Teaching Math to Children with Special Needs

Joan A. Cotter, Ph.D.
JoanCotter@RightStartMath.com

Sioux Empire Christian Home Educators
Homeschool Conference
Sioux Falls, SD
Saturday, May 2, 2015
3:00 p.m.– 4:00 p.m.

Myths about LD

- People with LD have lower intelligence.
  (33% are gifted.)
- They are lazy or stubborn.
- Children with LD can be cured or will outgrow it.
- Boys are more likely to be affected.
  (Girls are equally affected.)
- Dyslexia and learning disability are the same thing.
- Students with LD and ADHD cannot succeed in higher education.

Characteristics of LD

Child usually:

- Is more creative.
- Cannot learn by rote.
- Must understand and make sense of a concept in order to remember it.
- Is more visual and hands-on.
- Dislikes worksheets.
- Finds it very difficult to unlearn.

Problems Occurring with Math

Dyscalculia

- Reversals in writing numbers
- Poor number sense
- Slow fact retrieval
- Errors in computation
- Difficulty in solving word problems

Dyscalculia mainly affects arithmetic, not other branches of math.

How Math is Traditionally Taught

- Counting
  - Learn sequence (number names by heart)
  - One-to-one correspondence (one count per object)
  - Cardinality principle (last number tells how many)
- Memorizing facts
  - Flash cards and timed tests
  - Rhymes and songs
- Memorizing algorithms (procedures)
- Using key words to solve story problems

Traditional Counting

From a child's perspective

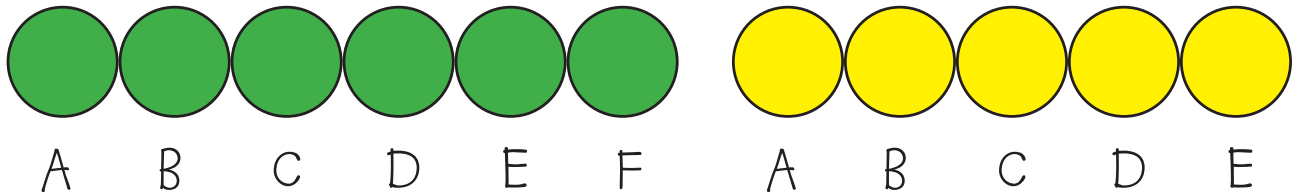
Because we're so familiar with 1, 2, 3, we'll use letters of the alphabet.

A = 1
B = 2
C = 3
D = 4
E = 5, and so forth

Traditional Counting

From a child's perspective

F + E =

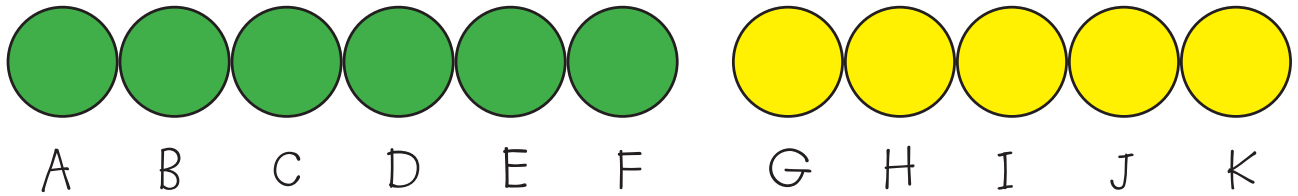


What is the sum?

Traditional Counting

From a child's perspective

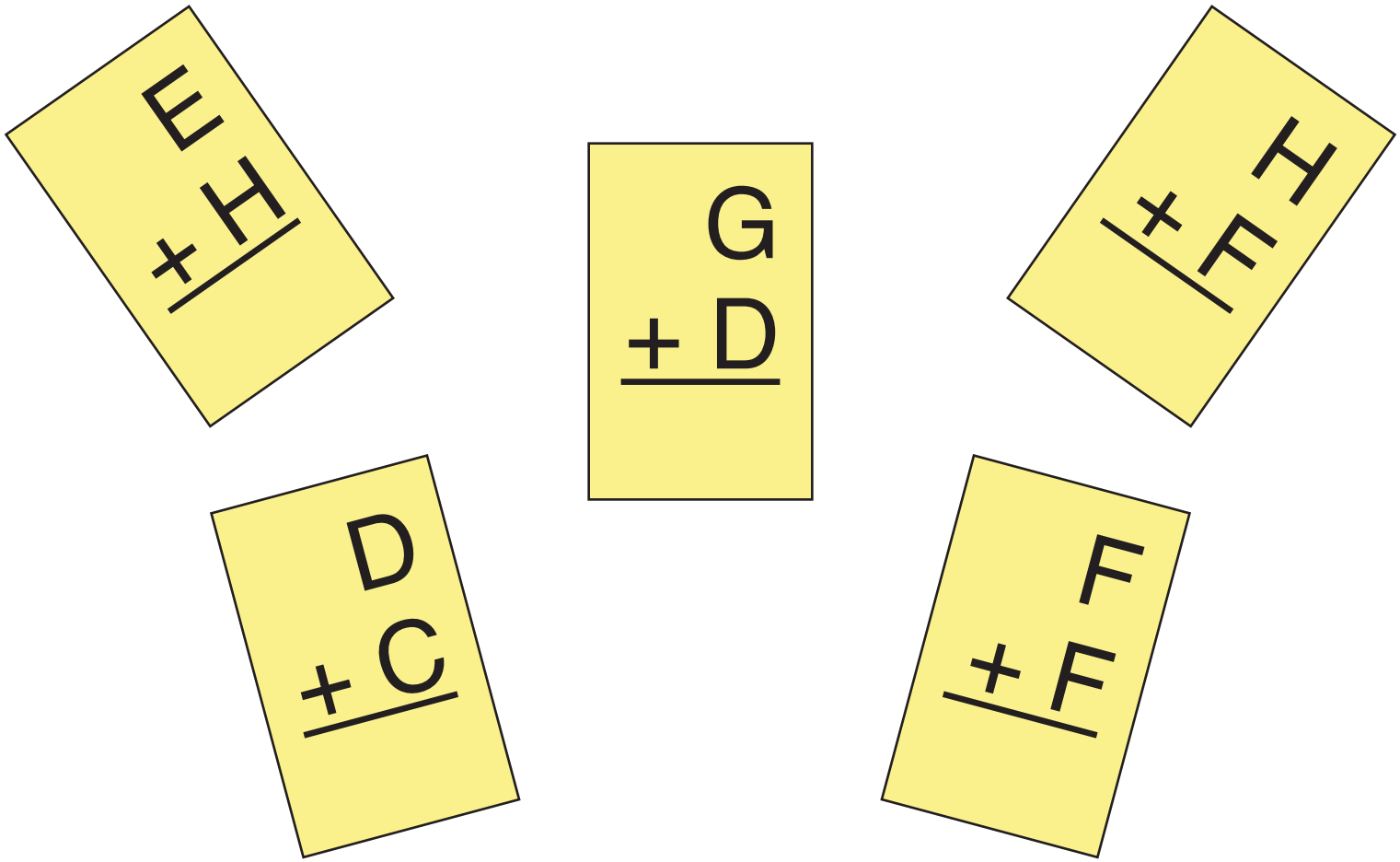
F + E = K



Traditional Counting

From a child's perspective

Now memorize the facts!!



Traditional Counting

From a child's perspective

H – C =

Try subtracting by "taking away."

Traditional Counting

From a child's perspective

Try skip counting by B's to T:

B, D, . . . , T.

What is D × E?

Compared to Reading

Just as reciting the alphabet doesn't teach reading, counting doesn't teach arithmetic.

Counting-Based Arithmetic

- Rote counting to 100 in kindergarten.
- Calendars misused to teach counting.
- Counting on for addition. (Jack and Jill)
- Counting back for subtraction.
- Number lines are abstract counting.
- Skip counting for multiplication facts.
- Does not work well for money or fractions or reading graphs.

Memorizing Math

	Percentage Recall		
	Immediately	After 1 day	After 4 weeks
Rote	32	23	8
Concept	69	69	58

Math needs to be taught so 95% is understood and only 5% memorized.

–Richard Skemp

Memorizing Math

According to a study with college students, it took them:

- 93 minutes to learn 200 nonsense syllables.
- 24 minutes to learn 200 words of text.
- 10 minutes to learn 200 words of poetry.

Flash Cards and Timed Tests

- Often used to teach rote.
- Liked only by those who don't need them.
- Give the false impression that math does not require thinking.
- Often produce stress – children under stress stop learning.
- Not concrete – use abstract symbols.
- Cause stress (may become physically ill).
- Result in short-term learning.
- May lead to math anxiety.

Published May, 2014

Visualizing

A pilot study of the effects of RightStart instruction on early numeracy skills of children with specific language impairment

Riikka Mononen, Pirjo Aunio, Tuire Koponen

Abstract:
....The children with SLI [specific language impairment] began kindergarten with significantly weaker early numeracy skills compared to their peers. Immediately after the instruction phase, there was no significant difference between the groups in counting skills....

Visualizing

- *Visual* is related to seeing.
- *Visualize* is to form a mental image.

Visualizing

Visualizing is also needed in other fields:

- Reading
- Sports
- Arts
- Geography
- Engineering
- Construction
- Biology
- Architecture
- Astronomy
- Archeology
- Chemistry
- Physics
- Surgery
- History

Visualizing

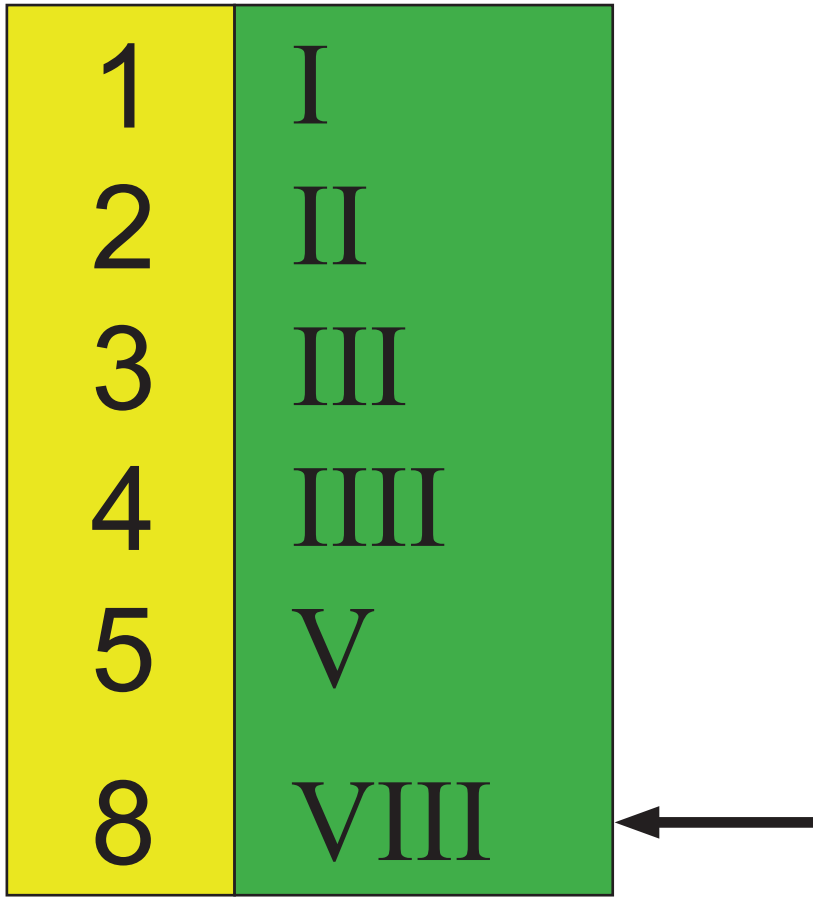
Try to visualize 8 identical apples without grouping.

Visualizing

Now try to visualize 8 apples: 5 red and 3 green.

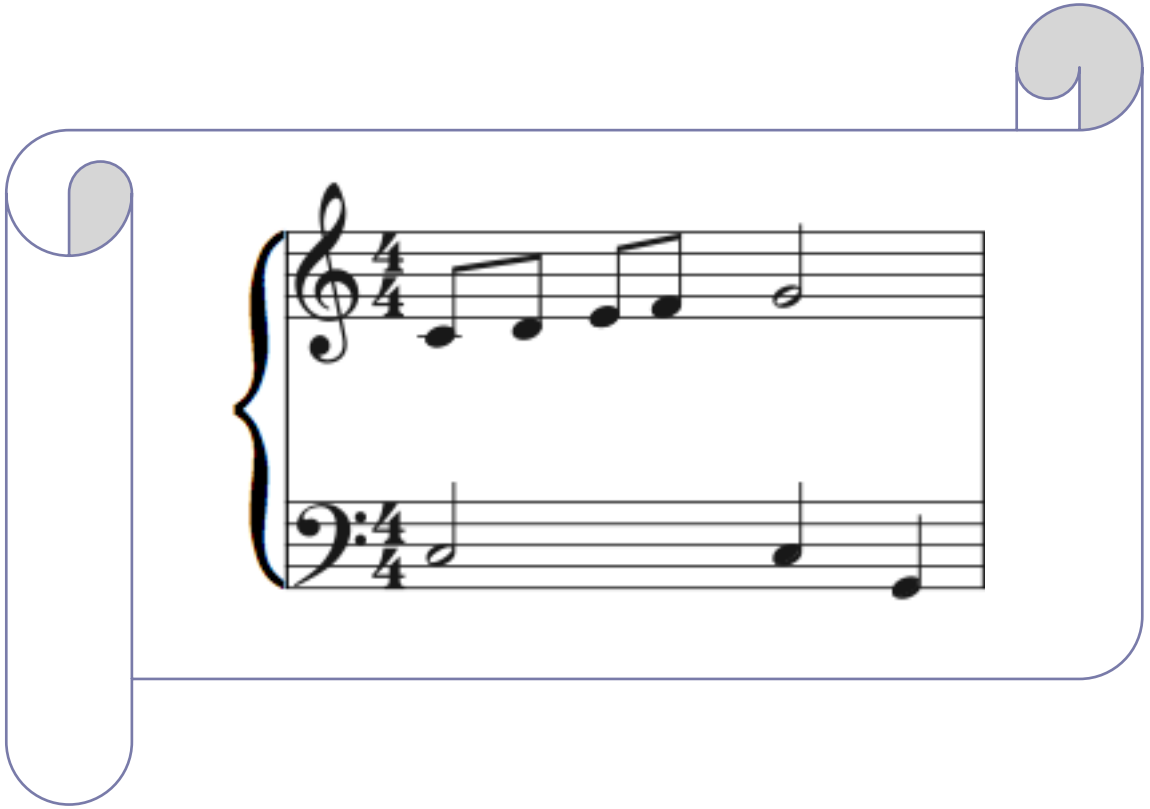
Grouping in Fives

Early Roman numerals



Grouping in Fives

Musical staff



Grouping in Fives

Clocks and nickels

Grouping in Fives

Tally marks

Grouping in Fives

Subitizing

- Instant recognition of quantity is called *subitizing*.
- Grouping in fives extends subitizing beyond five.

Subitizing

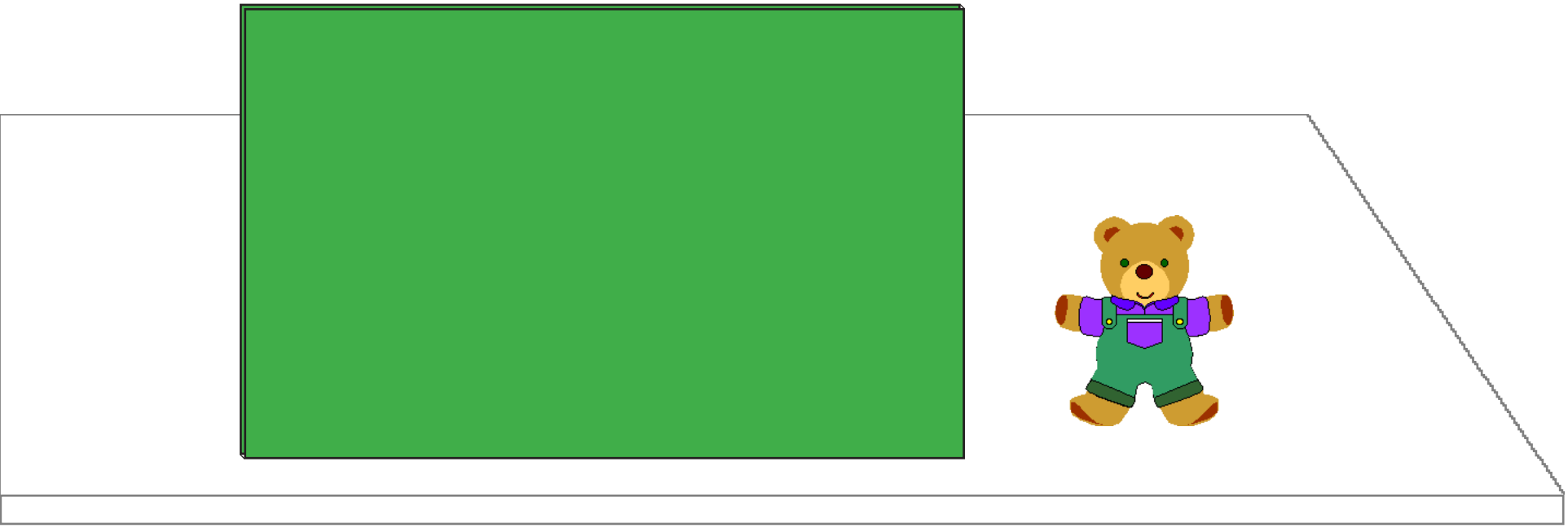
- Five-month-old infants can subitize to 1–3.
- Three-year-olds can subitize to 1–5.
- Four-year-olds can subitize 1–10 by grouping with five.

Research on Subitizing

Karen Wynn's research

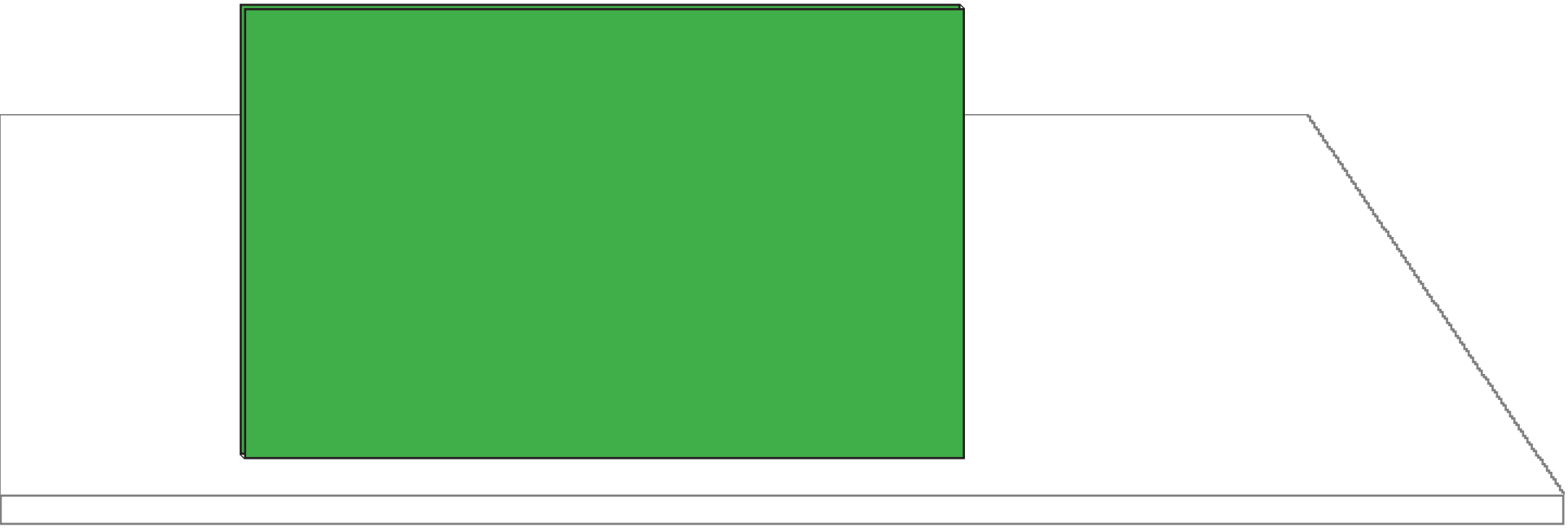
Research on Subitizing

Karen Wynn's research



Research on Subitizing

Karen Wynn's research



Research on Subitizing

Karen Wynn's research

Research on Subitizing

Karen Wynn's research

You could say subitizing is much more "natural" than counting.

Research on Subitizing

In Japanese schools

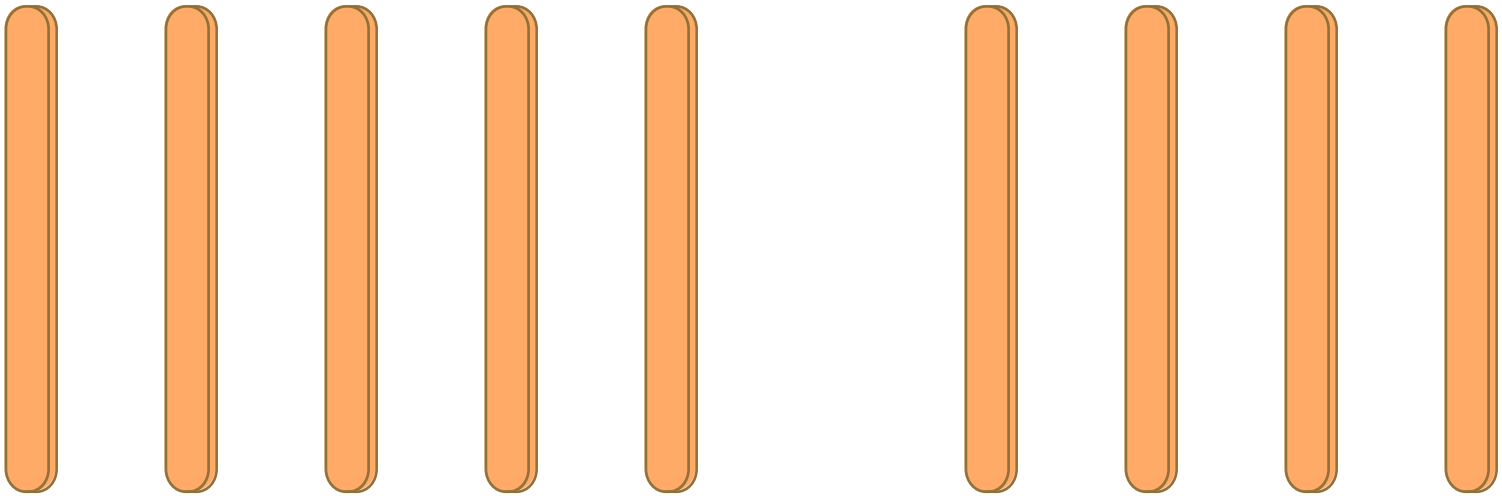
- Children are discouraged from using counting for adding.
- They consistently group in fives.

