Terry Chew B. Sc

Olympiad Maths Trainer

12-13 years old

6

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(12-13 years old)

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FOREWORD

I first met Terry when he approached SAP to explore the possibility of publishing Mathematical Olympiad type questions that he had researched, wrote and compiled. What struck me at our first meeting was not the elaborate work that he had consolidated over the years while teaching and training students, but his desire to make the materials accessible to *all* students, including those who deem themselves “not so good” in mathematics. Hence the title of the original series was most appropriate: *Maths Olympiad — Unleash the Maths Olympian in You!*

My understanding of his objective led us to endless discussions on how to make the book easy to understand and useful to students of various levels. It was in these discussions that Terry demonstrated his passion and creativity in solving non-routine questions. He was eager to share these techniques with his students and most importantly, he had also learned alternative methods of solving the same problems from his group of bright students.

This follow-up series is a result of his great enthusiasm to constantly sharpen his students' mathematical problem-solving skills. I am sure those who have worked through the first series, *Maths Olympiad — Unleash the Maths Olympian in You!*, have experienced significant improvement in their problem-solving skills. Terry himself is encouraged by the positive feedback and delighted that more and more children are now able to work through non-routine questions.

And we have something new to add to the growing interest in Mathematical Olympiad type questions — *Olympiad Maths Trainer* is now on *Facebook*! You can connect with Terry via this platform and share interesting problem-solving techniques with other students, parents and teachers.

I am sure the second series will benefit not only those who are preparing for mathematical competitions, but also all who are constantly looking for additional resources to hone their problem-solving skills.

Michelle Yoo
Chief Publisher
SAP

A word from the author …

Dear students, teachers and parents,

Welcome once more to the paradise of Mathematical Olympiad where the enthusiastic young minds are challenged by the non-routine and exciting mathematical problems!

My purpose of writing this sequel is twofold.

The old adage that "to do is to understand" is very true of mathematical learning. This series adopts a systematic approach to provide practice for the various types of mathematical problems introduced in my first series of books.

In the first two books of this new series, students are introduced to 5 different types of mathematical problems every 12 weeks. They can then apply different thinking skills to each problem type and gradually break certain mindsets in problem-solving. The remaining four books comprise 6 different types of mathematical problems in the same manner. In essence, students are exposed to stimulating and interesting mathematical problems where they can work on creatively.

Secondly, the depth of problems in the Mathematical Olympiad cannot be underestimated. The series contains additional topics such as the Konigsberg Bridge Problem, Maximum and Minimum Problem, and some others which are not covered in the first series, *Maths Olympiad – Unleash the Maths Olympian in You!*

Every student is unique, and so is his or her learning style. Teachers and parents should wholly embrace the strengths and weaknesses of each student in their learning of mathematics and constantly seek improvements.

I hope you will enjoy working on the mathematical problems in this series just as much as I enjoyed writing them.

Terry Chew

OLYMPIAD MATHS TRAINER 6

CONTENTS

WEEK 1

Name: ____________ **Date:** ____________

Class: ____________ **Marks:** ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Solve for the values of x and y in each of the following, given that x and y are whole numbers.

(a) $y = 3x$
$8x - 2y = 8$

(b) $4x + 5y = 23$
$9x - 5y = 3$

(c) $y = 5x$
$3x + 2y = 65$

(d) $2x + 5y = 23$
$6x - 5y = 9$

2. Among 64 students, 28 of them like Science, 41 like Mathematics and 20 like English. 24 of them like both Mathematics and English. 12 students like both Science and English. 10 students like both Science and Mathematics. How many students like all the three subjects?

3. What is the value of the digit in the ones place of the following?

$1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times \ldots \times 2007 \times 2009$

4. Evaluate

$$\frac{1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1}{7777777 \times 7777777}.$$

5. Evaluate each of the following.

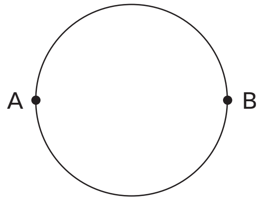
(a) ${}^{4}P_{3}$

(b) ${}^{5}P_{4}$

(c) ${}^{8}C_{2}$

(d) ${}^{12}C_{3}$

6. Jonathan and Cindy run on a circular track where AB is the diameter of the track, as shown below.



If Jonathan and Cindy run towards each other at the same time from Point A and Point B respectively, it will take them 40 seconds before they meet. If they start running at the same time but in the same direction, it will take Jonathan 280 seconds to catch up with Cindy. What is the ratio of their speeds?

