Teachers' Beliefs and Teaching Mathematics with Manipulatives

Nahid Golafshani
Laurentian University

Abstract

To promote the implementation of manipulatives into mathematics instruction, this research project examined how the instructional practices of four Grade 9 applied mathematics teachers related to their beliefs about the use of manipulatives in teaching mathematics, its effects on students' learning, and enabling and disabling factors. Teacher questionnaire and observation field notes were used to collect data concerning the teachers' beliefs and possible changes in their beliefs about the use of manipulatives after participating in training and practising their pilot lesson plans over the course of more than 20 weeks. The results have indicated that the teachers showed more desire to use manipulatives in their teaching as the project and the training were progressing. Teachers have also reported that the use of manipulatives had some direct effect on the students' learning, in particular, on the struggling students; however, its major effect was on creating an environment that facilitated students' learning through different methods of engagement.

Keywords: Mathematics, teaching, manipulatives, engagement.

Précis

Afin de promouvoir la mise en œuvre du matériel de manipulation dans l'enseignement des mathématiques, ce projet de recherche a permis d'examiner les pratiques d'enseignement de quatre enseignants de mathématiques appliquées de 9e année, quant à leur usage du matériel de manipulation dans l'enseignement des mathématiques, et l'impact de ce matériel sur l'apprentissage des élèves. Deux instruments ont servi à la cueillette des données : le Questionnaire de l'enseignant et des notes d'observation sur le terrain. Ces méthodes ont été utilisées pour recueillir des données sur l'efficacité avec laquelle les enseignants incorporent le matériel de manipulation dans leurs pratiques d'enseignement, après avoir suivi une formation et pratiqué leurs plans de cours pilote pendant plus de vingt semaines, ainsi que sur l'effet qu'a l'utilisation du matériel de manipulation sur l'apprentissage de leurs élèves. Les résultats ont démontré que les enseignants ont été en mesure d'intégrer dans leurs plans de cours quotidiens le matériel de manipulation qu'ils ont utilisé en pratiquant la prestation d'enseignement des cours modèles. Les enseignants ont indiqué avoir utilisé plus de matériel de manipulation virtuel que de matériel physique, à la suite du projet. L'utilisation du matériel de manipulation dans les classes de mathématiques observées a eu des effets directs sur l'apprentissage des élèves, en particulier, sur les élèves en difficulté. Toutefois, son principal effet fut la création d'un environnement facilitant l'apprentissage des apprenants par le biais de différentes méthodes d'engagement. L'apprentissage des mathématiques s'est fait par voie de partage des connaissances entre les élèves.

Introduction

The pedagogical context of this study is to promote effective learning through helpful teaching techniques using manipulatives in mathematics classrooms. Conventional professional development for teachers of mathematics is based on the idea that all students should build their own mathematical knowledge and acquire conceptual understanding. There has been an increasing interest in effective mathematics instructional strategies that allow, "all students to do, see, hear, and touch mathematics in a profound and meaningful way" (TIPS, 2004, p. 28) among educational agencies in Canadian government and in particular in the province of Ontario.

Mathematics reformers and educators suggest that using a variety of teaching strategies will lead to effective teaching and learning. They also believe that effective teaching includes teaching for understanding, teaching better mathematics, teaching mathematics better (Friesen, 2005), and reversing mathematics misconceptions (Green, Flowers, & Piel, 2008). That means the teacher should not be a transmitter of knowledge but should instead act as a facilitator to the construction of knowledge for all learners. The fact that every classroom consists of students with different levels of ability to understand mathematical concepts means that teachers should focus on using multiple teaching strategies so that all students can benefit. Thus, there is a need to investigate the effects of different teaching strategies such as the use of manipulatives in teaching mathematics. This study will broaden this line of investigation.

Background on Manipulatives

Teaching mathematics through manipulatives is not new. It is actually based on traditional techniques, using beans or counters, which have been replaced by more advanced items such as linking cubes, fraction circles, and technologies that are used in today's mathematics classrooms. The use of manipulatives can be traced to Piaget's (1952) suggestion that children cannot comprehend abstract math through explanations and lectures; therefore, they need experiences with models and instruments in order to grasp the mathematical concepts. Many researchers suggest the use of manipulatives in solving math problems (Lindorth, 2005; Reimer & Moyer, 2005; Sherman & Bisanz, 2009) as a result of their positive learning effects on all students, and in particular on struggling students. Depending on the student's identified learning ability, teachers may use appropriate manipulative objects to bring math to life and to make the invisible math concepts visible.