Quantities 1–10

Using fingers

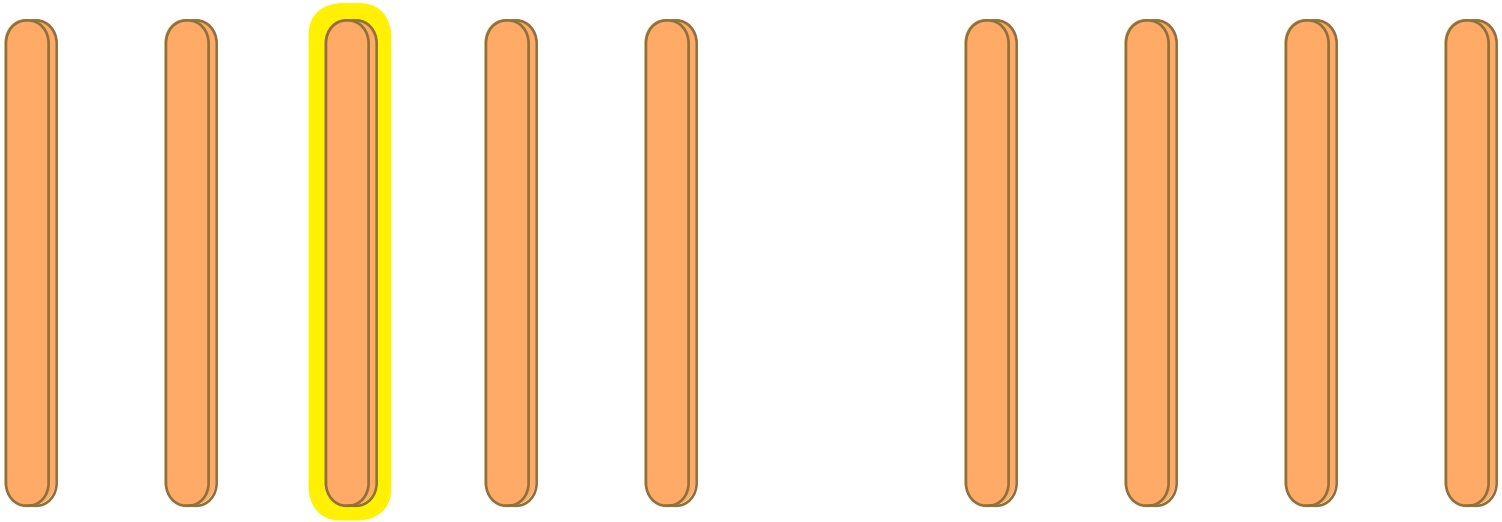
Quantities 1–10

Subitizing five



Quantities 1–10

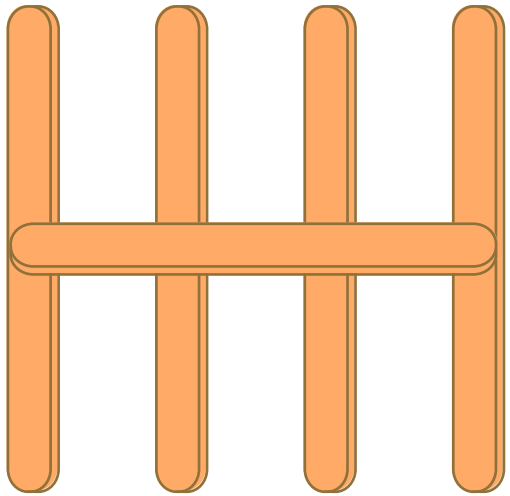
Subitizing five



5 has a middle; 4 does not.

Quantities 1–10

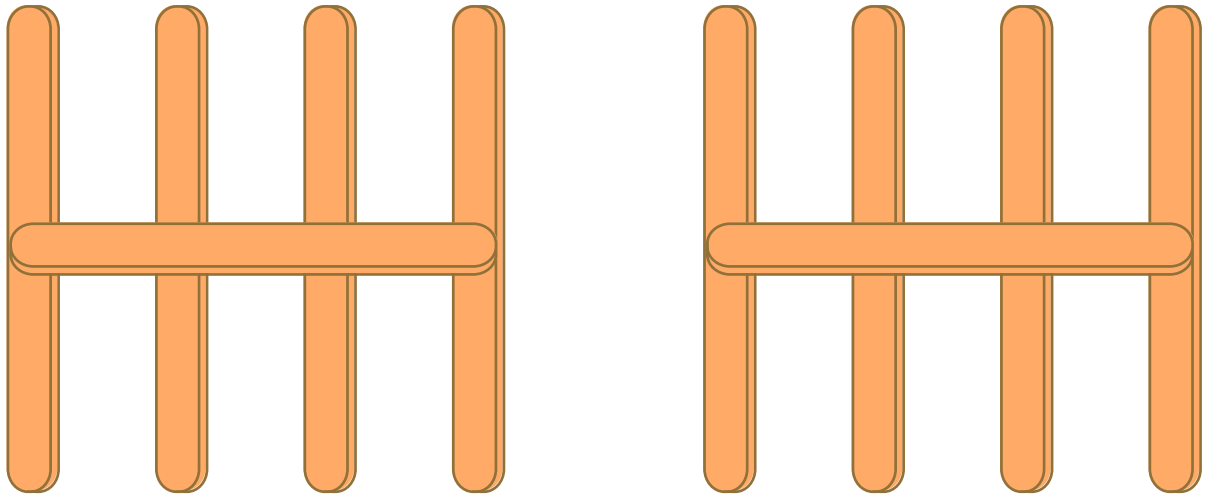
Tally sticks



Five as a group.

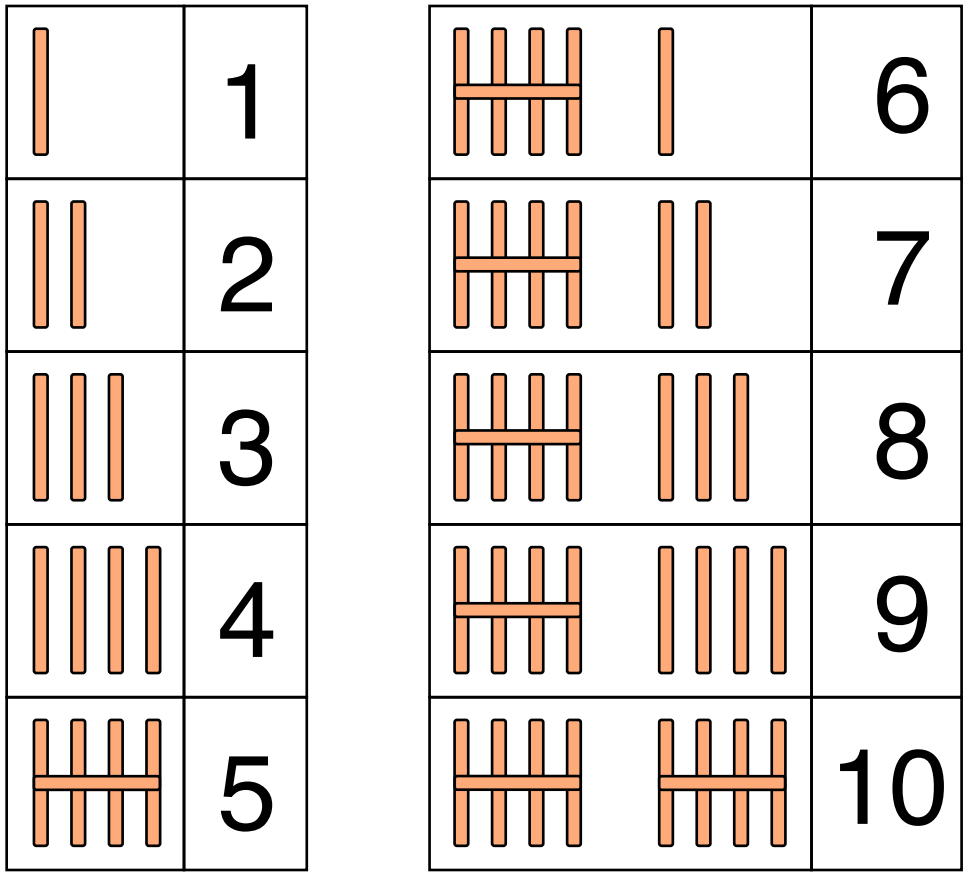
Quantities 1–10

Tally sticks



Quantities 1–10

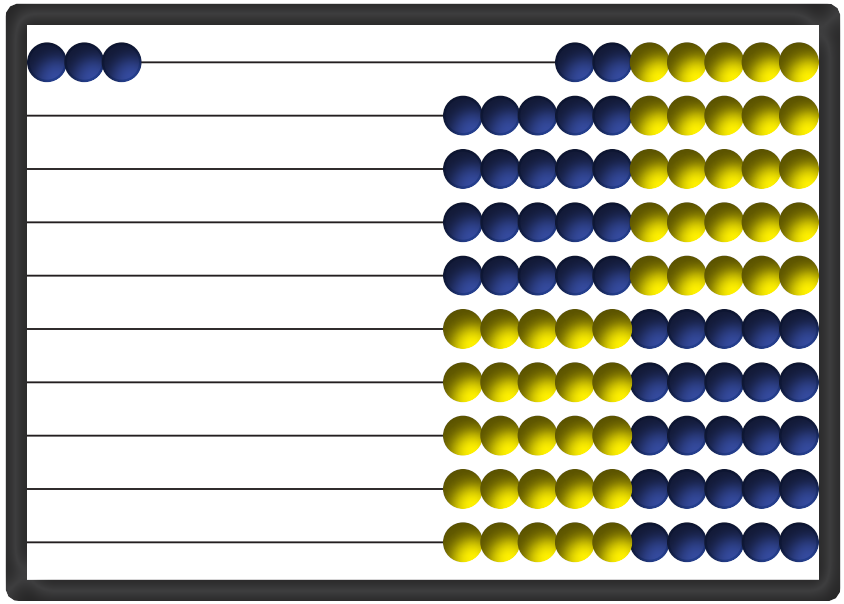
Number chart for remembering numerals



Quantities 1–10

Entering quantities

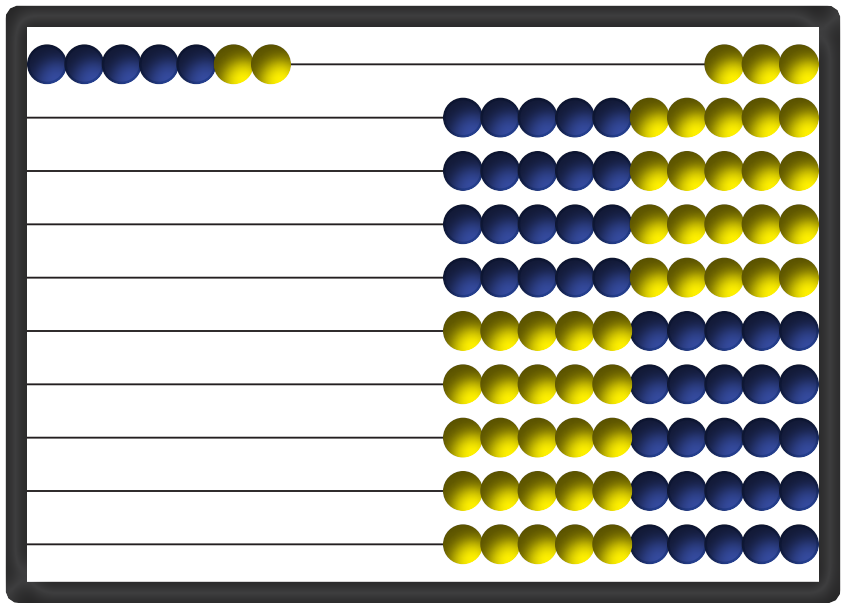
3



Quantities 1–10

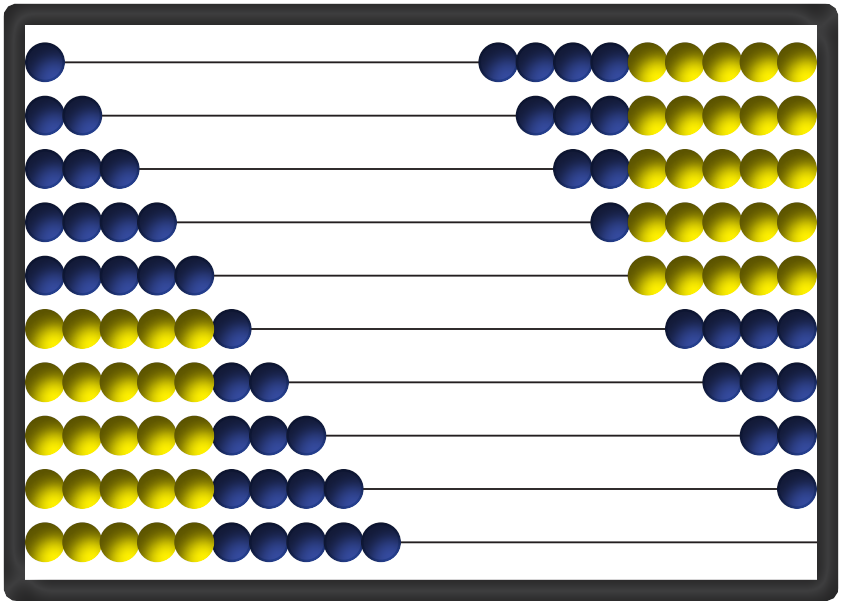
Entering quantities

7



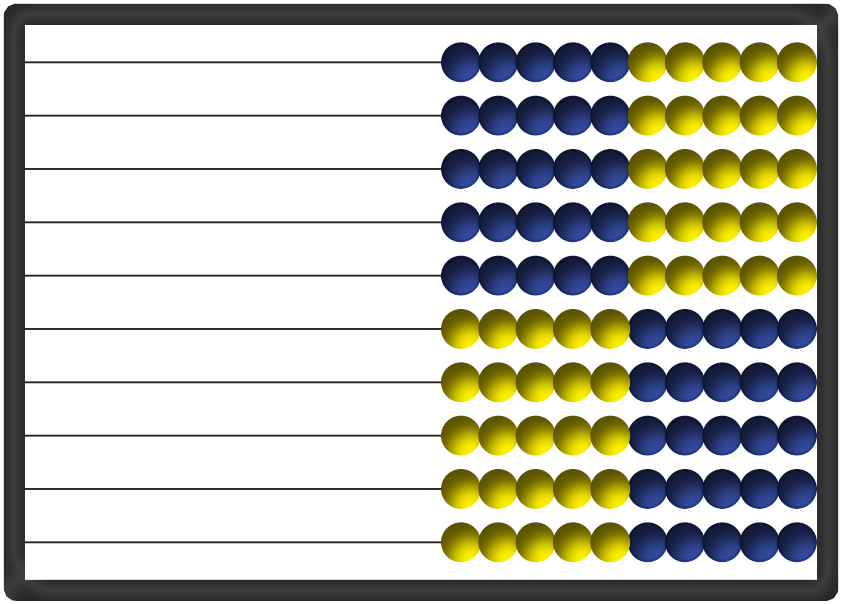
Quantities 1–10

Stairs



Quantities 1–10

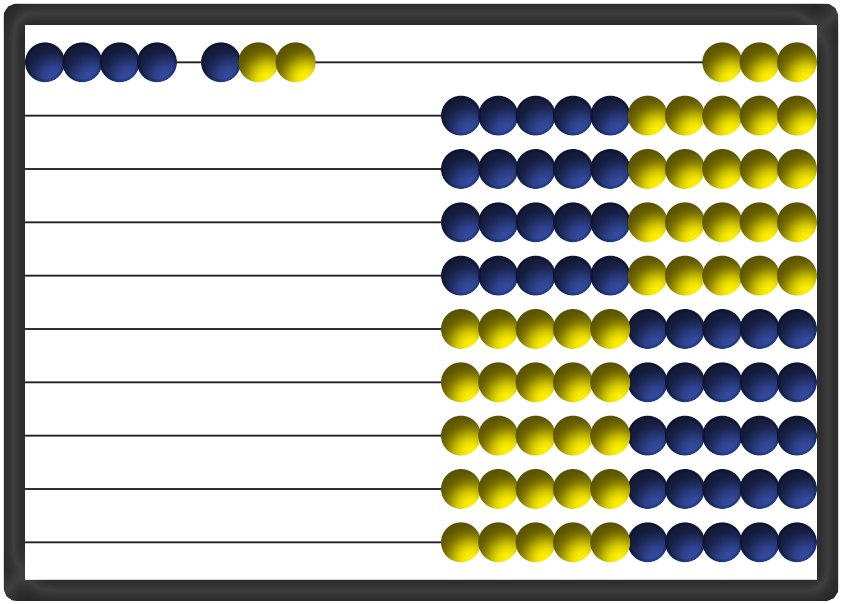
Adding



Quantities 1–10

Adding

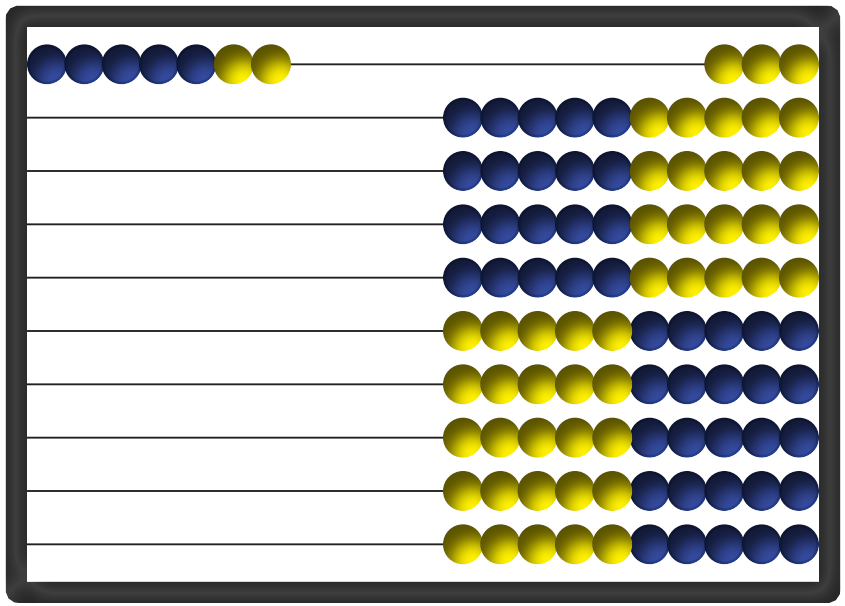
4 + 3 = __



Quantities 1–10

Adding

4 + 3 = 7



Japanese children learn to do this mentally.

Games

$$\frac{\text{Games}}{\text{Math}} = \frac{\text{Books}}{\text{Reading}}$$

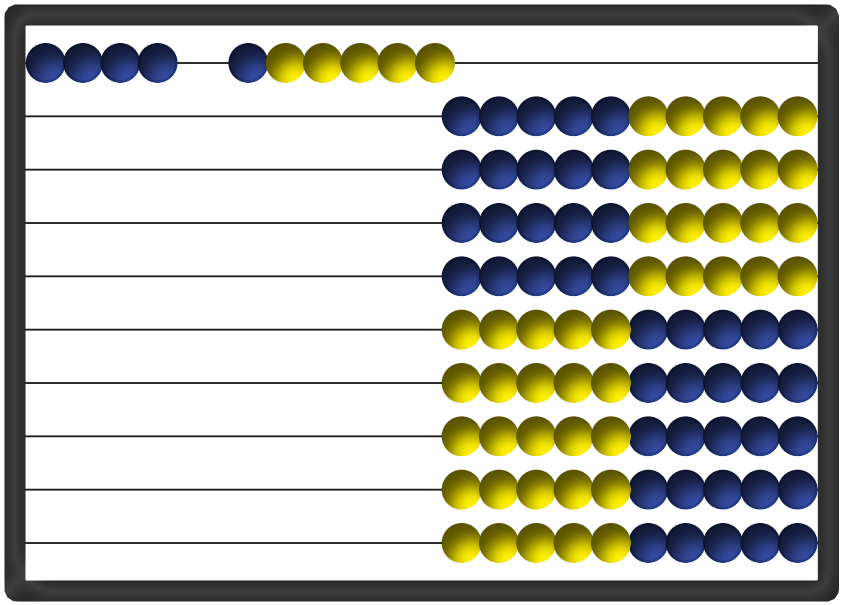
- Games provide interesting repetition needed for automatic responses in a social setting.
- Games provide an application for new information.
- Games provide instant feedback.

Games

Go to the Dump

Objective: To learn the facts that total 10:

1 + 9
2 + 8
3 + 7
4 + 6
5 + 5



It is played similar to Go Fish.

Place Value

The problem

- English-speaking children often think of 14 as 14 ones, not 10 and 4 ones.
- The pattern that is needed to make sense of tens and ones is hidden!

Place Value

Its importance

- Place value is the foundation of modern arithmetic.
- It is critical for understanding algorithms.
- It must be taught, not left for discovery.
- Children need the big picture, not tiny snapshots.

Transparent Number Naming

11 = ten 1
12 = ten 2
13 = ten 3
14 = ten 4
. . . .
19 = ten 9

20 = 2-ten
21 = 2-ten 1
22 = 2-ten 2
23 = 2-ten 3
. . . .
. . . .
99 = 9-ten 9

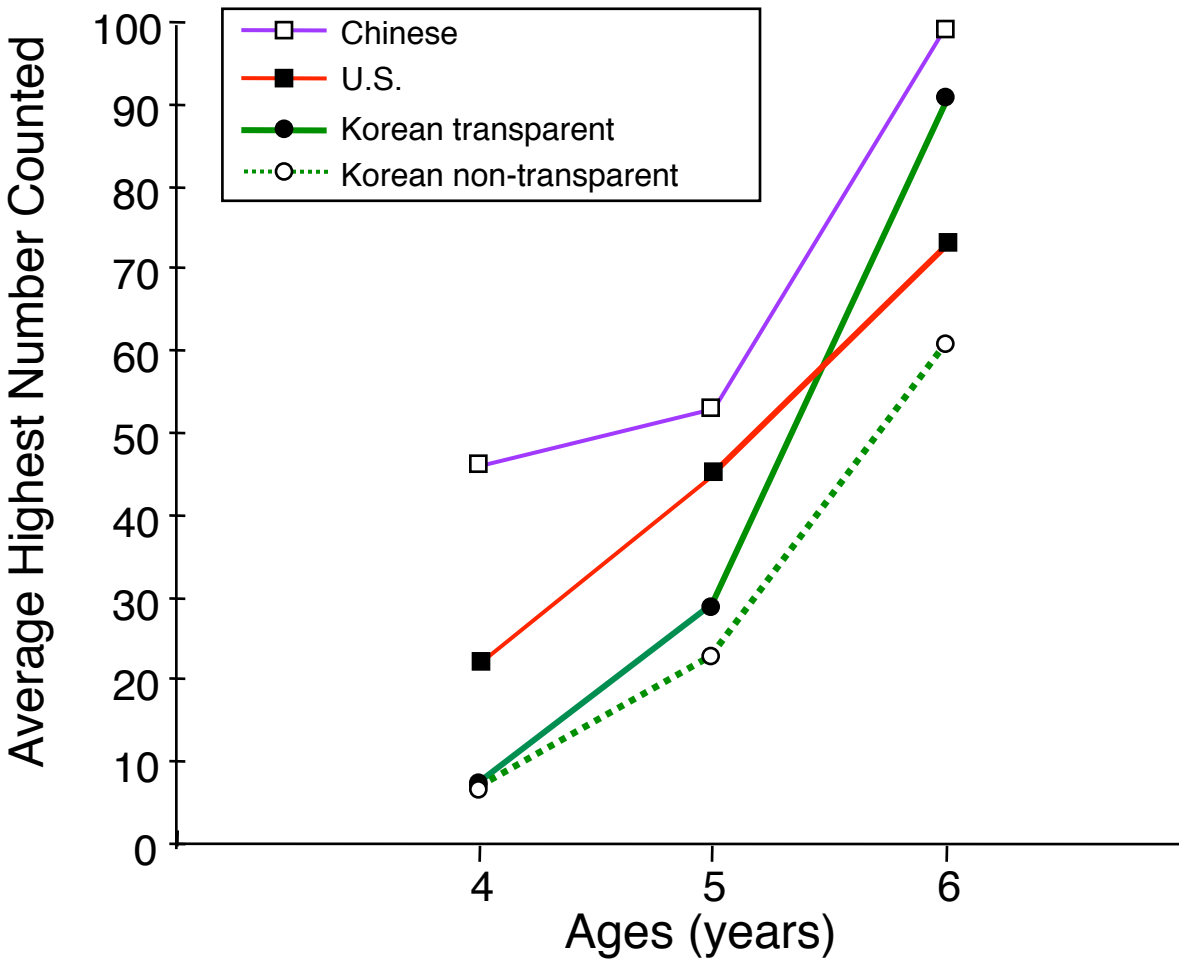
Transparent Number Naming

137 = 1 hundred 3-ten 7

or

137 = 1 hundred and 3-ten 7

Transparent Number Naming



Song, M., & Ginsburg, H. (1988). p. 326. The effect of the Korean number system on young children's counting: A natural experiment in numerical bilingualism. *International Journal of Psychology,* 23, 319-332.