WEEK 2

Name: __________ Date: __________

Class: __________ Marks: __________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Given that a and b are whole numbers, solve for a and b in each of the following.

(a) $2a + 3b = 18$

(b) $4a + 6b = 68$

(c) $4a + 7b = 73$

(d) $3a + 8b = 47$

2. Between 1 and 2009, how many numbers are multiples of 5 or 7?

3. The sum of the digits of a 3-digit number is 18. The tens digit is 1 more than the ones digit. If the hundreds digit and the ones digits are swapped, the difference between the new number and the original number is 396. What is the original number?

4. Evaluate $\left(1 + \frac{1}{3} + \frac{1}{5}\right) \times \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right) - \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right) \times \left(\frac{1}{3} + \frac{1}{5}\right)$ using a simple method.

5. **(a)** Choose any three letters from a, b, c, and d. In how many ways can we arrange the three letters?

(b) A teacher wants to choose a captain and vice-captain among 12 volleyball players. In how many ways can he do so?

6. A car travelled to Town B from Town A at a constant speed of 72 km/h. It then returned from Town B to Town A at a constant speed of 48 km/h. What was the average speed of the car for the whole journey?

WEEK 3

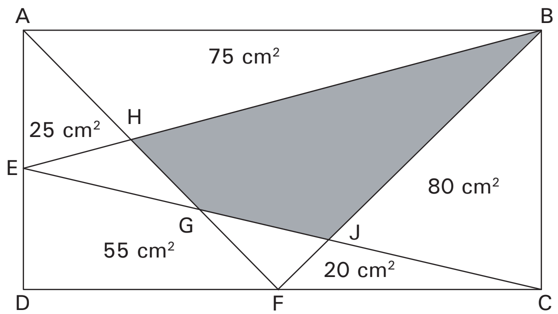
Name: ____________ Date: ____________

Class: ____________ Marks: ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Lucas multiplies his month of birth by 31. He then multiplies his day of birth by 12. The sum of the two products is 213. When is his birthday?

2. In the figure below, E and F are midpoints of AD and DC respectively. ABCD is a rectangle. Find the area of the shaded region.



3. Find the value of the following.

$20082009 \times 20092008 - 20082008 \times 20092009$

4. Evaluate $29\frac{1}{2} \times \frac{2}{3} + 39\frac{1}{3} \times \frac{3}{4} + 49\frac{1}{4} \times \frac{4}{5}$.

5. How many 3-digit numbers have the sum of the three digits equals to 4?

6. A car will travel from Town A to Town B. If it travels at a constant speed of 60 km/h, it will arrive at 3.00 pm. If it travels at a constant speed of 80 km/h, it will arrive at 1.00 pm. At what speed should it be travelling if the driver aims to arrive at Town B at 2.00 pm?

WEEK 4

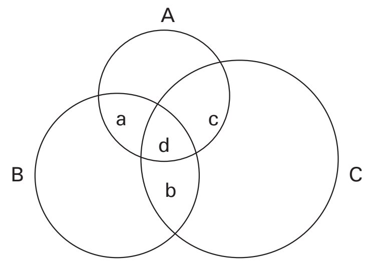
Name: ____________ Date: ____________

Class: ____________ Marks: ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. A big box can hold 48 marbles. A small box can hold 30 marbles. Find the number of big boxes and the number of small boxes that can hold a total of 372 marbles.

2. In the figure below, the area of three circles, A, B and C, are 40 cm^2, 50 cm^2 and 60 cm^2 respectively. Given that a + d = 12 cm^2, b + d = 14 cm^2, c + d = 16 cm^2 and d = 8 cm^2, find the area of the whole figure.



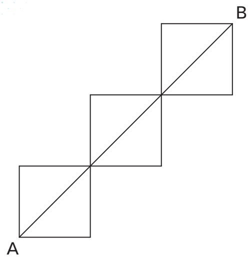
3. The sum of two numbers is 88. The product of the two numbers is 1612. What are the two numbers?

4. Evaluate

$$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)$$

using a simple method.

5. How many ways are there to reach B from A? Only movements →, ↑ and ↗ are allowed.



6. A tourist was travelling to a town which was 60 km away. He walked at a speed of 6 km/h at first. Then, he hitched a ride on a scooter travelling at 18 km/h. He arrived at the town 4 hours from the time he set off. How long had he walked?

WEEK 5

Name: ____________ **Date:** __________

Class: ____________ **Marks:** __________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Two fruit baskets contain some oranges. If an orange is transferred from the first basket to the second basket, both baskets will have the same number of oranges. If an orange is transferred from the second basket to the first basket, the number of oranges in the first basket becomes thrice the number of oranges in the second basket. How many oranges are in each basket at first?