Today's Classrooms

In today's mathematics classroom, using manipulatives is gradually becoming part of the instruction because either some teachers are beginning to notice its effects on student learning, or the curriculum is demanding their use. Studies show that teachers have been gradually incorporating both virtual and physical manipulatives; however, there are neither theoretical nor empirical grounds for predicting which one will be more effective in mathematics teaching and learning (Lindroth, 2005; Triona & Klahr, 2003). Current reform thinking in mathematics education indicates that the use of manipulatives in mathematics teaching will increase students' conceptual knowledge and improve their attitudes toward mathematics learning (Reimer & Moyer, 2005).

There is no doubt that everybody believes mathematics is important; however, many students have poor math skills, which indicates that changes are needed in the methodology of teaching mathematics. Related to the changes, there is a growing consensus around the use of manipulatives in instructional practices.

Much of the literature on teaching and learning with manipulatives has been positive and supportive. The literature states that manipulatives are well suited for mainstream mathematics instruction and in the professional teacher workshops because they give visual representations to abstract mathematical ideas. A few studies discuss some critical but constructive concerns about the use of manipulatives, as well as how teachers perceive the use of manipulatives in mathematics classrooms. Ball (1992) stated that manipulative materials are neither magic nor carriers of meaning or insight by themselves, but they are significant, "as potential tools [and] as a function of task for which a teacher conceives them being used" (Moyer, 2001, p. 176). If teachers perceive that the use of manipulatives in teaching math is just for fun but not necessary, or that it is simply a diversion in classrooms where teachers are not able to represent mathematics concepts themselves (Moyer, 2001), then students will learn to use manipulatives in an instrumental (step-by-step procedures) and/or rote learning manner (Hiebert & Wearne, 1992).

Thus, without considering teachers' beliefs about the use of concrete materials and its effects on learning, the use of manipulatives in classrooms will not promote constructive learning and will only be an add-on to the reform package. At the heart of any reforms is the issue of changing teachers' beliefs to concur with the curriculum revisions. The critical relationship between the beliefs of teachers regarding implementation of

reform efforts and instructional decisions needs more attention. Some research studies have been conducted regarding teachers' beliefs and practice (Crawley and Salyer, 1995; Hashweh, 1996), but more recent studies are needed to examine teachers' views (attitudes, beliefs) about teaching mathematics with either virtual or physical manipulatives, which is the line of inquiry of this study.

Teachers' Attitudes Toward Teaching and Learning with Manipulatives

Teachers' attitudes on the use of manipulatives in teaching are strongly associated with their beliefs about how effective manipulatives are on students' learning. A study suggests that teachers using manipulatives in their classroom develop self-efficacy and self-confidence; thus, they become less anxious about teaching mathematics (Vinson, 2001). A teacher's affective domain will direct her or his choice of teaching strategy in the classroom; then whether the choice is appropriate or not, the teacher's strategy will influence student learning and student attitudes about mathematics and mathematics learning. The association between teachers' beliefs about a new teaching strategy and their beliefs about effective student learning is so strong that one implies the other. It is like asking the question, "Which came first, the chicken or the egg?" This indicates that teachers would consider the use of manipulatives if, and only if, they believe manipulatives have positive effects on their students' learning ability. For example, teachers who view manipulatives as time wasting or secondary to the serious work of learning mathematics will inadvertently encourage their students to use these materials for play (Moyer & Jones, 2004), and as a result no learning will occur. Changing teachers' beliefs about manipulatives and encouraging them to bring manipulatives into their instructional practices seem to be related to how they view the usefulness of manipulatives in students' learning. Thus, the identification of ways to encourage and help teachers to make significant changes in their beliefs is very important because these beliefs will contribute to the success or failure of any changes made to their teaching practices that are consistent with the math curriculum reform.

Statement of the Problems and Our Hopes

Despite the recommendations of many curriculum standards regarding concrete exploration and representation of mathematical concepts, the traditional lecture format of teaching is still prevalent in our school system (Mura, 1995). Smith (1996) states that teachers in our schools teach mathematics as a fixed set of facts and procedures for computing numerical and symbolic expressions to find determinant "answers." Assuming that teachers have recently been using manipulatives in their mathematics classrooms, for effective learning, then why are our classrooms still mostly focusing on getting "correct answers to the problems at the end of the chapter, or on a work sheet or test?"

