

Technological Competition, Uncertainty, and Oligopoly

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In an oligopoly context the present technological choice of a firm which expects to receive private relevant information just prior to the uncertain market stage has both a *flexibility* value and a *strategic commitment* value. In contrast to some common wisdom ideas we provide a natural two-stage competition framework in which an increase in uncertainty always raises the commitment value of the technological choice of the firm and may decrease its flexibility value when the increased uncertainty takes the form of more variable beliefs. The first result tends to reinforce therefore the findings of the strategic commitment literature under certainty. *Journal of Economic Literature* Classification Numbers: 026, 621. © 1989 Academic Press, Inc.

1. INTRODUCTION

In this paper we analyze the effect of strategic competition in the product market on the technological choices of firms in an uncertain environment. In particular, we examine the impact of changes in the degree of uncertainty, be it in terms of more variability in the environment or in terms of more variable beliefs in an incomplete information context, on the value of flexibility and on the value of strategic commitment of the technological choice prior to the market stage. Received common wisdom provides us with two presumptions in this respect.

PRESUMPTION 1. *More uncertainty will induce firms to seek more flexible technological positions.*

PRESUMPTION 2. *Uncertainty will dilute the strategic commitment value of the technological choice.*

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In fact these two presumptions are linked together since it is thought that firms will gain flexibility by avoiding precommitment. For example, a firm may delay investment on the face of increased uncertainty and this way stay flexible (see Appelbaum and Lim [1] for a formalization of this argument). Nevertheless there are many situations where a firm, to gain production flexibility, does not need to delay decisions about plant design and investment. Instead, the firm may choose technologies which are flexible to respond to changes in the environment.

Firms when faced with uncertainty (and with the prospect of receiving private information) about demand or prices of inputs often have to choose between "multipurpose" technologies and "specific" technologies. The first ones are usually more expensive in terms of capital and maintenance costs but give the firm more flexibility in either the inputs that may be used in the production process and/or the capacity to meet the changing demand. Often flexibility comes at the cost of not being able to use the most efficient technology for any level of output. Some examples may illustrate this issue. (1) Consider an electric utility faced with fluctuating prices for oil, gas, and coal so that the least-cost fuel varies over time. The utility has the choice between installing a "multipurpose" (and relatively expensive) boiler which can use any type of fuel or installing a "specific" (and cheaper) technology which can use any type of fuel (see Fuss and McFadden [14, p. 312]). (2) Firms may be uncertain about the future level of demand, perhaps because there is a new product in the industry, like the case of the corn wet milling industry when confronted with the commercialization of high fructose corn syrup in the early seventies (see Porter and Spence [22]), and have to decide about productive capacity in advance of the realization of demand. Larger capacity choices will be more costly but will give the firms more flexibility to respond to demand conditions, to meet high demand for the new product for example. (3) Computer-integrated manufacturing (CIM) gives the firm flexibility to change output levels and alter its variety offer, perhaps even giving personalized design for customers. Automated factories with clusters of multipurpose machines (flexible machining centre) run by computers (computerised numerical control) require large investments of capital but are very far from the complete specialization of the transfer line.

In the process of making technological choices in an uncertain environment, firms have to consider not only the added production flexibility resulting but also the strategic commitment value of the choice made. That is, the fact that the technological choice fixes a short run cost function with which the firm has to compete in the market. For example, by investing a large amount on cost reduction a firm is credibly committing to an aggressive production strategy at the market stage. Four strands of the literature have dealt partially with these issues. The first has analyzed the effect of increased uncertainty on the flexibility choice of a *single* decision

maker (see Epstein [8], Jones and Ostroy [17] and Freixas and Laffont [11]). The second has studied the strategic value of capacity investment in the market under conditions of *certainty* (see, among others, Spence [23], Dixit [7], and Fudenberg and Tirole [12]). Nevertheless not much work has dealt with the effect of uncertainty on strategic investment decisions (the exceptions are Perrakis and Warskett [21] and the abovementioned Appelbaum and Lim [1]). The third strand of the literature includes the technological competition studies, which have not dealt at all with the *flexibility* issue (see Dasgupta [5] for a survey). Finally, the information oligopoly literature, with its analysis of information sharing and comparative statics issues of market competition (see Basar and Ho [3], Novshek and Sonnenschein [20], and Vives [25] as a sample).

An increase in the degree of uncertainty, be it in terms of more variability in the environment or of more variable beliefs in an incomplete information context, will affect the value of flexibility and the value of strategic commitment to the firms. In the paper we will explore this relationship decomposing the total (marginal) value of the technological position to the firm into a flexibility value and a strategic commitment value. This way we incorporate into the theory of technological competition the crucial desire for flexibility and analyze how the value of commitment is affected by uncertainty. All this in a small numbers framework where strategic interaction is unavoidable.

We envision the process of competition in two stages. Firms choose, at a first stage, their technological positions. Afterwards they receive private signals about uncertain payoff relevant parameters and compete in the marketplace, where production decisions are made. The technological choice may consist of either investment which lowers production costs or of plant design which affects the shape of the short run cost function. In practice, a mix of both is usual. Two extreme models will be analyzed in the paper. In the *cost reduction model* firms' investment at the first stage lowers the slope of the marginal cost of production,¹ keeping constant its intercept. In the *plant design model*, firms choose at the first stage a technological parameter, which corresponds to the slope of marginal production cost, but there is a trade-off: a lower slope means a higher intercept for marginal cost. A lower slope for marginal production costs corresponds to a more flexible technological position. This is basically Stigler's definition of flexibility regarding technologies: a technology is more flexible than another if average and marginal costs are flatter in the former than in the latter (Stigler [24]).² In the cost reduction model a higher investment at the first stage represents a higher degree of commit-

¹ Related cost reduction models are considered by Spence [23].

² This definition of flexibility was adopted also in Vives [27].

ment since it facilitates more aggressive production strategies at the market stage. In the plant design model a technology with a higher intercept and lower slope for marginal cost represents a higher degree of commitment since it also makes high outputs relatively cheaper to produce.

Firms will face more uncertainty either because the prior variability of the uncertain payoff-relevant parameter has increased or because they expect to receive more precise signals at the production stage, in which case their beliefs are more variable at the previous technological choice stage. Indeed, if a firm expected to receive no information at the production stage, its beliefs would not be variable at all.

According to Presumption 1, firms when confronted with more uncertainty will tend to choose more flexible first-stage positions. The reason why a firm may want to remain flexible when expecting to receive a more informative signal is to be able to take advantage of the increased information when the time of the production decision comes. Although this argument strictly only applies to a monopoly, since it does not account for market interaction, it provides a commonly used hypothesis to analyze the value of flexibility under uncertainty. Presumption 2 suggests that even under risk neutrality more uncertainty will lessen the value of strategic commitment. We will show that these presumptions do not hold in the presence of private information in a context where firms can gain production flexibility through plant design and investment activities.

In this article we analyze the subgame perfect equilibria of the two-stage game. In this situation a firm when making its technological choice takes into account the effects it will have on the subsequent production decisions of the firms in the market. An increase in uncertainty will affect then both the desire for flexibility and the desire for commitment in terms of the technological choice. We would like to separate the two effects. This can be accomplished by noting that the flexibility effect can be isolated considering open loop equilibria (OLE) of the game. In an open loop equilibrium a firm takes as given the technological positions *and* the output rules of the rival firms and therefore does not try to influence them via its first-stage technological choice. OLE isolate then the flexibility effect by abstracting from the strategic commitment value of the technology choice.³ With this technique we are able to define separate flexibility and strategic commitment effects which add up to the total effect on technological choice of a change in the degree of uncertainty faced by firms.

Assuming that the n firms in the market are ex-ante identical and restricting attention to symmetric equilibria we are able to obtain the following results for the cost reduction model. If the increase in uncertainty comes from an increase in prior variability, both the value of flexibility and the

³ A related analysis is provided in Fudenberg and Tirole [12].

value of strategic commitment are increased and therefore investment in cost reduction expands. If the increase in uncertainty comes from more variable beliefs (more precise signals) then the value of commitment is increased but the change in the value of flexibility is ambiguous. The basic reason is that by increasing the precision of the information from the point of view of a firm we are, at the same time, improving its precision and the precision of the rival firms. In our Cournot competition context, the first factor tends to increase the value of flexibility for the firm; the second, to decrease it. In fact, if the aggregate precision of the rival firms is high enough, the second factor will overwhelm the first and the value of flexibility will decrease. The sign of the total effect depends then on the relative strength of the changes in flexibility and strategic commitment values. Simulations with examples indicate that the negative changes in the value of flexibility may dominate the strategic effect for n as low as 5, in which case the investment in cost reduction as a function of the precision of the information is increasing first and decreasing afterwards.

The main message is that an increase in uncertainty always increases the value of strategic commitment, a result that hinges on the risk neutrality of firms, but that it may decrease the value of flexibility due to the interaction in the market when it is a consequence of more variable beliefs. The comparative static properties of the plant design model with respect to the information structure turn out to be qualitative identical to the cost reduction model.

Section 2 lays out our technology flexibility models and the information structure. Section 3 sets forth the structure of the two-stage game. Section 4 deals with the cost reduction model. The plant design model is analyzed in Section 5 and concluding remarks follow. In the Appendix we collect some notation and proofs.

2. TECHNOLOGY AND INFORMATION STRUCTURE

Technological Choice and Flexibility

When will we say that production plant A is more flexible than production plant B? According to Stigler [24] plant A is more flexible than plant B if the average and marginal costs associated with A are “flatter” than the average and marginal cost associated with B. Stigler considered the choice by firms of what degree of flexibility to incorporate in a plant, arguing that flexibility has value in an uncertain environment but comes at the cost of not being able to use the best technology for any given output level.

Fuss and McFadden [14] analyzed a two-stage process where first the firm makes a technological choice which involves design and investment

variables (different designs being associated with different parameters in a family of production functions and investments being in physical capital, for example), then receives a signal about the uncertain environment, and finally production decisions are made taking prices as given. In this paper we will consider two extreme types of technological choice which we label the cost reduction model and the plant design model.

