TEACHING GUIDE

Module 9: Parallelism and Perpendicularity

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of the key concepts of parallel and perpendicular lines.

Performance Standard:

The learner is able to communicate mathematical thinking with coherence and clarity in solving real-life problems involving parallelism and perpendicularity using appropriate and accurate representations.

<table>
<tr><td rowspan="4">SUBJECT:
Grade 8 Mathematics
QUARTER:
Third Quarter
STRAND:
Geometry
TOPIC:
Parallellism and Perpendicularity
LESSON:
Parallellism and Perpendicularity</td><td colspan="2">LEARNING COMPETENCIES</td></tr>
<tr><td colspan="2">Illustrates parallel and perpendicular lines.
* Illustrates and proves properties of parallel lines cut by a transversal.
* Determines and proves the conditions under which lines and segments are parallel or perpendicular.
* Determines the conditions that make a quadrilateral a parallelogram and prove that a quadrilateral is a parallelogram.
* Uses properties to find measures of angles, sides, and other quantities involving parallelograms.</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:
Students will understand that:

The establishment of parallelism and perpendicularity of lines in real life may be done through deductive reasoning.
• Lines can be related in different ways such as parallel, perpendicular, intersecting and skew.
• Concepts of parallel and perpendicular lines can be determined deductively.
When two parallel lines are cut by a transversal, pairs of angles can be related as either congruent or supplementary.</td><td>ESSENTIAL QUESTION:
How can parallelism or perpendicularity of lines be established?
• Are you sure that the given lines are parallel?
• Are you sure that the given lines are perpendicular?</td></tr>
<tr><td colspan="2">TRANSFER GOAL:
Students will on their own use the key concepts of parallelism and perpendicularity of lines in solving real-life problems.</td></tr>
</table>

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to accomplish after with in this module.

a. Active participation in the different activities presented in the module will be evident.
b. Real-life problems involving parallelism and perpendicularity of lines will be solved.
c. Proofs are completed and devised.
d. A model of a book case that displays students' understanding on the concepts of parallelism and perpendicularity of lines will be created.

Assessment Map

<table>
<tr><th>TYPE</th><th>KNOWLEDGE</th><th>PROCESS/SKILLS</th><th>UNDERSTANDING</th><th>PRODUCT</th></tr>
<tr><td rowspan="3">Pre-Assessment/ Diagnostic</td><td>Pre–Test
Items Nos. 1-4</td><td>Pre–Test
Items Nos. 5-8</td><td>Pre–Test
Items Nos. 9-14</td><td>Pre–Test
Items Nos. 15-20</td></tr>
<tr><td colspan="3">Generalization Table
(Self-Knowledge)</td><td></td></tr>
<tr><td>A-R Guide
(Self-Knowledge)</td><td></td><td></td><td></td></tr>
<tr><td rowspan="5">Formative Assessment</td><td></td><td colspan="2">Worksheet in Writing Proofs
Activity 14
(Explanation, Application, Interpretation)</td><td></td></tr>
<tr><td></td><td colspan="2">Open-Ended Exercises in Proving
Activities 15 & 16
(Explanation, Application, Interpretation)</td><td></td></tr>
<tr><td colspan="3">Generalization Table
(Explanation, Interpretation, Self-Knowledge)</td><td></td></tr>
<tr><td>Think Twice!</td><td colspan="2">Draw Me Right!
(Application)</td><td></td></tr>
<tr><td></td><td colspan="2">Problem Solving Worksheets
Activities 6 & 13
(Application, Interpretation)</td><td>Design Analysis
(Application)</td></tr>
</table>

Summative Assessment	Generalization Table *(Explanation, Interpretation, Self-Knowledge)*	
	A-R Guide (Revisit) *(Interpretation, Self-Knowledge)*	
	Concept Mapping *(Explanation, Interpretation)*	
	Summative Test *(Application, Interpretation, Explanation)*	Transfer Task *(Application, Empathy, Perspective)*
	Post Test *(Application, Interpretation)*	
Self-Assessment	Lesson Closure *(Explanation, Self- Knowledge)*	

Assessment Matrix

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	* Illustrates parallel and perpendicular lines. * Illustrates and proves properties of parallel lines cut by a transversal. * Determines and proves the conditions under which lines and segments are parallel or perpendicular. * Determines the conditions that make a quadrilateral a parallelogram and prove that a quadrilateral is a parallelogram. * Uses properties to find measures of angles, sides, and other quantities involving parallelograms	**Paper and Pencil Test** Activity 22 (Summative Test, Part A) Post Test (Item Nos. 1-3)	Every correct answer is given 1 point.
Process/Skills 25%		**Paper and Pencil Test** Activity 22 (Summative Test, Part B) Post Test (Item Nos. 4-8)	Every correct answer with correct solution is given 2 points. Every correct answer is given 1 point.
Understanding 30%		**Paper and Pencil Test** Activity 22 (Summative Test, Part C) Post Test (Item Nos. 9-14)	Refer to the Rubric in Writing Proof. Every correct answer is given 1 point.

Product 30%	* uses the key concepts of parallelism and perpendicularity of lines in solving real-life problems.	**Transfer Task** Activity 23 (Designer's Forum) Post Test (Item Nos. 15-20)	Refer to the Rubric for Performance Task Every correct answer is given 1 point.

C. Planning for Teaching-Learning

Introduction:

The module is all about Parallelism and Perpendicularity. It gives emphasis on the theorems involving parallel and perpendicular lines, proving properties of parallel lines cut by a transversal, the conditions to prove that a quadrilateral is a parallelogram and applications of parallelism and perpendicularity. The students are given various activities that will enable them to set up parallelism and perpendicularity and use the important concepts in solving real-life problems. These activities can be given in the form of a game or a worksheet to be done inside the classroom or these can be given as their assignment. These can also be done individually or collaboratively. Activities may or may not be graded depending on the teacher's discretion but all results will be recorded and kept for evaluation purposes. Some brain teasers are also presented in this guide and these can be used to catch students' attention.

As an introduction to the main lesson, ask the students the following questions:

Have you ever wondered how carpenters, architects and engineers design their work? What factors are being considered in making their designs?

The use of parallelism and perpendicularity of lines in real life necessitates the establishment of these concepts deductively.

This module seeks to answer the question: **"How can we establish parallelism or perpendicularity of lines"?**

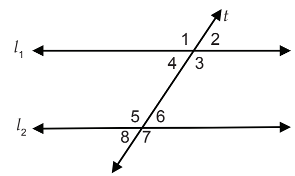
Let the students take the pre-assessment to find out how much they already know about the module. Instruct them to choose the letter that corresponds to the best answer and write it on a separate sheet. After taking the test, let them note the items that they were not able to answer correctly.

PRE - ASSESSMENT

Find out how much you already know about this module. Choose the letter that corresponds to the best answer and write it on a separate sheet. Please answer all items. After taking this short test, take note of the items that you were not able to answer correctly. Correct answers are provided as you go through the module.

(K)1. Using the figure below, if $l_1 \parallel l_2$ and t is a transversal, then which of the following are corresponding angles?

a. $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$
b. $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$
c. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$
d. $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$



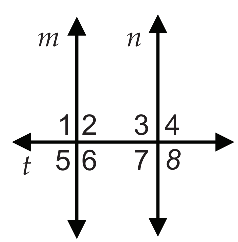
Answer: c, Corresponding angles are pair of non-adjacent angles on the same side of the transversal, one interior and one exterior.

(K)2. All of the following are properties of a parallelogram except:

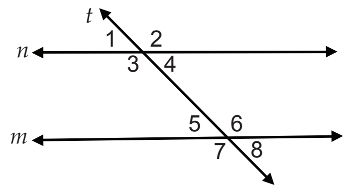
a. Diagonals bisect each other.
b. Opposite angles are congruent.
c. Opposite sides are congruent.
d. Opposite sides are not parallel.
Answer: d, A quadrilateral is a parallelogram if two pairs of opposite sides are parallel and congruent.

(K)3. Lines m and n are parallel cut by transversal t which is also perpendicular to m and n. Which statement is not correct?

a. $\angle 1$ and $\angle 6$ are congruent.
b. $\angle 2$ and $\angle 3$ are supplementary.
c. $\angle 3$ and $\angle 5$ are congruent angles.
d. $\angle 1$ and $\angle 4$ form a linear pair.
Answer: d, Linear pairs are adjacent angles.



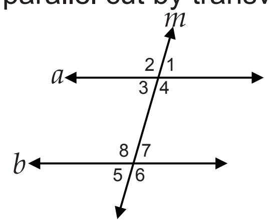
(K)4. Using the figure below, which of the following guarantees that $m \parallel n$?



a. $\angle 1 \cong \angle 7$
b. $\angle 3 \cong \angle 5$
c. $\angle 4 \cong \angle 5$
d. $\angle 4 \cong \angle 7$

Answer: c, If two parallel lines are cut by a transversal, alternate interior angles are congruent.

(S)5. Lines a and b are parallel cut by transversal m. If $m\angle 1 = 85$, what is the measure of $\angle 5$?

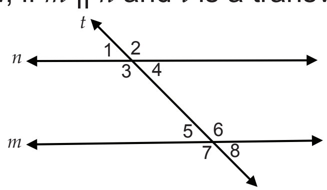


a. 80
b. 85
c. 95
d. 100

Answer: b, $\angle 1$ and $\angle 5$ are alternate-exterior angles and are congruent.

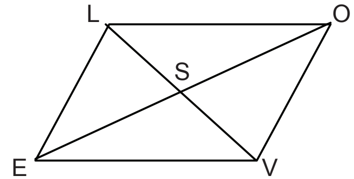
(S)6. ▭ JOSH is a parallelogram, $m\angle J = 57$, find the measure of $\angle H$.
a. 43
b. 57
c. 63
d. 123
Answer: b, $\angle J$ and $\angle H$ are consecutive angles of the parallelogram, therefore, they are supplementary.

(S)7. Using the figure below, if $m \parallel n$ and t is a transversal, which angles are congruent to $\angle 5$?



a. $\angle 1$, $\angle 2$ and $\angle 3$
b. $\angle 1$, $\angle 4$ and $\angle 8$
c. $\angle 1$, $\angle 4$ and $\angle 7$
d. $\angle 1$, $\angle 2$ and $\angle 8$
Answer: b, $\angle 5$ and $\angle 1$ are corresponding angles thus $\angle 5 \cong \angle 1$ and then $\angle 1$ and $\angle 4$ are vertical angles so $\angle 1 \cong \angle 4 \cong \angle 5$. Lastly, $\angle 5$ and $\angle 8$ are vertical angles therefore $\angle 5 \cong \angle 8$.

(S)8. LOVE is a parallelogram. If $SE = 6$, then what is SO?

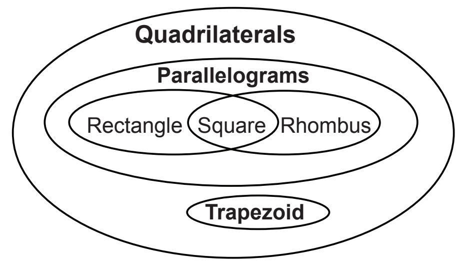


a. 3
b. 6
c. 12
d. 15

Answer: b, Diagonals of a parallelogram bisect each other. Thus, $\overline{SE} \cong \overline{SO}$ and the measures would be the same or equal.

(U)9. The Venn Diagram below shows the relationships of quadrilaterals. Which statements are true?

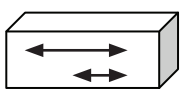
I - Squares are rectangles.
II- A trapezoid is a parallelogram.
III- A rhombus is a square.
IV- Some parallelograms are squares.

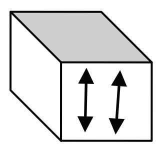


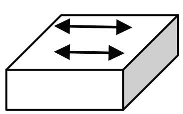
a. I and II
b. III and IV
c. I and IV
d. II and III

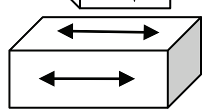
Answer: c, Only I and IV are true statements based on the properties of parallelograms

(U)10. All of the figures below illustrate parallel lines except:

a. 

