Teaching Mathematics Through Student Worked Examples

November 1, 2016

The Regional Educational Laboratory (REL) Program

- 10 Regions

- Bridging research, policy, and practice
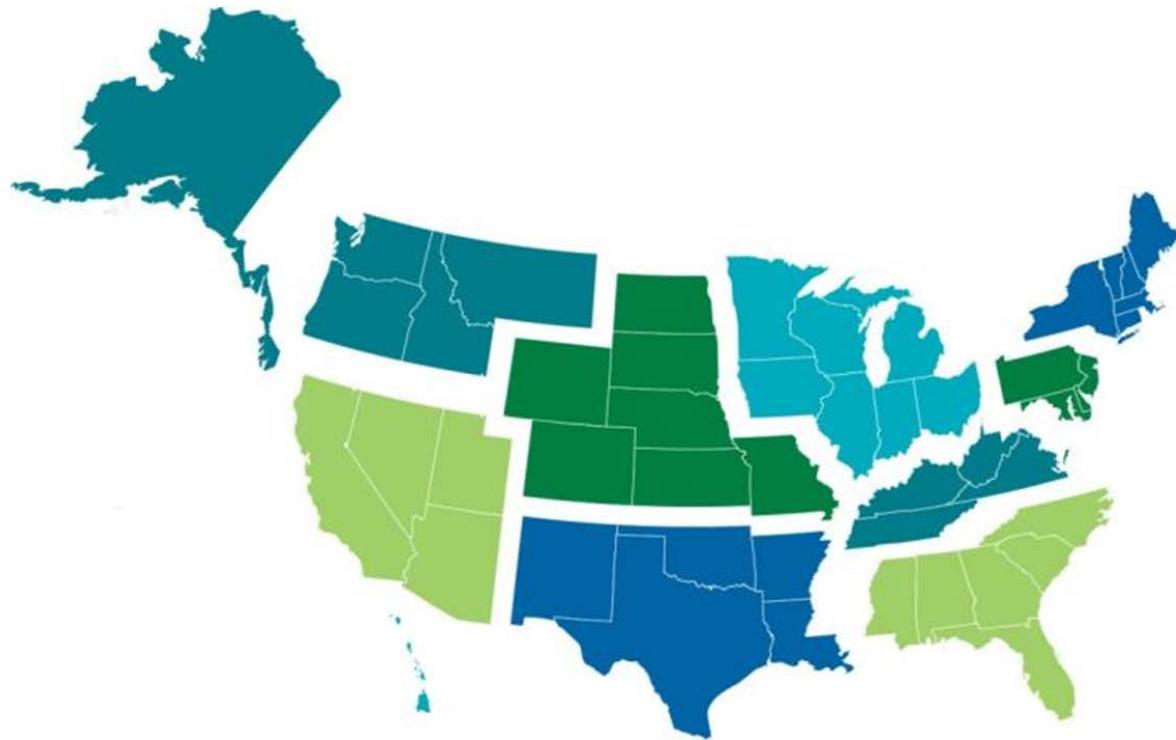
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About the Math Center

Applying findings from research on how people learn to improve classroom materials and instruction