In the *cost reduction model* a firm makes an investment at the first stage which lowers the marginal cost of production at the market stage. A firm by investing $F(\lambda)$ obtains a production technology characterized by the quadratic cost function λx^2 , where x is the output level and $\lambda > 0$. Investing more, the firm can lower the slope of marginal cost 2λ . We assume:

(ACR) $F: \mathbf{R}_{++} \rightarrow \mathbf{R}_{++}$, is twice-continuously differentiable, strictly decreasing ($F' < 0$) and strictly convex ($F'' > 0$) with $\lim_{\lambda \rightarrow 0} F(\lambda) = \infty$ and $\lim_{\lambda \rightarrow \infty} F(\lambda) = 0$.

Figure 1 depicts $F(\cdot)$. The total cost function of a firm is then $C(x; \lambda) = F(\lambda) + \lambda x^2$. For example, $F(\lambda) = \gamma \lambda^{-\epsilon}$, where γ and ϵ are positive parameters, $F(\cdot)$ can be thought of as being an innovation possibility curve showing diminishing returns to R & D expenditure. Alternatively, we could think that $F(\lambda)$ represents a capital investment which gives rise to the short run cost function $C(\cdot; \lambda)$. The parameter λ would be related then to the elasticity of substitution between capital and variable inputs. In any case by investing more a firm may get a more flexible production technology, i.e., a marginal cost curve with lower slope.

In the *plant design model* firms choose a technology parameter λ at the first stage. The parameter λ fixes the slope of marginal production cost but there is a trade-off; a lower slope means a higher intercept of marginal costs. The cost of design is fixed and independent of the level of λ chosen, we will forget about it. Given λ , production costs are given then by $C(x; \lambda) = c(\lambda)x + \lambda x^2$, where we assume:

(APD) $c: \mathbf{R}_{++} \rightarrow \mathbf{R}_{++}$, is a twice continuously differentiable, strictly decreasing ($c' < 0$) and strictly convex ($c'' > 0$) function with $\lim_{\lambda \rightarrow \infty} c(\lambda) = 0$ and $c(0) \equiv \bar{c} < \infty$.

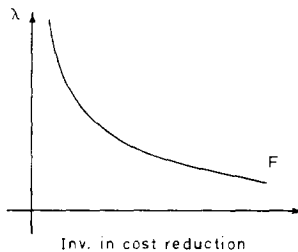


FIG. 1. Cost reduction model. Innovation possibility curve.

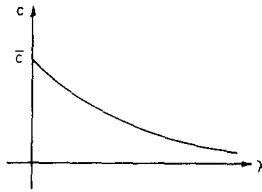


FIG. 2. Plant design model. Intercept of marginal cost as a function of λ .

Figure 2 depicts $c(\cdot)$. For example, $c(\lambda) = \bar{c}e^{-d\lambda}$, where \bar{c} and d are positive. Marginal cost is given by $MC(x; \lambda) = c(\lambda) + 2\lambda x$, a lower λ means a higher intercept $c(\lambda)$. In Fig. 3, $\lambda < \lambda'$ and $c(\lambda) > c(\lambda')$.

Information Structure

Firm i will receive a signal s_i about an uncertain payoff relevant parameter α distributed according to some prior density with mean μ and finite variance $V(\alpha)$. The signal received by firm i is of the “true state plus noise” variety:

$$s_i = \alpha + \varepsilon_i,$$

where $E\varepsilon_i = 0$, $E\varepsilon_i^2 = v$, and $\text{Cov}(\varepsilon_i, \varepsilon_j) = \text{Cov}(\alpha, \varepsilon_i) = 0$, $j \neq i$. Error terms are independent across firms and signals are independent and identically distributed conditional on the true state. The signal s_i is perfect if $v = 0$ and contains no information if $v = \infty$; $1/v$ represents the precision of signal. Assuming that $E(\alpha|s_i)$ is affine in s_i it follows (Ericson [9]) that

$$E(\alpha|s_i) = (1 - t)\mu + ts_i, \text{ where } t = V(\alpha)/(V(\alpha) + v)$$

and, furthermore,

$$E(s_j|s_i) = E(\alpha|s_i) \text{ and } \text{Cov}(s_i, s_j) = \text{Cov}(s_i, \alpha) = V(\alpha), \quad j \neq i.$$

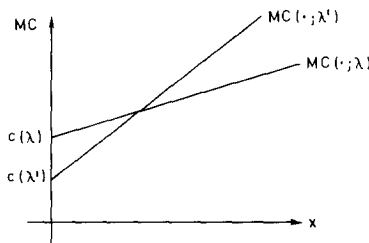


FIG. 3. Plant design model. Marginal costs for two different values of λ : $\lambda' > \lambda$.

Information structures which satisfy the assumptions above include the pairs prior-likelihood, normal-normal, beta-binomial, and gamma-Poisson (see Ericson [9] and DeGroot [6] for more examples). In these examples the sample mean is a sufficient statistic for α and a more precise signal just means a larger sample. Therefore signal s is more precise (weakly) than signal s' ($v \leq v'$) if and only if s is more informative than s' in the Blackwell sense (see Kihlstrom [18] and Marschak and Miyasawa [19]).

This affine information structure will allow us to give specific meaning to the statement that one situation is "more uncertain" than another from the point of view of the firm. Firms will choose first technological positions, and afterwards receive signals about the uncertain parameter α and compete in the marketplace. At the first stage "more uncertainty" can either be coming from an increase in the prior variance of α , $V(\alpha)$, or from the prospect of receiving a more informative signal, that is, with lower variance of the error term, v .

3. MARKET COMPETITION AS A TWO-STAGE GAME

There are n firms in the market. Demand is linear and given by $p = \alpha - \beta Z$, where α is possibly random, β is a positive parameter, and Z is total output. Firm i receives a private signal s_i about the uncertain α . (Firms are ex-ante identical, all face the same technological prospects and will receive signals of the same precision.)

The sequence of events is as follows: firms make their technological choice first, learn some (private) information about demand and finally compete in quantities. That is, firm i chooses, at the first stage, a technological parameter λ_i and, at the second, an output rule $z_i(\cdot)$ which yields productions contingent on the received signals. Firm i by choosing λ_i faces a total cost function $C(z; \lambda_i)$. We will consider subgame perfect equilibria (SPE) of this two-stage game. In this equilibrium the firms when making the first period decisions take into account the effects of their actions on the second stage equilibrium productions. As usual we solve the game starting with the second stage where a Bayesian-Cournot equilibrium obtains contingent on the chosen production cost schedules: $z_i^*(\cdot; A)$, $i = 1, \dots, n$; where $A = (\lambda_1, \dots, \lambda_n)$. This yields second-stage expected profits as a function of technological choices made by the firms. We obtain thus a reduced-form payoff function for every firm depending only on their first stage decisions:

$$P_i(A) = E\{pz_i^*(s_i; A) - C(z_i^*(s_i; A); \lambda_i)\}, \quad i = 1, \dots, n,$$

where $p = \alpha - \beta \sum_{j=1}^n z_j^*(s_j; A)$. We will restrict attention to symmetric

Nash equilibria of this reduced game. It is enough then to consider $A = (\lambda_i, \bar{\lambda})$, where $\lambda_j = \bar{\lambda}$, $j \neq i$. Under regularity conditions we will show that there is a unique interior symmetric equilibrium λ^* , characterized by the first-order condition (using the envelope result)

$$\frac{\partial P_i}{\partial \lambda_i}(A) \equiv E \left\{ -\beta z_i^*(s_i; A) \sum_{j \neq i} \frac{\partial z_j^*(s_j; A)}{\partial \lambda_i} - \frac{\partial C(z_i^*(s_i; A); \lambda)}{\partial \lambda_i} \right\} = 0,$$

where $A = (\lambda; \bar{\lambda})$.

In the SPE two phenomena are mixed: the desire for commitment and the desire for flexibility. A firm when making a technological choice takes into account both the production flexibility needed to face uncertainty and the strategic effect it will have on the output decisions of the rival firms.

Our purpose is to evaluate the impact of an increase in uncertainty on the equilibrium technological choices of firms. In particular, we would like to separate the flexibility from the strategic effects of a change in the variability of demand or in the beliefs held by firms. The pure flexibility effect is easy to isolate since it corresponds to open loop equilibria of our two-stage game. In an open loop equilibrium a firm does *not* take into account the effect of its technological choice on the output decisions of the other firms. In OLE firm i takes as given the technological decisions, λ_j , and the output rules, $z_j(\cdot)$, of the other firms, $j \neq i$, and optimizes accordingly, choosing an appropriate pair $(\lambda_i, z_i(\cdot))$. In SPE firm i takes as given the technological choices of the other firms but takes into account the effect of its choice of λ_i on the second stage equilibrium productions. Consideration of OLE will allow us to separate the flexibility from the strategic impact of changes in information parameters. In this case, given that firms $j \neq i$ use strategies $(\lambda_j, z_j(\cdot))$, firm i faces a payoff

$$E\{p z_i(s_i) - C(z_i(s_i); \lambda_i)\},$$

where $p = \alpha - \beta \sum_{j=1}^n z_j(s_j)$. Restricting attention again to symmetric equilibria and under regularity conditions, the open loop equilibrium technological choice λ will be characterized by

$$E \left\{ -\frac{\partial C(z^*(s_i); \lambda)}{\partial \lambda_i} \right\} = 0,$$

where $z^*(\cdot)$ is the symmetric Bayesian-Cournot equilibrium at the production stage (when all firms have λ as the technological parameter). This is so since production rules have, to the best responses to each other, given the technological choices of firms.