> [That] is no guarantee of problem-solving ability in real world situations. That ought not to be surprising, given that in traditional mathematics teaching, problems lack interesting real-world contexts; that memorization of algorithms by drill is not matched by learning when to use them; and that students receive little, if any, help in learning how to judge how good their answers are. (American Association for the Advancement of Science, 1993, cited in Reed, 2005, p. 56)

Assuming that the traditional way of teaching will teach students how to do math, why are educators still concerned about students' math results on provincial examinations and their achievement on international assessments? If many teachers are using manipulatives effectively, why do so many students either lack basic mathematical knowledge or are unable to transfer their mathematical knowledge to other fields and practice?

A reason for such issues may possibly be the disconnection between educational recommendations and teachers' beliefs. To address the concern around these issues, this study investigates how teachers incorporate the use of manipulatives into their teaching by focusing on teachers' beliefs about the use of manipulatives in teaching and about its affect on students' learning. The study finds teachers implementing issues and concerns regarding the use of manipulatives in mathematics classrooms. In light of these specific considerations, this investigation sought to answer the following research questions:

1. What are the teachers' attitudes about the use of manipulatives in teaching (focusing on notions of self-efficacy in teaching with manipulatives)?
2. What are the teachers' attitudes toward the effects of manipulatives on students learning?
3. What implementation issues are there regarding manipulatives (particularly enabling and disabling factors)?

Methods

The participants for this study were four Grade 9 mathematics teachers and their students. The study took place in the teachers' mathematics classrooms while they were teaching the various topics in Grade 9 mathematics to students with diverse learning abilities. The study began in mid-October and continued over a period of 21 weeks.

It was through various meetings with the teachers, an experienced district math curriculum coordinator, and a district literacy strategist for mathematics that the planning to launch a successful study for Grade 9 mathematics was decided upon. The teachers were provided with a variety of resources such as physical and virtual manipulatives, a literacy toolkit for math, and the opportunity for professional learning, training, and dialogue to plan five lessons with manipulatives. For each lesson plan, the teachers focused on planning effective lessons that included the key elements such as meeting the set learning goals, engaging students, and helping close the gap for struggling learners.

In January, the teachers were ready to deliver the lessons as pilot lessons to their classrooms. During the delivery of the pilot lessons, some visiting teachers, the project math leader, and the researcher were present as observers. With each pilot lesson, pre- and post-discussions were held with visiting teachers and the participating teacher about the planning of the lessons and why the teachers chose their strategies and handled them the way they did in the classroom. For example, in the pre-discussion period, the teacher described the lesson with references to the use of physical or virtual manipulatives, connecting to real-life contexts and math learning goals. In the post-discussion period, the teacher reflected on the lesson delivered and identified some other strategies that could be used for different classroom situations. They also decided if modifications or revisions to the lesson were needed to achieve a greater effect on student learning. The aim was to create model lessons that would be made available to the district's Grade 9 math teachers.

Data Collection

For the purpose of the study, a teacher questionnaire and an observation sheet were developed. The questionnaire was self-administered and it was given to the participating teachers twice, pre- and post-lessons. The first set of questionnaire data was considered as a pre-test and was administered before the launch of the pilot lessons. The second set

of questionnaire data was considered a post-test and was administered at the end of the study. The questionnaire had statements related to the research questions. The statements were generated following extensive discussion with the district math curriculum coordinator about the study and about a review of relevant literature, and discussion was guided by Golafshani (2005) and Ross, McDougall, & Hogaboam-Gray (2001).

The researcher also collected data through observations of the teachers in the classrooms. The observation sheets were constructed to focus on the use of manipulatives in the classrooms, the availability of manipulatives in the classrooms, and teacher's understanding of and comfort with using manipulatives. The structure of the observation sheets was guided by Center for Applied Research and Educational Improvement (2002) and Shaw and Hatfield (1996).

Data Analysis

A qualitative approach seeks to understand phenomena in context-specific settings, such as a "real-world setting [where] the researcher does not attempt to manipulate the phenomenon of interest" (Patton, 2001, p. 39), and it "produces findings arrived from real-world settings" (Golafshani, 2003, p. 600) where the "phenomenon of interest unfolds naturally" (Patton, 2001, p. 39). In this study, the teachers' answers to the questionnaires and the notes from the observation sheets were analyzed qualitatively.

For the data analysis, an identification number—PMT1, PMT2, PMT3, PMT4—was assigned to each participating math teacher to secure their anonymous identity. This identification number is used throughout this report.

Findings

The findings are focused on the responses to the research questions. The questions pertain to the teachers' self-reports to the questionnaire statements and to the researcher's observation notes. The questionnaire statements deal with affective domain, factors to be considered in using manipulatives to teach mathematics, and the teachers' answers to the open questions. The observation notes deal with what occured in the classrooms related to the research questions.

