MPACT

Math and Computational Thinking Through 3D Making

Teacher Notes: Grade 6 Module 2

Goals for the Module

Mathematics

Measure volume in cubic units and standard measurements: a review of 5th grade volume standards included.

Compare volume of successively smaller fractional unit cubes in order to see patterns and express growth.

Understand and use nets of 3D shapes to find surface area.

Spatial Reasoning

Rotate 3D objects mentally to decide if they are the same.

Interpret 2D drawings of 3D objects.

Computational Thinking (CT)

Calculate volume in different ways by creating algorithms.

Group objects to make pieces efficiently.

Decompose bigger problems into smaller parts.

Recognize and use patterns in comparing volumes.

Identify and correct errors in puzzle designs (debugging).

Materials

- 27 linking cubes in 7 colors for each student (4 of each of 6 colors, 3 of 1 color)
- Cardstock
- Tape or glue

Timing

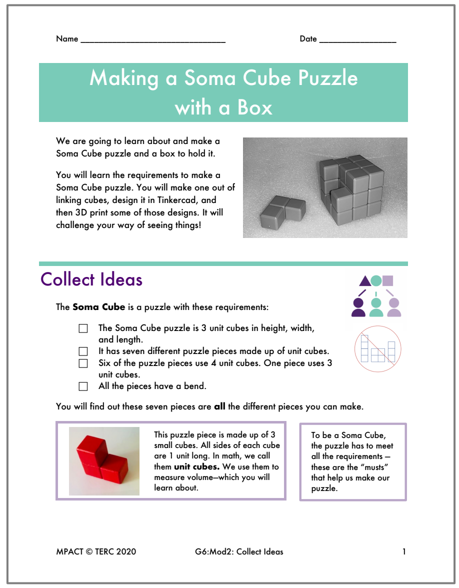
- Six or seven 45-minute lessons
- Times may vary. Times given below are for phases of design (such as prototyping) and you can break those into lessons for what works best for you.

Introduction 10 min

Goals

Design and Making

Understand the idea of design requirements and make sure students understand the requirements for the Soma Cube.



Name ______________________ Date ______________

Making a Soma Cube Puzzle with a Box

We are going to learn about and make a Soma Cube puzzle and a box to hold it.

You will learn the requirements to make a Soma Cube puzzle. You will make one out of linking cubes, design it in Tinkercad, and then 3D print some of those designs. It will challenge your way of seeing things!

Collect Ideas

The **Soma Cube** is a puzzle with these requirements:

- ☐ The Soma Cube puzzle is 3 unit cubes in height, width, and length.
- ☐ It has seven different puzzle pieces made up of unit cubes.
- ☐ Six of the puzzle pieces use 4 unit cubes. One piece uses 3 unit cubes.
- ☐ All the pieces have a bend.

You will find out these seven pieces are **all** the different pieces you can make.

This puzzle piece is made up of 3 small cubes. All sides of each cube are 1 unit long. In math, we call them **unit cubes.** We use them to measure volume—which you will learn about.

To be a Soma Cube, the puzzle has to meet all the requirements – these are the "musts" that help us make our puzzle.

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Materials

- Presentation slides
- Design process poster

Introduce the module

10 min

Introduce the module using the slides to provide a context for the Soma Cube activities.

Ask students to restate what their task is. Accept all answers.

Sample answers:

- *We will make a Soma Cube puzzle.*
- *I will be a puzzle maker.*

Ask: What is a cube? Can you give examples? Accept all answers.

Sample answers:

- *It looks like a box.*

Collect Ideas 35 min

Goals

Learn about the Soma Cube puzzle and the relationship between its pieces.

Math

Understand volume and the formula (Review).