# Organizing Instruction and Study to Improve Student Learning

Organizing Instruction and Study to Improve Student Learning
A Practice Guide

NCER 2007-2004
U.S. DEPARTMENT OF EDUCATION

ies NATIONAL CENTER FOR EDUCATION RESEARCH
Institute of Education Sciences
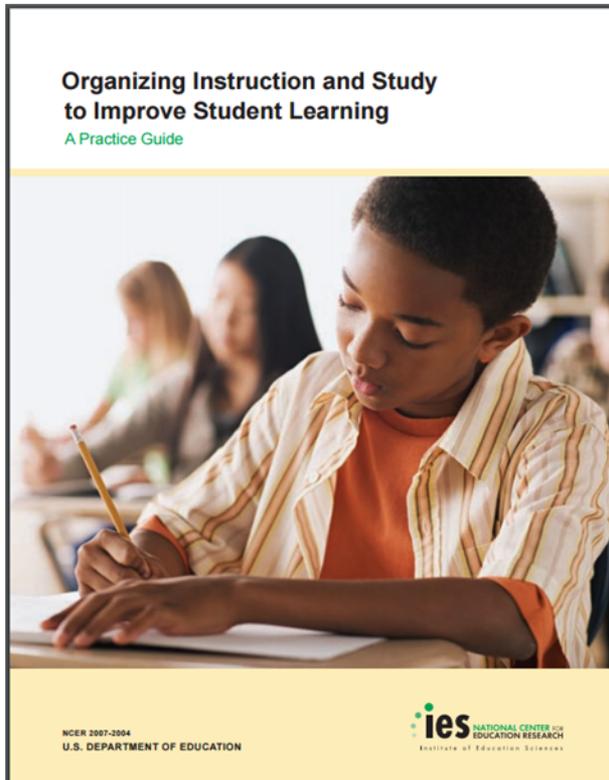
Table 2. Recommendations and corresponding Level of Evidence to support each	
Recommendation	**Level of Evidence**
1. Space learning over time. *Arrange to review key elements of course content after a delay of several weeks to several months after initial presentation.*	**Moderate**
2. Interleave worked example solutions with problem-solving exercises. *Have students alternate between reading already worked solutions and trying to solve problems on their own.*	**Moderate**
3. Combine graphics with verbal descriptions. *Combine graphical presentations (e.g., graphs, figures) that illustrate key processes and procedures with verbal descriptions.*	**Moderate**
4. Connect and integrate abstract and concrete representations of concepts. *Connect and integrate abstract representations of a concept with concrete representations of the same concept.*	**Moderate**
5. Use quizzing to promote learning. *Use quizzing with active retrieval of information at all phases of the learning process to exploit the ability of retrieval directly to facilitate long-lasting memory traces.*	**5a. Low**
5a. *Use pre-questions to introduce a new topic.* 5b. *Use quizzes to re-expose students to key content.*	**5b. Strong**
6. Help students allocate study time efficiently. *Assist students in identifying what material they know well, and what needs further study, by teaching children how to judge what they have learned.*	**6a. Low**
6a. *Teach students how to use delayed judgments of learning to identify content that needs further study.* 6b. *Use tests and quizzes to identify content that needs to be learned.*	**6b. Low**
7. Ask deep explanatory questions. *Use instructional prompts that encourage students to pose and answer "deep-level" questions on course material. These questions enable students to respond with explanations and supports deep understanding of taught material.*	**7. Strong**

http://ies.ed.gov/ncee/wwc/PracticeGuide/1

Learning goals

- Learn about what worked examples are, and the research that supports their use in classroom settings.

- Identify when worked examples may be particularly effective

- Understand how to use worked examples in math instruction

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Worked Examples

Worked Example Principle

Self-Explanation Principle

Worked Example Principle
(e.g., Sweller, 1999)
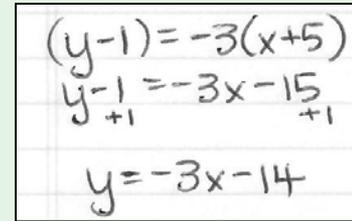
$$(y-1) = -3(x+5)$$
$$y-1 = -3x-15$$
$$y = -3x-14$$

Replacing some problems in a practice session with an example of how to solve a problem

- Reduces working memory load (compared with long strings of practice problems), so learners can focus on learning the steps in problem solving

- Allows students to process the information more deeply when not just routinely applying procedures

- May naturally generate more self-explanations

Self-Explanation Principle

"What does this mean?"

(e.g., Chi et al., 1994)

Prompting learners to explain information as they read or study

- Facilitates integration of new information with prior knowledge

- Forces learners to make their knowledge explicit

- Prompts learners to generate inferences to fill gaps in their knowledge

Also applicable in problem solving:

- How did you get your answer?

Three Kinds of Worked Examples

- Examining Correct Work

- Examining Partially Completed Work

- Examining Incorrect Work

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Implementing **Correct** Worked Examples

Students might be asked to:

- Explain the work shown for a particular task
- Extend the work to a second part
- Answer a question about the response
- Offer an alternative approach
- Explain how they know it is correct
- Determine how the correct response might have been found

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Examining **Correct** Worked Examples

5. Draw a diagram to show how to find the solution to $2.6 \div 0.4$. Explain what the quotient means.

Gary drew a correct diagram and provided a good explanation. Look at his work, and then answer the questions below.
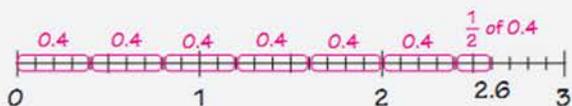
$2.6 \div 0.4$ can be thought of as how many 0.4s are in 2.6. The diagram shows that there are 6 whole 0.4s in 2.6, with a remainder of 0.2, which is half of the divisor (0.4). The solution means that there are 6.5 sets of 0.4 in 2.6.

