EURASIA Journal of Mathematics, Science and Technology Education
ISSN: 1305-8223 (online) 1305-8215 (print)
OPEN ACCESS 2018 14(4):1305-1315 DOI: 10.29333/ejmste/83561

Teaching of Mathematical Modeling Elements in the Mathematics Course of the Secondary School

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Received 6 October 2017 ▪ Revised 24 November 2017 ▪ Accepted 31 December 2017

ABSTRACT

The urgency of the problem under investigation is due to the role of mathematical modeling in modern science and human practice, which requires the acquaintance of students with the elements of this process at the early stages of education. Training of mathematical modeling shows the students how to apply mathematics in real life, which is also a motivation for learning the subject. The purpose of the research is to identify elements of mathematical modeling that can and should be appropriately formed at the secondary school. The leading method of the research is the analysis of the structure of the mathematical modeling process and the development of a system of tasks aimed to form training activities that are adequate to the identified elements. The authors offer to use the system of changed tasks contained in school mathematics textbooks. The article proves the necessity of acquaintance of schoolchildren with the structure of the process of mathematical modeling, features of models, purpose of their use. As a result of the research, the authors present a model of the system of problems aimed to form elements of mathematical modeling relating to the stages of formalization and interpretation. The methodology proposed in the article can be used by mathematics teachers at lessons and elective courses, authors of textbooks and manuals, and also can be the basis for special courses for students of pedagogical universities.

Keywords: elements of mathematical modeling, stages of formalization and interpretation, tasks for the formation of elements of mathematical modeling

INTRODUCTION

The study and teaching of mathematics plays a system-forming role, significantly influences the intellectual development of schoolchildren and students, ensures the readiness of students to apply mathematics in other fields. According to the Concept of the development of mathematical education in the Russian Federation, the speedy development of mathematical education, the appeal to mathematical research, including its practical application, has to provide a breakthrough in such strategic areas as information technology, modeling in engineering, forecasting natural and man-made disasters, energy, economy, biomedicine. In connection with this, among the tasks defined by the Concept, the primary role is played by the "modernization of the content of the curricula of mathematical education at all levels <...>, based on the needs of students and the needs of society in general mathematical literacy, in specialists of various profiles and levels of mathematical training, achievements of science and practice" (The Concept of the development of mathematical education in the Russian Federation, 2013). The federal state educational standard clearly indicates the need for students to get ideas about mathematical modeling, to master mathematics as a universal language of science (Federal state educational standard of secondary (complete) general education, 2012). The need for explicit introduction of the concepts of the mathematical model

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Contribution of this paper to the literature

- The authors have defined elements of mathematical modeling and actions characterizing the process of applying mathematics for describing and studying real processes and phenomena that can and should be appropriately formed in the main school.
- The authors have developed a model of the system of educational and applied problems aimed to form selected elements of mathematical modeling. This model includes tasks for testing the actions typical for both the formalization stage and the interpretation stage, which allows a more meaningful demonstration of the application of mathematics in practice.
- The technique described in the article makes it possible to use tasks from school textbooks of mathematics in the work with students, subjecting them to minor processing, which greatly facilitates the work of the teacher.

and mathematical modeling into the school course has been discussed by the pedagogical community for a long time. The works of Firsov (1974), Stukalov (1975), Morozov (1978), Fridman (1984), etc. are devoted to this problem. Arguments at different stages of the development of society were the didactic principle of linking education with life, the implementation of the applied orientation in the school course, strengthening the humanization of teaching mathematics, the introduction of specialized education at Russian schools. As a result, Zubareva and Mordkovich (2013), Dorofeeva and Peterson (2013) consider these concepts in their textbooks at the earliest stages of education. Mathematical modeling is associated with the solution of problems, and throughout the course of 5-11 grades the stages of formalization, intramodel solution and interpretation are clearly distinguished when studying each new mathematical model. However, the twenty-year practice of working with mentioned textbooks showed that teachers are very reluctant to study the material with the students, or they treat it formally. Our survey showed that students still connect the concept of "model" mainly with material, in particular, with models of geometric bodies. The solution of the tasks has not practically changed, and the problems have not changed either. Obviously, to teach elements of mathematical modeling, it is necessary to include special tasks in the content which are close to those tasks that people face in practice. Such tasks in the practice of teaching are called applied. The analysis of textbooks shows that they have very few such tasks, as a result of which the stages of formalization and interpretation are sufficiently closed. The purpose of our research is to identify elements and actions, relating to the stages of formalization and interpretation, which can be formed already in the secondary school. The means of formation are the education and applied tasks that can easily be obtained by the teacher from the tasks contained in the school textbooks by means of minor changes.