We see thus that the expression for OLE only involves the term $E\{-\partial C/\partial \lambda_i\}$, the expected marginal effect on total costs of the technologi-

cal choice of the firm, while the expression for SPE involves also the term $E\{-\beta z_i^* \sum_{j \neq i} \partial z_j^* / \partial \lambda_i\}$, the expected marginal effect on the firm's revenue due to the induced change in output decisions of other firms. The marginal profitability of the technological choice of a firm $\partial P_i / \partial \lambda_i$ can be decomposed then between a *flexibility effect*

$$E \left\{ -\frac{\partial C}{\partial \lambda_i} \right\}$$

and a *strategic commitment effect*

$$E \left\{ -\beta z_i^* \sum_{j \neq i} \frac{\partial z_j^*}{\partial \lambda_i} \right\}.$$

We examine now in the context of the cost reduction and the plant design models how changes in the information parameters impact the flexibility and the strategic effects and consequently affect the technological choices of firms.

4. COST REDUCTION

In this model every firm faces a total cost function⁴ $C(x; \lambda) = \lambda x^2 + F(\lambda)$. $F(\cdot)$ satisfies ACR (see Fig. 1). A firm by investing $F(\lambda)$ at the first stage can produce a level of output x at a cost λx^2 . Since we restrict attention to *symmetric* subgame perfect equilibria when solving for the second stage Bayesian-Cournot equilibrium we need only consider the situation where, say, firm 1 has chosen λ_1 and all the other firms $\tilde{\lambda}$. Lemma 1 characterizes the unique equilibrium for this class of subgames.

LEMMA 1. *Given $t \in [0, 1]$ and n firms in the market with $(\lambda_1, \tilde{\lambda}, \dots, \tilde{\lambda})$, λ_1 , and $\tilde{\lambda}$, in $[0, \infty)$, there is a unique Bayesian-Cournot equilibrium⁵ $(x_1(\cdot), \tilde{x}(\cdot), \dots, \tilde{x}(\cdot))$, where*

$$x_1(s_1) = a_1(s_1 - \mu) + b_1\mu \quad \text{and} \quad \tilde{x}(s_j) = \tilde{a}(s_j - \mu) + \tilde{b}\mu, \quad j \neq 1,$$

with $a_1 = (2(\beta + \tilde{\lambda}) - \beta t)t/D_1$, $\tilde{a} = (2(\beta + \lambda_1) - \beta t)t/D_1$, $b_1 = a_1|_{t=1}$, $\tilde{b} = \tilde{a}|_{t=1}$, and $D_1 = 2(\beta + \lambda_1)(2(\beta + \tilde{\lambda}) + (n - 2)\beta t) - (n - 1)\beta^2 t^2$.

⁴ Uncertainty could be also about a common and constant part of marginal production cost: $MC_i = mx_i + \lambda_i x_i^2$ with m random. Profits of firm i in a quantity setting context would then be $\pi_i = (\alpha - m - \beta x)x^i - \lambda x_i^2 - F(\lambda_i)$. Redefining appropriately α we go back to our original formulation. Notice nevertheless that this uncertainty cost formulation does not cover the complexity of our electric utility example.

⁵ To simplify notation we do not make explicit the dependence of equilibrium productions $x_i(\cdot)$ and $\tilde{x}(\cdot)$ on $(\lambda_1, \tilde{\lambda})$.

Proof. Similar to Basar and Ho [3].

Remark 1. The equilibrium strategies are certainty equivalent. $Ex_1 = b_1\mu$ and $E\tilde{x} = \tilde{b}\mu$ are the Cournot equilibrium strategies when $\alpha = \mu$ almost surely.

Remark 2. In the symmetric situations where $\lambda_1 = \tilde{\lambda} = \lambda$ equilibrium production is $x(s_i) = a(s_i - \mu) + b\mu$. The equilibrium slope is $a = t / ((2(\beta + \lambda) + (n - 1)(\beta(t)))$, which is increasing in t ($t = V(\alpha) / (v + V(\alpha))$). If the common precision of information ($1/v$) or the prior variance ($V(\alpha)$) increase firms trust more the signal and output is more responsive to the appraisals of demand. It is worth noticing nevertheless that by increasing $1/v$ from the point of view of a firm, we are increasing both the firms and the rivals' precision of information. In fact, these are likely to have opposite effects. By increasing their own precision the firm will tend to respond more to its signal but by increasing the rivals' precision, the opposite will be true in a Cournot context. This phenomenon is easily understood. Suppose that firm 1 is perfectly informed. If the other firms have no information then they will produce a constant amount (equal to $\tilde{b}\mu$). If the rival firms have also perfect information then when demand is high they will produce more than $\tilde{b}\mu$ and less when it is low. Consequently when the other firms are informed, firm 1 will produce less when demand is high and more when demand is low since in our Cournot context production best responses are decreasing. In other words, the slope of the equilibrium strategy of firm 1 is decreasing in the precision of the information of rival firms.

The implied short run profits for firm 1 when the rival firms choose $\tilde{\lambda}$ are given immediately from Lemma 1 by $E\pi_1 = (\beta + \lambda_1) E(x_1(s_1))^2$. The first stage payoff to firm 1 is therefore $P_1(\lambda_1, \tilde{\lambda}) = (\beta + \lambda_1) E(x_1(s_1))^2 - F(\lambda_1)$. Let $r(\cdot) \equiv \lambda F'' / |F'|$; that is, $r(\cdot)$ is the relative index of convexity of $F(\cdot)$. We will say that $r(\cdot)$ is large enough if $r(\lambda) \geq 6$ for all λ . We are now ready to characterize the (symmetric) SPE of our two-stage game.

PROPOSITION 1. *If the relative index of convexity of the innovation possibility curve $F(\cdot)$ is large enough then there is a unique symmetric subgame perfect equilibrium. The equilibrium λ^* is the unique solution to*

$$\varphi(\lambda) \equiv -[AV(\alpha) + B\mu^2] - F'(\lambda) = 0,$$

where

$$A \equiv \left\{ 1 + \frac{2(n-1)\beta^2 t^2}{D} \right\} \frac{a^2}{t} \quad \text{and} \quad B = A|_{t=1}.$$

Furthermore the equilibrium is regular: $\varphi'(\lambda^*) < 0$.

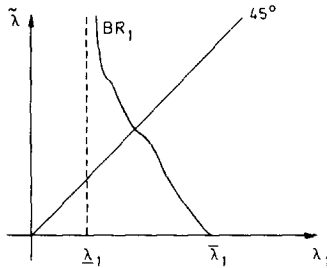


FIG. 4. Equilibrium in the cost reduction model.

Proof. Let $BR_1(\tilde{\lambda})$ be the best response function for firm 1 when the other firms choose $\tilde{\lambda}$. In the Appendix, under the assumption for $r \geq 6$, we show:

(i) $BR_1(\cdot)$ is well defined as the unique solution to

$$\frac{\partial P_1}{\partial \lambda_1}(\lambda_1, \tilde{\lambda}) = 0,$$

since $P_1(\lambda_1, \tilde{\lambda})$ is strongly quasiconcave⁶ in λ_1 for all $\tilde{\lambda} > 0$.

(ii) $BR_1(\cdot)$ is continuously differentiable and decreasing.

(iii) Letting $\underline{\lambda}_1 = \inf_{\tilde{\lambda}} BR_1(\tilde{\lambda})$ and $\bar{\lambda}_1 = \sup_{\tilde{\lambda}} BR_1(\tilde{\lambda})$, we have $\bar{\lambda}_1 > \underline{\lambda}_1 > 0$.

Therefore $BR_1(\cdot)$ is as depicted in Fig. 4 and uniqueness of a symmetric equilibrium follows immediately since the equilibrium is given by the intersection of $BR_1(\cdot)$ and the 45° line.

The equilibrium λ^* will be given by the unique solution to

$$\varphi(\lambda) \equiv \left. \frac{\partial E\pi_1}{\partial \lambda_1} \right|_{\lambda_1 = \tilde{\lambda} = \lambda} - F'(\lambda) = 0.$$

Some computations show that

$$-\left. \frac{\partial E\pi_1}{\partial \lambda_1} \right|_{\lambda_1 = \tilde{\lambda} = \lambda} = AV(\alpha) + B\mu^2,$$

as asserted.

(iv) $\varphi' < 0$ when $\varphi = 0$. Therefore $\varphi'(\lambda^*) < 0$.

Q.E.D.

⁶ We say that the twice-continuously differentiable function $f: \mathbf{R} \rightarrow \mathbf{R}$ is *strongly quasiconcave* if $f' = 0$ implies that $f'' < 0$.

TABLE I
Cost Reduction Model SP Equilibrium $(\lambda^*, x(\cdot))$

Technology: $\varphi(\lambda^*) \equiv -[AV(\alpha) + B\mu^2] - F'(\lambda^*) = 0;$
Production: $x(s_i) = a(s_i - \mu) + b\mu$, where

$A = (1 + \Delta)a^2/t$	$B = A _{t=1} = (1 + \bar{\Delta})b^2$
$\Delta = 2(n-1)\beta^2 t^2/D$	$\bar{\Delta} = \Delta _{t=1} = 2(n-1)\beta^2/\bar{D}$
$D = (2(\beta + \lambda^*) - \beta t)t/a$	$\bar{D} = D _{t=1} = (\beta + 2\lambda^*)/b$
$a = t/(2(\beta + \lambda^*) + (n-1)\beta t)$	$b = a _{t=1} = 1/((n+1)\beta + 2\lambda^*)$

Following the analysis of the section before we can decompose the incentives to invest in cost reduction between flexibility and strategic effects. Firm 1 by selecting $(\lambda_1, z_1(\cdot))$ when other firms select $(\bar{\lambda}, \bar{z}(\cdot))$ faces a payoff:

$$E \left\{ \left(\alpha - \beta z_1(s_1) - \beta \sum_{j>1} \bar{z}(s_j) \right) z_1(s_1) \right\} - \lambda_1 E(z_1(s_1))^2 - F(\lambda_1).$$

In a symmetric situation with all λ 's equal, the flexibility effect is then

$$\begin{aligned} \varphi^f &= E - \frac{\partial C}{\partial \lambda_1}(x(s_1), \lambda) = -E(x(s_1))^2 - F'(\lambda) \\ &= - \left[\frac{a^2}{t} V(\alpha) + b^2 \mu^2 \right] - F'(\lambda) \end{aligned}$$

and the strategic effect,

$$\varphi^s \equiv -(n-1)\beta E x(s_1) \frac{\partial \bar{x}(s_j)}{\partial \lambda_1} = - \left[\Delta \frac{a^2}{t} V(\alpha) + \bar{\Delta} b^2 \mu^2 \right].$$

These add up to the total effect:

$$\varphi = \varphi^f + \varphi^s.$$

The subgame perfect equilibrium λ^* solves

$$\varphi(\lambda) = 0$$

and the open loop equilibrium⁷ λ^{OL} ,

$$\varphi^f(\lambda) = 0.$$

⁷ Uniqueness and regularity of the symmetric open loop equilibrium follows similarly to the SPE case.