How does the number line show Gary's answer of 6.5? Where is the 6? Where is the .5?

6. Without doing any calculations, determine whether each number sentence is true. Explain. Then check your answer.

a. $50 \times 432 = (50 \times 400) + (50 \times 32)$

Nala solved this problem in a correct way. Look at her work. and then answer the question below:

True. Using the Distributive Property, 432 groups of 50 is the same as 400 groups of 50 plus 32 groups of 50.

$$50 \times 432 = 50 \times (400 + 32)$$
$$= (50 \times 400) + (50 \times 32)$$

Is the following number sentence also true:

$$50 \times 432 = (50 \times 400) + (50 \times 30) + (50 \times 2)?$$

Explain why or why not.

Implementing **Partially Completed** Worked Examples

Students might be asked to:

- Complete work where a student was stuck and explain the reasoning behind the steps to complete the work

- Complete a student's work and answer a question about the work

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Examining **Partially Completed** Worked Examples

5. Guests at a pizza party are seated at 3 tables. The small table has 5 seats and 2 pizzas. The medium table has 7 seats and 3 pizzas. The large table has 12 seats and 5 pizzas. The pizzas at each table are shared equally. At which table does a guest get the most pizza?

Denise started to solve this problem, but she got stuck. Complete the problem, and then explain your reasoning to Denise.

Small table	Medium table	Large table
$\dfrac{2\ \text{pizzas}}{5\ \text{seats}}$	$\dfrac{3\ \text{pizzas}}{7\ \text{seats}}$	$\dfrac{5\ \text{pizzas}}{12\ \text{seats}}$

b.

1   1.1
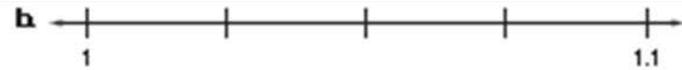
Marco started to label this number line, but he got stuck. Help him fill in the rest of the missing labels.

1   ■   1.05   ■   1.1

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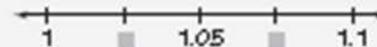
Incorrect Worked Examples

(e.g., Siegler, 2002)

$$K - 6 = 3$$
$$-6 \quad -6$$
$$K = -3$$

Showing students common **incorrect** ways to solve problems, and having them explain why the procedure is inappropriate

- Provides negative feedback, which reduces the relative strength of incorrect strategies

    - Helps them accept that the procedure is wrong

- Forces students to see the differences between the presented problem and others where a procedure does work

    - Exposes and fixes misconceptions

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Implementing **Incorrect** Worked Examples

Students might be asked to:

- Find the error, fix it, and explain the fix

- Fix the incorrect response and complete the work

- Explain why an incorrect response is incorrect

- Explain why the item is incorrect; then complete it correctly

- Answer a question about the incorrect work on the item

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Examining **Incorrect** Worked Examples

30. The greatest one-day temperature change in world records occurred at Browning, Montana (bordering Glacier National Park), from January 23–24 in 1916. The temperature fell from 44°F to ⁻56°F in less than 24 hours.

a. What was the temperature change that day?

Syneef tried to solve this problem, but he didn't do it correctly. Look at his work, and then answer the question below.

44 to ⁻56

44 − 56 = ⁻12 ✗

What is Syneef's error, and why is it a mistake?

4. The organizers of an environmental conference order buttons for the participants. They pay $18 for 12 dozen buttons. Write and solve proportions to answer each question. Assume that price is proportional to the size of the order.

c. How many dozens can the organizers buy for $27?

Anthony tried to set up a proportion, but he didn't do it correctly. Look at his work, and then follow the directions below.

$$\frac{18}{12} = \frac{d}{27} \quad ✗$$

Explain one way that you could fix Anthony's proportion to make it correct. Then, complete the problem.