Teachers' Views About Teaching with Manipulatives

The first set of statements in the questionnaire was related to the first research question that was concerned with the teachers' beliefs about the use of manipulatives in teaching mathematics. The teachers were required to choose one of the options—strongly disagree, disagree, neutral, agree, or strongly agree—for each statement in the questionnaire. The questionnaire was administered for the pre- and post-tests. Their responses to each statement were compared. The analysis of the comparison has revealed that teachers held views about three central aspects of teaching with manipulatives:

1. About the use of manipulatives
2. About no-use of manipulatives
3. About the use of physical manipulatives over virtual manipulatives.

The strengths of their agreement or disagreement with the above views in both pre- and post-tests are shown in Table 1.

Table 1: Teachers' Views About Teaching With Manipulatives

	Strength	
Central Views	**Pre-test**	**Post-test**
Use of manipulatives	Agree	Strongly agree
No use of manipulatives	Disagree	Disagree
Use of physical manipulatives over virtual manipulatives	Agree	Neutral

As shown in Table 1, the teachers have suggested the use of manipulatives in teaching mathematics in the pre-test; however, they showed more interest in the use of manipulatives in the post-test. The teachers disagreed with no use of manipulatives in teaching from the beginning to the end of the project. Interestingly, the teachers showed a stronger preference for the use of physical manipulatives over virtual manipulatives in the pre-test than they showed in the post-test. In other words, it showed teachers' preference for physical manipulatives reduced to neutral in the post-test.

Teachers' Views About Students' Learning With Manipulatives

The second set of statements in the questionnaire was related to the second research question that was concerned with teachers' attitudes toward the effects of manipulatives on students' learning. The strengths of teachers' agreements or disagreements with each statement were compared. The analysis of the comparison has revealed that the teachers held the following views about three aspects of learning with manipulatives:

1. Learning with manipulatives
2. Learning without manipulatives
3. Whether learning with physical manipulatives is more effective than with virtual manipulatives.

The strengths of their agreement to or disagreement with the above views in both pre- and post-tests are shown in Table 2.

Table 2: Teachers Views About Students Learning With Manipulatives

	Strength	
Central Views	**Pre-test**	**Post-test**
Learning of manipulatives	Strongly agree	Strongly agree
Learning without manipulatives	Strongly disagree	Strongly disagree
Learning with physical manipulatives versus virtual manipulatives	Agree	Disagree

As shown in Table 2, throughout the research process the teachers showed a strong agreement with the positive affects of manipulatives on students' learning and a strong disagreement that there was no effect on learning without the use of manipulatives. The teachers agreed with the idea that physical manipulatives are more effective than virtual manipulatives on students' learning in the pre-test but they changed their views in the post-test. The change is aligned with the teachers' change of view and it substantiates the teachers' strong agreement to the teaching with virtual manipulatives in the post-test in Table 1. Thus, it could be interpreted that teachers changed their views toward using virtual manipulatives in teaching and learning at the end of the project because they became more familiar or comfortable with manipulatives.

Factors to be Considered in Using Manipulatives to Teach Math

In this part of the questionnaire, the teachers were given a list of factors to choose from, which are commonly considered when implementing manipulatives in mathematics classrooms. The list of factors ranged from classroom control to guideline mandates. As shown in Table 3, the teachers considered more factors in the post-test than the pre-test. All the teachers expressed concerns about classroom control, availability of a class set of manipulatives, time factors, and teacher competency and the management of clean up of the manipulatives in both pre- and post-tests. In the post-test, the teachers stated that the students benefited and enjoyed manipulatives in the lessons. None of the teachers indicated that they have considered using manipulatives because it was the school's policy.

Table 3: Factors in Using Manipulatives

Most Prevalent Factors	
Pre-test	**Post-test**
Difficulty of classroom control and noise level	Difficulty of classroom control and noise level
Lack of availability of manipulatives	Lack of availability of manipulatives
Time factors	Time factors
Lack of teacher's competency	Lack of teacher's competency
Difficulty with the clean up of the manipulatives	Difficulty with the clean up of the manipulatives
	Benefits student learning
	Students enjoy manipulatives

The results of the prevalent factors will be discussed along with the results found from the open questions of the questionnaire.

Open Questions

The last part of the questionnaire had two open questions for the teachers to respond to. Their answers were analyzed and then categorized as themes. The researcher used the

strategy, constant comparison, to construct the categories of the themes for the open comments. The first and second questions were used to identify the enabling/supporting and disabling/hindering factors to the teachers' desired way of teaching with manipulatives.