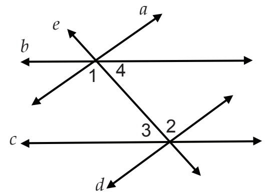
c. 

b. 

d. 

Answer: d, Lines that are not coplanar would never be parallel.

(U)11. In the figure below, $a \parallel d$ with e as the transversal. What must be true about ∠3 and ∠4 if $b \parallel c$ with e, also as the transversal?



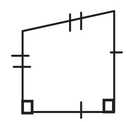
a. ∠3 is a complement of ∠4.
b. ∠3 is congruent to ∠4.
c. ∠3 is a supplement of ∠4.
d. ∠3 is greater than ∠4.

Answer: b, To make b║c ∠3 should be congruent to ∠4, since alternate interior angles of parallel lines cut by a transversal are congruent.

(U)12. Which of the following statements ensures that a quadrilateral is a parallelogram?

a. Diagonals bisect each other.
b. The two diagonals are congruent.
c. The consecutive sides are congruent.
d. Two consecutive angles are congruent.

Answer: a, The other statements don't guarantee that the quadrilateral is a parallelogram. An isosceles trapezoid has congruent diagonals. A trapezoid can also have pairs of consecutive sides that are congruent as well as consecutive angles.

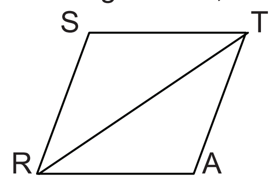


(U) 13. Which of the following statements is always true?

a. Lines that do not intersect are parallel lines.
b. Two coplanar lines that do not intersect are parallel lines.
c. Lines that form a right angle are parallel lines.
d. Skew lines are parallel lines.

Answer: b, Parallel lines are coplanar lines that do not intersect.

(U)14. $\square$ STAR is a rhombus with diagonal $\overline{RT}$, if $m\angle STR = 3x - 5$ and $m\angle ART = x + 21$. What is $m\angle RAT$?



a. 13
b. 34
c. 68
d. 112

Answer: d, $\square$ STAR is a rhombus, therefore $\overline{ST}$ is parallel to $\overline{RA}$ and $\overline{RT}$ is a transversal. ∠STR and ∠ART are congruent because they are alternate interior angles thus x = 13. Since a diagonal of a rhombus bisects opposite angles, the measure of ∠STA and ∠ARS is 34x2 or 68 each. ∠SRA and ∠RAT are consecutive angles and so they are supplementary, so $m\angle$RAT is 112.

(P)15. You are tasked to divide a blank card into three equal rows/pieces but you do not have a ruler. Instead, you will use a piece of equally lined paper and a straight edge. What is the sequence of the steps you are going to undertake in order to apply the theorem on parallel lines?

I – Mark the points where the second and third lines intersect the card.
II – Place a corner of the top edge of the card on the first line of the paper.
III – Repeat for the other side of the card and connect the marks.
IV – Place the corner of the bottom edge on the fourth line.

a. I, II, III, IV
b. II, III, IV, I
c. I, III, IV, II
d. II, IV, I, III

Answer: d, Since the lines of the paper are equally spaced, the lines are parallel.

(P)16. You are a student council president. You want to request for financial assistance for the installation of a book shelf for the improvement of your school's library. Your student council moderator asked you to submit a proposal for their approval. Which of the following will you prepare to ensure that your request will be granted?

I. design proposal of the book shelf
II. research on the importance of book shelf

III. estimated cost of the project
IV. pictures of the different libraries

a. I only
b. I and II only
c. I and III only
d. II and IV only

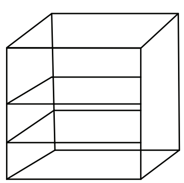
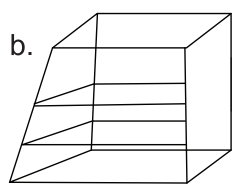
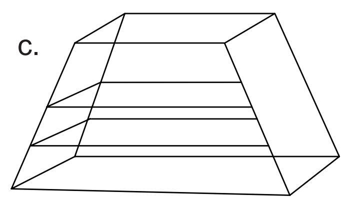
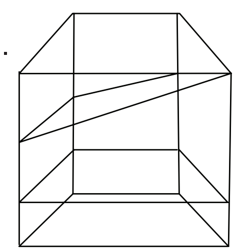
Answer: c, The cost of the project is one of the important thing to consider and of course the cost is dependent upon the proposed design of the book shelf.

(P)17. Based on your answer in item no. 16, which of the following standards should be the basis of your moderator in approving or granting your request?

a. accuracy, creativity and mathematical reasoning
b. practicality, creativity and cost
c. accuracy, originality and mathematical reasoning
d. organization, mathematical reasoning and cost

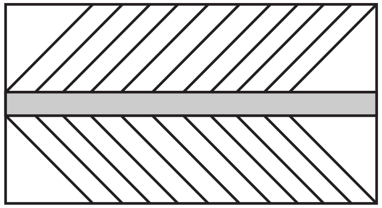
Answer: b, Since financial aspect is involved thus there is a need to consider cost of the project. But despite of its cost it should still be creative and feasible or realistic.

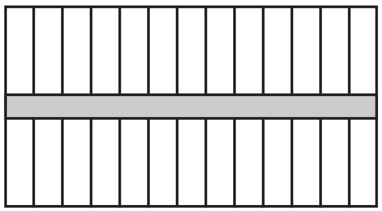
(P)18. Based on item no. 16, design is common to all the four given options. If you were to make the design, which of the illustrations below will you make to ensure stability?

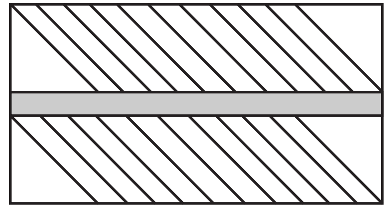
a. 
b. 
c. 
d. 

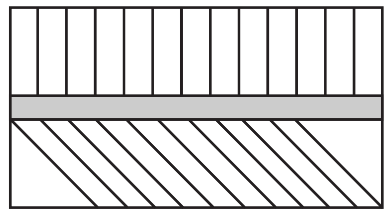
Answer: a, The key concepts about parallelism will ensure its stability.

(P)19. You are an architect of the design department of a mall. Considering the increasing number of mall-goers, the management decided to restructure their parking lot so as to maximize the use of the space. As the head architect, you are tasked to make a design of the parking area and this design is to be presented to the mall administrators for approval. Which of the following are you going to make so as to maximize the use of the available lot?

a. 

b. 

c. 

d. 

Answer: b, Vertical parallel parking will surely maximize the available lot.

(P)20. Based on your answer in item no. 19, how will your immediate supervisor know that you have a good design?

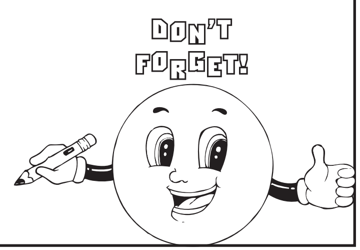
a. The design should be realistic.
b. The design should be creative and accurate.
c. The design should be accurate and practical.
d. The design shows a depth application of mathematical reasoning and it is practical.

Answer: d, A good design should apply mathematical reasoning aside from its practicality.

What to Know

The module starts with the hook activity on Optical Illusions where students are encouraged to answer certain questions. Students may be deceived by the pictures when presented in larger scale. This can be done as a class activity. Encourage students to explain their answers.

Teacher's Note and Reminders



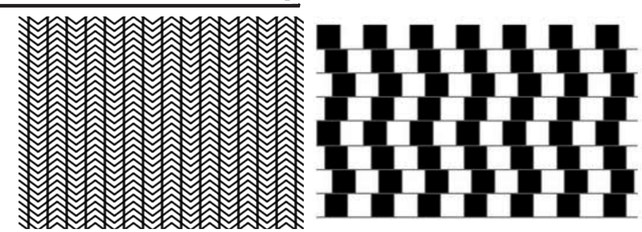
LEARNING GOALS AND TARGET:

- The learner demonstrates understanding of the key concepts of parallel and perpendicular lines.
- The learner is able to communicate mathematical thinking with coherence and clarity in solving real-life problems involving parallelism and perpendicularity using appropriate and accurate representations.

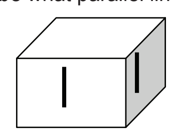
What to Know

Start the module by taking a look at the figures below and then answer the succeeding questions.

Activity 1 OPTICAL ILLUSION



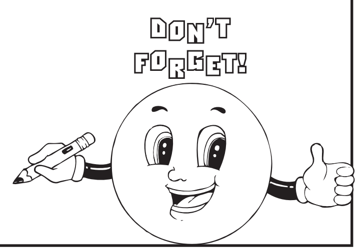
- Can you see straight lines in the pictures above? ____________________
- Do these lines meet/intersect? ____________________
- Are these lines parallel? Why? ____________________
- Are the segments on the faces of the prism below parallel? Why?_________
- Can you describe what parallel lines are? ____________________



- What can you say about the edges of the prism? ____________________
- Are these lines perpendicular? Why? ____________________
- Can you describe what perpendicular lines are? ____________________

Students have just tried describing parallel and perpendicular lines. In their next activities, their prior knowledge on parallelism and perpendicularity will be elicited.
They will be given individual activities such as Generalization Table and Anticipation-Reaction Guide.
Copy of each activity can be reproduced or be copied on a separate notebook where they can write their answers.
Keep the answer sheets of the students in Activities 2 and 3 for future use.

Teacher's Note and Reminders



You have just tried describing parallel and perpendicular lines. In Activities 2 and 3, your prior knowledge on parallelism and perpendicularity will be extracted.

Activity 2 GENERALIZATION TABLE

Direction: Fill in the first column of the generalization table below by stating your initial thoughts on the question.

"How can parallelism or perpendicularity of lines be established?"

My Initial Thoughts				

Activity 3 AGREE OR DISAGREE!

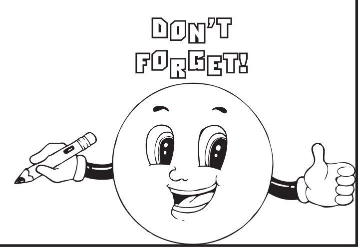
ANTICIPATION-REACTION GUIDE

Read each statement under the column TOPIC and write **A** if you agree with the statement; otherwise, write **D**.

Before-Lesson Response	TOPIC: Parallelism and Perpendicularity
	1. Lines that do not intersect are parallel lines.
	2. Skew lines are coplanar.
	3. Transversal is a line that intersects two or more lines.
	4. Perpendicular lines are intersecting lines.
	5. If two lines are parallel to a third line, then the two lines are parallel.

Instead of presenting Activity 4 as a worksheet, the teacher may present it as an outdoor activity where the figure will be drawn on the ground and pairs of students will step on the pairs of angles mentioned by the teacher. For classes that are technology-ready, they may use the link for the activity. http://www.mathwarehouse.com/geometry/angle/transveral-and-angles.php

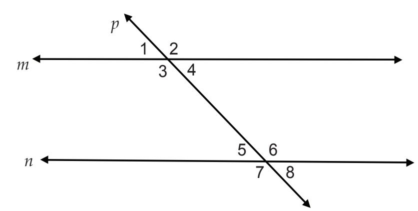
Teacher's Note and Reminders



	6. If two lines are perpendicular to the same line, then the two lines are parallel.
	7. If one side of a quadrilateral is congruent to its opposite side, then the quadrilateral is a parallelogram.
	8. Diagonals of a parallelogram bisect each other.
	9. Diagonals of a parallelogram are congruent.
	10. Diagonals of a parallelogram are perpendicular.
	11. Opposite sides of a parallelogram are parallel.
	12. Opposite angles of a parallelogram are congruent.
	13. Consecutive angles of a parallelogram are congruent.
	14. Squares are rectangles.
	15. Squares are rhombi.

Activity 4 NAME IT! A RECALL...

