front, side and top). Give each group a new set of pre-made (more challenging) examples and repeat the process.	Volunteers may share their sketches with the class. Trainer can choose any of the remaining sample shapes, or create their own.

Procedures	Notes
Part III:	
6. Distribute Handout 3, **Isometric Drawings**, (pages 5-9 – 5-10). Have participants practice creating isometric drawings from orthographic views without the aid of three-dimensional models. Bring the participants back together for a whole class discussion and to analyze their drawings. Questions to ask: ▪ "What difference do you see between the orthographic drawings of diagram 1 and diagram 2?" ▪ "How can the orthographic drawings be changed to include additional information differentiating diagram 1 and diagram 2?"	On Handout 3, what is the difference between the orthographic drawings of diagram 1 and diagram 2? (see page 5-10) *They are the same!* There are easy solutions to this dilemma. Try adding segments in the drawing to outline the cube faces seen in each view. Indicate different distances from the viewer to different parts of the orthogonal drawings by using different line weights. Faces nearest the viewer can be outlined with dark segments. Faces one cube farther away can be a medium weight and faces two cubes farther away can be a light or dashed segment. Alternatively, add segments in the top view to outline the cube faces and number the squares to indicate the number of cubes in the "stack." (See Transparency 8 (page 5-18) for solutions) These drawings are called "mats." Show a Transparency 8 after discussion.

Extensions: After students have learned to draw orthographic and isometric figures, they can investigate surface area and volume. Initially, they can use snap cube models, but as visualization skills improve, there should be less dependence on models and more interpretation from the diagrams.

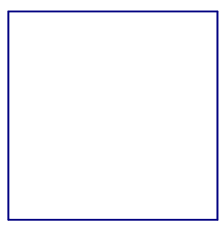
PICTURE THIS!
Part 1 Diagrams

Which of these two-dimensional pictures could be used to represent a cube?

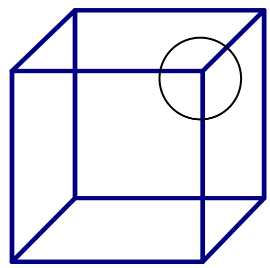







PICTURE THIS!
Part 1 Diagram Solutions

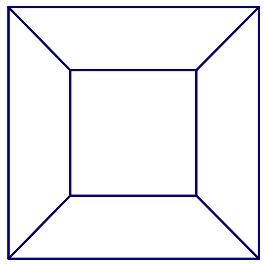
Which of these two-dimensional pictures could be used to represent a cube?



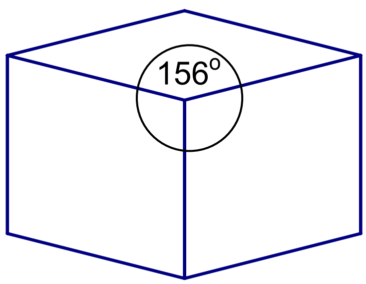
Orthographic



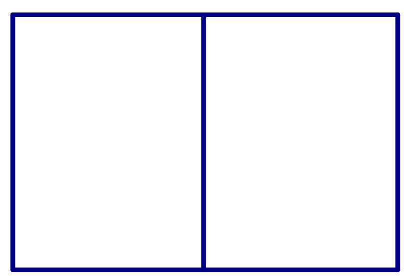
Dimetric



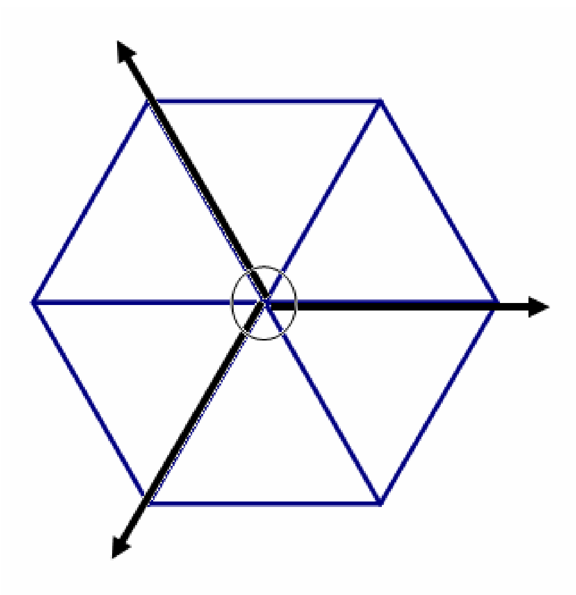
Perspective



Dimetric



Orthographic



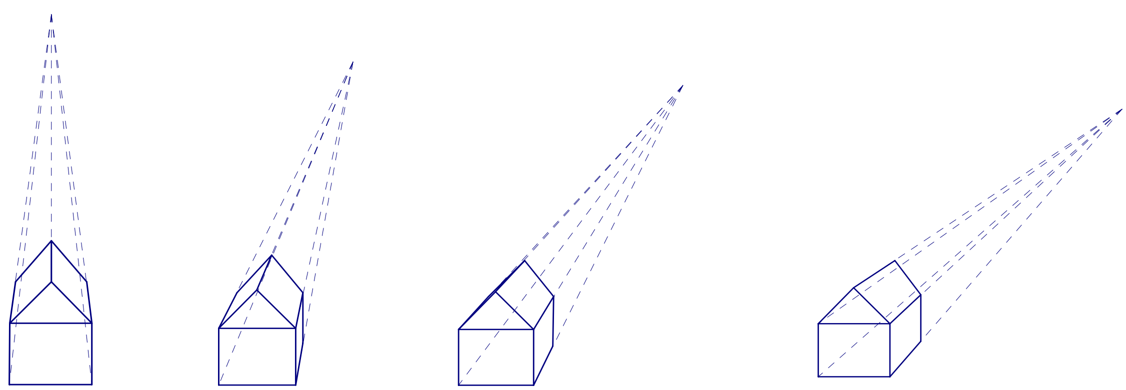
Isometric

PICTURE THIS!
Drawings in Two-Dimensions

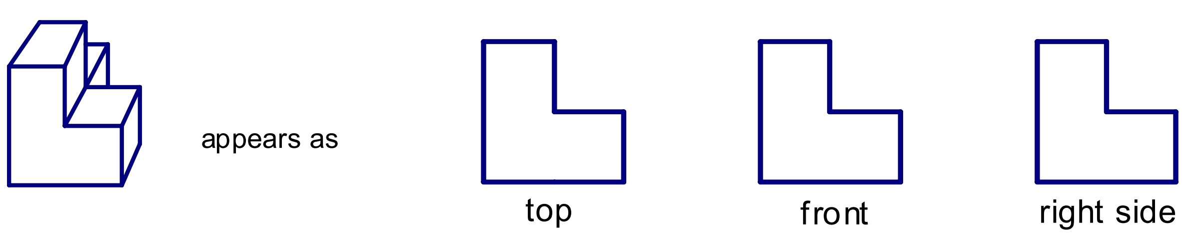
Use the definitions below to determine which of these descriptions best fits each of the sketches given in Part I.

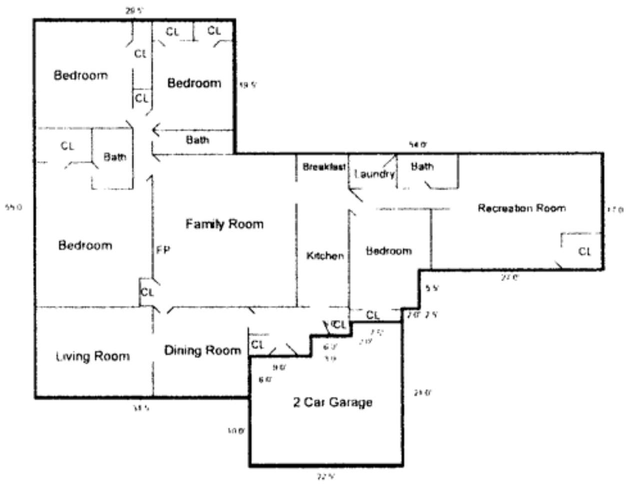
Perspective View: The technique of portraying solid objects and spatial relationships on a flat surface. There are numerous methods of depicting an object, depending upon the purpose of the drawing.