Since $\varphi^f(\lambda) > \varphi(\lambda)$ for all $\lambda > 0$ and both λ^{OL} and λ^* are regular equilibria, it follows immediately that $\lambda^{OL} > \lambda^*$. There is more investment in cost reduction with SPE. This result is in line with Dixit [7] and Brander and Spencer [4] which show that there is a strategic incentive for firms to invest more in capital than would be necessary for efficiency reasons.

The comparative static properties of the equilibrium investment in cost reduction with respect to the parameters $1/v$ and $V(\alpha)$ of the information structure will depend on how these parameters effect the marginal profitability of the investment φ . The information parameters do not affect the innovation possibility curve described by $F(\lambda)$. They affect only the marginal profitability at the production stage of investment in cost reduction, i.e., of a lower λ :

$$\Psi(\lambda; v^{-1}, V(\alpha)) \equiv \left. \frac{\partial E\pi_j}{\partial \lambda_i} \right|_{\lambda_1 = \lambda = \lambda}$$

Obviously, $\varphi = \Psi - F'$.

From Proposition 1 it follows immediately that

$$\frac{\partial \lambda^*}{\partial v^{-1}} = -\frac{\partial \Psi / \partial v^{-1}}{\varphi'} \quad \text{and} \quad \frac{\partial \lambda^*}{\partial V(\alpha)} = -\frac{\partial \Psi / \partial V(\alpha)}{\varphi'}$$

Since $\varphi'(\lambda^*) < 0$,

$$\text{sign} \left\{ \frac{\partial \lambda^*}{\partial v^{-1}} \right\} = \text{sign} \left\{ \frac{\partial \Psi}{\partial v^{-1}} \right\} \quad \text{and} \quad \text{sign} \left\{ \frac{\partial \lambda^*}{\partial V(\alpha)} \right\} = \text{sign} \left\{ \frac{\partial \Psi}{\partial V(\alpha)} \right\}$$

The marginal profitability Ψ reflects both flexibility and strategic effects: $\Psi = \Psi^f + \Psi^s$, where $\Psi^f = \varphi^f + F'$ and $\Psi^s = \varphi^s$. Changes in the information parameters $V(\alpha)$ and v^{-1} will affect both Ψ^f and Ψ^s . The next proposition tells us how.

PROPOSITION 2. *In a symmetric situation where all firms choose the same technological position λ ,*

(a) *an increase in uncertainty, be it in terms of $V(\alpha)$ or of v^{-1} , always magnifies the strategic effect, provided signals are informative, and*

(b) *an increase in prior uncertainty $V(\alpha)$ induces a larger flexibility effect but changes in the precision of information v^{-1} have an ambiguous impact:*

$$\text{sign} \left\{ -\frac{\partial \Psi^f}{\partial v^{-1}} \right\} = \text{sign} \{ 2\lambda - \beta(t(n-1) - 2) \}$$

Proof. See Appendix.

Remark. The inequality $2\lambda \geq \beta(t(n-1)-2)$ holds for all t if and only if $2\lambda \geq \beta(n-3)$. (As a curiosity it is interesting to notice that when $2\lambda > \beta(n-3)$ the Cournot equilibrium of the uncertainty game is stable with respect to the usual Cournot tatonnement (See Fisher [10])). It follows therefore that $-\partial\Psi^f/\partial v^{-1}$ is always positive if n is less than or equal to 3.

The total impact on the marginal profitability of investment of an increase in the variance of demand will be positive and of an increase in the precision of information will be ambiguous and will depend on the flexibility effect. This means that investment in cost reduction will always increase with $V(\alpha)$ but may increase or decrease with v^{-1} . The intuition for these results is not very complex. Let us start with the flexibility effect. We know from Remark 2 that the slope of the equilibrium production strategy increases with $V(\alpha)$ and with v^{-1} . Both make a firm more responsive to its signal. Nevertheless there is a difference between increasing $V(\alpha)$ and v^{-1} from the point of view of a firm. When increasing the common precision of information we are giving the rival firms more accurate signals and this we have argued will tend to decrease the slope of the production strategy, decreasing in turn the value of flexibility for the firm. When $2\lambda < \beta(t(n-1)-2)$ the flexibility effect is decreasing in v^{-1} , reflecting the fact that the aggregate precision of the rivals $t(n-1)$ is so high that a further increase in precision reduces the value of flexibility to the Cournotian firm; that is, it contributes to reducing the marginal profitability of investing in cost reduction. This explains the ambiguous comparative statics on the flexibility effect. In contrast, the induced changes on the strategic effect of an increase in uncertainty, in both of its forms, will always tend to increase investment. More uncertainty means that the strategic or commitment value of investment increases with risk-neutral firms. When $2\lambda \geq \beta(t(n-1)-2)$ the total impact of an increase in the precision of information $\partial\Psi^f/\partial v^{-1}$ will be unambiguous, since both $-\partial\Psi^f/\partial v^{-1}$ and $-\partial\Psi^s/\partial v^{-1}$ are positive. Otherwise ($2\lambda < \beta(t(n-1)-2)$) it is possible to show that the negative flexibility effect dominates⁸ and the marginal profitability of a firm is decreasing in the precision of the information if n is large and the common precision is high enough, provided that λ is not too small. The following proposition summarizes the comparative static results obtained.

PROPOSITION 3. *Under the assumptions of Proposition 1 the equilibrium investment in cost reduction $F(\lambda^*)$ is increasing in the variance of the demand intercept provided the signals are not pure noise and in the precision of information if $2\lambda^* \geq \beta(t(n-1)-2)$.*

⁸ Letting $t = \beta = 1$ we get that the coefficient of the n^2 in the expression for $-\partial\Psi^f/\partial v^{-1}$ is $2 - (1 + 2\lambda)^2$, which is negative if λ is not too small. For n large the n^2 term will dominate and therefore $-\partial\Psi^f/\partial v^{-1} < 0$ if t is large enough.

A graphical summary of the argument follows. The unique symmetric equilibrium is given by the intersection of the 45° line and BR_1 in Fig. 4. If the variance of the demand intercept increases then the marginal profitability from investing in cost reduction increases also and BR_1 shifts to the left and a new equilibrium obtains with a lower λ . This is also the case when the common precision of information increases, provided that at the initial equilibrium $2\lambda^* \geq \beta(t(n-1)-2)$. If there are many firms in the market and the precision of information is good enough then the inequality may not hold and an increase in the common precision of information may have the opposite effect by giving the rival firms so much information as to decrease the marginal profitability to the firm's investment in cost reduction.

How soon will we get the investment $F(\lambda^*)$ decreasing in the precision of the information? To get an analytic answer is quite messy but simulations with the canonical example $F(\lambda) = \gamma\lambda^{-\varepsilon}$, where γ and ε are positive parameters, suggest that this phenomenon occurs as early as $n = 5$ for some parameter configurations. Figure 5 shows λ^* as a function of t for $n = 8$, $\beta = 2$, $\varepsilon = 5$, $\gamma = 0.002$, $\mu = 2$, and $V(\alpha) = 1$. Investment in cost reduction is first increasing and then decreasing with the precision of information, peaking at a value of t between 0 and 1.

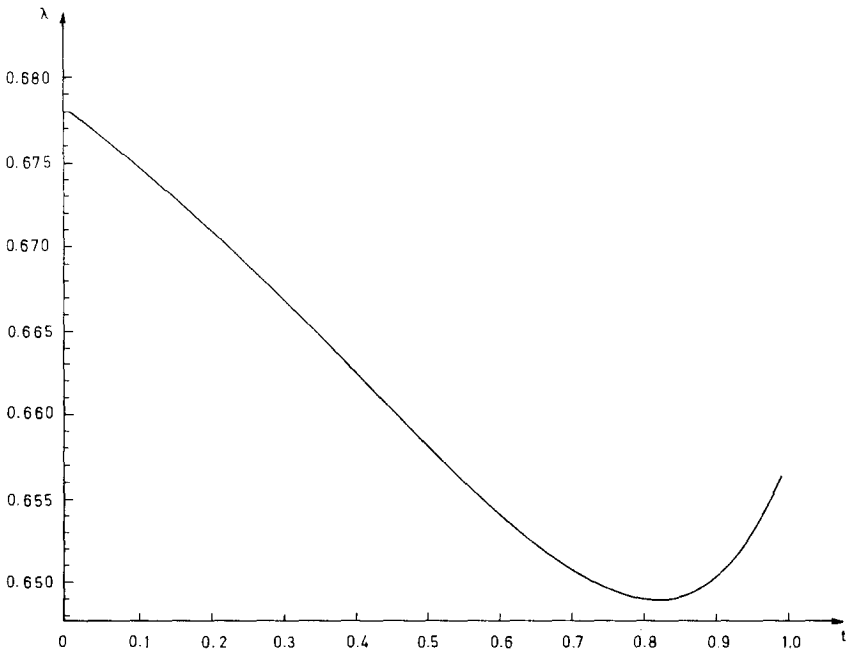


FIG. 5. The equilibrium λ in the cost reduction model as a function of t when $F(\lambda) = \gamma\lambda^{-\varepsilon}$ and $n = 8$, $\beta = 2$, $\mu = 2$, $V(\alpha) = 1$, $\gamma = 0.002$, and $\varepsilon = 5$.