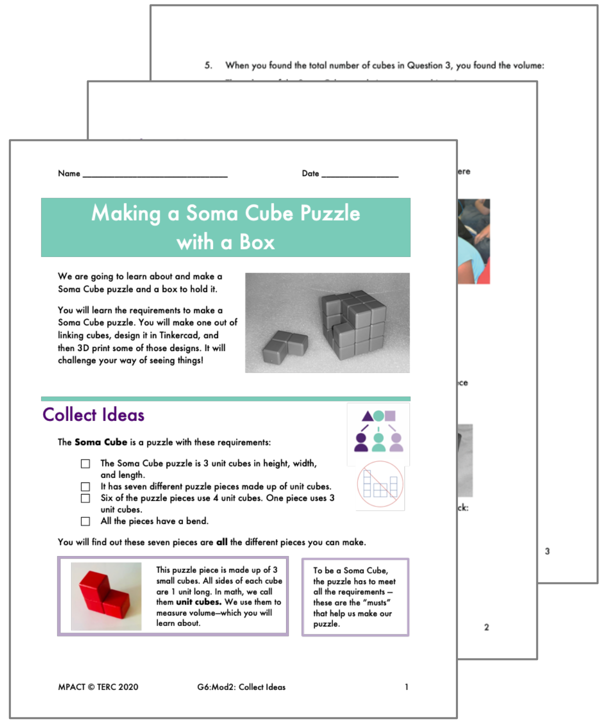
Spatial Reasoning

Rotate objects mentally to determine if they are “the same.”

Build real-world 3D objects to match a picture.

Computational Thinking

Recognize that a large cube is comprised of smaller pieces and smaller pieces fit together to create a large cube.



Materials

- Linking cubes—for each student, four of one each of six colors, three of another color

Introduce Soma Cube Puzzle requirements 10 min

Read aloud the design requirements as a whole class.

- As you read each bullet together, stop and ask them to explain each of the requirements.
- Use the picture and the linking cubes in front of them to help them interpret the requirements.
- Explain vocabulary: Design requirements: “Our design has to follow these rules or requirements.”
- Have students make a running list of requirements on a special page in their notebooks, if used, so they can return to them for reference and updating.

Read the definition for unit cubes and explain what it is: “We call each individual linking cube a unit cube.”

Helping Yesi: Present the context and make a prediction 25 min

This activity sets students up to do spatial reasoning and the computational thinking practice of seeing patterns. The students have an opportunity to measure volume in different ways.

Provide time for the students to rotate the pieces physically and mentally as they try to make the puzzle. The students should recognize patterns of how each unit cube is positioned in relation to another and check whether they are distinct from each other (CT).

Read the top part of page 2 together as a class.

Q1. *Accept all answers.*

Students make a prediction. Do not evaluate student work at this point. Students will check their guesses in Q2 using linking cubes.

Pass out cubes.

Have students answer Q2–6 in a whole class discussion. Encourage students to make the pieces to confirm their answers.

Q2. Let students make the puzzle piece to verify their answer.

- When students have different answers, ask them to explain to each other why theirs is right.
- Allow students to use informal language to address the mathematical ideas of "rotation" and "orientation."

Answer: K

- **Shape K is the same** as the piece Yesi had but rotated.
- **Shape M** is a mirror image of Yesi's piece and so **not the same.**
- Shape L is also not the same.

Q3–Q5: Individual or pair work. Assign students to answer the questions about unit cubes and to find the volume of the complete Soma Cube Puzzle.

Q3. Students may use different strategies:

- Count cubes individually.
- Count the number in a layer of cubes, then multiplying by 3.
- Multiply 3 • 3 • 3.

Answer: 27

Q4. Encourage students to think of more than one way to find out the answer for Q4: Students can use information in the requirements to find the number of unit cubes.

Possible Answers:

- *The Soma puzzle has 6 pieces of 4 unit cubes and 1 piece of 3 unit cubes. Since I have 1 with 4 unit cubes, I need 5x4 unit cubes and 1x3 unit cubes, which is 23 unit cubes.*
- *Some students may use the answer in Q3 by subtracting 4 unit cubes for Yesi's piece from the total and find 23 unit cubes. 27 – 4 = 23*

Q5. *27 cubic units*

Q6. $V=s^3$ *where V is the volume of a cube and s is the length of each side (or edge).*

- Students might describe their formula as multiplying the length by itself three times, or the length cubed, because in a cube, length, width, and height are all same.

Read the volume box aloud.

Emphasize the volume is the amount of space a shape takes up, and that it can be measured in cubic units.