Vanishing Point: In an artistic perspective drawing, receding parallel lines (lines that run away from the viewer) converge at a vanishing point on the horizon line. This maintains a realistic appearance of the object depicted, even as the vantage point changes.



Orthographic Drawing: Ortho means "straight" and the views in orthographic drawings show the faces of a solid as though you are looking at them "head-on" from the top, front or side. Orthographic drawings are used in engineering drawings to convey all the necessary information of how to make the part to the manufacturing department. The line of sight is perpendicular to the surfaces of the object. Two conventions are used in technical drawings. These are *first angle* and *third angle*, which differ only in position of the plan (top, front, and side views). They are derived from the method of projection used to transfer two-dimensional views onto an imaginary, transparent box surrounding the object being drawn. This is **not** a distinction the state of Texas has seen fit to recognize in the TEKS.

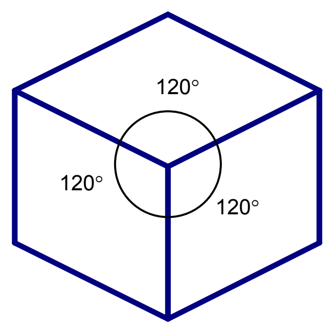




A common use of a “top” orthographic view is the floor plan of a house or other building.

Axonometric drawing: A two-dimensional drawing showing three dimensions of an object. The vantage point is not perpendicular to a surface of the object being drawn. Axonometric means, “to measure along the axes.” There are three types: isometric, dimetric, and trimetric.

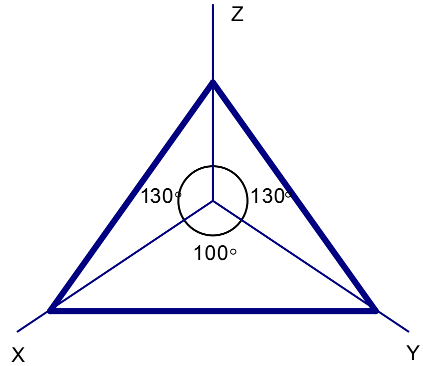
Isometric Drawing: A two-dimensional drawing that shows three sides of an object in one view. The vantage point is 45° to the side and above the object being viewed. The resulting angle between any two axes appears to be 120°. Isometric means, “one measure.” In this view, 90° angles appear to be 120° or 60°.



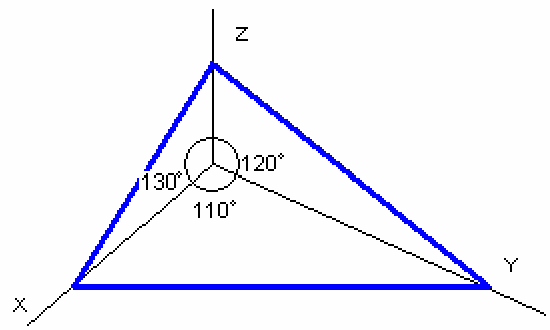

Courtesy Electronic Arts, Inc.

Video games, such as *SimCity 2000*, frequently use isometric drawings

Dimetric drawing: A two-dimensional drawing similar to an isometric drawing, but with only two of the resulting angles between the axes having the same apparent measure. Dimetric means, “two measures.”



Trimetric drawing: Again, A two-dimensional drawing similar to an isometric drawing, but with none of the resulting angles between the axes having the same apparent measure. Trimetric means, “three measures.”



PICTURE THIS!
Isometric Drawings

Make orthographic drawings of each diagram. You should include drawings from the top, front, and right side.

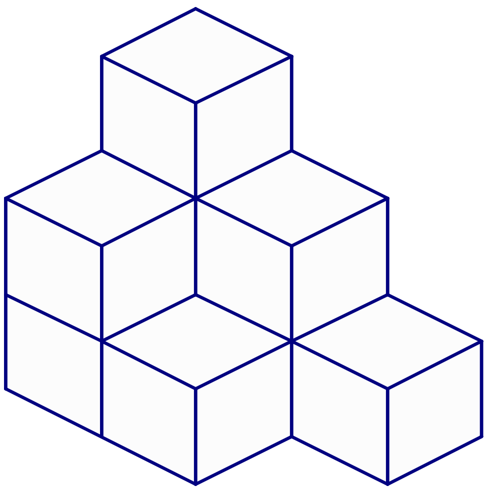


Diagram 1

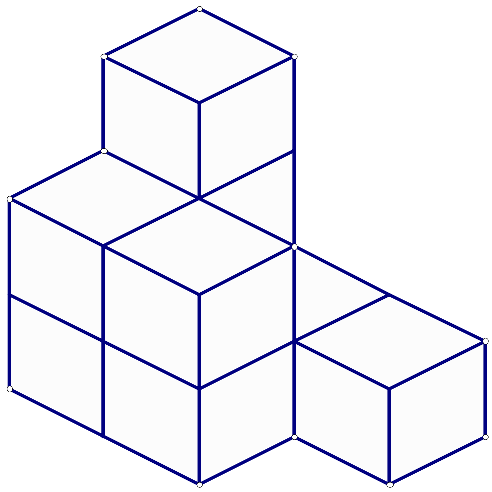
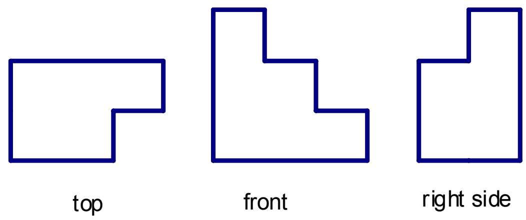
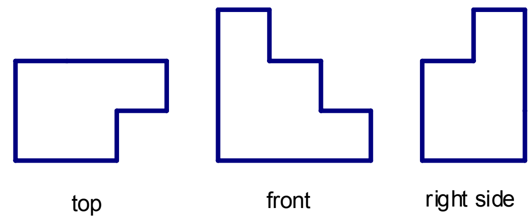
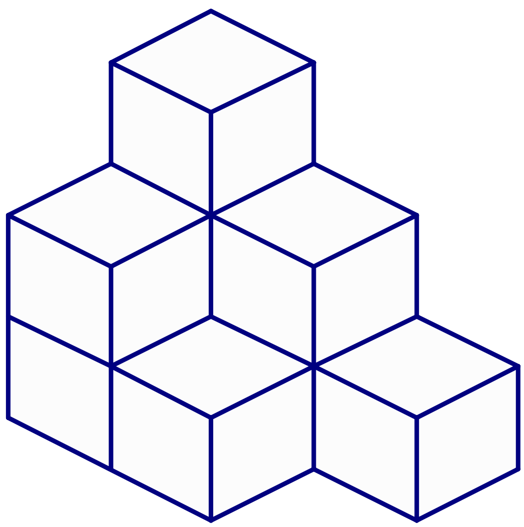