5. PLANT DESIGN

In this model a firm faces a total cost function $C(x; \lambda) = c(\lambda)x + \lambda x^2$. $c(\cdot)$ satisfies APD (see Fig. 2). By the choice of λ the firm trades off the slope (2λ) with the intercept ($c(\lambda)$) of marginal production cost (see Fig. 3). The λ parameter may be interpreted as a design parameter of the production plant, which is chosen at a first stage.

The game proceeds as before. Firms choose (noncooperatively) their levels of the λ parameter first and then, contingent on the choices made, they compete in quantities and a Bayesian–Cournot equilibrium obtains. We will restrict attention to symmetric subgame perfect equilibria of this two-stage game. As usual we check first that for any contemplated deviation of firm 1 from the candidate equilibrium technological position $\tilde{\lambda}$ a unique Bayesian–Cournot equilibrium is obtained at the second stage with firm 1 producing according to $y_1(\cdot)$ and the other firms according to $\tilde{y}(\cdot)$. To ensure positive production for any choices of λ_1 and $\tilde{\lambda}$ we will assume that demand is large enough relative to the intercepts of the marginal cost functions. In particular we will assume that $\mu - n\bar{c} > 0$. This implies that Ey_1 and $E\tilde{y}$ are positive. Lemma 3 describes the equilibrium strategies. They can be decomposed into two terms: the first term, as in the cost reduction model, plus an additional term reflecting the intercept of the marginal cost schedules.

LEMMA 3. *Given $t \in [0, 1]$ and n firm in the market with choices $(\lambda_1, \tilde{\lambda}, \dots, \tilde{\lambda})$, all nonnegative, there is a unique Bayesian–Cournot equilibrium $(y_1(\cdot), \tilde{y}(\cdot), \dots, \tilde{y}(\cdot))$, where*

$$y_1(s_1) = x(s_1) + d_1 \quad \text{and} \quad \tilde{y}(s_j) = \tilde{x}(s_j) + \tilde{d}, \quad j \neq 1,$$

with $x_1(\cdot)$ and $\tilde{x}(\cdot)$ as in the cost reduction model and

$$d_1 = [c(\tilde{\lambda})(n - 1)\beta - c(\lambda_1)(2\tilde{\lambda} + n\beta)]/\bar{D}_1,$$

$$\tilde{d} = [c(\lambda_1)\beta - 2c(\tilde{\lambda})(\beta + \lambda_1)]/\bar{D}_1, \text{ and}$$

$$\bar{D}_1 = D_1|_{t=1}.$$

Proof. Standard and similar to the proof of Lemma 1.

Remark. It is easily checked that $\mu - n\bar{c} > 0$ is a sufficient condition for expected equilibrium outputs

$$Ey_1 = [(2\tilde{\lambda} + n\beta)(\mu - c(\lambda_1)) - (n - 1)\beta(\mu - c(\tilde{\lambda}))]/\bar{D}_1,$$

$$E\tilde{y} = [2\lambda\beta + \lambda_1)(\mu - c(\tilde{\lambda})) - \beta(\mu - c(\lambda_1))]/\bar{D}_1$$

to be positive. In the symmetric case, where $\lambda_1 = \bar{\lambda} = \lambda$ equilibrium output equals $y(s_i) = a(s_i - \mu) + \bar{y}$, where expected output \bar{y} is given by

$$\bar{y} = (\mu - c)(2\lambda + \beta)/\bar{D}, \text{ where } \bar{D} = 2\lambda(2\lambda + (n + 2)\beta) + (n + 1)\beta^2.$$

From Lemma 3 we get the payoff to firm 1 at the first stage if the firm chooses λ_1 and the rivals $\bar{\lambda}$:

$$P_1(\lambda_1, \bar{\lambda}) = (\beta + \lambda_1) E(y_1(s_1))^2.$$

Let $\bar{r}(\lambda) = c''(\lambda)/|c'(\lambda)|$; that is, $\bar{r}(\cdot)$ is the absolute index of convexity of $c(\cdot)$. Given the parameters μ, β , and n we will say that $\bar{r}(\cdot)$ is large enough if

$$\bar{r}(\lambda) \geq \max \left\{ \frac{n |c'(0)|}{\mu - n\bar{c}}, \frac{|c'(0)|}{\mu - c} + \frac{5}{\beta} \right\} \text{ for all } \lambda \geq 0.$$

Proposition 4 characterizes the (symmetric) SPE of the game under this assumption.

PROPOSITION 4. *If the absolute index of convexity of $c(\cdot)$ is large enough then there is a unique symmetric subgame perfect equilibrium. If $\omega(0) > 0$ the equilibrium $\hat{\lambda}$ is the unique solution to*

$$\omega(\lambda) = - [AV(\alpha) + H] = 0,$$

where $H = \bar{y}(\bar{y}(n - 1)\beta^2 + 2(\beta + \lambda)(2\lambda + n\beta)(\bar{y} + c'))/\bar{D}$. If $\omega(0) > 0$, then $\hat{\lambda} > 0$ and $\omega'(\hat{\lambda}) < 0$. Otherwise, $\hat{\lambda} = 0$.

Proof. In the Appendix we show

(i) If $\bar{r}(\lambda) \geq n |c'(0)|/(\mu - n\bar{c})$ for all $\lambda \geq 0$ then $P_1(\lambda_1, \bar{\lambda})$ is strongly quasiconcave in λ_1 for all $\bar{\lambda} \geq 0$. Let $\omega(\lambda) = (\partial P_1/\partial \lambda_1)|_{\lambda_1 = \bar{\lambda} = \lambda}$.

(ii) If $\bar{r}(\lambda) \geq |c'(0)|/(\mu - \bar{c}) + 5/\beta$ then $\omega(\lambda) < 0$ for λ large enough and $\omega' < 0$, provided that $\omega = 0$. If $\omega(0) > 0$ then there will be a unique symmetric equilibrium $\hat{\lambda} > 0$. Otherwise, the unique symmetric equilibrium is $\hat{\lambda} = 0$.

(iii) $-\omega(\lambda) = AV(\alpha) + H$, where $A = (1 + A)a^2/t$ and $H = \bar{y}^2(1 + \bar{A}) + c'(\lambda) \bar{y}(1 + \bar{A}/2)$. Q.E.D.

EXAMPLE. Let $c(\lambda) = \bar{c}e^{-\delta\lambda}$, then $\bar{r}(\lambda) = \delta$ and $|c'(0)| = \delta\bar{c}$. In order for the convexity assumption on $\bar{r}(\cdot)$ to be satisfied we need $\mu \geq 2n\bar{c}$ and $\delta \geq 5(\mu - \bar{c})/(\mu - 2\bar{c})\beta$.

Under what circumstances will the equilibrium be interior? It seems plausible to conjecture that at least two factors will play an important role:

$c'(0)$, the slope at zero of the function which gives the trade-off between the slope and the intercept of marginal costs, and $V(\alpha)$, the variance of demand. If $|c'(0)|$ is high and $V(\alpha)$ is low, firms will choose a positive λ , since by increasing λ from zero they can reduce substantially the intercept of marginal costs and there is not much basic uncertainty in the model. In fact, in the Appendix it is checked that a sufficient condition for $\omega(0) > 0$, and therefore $\hat{\lambda} > 0$, is that

$$|c'(0)| > \frac{(\mu - \bar{c})^2 + \Omega(n) V(\alpha)}{\mu - \bar{c}} \frac{3n - 1}{2n(n + 1)\beta}$$

where

$$\Omega(n) = \frac{(n + 1)^3}{3n - 1} \max_{t \in [0, 1]} \left\{ t(2 - t)^2 \frac{4 + 2(n - 2)t + (n - 1)t^2}{[4 + 2(n - 2)t - (n - 1)t^2]^3} \right\} > 0.$$

Like in the cost reduction model, we can decompose changes in the degree of uncertainty between flexibility and strategic effects. The rationalization of the decomposition comes again from the open loop benchmark. Firm 1, by selecting a strategy $(\lambda_1, z_1(\cdot))$ when the rival firms select $(\tilde{\lambda}, \tilde{z}(\cdot))$, faces a payoff

$$E \left[\left(\alpha - \beta z_1(s_1) - \beta \sum_{j > 1} \tilde{z}(s_j) \right) z_1(s_1) \right] - E [c(\lambda_1) z_1(s_1) + \lambda(z_1(s_1))^2].$$

With symmetry the flexibility effect will be

$$\begin{aligned} \omega^f &\equiv -E \left[\frac{\partial C}{\partial \lambda_1} (y(s_1), \lambda) \right] = - [c'(\lambda) \bar{y} + E(y(s_1))^2] \\ &= - \left[\frac{a^2}{t} V(\alpha) + \bar{y}^2 c'(\lambda) \right] \end{aligned}$$

TABLE II

Plant Design Model SP Equilibrium $(\hat{\lambda}, y(\cdot))$

<i>Technology:</i>	$\omega(\hat{\lambda}) = -[AV(\alpha) + H] = 0;$
<i>Production:</i>	$y(s_i) = a(s_i - \mu) + b\mu + d$, where
	A, a, b , and \bar{D} are as in Table I replacing λ^* by $\hat{\lambda}$,
	$H = \bar{y}(\bar{y}(n - 1)\beta^2 + 2(\beta + \hat{\lambda})(2\hat{\lambda} + n\beta)(\bar{y} + c'(\hat{\lambda}))) / \bar{D}$,
	$\bar{y} = b\mu + d$, and
	$d = -(2\hat{\lambda} + \beta) c(\hat{\lambda}) / \bar{D}$

and the strategic effect,

$$\begin{aligned}\omega^s &\equiv -\beta(n-1) E y(s_1) \frac{\partial \bar{y}(s_j)}{\partial \lambda_1} \\ &= -\left[\Delta \frac{a^2}{t} V(\alpha) + \bar{\Delta} \bar{y}^2 + \frac{\bar{\Delta}}{2} \bar{y} c'(\lambda) \right],\end{aligned}$$

where $y(\cdot)$ is the symmetric Bayesian-Cournot equilibrium when all firms choose $\lambda_1 = \lambda$.

