Math 2 Lesson Title: Understanding Expressions with Rational Exponents and Radicals

Unit 1: Extending the Number System *(Lesson 1 of 4)* **Time Frame:** 1 week

Essential Question:

- How can radical and rational exponents be written equivalently?
- Do the properties of integer exponents apply to rational exponents?

Targeted Content Standard(s):

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, *we define* $(5^{1/3})^3$ *must equal 5 because we want* $(5^{1/3})^3=5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Targeted Mathematical Practice(s):

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☒ 3 Construct viable arguments and critique the reasoning of others.
- ☒ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
- ☒ 7 Look for and make use of structure.
- ☒ 8 Look for an express regularity in repeated reasoning.

Supporting Content Standard(s): *(optional)*

Student Friendly Learning Targets

I can…

- Use properties of integer exponents and apply those to rational exponents.
- Convert between exponential and radical form.

Purpose of Lesson:

The purpose of this lesson is to guide students in making connections between integer and rational exponents and radical expressions and expressions with rational exponents.

Explanation of Rigor: *(Fill in those that are appropriate.)*

Conceptual:	**Procedural:**	**Application:**
Students will extend properties of integer exponents to rational exponents. (N-RN.1)	Students will convert between exponential and radical form, write exponential equations and inequalities to model situations, and solve exponential equations and inequalities. (N-RN.2)	

Vocabulary:

Integer exponent	Radical expression
Rational exponent	Equivalent forms

Evidence of Learning (Assessment):

Pre-Assessment: Properties of Exponents for Integers

Formative Assessment(s): Check Their Work-Rational Exponents

Summative Assessment: Unit 1 will be assessed as a whole with a Summative Assessment

Self-Assessment: On the 2nd day, use Quick Quiz to have students self-assess

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Lesson Procedures:

Segment 1

Approximate Time Frame: 45-50 minutes	**Lesson Format:** ☐ Whole Group ☒ Small Group ☐ Independent ☒ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Resources:** Properties of Exponents Pre-Assessment
Focus: Students can show their knowledge of the properties of integer exponents.		**Modalities Represented:** ☐ Concrete/Manipulative ☐ Picture/Graph ☒ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☐ Real-Life Situation

Math Practice Look For(s): • **MP#8 - Look for and express regularity in repeated reasoning.** Students will see that they are using the same processes for rational exponents as they used previously with integer exponents.	**Differentiation for Remediation:** May need to do another example on the tables to help them move along. **Differentiation for English Language Learners:**
Potential Pitfall(s): Students may have forgotten some of their properties of exponents.	**Differentiation for Enrichment:** Have them attempt to do this with rational numbers and see if they can complete the assessment. **Independent Practice (Homework):**
Steps: 1. As a class we will be talking about the examples that have been filled into the tables and giving the students a direction to go.	**Teacher Notes/Reflections:**
2. Have the class then complete the tables and questions as directed on the work sheet.	**Teacher Notes/Reflections:**
3. Before the students move on to the problem set, have a discussion about the answers they found and their explanations.	**Teacher Notes/Reflections:**

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Lesson Procedures:

Segment 2

Approximate Time Frame:

45-50 minutes

Lesson Format:

☒ Whole Group
☐ Small Group
☐ Independent

☐ Modeled
☒ Guided
☒ Collaborative
☐ Assessment

Resources:

Focus:

Reviewing properties of exponents and extending those properties to rational exponents.

Modalities Represented:

☐ Concrete/Manipulative
☐ Picture/Graph
☐ Table/Chart
☒ Symbolic
☒ Oral/Written Language
☐ Real-Life Situation

Math Practice Look For(s):

- **MP#8 - Look for and express regularity in repeated reasoning.** Students will see that they are using the same processes for rational exponents as they used previously with integer exponents.

Potential Pitfall(s):

Students may have difficulty with addition and multiplication of rational numbers.

Independent Practice (Homework):

Differentiation for Remediation:

Provide students with guided practice for operating on rational numbers including fractions with common denominators and fractions with non-common denominators.

Differentiation for English Language Learners:

Allow students to work in small groups with peers that also speak their native language and assist these groups in translating prior to sharing with their peers.