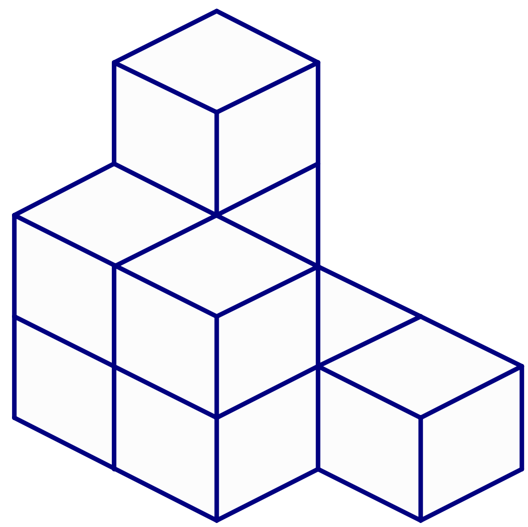
Diagram 2

PICTURE THIS!
Isometric Drawings Solutions

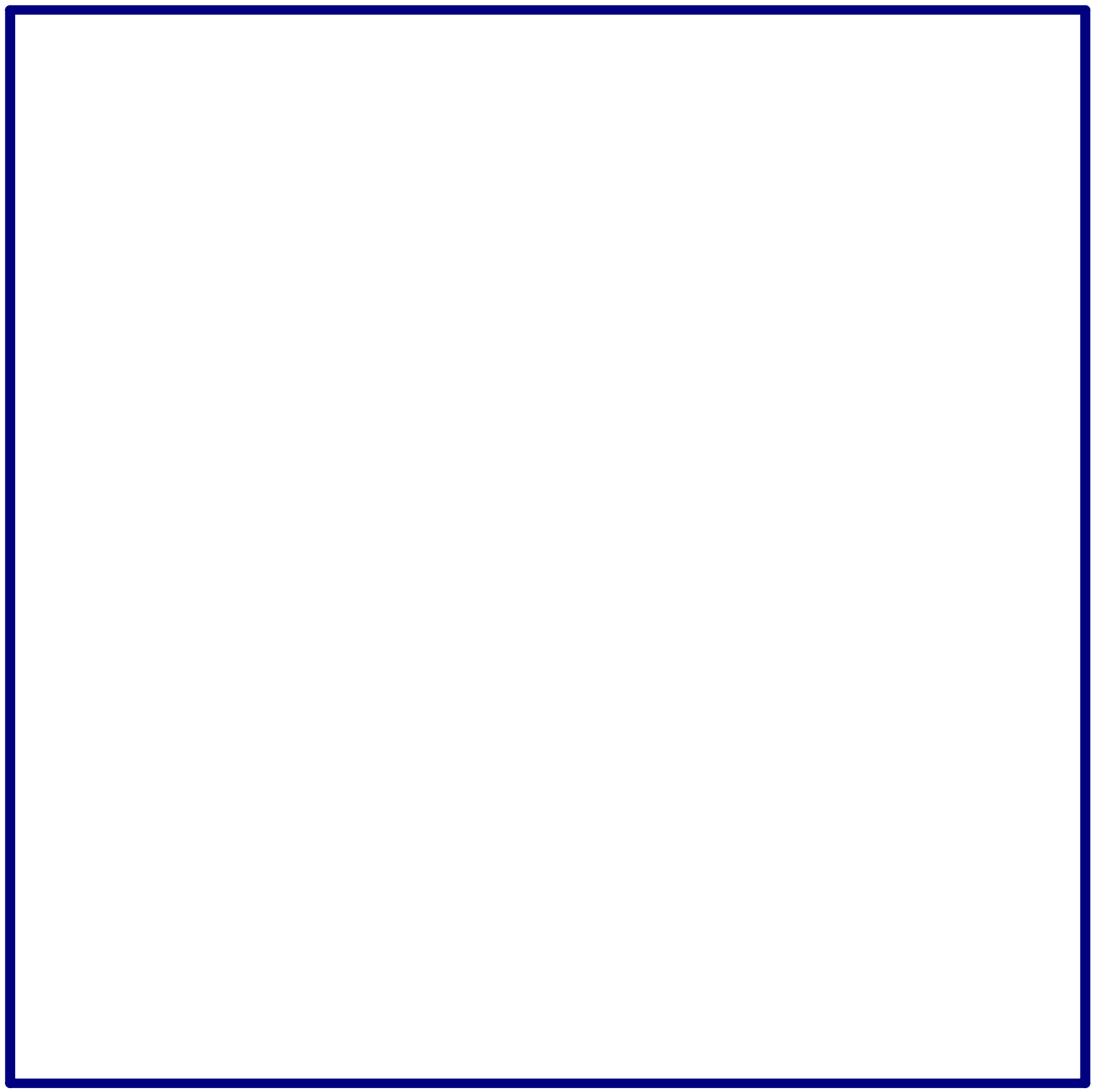


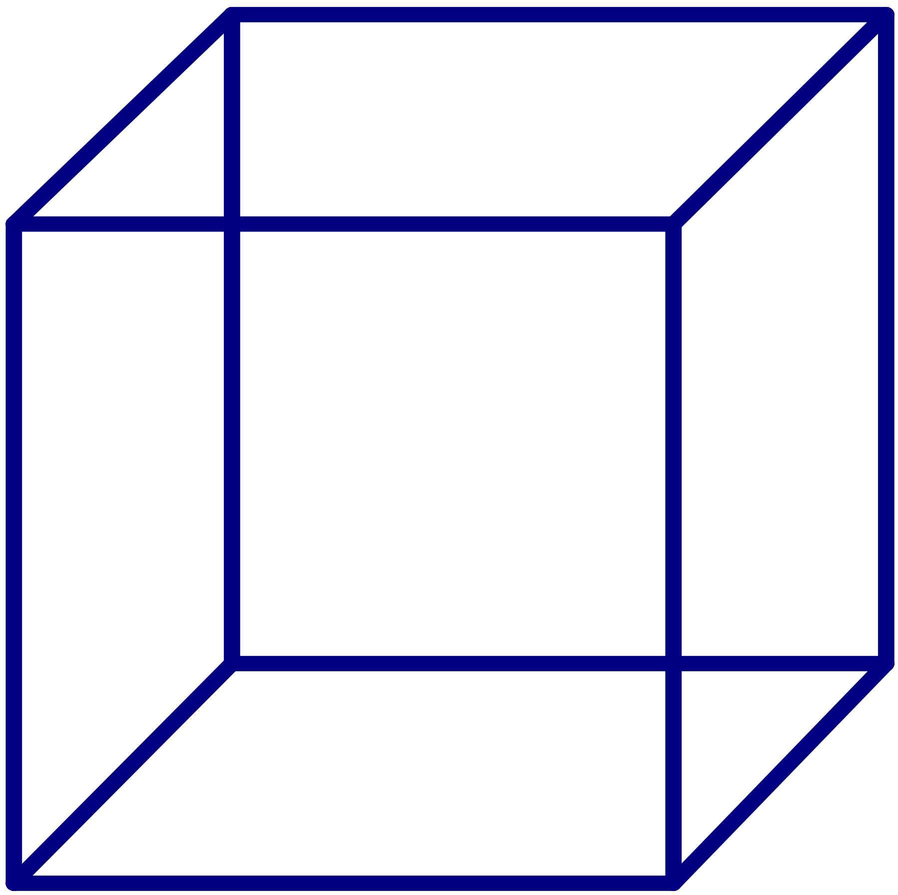


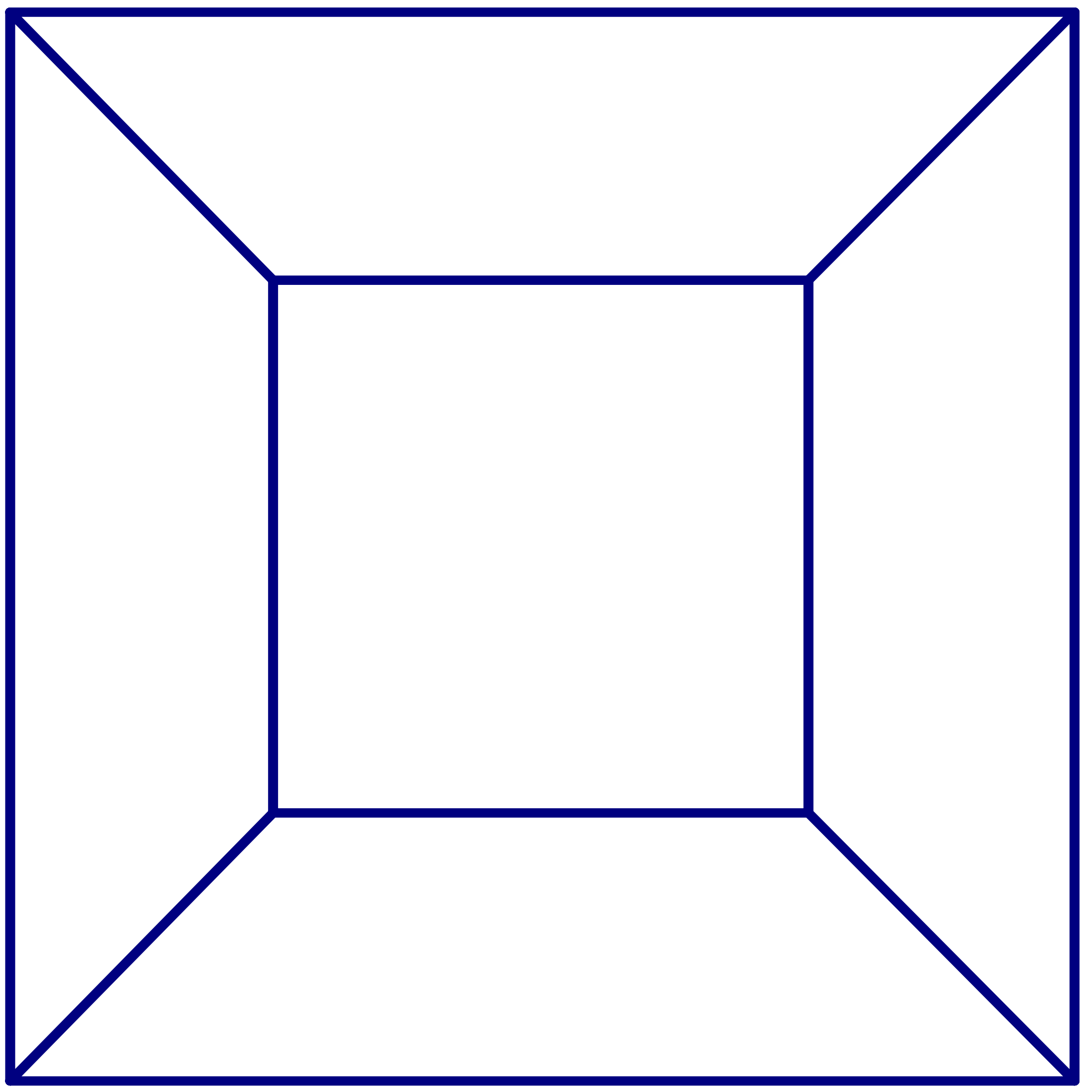


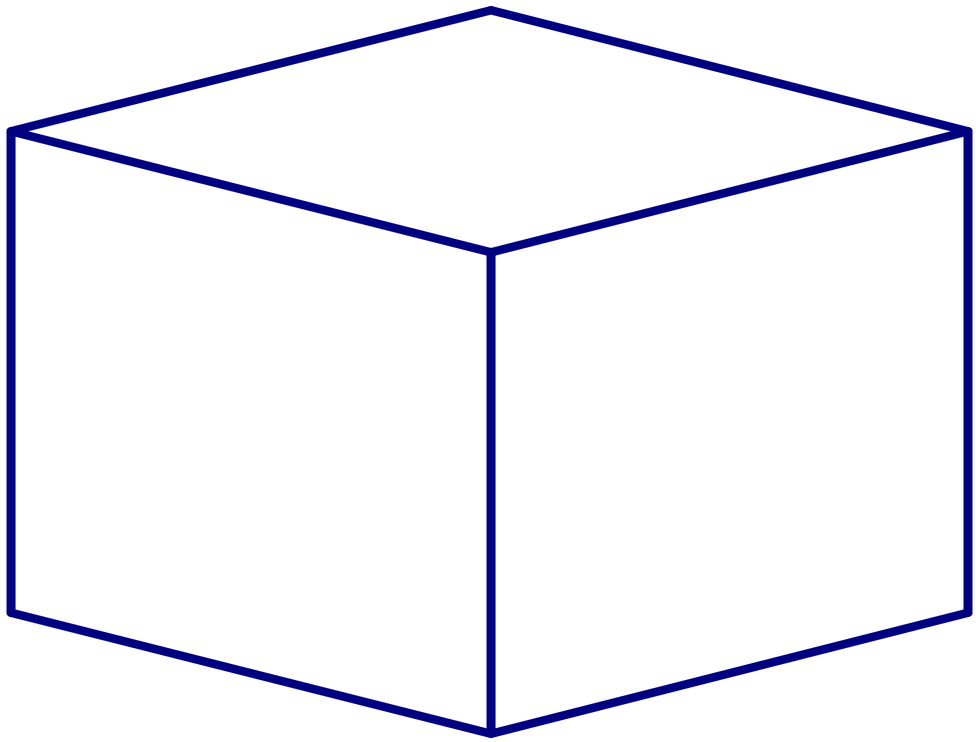


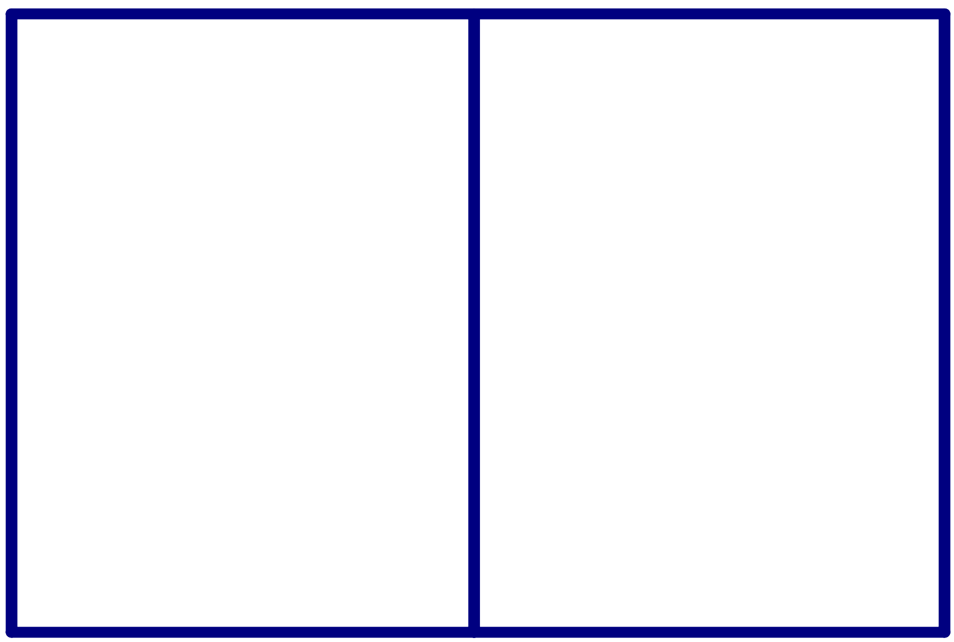
The orthogonal drawings are the same!
There are easy solutions to this dilemma. Try adding segments in the drawing to outline the cube faces seen in each view. Indicate different distances from the viewer to different parts of the orthogonal drawings by using different line weights. Faces nearest the viewer can be outlined with dark segments. Faces one cube farther away can be a medium weight and faces two cubes farther away can be a light or dashed segment. Alternatively, add segments in the top view to outline the cube faces and number the squares to indicate the number of cubes in the "stack."