A key observation is that the terms which involve the information parameters in the equations characterizing ω^f and ω^s are both the *same* than in the cost reduction case, namely $a^2 V(\alpha)/t$ for ω^f and $\Delta a^2 V(\alpha)/t$ for ω^s . Therefore, the decomposition into flexibility and strategic effects and the comparative static properties of the interior equilibria of the plant design model, with respect to changes in the information structure, will be identical to the cost reduction model. Proposition 2 holds as stated and Proposition 5 restates Proposition 3 for the plant design case.

PROPOSITION 5. *Under the assumption of Proposition 4, if the equilibrium calls for a positive choice of the flexibility parameter λ , say $\hat{\lambda}$, $\hat{\lambda}$ will be decreasing with the variance of the demand intercept provided the signals are informative and with the precision of the information if $2\hat{\lambda} \geq \beta(t(n-1) - 2)$.*

6. EXTENSIONS

The models in this article are quite specific in their assumptions: risk neutral firms, quadratic payoffs, affine information structure, Cournot competition at the market stage, and the timing of the information. Many variations and extensions could be thought of. Let us mention a few.

Firms could receive information twice: first, before making the technological choice and, second, before production decisions are made. Information should be more accurate at the second stage. In this situation, if first stage choices are observable, firms may try to infer information from the actions of the rivals and signalling becomes then a possibility.⁹ A further complication would be to make information acquisition endogenous. Firms should decide how much information to purchase at each stage.

The phenomena studied in this paper are not exclusive of oligopolistic situations. What would happen if firms had no market power? In a large market where firms are negligible open loop and subgame perfect equilibria will coincide¹⁰ and the strategic commitment effect will disappear. Further,

⁹ Gal-Or [15] analyzes an R & D competition model where this type of signalling occurs.

¹⁰ Fudenberg and Levine [13] analyze this issue.

the same market process may aggregate private information. These issues are explored in Vives [27, and 28].

Bertrand competition at the market stage will modify the results with respect to the value of flexibility since it changes the nature of the strategic interaction. The effect of an increased precision of information of rival firms on the equilibrium strategy of a firm is opposite to the one in the Cournot world (see Vives [25]). Therefore we may conjecture that received common wisdom about the relationship between uncertainty and flexibility holds in the Bertrand world. The general results on the value of strategic commitment should be robust with respect to the competition mode. Risk aversion may make the firms turn conservative with increases in uncertainty and therefore make them invest less in cost reduction, for example. If the degree of risk aversion is high enough more uncertainty could be associated in general with less flexible positions and a lower degree of commitment being chosen. A similar phenomenon is described by Porter and Spence [22] in the capacity expansion process of the corn wet milling industry. These issues are still awaiting further research.

7. CONCLUDING REMARKS

We have analyzed how changes in the precision of information and in the basic uncertainty in demand or cost conditions affect the technological choices of firms in an oligopoly framework. The questions involved in the common wisdom ideas stated in the Introduction were:

- (1) Will more uncertainty induce firms to choose a more flexible technological position?
- (2) How will an increase in uncertainty affect the strategic positioning of the production technology for the market battle?

With respect to the first question we have found that an increase in basic uncertainty leads the firms to seek a more flexible technological position provided the signals are informative. Nevertheless, an increase in the common precision need not be associated with more flexible technological positions and in fact the opposite can be true if the precision of the information is good enough and the number of firms in the industry is large enough.¹¹ Thus the common presumption that the expectation of more informative signals will lead firms to choose more flexible initial positions, to take advantage of the improved accuracy of information, will only hold when the number of firms in the market is not too large. The presumption always will be true when the number of firms is no more than three in our

¹¹ See Vives [27] for an analysis of the limit case with a continuum of firms.

model, in particular for the monopoly case. Otherwise, when the aggregate precision in the market is high, a further increase in precision may decrease the marginal profitability of adding flexibility to a firm since it gives too much information accuracy to the rival firms. Therefore oligopolistic interaction is seen to change the conclusions reached when analyzing the monopoly case.

With respect to the second question we have concluded that an increase in uncertainty always raises the value of strategic commitment for risk-neutral firms. Therefore the suspicion that the importance of strategic commitment may fade under uncertainty is shown not to be well founded. This, we think, adds to the robustness of the results obtained in the certainty literature on capacity commitment.

APPENDIX

Notation

$a_1 = (2(\beta + \tilde{\lambda}) - \beta t)t/D_1$	$b_1 = a_1 _{t=1}$
$\tilde{a} = (2(\beta + \lambda_1) - \beta t)t/D_1$	$\tilde{b} = \tilde{a} _{t=1}$
$D_1 = 2(\beta + \lambda_1)(2(\beta + \tilde{\lambda}) + (n-2)\beta t) - (n-1)\beta^2 t^2$	$\bar{D}_1 = D_1 _{t=1}$
$a = a_1 _{\lambda_1 = \tilde{\lambda} = \lambda}$	$b = a _{t=1}$
$D = D_1 _{\lambda_1 = \tilde{\lambda} = \lambda}$	$\bar{D} = D _{t=1}$
$\Delta = (2(n-1)\beta^2 t^2)/D$	$\bar{\Delta} = \Delta _{t=1}$
$A_1 = (1 + 2(n-1)\beta^2 t^2/D_1)a_1^2/t$	$B_1 = A_1 _{t=1}$
$A = A_1 _{\lambda_1 = \tilde{\lambda} = \lambda}$	$B = A _{t=1}$
$E_1 = 2(\beta + \lambda_1)(2(\beta + \tilde{\lambda}) + (n-2)\beta t) + (n-1)\beta^2 t^2$	$\bar{E}_1 = E_1 _{t=1}$
$M_1 = (\beta + \lambda_1)(2\tilde{\lambda} + n\beta)/2$	
$M = M_1 _{\lambda_1 = \tilde{\lambda} = \lambda}$	
$E = E_1 _{\lambda_1 = \tilde{\lambda} = \lambda}$	$\bar{E} = E _{t=1}$
$E/D = 1 + \Delta, \bar{E}/\bar{D} = 1 + \bar{\Delta}$	$M/\bar{D} = 1 + \bar{\Delta}/2$
$g = 2(\beta + \lambda) - \beta t$	$\bar{g} = g _{t=1}$
$h = 2(\beta + \lambda) + (n-1)\beta t$	$\bar{h} = h _{t=1}$
$d = -(2\lambda + \beta)c(\lambda)/\bar{D}$	
$\bar{y} = b\mu + d$	
$H = \bar{y}(\bar{y}(n-1)\beta^2 + 2(\beta + \lambda)(2\tilde{\lambda} + n\beta)(\bar{y} + c'(\lambda)))/\bar{D}.$	

Proofs

Proof of Proposition 1. Let $P_1(\lambda_1, \tilde{\lambda}) = (\beta + \lambda_1) E(x_1(s_1))^2 - F(\lambda_1)$. Assume that $\lambda(F''/|F'|) \geq 6$.

(i) $P_1(\lambda_1, \tilde{\lambda})$ is strongly quasiconcave in λ_1 for all $\tilde{\lambda} > 0$. We show that $\partial P_1/\partial \lambda_1 = 0$ implies that $\partial^2 P_1/\partial \lambda_1^2 < 0$. $\partial P_1/\partial \lambda_1 = \Psi_1(\lambda_1, \tilde{\lambda}) - F'(\lambda_1)$, where

$$\begin{aligned} \Psi_1(\lambda_1, \tilde{\lambda}) &= -[A_1(\lambda_1, \tilde{\lambda}) V(\alpha) + B_1(\lambda_1, \tilde{\lambda}) \mu^2] \\ &= -\frac{a_1^2}{t} \left[1 + \frac{2(n-1)\beta^2 t^2}{D_1} \right] V(\alpha) \\ &\quad + b_1^2 \left(1 + \frac{2(n-1)\beta^2}{\bar{D}_1} \right) \mu^2 \end{aligned}$$

and a_1, b_1 , and D_1 are as in Lemma 1, $\bar{D}_1 = D_1|_{t=1}$. $\partial^2 P_1/\partial \lambda_1^2 = \partial \Psi_1/\partial \lambda_1 - F''$ will be negative when $\partial P_1/\partial \lambda_1 = \Psi_1 - F' = 0$ if and only if

$$\lambda_1 \left| \frac{\partial \Psi_1/\partial \lambda_1}{\Psi_1} \right| < \lambda_1 \frac{F''}{|F'|}.$$

We have assumed $\lambda_1(F''/|F'|) \geq 6$; we show now that $\lambda_1 |\partial A_1/\partial \lambda_1/A_1| < 6$ for all $t > 0$, since this implies that $\lambda_1 |\partial B_1/\partial \lambda_1/B_1| < 6$ (since $B_1 = A_1|_{t=1}$). Rewriting the inequalities as $\lambda_1 |\partial A_1/\partial \lambda_1| < 6|A_1|$, $\lambda_1 |\partial B_1/\partial \lambda_1| < 6|B_1|$ ($B_1 > 0$ always and $A_1 > 0$ if $t > 0$), we get that $\lambda_1(|\partial A_1/\partial \lambda_1| V(\alpha) + |\partial B_1/\partial \lambda_1| \mu^2) < 6(|A_1| V(\alpha) + |B_1| \mu^2)$ and therefore $\lambda_1 |\partial \Psi_1/\partial \lambda_1/\Psi_1| < 6$, since

$$\frac{\partial \Psi_1}{\partial \lambda_1} = -\left(\frac{\partial A_1}{\partial \lambda_1} V(\alpha) + \frac{\partial B_1}{\partial \lambda_1} \mu^2 \right).$$

If $t = 0$ then $A_1 = 0$ and $\Psi_1 = -B_1 \mu^2$.