From the Lab to the Classroom

There is ample evidence that this approach works in laboratory settings, but will it make a meaningful difference in real-world classrooms?

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Example-based condition outscored control on all measures
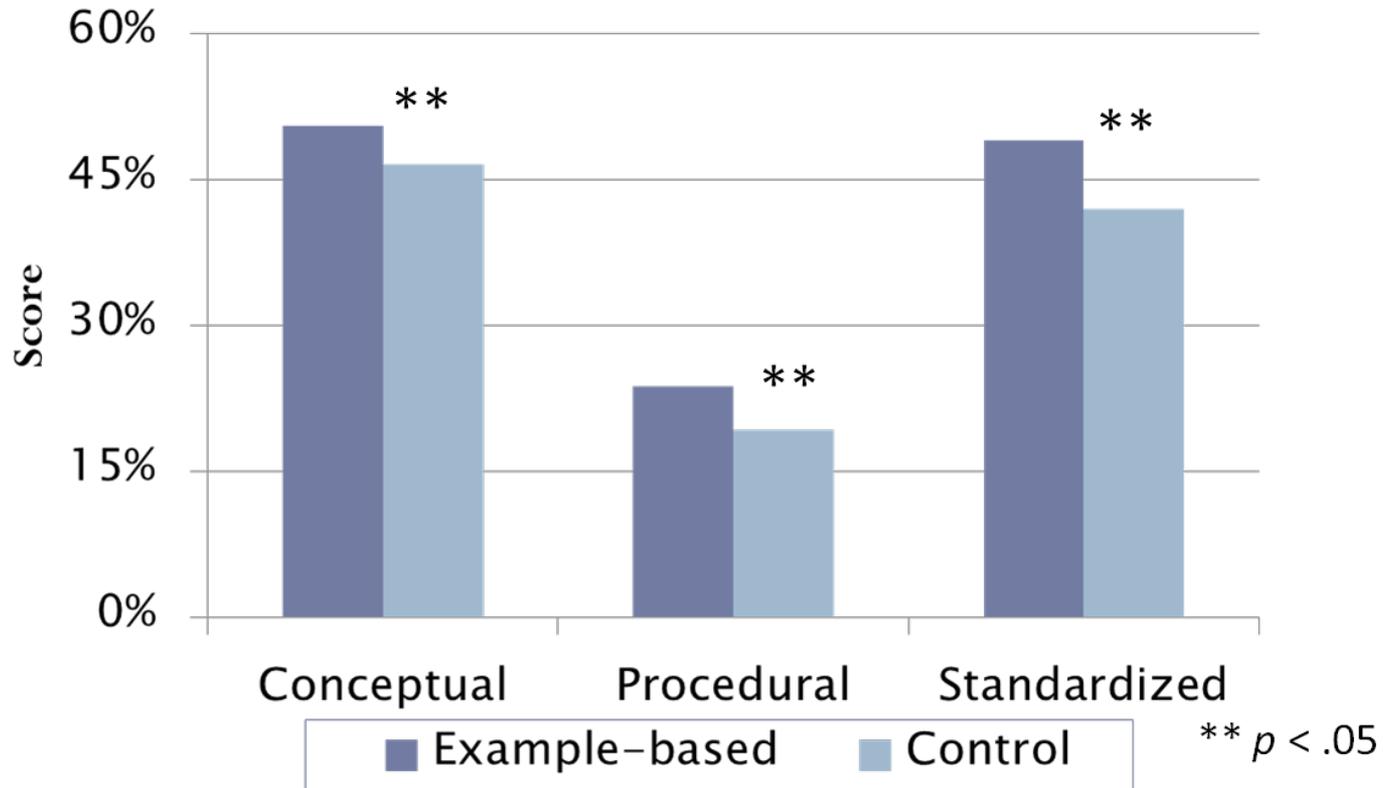
Booth, J. L., Cooper, L., Donovan, M. S., Huyghe, A., Koedinger, K. R., & Paré-Blagoev, E. J. (2015). Design-based research within the constraints of practice: AlgebraByExample. *Journal of Education for Students Placed at Risk, 20*(1–2), 79–100.