LITERATURE REVIEW

Theoretical and psychological bases of teaching schoolchildren of mathematical modeling have been studied by Salmina (1981), Fridman (1983), Bavrin (1993), Melnikov, Solovyanov, and Shirpuzhev (2017), Edelstein-Keshet (1988) etc. Methodical literature considers mathematical modeling from different points of view. First of all, let us single out a group of works where knowledge of mathematical modeling acts as a way of realizing the applied orientation of mathematics. Firsov (1977), Kolyagin and Pikan (1985), Tereshin (1990), Wozniak and Gusev (1985), Shapiro (1990), Egupova (2014), etc. studied mathematical modeling from this perspective. Another aspect of mathematical modeling is devoted to the study of intersubject connections and the integration of mathematics with other school disciplines (natural and humanitarian sciences). Here we can point out works of Usova (1980), Khomutsky (1981), Polyakova (1999), Abaturova (2010), Luneeva and Zakirova (2017), etc. In the works of these authors, the term "a mathematical model" is studied mainly in the upper grades of secondary school and serves as an illustration of the application of mathematics in various fields of human activity. This approach is supported by authors who study possibilities of teaching mathematics in specialized non-mathematical classes (chemico-biological, medical, etc.): Dvoritkina (1998), Ivanova (2003, 2004), Voznyuk (2013), Zelenina and Krutikhina (2011, 2014).

In Mordkovich's course of mathematics for the secondary school (1996), mathematics is considered to be the science of mathematical models. Therefore, the concept of a model is a core in the gradual unfolding of discipline. Therefore, it should be taught if not in the primary school, then at least in 5-6 grades. At the same time, the authors emphasizes the importance of mathematical modeling in the realization of the humanitarian (cultural) potential of the school course of algebra. However, as mentioned above, some essential properties of the mathematical model are not considered and, therefore, require a more meaningful disclosure of the stages of formalization and interpretation.

In foreign literature, the widespread use of math applications in teaching has been perceived ambiguously up to the present time. Thus, Ocampo, Santos, and Folmer (2016) discuss the problem of interdisciplinary learning. A survey of a large number of teachers showed that many of them are afraid of devaluation of mathematics. However, the authors argue that initial and continuing education within the framework of the interdisciplinary approach will facilitate the acceptance of this process by teachers and their self-education. Alongside the subject of integration of mathematics and natural sciences is widely studied in the literature. For example, Kim and Aktan (2014) showed that all units in the mathematics curriculum in Turkey can be integrated with physics, chemistry or biology. This integration will promote both motivation for learning and social needs. At the same time, it is underlined that there are difficulties in harmonizing the curricula of different disciplines and choosing the means for their effective implementation. Many scientists consider mathematical modeling to be a promising way of improving education. Michelsen (2006) emphasizes the importance of modeling in teaching, in particular, in the study of the term "function" in the upper grades. Also there is a didactic model of interdisciplinary learning in the field of mathematics and natural sciences. Burkhardt and Pollak (2006), considering mathematical modeling as an element of school curricula, show that modeling makes a significant contribution to the mathematical literacy of future citizens and professionals. The problems of modeling are also discussed by Lester and Kehle (2003), Lesh and Doerr (2003), Lingefjard (2006), Zeytun, Cetinkaya and Erbas (2017). Blomhoj and Hoff Kjeldsen (2006) describe the experiment on conducting a series of seminars on the creation of projects in the field of mathematical modeling in the upper grades of the gymnasium. In the work of Kaiser and Schwarz (2006) modeling is a bridge between school and university. Thus, the main direction of integration of mathematics and natural sciences is the teaching of mathematical modeling in high school. At the same time, there are difficulties both in the organization of the learning process and in the selection of the necessary content.