We show now that $\lambda_1 |\partial A_1/\partial \lambda_1/A_1| < 6$ for all $t > 0$ and $\tilde{\lambda} > 0$. Let $E_1 = 2(\beta + \lambda_1)(2(\beta + \tilde{\lambda}) + (n-2)\beta t) + (n-1)\beta^2 t^2$, then $A_1 = (2(\beta + \tilde{\lambda}) - \beta t)^2 t E_1/D_1^3$ and

$$\begin{aligned} &\lambda_1 \left| \frac{\partial A_1/\partial \lambda_1}{A_1} \right| \\ &= \lambda_1 \frac{2(2(\beta + \tilde{\lambda}) + (n-2)\beta t) |D_1 - 3E_1|}{D_1 E_1} \\ &= \frac{2\lambda_1 [2(\beta + \tilde{\lambda}) + (n-2)\beta t] [4(\beta + \lambda_1)(2(\beta + \tilde{\lambda}) + (n-2)\beta t) + 4(n-1)\beta^2 t^2]}{4(\beta + \lambda_1)^2 [2(\beta + \tilde{\lambda}) + (n-2)\beta t]^2 - (n-1)^2 \beta^4 t^4} \\ &\leq \frac{2\lambda_1 [2(\beta + \tilde{\lambda}) + (n-2)\beta t] 8(\beta + \lambda_1) [2(\beta + \tilde{\lambda}) + (n-2)\beta t]}{3(\beta + \lambda_1)^2 [2(\beta + \tilde{\lambda}) + (n-2)\beta t]^2}, \end{aligned}$$

since $(\beta + \lambda_1)[2(\beta + \bar{\lambda}) + (n - 2)\beta t] \geq \beta^2(n - 1)t^2$. Therefore

$$\lambda_1 \left| \frac{\partial A_1 / \partial \lambda_1}{A_1} \right| \leq \frac{16\lambda_1}{3(\beta + \lambda_1)} < \frac{16}{3} < 6.$$

(ii) The best response function of firm 1, $BR_1(\bar{\lambda})$, is decreasing in $\bar{\lambda}$. $BR_1(\bar{\lambda})$ is the unique solution to $(\partial P_1 / \partial \lambda_1)(\lambda, \bar{\lambda}) = \Psi_1(\lambda_1, \bar{\lambda}) - F'(\lambda_1) = 0$:

$$\frac{dBR_1}{d\bar{\lambda}} = - \frac{\partial \Psi_1 / \partial \bar{\lambda}}{\partial^2 P_1 / \partial \lambda_1^2}.$$

We check that $\partial \Psi_1 / \partial \bar{\lambda} < 0$. As before, it is sufficient to check that $\partial A_1 / \partial \bar{\lambda} > 0$ for all $t > 0$,

$$\begin{aligned} \frac{\partial A_1}{\partial \bar{\lambda}} &= \frac{4a_1}{tD_1} \left[\left(1 + \frac{2(n-1)\beta^2 t^2}{D_1} \right) \right. \\ &\quad \left. \times (t - 2(\beta + \lambda_1)a_1) - \frac{2(n-1)(\beta + \lambda_1)\beta^2 t^2 a_1}{D_1} \right] \\ \frac{\partial A_1}{\partial \bar{\lambda}} > 0 &\quad \text{if } t - 2(\beta + \lambda_1)a_1 \geq \frac{2(n-1)(\beta + \lambda_1)\beta^2 t^2 a_1}{D_1}. \end{aligned}$$

But $LHS \geq (n - 1)\beta^2 t^2 / D \geq RHS$. The first inequality is true since $2(\beta + \lambda_1)[2(\beta + \bar{\lambda}) - \beta t] - (n - 1)\beta^2 t^2 \leq D_1$ and the second follows immediately.

(iii) $\underline{\lambda}_1 = \inf_{\bar{\lambda}} BR_1(\bar{\lambda})$ and $\bar{\lambda}_1 = \sup_{\bar{\lambda}} BR_1(\bar{\lambda})$ solve respectively $(\partial P_1 / \partial \lambda_1)(\lambda_1, \infty) = 0$ and $(\partial P_1 / \partial \lambda_1)(\lambda_1, 0) = 0$. To check this just notice that $\lim_{\bar{\lambda} \rightarrow 0} \Psi_1(\lambda_1, \bar{\lambda})$ and $\lim_{\bar{\lambda} \rightarrow \infty} \Psi_1(\lambda_1, \bar{\lambda})$ are both negative numbers.

Let now $\varphi(\lambda) = -[AV(\alpha) + B\mu^2] - F'(\lambda)$.

(iv) $\varphi'(\lambda) < 0$ whenever $\varphi(\lambda) = 0$. This is shown as in the proof of (i). It is sufficient to show that $\lambda |\partial A / \partial \lambda / A| < 6$. After some computations, we get that

$$\left| \frac{\partial A / \partial \lambda}{A} \right| = \frac{2}{gh + 2(n-1)\beta^2 t^2} \left(2g + \frac{2(n-1)\beta^2 t^2}{g} + \frac{6(n-1)\beta^2 t^2}{h} \right),$$

where $g = 2(\beta + \lambda) - \beta t$ and $h = (\beta + \lambda) + (n - 1)\beta t$. Therefore,

$$\left| \frac{\partial A / \partial \lambda}{A} \right| \leq 2 \left(\frac{2}{h} + \frac{1}{g} + \frac{3}{h} \right) \leq \frac{5}{\beta + \lambda} + \frac{2}{\beta + 2\lambda}$$

and $\lambda |(\partial A / \partial \lambda) / A| < 6$.

Q.E.D.

Proof of Proposition 2. Without loss of generality let $\mu = 0$. Then

$$-\Psi^r = \frac{a^2}{t} V(\alpha) \quad \text{and} \quad -\Psi^s = \Delta \frac{a^2}{t} V(\alpha).$$

Now, $\Delta = (2(n-1)\beta^2 t^2)/D$ and $D = (2(\beta + \lambda) - \beta t)t/a$, $a = t/(2(\beta + \lambda) + (n-1)\beta t)$. We can rewrite Δ as

$$\Delta = \frac{2(n-1)\beta^2 t}{2(\beta + \lambda) - \beta t} a.$$

Therefore,

$$-\Psi^s = \frac{2(n+1)\beta^2}{2(\beta + \lambda) - \beta t} a^3 V(\alpha),$$

which is increasing in both v^{-1} and $V(\alpha)$, since t and a are both increasing in v^{-1} and $V(\alpha)$. This takes care of (a).

To show (b), let $v < \infty$. Noting that $V(\alpha)(\partial t/V(\alpha)) = t(1-t)$ we get

$$\begin{aligned} \frac{\partial t}{\partial V(\alpha)} \left[\frac{a^2}{t} V(\alpha) \right] &= \frac{a^2}{t} + \frac{\partial(a^2/t)}{\partial t} \frac{\partial t}{\partial V(\alpha)} V(\alpha) \\ &= \frac{a^2}{t} \left(1 + (1-t) \frac{2(\beta + \lambda) - (n-1)\beta t}{2(\beta + \lambda) + (n-1)\beta t} \right) \\ &= \frac{a^2}{t} \frac{2(\beta + \lambda)(2-t) + (n-1)\beta t^2}{2(\beta + \lambda) + (n-1)\beta t} > 0. \end{aligned}$$

Fix now $V(\alpha)$, then v^{-1} moves as t . We obtain

$$\frac{\partial}{\partial t} \left[\frac{a^2}{t} \right] = \frac{2(\beta + \lambda) - (n-1)\beta t}{2(\beta + \lambda) + (n-1)\beta t} \left(\frac{a}{t} \right)^2.$$

Therefore $(\partial/\partial t)[(a^2/t) V(\alpha)] \geq 0$ if and only if $2(\beta + \lambda) - (n-1)\beta t \geq 0$.

Q.E.D.

Proof of Proposition 4. (i) If $\bar{r}(\lambda) \geq n|c'(0)|/(\mu - n\bar{c})$ then $\partial^2 P_1/\partial \lambda_1^2 < 0$ whenever $\partial P_1/\partial \lambda_1 = 0$. Let $\bar{y}_1 = E y_1(s_1)$. Some computations yield that $\partial P_1/\partial \lambda_1 = -[A_1 V(\alpha) + \bar{y}_1^2(\bar{E}_1/\bar{D}_1) + c'(\lambda_1)(M_1/\bar{D}_1)\bar{y}_1]$, where A_1 and E_1 are as in the proof of Proposition 1, $\bar{E}_1 = E_1|_{t_{-1}}$, and $M_1 = 2(\beta + \lambda_1)(2\bar{\lambda} + n\beta)$. Differentiating again with respect to λ_1 , we get

$$\begin{aligned} \frac{\partial^2 P_1}{\partial \lambda_1^2} &= \frac{a_1^2 (2E_1 + 2(n-1)\beta^2 t^2)}{t D_1^2} E_1' V(\alpha) \\ &+ \bar{y}_1^2 \frac{2E_1 + 2(n-1)\beta^2}{\bar{D}_1^2} \bar{E}_1' - c''(\lambda_1) \bar{y}_1 \frac{M_1}{\bar{D}} \\ &+ (c'(\lambda_1))^2 \frac{(2\bar{\lambda} + n\beta)M_1}{\bar{D}_1^2} + c'(\lambda_1) \bar{y}_1 \frac{2\bar{D}_1' \bar{E}_1}{\bar{D}_1^2}, \end{aligned}$$

where primes denote derivative with respect to λ_1 . The FOC is $\partial P_1 / \partial \lambda_1 = 0$; we get, by substitution,

$$\begin{aligned} \left. \frac{\partial^2 P}{\partial \lambda_1^2} \right|_{\text{FOC}} &= \frac{2a_1^2}{t D_1} \left[\frac{E_1'(E_1 + (n-1)\beta^2 t^2)}{D_1} - \frac{\bar{D}_1' E_1 \bar{E}_1}{M_1 \bar{D}_1} \right] V(\alpha) \\ &+ \bar{y}_1^2 \frac{2\bar{E}_1'}{\bar{D}_1^2} \left[\bar{E}_1 + (n-1)\beta^2 - \frac{\bar{E}_1^2}{M_1} \right] \\ &- \frac{M_1}{\bar{D}_1} \left(c''(\lambda_1) \bar{y}_1 - \frac{(2\bar{\lambda} + n\beta)}{\bar{D}_1} (c'(\lambda_1))^2 \right). \end{aligned}$$