Gains in conceptual knowledge even greater for struggling students (up to 10 percentage points)!
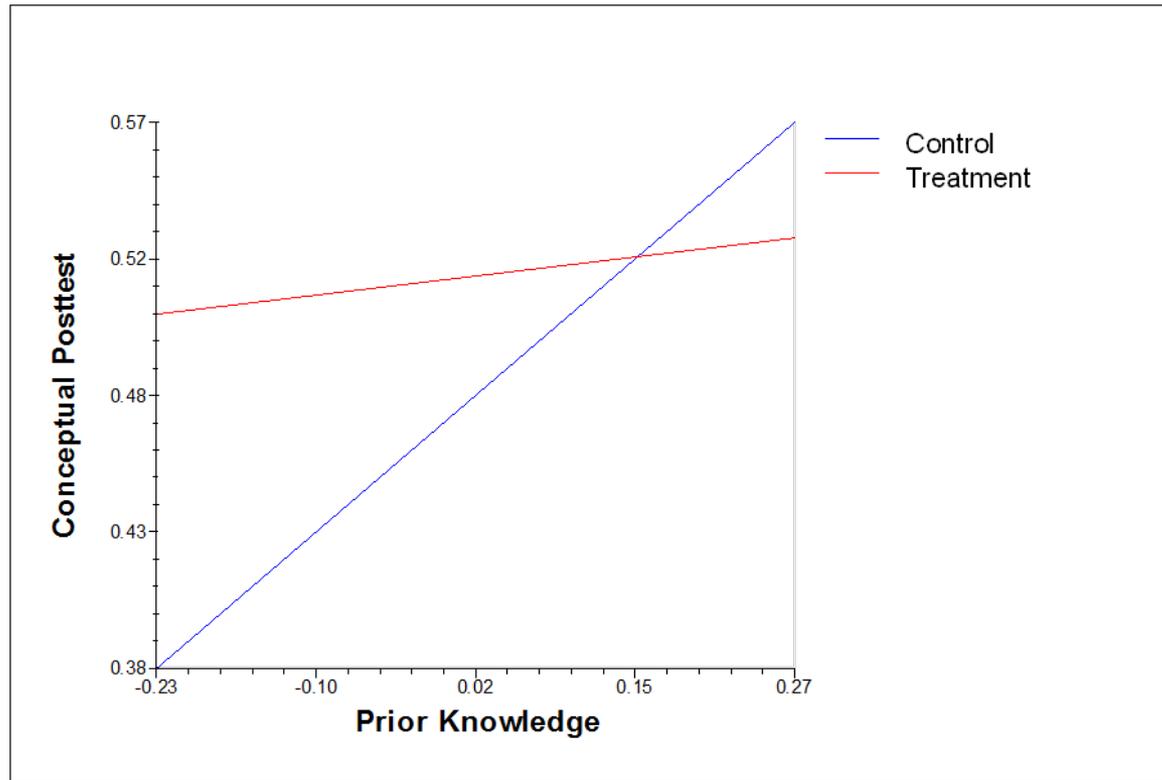
Booth et al., (2015), *Journal of Education for Students Placed at Risk (JESPAR)*