First Open Question: Themes of Disabling Factors

The first open question in the questionnaire was, "What prevents teachers from teaching with manipulatives the way they like to teach?" This question helps reveal some of the teachers' implementation issues regarding manipulatives, issues that might be related to the origin of their beliefs. Table 4, below, indicates the themes of the disabling factors that grew out of teachers' answers to the first open question in the questionnaire in pre- and post-tests. Elaboration on the disabling factors in the pre- and post-tests is presented by providing some quotes from the teachers' written answers to the first open question.

Table 4: Disabling Factors

Strength	
Pre-test	**Post-test**
Lack of confidence	Low comfort level
Lack of time to practise	Lack of time to prepare
Lack of space	Lack of own classroom
Lack of a class set of manipulatives	Lack of knowledge of variety of uses
Difficulty with classroom management	Difficulty with classroom management

In the pre-test, all the teachers reported that their lack of confidence in using manipulatives was problematic and that they had "not enough confidence to use certain manipulatives." All the teachers expressed a lack of time to practise with students. PMT4 mentioned two issues related to time. The teacher stated, "It takes time to learn how to use [manipulatives] and what lesson to use them in . . . [and it] takes more class time to introduce a concept using manipulatives," which showed that, for the teacher, the concern about time was an important factor in the pre-test. PMT3 was concerned about small classrooms, which often impeded the teacher from having "good group work situations and interactions." Obviously, the lack of space was not a quick fix, and the teacher raised

concern again in the post-test by stating, "Sometimes the class itself will prevent me from using the manipulatives. Not having your own classroom really puts a strain on accessing and organizing the manipulatives." PMT2 did not have enough manipulatives for the entire class in the pre-test but the problem was probably solved in the post-test because all the teachers received several versions of different manipulative packs from the board. The fear of disruption in the classroom while using manipulatives and the issue of classroom management stayed with the teachers throughout the project. PMT4 reported that manipulatives made the classroom management more difficult.

Lack of enough time for delivering the lesson with manipulatives in the pre-test was replaced by another time-related disabling factor in the post-test: The lack of time to prepare and plan the lessons with manipulatives. PMT3 stated, "I need to put some extra planning in place when I want to use manipulatives in order for it to be successful." PMT1 explained that "manipulatives require a lot of extra time and planning prior to the actual class" and that was not always possible for her to do. The most reported factor —lack of confidence—was changed to the phrase, "low comfort level" by the teachers in the post-test. The need to acquire more knowledge of the variety of uses of a set of manipulatives in different lessons was added to the factors in the post-test.

Second Open Question: Themes of Enabling and Supporting Factors

The second open question in the questionnaire was "What kinds of supports will enable teachers to teach with manipulatives in order to achieve success with their students?" This question was also used to find out what the teachers wished for regarding success in the use of manipulatives in the pre-test and to see if they had obtained their wishes in the post-test. Table 5 indicates the themes of the enabling factors that grew out of teachers' answers to the second open question in the questionnaire in both the pre- and post-tests. Elaboration on these enabling factors in pre- and post-tests is presented by providing some quotes from the teachers' written answers to the second open question.

Table 5: Enabling and Supporting Factors

Themes	
Pre-test	**Post-test**
Availability of a class set of manipulatives	Availability of a class set of manipulatives
Specific training on when and how to use the manipulatives	Specific training on when and how to use the manipulatives
Support from administration	Support from administration
Specific training on the use of manipulatives with the Grade 9 curriculum	Presence of a student helper
Administrative support in training for and purchasing the manipulatives	Additional training
Methods for keeping all students on track (especially students who missed a critical class)	

Interestingly, some of the enabling factors for both the pre- and post-tests were the same but in two different contexts. In the pre-test, the teachers reflected on the ideas of the enabling factors that would facilitate teaching the way they would like to teach: effectively, using manipulatives, through manipulative availability, with proper training, and with administrative support. In the post-test, the teachers reported the same enabling factors because the teachers realized or became aware of their needs or supports for effective and successful teaching experiences with manipulatives in the classrooms during their limited time training with manipulatives for this study. For example, one teacher mentioned the school principal's support by giving the teacher release time to attend the teachers' training sessions. In fact, the teachers' success in teaching with manipulatives was very much dependent on the different supports they received from different sources.

The teachers emphasized their need for different kinds of training related to manipulatives. Whether the training was concerned with instruction and manipulatives, or management and manipulatives, the teachers asked for it in both the pre- and post-tests. In fact, they asked for more knowledge in using manipulatives in their responses to each open question. For example, PMT1 expressed a need for more training on "specific uses of certain manipulatives" in both the pre- and post-tests. PMT4 expressed need for a student helper in the class "to help with handing out and collecting the supplies," which would help alleviate the classroom disruption.