We see parallel lines everywhere. Lines on a pad paper, railways, edges of a door or window, fence, etc. suggest parallel lines. Complete the table below using the given figure as your reference:



Corresponding Angles	Alternate Interior Angles	Alternate Exterior Angles	Same Side Interior Angles	Same Side Exterior Angles

What to Process

Your goal in this section is to let the students learn, and understand the key concepts of measurement of angles formed by parallel lines cut by a transversal and basic concepts about perpendicularity and the properties of parallelogram. Towards the end of this section, the students will be encouraged to learn the different ways of proving.

Students will be given activities such as investigation, worksheets and some formative assessments where the decision of whether they will be graded or not will depend on the teacher.
Discussions on parallelism, perpendicularity, properties of parallelogram, and writing proofs will also be done in this section. Assessments will also follow after the discussion.

The generalization table will again be presented in order to keep track and evaluate students' understanding about the lesson. Make sure to keep their answer sheets or let the students keep theirs.

Activity 5 needs advance preparation for the teacher. Reproduce the activity sheet and assign the students to bring their own protractors. This activity can be done in pairs. Students are expected that they already know how to measure an angle. If possible, students may also visit the link. http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php

Teacher's Note and Reminders

You gave your initial ideas on naming angle pairs formed by two lines cut by a transversal. What you will learn in the next sections will enable you to do the final project which involves integrating the key concepts of parallelism and perpendicularity of lines in model making of a book case. Now find out how these pairs of angles are related in terms of their measures by doing the first activity on investigating the relationship between the angles formed by parallel lines cut by a transversal.

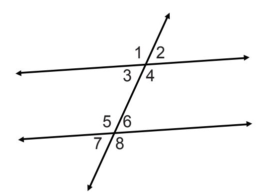
What to Process

Your goal in this section is to learn and understand key concepts of measurement of angles formed by parallel lines cut by a transversal and basic concepts about perpendicularity and the properties of parallelogram. Towards the end of this section, you will be encouraged to learn the different ways of proving deductively. You may also visit the link for this investigation activity. http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php

Activity 5 LET'S INVESTIGATE!

Two parallel lines when cut by a transversal form eight angles. This activity will lead you to investigate the relationship between and among angles formed.

Measure the eight angles using your protractor and list all inferences or observations in the activity.



$m\angle 1 =$ ________
$m\angle 2 =$ ________
$m\angle 3 =$ ________
$m\angle 4 =$ ________
$m\angle 5 =$ ________
$m\angle 6 =$ ________
$m\angle 7 =$ ________
$m\angle 8 =$ ________

OBSERVATIONS:

____
____
____.

Now, think about the answers to the following questions. Write your answers in your answer sheet.

After doing the investigation, encourage the students to answer the questions. This can be done orally or they may continue working in pairs.

Students can list any pair of angles and then classify which pairs have equal measures and which are supplementary.

Students can list any pair of angles and then classify which pairs have equal measures and which are supplementary.

At this point, discuss important concepts about parallelism and perpendicularity of lines. Always keep on asking their ideas first from time to time and make sure to clarify/explain the misconceptions. For example, NOT ALL LINES THAT DO NOT INTERSECT ARE PARALLEL LINES. That is because we have the SKEW LINES. These lines are non-coplanar lines that do not intersect. Another misconception is on transversal. A TRANSVERSAL IS NOT JUST A LINE THAT INTERSECTS TWO OR MORE LINES. It should intersect those lines at DIFFERENT POINTS.

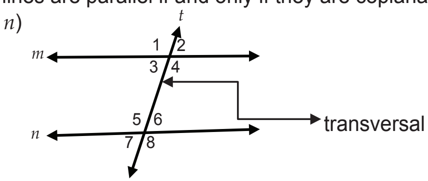
Teacher's Note and Reminders

QUESTIONS

1. What pairs of angles are formed when two lines are cut by a transversal line?
2. What pairs of angles have equal measures? What pairs of angles are supplementary?
3. Can the measures of any pair of angles (supplementary or equal) guarantee the parallelism of lines? Support your answer.
4. How can the key concepts of parallel lines facilitate solving real-life problems using deductive reasoning?

Discussion: ***Parallelism***

1. Two lines are parallel if and only if they are coplanar and they do not intersect. ($m \parallel n$)



2. A line that intersects two or more lines at different points is called a **transversal**.

   a. The angles formed by the transversal with the two other lines are called:
      - *exterior angles* (∠1, ∠2, ∠7 and ∠8)
      - *interior angles* (∠3, ∠4, ∠5 and ∠6).

   b. The pairs of angles formed by the transversal with the other two lines are called:
      - *corresponding angles* (∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8)
      - *alternate-interior angles* (∠3 and ∠6, ∠4 and ∠5)
      - *alternate-exterior angles* (∠1 and ∠8, ∠2 and ∠7)
      - *interior angles* on the same side of the transversal (∠3 and ∠5, ∠4 and ∠6)
      - *exterior angles* on the same side of the transversal (∠1 and ∠7, ∠2 and ∠8)

3. If two lines are cut by a transversal, then the two lines are parallel if:
   a. corresponding angles are congruent.
   b. alternate-interior angles are congruent.
   c. alternate-exterior angles are congruent.
   d. interior angles on the same side of the transversal are supplementary.
   e. exterior angles on the same side of the transversal are supplementary.

To strengthen your knowledge regarding the different angles formed by parallel lines cut by a transversal line and how they are related with one another, you may visit the following sites:

http://www.youtube.com/watch?v=AE3Pqhlvqw0&feature=related
http://www.youtube.com/watch?v=VA92EWf9SRI&feature=relmfu

To check their understanding in the previous discussion, encourage them to answer Activity 6.

Answer Key

Activity 6

1. Each pair of angles is supplementary. They are interior angles on the same side of the transversals MA and HT.
2. Equate each pair of angles to 180^0. This will result to z = 33^0 and y = 82^0.
   $m\angle M$ = 81
   $m\angle A$ = 99
   $m\angle MHT$ = 98
   $m\angle ATH$= 82
3. Yes, because pairs of interior angles on the same side of each transversal are supplementary.
4. No, because pairs of interior angles on the same side of each transversal (tower) are not supplementary.

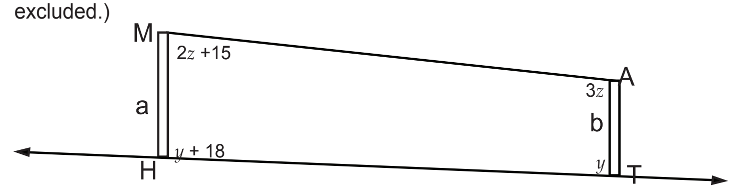
To strengthen their discoveries regarding the different angles formed by parallel lines cut by a transversal and how they are related with one another, they may visit the following sites:
http://www.youtube.com/watch?v=AE3Pqhlvqw0&feature=related
http://www.youtube.com/watch?v=VA92EWf9SRI&feature=relmfu

For practice, students may proceed to this link:
http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm

Activity 6 UNCOVERING THE MYSTERY OF PARALLEL LINES CUT BY A TRANSVERSAL

Study the problem situation below and answer the succeeding questions:

A zip line is a rope or a cable that you can ride down on a pulley. The pair of zip lines below goes from a 20- foot tall tower to a 15- foot tower 150 meters away in a slightly inclined ground as shown in the sketch. (Note: Tension of the rope is excluded.)



1. What kind of angle pairs are $\angle M$ and $\angle A$? $\angle MHT$ and $\angle ATH$?
   ____
   ____

2. Using the given information stated in the figure, what are the measures of the four angles?

Solution:	Answers:
	$m\angle M$ = ________ $m\angle A$ = ________ $m\angle MHT$ = ________ $m\angle ATH$ = ________

3. Are the two towers parallel? Why do you say so?

4. Is the zip line parallel to the ground? Why do you say so?

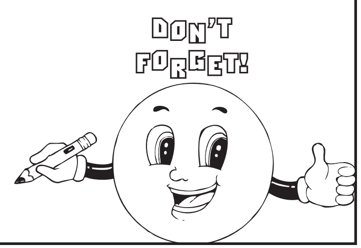
For practice you may proceed to this link:
http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm

Activity 7 can given as an individual activity using paper and pen.

Answers to Part III

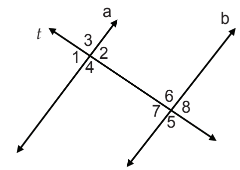
1. x = 50^0 2. x = 55^0 3. x = 34^0

Teacher's Note and Reminders



Activity 7 LINES AND ANGLES

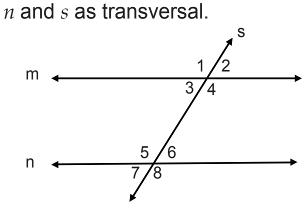
I. Study the figure and answer the following questions as accurate as you can. The figure below shows $a \parallel b$ with t as transversal.



Name:

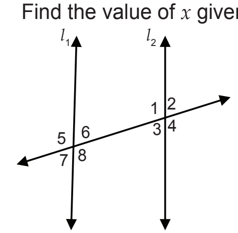
1. 2 pairs of corresponding angles __________ __________
2. 2 pairs of alternate interior angles __________ __________
3. 2 pairs of alternate exterior angles __________ __________
4. 2 pairs of interior angles on the same side of the transversal __________ __________
5. 2 pairs of exterior angles on the same side of the transversal __________ __________

II. Given $m \parallel n$ and s as transversal.



1. Name all the angles that are congruent to ∠1. ______________
2. Name all the angles that are supplement of ∠2. ______________

III. Find the value of x given that $l_1 \parallel l_2$.

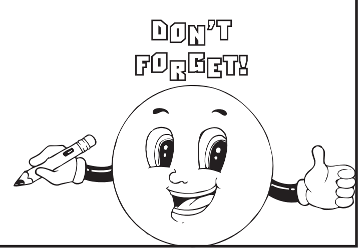


1. $m\angle 1 = 2x + 25$ and $m\angle 8 = x + 75$ ________
2. $m\angle 2 = 3x - 10$ and $m\angle 6 = 2x + 45$ ________
3. $m\angle 3 = 4v - 31$ and $m\angle 8 = 2x + 7$ ________

Before discussing Perpendicularity, present Activity 8. You can think of a way to make it more interactive. The process questions can be done as a class activity.

You may suggest to the students the following link to learn more about perpendicular lines: http://www.mathsisfun.com/geometry/parallel-perpendicular-lines-planes.html

Teacher's Note and Reminders



Activity 8 AM I PERPENDICULAR? LET'S FIND OUT...!

Given any two distinct lines on a plane, the lines either intersect or are parallel. If two lines intersect, then they form four angles. Consider the figures below to answer the questions that follow.

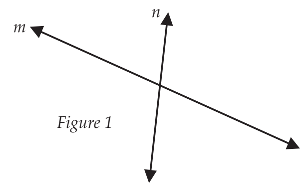


Figure 1

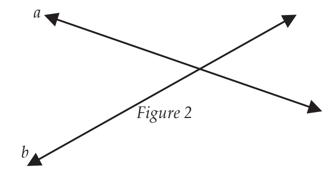


Figure 2

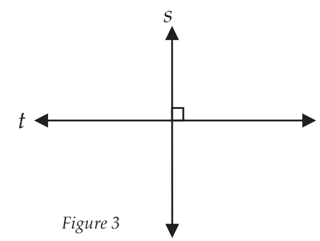


Figure 3

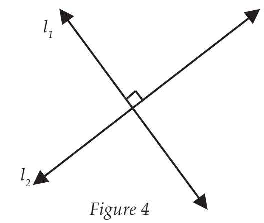


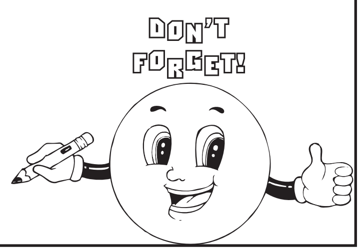
Figure 4

QUESTIONS ?