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Inclusion of incorrect examples appears especially important
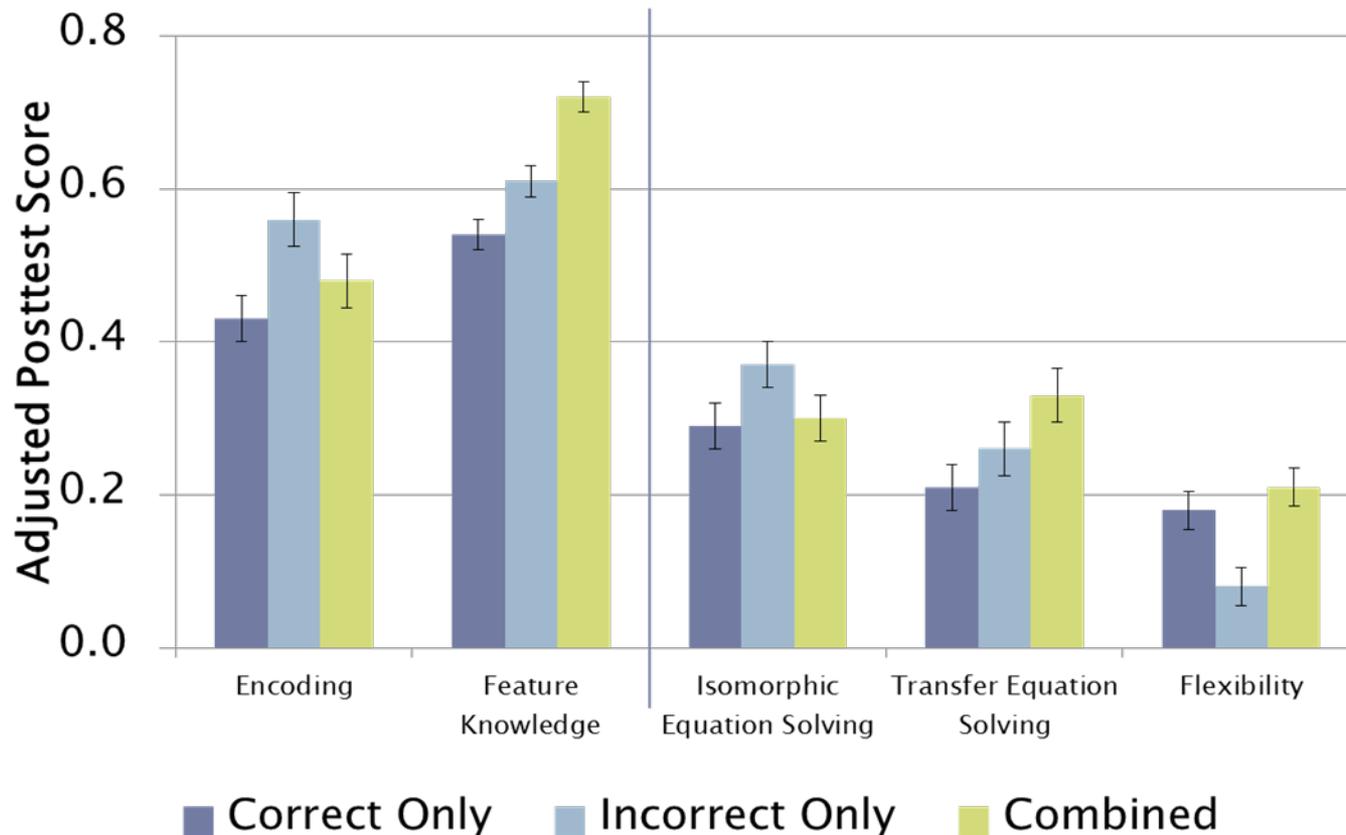
Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Example problems that improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24–34.

Incorrect examples are particularly beneficial for struggling students
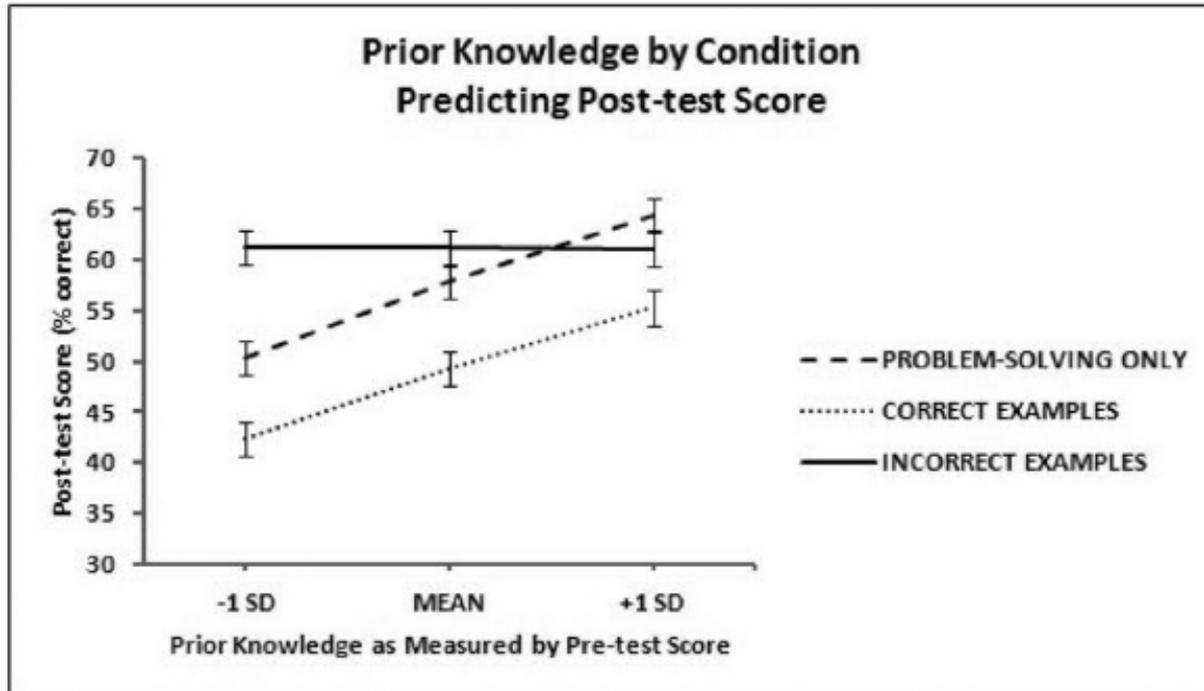
Barbieri, C., & Booth, J. L. (in press). Support for struggling students in algebra: Contributions of incorrect worked examples. *Learning and Individual Differences.*