Math Way of Number Naming

- Only 11 words are needed to count to 100 the math way, 28 in English. (All Indo-European languages are non-standard in number naming.)
- Asian children learn mathematics using transparent number naming.
- Mathematics is the science of patterns. Number names need to be an example.
- Children who are hearing-impaired can distinguish between 14 and 40, and 13 and 30.
- Learning two languages helps brain development.

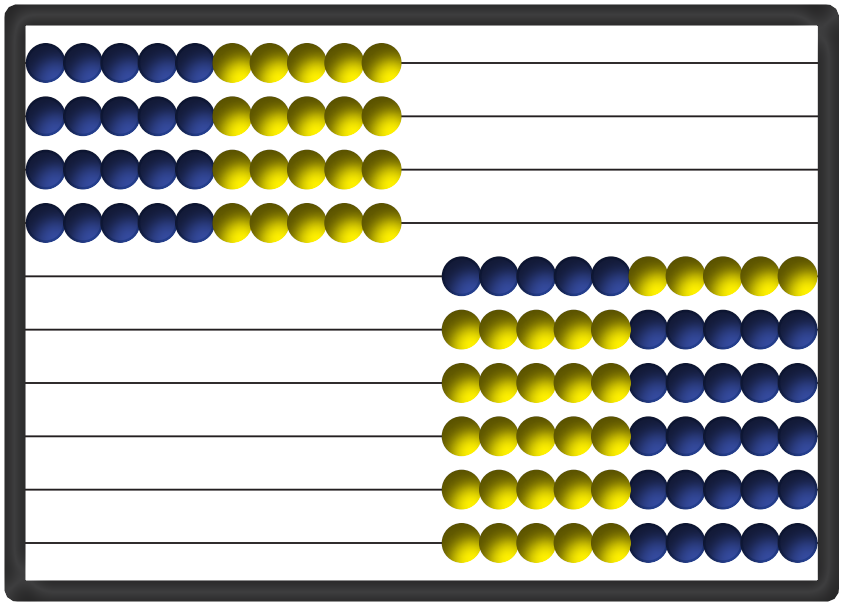
Compared to Reading

Just as we first teach the *sound* of the letters, we must first teach the *name* of the quantity, the math way.

Math Way of Number Naming

Regular names

4-ten = forty

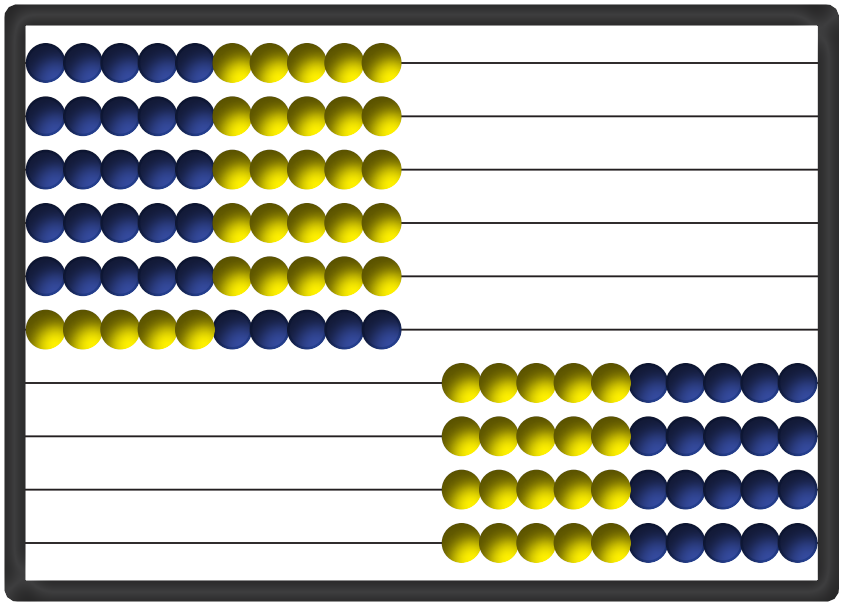


The “ty”
means tens.

Math Way of Number Naming

Regular names

6-ten = sixty

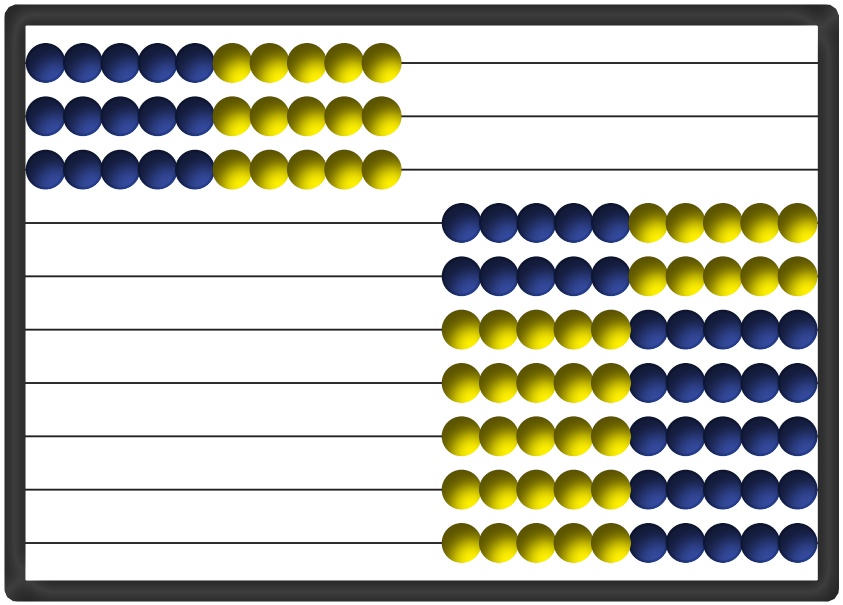


The “ty”
means tens.

Math Way of Number Naming

Regular names

3-ten = thirty

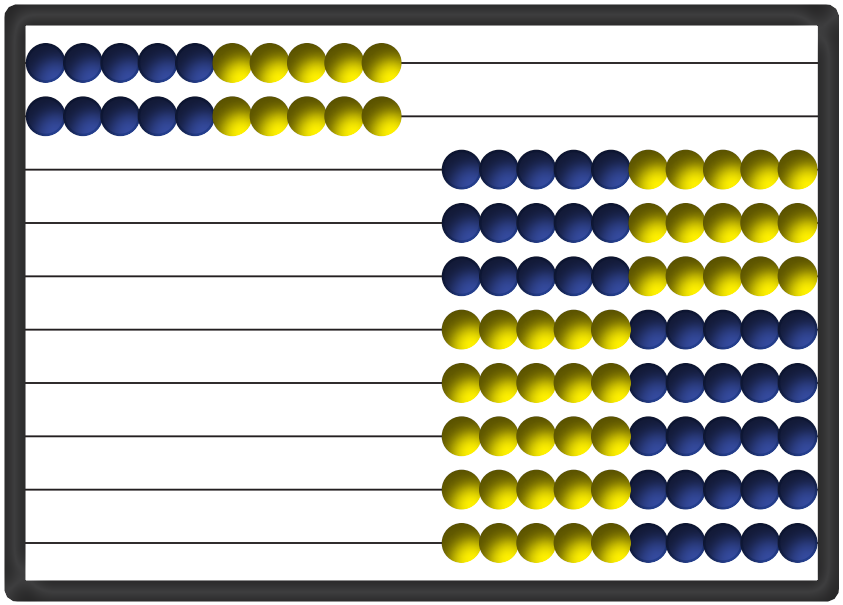


“Thir” also used
in 1/3, 13 and 30.

Math Way of Number Naming

Regular names

2-ten = twenty



Two used to
be pronounced
“twoo.”

Math Way of Number Naming

Regular names

A word game

fireplace → place-fire

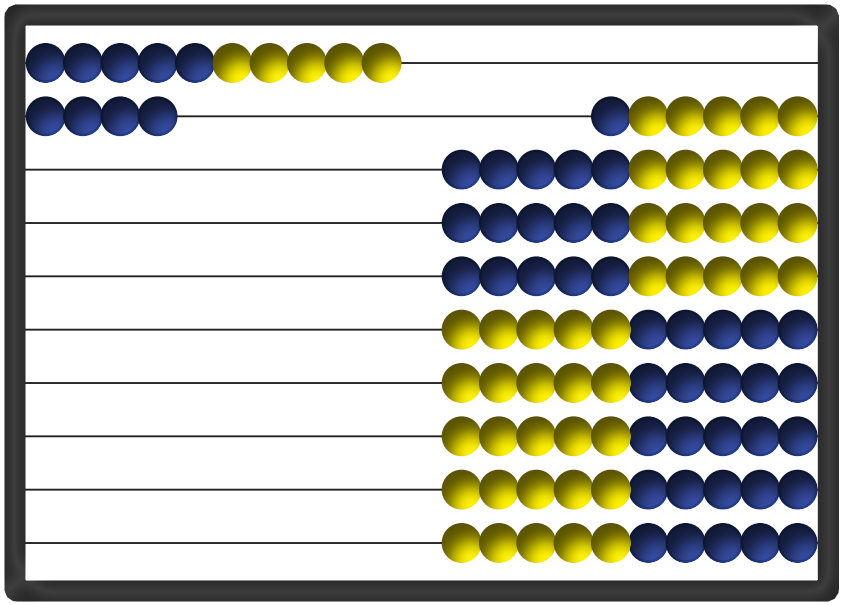
newspaper → paper-news

box-mail → mailbox

Math Way of Number Naming

Regular names

ten 4 → teen 4 → fourteen

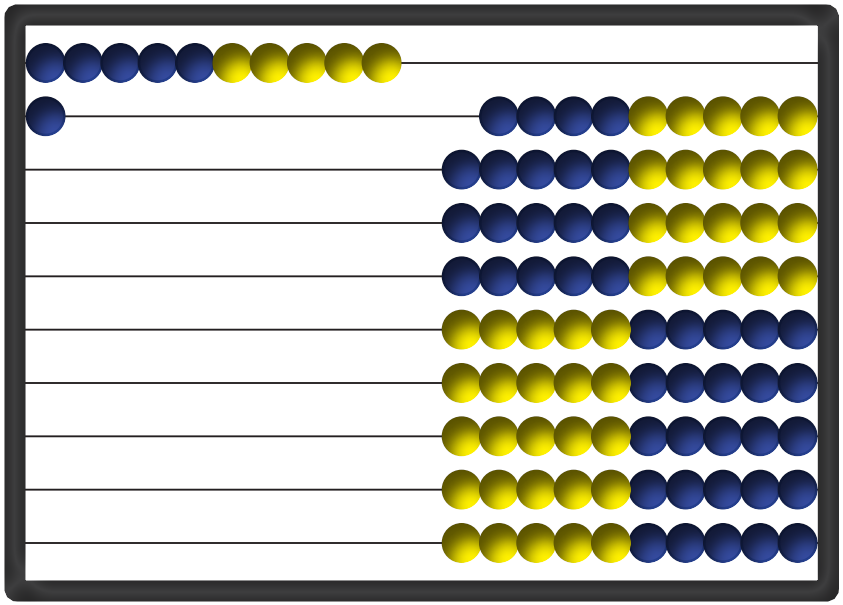


Suffix *-teen* means ten.

Math Way of Number Naming

Regular names

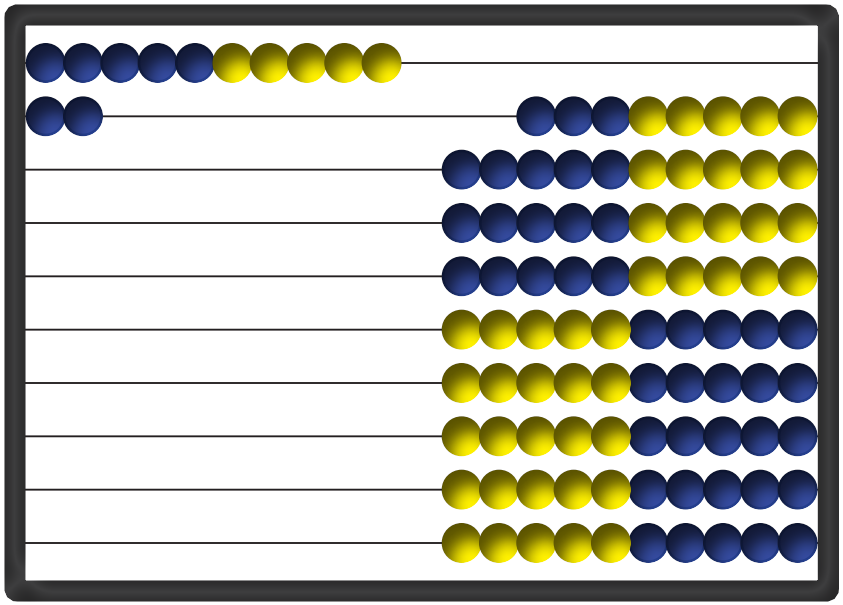
a one left → a left-one → eleven



Math Way of Number Naming

Regular names

two left → twelve

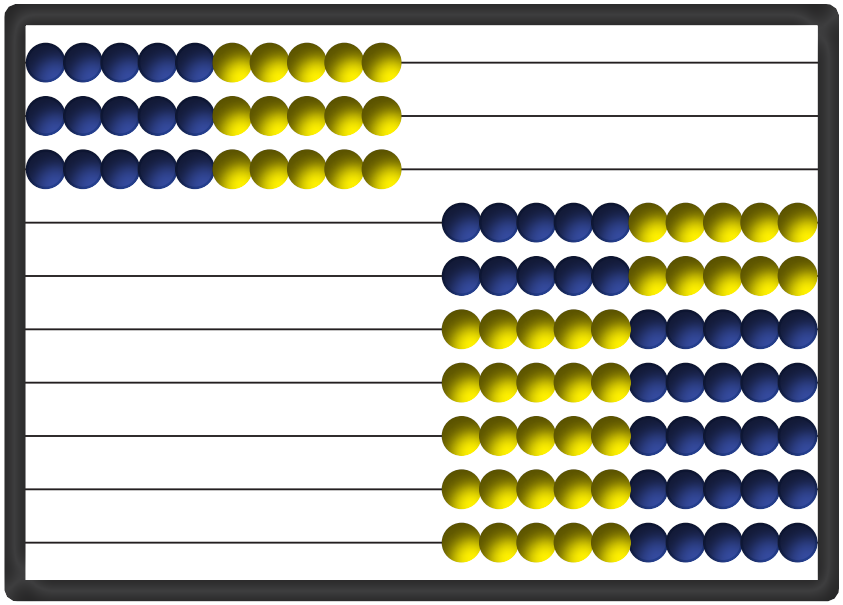


Two said as
“twoo.”

Composing Numbers

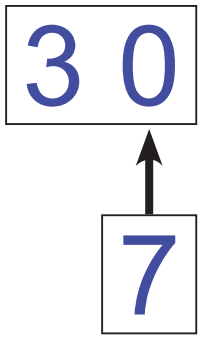
3-ten

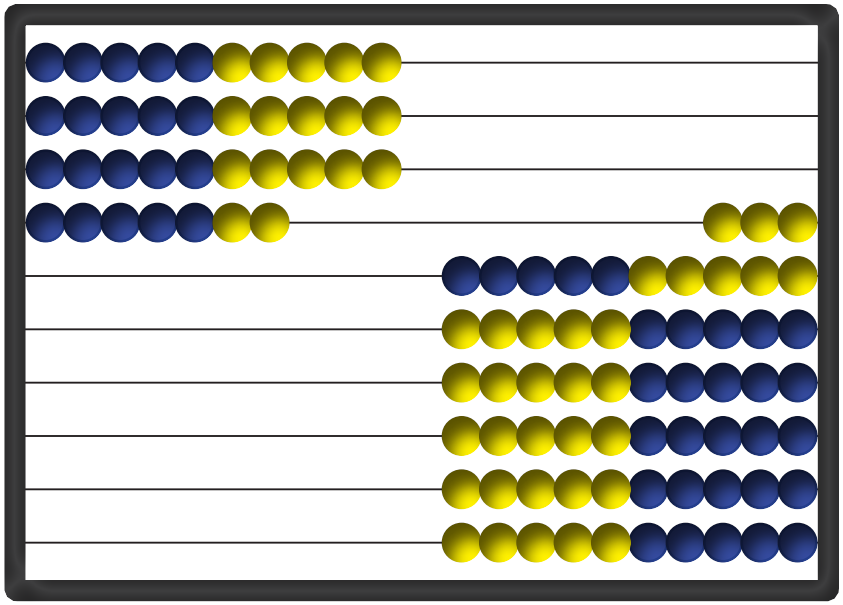
3	0



Composing Numbers

3-ten 7

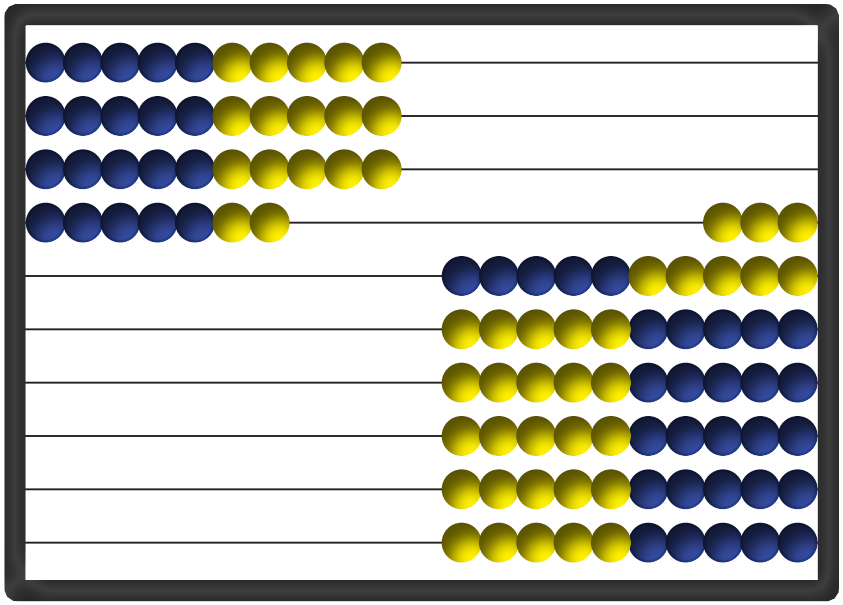




Composing Numbers

3-ten 7

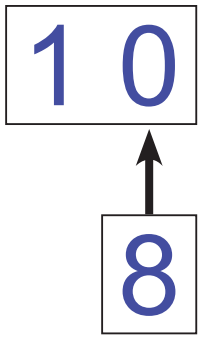
3	7

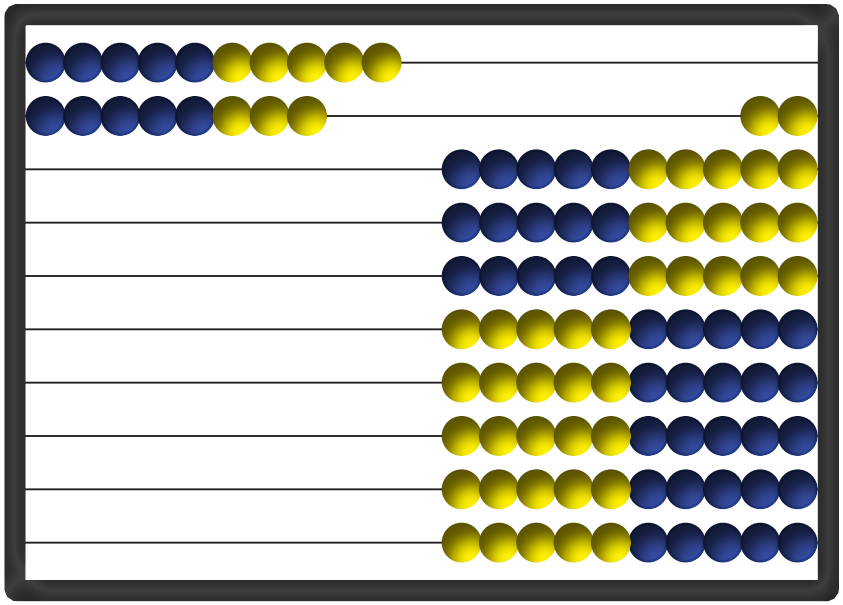


Note the congruence in how we say the number, represent the number, and write the number.