The first term is negative if and only if

$$E_1'(E_1 + (n-1)\beta^2 t^2) \bar{D}_1 M_1 - \bar{D}_1' D_1 E_1 \bar{E}_1 < 0.$$

Multiplying by $(\beta + \lambda_1)$ and dividing by M_1 , this expression equals

$$\begin{aligned} &[8(n-1)\beta^2(\beta + \lambda_1)^2 (2(\beta + \bar{\lambda}) + (n-2)\beta t) + 2(n-1)^2 \beta^4 t^2 (\beta + \lambda_1)] \\ &\times [t^2(2\bar{\lambda} + n\beta) - (2(\beta + \bar{\lambda}) + (n-2)\beta t)] + (n-1)^2 \beta^4 t^2 \\ &\times [(n-1)\beta^2 t^2 - 2(\beta + \lambda_1)(2(\beta + \bar{\lambda}) + (n-2)\beta t)], \end{aligned}$$

which is negative since $t^2(2\bar{\lambda} + n\beta) \leq 2(\beta + \bar{\lambda}) + (n-2)\beta t$ and $(n-1)\beta^2 t^2 < 2(\beta + \lambda_1)(2(\beta + \bar{\lambda}) + (n-2)\beta t)$. The second is negative if and only if $(\bar{E}_1 + (n-1)\beta^2 M_1 - \bar{E}_1^2) < 0$. The LHS equals $-(n-1)^2 \beta^4 < 0$. The third term is negative provided that $\bar{r} \geq n |c'(0)| / (\mu - n\bar{c})$,

$$\begin{aligned} &c''(\lambda_1) \bar{y}_1 - \frac{2\bar{\lambda} + n\beta}{\bar{D}} (c'(\lambda_1))^2 \\ &\geq \frac{|c'(\lambda_1)|}{\bar{D}_1} [\bar{r}(2\bar{\lambda} + n\beta)(\mu - c(\lambda_1)) - (n-1)\beta\mu - (2\bar{\lambda} + n\beta) |c'(\lambda_1)|] \\ &\geq \frac{|c'(\lambda_1)| (2\bar{\lambda} + \beta)}{\bar{D}_1} [\bar{r}(\mu - n\bar{c}) - n |c'(0)|] \geq 0. \end{aligned}$$

(ii) Let $\omega(\lambda) = (\partial P_1 / \partial \lambda_1)|_{\lambda_1 = \lambda} = \lambda$. If $\bar{r}(\lambda) \geq (|c'(0)|/\mu - \bar{c}) + (5/\beta)$, then $\omega(\lambda) < 0$ for λ large enough and $\omega'(\lambda) < 0$ provided that $\omega(\lambda) = 0$,

$$\omega(\lambda) = - \left[a^2 \frac{E}{D} V(\alpha) + \bar{y}^2 \frac{\bar{E}}{\bar{D}} + c'(\lambda) \bar{y} \frac{M}{\bar{D}} \right],$$

where \bar{y} , a , E , D , \bar{D} , and M are as \bar{y}_1 , a_1 , E_1 , D_1 , \bar{D}_1 , and M_1 letting $\lambda_1 = \lambda = \lambda$. It $\bar{r}(\lambda)$ is bounded away from zero it follows that $\lim_{\lambda \rightarrow \infty} \lambda^2 |c'(\lambda)| = 0$. It is easily seen that M/\bar{D} , E/D , and \bar{E}/\bar{D} converge to a positive constant as $\lambda \rightarrow \infty$ and so do $\lambda^2 a^2$ and $\lambda^2 \bar{y}^2$. Therefore $\lambda^2 \omega(\lambda)$ will be negative for λ large, since the first two terms in $\lambda^2 \omega(\lambda)$ will be positive (and bounded away from zero) while the third will be negligible.

We show now that $\omega' < 0$ if $\omega = 0$. Let $c = c(\lambda)$,

$$\begin{aligned} -\omega(\lambda) &= \frac{gh + 2(n-1)\beta^2 t^2}{gh^3} tV(\alpha) \\ &+ \frac{(\mu - c)^2 (\bar{g}\bar{h} + 2(n-1)\beta^2) + (\mu - c) c' \bar{h} (\bar{g}\bar{h} + (n-1)\beta^2)}{\bar{g}\bar{h}^3} \end{aligned}$$

where $g = 2(\beta + \lambda) - \beta t$, $\bar{g} = g|_{t=1}$, $h = 2(\beta + \lambda) + (n-1)\beta t$ and $\bar{h} = h|_{t=1}$;

$$\begin{aligned} -\omega'(\lambda)|_{\text{FOC}} &= \frac{gh + 2(n-1)\beta^2 t^2}{gh^3} t \left[\frac{2}{\bar{g}} + \frac{6}{\bar{h}} - \frac{2}{g} - \frac{6}{h} \right] V(\alpha) \\ &+ \frac{(\mu - c) c'' \bar{h} (\bar{g}\bar{h} + (n-1)\beta^2)}{\bar{g}\bar{h}^3} \\ &+ \frac{2c'(\mu - c)(\bar{h}^2 - 2(n-1)\beta^2)}{\bar{g}\bar{h}^3} \\ &- \frac{(c')^2 \bar{h} (\bar{g}\bar{h} + (n-1)\beta^2)}{\bar{g}\bar{h}^3} \\ &+ \frac{2(\mu - c)^2}{\bar{g}\bar{h}^4} (\bar{h}^2 - 2(n-1)\beta^2) \\ &+ \frac{2tV(\alpha)}{gh^3 \bar{h}} (g\bar{h} + h\bar{h} - gh - 2(n-1)\beta^2 t^2), \end{aligned}$$

where we have used the FOC $\omega(\lambda) = 0$ in deriving the last two terms. Now, the last term is nonnegative since $\bar{h} \geq h$ and

$$h\bar{h} \geq h^2 \geq (2\lambda + (n+1)\beta t)^2 \geq 2(n-1)\beta^2 t^2.$$

The next to the last term is also nonnegative, since

$$\bar{h}^2 = (2\lambda + (n+1)\beta)^2 \geq (n+1)^2 \beta^2 \geq 2(n-1)\beta^2.$$

The second term is larger than or equal to $((\mu - c) \bar{h}(\bar{g}\bar{h} + (n - 1)\beta^2))/\bar{g}\bar{h}^3$ $\bar{r} |c'|$, which equals

$$\bar{r} \left[t \frac{gh + 2(n - 1) \beta^2 t^2}{gh^3} V(\alpha) + \frac{(\mu - c)^2 (\bar{g}\bar{h} + 2(n - 1)\beta^2)}{\bar{g}\bar{h}^3} \right],$$

using the FOC. It follows that the sum of the first two terms is larger than or equal to

$$\left(\bar{r} + \frac{6}{\bar{h}} - \frac{6}{h} \right) \left(\frac{(\mu - c) \bar{h}(\bar{g}\bar{h} + (n - 1)\beta^2) |c'|}{\bar{g}\bar{h}^3} \right).$$

The sum of the first four terms is then no less than

$$\frac{(\mu - c) |c'| \bar{h}(\bar{g}\bar{h} + (n - 1)\beta^2)}{\bar{g}\bar{h}^3} \left(\bar{r} + \frac{6}{\bar{h}} - \frac{6}{h} + \frac{c'}{\mu - c} - \frac{2\bar{h}}{\bar{g}\bar{h} + (n - 1)\beta^2} \right).$$

Let Z represent the expression in the brackets; then

$$Z \geq \bar{r} - \frac{6}{h} + \frac{c'(0)}{\mu - \bar{c}} - \frac{2}{\bar{g}} \geq \bar{r} - \frac{|c'(0)|}{\mu - \bar{c}} - \frac{5}{\beta}$$

which is in turn positive, if our assumption is satisfied.

Q.E.D.

CLAIM. $\omega(0) > 0$ if

$$|c'(0)| > \frac{(\mu - \bar{c})^2 + \Omega(n) V(\alpha)}{\mu - \bar{c}} \frac{3n - 1}{2n(n + 1)\beta},$$

where

$$\Omega(n) = \frac{(n + 1)^3}{3n - 1} \max_{t \in [0, 1]} \left(t(2 - t)^2 \frac{4 + 2(n - 2)t + (n - 1)t^2}{[4 + 2(n - 2)t - (n - 1)t^2]} \right).$$

Proof. From the proof of Proposition 5,

$$\begin{aligned} -\omega(0) &= \left(t(2 - t)^2 \frac{E}{D^3} V(\alpha) + (\mu - \bar{c})^2 - \frac{E}{D^3} \right) \beta^2 \\ &\quad - |c'(0)| (\mu - \bar{c}) \frac{M}{D^2} \beta. \end{aligned}$$

Now $\omega(0) > 0$ if

$$|c'(0)| (\mu - \bar{c}) \frac{2n}{(n+1)^2} < \left(t(2-t)^2 \frac{4+2(n-2)t+(n-1)t^2}{(4+2(n-2)t-(n-1)t^2)^3} V(\alpha) + (\mu - \bar{c})^2 \frac{3n-1}{(n+1)^3} \right) \frac{1}{\beta}.$$

This inequality is satisfied if our condition is satisfied.

Q.E.D.

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