Differentiation for Enrichment:

Steps:

1. **Review of Integer Exponent Properties**
   - Provide expression and ask the student to make recommendations for how to simplify.
   - For example, use $x^2 * x^3 =?$. What do the individual parts of this equation stand for? What does x^2 mean? x^3? Can we rewrite this expression to demonstrate that the property we learned is true?
   - Provide multiple examples of these including products, quotients, powers raised to powers, and equivalent exponents to generate discussion about each property of exponents.

Teacher Notes/Reflections:

2. **Extend to Rational Exponents**
   - Give students an example with rational exponents.
   - For example, $x^{\frac{1}{5}} * x^{\frac{3}{5}} =?$
   - Ask students to discuss possible approaches to

Teacher Notes/Reflections:

Essential Question:

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simplify this expression with a partner or small group. - Give 2-3 more examples addressing the other properties stated above. - Ask groups to volunteer to share for one of the given problems and ask the rest of the students to give feedback. - Ask students to draw conclusions from the day's discussion.	

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Segment 3		
Approximate Time Frame: 45-50 minutes	**Lesson Format:** ☒ Whole Group ☐ Small Group ☐ Independent ☒ Modeled ☒ Guided ☒ Collaborative ☒ Assessment	**Resources:**
Focus: Reviewing properties of exponents and extending those properties to rational exponents.		**Modalities Represented:** ☐ Concrete/Manipulative ☐ Picture/Graph ☐ Table/Chart ☒ Symbolic ☐ Oral/Written Language ☐ Real-Life Situation

Math Practice Look For(s): • **MP#7: Look for and make use of structure.** Students will see the relationship between radical and exponential notation. • **MP#8: Look for and express regularity in repeated reasoning.** Students will see that they are using the same processes for rational exponents as they used previously with integer exponents.	**Differentiation for Remediation:** **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s): Students may arrive at incorrect conclusions when trying to find a pattern. (i.e. 4 raised to the ½ is equivalent to 4 divided by 2)	
Independent Practice (Homework):	
Steps: 1. **Reinforcement of Properties of Rational Exponents** - Create an entrance "quick quiz" to assess students' level of understanding of the properties used in the previous day. - Be sure to include examples of each property from Day 1. - Self-grade or partner grade for correctness.	**Teacher Notes/Reflections:**
2. **Exploration of Rewriting Expressions Involving Rational Exponents into Radical Expressions (numerator =1)** - Each student will need a calculator (additional scaffolding may be needed for use of calculator to input rational exponents). - Have students compute the following:	**Teacher Notes/Reflections:**

  - 4 raised to the ½
  - 9 raised to the ½
  - 16 raised to the ½
- As a large group, ask for any patterns students see.
  - Goal is for students to see the pattern that the number from their calculator is the square root of the original number.
  - Write equivalent expressions and predict what 25 raised to the ½ would look like.
    - Ex. $4^{1/2} = \sqrt{4}$
- Following the same process from above, have students compute the following:
  - 8 raised to the 1/3
  - 27 raised to the 1/3
  - 64 raised to the 1/3
- If more reinforcement is needed, continue with numbers raised to the ¼ power.

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Segment 4		
Approximate Time Frame: 45-50 minutes	**Lesson Format:** ☒ Whole Group ☒ Small Group ☐ Independent ☐ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Resources:** Create an exit slip for the end of the lesson.
Focus: Rewriting expressions involving rational exponents as equivalent radical expressions (numerator >1).		**Modalities Represented:** ☐ Concrete/Manipulative ☐ Picture/Graph ☐ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☐ Real-Life Situation

Math Practice Look For(s): • **MP#7: Look for and make use of structure.** Students will see the relationship between radical and exponential notation.	**Differentiation for Remediation:** Teacher can provide additional examples with a small group to assist with enforcement of the relationship. **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s): Students may incorrectly multiply fractions by whole numbers. Students may write the numerator of the exponent as the index of the radical or as the radicand.	
Independent Practice (Homework):	
Steps: 1. Give students the following example: $(8^2)^{1/3}$ & $(8^{1/3})^2$ - In partners or small groups, have students show work or write 1-2 sentences explaining why these two expressions are equivalent.	**Teacher Notes/Reflections:**
2. Instruct students to rewrite these expressions in as many different ways as possible. ○ The goal is for students to connect Day 2 learning to this lesson. - Use student response to highlight the following: ○ $(8^{1/3})^2 = (8^2)^{1/3} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 8^{2/3}$	**Teacher Notes/Reflections:**
3. With their partners, have students come up with their own examples writing equivalent expressions (no evaluating) using exponents involving numbers other than 1,2,3 as an "exit slip" activity.	**Teacher Notes/Reflections:**