Make and Re-make Prototypes 25 min

Goals

Design and Making

Make the Soma Cube puzzle pieces from linking cubes (a prototype.)

Math

Find volume in cubic units.

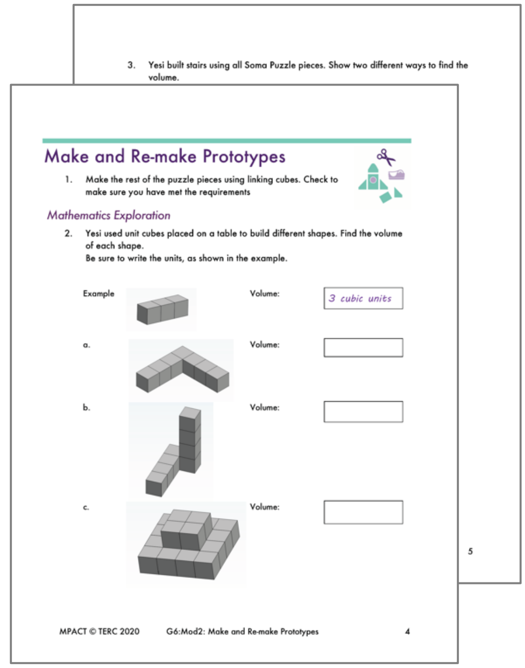
Spatial Reasoning

Rotate puzzle pieces with physical objects and then mentally to see if they are the same.

Envision missing pieces in a cube.

Computational Thinking

Ensure that the pieces meet the design requirements (*debugging*).



3. Yesi built stairs using all Soma Puzzle pieces. Show two different ways to find the volume.

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Make and Re-make Prototypes

1. Make the rest of the puzzle pieces using linking cubes. Check to make sure you have met the requirements

Mathematics Exploration

2. Yesi used unit cubes placed on a table to build different shapes. Find the volume of each shape.
Be sure to write the units, as shown in the example.

Example Volume: 3 cubic units

a. Volume:

b. Volume:

c. Volume:

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Materials

- Linking cubes

Make the remaining pieces

20 min

Pass out linking cubes.

Students use linking cubes to make the rest of the puzzle pieces.

Finding the remaining 6 pieces can be challenging.

Circulate:

- **Remind students of the requirements.**
- **If students struggle, have them start with the 3-cube piece and ask where another unit cube can go.**
- **Have them find all the places to make different 4-cube pieces.**
- **Ask if there are other 4-unit pieces that can be made.**

The hardest piece to find are the mirror image of the piece that Yesi found and S-shape.

- **Remind them to look at the piece they made as the piece that Yesi found.**
- **To check if they have made duplicates of pieces, have them rotate pieces and compare.**
- **Ask if they have made all those pieces.**
- After students have found all the missing pieces, have them play with the pieces for about 10 minutes to try to complete the puzzle. If no one comes up with a way to complete a puzzle, then save that activity for later.

NOTE: students can find more solutions for making a cube (240 in all) and can also make different shapes with the puzzle pieces (see MPACT community resources). Extra time to solve these puzzle is great for spatial reasoning,

Students should mark off the requirements they have met. They may need to remake some of the pieces to meet all the requirements.

Mathematics Exploration: Find volumes of more 3D shapes

5 min

Q2. Students find the volume of shapes that are not cubes.

Answers

a. 7 cubic units

b. 8 cubic units

c. 20 cubic units

For c, some students might count 16, ignoring the ones in the middle of the bottom layer. Remind them these are composed of building blocks and blocks on the top layer can't stay at top without blocks below.

Q3. *Answer: 27 cubic units.*

- Students may count layer by layer.
- Others may mentally form the cube and subtract the pieces not shown.
- Other may immediately know it is 27, since Yesi used all the puzzle pieces.

Design the Soma Cube Puzzle 95 min

Goals

Use spatial reasoning to design the puzzle pieces in Tinkercad.

Math

Explore the relationship between volumes of increasingly small unit cubes with fractional length sides.

Find volume in standard measurements.

