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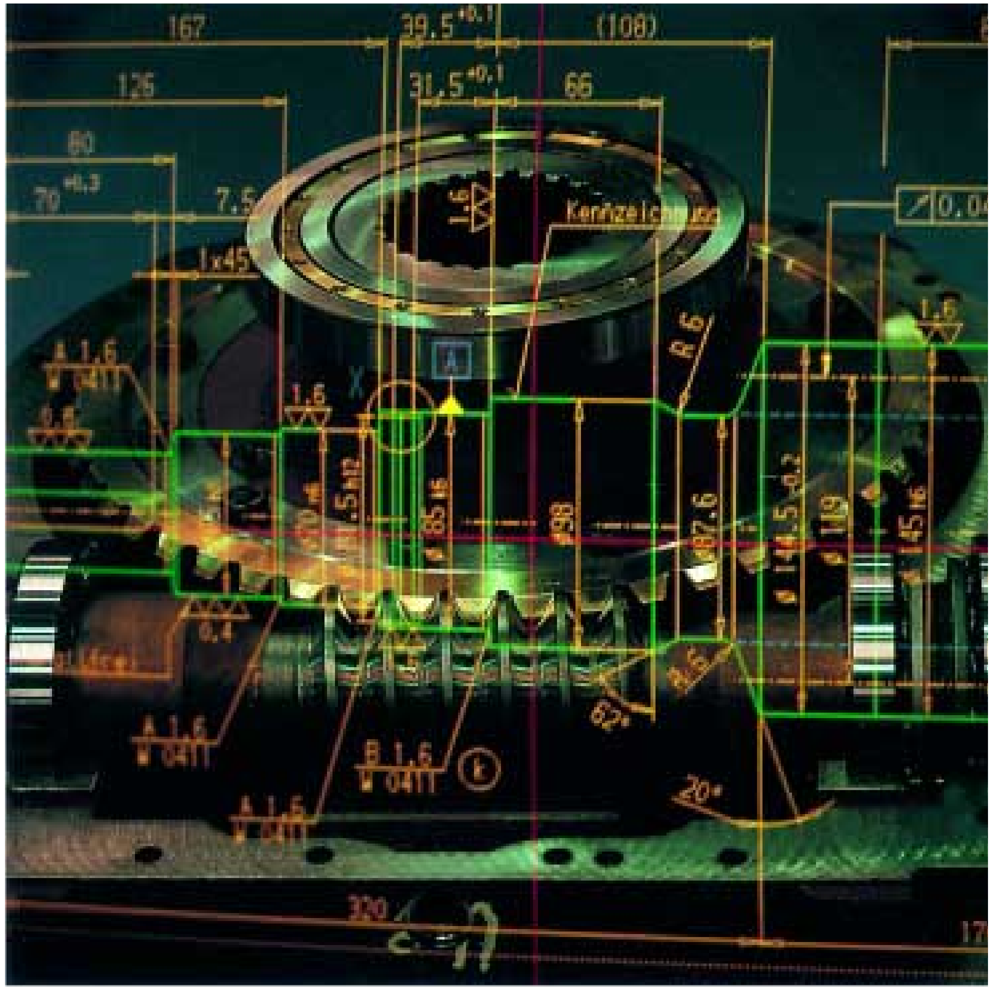

TECHNICAL HANDBOOK

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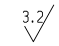
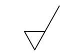
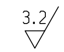
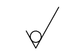
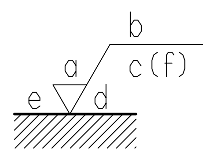
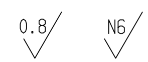
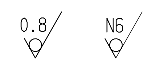
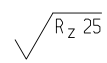
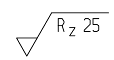
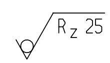
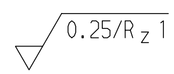
1. Method of indicating surface texture on drawings acc. to DIN 1302

1.1 Symbols	
Symbol without additional indications. Basic symbol. The meaning must be explained by additional indications.	
Symbol with additional indications. Any production method, with specified roughness.	
Symbol without additional indications. Removal of material by machining, without specified roughness.	
Symbol with additional indications. Removal of material by machining, with specified roughness.	
Symbol without additional indications. Removal of material is not permitted (surface remains in state as supplied).	
Symbol with additional indications. Made without removal of material (non-cutting), with specified roughness.	

1.2 Position of the specifications of surface texture in the symbol



a = Roughness value R_a in micrometres or microinches or roughness grade number N1 to N12
b = Production method, surface treatment or coating
c = Sampling length
d = Direction of lay
e = Machining allowance
f = Other roughness values, e.g. R_z

Examples			Explanation
Production method			
Any	Material removing	Non-cutting	
			Centre line average height R_a: maximum value = 0.8 µm
			Mean peak-to-valley height R_z: maximum value = 25 µm
			Mean peak-to-valley height R_z: maximum value = 1 µm at cut-off = 0.25 mm

2. Explanation of the usual surface roughness parameters

2.1 Centre line average height R_a acc. to DIN 4768

The centre line average height R_a is the arithmetic average of the absolute values of the distances y between the profile heights and the centre line within the measuring length. This is equivalent to the height of a rectangle (A_g) with a length equal to the evaluation length l_m and with an area equal to the sum of the areas enclosed between the roughness profile and the centre line (A_{oi} and A_{ui}) (see figure 1).

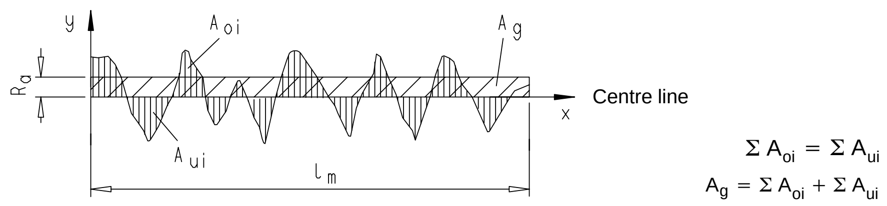


Figure 1

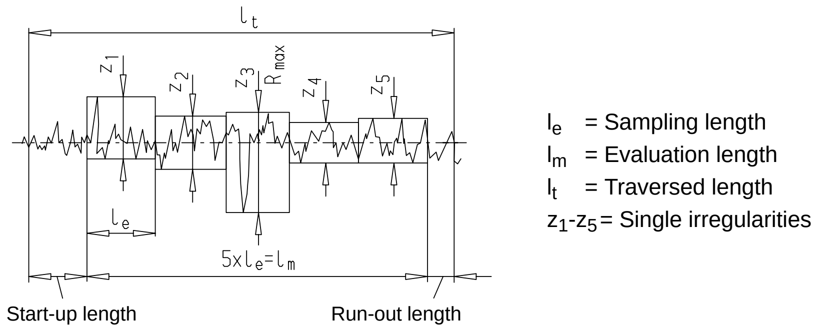


Figure 2

2.2 Mean peak-to-valley height R_z acc. to DIN 4768

The mean peak-to-valley height R_z is the arithmetic average of the single irregularities of five consecutive sampling lengths (see figure 2).

Note:
An exact conversion of the peak-to-valley height R_z and the centre line average height R_a can neither be theoretically justified nor empirically proved. For surfaces which are generated by manufacturing methods of the group "metal cutting", a diagram for the conversion from R_a to R_z and vice versa is shown in supplement 1 to DIN 4768 Part 1, based on comparison measurements (see table "Comparison of roughness values").

2.3 Maximum roughness height R_{max} acc. to DIN 4768 (see figure 2)

The maximum roughness height R_{max} is the largest of the single irregularities z occurring over the evaluation length l_m (in figure 2: z_3). R_{max} is stated in cases where the largest single irregularity ("runaway") is to be recorded for reasons important for function.

2.4 Roughness grade numbers N.. acc. to DIN ISO 1302

In supplement 1 to DIN ISO 1302 it is recommended not to use roughness grade numbers. The N-grade numbers are most frequently used in America (see also table "Comparison of roughness values").

3. Comparison of roughness values

DIN ISO 1302	Roughness values R_a	µm	**0.025**	**0.05**	**0.1**	**0.2**	**0.4**	**0.8**	**1.6**	**3.2**	**6.3**	**12.5**	**25**	**50**
		µin	1	2	4	8	16	32	63	125	250	500	1000	2000
	Roughness grade number		N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
Suppl. 1 to DIN 4768/1	Roughness values R_z in µm	from	0.1	0.25	0.4	0.8	1.6	3.15	6.3	12.5	25	40	80	160
		to	0.8	1.6	2.5	4	6.3	12.5	20	31.5	63	100	160	250

4. General

4.1 The particulars given are in accordance with the international standard DIN ISO 1101, March 1985 edition.
This standard gives the principles of symbolization and indication on technical drawings of tolerances of form, orientation, location and run-out, and establishes the appropriate geometrical definitions. The term "geometrical tolerances" is used in this standard as generic term for these tolerances.

4.2 Relationship between tolerances of size, form and position

According to current standards there are two possibilities of making indications on technical drawings in accordance with:

a) the principle of independence according to DIN ISO 8015 where tolerances of size, form and position must be adhered to independent of each other, i.e. there is no direct relation between them. In this case reference must be made on the drawing to DIN ISO 8015.

b) the envelope requirements according to DIN 7167, according to which the tolerances of size, form and parallelism are in direct relation with each other, i.e. that the size tolerances limit the form and parallelism tolerances. In this case no special reference to DIN 7167 is required on the drawing.

5. Application; general explanations

5.1 Geometrical tolerances shall be specified on drawings only if they are imperative for the functioning and/or economical manufacture of the respective workpiece. Otherwise, the general tolerances according to DIN 7168 apply.

5.2 Indicating geometrical tolerances does not necessarily imply the use of any particular method of production, measurement or gauging.

5.3 A geometrical tolerance applied to a feature defines the tolerance zone within which the feature (surface, axis, or median plane) is to be contained.

According to the characteristic which is to be tolerated and the manner in which it is dimensioned, the tolerance zone is one of the following:
- the area within a circle;
- the area between two concentric circles;
- the area between two equidistant lines or two parallel straight lines;
- the space within a cylinder;
- the space between two coaxial cylinders;
- the space between two parallel planes;
- the space within a parallelepiped.

The toleranced feature may be of any form or orientation within this tolerance zone, unless a more restrictive indication is given.

5.4 Unless otherwise specified, the tolerance applies to the whole length or surface of the considered feature.

5.5 The datum feature is a real feature of a part, which is used to establish the location of a datum.

5.6 Geometrical tolerances which are assigned to features referred to a datum do not limit the form deviations of the datum feature itself. The form of a datum feature shall be sufficiently accurate for its purpose and it may therefore be necessary to specify tolerances of form for the datum features.

5.7 See Page 26

5.8 Tolerance frame

The tolerance requirements are shown in a rectangular frame which is divided into two or more compartments. These compartments contain, from left to right, in the following order (see figures 3, 4 and 5):
- the symbol for the characteristic to be toleranced;
- the tolerance value in the unit used for linear dimensions. This value is preceded by the sign ∅ if the tolerance zone is circular or cylindrical;
- if appropriate, the capital letter or letters identifying the datum feature or features (see figures 4 and 5)

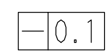


Figure 3

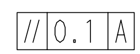


Figure 4

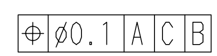


Figure 5

Remarks referred to the tolerance, for example "6 holes", "4 surfaces", or "6 x" shall be written above the frame (see figures 6 and 7).

If it is necessary to specify more than one tolerance characteristic for a feature, the tolerance specifications are given in tolerance frames one below the other (see figure 8).

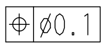


Figure 6

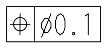


Figure 7

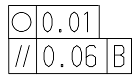


Figure 8

5.7 Table 1: Kinds of tolerances; symbols; included tolerances

Tolerances		Symbols	Toleranced characteristics	Included tolerances
Form tolerances		⏤	Straightness	–
		⏥	Flatness	Straightness
		○	Circularity (Roundness)	–
		⌭	Cylindricity	Straightness, Parallelism, Circularity
Tolerances of position [1]	Orientation tolerances	//	Parallelism	Flatness
		⊥	Perpendicularity	Flatness
		∠	Angularity	Flatness
	Location tolerances	⌖	Position	–
		◎	Concentricity, Coaxiality	–
		⌯	Symmetry	Straightness, Flatness, Parallelism
	Runout tolerances	↗	Circular runout, Axial runout	Circularity, Coaxiality

1) Tolerances of position always refer to a datum feature or theoretically exact dimensions.

Table 2: Additional symbols

Description		Symbols
Toleranced feature indications	direct	
Datum indications	direct	
	by capital letter	A
Theoretically exact dimension		$\boxed{50}$

Technical Drawings
Geometrical Tolerancing

5.9 Toleranced features

The tolerance frame is connected to the toler-anced feature by a leader line terminating with an arrow in the following way:

- on the outline of the feature or an extension of the outline (but clearly separated from the dimension line) when the tolerance refers to the line or surface itself (see figures 9 and 10).

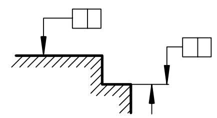


Figure 9

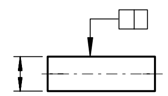


Figure 10

- as an extension of a dimension line when the tolerance refers to the axis or median plane defined by the feature so dimensioned (see figures 11 to 13).

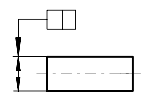


Figure 11

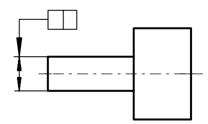


Figure 12

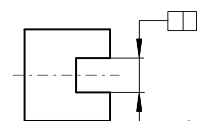


Figure 13

- on the axis or the median plane when the tolerance refers to the common axis or median plane of <u>two</u> features (see figure 14).

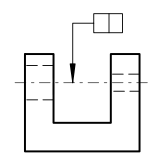


Figure 14

Note:
Whether a tolerance should be applied to the contour of a cylindrical or symmetrical feature or to its axis or median plane, depends on the functional requirements.

5.10 Tolerance zones

The tolerance zone is the zone within which all the points of a geometric feature (point, line, surface, median plane) must lie. The width of the tolerance zone is in the direction of the arrow of the leader line joining the tolerance frame to the feature which is toleranced, unless the tolerance value is preceded by the sign ∅ (see figures 15 and 16).

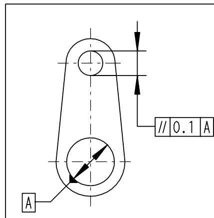


Figure 15

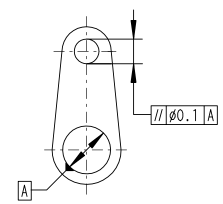


Figure 16

Where a <u>common tolerance zone</u> is applied to several separate features, the requirement is indicated by the words "common zone" above the tolerance frame (see figure 17).

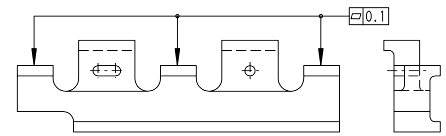


Figure 17

5.11 Datums and datum systems

Datum features are features according to which a workpiece is aligned for recording the tolerated deviations.

5.11.1 When a toleranced feature is referred to a datum, this is generally shown by datum letters. The same letter which defines the datum is repeated in the tolerance frame.
To identify the datum, a capital letter enclosed in a frame is connected to a solid datum triangle (see figure 18).

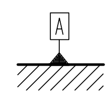


Figure 18

The datum triangle with the datum letter is placed:
- on the outline of the feature or an extension of the outline (but clearly separated from the dimension line), when the datum feature is the line or surface itself (see figure 19).

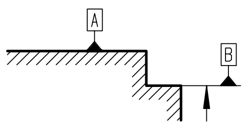


Figure 19

- as an extension of the dimension line when the datum feature is the axis or median plane (see figures 20 and 21).

Note:
If there is not enough space for two arrows, one of them may be replaced by the datum triangle (see figure 21).

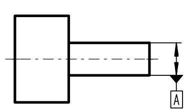


Figure 20

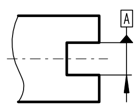


Figure 21

- on the axis or median plane when the datum is:

a) the axis or median plane of a single feature (for example a cylinder);
b) the common axis or median plane formed by <u>two</u> features (see figure 22).

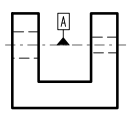


Figure 22

If the tolerance frame can be directly connected with the datum feature by a leader line, the datum letter may be omitted (see figures 23 and 24).

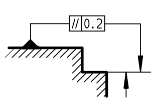


Figure 23

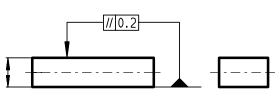


Figure 24

A single datum is identified by a capital letter (see figure 25).
A common datum formed by two datum features is identified by two datum letters separated by a hyphen (see figures 26 and 28).
In a datum system (see also 5.11.2) the sequence of two or more datum features is important. The datum letters are to be placed in different compartments, where the sequence from left to right shows the order of priority, and <u>the datum letter placed first should refer to the directional datum feature</u> (see figures 27, 29 and 30).

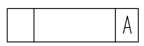


Figure 25

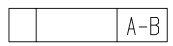


Figure 26

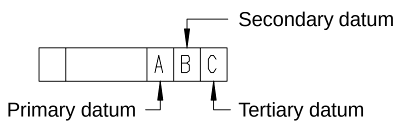


Figure 27

5.11.2 Datum system

A datum system is a group of two or more datums to which one toleranced feature refers in common. A datum system is frequently required because the <u>direction</u> of a <u>short axis</u> cannot be determined alone.
Datum formed by two form features (common datum):

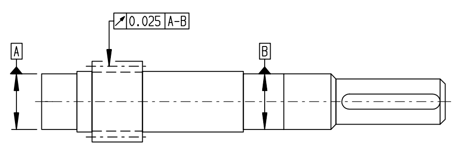


Figure 28

<u>Datum system formed by two datums (short axis "A" and directional datum "B"):</u>

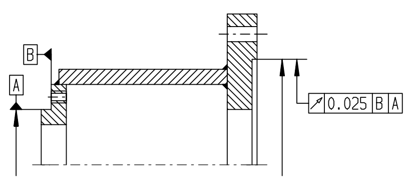


Figure 29

<u>Datum system formed by one plane and one perpendicular axis of a cylinder:</u>
Datum "A" is the plane formed by the plane contact surface. Datum "B" is the axis of the largest inscribed cylinder, the axis being at right angles with datum "A" (see figure 30).

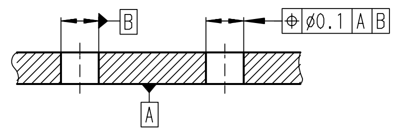


Figure 30

5.12 Theoretically exact dimensions

If tolerances of position or angularity are prescribed for a feature, the dimensions determining the theoretically exact position or angle shall not be toleranced.
These dimensions are enclosed, for example $\boxed{30}$.
The corresponding actual dimensions of the part are subject only to the position tolerance or angularity tolerance specified within the tolerance frame (see figures 31 and 32).

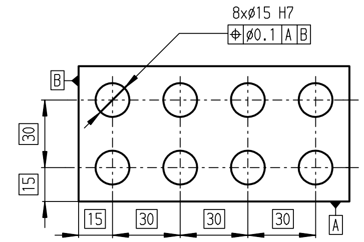


Figure 31

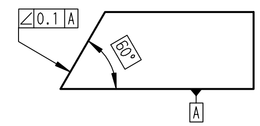
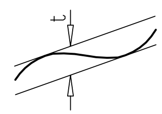
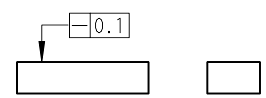
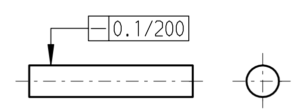
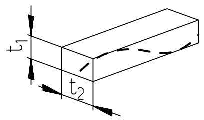
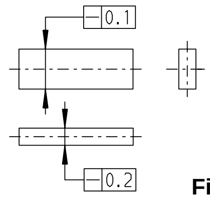
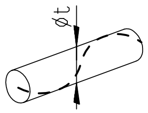
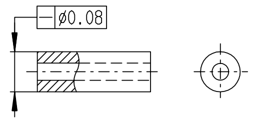
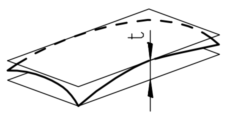
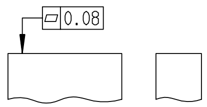
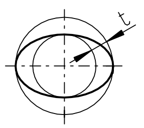
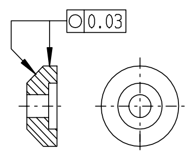
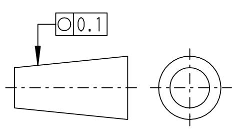
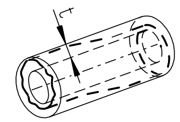
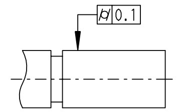
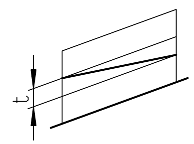
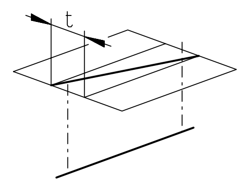
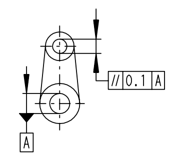
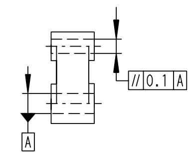
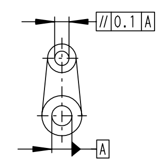
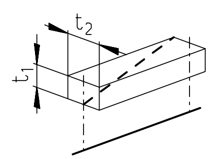
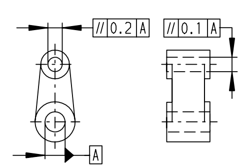
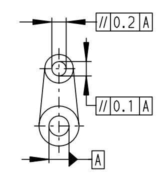


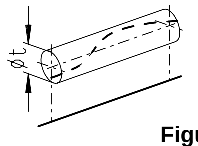
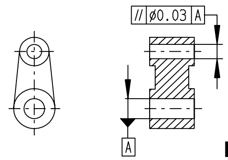
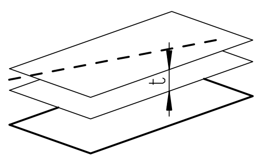
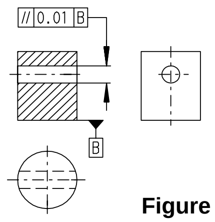
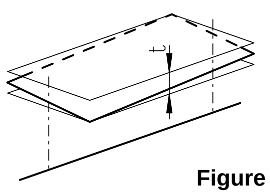
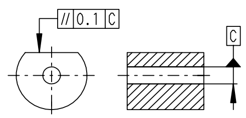
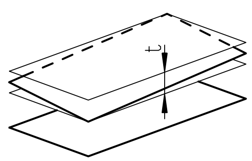
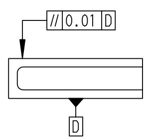
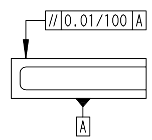
Figure 32

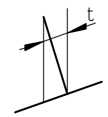
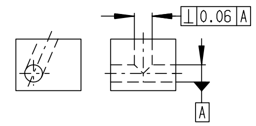
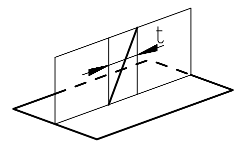
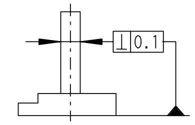
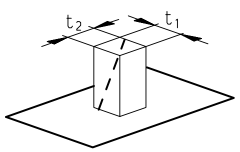
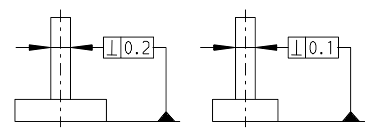
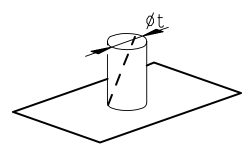
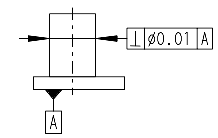
5.13 Detailed definitions of tolerances

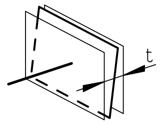
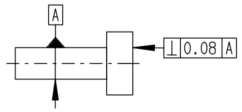
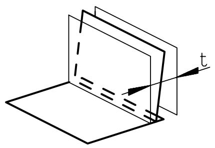
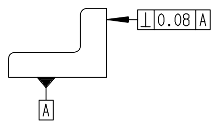
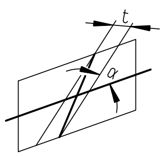
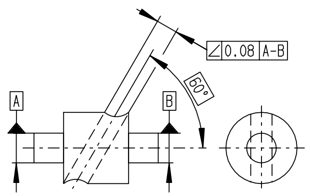
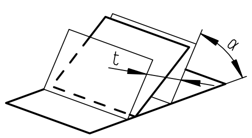
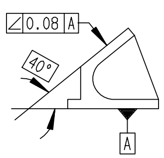
Symbol	Definition of the tolerance zone	Indication and interpretation
	5.13.1 Straightness tolerance	
—	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart.  **Figure 33**	Any line on the upper surface parallel to the plane of projection in which the indication is shown shall be contained between two parallel straight lines 0.1 apart.  **Figure 34** Any portion of length 200 of any generator of the cylindrical surface indicated by the arrow shall be contained between two parallel straight lines 0.1 apart in a plane containing the axis.  **Figure 35**

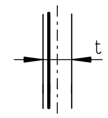
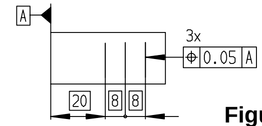
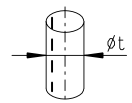
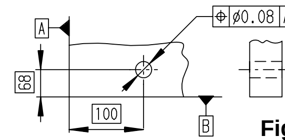
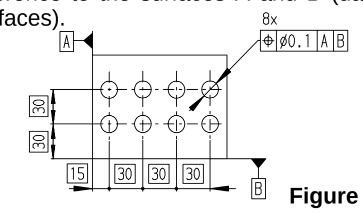
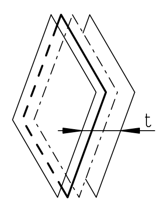
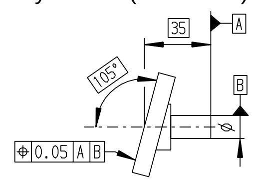
Symbol	Definition of the tolerance zone	Indication and interpretation
—	The tolerance zone is limited by a parallelepiped of section $t_1 \cdot t_2$ if the tolerance is specified in two directions perpendicular to each other.  **Figure 36**	The axis of the bar shall be contained within a parallelepipedic zone of width 0.1 in the vertical and 0.2 in the horizontal direction.  **Figure 37**
	The tolerance zone is limited by a cylinder of diameter t if the tolerance value is preceded by the sign ∅.  **Figure 38**	The axis of the cylinder to which the tolerance frame is connected shall be contained in a cylindrical zone of diameter 0.08.  **Figure 39**
▱	**5.13.2 Flatness tolerance**	
	The tolerance zone is limited by two parallel planes a distance t apart.  **Figure 40**	The surface shall be contained between two parallel planes 0.08 apart.  **Figure 41**
○	**5.13.3 Circularity tolerance**	
	The tolerance zone in the considered plane is limited by two concentric circles a distance t apart.  **Figure 42**	The circumference of each cross-section of the outside diameter shall be contained between two co-planar concentric circles 0.03 apart.  **Figure 43** The circumference of each cross-section shall be contained between two co-planar concentric circles 0.1 apart.  **Figure 44**

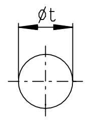
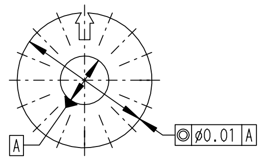
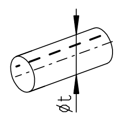
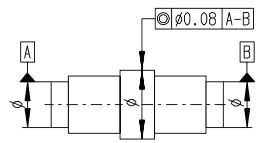
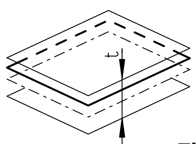
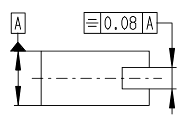
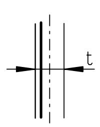
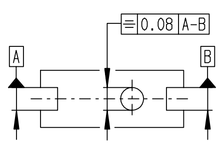
Symbol	Definition of the tolerance zone	Indication and interpretation
⌭	**5.13.4 Cylindricity tolerance**	
	The tolerance zone is limited by two coaxial cylinders a distance t apart.  **Figure 45**	The considered surface area shall be contained between two coaxial cylinders 0.1 apart.  **Figure 46**
//	**5.13.5 Parallelism tolerance**	
	Parallelism tolerance of a line with reference to a datum line	
	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and parallel to the datum line, if the tolerance zone is only specified in one direction.  **Figure 47**  **Figure 50**	The toleranced axis shall be contained between two straight lines 0.1 apart, which are parallel to the datum axis A and lie in the vertical direction (see figures 48 and 49).   **Figure 48** **Figure 49** The toleranced axis shall be contained between two straight lines 0.1 apart, which are parallel to the datum axis A and lie in the horizontal direction.  **Figure 51**
	The tolerance zone is limited by a parallelepiped of section $t_1 \cdot t_2$ and parallel to the datum line if the tolerance is specified in two planes perpendicular to each other.  **Figure 52**	The toleranced axis shall be contained in a parallelepipedic tolerance zone having a width of 0.2 in the horizontal and 0.1 in the vertical direction and which is parallel to the datum axis A (see figures 53 and 54).   **Figure 53** **Figure 54**

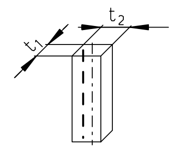
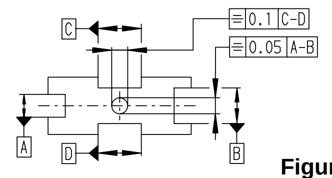
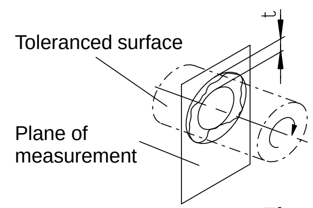
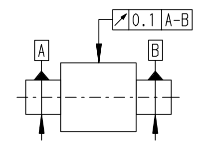
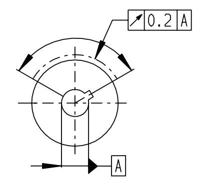
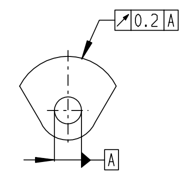
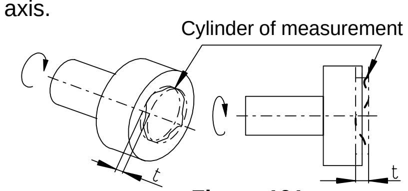
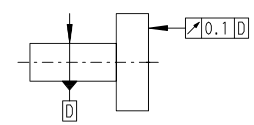
Symbol	Definition of the tolerance zone	Indication and interpretation
//	Parallelism tolerance of a line with reference to a datum line	
	The tolerance zone is limited by a cylinder of diameter t parallel to the datum line if the tolerance value is preceded by the sign ∅.  **Figure 55**	The toleranced axis shall be contained in a cylindrical zone of diameter 0.03 parallel to the datum axis A (datum line).  **Figure 56**
	Parallelism tolerance of a line with reference to a datum surface	
	The tolerance zone is limited by two parallel planes a distance t apart and parallel to the datum surface.  **Figure 57**	The toleranced axis of the hole shall be contained between two planes 0.01 apart and parallel to the datum surface B.  **Figure 58**
	Parallelism tolerance of a surface with reference to a datum line	
	The tolerance zone is limited by two parallel planes a distance t apart and parallel to the datum line.  **Figure 59**	The toleranced surface shall be contained between two planes 0.1 apart and parallel to the datum axis C of the hole.  **Figure 60**
	Parallelism tolerance of a surface with reference to a datum surface	
	The tolerance zone is limited by two parallel planes a distance t apart and parallel to the datum surface.  **Figure 61**	The toleranced surface shall be contained between two parallel planes 0.01 apart and parallel to the datum surface D (figure 62).  **Figure 62**  **Figure 63** All the points of the toleranced surface in a length of 100, placed anywhere on this surface, shall be contained between two parallel planes 0.01 apart and parallel to the datum surface A (figure 63).

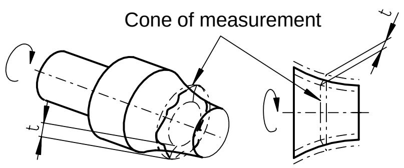
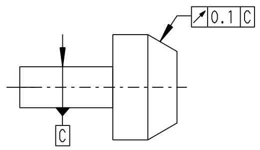
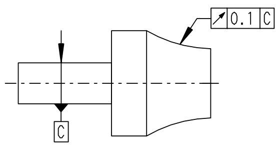
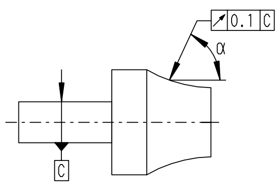
Symbol	Definition of the tolerance zone	Indication and interpretation
⊥	**5.13.6 Perpendicularity tolerance**	
	Perpendicularity tolerance of a line with reference to a datum line	
	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and perpendicular to the datum line.  **Figure 64**	The toleranced axis of the inclined hole shall be contained between two parallel planes 0.06 apart and perpendicular to the axis of the horizontal hole A (datum line).  **Figure 65**
	Perpendicularity tolerance of a line with reference to a datum surface	
	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and perpendicular to the datum plane if the tolerance is specified only in one direction.  **Figure 66**	The toleranced axis of the cylinder, to which the tolerance frame is connected, shall be contained between two parallel planes 0.1 apart, perpendicular to the datum surface.  **Figure 67**
	The tolerance zone is limited by a parallelepiped of section $t_1 \cdot t_2$ and perpendicular to the datum surface if the tolerance is specified in two directions perpendicular to each other.  **Figure 68**	The toleranced axis of the cylinder shall be contained in a parallelepipedic tolerance zone of $0.1 \cdot 0.2$ which is perpendicular to the datum surface.  **Figure 69**
	The tolerance zone is limited by a cylinder of diameter t perpendicular to the datum surface if the tolerance value is preceded by the sign ∅.  **Figure 70**	The toleranced axis of the cylinder to which the tolerance frame is connected shall be contained in a cylindrical zone of diameter 0.01 perpendicular to the datum surface A.  **Figure 71**

Symbol	Definition of the tolerance zone	Indication and interpretation
⊥	Perpendicularity tolerance of a surface with reference to a datum line	
	The tolerance zone is limited by two parallel planes a distance t apart and perpendicular to the datum line.  **Figure 72**	The toleranced face of the workpiece shall be contained between two parallel planes 0.08 apart and perpendicular to the axis A (datum line).  **Figure 73**
	Perpendicularity tolerance of a surface with reference to a datum surface	
	The tolerance zone is limited by two parallel planes a distance t apart and perpendicular to the datum surface.  **Figure 74**	The toleranced surface shall be contained between two parallel planes 0.08 apart and perpendicular to the horizontal datum surface A.  **Figure 75**
∠	**5.13.7 Angularity tolerance**	
	Angularity tolerance of a line with reference to a datum line	
	Line and datum line in the same plane. The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and inclined at the specified angle to the datum line.  **Figure 76**	The toleranced axis of the hole shall be contained between two parallel straight lines 0.08 apart which are inclined at 60° to the horizontal axis A-B (datum line).  **Figure 77**
	Angularity tolerance of a surface with reference to a datum surface	
	The tolerance zone is limited by two parallel planes a distance t apart and inclined at the specified angle to the datum surface.  **Figure 78**	The toleranced surface shall be contained between two parallel planes 0.08 apart which are inclined at 40° to the datum surface A.  **Figure 79**

Symbol	Definition of the tolerance zone	Indication and interpretation
⌖	**5.13.8 Positional tolerance**	
	Positional tolerance of a line	
	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and disposed symmetrically with respect to the theoretically exact position of the considered line if the tolerance is specified only in one direction.  **Figure 80**	Each of the toleranced lines shall be contained between two parallel straight lines 0.05 apart which are symmetrically disposed about the theoretically exact position of the considered line, with reference to the surface A (datum surface).  **Figure 81**
	The tolerance zone is limited by a cylinder of diameter t the axis of which is in the theoretically exact position of the considered line if the tolerance value is preceded by the sign ∅.  **Figure 82**	The axis of the hole shall be contained within a cylindrical zone of diameter 0.08 the axis of which is in the theoretically exact position of the considered line, with reference to the surfaces A and B (datum surfaces).  **Figure 83**
		Each of the axes of the eight holes shall be contained within a cylindrical zone of diameter 0.1 the axis of which is in the theoretically exact position of the considered hole, with reference to the surfaces A and B (datum surfaces).  **Figure 84**
	Positional tolerance of a flat surface or a median plane	
	The tolerance zone is limited by two parallel planes a distance t apart and disposed symmetrically with respect to the theoretically exact position of the considered surface.  **Figure 85**	The inclined surface shall be contained between two parallel planes which are 0.05 apart and which are symmetrically disposed with respect to the theoretically exact position of the considered surface with reference to the datum surface A and the axis of the datum cylinder B (datum line).  **Figure 86**

Symbol	Definition of the tolerance zone	Indication and interpretation
◎	**5.13.9 Concentricity and coaxiality tolerance**	
	Concentricity tolerance of a point	
	The tolerance zone is limited by a circle of diameter t the centre of which coincides with the datum point.  **Figure 87**	The centre of the circle, to which the tolerance frame is connected, shall be contained in a circle of diameter 0.01 concentric with the centre of the datum circle A.  **Figure 88**
	Coaxiality tolerance of an axis	
	The tolerance zone is limited by a cylinder of diameter t, the axis of which coincides with the datum axis if the tolerance value is preceded by the sign ∅.  **Figure 89**	The axis of the cylinder, to which the tolerance frame is connected, shall be contained in a cylindrical zone of diameter 0.08 coaxial with the datum axis A-B.  **Figure 90**
≡	**5.13.10 Symmetry tolerance**	
	Symmetry tolerance of a median plane	
	The tolerance zone is limited by two parallel planes a distance t apart and disposed symmetrically to the median plane with respect to the datum axis or datum plane.  **Figure 91**	The median plane of the slot shall be contained between two parallel planes, which are 0.08 apart and symmetrically disposed about the median plane with respect to the datum feature A.  **Figure 92**
	Symmetry tolerance of a line or an axis	
	The tolerance zone when projected in a plane is limited by two parallel straight lines a distance t apart and disposed symmetrically with respect to the datum axis (or datum plane) if the tolerance is specified only in one direction.  **Figure 93**	The axis of the hole shall be contained between two parallel planes which are 0.08 apart and symmetrically disposed with respect to the actual common median plane of the datum slots A and B.  **Figure 94**

Symbol	Definition of the tolerance zone	Indication and interpretation
≡	Symmetry tolerance of a line or an axis	
	The tolerance zone is limited by a parallelepiped of section $t_1 \cdot t_2$, the axis of which coincides with the datum axis if the tolerance is specified in two directions perpendicular to each other.  **Figure 95**	The axis of the hole shall be contained in a parallelepipedic zone of width 0.1 in the horizontal and 0.05 in the vertical direction and the axis of which coincides with the datum axis formed by the intersection of the two median planes of the datum slots A-B and C-D.  **Figure 96**
↗	**5.13.11 Circular runout tolerance**	
	Circular runout tolerance - radial	
	The tolerance zone is limited within any plane of measurement perpendicular to the axis by two concentric circles a distance t apart, the centre of which coincides with the datum axis.  **Figure 97** Runout normally applies to complete revolutions about the axis but could be limited to apply to a part of a revolution.	The radial runout shall not be greater than 0.1 in any plane of measurement during one revolution about the datum axis A-B.  **Figure 98** The radial runout shall not be greater than 0.2 in any plane of measurement when measuring the toleranced part of a revolution about the centre line of hole A (datum axis).   **Figure 99** **Figure 100**
	Circular runout tolerance - axial	
	The tolerance zone is limited at any radial position by two circles a distance t apart lying in a cylinder of measurement, the axis of which coincides with the datum axis.  **Figure 101**	The axial runout shall not be greater than 0.1 at any position of measurement during one revolution about the datum axis D.  **Figure 102**

Symbol	Definition of the tolerance zone	Indication and interpretation
↗	Circular runout tolerance in any direction	
	The tolerance zone is limited within any cone of measurement, the axis of which coincides with the datum axis by two circles a distance t apart. Unless otherwise specified the measuring direction is normal to the surface.  **Figure 103**	The runout in the direction indicated by the arrow shall not be greater than 0.1 in any cone of measurement during one revolution about the datum axis C.  **Figure 104** The runout in the direction perpendicular to the tangent of a curved surface shall not be greater than 0.1 in any cone of measurement during one revolution about the datum axis C.  **Figure 105**
	Circular runout tolerance in a specified direction	
	The tolerance zone is limited within any cone of measurement of the specified angle, the axis of which coincides with the datum axis by two circles a distance t apart.	The runout in the specified direction shall not be greater than 0.1 in any cone of measurement during one revolution about the datum axis C.  **Figure 106**

Technical drawings [extract from DIN 476 (10.76) and DIN 6671 Part 6 (04.88)]

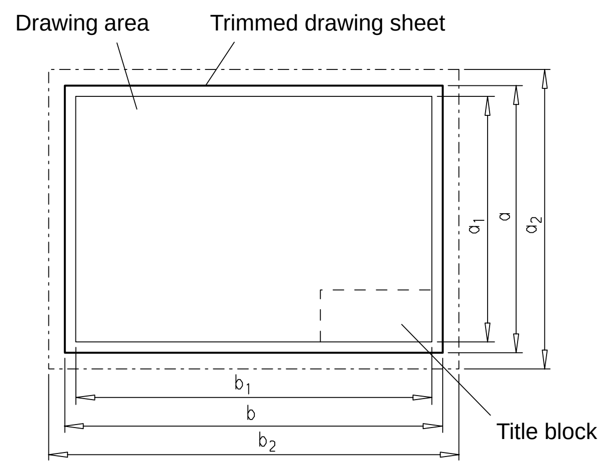
6. Sheet sizes
The DIN 6771 standard Part 6 applies to the presentation of drawing forms even if they are created by CAD. This standard may also be used for other technical documents. The sheet sizes listed below have been taken from DIN 476 and DIN 6771 Part 6.

Table 3

Sheet sizes acc. to DIN 476, A series	Trimmed sheet a x b	Drawing area 1) a_1 x b_1	Untrimmed sheet a_2 x b_2
A0	841 x 1189	831 x 1179	880 x 1230
A1	594 x 841	584 x 831	625 x 880
A2	420 x 594	410 x 584	450 x 625
A3	297 x 420	287 x 410	330 x 450
A4	210 x 297	200 x 287	240 x 330

1) The actually available drawing area is reduced by the title block, the filing margin, the possible sectioning margin, etc.

6.1 Title block
Formats ≥ A3 are produced in broadside. The title block area is in the bottom right corner of the trimmed sheet. For the A4 format the title block area is at the bottom of the short side (upright format).



6.2 Non-standard formats
Non-standard formats should be avoided. When necessary they should be created using the dimensions of the short side of an A-format with the long side of a greater A-format.

7. General
In order to obtain perfect microfilm prints the following recommendations should be adhered to:

7.1 Indian ink drawings and CAD drawings show the best contrasts and should be preferred for this reason.

7.2 Pencil drawings should be made in special cases only, for example for drafts.
Recommendation:
- 2H-lead pencils for visible edges, letters and dimensions;
- 3H-lead pencils for hatching, dimension lines and hidden edges.

8. Lettering
For the lettering - especially with stencil - the vertical style standard lettering has to be used acc. to DIN 6776 Part 1, lettering style B, vertical (ISO 3098). In case of manual lettering the vertical style or sloping style standard lettering may be used according to DIN 6776 Part 1, lettering style B (ISO 3098).

8.1 The minimum space between two lines in a drawing as well as for lettering should be at least once, but better twice the width of a line in order to avoid merging of letters and lines in case of reductions.

9. Type sizes

Table 4: Type sizes for drawing formats (h = type height, b = line width)

Application range for lettering	Paper sizes			
	A0 and A1		A2, A3 and A4	
	h	b	h	b
Type, drawing no.	10	1	7	0.7
Texts and nominal dimensions	5	0.5	3.5	0.35
Tolerances, roughness values, symbols	3.5	0.35	2.5	0.25

9.1 The type sizes as assigned to the paper sizes in table 4 must be adhered to with regard to their application range. Larger type heights are also permissible. Type heights smaller by approx. 20% will be accepted if this is required in a drawing because of restricted circumstances.

10. Lines according to DIN 15 Part 1 and Part 2

Table 5: Line groups, line types and line widths

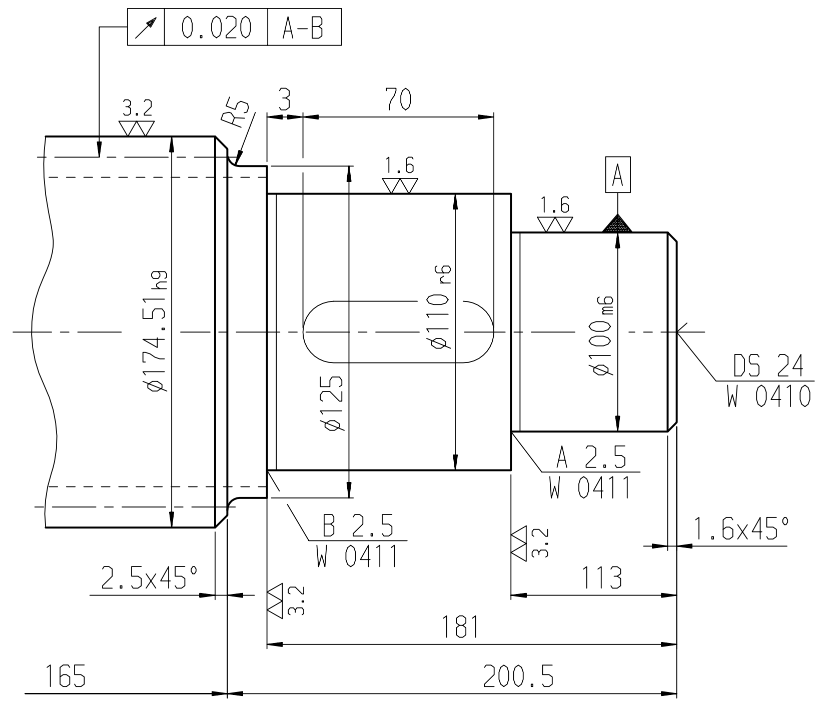
Line group	0.5	0.7
Drawing format	A4, A3, A2	A1, A0
Line type	Line width	
Solid line (thick)	0.5	0.7
Solid line (thin)	0.25	0.35
Short dashes (thin)	0.25	0.35
Dot-dash line (thick)	0.5	0.7
Dot-dash line (thin)	0.25	0.35
Dash/double-dot line (thin)	0.25	0.35
Freehand (thin)	0.25	0.35

10.1 Line groups 0.5 and 0.7 with the pertaining line width according to table 5 may only be used. Assignment to the drawing formats A1 and A0 is prescribed. For the A4, A3 and A2 formats, line group 0.7 may be used as well.

11. Indian ink fountain pen
The use of the type sizes according to table 4 and the lines according to table 5 permits a restricted number of 5 different fountain pens (line widths 0.25; 0.35; 0.5; 0.7; 1 mm).