MATERIALS AND METHODS

Goals and Objectives of the Study

The purpose of the research is to develop theoretical and methodological bases for teaching of mathematical modeling elements in the secondary school. The tasks are: to identify the elements of mathematical modeling, which can be studied at school; to develop a model of a system of tasks aimed at the formation of selected actions; to prove the introduction of concepts of mathematical model and mathematical modeling, properties of mathematical model in the secondary school; to develop methodological recommendations for the implementation of the research results.

Methods of the Research

To carry out the research the authors used the following methods: analysis of psycho-pedagogical and mathematical-methodical literature on the topic, analysis and generalization of the experience of teachers and their own experience, analysis of educational products, method of mental experimentation, forecasting, systematization and generalization of facts and concepts , modeling, method of expert assessments, analysis of the results of educational activity, development and application of teaching materials.

Experimental Research Base

Approbation, generalization and implementation of research results are carried out:

- during the authors' teaching, in practical classes on the methodology of teaching mathematics and during the period of pedagogical practice with students of the Mathematics Faculty of Vyatka State University, the Faculty of Computer and Physico-Mathematical Sciences of the VSU, in the practice of mathematics teachers in Kirov and Kirov region;
- as reports and speeches at scientific conferences and seminars of various levels, including international ones, publications in collections of scientific articles and scientific and methodical periodicals.

Stages of the Research

The research had three stages.

The first stage revealed the state of the problem in the theory and practice of teaching. For this purpose, the authors studied and analyzed psychological-pedagogical and mathematical-methodical literature on the problem, observed and analyzed the experience of teachers of mathematics on the subject of learning the elements of modeling in the process of solving the problems.

At the second stage, the authors developed methodical recommendations for teaching selected elements of mathematical modeling. Discussion of the implementation of methodological recommendations has been carried out through feedback from teachers of mathematics and at conferences and seminars of various levels. It has led to a consistent improvement of the proposed methodology.

In parallel with the second stage, the third stage has been carried out, during which the authors and teachers of the mathematics of schools in Kirov and Kirov region conduct experiential teaching and approbation of the proposed recommendations.

RESULTS

To identify the elements of mathematical modeling that can and should be appropriately formed in the secondary school, let us turn to the heuristic block diagram of constructing a mathematical model of the applied problem, proposed by Stukalov (1976). Under the applied problem, we understand a problem that is outside mathematics and solved by mathematical means (Tereshin, 1990). In the methodological literature, many authors consider the process of solving the applied problem as a series of constructing various models (Bylkov, 1986; Fridman, 1984; Lozhkina, 2008). In practice, tasks are usually formulated as a question and do not contain or contain a minimum number of numerical data. Such problems are called tasks-problems. To solve the task-problem, it is necessary to find out which quantities and their values are necessary for constructing a mathematical model. The task, the text of which contains these components, is called the verbal model of the task-problem. A mathematical (solving) model is compiled using a verbal model, besides auxiliary visual diagram models, drawings, tables, drawings, etc. can be used during the compilation (Friedman, 1984; Krutikhina, 2004; Luneeva, 2011). School textbooks, as a rule, do not contain tasks-problems, students are immediately faced a verbal model of the task-problem. Thus, the formalization stage is presented too narrowly, that is, there are no conditions for a meaningful disclosure of the activities taking place at this stage of mathematical modeling. Of course, within the basic general education it is impossible and it is not necessary to teach all schoolchildren to solve applied problems. However, individual actions that reveal the essential properties of the mathematical model and the features of its construction, which characterize precisely the transition from the task-problem to the verbal model, can be successfully formed with the help of educational and applied problems. Educational and applied tasks are called subject tasks aimed to form individual actions necessary to ensure the activities of students at the stages of formalization and interpretation in solving applied problems. Using the block diagram of Stukalov (1976), the results of the study by Morozov (1978), Skvortsova (2003), Tselishcheva and Zaitseva (2008), we identified the following elements of mathematical modeling, which allow more meaningfully to disclose the stages of formalization and interpretation: replacement of the original terms with selected mathematical equivalents; evaluation of the completeness of the initial information and if necessary, the introduction of missing numerical data; choice of the accuracy of numerical values corresponding to the meaning of the problem; identification of the possibility of obtaining data for solving the problem in practice.