Spatial Reasoning

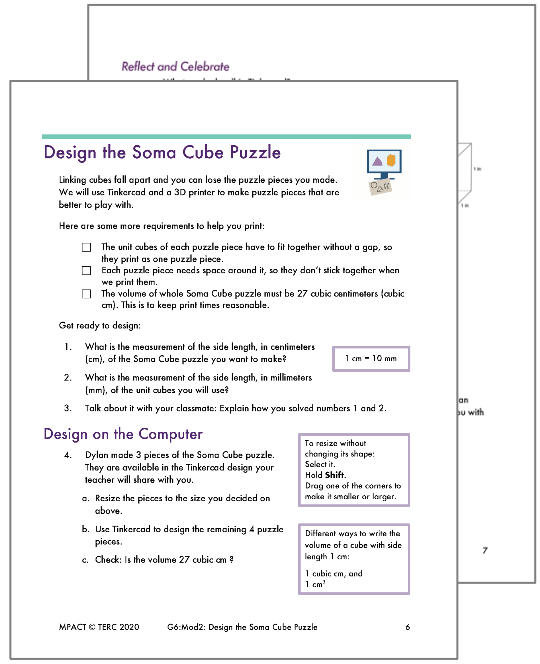
Manipulate and interpret 2D objects as 3D objects.

Computational Thinking

Demonstrate different ways to find the volume (*algorithmic thinking*).

Understand *modular component*s of the design in order to solve problems.

Debug their Soma Cube puzzle design.


Reflect and Celebrate

Design the Soma Cube Puzzle

Linking cubes fall apart and you can lose the puzzle pieces you made. We will use Tinkercad and a 3D printer to make puzzle pieces that are better to play with.

Here are some more requirements to help you print:

- ☐ The unit cubes of each puzzle piece have to fit together without a gap, so they print as one puzzle piece.
- ☐ Each puzzle piece needs space around it, so they don't stick together when we print them.
- ☐ The volume of whole Soma Cube puzzle must be 27 cubic centimeters (cubic cm). This is to keep print times reasonable.

Get ready to design:

1. What is the measurement of the side length, in centimeters (cm), of the Soma Cube puzzle you want to make?
2. What is the measurement of the side length, in millimeters (mm), of the unit cubes you will use?
3. Talk about it with your classmate: Explain how you solved numbers 1 and 2.

1 cm = 10 mm

Design on the Computer

4. Dylan made 3 pieces of the Soma Cube puzzle. They are available in the Tinkercad design your teacher will share with you.
   a. Resize the pieces to the size you decided on above.
   b. Use Tinkercad to design the remaining 4 puzzle pieces.
   c. Check: Is the volume 27 cubic cm ?

To resize without changing its shape: Select it. Hold **Shift**. Drag one of the corners to make it smaller or larger.

Different ways to write the volume of a cube with side length 1 cm: 1 cubic cm, and 1 cm³

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Materials

- Tinkercad, Tinkercad shortcut keys, pre-made file with three Soma Cube pieces
- Linking cubes.

Understand new design requirements — 5 min

Students read the new design requirements.
Ask how they will help with printing.

Get ready to design: measurement — 10 min

Students review cm and mm by determining the measurements of the puzzle they will design.

Answers

Q1. *3 cm*

Q2. *10mm*

Q3. *Students may*

- *Figure out that* 3^3 *is equal to 27.*
- *Say that a side length of soma cube is made up of three 1 cm cubes.*

Make all the pieces — 40 min

Students can find the file by searching for "MPACTDylanGivenSoma" when they are in Tinkercad.

Q4a, b, c:

Circulate and recommend these strategies if students are stuck:

- **Create 27 individual unit cubes and try to put them together.**
- **Change measurements by typing in values, rather than dragging.**
- **Create one 2-unit-cube part, then duplicate it to create other pieces to make the process efficient. CT practice: *decomposition.***
- **To check if pieces fit together for printing, group them, and see if they become a whole piece with no spaces.**

Good place to end the lesson or transition

Reflect and Celebrate

10 min

Make it a celebration! Students have learned a lot about Tinkercad.