12. Lettering examples for stenciling and handwritten entries

12.1 Example for formats A4 to A2



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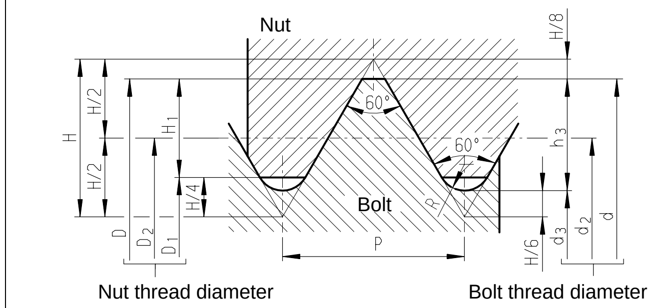
ISO metric screw threads (coarse pitch threads) following DIN 13 Part 1, 12.86 edition



$D_1 \quad d = 2\,H_1$

$d_2 \quad D_2 \quad d = 0.64952\,P$

$d_3 \quad d = 1.22687\,P$

$H \quad 0.86603\,P$

$H_1 \quad 0.54127\,P$

$h_3 \quad 0.61343\,P$

$R \quad \frac{H}{6} \quad 0.14434\,P$

Diameters of series 1 should be preferred to those of series 2, and these again to those of series 3.

Nominal thread diameter d = D Series 1	Series 2	Series 3	Pitch P mm	Pitch diameter $d_2 = D_2$ mm	Core diameter d_3 mm	Core diameter D_1 mm	Depth of thread h_3 mm	Depth of thread H_1 mm	Round R mm	Tensile stress cross-section A_s 1) mm^2
3			0.5	2.675	2.387	2.459	0.307	0.271	0.072	5.03
	3.5		0.6	3.110	2.764	2.850	0.368	0.325	0.087	6.78
4			0.7	3.545	3.141	3.242	0.429	0.379	0.101	8.78
	4.5		0.75	4.013	3.580	3.688	0.460	0.406	0.108	11.3
5			0.8	4.480	4.019	4.134	0.491	0.433	0.115	14.2
6			1	5.350	4.773	4.917	0.613	0.541	0.144	20.1
		7	1	6.350	5.773	5.917	0.613	0.541	0.144	28.9
8			1.25	7.188	6.466	6.647	0.767	0.677	0.180	36.6
		9	1.25	8.188	7.466	7.647	0.767	0.677	0.180	48.1
10			1.5	9.026	8.160	8.376	0.920	0.812	0.217	58.0
		11	1.5	10.026	9.160	9.376	0.920	0.812	0.217	72.3
12			1.75	10.863	9.853	10.106	1.074	0.947	0.253	84.3
	14		2	12.701	11.546	11.835	1.227	1.083	0.289	115
16			2	14.701	13.546	13.835	1.227	1.083	0.289	157
	18		2.5	16.376	14.933	15.294	1.534	1.353	0.361	193
20			2.5	18.376	16.933	17.294	1.534	1.353	0.361	245
	22		2.5	20.376	18.933	19.294	1.534	1.353	0.361	303
24			3	22.051	20.319	20.752	1.840	1.624	0.433	353
	27		3	25.051	23.319	23.752	1.840	1.624	0.433	459
30			3.5	27.727	25.706	26.211	2.147	1.894	0.505	561
	33		3.5	30.727	28.706	29.211	2.147	1.894	0.505	694
36			4	33.402	31.093	31.670	2.454	2.165	0.577	817
	39		4	36.402	34.093	34.670	2.454	2.165	0.577	976
42			4.5	39.077	36.479	37.129	2.760	2.436	0.650	1121
	45		4.5	42.077	39.479	40.129	2.760	2.436	0.650	1306
48			5	44.752	41.866	42.587	3.067	2.706	0.722	1473
	52		5	48.752	45.866	46.587	3.067	2.706	0.722	1758
56			5.5	52.428	49.252	50.046	3.374	2.977	0.794	2030
	60		5.5	56.428	53.252	54.046	3.374	2.977	0.794	2362
64			6	60.103	56.639	57.505	3.681	3.248	0.866	2676
	68		6	64.103	60.639	61.505	3.681	3.248	0.866	3055

1) The tensile stress cross-section is calculated acc. to DIN 13 Part 28 with formula $A_s \quad \frac{\pi}{4} \quad \left(\frac{d_2 + d_3}{2}\right)^{2}$

Standardization
ISO Metric Screw Threads
(Coarse and Fine Pitch Threads)

Selection of nominal thread diameters and pitches for coarse and fine pitch threads from 1 mm to 68 mm diameter, following DIN 13 Part 12, 10.88 edition

Nominal thread diameter d = D Series 1	Series 2	Series 3	Coarse pitch thread	Pitches P for fine pitch threads 4	3	2	1.5	1.25	1	0.75	0.5
1			0.25								
1.2			0.25								
	1.4		0.3								
1.6			0.35								
	1.8		0.35								
2			0.4								
	2.2		0.45								
2.5			0.45								
3			0.5								
	3.5		0.6								
4			0.7								0.5
5			0.8								0.5
6			1							0.75	0.5
8			1.25						1	0.75	0.5
10			1.5					1.25	1	0.75	
12			1.75				1.5	1.25	1		
	14		2				1.5	1.25	1		
		15					1.5		1		
16			2				1.5		1		
		17							1		
	18		2.5			2	1.5		1		
20			2.5			2	1.5		1		
	22		2.5			2	1.5		1		
24			3			2	1.5		1		
		25					1.5				
		26					1.5				
	27		3			2	1.5				
		28					1.5				
30			3.5			2	1.5				
		32					1.5				
	33		3.5			2	1.5				
		35					1.5				
36			4		3	2	1.5				
		38					1.5				
	39		4		3	2					
		40					1.5				
42			4.5		3	2	1.5				
	45		4.5		3	2	1.5				
48			5		3	2	1.5				
		50					1.5				
	52		5		3	2	1.5				
		55				2	1.5				
56			5.5	4	3	2	1.5				
		58					1.5				
	60		5.5	4	3	2	1.5				
64			6	4	3	2					
		65				2					
	68		6	4	3	2					

Standardization
Cylindrical Shaft Ends

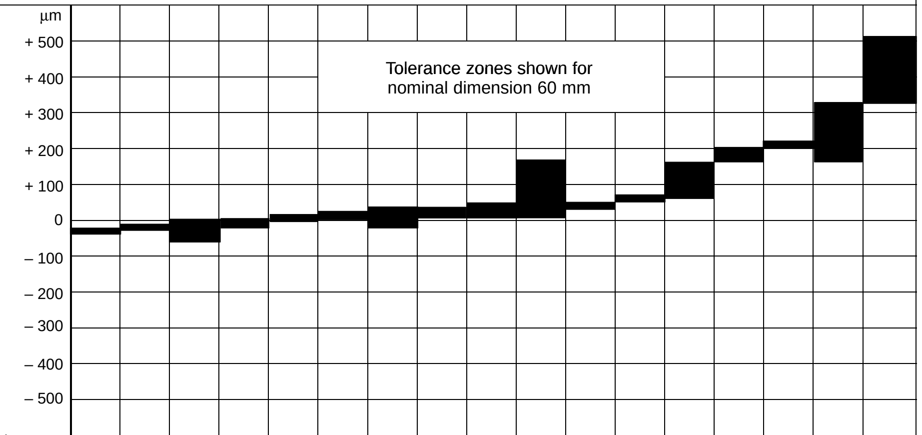
Cylindrical shaft ends

Acc. to DIN 748/1, 1.70 edition: Diameter Series 1 mm	Series 2 mm	ISO tolerance zone	Length Long mm	Length Short mm	FLENDER works standard W 0470, 5.82 edition: Diameter mm	Length mm	ISO tolerance zone
6		k6	16				
7			16				
8			20				
9			20				
10			23	15			
11			23	15			
12			30	18			
14			30	18	14	30	k6
16			40	28	16		
19			40	28	19	35	
20			50	36	20		
22			50	36	22		
24			50	36	24	40	
25			60	42	25		
28			60	42	28	50	m6
30			80	58	30		
32			80	58	32	60	
35			80	58	35		
38			80	58	38		
40			110	82	40	70	
42			110	82	42		
45			110	82	45	80	
48			110	82	48		
50			110	82	50		
55		m6	110	82	55	90	
60			140	105	60	105	
65			140	105	65		
70			140	105	70	120	
75			140	105	75		
80			170	130	80	140	
85			170	130	85		
90			170	130	90	160	
95			170	130	95		

Cylindrical shaft ends

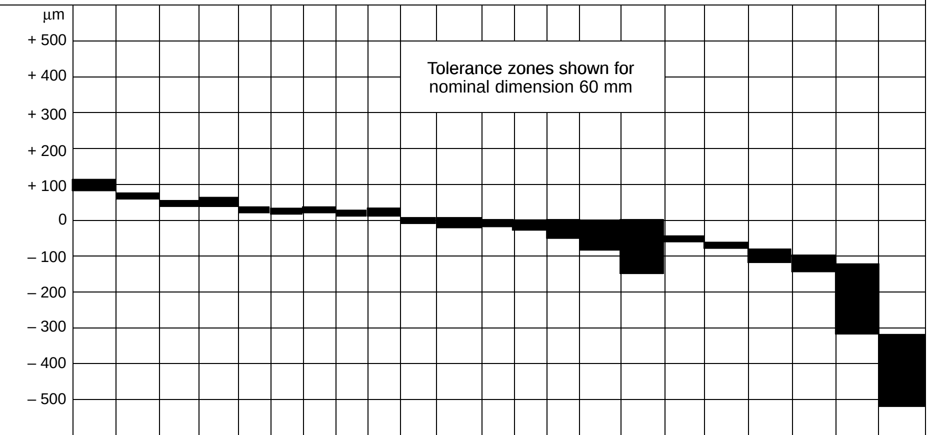
Acc. to DIN 748/1, 1.70 edition: Diameter Series 1 mm	Series 2 mm	ISO tolerance zone	Length Long mm	Length Short mm	FLENDER works standard W 0470, 5.82 edition: Diameter mm	Length mm	ISO tolerance zone
100		m6	210	165	100	180	m6
110			210	165	110		n6
120			210	165	120	210	
	130		250	200	130		
140			250	200	140	240	
	150		250	200	150		
160			300	240	160	270	
	170		300	240	170		
180			300	240	180	310	
	190		350	280	190		
200			350	280	200		
220			350	280	220	350	
	240		410	330	240	400	
250			410	330	250		
	260		410	330	260		
280			470	380	280	450	
	300		470	380	300	500	
320			470	380	320		
	340		550	450	340	550	
360			550	450	360	590	
	380		550	450	380		
400			650	540	400	650	
	420		650	540	420		
	440		650	540	440	690	
450			650	540	450	750	
	460		650	540	460		
	480		650	540	480	790	
500			650	540	500		
	530		800	680			
560			800	680			
	600		800	680			
630			800	680			

ISO tolerance zones, allowances, fit tolerances; Inside dimensions (holes) acc. to DIN 7157, 1.66 edition; DIN ISO 286 Part 2, 11.90 edition



ISO abbrev.		Series 1							H7	H8			F8	E9		D10	C11	
		Series 2	P7	N7	N9	M7	K7	J6	J7			H11	G7			D9		A11
Nominal dimensions in mm	from to	1 3	– 6 –16	– 4 –14	– 4 –29	– 2 –12	0 –10	+ 2 – 4	+ 4 – 6	+10 0	+14 0	+ 60 0	+12 + 2	+ 20 + 6	+ 39 + 14	+ 45 + 20	+ 60 + 20	+120 + 60	+330 +270
	above to	3 6	– 8 –20	– 4 –16	0 –30	0 –12	+ 3 – 9	+ 5 – 3	+ 6 – 6	+12 0	+18 0	+ 75 0	+16 4	+ 28 + 10	+ 50 + 20	+ 60 + 30	+ 78 + 30	+145 + 70	+345 +270
	above to	6 10	– 9 –24	– 4 –19	0 –36	0 –15	+ 5 –10	+ 5 – 4	+ 8 – 7	+15 0	+22 0	+ 90 0	+20 + 5	+ 35 + 13	+ 61 + 25	+ 76 + 40	+ 98 + 40	+170 + 80	+370 +280
	above to	10 14	–11 –29	– 5 –23	0 –43	0 –18	+ 6 –12	+ 6 – 5	+10 – 8	+18 0	+27 0	+110 0	+24 + 6	+ 43 + 16	+ 75 + 32	+ 93 + 50	+120 + 50	+205 + 95	+400 +290
	above to	14 18																	
	above to	18 24	–14 –35	– 7 –28	0 –52	0 –21	+ 6 –15	+ 8 – 5	+12 – 9	+21 0	+33 0	+130 0	+28 + 7	+ 53 + 20	+ 92 + 40	+117 + 65	+149 + 65	+240 +110	+430 +300
	above to	24 30																	
	above to	30 40	–17 –42	– 8 –33	0 –62	0 –25	+ 7 –18	+10 – 6	+14 –11	+25 0	+39 0	+160 0	+34 + 9	+ 64 + 25	+112 + 50	+142 + 80	+180 + 80	+280 +120	+470 +310
	above to	40 50																+290 +130	+480 +320
	above to	50 65	–21 –51	– 9 –39	0 –74	0 –30	+ 9 –21	+13 – 6	+18 –12	+30 0	+46 0	+190 0	+40 +10	+ 76 + 30	+134 + 60	+174 +100	+220 +100	+330 +140	+530 +340
	above to	65 80																+340 +150	+550 +360
	above to	80 100	–24 –59	–10 –45	0 –87	0 –35	+10 –25	+16 – 6	+22 –13	+35 0	+54 0	+220 0	+47 +12	+ 90 + 36	+159 + 72	+207 +120	+260 +120	+390 +170	+600 +380
	above to	100 120																+400 +180	+630 +410
	above to	120 140	–28 –68	–12 –52	0 –100	0 –40	+12 –28	+18 – 7	+26 –14	+40 0	+63 0	+250 0	+54 +14	+106 + 43	+185 + 85	+245 +145	+305 +145	+450 +200	+710 +460
	above to	140 160																+460 +210	+770 +520
	above to	160 180																+480 +230	+830 +580
	above to	180 200	–33 –79	–14 –60	0 –115	0 –46	+13 –33	+22 – 7	+30 –16	+46 0	+72 0	+290 0	+61 +15	+122 + 50	+215 +100	+285 +170	+355 +170	+530 +240	+950 +660
	above to	200 225																+550 +260	+1030 + 740
	above to	225 250																+570 +280	+1110 + 820
	above to	250 280	–36 –88	–14 –66	0 –130	0 –52	+16 –36	+25 – 7	+36 –16	+52 0	+81 0	+320 0	+69 +17	+137 + 56	+240 +110	+320 +190	+400 +190	+620 +300	+1240 + 920
	above to	280 315																+650 +330	+1370 +1050
	above to	315 355	–41 –98	–16 –73	0 –140	0 –57	+17 –40	+29 – 7	+39 –18	+57 0	+89 0	+360 0	+75 +18	+151 + 62	+265 +125	+350 +210	+440 +210	+720 +360	+1560 +1200
	above to	355 400																+760 +400	+1710 +1350
	above to	400 450	– 45 –108	–17 –80	0 –155	0 –63	+18 –45	+33 – 7	+43 –20	+63 0	+97 0	+400 0	+83 +20	+165 + 68	+290 +135	+385 +230	+480 +230	+840 +440	+1900 +1500
	above to	450 500																+880 +480	+2050 +1650
ISO abbrev.		Series 1							H7	H8			F8	E9		D10	C11	
		Series 2	P7	N7	N9	M7	K7	J6	J7			H11	G7			D9		A11

ISO tolerance zones, allowances, fit tolerances; Outside dimensions (shafts) acc. to DIN 7157, 1.66 edition; DIN ISO 286 Part 2, 11.90 edition



ISO abbrev.		Series 1	x8/u8			r6	n6							h6			h9			f7				
		Series 2	1)	s6	r5			m5	m6	k5	k6	j6	js6		h7	h8		h11	g6		e8	d9	c11	a11
Nominal dimensions in mm	from to	1 3	+ 34 + 20	+ 20 + 14	+ 14 + 10	+ 16 + 10	+10 + 4	+ 6 + 2	+ 8 + 2	+ 4 0	+ 6 0	+ 4 – 2	+ 3 – 3	0 – 6	0 –10	0 –14	0 – 25	0 – 60	– 2 – 8	– 6 – 16	– 14 – 28	– 20 – 45	– 60 –120	–270 –330
	above to	3 6	+ 46 + 28	+ 27 + 19	+ 20 + 15	+ 23 + 15	+16 + 8	+ 9 + 4	+12 + 4	+ 6 + 1	+ 9 + 1	+ 6 – 2	+ 4 – 4	0 – 8	0 –12	0 –18	0 – 30	0 – 75	– 4 –12	– 10 – 22	– 20 – 38	– 30 – 60	– 70 –145	–270 –345
	above to	6 10	+ 56 + 34	+ 32 + 23	+ 25 + 19	+ 28 + 19	+19 +10	+12 + 6	+15 + 6	+ 7 + 1	+10 + 1	+ 7 – 2	+4.5 –4.5	0 – 9	0 –15	0 –22	0 – 36	0 – 90	– 5 –14	– 13 – 28	– 25 – 47	– 40 – 76	– 80 –170	–280 –370
	above to	10 14	+ 67 + 40	+ 39 + 28	+ 31 + 23	+ 34 + 23	+23 +12	+15 + 7	+18 + 7	+ 9 + 1	+12 + 1	+ 8 – 3	+5.5 –5.5	0 –11	0 –18	0 –27	0 – 43	0 –110	– 6 –17	– 16 – 34	– 32 – 59	– 50 – 93	– 95 –205	–290 –400
	above to	14 18	+ 72 + 45																					
	above to	18 24	+ 87 + 54	+ 48 + 35	+ 37 + 28	+ 41 + 28	+28 +15	+17 + 8	+21 + 8	+11 + 2	+15 + 2	+ 9 – 4	+6.5 –6.5	0 –13	0 –21	0 –33	0 – 52	0 –130	– 7 –20	– 20 – 41	– 40 – 73	– 65 –117	–110 –240	–300 –430
	above to	24 30	+ 81 + 48																					
	above to	30 40	+ 99 + 60	+ 59 + 43	+ 45 + 34	+ 50 + 34	+33 +17	+20 + 9	+25 + 9	+13 + 2	+18 + 2	+11 – 5	+8 –8	0 –16	0 –25	0 –39	0 – 62	0 –160	– 9 –25	– 25 – 50	– 50 – 89	– 80 –142	–120 –280	–310 –470
	above to	40 50	+109 + 70																				–130 –290	–320 –480
	above to	50 65	+133 + 87	+ 72 + 53	+ 54 + 41	+ 60 + 41	+39 +20	+24 +11	+30 +11	+15 + 2	+21 + 2	+12 – 7	+9.5 –9.5	0 –19	0 –30	0 –46	0 – 74	0 –190	–10 –29	– 30 – 60	– 60 –106	–100 –174	–140 –330	–340 –530
	above to	65 80	+148 +102	+ 78 + 59	+ 56 + 43	+ 62 + 43																	–150 –340	–360 –550
	above to	80 100	+178 +124	+ 93 + 71	+ 66 + 51	+ 73 + 51	+45 +23	+28 +13	+35 +13	+18 + 3	+25 + 3	+13 – 9	+11 –11	0 –22	0 –35	0 –54	0 – 87	0 –220	–12 –34	– 36 – 71	– 72 –126	–120 –207	–170 –390	–380 –600
	above to	100 120	+198 +144	+101 + 79	+ 69 + 54	+ 76 + 54																	–180 –400	–410 –630
	above to	120 140	+233 +170	+117 + 92	+ 81 + 63	+ 88 + 63	+52 +27	+33 +15	+40 +15	+21 + 3	+28 + 3	+14 –11	+12.5 –12.5	0 –25	0 –40	0 –63	0 –100	0 –250	–14 –39	– 43 – 83	– 85 –148	–145 –245	–200 –450	–460 –710
	above to	140 160	+253 +190	+125 +100	+ 83 + 65	+ 90 + 65																	–210 –460	–520 –770
	above to	160 180	+273 +210	+133 +108	+ 86 + 68	+ 93 + 68																	–230 –480	–580 –830
	above to	180 200	+308 +236	+151 +122	+ 97 + 77	+106 + 77	+60 +31	+37 +17	+46 +17	+24 + 4	+33 + 4	+16 –13	+14.5 –14.5	0 –29	0 –46	0 –72	0 –115	0 –290	–15 –44	– 50 – 96	–100 –172	–170 –285	–240 –530	–660 –950
	above to	200 225	+330 +258	+159 +130	+100 + 80	+109 + 80																	–260 –550	– 740 –1030
	above to	225 250	+356 +284	+169 +140	+104 + 84	+113 + 84																	–280 –570	– 820 –1100
	above to	250 280	+396 +315	+190 +158	+117 + 94	+126 + 94	+66 +34	+43 +20	+52 +20	+27 + 4	+36 + 4	+16 –16	+16 –16	0 –32	0 –52	0 –81	0 –130	0 –320	–17 –49	– 56 –108	–110 –191	–190 –320	–300 –620	– 920 –1240
	above to	280 315	+431 +350	+202 +170	+121 + 98	+130 + 98																	–330 –650	–1050 –1370
	above to	315 355	+479 +390	+226 +190	+133 +108	+144 +108	+73 +37	+46 +21	+57 +21	+29 + 4	+40 + 4	+18 –18	+18 –18	0 –36	0 –57	0 –89	0 –140	0 –360	–18 –54	– 62 –119	–125 –214	–210 –350	–360 –720	–1200 –1560
	above to	355 400	+524 +435	+244 +208	+139 +114	+150 +114																	–400 –760	–1350 –1710
	above to	400 450	+587 +490	+272 +232	+153 +126	+166 +126	+80 +40	+50 +23	+63 +23	+32 + 5	+45 + 5	+20 –20	+20 –20	0 –40	0 –63	0 –97	0 –155	0 –400	–20 –60	– 68 –131	–135 –232	–230 –385	–440 –840	–1500 –1900
	above to	450 500	+637 +540	+292 +252	+159 +132	+172 +132																	–480 –880	–1650 –2050
ISO abbrev.		Series 1	x8/u8			r6	n6							h6			h9			f7				
		Series 2	1)	s6	r5			m5	m6	k5	k6	j6	js6		h7	h8		h11	g6		e8	d9	c11	a11

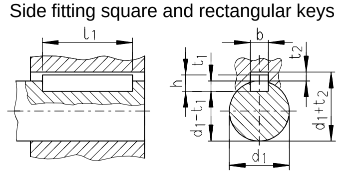
1) Up to nominal dimension 24 mm: x8; above nominal dimension 24 mm: u8

Dimensions of parallel keys and taper keys										
Diameter d		Width b	Height h	Depth of keyway in shaft t_1	Depth of keyway in hub t_2 DIN		Lengths, see below l_1 DIN		l	
above	to	1)	2)		6885/1	6886/6887 2)	6885/1		6886	
							from	to	from	to
mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
6	8	2	2	1.2	1.0	0.5	6	20	6	20
8	10	3	3	1.8	1.4	0.9	6	36	8	36
10	12	4	4	2.5	1.8	1.2	8	45	10	45
12	17	5	5	3	2.3	1.7	10	56	12	56
17	22	6	6	3.5	2.8	2.2	14	70	16	70
22	30	8	7	4	3.3	2.4	18	90	20	90
30	38	10	8	5	3.3	2.4	22	110	25	110
38	44	12	8	5	3.3	2.4	28	140	32	140
44	50	14	9	5.5	3.8	2.9	36	160	40	160
50	58	16	10	6	4.3	3.4	45	180	45	180
58	65	18	11	7	4.4	3.4	50	200	50	200
65	75	20	12	7.5	4.9	3.9	56	220	56	220
75	85	22	14	9	5.4	4.4	63	250	63	250
85	95	25	14	9	5.4	4.4	70	280	70	280
95	110	28	16	10	6.4	5.4	80	320	80	320
110	130	32	18	11	7.4	6.4	90	360	90	360
130	150	36	20	12	8.4	7.1	100	400	100	400
150	170	40	22	13	9.4	8.1	110	400	110	400
170	200	45	25	15	10.4	9.1	125	400	125	400
200	230	50	28	17	11.4	10.1	140	400	140	400
230	260	56	32	20	12.4	11.1	160	400		
260	290	63	32	20	12.4	11.1	180	400	Lengths not determined	
290	330	70	36	22	14.4	13.1	200	400		
330	380	80	40	25	15.4	14.1	220	400		
380	440	90	45	28	17.4	16.1	250	400		
440	500	100	50	31	19.5	18.1	280	400		

Lengths mm l_1 or l: 6 8 10 12 14 16 18 20 22 25 28 32 36 40 45 50 56 63 70 80 90 100 110 125 140 160 180 200 220 250 280 320 360 400

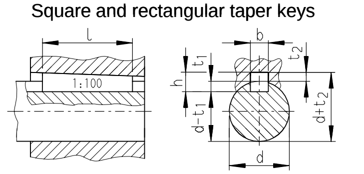
Parallel keys and taper keys
acc. to DIN 6885 Part 1, 6886 and 6887
Editions: 08.68 12.67 4.68

Side fitting square and rectangular keys



Parallel key and keyway acc. to DIN 6885 Part 1

Square and rectangular taper keys



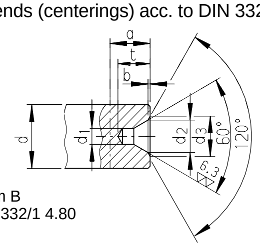
Taper and round-ended sunk key and keyway acc. to DIN 6886

1) The tolerance zone for hub keyway width b for parallel keys with normal fit is ISO JS9 and with close fit ISO P9. The tolerance zone for shaft keyway width b with normal fit is ISO N9 and with close fit ISO P9.

2) Dimension h of the taper key names the largest height of the key, and dimension t_z the largest depth of the hub keyway. The shaft keyway and hub keyway dimensions according to DIN 6887 - taper keys with gib head - are equal to those of DIN 6886.

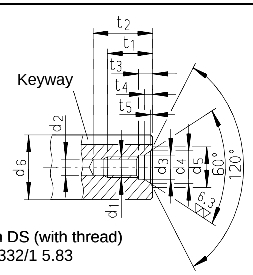
Dimensions of 60° centre holes							
Recommended diameters d 2)		Bore diameter d_1	Form B a 1)	b	d_2	d_3	Minimum dimensions t
above	to						
mm	mm	mm	mm	mm	mm	mm	mm
6	10	1.6	5.5	0.5	3.35	5	3.4
10	25	2	6.6	0.6	4.25	6.3	4.3
		2.5	8.3	0.8	5.3	8	5.4
25	63	3.15	10	0.9	6.7	10	6.8
		4	12.7	1.2	8.5	12.5	8.6
63	100	5	15.6	1.6	10.6	16	10.8
		6.3	20	1.4	13.2	18	12.9

Centre holes
in shaft ends (centerings) acc. to DIN 332 Part 1



Form B
DIN 332/1 4.80

Recommended diameters d_6 2)		Form DS d_1	d_2	d_3	d_4	d_5	t_1	t_2	t_3	t_4	t_5
above	to		3)				+2	min.	+1	≈	≈
mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
7	10	M3	2.5	3.2	5.3	5.8	9	12	2.6	1.8	0.2
10	13	M4	3.3	4.3	6.7	7.4	10	14	3.2	2.1	0.3
13	16	M5	4.2	5.3	8.1	8.8	12.5	17	4	2.4	0.3
16	21	M6	5	6.4	9.6	10.5	16	21	5	2.8	0.4
21	24	M8	6.8	8.4	12.2	13.2	19	25	6	3.3	0.4
24	30	M10	8.5	10.5	14.9	16.3	22	30	7.5	3.8	0.6
30	38	M12	10.2	13	18.1	19.8	28	37	9.5	4.4	0.7
38	50	M16	14	17	23	25.3	36	45	12	5.2	1.0
50	85	M20	17.5	21	28.4	31.3	42	53	15	6.4	1.3
85	130	M24	21	25	34.2	38	50	63	18	8	1.6
130	225	M30*	26	31	44	48	60	77	17	11	1.9
225	320	M36*	31.5	37	55	60	74	93	22	15	2.3
320	500	M42*	37	43	65	71	84	105	26	19	2.7



Form DS (with thread)
DIN 332/1 5.83

1) Cutting-off dimension in case of no centering
2) Diameter applies to finished workpiece
* Dimensions not acc. to DIN 332 Part 2
3) Drill diameter for tapping-size holes acc. to DIN 336 Part 1

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Internationally determined prefixes

Decimal multiples and sub-multiples of units are represented with prefixes and symbols. Prefixes and symbols are used only in combination with unit names and unit symbols.

Factor by which the unit is multiplied	Prefix	Symbol	Factor by which the unit is multiplied	Prefix	Symbol
10^{-18}	Atto	a	10^{1}	Deka	da
10^{-15}	Femto	f	10^{2}	Hecto	h
10^{-12}	Pico	p	10^{3}	Kilo	k
10^{-9}	Nano	n	10^{6}	Mega	M
10^{-6}	Micro	µ	10^{9}	Giga	G
10^{-3}	Milli	m	10^{12}	Tera	T
10^{-2}	Centi	c	10^{15}	Peta	P
10^{-1}	Deci	d	10^{18}	Exa	E

– Prefix symbols and unit symbols are written without blanks and together they form the symbol for a new unit. An exponent on the unit symbol also applies to the prefix symbol.

Example:
$1\ \mathrm{cm}^3 = 1 \cdot (10^{-2}\mathrm{m})^3 = 1 \cdot 10^{-6}\mathrm{m}^3$
$1\ \mu\mathrm{s} = 1 \cdot 10^{-6}\mathrm{s}$
$10^6\mathrm{s}^{-1} = 10^6\mathrm{Hz} = 1\ \mathrm{MHz}$

– Prefixes are not used with the basic SI unit kilogram (kg) but with the unit gram (g).

Example:
Milligram (mg), NOT microkilogram (µkg).

– When giving sizes by using prefix symbols and unit symbols, the prefixes should be chosen in such a way that the numerical values are between 0.1 and 1000.

Example:

12 kN	instead of	$1.2 \cdot 10^4$N
3.94 mm	instead of	0.00394 m
1.401 kPa	instead of	1401 Pa
31 ns	instead of	$3.1 \cdot 10^{-8}$s

– Combinations of prefixes and the following units are not allowed:

Units of angularity: degree, minute, second
Units of time: minute, hour, year, day
Unit of temperature: degree Celsius

Basic SI units

Physical quantity	Basic SI unit		Physical quantity	Basic SI unit	
	Name	Symbol		Name	Symbol
Length	Metre	m	Thermodynamic temperature	Kelvin	K
Mass	Kilo-gram	kg			
Time	Second	s	Amount of substance	Mole	mol
Electric current	Ampere	A	Luminous intensity	Candela	cd

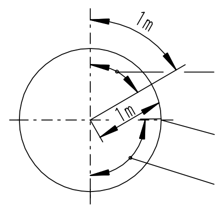
Derived SI units having special names and special unit symbols

Physical quantity	SI unit		Relation
	Name	Symbol	
Plane angle	Radian	rad	1 rad = 1 m/m
Solid angle	Steradian	sr	1 sr = 1 m^2/m^2
Frequency, cycles per second	Hertz	Hz	1 Hz = 1 s^{-1}
Force	Newton	N	1 N = 1 kg · m/s^2
Pressure, mechanical stress	Pascal	Pa	1 Pa = 1 N/m^2 = 1 kg/ (m · s^2)
Energy; work; quantity of heat	Joule	J	1 J = 1 N · m = 1 W · s = 1 kg · m^2/m^2
Power, heat flow	Watt	W	1 W = 1 J/s = 1 kg · m^2/s^3
Electric charge	Coulomb	C	1 C = 1 A · s
Electric potential	Volt	V	1 V = 1 J/C = 1 (kg · m^2)/(A · s^3)
Electric capacitance	Farad	F	1 F = 1 C/V = 1 (A^2 · s^4)/(kg · m^2)
Electric resistance	Ohm	Ω	1 Ω = 1 V/A = 1 (kg · m^2)/A^2 · s^3)
Electric conductance	Siemens	S	1 S = 1 Ω^{-1} = 1 (A^2 · s^3)/(kg · m^2)
Celsius temperature	degrees Celsius	°C	1 °C = 1 K
Inductance	Henry	H	1 H = 1 V · s/A

Legal units outside the SI

Physical quantity	Unit name	Unit symbol	Definition
Plane angle	Round angle Gon Degree Minute Second	1) gon ° 2) ' 2) " 2)	1 perigon = 2 π rad 1 gon = (π/200)rad 1° = (π/180)rad 1' = (1/60)° 1" = (1/60)'
Volume	Litre	l	1 l = 1 dm^3 = (1/1000) m^3
Time	Minute Hour Day Year	min 2) h 2) d 2) a 2)	1 min = 60 s 1 h = 60 min = 3600 s 1 d = 24 h = 86 400 s 1 a = 365 d = 8 760 h
Mass	Ton	t	1 t = 10^3 kg = 1 Mg
Pressure	Bar	bar	1 bar = 10^5 Pa

1) A symbol for the round angle has not yet been internationally determined
2) Do not use with prefixes

Physical quantities and units of lengths and their powers			
Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
l	Length	m (metre)	N.: Basic unit L.U.: µm; mm; cm; dm; km; etc. N.A.: micron (µ): 1 µ = 1 µm Ångström unit (Å): 1 Å = 10^{-10} m
A	Area	m^2 (square metre)	L.U.: mm^2; cm^2; dm^2; km^2 are (a): 1 a = 10^2 m^2 hectare (ha): 1 ha = 10^4 m^2
V	Volume	m^3 (cubic metre)	L.U.: mm^3; cm^3; dm^3 litre (l): 1 l = dm^3
H	Moment of area	m^3	N.: moment of a force; moment of resistance L.U.: mm^3; cm^3
I	Second moment of area	m^4	N.: formerly: geometrical moment of inertia L.U.: mm^4; cm^4
α,β. γ	Plane angle	rad (radian)	N. : $1\ rad + \frac{1\ m\ (arc)}{1\ m\ (radius)} + \frac{1\ m}{1\ m} + 1m\ m$  L.U. : µrad, mrad Degree (°) : $1^o + \frac{\pi}{180}$ rad Minute ( ) : $1 + \frac{1^o}{60}$ Second ( ) : $1 + \frac{1}{60}$ Gon (gon) : 1 gon $+ \frac{\pi}{200}$ rad N.A. : Right angle = (L) : 1L $+ \frac{\pi}{2}$ rad Centesimal degree (g) : 1g + 1 gon Centesimal minute (c) : $1^c + \frac{1}{100}$ gon Centesimal second (cc) : $1^{cc} + \frac{1^c}{100}$
Ω, ω	Solid angle	sr (steradian)	N. : $1\ sr + \frac{1\ m^2\ (spherical\ surface)}{1\ m^2\ (square\ of\ spherical\ radius)} + 1\ \frac{m^2}{m^2}$

Physical quantities and units of time			
Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
t	Time, Period, Duration	s (second)	N.: Basic unit L.U.: ns; µs; ms; ks Minute (min): 1 min = 60 s Hour (h): 1 h = 60 min Day (d): 1 d = 24 h Year (a): 1 a = 365 d (Do not use prefixes for decimal multiples and sub-multiples of min, h, d, a)
f	Frequency, Periodic frequency	Hz (Hertz)	L.U.: kHz; MHz; GHz; THz Hertz (Hz): 1 Hz = 1/s
n	Rotational frequency (speed)	s^{-1}	N.: Reciprocal value of the duration of one revolution L.U.: min^{-1} = 1/min
v	Velocity	m/s	L.U.: cm/s; m/h; km/s; km/h $1\ km\ h + \frac{1}{3.6}\ m\ s$
a	Acceleration, linear	m/s^2	N.: Time-related velocity L.U.: cm/s^2
g	Gravity	m/s^2	N.: Gravity varies locally. Normal gravity (g_n): g_n = 9.80665 m/s^2 ≈ 9.81 m/s^2
ω	Angular velocity	rad/s	L.U.: rad/min
α	Angular acceleration	rad/s^2	L.U.: $°/s^2$
$\dot{V}$	Volume flow rate	m^3/s	L.U.: l/s; l/min; dm^3/s; l/h; m^3/h; etc.

Physical quantities and units of mechanics			
Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
m	Mass	kg (kilogram)	N.: Basic unit L.U.: µg; mg; g; Mg ton (t): 1 t = 1000 kg
m’	Mass per unit length	kg/m	N.: m’ = m/l L.U.: mg/m; g/km; In the textile industry: Tex (tex):1 tex = 10^{-6} kg/m = 1 g/km
m’’	Mass in relation to the surface	kg/m^2	N.: m’’ = m/A L.U.: g/mm^2; g/m^2; t/m^2
ϱ	Density	kg/m^3	N.: ϱ = m/V L.U.: g/cm^3, kg/dm^3, Mg/m^3, t/m^3, kg/l $1 g/cm^3 = 1 kg/dm^3 = 1 Mg/m^3 =$ $1 t/m^3$ = 1 kg/l

Physical quantities and units of mechanics (continued)

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
J	Mass moment of inertia; second mass moment	$kg \cdot m^2$	N.: Instead of the former flywheel effect GD^2 GD^2 in kpm^2 now : $J + \frac{GD^2}{4}$ L.U.: $g \cdot m^2$; $t \cdot m^2$
$\dot{m}$	Rate of mass flow	kg/s	L.U.: kg/h; t/h
F	Force	N (Newton)	L.U.: µN; mN; kN; MN; etc.; 1 N = 1 kg m/s^2 N.A.: kp (1 kp = 9.80665 N)
G	Weight	N (Newton)	N.: Weight = mass acceleration due to gravity L.U.: kN; MN; GN; etc.
M, T	Torque	Nm	L.U.: µNm; mNm; kNm; MNm; etc. N.A.: kpm; pcm; pmm; etc.
M_b	Bending moment	Nm	L.U.: Nmm; Ncm; kNm etc. N.A.: kpm; kpcm; kpmm etc.
p	Pressure	Pa (Pascal)	N.: 1 Pa = 1 N/m^2 L.U.: Bar (bar): 1 bar = 100 000 Pa = 10^5 Pa µbar, mbar N.A.: kp/cm^2; at; ata; atü; mmWS; mmHg; Torr 1kp/cm^2 = 1 at = 0.980665 bar 1 atm = 101 325 Pa = 1.01325 bar 1 Torr $+ \frac{101325}{760}$ Pa $+$ 133.322 Pa 1 mWS = 9806.65 Pa = 9806.65 N/m^2 1 mmHg = 133.322 Pa = 133.322 N/m^2
p_{abs}	Absolute pressure	Pa (Pascal)	
p_{amb}	Ambient atmospheric pressure	Pa (Pascal)	
p_e	Pressure above atmospheric	Pa (Pascal)	$p_e = p_{abs} - p_{amb}$
σ	Direct stress (tensile and compressive stress)	N/m^2	L.U.: N/mm^2 1 N/mm^2 = 10^6 N/m^2
τ	Shearing stress	N/m^2	L.U.: N/mm^2
ε	Extension	m/m	N.: Δl / l L.U.: µm/m; cm/m; mm/m
W, A	Work	J (Joule)	N.: 1 J = 1 Nm = 1 Ws L.U.: mJ; kJ; MJ; GJ; TJ; kWh 1 kWh = 3.6 MJ N.A.: kpm; cal; kcal 1 cal = 4.1868 J; 860 kcal = 1 kWh
E, W	Energy		

Physical quantities and units of mechanics (continued)

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
P	·Power	W Watt)	N.: 1 W = 1 J/s = 1 Nm/s L.U.: µW; mW; kW; MW; etc. kJ/s; kJ/h; MJ/h, etc. N.A.: PS; kpm/s; kcal/h 1 PS = 735.49875 W 1 kpm/s = 9.81 W 1 kcal/h = 1.16 W 1 hp = 745.70 W
$\dot{Q}$	Heat flow		
η	Dynamic viscosity	Pa · s	N.: 1 Pa · s = 1 Ns/m^2 L.U.: dPa · s, mPa · s N.A.: Poise (P): 1 P = 0.1 Pa · s
ν	Kinematic viscosity	m^2/s	L.U.: mm^2/s; cm^2/s N.A.: Stokes (St): 1 St = 1/10000 m^2/s 1cSt = 1 mm^2/s

Physical quantities and units of thermodynamics and heat transfer

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
T	Thermodynamic temperature	K (Kelvin)	N.: Basic unit 273.15 K = 0 °C 373.15 K = 100 °C L.U.: mK
t	Celsius temperature	°C	N.: The degrees Celsius (°C) is a special name for the degrees Kelvin (K) when stating Celsius temperatures. The temperature interval of 1 K equals that of 1 °C.
Q	Heat Quantity of heat	J	1 J = 1 Nm = 1 Ws L.U.: mJ; kJ; MJ; GJ; TJ N.A.: cal; kcal
a	Temperature conductivity	m^2/s	$a + \frac{}{\mu = c_p}$ λ [W/(m · K)] = thermal conductivity µ [kg/m^3] = density of the body c_p [J/(kg · K)] = specific heat capacity at constant pressure
H	Enthalpy (Heat content)	J	N.: Quantity of heat absorbed under certain conditions L.U.: kJ; MJ; etc. N.A.: kcal; Mcal; etc.
s	Entropy	J/K	1 J/K = 1 Ws/K = 1 Nm/K L.U.: kJ/K N.A.: kcal/deg; kcal/°K
α,h	Heat transfer coefficient	W/(m^2 · K)	L.U.: W/(cm^2 · K); kJ/(m^2 · h · K) N.A.: cal/(cm^2 · s · grd) kal/(m^2 · h · grd) ≈ 4.2 kJ/(m^2 · h · K)

Physical quantities and units of thermodynamics and heat transfer (continued)

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
c	Specific heat capacity	J/(K · kg)	1 J/(K · kg) = W · s / (kg · K) N.: Heat capacity referred to mass N.A.: cal / (g · deg); kcal / (kg · deg); etc.
α_l	Coefficient of linear thermal expansion	K^{-1}	m / (m · K) = K^{-1} N.: Temperature unit/length unit ratio L.U.: µm / (m · K); cm / (m · K); mm / (m · K)
α_v, γ	Coefficient of volumetric expansion	K^{-1}	m^3 / (m^3 · K) = K^{-1} N.: Temperature unit/volume ratio N.A.: m^3 / (m^3 · deg)

Physical quantities and units of electrical engineering

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
I	Current strength	A (Ampere)	N.: Basic unit L.U.: pA; nA; µA; mA; kA; etc.
Q	Electric-charge; Quantity of electricity	C (Coloumb)	1 C = 1 A · s 1 Ah = 3600 As L.U.: pC; nC; µC; kC
U	Electric voltage	V (Volt)	1 V = 1 W / A = 1 J / (s · A) = 1 A · Ω = 1 N · m / (s · A) L.U.: µV; mV; kV; MV; etc.
R	Electric resistance	Ω (Ohm)	1 Ω = 1 V / A = 1 W / A^2 1 J / (s · A^2) = 1 N · m / (s · A^2) L.U.: µΩ; mΩ; kΩ; etc.
G	Electric conductance	S (Siemens)	N.: Reciprocal of electric resistance 1 S = 1 Ω^{-1} = 1 / Ω; G = 1 / R L.U.: µS; mS; kS
C	Electrostatic capacitance	F (Farad)	1 F = 1 C / V = 1 A · s / V = 1 A^2 · s / W = 1 A^2 · s^2 / J = 1 A^2 · s^2/ (N · m) L.U.: pF; µF; etc.

Physical quantities and units of lighting engineering

Symbol	Physical quantity	SI unit Symbol Name	N.: Note L.U.: Further legal units N.A.: Units no longer allowed
I	Luminous intensity	cd (Candela)	N.: Basic unit 1 cd = 1 lm (lumen)/sr (Steradian) L.U.: mcd; kcd
L	Luminous density; Luminance	cd / m^2	L.U.: cd / cm^2; mcd/m^2; etc. N.A.: Apostilb (asb); 1 asb $\frac{1}{\pi}$ cd m^2 Nit (nt): 1 nt = 1 cd / m^2 Stilb (sb): 1 sb = 10^4 cd / m^2
Φ	Luminous flux	lm (Lumen)	1 lm = 1 cd · sr L.U.: klm
E	Illuminance	lx (Lux)	1 lx = 1 lm / m^2

Different measuring units of temperature

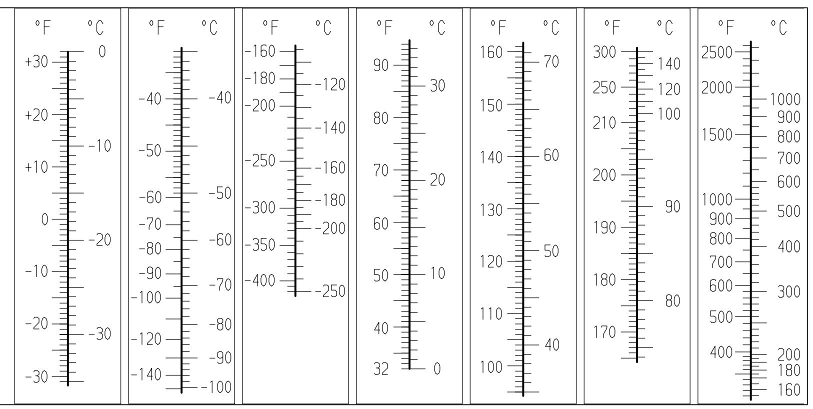
Kelvin K T_K	Degrees Celsius °C t_C	Degrees Fahrenheit °F t_F	Degrees Rankine °R T_R
$T_K \quad 273.15 + t_c$	$t_C \quad T_K = 273.15$	$t_F \quad \frac{9}{5} \; T_K = 459.67$	$T_R \quad \frac{9}{5} \; T_K$
$T_K \quad 255.38 + \frac{5}{9} \; t_F$	$t_C \quad \frac{5}{9} \; t_F = 32$	$t_F \quad 32 + \frac{9}{5} \; t_C$	$T_R \quad \frac{9}{5} \; t_c + 273.15$
$T_K \quad \frac{5}{9} \; T_R$	$t_C \quad \frac{5}{9} \; T_R = 273.15$	$t_F \quad T_R = 459.67$	$T_R \quad 459.67 + t_F$

Comparison of some temperatures

0.00 + 255.37 + 273.15 + 273.16 1) + 373.15	– 273.15 – 17.78 0.00 + 0.01 1) + 100.00	– 459.67 0.00 + 32.00 + 32.02 + 212.00	0.00 + 459.67 + 491.67 + 491.69 + 671.67

1) The triple point of water is +0.01 °C. The triple point of pure water is the equilibrium point between pure ice, air-free water and water vapour (at 1013.25 hPa).