Composing Numbers

1-ten 8

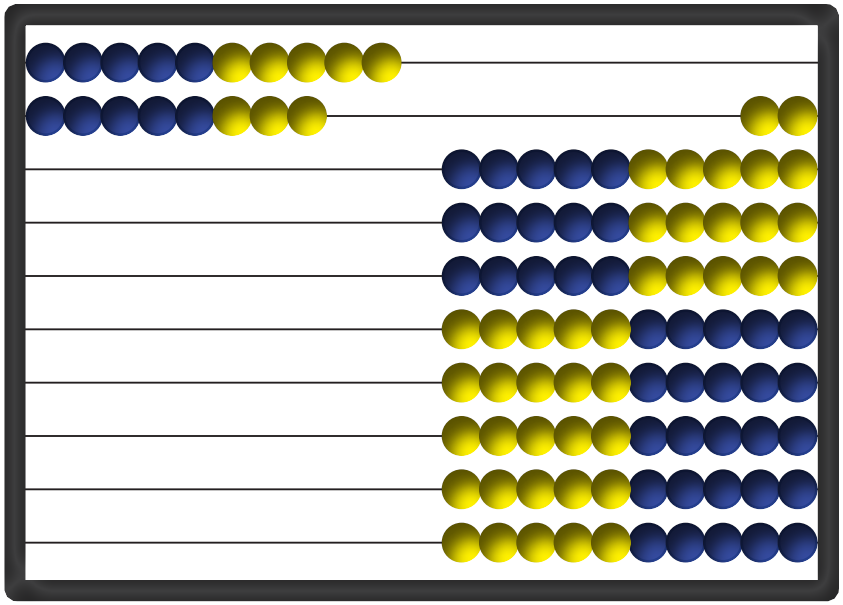




Composing Numbers

1-ten 8

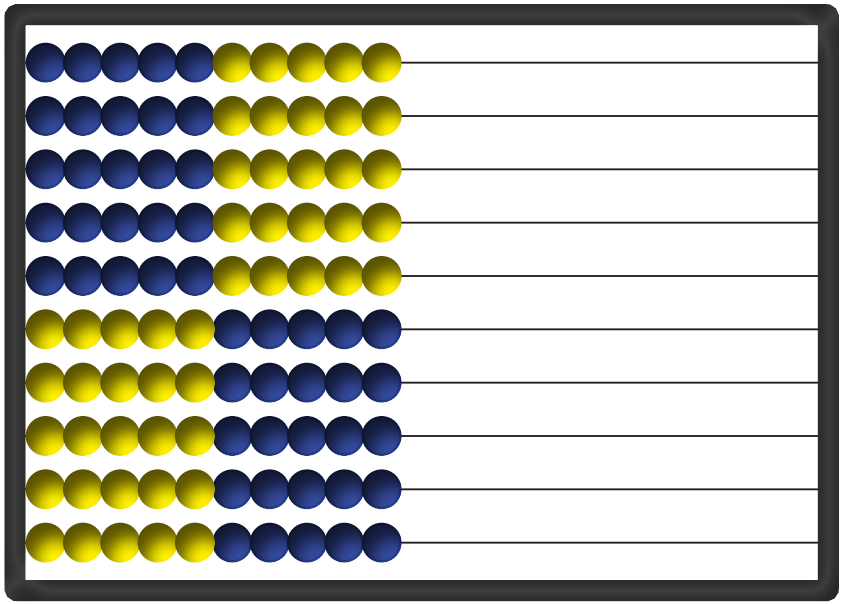
1	8



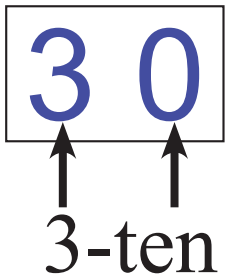
Composing Numbers

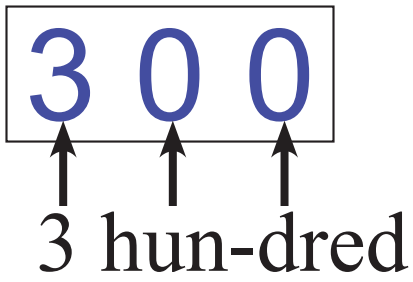
10-ten

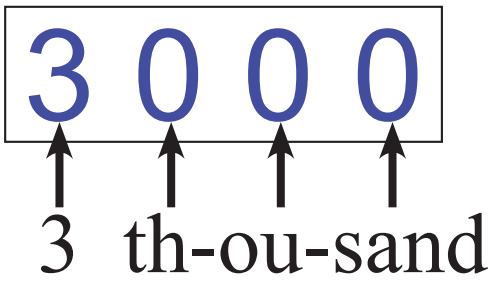
1 0 0



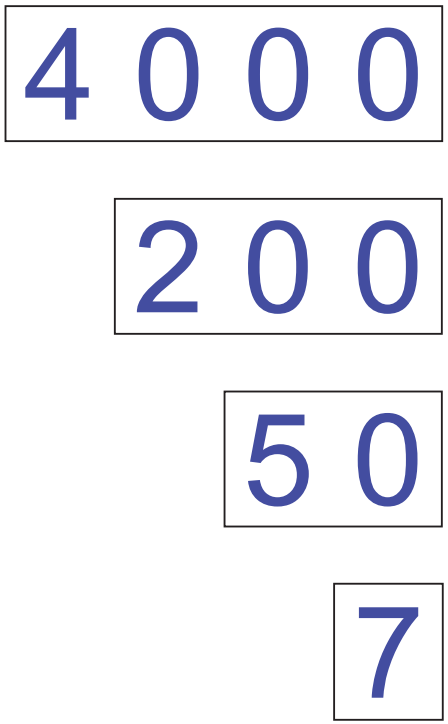
Place-Value Cards

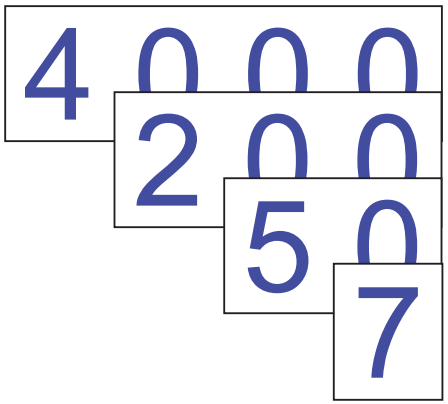


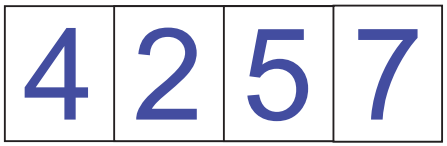




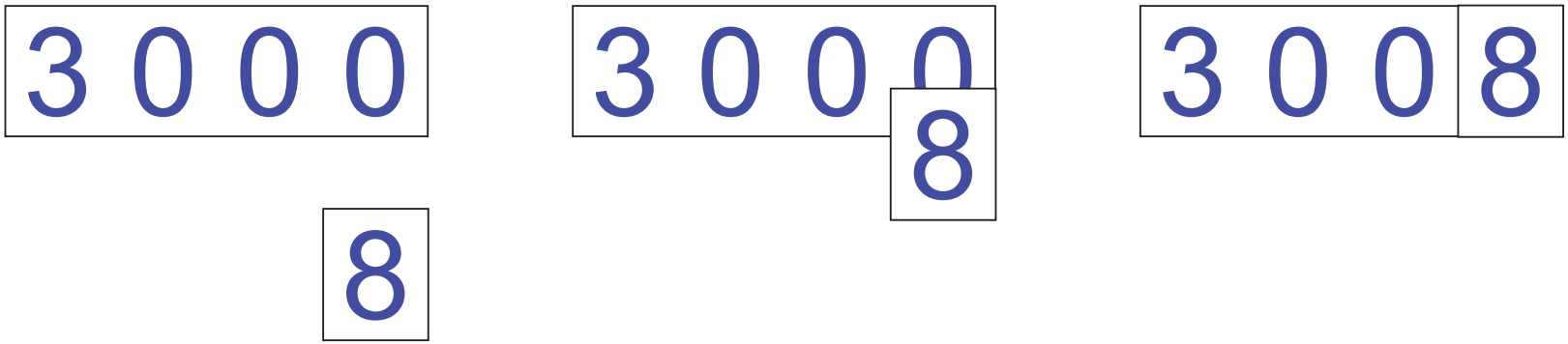
Place-Value Cards







Place-Value Cards



Place Value

Two aspects

Static (Recording)

- Value of a digit is determined by position.
- No position may have more than nine.
- As you progress to the left, value at each position is ten times greater than previous position.
- Represented by the place-value cards.

Dynamic (Trading)

- 10 ones = 1 ten;
  10 tens = 1 hundred;
  10 hundreds = 1 thousand, ….
- Represented with abacus and other materials.

Learning the Facts

Limited success, especially for struggling children, when learning is:

- Based on counting: whether dots, fingers, number lines, or counting words.
- Based on rote memory: whether flash cards, timed tests, or computer games.
- Based on skip counting: whether fingers or songs.

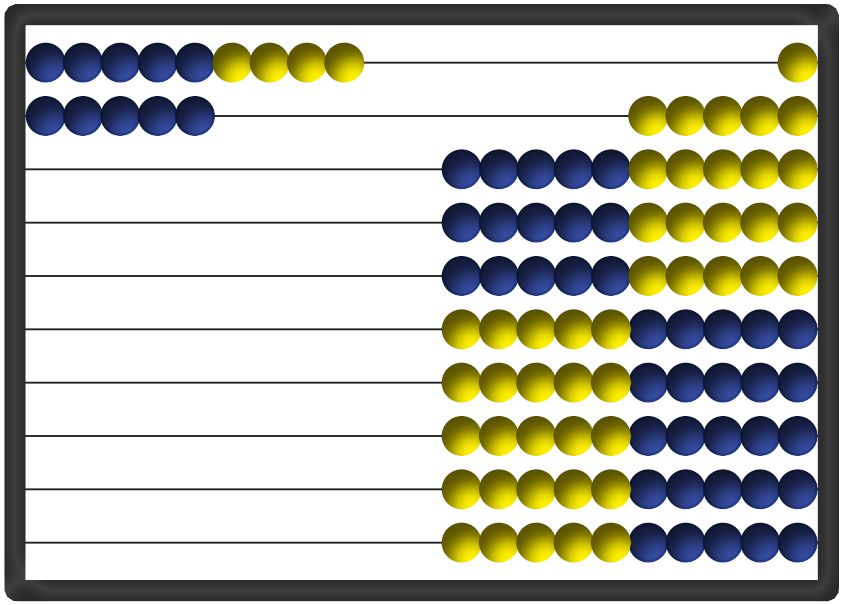
A child is considered to know a fact if they can give it in 2–3 seconds.

Fact Strategies

Complete the Ten

9 + 5 = ___

Take 1 from the 5 and give it to the 9.

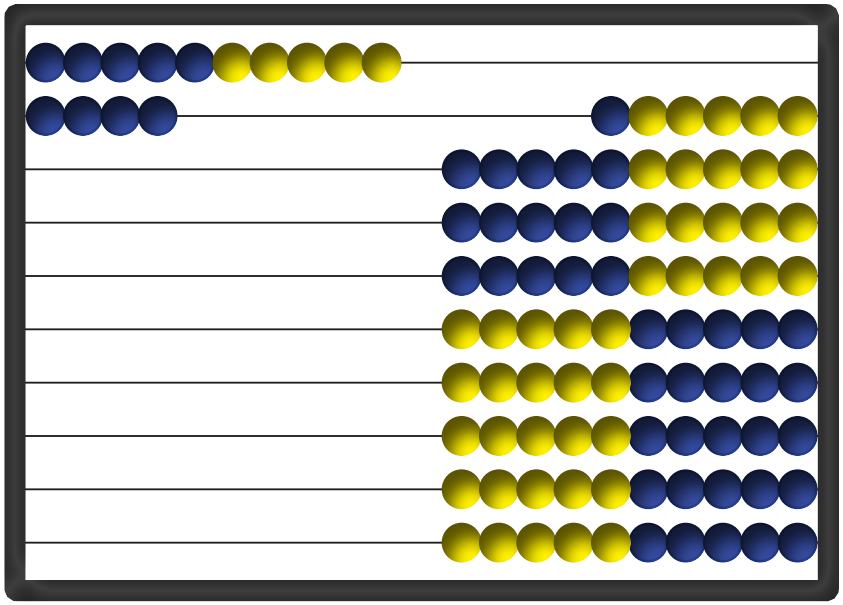


Fact Strategies

Complete the Ten

9 + 5 = ___

Take 1 from the 5 and give it to the 9.

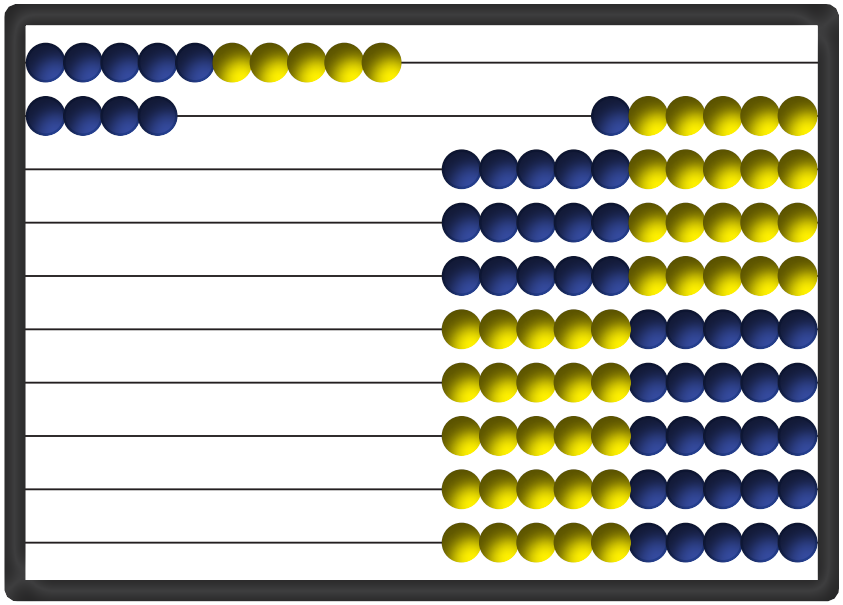


Fact Strategies

Complete the Ten

9 + 5 = 14

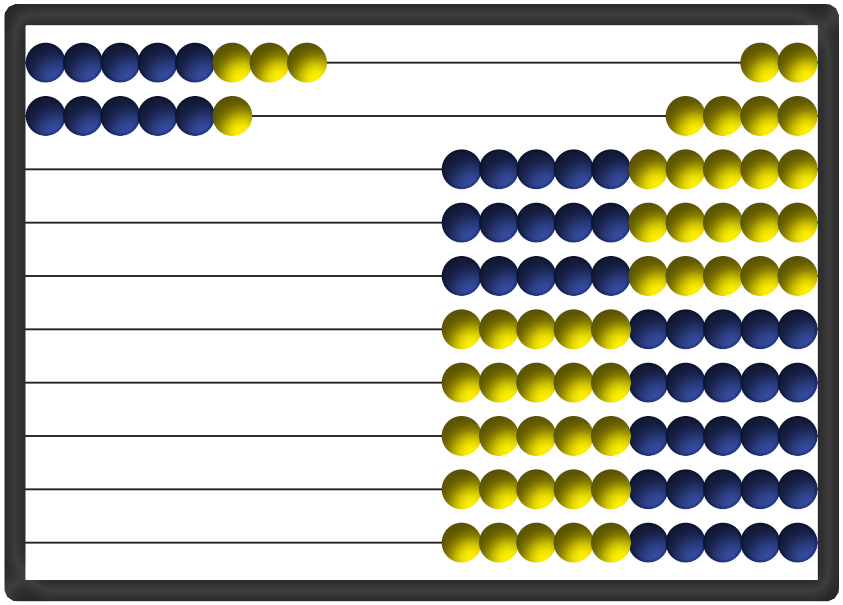
Take 1 from the 5 and give it to the 9.



Fact Strategies

Two Fives

8 + 6 =



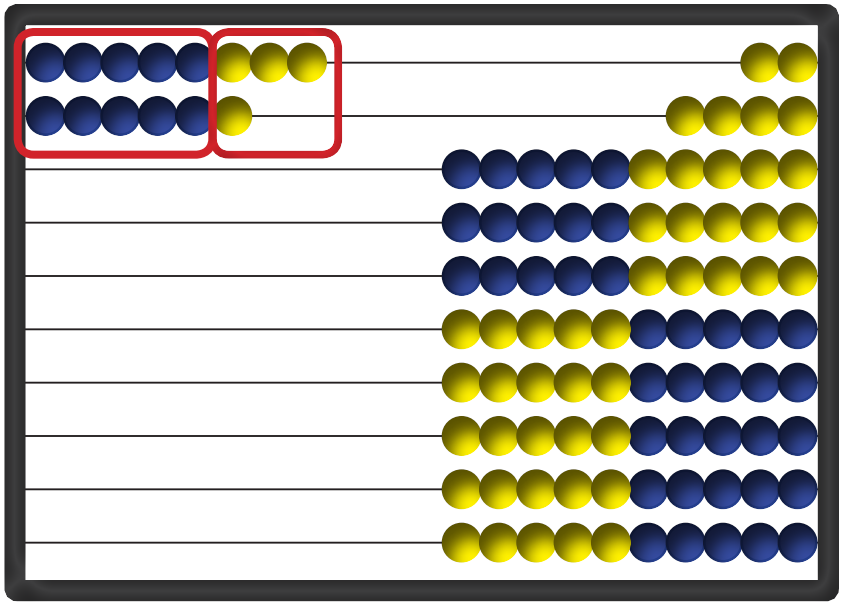
Fact Strategies

Two Fives

8 + 6 =

10 + 4 = 14

The two fives make 10.

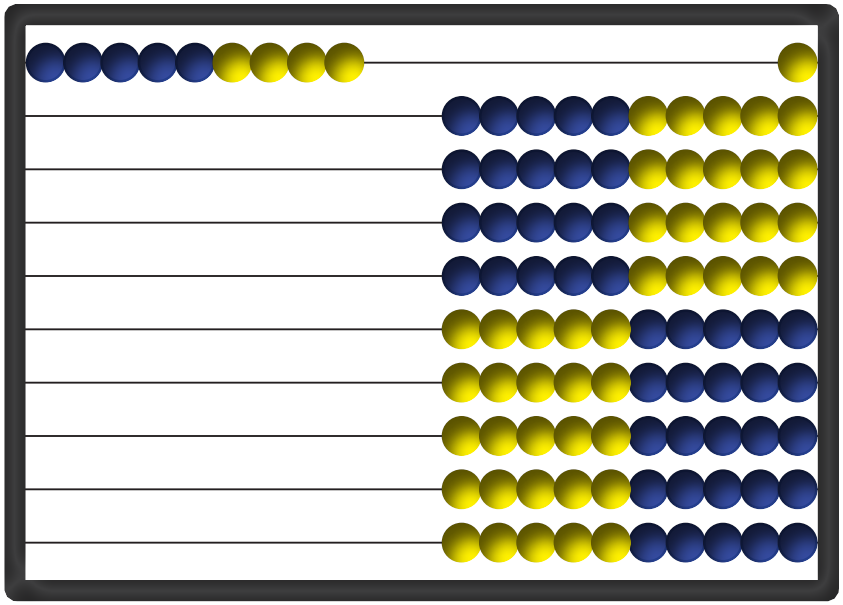


Fact Strategies

Missing Addend

9 + __ = 15

Start with 9;
go up to 15.

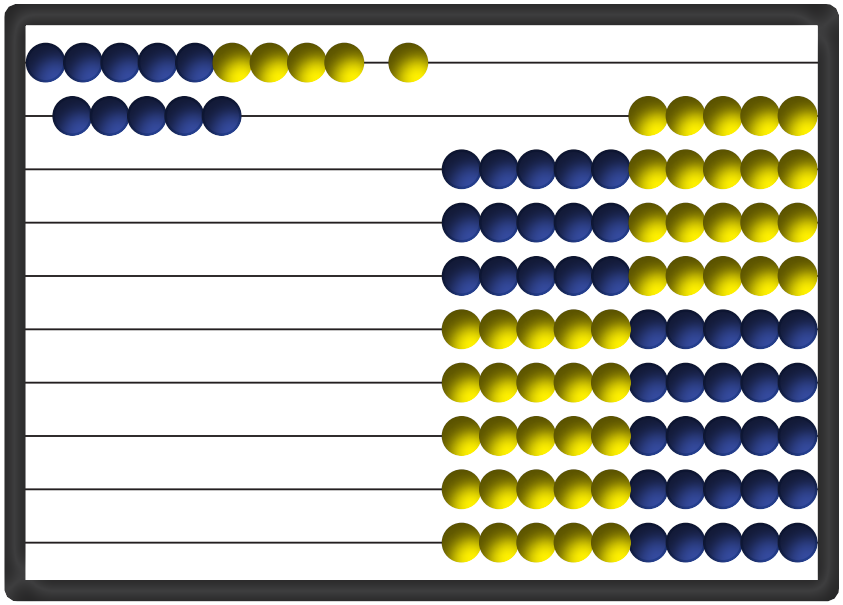


Fact Strategies

Missing Addend

9 + 6 = 15

Start with 9;
go up to 15.

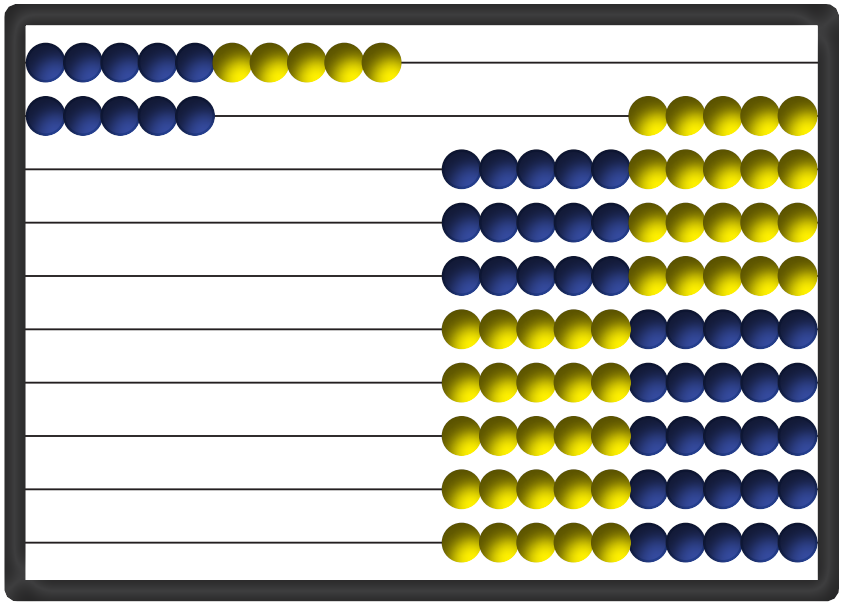


Fact Strategies

Subtracting Part from Ten

15 – 9 = __

Subtract 5 from 5 and 4 from 10.

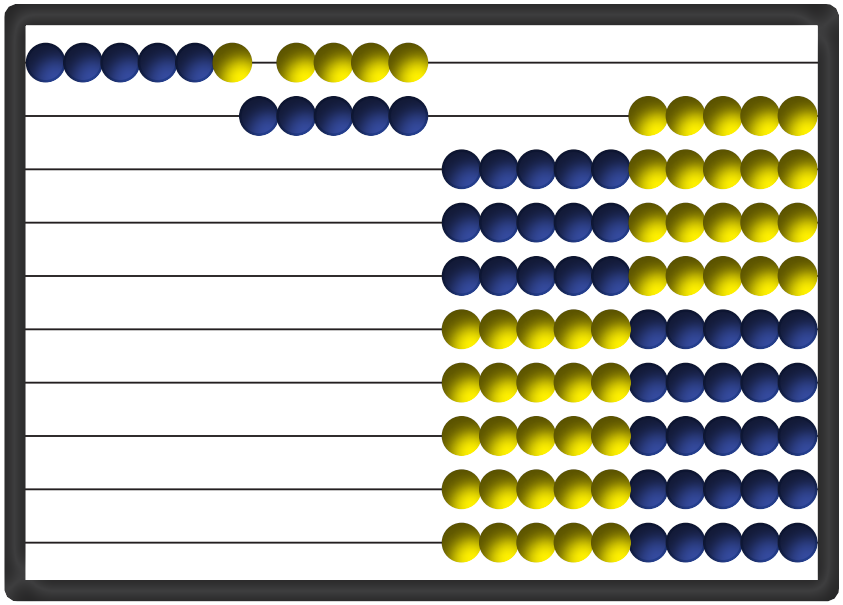


Fact Strategies

Subtracting Part from Ten

15 – 9 = 6

Subtract 5 from 5 and 4 from 10.

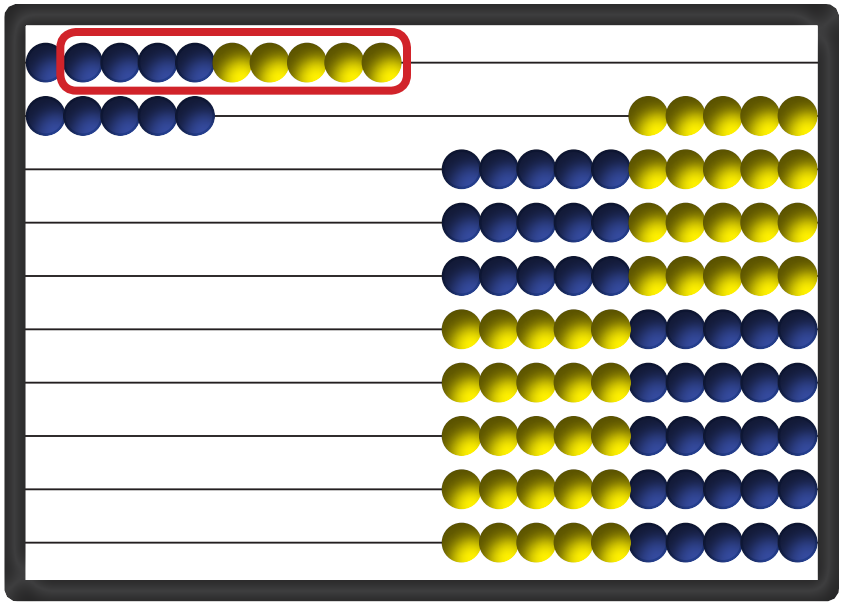


Fact Strategies

Subtracting All from Ten

15 – 9 = __

Subtract 9
from 10.