1. What is common in the four figures given above?
   ____
2. What makes figures 3 and 4 different from the first two figures?
   ____
3. What does this symbol ∟ indicate?
   ____
4. Which among the four figures show perpendicularity? Check by using your protractor.
   ____
5. When are the lines said to be perpendicular to each other?
   ____
6. How useful is the knowledge on perpendicularity in real-life? Cite an example in which perpendicularity is said to be important in real-life.
   ____
   ____

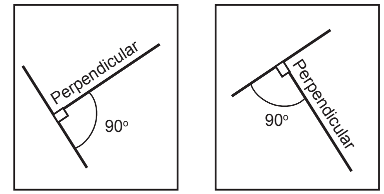
Students may watch the video lessons using the given links. These videos will explain how to construct a perpendicular line to a point and a perpendicular line through a point not on a line.
http://www.youtube.com/watch?v=dK3S78SjPDw&feature=player_embedded
http://www.youtube.com/watch?feature=player_embedded&v=_jWw_nQTtCw
http://www.youtube.com/watch?v=9ZjSz199Huc&feature=player_embedded:

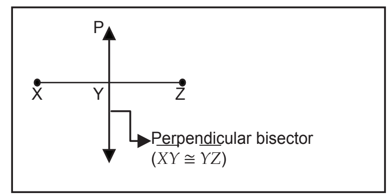
Teacher's Note and Reminders

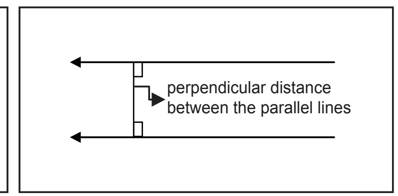


Discussion: *Perpendicularity*

Two lines that intersect to form right angles are said to be **perpendicular**. This is not limited to lines only. Segments and rays can also be perpendicular. A **perpendicular bisector** of a segment is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint. The distance between two parallel lines is the **perpendicular distance** between one of the lines and any point on the other line.



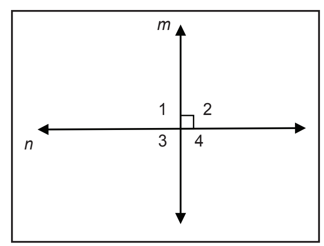


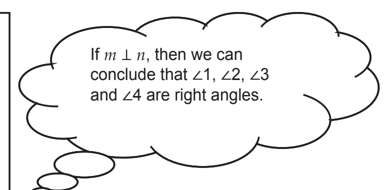


The small rectangle drawn in the corner indicates "right angle". Whereas, ⊥ is a symbol use to indicate perpendicularity of lines as in $\overline{XZ} \perp \overline{PY}$.

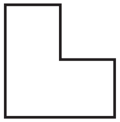
To prove that two lines are perpendicular, you must show that one of the following theorems is true:

1. If two lines are perpendicular to each other, then they form four right angles.

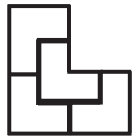




BRAIN TEASER! (PERPENDICULAR-LIKE)
Dissect the polygon into 4 identical polygons.

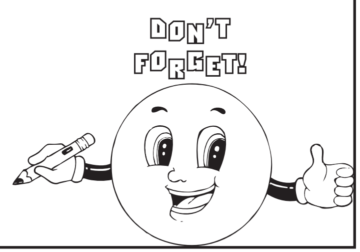


Answer:

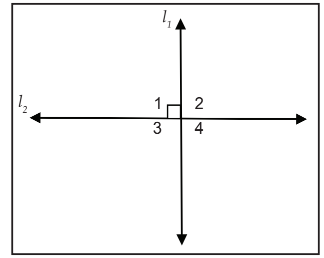


Reference: http://brainden.com/geometry-puzzles.htm

Teacher's Note and Reminders

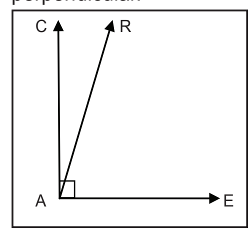


2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.



If $\angle 1$ and $\angle 2$ form a linear pair and $\angle 1 \cong \angle 2$, then $l_1 \perp l_2$.

3. If two angles are adjacent and complementary, the non-common sides are perpendicular.



If $\angle CAR$ and $\angle EAR$ are complementary and adjacent, then $\overline{AC} \perp \overline{AE}$.

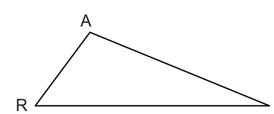
You may watch the video lesson using the given links. These videos will explain how to construct a perpendicular line to a point and a perpendicular line through a point not on a line.
http://www.youtube.com/watch?v=dK3S78SjPDw&feature=player_embedded

Activity 9 will test your skill and knowledge about perpendicular lines. This will prepare you also to understand the final task for this module.

Activity 9 DRAW ME RIGHT!

Directions: Copy each figure in a separate sheet of bond paper. Draw the segment that is perpendicular from the given point to the identified side. Extend the sides if necessary.

1. A to $\overline{RH}$



Prior to Activity 10, let the students prepare simple YES and NO CARDS.

As you go through the activity, let the students justify their answer for each item. Guide them if necessary.

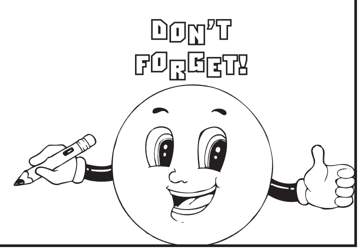
Answer Key

Prior to Activity 10

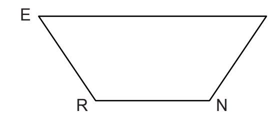
1. Yes	5. No	8. Yes
2. Yes	6. Yes	9. No
3. No	7. Yes	10. Yes
4. Yes		

For nos. 9 and 10, take note of perpendicular distance between $\overline{SM}$ and $\overline{EI}$.

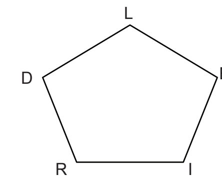
Teacher's Note and Reminders



2. E to $\overline{RN}$



3. D to $\overline{IE}$



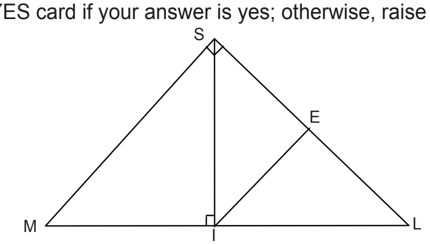
QUESTIONS ?

1. What did you use to draw the perpendicular segments?
____
____
2. How sure are you that the segments you drawn are really perpendicular to the indicated side?
____
____

Activity 10 THINK TWICE!

Refer to the given figure and the given conditions in answering the succeeding questions. Raise your YES card if your answer is yes; otherwise, raise your NO card.

Given:
$\overline{MI} \cong \overline{IL}$
$\overline{SE} \cong \overline{EL}$
$m\angle SEI = 90$



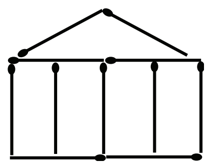
	YES	NO
1. Is $\overline{ML} \perp \overline{IS}$?		
2. Is $\overline{MS} \perp \overline{SL}$?		
3. Is $SL \perp \overline{ML}$?		
4. Are $\angle MSI$ and $\angle ISL$ complementary angles?		

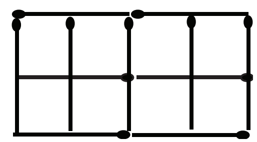
The generalization table can be given as their assignment. Keep their answer sheets.

BRAIN TEASER! (SQUARES)

Move 2 match sticks to form 11 squares.

ANSWER:

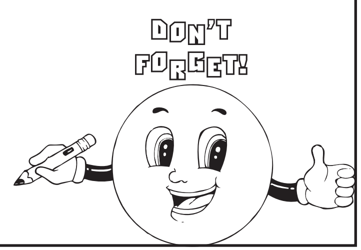




(eight 1x1 squares and three 2x2 squares)

Reference: http://brainden.com/matchstick-puzzles.htm

Teacher's Note and Reminders



5. Are $\angle MIS$ and $\angle SIE$ complementary angles?
6. Is $\overline{IE}$ a perpendicular bisector of $\overline{SL}$?
7. Do $\angle MIS$ and $\angle SIL$ form a linear pair?
8. Is the $m\angle MIS$ = 90?
9. Is $\overline{SI}$ shorter than $\overline{SE}$?
10. Is $\overline{SE}$ shorter than $\overline{MI}$?

Activity 11 GENERALIZATION TABLE

Fill in the second, third, and fourth columns of the generalization table below by stating your present thoughts on the question.

"How can parallelism or perpendicularity of lines be established?"

	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	

Discussion: ***KINDS OF QUADRILATERALS: A review***

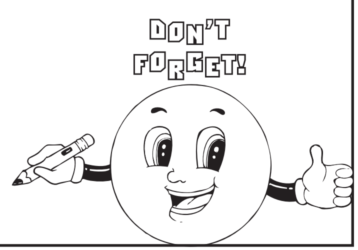
Quadrilateral is a polygon with four sides. The symbol ▭ is used in this module to indicate a quadrilateral. For example, ▭ABCD, this is read as "quadrilateral ABCD".

Quadrilaterals are classified as follows:

1. Trapezium – a quadrilateral with no pair of parallel sides.
2. Trapezoid – a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides are congruent, the trapezoid is said to be isosceles.
3. Parallelogram – a quadrilateral with two pairs of parallel and congruent sides. There are two special kinds of parallelogram: the rectangle which has four right angles and the rhombus which has four congruent sides. A square which has four congruent angles and four congruent sides can be a rectangle or a rhombus because it satisfies the definition for a rectangle and a rhombus.

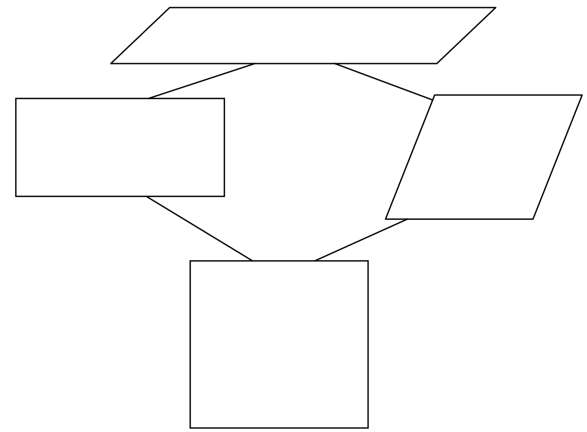
The activity on completing the table can be presented in the form of a game. The parallelograms will be the four choices. The properties will serve as the questions. After asking the questions, let the students choose the answer and stand behind the name of the parallelogram. It is possible that there are more than one answer. The last five (or may be ten) will be declared winners. After the game, the process questions can be given as their assignment or if there will be enough time, just ask them the questions and discuss their answers.

Teacher's Note and Reminders



Activity 12 SPECIAL QUADRILATERALS

Study the blank diagram below. Write the appropriate quadrilateral in the box. After which, complete the table below.



Direction: Place a check mark (√) in the boxes below if the quadrilateral listed along the top row has the properties listed in the left column.

Properties	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are congruent.				
Opposite angles are congruent.				
Sum of the measures of the consecutive angles is 180°.				
Diagonals are congruent.				
Diagonals are perpendicular.				
Diagonals bisect each other				

If possible, you can let the students watch the lessons on properties of parallelograms using the following link:
http://www.youtube.com/watch?feature=player_detailpage&v=0rNjGNI1Uzo

Teacher's Note and Reminders

QUESTIONS

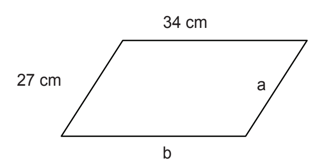
1. What properties are common to rectangles, rhombi, and squares, if any?
   ____
   ____
2. What makes a rectangle different from a rhombus? A rectangle from a square? A rhombus from a square?
   ____
   ____
3. What do you think makes parallelograms special in relation to other quadrilaterals?
   ____
   ____
4. Are the properties of parallelograms helpful in establishing parallelism and perpendicularity of lines?
   ____
   ____

You may visit this URL to have more understanding of the properties of parallelogram.
http://www.youtube.com/watch?feature=player_detailpage&v=0rNjGNI1Uzo

Activity 13 HIDE AND SEEK!