Temperature comparison of °F with °C



Measures of length

Unit		Inch in	Foot ft	Yard yd	Stat mile	Naut mile	mm	m	km
1 in	=	1	0.08333	0.02778	–	–	25.4	0.0254	–
1 ft	=	12	1	0.3333	–	–	304.8	0.3048	–
1 yd	=	36	3	1	–	–	914.4	0.9144	–
1 stat mile	=	63 360	5280	1760	1	0.8684	–	1609.3	1.609
1 naut mile	=	72 960	6080	2027	1.152	1	–	1853.2	1.853
1 mm	=	0.03937	$3.281 \cdot 10^{-3}$	$1.094 \cdot 10^{-3}$	–	–	1	0.001	10^{-6}
1 m	=	39.37	3.281	1.094	–	–	1000	1	0.001
1 km	=	39 370	3281	1094	0.6214	0.5396	10^6	1000	1

1 German statute mile = 7500 m
1 geograph. mile = 7420.4 m = 4 arc minutes at the equator (1° at the equator = 111.307 km)

1 internat. nautical mile
1 German nautical mile (sm)
1 mille marin (French)
} =1852 m = 1 arc minute at the degree of longitude (1° at the meridian = 111.121 km)

Astronomical units of measure
1 light-second = 300 000 km
1 l.y. (light-year) = $9.46 \cdot 10^{12}$ km
1 parsec (parallax second, distances to the stars) = 3.26 l.y.
1 astronomical unit (mean distance of the earth from the sun) = $1.496 \cdot 10^8$ km
Typographical unit of measure: 1 point (p) = 0.376 mm

Other measures of length of the Imperial system
1 micro-in = 10^{-6} in = 0.0254 µm
1 mil = 1 thou = 0.001 in = 0.0254 mm
1 line = 0.1 in = 2,54 mm
1 fathom = 2 yd = 1.829 m
1 engineer's chain = 100 eng link = 100 ft = 30.48 m
1 rod = 1 perch = 1 pole = 25 surv link = 5.029 m
1 surveyor's chain = 100 surv link = 20.12 m
1 furlong = 1000 surv link = 201.2 m
1 stat league = 3 stat miles = 4.828 km

Other measures of length of the metric system
France:
1 toise = 1.949 m
1 myriametre = 10 000 m
Russia:
1 werschok = 44.45 mm
1 arschin = 0.7112 m
1 saschen = 2.1336 m
1 werst = 1.0668 km
Japan:
1 shaku = 0.3030 m
1 ken = 1.818 m
1 ri = 3.927 km

Square measures

Unit		sq in	sq ft	sq yd	sq mile	cm²	dm²	m²	a	ha	km²
1 square inch	=	1	–	–	–	6.452	0.06452	–	–	–	–
1 square foot	=	144	1	0.1111	–	929	9.29	0.0929	–	–	–
1 square yard	=	1296	9	1	–	8361	83.61	0.8361	–	–	–
1 square mile	=	–	–	–	1	–	–	–	–	259	2.59
1 cm²	=	0.155	–	–	–	1	0.01	–	–	–	–
1 dm²	=	15.5	0.1076	0.01196	–	100	1	0.01	–	–	–
1 m²	=	1550	10.76	1.196	–	10000	100	1	0.01	–	–
1 a	=	–	1076	119.6	–	–	10000	100	1	0.01	–
1 ha	=	–	–	–	–	–	–	10000	100	1	0.01
1 km²	=	–	–	–	0.3861	–	–	–	10000	100	1

Other square measures of the Imperial system
1 sq mil = $1 \cdot 10^{-6}$ sq in = 0.0006452 mm²
1 sq line = 0.01 sq in = 6.452 mm²
1 sq surveyor's link = 0.04047 m_2
1 sq rod = 1 sq perch = 1 sq pole = 625 sq surv link = 25.29 m²
1 sq chain = 16 sq rod = 4.047 a
1 acre = 4 rood = 40.47 a
1 township (US) = 36 sq miles = 3.24 km²
1 circular in $= \frac{\pi}{4}$ sq in $=$ 5.067cm² (circular area with 1 in dia.)
1 circular mil $= \frac{\pi}{4}$ sq mil $=$ 0.0005067mm² (circular area with 1 mil dia.)

Other square measures of the metric system
Russia:
1 kwadr. archin = 0.5058 m²
1 kwadr. saschen = 4.5522 m²
1 dessjatine = 1.0925 ha
1 kwadr. werst = 1.138 km²
Japan:
1 tsubo = 3.306 m²
1 se = 0.9917a
1 ho-ri = 15.42 km²

Cubic measures

Unit		cu in	cu ft	US liquid quart	US gallon	Imp quart	Imp gallon	cm³	dm³ (l)	m³
1 cu in	=	1	–	0.01732	–	0.01442	–	16.39	0.01639	–
1 cu ft	=	1728	1	29.92	7.481	24.92	6.229	–	28.32	0.02832
1 cu yd	=	46656	27	807.9	202	672.8	168.2	–	764.6	0.7646
1 US liquid quart	=	57.75	0.03342	1	0.25	0.8326	0.2082	946.4	0.9464	–
1 US gallon	=	231	0.1337	4	1	3.331	0.8326	3785	3.785	–
1 imp quart	=	69.36	0.04014	1.201	0.3002	1	0.25	1136	1.136	–
1 imp gallon	=	277.4	0.1605	4.804	1.201	4	1	4546	4.546	–
1 cm³	=	0.06102	–	–	–	–	–	1	0.001	10^6
1 dm³ (l)	=	61.02	0.03531	1.057	0.2642	0.88	0.22	1000	1	0.001
1 m³	=	61023	35.31	1057	264.2	880	220	10^6	1000	1

1 US minim = 0.0616 cm³ (USA)
1 US fl dram = 60 minims = 3.696 cm³
1 US fl oz = 8 fl drams = 0,02957 l
1 US gill = 4 fl oz = 0.1183 l
1 US liquid pint = 4 gills = 0.4732 l
1 US liquid quart = 2 liquid pints = 0.9464 l
1 US gallon = 4 liquid quarts = 3.785 l
1 US dry pint = 0.5506 l
1 US dry quart = 2 dry pints = 1.101 l
1 US peck = 8 dry quarts = 8.811 l
1 US bushel = 4 pecks = 35.24 l
1 US liquid barrel = 31.5 gallons = 119.2 l
1 US barrel = 42 gallons = 158.8 l (for crude oil)
1 US cord = 128 cu ft = 3.625 m²

1 Imp minim = 0.0592 cm³ (GB)
1 Imp ft drachm = 60 minims = 3.552 cm³
1 Imp ft oz = 8 ft drachm = 0,02841 l
1 Imp gill = 5 ft oz = 0.142 l
1 Imp pint = 4 gills = 0.5682 l
1 Imp quart = 2 pints = 1.1365 l
1 imp gallon = 4 quarts = 4.5461 l
1 iImp pottle = 2 quarts = 2.273 l
1 Imp peck = 4 pottles = 9.092 l
1 Imp bushel = 4 pecks = 36.37 l
1 Imp quarter = 8 bushels = 64 gallons = 290.94 l

Weights

Unit		dram	oz	lb	short cwt	long cwt	short ton	long ton	g	kg	t
1 dram	=	1	0.0625	0.003906	–	–	–	–	1.772	0.00177	–
1 oz (ounze)	=	16	1	0.0625	–	–	–	–	28.35	0.02835	–
1 lb (pound)	=	256	16	1	0.01	0.008929	–	–	453.6	0.4536	–
1 short cwt (US)	=	25600	1600	100	1	0.8929	0.05	0.04464	45359	45.36	0.04536
1 long cwt (GB/US)	=	28672	1792	112	1.12	1	0.056	0.05	50802	50.8	0.0508
1 short ton (US)	=	–	32000	2000	20	17.87	1	0.8929	–	907.2	0.9072
1 long ton (GB/US)	=	–	35840	2240	22.4	20	1.12	1	–	1016	1.016
1g	=	0.5643	0.03527	0.002205	–	–	–	–	1	0.001	10^{-6}
1kg	=	564.3	35.27	2.205	0.02205	0.01968	–	–	1000	1	0.001
1t	=	–	35270	2205	22.05	19.68	1.102	0.9842	10^6	1000	1

1 grain = 1 / 7000 lb = 0.0648 g (GB)
1 stone = 14 lb = 6.35 kg (GB)
1 short quarter = 1/4 short cwt = 11.34 kg (USA)
1 long quarter = 1/4 long cwt = 12.7 kg (GB / USA)
1 quintal or 1 cental = 100 lb = 45.36 kg (USA)
1 quintal = 100 livres = 48.95 kg (F)
1 kilopound = 1kp = 1000 lb = 453.6 kg (USA)

1 solotnik = 96 dol = 4.2659 g (CIS)
1 lot = 3 solotnik = 12.7978 g (CIS)
1 funt = 32 lot = 0.409 kg (CIS)
1 pud = 40 funt = 16.38 kg (CIS)
1 berkowetz = 163.8 kg (CIS)
1 kwan = 100 tael = 1000 momme = 10000 fun = 3.75 kg (J) (J)
1 hyaku kin = 1 picul = 16 kwan = 60 kg (J)

tdw = tons dead weight = lading capacity of a cargo vessel (cargo + ballast + fuel + stores), mostly given in long tons, i.e. 1 tdw = 1016 kg

Energy, work, quantity of heat

Work		ft lb	erg	J = Nm = Ws	kpm	PSh	hph	kWh	kcal	Btu
1 ft lb	=	1	$1.356 \cdot 10^7$	1.356	0.1383	$0.5121 \cdot 10^{-6}$	$0.505 \cdot 10–6$	$0.3768 \cdot 10^{-6}$	$0.324 \cdot 10^{-3}$	$1.286 \cdot 10^{-3}$
1 erg	=	$0.7376 \cdot 10^7$	1	10^{-7}	$0.102 \cdot 10^{-7}$	$37.77 \cdot 10^{-15}$	$37.25 \cdot 10^{15}$	$27.78 \cdot 10^{-15}$	$23.9 \cdot 10^{-12}$	$94.84 \cdot 10^{-12}$
1 Joule (WS)	=	0.7376	10^7	1	0.102	$377.7 \cdot 10^{-9}$	$372.5 \cdot 10^{-9}$	$277.8 \cdot 10^{-9}$	$238 \cdot 10^{-6}$	$948.4 \cdot 10^{-6}$
1 kpm	=	7.233	$9.807 \cdot 10^7$	9.807	1	$3.704 \cdot 10^{-6}$	$3.653 \cdot 10^{-6}$	$2.725 \cdot 10^{-6}$	$2.344 \cdot 10^{-3}$	$9.301 \cdot 10^{-3}$
1 PSh	=	$1.953 \cdot 10^6$	$26.48 \cdot 10^{12}$	$2.648 \cdot 10^6$	$270 \cdot 10^3$	1	0.9863	0.7355	632.5	2510
1 hph	=	$1.98 \cdot 10^6$	$26.85 \cdot 10^{12}$	$2.685 \cdot 10^6$	$273.8 \cdot 10^3$	1.014	1	0.7457	641.3	2545
1 kWh	=	$2.655 \cdot 10^6$	$36 \cdot 10^{12}$	$3.6 \cdot 10^6$	$367.1 \cdot 10^3$	1.36	1.341	1	860	3413
1 kcal	=	$3.087 \cdot 10^3$	$41.87 \cdot 10^9$	4186.8	426.9	$1.581 \cdot 10^{-3}$	$1.559 \cdot 10^{-3}$	$1.163 \cdot 10^3$	1	3.968
1 Btu	=	778.6	$10.55 \cdot 10^9$	1055	107.6	$398.4 \cdot 10^{-6}$	$392.9 \cdot 10^{-6}$	$293 \cdot 10^{-6}$	0.252	1

1 in oz = 0.072 kpcm; 1 in lb = 0.0833ft lb = 0.113 Nm, 1 thermi (French) = $4.1855 \cdot 10^6$ J; 1 therm (English) = $105.51 \cdot 10^6$ J
Common in case of piston engines: 1 litre-atmosphere (litre · atmosphere) = 98.067 J

Power, energy flow, heat flow									
Power	erg/s	W	kpm/s	PS	hp	kW	kcal/s	Btu/s
1 erg/s =	1	10^{-7}	$0.102 \cdot 10^{-7}$	$0.136 \cdot 10^{-9}$	$0.1341 \cdot 10^{-9}$	10^{-10}	$23.9 \cdot 10^{-12}$	$94.84 \cdot 10^{-12}$
1W =	10^7	1	0.102	$1.36 \cdot 10^{-3}$	$1.341 \cdot 10^{-3}$	10^{-3}	$239 \cdot 10^{-6}$	$948.4 \cdot 10^{-6}$
1kpm/s =	$9.807 \cdot 10^7$	9.807	1	$13.33 \cdot 10^{-3}$	$13.15 \cdot 10^{-3}$	$9.804 \cdot 10^{-3}$	$2.344 \cdot 10^{-3}$	$9.296 \cdot 10^{-3}$
1 PS (ch) 2) =	$7.355 \cdot 10^9$	735.5	75	1	0.9863	0.7355	0.1758	0.6972
1hp =	$7.457 \cdot 10^9$	745.7	76.04	1.014	1	0.7457	0.1782	0.7068
1 kW =	10^{10}	1000	102	1.36	1.341	1	0.239	0.9484
1 kcal/s =	$41.87 \cdot 10^8$	4187	426.9	5.692	5.614	4.187	1	3.968
1 Btu/s =	$10.55 \cdot 10^9$	1055	107.6	1.434	1.415	1.055	0.252	1

1 poncelet (French) = 980.665 W; flywheel effect: 1 kgm^2 = 3418 lb in ^{2}

Pressure and tension													
Unit	µbar = dN/m^2	mbar = cN/ cm^2	bar = daN/ cm^2	kp/m^2 mm WS	p/cm^2	kp/cm^2 = at	kp/mm^2	Torr= mm QS	atm	lb/sq ft	lb/sq in	long ton/sq in	{sh ton}/sq in
1 µb=daN =	1	0.001	–	0.0102	–	–	–	–	–	–	–	–	–
1mbar=cN/cm^2 =	1000	1	0.001	10.2	1.02	–	–	0.7501	–	2.089	0.0145	–	–
1 bar = daN/cm^2 =	10^6	1000	1	10197	1020	1.02	0.0102	750.1	0.9869	2089	14.5	0.0064	0.0072
1 kp/m^2=1mm WS at 4 °C =	98.07	–	–	1	0.1	0.0001	–	–	–	0.2048	–	–	–
1 p/cm^2 =	980.7	0.9807	–	10	1	0.001	–	0.7356	–	2.048	0.0142	–	–
1 kp/cm^2=1at (techn. atmosph.) =	–	980.7	0.9807	10000	1000	1	0.01	735.6	0.9678	2048	14.22	–	–
1 kp/mm^2 =	–	98067	98.07	10^6	10^5	100	1	73556	96.78	–	1422	0.635	0.7112
1 Torr = 1 mm QS at 0 °C =	1333	1.333	0.00133	13.6	1.36	0.00136	–	1	–	2.785	0.01934	–	–
1 atm (pressure of the atmosphere) =	–	1013	1.013	10332	1033	1.033	–	760	1	2116	14.7	–	–
1 lb/sq ft =	478.8	0.4788	–	4.882	0.4882	–	–	0.3591	–	1	–	–	–
1 lb/sq in=1 psi =	68948	68.95	0.0689	703.1	70.31	0.0703	–	51.71	0.068	144	1	–	0.0005
1 long ton/sq in (GB) =	–	–	154.4	–	–	157.5	1.575	–	152.4	–	2240	1	1.12
1 short ton/sq in (US) =	–	–	137.9	–	–	140.6	1.406	–	136.1	–	2000	0.8929	1

1 psi = 0.00689 N / mm^2

1 N/m^2 (Newton/m^2) = 10 µb, 1 barye (French) = 1 µb, 1 piece (pz) (French) = 1 sn/m^2 ≈ 102 kp/m^2. 1 hpz = 100 pz = 1.02 kp/m^2.
In the USA, "inches Hg" are calculated from the top, i.e. 0 inches Hg = 760 mm QS and 29.92 inches Hg = 0 mm QS = absolute vacuum.
The specific gravity of mercury is assumed to be 13.595 kg/dm^3.

Velocity					
Unit	m/s	m/min	km/h	ft/min	mile/h
m/s =	1	60	3.6	196.72	2.237
m/min =	0.0167	1	0.06	3.279	0.0373
km/h =	0.278	16.67	1	54.645	0.622
ft/min =	0.0051	0.305	0.0183	1	0.0114
mile/h =	0.447	26.82	1.609	87.92	1

Definition	SI unit	Symbol	Basic formulae: Linear motion	Basic formulae: Rotary motion
Uniform motion			distance moved divided by time	angular velocity = angle of rotation in radian measure/time
Velocity	m/s	v	$v = \frac{s_2 + s_1}{t_2 + t_1} = \frac{s}{t} = \text{const.}$	$\pi = \frac{\varrho_2 + \varrho_1}{t_2 + t_1} = \frac{\varrho}{t} = \text{const.}$
Angular velocity	rad/s	ω	motion accelerated from rest:	
Angle of rotation	rad m/s	ϱ v	$v = \frac{s}{t}$	$\varrho = \frac{\varrho}{t}$
Distance moved	m	s	s = v · t	angle of rotation φ = ω · t
Uniformly accelerated motion			acceleration equals change of velocity divided by time	angular acceleration equals change of angular velocity divided by time
Acceleration	m/s^2	a	$a = \frac{v_2 + v_1}{t_2 + t_1} = \frac{v}{t} = \text{const.}$	$\mu = \frac{\pi_2 + \pi_1}{t_2 + t_1} = \frac{\pi}{t} = \text{const.}$
Angular acceleration	rad/s^2	α	motion accelerated from rest:	
	m/s^2	a	$a = \frac{v}{t} = \frac{v^2}{2s} = \frac{2s}{t^2}$	$\mu = \frac{\pi}{t} = \frac{\pi^2}{2\varrho} = \frac{2\varrho}{t^2}$
Velocity	m/s	v	$v = a \quad t = \overline{2}\ a \quad s$	$\pi = \mu \quad t$
Circumferential speed	m/s	v		$v = r \quad \pi = r \quad \mu \quad t$
Distance moved	m	s	$s = \frac{v}{2} \quad t = \frac{a}{2} \quad t^2 = \frac{v^2}{2a}$	angle of rotation $\varrho = \frac{\pi}{2} \quad t = \frac{\mu}{2} \quad t^2 = \frac{\pi^2}{2\mu}$
Uniform motion and constant force or constant torque			force · distance moved	torque · angle of rotation in radian measure
Work	J	W	W = F · s	W = M · φ
			work in unit of time = force · velocity	work in unit of time = torque · angular velocity
Power	W	P	$P = \frac{W}{t} = F \quad v$	$P = \frac{W}{t} = M \quad \pi$
Non-uniform (accelerated) motion			accelerating force = mass · acceleration	accel. torque = second mass moment · angular acceleration
Force	N	F	F = m · a	M = J · α
In case of any motion			*)	* *)
Energy	J	E_k	$E_k \quad \frac{m}{2} \quad v^2$	$E_k = \frac{J}{2} \quad \pi^2$
Potential energy (due to force of gravity)	J	E_p	weight · height $E_p = G \cdot h = m \cdot g \cdot h$	
Centrifugal force	N	F_F	$F_F = m \cdot r_s \cdot \omega^2$ (r_s = centre-of-gravity radius)	

*) Momentum (kinetic energy) equals half the mass · second power of velocity.
**) Kinetic energy due to rotation equals half the mass moment of inertia · second power of the angular velocity.

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Calculation of Areas	63
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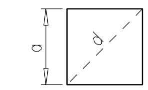
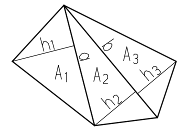
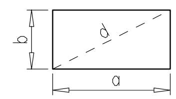
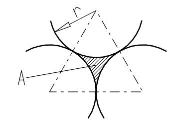
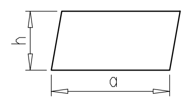
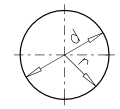
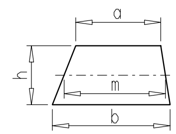
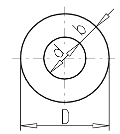
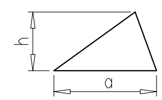
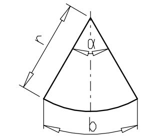
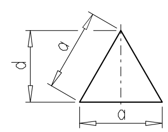
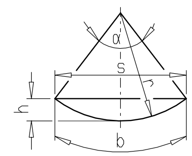
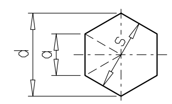
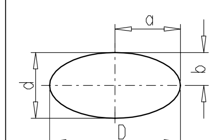
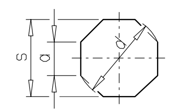
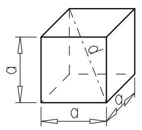
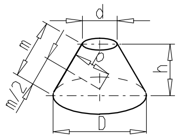
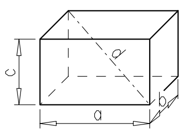
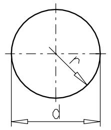
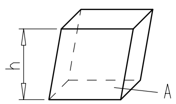
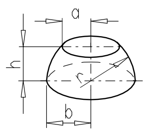
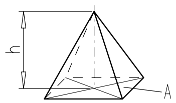
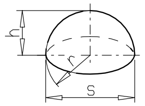
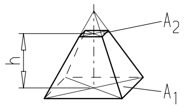
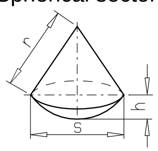
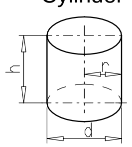
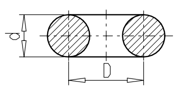
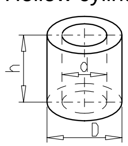
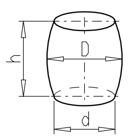
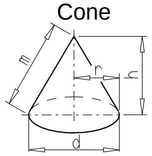
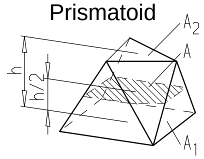
A = area U = circumference

Shape	Figure	Formulas	Shape	Figure	Formulas
Square		$A = a^2$ $a \quad \overline{A}$ $d \quad a \ \overline{2}$	Polygon		$A \quad A_1 + A_2 + A_3$ $\frac{a \quad h_1 + b \quad h_2 + b \quad h_3}{2}$
Rectangle		$A \quad a \quad b$ $d \quad \overline{a^2 + b^2}$	Formed area		$A \quad \frac{r^2}{2}(2 \ \overline{3} = \mu)$ $0.16 \quad r^2$
Parallelogram		$A \quad a \quad h$ $a \quad \frac{A}{h}$	Circle		$A \quad \frac{d^2 \quad \mu}{4} \quad r^2 \quad \mu$ $0.785 \quad d^2$ $U \quad 2r\mu \quad d \quad \mu$
Trapezium		$A \quad m \quad h$ $m \quad \frac{\{a + b\}}{2}$	Circular ring		$A \quad \frac{\mu}{4} \quad (D^2 = d^2)$ $(d + b)\ b \quad \mu$ $b \quad \frac{\{D = d\}}{2}$
Triangle		$A \quad \frac{\{a \quad h\}}{2}$ $a \quad \frac{\{2 \quad A\}}{h}$	Circular sector		$A \frac{r^2 \quad \mu \quad {}^o}{360^o}$ $\frac{\{b \quad r\}}{2}$ $b \quad \frac{\{r \quad \mu \quad {}^o\}}{180^o}$
Equilateral triangle		$A \quad \frac{a^2}{4} \ \overline{3}$ $d \quad \frac{a}{2} \ \overline{3}$	Circular segment		$A \quad \frac{r^2}{2} \ \frac{\{ \ {}^o \quad \mu\}}{180} = \sin$ $\frac{1}{2}[\ r(b = s) + sh]$ $s \quad 2\ r\ \sin\frac{}{2}$ $h \quad r\,(1 = \cos\frac{a}{2}) \quad \frac{s}{2}\tan\frac{}{4}$ $\hat{} \quad \frac{\{ \ {}^o \quad \mu\}}{180}$ $b \quad r \ \hat{}$
Hexagon		$A \quad \frac{3 \quad a^2 \quad \overline{3}}{2}$ $d \quad 2 \quad a$ $s \quad \overline{3} \quad a$	Ellipse		$A \quad \frac{\{D \quad d \quad \mu\}}{4} \quad a \quad b \quad \mu$ $U \quad \frac{\{D + d\}}{2} \quad \mu$ $U \quad \mu\,(a + b)\,[\,1 +$ $\frac{1}{4}\left(\frac{\{a = b\}}{\{a + b\}}\right)^2 + \frac{1}{64}\left(\frac{\{a = b\}}{\{a + b\}}\right)^4$ $+ \frac{1}{256}\left(\frac{\{a = b\}}{\{a + b\}}\right)^6 \ ..\,]$
Octagon		$A \quad 2a^2(\ \overline{2} + 1)$ $d \quad a \ \overline{4 + 2 \ \overline{2}}$ $s \quad a(\ \overline{2} + 1)$			

V = volume	O = surface	M = generated surface	
Cube 	$V \quad a^3$ $O \quad 6 \quad a^2$ $d \quad a \quad \overline{3}$	Frustum of cone 	$V \quad \frac{\{\pi \quad h\}}{12} (D^2 + Dd + d^2)$ $M \quad \frac{\{\pi \quad m\}}{2} (D + d)$ $2\pi \quad p \quad h$ $m \quad \overline{\frac{\{D = d\}}{2}^{2} + h^2}$
Parallelepiped 	$V \quad a \quad b \quad c$ $O \quad 2 (ab + ac + bc)$ $d \quad \overline{a^2 + b^2 + c^2}$	Sphere 	$V \quad \frac{4}{3} r^3 \pi \quad \frac{1}{6} \quad d^3\pi$ $4.189 \quad r^3$ $O \quad 4\pi \quad r^2 \quad \pi \quad d^2$
Rectangular block 	$V \quad A \quad h$ (Cavalier principle)	Spherical zone 	$V \quad \frac{\{\pi \quad h\}}{6} (3a^2 + 3b^2 + h^2)$ $M \quad 2 \quad r \quad \pi \quad h$
Pyramid 	$V \quad \frac{\{A \quad h\}}{3}$	Spherical segment 	$V \quad \frac{\{\pi \quad h\}}{6} \quad \frac{3}{4} s^2 + h^2$ $\pi h^2 \quad r = \frac{h}{3}$ $M \quad 2 \quad r \quad \pi \quad h$ $\frac{\pi}{4} (s^2 + 4h^2)$
Frustum of pyramid 	$V \quad \frac{h}{3} (A_1 + A_2 + \quad \overline{A_1 \quad A_2})$ $h \quad \frac{A_1 + A_2}{2}$	Spherical sector 	$V \quad \frac{2}{3} \quad h \quad r^2 \quad \pi$ $O \quad \frac{\{\pi \quad r\}}{2} (4h + s)$
Cylinder 	$V \quad \frac{d^2 \quad \pi}{4} h$ $M \quad 2 \quad r \quad \pi \quad h$ $O \quad 2 \quad r \quad \pi \quad (r + h)$	Cylindrical ring 	$V \quad \frac{D \quad \pi^2 \quad d^2}{4}$ $O \quad D \quad d \quad \pi^2$
Hollow cylinder 	$V \quad \frac{\{h \quad \pi\}}{4} (D^2 = d^2)$	Cylindrical barrel 	$V \quad \frac{\{h \quad \pi\}}{12} (2D^2 + d^2)$
Cone 	$V \quad \frac{r^2 \quad \pi \quad h}{3}$ $M \quad r \quad \pi \quad m$ $O \quad r \quad \pi \quad (r + m)$ $m \quad \sqrt{h^2 + \frac{d}{2}^{2}}$	Prismatoid 	$V \quad \frac{h}{6} (A_1 + A_2 + 4A)$

Mechanics / Strength of Materials	Page
Axial Section Moduli and Axial Second Moments of Area (Moments of Inertia) of Different Profiles	66
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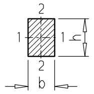
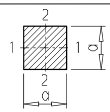
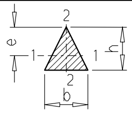
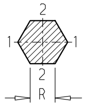
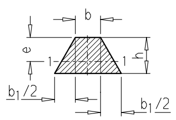
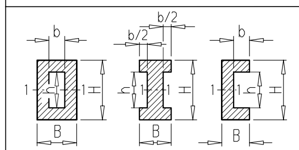
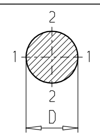
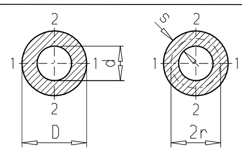
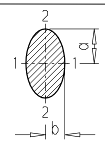
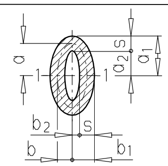
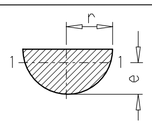
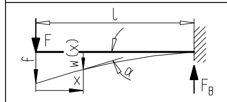
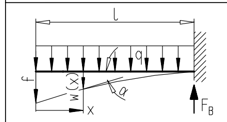
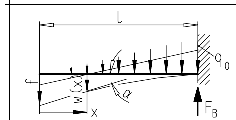
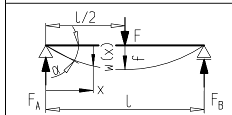
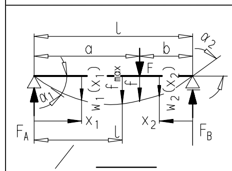
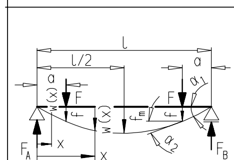
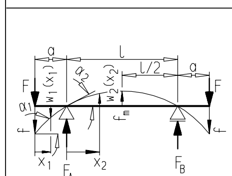
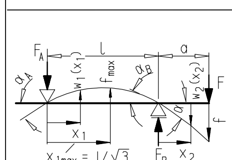
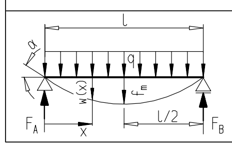
Values for Circular Sections	68
Stresses on Structural Members and Fatigue Strength of Structures	69

Axial Section Moduli and Axial Second Moments of Area (Moments of Inertia) of Different Profiles

Cross-sectional area	Section modulus	Second moment of area
	W_1 bh^2 6 W_2 hb^2 6	1 bh^3 12 2 hb^3 12
	W_1 W_2 a^3 6	1 2 a^4 12
	W_1 bh^2 24 for e $\frac{2}{3}$ h W_2 hb^2 24	1 bh^3 36 2 hb^3 48
	W_1 $\frac{5}{8}$ R^3 0.625 R^3 W_2 0.5413 R^3	1 2 $\frac{5}{16}$ $\overline{3}$ R^4 0.5413 R^4
	W_1 $\frac{6b^2 + 6bb_1 + b^2{}_1}{12(3b + 2b_1)}h^2$ for e $\frac{1}{3}\frac{3b + 2b_1}{2b + b_1}$ h	1 $\frac{6b^2 + 6bb_1 + b^2{}_1}{36(2b + b_1)}h^3$
	W_1 $\frac{BH^3 = bh^3}{6H}$	1 $\frac{BH^3 = bh^3}{12}$
	W_1 W_2 ϱ D^3 32 D^3 10	1 2 ϱD^4 64 D^4 20
	W_1 W_2 $\frac{ϱ}{32}\frac{D^4 = d^4}{D}$ or in case of thin wall thickness s: W_1 W_2 (r + s 2) ϱsr^2	1 2 $\frac{ϱ}{64}$ (D^4 = d^4) 1 2 ϱsr^3 1 + (s 2r)^2 ϱsr^3
	W_1 ϱa^2b 4 W_2 ϱb^2a 4	1 ϱa^3b 4 2 ϱb^3a 4
	W_1 1 a_1 or if the wall thickness is s $a_1 = a_2$ $b_1 = b_2$ 2 (a = a_2) 2 (b = b_2) thin W_1 $\frac{ϱ}{4}$ a (a + 3b) s	1 $\frac{ϱ}{4}$ ($a^3$1b_1 = $a^3$2b_2) 1 $\frac{ϱ}{4}$ a^2 (a + 3b) s
	W_1 1 e 0.1908 r^3 with e r 1 = $\frac{4}{\{3ϱ\}}$ 0.5756 r axis 1-1 = axis of centre of gravity	1 [ϱ 8 = 8 (9 ϱ) r^4 0.1098 r^4

Deflections in Beams

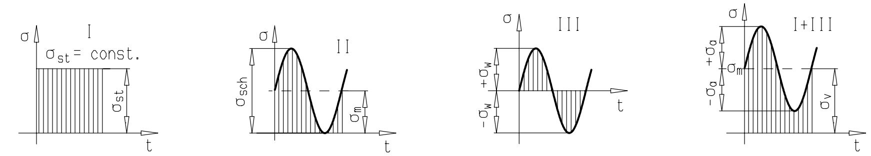
f, f_{max}, f_m, w, w_1, w_2 a, b, l, x_1, x_{1max}, x_2 E q, q_o	Deflection (mm) Lengths (mm) Modulus of elasticity (N/mm^2) Line load (N/mm)	α, $α_1$, $α_2$, $α_A$, $α_B$, F, F_A, F_B I Second moment of area (mm^4) (moment of inertia)	Angle (°) Forces (N)

	w(x) $\frac{F\ 3}{\{3E\}}$ 1 = $\frac{3}{2}\frac{x}{}$ + $\frac{1}{2}\frac{x}{}$ 3 f $\frac{F\ 3}{\{3E\}}$ tan μ $\frac{F\ 2}{\{2E\}}$ $F_π$ F
	w(x) $\frac{q\ 4}{\{8E\}}$ 1 = $\frac{4}{3}\frac{x}{}$ + $\frac{1}{3}\frac{x}{}$ 4 f $\frac{q\ 4}{\{8E\}}$ tan μ $\frac{q\ 3}{\{6E\}}$ $F_π$ q
	w(x) $\frac{q_o\ 4}{\{120E\}}$ 4 = 5 $\frac{x}{}$ + $\frac{x}{}$ 5 f $\frac{q_o\ 4}{\{30E\}}$ tan μ $\frac{q_o\ 3}{\{24E\}}$ $F_π$ $\frac{q_o}{2}$
	w(x) $\frac{F\ 3}{\{16E\}}$ $\frac{x}{}$ 1 = $\frac{4}{3}\frac{x}{l}$ 2 x $\frac{}{2}$ f $\frac{F\ 3}{\{48E\}}$ tan μ $\frac{F\ 2}{\{16E\}}$ F_A F_B $\frac{F}{2}$
 x_{1max} a $\overline{(l + b)\ 3a}$ for a > b change a and b for a < b	$w_1(x_1)$ $\frac{F\ 3}{\{6E\}}$ $\frac{a}{}$ $\frac{b}{}$ 2 $\frac{x_1}{}$ 1 + $\frac{l}{b}$ = $\frac{x_1^2}{ab}$ x_1 a f $\frac{F\ 3}{\{3E\}}$ $\frac{a}{}$ 2 $\frac{b}{}$ 2 tan μ$_1$ $\frac{f}{2a}$ 1 + $\frac{}{b}$ $w_2(x_2)$ $\frac{F\ 3}{\{6E\}}$ $\frac{b}{}$ $\frac{a}{}$ 2 $\frac{x_2}{}$ 1 + $\frac{}{a}$ = $\frac{x_2^2}{ab}$ x_2 b f_{max} f $\frac{\{+\ b\}}{3b}$ $\frac{\overline{+\ b}}{3a}$ tan μ$_2$ $\frac{f}{2b}$ 1 + $\frac{}{a}$ F_A F $\frac{b}{}$ F_B F $\frac{a}{}$
	w(x) $\frac{F\ 3}{\{2E\}}$ $\frac{x}{}$ $\frac{a}{}$ 1 = $\frac{a}{}$ = $\frac{1}{3}$ $\frac{x}{}$ 2 f $\frac{F\ 3}{\{2E\}}$ $\frac{a}{l}$ 2 1 = $\frac{4}{3}$ $\frac{a}{}$ tan μ$_1$ $\frac{F\ 2}{\{2E\}}$ $\frac{a}{}$ 1 = $\frac{a}{}$ x a 2 w(x) $\frac{F\ 3}{\{2E\}}$ $\frac{a}{}$ $\frac{x}{}$ 1 = $\frac{x}{}$ = $\frac{1}{3}$ $\frac{a}{}$ 2 f_m $\frac{F\ 3}{\{8E\}}$ $\frac{a}{}$ 1 = $\frac{4}{3}$ $\frac{a}{}$ 2 tan μ$_2$ $\frac{F\ 2}{\{2E\}}$ $\frac{a}{}$ 1 = 2$\frac{a}{}$ a x 2 $F_A = F_B = F$
	x_1 a $w_1(x_1)$ $\frac{F\ 3}{\{2E\}}$ $\frac{1}{3}$ $\frac{x_1}{}$ 3 = $\frac{a}{}$ 1 + $\frac{a}{}$ $\frac{x_1}{}$ + $\frac{a}{}$ 2 1 + $\frac{2}{3}$ $\frac{a}{}$ tan μ$_1$ $\frac{F\ 2}{\{2E\}}$ $\frac{a}{}$ 1 + $\frac{a}{}$ f $\frac{F\ 3}{\{2E\}}$ $\frac{a}{}$ 2 1 + $\frac{2}{3}$ $\frac{a}{}$ x_2 $w_2(x_2)$ $\frac{F\ 3}{\{2E\}}$ $\frac{a}{}$ $\frac{x_2}{}$ 1 = $\frac{x_2}{}$ f_m $\frac{F\ 3}{\{8E\}}$ $\frac{a}{}$ tan μ$_2$ $\frac{F\ 2}{\{2E\}}$ $\frac{a}{}$ $F_A = F_B = F$
	$w_1(x_1)$ $\frac{F\ 3}{\{6E\}}$ $\frac{a}{}$ $\frac{x_1}{}$ 1 = $\frac{x_1}{}$ 2 x_1 f $\frac{F\ 3}{\{3E\}}$ $\frac{a}{}$ 2 1 + $\frac{a}{}$ tan μ$_A$ $\frac{F\ 2}{\{6E\}}$ $\frac{a}{l}$ $w_2(x_2)$ $\frac{F\ 3}{\{6E\}}$ $\frac{x_2}{}$ $\frac{2a}{}$ + $\frac{3a}{}$ $\frac{x_2}{}$ = $\frac{x_2}{}$ 2 x_2 a f_{max} $\frac{F\ 3}{9\ \overline{3}E}$ $\frac{a}{}$ tan μ$_B$ 2 tan μ$_A$ F_A F $\frac{a}{}$ F_B F 1 + $\frac{a}{}$ tan μ $\frac{F\ 2}{\{6E\}}$ $\frac{a}{}$ 2 + 3 $\frac{a}{}$
	w(x) $\frac{q\ 4}{\{24E\}}$ $\frac{x}{}$ 1 = 2 $\frac{x}{}$ 2 + $\frac{x}{}$ 3 0 x f_m $\frac{5q\ 4}{\{384E\}}$ tan μ $\frac{q\ 3}{\{24E\}}$ F_A $\frac{q}{2}$ F_B $\frac{q}{2}$

Axial section modulus: W_a $\frac{\pi\ d^3}{32}$	Area: A $\frac{\pi\ d^2}{4}$
Polar section modulus: W_p $\frac{\pi\ d^3}{16}$	Mass: m $\frac{\pi\ d^2}{4}\ l\ \varrho$
Axial second moment of area (axial moment of inertia): $_a$ $\frac{\pi\ d^4}{64}$	Density of steel: ϱ $7,85\ \frac{kg}{dm^3}$
Polar second moment of area (polar moment of area): $_p$ $\frac{\pi\ d^4}{32}$	Second mass moment of inertia (mass moment of inertia): J $\frac{\pi\ d^4\ l\ \varrho}{32}$

d mm	A cm^2	W_a cm^3	I_a cm^4	Mass /l kg/m	J / l kgm^2/m	d mm	A cm^2	W_a cm^3	I_a cm^4	Mass/ l kg/m	J / l kgm^2/m
6.	0.293	0.0212	0.0064	0.222	0.000001	115.	103.869	149.3116	858.5414	81.537	0.134791
7.	0.385	0.0337	0.0118	0.302	0.000002	120.	113.097	169.6460	1017.8760	88.781	0.159807
8.	0.503	0.0503	0.0201	0.395	0.000003	125.	122.718	191.7476	1198.4225	96.334	0.188152
9.	0.636	0.0716	0.0322	0.499	0.000005	130.	132.732	215.6900	1401.9848	104.195	0.220112
10.	0.785	0.0982	0.0491	0.617	0.000008	135.	143 139	241.5468	1630.4406	112.364	0.255979
11.	0.950	0.1307	0.0719	0.746	0.000011	140.	153.938	269.3916	1895.7410	120.841	0.296061
12.	1.131	0.1696	0.1018	0.888	0.000016	145.	165.130	299.2981	2169.9109	129.627	0.340676
13.	1.327	0.2157	0.1402	1.042	0.000022	150.	176.715	331.3398	2485.0489	138.721	0.390153
14.	1.539	0.2694	0.1986	1.208	0.000030	155.	188.692	365.5906	2833.3269	148.123	0.444832
15.	1.767	0.3313	0.2485	1.387	0.000039	160.	201.062	402.1239	3216.9909	157.834	0.505068
16.	2.011	0.4021	0.3217	1.578	0.000051	165.	213.825	441.0133	3638.3601	167.852	0.571223
17.	2.270	0.4823	0.4100	1.782	0.000064	170.	226.980	482.3326	4099.8275	178.179	0.643673
18.	2.545	0.5726	0.5153	1.998	0.000081	175.	240.528	526.1554	4603.8598	188.815	0.722806
19.	2.835	0.6734	0.6397	2.226	0.000100	180.	254.469	572.5553	5152.9973	199.758	0.809021
20.	3.142	0.7854	0.7854	2.466	0.000123	185.	268.803	621.6058	5749.8539	211.010	0.902727
21.	3.464	0.9092	0.9547	2.719	0.000150	190.	283.529	673.3807	6397.1171	222.570	1.004347
22.	3.801	1.0454	1.1499	2.984	0.000181	195.	298.648	727.9537	7097.5481	234.438	1.114315
23.	4.155	1.1945	1.3737	3.261	0.000216	200.	314.159	785.3982	7853.9816	246.615	1.233075
24.	4.524	1.3572	1.6286	3.551	0.000256	210.	346.361	909.1965	9546.5638	271.893	1.498811
25.	4.909	1.5340	1.9175	3.853	0.000301	220.	380.133	1045.3650	11499.0145	298.404	1.805345
26.	5.309	1.7255	2.2432	4.168	0.000352	230.	415.476	1194.4924	13736.6629	326.148	2.156656
27.	5.726	1.9324	2.6087	4.495	0.000410	240.	452.389	1357.1680	16286.0163	355.126	2.556905
28.	6.158	2.1551	3.0172	4.834	0.000474	250.	490.874	1533.9808	19174.7598	385.336	3.010437
29.	6.605	2.3944	3.4719	5.185	0.000545	260.	530.929	1725.5198	22431.7569	416.779	3.521786
30.	7.069	2.6507	3.9761	5.549	0.000624	270.	572.555	1932.3740	26087.0491	449.456	4.095667
32.	8.042	3.2170	5.1472	6.313	0.000808	280.	615.752	2155.1326	30171.8558	483.365	4.736981
34.	9.079	3.8587	6.5597	7.127	0.001030	300.	706.858	2650.7188	39760.7820	554.884	6.242443
36.	10.179	4.5804	8.2448	7.990	0.001294	320.	804.248	3216.9909	51471.8540	631.334	8.081081
38.	11.341	5.3870	10.2354	8.903	0.001607	340.	907.920	3858.6612	65597.2399	712.717	10.298767
40.	12.566	6.2832	12.5664	9.865	0.001973	360.	1017.876	4580.4421	82447.9575	799.033	12.944329
42.	13.854	7.2736	15.2745	10.876	0.002398	380.	1134.115	5387.0460	102353.8739	890.280	16.069558
44.	15.205	8.3629	18.3984	11.936	0.002889	400.	1256.637	6283.1853	125663.7060	986.460	19.729202
46.	16.619	9.5559	21.9787	13.046	0.003451	420.	1385.442	7273.5724	152745.0200	1087.572	23.980968
48.	18.096	10.8573	26.0576	14.205	0.004091	440.	1520.531	8362.9196	183984.2320	1193.617	28.885524
50.	19.635	12.2718	30.6796	15.413	0.004817	460.	1661.903	9555.9364	219786.6072	1304.593	34.506497
52.	21.237	13.9042	35.8908	16.671	0.005635	480.	1809.557	10857.3442	260576.2608	1420.503	40.910473
54.	22.902	15.4590	41.7393	17.978	0.006553	500.	1693.495	12271.8463	306796.1572	1541.344	48.166997
56.	24.630	17.2411	48.2750	19.335	0.007579	520.	2123.717	13804.1581	358908.1107	1667.118	56.348573
58.	26.421	19.1551	55.5497	20.740	0.008721	540.	2290.221	15458.9920	417392.7849	1797.824	65.530667
60.	28.274	21.2058	63.6173	22.195	0.009988	560.	2463.009	17241.0605	482749.6930	1933.462	75.791702
62.	30.191	23.3978	72.5332	23.700	0.011388	580.	2642.079	19155.0758	555497.1978	2074.032	87.213060
64.	32.170	25.7359	82.3550	25.253	0.012930	600.	2827.433	21205.7504	636172.5116	2219.535	99.879084
66.	34.212	28.2249	93.1420	26.856	0.014623	620.	3019.071	23397.7967	725331.6994	2369.970	113.877076
68.	36.317	30.8693	104.9556	28.509	0.016478	640.	3216.991	25735.9270	823549.6636	2525.338	129.297297
70.	38.485	33.6739	117.8588	30.210	0.018504	660.	3421.194	28224.8538	931420.1743	2685.638	146.232967
72.	40.715	36.6435	131.9167	31.961	0.020711	680.	3631.681	30869.2894	1049555.8389	2850.870	164.780267
74.	43.008	39.7828	147.1963	33.762	0.023110	700.	3848.451	33673.9462	1178588.1176	3021.034	185.038334
76.	45.365	43.0964	163.7662	35.611	0.025711	720.	4071.504	36643.5367	1319167.3201	3196.131	207.109269
78.	47.784	46.5890	181.6972	37.510	0.028526	740.	4300.840	39782.7731	1471962.6056	3376.160	231.098129
80.	50.265	50.2655	201.0619	39.458	0.031567	760.	4536.460	43096.3680	1637661.9830	3561.121	257.112931
82.	52.810	54.1304	221.9347	41.456	0.034844	780.	4778.362	46589.0336	1816972.3105	3751.015	285.264653
84.	55.418	58.1886	244.3920	43.503	0.038370	800.	5026.548	50265.4824	2010619.2960	3945.840	315.667229
86.	58.088	62.4447	268.5120	45.599	0.042156	820.	5281.017	54130.4268	2219347.4971	4145.599	348.437557
88.	60.821	66.9034	294.3748	47.745	0.046217	840.	5541.769	58188.5791	2443920.3207	4350.289	383.695490
90.	63.617	71.5694	322.0623	49.940	0.050564	860.	5808.805	62444.6517	2685120.0234	4559.912	421.563844
92.	66.476	76.4475	351.6586	52.184	0.055210	880.	6082.123	66903.3571	2943747.7113	4774.467	462.168391
95.	70.882	84.1726	399.8198	55.643	0.062772	900.	6361.725	71569.4076	3220623.3401	4993.954	505.637864
100.	78.540	98.1748	490.8739	61.654	0.077067	920.	6647.610	76447.5155	3516585.7151	5218.374	552.103957
105.	86.590	113.6496	596.6602	67.973	0.093676	940.	6939.778	81542.3934	3832492.4910	5447.726	601.701321
110.	95.033	130.6706	718.6884	74.601	0.112834	960.	7238.229	86858.7536	4169220.1722	5682.010	654.567567
						980.	7542.964	92401.3084	4527664.1126	5921.227	710.843266
						1000.	7853.982	98174.7703	4908738.5156	6165.376	770.671947

Diffusion of stress in structural members: loading types

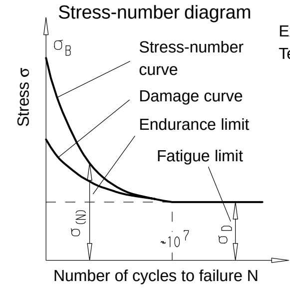


	static	dynamic	alternating	oscillating
Maximum stress limit:		$_o$ $_{sch}$	$_o$ + $_w$	$_o$ $_m$ + $_a$
Mean stress:		$_m$ $_{sch}$ 2	$_m$ 0	$_m$ $_v$ (initial stress)
Minimum stress limit:		$_u$ 0	$_u$ = $_w$	$_u$ $_m$ = $_a$

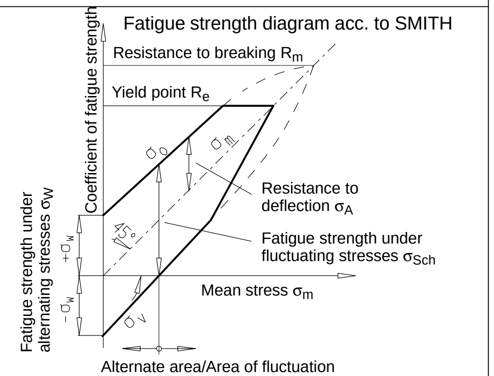
Ruling coefficient of strength of material for the calculation of structural members:

Resistance to breaking R_m — Fatigue strength under fluctuating stresses σ_{Sch} — Fatigue strength under alternating stresses σ_W — Resistance to deflection σ_A

Yield point R_e; $R_{p0.2}$ — Coefficients of fatigue strength σ_D



Example:
Tension-Compression



In case of stresses below the damage curve initial damage will not occur to the material.