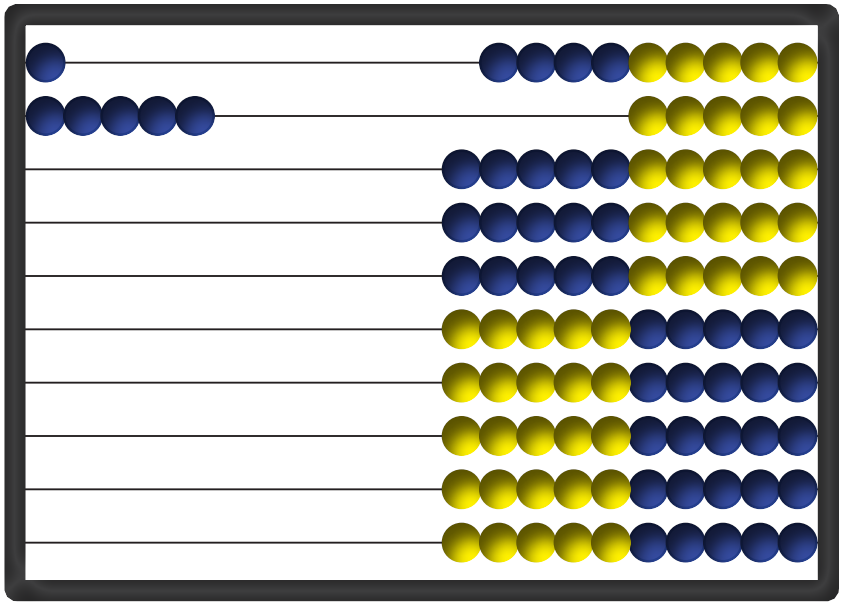


Fact Strategies

Subtracting All from Ten

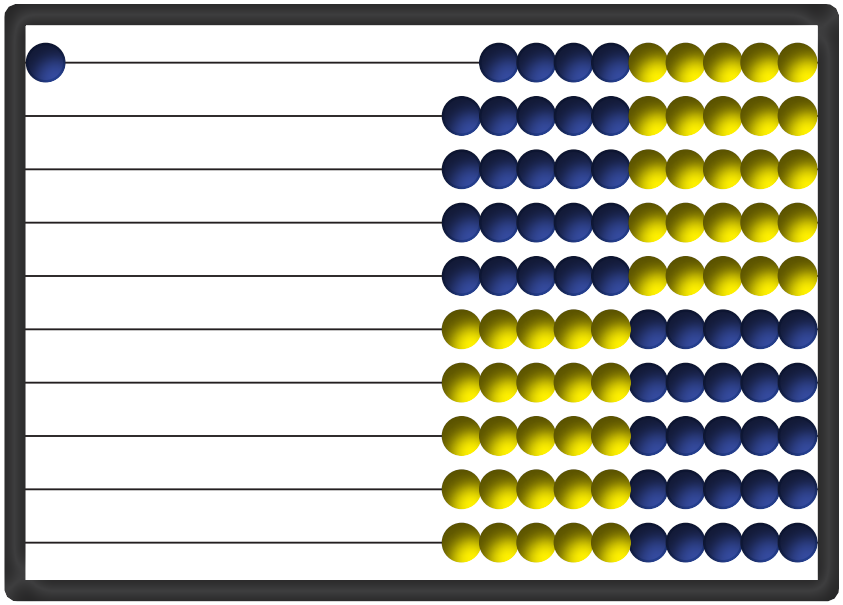
15 – 9 = 6

Subtract 9
from 10.



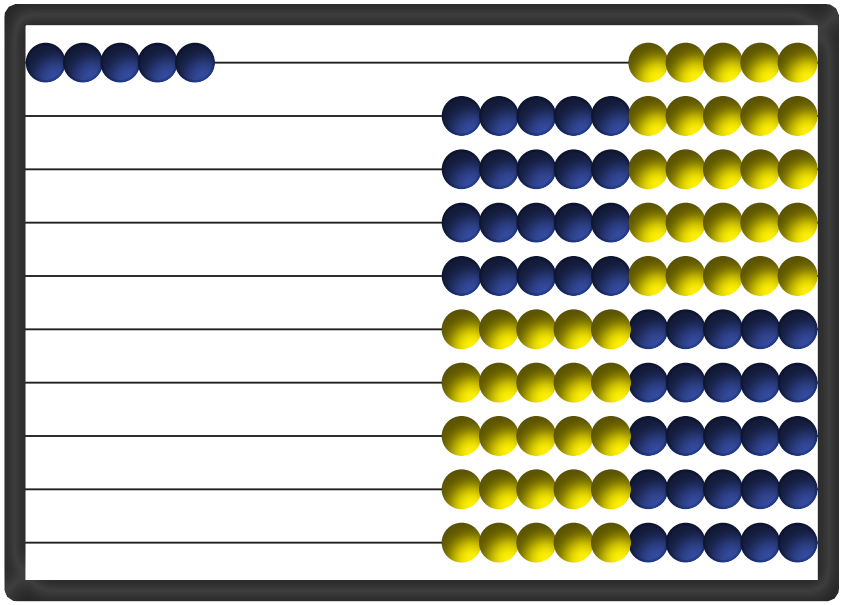
Money

Penny



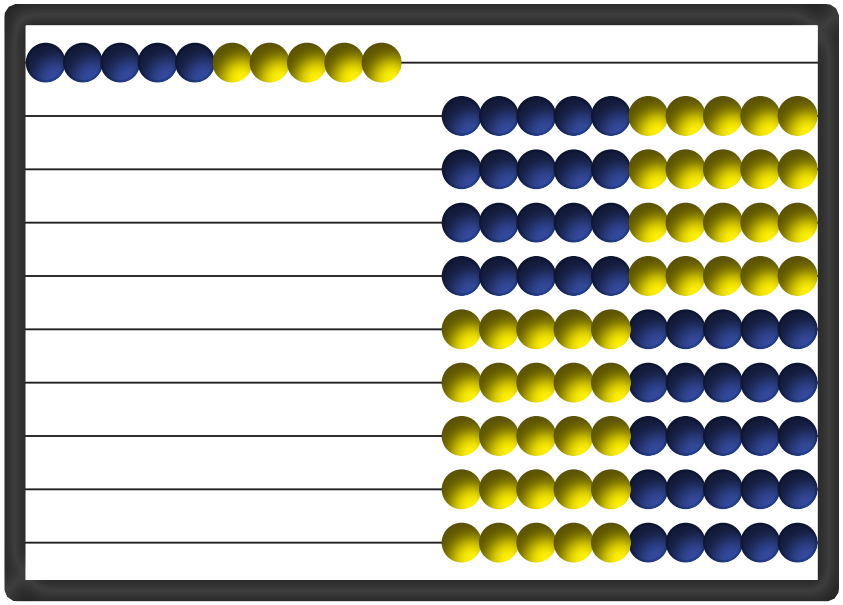
Money

Nickel



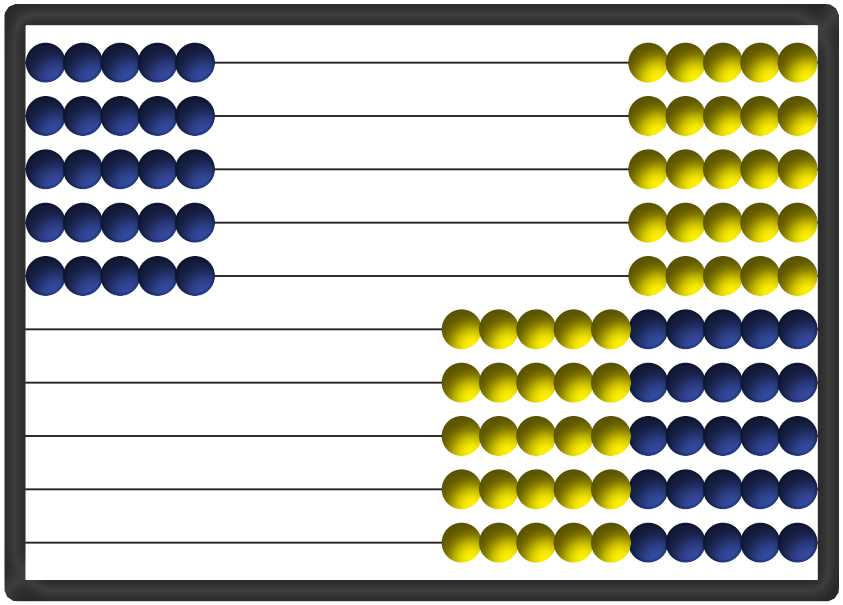
Money

Dime



Money

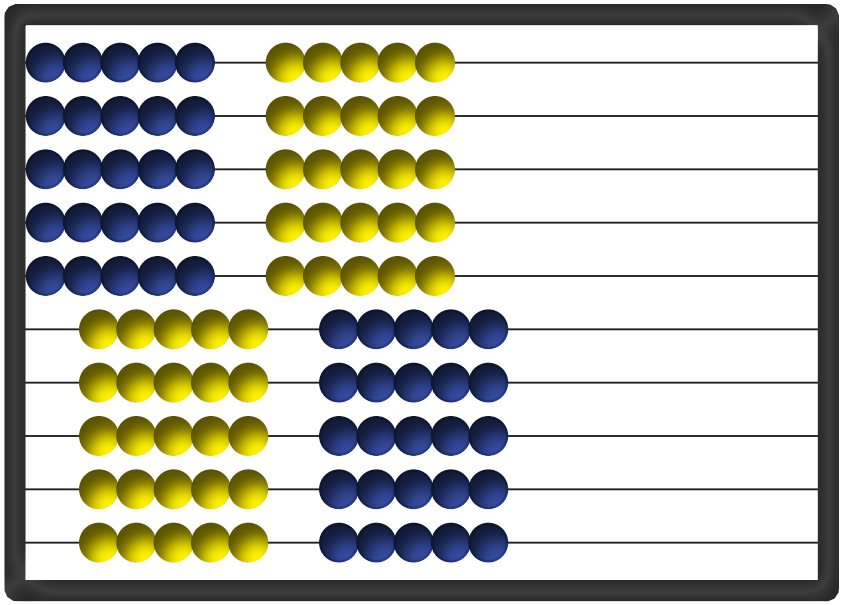
Quarter



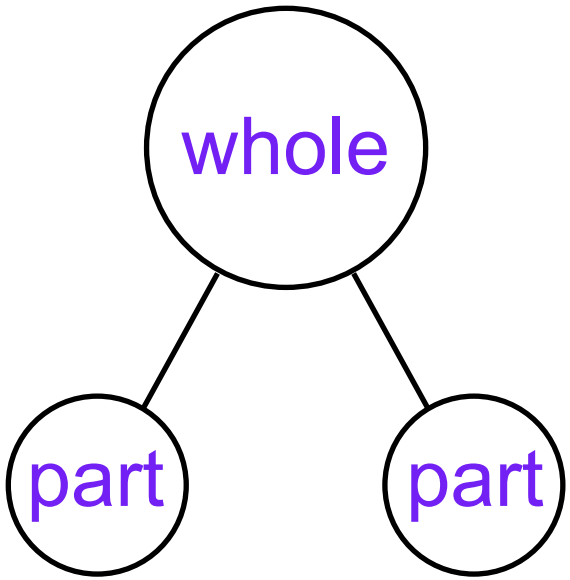
Money

Quarter

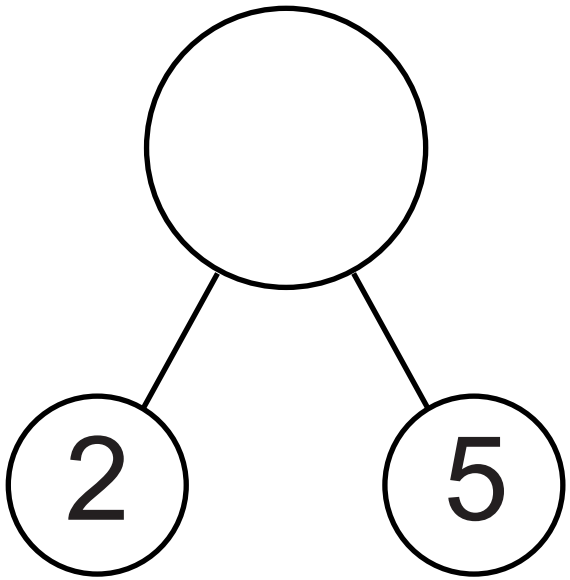
Four quarters.



Part-Whole Circles

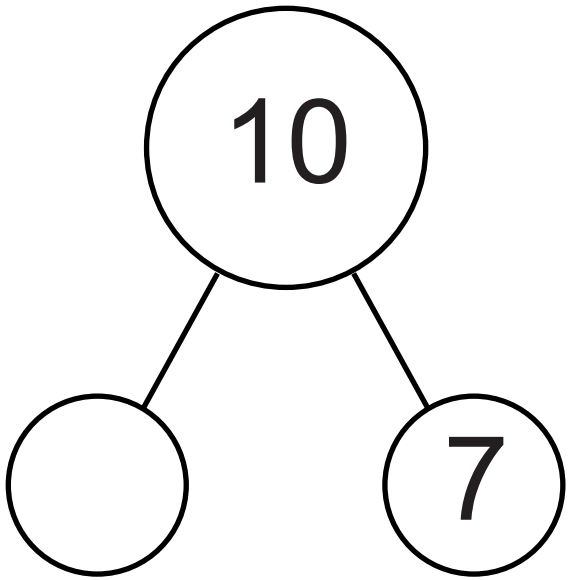


Part-Whole Circles



What is the whole?

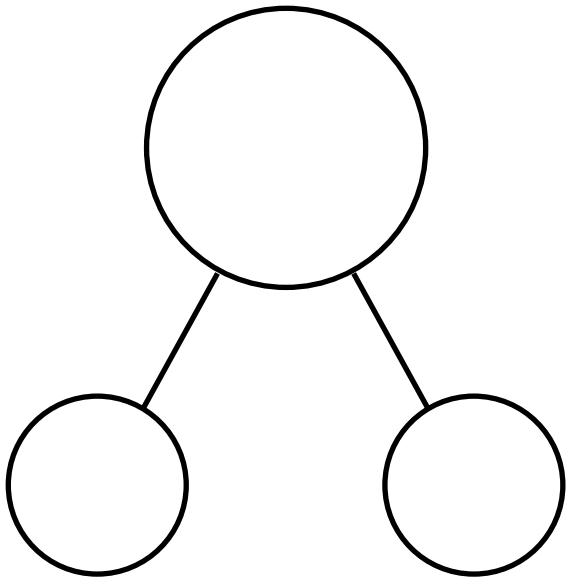
Part-Whole Circles



What is the other part?

Part-Whole Circles

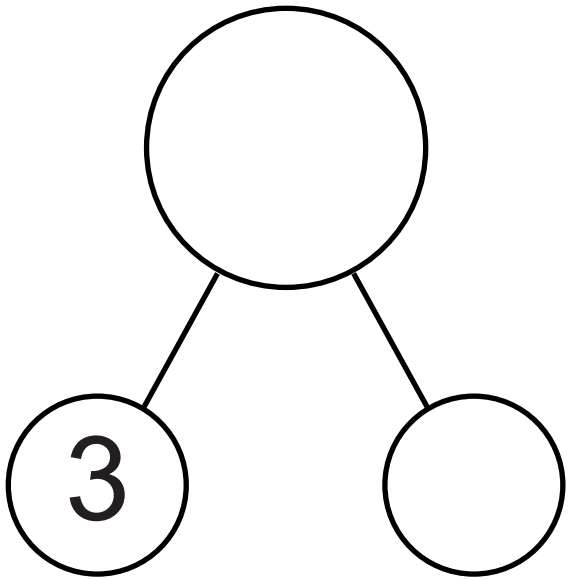
Missing addend problem



Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

Part-Whole Circles

Missing addend problem

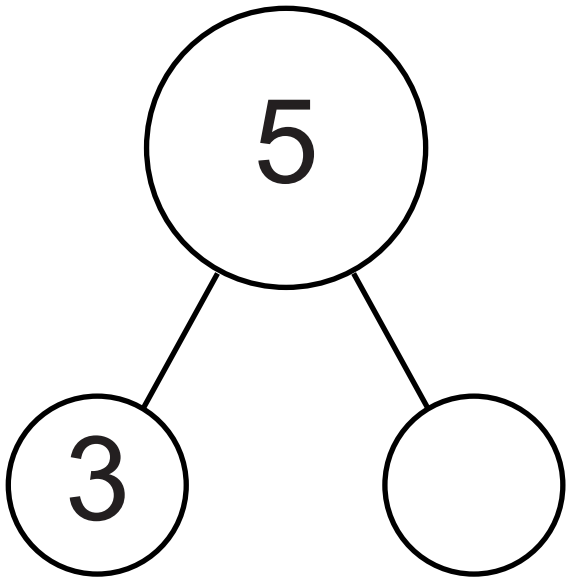


Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

Is 3 the whole or a part?

Part-Whole Circles

Missing addend problem

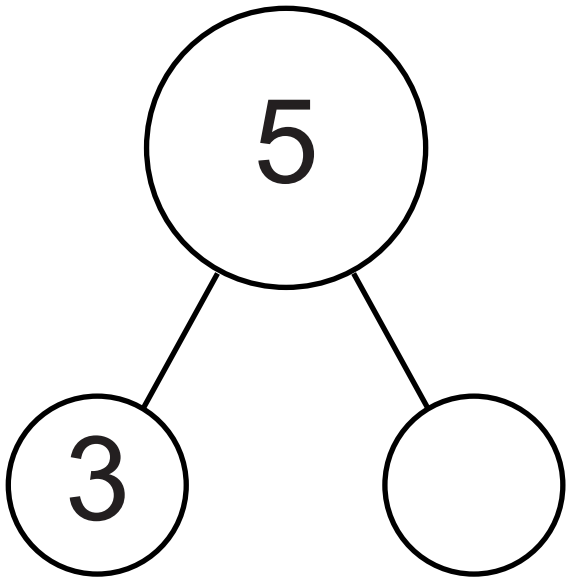


Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

Is 5 the whole or a part?

Part-Whole Circles

Missing addend problem

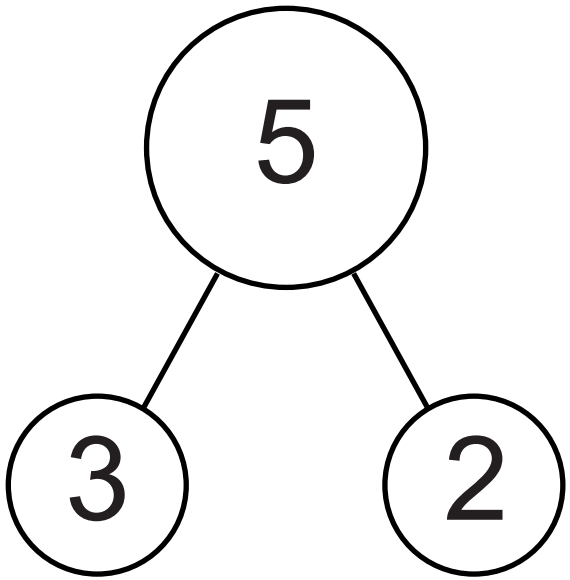


Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

What is the missing part?

Part-Whole Circles

Missing addend problem

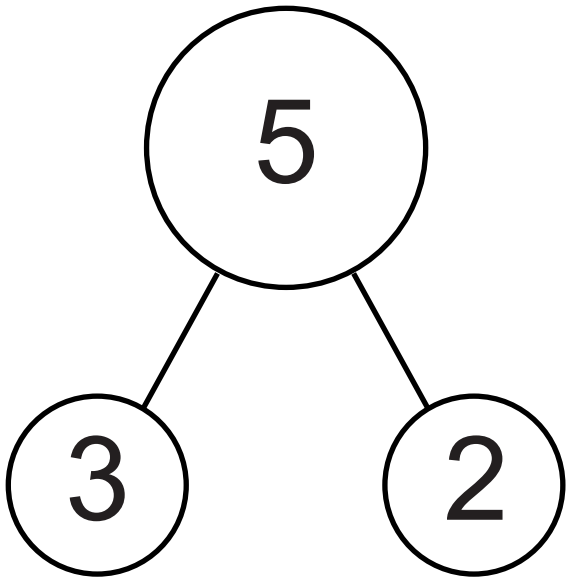


Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

What is the missing part?

Part-Whole Circles

Missing addend problem



Lee received 3 goldfish as a gift. Now Lee has 5. How many goldfish did Lee have to start with?

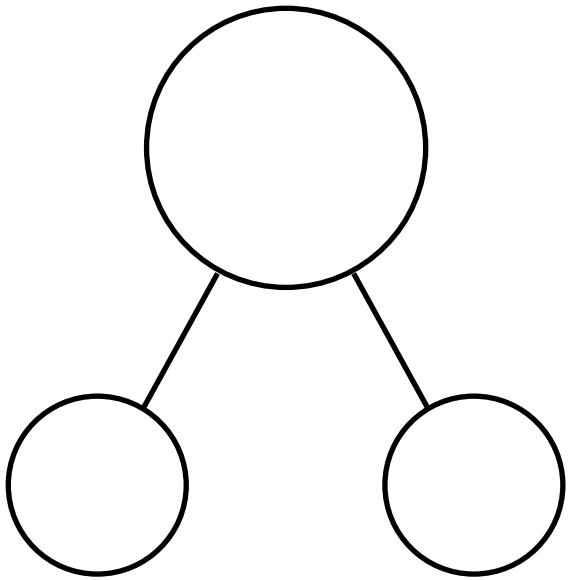
Write the equation.

2 + 3 = 5

3 + 2 = 5

5 – 3 = 2

Part-Whole Circles



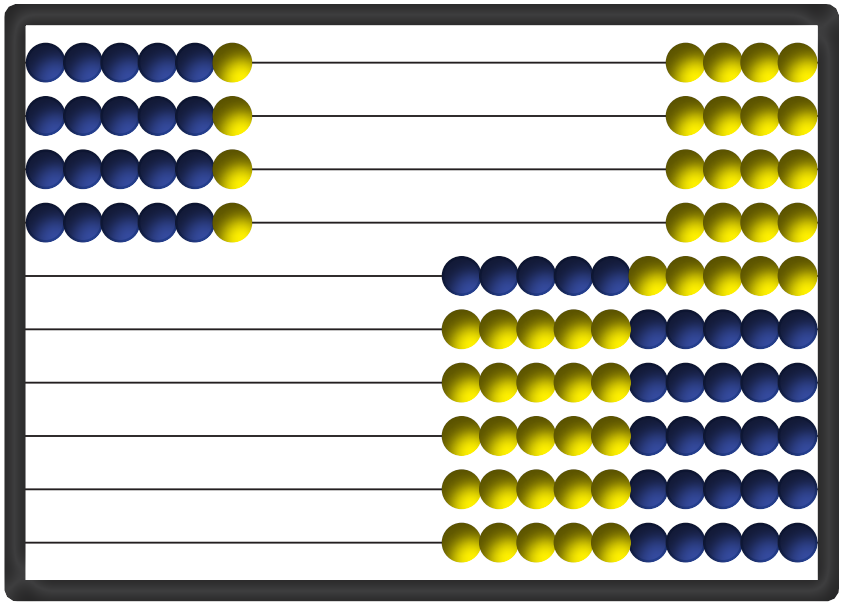
- Research shows part-whole circles help young children solve problems. Writing equations do not.
- Do not teach “key” words. The child needs to focus on the situation, not look for specific words.