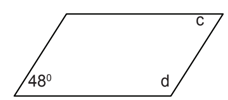
Each figure below is a parallelogram. Use your observations in the previous activity to find the value of the unknown parts.

1.



YOUR ANSWER

a = __________
b = __________



c = __________
d = __________

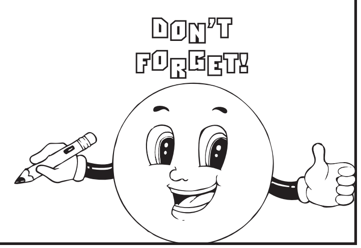
Answer Key

Activity 13

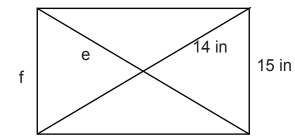
a = 27 cm	b = 34 cm	c = 48^0	d = 132^0
e = 14 in	f = 15 in	g = 63^0	h = 78^0

In presenting the answers, it is advisable to discuss also the theorems used in finding the value of the unknown parts.

Teacher's Note and Reminders

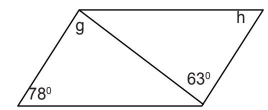


3.



e = ___________
f = ___________

4.



g = ___________
h = ___________

Discussion: ***Writing Proofs/Proving (A review)***

In the previous discussions, you have solved a lot of equations and inequalities by applying the different properties of equality and inequality. To name some, you have the APE (Addition Property of Equality), MPE (Multiplication Property of Equality) and TPE (Transitive Property of Equality). Now, you will use the same properties with some geometric definitions, postulates, and theorems to write a complete proof.

One of the tools used in proving is reasoning, specifically deductive reasoning. **Deductive reasoning** is a type of logical reasoning that uses accepted facts as reasons in a step-by-step manner until the desired statement is established.

A **proof** is a logical argument in which each statement you make is supported/ justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

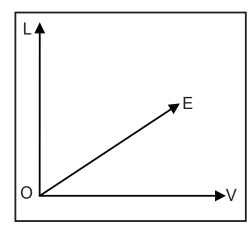
Proofs can be written in three different ways:

1. **Paragraph Form/ Informal Proof:**

   The paragraph or informal proof is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

Given: $\angle LOE$ and $\angle EOV$ are complementary

Prove: $\overline{LO} \perp \overline{OV}$



You may suggest to the students to watch the following video lessons about writing proofs:
http://www.youtube.com/watch?feature=player_embedded&v=3Ti7-Ojr7Cg
http://www.youtube.com/watch?feature=player_embedded&v=jgylP7yPgFY

Teacher's Note and Reminders

The Paragraph Proof:

Since $\angle LOE$ and $\angle EOV$ are complementary, then $m\angle LOE + m\angle EOV = 90$ by definition of complementary angles. Thus, $m\angle LOE + m\angle EOV = m\angle LOV$ by angle addition postulate and $m\angle LOV = 90$ by transitive property of equality. So, $\angle LOV$ is a right angle by definition of right angles. Therefore, $\overline{LO} \perp \overline{OV}$ by definition of perpendicularity.

2. Two-Column Form/ Formal Proof:

Two-column form is a proof with statements and reasons. The first column is for the statements and the other column for the reasons.

Using the same problem in #1, the proof is as follows:

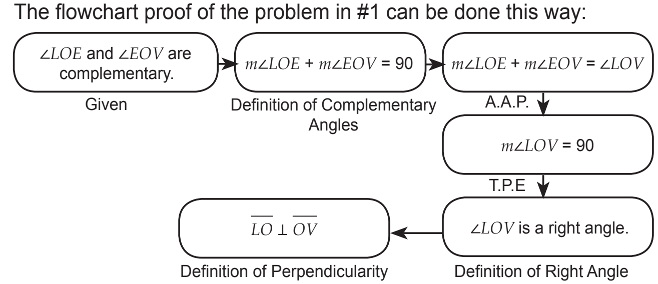
Statements	Reasons
1. $\angle LOE$ and $\angle EOV$ are complementary.	1. Given
2. $m\angle LOE + m\angle EOV = 90$	2. Definition of Complementary Angles
3. $m\angle LOE + m\angle EOV = m\angle LOV$	3. Angle Addition Postulate (AAP)
4. $m\angle LOV = 90$	4. Transitive Property of Equality (TPE)
5. $\angle LOV$ is a right angle.	5. Definition of Right Angle
6. $\overline{LO} \perp \overline{OV}$	6. Definition of Perpendicularity

You may watch the video lesson on this kind of proof using the following link:
http://www.youtube.com/watch?feature=player_embedded&v=3Ti7-Ojr7Cg

3. Flowchart Form:

A flowchart-proof organizes a series of statements in a logical order, starting with the given statements. Each statement together with its justification is written in a box and arrows are used to show how each statement leads to another. It can make one's logic visible and help others follow the reasoning.

The flowchart proof of the problem in #1 can be done this way:



Discussions about the key concepts on parallelism and perpendicularity and on parallelograms were presented. Relationships of the different angle pairs formed by parallel lines cut by a transversal and the properties of parallelograms were also given emphasis. The different ways of proving through deductive reasoning were discussed with examples Encourage the students to answer all the remaining activities.

Teacher's Note and Reminders

BRAIN TEASER! (REASONING)

How would you measure exactly 4 liters of water if you only have a 5-liter container and a 3-liter container and an unlimitted supply of water.

Answer: *Fill the 5-liter container and pour water to the 3-liter container, which you empty afterwards. From the 5-liter container pour the 2 remaining liters to the 3-liter container. Refill the 5-liter container and fill in the 3-liter container (with 1 liter), so there stay the 4 required liters in the 5-liter container.*
Reference: http://brainden.com/weighing-puzzles.htm

This URL shows you a video lessons in proving using flow chart.
http://www.youtube.com/watch?feature=player_embedded&v=jgylP7yPgFY

The following rubric will be used in giving grades in writing proofs.

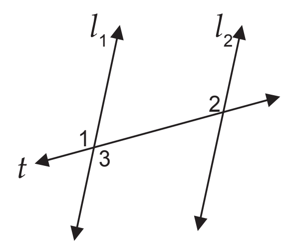
	4	3	2	1
Logic and Reasoning	The mathematical reasoning is sound and cohesive.	The mathematical reasoning is mostly sound, but lacking in some minor way.	The proof contains some flaws or omissions in mathematical reasoning.	The mathematical reasoning is either absent or seriously flawed. Use of mathematical terminology and notation
Use of mathematical terminology and notation	Notation is skillfully used; terminology is used flawlessly	Notation and terminology are used correctly with only a few exceptions.	There is a clear need for improvement in the use of terminology or notation	Terminology and notation are incorrectly and inconsistently used.
Correctness	The proof is complete and correct.	The proof is mostly correct, but has a minor flaw.	More than one correction is needed for a proper proof.	The argument given does not prove the desired result.

It's your turn. Accomplish Activity 14 and for sure you will enjoy!

Activity 14 COMPLETE ME!

Complete each proof below:

1. Given: Line t intersects l_1 and l_2 such that $\angle 1 \cong \angle 2$.
Prove: $l_1 \parallel l_2$
Proof:



	Reasons
1. $\angle 1 \cong \angle 2$	1. ________________
2. ________________	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 2$	3. Transitive Property of Congruence
4. $l_1 \parallel l_2$	4. ________________

Answer Key

Activity 14

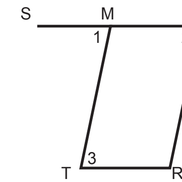
1. Given
   $\angle 1 \cong \angle 3$
   Converse of Alternate Interior Angles Theorem

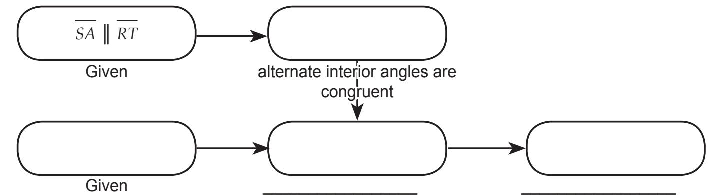
2. $\angle 1 \cong \angle 3$
   $\angle 2 \cong \angle 3$
   $\angle 1 \cong \angle 2$ Transitive Property of Congruence
   $\overline{MT} \parallel \overline{AR}$ Converse of Corresponding Angles Theorem

3. Given
   Definition of Parallelogram
   Same Side Interior Angles are Supplementary

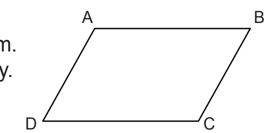
4. $\overline{AC}$ and $\overline{BD}$ bisect each other at E.
   Definition of Segment Bisector
   Vertical angles are congruent.
   $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
   Definition of Parallelogram (A parallelogram is a quadrilateral with both pairs of opposite sides parallel.)

2. Given: $\overline{SA} \parallel \overline{RT}$
   $\angle 2 \cong \angle 3$
   Prove: $\overline{MT} \parallel \overline{AR}$
   Proof:



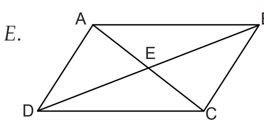


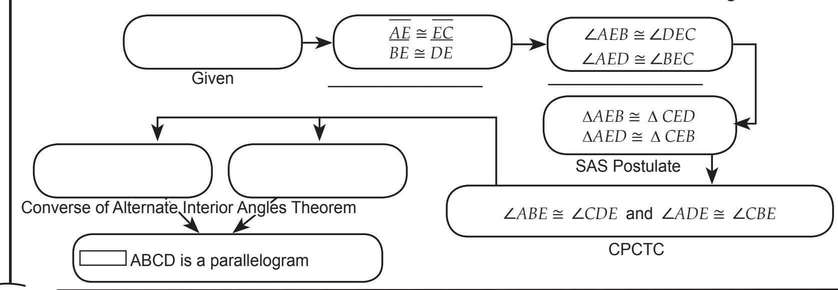
3. Given: ▭ $ABCD$ is a parallelogram.
   Prove: $\angle A$ and $\angle B$ are supplementary.
   Proof:



Statements	Reasons
1. ▭ ABCD is a parallelogram.	1.
2. $\overline{BC} \parallel \overline{AD}$	2.
3. $\angle A$ and $\angle B$ are supplementary.	3.

4. Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.
   Prove: ▭ ABCD is a parallelogram.





What to Understand

In the previous section the different ways in proving were discussed. Exercises in completing the proof were also given. Encourage students to do their own now. They may use any form of proof.

For Activity 15 answers may vary. Students may use any form of proving but the paragraph form may be the simplest/easiest one for this activity.

For #1, students may use the corresponding angles 1 and 5, then angles 5 and 7 which form linear pair or the alternate exterior angles 1 and 8, then angles 7 and 8 which also form linear pair. By substitution, they can already prove that angles 1 and 7 are supplementary.

For #2, students can prove that $\overline{CT} \perp \overline{UE}$ if they can show that the measure of $\angle 1$ and $\angle 2$ is equal to 90^0 each.

Teacher's Note and Reminders

In this section, the discussion was about the key concepts on parallelism and perpendicularity. Relationships of the different angle pairs formed by parallel lines cut by a transversal and the properties of parallelograms were also given emphasis. The different ways of proving through deductive reasoning were discussed with examples presented.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, go deeper by moving on to the next section.

What to Understand

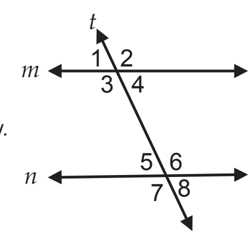
Your goal in this section is to take a closer look at some aspects of the topic. I hope that you are now ready to answer the exercises given in this section. Expectedly, the activities aim to intensify the application of the different concepts you have learned.

Activity 15 PROVE IT!