Interpretation of the Findings and Literature Sources

The analysis of the answers to the research questions revealed that the teachers had positive attitudes toward the use of manipulatives, and their agreement to use manipulatives increased at the end of the study. The teachers' positive view in the pre-test could be because the teachers experienced some success in their classroom after they previously attended workshops, which were in line with and organized by the newly revised mathematics curriculum of the Ontario Ministry of Education (2005) that encourages teachers to "select and use concrete learning [and teaching] tools to make models of mathematical ideas" (p. 18). The teachers' previous experience with manipulatives was evident by PMT1 who added fraction circles, and algebra tiles to the list of manipulative aids in the questionnaire in the pre-test. The change in the teachers' view about the use of manipulatives in the post-test was demonstrated when PMT1 added three more manipulative aids to the list. The change in the teachers' views about the use of manipulatives in the post-test was not the only view affected by their training and practicing during the study; their preferences for the types of manipulatives have also changed. Teachers' familiarity with virtual manipulatives did increase during the study, which led to the some but not complete agreement about the effectiveness of physical over virtual manipulatives at the end of the study. Perhaps, having access to more software could initiate such a change since PMT1 mentioned in the post-test about having unlimited access to the software, Geometer's Sketchpad (GSP) and Fathom. Yet, they indicated they needed more training in the area of virtual manipulatives.

The teachers showed agreement about the importance of students' learning with manipulatives; however, during an informal interview after the post-test, the teachers indicated a slightly less agreement and even some mixed views about the students learning with manipulatives.

A reason for the teachers' mixed views about students learning with manipulatives was that some students did not need to use manipulatives to learn math. For example, PMT4 did not completely agree with the use of manipulatives because most of the students were abstract learners and the teacher did not feel the need to use manipulatives in the classroom. On the other hand, PMT2 was not concerned about his/her students understanding abstract mathematics; the teacher was concerned about whether the students could relate the abstract content to some applications with the use of manipulatves.

Teachers have always believed that the use of manipulatives could help students to understand mathematics but it has been considered an option. Now, this study reveals that some teachers may see the use of manipulatives in teaching as an option but others may see it as a necessity. If the teachers believe that the students should understand the math content through its connection to real-life situations, then the use of manipulatives would be considered necessary in teaching mathematics. For example, one day PMT1 asked the students to convert 2.07 to a percentage. The teacher observed the student who could do abstract math quickly gave the right answer of 207% but had no understanding of its practical use in an everyday life situation because all previous examples yielded percentages of less than 100. Just a few minutes later, the same student asked why the answer was more than 100%. What the student wanted to know was what it meant to have 207% of something. The situation showed the student's ability to solve a math problem numerically but did not have the ability to see its concrete connections to the real-life situation until the teacher used a linking cube to show the concept visually.

This study shows that whether teachers view manipulatives as optional or necessary tools in teaching math, they definitely believe that manipulatives help both visual and abstract learners to acquire knowledge in different ways. For the abstract learners, the use of manipulatives offers them a new and practical venue of perceiving math problems along with calculations of some numbers. For the visual learners, the use of manipulatives has duel purposes, learning the concept and then writing the concept abstractly in math language.

The teachers' views about students learning more effectively with physical manipulatives than virtual changed from agree to disagree in the post-test. The change was more significant than the variation of their view from agree to neutral for the teaching with physical over virtual manipulatives. One reason for the sudden change of view could be because the teachers experienced effective learning by using virtual manipulatives (Smartboard lessons) while teaching the pilot lessons, or because they have acquired more knowledge about virtual manipulatives during their training and used their knowledge in their classroom, where they witnessed improvements in their students' learning. Another reason might be linked to resolving the classroom management and clean-up issues of physical manipulatives, when with virtual manipulatives such issues would almost disappear.

In the analysis of the teachers' responses to the enabling factors in using manipulatives, some of the responses—such as availability of a class set of manipulatives, specific training on when and how to use the manipulatives, and support from administration—in the pre-test stayed with the teachers throughout and after the study (post-test). This persistence could be attributed to the fact that the study and the training have been implemented over a relatively short period of time. It would be reasonable to state that if the training period were longer, then the teachers would have had more time for classroom practice with manipulatives. Even in the short period of the study, the teachers witnessed the students' benefit and enjoyment with the use of manipulatives in their classrooms. This could indicate two effects. One, the teachers' method of delivering the lessons with manipulatives was significantly changed during the practising period and became more effective. The second effect may indicate that students learned they could understand previously difficult mathematical concepts and experience an enjoyable mathematics class. The students felt pleased at being able to understand some difficult mathematical concepts they previously felt were unattainable. According to PMT3, the students have felt a sense of self-confidence that, in-turn, led to higher achievements in mathematics. The researcher has also detected the teachers' positive attitude toward using manipulatives, and the students' comprehension and enjoyment of the concepts in their mathematics classrooms during the post-test observation.