Multiplication Strategies

Basic facts

$6 \times 4 = 24$

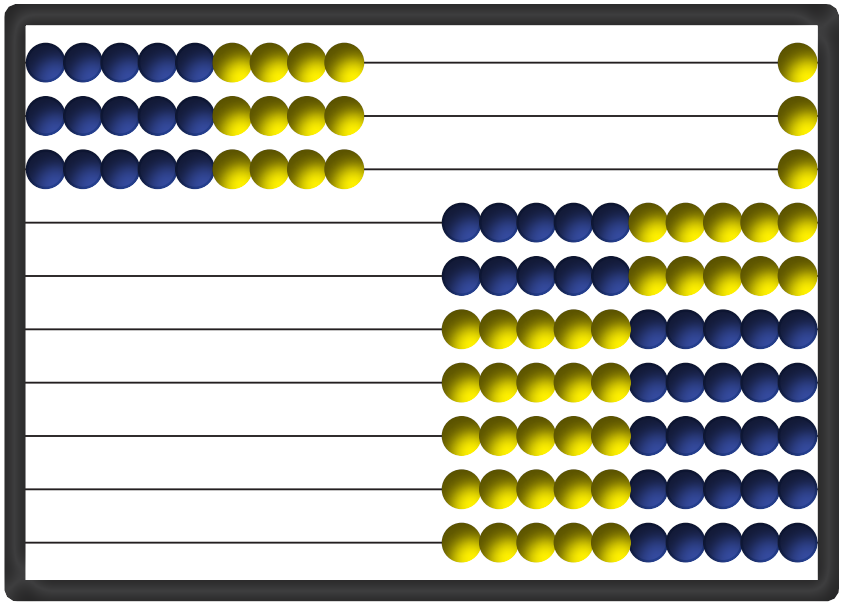
(6 taken 4 times.)



Multiplication Strategies

Basic facts

$9 \times 3 =$

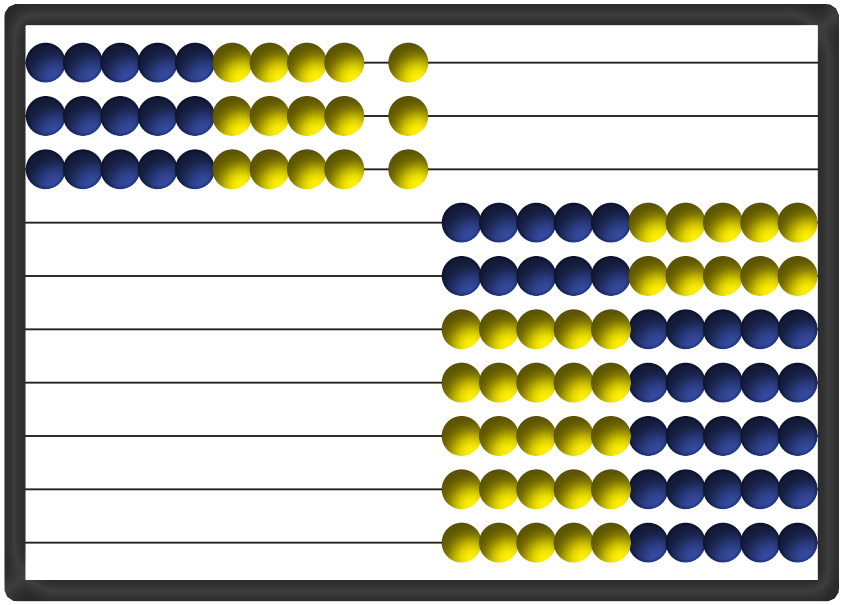


Multiplication Strategies

Basic facts

$9 \times 3 =$

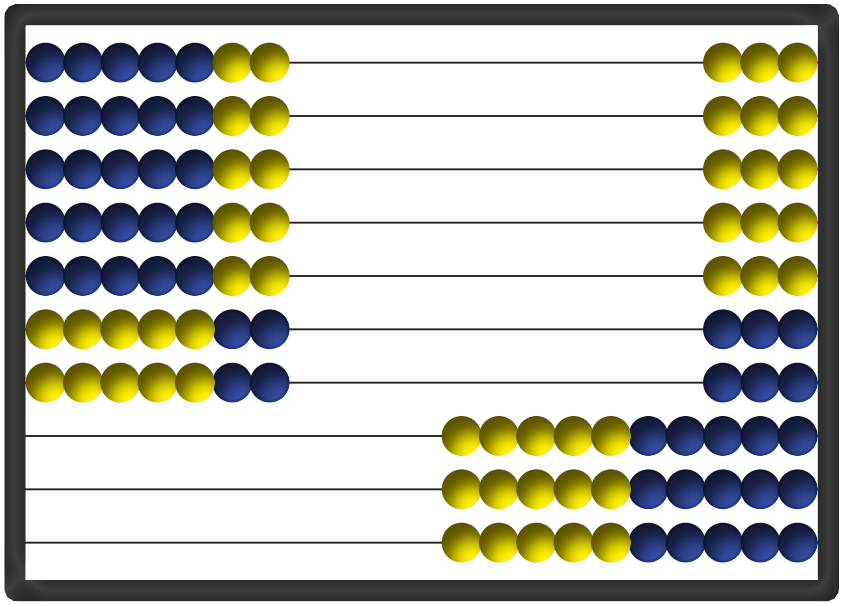
$30 - 3 = 27$



Multiplication Strategies

Basic facts

$7 \times 7 =$

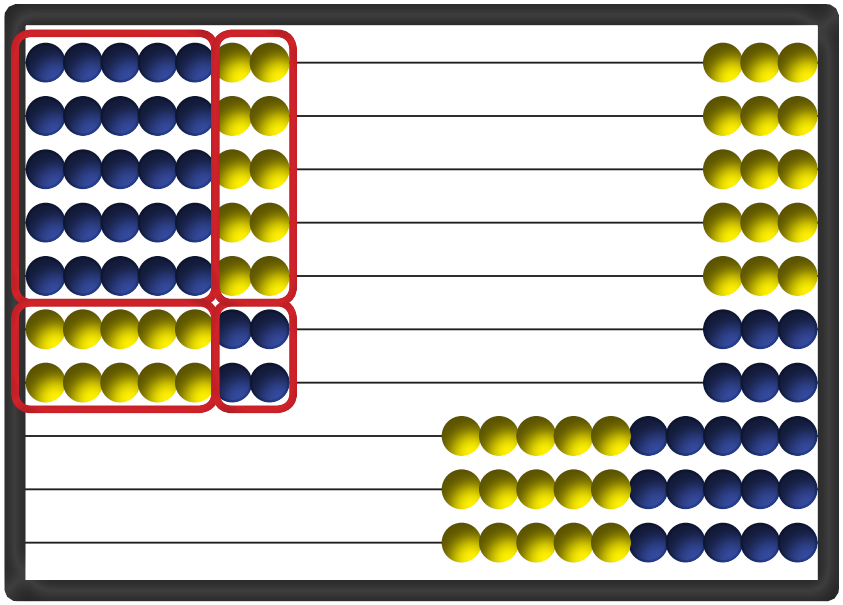


Multiplication Strategies

Basic facts

$7 \times 7 =$

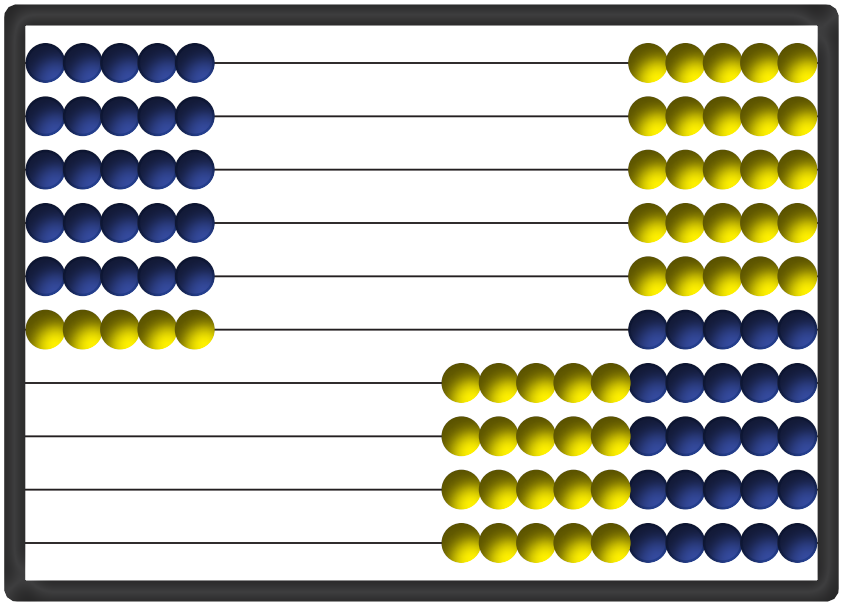
$25 + 10 + 10 + 4 = 49$

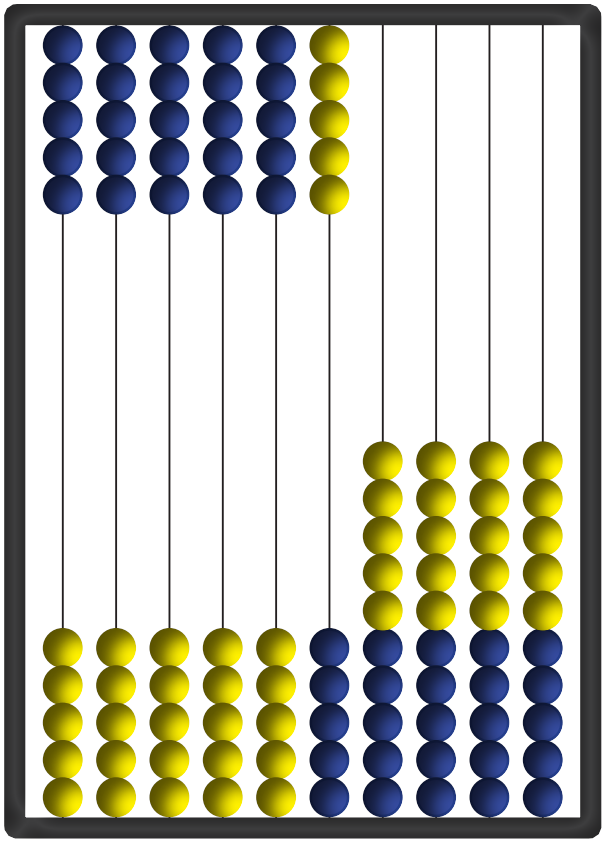


Multiplication Strategies

Commutative property

$5 \times 6 = 6 \times 5$





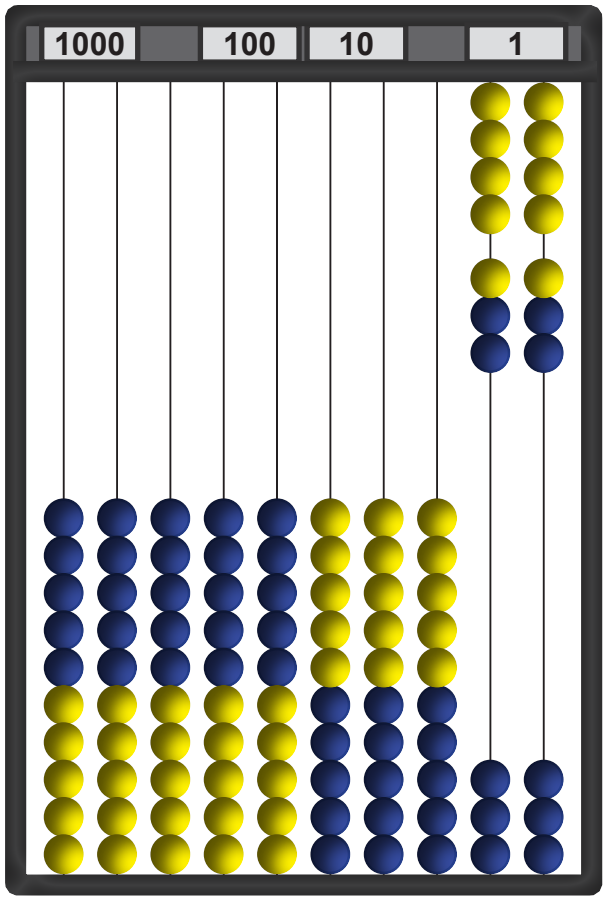
Multiplication Strategies

Multiplication table

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Trading

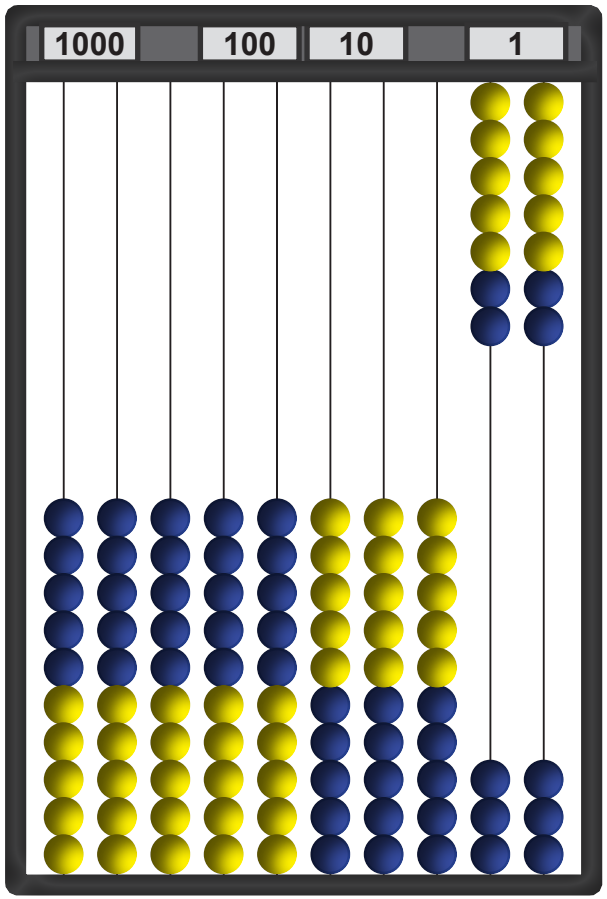
Simple adding



$$\begin{array}{r} 8 \\ +\ 6 \\ \hline \end{array}$$

Trading

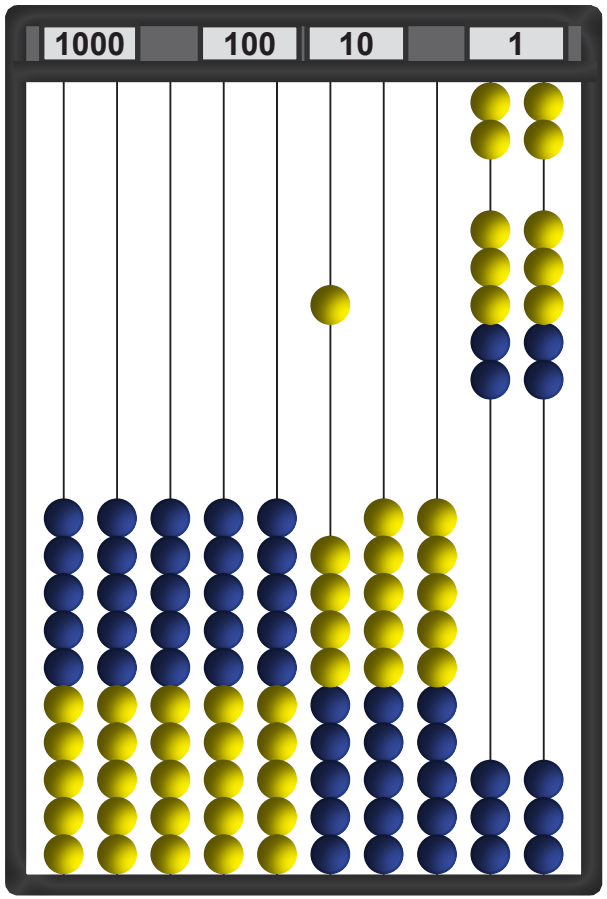
Simple adding



$$\begin{array}{r} 8 \\ +\ 6 \\ \hline 14 \end{array}$$

Trading

Simple adding

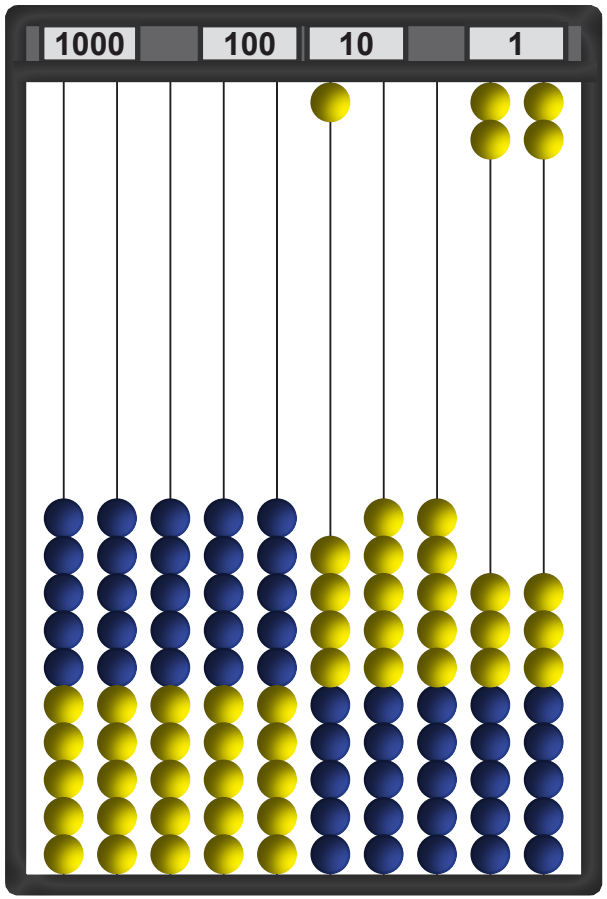


$$\begin{array}{r} 8 \\ +\ 6 \\ \hline 14 \end{array}$$

Too many ones; trade 10 ones for 1 ten.

Trading

Simple adding

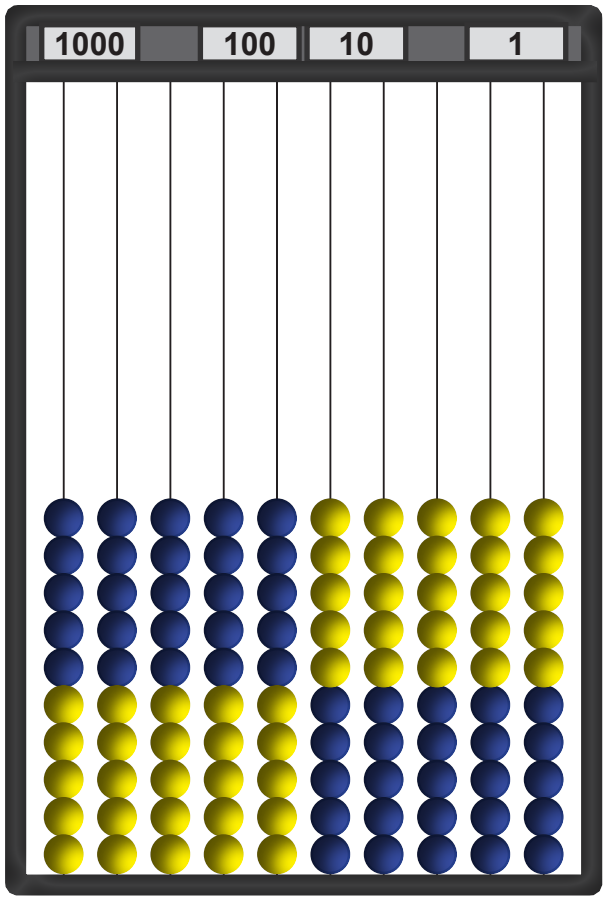


$$\begin{array}{r} 8 \\ +\ 6 \\ \hline 14 \end{array}$$

Same answer before and after trading.

Trading

Adding 4-digit numbers

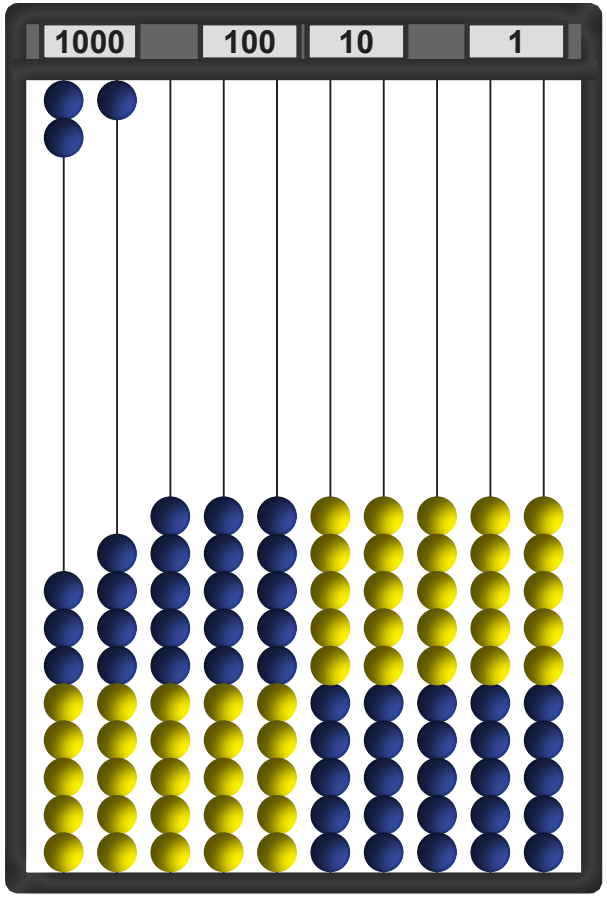


3658
+ 2738

Enter numbers from left to right.

Trading

Adding 4-digit numbers

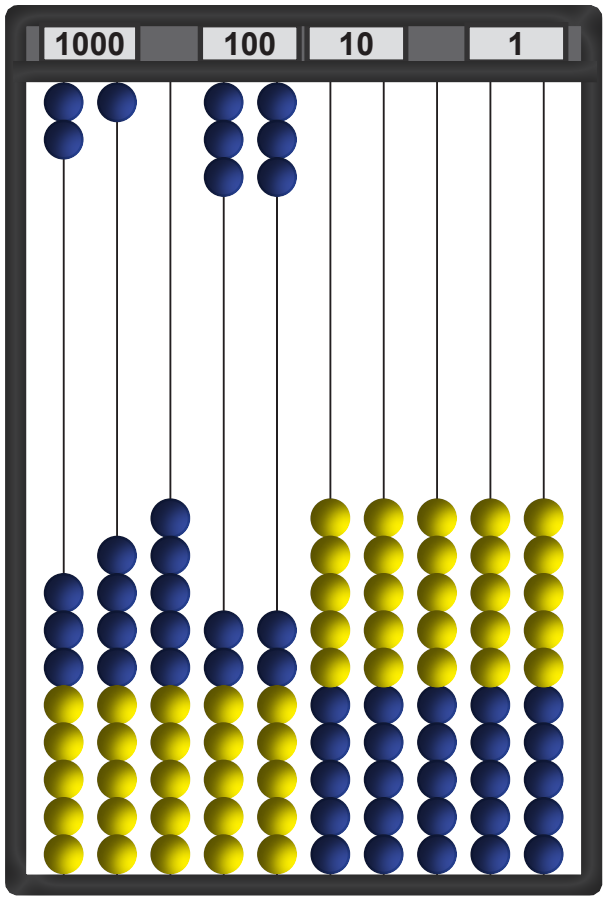


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline \end{array}$$

Enter numbers from left to right.