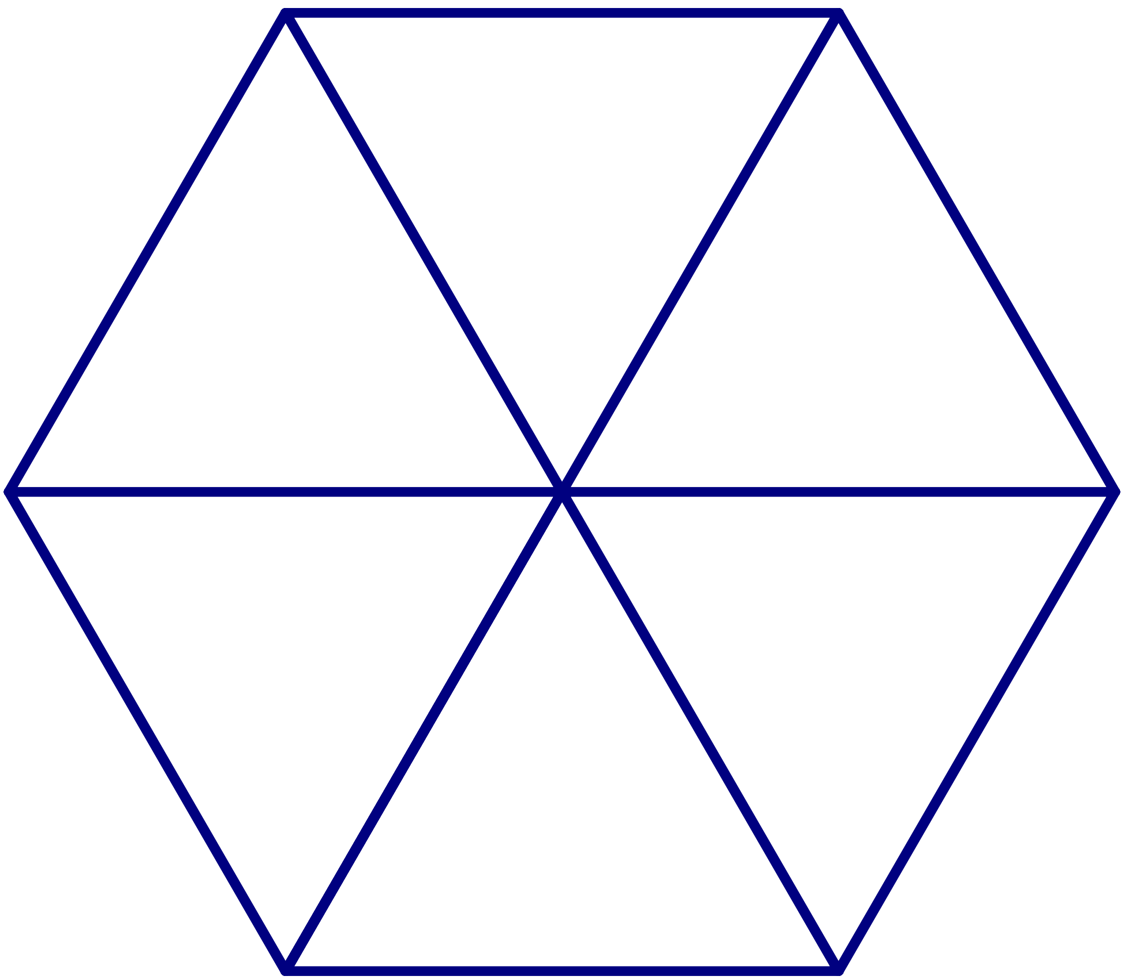










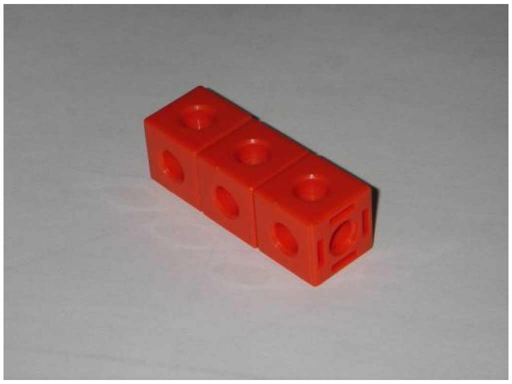


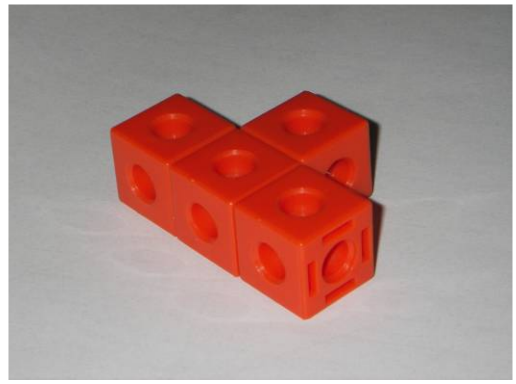
PICTURE THIS!

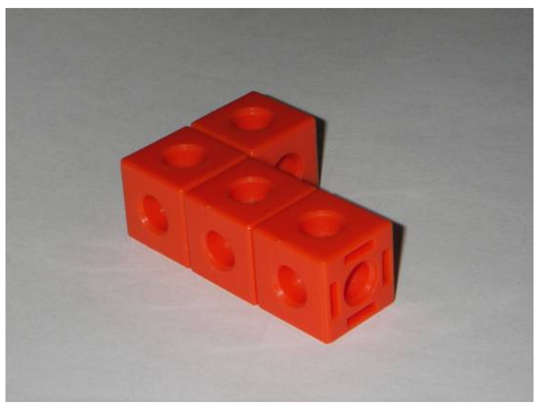
Materials: snap cube shapes, grid paper, isometric paper, and rulers

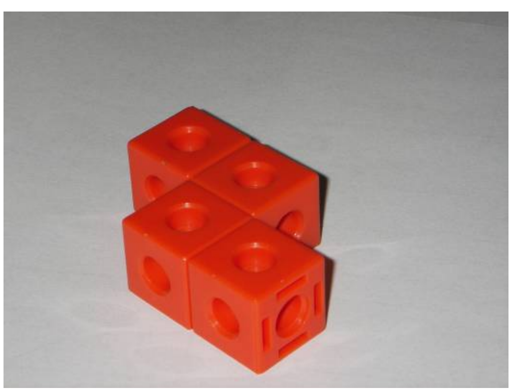
Prepare orthographic and isometric drawings of each shape.

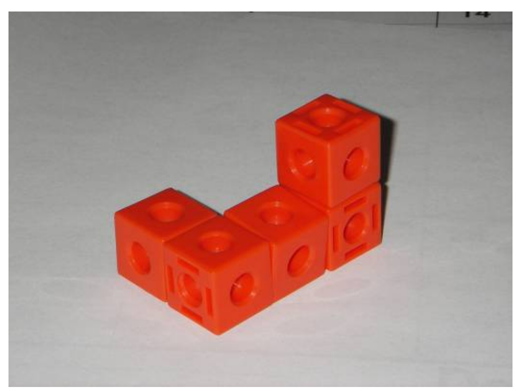
Sample shapes:



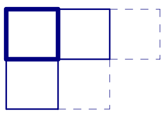








PICTURE THIS!
Isometric Drawings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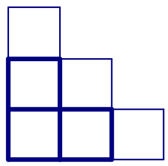
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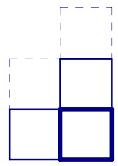
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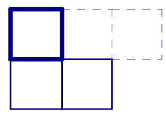
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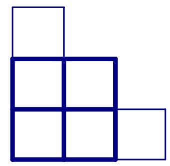
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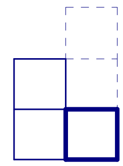
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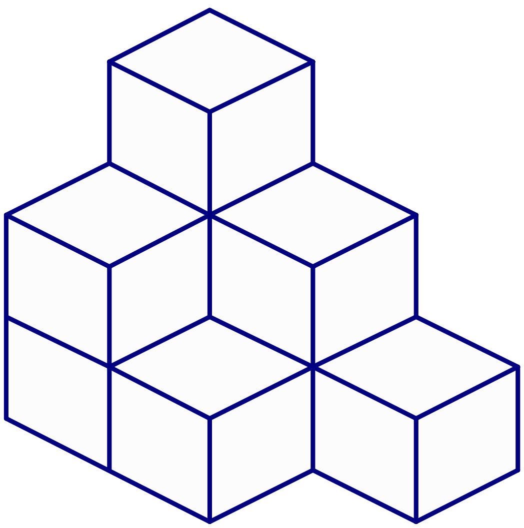
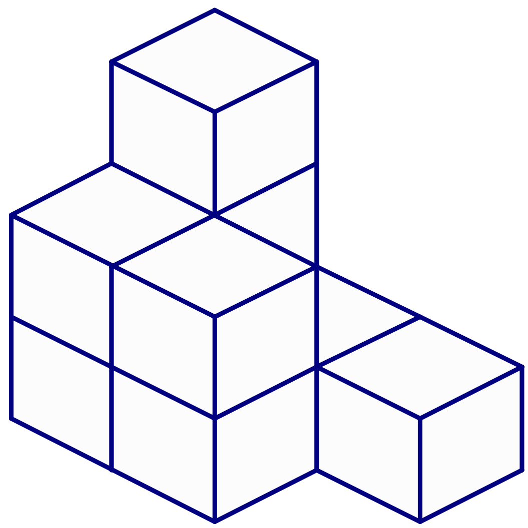




Mats

3	2	1
2	1	

3	1	1
2	2	

Activity: Texas "T" Activity

TEKS: This activity supports teacher content knowledge needed for:

(G.2) **Geometric Structure**. The student analyzes geometric relationships in order to make and verify conjectures.

The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.

(G.3) **Geometric Structure**. The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(D) use inductive reasoning to formulate a conjecture.

(G.5) **Geometric Patterns**. The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties.

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

(G.6) **Dimensionality and the geometry of location**. The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.

The student is expected to:

(B) use nets to represent and construct three-dimensional geometric figures.

(G.11) **Similarity and the geometry of shape**. The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.

(B) use ratios to solve problems involving similar figures.

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

Overview: This activity encourages participants to explore patterns in height, surface area, and volume of similar figures when one or more dimensions are changed.

Materials: Snap cubes
Calculator
Transparencies 1-6 (pages 5-24 – 5-29)
Markers
Handout 1 (page 5-22)
Handout 2 (page 5-23)
Handout 3 (pages 5-30 – 5-34)

Grouping: Groups of 3-4

Time: 1 ½ hours

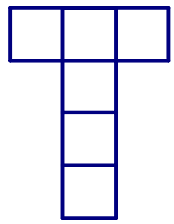
Lesson:

Procedures	Notes
Part I:	
1. Distribute Handout 1, Part I, (page 5-22) of the activity to each participant. Distribute approximately 250 snap cubes and a calculator to each group. Each participant builds his/her own "T" #1 and completes handout 1 of Part I of the activity. As a group, participants build additional "Ts" and complete Handout 2 of Part I (page 5-23).	This activity will help participants to discover the relationship between linear ratios, area ratios, and volume ratios when similar three-dimensional objects are built. Participants should work alone to build the first "T" and answer the questions, then discuss their answers in their groups. Participants should then discuss their answers to page 1 before beginning to work as a group to complete page 2.
2. Bring the class together for a whole group discussion of the findings for Part I. Questions to ask: ▪ "Did anyone build all 4 'Ts'? If not, why not?" ▪ "Were you able to complete the table without building all the models?" ▪ "Did you notice any patterns as you completed the table?" ▪ "How did you describe the patterns in terms of *n*?"	

Procedures	Notes
Part II:	
3. Using a jigsaw cooperative learning procedure, assign each group Activity A, Activity B, or Activity C [Transparencies 1 & 2 (pages 5-24 – 5-25), Transparencies 3 & 4 (pages 5-26 – 5-27), or Transparencies 5 & 6 (pages 5-28 – 5-29)] and provide each group with the appropriate transparencies for its assigned activity. Each group should select a reporter to complete the transparency for the assigned activity, and be prepared to present to the entire group.	Select one group per activity to actually present its results. Ask for discussion from other groups who completed the same activity.
Part III:	
4. Distribute Handout 3, Part III, (pages 5-30 – 5-34), “One and Two-Dimensional Change,” to each participant. Have participants work in their groups to complete Part III. Then bring the class back together to report their findings and discuss them as a whole group. Questions to ask: ▪ “How did what you discovered in Activities A, B, and C relate to what you did in Part III?” ▪ “When is the change in dimension(s) linear? Quadratic?”	Assign each group a piece of Part III to report out. Select groups that did not report out for the previous parts in step 3.
5. Lead participants to begin a discussion about the “Think About It” questions in Part III.	After the group determines that a change in the radius of a can would not be a linear change in volume, ask, “What would happen if we made a one-dimensional change in a sphere?”

TEXAS “T”: Part I

Using snap cubes with the edge of one cube representing 1 unit of length, build the first “T” like the one shown in the illustration. The “T” is 4 units tall and 3 units wide.



Use the “T” to complete the table below:

***N* = “T” NUMBER**	**HEIGHT OF “T”**	**TOTAL SURFACE AREA**	**VOLUME**
“T” #1			

What do you think will happen to the height of the “T” when the dimensions of each part are doubled? ____________________ tripled? _________________________

What do you think will happen to the surface area of the “T” when the dimensions of each part are doubled? __________________ tripled? _________________________

What do you think will happen to the volume of the “T” when the dimensions of each part are doubled? ____________________ tripled? _________________________

TEXAS “T”: Part I

In your group, build at least the next two “Ts”, where *n* represents the height of the “T.” To build each “T”, take the dimensions of “T” #1 and multiply by *n* (*n*=2, *n*=3, *n*=4).

N **= “T” NUMBER**	**HEIGHT OF “T”**	**TOTAL SURFACE AREA**	**VOLUME**
“T” #2			
“T” #3			
“T” #4			
“T” *N*			

TEXAS “T”: Part II
Activity A: Comparing Height in Similar Figures

Using the completed tables from Part I, fill in the ratios for the following table. Record fractions in lowest terms. Use the table data to answer the questions below.

RATIO OF FIGURES	RATIO OF HEIGHT
“T” #1:”T” #2	
“T” #2:”T” #3	
“T” #1:”T” #3	
“T” #3:”T” #4	
“T” #1:”T” #4	
“T” #2:”T” #5	

1. Was your prediction in Part I about the height of the “Ts” accurate? Explain your answer.

2. What pattern do you observe about the simplified ratios of the heights?

3. Using what you observed about the patterns, can you determine the ratio of the height of “T” #20 to the height of “T” #12?

4. If the heights of the “Ts” were in the ratio of 100/40, which “Ts” would you be comparing? Explain your answer.

5. Determine which “T” has a height of 60 units. Explain your answer.

6. What is the ratio of the height of “T” #48 to “T” #16?

7. If you know the height of “T” #1, how can you find the height of “T” #*N*?

Based upon your results, if the ratio of two similar figures is 4/5, the ratio of the heights would be _______________.

Write a rule based on your results:

If the ratio of the heights in two similar figures is *m/n*, the ratio of their heights would be ______________.

TEXAS "T": Part II
Activity B: Comparing Height and Surface Area in Similar Figures

Using the completed table from Part I, fill in the ratios for the following table. Record fractions in lowest terms. Use the table data to answer the questions below.