The participating teachers stated that they had access to more manipulatives during and after the study; however, PMT2 raised a concern about not having enough manipulatives for the entire classroom. The teacher had to ask the students to share some of the manipulative objects during group work, which created a time constraint and disruption in the classroom. This indicates that providing teachers with different manipulative aids will help teachers familiarize themselves with the varieties of available manipulatives; however, providing the teachers with a large enough set of manipulatives for their classrooms, not only will enhance students' learning but also will help teachers with their classroom management and time constraints when implementing manipulative lessons.

A comparative analysis of the responses revealed that the teachers did not use particular manipulative aids because either they were not available or they were not familiar with them. Such responses showed the connections among the enabling factors, such as the familiarity with manipulatives, availability of manipulatives, and the frequency of use. For example, before the start of the study, PMT1 did not have access to Base 10 ma-

terials, and he/she was not familiar with these materials. In contrast, another teacher has stated that since he/she became familiar with a manipulative aid, Motion Sensor, through various meetings, training, and debriefing sessions, he/she has been using the device for related topics in math.

The analysis of the disabling and enabling factors, which grew out of the teachers' responses to the first and second open questions, revealed the consistencies between their reported disabling and enabling factors. A close examination of the enabling factors could be interpreted as teachers addressing not only their need to become better at teaching with manipulatives but also providing some suggestions to alleviate the disabling factors in their teaching practices. The enabling and disabling factors are really two sides of the same coin and must be investigated together, rather than in isolation of each other. Figure 1 below shows how the enabling factors could overcome the effects of the disabling factors in the teachers' classrooms. The figure also shows the links among the themes of the two factors.

The association between the availability of manipulatives and teachers' familiarity was discussed earlier. The teachers' familiarity with the manipulative objects are embedded in their knowledge of the use of manipulatives. Without the knowledge of how to use specific manipulatives, the teachers could not implement available manipulative objects successfully in their classrooms. PMT1's lesson on parallel line theorem and corresponding angles using GSP was an example of a successful implementation of a revised lesson plan, which could not have been possible if the teacher had no knowledge of the software. Teaching is for understanding. But how can teachers who themselves do not have the knowledge of how to use manipulatives be expected to teach the concept convincingly to their students and help them to construct concrete knowledge from the abstract? The association between teachers' knowledge and students' learning is well documented (Goya, 2006; Liping Ma, 1999).

Studies show there is a strong association between teachers' knowledge and teachers' confidence especially for those who are committed to constructivist teaching (Ross, Hogaboam-Gray, & Hannay, 1999, Ross, McDougall, Hogaboam-Gray, & Bruce, 2002). These findings support our participating teachers' desire to gain more knowledge about the use of manipulatives and to increase their confidence level in using them in their classroom. For example, PMT2 stressed that knowledge about a software program in mathematics gave the teacher enough confidence to use that specific virtual manipulative