Reduced stress on the member — Permissible stress — Design strength of the member

$_v$ perm. $\frac{_D\ b_o\ b_d}{S\ \ ß_k}$

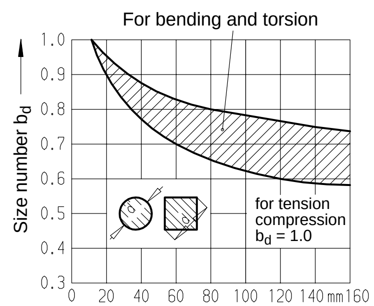
with: σ_D = ruling fatigue strength value of the material
b_o = surface number (≤ 1)
b_d = size number (≤ 1)
$ß_k$ = stress concentration factor (≥ 1)
S = safety (1.2 ... 2)

Reduced stress σ_v
For the frequently occurring case of combined bending and torsion, according to the distortion energy theory:

$_v$ $\overline{\ ^2 + 3\ (\mu_o\ )^2}$

with:
σ = single axis bending stress
τ = torsional stress
α_0 = constraint ratio according to Bach

Alternating bending, dynamic torsion: $\alpha_0 \approx 0.7$
Alternating bending, alternating torsion: $\alpha_0 \approx 1.0$
Static bending, alternating torsion: $\alpha_0 \approx 1.6$



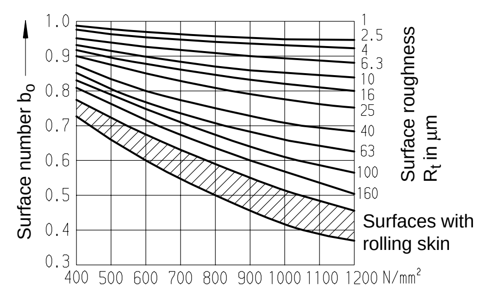


Table of Contents Section 6

Hydraulics Page

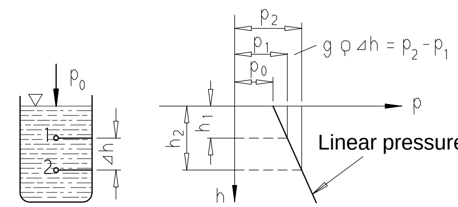
Hydrostatics 71
(Source: K. Gieck, Technische Formelsammlung, 29th Edition, Gieck Verlag, D-7100 Heilbronn)

Hydrodynamics 72
(Source: K. Gieck, Technische Formelsammlung, 29th Edition, Gieck Verlag, D-7100 Heilbronn)

Pressure distribution in a fluid

$p_1 \quad p_o + g\ h_1$

$P_2 \quad p_1 + g\ (h_2 = h_1) \quad p_1 + g \quad h$

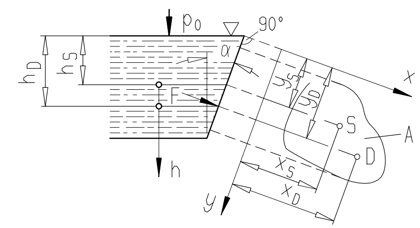


Hydrostatic force of pressure on planes

The hydrostatic force of pressure F is that force which is exerted on the wall by the fluid only - i.e. without consideration of pressure p_o.

$F \quad g \quad y_s\ A\ \cos\pi \quad g \quad h_s\ A$

$y_D \quad \frac{\mu_x}{y_s A} \quad y_s + \frac{\mu_s}{y_s A} \quad ; \quad x_D \quad \frac{\mu_{xy}}{y_s A} \quad$ m, mm

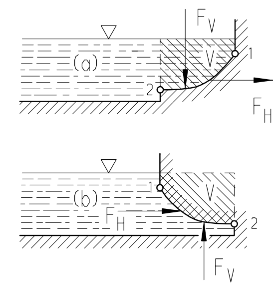


Hydrostatic force of pressure on curved surfaces

The hydrostatic force of pressure on the curved surface 1, 2 is resolved into a horizontal component F_H and a vertical component F_V.
F_V is equal to the weight of the fluid having a volume V located (a) or thought to be located (b) over the surface 1, 2. The line of application runs through the centre of gravity.

$F_v \quad g \quad V \qquad$ (N, kN)

F_H is equal to the hydrostatic force of pressure on the projection of the considered surface 1, 2 on the plane perpendicular to F_H.



Buoyance

The buoyant force F_A is equal to the weight of the displaced fluids having densities and ’.

$F_A \quad g \quad V + g \quad V \qquad$ (N, kN)

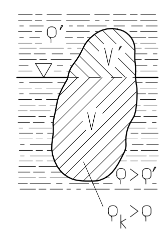
If the fluid with density ’ is a gas, the following applies:

$F_A \quad g \quad V \qquad$ (N, kN)

For $_k$ density of the body applies:

- $> \ _k$ the body floats
- $= \ _k$ the body is suspended
- $< \ _k$ the body sinks

} in a heavy liquid



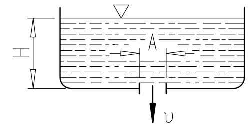
S = centre of gravity of plane A
D = centre of pressure
I_x, I_s = moments of inertia
I_{xy} = product of inertia of plane A referred to the x- and y-axes

Discharge of liquids from vessels

Vessel with bottom opening

$v \qquad \sqrt{2\,g\,H}$

$\dot{V} \quad \varrho \;\; A \;\; \sqrt{2\,g\,H}$



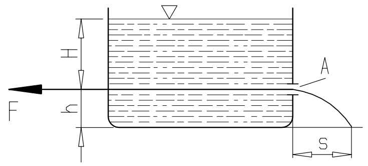
Vessel with small lateral opening

$v \qquad \sqrt{2\,g\,H}$

$s \quad 2 \;\; \sqrt{H\,h}$
(without any coefficient of friction)

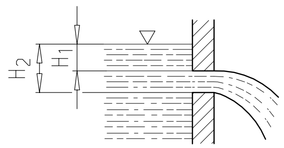
$\dot{V} \quad \varrho \;\; A \;\; \sqrt{2\,g\,H}$

$F \qquad \dot{V}\,v$



Vessel with wide lateral opening

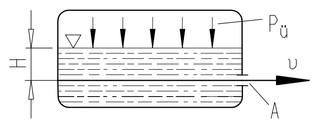
$\dot{V} \quad \frac{2}{3}\,\varrho\,b \;\; \sqrt{2\,g}\;\;(H_2{}^{3\;2} = H_1{}^{3\;2})$



Vessel with excess pressure on liquid level

$v \qquad \sqrt{2\;(g\,H \;+\; \frac{p_{ü}}{})}$

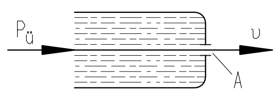
$\dot{V} \qquad A \quad \sqrt{2\;(g\,H \;+\; \frac{p_{ü}}{})}$



Vessel with excess pressure on outlet

$v \qquad \sqrt{2\;\frac{p_{ü}}{}}$

$\dot{V} \qquad A \quad \sqrt{2\;\frac{p_{ü}}{}}$



v: discharge velocity
g: gravity
: density
$p_{ü}$: excess pressure compared to external pressure
φ: coefficient of friction (for water φ = 0.97)
ε: coefficient of contraction (ε = 0.62 for sharp-edged openings)
(ε = 0.97 for smooth-rounded openings)
F: force of reaction
$\dot{V}$: volume flow rate
b: width of opening

Table of Contents Section 7

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Types of Construction and Mounting Arrangements of Rotating Electrical Machinery	76
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Types of Protection for Electrical Equipment (Protection Against Water)	78
Explosion Protection of Electrical Switchgear (Types of protection)	79
Explosion Protection of Electrical Switchgear (Gases and vapours)	80

Ohm's law:

U   R $\frac{U}{R}$ R   U

Series connection of resistors:

R $R_1 + R_2 + R_3 +$ + R_n

R   total resistance μ

R_n individual resistance μ

Shunt connection of resistors:

$\frac{1}{R}$ $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} +$ + $\frac{1}{R_n}$

R   total resistance μ

R_n individual resistance μ

Electric power:

	Power	Current consumption
Direct current	P U	$\frac{P}{U}$
Single-phase alternating current	P U cos	$\frac{P}{U\ \ \cos}$
Three-phase current	P 1.73 U cos	$\frac{P}{1.73\ \ U\ \ \cos}$

Material	π $\frac{m}{\mu\ mm^2}$	$\frac{\mu\ mm^2}{m}$
a) Metals		
Aluminium	36	0.0278
Bismuth	0.83	1.2
Lead	4.84	0.2066
Cadmium	13	0.0769
Iron wire	6.7...10	0.15..0.1
Gold	43.5	0.023
Copper	58	0.01724
Magnesium	22	0.045
Nickel	14.5	0.069
Platinum	9.35	0.107
Mercury	1.04	0.962
Silver	61	0.0164
Tantalum	7.4	0.135
Tungsten	18.2	0.055
Zinc	16.5	0.061
Tin	8.3	0.12
b) Alloys		
Aldrey (AlMgSi)	30.0	0.033
Bronze I	48	0.02083
Bronze II	36	0.02778
Bronze III	18	0.05556
Constantan (WM 50)	2.0	0.50
Manganin	2.32	0.43
Brass	15.9	0.063
Nickel silver (WM 30)	3.33	0.30
Nickel chromium	0.92	1.09
Niccolite (WM 43)	2.32	0.43
Platinum rhodium	5.0	0.20
Steel wire (WM 13)	7.7	0.13
Wood's metal	1.85	0.54
c) Other conductors		
Graphite	0.046	22
Carbon, homog.	0.015	65
Retort graphite	0.014	70

Resistance of a conductor:

R $\frac{l}{\pi\ \ A}$ $\frac{l}{A}$

R = resistance (Ω)
l = length of conductor (m)
γ = electric conductivity (m/Ω mm^2)
A = cross section of conductor (mm^2)
  = specific electrical resistance (Ω mm^2)/m)

Speed:

n $\frac{f\ \ 60}{p}$

n = speed (min^{-1})
f = frequency (Hz)
p = number of pole pairs

Example: f = 50 Hz, p = 2

n $\frac{50\ \ 60}{2}$ 1500 min^{-1}

Efficiency:

ϱ $\frac{P_{ab}}{P_{zu}}$ 100 % [1)]

Example:
Efficiency and power factor of a four-pole 1.1-kW motor and a 132-kW motor dependent on the load

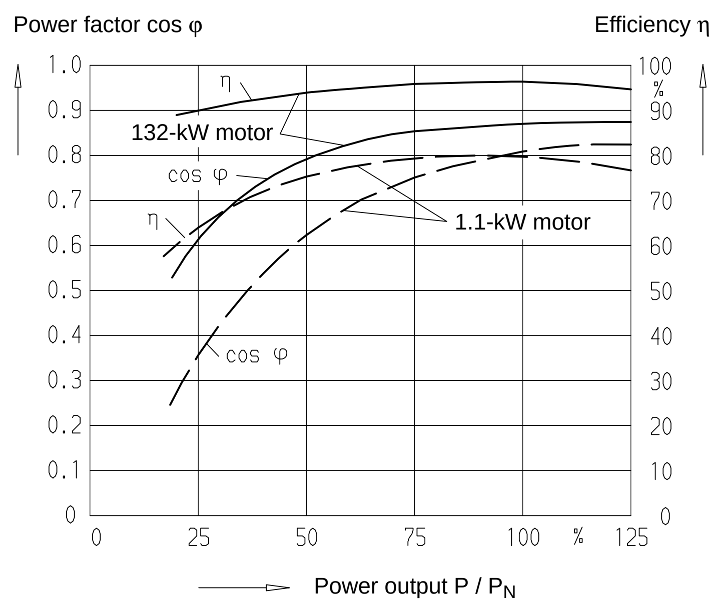
Power rating:

Output power 1)

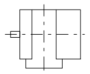
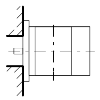
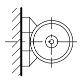
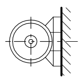
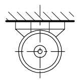
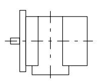
Direct current: ·
P_{ab} = U ·   · η

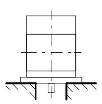
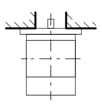
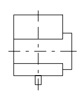
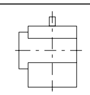
Single-phase alternating current:
P_{ab} = U ·   · cos   · ϱ

Three-phase current:
P_{ab} = 1.73 · U ·   · cos   · ϱ



1) P_{ab} = mechanical output power on the motor shaft
P_{zu} = absorbed electric power

Types of construction and mounting arrangements of rotating electrical machinery [**Extract from DIN/IEC 34, Part 7 (4.83)**]						
Machines with end shields, horizontal arrangement						
Design		Explanation				
Sym-bol	Figure	Bearings	Stator (Housing)	Shaft	General design	Design/Explanation Fastening or Installation
B3		2 end shields	with feet	free shaft end	–	installation on substructure
B5		2 end shields	without feet	free shaft end	mounting flange close to bearing, access from housing side	flanged
B6		2 end shields	with feet	free shaft end	design B3, if necessary end shields turned through -90°	wall fastening, feet on LH side when looking at input side
B7		2 end shields	with feet	free shaft end	design B3, if necessary end shields turned through 90°	wall fastening, feet on RH side when looking at input side
B8		2 end shields	with feet	free shaft end	design B3, if necessary end shields turned through 180°	fastening on ceiling
B 35		2 end shields	with feet	free shaft end	mounting flange close to bearing, access from housing side	installation on substructure with additional flange

Machines with end shields, vertical arrangement						
Design		Explanation				
Sym-bol	Figure	Bearings	Stator (Housing)	Shaft	General design	Design/Explanation Fastening or Installation
V 1		2 end shields	without feet	free shaft end at the bottom	mounting flange close to bearing on input side, access from housing side	flanged at the bottom
V 3		2 end shields	without feet	free shaft end at the top	mounting flange close to bearing on input side, access from housing side	flanged at the top
V 5		2 end shields	with feet	free shaft end at the bottom	–	fastening to wall or on substructure
V 6		2 end shields	with feet	free shaft end at the top	–	fastening to wall or on substructure

Types of protection for electrical equipment
[**Extract from DIN 40050 (7.80)**]

Example of designation Type of protection DIN 40050 IP 4 4

Designation
DIN number
Code letters
First type number
Second type number

An enclosure with this designation is protected against the ingress of solid foreign bodies having a diameter above 1 mm and of splashing water.

Degrees of protection for protection against contact and foreign bodies (first type number)

First type number	Degree of protection (Protection against contact and foreign bodies)
0	No special protection
1	Protection against the ingress of solid foreign bodies having a diameter above 50 mm (large foreign bodies) 1) No protection against intended access, e.g. by hand, however, protection of persons against contact with live parts
2	Protection against the ingress of solid foreign bodies having a diameter above 12 mm (medium-sized foreign bodies) 1) Keeping away of fingers or similar objects
3	Protection against the ingress of solid foreign bodies having a diameter above 2.5 mm (small foreign bodies) 1) 2) Keeping away tools, wires or similar objects having a thickness above 2.5 mm
4	Protection against the ingress of solid foreign bodies having a diameter above 1 mm (grain sized foreign bodies) 1) 2) Keeping away tools, wires or similar objects having a thickness above 1 mm
5	Protection against harmful dust covers. The ingress of dust is not entirely prevented, however, dust may not enter to such an amount that operation of the equipment is impaired (dustproof). 3) Complete protection against contact
6	Protection against the ingress of dust (dust-tight) Complete protection against contact

1) For equipment with degrees of protection from 1 to 4, uniformly or non-uniformly shaped foreign bodies with three dimensions perpendicular to each other and above the corresponding diameter values are prevented from ingress.
2) For degrees of protection 3 and 4, the respective expert commission is responsible for the application of this table for equipment with drain holes or cooling air slots.
3) For degree of protection 5, the respective expert commission is responsible for the application of this table for equipment with drain holes.

Types of protection for electrical equipment
[**Extract from DIN 40050 (7.80)**]

Example of designation	Type of protection	DIN 40050	IP	4	4
Designation	Type of protection				
DIN number		DIN 40050			
Code letters			IP		
First type number				4	
Second type number					4

An enclosure with this designation is protected against the ingress of solid foreign bodies having a diameter above 1 mm and of splashing water.

Degrees of protection for protection against water (second type number)

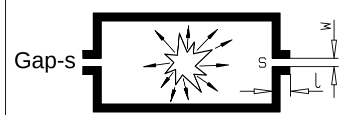
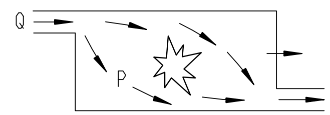
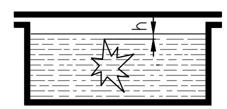
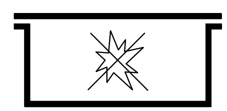
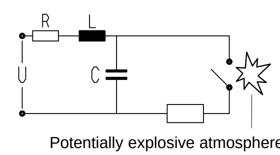
Second type number	Degree of protection (Protection against water)
0	No special protection
1	Protection against dripping water falling vertically. It may not have any harmful effect (dripping water).
2	Protection against dripping water falling vertically. It may not have any harmful effect on equipment (enclosure) inclined by up to 15° relative to its normal position (diagonally falling dripping water).
3	Protection against water falling at any angle up to 60° relative to the perpendicular. It may not have any harmful effect (spraying water).
4	Protection against water spraying on the equipment (enclosure) from all directions. It may not have any harmful effect (splashing water).
5	Protection against a water jet from a nozzle which is directed on the equipment (enclosure) from all directions. It may not have any harmful effect (hose-directed water).
6	Protection against heavy sea or strong water jet. No harmful quantities of water may enter the equipment (enclosure) (flooding).
7	Protection against water if the equipment (enclosure) is immersed under determined pressure and time conditions. No harmful quantities of water may enter the equipment (enclosure) (immersion).
8	The equipment (enclosure) is suitable for permanent submersion under conditions to be described by the manufacturer (submersion). 1)

1) This degree of protection is normally for air-tight enclosed equipment. For certain equipment, however, water may enter provided that it has no harmful effect.

Explosion protection of electrical switchgear
Example of designation / Type of protection [**Extract from DIN EN 50014 ... 50020**]

Example of designation	Ex	EEx	d	II B	T 3
Symbol for equipment certified by an EC testing authority	Ex				
Symbol for equipment made according to European Standards		EEx			
Type of protection			d		
Explosion group				II B	
Temperature class					T 3

Types of protection

Type of protection	Symbol	Scheme	Application
Flameproof enclosure	d		Heavy-current engineering (commutator) motors, transformers, switchgear, lighting fittings, and other spark generating parts
Pressurized enclosure	p		Especially for large apparata, switchgears, motors, generators
Oil-immersion enclosure	o		Switchgears, transformers
Sand-filled enclosure	q		Capacitors
Increased safety	e		Squirrel-cage motors, terminal and junction boxes, lighting fittings, current transformers, measuring and control devices
Intrinsic safety	i		Low-voltage engineering: measuring and control devices (electrical equipment and circuits)

Explosion protection of electrical switchgear
Designation of electrical equipment / Classification of areas acc. to gases and vapours
[**Extract from DIN EN 50014 ... 50020**]

Designation of electrical equipment

Designation acc. to	VDE 0170/0171/2.61	EN 50014 ... 50020
Firedamp protection Explosion protection	Sch Ex	EEx..I EEx..II

Classification according to gases and vapours		Explosion class	Explosion group
For flame proof enclosures: maximum width of gap	For intrinsically safe circuits: minimum ignition current ratio referred to methane 1)		
> 0.9 mm	> 0.8 mm	1	A
≥ 0.5 - 0.9mm	≥ 0.45 - 0.8mm	2	B
< 0.5 mm	< 0.45 mm	3a ... 3n	C

Ignition temperature of gases and vapours in °C	Ignition group	Ignition temperature °C	Permissible limiting temperature °C	Temperature class	Ignition temperature °C	Maximum surface temperature °C
	G1>	450	360	T1	> 450	450
	G2>	300...450	240	T2	> 300	300
	G3>	200...300	160	T3	> 200	200
	G4>	135...200	110	T4	> 135	135
	G5 from	100...135	80	T5	> 100	100
				T6	> 85	85

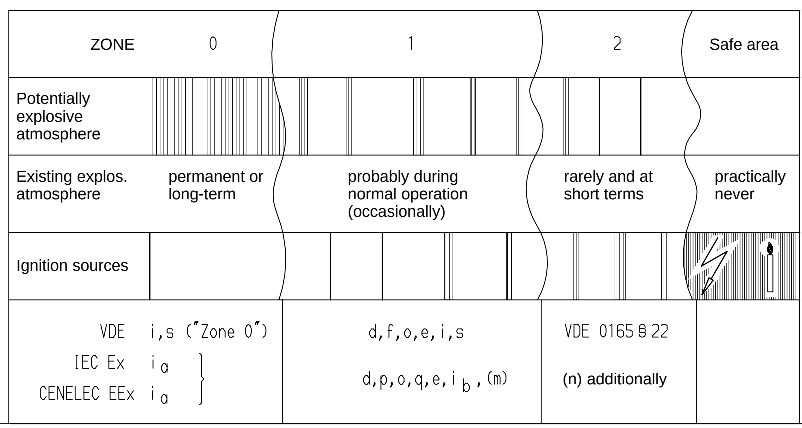
1) For definition, see EN 50014, Annex A

Classification of areas according to gases and vapours

Zone 0
Areas with **permanent** or **long-term** potentially explosive atmospheres.

Zone 1
Areas where potentially explosive atmospheres are expected to occur **occasionally.**

Zone 2
Areas where potentially explosive atmospheres are expected to occur only **rarely** and then only for **short periods.**



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Materials

Conversion of Fatigue Strength Values of Miscellaneous Materials

Conversion of fatigue strength values of miscellaneous materials								
Material	Tension [3]		Bending [1]			Torsion [1]		
	σ_W	σ_{Sch}	σ_{bW}	σ_{bSch}	σ_{bF}	τ_{tW}	τ_{tSch}	τ_F
Structural steel	0.45 R_m	1.3 σ_W	0.49 R_m	1.5 σ_{bW}	1.5 R_e	0.35 R_m	1.1 τ_{tW}	0.7 R_e
Quenched and temper-ed steel	0.41 R_m	1.7 σ_W	0.44 R_m	1.7 σ_{bW}	1.4 R_e	0.30 R_m	1.6τ_{tW}	0.7 R_e
Case harden-ing steel [2]	0.40 R_m	1.6 σ_W	0.41 R_m	1.7 σ_{bW}	1.4 R_e	0.30 R_m	1.4τ_{tW}	0.7 R_e
Grey cast iron	0.25 R_m	1.6 σW	0.37 R_m	1.8 σ_{bW}	–	0.36 R_m	1.6τ_{tW}	–
Light metal	0.30 R_m	–	0.40 R_m	–	–	0.25 R_m	–	–

1) For polished round section test piece of about 10 mm diameter.
2) Case-hardened; determined on round section test piece of about 30 mm diameter. R_m and R_e of core material.
3) For compression, σ_{Sch} is larger, e.g. for spring steel $\sigma_{dSch} \approx 1.3 \cdot \sigma_{Sch}$
For grey cast iron $\sigma_{dSch} \approx 3 \cdot \sigma_{Sch}$

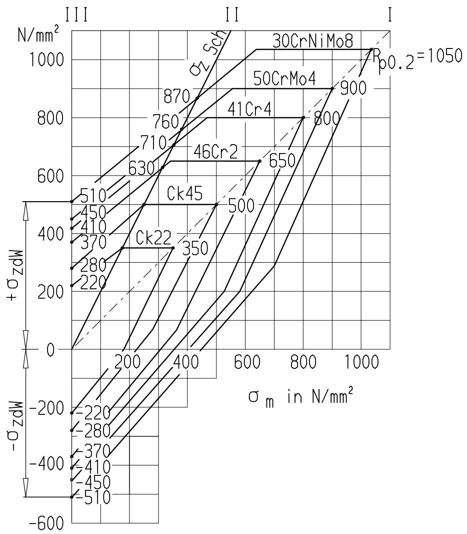
Ultimate stress values		Type of load
R_m	Tensile strength	Tension
R_e	Yield point	Tension
σ_W	Fatigue strength under alternating stresses	Tension
σ_{Sch}	Fatigue strength under fluctuating stresses	Tension
σ_{bW}	Fatigue strength under alternating stresses	Bending
σ_{bSch}	Fatigue strength under fluctuating stresses	Bending
σ_{bF}	Yield point	Bending
τ_{tW}	Fatigue strength under alternating stresses	Torsion
τ_{tSch}	Fatigue strength under fluctuating stresses	Torsion
τ_{tF}	Yield point	Torsion

Materials

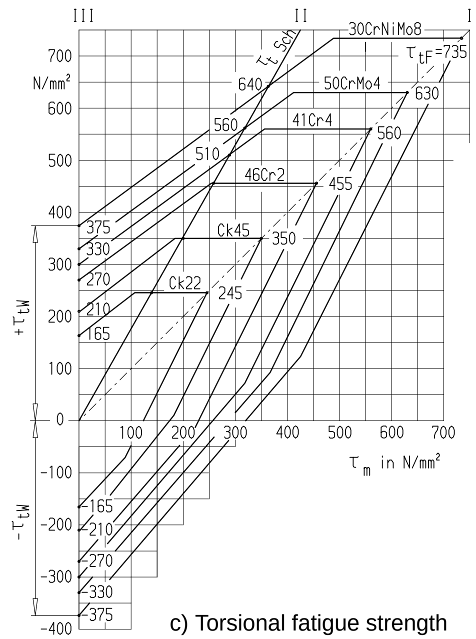
Mechanical Properties of Quenched and Tempered Steels

Quenched and tempered steels [Extract from DIN 17200 (3.87)] Mechanical properties of steels in quenched and tempered condition (Code letter V)											
Steel grade		Diameter									
		up to 16 mm		above 16 up to 40 mm		above 40 up to 100 mm		above 100 up to 160 mm		above 160 up to 250 mm	
Symbol	Mate-rial no.	Yield point (0.2 Gr) N/mm² min. R_e, $R_{p\ 0.2}$	Tensile strength N/mm² R_m	Yield point (0.2 Gr) N/mm² min. R_e, $R_{p\ 0.2}$	Tensile strength N/mm² R_m	Yield point (0.2 Gr) N/mm² min. R_e, $R_{p\ 0.2}$	Tensile strength N/mm² R_m	Yield point (0.2 Gr) N/mm² min. R_e, $R_{p\ 0.2}$	Tensile strength N/mm² R_m	Yield point (0.2 Gr) N/mm² min. R_e, $R_{p\ 0.2}$	Tensile strength N/mm² R_m
C 22	1.0402	350	550– 700	300	500– 650	–	–	–	–	–	–
C 35	1.0501	430	630– 780	370	600– 750	320	550– 700	–	–	–	–
C 45	1.0503	500	700– 850	430	650– 800	370	630– 780	–	–	–	–
C 55	1.0535	550	800– 950	500	750– 900	430	700– 850	–	–	–	–
C 60	1.0601	580	850–1000	520	800– 950	450	750– 900	–	–	–	–
Ck 22	1.1151	350	550– 700	300	500– 650	–	–	–	–	–	–
Ck 35	1.1181	430	630– 780	370	600– 750	320	550– 700	–	–	–	–
Cm 35	1.1180	430	630– 780	370	600– 750	320	550– 700	–	–	–	–
Ck 45	1.1191	500	700– 850	430	650– 800	370	630– 780	–	–	–	–
Cm 45	1.1201	500	700– 850	430	650– 800	370	630– 780	–	–	–	–
Ck 55	1.1203	550	800– 950	500	750– 900	430	700– 850	–	–	–	–
Cm 55	1.1209	550	800– 950	500	750– 900	430	700– 850	–	–	–	–
Ck 60	1.1221	580	850–1000	520	800– 950	450	750– 900	–	–	–	–
Cm 60	1.1223	580	850–1000	520	800– 950	450	750– 900	–	–		
28 Mn 6	1.1170	590	780– 930	490	690– 840	440	640– 790	–	–	–	–
38 Cr 2	1.7003	550	800– 950	450	700– 850	350	600– 750	–	–	–	–
46 Cr 2	1.7006	650	900–1100	550	800– 950	400	650– 800	–	–	–	–
34 Cr 4	1.7033	700	900–1100	590	800– 950	460	700– 850	–	–	–	–
34 Cr S4	1.7037	700	900–1100	590	800– 950	460	700– 850	–	–	–	–
37 Cr 4	1.7034	750	950–1150	630	850–1000	510	750– 900	–	–	–	–
37 Cr S4	1.7038	750	950–1150	630	850–1000	510	750– 900	–	–	–	–
41 Cr 4	1.7035	800	1000–1200	660	900–1100	560	800– 950	–	–	–	–
41 Cr S4	1.7039	800	1000–1200	660	900–1100	560	800– 950	–	–	–	–
25 CrMo 4	1.7218	700	900–1100	600	800– 950	450	700– 850	400	650– 800	–	–
34 CrMo 4	1.7220	800	1000–1200	650	900–1100	550	800– 950	500	750– 900	450	700– 850
34 CrMo S4	1.7226	800	1000–1200	650	900–1100	550	800– 950	500	750– 900	450	700– 850
42 CrMo 4	1.7225	900	1100–1300	750	1000–1200	650	900–1100	550	800– 950	500	750– 900
42 CrMo S4	1.7227	900	1100–1300	750	1000–1200	650	900–1100	550	800– 950	500	750– 900
50 CrMo 4	1.7228	900	1100–1300	780	1000–1200	700	900–1100	650	850–1000	550	800– 950
36 CrNiMo 4	1.6511	900	1100–1300	800	1000–1200	700	900–1100	600	800– 950	550	750– 900
34 CrNiMo 6	1.6582	1000	1200–1400	900	1100–1300	800	1000–1200	700	900–1100	600	800– 950
30 CrNiMo 6	1.6580	1050	1250–1450	1050	1250–1450	900	1100–1300	800	1000–1200	700	900–1100
50 CrV 4	1.8159	900	1100–1300	800	1000–1200	700	900–1100	650	850–1000	600	800– 950
30 CrMoV9	1.7707	1050	1250–1450	1020	1200–1450	900	1100–1300	800	1000–1200	700	900–1100

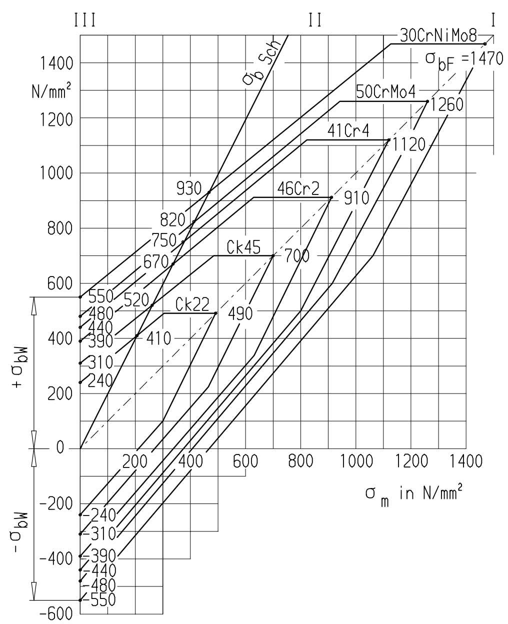
Fatigue strength diagrams of quenched and tempered steels, DIN 17200
(in quenched and tempered condition, test piece diameter d = 10 mm)



a) Tension/compression fatigue strength



c) Torsional fatigue strength



b) Bending fatigue strength

Quenched and tempered steels not illustrated may be used as follows:

34 CrNiMo 6	like 30 CrNiMo 8
30 CrMoV 4	like 30 CrNiMo 8
42 CrMo 4	like 50 CrMo 4
36 CrNiMo 4	like 50 CrMo 4
50 CrV 4	like 50 CrMo 4
34 CrMo 4	like 41 Cr 4
28 Cr 4	like 46 Cr 2
C 45	like Ck 45
C 22	like Ck 22

C 60 and C 50 lie approximately between Ck 45 and 46 Cr 2.

C 40, 32 Cr 2, C 35 , C 30 and C 25 lie approximately between Ck 22 and Ck 45.

Loading type I: static

Loading type II: dynamic

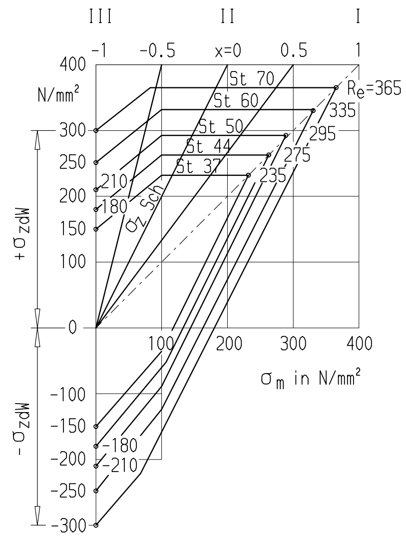
Loading type III: alternating

General-purpose structural steels [Extract from DIN 17100 (1.80)]

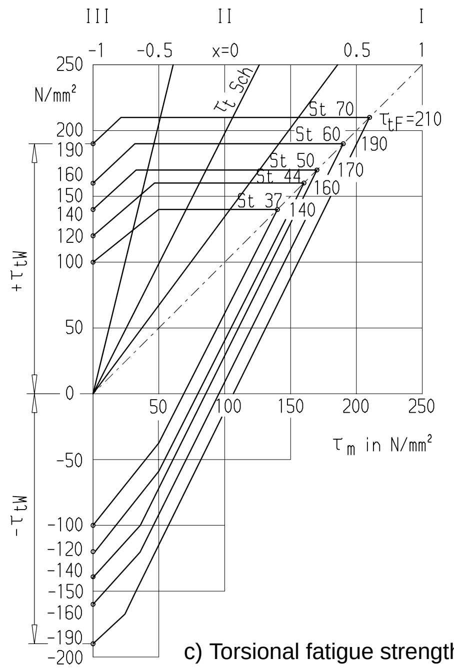
Steel grade Symbol	Material no.	Treatment condition 1)	Similar steel grades EURON. 25	Tensile strength R_m in N/mm² for product thickness in mm: <3	≥3 ≤100	>100	Upper yield point R_{eH} in N/mm² (minimum) for product thickness in mm: ≤16	>16 ≤40	>40 ≤63	>63 ≤80	>80 ≤100	>100
St 33	1.0035	U, N	Fe 310-0	310... 540	290	To be agreed upon	185	175 2)	–	–	–	To be agreed upon
St 37-2	1.0037	U, N	–	360... 510	340... 470		235	225	215	205	195	
U St 37-2	1.0036	U, N	Fe 360-BFU									
R St 37-2	1.0038	U, N,	Fe 360-BFN				235	225	215	215	215	
St 37-3	1.0116	U N	Fe 360-C Fe 360-D									
St 44-2	1.0044	U, N	Fe 430-B	430... 580	410... 540		275	265	255	245	235	
St 44-3 St 44-3	1.0144	U N	Fe 430-C Fe 430-D									
St 52-3	1.0570	U N	Fe 510-C Fe 510-D	510... 680	490... 630		355	345	335	325	315	
St 50-2	1.0050	U, N	Fe 490-2	490... 660	470... 610		295	285	275	265	255	
St 60-2	1.0060	U, N	Fe 590-2	590... 770	570... 710		335	325	315	305	295	
St 70-2	1.0070	U, N	Fe 690-2	690... 900	670... 830		365	355	345	335	325	

1) N normalized; U hot-rolled, untreated
2) This value applies to thicknesses up to 25 mm only

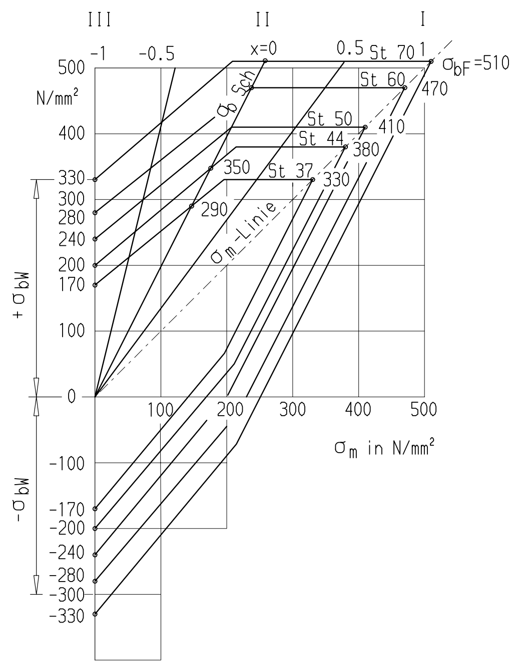
Fatigue strength diagrams of general-purpose structural steels, DIN 17100
(test piece diameter d = 10 mm)



a) Tension/compression fatigue strength



c) Torsional fatigue strength



b) Bending fatigue strength

Loading type I: static
Loading type II: dynamic
Loading type III: alternating

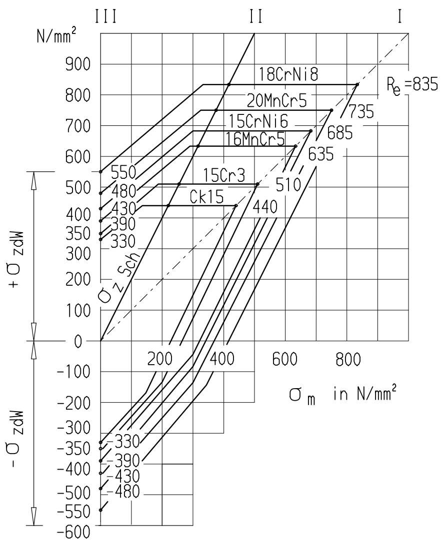
Case hardening steels; Quality specifications to DIN 17210 (12.69) from SI tables (2.1974) of VDEh

Steel grade		Treatment condition 1)	For dia. 11		For dia. 30		For dia. 63	
Symbol	Material no.		Yield point R_e N/mm² min.	Tensile strength R_m N/mm²	Yield point R_e N/mm² min.	Tensile strength R_m N/mm²	Yield point R_e N/mm² min.	Tensile strength R_m N/mm²
C 10 Ck 10	1.0301 1.1121	For details, see DIN 17210	390 390	640– 790 640– 790	295 295	490– 640 490– 640	– –	– –
C 15 Ck 15 Cm 15	1.0401 1.1141 1.1140		440 440 440	740– 890 740– 890 740– 890	355 355 355	590– 790 590– 790 590– 790	– – –	– – –
15 Cr 13	1.7015		510	780–1030	440	690– 890	–	
16 MnCr 5 16 MnCrS 5 20 MnCr 5 20 MnCrS5	1.7131 1.7139 1.7147 1.7149		635 635 735 735	880–1180 880–1180 1080–1380 1080–1380	590 590 685 685	780–1080 780–1080 980–1280 980–1280	440 440 540 540	640– 940 640– 940 780–1080 780–1080
20 MoCr 4 20 MoCrS 4 25 MoCrS4 25 MoCrS 4	1.7321 1.7323 1.7325 1.7326		635 635 735 735	880–1180 880–1180 1080–1380 1080–1380	590 590 685 685	780–1080 780–1080 980–1280 980–1280	– – – –	– – – –
15 CrNi 6 18 CrNi 8	1.5919 1.5920		685 835	960–1280 1230–1480	635 785	880–1180 1180–1430	540 685	780–1080 1080–1330
17 CrNiMo 6	1.6587		835	1180–1430	785	1080–1330	685	980–1280

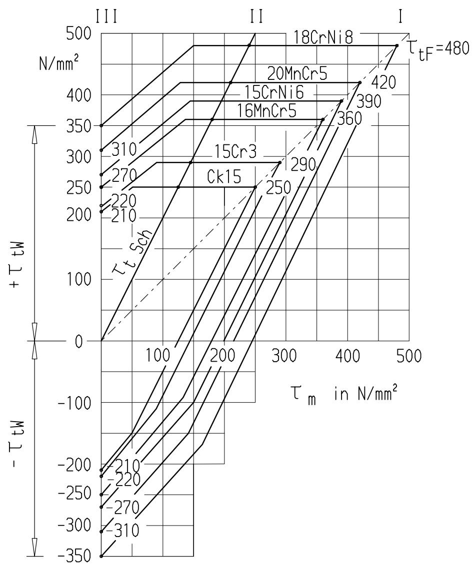
1) Dependent on treatment, the Brinell hardness is different.

Treatment condition	Meaning
C	treated for shearing load
G	soft annealed
BF	treated for strength
BG	treated for ferrite/pearlite structure

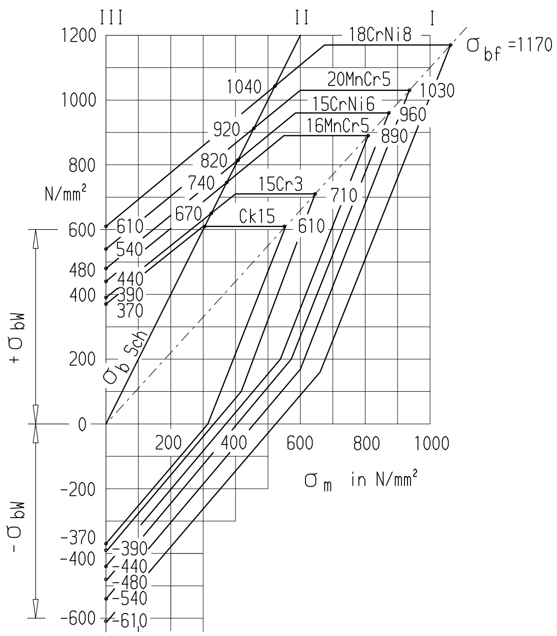
Fatigue strength diagrams of case hardening steels, DIN 17210
(Core strength after case hardening, test piece diameter d = 10 mm)



a) Tension/compression fatigue strength



c) Torsional fatigue strength



b) Bending fatigue strength

Case hardening steels not illustrated may be used as follows:

25 MoCr 4 like 20 MnCr 5
17 CrNiMo 6 like 18 CrNi 8

Loading type I: static
Loading type II: dynamic
Loading type III: alternating

Cold rolled steel strips for springs [Extract from DIN 17222 (8.79)]

Steel grade		Comparable grade acc. to EURONORM 132	Degree of conformity 1)	Tensile strength R_m 2)
Symbol	Material no.			N/mm² maximum
C 55 Ck 55	1.0535 1.1203	1 CS 55 2 CS 55	● ●	610
C 60 Ck 60	1.0601 1.1221	1 CS 60 2 CS 60	● ●	620
C 67 Ck 67	1.0603 1.1231	1 CS 67 2 CS 67	● ●	640
C 75 CK75	1.0605 1.1248	1 CS 75 2 CS 75	● ●	640
Ck 85 CK 101	1.1269 1.1274	2 CS 85 CS 100	● ●	670 690
55 Si 7	1.0904	–	–	740
71 Si 7	1.5029	–	–	800
67 SiCr 5	1.7103	67 SiCr 5	○	800
50 CrV 4	1.8159	50 CrV 4	●	740

1) ● = minor deviations
○ = substantial deviations

2) R_m for cold rolled and soft-annealed condition; for strip thicknesses up to 3 mm

Cast steels for general engineering purposes [Extract from DIN 1681 (6.85)]

Cast steel grade		Yield point R_e, $R_{p\,0.2}$	Tensile strength R_m	Notched bar impact work (ISO-V-notch specimens) A_v	
				≤ 30 mm	> 30 mm
Symbol	Material no.	N/mm² min.	N/mm² min.	Mean value 1) J min.	
GS-38	1.0420	200	380	35	35
GS-45	1.0446	230	450	27	27
GS-52	1.0552	260	520	27	22
GS-60	1.0558	300	600	27	20

The mechanical properties apply to specimens which are taken from test pieces with thicknesses up to 100 mm. Furthermore, the yield point values also apply to the casting itself, in so far as the wall thickness is ≤ 100 mm.

1) Determined from three individual values each.