RATIO OF FIGURES	RATIO OF HEIGHT	RATIO OF SURFACE AREA
"T" #1:"T" #2		
"T" #2:"T" #3		
"T" #1:"T" #3		
"T" #3:"T" #4		
"T" #1:"T" #4		
"T" #2:"T" #5		

1. Was your prediction in Part I about the surface areas of the "Ts" accurate? Explain your answer.

2. What pattern do you observe about the simplified ratios of the heights and surface areas?

3. Using what you observed about the patterns, can you determine the ratio of the surface area of "T" #20 to the surface area of "T" #12?

4. If the surface area of the "Ts" were in the ratio of 256/49, which "Ts" would you be comparing? Explain your answer.

5. Determine which "T" has a surface area of 3,744 square units. Explain your answer.

6. What is the ratio of the surface area of “T” #50 to “T” #15?

7. If you know the surface area of “T” #1, how can you find the surface area of “T” #*N*?

Based upon your results, if the ratio of two similar figures is 4/5, the ratio of the heights would be _______________ and the ratio of the surface area would be ______________.

Write a rule based on your results:

If the ratio of the heights in two similar figures is *m/n*, then the ratio of their surface area would be _________________.

TEXAS "T": Part II
Activity C: Comparing Total Surface Area and Volume in Similar Figures

Using the completed table from Part I, fill in the ratios for the following table. Record fractions in lowest terms. Use the table data to answer the questions below.

RATIO OF FIGURES	RATIO OF HEIGHT	RATIO OF SURFACE AREA	RATIO OF VOLUME
"T" #1:"T" #2			
"T" #2:"T" #3			
"T" #1:"T" #3			
"T" #3:"T" #4			
"T" #1:"T" #4			
"T" #2:"T" #5			

1. Was your prediction in Part I about the volumes of the "Ts" accurate? Explain your answer.
2. What pattern do you observe about the simplified ratios of the heights, surface areas and volumes?
3. Using what you observed about the patterns, can you determine the ratio of the volume of "T" #20 to the height of "T" #12?
4. If the volume of the "Ts" were in the ratio of 1000/512, which "Ts" would you be comparing? Explain your answer.

5. Determine which “T” has a volume of 34,992 cubic units? Explain your answer.

6. What is the ratio of the volume of “T” #50 to “T” #15?

7. If you know the volume of “T” #1, how can you find the volume of “T” #*N*?

8. What happens to the volume if all the dimensions of the “T” are reduced by one-half? Explain your answer.

Based upon your results, if the ratio of the heights of two similar figures is 4/5, the ratio of the surface area would be ________________, and the ratio of the volume would be ___________________.

Write a rule based on your results:

If the ratio of the heights in two similar figures is m/n, the ratio of their surface area would be _______________ and the ratio of their volume would be ______________.

TEXAS “T”: Part III
One and Two-Dimensional Change

Thus far we have examined the effect on surface area and volume of a figure if all dimensions are changed by the same factor. What do you suppose will happen to surface area and volume if only one dimension is changed?

For this exercise, let’s use a simpler figure.

l = length *w* = width *h* = height

Volume of a rectangular prism: $V = lwh$

Surface area of a rectangular prism: $SA = 2(lw) + 2(wh) + 2(hl)$

Volume:

A. Change in one dimension:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length and width, **change the height by a factor of *n*** and complete the following table.

n	Volume
1	**4**
2	
3	
4	
5	
6	
n	

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the volumes.

____

____

4. Write an algebraic function for the pattern. ____________________

5. Without changing the length and height, **change the width by a factor of *n*** and complete the following table.

n	Volume
1	**4**
2	
3	
4	
5	
6	
n	

6. Write in words the pattern you observe in the volumes.

____

____

7. Write an algebraic function for the pattern.____________________

8. Why is the pattern the same as #3? ____________________

B. Change in two dimensions

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length, **change the height and width by a factor of *n*** and complete the following table.

n	Volume
1	**4**
2	
3	
4	
5	
6	
n	

2. Enter the data into lists in your calculator as L1 and L3 and plot the data.

3. Write in words the pattern you observe in the volumes.

____

____

4. Write an algebraic function for the pattern.______________________________

5. Write a function to predict the volume of our figure if all three dimension change by factor of ***n.*** ____

Surface Area:

A. Change in one dimension:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length and width, **change the height by a factor of *n*** and complete the following table.

n	Surface Area
1	
2	
3	
4	
5	
6	
n	

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the surface area.

____

____

4. Write an algebraic function for the pattern.______________________________

B. Change in two dimensions:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length, **change the height and width by a factor of *n*** and complete the following table.

n	Surface Area
1	
2	
3	
4	
5	
6	
n	

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the volumes.

____

____

4. Write an algebraic function for the pattern.________________________________

Think about it:

Does a one dimensional change in any 3-dimensional figure create a linear change in volume? What if the figure were a can?

TEXAS “T”: Part I SOLUTIONS

***N* = “T” NUMBER**	**HEIGHT OF “T”**	**TOTAL SURFACE AREA**	**VOLUME**
“T” #1	4	26	6
“T” #2	8	104 $(26*2^2)$	48 $(6*2^3)$
“T” #3	12	234 $(26*3^2)$	162 $(6*3^3)$
“T” #4	16	416 $(26*4^2)$	384 $(6*4^3)$
“T” N	$4N$	$26N^2$	$6N^3$

Part II
Activity A: Comparing Height in Similar Figures

RATIO OF FIGURES	RATIO OF HEIGHT
“T” #1:”T” #2	4/8 = 1/2
“T” #2:”T” #3	8/12 = 2/3
“T” #1:”T” #3	4/12 = 1/3
“T” #3:”T” #4	12/16 = 3/4
“T” #1:”T” #4	4/16 = 1/4
“T” #2:”T” #5	8/20=2/5

1. **Was your prediction about the height of the “Ts” accurate? Explain your answer.** *Participants should predict that the height doubles when the dimensions are doubled and triples when the dimensions are tripled.*

2. **What pattern do you observe about the simplified ratios of the heights?** *The simplified ratio is the same as the ratio of the “Ts”. For example, “T” 2:”T”3 = 8/12 = 2/3*

3. **Using what you observed about the patterns, can you determine the ratio of the height of “T” 20 to the height of “T” 12?** *The ratio is 20/12.*

4. **If the heights of the “Ts” were in the ratio of 100/40, which “Ts” would you be comparing? Explain your answer.** *You would be comparing “T” 25 to “T” 10. 100/4 = 25 and 40/4 = 10.*

5. **Determine which “T” has a height of 60 units. Explain your answer.** *“T” 15 has a height of 60 since 60 divided by 4 equals 15.*

6. **What is the ratio of the perimeters of “T” 48 to “T” 16?** *The ratio is 48/16 which simplifies to 3/1.*

7. **If you know the height of "T" 1, how can you find the height of "T" N?** *N times the height of "T" 1*

Based upon your results, if the ratio of two similar figures is 4/5, the ratio of the heights would be ____*4/5*___________.

Write a rule based on your results:

If the ratio of two similar figures is m/n, the ratio of their heights would be ____*m/n*__________.