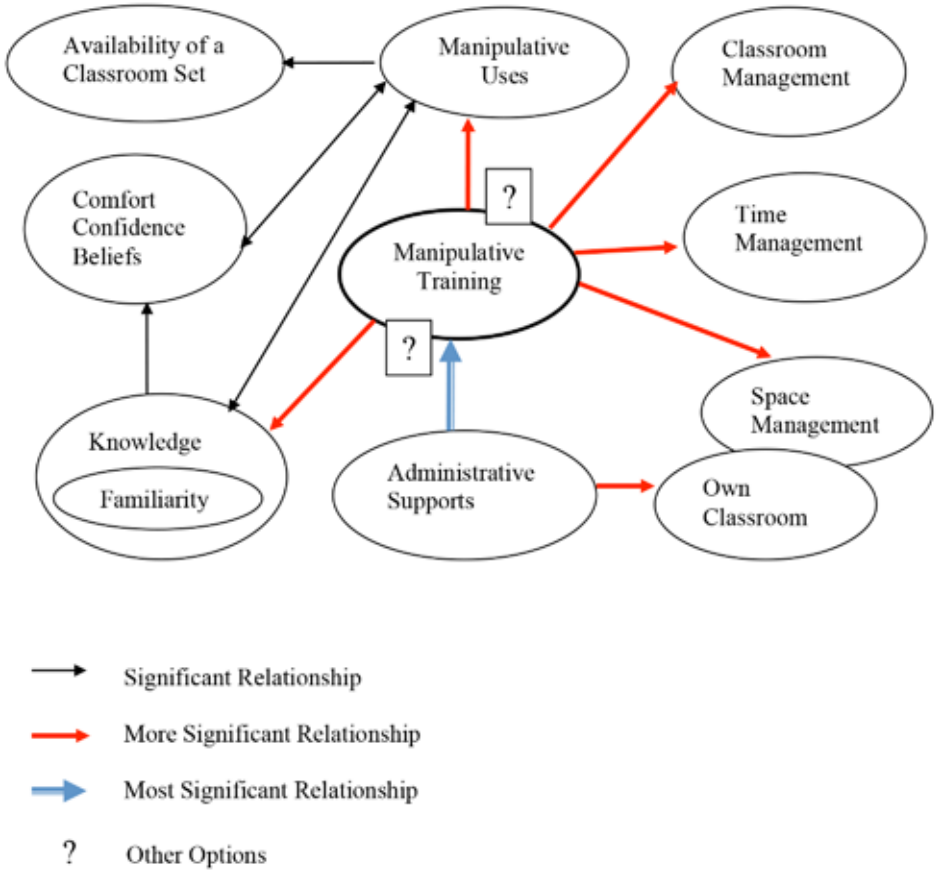

Figure 1: The Links Among the Disabling Factors, Enabling Factors and the Themes

in a variety of lessons with some different applications. However, without training, no knowledge would be gained and no confidence would be developed.

One of the enabling factors mentioned was the need for teacher training on how and when to use manipulatives. Figure 1 above shows that teacher training is the core of all links. It is the main contributing centre of different knowledge such as how and when to use manipulatives, classroom management techniques, time management skills, and space management methods when manipulatives are involved. But without administrative support, the training of our teachers would be hindered. This support could take many directions: providing teachers with classroom manipulative sets, organizing training sessions, and encouraging learning communities to increase teachers' knowledge. The support could also foster a positive attitude toward the use of manipulatives that in turn

would develop teachers' self-confidence. In this study, it has been observed that teachers who have more confidence in their teaching ability also have more success in promoting effective learning. At the heart of such support lies the need to organize and generate more professional learning communities to help teachers receive more training via their own teams of collaborating teachers.

The significant finding of the study has been about changing one set of teachers' beliefs to an alternative set of beliefs. The study shows that changing teachers' belief is difficult, but it is not impossible if the foundation of a solidified belief is revealed. It seemed that the disabling factors were contributing causes to the teachers' less favourable view about the use of manipulatives in the pre-test. Their views toward the use of manipulatives in the classroom gradually changed during the research project because the hindering factors were detected and then progressively reduced through the collaboration and support of professional and administrative communities.

On the Side

A few months after the completion of the research study, the researcher had the opportunity to observe the participating teachers in their classrooms and found that the they were incorporating manipulatives in their daily lesson plans effectively. However, they limited the manipulatives they used to those they had learned about during the research project. The comparative analysis of the collected data before, during, and after the study revealed that the teachers were continuing to use manipulatives in their classrooms and implementing the strategies they had learned during training. The participating teachers have also found another strategy to be productive and effective in the implementation process. The strategy that helped them to effectively incorporate the use of manipulatives in their classrooms was the math strategies, such as modelling, worksheets, word walls, reading, and writing, which were included in the training sessions.

Recommendations

There are unique challenges with teachers' use of manipulatives in their classroom instruction because of their multidimensional aspects and because of their demand on teachers. Teachers' knowledge of manipulatives, teachers' confidence with manipulatives,

time and classroom management are a few of the concerns found during this study. Finding answers to these challenges is difficult but not impossible. The following recommendations to educators, officials, and administrators are intended to ease the challenges for teachers and students when implementing manipulatives in mathematics classrooms.

- Expand this kind of research study, to identify if any other challenges may exist in teachers' use of manipulatives in math classrooms in order to take further appropriate actions.
- Provide more resources to increase not only the alignment between the content of the resource material and the use of manipulatives but also to give more choices to the teachers for finding strategies that will fit their classroom situations.
- Organize professional learning communities, like workshops, that promote and encourage the effective use of manipulatives.
- Recognize that teachers' teaching styles will become more constructivist through instructional practices with manipulatives.
- Use a collaborating team of mathematics teachers to provide professional learning communities to share different instructional strategies with teachers in training sessions.

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