Trading

Adding 4-digit numbers

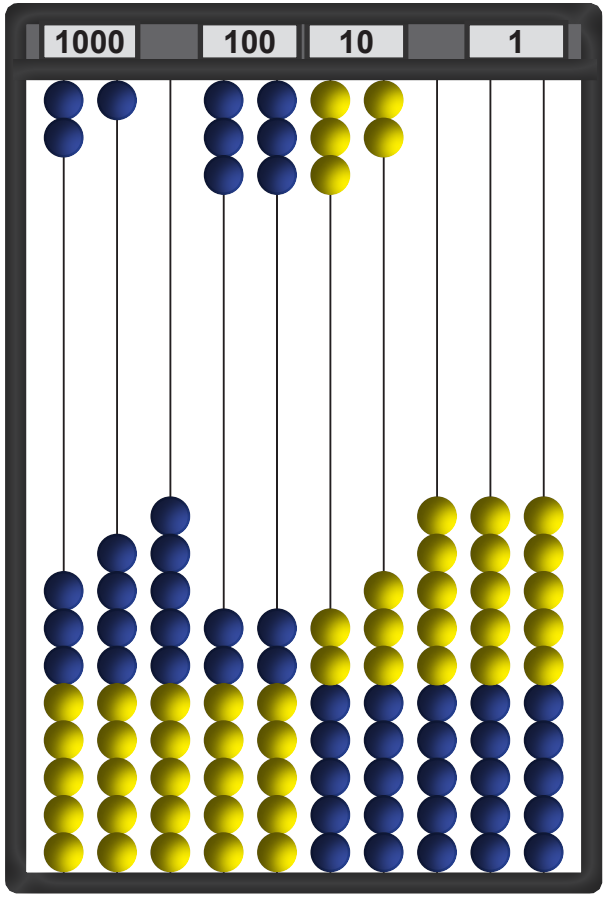


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline \end{array}$$

Enter numbers from left to right.

Trading

Adding 4-digit numbers

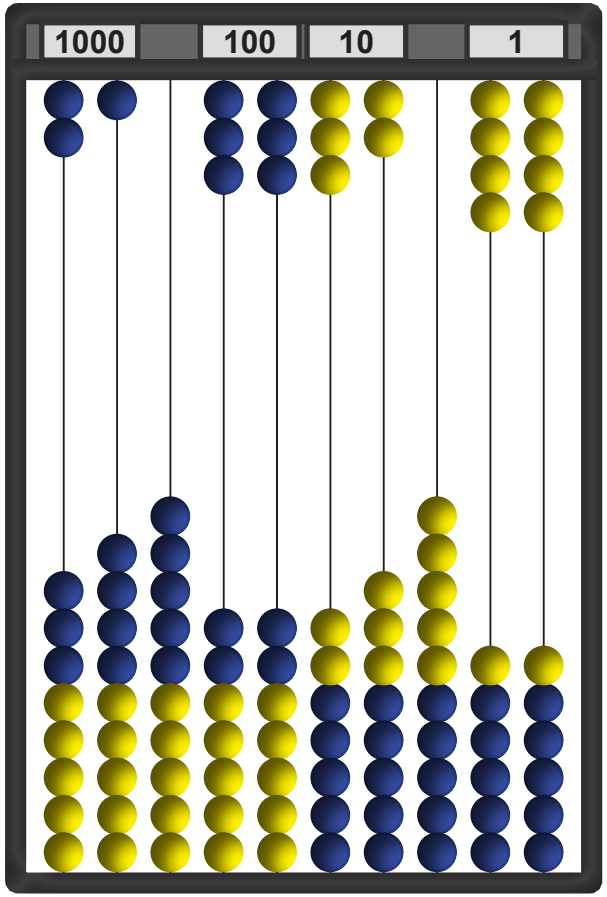


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline \end{array}$$

Enter numbers from left to right.

Trading

Adding 4-digit numbers

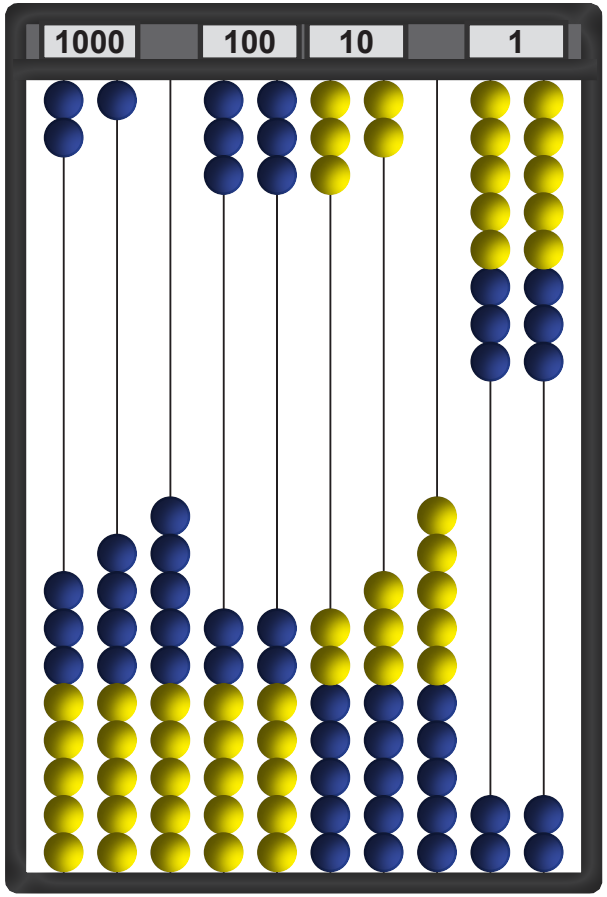


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline \end{array}$$

Enter numbers from left to right.

Trading

Adding 4-digit numbers

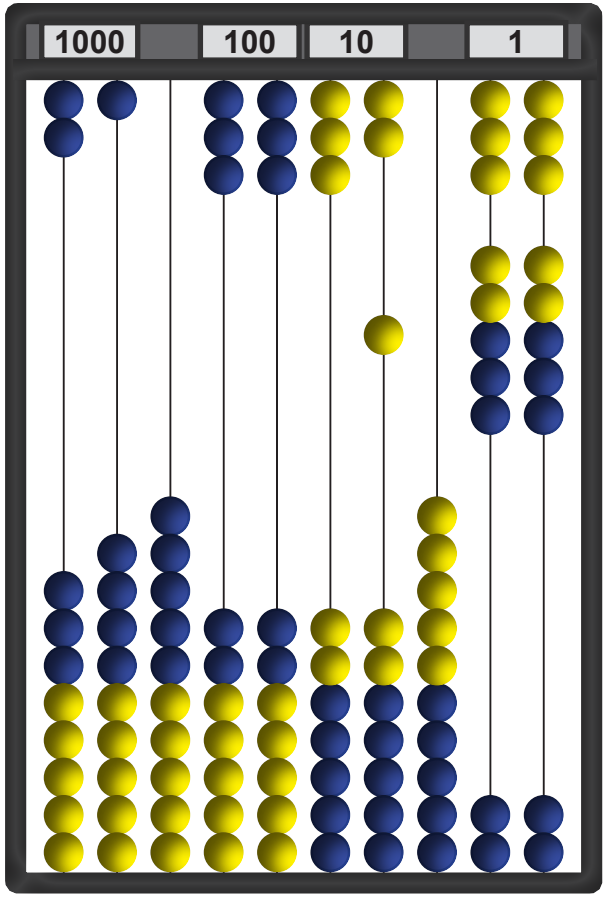


3658
+ 2738

Add starting at the right. Write results after each step.

Trading

Adding 4-digit numbers

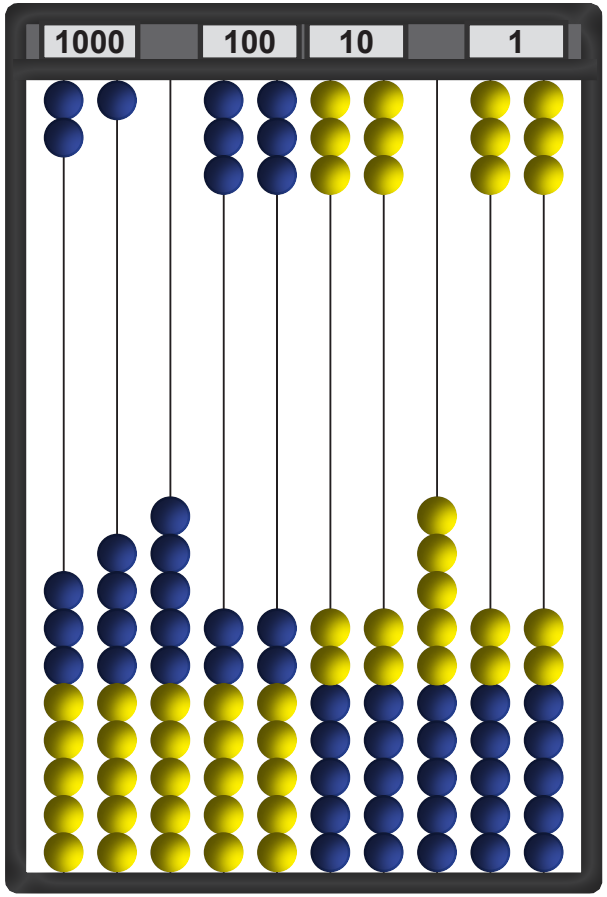


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline \end{array}$$

Trade 10 ones for 1 ten.

Trading

Adding 4-digit numbers

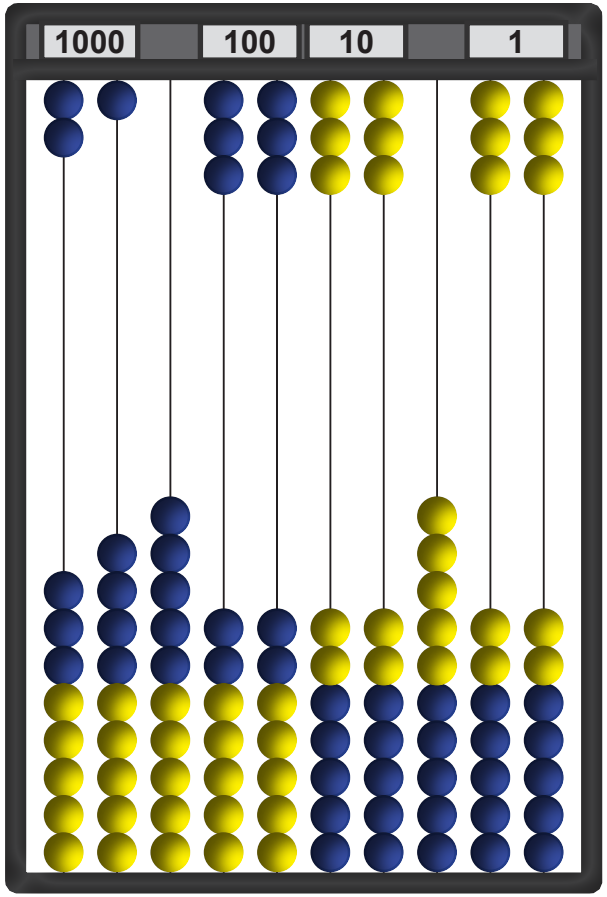


$$\begin{array}{r} 3658 \\ +\ 2738 \\ \hline 6 \end{array}$$

Write 6.

Trading

Adding 4-digit numbers

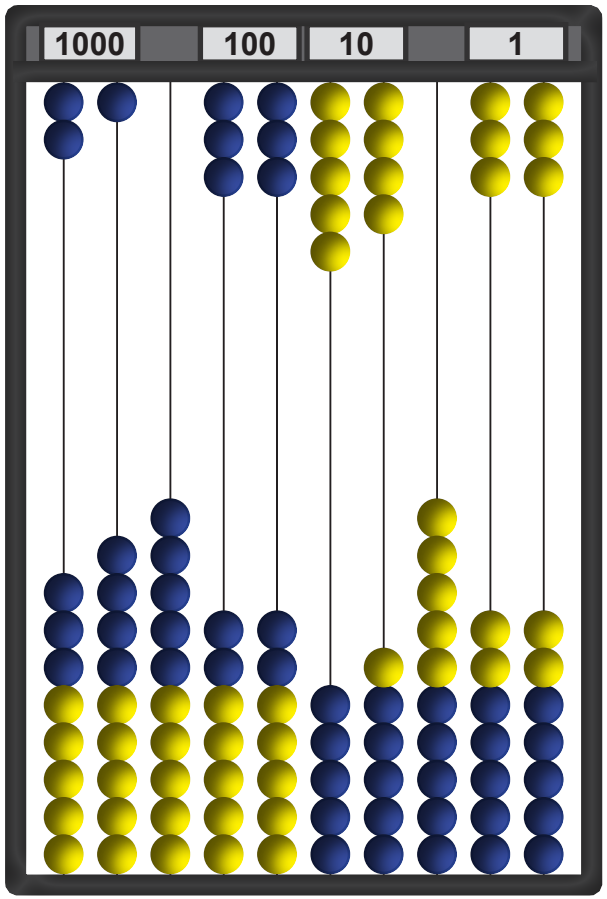


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 6 \end{array}$$

Write 1 for the extra 10.

Trading

Adding 4-digit numbers

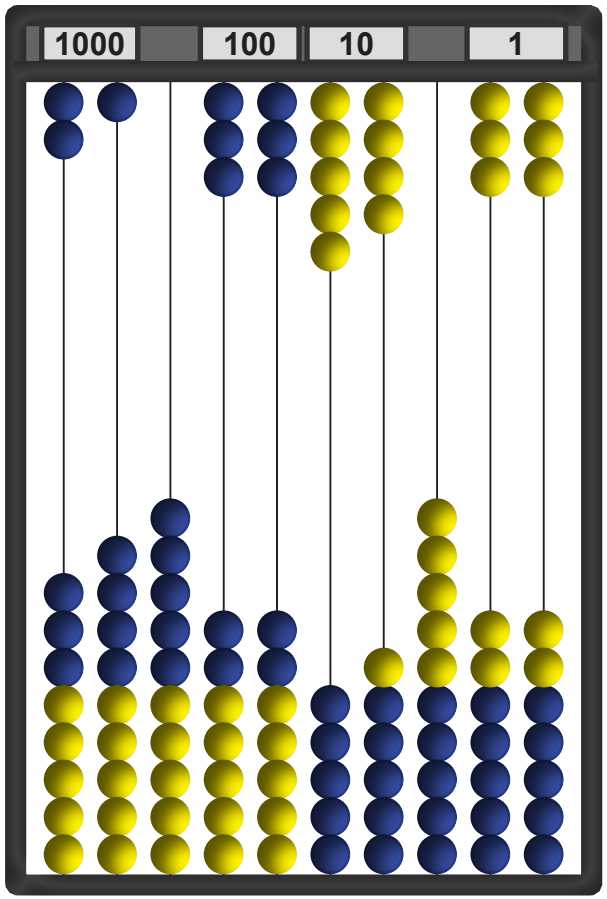


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 6 \end{array}$$

Add the tens.

Trading

Adding 4-digit numbers

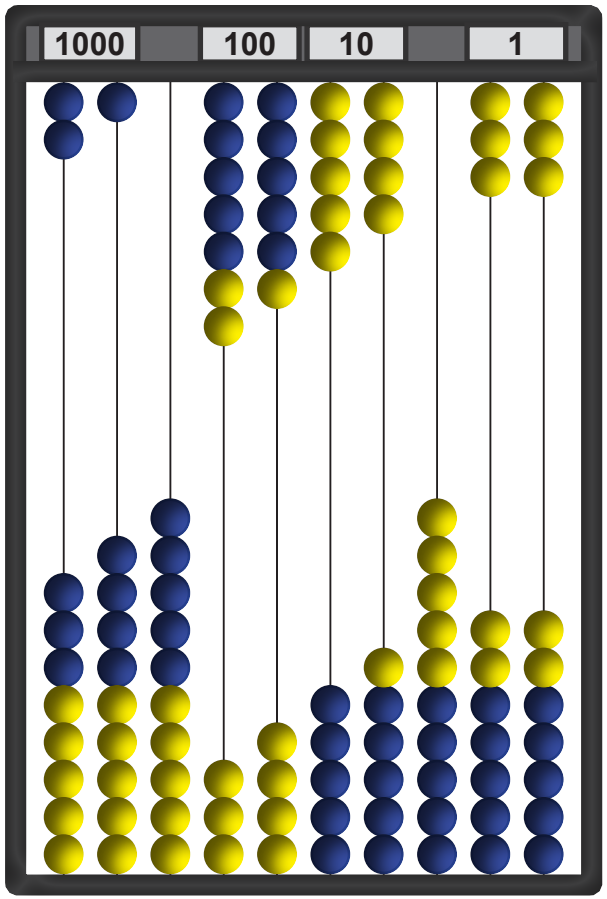


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 96 \end{array}$$

Write the tens.

Trading

Adding 4-digit numbers

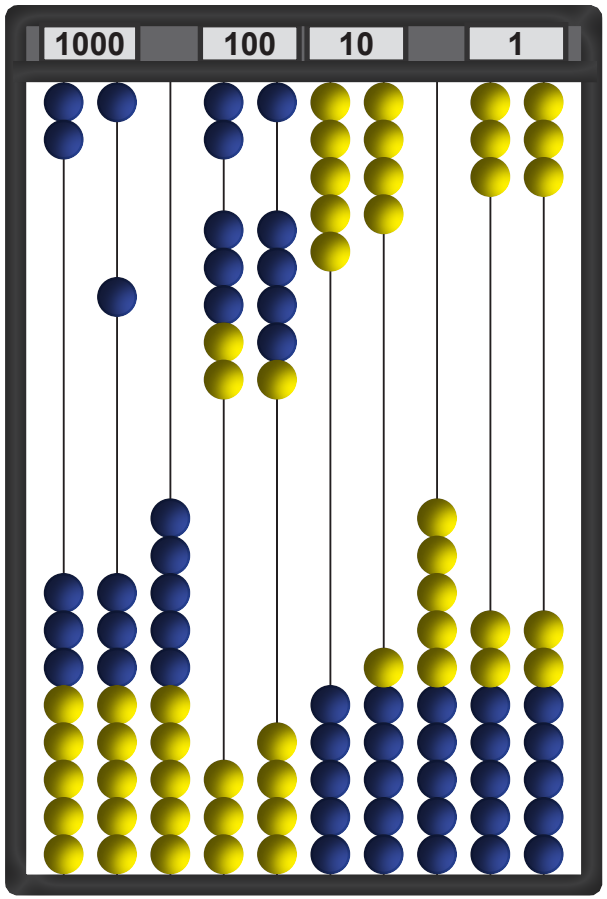


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 96 \end{array}$$

Add the hundreds.

Trading

Adding 4-digit numbers

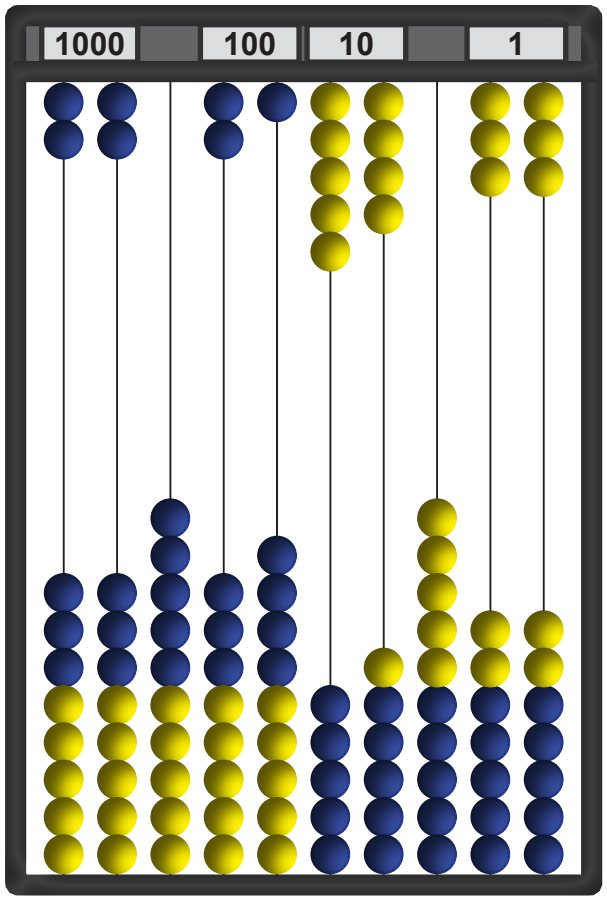


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 96 \end{array}$$

Trade 10 hundreds
for 1 thousand.

Trading

Adding 4-digit numbers

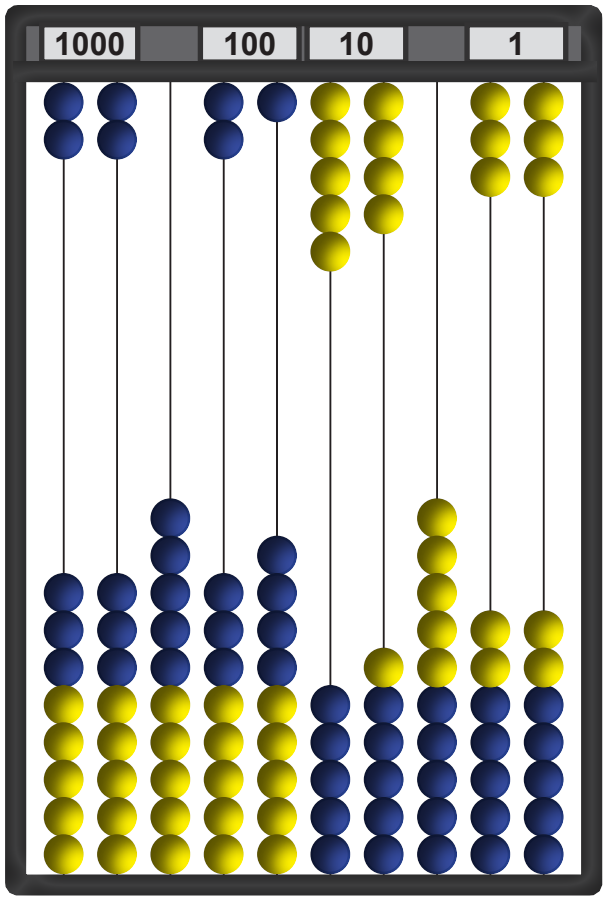


$$\begin{array}{r} {\scriptstyle 1} \\ 3658 \\ +\ 2738 \\ \hline 396 \end{array}$$

Write the hundreds.