We give the detailed description of the above-mentioned actions and applied problems, with the help of which they can be formed.

The replacement of the original terms with the selected mathematical equivalents is based, first of all, on the students' life experience. Pupils should know the terms that occur in everyday life or when studying other subjects that can be replaced by mathematical concepts or relations. Therefore, the system of tasks of school textbooks should have as many problems as possible with terms from different scientific fields, which do not require a long and cumbersome explanation of their meaning. Moreover, such tasks enlarge the students' vocabulary, introduce new interesting facts of some science. Teaching how to replace the original terms with their mathematical equivalents can happen while forming concepts. For example, existing school textbooks, when studying the circle in grades in the explanatory text and in the source material, have very few examples of analogues of the circle in practice. The overwhelming majority of tasks require only the image of the circle, the calculation of the radius or its length, although tasks using such terms as "parallel", "meridian", "rim of the wheel", "girth", "rim", etc., will be of great use. Students can be offered the following tasks.

1. What is the length of the wheel rim of the bike, if the length of the spoke is 35 cm?
2. The girth of the tree is 1.5 m. Find the thickness of the tree.
3. The length of the diameter of the globe is approximately 12.7 thousand km. How many thousands of kilometers is the length of the radius and length of the equator of the Earth?

By substituting the terms "rim", "thickness", "girth", etc., students actually deal with the mathematical model of the circle and its elements. Here it is appropriate to clarify the fact that the model is always close to real objects, takes into account their most essential properties, and therefore has a certain error. Thus, when studying mathematical concepts, it is advisable to offer students examples of objects for which this concept is a mathematical model. At the same time, while solving problems, it is necessary to specify significant and insignificant properties of models.

When teaching the evaluation of the completeness of the initial information and the introduction of missing numerical data, it is necessary to take into account components that may be in the condition of these tasks: the plot (objects), quantities characterizing them, values of these quantities. Combining different components in the condition, you can get different types of tasks. Besides, tasks within one type can differ by the form of the task: a table, a diagram, a drawing, a short entry, and so on. Let us give some examples.

1. The cyclist and the pedestrian left the village at the same time and went to the city along the same road. The cyclist is moving at a speed of ... km / h, the pedestrian is moving at a speed of... km / h. What will the distance be between them in 1.5 hours? The task has the plot, quantities necessary for the decision. A student should introduce real numerical values of quantities independently and perform rounding in accordance with the meaning of the task.
2. According to the annual report of the school:

- number of students at the beginning of the academic year:	643
- arrived during the year:	24
- transferred to other classes:	4
- left the school:	8
- left for re-training:	2
- finished school:	78

How many students are at school at the end of the school year?

The task has the plot, an excessive number of quantities and their values. It is necessary to evaluate the completeness of the information and choose the quantities necessary for answering the question.

3. Write a problem by numerical expression 1230: (65 + (65 - 7)). Only numerical data of certain quantities are known. We need to create the plot and choose values that can be characterized by such numerical data.
4. Compose the task using the values of distance, speed, time. Only quantities are known. It is necessary to create the plot, including in it a necessary and sufficient number of quantities and their values. You can create the task with too much data, but it should not be contradictory.
5. *M* liters of gasoline for *N* cars comes to the garage during *t* days. Which of the given values shows how many liters of gasoline is used by one machine per day? What do other values show?

A) $\frac{m}{t}$; B) $\frac{m}{N}$; C) $\frac{m}{Nt}$.

In the proposed problem, a mathematical model is already given, its interpretation is required.

Speaking about teaching of choosing the accuracy of numerical values corresponding to the meaning of the problem, we do not mean the formation of concepts related to approximate calculations. When solving problems in practice, we seldom get "round" answers. For example, it is unreasonable to calculate the mass of paint to cover a large surface with an accuracy of a gram, so it is necessary to be able to round off numerical data in accordance with the meaning of the problem. The formation of this action should begin already with the acquaintance of schoolchildren with units of measurement in primary school. When studying all the units, it is necessary to consider what objects in practice are measured by the given unit. For this you can use tasks of the following types.