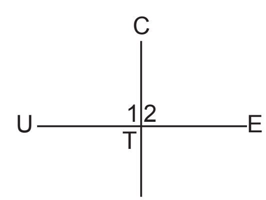
Prove the given statements below using any form of writing proofs.

1. Given:
   $m \parallel n$ and t is a transversal.

   Prove:
   $\angle 1$ and $\angle 7$ are supplementary.



2. In the figure, if $m\angle 1 = 3x + 15$, $m\angle 2 = 4x - 10$ prove that $\overline{CT}$ is perpendicular to $\overline{UE}$ if $x = 25°$.

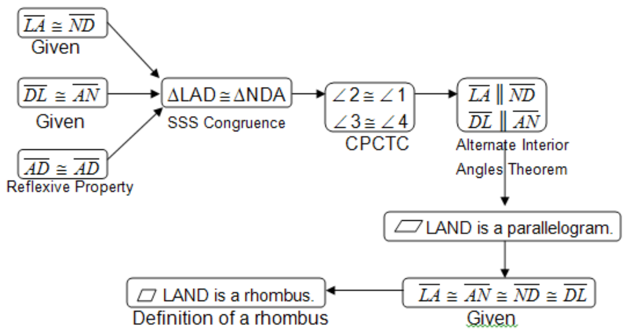


To have more practice in proving, give the following activity."
to "let the students answer Activity 16.

Answer Key

Activity 16
Students may use any form of proving even though the use flowchart is highly recommended in the instruction.

1. Proof:



2. Proof:

Statements	Reasons
1. ▭ BEAD is a rectangle.	1. Given
2. $\overline{BD} \cong \overline{EA}$	2. Opposite sides of a rectangle are congruent.
3. $\overline{AD} \cong \overline{AD}$	3. Reflexive Property of Congruence.
4. ∠BDA and ∠EAD are right angles.	4. Angles of a rectangle are right angles.
5. ∠BDA ≅ ∠EAD	5. Right angles are congruent.
6. ΔBDA ≅ ΔEAD	6. SAS Congruence
7. $\overline{AB} \cong \overline{DE}$	7. CPCTC

Statements 4 and 5 may be skipped if students will use the LL Congruence in proving the congruency of the stated triangles. The segments in statements 2 and 3 are the legs of the right triangles BDA and EAD.

QUESTIONS

1. What are the three different ways of proving deductively?
____
____

2. Which of the three ways is the best? Why?
____
____

3. How can one reason out deductively?
____
____

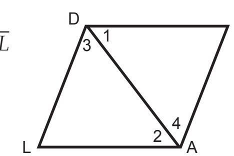
4. Why is there a need to study deductive reasoning? How is it related to real life? Cite a situation where deductive reasoning is applied.
____
____

Activity 16 PROVE SOME MORE... OKAY!

To strengthen your skill in proving deductively, provide a complete proof for each problem below. The use of flowchart is highly recommended.

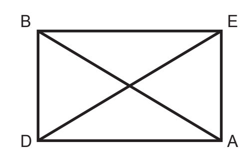
1. Given:
▱ LAND has $\overline{LA} \cong \overline{AN} \cong \overline{ND} \cong \overline{DL}$ with diagonal $\overline{AD}$.

Prove: ▱ LAND is a rhombus.



2. Given:
▭ BEAD is a rectangle.

Prove: $\overline{AB} \cong \overline{DE}$



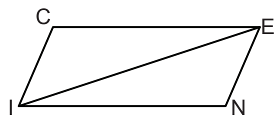
Answer Key

Activity 17

I. Both are parallelograms. Diagonals of parallelograms bisect each other. Opposite angles of parallelograms are equal and pairs of consecutive angles are supplementary.

II. 1. x = 75; 2. x = 6

III. Figure:



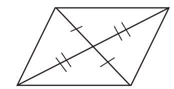
Proof:

Statements	Reasons
1. $\overline{CE} \parallel \overline{NI}$, $\overline{CE} \cong \overline{NI}$	1. Given
2. Draw segment from I to E	2. Two points determine a line.
3. $\angle CEI \cong \angle EIN$;	3. If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
4. $\overline{IE} \cong \overline{IE}$	4. Reflexive Property of Congruence
5. $\Delta CEI \cong \Delta NIE$	5. SAS Congruence
6. $\angle CIE \cong \angle NEI$	6. CPCTC
7. $\overline{AB} \cong \overline{DE}$	7. If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.
8. ▭ NICE is a parallelogram.	8. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

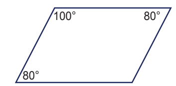
Remember that when proving a theorem, you cannot use that theorem as a reason in your proof.

Activity 17 PARALLELOGRAMS

I. Study the markings on the given figures and shade ☺ if it is a parallelogram and ⊘ if it is not. If your answer is ☺ state the definition or theorem that justifies your answer.

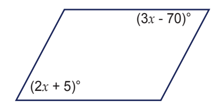
1. 

☺ ____________________
⊘ ____________________

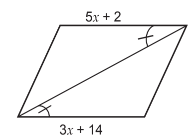
2. 

☺ ____________________
⊘ ____________________

II. What value of x will make each quadrilateral a parallelogram?

1. 

Solution:

2. 

Solution:

III. Show a complete proof:

Given: $\overline{CE} \parallel \overline{NI}$, $\overline{CE} \cong \overline{NI}$
Prove: ▭ NICE is a parallelogram.

Proof:

Answer Key

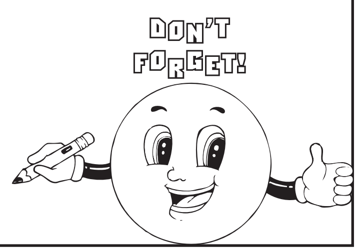
Activity 18

1. D	6. A	11. A
2. D	7. D	12. A
3. D	8. A	13. D
4. A	9. D	14. A
5. A	10. D	15. A

For #7, the isosceles trapezoid is a counterexample.

Check their answers and let them compare their answer with the first answer sheets they had. Do they have improvement in terms of their score? Again, let them keep the two answer sheets which will serve as an attachment for their Reflection Organizer.

Teacher's Note and Reminders



Activity 18 (REVISIT) AGREE OR DISAGREE!

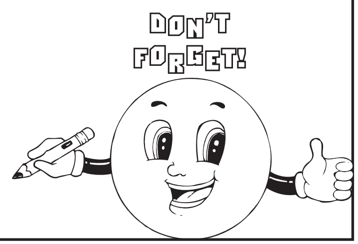
ANTICIPATION-REACTION GUIDE

Instruction: You were tasked to answer the first column during the earlier part of this module. Now, see how well you understood the lessons presented. Write A if you agree with the statement and write D if you disagree.

After-Lesson Response	TOPIC: Parallelism and Perpendicularity
	1. Lines that do not intersect are parallel lines.
	2. Skew lines are coplanar.
	3. Transversal lines are lines that intersects two or more lines.
	4. Perpendicular lines are intersecting lines.
	5. If two lines are parallel to a third line, then the two lines are parallel.
	6. If two lines are perpendicular to the same line, then the two lines are parallel.
	7. If one side of a quadrilateral is congruent to its opposite side, then the quadrilateral is a parallelogram.
	8. Diagonals of parallelogram bisect each other.
	9. Diagonals of parallelograms are congruent.
	10. Diagonals of parallelograms are perpendicular.
	11. Opposite sides of parallelograms are parallel.
	12. Opposite angles of a parallelogram are congruent.
	13. Consecutive angles of a parallelogram are congruent.
	14. Squares are rectangles.
	15. Squares are rhombi.

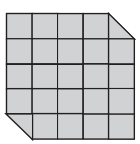
You can reproduce the map and let the students fill in the boxes, or you might want the students to create their own map reflecting the summary of the lesson on properties of parallelograms.

Teacher's Note and Reminders

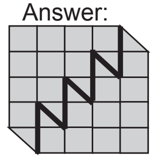


For the third time, let them complete the generalization table. Let them keep the three answer sheets for the Generalization Table and make sure they will keep all. These will also be attached to their Reflection Journal.

BRAIN TEASER ! (PARALLELOGRAM)
Dissect the picture into two sections from which you could rearrange the pieces to form a rectangle 6 x 4 squares.



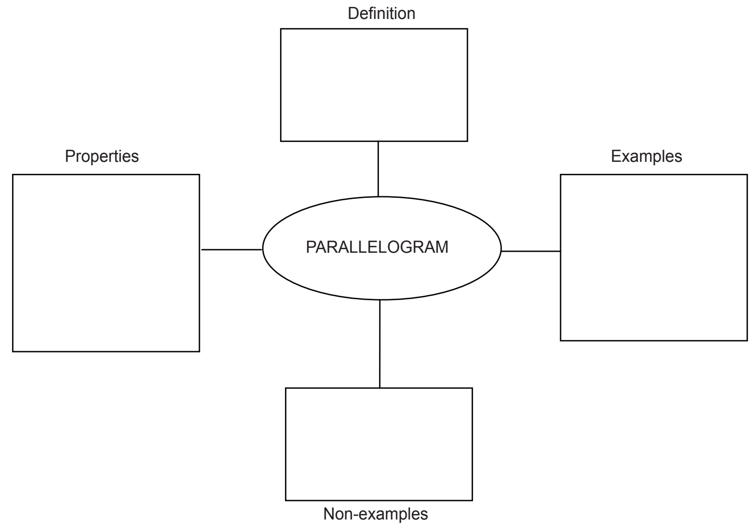
Answer:



Reference: http://brainden.com/geometry-puzzles.htm

Activity 19 CONCEPT MAPPING

Group Activity: Summarize the important concepts about parallelograms by completing the concept map below. Present and discuss them in a large group.



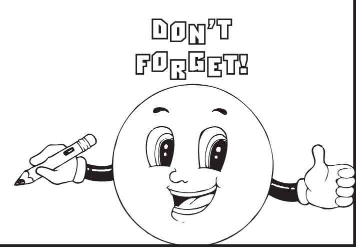
Activity 20 GENERALIZATION TABLE

After a lot of exercises, it's now time for you to fill in the last column of the generalization table below by stating your conclusions or insights about parallelism and perpendicularity.

"How can parallelism or perpendicularity of lines be established?"

				My Generalizations

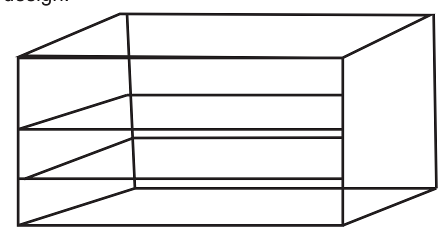
Teacher's Note and Reminders



Activity 21 DESIGN IT!

You are working in a furniture shop as designer. One day, your immediate supervisor asked you to make a design of a wooden shoe rack for a new client, who is a well-known artist in the film industry. In as much as you don't want to disappoint your boss, you immediately think of the design and try to research on the different designs available on the internet.

Below is your design:



QUESTIONS

1. Based on your design, how will you ensure that the compartments of the shoe rack are parallel? Describe the different ways to ensure that the compartments are parallel.

2. Why is there a need to ensure parallelism on the compartments? What would happen if the compartments are not parallel?

3. How should the sides be positioned in relation to the base of the shoe rack? Does positioning of the sides in relation to the base matter?

The copy of this summative test should be reproduced and distributed to each student. This test is a graded assessment and should be done individually.

Activity 22 SUMMATIVE TEST

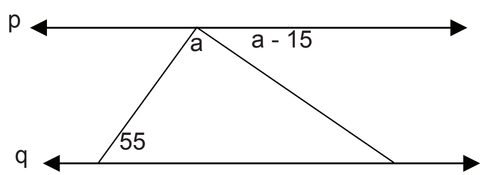
A. LEVEL 1! Determine whether the statement is ALWAYS TRUE (AT), SOMETIMES TRUE (ST) or NEVER TRUE (NT).