Trading

Adding 4-digit numbers

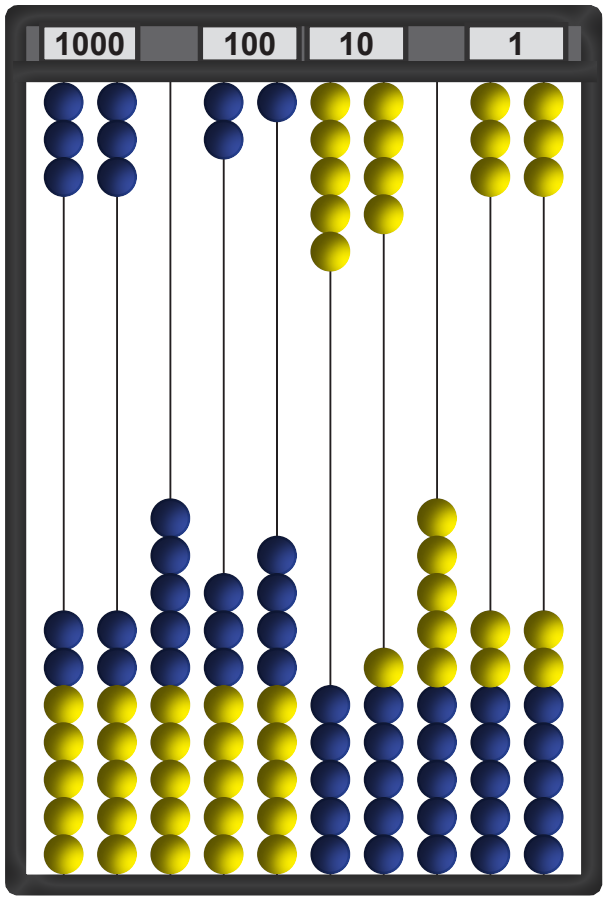


$$\begin{array}{r} {\scriptstyle 1\quad 1} \\ 3658 \\ +\ 2738 \\ \hline 396 \end{array}$$

Write 1 for the extra thousand.

Trading

Adding 4-digit numbers

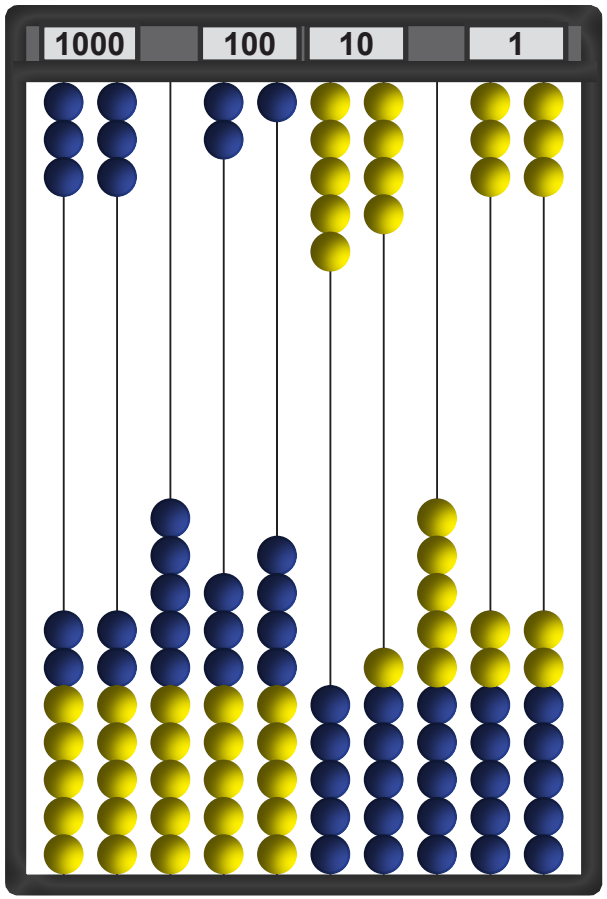


$$\begin{array}{r} {\scriptstyle 1\quad 1} \\ 3658 \\ +\ 2738 \\ \hline 396 \end{array}$$

Add the thousands.

Trading

Adding 4-digit numbers

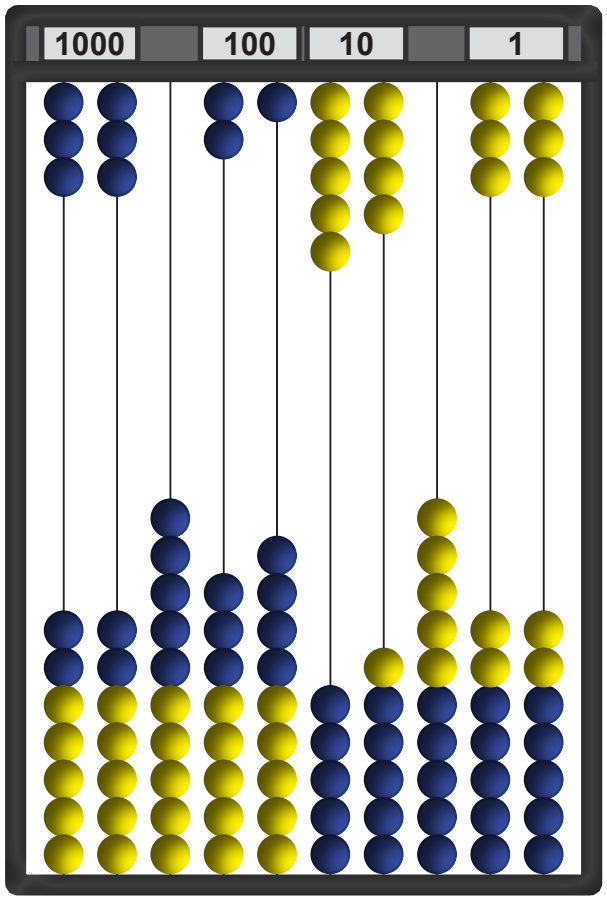


$$\begin{array}{r} {\scriptstyle 1\quad 1} \\ 3658 \\ +\ 2738 \\ \hline 6396 \end{array}$$

Write the thousands.

Trading

Adding 4-digit numbers



$$\begin{array}{r} {\scriptstyle 1\quad 1} \\ 3658 \\ +\ 2738 \\ \hline 6396 \end{array}$$

Short Division

- Means we don’t write the stuff underneath.
- Should always be used for single-digit divisors.
- Is easier to understand than long division.
- Needs to be taught before long division.
- Is much more useful in real life.
- Is much quicker to perform for tests.

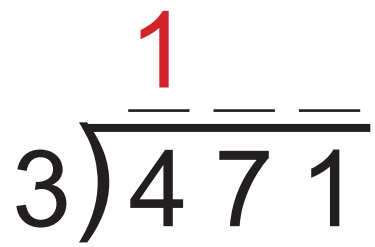
Short Division

$$3\overline{)471}$$

Short Division

The little lines help keep track of place value.

Short Division



400 ÷ 3 = ? [100] Write the 1 on the line.
What is the remainder? [100]
How many tens is that? [10]
How many total tens do we have? [10 + 7 = 17]

Short Division

$$3\overline{)4^{1}7\ 1}\qquad \underline{\ 1\ }\ \underline{\ \ }\ \underline{\ \ }$$

Show the 17 tens by writing a 1 before the 7.

Short Division

$$3\overline{)4^{1}7\ 1}\qquad 1\ \_\ \_\ \_$$

Divide the tens: 17 tens ÷ 3 = ? [5 tens]

Short Division

$$3\overline{)4^{1}7\ 1}\quad 1\ 5$$

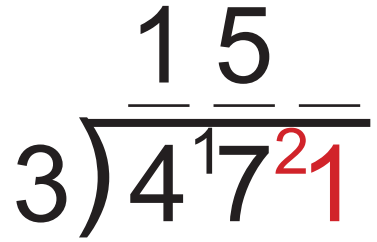
To find the remainder go up: $3 \times 5 = 15$.
How far is 15 from 17? [2]
How many ones do we have? [20 + 1 = 21]

Short Division

$$3\overline{)4^{1}7^{2}1}\quad \underline{\ }\,\underline{5}\,\underline{\ }$$

To find the remainder go up: $3 \times 5 = 15$.
How far is 15 from 17? [2]
How many ones do we have? [20 + 1 = 21]

Short Division



Divide the ones: 21 ÷ 3 = ? [7]

Short Division

$$3\overline{)4^{1}7^{2}1}\quad \underline{1}\,\underline{5}\,\underline{7}$$

Divide the ones: 21 ÷ 3 = ? [7]
Write 7 ones.

Short Division

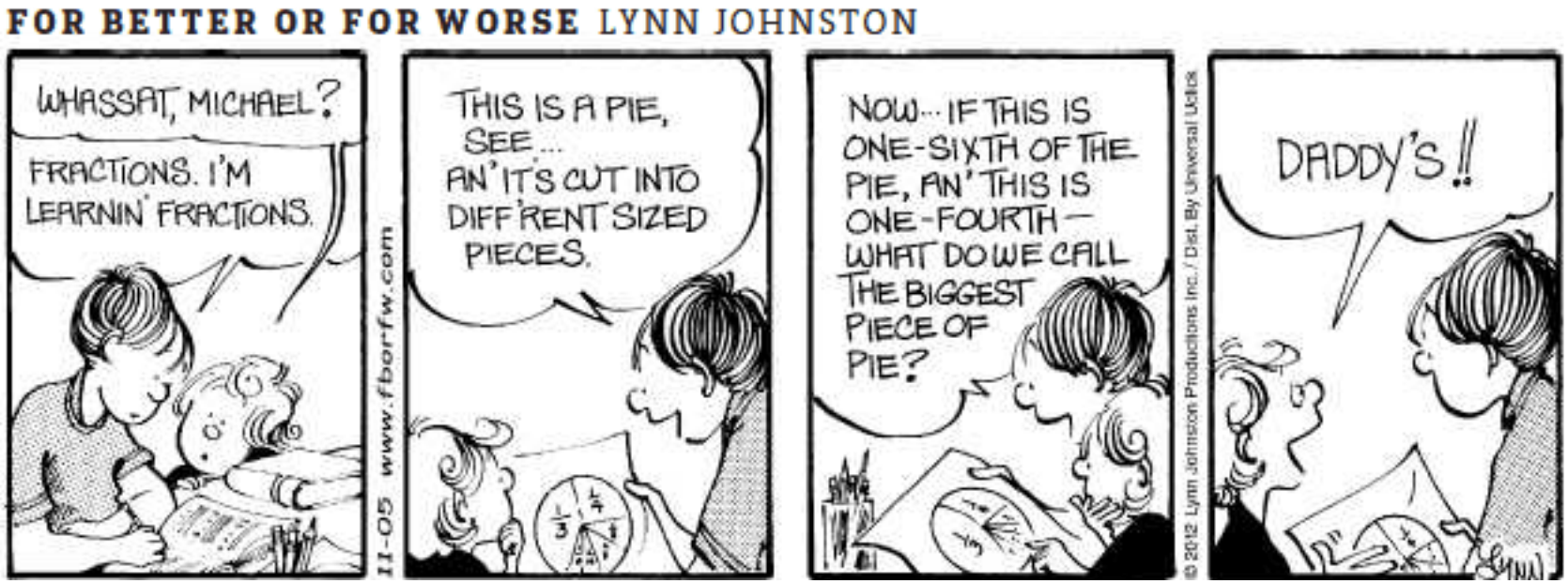
$$3\overline{)4^{1}7^{2}1}\quad 1\,5\,7$$

Long Division

- Children with learning disabilities should not be expected to learn long division. It can take months to learn it — an unwise use of time.
- It is a mostly memorizing a number of steps with little understanding.
- It is not a skill needed in advanced math. Dividing polynomials does not entail guessing a trial divisor.
- Rarely performed on the job or in everyday life. What is important is estimating an answer and knowing what to do with any remainders.

Fractions in the Comics

Fractions in the Comics



Meaning of a Fraction

- One or more equal parts of a whole.
- One or more equal parts of a set.
- Division of two whole numbers.
- Location on a number line.
- Ratio of two numbers.

Meaning of a Fraction

Which meanings are most mathematical?

- One or more equal parts of a whole.
- One or more equal parts of a set.
- Division of two whole numbers.
- Location on a number line.
- Ratio of two numbers.

Meaning of a Fraction

Which meanings are used in everyday life?

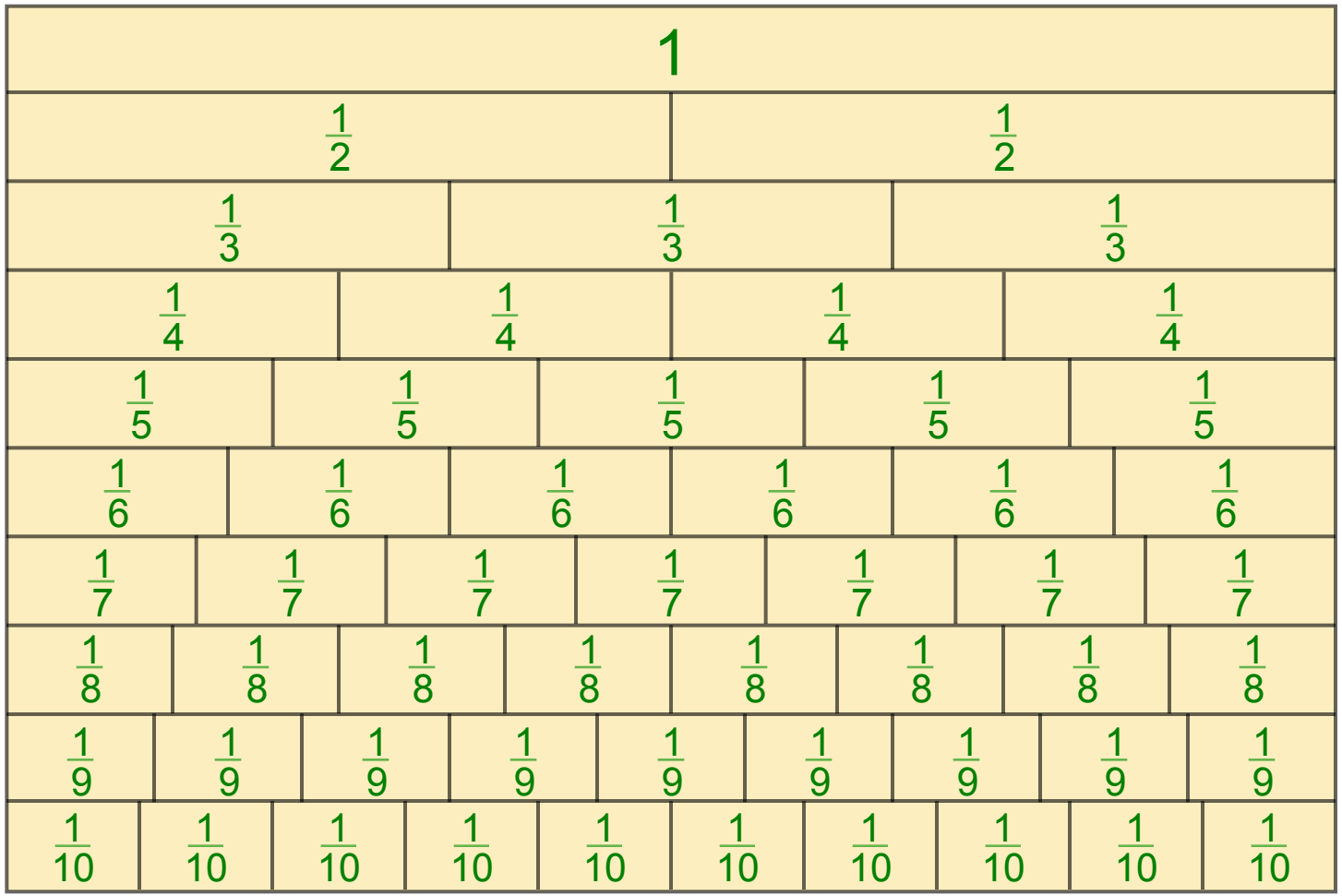
- One or more equal parts of a whole.
- One or more equal parts of a set.
- Division of two whole numbers.
- Location on a number line.
- Ratio of two numbers.

Meaning of a Fraction

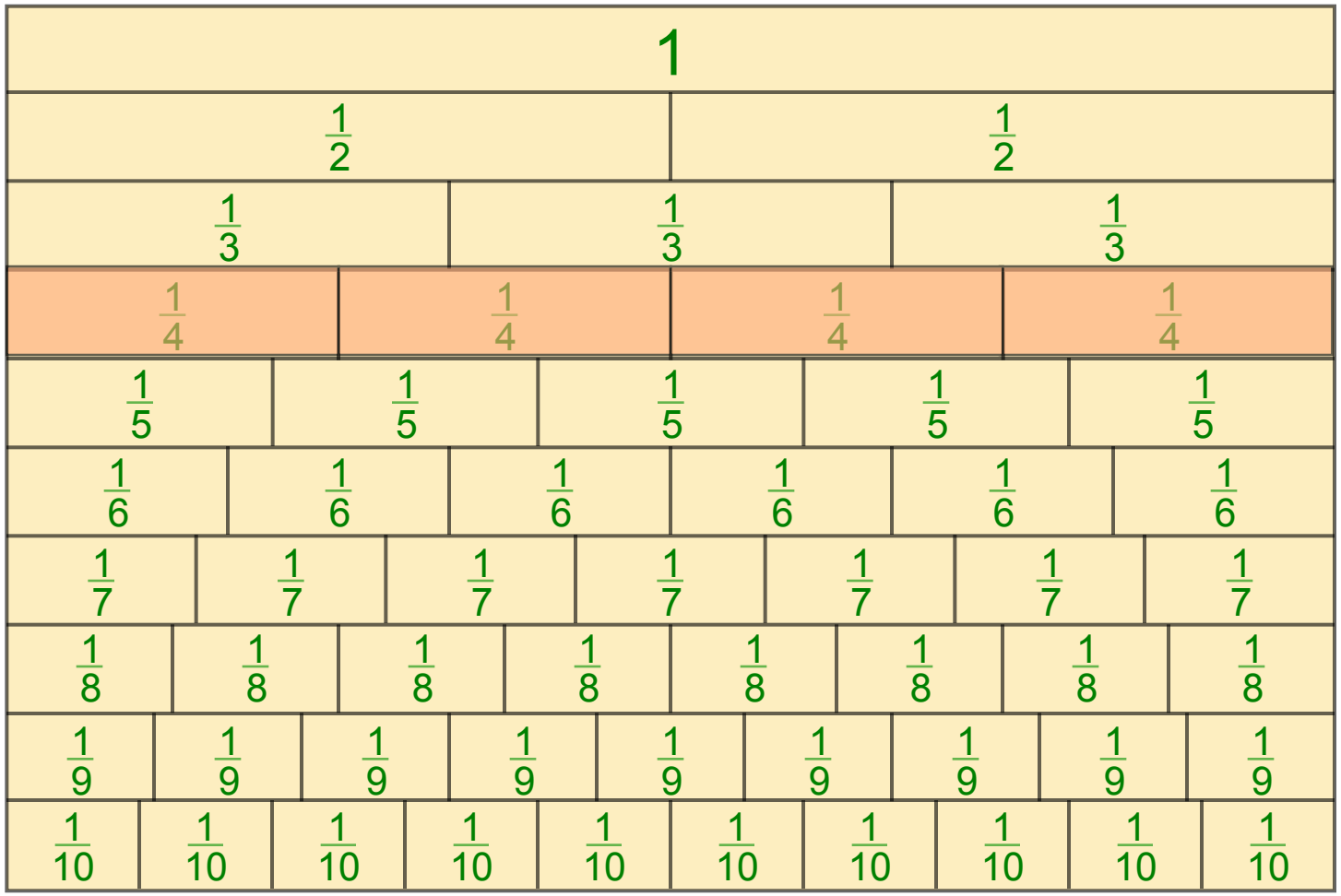
Which meaning is used in elementary texts?

- One or more equal parts of a whole.
- One or more equal parts of a set.
- Division of two whole numbers.
- Location on a number line.
- Ratio of two numbers.

Fraction Chart

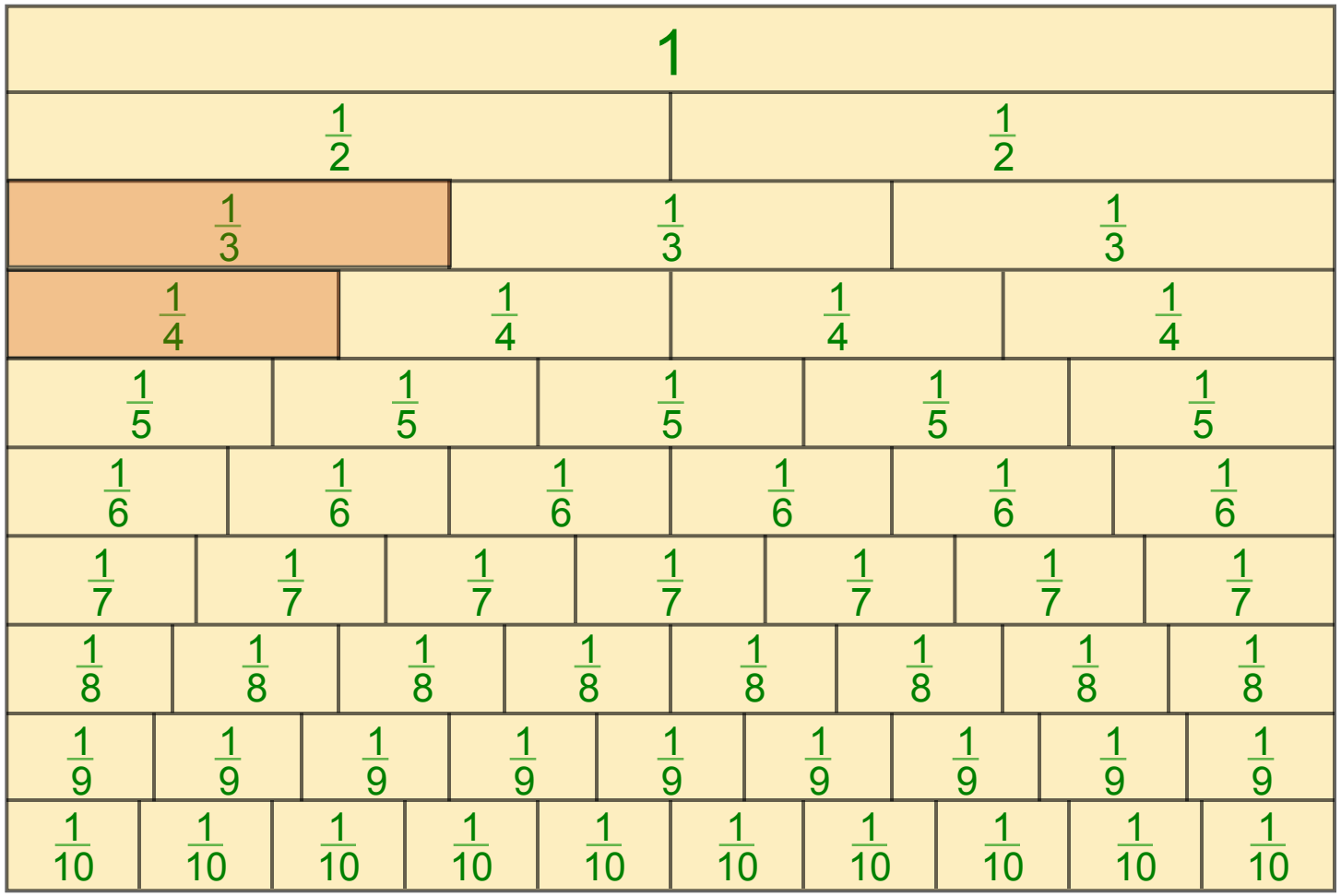


Fraction Chart



How many fourths are in a whole?

Fraction Chart



Which is more, one-third or one-fourth?

Compared to Reading

- Just as reading is much more than decoding, phonics, and word attack skills, mathematics is much more memorizing facts and learning algorithms.
- Just as the goal of learning to read is reading to learn and enjoyment, the goal of math is solving problems and experiencing wonder.

Teaching Math to Children with Special Needs

Joan A. Cotter, Ph.D.
JoanCotter@RightStartMath.com

Sioux Empire Christian Home Educators
Homeschool Conference
Sioux Falls, SD
Saturday, May 2, 2015
3:00 p.m.– 4:00 p.m.