Then you can ask:

- Were there places you got stuck when using the CAD?
- Was there a time when you realized you hadn't met a requirement?
- Did you have any other problems?
- What did you do to get unstuck/meet requirements/solve problems?

Mathematics Exploration: Find patterns in volumes of cubes with fractional side lengths

20 min

Students solve problems involving cubes with fractional dimensions.

Students see a relationship between the fractional side length and the volume of a cube.

Answers

Q5. *1* in^3

Q6. Students can draw the ½ inch cubes within the 1 inch cube if they cannot visualize them.

Answer: 8

Q7. Ask students if the volume of the ½ inch cube should be greater or less than the inch cube. How can they be sure?

Answer:1/8 in^3

Q8.Students may reason in different ways. *Answers:*

First way: divide the volume of the large cube by 8.

Second way: cube ½

Q9. Suggest that students use the same methods that they used for ½. *(creating algorithms)*

a. Answer:27

b. Answer: 1/27 in^3

c. First way: divide the volume of the large cube by 27.

Second way: find the cube of 1/3

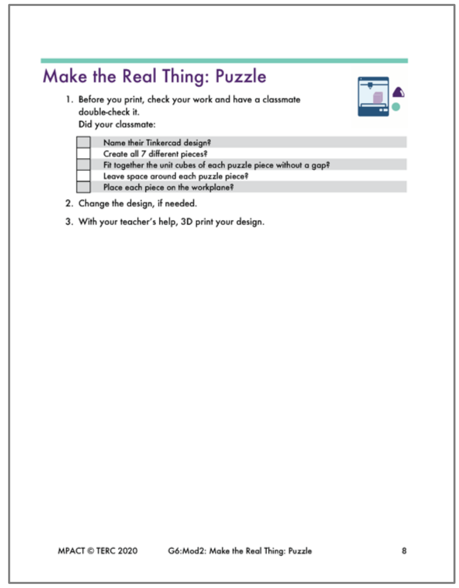
Ask students to describe and explain the patterns they see across Q5–9.

Ask your mentor

10 min

- Help students communicate their design experience with mentors and learn about how professionals solve problems in the face of challenges.

Make the Real Thing: Puzzle 15 min



Make the Real Thing: Puzzle

1. Before you print, check your work and have a classmate double-check it.
   Did your classmate:
   - [ ] Name their Tinkercad design?
   - [ ] Create all 7 different pieces?
   - [ ] Fit together the unit cubes of each puzzle piece without a gap?
   - [ ] Leave space around each puzzle piece?
   - [ ] Place each piece on the workplane?
2. Change the design, if needed.
3. With your teacher's help, 3D print your design.

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Goals

Check design and pick one to print.

Spatial Reasoning

Rotate physically and mentally a 3D shape in order to solve problems.

Computational Thinking

Check work and modify designs so they print properly (*debugging*).

Materials

- 3D Printer

Get ready to print

15 min

Make groups of four. Each group of four chooses one model of the pieces to print.

Before students do the Print Check, model it for them with one design you know will work, and one that won't.

Students use the checklist to see if they are ready to print.

- Note: in addition to Print Check items, "overhangs" can slow down printing by requiring supports to be printed—or could make the piece collapse.

Print one puzzle for each group.

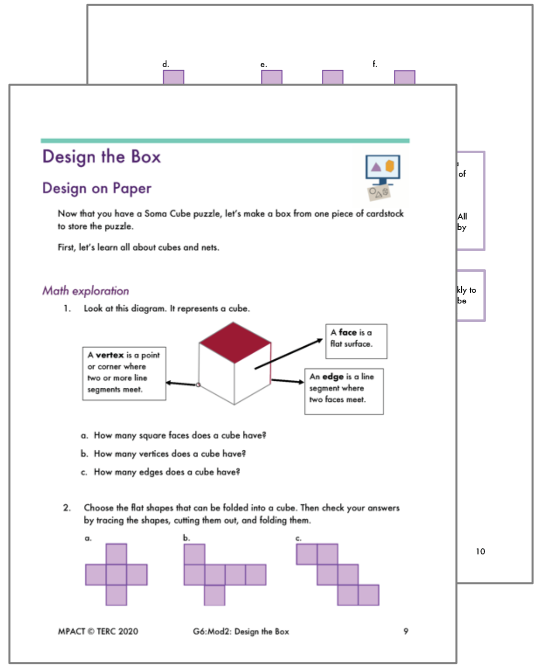
Print out more for each student to have one later, if you have time.