1. To manufacture labels for a matchbox, you should know the dimensions of the rectangle on which the label will be stuck. In what units should you measure the length and width of the rectangle?
2. You need to buy a cloth for the dress. In what units will the seller measure you the length of the fabric?

Let's pay attention to the fact that tasks for the correspondence of numerical values of quantities and objects measured by them in recent years have been included in the control measuring materials of the final school exam. It emphasizes the necessity and importance of such tasks.

Next, it is necessary to consider the tasks where you have to round the result, but the rounding accuracy is not specified. Such problems are very rare, but they occur in school textbooks. Let us give some examples.

1. Three teenagers were given the task: to calculate how many trees there are in the forest. Each of them counted the trees, and the following answers were received: 2574, 2588, 2583. Which approximate answer can teenagers give?

The number of trees in this task is reasonably rounded up to 2580, because, judging by the numbers obtained, the area is quite large, the accuracy to one tree will be redundant from a practical point of view.

2. The hay storage has the shape of a rectangular parallelepiped with measurements of 16.6 m, 5.2 m and 4 m. How many tons of hay can the storage keep if 1 $м^3$ of hay weighs 54 kg?

The answer is advisable to round to 18.5 tons.

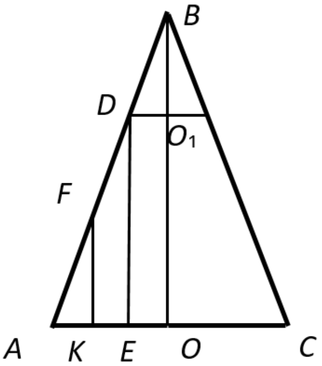

Figure 1. The trunk of the tree

When solving problems in practice, it is necessary to round not only the result (the stage of interpretation), but also the original numerical data. It can happen, for example, when using tabular data, where the accuracy is higher than it is required by the meaning of the problem. To teach how to choose the accuracy of the initial data can help the tasks: a) which require practical measurements, for example, in laboratory work; b) which are associated with reading and plotting graphs, if scales on the coordinate axes allow to get only approximate values; c) with excessive accuracy of numerical data. In the process of solving the proposed and similar tasks, students should understand that the choice of the accuracy of numerical values depends on the purpose of the problem, and on the qualities of the measured object. When students answer, they rely on their ideas about the real objects and processes described in the tasks. Approximate values of numerical data again allow us to pay attention to the fact that the mathematical model is not completely adequate to the original and describes only the properties of a process or phenomenon that are significant for a given situation.

The effect of evaluating the possibility of obtaining numerical values of quantities in practice is closely related to the action of estimating the completeness of the initial information and introducing the necessary numerical values. The formation of the first is possible mainly in the process of the formation of the second. Therefore, in order to place more emphasis on assessing the possibility of obtaining values of quantities in practice, we should offer problems, where the direct choice of the quantities, necessary for finding the desired one, do not cause difficulties. Let us give some examples.

1. How approximately to find the distance from your home to school?
2. The container, which has the form of a rectangular parallelepiped, contains wheat. How to calculate the mass of the wheat in the container without weighing it?
3. In the shed it is required to make a brick floor in one layer, the thickness of which is equal to the smallest brick size. How to determine the quantity of bricks you need?
4. The trunk of the tree becomes narrower evenly from the base to the apex. What measurements and calculations are sufficient to determine at what height the thickness of the tree reaches the predetermined value?

Experience shows that in solving this problem, students do not correlate the proposed method with its practical implementation. As a rule, they choose the following solution.

$$\Delta ABO \backsim \Delta DBO_1 \text{ (**Figure 1**)},\ \frac{DO_1}{AO} = \frac{BO_1}{BO},\ BO_1 = \frac{DO_1 \cdot BO}{AO},\ OO_1 = BO - BO_1$$

It is necessary to measure BO and AO.

Students do not realize that the height of the tree is not less difficult to measure than the desired value. For the solution it is necessary to use ΔAFK, where FK is accessible for measurement height (1,5-2 m).