1. Parallelograms have diagonals that bisect each other.
2. A parallelogram with diagonals that are congruent is a square.
3. A parallelogram has diagonals that are perpendicular.
4. The adjacent sides of a parallelogram are congruent.
5. A rhombus has congruent sides and congruent angles.
6. The diagonals of a square are perpendicular bisectors of each other.
7. If two lines are perpendicular to the same line, then those lines are perpendicular.
8. A transversal intersects two or more other lines at a single point.
9. Lines that do not intersect are parallel.
10. Intersecting lines that form 90^0-angle are perpendicular lines.

B. LEVEL UP! Answer the following:

1. Draw two lines and a transversal such that $\angle 1$ and $\angle 3$ are corresponding angles, $\angle 1$ and $\angle 2$ are alternate interior angles and $\angle 3$ and $\angle 4$ are alternate exterior angles. What type of angle pair is $\angle 2$ and $\angle 4$?

2. What value of a will make lines p and q parallel?



Activity 22 SUMMATIVE TEST

The copy of the summative test will be given to you by your teacher. Do your best to answer all the items correctly. The result will be one of the bases of your grade.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

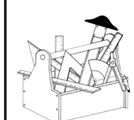
What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

This task challenges you to apply what you learned about parallel lines, perpendicular lines, parallelograms and the angles and segments related to these figures. Your work will be graded in accordance with the rubric presented.

Activity 23 DESIGNERS FORUM!

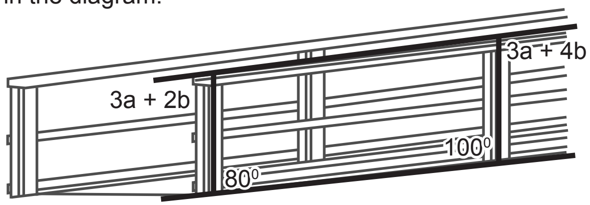
Scenario:



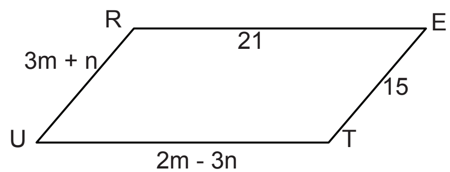
The Student Council of a school had a fund raising activity in order to put up a book case or shelf for the Student Council Office. You are a carpenter who is tasked to create a model of a book case/shelf using Euclidean tools (compass and a straight edge) and present it to the council adviser. Your output will be evaluated according to the following criteria: stability, accuracy, creativity and mathematical reasoning.

Goal – You are to create a model of a book case/shelf
Role – Carpenter
Audience – Council Adviser
Situation – The Student Council of a school had a fund raising activity in order to put up a book case or shelf for the Student Council Office.
Product – Book Case/Shelf
Standards – stability, accuracy, creativity, and mathematical reasoning.

3. The railing of a wheel chair ramp is parallel to the ramp. Find the value of a and b in the diagram.



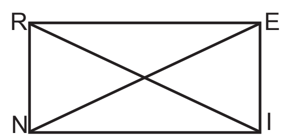
4. What values of m and n will make ▭ TRUE a parallelogram?



C. THE HIGHEST LEVEL! Present a proof (in any way you want) for the following problem.

Given: ▭ REIN is a rectangle with diagonals $\overline{RI}$ and $\overline{EN}$.

Prove: $\overline{RI} \cong \overline{EN}$.



Answer Key

Activity 22

A. LEVEL 1

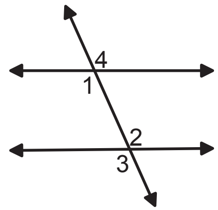
1. Always True
2. Sometimes True (Rectangles have congruent diagonals too.)
3. Sometimes True (Not all parallelograms have perpendicular diagonals.)
4. Sometimes True (True only for rhombus)
5. Sometimes True (True only for squares)
6. Always True
7. Never True
8. Never True
9. Sometimes True (Skew lines do not intersect yet not parallel.)
10. Always True

RUBRIC FOR THE PERFORMANCE TASK

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the key concepts of parallelism and perpendicularity of lines.	The computations are accurate and show the use of key concepts of parallelism and perpendicularity of lines.	The computations are erroneous and show some use of the key concepts of, parallelism and perpendicularity of lines.	The computations are erroneous and do not show the use of key concepts of parallelism and perpendicularity of lines.	
Stability	The model is well fixed and in its place.	The model is firm and stationary.	The model is less firm and show slight movement.	The model is not firm and has the tendency to collapse.	
Creativity	The design is comprehensive and displays the aesthetic aspects of the mathematical concepts learned.	The design is presentable and makes use of the concepts of geometric representations.	The design makes use of the geometric representations but not presentable.	The design doesn't use geometric representations and not presentable.	
Mathematical Reasoning	The explanation is clear, exhaustive or thorough and coherent. It includes interesting facts and principles. It uses complex and refined mathematical reasoning.	The explanation is clear and coherent. It covers the important concepts. It uses effective mathematical reasoning.	The explanation is understandable but not logical. Some evidence of mathematical reasoning.	The explanation is incomplete and inconsistent. Little evidence of mathematical reasoning.	
				OVERALL RATING	

B. LEVEL UP!

1.



(The two lines may not be parallel.)

$\angle 2$ and $\angle 4$ are corresponding angles.

2. $55 + a + a - 15 = 180$
   $a = 70$

3. Solving the two equations simultaneously, $3a + 2b = 80$ and $3a + 4b = 180$ will result to $b = 10$ and $a = 20$.

4. Do the same in no. 3 for the equations $3m + n = 15$ and $2m – 3n = 21$. This will result to $m = 6$ and $n = -3$.

C. TO THE HIGHEST LEVEL!

Statements	Reasons
1. ▭ REIN with diagonals $\overline{RI}$ and $\overline{EN}$.	Given
2. $\overline{RI} \cong \overline{EN}$	Opposite sides of a rectangle are congruent.
3. $\angle$RNI and $\angle$EIN are right angles.	Definition of a rectangle
4. $\angle$RNI $\cong$ $\angle$EIN	Right angles are congruent.
5. $\overline{NI} \cong \overline{NI}$	Reflexive Property
6. ΔRNI $\cong$ ΔEIN	SAS Postulate
7. $\overline{RI} \cong \overline{EN}$	Congruent Parts of Congruent Triangles are Congruent (CPCTC)

The proof can also be presented in a flowchart or paragraph form. Students may use other pairs of congruent triangles like ΔERN $\cong$ ΔINR, ΔNRE $\cong$ ΔIER, or ΔREI $\cong$ ΔNIE, as long as they have proven the corresponding parts to be congruent.

Activity 24 LESSON CLOSURE – REFLECTION ORGANIZER

You have accomplished the task successfully. This shows that you learned the important concepts in this module. To end this lesson meaningfully and to welcome you to the next module, I want you to accomplish this activity.

In this unit I learned about

These concepts can be used in

I understand that

These are important because

I can use the concepts of parallelism and perpendicularity in my life by

In this section, your task was to create a model of a book case using protractor compass and a straight edge and present it to the council adviser.

How did you find the performance task? How did the task help you see the real-world application of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the post assessment to evaluate your learning. Take time to answer the post assessment which will be given to you. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and study the module again.

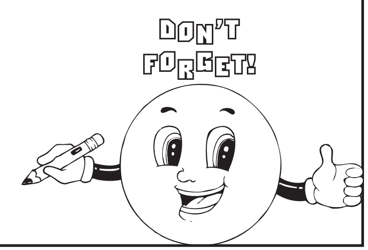
What to Transfer

The goal in this section is to apply their learning to real life situations. They will be given a practical task which will demonstrate their understanding. It is recommended that this activity will be done in pairs or in small groups. Provide a deadline for the activity and discuss also the rubric to be used in giving grade to their product.

The remaining days for this module will be allotted in making the performance task, accomplishing the Reflection Organizer and administering the Post Test. The copy of the Reflection Organizer should be reproduced and it is recommended that it will be submitted together with their answer sheets in Generalization Table (3 phases) and the Anticipation-Reaction Guide (2 phases) which were kept.

It’s now time to evaluate student's learning. In the post test let them choose the letter of the correct answer and write on a separate sheet. If they do well, they may move on to the next module. If their score does not meet expected level, they have to go back and take the module again.

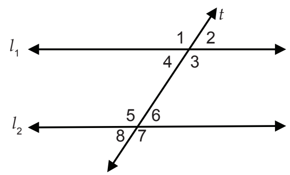
Teacher’s Note and Reminders



POST-ASSESSMENT:

Read each statement carefully. Choose the letter of the correct answer and write it on a separate sheet.

(K)1. Using the figure below, if $l_1 \parallel l_2$ and t is a transversal, then which of the following is true about the measures of ∠4 and ∠6?



a. The sum of the measures of ∠4 and ∠6 is 180°.
b. The measure of ∠4 is equal to the measure of ∠6.
c. The measure of ∠4 is greater than the ∠6.
d. The measure of ∠4 is less than the measure of ∠6.

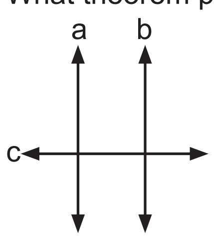
Answer: (b) ∠4 and ∠6 are alternate interior angle and there is a postulate which states that if two lines are cut by a transversal, the alternate interior angles are congruent.

(K)2. Which of the following statements is true?

a. A rhombus is a square.
b. A diagonal divides a square into two isosceles right triangles.
c. A diagonal divides a square into two congruent equilateral triangles.
d. A rectangle is a square.

Answer: (b) A diagonal divides a square into two isosceles right triangles.

(K)3. What theorem proves the following?



If $a \perp c$ and $b \perp c$, then $a \parallel b$.

a. Given a line and a point on the line, there is only one line through the given point that is perpendicular to the given line.
b. In a plane, if two lines are perpendicular to the same line, then the two lines are parallel.
c. Two lines are parallel if they do not intersect.
d. Two lines are perpendicular if they intersect at right angles.

Answer: (B) This is the theorem that explains the situation.

(S)4. Lines m and n are cut by transversal q. What value of x will make $m \parallel n$, given that $\angle 1$ and $\angle 4$ are corresponding angles and
$m\angle 1 = 5x - 11$ and
$m\angle 4 = 3x + 5$?
a. 6
b. 8
c. 10
d. 12
Answer: (b) Since the measures of corresponding angles are equal, thus
$m\angle 1 = m\angle 4$
$5x - 11 = 3x + 5$
$x = 8$

(S)5. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point E. If $m\angle BEC = 2x + 3$, then what is the value of x?

a. 43.5
b. 55
c. 77.5
d. 90

Answer: (a) If lines are perpendicular, they form right angles, thus
$2x + 3 = 90$
$x = 43.5$

(S)6. ▭ ALYS is a parallelogram. If $m\angle A$ is twice the measure of $m\angle L$, find the measure of $\angle Y$.

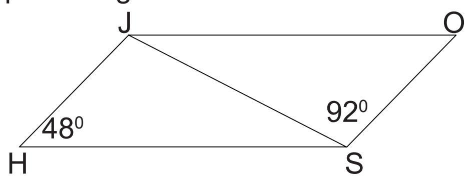
a. 60^0
b. 90^0
c. 120^0
d. 150^0
Answer: (c) $\angle A$ and $\angle L$ are supplementary as implied in the properties of parallelogram, we have $2m\angle L + m\angle L = 180$, $m\angle L = 60$ and $m\angle A = 120$. $\angle A$ and $\angle Y$ are opposite angles, therefore they have the same measure.

(S)7. $\angle 1$ and $\angle 2$ are non-adjacent exterior angles on the same side of a transversal. If $m\angle 1 = 2x + 25$ and $m\angle 2 = 3x + 15$, find the measure of $\angle 2$.

a. 28^0
b. 56^0
c. 81^0
d. 99^0

Answer: (d) $\angle 1$ and $\angle 2$ are supplementary angles.

(S)8. ▱ JOSH is a parallelogram.