- Using the printer software, place as many pieces as you can on the print bed, following the Print Check rules.
- Find out the estimated time to print from the printer software.
- If this is more time than you have, send some of the designs to the MPACT print farm.

Continue with the following activities while waiting for the cubes to be printed.

Design the box 30 min

Goals



Create a pattern for a box to hold their Soma Cube Puzzle, considering the net of a cube, through the *Math Exploration.*

Math

Understand the attributes of a cube and use them in solving math problems.

Use nets of cubes.

Spatial Reasoning

Rotate physically and mentally a 3D shape in order to solve problems.

Imagine how a 2D shape can fold to make a 3D shape.

Computational Thinking

Check work and modify designs so they print properly (*debugging*).

Use nets to understand the components of a design and how the components interact (*decomposition*).

Materials

- Making kit: Cardstock, tape, markers
- Scrap paper

Mathematics Exploration: Understand parts of a cube

30 min

Q1. Students must count unseen parts to get the answers.

Circulate:

- Ask:
  - Are there faces on the box you can't see?
  - What would the cube look like if you turned it around?
- Offer a linking cube to help them interpret the drawing.

Q1. *Answers: a. 6 faces; b. 8 vertices; c.12 edges.*

Pass out scrap paper.

Q2. Students must determine which shapes are really nets of a cube.

Circulate and advise students to try copying and cutting out the nets if they are having trouble visualizing the folded up net. Help them analyze nets. For example, for *a*, it has only 5 squares and a cube needs 6 faces. For b, consider rolling a net from left to right. Then the middle four match the top, right, bottom and left face. And the top and bottom on the net make back and front face of the cube.

Answers: b, c, f

Q3. Students make a prototype of their box out of scrap paper by making a different net of a cube.

Have students make a box of the right size with scrap paper.

- Students may do this by cutting out individual pieces or cutting out a net, tabs or no tabs.
- Tell students to imagine unwrapping a box and cutting along its edges, to envision the net.
- Suggest that students use one of the nets above and modify it to create their own net.

Q4. *Possible student strategies:*

- *I modified the net in the previous problem by moving the top left square to the right. It still stays in the same position on the cube.*
- *I imagined it as if peeling the skin off a cube.*

Make the Real Thing: Box 75 min + print time

Goals

Make a box using a net, which can hold their puzzle.

Math

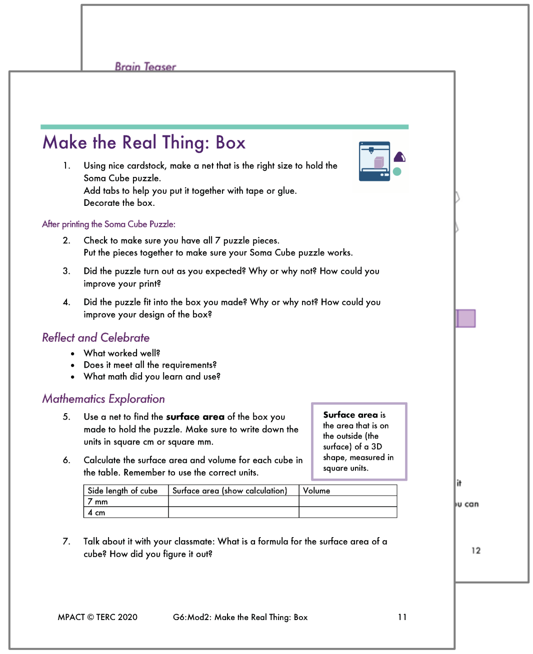
Find surface areas of shapes.
Derive the formula for the surface area of a cube.

Spatial Reasoning

Mentally rotate complex 3D shapes.
Imagine if possible nets make a cube.

Computational Thinking

Create efficient counting strategies for finding surface area. (*Algorithmic thinking*).