Hands-on: How to Create Incorrect Examples

1. Write the objective.

2. List a few common misconceptions associated with this objective.

3. Choose **one misconception** for each example.

4. Create the incorrect worked example using that misconception.

   a) Clearly mark the problem as incorrect.

   b) Use a student name — remember to be gender and culturally diverse.

5. Write the self-explanation prompt focusing on the misconception.

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Self-Explanation Prompt Tips

Do…

- Ask the "why" questions.

- Have students explain their reasoning.

- Call students attention to the features of the problem you think are important.

Don't just …

- Ask questions that instruct students to state the procedure.

- Ask "what is wrong with the example", "what mistake was made," or "what is the correct answer."

Let's Practice: Step 1

Step 1: Write the objective.

Students will be able to simplify an expression by combining like terms

Let's Practice: Steps 2 and 3

Step 2: List a few common misconceptions associated with this objective.

Step 3: Choose **one misconception** for each example.

"Students do not move negative sign with the term"

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Let's Practice: Step 4

Step 4: Create the incorrect worked example using that misconception.

1. Clearly mark the problem as incorrect.

2. Use a student name – remember to be gender and culturally diverse.
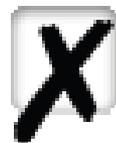
Joseph tried to simplify this problem but **didn't** do it correctly. Here is his first step:

$(5-4x)+12x$

$$(5-4x)+12x$$
$$5-(4x+12x)$$

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Let's Practice: Step 5

Step 5: Write the self-explanation prompt focusing on the misconception.
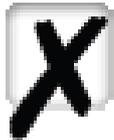
X Joseph tried to simplify this problem but **didn't** do it correctly. Here is his first step:

$(5-4x)+12x$

$$(5-4x)+12x$$
$$5-(4x+12x)$$

✎ Why can't Joseph just move the parentheses in this problem?

✎ What should Joseph have written in order to write the answer in simplest form?

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Activity

Use the worksheet to create an incorrect worked-example with a self-explanation prompt.

Things to remember…

- ✓ Remember to focus on a specific misconception.
- ✓ Ask students to explain their reasoning.
- ✓ Avoid simply asking students what is wrong with the example.

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Instructions

1. Write the objective.

2. List a few common misconceptions associated with this objective.

3. Choose **one misconception** for each example.

4. Create the incorrect worked example using that misconception.

    a) Clearly mark the problem as incorrect.

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5. Write the self-explanation prompt focusing on the misconception.

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REL West Resources

- Follow us on Twitter
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- Ask A REL: http://ies.ed.gov/ncee/edlabs/askarel/index.asp
- This webinar will be archived in the coming weeks at https://relwest.wested.org/events/341

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Math Center Resources

- Website: http://iesmathcenter.org
- *A Worked Example for Creating Worked Examples*: http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/2015/Vol21/Issue1/A-Worked-Example-for-Creating-Worked-Examples/