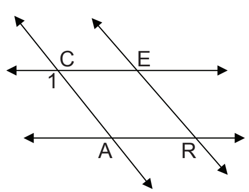
What is the measure of $\angle$JSH?

a. 40°
b. 48°
c. 92°
d. 94°

Answer: (a) If a quadrilateral is a parallelogram, then the sum of the measures of consecutive angles is 180°. Let y be the measure of $\angle$JSH, thus

$$48° + y + 92° = 180°$$
$$y = 40°$$

(U)9. In the figure below, $\overleftrightarrow{AR} \parallel \overleftrightarrow{CE}$ and $\overleftrightarrow{CA} \parallel \overleftrightarrow{RE}$. If $m\angle 1 = 110°$, then what is the measure of $\angle$ERA?



a. 10°
b. 20°
c. 70°
d. 180°

Answer: (c) $\angle$ERA is a supplement of $\angle$CAR which forms alternate interior angles with $\angle 1$.

(U)10. Which of the following statements is not sufficient to prove that a quadrilateral is a parallelogram?

a. The diagonals are perpendicular.
b. The diagonals bisect each other.
c. Pairs of opposite angles are congruent.
d. Pair of opposite sides is congruent and parallel.

Answer: (a) In the case of a kite, though its diagonals are perpendicular, kite is not a parallelogram. Moreover, a rectangle is a parallelogram but its diagonals are not perpendicular.

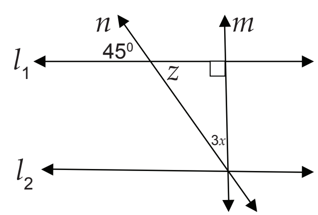
(U)11. In ▭ RICH, $\overline{RI} = 4x - 7$ cm, $\overline{IC} = 4x - 9$ cm, $\overline{CH} = 3x + 2$ cm and $\overline{RH} = 2x + 9$ cm. What value of x will make quadrilateral RICH a parallelogram?

a. 3
b. 5
c. 7
d. 9

Answer: (d) Since opposite sides of a parallelogram are congruent, equate any two opposite side then solve for x.

(U)12. In the figure below, l_1 and l_2 are cut by transversals m and n. What value of x will make $m \perp l_2$?

a. 6
b. 9
c. 12
d. 15



Answer: (d) $m\angle z$ = 45 also because it forms vertical angles with the given angle, thus
$3x + 45 = 90$ (acute angles of a right triangle are complementary)
$x = 15$

(U) 13. All of the following figures illustrate parallel lines except:

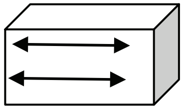


Figure 1

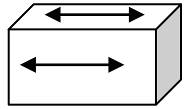


Figure 2

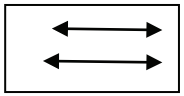


Figure 3

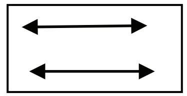


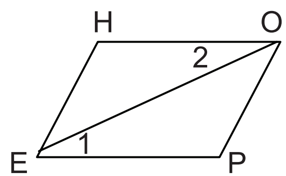
Figure 4

a. Figure 1
b. Figure 2
c. Figure 3
d. Figure 4

Answer: (b) The lines are not on the same plane.

(U)14. Choose the correct reason for the last statement to complete the two-column proof.

Given:
$\overline{HO} \cong \overline{EP}$
$\angle 1 \cong \angle 2$



Prove: Quadrilateral HOPE is a parallelogram.

Statements	Reasons
1. $\overline{HO} \cong \overline{EP}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
2. $\overline{HO} \parallel \overline{EP}$	2. If 2 lines cut by a transversal form congruent alternate interior angles then the 2 lines are parallel.
3. ▭ HOPE is a parallelogram.	3. If a quadrilateral ___.

a. has a pair of opposite sides that are congruent and parallel, then it is a parallelogram.
b. has a pair of congruent interior angles, then the quadrilateral is a parallelogram.
c. has a pair of congruent opposite angles, then the quadrilateral is a parallelogram.
d. has a diagonal that divides the quadrilateral into two congruent triangles, then the quadrilateral is a parallelogram.

Answer: (a) Since it is proven that $\overline{HO} \parallel \overline{EP}$ and it is given that they are equal, therefore the quadrilateral is a parallelogram.

(P)15. A contractor tacked one end of a string to each vertical edge of a window. He then handed a protractor to his apprentice to find out if the vertical edges are parallel. What should the apprentice do?

a. Measure the angles formed by the string and the vertical edge on both ends.
b. Measure the length of the string and the edge of the window.
c. Measure the length of the string and the horizontal edge of the window.
d. Measure the diagonal of the window and the angle formed by the edges of the window.

Answer: (a) Measure the angles formed by the string and the vertical edge on both ends.

(P)16. How would one construct a rhombus by using a protractor and a ruler or a double-edged straightedge?
a. Draw two intersecting segments and connect their endpoints.
b. Draw two perpendicular segments and connect their endpoints.
c. Draw two bisecting segments and connect their endpoints.
d. Draw two perpendicular and bisecting segments and connect their endpoints.

Answer: (d) The diagonals of a rhombus are perpendicular and bisect each other.

(P)17. As a design expert, a certain furniture shop invited you to conduct a mini-seminar on a topic entitled: *"Ensuring Stability of Furniture."* This seminar aims to orient the workers of the furniture shop on how they will ensure the stability of their product. Which one should you give emphasis in your talk?
a. accuracy of measures, parallelism, and perpendicularity of parts
b. attractive colors and accuracy of measures
c. parallelism of parts and quality of materials
d. perpendicularity of parts and quality of materials

Answer: (a) Accuracy of measures, parallelism and perpendicularity of parts ensure stability of a furniture.

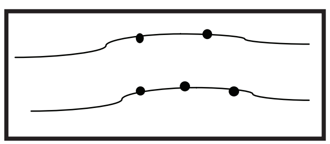
(P)18. You are tasked to sketch a plan of a parking lot of a mall. Which of the following should you include in the plan in order to maximize the use of the area?
  a. landscaping designs
  b. use of parallel lines
  c. entrance art design
  d. use of different shapes

Answer: (b) Key concepts about parallelism ensure maximizing the space allotted.

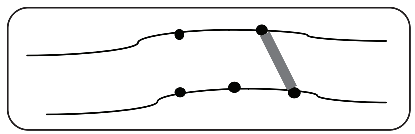
(P)19. Michael is repairing a wooden clothes stand with damaged legs. Which action should he consider?
  a. Check if the clothes stand is high enough for the lengthy garments.
  b. Check if the legs of the clothes stand are parallel to one another.
  c. Check if the distance between legs is greater than the length of the base.
  d. Check if the length of the base is the same as the length of the legs.

Answer: (b) Placing the legs of the clothes stand parallel to one another will make it more stable.

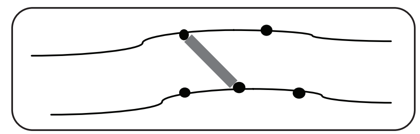
(P)20. An engineer is tasked to submit a design of a two-lane bridge in one of the *barangays* of General Santos City. The length of the bridge affects the entire construction cost. Considering the sketch below, which of the following drawings would he make?



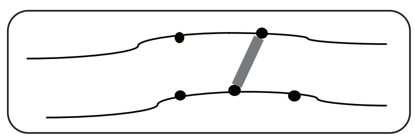
a.



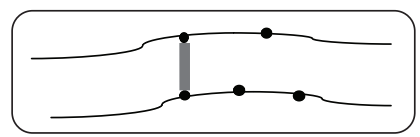
c.



b.



d.



Answer: (d) The shortest segment (bridge) is the distance perpendicular to the river banks.

GLOSSARY OF TERMS USED IN THIS LESSON:

1. Adjacent Sides

   These are two non-collinear sides with a common endpoint.

2. Alternate Exterior Angles

   These are non-adjacent exterior angles that lie on opposite sides of the transversal.

3. Alternate Interior Angles

   These are non-adjacent interior angles that lie on opposite sides of the transversal.

4. Consecutive Angles

   These are two angles whose vertices are the endpoints of a common(included) side.

5. Consecutive Vertices

   These are the vertices which are endpoints of a side.

6. Corresponding Angles

   These are non-adjacent angles that lie on the same side of the transversal, one interior angle and one exterior angle.

7. Deductive Reasoning

   It is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

8. Flowchart-Proof

   It is a series of statements in a logical order, starting with the given statements. Each statement together with its reason written in a box, and arrows are used to show how each statement lead to another. It can make ones logic visible and help others follow the reasoning.

9. Kite

   It is a quadrilateral with two distinct pairs of adjacent congruent sides and no opposite sides congruent.

10. Opposite Angles of a quadrilateral
    These are two angles which do not have a common side.

11. Opposite Sides of a quadrilateral
    These are the two sides that do not have a common endpoint.

12. Paragraph or Informal Proof
    It is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

13. Parallel lines
    Parallel lines are coplanar lines that do not intersect.

14. Parallelogram
    It is a quadrilateral with both pairs of sides parallel and congruent.

15. Perpendicular Bisector
    It is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint.

16. Perpendicular lines
    These are lines that intersect at 90^0- angle.

17. Proof
    It is a logical argument in which each statement you make is justified by a statement that is accepted as true.

18. Rectangle
    It is a parallelogram with four right angles.

19. Rhombus
    It is a parallelogram with four congruent sides.

20. Same-Side Interior Angles
    These are consecutive interior angles that lie on the same side of the transversal.

21. Same-Side Exterior Angles

    These are consecutive exterior angles that lie on the same side of the transversal.

22. Skew Lines

    Skew lines are non-coplanar lines that do not intersect.

23. Square

    It is a parallelogram with four congruent sides and four right angles.

24. Transversal

    It is a line that intersects two or more coplanar lines at different points.

25. Trapezoid

    It is a quadrilateral with exactly one pair of parallel sides.

26. Two-Column Form/Formal Proof

    It is the most formal proof with statements and reasons. The first column is for the statements and the other column for the reason.

POSTULATES OR THEOREMS IN PROVING LINES PARALLEL:

1. Given two coplanar lines cut by a transversal, if corresponding angles are congruent, then the two lines are parallel. (CACP)

2. Given two lines cut by a transversal, if alternate interior angles are congruent, then the lines are parallel. (AICP)

3. If two lines are cut by a transversal such that the alternate exterior angles are congruent, then the lines are parallel. (AECP)

4. Given two lines cut by a transversal, if same side interior angles are supplementary, then the lines are parallel. (SSIASP)

5. If two lines are cut by a transversal so that exterior angles on the same side of the transversal are supplementary, then the lines are parallel. (SSEASP)

6. In a plane, if two lines are both parallel to a third line, then they are parallel.

7. If two coplanar lines are perpendicular to a third line, then they are parallel to each other.

THEOREMS INVOLVING PERPENDICULAR LINES:

1. If two lines are perpendicular, then they form four right angles.

2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.

3. In a plane, through a point on a given line there is one and only one line perpendicular to the given line.

4. In a plane, a segment has a unique perpendicular bisector.

5. If two angles are adjacent and complementary, then their non-common sides are perpendicular.

6. In a plane, if the non-common sides of adjacent angles are perpendicular, then the angles are complementary.

DEFINITIONS AND THEOREMS INVOLVING PARALLELOGRAMS

Given a parallelogram, related definition and theorems are stated as follows:

1. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. If a quadrilateral is a parallelogram, then two pairs of opposite sides are congruent.
3. If a quadrilateral is a parallelogram, then two pairs of opposite angles are congruent.
4. If a quadrilateral is a parallelogram, then the consecutive angles are supplementary.
5. If a quadrilateral is a parallelogram, then the diagonals bisect each other.
6. If a quadrilateral is a parallelogram, then the diagonals form two congruent triangles.

To prove a parallelogram, related definition and theorems are stated as follows: (Many of these theorems are converses of the previous theorems.)

1. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

3. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
4. If one angle is supplementary to both consecutive angles in a quadrilateral, then the quadrilateral is a parallelogram.
5. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
6. If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

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These sites provide exercises and review in the relationships of the different angles formed by parallel lines cut by a transversal.

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These sites provide an educational video presentation about parallel lines.

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Author: Lois Edward
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This site provides a reference of the concept map.

Teacher's Note and Reminders