Part III
Activity B: Comparing Height and Surface Area in Similar Figures

RATIO OF FIGURES	RATIO OF HEIGHT	RATIO OF SURFACE AREA
"T" 1:"T" 2	4/8 = 1/2	26/104= 1/4
"T" 2:"T" 3	8/12 = 2/3	104/234 = 4/9
"T" 1:"T" 3	4/12 = 1/3	26/234 = 1/9
"T" 3:"T" 4	12/16 = 3/4	234/416 = 9/16
"T" 1:"T" 4	4/16 = 1/4	26/416 = 1/16
"T" 2:"T" 5	8/20 = 2/5	104/650 = 4/25

1. **Was your prediction about the surface areas of the "Ts" accurate? Explain your answer.** *The participants should predict that the surface area will increase by the original surface area times the square of the dimensional change (height).*

2. **What pattern do you observe about the simplified ratios of the heights and surface areas?** *The ratio of the surface area is equal to the ratio of the heights squared.*

3. **Using what you observed about the patterns, can you determine the ratio of the surface area of "T" 20 to the surface area of "T" 12?** *400/144 = 25/9*

4. **If the surface area of the "Ts" were in the unsimplified ratio of 256/49, which "Ts" would you be comparing? Explain your answer.** *You would be comparing "T" 16 to "T" 7. You take the square root of 256 and 49 to determine the answer.*

5. **Determine which "T" has a surface area of 3,744 square units. Explain your answer.** *3,744 = 26 N^2 Divide by 26 and take the square root. "T" 12 has a surface are of 3,744.*

6. **What is the ratio of the surface area of "T" 50 to "T" 15?** *2500/225 = 100/9*

7. **If you know the surface area of "T" 1, how can you find the surface area of "T" N?** *$26N^2$*

Based upon your results, if the ratio of two similar figures is 4/5, the ratio of the heights would be ____*4/5*___ and the ratio of the surface area would be ___*16/25*___________.

Write a rule based on your results:

If the ratio of two similar figures is m/n, the ratio of their heights would be ___*m/n*____ and the ratio of their surface area would be __*m^2/n^2*____.

Part II
Activity C: Comparing Total Surface Area and Volume of Similar Figures

RATIO OF FIGURES	RATIO OF HEIGHT	RATIO OF SURFACE AREA	RATIO OF VOLUME
"T" 1:"T" 2	4/8 = 1/2	26/104 = 1/4	6/48 = 1/8
"T" 2:"T" 3	8/12 = 2/3	104/234 = 4/9	48/162 = 8/27
"T" 1:"T" 3	4/12 = 1/3	26/234 = 1/9	6/162 = 1/27
"T" 3:"T" 4	12/16 = 3/4	234/416 = 9/16	162/384 = 27/64
"T" 1:"T" 4	4/16 = 1/4	26/416= 1/16	6/384 = 1/64
"T" 2:"T" 5	8/20 = 2/5	104/650 = 4/25	48/750 = 8/125

1. **Was your prediction about the volumes of the "Ts" accurate? Explain your answer.** *The participants should predict that the volume will increase by the original volume times the cube of the dimensional change (height).*

2. **What pattern do you observe about the simplified ratios of the heights, surface areas and volumes?** *The ratio of the surface area is equal to the ratio of the heights squared. The ratio of the volume is equal to the ratio of the heights cubed.*

3. **Using what you observed about the patterns, can you determine the ratio of the volume of "T" 20 to the volume of "T" 12?** *8000/1728 = 125/27*

4. **If the volume of the "Ts" were in the unsimplified ratio of 1000/512, which "Ts" would you be comparing? Explain your answer.** *You would be comparing "T" 10 to "T" 8. You take the cube root of 1000 and 512 to determine the answer.*

5. **Determine which "T" has a volume of 34,992 cubic units. Explain your answer.** $34{,}992 = 6N^3$ *Divide by 6 and take the cube root. "T" 18 has a volume of 34,992.*

6. **What is the ratio of the volume of "T" 50 to "T" 15?** *125,000/3375 = 1000/27*

7. **If you know the volume of "T" 1, how can you find the volume of "T" N?** *$6N^3$*

8. **What happens to the volume if all the dimensions of the "T" are reduced by one-half? Explain your answer.** *The volume will reduce to 1/8 of the original volume. $(1/2)^3 = 1/8$*

Based upon your results, if the ratio of the heights of two similar figures is 4/5, the ratio of the surface area would be ___*16/25*___, and the ratio of the volume would be ___*64/125*______.

Write a rule based on your results:

If the ratio of the heights in two similar figures is m/n, the ratio of their surface area would be __m^2/n^2___ and the ratio of their volume would be ___m^3/n^3___.

TEXAS "T"
Part III
One and Two-Dimensional Change

Thus far we have examined the effect on surface area and volume of a figure if all dimensions are changed by the same factor. What do you suppose will happen to surface area and volume if only one dimension is changed?

For this exercise, let us use a simpler figure.

l = length *w* = width *h* = height

Volume of a rectangular prism: $V = lwh$

Surface area of a rectangular prism: $SA = 2(lw) + 2(wh) + 2(hl)$

Volume:

A. Change in one dimension:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length and width, change the height by a factor of *n* and complete the following table.

n	Volume
1	**4**
2	**8**
3	**12**
4	**16**
5	**20**
6	**24**
n	**4*n***

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the volumes.

Each increase of n causes the volume to increase by a factor of 4.

4. Write an algebraic function for the pattern. $V = 4n$

5. Without changing the length and height, change the width by a factor of *n* and complete the following table.

n	Volume
1	**4**
2	**8**
3	**12**
4	**16**
5	**20**
6	**24**
n	**4*n***

6. Write in words the pattern you observe in the volumes.

Each increase of n causes the volume to increase by a factor of 4.

7. Write an algebraic function for the pattern. $V = 4n$

8. Why is the pattern the same as #3?

Because the change is only in one dimension, the effect on volume is linear. It does not matter which dimension is changed.

B. Change in two dimensions

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length, change the height and width by a factor of *n* and complete the following table.

n	Volume
1	**4**
2	**16**
3	**36**
4	**64**
5	**100**
6	**144**
n	$4n^2$

2. Enter the data into lists in your calculator as L1 and L3 and plot the data.

3. Write in words the pattern you observe in the volumes.

Volume increases by $4n^2$

4. Write an algebraic function for the pattern. $V = 4n^2$

5. Write a function to predict the volume of our figure if all three dimensions change by a factor of *n*: $V=4n^3$

Surface Area:

A. Change in one dimension:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length and width, change the height by a factor of *n* and complete the following table.

n	Surface Area
1	**16**
2	**24**
3	**32**
4	**40**
5	**48**
6	**54**
n	**8*n*+8**

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the surface area.

The change is linear, increasing by eight each time the factor increases. Given that the surface area equals 2(lw)+2(wh)+2(hl), the only parenthesis that does not change when the height is increased by a given factor is 2(lw). This is your constant. The sum of the remaining parenthesis [{(wh)+(hl)] is multiplied times the factor. The surface area for a 2x2x1 rectangular prism where only the height is changed by a given factor can be found by the equation: n[2((wh)+(hl))] + 2(lw), where n is the given factor. For problems 1-4, the equation is SA = n[2((2x1)+(1x2))]+2(2x2) = 8n + 8.

4. Write an algebraic function for the pattern. *SA = 8n+8*

B. Change in two dimensions:

Given a rectangular prism with a length of 2, width of 2 and a height of 1;

1. Without changing the length, change the height and width by a factor of *n* and complete the following table.

n	Surface Area
1	**16**
2	**40**
3	**72**
4	**112**
5	**160**
6	**216**
n	$4n^2 + 12n$

2. Enter the data into lists in your calculator as L1 and L2 and plot the data.

3. Write in words the pattern you observe in the surface areas.

The change is quadratic. The parenthesis where both dimensions change is multiplied times the square of the given factor. The sum of the remaining parenthesis is multiplied times the given factor. The surface area for a 2x2x1 rectangular prism where the width and height are changed by the same given factor can be found by the equation: $n^2 (2(wh))+n[2((lw+hl))]$*, where n is the given factor. For problems 1-4, the equation is* $SA = n^2(2(2x1)) + n[2((2x2) + (1x2))] = 4n^2+12n$.

4. Write an algebraic function for the pattern. $\underline{SA = 4n^2 + 12n}$

Think about it:

Does a one dimensional change in any 3-dimensional figure create a linear change in volume? What if the figure was a can?

No. For example, a change in the radius of a cylinder produces a quadratic change in volume.