5. **Figure 2** shows a pattern for sewing a circle skirt. What measurements and calculations need to be performed to make such a pattern?

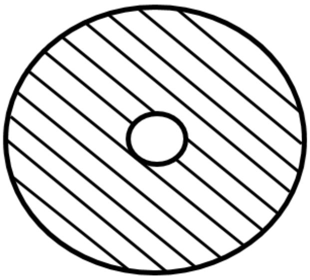

Figure 2. A pattern for sewing a circle

To solve this problem, students need first of all to correlate the mathematical concepts of "circumference", "radius" with the terms "waist circumference", "skirt length". The construction of a smaller circle requires knowledge of a radius that cannot directly be measured. It is possible to measure it knowing the length of the circle, i.e. girth of the waist. To find the radius of a larger circle, it is necessary to take into account that the width of the ring is the length of the skirt. Thus, the initial terms are also replaced by mathematical equivalents.

6. It is necessary to approximately count the number of trees in the forest in order to determine the wood contained in it. What is the easiest way to do this?

This problem can be proposed when studying the property of a directly proportional relation. For the solution it is necessary to count the number of trees in a small area of medium density and multiply the result by the ratio of the forest area to the area of the specified area. After solving problems in general form, you can give the task: to enter the necessary numerical data and perform calculations.

Thus, we studied the elements of mathematical modeling relating to the stages of formalization and interpretation, and presented a model of the system of applied problems, through which actions that are adequate to the selected elements can be formed.

Let us analyze in more detail the introduction of the concept of a mathematical model and its properties, which can be implemented at one of the program lessons for solving plot problems. Let us give it a brief summary. First of all, the teacher points out that the term "model" is quite often met in everyday life and at lessons about the surrounding world ("clothes or shoes" models, the globe is a model of the globe, etc.). The given examples are material models. The main property of each model is that it reflects the essential properties of its original. The mathematical model also reflects the main properties of any process, phenomenon, but it is constructed in a logical way and exists in our consciousness. Mathematical models can be numerical expressions, equations, inequalities and their systems, functions. A mathematical model is a description of any real process in the language of mathematical concepts, formulas and relations. Students often face mathematical modeling while solving plot problems. Then the following task can be considered: Calculate the necessary number of cash registers in the supermarket so that there are no queues in them. The first stage of mathematical modeling - the stage of formalization - consists in translating the condition of the problem into a mathematical language. In this case, the data necessary for the solution are found, and the relations between them are described by means of mathematical relations. To solve the problem you need to enter the following characteristics: κ is the required number of cash registers; $в$ is time of service of one buyer; T is the working hours of the shop; N is the number of buyers a day. During the working day, one cashier can serve $(\mathrm{T}/\mathrm{в}) \cdot \mathrm{к} = N$ people. The resulting relation is the mathematical model of the problem under consideration. The next stage of modeling is the intramodel solution. At this stage we find from the given equality the required number of cash registers: $\mathrm{к} = (N/\mathrm{T})\mathrm{в}$. The third stage of mathematical modeling is interpretation; translation of the resulting solution into the language in which the original problem was formulated. To ensure that there is no queue in the supermarket, the number of cash desks must be an integer equal to or larger than the received value. Further, we draw the attention of students to the simplifying assumptions made in constructing the model: the *average* time of passage of one person through the cash desk. But cashiers work at different speeds. In addition, every day in the supermarket there is a different number of buyers N. The intensity of the flow of customers at different times of the day is different too, i.e. the number of people passing through the cash desk per unit of time. For more accurate calculations in the resulting formula, we need to take the maximum value of this quantity instead of the mean value N / T. The teacher emphasizes that any mathematical model is based on simplification. It does not coincide with the concrete real situation, but it is its approximate description. Hence, some error in the results is obvious. However, it is due to the replacement of a real process by the mathematical model it is possible to use mathematical methods in its study. Teacher must also explain that mathematical modeling is important as the same model can describe different situations, different processes of real human practice. Having investigated one model, the results can be applied in another situation. So, the result obtained in solving this problem can be used in the following: Calculate the number of throughput turnstiles at the stadium. The same model can describe other situations in queuing systems. Thus, at this lesson the teacher considers the concepts of the mathematical model, the structure of the mathematical modeling process, some properties of models and the value of their use. Experimental studies have shown that the material described is interesting and accessible to primary school students.