Brain Teaser

Make the Real Thing: Box

1. Using nice cardstock, make a net that is the right size to hold the Soma Cube puzzle.
   Add tabs to help you put it together with tape or glue.
   Decorate the box.

After printing the Soma Cube Puzzle:

2. Check to make sure you have all 7 puzzle pieces.
   Put the pieces together to make sure your Soma Cube puzzle works.
3. Did the puzzle turn out as you expected? Why or why not? How could you improve your print?
4. Did the puzzle fit into the box you made? Why or why not? How could you improve your design of the box?

Reflect and Celebrate

- What worked well?
- Does it meet all the requirements?
- What math did you learn and use?

Mathematics Exploration

5. Use a net to find the **surface area** of the box you made to hold the puzzle. Make sure to write down the units in square cm or square mm.
6. Calculate the surface area and volume for each cube in the table. Remember to use the correct units.

Surface area is the area that is on the outside (the surface) of a 3D shape, measured in square units.

Side length of cube	Surface area (show calculation)	Volume
7 mm		
4 cm		

7. Talk about it with your classmate: What is a formula for the surface area of a cube? How did you figure it out?

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Materials

Making kit: cardstock, tape, markers

Make the real box for the puzzle 30 min

Pass out cardstock, tape, markers.

Q1. **While students are working on their boxes, circulate:**

- **Elicit that the box must be a bit bigger than the Soma Cube in order for it to fit inside. Ask how this affects the net.**
- **If students add tabs to help fasten the cube together, ask if they are part of the net of a cube (they are not).**
- **Encourage students to test and refine their boxes if necessary and there is sufficient time.**

Q2–4. Pass out printed puzzles. These questions are about the printed Soma Cube, once they get it.

Have students check to see whether their boxes hold their puzzles.

Ask why or why not the puzzles fit into the boxes.

Reflect and Celebrate 10 min

Use the prompts in the student handout.

Ask students what may have gone wrong with some prints and how they could be avoided in the future. For example:

- The overhangs were too big, so pieces collapsed.
- Not all the unit cubes were connected, so one piece printed as two.

Mathematics Exploration: Use nets to find surfaces areas

20 min

With whole class, review the definition of surface area. Have students work in pairs to complete the questions. Encourage students to talk about question 3 with their partner.

Q5. **While students make their net and calculate surface area, circulate:**

- **Suggest students use the pattern for their box, as it was a net.**
- **Remind students to use units; ask them about which kind of units are used for surface area (square units).**

Q6. *Answers:*

- *For 7mm, SA = 6 faces x 49 square mm = 294 square mm, V = 7x7x7 = 343 cubic mm*
- *For 4cm, SA=6 faces x 16 square cm = 96 square cm, V = 64 cubic cm*

Q7. *Possible answer:*

SA of a cube = number of faces x area of a square face = 6 x L x L, where L is the side length of a cube.

Encourage students to explain other ways to find surface area. For example, they can find surface area by saying that all faces are the exact same sized square and each area is, for example, 8 square cm. There are 6 faces. So, 6 x 8 cm^2 is 48 cm^2.

Remind students that area is a measurement in two dimensions, so square units are usually used.

Brain Teaser: Reason spatially with 3D shapes

5 min

These more complex geometric shapes are intended to challenge students and enhance their spatial reasoning skills. Q8 is an item from a classic mental rotation task modified from Peters & Battista (2008).

Q8. The students mentally rotate the solid shapes. They may build the shapes from linking cubes to help them develop mental rotation skills.

Q9. *Possible answer: I chose those pieces because each had an L shape with a smaller L hanging off of it in the same direction. Mentally I rotated the target shape and it matched with A and C.*

Q10. Circulate:

- **Ask, what would happen if you folded up these two sides? And two more? And two more? Does it make a cube?**
- **You can suggest students copy and cut out the shapes if they are having troubles visualizing the cubes.**

Answers: a, d

Try this GeoGebra resource: https://www.geogebra.org/m/FQXxW67R

Ask Your Mentor

10 min

Help students communicate their design experience with mentors and learn about how professionals solve problems in the face of challenges.