Round steel wire for springs [Extract from DIN 17223, Part 1 (12.84)]				
Grade of wire	A	B	C	D
Diameter of wire mm	Tensile strength R_m in N/mm²			
0.07	–	–	–	2800–3100
0.3	–	2370–2650	–	2660–2940
1	1720–1970	1980–2220	–	2230–2470
2	1520–1750	1760–1970	1980–2200	1980–2200
3	1410–1620	1630–1830	1840–2040	1840–2040
4	1320–1520	1530–1730	1740–1930	1740–1930
5	1260–1450	1460–1650	1660–1840	1660–1840
6	1210–1390	1400–1580	1590–1770	1590–1770
7	1160–1340	1350–1530	1540–1710	1540–1710
8	1120–1300	1310–1480	1490–1660	1490–1660
9	1090–1260	1270–1440	1450–1610	1450–1610
10	1060–1230	1240–1400	1410–1570	1410–1570
11	–	1210–1370	1380–1530	1380–1530
12	–	1180–1340	1350–1500	1350–1500
13	–	1160–1310	1320–1470	1320–1470
14	–	1130–1280	1290–1440	1290–1440
15	–	1110–1260	1270–1410	1270–1410
16	–	1090–1230	1240–1390	1240–1390
17	–	1070–1210	1220–1360	1220–1360
18	–	1050–1190	1200–1340	1200–1340
19	–	1030–1170	1180–1320	1180–1320
20	–	1020–1150	1160–1300	1160–1300

Lamellar graphite cast iron [Extract from DIN 1691 (5.85)]						
Grade Material		Wall thicknesses in mm		Tensile strength 1) R_m	Brinell hardness 1)	Compres-sive strength 2) σ_{dB}
Symbol	Number	above	up to	N/mm²	HB 30	N/mm²
GG-10	0.6010	5	40	min. 100 2)	–	–
GG-15	0.6015	10 20 40 80	20 40 80 150	130 110 95 80	225 205 – –	600
GG-20	0.6020	10 20 40 80	20 40 80 150	180 155 130 115	250 235 – –	720
GG-25	0.6025	10 20 40 80	20 40 80 150	225 195 170 155	265 250 – –	840
GG-30	0.6030	10 20 40 80	20 40 80 150	270 240 210 195	285 265 – –	960
GG-35	0.6035	10 20 40 80	20 40 80 150	315 280 250 225	285 275 – –	1080

The values apply to castings which are made in sand moulds or moulds with comparable heat diffusibility.

1) These values are reference values.
2) Values in the separately cast test piece with 30 mm diameter of the unfinished casting.

Nodular graphite cast iron [Extract from DIN 1693, Part 2 (10.77)]						
Properties in cast-on test pieces						
Grade Material		Wall thickness of casting		Thickness of cast-on test piece	Tensile strength R_m	0.2% proof stress $Rp_{0.2}$
Symbol	Number	mm	mm	mm	N/mm²	N/mm²
GGG-40.3	0.7043	from 30 above 60	up to 60 up to 200	40 70	390 370	250 240
GGG-40	0.7040	from 30 above 60	up to 60 up to 200	40 70	390 370	250 240
GGG-50	0.7050	from 30 above 60	up to 60 up to 200	40 70	450 420	300 290
GGG-60	0.7060	from 30 above 60	up to 60 up to 200	40 70	600 550	360 340
GGG-70	0.7070	from 30 above 60	up to 60 up to 200	40 70	700 650	400 380

Copper-tin- and copper-zinc-tin casting alloys [Extract from DIN 1705 (11.81)]				
Material		Condition on delivery	0.2% proof stress 1) $R_{p0.2}$	Tensile strength 1) R_m
Symbol	Number		min. in N/mm^2	min. in N/mm^2
G-CuSn 12 GZ-CuSn 12 GC-CuSn12	2.1052.01 2.1052.03 2.1052.04	Sand-mould cast iron Centrifugally cast iron Continuously cast iron	140 150 140	260 280 280
G-CuSn 12 Ni GZ-CuSn 12 Ni GC-CuSn 12 Ni	2.1060.01 2.1060.03 2.1060.04	Sand-mould cast iron Centrifugally cast iron Continuously cast iron	160 180 170	280 300 300
G-CuSn 12 Pb GZ-CuSn 12 Pb GC-CuSn 12 Pb	2.1061.01 2.1061.03 2.1061.04	Sand-mould cast iron Centrifugally cast iron Continuously cast iron	140 150 140	260 280 280
G-CuSn 10	2.1050.01	Sand-mould cast iron	130	270
G-CuSn 10 Zn	2,1086.01	Sand-mould cast iron	130	260
G-CuSn 7 ZnPb GZ-CuSn 7 ZnPb GC-CuSn 7 ZnPb	2.1090.01 2.1090.03 2.1090.04	Sand-mould cast iron Centrifugally cast iron Continuously cast iron	120 130 120	240 270 270
G-CuSn 6 ZnNi	2.1093.01	Sand-mould cast iron	140	270
G-CuSn 5 ZnPb	2.1096.01	Sand-mould cast iron	90	220
G-CuSn 2 ZnPb	2.1098.01	Sand-mould cast iron	90	210

1) Material properties in the test bar

Copper-aluminium casting alloys [Extract from DIN 1714 (11.81)]				
Material		Condition on delivery	0.2% proof stress 1) $R_{p0.2}$	Tensile strength 1) R_m
Symbol	Number		min. in N/mm^2	min. in N/mm^2
G-CuAl 10 Fe GK-CuAl 10 Fe GZ-CuAl 10 Fe	2.0940.01 2.0940.02 2.0940.03	Sand-mould cast iron Chilled casting Centrifugally cast iron	180 200 200	500 550 550
G-CuAl 9 Ni GK-CuAl 9 Ni GZ-CuAl 9 Ni	2.0970.01 2.0970.02 2.0970.03	Sand-mould cast iron Chilled casting Centrifugally cast iron	200 230 250	500 530 600
G-CuAl 10 Ni GK-CuAl 10 Ni GZ-CuAl 10 Ni GC-CuAl 10 Ni	2.0975.01 2.0975.02 2.0975.03 2.0975.04	Sand-mould cast iron Chilled casting Centrifugally cast iron Continuously cast iron	270 300 300 300	600 600 700 700
G-CuAl 11 Ni GK-CuAl 11 Ni GZ-CuAl 11 Ni	2.0980.01 2.0980.02 2.0980.03	Sand-mould cast iron Chilled casting Centrifugally cast iron	320 400 400	680 680 750
G-CuAl 8 Mn GK-CuAl 8 Mn	2.0962.01 2.0962.02	Sand-mould cast iron Chilled casting	180 200	440 450

1) Material properties in the test bar

Aluminium casting alloys [Extract from DIN 1725 (2.86)]				
Material		Casting method and condition on delivery	0.2 proof stress $R_{p0.2}$	Tensile strength R_m
Symbol	Number		in N/mm^2	in N/mm^2
G-AlSi 12	3.2581.01	Sand-mould cast iron as cast	70 up to 100	150 up to 200
G-AlSi 12 g	3.2581.44	Sand-mould cast iron annealed and quenched	70 up to 100	150 up to 200
GK-AlSi 12	3.2581.02	Chilled casting as cast	80 up to 110	170 up to 230
GK-AlSi 12 g	3.2581.45	Chilled casting annealed and quenched	80 up to 110	170 up to 230
G-AlSi 10 Mg	3.2381.01	Sand-mould cast iron as cast	80 up to 110	160 up to 210
G-AlSi 10 Mg wa	3.2381.61	Sand-mould cast iron temper-hardened	180 up to 260	220 up to 320
GK-AlSi 10 Mg	3.2381.02	Chilled casting as cast	90 up to 120	180 up to 240
GK-AlSi 10 Mg wa	3.2381.62	Chilled casting temper-hardened	210 up to 280	240 up to 320
G-AlSi 11	3.2211.01	Sand-mould cast iron as cast	70 up to 100	150 up to 200
G-AlSi 11 g	3.2211.81	annealed	70 up to 100	150 up to 200
GK-AlSi 11	3.2211.02	Chilled casting as cast	80 up to 110	170 up to 230
GK-AlSi 11g	3.2211.82	annealed	80 up to 110	170 up to 230
G-AlSi 7 Mg wa	3.2371.61	Sand-mould cast iron temper-hardened	190 up to 240	230 up to 310
GK-AlSi 7 Mg wa	3.2371.62	Chilled casting temper-hardened	200 up to 280	250 up to 340
GF-AlSi 7 Mg wa	3.2371.63	High-quality casting temper-hardened	200 up to 260	260 up to 320
G-AlMg 3 Si	3.3241.01	Sand-mould cast iron as cast	80 up to 100	140 up to 190
G-AlMg 3 Si wa	3.3241.61	Sand-mould cast iron temper-hardened	120 up to 160	200 up to 280
GK-AlMg 3 Si	3.3241.02	Chilled casting as cast	80 up to 100	150 up to 200
GK-AlMg 3 Si wa	3.3241.62	Chilled casting temper-hardened	120 up to 180	220 up to 300
GF-AlMg 3 Si wa	3.3241.63	Chilled casting temper-hardened	120 up to 160	200 up to 280

Lead and Tin Casting Alloys for Babbit Sleeve Bearings

Lead and tin casting alloys for babbit sleeve bearings [Extract from DIN ISO 4381 (10.82)]							
Grade Material		Brinell hardness 1) HB 10/250/180			0.2% proof stress 1) $R_{p\,0.2}$ in N/mm²		
Symbol	Number	20 °C	50 °C	120 °C	20 °C	50 °C	100 °C
PbSb 15 SnAs	2.3390	18	15	14	39	37	25
PbSb 15 Sn 10	2.3391	21	16	14	43	32	30
PbSb 14 Sn 9 CuAs	2.3392	22	22	16	46	39	27
PbSb 10 Sn 6	2.3393	16	16	14	39	32	27
SnSb 12 Cu 6 Pb	2.3790	25	20	12	61	60	36
SnSb 8 Cu 4	2.3791	22	17	11	47	44	27
SnSb 8 Cu 4 Cd	2.3792	28	25	19	62	44	30

1) Material properties in the test bar

Comparison of Tensile Strength and Miscellaneous Hardness Values

Tensile strength N/mm²	Vickers hardness (F>98N)	Brinell hardness 2) $0.102 = \frac{F}{D^2} + 30 \frac{N}{mm^2}$	Rockwell hardness HRB	HRC	HRA	HRD 1)
255	80	76.0				
270	85	80.7	41.0			
285	90	85.5	48.0			
305	95	90.2	52.0			
320	100	95.0	56.2			
335	105	99.8				
350	110	105	62.3			
370	115	109				
385	120	114	66.7			
400	125	119				
415	130	124	71.2			
430	135	128				
450	140	133	75.0			
465	145	138				
480	150	143	78.7			
495	155	147				
510	160	152	81.7			
530	165	156				
545	170	162	85.0			
560	175	166				
575	180	171	87.1			
595	185	176				
610	190	181	89.5			
625	195	185				
640	200	190	91.5			
660	205	195	92.5			
675	210	199	93.5			
690	215	204	94.0			
705	220	209	95.0			
720	225	214	96.0			
740	230	219	96.7			
755	235	223				
770	240	228	98.1	20.3	60.7	40.3
785	245	233		21.3	61.2	41.1
800	250	238	99.5	22.2	61.6	41.7
820	255	242		23.1	62.0	42.2
835	260	247	(101)	24.0	62.4	43.1
850	265	252		24.8	62.7	43.7
865	270	257	(102)	25.6	63.1	44.3
880	275	261		26.4	63.5	44.9
900	280	266	(104)	27.1	63.8	45.3
915	285	271		27.8	64.2	46.0
930	290	276	(105)	28.5	64.5	46.5
950	295	280		29.2	64.8	47.1
965	300	285		29.8	65.2	47.5
995	310	295		31.0	65.8	48.4
1030	320	304		32.3	66.4	49.4
1060	330	314		33.3	67.0	50.2
1095	340	323		34.4	67.6	51.1
1125	350	333		35.5	68.1	51.9

Tensile strength N/mm²	Vickers hardness (F>98N)	Brinell hardness 2) $0.102 = \frac{F}{D^2} + 30 \frac{N}{mm^2}$	Rockwell hardness HRC	HRA	HRD 1)
1155	360	342	36.6	68.7	52.8
1190	370	352	37.7	69.2	53.6
1220	380	361	38.8	69.8	54.4
1255	390	371	39.8	70.3	55.3
1290	400	380	40.8	70.8	56.0
1320	410	390	41.8	71.4	56.8
1350	420	399	42.7	71.8	57.5
1385	430	409	43.6	72.3	58.2
1420	440	418	44.5	72.8	58.8
1455	450	428	45.3	73.3	59.4
1485	460	437	46.1	73.6	60.1
1520	470	447	46.9	74.1	60.7
1555	480	(456)	47.7	74.5	61.3
1595	490	(466)	48.4	74.9	61.6
1630	500	(475)	49.1	75.3	62.2
1665	510	(485)	49.8	75.7	62.9
1700	520	(494)	50.5	76.1	63.5
1740	530	(504)	51.1	76.4	63.9
1775	540	(513)	51.7	76.7	64.5
1810	550	(523)	52.3	77.0	64.8
1845	560	(532)	53.0	77.4	65.4
1880	570	(542)	53.6	77.8	65.8
1920	580	(551)	54.1	78.0	66.2
1955	590	(561)	54.7	78.4	66.7
1995	600	(570)	55.2	78.6	67.0
2030	610	(580)	55.7	78.9	67.5
2070	620	(589)	56.3	79.2	67.9
2105	630	(599)	56.8	79.5	68.3
2145	640	(608)	57.3	79.8	68.7
2180	650	(618)	57.8	80.0	69.0
	660		58.3	80.3	69.4
	670		58.8	80.6	69.8
	680		59.2	80.8	70.1
	690		59.7	81.1	70.5
	700		60.1	81.3	70.8
	720		61.0	81.8	71.5
	740		61.8	82.2	72.1
	760		62.5	82.6	72.6
	780		63.3	83.0	73.3
	800		64.0	83.4	73.8
	820		64.7	83.8	74.3
	840		65.3	84.1	74.8
	860		65.9	84.4	75.3
	880		66.4	84.7	75.7
	900		67.0	85.0	76.1
	920		67.5	85.3	76.5
	940		68.0	85.6	76.9

The figures in brackets are hardness values outside the domain of definition of standard hardness test methods which, however, in practice are frequently used as approximate values. Furthermore, the Brinell hardness values in brackets apply only if the test was carried out with a carbide ball.

1) Internationally usual, e.g. ASTM E 18-74 (American Society for Testing and Materials)
2) Calculated from HB = 0.95 HV (Vickers hardness)

Determination of Rockwell hardness HRA, HRB, HRC, and HRD acc. to DIN 50103 Part 1 and 2
Determination of Vickers hardness acc. to DIN 50133 Part 1
Determination of Brinell hardness acc. to DIN 50351
Determination of tensile strength acc. to DIN 50145

Values of solids and liquids

Mean density of the earth = 5.517 g/cm³

Substance (solid)	Symbol	Density π g/cm³	Melting point t in °C	Thermal conductivity λ at 20 °C W/(mK)
Agate		2.5...2.8	≈1600	11.20
Aluminium	Al	2.7	658	204
Aluminium bronze		7.7	1040	128
Antimony	Sb	6.67	630	22.5
Arsenic	As	5.72	–	–
Asbestos		≈2.5	≈1300	–
Asphaltum		1.1...1.5	80...100	0.698
Barium	Ba	3.59	704	–
Barium chloride		3.1	960	–
Basalt, natural		2.7...3.2	–	1.67
Beryllium	Be	1.85	1280	1.65
Concrete		≈2	–	≈1
Lead	Pb	11.3	327.4	34.7
Boron (amorph.)	B	1.73	2300	–
Borax		1.72	740	–
Limonite		3.4...3.9	1565	–
Bronze (CuSn6)		8.83	910	64
Chlorine calcium		2.2	774	–
Chromium	Cr	7.1	1800	69
Chromium nickel (NiCr 8020)		7.4	1430	52.335
Delta metal		8.6	950	104.7
Diamond	C	3.5	–	–
Iron, pure	Fe	7.86	1530	81
Grease		0.92...0.94	30...175	0.209
Gallium	Ga	5.9	29.75	–
Germanium	Ge	5.32	936	58.615
Gypsum		2.3	1200	0.45
Glass, window		≈2.5	≈700	0.81
Mica		≈2.8	≈1300	0.35
Gold	Au	19.29	1063	310
Granite		2.6...2.8	–	3.5
Graphite	C	2.24	≈3800	168
Grey cast iron		7.25	1200	58
Laminated fabric		1.3...1.42	–	0.34...0.35
Hard rubber		≈1.4	–	0.17
Hard metal K20		14.8	2000	81
Woods		0.45...0.85	–	0.12...0.17
Indium	In	7.31	156	24
Iridium	Ir	22.5	2450	59.3
Cadmium	Cd	8.64	321	92.1
Potassium	K	0.86	63.6	110
Limestone		2.6	–	2.2
Calcium	Ca	1.55	850	–
Calcium oxide (lime)		3.4	2572	–
Caoutchouc, crude		0.95	125	0.2
Cobalt	Co	8.8	1490	69.4
Salt, common		2.15	802	–
Coke		1.6...1.9	–	0.184
Constantan		8.89	1600	23.3
Corundum (AL_2O_3)		3.9...4	2050	12...23
Chalk		1.8...2.6	–	0.92
Copper	Cu	8.9	1083	384
Leather, dry		0.9....1	–	0.15
Lithium	Li	0.53	179	71
Magnesium	Mg	1.74	657	157
Magnesium, alloyed		1.8...1.83	650	69.8..145.4
Manganese	Mn	7.43	1250	30
Marble		2.6...2.8	1290	2.8
Red lead oxide		8.6...9.1	–	0.7
Brass (63Cu37Zn)		8.5	900	116
Molybdenum	Mo	10.2	2600	145
Monel metal		8.8	≈1300	19.7
Sodium	Na	0.98	97.5	126
Nickel silver		8.7	1020	48
Nickel	Ni	8.9	1452	59
Niobium	Nb	8.6	2415	54.43
Osmium	Os	22.5	2500	–
Palladium	Pd	12	1552	70.9
Paraffin		0.9	52	0.26
Pitch		1.25	–	0.13
Phosphorus (white)	P	1.83	44	–
Platinum	Pt	21.5	1770	70
Polyamide A, B		1.13	≈250	0.34

Substance (solid)	Symbol	Density π g/cm³	Melting point t in °C	Thermal conductivity λ at 20 °C W/(mK)
Porcelain		2.2...2.5	≈1650	≈1
Pyranite		3.3	1800	8.14
Quartz-flint		2.5...2.8	1480	9.89
Radium	Ra	5	700	–
Rhenium	Re	21	3175	71
Rhodium	Rh	12.3	1960	88
Gunmetal (CuSn5ZnPb)		8.8	950	38
Rubidium	Rb	1.52	39	58
Ruthenium	Ru	12.2	2300	106
Sand, dry		1.4...1.6	1480	0.58
Sandstone		2.1...2.5	≈1500	2.3
Brick, fire		1.8...2.3	≈2000	≈1.2
Slate		2.6...2.7	≈2000	≈0.5
Emery		4	2200	11.6
Sulphur, rhombic	S	2.07	112.8	0.27
Sulphur, monoclinic	S	1.96	119	0.13
Barytes		4.5	1580	–
Selenium, red	Se	4.4	220	0.2
Silver	Ag	10.5	960	407
Silicon	Si	2.33	1420	83
Silicon carbide		3.12	–	15.2
Sillimanite		2.4	1816	1.69
Soapstone (talcous)		2.7	–	3.26
Steel, plain + low-alloy		7.9	1460	47...58
stainless 18Cr8Ni		7.9	1450	14
non-magnetic 15Ni7Mn		8	1450	16.28
Tungsten steel 18W		8.7	1450	26
Steanit		2.6...2.7	≈1520	1.63
Hard coal		1.35	–	0.24
Strontium	Sr	2.54	797	0.23
Tantalum	Ta	16.6	2990	54
Tellurium	Te	6.25	455	4.9
Thorium	Th	11.7	≈1800	38
Titanium	Ti	4.5	1670	15.5
Tombac		8.65	1000	159
Uranium 99.99%		1.8...2.6	1500..1700	0.93...1.28
Uranium 99.99%	U	18.7	1133	28
Vanadium	V	6.1	1890	31.4
Soft rubber		1...1.8	–	0.14...0.23
White metal		7.5...10.1	300...400	34.9...69.8
Bismuth	Bi	9.8	271	8.1
Wolfram	W	19.2	3410	130
Cesium	Cs	1.87	29	–
Cement, hard		2...2.2	–	0.9...1.2
Cerium	Ce	6.79	630	–
Zinc	Zn	6.86	419	110
Tin	Sn	7.2	232	65
Zirconium	Zr	6.5	1850	22

Substance (liquid)	Symbol	Density π g/cm³	at °C	Boiling point at 1.013MPa °C	Thermal conductivity λ at 20 °C W/(mK)
Ether		0.72	20	35	0.14
Benzine		≈0.73	15	25...210	0.13
Benzole, pure		0.83	15	80	0.14
Diesel oil		0.83	15	210...380	0.15
Glycerine		1.26	20	290	0.29
Resin oil		0.96	20	150...300	0.15
Fuel oil EL		≈0.83	20	> 175	0.14
Linseed oil		0.93	20	316	0.17
Machinery oil		0.91	15	380...400	0.125
Methanol		0.8	15	65	0.21
Methyl chloride		0.95	15	24	0.16
Mineral oil		0.91	20	> 360	0.13
Petroleum ether		0.66	20	> 40	0.14
Petroleum		0.81	20	> 150	0.13
Mercury	Hg	13.55	20	357	10
Hydrochloric acid 10%		1.05	15	102	0.5
Sulphuric acid, strong		1.84	15	338	0.47
Silicon fluid		0.94	20	–	0.22

Coefficient of linear expansion α

The coefficient of linear expansion α gives the fractional expansion of the unit of length of a substance per 1 degree K rise in temperature. For the linear expansion of a body applies:

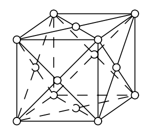
$$l + l_o = \mu = T$$

where
Δl: change of length
l_o: original length
α: coefficient of linear expansion
ΔT: rise of temperature

Coefficients of linear expansion of some substances at 0 ... 100 °C

Substance	α [10⁻⁶/K]
Aluminium alloys	21 ... 24
Grey cast iron (e.g. GG-20, GG-25)	10.5
Steel, plain and low-alloy	11.5
Steel, stainless (18Cr 8Ni)	16
Steel, rapid machining steel	11.5
Copper	17
Brass CuZn37	18.5
Bronze CuSn8	17.5

Iron-carbon diagram



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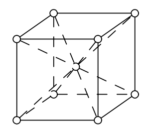

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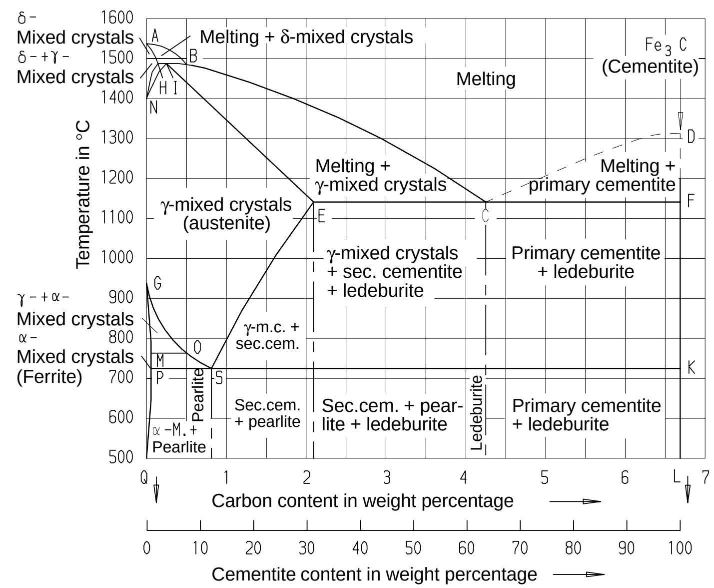
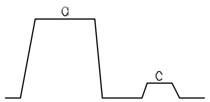
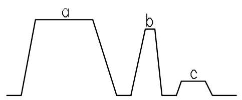
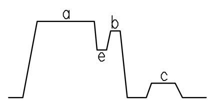
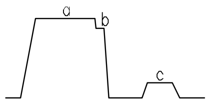
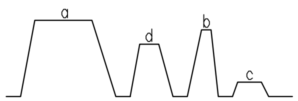
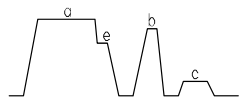
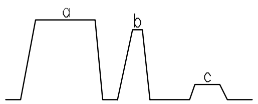


Pitting and tooth root fatigue strength of case hardening steels, DIN 17210

Symbol	Hardness on finished gear HV1	σ_{Hlim} N/mm²	σ_{Flim} N/mm²
16 MnCr 5	720	1470	430
15 CrNi 6	730	1490	460
17 CrNiMo 6	740	1510	500

Heat treatment during case hardening of case hardening steels acc. to DIN 17210		
Usual heat treatment during case hardening		
A. Direct hardening or double hardening	B. Single hardening	C. Hardening after isothermal transformation
 Direct hardening from carburizing temperature	 Single hardening from core or case hardening temperature	 Hardening after isothermal transformation in the pearlite stage (e)
 Direct hardening after lowering to hardening temperature	 Single hardening after intermediate annealing (soft annealing) (d)	 Hardening after isothermal transformation in the pearlite stage (e) and cooling-down to room temperature
 Double hardening	a carburizing temperature b hardening temperature c tempering temperature d intermediate annealing (soft annealing) temperature e transformation temperature in the pearlite stage	

Usual case hardening temperatures						
Grade of steel		a	b			c
Symbol	Material number	Carburizing temperature 1) °C	Core hardening temperature 2) °C	Case hardening temperature 2) °C	Quenchant	Tempering °C
C 10 Ck 10 Ck 15 Cm 15	1.0301 1.1121 1.0401 1.1141 1.1140	880 up to 980	880 up to 920	780 up to 820	With regard to the properties of the component, the selection of the quenchant depends on the hardenability or case-hardenability of the steel, the shape and cross section of the work piece to be hardened, as well as on the effect of the quenchant.	150 up to 200
17 Cr 3 20 Cr 4 20 CrS 4 16 MnCr 5 16 MnCrS5 20 MnCr 5 20 MnCrS 5 20 MoCr 4 20 MoCrS 4 22 CrMoS 3 5 21 NiCrMo 2 21 NiCrMoS 2	1.7016 1.7027 1.7028 1.7131 1.7139 1.7147 1.7149 1.7321 1.7323 1.7333 1.6523 1.6526		860 up to 900			
15 CrNi 6 17 CrNiMo 6	1.5919 1.6587		830 up to 870			

1) Decisive criteria for the determination of the carburizing temperature are mainly the required time of carburizing, the chosen carburizing agent, and the plant available, the provided course of process, as well as the required structural constitution. For direct hardening, carburizing usually is carried out at temperatures below 950 °C. In special cases, carburizing temperatures up to above 1000 °C are applied.

2) In case of direct hardening, quenching is carried out either from the carburizing temperature or any lower temperature. In particular if there is a risk of warping, lower hardening temperatures are preferred.

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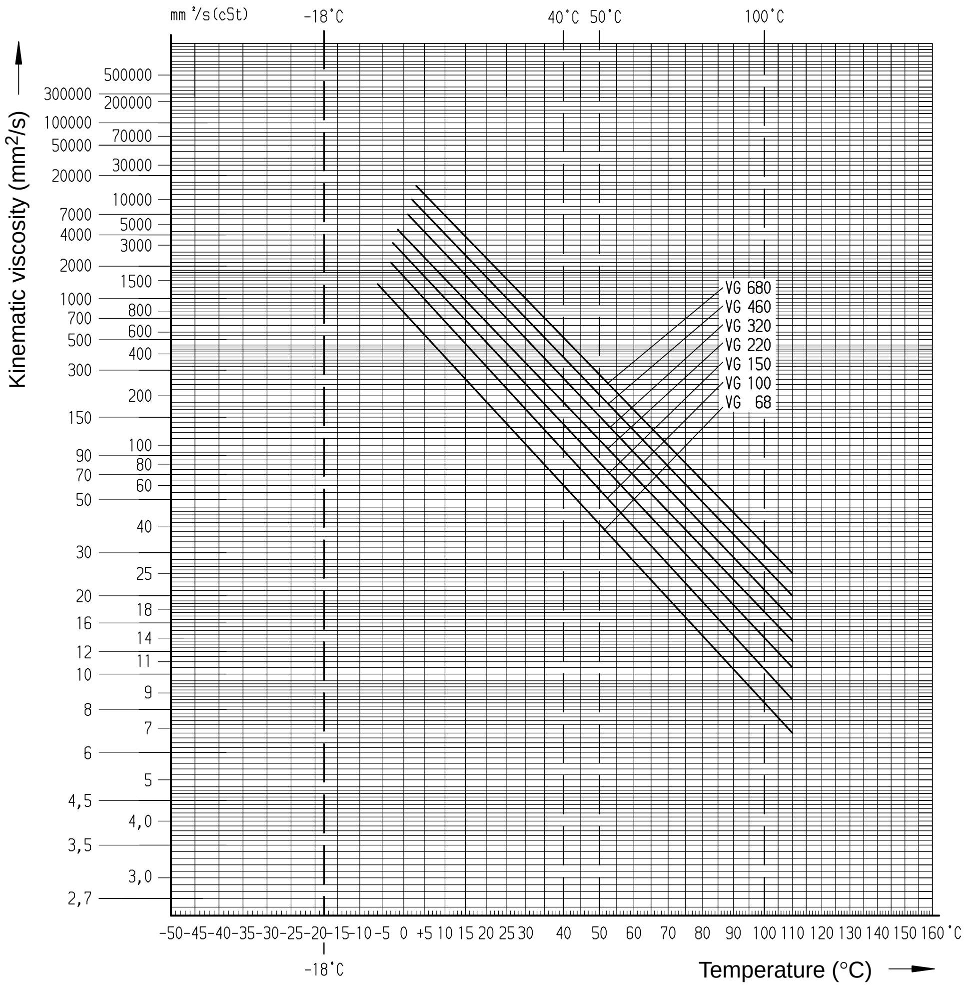
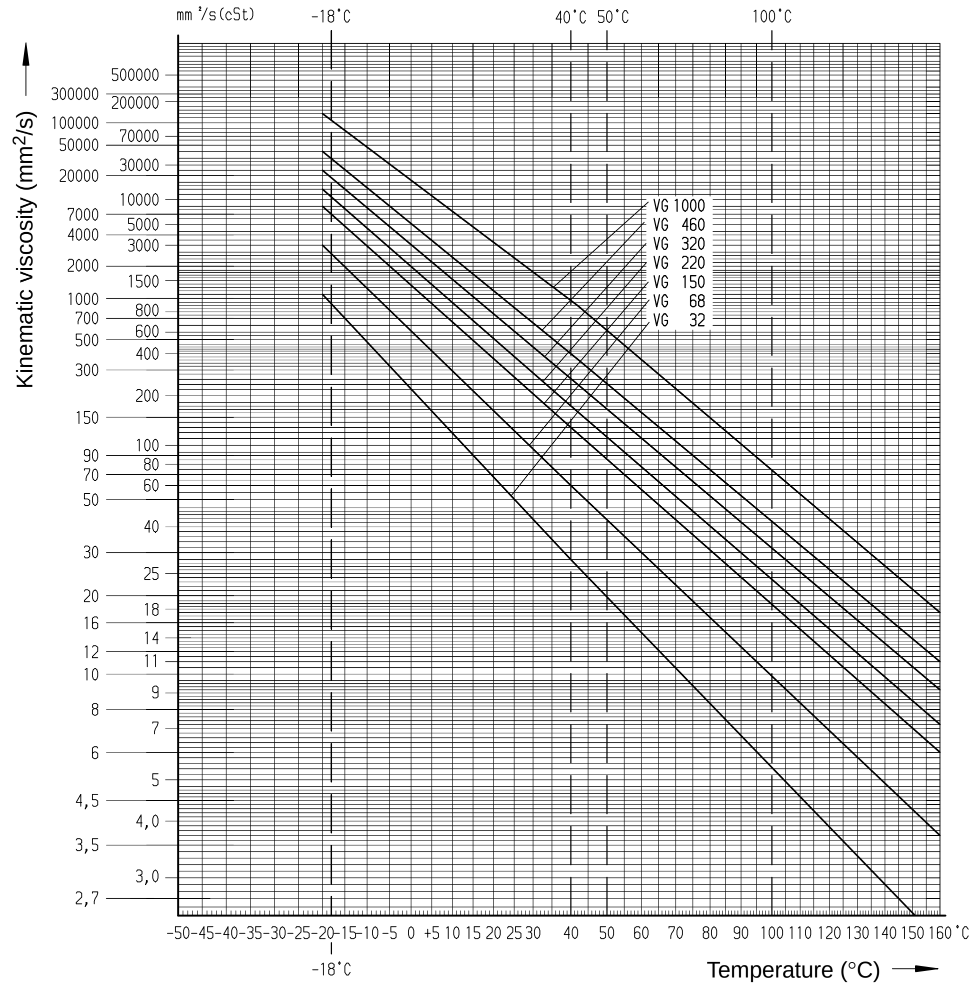
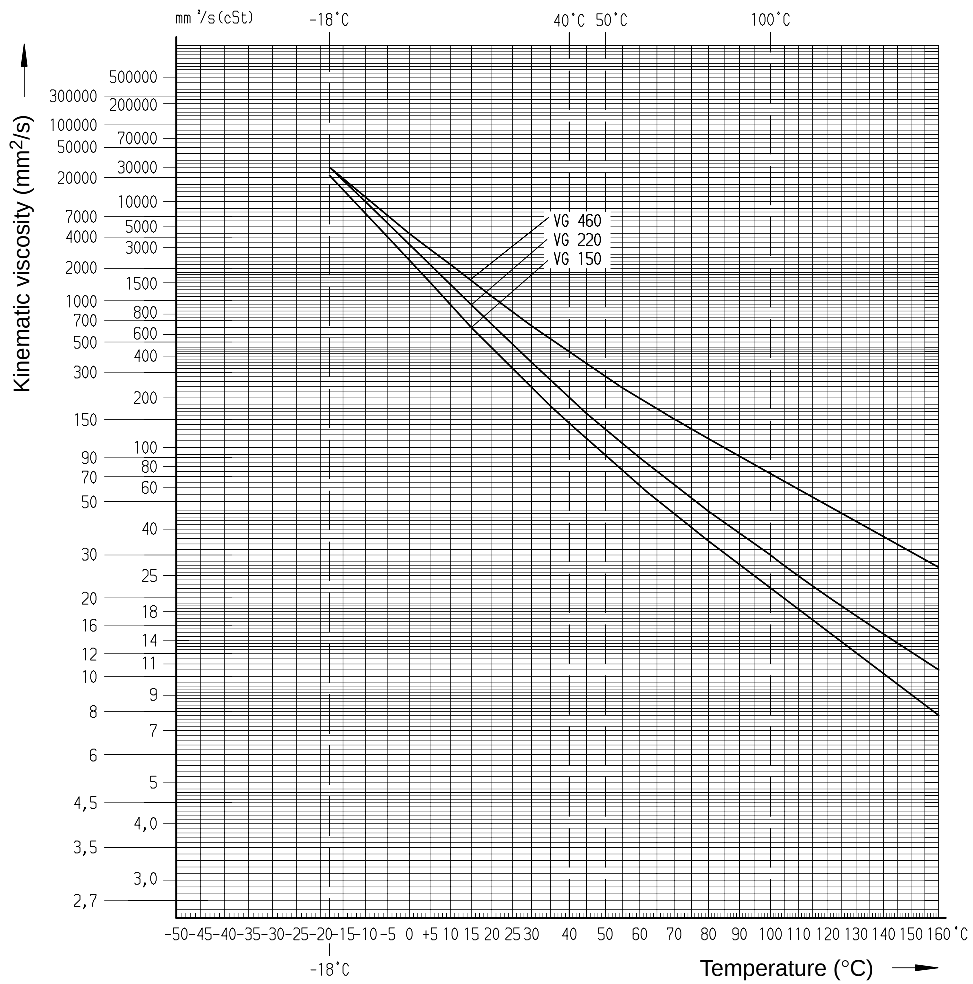
Viscosity-temperature-diagram for mineral oils



Viscosity-temperature-diagram for synthetic oils of poly-α-olefine base



Viscosity-temperature-diagram for synthetic oils of polyglycole base



Kinematic viscosity υ		
Quantities for the determination of the kinematic viscosity		
VG grade	W_{40} [–]	m [–]
32	0.18066	3.7664
46	0.22278	3.7231
68	0.26424	3.6214
100	0.30178	3.5562
150	0.33813	3.4610
220	0.36990	3.4020
320	0.39900	3.3201
460	0.42540	3.3151
680	0.45225	3.2958
1000	0.47717	3.2143
1500	0.50192	3.1775

$$W = m\,(2.49575 - \lg T) + W_{40} \qquad (1)$$

$$\mu = 10^{10^{W}} + 0.8 \qquad (2)$$

- m [-]: slope
- T [K]: thermodynamic temperature [1)]
- W_{40} [-]: auxiliary quantity at 40 °C
- W [-]: auxiliary quantity
- υ [cSt]: kinematic viscosity

1) T = t + 273.15 [K]

Dynamic viscosity η

$$\eta = \upsilon \cdot \quad \cdot 0.001 \qquad (3)$$

$$= \ _{15} - (t - 15) \cdot 0.0007 \qquad (4)$$

- t [°C]: temperature
- $_{15}$ [kg/dm^3]: density at 15 °C
- [kg/dm^3]: density
- υ [cSt]: kinematic viscosity
- η [Ns/m^2]: dynamic viscosity

Density $_{15}$ in kg/dm^3 of lubricating oils for gear units) (Example) [2)]

VG grade	**68**	**100**	**150**	**220**	**320**	**460**	**680**
ARAL Degol BG	0.890	0.890	0.895	0.895	0.900	0.900	0.905
ESSO Spartan EP	0.880	0.885	0.890	0.895	0.900	0.905	0.920
MOBIL OIL Mobilgear 626 ... 636	0.882	0.885	0.889	0.876	0.900	0.905	0.910
OPTIMOL Optigear BM	0.890	0.901	0.904	0.910	0.917	0.920	0.930
TRIBOL Tribol 1100	0.890	0.895	0.901	0.907	0.912	0.920	0.934

2) Mineral base gear oils in accordance with designation CLP as per DIN 51502. These oils comply with the minimum requirements as specified in DIN 51517 Part 3. They are suitable for operating temperatures from -10 °C up to +90 °C (briefly +100 °C).

ISO-VG DIN 51519	Approx. assignment to previous DIN 51502	Mean viscosity (40 °C) and approx. viscosities in mm^2/s (cSt) at					Saybolt universal seconds (SSU) at 40 °C (mean value) 1)	AGMA lubricant	Approx. assignment to	
		20 C	40 C	50 C		100 C		N° at 40 °C 1)	motor oils	motor-car gear oils
		cSt	cSt	cSt	Engler	cSt			SAE	SAE
5	2	8 (1.7 E)	4.6	4	1.3	1.5				
7	4	12 (2 E)	6.8	5	1.4	2.0				
10	9	21 (3 E)	10	8	1.7	2.5				
15	–	34	15	11	1.9	3.5			5W	
22	16	55	22	15	2.3	4.5			10 W	70 W 75 W
32	25	88	32	21	3	5.5				
46	36	137	46	30	4	6.5	214	1 EP	15 W 20 W 20	
68	49	219	68	43	6	8.5	316	2.2 EP		80 W
100	68	345	100	61	8	11	464	3.3 EP	30	85 W
150	92	550	150	90	12	15	696	4.4 EP	40	
220	114 144	865	220	125	16	19	1020	5.5 EP	50	90
320	169	1340	320	180	24	24	1484	6.6 EP		
460	225	2060	460	250	33	30	2132	7 EP		140
680	324	3270	680	360	47	40	3152	8 EP		
1000		5170	1000	510	67	50				250
1500		8400	1500	740	98	65				

1) Approximate comparative value to ISO VG grades

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a	mm	Centre distance
a_d	mm	Reference centre distance
b	mm	Facewidth
c_p	mm	Bottom clearance between standard basic rack tooth profile and counter profile
d	mm	Reference diameter
d_a	mm	Tip diameter
d_b	mm	Base diameter
d_f	mm	Root diameter
d_w	mm	Pitch diameter
e	mm	Spacewidth on the reference cylinder
e_p	mm	Spacewidth on the standard basic rack tooth profile
f	Hz	Frequency
g_α	mm	Length of path of contact
h	mm	Tooth depth
h_a	mm	Addendum
h_{aP}	mm	Addendum of the standard basic rack tooth profile
h_{aPO}	mm	Addendum of the tool's standard basic rack tooth profile
h_f	mm	Dedendum
h_{fP}	mm	Dedendum of the standard basic rack tooth profile
h_{fPO}	mm	Dedendum of the tool's standard basic rack tooth profile
h_p	mm	Tooth depth of the standard basic rack tooth profile
h_{PO}	mm	Tooth depth of the tool's standard basic rack tooth profile
h_{prPO}	mm	Protuberance height of the tool's standard basic rack tooth profile
h_{wP}	mm	Working depth of the standard basic rack tooth profile and the counter profile
k	–	Tip diameter modification coefficient
m	mm	Module
m_n	mm	Normal module
m_t	mm	Transverse module

n	1/min	Speed
p	N/mm^2	Sound pressure
p	mm	Pitch on the reference circle
p_{bt}	mm	Pitch on the base circle
p_e	mm	Normal base pitch
p_{en}	mm	Normal base pitch at a point
p_{et}	mm	Normal transverse pitch
p_{ex}	mm	Axial pitch
p_t	mm	Transverse base pitch, reference circle pitch
p_{rPO}	mm	Protuberance value on the tool's standard basic rack tooth profile
q	mm	Machining allowance on the cylindrical gear tooth flanks
r	mm	Reference circle radius, radius
r_a	mm	Tip radius
r_b	mm	Base radius
r_w	mm	Radius of the working pitch circle
s	mm	Tooth thickness on the reference circle
s_{an}	mm	Tooth thickness on the tip circle
s_p	mm	Tooth thickness of the standard basic rack tooth profile
s_{PO}	mm	Tooth thickness of the tool's standard basic rack tooth profile
u	–	Gear ratio
v	m/s	Circumferential speed on the reference circle
w	N/mm	Line load
x	–	Addendum modification coefficient
x_E	–	Generating addendum modification coefficient
z	–	Number of teeth
A	m^2	Gear teeth surface
A_s	mm	Tooth thickness deviation
B_L	N/mm^2	Load value

D	mm	Construction dimension
F_n	N	Load
F_t	N	Nominal peripheral force at the reference circle
G	kg	Gear unit weight
HV1	–	Vickers hardness at F = 9.81 N
K_A	–	Application factor
$K_{F\alpha}$	–	Transverse load factor (for tooth root stress)
$K_{F\beta}$	–	Face load factor (for tooth root stress)
$K_{H\alpha}$	–	Transverse load factor (for contact stress)
$K_{H\beta}$	–	Face load factor (for contact stress)
K_v	–	Dynamic factor
L_{pA}	dB	Sound pressure level A
L_{WA}	dB	Sound power level A
P	kW	Nominal power rating of driven machine
R_Z	μm	Mean peak-to-valley roughness
S_F	–	Factor of safety from tooth breakage
S_H	–	Factor of safety from pitting
S	m^2	Enveloping surface
T	Nm	Torque
V_{40}	mm^2/s	Lubricating oil viscosity at 40 °C
Y_β	–	Helix angle factor
Y_ε	–	Contact ratio factor
Y_{FS}	–	Tip factor
Y_R	–	Roughness factor
Y_X	–	Size factor
Z_β	–	Helix angle factor
Z_ε	–	Contact ratio factor
Z_H	–	Zone factor
Z_L	–	Lubricant factor
Z_V	–	Speed factor

Note: The unit rad may be replaced by 1.

Z_X	–	Size factor
α	Degree	Transverse pressure angle at a point; Pressure angle
^	rad	Angle α in the circular measure ^ + μ = 180
α_{at}	Degree	Transverse pressure angle at the tip circle
α_n	Degree	Normal pressure angle
α_P	Degree	Pressure angle at a point of the standard basic rack tooth profile
α_{PO}	Degree	Pressure angle at a point of the tool's standard basic rack tooth profile
α_{prPO}	Degree	Protuberance pressure angle at a point
α_t	Degree	Transverse pressure angle at the reference circle
α_{wt}	Degree	Working transverse pressure angle at the pitch circle
β	Degree	Helix angle at the reference circle
β_b	Degree	Base helix angle
ε_α	–	Transverse contact ratio
ε_β	–	Overlap ratio
ε_γ	–	Total contact ratio
η	–	Efficiency
ζ	Degree	Working angle of the involute
π	mm	Radius of curvature
π_{aPO}	mm	Tip radius of curvature of the tool's standard basic rack tooth profile
π_{fPO}	mm	Root radius of curvature of the tool's standard basic rack tooth profile
σ_H	N/mm^2	Effective Hertzian pressure
σ_{Hlim}	N/mm^2	Allowable stress number for contact stress
σ_{HP}	N/mm^2	Allowable Hertzian pressure
σ_F	N/mm^2	Effective tooth root stress
σ_{Flim}	N/mm^2	Bending stress number
σ_{FB}	N/mm^2	Allowable tooth root stress

1. Cylindrical gear units

1.1 Introduction

In the industry, mainly gear units with case hardened and fine-machined gears are used for torque and speed adaptation of prime movers and driven machines. After carburising and hardening, the tooth flanks are fine-machined by hobbing or profile grinding or removing material (by means of shaping or generating tools coated with mechanically resistant material). In comparison with other gear units, which, for example, have quenched and tempered or nitrided gears, gear units with case hardened gears have higher power capacities, i.e. they require less space for the same speeds and torques. Further, gear units have the best efficiencies. Motion is transmitted without slip at constant speed. As a rule, an infinitely variable change-speed gear unit with primary or secondary gear stages presents the most economical solution even in case of variable speed control.

In industrial gear units mainly involute gears are used. Compared with other tooth profiles, the technical and economical advantages are basically:

- Simple manufacture with straight-sided flanked tools;
- The same tool for all numbers of teeth;
- Generating different tooth profiles and centre distances with the same number of teeth by means of the same tool by addendum modification;
- Uniform transmission of motion even in case of centre distance errors from the nominal value;
- The direction of the normal force of teeth remains constant during meshing;
- Advanced stage of development;
- Good availability on the market.

When load sharing gear units are used, output torques can be doubled or tripled in comparison with gear units without load sharing. Load sharing gear units mostly have one input and one output shaft. Inside the gear unit the load is distributed and then brought together again on the output shaft gear. The uniform sharing of the load between the individual branches is achieved by special design measures.

1.2 Geometry of involute gears

The most important concepts and parameters associated with cylindrical gears and cylindrical gear pairs with involute teeth in accordance with DIN 3960 are represented in sections 1.2.1 to 1.2.4. /1/

1.2.1 Concepts and parameters associated with involute teeth

1.2.1.1 Standard basic rack tooth profile

The standard basic rack tooth profile is the normal section through the teeth of the basic rack which is produced from an external gear tooth system with an infinitely large diameter and an infinitely large number of teeth. From figure 1 follows:

- The flanks of the standard basic rack tooth profile are straight lines and are located symmetrically below the pressure angle at a point α_P to the tooth centre line;
- Between module m and pitch p the relation is $p = \pi m$;
- The nominal dimensions of tooth thickness and spacewidth on the datum line are equal, i.e. $s_P = e_P = p/2$;
- The bottom clearance c_P between basic rack tooth profile and counter profile is 0.1 m up to 0.4 m;
- The addendum is fixed by $h_{aP} = m$, the dedendum by $h_{fP} = m + c_P$ and thus, the tooth depth by $h_P = 2\,m + c_P$;
- The working depth of basic rack tooth profile and counter profile is $h_{wP} = 2$ m.