DISCUSSIONS

The research showed that the problems associated with the introduction of the concept of a mathematical model and teaching of elements of modeling are widely discussed in the scientific and methodological literature. Although the need for these steps is almost unquestionable, but the place of study and the depth of consideration of applications within the school discipline "Mathematics" remains controversial. Practice shows that the formal introduction of the term "mathematical model" in the early stages of education (grades 5-6) (Mordkovich, 2013; Dorofeev & Peterson, 2013) practically does not contribute to the formation of correct ideas about the role and significance of the application of mathematics in the modern world. An insignificant number of applied problems in school textbooks does not give the possibility of learning such elements of modeling that are typical for solving problems in practice. However, our study shows that minor changes of the problem material (reformulation of tasks in terms that are used in life, including in the condition extra data or withdrawal some numerical data that need to be entered, the use of different ways of setting quantities or their values - tables, graphs, drawings, diagrams, etc.) will allow the teacher to substantially bring students closer to real practice. At the same time, it will increase the motivation for learning and the humanitarian aspect of teaching the subject through demonstrating the value of mathematics in various areas of human activity. The main feature of constructing a mathematical model is abstraction and, therefore, the presence of a certain error is accessible to the understanding of schoolchildren of 8-9 grades. By this time, students already know a large number of mathematical models, which allows us to raise the question of their comparison and the choice of the most appropriate. This problem is investigated by Ovezov (1991), where it is shown that when solving an applied problem, a complex model (quadratic equation) can be replaced by a simpler one (linear equation) based on so-called rational reasoning. Such reasoning shows that the degree of accuracy of the result, provided by a more complex model, does not correspond to the problem that is solved in the task. Lozhkina (2008) emphasizes that the compulsory component of the methodology of teaching mathematical modeling is the purposeful work on the formation ability to perform dismembering abstraction. This again emphasizes the idea of an approximate description by the model of the process, which is available for students of 7-9 grades.

CONCLUSION

Based on the analysis of normative documents, scientific and pedagogical and methodological literature, the study of the experience of teachers' work and own teaching experience, the authors have developed and introduced a method of teaching of mathematical modeling elements, typical for the stages of formalization and interpretation: the replacement of the original terms with mathematical equivalents; evaluation of the completeness of the initial information and if necessary, the introduction of missing numerical data; choice of the accuracy of numerical values corresponding to the meaning of the problem; identification of the possibility of obtaining data for solving the problem in practice. These elements can be formed in the process of solving applied problems from the 5th grade. Based on experience in solving such problems in 8-9 grades, it is important to introduce the term "mathematical model", its properties and the necessity of constructing, the structure of the process of mathematical modeling. Approbation of the proposed methodology showed both the possibility of its implementation, and the appropriateness. It is confirmed by students' results, especially at final school exams, since such tasks are included in the control materials of the final exams in mathematics. Observations of teachers of mathematics, as well as the authors' own observations show that students begin to give more meaningful descriptions of the application of mathematics in practice. The proposed methodology does not require a special organization of the teaching process, adjusting the content of teaching mathematics and additional time for classes. Applied tasks can easily be obtained by insignificant changes of the tasks from school textbooks. The content connected with the term "mathematical model and modeling" is considered at lessons devoted to the solution of plot tasks.

The article can be useful for teachers of mathematics, teachers of additional mathematical education, teachers of higher educational institutions. Mathematical education allows us to understand the general cultural role and importance of mathematics in the development of modern society, apply mathematical knowledge in various spheres of human activity. The ways to improve the proposed methodology are to createy sets of applied problems on various topics during the school course of mathematics, interesting and meaningful applications in the natural sciences and humanities. Questions of teaching mathematical modeling in profile classes require further research.

ACKNOWLEDGEMENT

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

The work was carried out with the financial support of the Ministry of Education and Science of the Russian Federation under the Peoples' Friendship University of Russia (RUDN University) Program "5-100" among the world's leading research and educational centers for 2016-2020.

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