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Unit 1: Extending the Number System *(Lesson 1 of 4)* **Time Frame:** 1 week

Essential Question:

- How can radical and rational exponents be written equivalently?
- Do the properties of integer exponents apply to rational exponents?

Segment 5

Approximate Time Frame:

45-50 minutes

Lesson Format:

☒ Whole Group
☐ Small Group
☒ Independent

☐ Modeled
☒ Guided
☒ Collaborative
☒ Assessment

Resources:

Create practice problems worksheet

Focus:

Combining expressions involving rational exponents and radicals.

Modalities Represented:

☐ Concrete/Manipulative
☐ Picture/Graph
☐ Table/Chart
☒ Symbolic
☒ Oral/Written Language
☐ Real-Life Situation

Math Practice Look For(s):

- **MP#7: Look for and make use of structure.** Students will see the relationship between radical and exponential notation.
- **MP#8: Look for and express regularity in repeated reasoning.** Students will see that they are using the same processes for rational exponents as they used previously with integer exponents.

Differentiation for Remediation:

Reference Grade 5/6 for additional practice.
May need more practice problems to work on.

Differentiation for English Language Learners:

Differentiation for Enrichment:

Have student peer review their practice problems, discuss common errors, and rework problems together.

Potential Pitfall(s):

Students may still struggle with operations on rational numbers, especially adding, subtracting, and multiplying fractions.

Independent Practice (Homework):

Finish practice problems.

Steps:

- Reflect on and discuss the previous day's exit slip activity.
  - Have students share responses with another student, then have the other student share with the large group.
- As a large group, work several examples where students are asked to combine/simplify expressions involving rational exponents and radicals.
  - Potential example problems:
    - $\frac{y^{5/8}}{y^{2/3}}$
    - $(x^{1/2} * y^{-2/3})^6$

Teacher Notes/Reflections:

Essential Question:

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▪ $\sqrt[4]{18} * \sqrt{12}$ - Provide additional practice problems for students to work independently during a guided practice time.	

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Essential Question:

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Segment 6

Approximate Time Frame:

35-40 minutes

Lesson Format:

- ☐ Whole Group
- ☒ Small Group
- ☐ Independent

- ☐ Modeled
- ☐ Guided
- ☐ Collaborative
- ☒ Assessment

Resources:

Check their work rational exponents

Focus:

To assess students' ability to extend the properties of integer exponents to rational exponents and students' ability to rewrite radical expressions as expressions with rational exponents.

Modalities Represented:

- ☐ Concrete/Manipulative
- ☐ Picture/Graph
- ☐ Table/Chart
- ☒ Symbolic
- ☒ Oral/Written Language
- ☐ Real-Life Situation

Math Practice Look For(s):

- **MP#3: Construct viable arguments and critique the reasoning of others.** Students will analyze written work of others to make decisions about correct procedures and explain their reasoning.
- **MP#7: Look for and make use of structure.** Students will see the relationship between radical and exponential notation.
- **MP#8: Look for and express regularity in repeated reasoning.** Students will see that they are using the same processes for rational exponents as they used previously with integer exponents.

Differentiation for Remediation:

Based on the results of this assessment, students may need an additional activity with similar concepts.

Differentiation for English Language Learners:

Differentiation for Enrichment:

Potential Pitfall(s):

Students may have trouble putting into words their mathematical justifications.

Independent Practice (Homework):

Finish assignment individually if not finished at the end of class.

Steps:

1. **"Check Their Work" Assessment**

- Follow up on previous day's guided practice assignment.
- Go over problems that students struggled with.
- Assign students to groups of two and hand out the assessment:
  - ***"Check Their Work Rational Exponents"***

Teacher Notes/Reflections:

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