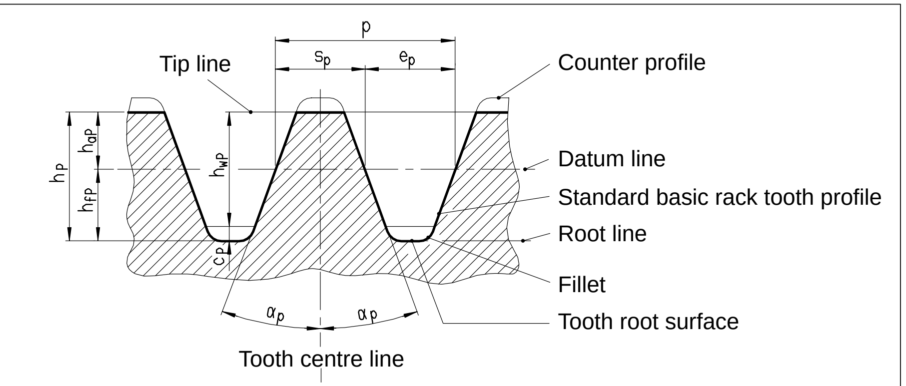


Figure 1 Basic rack tooth profiles for involute teeth of cylindrical gears (acc. to DIN 867)

1.2.1.2 Module

The module m of the standard basic rack tooth profile is the module in the normal section m_n of the gear teeth. For a helical gear with helix angle β on the reference circle, the transverse module in a transverse section is $m_t = m_n/\cos\beta$. For a spur gear β = 0 and the module is $m = m_n = m_t$. In order to limit the number of the required gear cutting tools, module m has been standardized in preferred series 1 and 2, see table 1.

Table 1 Selection of some modules m in mm (acc. to DIN 780)

Series 1	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32
Series 2			1.75			3.5	4.5		7	9		14	18	22	28	

1.2.1.3 Tool reference profile

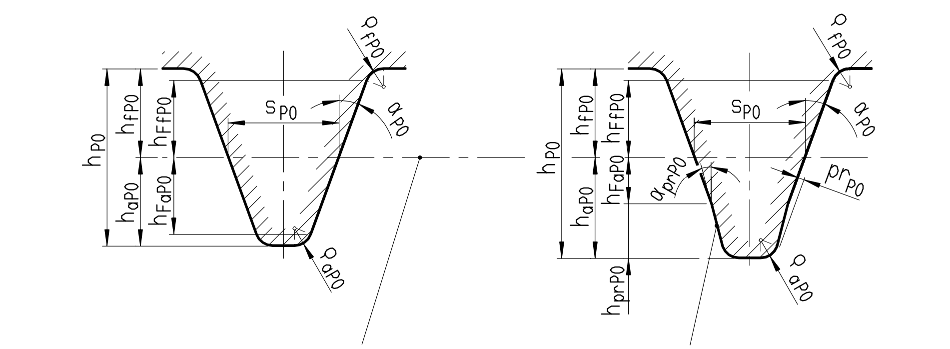
The tool reference profile according to figure 2a is the counter profile of the standard basic rack tooth profile according to figure 1. For industrial gear units, the pressure angle at a point of the tool reference profile $\alpha_{PO} = \alpha_P$ is 20°, as a rule. The tooth thickness s_{PO} of the tool on the tool datum line depends on the stage of machining. The pre-machining tool leaves on both flanks of the teeth a machining allowance q for finish-machining. Therefore, the tooth thickness for pre-machining tools is $s_{PO} < p/2$, and for finish-machining tools $s_{PO} = p/2$.

The pre-machining tool generates the root diameter and the fillet on a cylindrical gear. The finish-machining tool removes the machining allowance on the flanks, however, normally it does not touch the root circle - like on the tooth profile in figure 3a.

Between pre- and finish- machining, cylindrical gears are subjected to a heat treatment which, as a rule, leads to warping of the teeth and growing of the root and tip circles.

Especially for cylindrical gears with a relatively large number of teeth or a small module there is a risk of generating a notch in the root on finish machining. To avoid this, pre-machining tools are provided with protuberance flanks as shown in figure 2b. They generate a root undercut on the gear, see figure 3b. On the tool, protuberance value pr_{PO}, protuberance pressure angle at a point α_{prPO}, as well as the tip radius of curvature μ_{aPO} must be so dimensioned that the active tooth profile on the gear will not be reduced and the tooth root will not be weakened too much.

On cylindrical gears with small modules one often accepts on purpose a notch in the root if its distance to the root circle is large enough and thus the tooth root load carrying capacity is not impaired by a notch effect, figure 3c. In order to prevent the tip circle of the mating gear from touching the fillet it is necessary that a check for meshing interferences is carried out on the gear pair. /1/



a) Tool datum line b) Protuberance flank

Figure 2 Reference profiles of gear cutting tools for involute teeth of cylindrical gears

a) For pre-machining and finish-machining
b) For pre-machining with root undercut (protuberance)

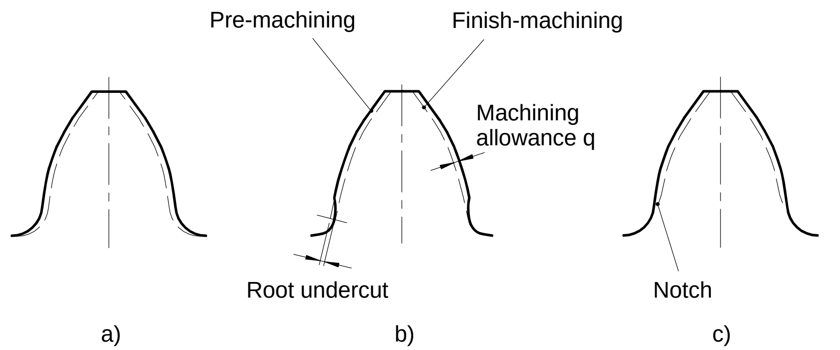


Figure 3 Tooth profiles of cylindrical gears during pre- and finish-machining

a) Pre- and finish-machining down to the root circle
b) Pre-machining with root undercut (protuberance)
c) Finish-machining with notch

1.2.1.4 Generating tooth flanks

With the development of the envelope, an envelope line of the base cylinder with the base diameter d_b generates the involute surface of a spur gear.
A straight line inclined by a base helix angle β_b to the envelope line in the developed envelope is the generator of an involute surface (involute helicoid) of a helical gear, figure 4.
The involute which is always lying in a transverse section, figure 5, is described by the transverse pressure angle at a point α and radius r in the equations

$\mathrm{inv}\alpha = \tan\alpha - \hat{}$ (1)

$r = r_b / \cos\alpha$ (2)

$r_b = d_b/2$ is the base radius. The angle $\mathrm{inv}\alpha$ is termed involute function, and the angle

$\zeta = \hat{} + \mathrm{inv}\alpha = \tan\alpha$ is termed working angle.

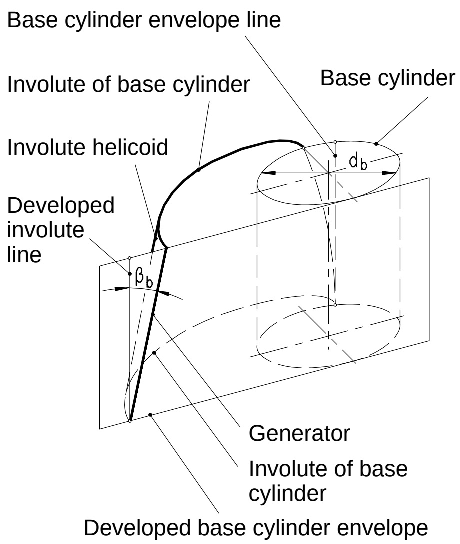


Figure 4 Base cylinder with involute helicoid and generator

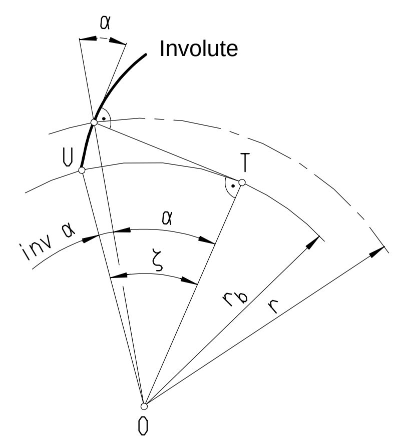
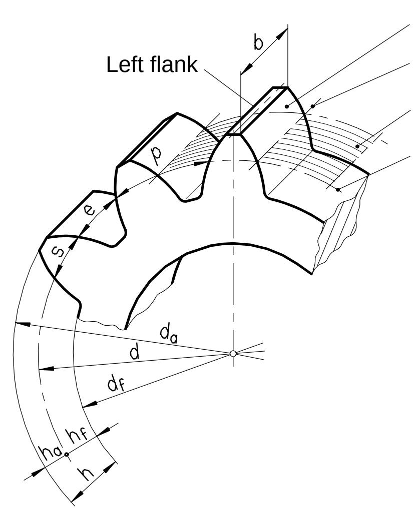


Figure 5 Involute in a transverse section

1.2.2 Concepts and parameters associated with cylindrical gears

1.2.2.1 Geometric definitions

In figure 6 the most important geometric quantities of a cylindrical gear are shown.
The reference circle is the intersection of the reference cylinder with a plane of transverse section. When generating tooth flanks, the straight pitch line of the tool rolls off at the reference circle. Therefore, the reference circle periphery corresponds to the product of pitch p and number of teeth z, i.e. π d = p z. Since $m_t = p/\pi$, the equation for the reference diameter thus is $d = m_t$ z.
Many geometric quantities of the cylindrical gear are referred to the reference circle.
For a helical gear, at the point of intersection of the involute with the reference circle, the transverse pressure angle at a point α in the transverse section is termed transverse pressure angle α_t, see figures 5 and 7. If a tangent line is put against the involute surface in the normal section at the point of intersection with the reference circle, the corresponding angle is termed normal pressure angle α_n; this is equal to the pressure angle α_{PO} of the tool. The interrelationship with the helix angle β at the reference circle is $\tan\alpha_n = \cos\beta \tan\alpha_t$. On a spur gear $\alpha_n = \alpha_t$.
Between the base helix angle β_b and the helix angle β on the reference circle the relationship is $\sin\beta_b = \cos\alpha_n \sin\beta$. The base diameter d_b is given by the reference diameter d, by $d_b = d \cos\alpha_t$.
In the case of internal gears, the number of teeth z and thus also the diameters d, d_b, d_a, d_f are negative values.



d Reference diameter
d_a Tip diameter
d_f Root diameter
b Facewidth
h Tooth depth
h_a Addendum
h_f Dedendum
s Tooth thickness on the reference circle
e Spacewidth on the reference circle
p Pitch on the reference circle

Figure 6 Definitions on the cylindrical gear

1.2.2.2 Pitches

The pitch p_t of a helical gear (p in the case of a spur gear) lying in a transverse section is the length of the reference circle arc between two successive right or left flanks, see figures 6 and 7. With the number of teeth z results $p_t = \pi d/z = \pi m_t$.
The normal transverse pitch p_{et} of a helical gear is equal to the pitch on the basic circle p_{bt}, thus $p_{et} = p_{bt} = \pi d_b/z$. Hence, in the normal section the normal base pitch at a point $p_{en} = p_{et} \cos\beta_b$ is resulting from it, and in the axial section the axial pitch $p_{ex} = p_{et}/\tan\beta_b$, see figure 13.

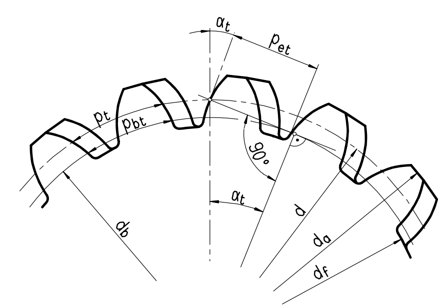


Figure 7 Pitches in the transverse section of a helical gear

1.2.2.3 Addendum modification

When generating tooth flanks on a cylindrical gear by means of a tooth-rack-like tool (e.g. a hob), a straight pitch line parallel to the datum line of tool rolls off on the reference circle. The distance ($x \cdot m_n$) between the straight pitch line and the datum line of tool is the addendum modification, and x is the addendum modification coefficient, see figure 8.

An addendum modification is positive, if the datum line of tool is displaced from the reference circle towards the tip, and it is negative if the datum line is displaced towards the root of the gear. This is true for both external and internal gears. In the case of internal gears the tip points to the inside. An addendum modification for external gears should be carried through approximately within the limits as shown in figure 9.

The addendum modification limits x_{min} and x_{max} are represented dependent on the virtual number of teeth $z_n = z/(\cos\beta \cos^2\beta_b)$. The upper limit x_{max} takes into account the intersection circle of the teeth and applies to a normal crest width in the normal section of $s_{an} = 0.25\ m_n$. When falling below the lower limit x_{min} this results in an undercut which shortens the usable involute and weakens the tooth root.

A positive addendum modification results in a greater tooth root width and thus in an increase in the tooth root carrying capacity. In the case of small numbers of teeth this has a considerably stronger effect than in the case of larger ones. One mostly strives for a greater addendum modification on pinions than on gears in order to achieve equal tooth root carrying capacities for both gears, see figure 19.

Further criteria for the determination of addendum modification are contained in /2/, /3/, and /4/. The addendum modification coefficient x refers to gear teeth free of backlash and deviations. In order to take into account tooth thickness deviation A_s (for backlash and manufacturing tolerances) and machining allowances q (for premachining), one has to give the following generating addendum modification coefficient for the manufacture of a cylindrical gear:

$$X_E = x + \frac{A_s}{2m_n \tan\alpha_n} + \frac{q}{m_n \sin\alpha_n} \qquad (3)$$

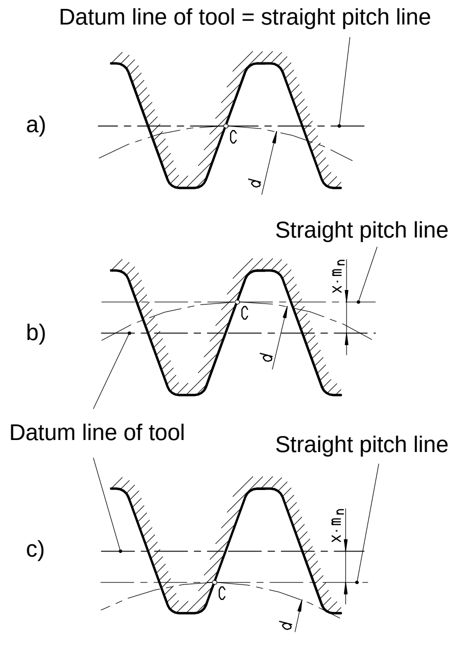


Figure 8 Different positions of the datum line of tool in relation to the straight pitch line through pitch point C.

a) Zero addendum modification; x = 0
b) Negative addendum modification; x < 0
c) Positive addendum modification; x > 0

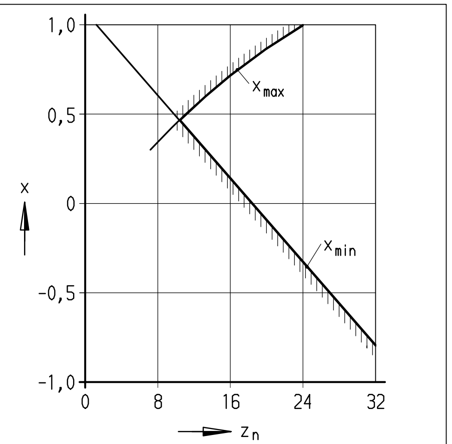


Figure 9 Addendum modification limit x_{max} (intersection circle) and x_{min} (undercut limit) for external gears dependent on the virtual number of teeth zn (for internal gears, see /1/ and /3/).

1.2.3 Concepts and parameters associated with a cylindrical gear pair

1.2.3.1 Terms

The mating of two external cylindrical gears (external gears) gives an external gear pair. In the case of a helical external gear pair one gear has left-handed and the other one right-handed flank direction.

The mating of an external cylindrical gear with an internal cylindrical gear (internal gear) gives an internal gear pair. In the case of a helical internal gear pair, both gears have the same flank direction, that is either right-handed or left-handed. The subscript 1 is used for the size of the smaller gear (pinion), and the subscript 2 for the larger gear (wheel or internal gear).

In the case of a zero gear pair both gears have as addendum modification coefficient $x_1 = x_2 = 0$ (zero gears).

In the case of a V-zero gear pair, both gears have addendum modifications (V-gears), that is with $x_1 + x_2 = 0$, i.e. $x_1 = -x_2$.

For a V-gear pair, the sum is not equal to zero, i.e. $x_1 + x_2 \neq 0$. One of the cylindrical gears in this case may, however, have an addendum modification x = 0.

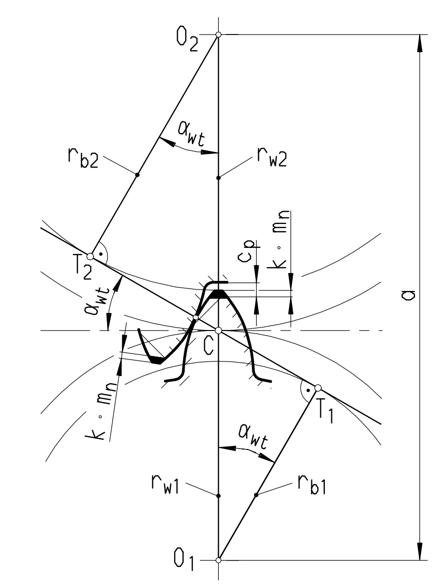


Figure 10 Transverse section of an external gear pair with contacting left-handed flanks

1.2.3.2 Mating quantities

The gear ratio of a gear pair is the ratio of the number of teeth of the gear z_2 to the number of teeth of the pinion z_1, thus $u = z_2/z_1$. Working pitch circles with diameter $d_w = 2r_w$ are those transverse intersection circles of a cylindrical gear pair, which have the same circumferential speed at their mutual contact point (pitch point C), figure 10. The working pitch circles divide the centre distance $a = r_{w1} + r_{w2}$ in the ratio of the tooth numbers, thus $d_{w1} = 2\ a/(u + 1)$ and $d_{w2} = 2\ a\ u/(u + 1)$.

In the case of both a zero gear pair and a V-zero gear pair, the centre distance is equal to the zero centre distance $a_d = (d_1 + d_2)/2$, and the pitch circles are simultaneously the reference circles, i.e. $d_w = d$. However, in the case of a V-gear pair the centre distance is not equal to the zero centre distance, and the pitch circles are not simultaneously the reference circles.

If in the case of V-gear pairs the bottom clearance c_p corresponding to the standard basic rack tooth profile is to be retained (which is not absolutely necessary), then an addendum modification is to be carried out. The addendum modification factor is $k = (a - a_d)/m_n - (x_1 + x_2)$. For zero gear pairs and V-zero gear pairs k = 0. In the case of external gear pairs k < 0, i.e. the tip diameters of both gears become smaller. In the case of internal gear pairs k > 0, i.e. the tip diameters of both gears become larger (on an internal gear with negative tip diameter the absolute value becomes smaller).

In a cylindrical gear pair either the left or the right flanks of the teeth contact each other on the line of action. Changing the flanks results in a line of action each lying symmetrical in relation to the centre line through O_1O_2. The line of action with contacting left flanks in figure 10 is the tangent to the two base circles at points T_1 and T_2. With the common tangent on the pitch circles it includes the working pressure angle α_{wt}.

The working pressure angle α_{wt} is the transverse pressure angle at a point belonging to the working pitch circle. According to figure 10 it is determined by $\cos\alpha_{wt} = d_{b1}/d_{w1} = d_{b2}/d_{w2}$. In the case of zero gear pairs and V-zero gear pairs, the working pressure angle is equal to the transverse pressure angle on the reference circle, i.e. $\alpha_{wt} = \alpha_t$.

The length of path of contact g_α is that part of the line of action which is limited by the two tip circles of the cylindrical gears, figure 11.

The starting point A of the length of path of contact is the point at which the line of action intersects the tip circle of the driven gear, and the finishing point E is the point at which the line of action intersects the tip circle of the driving gear.

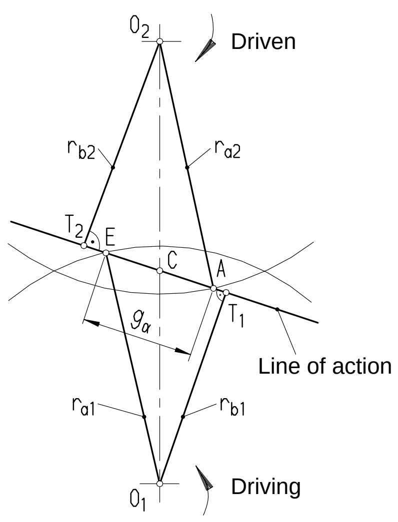


Figure 11 Length of path of contact $\overline{AE}$ in the transverse section of an external gear pair

A Starting point of engagement
E Finishing point of engagement
C Pitch point

1.2.3.3 Contact ratios

The transverse contact ratio ε_α in the transverse section is the ratio of the length of path of contact g_α to the normal transverse pitch p_{et}, i.e. $\varepsilon_\alpha = g_\alpha/p_{et}$, see figure 12.
In the case of spur gear pairs, the transverse contact ratio gives the average number of pairs of teeth meshing during the time of contact of a tooth pair. According to figure 12, the left-hand tooth pair is in the individual point of contact D while the right-hand tooth pair gets into mesh at the starting point of engagement A. The right-hand tooth pair is in the individual point of contact B when the left-hand tooth pair leaves the mesh at the finishing point of engagement E. Along the individual length of path of contact $\overline{BD}$ one tooth pair is in mesh, and along the double lengths of paths of contact $\overline{AB}$ and $\overline{DE}$ two pairs of teeth are simultaneously in mesh.
In the case of helical gear pairs it is possible to achieve that always two or more pairs of teeth are in mesh simultaneously. The overlap ratio ε_β gives the contact ratio, owing to the helix of the teeth, as the ratio of the facewidth b to the axial pitch p_{ex}, i.e. $\varepsilon_\beta = b/p_{ex}$, see figure 13.
The total contact ratio ε_γ is the sum of transverse contact ratio and overlap ratio, i.e. $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$.
With an increasing total contact ratio, the load carrying capacity increases, as a rule, while the generation of noise is reduced.

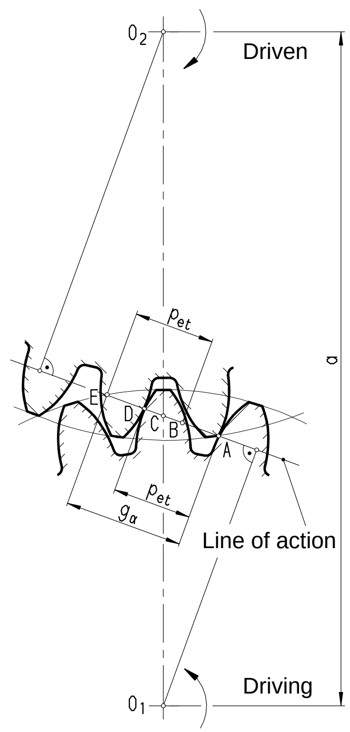


Figure 12 Single and double contact region in the transverse section of an external gear pair

B, D Individual points of contact
A, E Starting and finishing point of engagement, respectively
C Pitch point

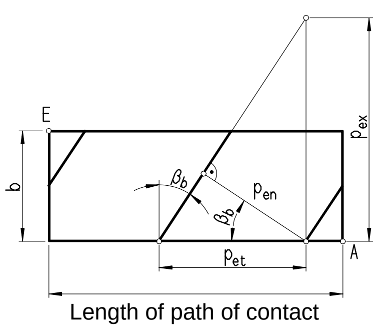


Figure 13 Pitches in the plane of action

A Starting point of engagement
E Finishing point of engagement

1.2.4 Summary of the most important formulae

Tables 2 and 3 contain the most important formulae for the determination of sizes of a cylindrical gear and a cylindrical gear pair, and this for both external and internal gear pairs.
The following rules for signs are to be observed: In the case of internal gear pairs the number of teeth z_2 of the internal gear is a negative quantity. Thus, also the centre distance a or a_d and the gear ratio u as well as the diameters d_2, d_{a2}, d_{b2}, d_{f2}, d_{w2} and the virtual number of teeth z_{n2} are negative.
When designing a cylindrical gear pair for a gear stage, from the output quantities of tables 2 and 3 only the normal pressure angle α_n and the gear ratio u are given, as a rule. The number of teeth of the pinion is determined with regard to silent running and a balanced foot and flank load carrying capacity, at approx. z_1 = 18 ... 23. If a high foot load carrying capacity is required, the number may be reduced to z_1 = 10. For the helix angle, β = 10 up to 15 degree is given, in exceptional cases also up to 30 degree. The addendum modification limits as shown in figure 9 are to be observed. On the pinion, the addendum modification coefficient should be within the range of x_1 = 0.2 to 0.6 and from $|u| > 2$ the width within the range b_1 = (0.35 to 0.45) a. Centre distance a is determined either by the required power to be transmitted or by the constructional conditions.

Table 2 Parameters for a cylindrical gear *)	
Output quantities: m_n mm normal module α_n degree normal pressure angle β degree reference helix angle z – number of teeth *) x – addendum modification coefficient x_E – generating addendum modification coefficient, see equation (3) h_{aPO} mm addendum of the tool	
Item	**Formula**
Transverse module	$m_t = \dfrac{m_n}{\cos\beta}$
Transverse pressure angle	$\tan\alpha_t = \dfrac{\tan\alpha_n}{\cos\beta}$
Base helix angle	$\sin\beta_b = \sin\beta \cos\alpha_n$
Reference diameter	$d = m_t\ z$
Tip diameter (k see table 3)	$d_a = d + 2\,m_n\ (1 + x + k)$
Root diameter	$d_f = d - 2\ (h_{aPO} - m_n\,x_E)$
Base diameter	$d_b = d \cos \alpha_t$
Transverse pitch	$p_t = \dfrac{\pi\,d}{z} = \pi\,m_t$
Transverse pitch on path of contact; Transverse base pitch	$p_{et} = p_{bt} = \dfrac{\pi\,d_b}{z} = p_t \cos\alpha_t$
Transverse pressure angle at tip circle	$\cos \alpha_{at} = \dfrac{d_b}{d_a}$
Transverse tooth thickness on the pitch circle	$s_t = m_t \left(\dfrac{\pi}{2} + 2\,x \tan\alpha_n\right)$
Normal tooth thickness on the pitch circle	$s_n = s_t \cos\beta$
Transverse tooth thickness on the addendum circle	$s_{at} = d_a \left(\dfrac{s_t}{d} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at}\right)$ **)
Virtual number of teeth	$z_n = \dfrac{z}{\cos\beta\ \cos^2\beta_b}$

*) For an internal gear, z is to be used as a negative quantity. **) For invα, see equation (1).

Table 3 Parameters for a cylindrical gear pair *)	
Output quantities: The parameters for pinion and wheel according to table 2 must be given, further the facewidths b_1 and b_2, as well as either the centre distance a or the sum of the addendum modification coefficients $x_1 + x_2$.	
Item	**Formula**
Gear ratio	$u = \dfrac{z_2}{z_1}$
Working transverse pressure angle ("a" given)	$\cos\alpha_{wt} = \dfrac{m_t}{2a}\ (z_1 + z_2)\ \cos\alpha_t$
Sum of the addendum modification coefficients ("a" given)	$x_1 + x_2 = \dfrac{z_1 + z_2}{2 \tan\alpha_n}\ (\mathrm{inv}\alpha_{wt} - \mathrm{inv}\alpha_t)$
Working transverse pressure angle ($x_1 + x_2$ given)	$\mathrm{inv}\alpha_{wt} = 2\ \dfrac{x_1 + x_2}{z_1 + z_2}\ \tan\alpha_n + \mathrm{inv}\alpha_t$
Centre distance ($x_1 + x_2$ given)	$a = \dfrac{m_t}{2}\ (z_1 + z_2)\ \dfrac{\cos\alpha_t}{\cos\alpha_{wt}}$
Reference centre distance	$a_d = \dfrac{m_t}{2}\ (z_1 + z_2)$
Addendum modification factor **)	$k = \dfrac{a - a_d}{m_n} - (x_1 + x_2)$
Working pitch circle diameter of the pinion	$d_{w1} = \dfrac{2a}{u + 1} = d_1\ \dfrac{\cos\alpha_t}{\cos\alpha_{wt}}$
Working pitch circle diameter of the gear	$d_{w2} = \dfrac{2au}{u + 1} = d_2\ \dfrac{\cos\alpha_t}{\cos\alpha_{wt}}$
Length of path of contact	$g_\alpha = \dfrac{1}{2}\left(\sqrt{d_{a1}^2 - d_{b1}^2} + \dfrac{u}{\lvert u\rvert}\sqrt{d_{a2}^2 - d_{b2}^2}\right) - a\sin\alpha_{wt}$
Transverse contact ratio	$\varepsilon_\alpha = \dfrac{g_\alpha}{p_{et}}$
Overlap ratio	$\varepsilon_\beta = \dfrac{b \tan\beta_b}{p_{et}}$ $b = \min\ (b_1,\ b_2)$
Total contact ratio	$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$

*) For internal gear pairs, z_2 and a are to be used as negative quantities.
**) See subsection 1.2.3.2.

1.2.5 Tooth corrections

The parameters given in the above subsections 1.2.1 to 1.2.4 refer to non-deviating cylindrical gears. Because of the high-tensile gear materials, however, a high load utilization of the gear units is possible. Noticeable deformations of the elastic gear unit components result from it.
The deflection at the tooth tips is, as a rule, a multiple of the manufacturing form errors. This leads to meshing interferences at the entering and leaving sides, see figure 14. There is a negative effect on the load carrying capacity and generation of noise.

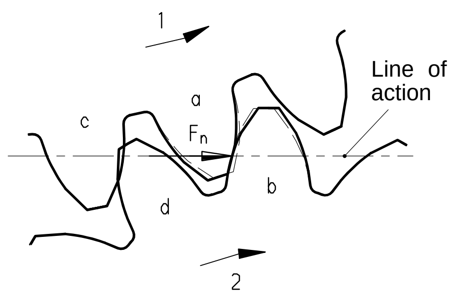


Figure 14 Cylindrical gear pair under load
1 Driving gear
2 Driven gear
a, b Tooth pair being in engagement
c, d Tooth pair getting into engagement

Further, the load causes bending and twisting of pinion and wheel shaft, pinion and wheel body, as well as settling of bearings, and housing deformations. This results in skewing of the tooth flanks which often amounts considerably higher than the tooth trace deviations caused by manufacture, see figure 15. Non-uniform load carrying occurs along the face width which also has a negative effect on the load carrying capacity and generation of noise.
The running-in wear of case hardened gears amounts to a few micrometers only and cannot compensate the mentioned deviations. In order to restore the high load carrying capacity of case hardened gears and reduce the generation of noise, intentional deviations from the involute (profile correction) and from the theoretical tooth trace (longitudinal correction) are produced in order to attain nearly ideal geometries with uniform load distribution under load again.
The load-related form corrections are calculated and made for one load only - as a rule for 70 ... 100% of the permanently acting nominal load - /5, 6, 7/. At low partial load, contact patterns which do not cover the entire tooth depth and facewidth are achieved. This has to be taken into consideration especially in the case of checks of contact patterns carried out under low loads. Under partial load, however, the local maximum load rise is always lower than the theoretical uniform load distribution under full load. In the case of modified gear teeth, the contact ratio is reduced under partial load because of incomplete carrying portions, making the noise generating levels increase in the lower part load range. With increasing load, the carrying portions and thus the contact ratio increase so that the generating levels drop. Gear pairs which are only slightly loaded do not require any modification.

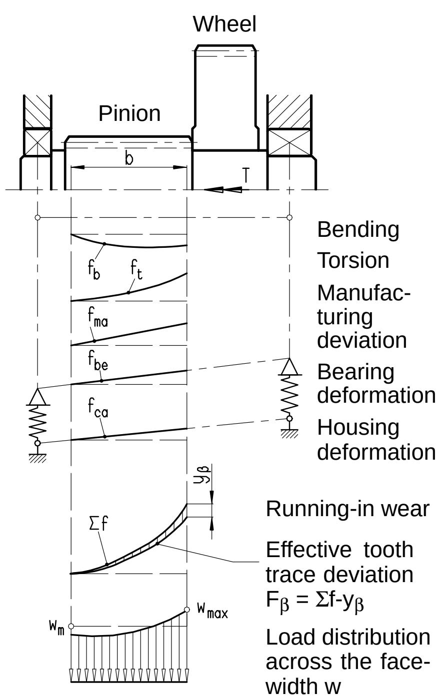


Figure 15 Deformations and manufacturing deviations on a gear unit shaft

In figure 16, usual profile and longitudinal corrections are illustrated. In the case of profile correction, the flanks on pinion and wheel are relieved at the tips by an amount equal to the length they are protruding at the entering and leaving sides due to the bending deflection of the teeth. Root relief may be applied instead of tip relief which, however, is much more expensive. Thus, a gradual load increase is achieved on the tooth get-

ting into engagement, and a load reduction on the tooth leaving the engagement. In the case of longitudinal correction, the tooth trace relief often is superposed by a symmetric longitudinal crowning. With it, uniform load carrying along the face width and a reduction in load concentration at the tooth ends during axial displacements is attained.

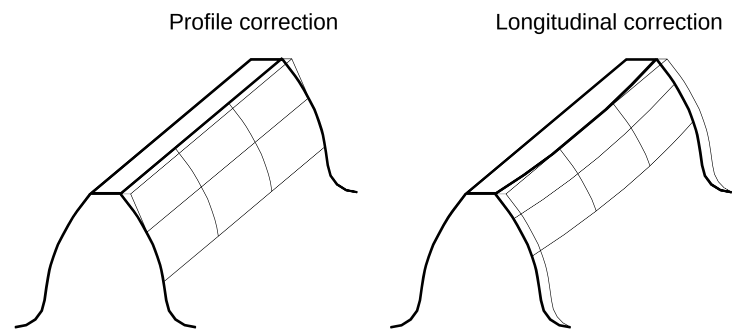


Figure 16
Tooth corrections designed for removing local load increases due to deformations under nominal load

1.3 Load carrying capacity of involute gears

1.3.1 Scope of application and purpose

The calculation of the load carrying capacity of cylindrical gears is generally carried out in accordance with the calculation method according to DIN 3990 /8/ (identical with ISO 6336) which takes into account pitting, tooth root bending stress and scoring as load carrying limits. Because of the relatively large scope of standards, the calculation in accordance with this method may be carried out only by using EDP programs. As a rule, gear unit manufacturers have such a tool at hand. The standard work is the FVA-Stirnradprogramm /9/ which includes further calculation methods, for instance, according to Niemann, AGMA, British Standard, and other.
In DIN 3990, different methods A, B, C ... are suggested for the determination of individual factors, where method A is more exact but also more time-consuming than method B, etc. The application standard /10/ according to DIN 3990 is based on simplified methods.
Because of its - even though limited - universal validity it still is relatively time-consuming.
The following calculation method for pitting resistance and tooth strength of case-hardened cylindrical gears is a further simplification if compared with the application standard, however, without losing some of its meaning. Certain conditions must be adhered to in order to attain high load carrying capacities which also results in preventing scuffing. Therefore, a calculation of load carrying capacity for scuffing will not be considered in the following.

It has to be expressly emphasized that for the load carrying capacity of gear units the exact calculation method - compared with the simplified one - is always more meaningful and therefore is exclusively decisive in borderline cases.
Design, selection of material, manufacture, heat treatment and operation of industrial gear units are subject to certain rules which lead to a long service life of the cylindrical gears. Those rules are:

- Gear teeth geometry acc. to DIN 3960;
- Cylindrical gears out of case-hardened steel; Tooth flanks in DIN quality 6 or better, fine machined;
- Quality of material and heat treatment proved by quality inspections acc. to DIN 3990 /11/;
- Effective case depth after carburizing according to instructions /12/ with surface hardnesses of 58 ... 62 HRC;
- Gears with required tooth corrections and without harmful notches in the tooth root;
- Gear unit designed for fatigue strength, i.e. life factors $Z_{NT} = Y_{NT} = 1.0$;
- Flank fatigue strength $\sigma_{Hlim} \geqq 1{,}200$ N/mm^2;
- Subcritical operating range, i.e. pitch circle velocity lower than approx. 35 m/s;
- Sufficient supply of lubricating oil;
- Use of prescribed gear oils with sufficient scuffing load capacity (criteria stage ≥ 12) and grey staining load capacity (criteria stage ≥ 10);
- Maximum operating temperature 95 °C.

If these requirements are met, a number of factors can be definitely given for the calculation of the load carrying capacity according to DIN 3990, so that the calculation procedure is partly considerably simplified. Non-observance of the above requirements, however, does not necessarily mean that the load carrying capacity is reduced. In case of doubt one should, however, carry out the calculation in accordance with the more exact method.

1.3.2 Basic details

The calculation of the load carrying capacity is based on the nominal torque of the driven machine. Alternatively, one can also start from the nominal torque of the prime mover if this corresponds with the torque requirement of the driven machine.
In order to be able to carry out the calculation for a cylindrical gear stage, the details listed in table 4 must be given in the units mentioned in the table. The geometric quantities are calculated according to tables 2 and 3. Usually, they are contained in the workshop drawings for cylindrical gears.

Table 4 Basic details

Abbreviation	Meaning	Unit
P	Power rating	kW
n_1	Pinion speed	1/min
a	Centre distance	mm
m_n	Normal module	mm
d_{a1}	Tip diameter of the pinion	mm
d_{a2}	Tip diameter of the wheel	mm
b_1	Facewidth of the pinion	mm
b_2	Facewidth of the wheel	mm
z_1	Number of teeth of the pinion	–
z_2	Number of teeth of the wheel	–
x_1	Addendum modification coefficient of the pinion	–
x_2	Addendum modification coefficient of the wheel	–
α_n	Normal pressure angle	Degree
β	Reference helix angle	Degree
V_{40}	Kinematic viscosity of lubricating oil at 40 °C	cSt
R_{z1}	Peak-to-valley height on pinion flank	µm
R_{z2}	Peak-to-valley height on wheel flank	µm

In the further course of the calculation, the quantities listed in table 5 are required. They are derived from the basic details according to table 4.

Table 5 Derived quantities

Designation	Relation	Unit
Gear ratio	$u = z_2/z_1$	–
Reference diameter of the pinion	$d_1 = z_1 \, m_n/\cos\beta$	mm
Transverse tangential force at pinion reference circle	$F_t = 19.1 \cdot 10^6 \, P/(d_1 \, n_1)$	N
Circumferential speed at reference circle	$v = \pi \, d_1 \, n_1/60\,000$	m/s
Base helix angle	$\beta_b = \arc \sin(\cos\alpha_n \sin\beta)$	Degree
Virtual number of teeth of the pinion	$z_{n1} = z_1 / (\cos\beta \cos^2\beta_b)$	–
Virtual number of teeth of the wheel	$z_{n2} = z_2 / (\cos\beta \cos^2\beta_b)$	–
Transverse module	$m_t = m_n / \cos\beta$	mm
Transverse pressure angle	$\alpha_t = \arc \tan (\tan\alpha_n / \cos\beta)$	Degree
Working transverse pressure angle	$\alpha_{wt} = \arc \cos \left[(z_1 + z_2) \, m_t \cos\alpha_t / (2a)\right]$	Degree
Transverse pitch	$p_{et} = \pi m_t \cos\alpha_t$	mm
Base diameter of the pinion	$d_{b1} = z_1 m_t \cos\alpha_t$	mm
Base diameter of the wheel	$d_{b2} = z_2 m_t \cos\alpha_t$	mm
Length of path of contact	$g_\alpha = \frac{1}{2}\left(\sqrt{d_{a1}^2 - d_{b1}^2} + \frac{u}{\lvert u \rvert}\sqrt{d_{a2}^2 - d_{b2}^2}\right) - a\sin\alpha_{wt}$	mm
Transverse contact ratio	$\varepsilon_\alpha = g_\alpha / p_{et}$	–
Overlap ratio	$\varepsilon_\beta = b \tan\beta_b / p_{et} \quad b = \min(b_1, b_2)$	–

1.3.3 General factors

1.3.3.1 Application factor

With the application factor K_A, all additional forces acting on the gears from external sources are taken into consideration. It is dependent on the characteristics of the driving and driven machines, as well as the couplings, the masses and stiffness of the system, and the operating conditions.

The application factor is determined by the service classification of the individual gear. If possible, the factor K_A should be determined by means of a careful measurement or a comprehensive analysis of the system. Since very often it is not possible to carry out the one or other method without great expenditure, reference values are given in table 6 which equally apply to all gears in a gear unit.

Table 6 Application factor K_A

Working mode of prime mover	Working mode of the driven machine			
	Uniform	Moderate shock loads	Average shock loads	Heavy shock loads
Uniform	1.00	1.25	1.50	1.75
Moderate shock loads	1.10	1.35	1.60	1.85
Average shock loads	1.25	1.50	1.75	2.00 or higher
Heavy shock loads	1.50	1.75	2.00	2.25 or higher

1.3.3.2 Dynamic factor

With the dynamic factor K_V, additional dynamic forces caused in the meshing itself are taken into consideration. Taking z_1, v and u from tables 4 and 5, it is calculated from

$$K_v = 1 + 0.0003\, z_1\, v \sqrt{\frac{u^2}{1 + u^2}} \qquad (4)$$

1.3.3.3 Face load factor

The face load factor $K_{H\beta}$ takes into account the increase in the load on the tooth flanks caused by non-uniform load distribution over the facewidth. According to /8/, it can be determined by means of different methods. Exact methods based on comprehensive measurements or calculations or on a combination of both are very expensive. Simple methods, however, are not exact, as a consequence of which estimations made to be on the safe side mostly result in higher factors.

For normal cylindrical gear teeth without longitudinal correction, the face load factor can be calculated according to method D in accordance with /8/ dependent on facewidth b and reference diameter d_1 of the pinion, as follows:

$$K_{H\beta} = 1.15 + 0.18\,(b/d_1)^2 + 0.0003\, b \qquad (5)$$

with b = min (b_1, b_2). As a rule, the gear unit manufacturer carries out an analysis of the load distribution over the facewidth in accordance with an exact calculation method /13/. If required, he makes longitudinal corrections in order to attain uniform load carrying over the facewidth, see subsection 1.2.5. Under such conditions, the face load factor lies within the range of $K_{H\beta}$ = 1.1 ... 1.25. As a rough rule applies: A sensibly selected crowning symmetrical in length reduces the amount of $K_{H\beta}$ lying above 1.0 by approx. 40 to 50%, and a directly made longitudinal correction by approx. 60 to 70%.

In the case of slim shafts with gears arranged on one side, or in the case of lateral forces or moments acting on the shafts from external sources, for the face load factors for gears without longitudinal correction the values may lie between 1.5 and 2.0 and in extreme cases even at 2.5.

Face load factor $K_{F\beta}$ for the determination of increased tooth root stress can approximately be deduced from face load factor $K_{H\beta}$ according to the relation

$$K_{F\beta} = (K_{H\beta})^{0.9} \qquad (6)$$

1.3.3.4 Transverse load factors

The transverse load factors $K_{H\alpha}$ and $K_{F\alpha}$ take into account the effect of the non-uniform distribution of load between several pairs of simultaneously contacting gear teeth. Under the conditions as laid down in subsection 1.3.1, the result for surface stress and for tooth root stress according to method B in accordance with /8/ is

$$K_{H\alpha} = K_{F\alpha} = 1.0 \qquad (7)$$

1.3.4 Tooth flank load carrying capacity

The calculation of surface durability against pitting is based on the Hertzian pressure at the pitch circle. For pinion and wheel the same effective Hertzian pressure σ_H is assumed. It must not exceed the permissible Hertzian pressure σ_{Hp}, i.e. $\sigma_H + \sigma_{Hp}$.

1.3.4.1 Effective Hertzian pressure

The effective Hertzian pressure is dependent on the load, and for pinion and wheel is equally derived from the equation

$$\sigma_H = Z_E\; Z_H\; Z_\beta\; Z_\varepsilon \sqrt{K_A\; K_v\; K_{H\alpha}\; K_{H\beta}\; \frac{u+1}{u}\; \frac{F_t}{d_1\; b}} \qquad (8)$$

σ_H Effective Hertzian pressure in N/mm^2

Further:

- b is the smallest facewidth b_1 or b_2 of pinion or wheel acc. to table 4
- F_t, u, d_1 acc. to table 5
- K_A Application factor acc. to table 6
- K_V Dynamic factor acc. to equation (4)
- $K_{H\beta}$ Face load factor acc. to equ. (5)
- $K_{H\alpha}$ Transverse load factor acc. to equ. (7)
- Z_E Elasticity factor; $Z_E = 190 \sqrt{N/mm^2}$ for gears out of steel
- Z_H Zone factor acc. to figure 17
- Z_β Helix angle factor acc. to equ. (9)
- Z_ε Contact ratio factor acc. to equ. (10) or (11)

With ß according to table 4 applies:

$$Z_\beta = \sqrt{\cos\beta} \qquad (9)$$

With ε_α and ε_β according to table 5 applies:

$$Z_\varepsilon = \sqrt{\frac{4 - \varepsilon_\alpha}{3}\,(1 - \varepsilon_\alpha) + \frac{\varepsilon_\beta}{\varepsilon_\alpha}} \quad \text{for } \varepsilon_\beta < 1 \qquad (10)$$

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \quad \text{for } \varepsilon_\beta = 1 \qquad (11)$$

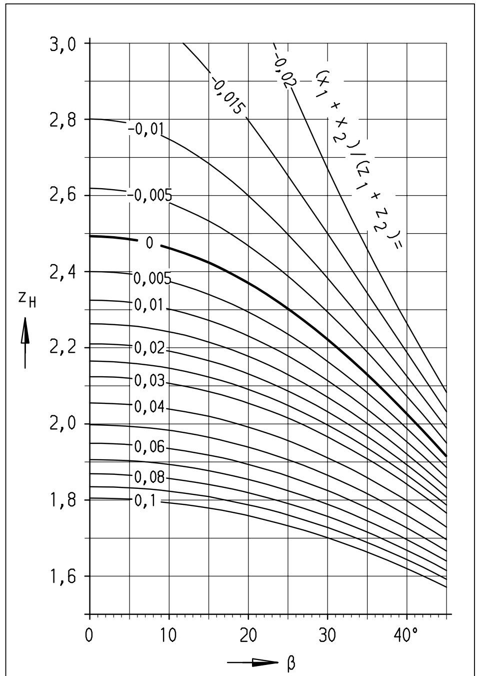


Figure 17
Zone factor Z_H depending on helix angle β as well as on the numbers of teeth z_1, z_2, and addendum modification coefficients x_1, x_2; see table 4.

1.3.4.2 Permissible Hertzian pressure

The permissible Hertzian pressure is determined by

$$\sigma_{HP} = Z_L\; Z_v\; Z_X\; Z_R\; Z_W\; \frac{\sigma_{Hlim}}{S_H} \qquad (12)$$

σ_{HP} permissible Hertzian pressure in N/mm^2. It is of different size for pinion and wheel if the strengths of materials σ_{Hlim} are different. Factors Z_L, Z_v, Z_R, Z_W and Z_X are the same for pinion and wheel and are determined in the following.

The lubricant factor is computed from the lubricating oil viscosity V_{40} according to table 4 using the following formula:

$$Z_L = 0.91 + \frac{0.25}{\left(1 + \frac{112}{V_{40}}\right)^2} \qquad (13)$$

For the speed factor, the following applies using the circumferential speed v according to table 5:

$$Z_v = 0.93 + \frac{0.157}{\sqrt{1 + \frac{40}{v}}} \qquad (14)$$

The roughness factor can be determined as a function of the mean peak-to-valley height $R_Z = (R_{Z1} + R_{Z2})/2$ of the gear pair as well as the gear ratio u and the reference diameter d_1 of the pinion, see tables 4 and 5, from

$$Z_R = \left[\frac{0.513}{R_z} \sqrt[3]{(1 + |u|)\, d_1} \right]^{0.08} \qquad (15)$$

For a gear pair with the same tooth flank hardness on pinion and wheel, the work hardening factor is

$$Z_W = 1.0 \qquad (16)$$

The size factor is computed from module m_n according to table 4 using the following formula:

$$Z_X = 1.05 - 0.005\, m_n \qquad (17)$$

with the restriction $0.9 \leqq Z_X \leqq 1$.

σ_{Hlim} Endurance strength of the gear material. For gears made out of case hardening steel, case hardened, figure 18 shows a range from 1300 ... 1650 N/mm² depending on the surface hardness of the tooth flanks and the quality of the material. Under the conditions as described in subsection 1.3.1, material quality MQ may be selected for pinion and wheel, see table on page 97.

S_H required safety factor against pitting, see subsection 1.3.6.

1.3.5 Tooth strength

The maximum load in the root fillet at the 30-degree tangent is the basis for rating the tooth strength. For pinion and wheel it shall be shown separately that the effective tooth root stress σ_F does not exceed the permissible tooth root stress σ_{FP}, i.e. $\sigma_F < \sigma_{FP}$.

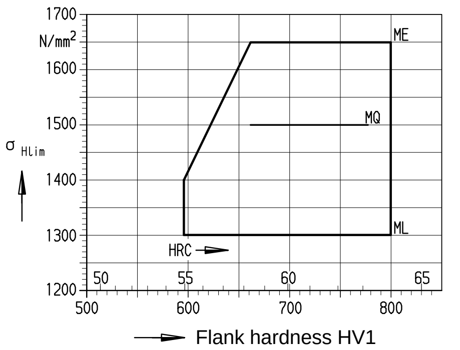


Figure 18
Allowable stress number for contact stress σ_{Hlim} of alloyed case hardening steels, case hardened, depending on the surface hardness HV1 of the tooth flanks and the material quality.
ML modest demands on the material quality
MQ normal demands on the material quality
ME high demands on the material quality, see /11/

1.3.5.1 Effective tooth root stress

As a rule, the load-dependent tooth root stresses for pinion and wheel are different. They are calculated from the following equation:

$$\sigma_F = Y_\varepsilon\, Y_\beta\, Y_{FS}\, K_A\, K_v\, K_{F\alpha}\, K_{F\beta}\, \frac{F_t}{b\, m_n} \qquad (18)$$

σ_F Effective tooth root stress in N/mm²

The following factors are of equal size for pinion and wheel:

- m_n, F_t acc. to tables 4 and 5
- K_A Application factor acc. to table 6
- K_V Dynamic factor acc. to equation (4)
- $K_{F\beta}$ Face load factor acc. to equation (6)
- $K_{F\alpha}$ Transverse load factor acc. to equ. (7)
- Y_ε Contact ratio factor acc. to equ. (19)
- Y_β Helix angle factor acc. to equ. (20)

The following factors are of different size for pinion and wheel:

b_1, b_2 Facewidths of pinion and wheel acc. to table 4. If the facewidths of pinion and wheel are different, it may be assumed that the load bearing width of the wider facewidth is equal to the smaller facewidth plus such extension of the wider that does not exceed one times the module at each end of the teeth.

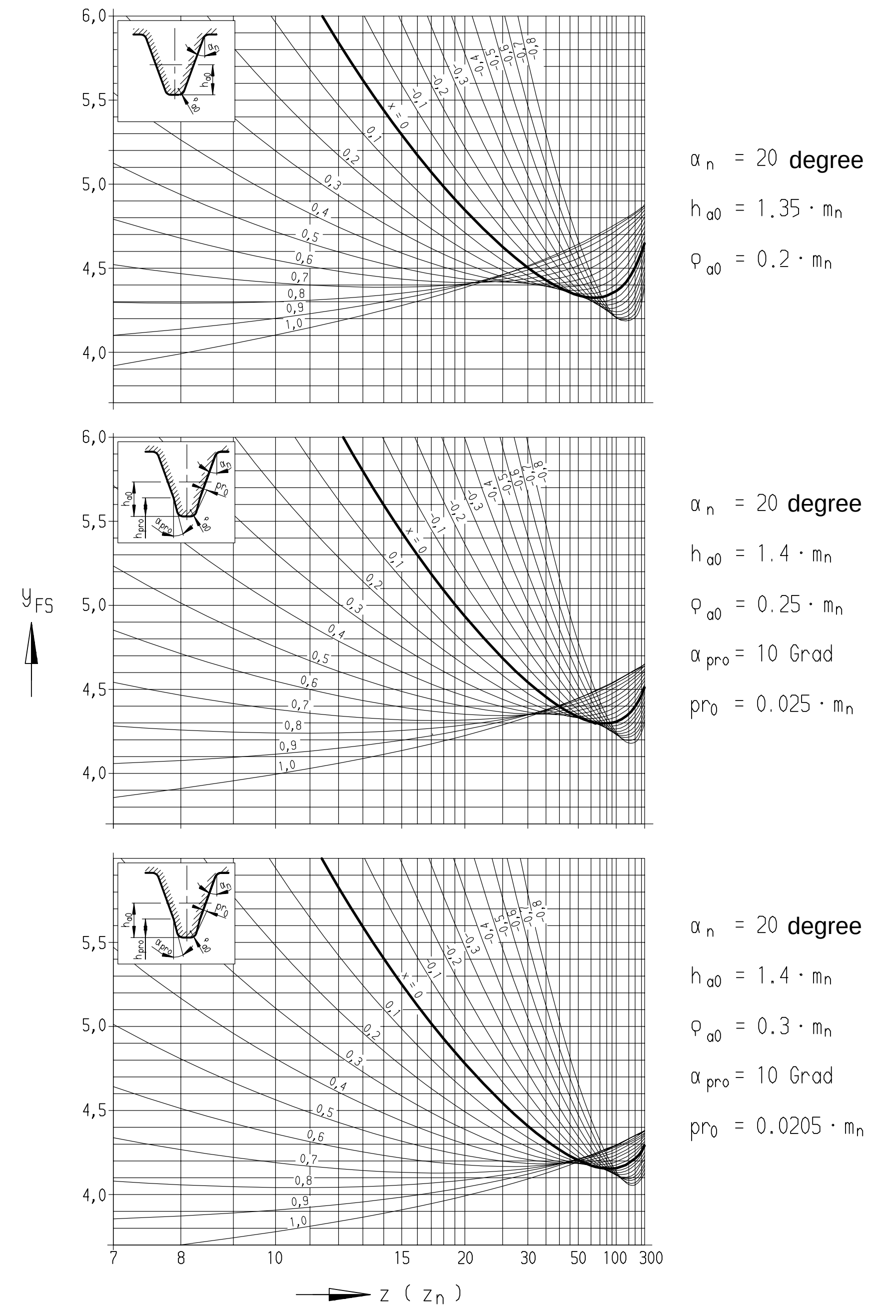


Figure 19
Tip factor Y_{FS} for external gears with standard basic rack tooth profile acc. to DIN 867 depending on the number of teeth z (or z_n in case of helical gears) and addendum modification coefficient x, see tables 4 and 5. The following only approximately applies to internal gears: $Y_{FS} = Y_{FS\infty}$ (≈ value for x = 1.0 and z = 300).

Y_{FS1}, Y_{FS2} Tip factors acc. to figure 19. They account for the complex stress condition inclusive of the notch effect in the root fillet.

With the helix angle β acc. to table 4 and the overlap ratio ε_β acc. to table 5 follows:

$$Y\varepsilon = 0.25 + \frac{0.75}{\varepsilon_\alpha} \cos^2\beta \qquad (19)$$

with the restriction 0.625 + Y_ε + 1

$$Y_\beta = 1 - \frac{\varepsilon_\beta\, \beta}{120} \qquad (20)$$

with the restriction
Y_β = max [(1 - 0.25 ε_β); (1– β/120)].

1.3.5.2 Permissible tooth root stress

The permissible tooth root stress for pinion and wheel is determined by

$$\sigma_{FP} = Y_{ST}\ Y_{\delta relT}\ Y_{RrelT}\ Y_X\ \frac{\sigma_{Flim}}{(S_F)} \qquad (21)$$

σ_{FP} permissible tooth root stress in N/mm².
It is not equal for pinion and wheel if the material strengths σ_{Flim} are not equal. Factors Y_{ST}, $Y_{\delta relT}$, Y_{RrelT} and Y_X may be approximately equal for pinion and wheel.

Y_{ST} is the stress correction factor of the reference test gears for the determination of the bending stress number σ_{Flim}. For standard reference test gears, Y_{ST} = 2.0 has been fixed in the standard.

$Y_{\delta relT}$ is the notch relative sensitivity factor (notch sensitivity of the material) referring to the standard reference test gear. By approximation $Y_{\delta relT}$ = 1.0.

For the relative surface factor (surface roughness factor of the tooth root fillet) referring to the standard reference test gear the following applies by approximation, depending on module m_n:

$$\begin{aligned} Y_{RrelT} &= 1.00 \text{ for } && m_n + 8 \text{ mm} \\ &= 0.98 \text{ for } 8 \text{ mm} < && m_n + 16 \text{ mm} \qquad (22) \\ &= 0.96 \text{ for } && m_n > 16 \text{ mm} \end{aligned}$$

and for the size factor

$$Y_X = 1.05 - 0.01\ m_n \qquad (23)$$

with the restriction 0.8 + Y_X + 1.

σ_{Flim} Bending stress number of the gear material. For gears out of case hardening steel, case hardened, a range from 310 ... 520 N/mm² is shown in figure 20 depending on the surface hardness of the tooth flanks and the material quality. Under the conditions according to subsection 1.3.1, a strength pertaining to quality MQ may be used as a basis for pinion and wheel see table on page 97.

S_F Safety factor required against tooth breakage, see subsection 1.3.6.

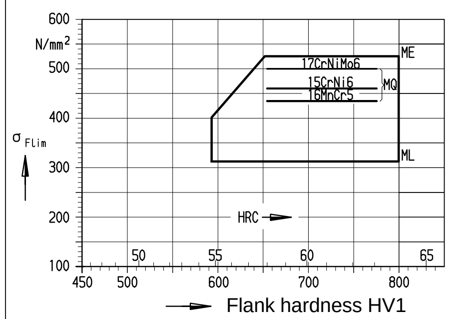


Figure 20
Bending stress number σ_{Flim} of alloyed case hardening steel, case hardened, depending on the surface hardness HV1 of the tooth flanks and the material quality.
ML modest demands on the material quality
MQ normal demands on the material quality
ME high demands on the material quality, see /11/

1.3.6 Safety factors

The minimum required safety factors according to DIN are:
against pitting S_H = 1.0
against tooth breakage S_F = 1.3.

In practice, higher safety factors are usual. For multistage gear units, the safety factors are determined about 10 to 20% higher for the expensive final stages, and in most cases even higher for the cheaper preliminary stages.

Also for risky applications a higher safety factor is given.

1.3.7 Calculation example

An electric motor drives a coal mill via a multi-stage cylindrical gear unit. The low speed gear stage is to be calculated.

Given: Nominal power rating P = 3300 kW; pinion speed n_1 = 141 1/min.; centre distance a = 815 mm; normal module m_n = 22 mm; tip diameter d_{a1} = 615.5 mm and d_{a2} = 1100 mm; pinion and wheel widths b_1 = 360 mm and b_2 = 350 mm; numbers of teeth z_1 = 25 and z_2 = 47; addendum modification coefficients x_1 = 0.310 and x_2 = 0.203; normal pressure angle α_n = 20 degree;

helix angle β = 10 degree; kinematic viscosity of the lubricating oil V_{40} = 320 cSt; mean peak-to-valley roughness R_{z1} = R_{z2} = 4.8 µm.
The cylindrical gears are made out of the material 17 CrNiMo 6. They are case hardened and ground with profile corrections and width-symmetrical crowning.

Calculation (values partly rounded):
Gear ratio u = 1.88; reference diameter of the pinion d_1 = 558.485 mm; nominal circumferential force on the reference circle F_t = 800,425 N; circumferential speed on the reference circle v = 4.123 m/s; base helix angle β_b = 9.391 degree; virtual numbers of teeth z_{n1} = 26.08 and z_{n2} = 49.03; transverse module m_t = 22.339 mm; transverse pressure angle α_t = 20.284 degree; working transverse pressure angle α_{wt} = 22.244 degree; normal transverse pitch p_{et} = 65.829; base diameters d_{b1} = 523.852 mm and d_{b2} = 984.842 mm; length of path of contact g_α = 98.041 mm; transverse contact ratio ε_α = 1.489; overlap ratio ε_β = 0.879.
Application factor K_A = 1.50 (electric motor with uniform mode of operation, coal mill with medium shock load); dynamic factor K_V = 1.027; face load factor $K_{H\beta}$ = 1.20 [acc. to equation (5) follows $K_{H\beta}$ = 1.326, however, because of symmetrical crowning the calculation may be made with a smaller value]; $K_{F\beta}$ = 1.178; $K_{H\alpha}$ = $K_{F\alpha}$ = 1.0.

Load carrying capacity of the tooth flanks:
Elasticity factor $Z_E = 190\ \sqrt{N\ mm^2}$; zone factor Z_H = 2.342; helix angle factor Z_β = 0.992; contact ratio factor Z_ε = 0.832. According to equation (8), the Hertzian pressure for pinion and wheel is σ_H = 1251 N/mm².
Lubricant factor Z_L = 1.047; speed factor Z_V = 0.978; roughness factor Z_R = 1.018; work hardening factor Z_W = 1.0; size factor Z_X = 0.94. With the allowable stress number for contact stress (pitting) σ_{Hlim} = 1500 N/mm², first the permissible Hertzian pressure σ_{HP} = 1470 N/mm² is determined from equation (12) without taking into account the safety factor.
The safety factor against pitting is found by S_H = σ_{HP}/σ_H = 1470/1251 = 1.18. The safety factor referring to the torque is S_H^2 = 1.38.

Load carrying capacity of the tooth root:
Contact ratio factor Y_ε = 0.738; helix angle factor Y_β = 0.927; tip factors Y_{FS1} = 4.28 and Y_{FS2} = 4.18 (for h_{a0} = 1.4 m_n; φ_{a0} = 0.3 m_n; α_{pro} = 10 degree; p_{rO} = 0.0205 m_n). The effective tooth root stresses σ_{F1} = 537 N/mm² for the pinion and σ_{F2} = 540 N/mm² for the wheel can be obtained from equation (18).
Stress correction factor Y_{ST} = 2.0; relative sensitivity factor $Y_{\delta relT}$ = 1.0; relative surface factor Y_{RelT} = 0.96; size factor Y_X = 0.83. Without taking into consideration the safety factor, the permissible tooth root stresses for pinion and wheel σ_{FP1} = σ_{FP2} = 797 N/mm² can be obtained from equation (21) with the bending stress number σ_{Flim} = 500 N/mm².
The safety factors against tooth breakage referring to the torque are $S_F = \sigma_{FP}/\sigma_F$: for the pinion S_{F1} = 797/537 = 1.48 and for the wheel S_{F2} = 797/540 = 1.48.

1.4 Gear unit types

1.4.1 Standard designs

In the industrial practice, different types of gear units are used. Preferably, standard helical and bevel-helical gear units with fixed transmission ratio and size gradation are applied. These single-stage to four-stage gear units according to the modular construction system cover a wide range of speeds and torques required by the driven machines. Combined with a standard electric motor such gear units are, as a rule, the most economical drive solution.
But there are also cases where no standard drives are used. Among others, this is true for high torques above the range of standard gear units. In such cases, special design gear units are used, load sharing gear units playing an important role there.

1.4.2 Load sharing gear units

In principle, the highest output torques of gear units are limited by the manufacturing facilities, since gear cutting machines can make gears up to a maximum diameter only. Then, the output torque can be increased further only by means of load sharing in the gear unit. Load sharing gear units are, however, also widely used for lower torques as they provide certain advantages in spite of the larger number of internal components, among others they are also used in standard design. Some typical features of the one or other type are described in the following.

1.4.3 Comparisons

In the following, single-stage and two-stage gear units up to a ratio of i = 16 are examined. For common gear units the last or the last and the last but one gear stage usually come to approx. 70 to 80% of the total weight and also of the manufacturing expenditure. Adding further gear stages in order to achieve higher transmission ratios thus does not change anything about the following fundamental description.
In figure 21, gear units without and with load sharing are shown, shaft 1 each being the HSS and shaft 2 being the LSS. With speeds n_1 and n_2, the transmission ratio can be obtained from the formula

$$i = n_1 / n_2 \qquad (24)$$

The diameter ratios of the gears shown in figure 21 correspond to the transmission ratio i = 7. The gear units have the same output torques, so that in figure 21 a size comparison to scale is illustrated. Gear units A, B, and C are with offset shaft arrangement, and gear units D, E, F, and G with coaxial shaft arrangement.

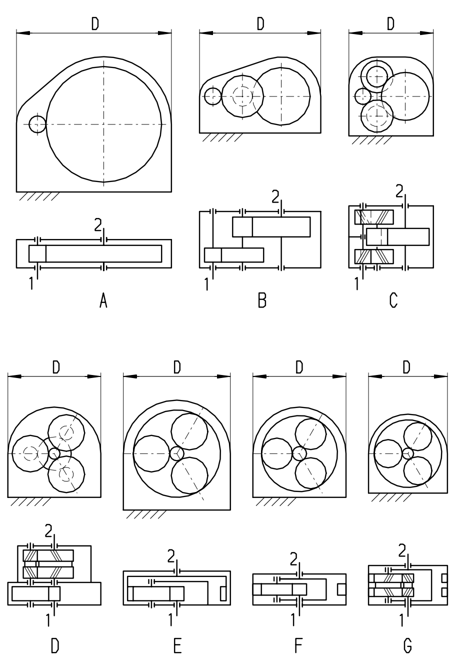


Figure 21
Diagrammatic view of cylindrical gear unit types without and with load sharing. Transmission ratio i = 7. Size comparison to scale of gear units with the same output torque.

Gear unit A has one stage, gear unit B has two stages. Both gear units are without load sharing. Gear units C, D, E, F, and G have two stages and are load sharing. The idler gears in gear units C and D have different diameters. In gear units E, F, and G the idler gears of one shaft have been joined to one gear so that they are also considered to be single-stage gear units.
Gear unit C has double load sharing. Uniform load distribution is achieved in the high-speed gear stage by double helical teeth and the axial movability of shaft 1.

In gear unit D the load of the high-speed gear stage is equally shared between three intermediate gears which is achieved by the radial movability of the sun gear on shaft 1. In the low-speed gear stage the load is shared six times altogether by means of the double helical teeth and the axial movability of the intermediate shaft.
In order to achieve equal load distribution between the three intermediate gears of gear units E, F, and G the sun gear on shaft 1 mostly is radially movable. The large internal gear is an annulus gear which in the case of gear unit E is connected with shaft 2, and in the case of gear units F and G with the housing. In gear units F and G, web and shaft 2 form an integrated whole. The idler gears rotate as planets around the central axle. In gear unit G, double helical teeth and axial movability of the idler gears guarantee equal load distribution between six branches.

1.4.3.1 Load value

By means of load value B_L, it is possible to compare cylindrical gear units with different ultimate stress values of the gear materials with each other in the following examinations.
According to /14/, the load value is the tooth peripheral force F_u referred to the pinion pitch diameter d_w and the carrying facewidth b, i.e.

$$B_L = \frac{F_u}{b\ d_w} \qquad (25)$$

The permissible load values of the meshings of the cylindrical gear units can be computed from the pitting resistance by approximation, as shown in /15/ (see section 1.3.4), using the following formula:

$$B_L \approx 7 \cdot 10^{-6}\ \frac{u}{u+1}\ \frac{\sigma^2_{Hlim}}{K_A\ S_H^2} \qquad (26)$$

with B_L in N/mm^2 and allowable stress number for contact stress (pitting) σ_{Hlim} in N/mm^2 as well as gear ratio u, application factor K_A and factor of safety from pitting S_H. The value of the gear ratio u is always greater than 1, and is negative for internal gear pairs (see table 3).
Load value B_L is a specific quantity and independent of the size of the cylindrical gear unit. The following applies for practically executed gear units: cylindrical gears out of case hardening steel B_L = 4...6 N/mm^2; cylindrical gears out of quenched and tempered steel B_L = 1...1.5 N/mm^2; planetary gear stages with annulus gears out of quenched and tempered steel, planet gears and sun gears out of case hardening steel B_L = 2.0...3.5 N/mm^2.

1.4.3.2 Referred torques

In figure 22, referred torques for the gear units shown in figure 21 are represented, dependent on the transmission ratio i. Further explanations are given in table 7. The torque T_2 is referred to the construction dimension D when comparing the sizes, to the weight of the gear unit G when comparing the weights, and to the generated surface A of the pitch circle cylinders when comparing the gear teeth surfaces. Gear unit weight G and gear teeth surface A (= generated surface) are one measure for the manufacturing cost. The higher a curve, in figure 22, the better the respective gear unit in comparison with the others.

Table 7 Referred Torques

Comparison criteria	Definition	Dimension	Units of the basic details
Size	$\delta = \frac{T_2}{D^3\ B_L}$	$\frac{m}{mm}$	T_2 in mm B_L in N/mm^2 D in mm G in kg A in m^2
Weight	$\gamma = \frac{T_2}{G\ B_L}$	$\frac{m\ mm^2}{kg}$	
Gear teeth surface	$\alpha = \frac{T_2}{A^{3/2}\ B_L}$	$\frac{mm^2}{m^2}$	

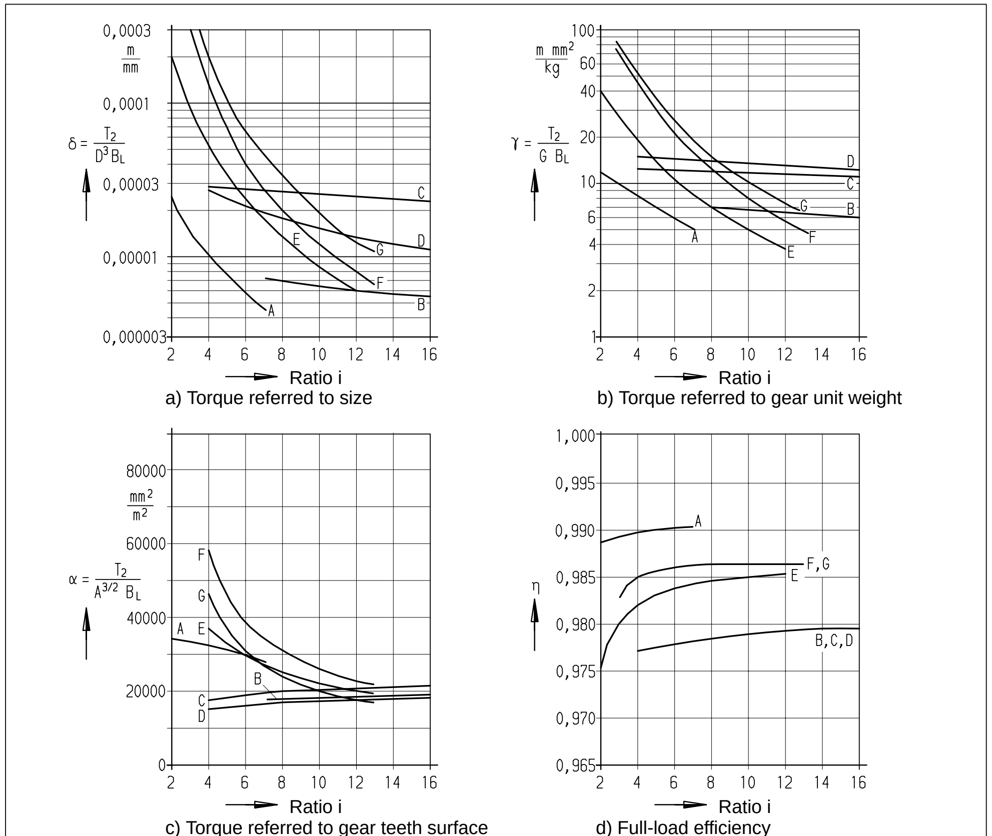


Figure 22
Comparisons of cylindrical gear unit types in figure 21 dependent on the transmission ratio i. Explanations are given in table 7 as well as in the text.

For all gear units explained in figures 21 and 22, the same prerequisites are valid. For all gear units, the construction dimension D is larger than the sum of the pitch diameters by the factor 1.15. Similar definitions are valid for gear unit height and width. Also the wall thickness of the housing is in a fixed relation to the construction dimension D /15/.
With a given torque T_2 and with a load value B_L computed according to equation (26), the construction dimension D, the gear unit weight G, and the gear teeth surface A can be determined by approximation by figure 22 for a given transmission ratio i. However, the weights of modular-type gear units are usually higher, since the housing dimensions are determined according to different points of view.
Referred to size and weight, planetary gear units F and G have the highest torques at small ratios i. For ratios i < 4, the planetary gear becomes the pinion instead of the sun gear. Space requirement and load carrying capacity of the planetary gear bearings decrease considerably. Usually, the planetary gear bearings are arranged in the planet carrier for ratio i < 4.5.
Gear units C and D, which have only external gears, have the highest torque referred to size and weight for ratios above $i \approx 7$. For planetary gear units, the torque referred to the gear teeth surface is more favourable only in case of small ratios, if compared with other gear units. It is to be taken into consideration, however, that internal gears require higher manufacturing expenditure than external gears for the same quality of manufacture.
The comparisons show that there is no optimal gear unit available which combines all advantages over the entire transmission ratio range. Thus, the output torque referred to size and weight is the most favourable for the planetary gear unit, and this all the more, the smaller the transmission ratio in the planetary gear stage. With increasing ratio, however, the referred torque decreases considerably. For ratios above i = 8, load sharing gear units having external gears only are more favourable because with increasing ratio the referred torque decreases only slightly.
With regard to the gear teeth surface, planetary gear units do not have such big advantages if compared to load sharing gear units having external gears only.

1.4.3.3 Efficiencies

When comparing the efficiency, figure 22d, only the power losses in the meshings are taken into consideration. Under full load, they come to approx. 85% of the total power loss for common cylindrical gear units with rolling bearings. The efficiency as a quantity of energy losses results from the following relation with the input power at shaft 1 and the torques T_1 and T_2

$$\eta + \frac{1}{i} = \frac{T_2}{T_1} = \qquad (27)$$

All gear units shown in figure 21 are based on the same coefficient of friction of tooth profile μz = 0.06. Furthermore, gears without addendum modification and numbers of teeth of the pinion z = 17 are uniformly assumed for all gear units /15/, so that a comparison is possible.
The single stage gear unit A has the best efficiency. The efficiencies of the two stage gear units B, C, D, E, F, and G are lower because the power flow passes two meshings. The internal gear pairs in gear units E, F, and G show better efficiencies owing to lower sliding velocities in the meshings compared to gear units B, C, and D which only have external gear pairs.
The lossfree coupling performance of planetary gear units F and G results in a further improvement of the efficiency. It is therefore higher than that of other comparable load sharing gear units. For higher transmission ratios, however, more planetary gear stages are to be arranged in series so that the advantage of a better efficiency compared to gear units B, C, and D is lost.

1.4.3.4 Example

Given: Two planetary gear stages of type F arranged in series, total transmission ratio i = 20, output torque $T_2 = 3 \cdot 10^6$ Nm, load value $B_L = 2.3$ N/mm^2. A minimum of weight is approximately achieved by a transmission ratio division of $i = 5 \cdot 4$ of the HS and LS stage. At $\gamma_1 = 30$ m mm^2/kg and $\gamma_2 = 45$ m mm^2/kg according to figure 22b, the weight for the HS stage is approximately 10.9 t and for the LS stage approximately 30 t, which is a total 40.9 t. The total efficiency according to figure 22d is $\eta = 0.986 \cdot 0.985 = 0.971$.
In comparison to a gear unit of type D with the same transmission ratio i = 20 and the same output torque $T_2 = 3 \cdot 10^6$ Nm, however, with a better load value $B_L = 4$ N/mm^2 this gear unit has a weight of 68.2 t according to figure 22 with $\gamma = 11$ m mm^2/kg and is thus heavier by 67%. The advantage is a better efficiency of $\eta = 0.98$. The two planetary gear stages of type F together have a power loss which is by 45% higher than that of the gear unit of type D. In addition, there is not enough space for the rolling bearings of the planet gears in the stage with i = 4.

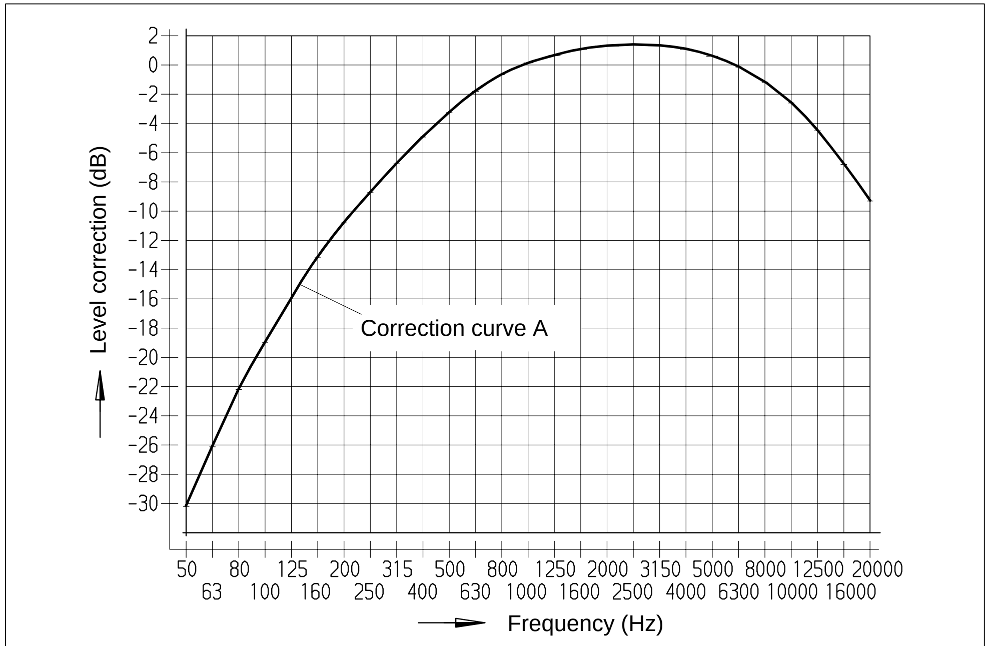


Figure 23
Correction curve according to DIN 45635 /16/ for the A-weighted sound power level or sound pressure level

1.5 Noise emitted by gear units

1.5.1 Definitions

Noise emitted by a gear unit - like all other noises - is composed of tones having different frequencies f.
Measure of intensity is the sound pressure p which is the difference between the highest (or lowest) and the mean pressure in a sound wave detected by the human ear.
The sound pressure can be determined for a single frequency or - as a combination - for a frequency range (single-number rating). It is dependent on the distance to the source of sound.
In general, no absolute values are used but amplification or level quantities in bel (B) or decibel (dB). Reference value is, for instance, the sound pressure at a threshold of audibility $p_o = 2 \cdot 10^{-5}$ N/m^2.
In order to take into consideration the different sensitivities of the human ear at different frequencies, the physical sound pressure value at the different frequencies is corrected according to rating curve A, see figure 23.
Apart from sound pressures at certain places, sound powers and sound intensities of a whole system can be determined.
From the gear unit power a very small part is turned into sound power. This mainly occurs in the meshings, but also on bearings, fan blades, or by oil movements. The sound power is transmitted from the sources to the outside gear unit surfaces mainly by structure-borne noise (material vibrations). From the outside surfaces, air borne noise is emitted.
The sound power L_{WA} is the A-weighted sound power emitted from the source of sound and thus a quantity independent of the distance. The sound power can be converted to an average sound pressure for a certain place. The sound pressure decreases with increasing distance from the source of sound.
The sound intensity is the flux of sound power through a unit area normal to the direction of propagation. For a point source of sound it results from the sound power L_W divided by the spherical enveloping surface $4\pi r^2$, concentrically enveloping the source of sound. Like the sound pressure, the sound intensity is dependent on the distance to the source of sound, however, unlike the sound pressure it is a directional quantity.
The recording instrument stores the sound pressure or sound intensity over a certain period of time and writes the dB values in frequency ranges (bands) into the spectrum (system of coordinates).
Very small frequency ranges, e.g. 10 Hz or 1/12 octaves are termed narrow bands, see figure 24.

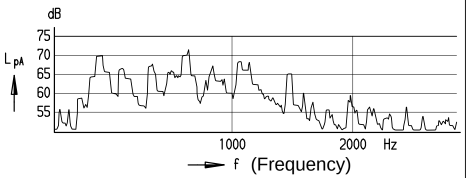


Figure 24
Narrow band frequency spectrum for L_{pA} (A-weighted sound pressure level) at a distance of 1 m from a gear unit.

Histograms occur in the one-third octave spectrum and in the octave spectrum, see figures 25 and 26. In the one-third octave spectrum (spectrum with 1/3 octaves), the bandwidth results from

$$f_o / f_u = \sqrt[3]{2}\text{, i.e. } f_o / f_u = 1.26,$$
$$f_o = f_m \cdot 1.12 \text{ and } f_u = f_m / 1.12;$$

f_m = mean band frequency, f_o = upper band frequency, f_u = lower band frequency. In case of octaves, the upper frequency is as twice as big as the lower one, or $f_o = f_m \cdot 1.41$ and $f_u = f_m / 1.41$.

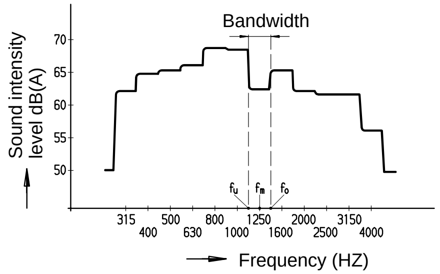


Figure 25
One-third octave spectrum of a gear unit (sound intensity level, A-weighted)

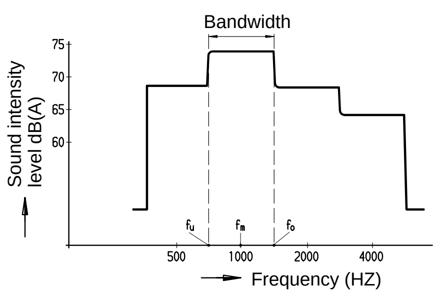


Figure 26
Octave spectrum of a gear unit (sound intensity level, A-weighted)

The total level (resulting from logarithmic addition of individual levels of the recorded frequency range) is a single-number rating. The total level is the common logical value for gear unit noises. The pressure level is valid for a certain distance, in general 1 m from the housing surface as an ideal parallelepiped.

1.5.2 Measurements

The main noise emission parameter is the sound power level.

1.5.2.1 Determination via sound pressure

DIN 45635 Part 1 and Part 23 describe how to determine the sound power levels of a given gear unit /16/. For this purpose, sound pressure levels L_{pA} are measured at fixed points surrounding the gear unit and converted to sound power levels L_{WA}. The measurement surface ratio L_S is an auxiliary quantity which is dependent on the sum of the measurement surfaces. When the gear unit is placed on a reverberant base, the bottom is not taken into consideration, see example in figure 27.

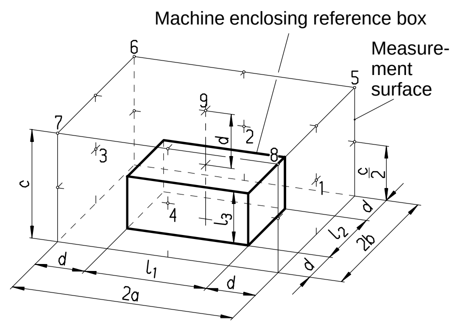


Figure 27
Example of arrangement of measuring points according to DIN 45 635 /16/

In order to really detect the noise radiated by the gear unit alone, corrections for background noise and environmental influences are to be made. It is not easy to find the correct correction values, because in general, other noise radiating machines are in operation in the vicinity.

1.5.2.2 Determination via sound intensity

The gear unit surface is scanned manually all around at a distance of, for instance, 10 cm, by means of a special measuring device containing two opposing microphones. The mean of the levels is taken via the specified time, e.g. two minutes. An analyzer computes the intensity or power levels in one-third octave or octave bands. The results can be seen on a display screen. In most cases, they can also be recorded or printed, see figures 25 and 26.

Cylindrical Gear Units
Noise Emitted by Gear Units

The results correspond to the sound power levels as determined in accordance with DIN 45635. This procedure requires a larger number of devices to be used, however, it is a very quick one. Above all, foreign influences are eliminated in the simplest way.

1.5.3 Prediction

It is not possible to exactly calculate in advance the sound power level of a gear unit to be made. However, one can base the calculations on experience. In the VDI guidelines 2159 /17/, for example, reference values are given. Gear unit manufacturers, too, mostly have own records.
The VDI guidelines are based on measurements carried out on a large number of industrial gear units. Main influence parameters for gear unit noises are gear unit type, transmitted power, manufacturing quality and speed. In VDI 2159, a distinction is made between cylindrical gear units with rolling bearings, see figure 28, cylindrical gear units with sliding bearings (high-speed gear units), bevel gear and bevel-helical gear units, planetary gear units and worm gear units. Furthermore, information on speed variators can be found in the guidelines.
Figure 28 exemplary illustrates a characteristic diagram of emissions for cylindrical gear units. Similar characteristic diagrams are also available for the other gear unit types mentioned. Within the characteristic diagrams, 50%- and 80%-lines are drawn. The 80%-line means, for example, that 80% of the recorded industrial gear units radiate lower noises.
The lines are determined by mathematical equations. For the 80%-lines, the equations according to VDI 2159 are:

Gear units	Total sound power level L_{WA}
Cylindrical gear units (rolling bearings)	$77.1 + 12.3 \cdot \log P$ / kW (dB)
Cylindrical gear units (sliding bearings)	$85.6 + 6.4 \cdot \log P$ / kW (dB)
Bevel gear and bevel-helical gear units	$71.7 + 15.9 \cdot \log P$ / kW (dB)
Planetary gear units	$87.7 + 4.4 \cdot \log P$ / kW (dB)
Worm gear units	$65.0 + 15.9 \cdot \log P$ / kW (dB)

For restrictions, see VDI 2159.

Type: Cylindrical gear units with external teeth mainly (> 80%) having the following characteristic features:
Housing:
Cast iron housing
Bearing arrangement:
Rolling bearings
Lubrication:
Dip lubrication
Installation:
Rigid on steel or concrete
Power rating: 0.7 up to 2400 kW
Input speed (= max. speed):
1000 up to 5000 min^{-1}
(mostly 1500 min^{-1})
Max. circumferential speed:
1 up to 20 ms^{-1}
Output torque:
100 up to 200000 Nm
No. of gear stages: 1 to 3
Information on gear teeth:
HS gear stage with helical teeth
(β = 10° up to 30°), hardened,
fine-machined, DIN quality 5 to 8

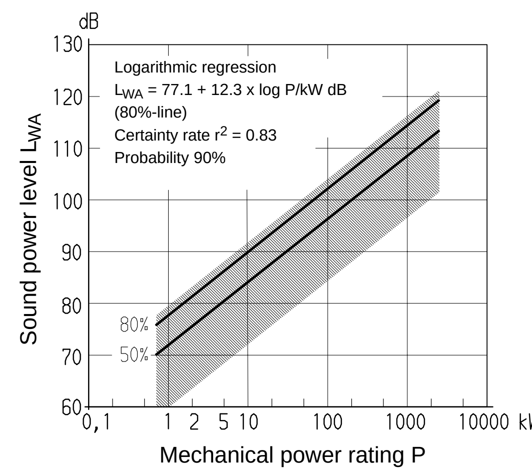


Figure 28
Characteristic diagram of emissions for cylindrical gear units (industrial gear units) acc. to VDI 2159 /17/

The measurement surface sound pressure level L_{pA} at a distance of 1 m is calculated from the total sound power level

$$L_{pA} = L_{WA} - L_s \text{ (dB)} \qquad (28)$$

$$L_S = 10 \cdot \log\ S \text{ (dB)} \qquad (29)$$

S = Sum of the hypothetical surfaces (m^2) enveloping the gear unit at a distance of 1 m (ideal parallelepiped)

Example of information for P = 100 kW in a 2-stage cylindrical gear unit of size 200 (centre distance in the 2nd gear stage in mm), with rolling bearings, of standard quality:

“The sound power level, determined in accordance with DIN 45635 (sound pressure measurement) or according to the sound intensity measurement method, is 102 + 2 dB (A). Room and connection influences have not been taken into consideration. If it is agreed that measurements are to be made they will be carried out on the manufacturer’s test stand.”

Note:
For this example, a measurement surface sound pressure level of 102 - 13.2 ≈ 89 db (A), tolerance + 2 dB, is calculated at a distance of 1 m with a measurement surface S = 21 m^2 and a measurement surface ratio L_S = 13.2 dB.

Individual levels in a frequency spectrum cannot safely be predicted for gear units because of the multitude of influence parameters.

1.5.4 Possibilities of influencing

With the selection of other than standard geometries and with special tooth modifications (see section 1.2.5), gear unit noises can be positively influenced. In some cases, such a procedure results in a reduction in the performance (e.g. module reduction) for the same size, in any case, however, in special design and manufacturing expenditure. Housing design, distribution of masses, type of rolling bearing, lubrication and cooling are also important.

Sometimes, the only way is to enclose the gear units which makes possible that the total level is reduced by 10 to 25 dB, dependent on the conditions.

Attention has to be paid to it, that no structure-borne noise is radiated via coupled elements (couplings, connections) to other places from where then airborne noise will be emitted.

A sound screen does not only hinder the propagation of airborne noise but also the heat dissipation of a gear unit, and it requires more space.

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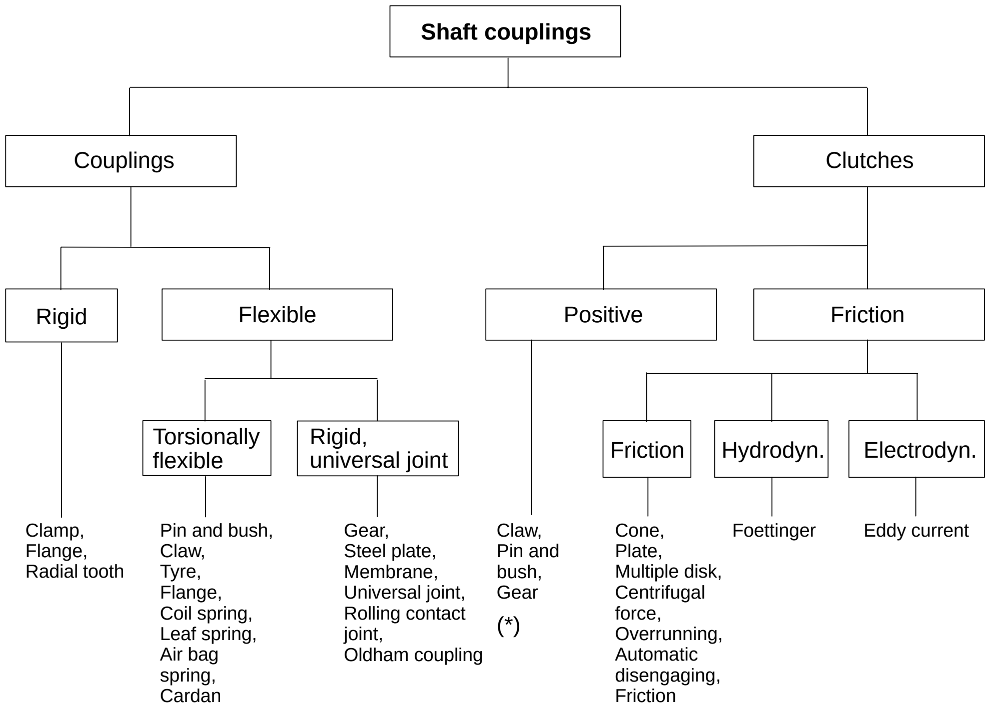
2. Shaft couplings

2.1 General fundamental principles

In mechanical equipment, drives are consisting of components like prime mover, gear unit, shafts and driven machine. Such components are connected by couplings which have the following tasks:

- Transmitting an as slip-free as possible motion of rotation, and torques;
- Compensating shaft misalignments (radial, axial, angular);
- Reducing the torsional vibration load, influencing and displacing the resonant ranges;
- Damping torque and speed impulses;
- Interrupting the motion of rotation (clutches);
- Limiting the torque;
- Sound isolation;
- Electrical insulation.

The diversity of possible coupling variants is shown in the overview in figure 29. A distinction is made between the two main groups couplings and clutches, and the subgroups rigid/flexible couplings and positive/friction clutches.



(*) In case of additional gearing, all clutches are disengageable when stationary.

Figure 29
Overview of possible shaft coupling designs

2.2 Rigid couplings

Rigid couplings connect two shaft ends and do practically not allow any shaft misalignment. They are designed as clamp, flange and radial tooth couplings and allow the transmission of high torques requiring only small space. The coupling halves are connected by means of bolts (close fitting bolts). In case of clamp and flange couplings (with split spacer ring), radial disassembly is possible. Radial tooth couplings are self-centering and transmit both high and alternating torques.

2.3 Torsionally flexible couplings

Torsionally flexible couplings are offered in many designs. Main functions are the reduction of torque impulses by elastic reaction, damping of torsional vibrations by internal damping in case of couplings with flexible rubber elements, and frictional damping in case of couplings with flexible metal elements, transfer of resonance frequencies by variation of the torsional stiffness, and compensation of shaft misalignments with low restoring forces.

The flexible properties of the couplings are generated by means of metal springs (coil springs, leaf springs) or by means of elastomers (rubber, plastics). For couplings incorporating flexible metal elements, the torsional flexibility is between 2 and 25 degree, depending on the type. The stiffness characteristics, as a rule, show a linear behaviour, unless a progressive characteristic has intentionally been aimed for by design measures. Damping is achieved by means of friction and viscous damping means.

In case of couplings incorporating elastomer elements, a distinction is made between couplings of average flexibility with torsion angles of 2 up to 5 degree and couplings of high flexibility with torsion angles of 5 up to 30 degree. Depending on the type, the flexible elements of the coupling are subjected to compression (tension), bending and shearing, or to a combined form of stressing. In some couplings (e.g. tyre couplings), the flexible elements are reinforced by fabric or thread inserts. Such inserts absorb the coupling forces and prevent the elastic-viscous flow of the elastomer.

Couplings with elastomer elements primarily subjected to compression and bending have non-linear progressive stiffness characteristics, while flexible elements (without fabric insert) merely subjected to shearing generate linear stiffness characteristics. The quasi-statical torsional stiffness of an elastomer coupling increases at dynamic load (up to approximately 30 Hz, test frequency 10 Hz) by approximately 30 to 50%. The dynamic stiffness of a coupling is influenced [(+) increased; (–) reduced] by the average load (+), the oscillation amplitude (–), temperature (–), oscillation frequency (+), and period of use (–).

For rubber-flexible couplings, the achievable damping values are around ψ = 0.8 up to 2 (damping coefficient ψ; DIN 740 /18/). Damping leads to heating of the coupling, and the heat loss has to be dissipated via the surface. The dynamic loading capacity of a coupling is determined by the damping power and the restricted operating temperature of elastomers of 80°C up to max. 100°C.

When designing drives with torsionally flexible couplings according to DIN 740 /18/, torsional vibrations are taken into account by reducing the drive to a two-mass vibration generating system, or by using torsional vibration simulation programs which can compute detailed vibration systems for both steady and unsteady conditions.
Examples of couplings incorporating elastomer elements of average flexibility are claw-, pin-, and pin and bush couplings.

The N-EUPEX coupling is a wear-resistant pin coupling for universal use (figure 30) absorbing large misalignments. The coupling is available as fail-safe coupling and as coupling without fail-safe device. In its three-part design it is suitable for simple assembly and simple replacement of flexible elements. The coupling is made in different types and sizes for torques up to 62,000 Nm.

The BIPEX coupling is a flexible fail-safe claw coupling in compact design for high power capacity and is offered in different sizes for maximum torques up to 3,700 Nm. The coupling is especially suitable for plug-in assembly and fitting into bell housings.

The RUPEX coupling is a flexible fail-safe pin and bush coupling which as a universal coupling is made in different sizes for low up to very high torques (10^6 Nm) (figure 31). The coupling is suitable for plug-in assembly and capable of absorbing large misalignments. The optimized shape of the barrelled buffers and the conical seat of the buffer bolts facilitate assembly and guarantee maintenance-free operation. Because of their capability to transmit high torques, large RUPEX couplings are often used on the output side between gear unit and driven machine. Since the coupling hubs are not only offered in grey cast iron but also in steel, the couplings are also suitable for high speeds.

Examples of highly flexible couplings incorporating elastomer elements are tyre couplings, flange couplings, ring couplings, and large-volume claw couplings with cellular elastic materials. Examples of flexible couplings incorporating metal elements are coil spring and leaf spring couplings.

The ELPEX coupling (figure 32) is a highly flexible ring coupling without torsional backlash which is suitable for high dynamic loads and has good damping properties. Rings of different elasticity are suitable for optimum dynamic tuning of drives. Torque transmitting thread inserts have been vulcanized into the rings out of high-quality natural rubber. Due to the symmetrical design the coupling is free from axial and radial forces and allows large shaft misalignments even under torque loads. Typical applications for ELPEX couplings which are available for torques up to 90,000 Nm are drives with periodically exciting aggregates (internal combustion engines, reciprocating engines) or extremely shock-loaded drives with large shaft misalignments.

Another highly flexible tyre coupling with a simple closed tyre as flexible element mounted between two flanges is **the ELPEX-B coupling**. It is available in different sizes for torques up to 20,000 Nm.

This coupling features high flexibility without torsional backlash, absorbs large shaft misalignments, and permits easy assembly and disassembly (radial).

The ELPEX-S coupling (figure 33) is a highly flexible, fail-safe claw coupling absorbing large shaft misalignments. The large-volume cellular flexible elements show very good damping properties with low heating and thus allow high dynamic loads. The couplings have linear stiffness characteristics, and with the use of different flexible elements they are suitable for optimum dynamic tuning of drives. The couplings are of compact design and are suitable for torques up to 80,000 Nm. Plug-in assembly is possible. This universal coupling can be used in drives with high dynamic loads which require low frequency with good damping.

2.4 Torsionally rigid couplings

Torsionally rigid couplings are used where the torsional vibration behaviour should not be changed and exact angular rotation is required, but shaft misalignment has to be absorbed at the same time. With the use of long floating shafts large radial misalignments can be allowed. Torsionally rigid couplings are very compact, however, they have to be greased with oil or grease (exception: steel plate and membrane couplings). Typical torsionally rigid couplings are universal joint, gear, membrane and steel plate couplings, which always have to be designed as double-jointed couplings with floating shafts (spacers) of different lengths.

Universal joints allow large angular misalignments (up to 40 degree), the dynamic load increasing with the diffraction angle. In order to avoid pulsating angular rotation (2 times the torsional frequency), universal joints must always be arranged in pairs (same diffraction angle, forks on the intermediate shaft in one plane, input and output shaft in one plane). Constant velocity joints, however, always transmit uniformly and are very short.

Gear couplings of the ZAPEX type (figure 34) are double-jointed steel couplings with crowned gears which are capable of absorbing shaft misalignments (axial, radial and angular up to 1 degree) without generating large restoring forces. The ZAPEX coupling is of compact design, suitable for high speeds, and transmits very high torques (depending on the size up to > 10^6 Nm), and in addition offers large safety reserves for the absorption of shock loads. It is lubricated with oil or grease. Fields of application are, among others, rolling mills, cement mills, conveyor drives, turbines.

The ARPEX coupling (figure 35) is a double-jointed, torsionally rigid plate coupling for the absorption of shaft misalignments (angular up to 1 degree). The coupling is maintenance-free (no lubrication) and wear-resistant and owing to its closed plate packs allows easy assembly. A wide range of ARPEX couplings is available - from the miniature coupling up to large-size couplings for torques up to > 10^6 Nm. The coupling transmits torques very uniformly, and owing to its all-steel design is suitable for high ambient temperatures (up to 280°C) and high speeds. Fields of application are, among others, paper machines, ventilators, pumps, drives for materials-handling equipment as well as for control systems.

2.5 Positive clutches

This type includes all clutches which can be actuated when stationary or during synchronous operation in order to engage or disengage a machine to or from a drive. Many claw, pin and bush, or gear couplings can be used as clutches by axially moving the driving member. With the additional design element of interlocking teeth, all flexible couplings can be used as clutches.

2.6 Friction clutches

In friction clutches, torques are generated by friction, hydrodynamic or electrodynamic effect. The clutch is actuated externally, even with the shaft rotating (mechanically, hydraulically, pneumatically, magnetical), speed-dependent (centrifugal force, hydrodynamic), torque-dependent (slip clutches, safety clutches), and dependent on the direction of rotation (overrunning clutches).

Of the different clutch types, friction clutches are most commonly used which may contain either dry- or wet- (oil-lubricated) friction elements. Dependent on the friction element and the number of friction surface areas, a distinction is made between cylindrical, cone, flange and disk clutches. The larger the number of friction surface areas, the smaller the size of the clutch. Further criteria are wear, service life, idle torque, cooling, cycle rate, and uniform friction effect (non-chattering).

The PLANOX clutch is a dry-friction multiple disk clutch with one up to three disks, which has been designed with overload protection for application in general mechanical engineering. It is actuated externally by mechanical, electrical, pneumatic or hydraulic force. Uniform transmission of torque is guaranteed by spring pressure even after high cylce rates. The clutch is made in different types and sizes for torques up to 3 • 10^5 Nm.

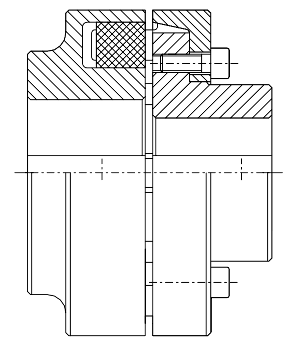


Figure 30
Flexible pin coupling,
N-EUPEX, in three parts

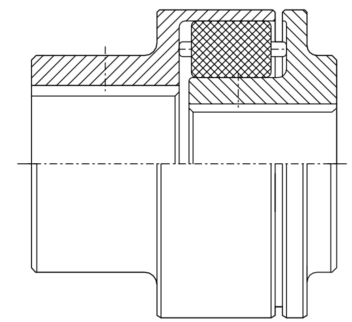


Figure 33
Highly flexible claw coupling with
cellular flexible elements, ELPEX-S

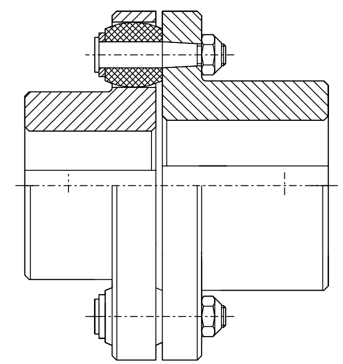


Figure 31
Flexible pin and bush coupling, RUPEX

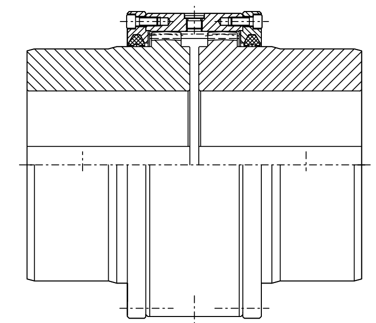


Figure 34
Gear coupling, ZAPEX

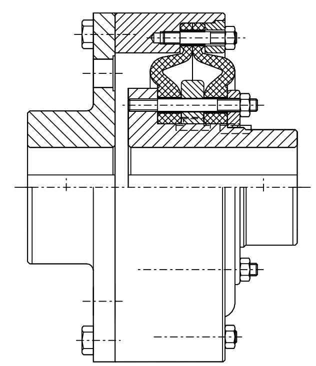


Figure 32
Highly flexible ring coupling, ELPEX

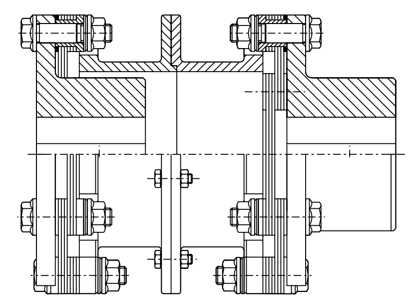


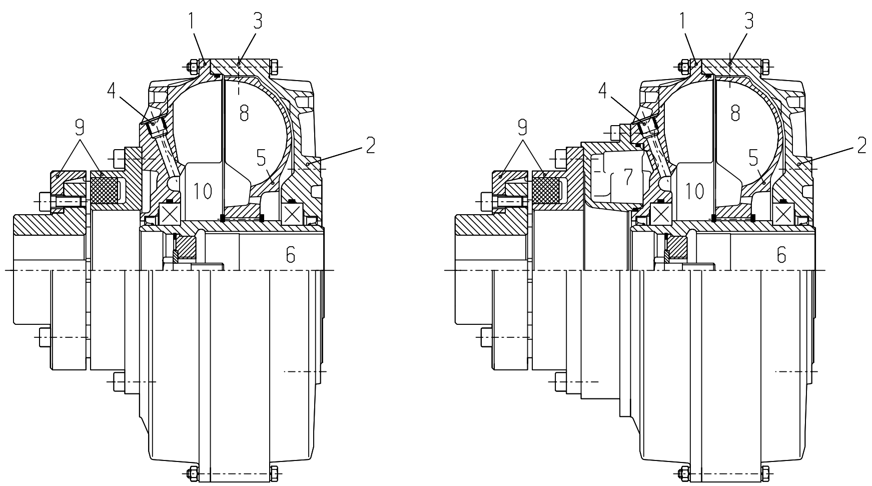
Figure 35
All-steel coupling, with plate packs, ARPEX

The AUTOGARD torque limiter is an automatically actuating safety clutch which disconnects driving and driven side by means of a high-accuracy ball-operated mechanism and interrupts the transmission of torque as soon as the set disengagement torque is exceeded. The torque limiter is ready for operation again when the mechanism has been re-engaged during standstill. The clutch is made in different sizes for disengagement torques up to 56,500 Nm.

Speed-controlled clutches allow soft starting of heavy-duty driven machines, the motor accelerating itself at first and then driving the machine. This permits the use of smaller dimensioned motors for high mass moments of inertia and a high number of starts. Speed-controlled clutches are designed as centrifugal clutches with segments, e.g. retaining springs which transmit torques only from a specified operating speed on, or with pellets (powder, balls, rollers). The torque which is generated by friction on the lateral area of the output part increases as the square of the input speed. After running up, the clutch operates without slip.

The FLUDEX coupling (figure 36) is a hydrodynamic fluid coupling operating according to the Föttinger principle without mechanical friction. The coupling parts on the input (pump) and output (turbine) side are not mechanically connected and thus wear-resistant.

Torque is transmitted by the rotating oil fluid in the coupling accelerated by the radial blades (pulse exchange). Fluid couplings have the same characteristics as turbines; torque increases with the second power, and power capacity is proportional to the third power of the input speed. During steady torque transmission little operating slip occurs which heats up the coupling. As safety elements for limiting the temperature, fusible safety plugs and electronically or mechanically controlled temperature monitors are used.
Fluid couplings are mainly used for starting great masses, for separating torsional vibrations, and for limiting overloads during starting and in case of blockages.



Outer wheel drive

1 Blade wheel housing (outer wheel)
2 Cover
3 Fusible safety plug
4 Filler plug
5 Impeller (inner wheel)
6 Hollow shaft
7 Delay chamber
8 Working chamber
9 Flexible coupling (N-EUPEX D)
10 Damming chamber

Figure 36
Basic design of a fluid coupling with and without delay chamber, FLUDEX type

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a	m	Length of overhanging end
A	m^2	Cross-sectional area
A	m, rad	Amplitude of oscillation
A_D; A_e		Damping energy; elastic energy
c	Nm/rad	Torsional stiffness
c’	N/m	Translational stiffness; bending stiffnes
d	m	Diameter
d_i	m	Inside diameter
d_a	m	Outside diameter
D	–	Attenuation ratio (Lehr’s damping)
D_m	m	Mean coil diameter (coil spring)
e =	2.718	Natural number
E	N/m^2	Modulus of elasticity
f, f_e	Hz	Frequency; natural frequency
f	m	Deformation
F	N	Force
F (t)	N	Time-variable force
G	N/m^2	Shear modulus
i	–	Transmission ratio
i_F	–	Number of windings (coil spring)
I_a	m^4	Axial moment of area
I_p	m^4	Polar moment of area
J, J_i	kgm^2	Mass moment of inertia
J*	kgm^2	Reduced mass moment of inertia of a two-mass vibration generating system
k	Nms/rad	Viscous damping in case of torsional vibrations
k’	Ns/m	Viscous damping in case of translational and bending vibrations
l	m	Length; distance between bearings
m, m_i	kg	Mass
M (t)	Nm	Time-variable excitation moment
M_o	Nm	Amplitude of moment
M_o*	Nm	Reduced amplitude of moment of a two-mass vibration generating system
n_e	1/min	Natural frequency (vibrations per minute)
n_1; n_2	1/min	Input speed; output speed
q	–	Influence factor for taking into account the mass of the shaft when calculating the natural bending frequency

t	s	Time
T	s	Period of a vibration
T	Nm	Torque
V	m^3	Volume
V	–	Magnification factor; Dynamic/static load ratio
x	m	Displacement co-ordinate (translational, bending)
$\hat{x}$	m	Displacement amplitude
α	rad	Phase angle
γ	rad	Phase angle with free vibration
δ	1/s	Damping constant
ε	rad	Phase displacement angle with forced vibration
η	–	Excitation frequency/natural frequency ratio
λ_i	–	Inherent value factor for i-th natural frequency
Λ	–	Logarithmic decrement
π =	3.142	Peripheral/diameter ratio
ϱ	kg/m^3	Specific density
φ, φ_i	rad	Angle of rotation
^	rad	Angular amplitude of a vibration
·	rad/s	Angular velocity (first time derivation of)
··	rad/s^2	Angular acceleration (second time derivation of)
$_h$	rad	Vibratory angle of the free vibration (homogeneous solution)
$_p$	rad	Vibratory angle of the forced vibration (particular solution)
$\hat{}_p$	rad	Angular amplitude of the forced vibration
$\hat{}_{stat}$	rad	Angular amplitude of the forced vibration under load (= 0)
ψ	–	Damping coefficient acc. to DIN 740 /18/
ω	rad/s	Angular velocity, natural radian frequency of the damped vibration
$_o$	rad/s	Natural radian frequency of the undamped vibration
Ω	rad/s	Radian frequency of the exciting vibration

Note: The unit “rad” may be replaced by “1”.

3. Vibrations

3.1 General fundamental principles

Vibrations are more or less regularly occurring temporary variations of state variables. The state of a vibrating system can be described by suitable variables, such as displacement, angle, velocity, pressure, temperature, electric voltage/current, and the like.
The simplest form of a mechanical vibrating system consists of a mass and a spring with fixed ends, the mass acting as kinetic energy store and the spring as potential energy store, see figure 37. During vibration, a periodic conversion of potential energy to kinetic energy takes place, and vice versa, i.e. the kinetic energy of the mass and the energy stored in the spring are converted at certain intervals of time. Dependent on the mode of motion of the mass, a distinction is made between translational (bending) and torsional vibrating systems as well as coupled vibrating systems in which translational and torsional vibrations occur at the same time, influencing each other.

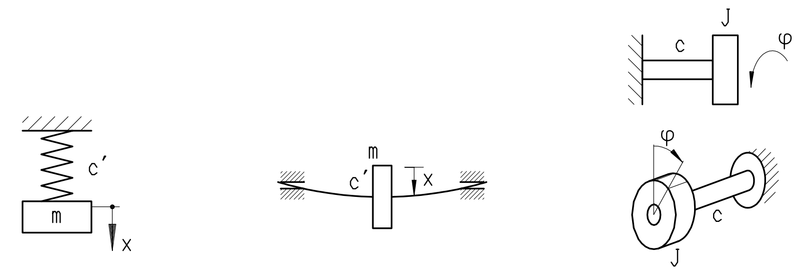
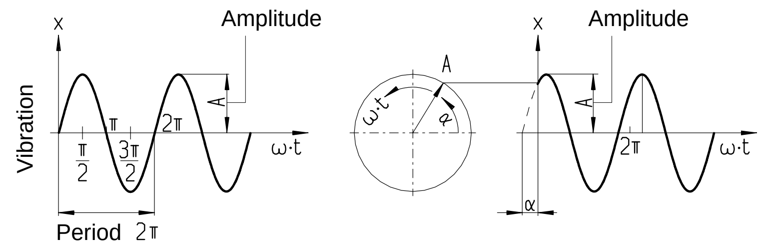


Figure 37
Different vibrating systems with one degree of freedom

Further, a distinction is made between free vibrations and externally forced vibrations, and whether the vibration takes place without energy losses (undamped) or with energy losses (damped).
A vibration is free and undamped if energy is neither supplied nor removed by internal friction so that the existing energy content of the vibration is maintained. In this case the system carries out steady-state natural vibrations the frequency of which is determined only by the characteristics of the spring/mass system (natural frequency), figure 39a.
The vibration variation with time x can be described by the constant amplitude of oscillation A and a harmonic function (sine, cosine) the arguments of which contain natural radian frequency $\omega = 2\pi \cdot f$ (f = natural frequency in Hertz) and time, see figure 38.



x = A · sinω · t
A = Amplitude
ω = Radian frequency
t = Time

x = A · sin (ω · t + α)
α = Phase angle

Figure 38
Mathematical description of an undamped vibration with and without phase angle

A damped vibration exists, if during each period of oscillation a certain amount of vibrational energy is removed from the vibration generating system by internal or external friction. If a constant viscous damping (Newton's friction) exists, the amplitudes of oscillation decrease in accordance with a geometric progression, figure 39b. All technical vibration generating systems are subject to more or less strong damping effects.

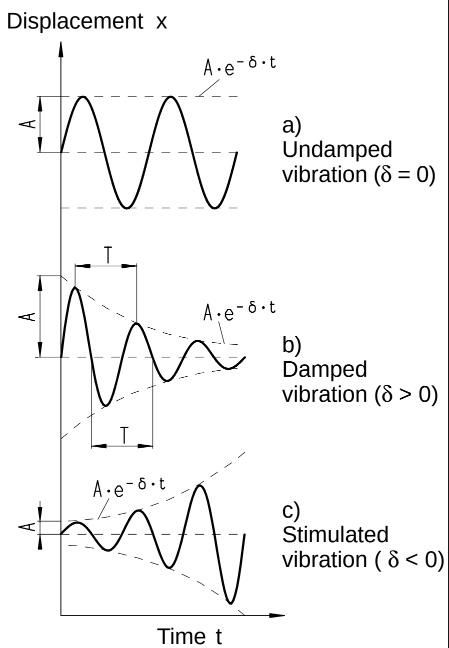


Figure 39
Vibration variations with time (A = initial amplitude at time t = 0; δ = damping constant)

If the vibrating system is excited by a periodic external force F(t) or moment M(t), this is a forced or stimulated vibration, figure 39c. With the periodic external excitation force, energy can be supplied to or removed from the vibrating system.
After a building-up period, a damped vibrating system does no longer vibrate with its natural frequency but with the frequency of the external excitation force.
Resonance exists, when the applied frequency is at the natural frequency of the system. Then, in undamped systems the amplitudes of oscillation grow at an unlimited degree. In damped systems, the amplitude of oscillation grows until the energy supplied by the excitation force and the energy converted into heat by the damping energy are in equilibrium. Resonance points may lead to high loads in the components and therefore are to be avoided or to be quickly traversed. (Example: natural bending frequency in high-speed gear units).
The range of the occurring amplitudes of oscillation is divided by the resonance point (natural frequency = excitation frequency, critical vibrations) into the subcritical and supercritical oscillation range. As a rule, for technical vibrating systems (e.g. drives), a minimum frequency distance of 15% or larger from a resonance point is required.
Technical vibrating systems often consist of several masses which are connected with each other by spring or damping elements. Such systems have as many natural frequencies with the corresponding natural vibration modes as degrees of freedom of motion. A free, i.e. unfixed torsional vibration system with n masses, for instance, has n-1 natural frequencies. All these natural frequencies can be excited to vibrate by periodic external or internal forces, where mostly only the lower natural frequencies and especially the basic frequency (first harmonic) are of importance.
In technical drive systems, vibrations are excited by the following mechanisms:

a) From the input side:
Starting processes of electric motors, system short circuits, Diesel Otto engines, turbines, unsteady processes, starting shock impulses, control actions.
b) From transmitting elements:
Meshing, unbalance, universal-joint shaft, alignment error, influences from bearings.
c) From the output side:
Principle of the driven machine, uniform, non-uniform, e.g. piston compressor, propeller.

As a rule, periodic excitation functions can be described by means of sine or cosine functions and the superpositions thereof. When analysing vibration processes, a Fourier analysis may often be helpful where periodic excitation processes are resolved into fundamental and harmonic oscillations and thus in comparison with the natural frequencies of a system show possible resonance points.
In case of simple vibrating systems with one or few (maximum 4) masses, analytic solutions for the natural frequencies and the vibration variation with time can be given for steady excitation. For unsteady loaded vibrating systems with one or more masses, however, solutions can be calculated only with the aid of numerical simulation programmes. This applies even more to vibrating systems with non-linear or periodic variable parameters (non-linear torsional stiffness of couplings; periodic meshing stiffnesses). With EDP

programmes, loads with steady as well as unsteady excitation can be simulated for complex vibrating systems (linear, non-linear, parameter-excited) and the results be represented in the form of frequency analyses, load as a function of time, and overvoltages of resonance. Drive systems with torsionally flexible couplings can be designed dynamically in accordance with DIN 740 /18/. In this standard, simplified solution proposals for shock-loaded and periodically loaded drives are made, the drive train having been reduced to a two-mass vibration generating system.

3.2 Solution proposal for simple torsional vibrators

Analytic solution for a periodically excited one- (fixed) or two-mass vibration generating system, figure 40.

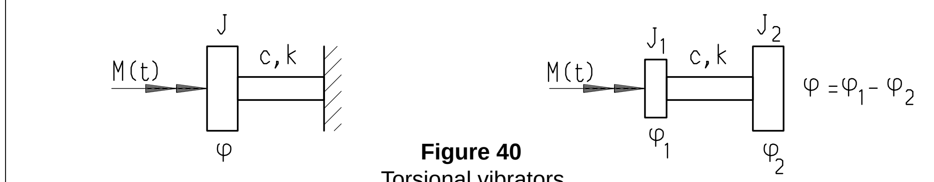


Figure 40
Torsional vibrators

J, J_1, J_2	= mass moment of inertia [kgm^2]
c	= torsional stiffness [Nm/rad]
k	= viscous damping [Nms/rad]
M (t)	= external excitation moment [Nm] , time-variable
φ	= angle of rotation [rad] , ($\varphi = \varphi_1 - \varphi_2$ for 2-mass vibration generating systems as relative angle)
$\dot{\varphi}$	= angular velocity [rad/s] (first time derivation of φ)
$\ddot{\varphi}$	= angular acceleration [rad/s^2] (second time derivation of φ)

Differential equation of motion:

<u>One-mass vibration generating system:</u>

$$\ddot{} + \underbrace{\frac{k}{J}}_{2}\,\dot{} + \underbrace{\frac{c}{J}}_{{}^{2}_{o}} \qquad \frac{M\,(t)}{J} \qquad (30)$$

<u>Two-mass vibration generating system with relative coordinate:</u>

$$\ddot{} + \underbrace{\frac{k}{J^*}}_{2}\,\dot{} + \underbrace{\frac{c}{J^*}}_{{}^{2}_{o}} \qquad \frac{M(t)}{J_1} \qquad (31)$$

with $\quad {}_1 = \quad {}_2 \qquad (32)$

$$J^* \quad \frac{J_1 \quad J_2}{J_1 + J_2} \qquad (33)$$

<u>Natural radian frequency</u> (undamped): ω_o

$${}_o \qquad \sqrt{\frac{c}{J}} \qquad [\text{ rad/s}] \qquad (34)$$

$${}_o \qquad \sqrt{c \quad \frac{J_1 + J_2}{J_1 \quad J_2}} \qquad \text{rad s} \qquad (35)$$

<u>Natural frequency:</u>

$$f_e \quad \frac{{}_o}{2} \qquad [\text{Hz}] \qquad (36)$$

$$n_e \quad \frac{{}_o \quad 30}{} \qquad [\text{1/min}] \qquad (37)$$

$$\frac{k}{J} \quad \text{damping constant} \qquad [\text{1/s}] \qquad (38)$$

ω_o = natural radian frequency of the undamped vibration in rad/s
f_e = natural frequency in Hertz
n_e = natural frequency in 1/min

<u>Damped natural radian frequency:</u>

$$\sqrt{{}^{2}_{o} = {}^{2}} \qquad {}_o \quad \sqrt{1 = D^2} \qquad (39)$$

<u>Attenuation ratio</u> (Lehr's damping): D

$$D \quad \frac{}{o} \quad \frac{k \quad o}{2 \quad c} \quad \frac{}{4} \qquad (40)$$

ψ = damping coefficient on torsionally flexible coupling, determined by a damping hysteresis of a period of oscillation acc. to DIN 740 /18/ and/or acc. to Flender brochure

$$\frac{\text{damping energy}}{\text{elastic deformation energy}} \quad \frac{A_D}{A_e}$$

<u>Reference values for some components:</u>

D = 0.001...0.01	shafts (material damping of steel)
D = 0.04...0.08	gear teeth in gear units
D = 0.04...0.15 (0.2)	torsionally flexible couplings
D = 0.01...0.04	gear couplings, all-steel couplings, universal joint shafts

Static spring characteristic for one load cycle

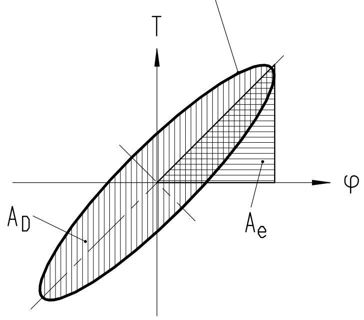


Figure 41
Damping hysteresis of a torsionally flexible component

3.3 Solution of the differential equation of motion

Periodic excitation moment

$$M(t) \quad M_o \quad \cos \mu \quad t \qquad (41)$$

M_o = amplitude of moment [Nm]
Ω = exciting circuit frequency [rad/s]

<u>Total solution:</u>

$$_h + \; _p \qquad (42)$$

a) <u>Free vibration</u> (homogeneous solution $_h$)

$$_h \quad A \quad e^{\{= \quad t\}} \quad \cos (\quad t = \quad) \qquad (43)$$

Constants A and γ are determined by the starting conditions, e.g. by $_h = 0$ and $\dot{}_h = 0$ (initial-value problem).
In damped vibrating systems (δ > 0) the free component of vibration disappears after a transient period.

b) <u>Forced vibration</u> (particular solution $_p$)

$$_p \quad \frac{M_o^*}{c} \quad \frac{1}{\overline{(1 = \; ^2)^2 \; + 4D^2 \quad ^2}}$$

$$\cos(\mu \quad t = \;) \qquad (44)$$

Phase angle: $\tan \quad \frac{2 \quad D}{1 = \quad ^2}$ (45)

Frequency ratio: $\frac{\mu}{o}$ (46)

One-mass vibration generating system:
$M_o{}^* \quad M_o$ (47)

Two-mass vibration generating system:
$M_o{}^* \quad \frac{J_2}{J_1 + J_2} \quad M_o$ (48)

c) <u>Magnification factor</u>

$$_p \quad \frac{M_o{}^*}{c} \quad V \quad \cos (\mu \quad t = \;) \qquad (49)$$

$$V \quad \frac{1}{\overline{(1 = \; ^2)^2 \; + 4D^2 \quad ^2}} \quad \frac{\hat{}_p}{\hat{}_{stat}} \quad \frac{M}{M_o^*} \qquad (50)$$

$\hat{}_p$ = vibration amplitude of forced vibration

$\hat{}_{stat}$ = vibration amplitude of forced vibration at a frequency ratio η = 0.

The magnification factor shows the ratio of the dynamic and static load and is a measure for the additional load caused by vibrations (figure 42).

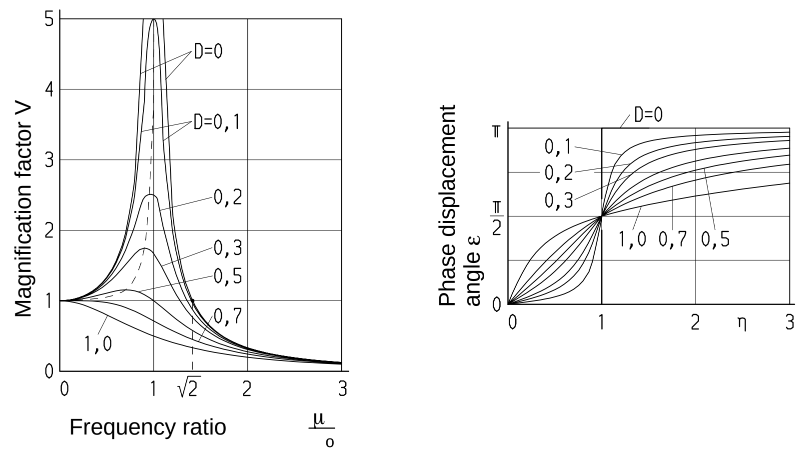


Figure 42
Magnification factors for forced, damped and undamped vibrations at periodic moment excitation (power excitation).

Magnification factors V and phase displacement angle ε.

3.4 Formulae for the calculation of vibrations

For the calculation of natural frequencies and vibrational loads, a general vibration generating system has to be converted to a calculable substitute system with point masses, spring and damping elements without mass.

3.4.1 Mass

$m = \varrho \cdot V$ [kg]

V = volume [m^3]
ϱ = specific density [kg/m^3]

3.4.2 Mass moment of inertia

$J = \quad r^2 dm$: general integral formula

Circular cylinder:
$J \quad \frac{1}{32}\varrho \quad d^4 \quad l \quad \pi(kgm^2\varrho$

d = diameter [m]
l = length of cylinder πmϱ

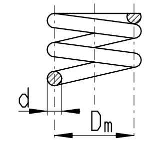
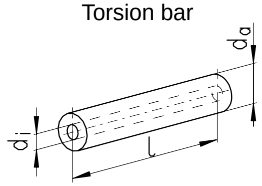
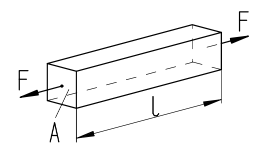
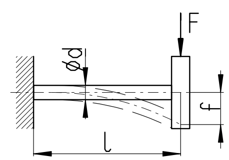
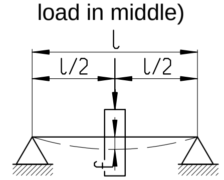
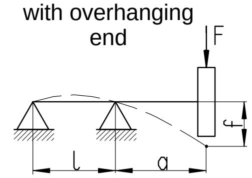
Table 8 Symbols and units of translational and torsional vibrations

Term	Quantity	Unit	Explanation
Mass, Mass moment of inertia	m J	kg $kg \cdot m^2$	Translatory vibrating mass m; Torsionally vibrating mass with mass moment of inertia J
Instantaneous value of vibration (displacement, angle)	x φ	m rad*)	Instantaneous, time-dependent value of vibration amplitude
Amplitude	x_{max}, $\hat{x}$, A $_{max}$, ^, A	m rad	Amplitude is the maximum instantaneous value (peak value) of a vibration.
Oscillating velocity	$\dot{x}$	m/s rad/s	Oscillating velocity; Velocity is the instantaneous value of the velocity of change in the direction of vibration.
Inertia force, Moment of inertia forces	m $\ddot{x}$ J	N N • m	The d'Alembert's inertia force or the moment of inertia force acts in the opposite direction of the positive acceleration.
Spring rate, Torsional spring rate	c' c	Nm N • m/rad	Linear springs
Spring force, Spring moment	$c' \cdot x$ $c \cdot \varphi$	N N • m	In case of linear springs, the spring recoil is proportional to deflection.
Attenuation constant (Damping coefficient), Attenuation constant for rotary motion	k' k	N • s/m Nms/rad	In case of Newton's friction, the damping force is proportional to velocity and attenuation constant (linear damping).
Damping factor (Decay coefficient)	$\delta = k'/(2 \cdot m)$ $\delta = k/(2 \cdot J)$	1/s 1/s	The damping factor is the damping coefficient referred to twice the mass.
Attenuation ratio (Lehr's damping)	$D = \delta/\omega_o$	–	For D < 1, a damped vibration exists; for D ≥ 1, an aperiodic case exists.
Damping ratio	$\hat{x}_n \quad \hat{x}_{n+1}$ $\hat{}_n \quad \hat{}_{n+1}$	– –	The damping ratio is the relation between two amplitudes, one cycle apart.
Logarithmic damping decrement	$\frac{2 \quad D}{\overline{1 = D^2}}$	–	$\pi \quad \ln(\hat{x}_n \quad \hat{x}_{n+1})$ $\pi \quad \ln(\hat{}_n \quad \hat{}_{n+1})$
Time	t	s	Coordinate of running time
Phase angle	α	rad	In case of a positive value, it is a lead angle.
Phase displacement angle	$\varepsilon = \alpha_1 - \alpha_2$	rad	Difference between phase angles of two vibration processes with same radian frequency.
Period of a vibration	$T = 2 \cdot \pi / \omega_o$	s	Time during which a single vibration occurs.
Frequency of natural vibration	$f = 1/T = \omega_o/(2 \cdot \pi)$	Hz	Frequency is the reciprocal value to a period of vibrations; vibrations per sec.
Radian frequency of natural vibration	$\omega_o = 2 \cdot \pi \cdot f$	rad/s	Radian frequency is the number of vibrations in $2 \cdot \pi$ seconds.
Natural radian frequency (Natural frequency)	$_o \quad \overline{c \;\; m}$ $_o \quad \overline{c \;\; J}$	rad/s rad/s	Vibration frequency of the natural vibration (undamped) of the system
Natural radian frequency when damped	$_d \quad \overline{^2_o = \varrho^2}$		For a very small attenuation ratio D < 1 becomes $\omega_d \approx \omega_o$.
Excitation frequency	Ω	rad/s	Radian frequency of excitation
Radian frequency ratio	$\eta = \Omega/\omega_o$	–	Resonance exists at η= 1.

*) The unit "rad" may be replaced by "1".

3.4.3 Determination of stiffness

Table 9 Calculation of stiffness (examples)

Example	Stiffness	Symbol
Coil spring 	c $\frac{G \quad d^4}{8 \quad D_m^3 \quad i_f}$ $\frac{N}{m}$	i_F = number of windings G = shear modulus 1) d = diameter of wire D_m = mean coil diameter
Torsion bar 	c $\frac{G \quad I_p}{l}$ $\frac{Nm}{rad}$ Shaft : I_p $\frac{d^4}{32}$ Hollow shaft : I_p $\frac{}{32}(d_a^4 = d_i^4)$	I_p = polar moment of inertia l = length d, d_i, d_a = diameters of shafts
Tension bar 	c $\frac{E \quad A}{l}$ $\frac{N}{m}$	E = modulus of elasticity 1) A = cross-sectional area
Cantilever beam 	c $\frac{F}{f}$ $\frac{3 \quad E \quad I_a}{l^3}$ $\frac{N}{m}$ Shaft : I_a $\frac{d^4}{64}$ Hollow shaft : I_a $\frac{}{64}(d_a^4 = d_i^4)$	F = force f = deformation at centre of mass under force F I_a = axial moment of area
Transverse beam (single load in middle) 	c $\frac{F}{f}$ $\frac{48 \quad E \quad I_a}{l^3}$ $\frac{N}{m}$	
Transverse beam with overhanging end 	c $\frac{F}{f}$ $\frac{3 \quad E \quad I_a}{a^2 \quad (l + a)}$ $\frac{N}{m}$	l = distance between bearings a = length of overhanging end

1) For steel: $E = 21 \cdot 10^{10}$ N/m²; $G = 8.1 \cdot 10^{10}$ N/m²

Measuring the stiffness:
In a test, stiffness can be determined by measuring the deformation. This is particularly helpful if the geometric structure is very complex and very difficult to acquire.

Translation:

$$c \quad \frac{F}{f} \quad N\ m \qquad (51)$$

F = applied force [N]
f = measured deformation [m]

Torsion:

$$c \quad \frac{T}{} \quad Nm\ rad \qquad (52)$$

T = applied torsion torque [Nm]
φ = measured torsion angle [rad]

Measurements of stiffness are furthermore required if the material properties of the spring material are very complex and it is difficult to rate them exactly. This applies, for instance, to rubber materials of which the resilient properties are dependent on temperature, load frequency, load, and mode of stress (tension, compression, shearing). Examples of application are torsionally flexible couplings and resilient buffers for vibration isolation of machines and internal combustion engines.
These components often have non-linear progressive stiffness characteristics, dependent on the direction of load of the rubber material.
For couplings the dynamic stiffness is given, as a rule, which is measured at a vibrational frequency of 10 Hz (vibrational amplitude = 25% of the nominal coupling torque). The dynamic torsional stiffness is greater than the static torsional stiffness, see figure 43.

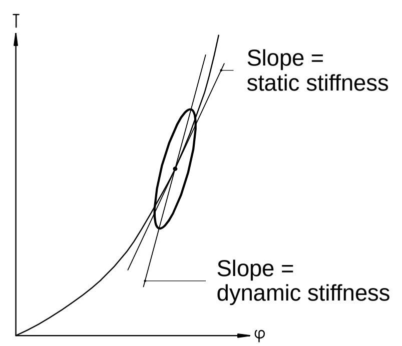


Figure 43
Static and dynamic torsional stiffness

3.4.4 Overlaying of different stiffnesses
To determine resulting stiffnesses, single stiffnesses are to be added where arrangements in series connection or parallel connection are possible.

Series connection:
Rule: The individual springs in a series connection carry the same load, however, they are subjected to different deformations.

$$\frac{1}{c_{ges}} \quad \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \quad + \quad \frac{1}{c_n} \qquad (53)$$

Parallel connection:
Rule: The individual springs in a parallel connection are always subject to the same deformation.

$$c_{ges} \quad c_1 + c_2 + c_3 + \quad + \quad c_n \qquad (54)$$

3.4.5 Conversions
If drives with different speeds or shafts are combined in one vibration generating system, the stiffnesses and masses are to be converted to a reference speed (input or output).
Conversion is carried out as a square of the transmission ratio:

Transmission ratio:

$$i \quad \frac{n_1}{n_2} \quad \frac{\text{reference speed}}{\text{speed}} \qquad (55)$$

Conversion of stiffnesses c_{n2} and masses J_{n2} with speed n_2 to the respective values c_{n1} and J_{n1} with reference speed n_1:

$$C_{n1} \quad c_{n2}\ i^2 \qquad (56)$$

$$J_{n1} \quad J_{n2}\ i^2 \qquad (57)$$

Before combining stiffnesses and masses with different inherent speeds, conversion to the common reference speed has to be carried out first.

3.4.6 Natural frequencies
a) Formulae for the calculation of the natural frequencies of a fixed one-mass vibration generating system and a free two-mass vibration generating system.
Natural frequency f in Hertz (1/s):

One-mass vibration generating system:

$$\text{Torsion}: \ f_e = \frac{1}{2\pi}\sqrt{\frac{c}{J}} \qquad (58)$$

Two-mass vibration generating system:

$$f_e = \frac{1}{2\pi}\sqrt{c\ \frac{J_1 + J_2}{J_1\ J_2}} \qquad (59)$$

c = torsional stiffness in [Nm/rad]
J, J_i = mass moments of inertia in [kgm^2]

$$\text{Translation, Bending}: \ f_e = \frac{1}{2\pi}\sqrt{\frac{c}{m}} \qquad (60)$$

$$f_e = \frac{1}{2\pi}\sqrt{c\ \frac{m_1 + m_2}{m_1\ m_2}} \qquad (61)$$

c' = translational stiffness (bending stiffness) in [N/m]
m, m_i = masses in [kg]

b) Natural bending frequencies of shafts supported at both ends with applied masses with known deformation f due to the dead weight

$$f_e = \frac{q}{2\pi}\sqrt{\frac{g}{f}} \quad [Hz \qquad (62)$$

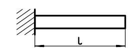
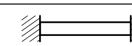
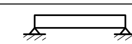
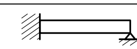
g = 9.81 m/s² gravity
f = deformation due to dead weight [m]
q = factor reflecting the effect of the shaft masses on the applied mass
q = 1 shaft mass is neglected compared with the applied mass
q = 1.03 ... 1.09 common values when considering the shaft masses
q = 1.13 solid shaft without pulley

c) Natural bending frequencies for shafts, taking into account dead weights (continuum); general formula for the natural frequency in the order $f_{e,i}$.

$$f_{e,i} = \frac{1}{2\pi}\left(\frac{\mu_i}{l}\right)^2\sqrt{\frac{E}{\varrho\ A}} \quad Hz \qquad (63)$$

λ_i = inherent value factor for the i-th natural frequency
l = length of shaft [m]
E = modulus of elasticity [N/m^2]
I = moment of area [m^4]
ϱ = density [kg/m^3]
A = cross-sectional area [m^2]
d = diameter of solid shaft [m]

Table 10 λ-values for the first three natural frequencies, dependent on mode of fixing

Bearing application	λ_1	λ_2	λ_3
	1.875	4.694	7.855
	4.730	7.853	10.966
	π	2π	3π
	3.927	7.069	10.210

For the solid shaft with free bearing support on both sides, equation (63) is simplified to:

$$f_{e,i} = \frac{\pi\ d}{8}\left(\frac{i}{l}\right)^2\sqrt{\frac{E}{\varrho}} \quad Hz \qquad (64)$$

i = 1st, 2nd, 3rd ... order of natural bending frequencies.

3.5 Evaluation of vibrations
The dynamic load of machines can be determined by means of different measurement methods. Torsional vibration loads in drives, for example, can be measured directly on the shafts by means of wire strain gauges. This requires, however, much time for fixing the strain gauges, for calibration, signal transmission and evaluation. Since torques in shafts are generated via bearing pressure in gear units, belt drives, etc., in case of dynamic loads, structure-borne noise is generated which can be acquired by sensing elements at the bearing points in different directions (axial, horizontal, vertical).

Dependent on the requirements, the amplitudes of vibration displacement, velocity and acceleration can be recorded and evaluated in a sum (effective vibration velocity) or frequency-selective. The structure-borne noise signal reflects besides the torque load in the shafts also unbalances, alignment errors, meshing impulses, bearing noises, and possibly developing machine damages.

To evaluate the actual state of a machine, VDI guideline 2056 1) or DIN ISO 10816-1 /19, 20/ is consulted for the effective vibration velocity, as a rule, taking into account structure-borne noise in the frequency range between 10 and 1,000 Hertz. Dependent on the machine support structure (resilient or rigid foundation) and power transmitted, a distinction is made between four machine groups (table 11). Dependent on the vibration velocity, the vibrational state of a machine is judged to be "good", "acceptable", "still permissible", and "non-permissible". If vibration velocities are in the "non-permissible" range, measures to improve the vibrational state of the machine (balancing, improving the alignment, replacing defective machine parts, displacing the resonance) are required, as a rule, or it has to be verified in detail that the vibrational state does not impair the service life of the machine (experience, verification by calculation). Structure-borne noise is emitted from the machine surface in the form of airborne noise and has an impact on the environment by the generated noises. For the evaluation of noise, sound pressure level and sound intensity are measured. Gear unit noises are evaluated according to VDI guideline 2159 or DIN 45635 /17, 16/, see subsection 1.5.

Table 11 Boundary limits acc. to VDI guideline 2056 1) for four machine groups

Machine groups	Including gear units and machines with input power ratings of ...	Range classification acc. to VDI 2056 ("Effective value of the vibration velocity" in mm/s)			
		Good	Acceptable	Still permissible	Non-permissible
K	... up to approx. 15 kW without special foundation.	up to 0.7	0.7 ... 1.8	1.8 ... 4.5	from 4.5 up
M	... from approx. 15 up to 75 kW without special foundation. ... from approx. 75 up to 300 kW and installation on highly tuned, rigid or heavy foundations.	up to 1.1	1.1 ... 2.8	2.8 ... 7.1	from 7.1 up
G	... over 300 kW and installation on highly tuned, rigid or heavy foundations.	up to 1.8	1.8 ... 4.5	4.5 ... 11	from 11 up
T	... over 75 kW and installation on broadly tuned resilient foundations (especially also steel foundations designed according to light-construction guidelines).	up to 2.8	2.8 ... 7	7 ... 18	from 18 up

1) 08/97 withdrawn without replacement; see /20/

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