2. A survey was made on 250 students on their preferred school activities: badminton, volleyball and basketball. 140 of them liked badminton, 120 of them liked volleyball and 100 of them liked basketball. 40 of them liked both badminton and volleyball but not basketball. 20 of them liked badminton and basketball but not volleyball. How many liked both volleyball and basketball but not badminton, given 10 liked all three activities?

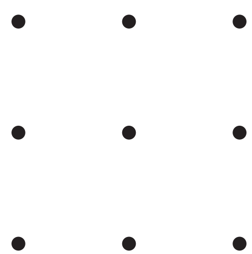
3. A contractor has 1088 square tiles. In how many ways can he form a rectangle using all the tiles each time?

4. Evaluate

$$\frac{1 \times 2 \times 3 + 2 \times 4 \times 6 + 3 \times 6 \times 9 + \ldots + 100 \times 200 \times 300}{1 \times 3 \times 5 + 2 \times 6 \times 10 + 3 \times 9 \times 15 + \ldots + 100 \times 300 \times 500}$$

by factorising.

5. In the figure below, how many triangles can be formed using any three points as the vertices?



6. A fighter plane had enough fuel to last a 6-hour flight. The speed of wind and the speed of the plane made up a total of 1500 km/h when the plane was flying in the direction of the wind during its mission. On its return trip, the total speed was reduced to 1200 km/h as the plane was travelling against the wind. How far could the plane travel before it made its return?

WEEK 6

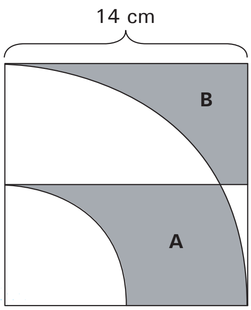
Name: ____________ Date: ____________

Class: ____________ Marks: ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Julie asked her teacher, "How old were you in 2008?" "My age in 2008 was the sum of all the digits of my year of birth," replied the teacher. How old was the teacher in 2008?

2. In the figure below, the side of the square is 14 cm. The radii of the two quadrants are 7 cm and 14 cm respectively. A and B represent the areas of the two shaded regions. Find (A − B). $\left(\text{Take } \pi = \frac{22}{7}.\right)$



3. For $1^2 + 2^2 + 3^2 + \ldots + n^2$, we can compute using $n(n + 1)(2n + 1) \div 6$. Find the value of $1^2 + 2^2 + 3^2 + \ldots + 15^2$.

4. Evaluate $\frac{12345678}{12345678^2 - 12345677 \times 12345679}$.

5. A 4-digit number is formed using 2, 3, 5, 7 or 9 without repeating any of the digits. How many 4-digit numbers are there if each number has a remainder of 2 when divided by either 3 or 5?

6. During a school walkathon, Alan complete[illegible]st half of the journey at a speed of 4.5 km/h. He then finished the second half of the journey at a speed of 5.5 km/h. On the other hand, Benny walked at a speed of 4.5 km/h for the first half of the time taken. He then completed the remaining journey at 5.5 km/h. Who would arrive at the finishing line first?

WEEK 7

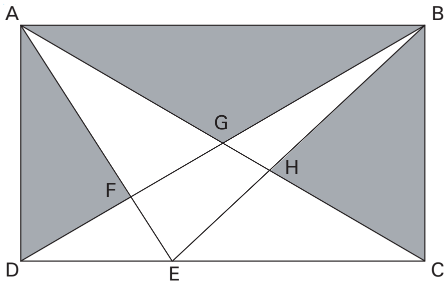
Name: ____________ Date: ____________

Class: ____________ Marks: ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. Don and Andy have some marbles. If Don gives some marbles to Andy, the number of marbles that Don has is twice what Andy has. If Andy gives the same number of marbles to Don, the number of marbles that Don has is 4 times what Andy has.
How many marbles does each of them have at first?

2. In the figure below, AB = 20 cm, AD = 10 cm and the area of quadrilateral EFGH is 15 cm^2. Find the area of the shaded region.



3. Alice, Bernard and Colin draw 3 cards each from nine cards numbered from 1 to 9.

Alice: The product of my numbers is 48.
Bernard: The sum of my numbers is 15.
Colin: The product of my numbers is 63.

Find the three cards that each of them draw.

4. Evaluate $31\frac{1}{2} \times \frac{2}{3} + 41\frac{1}{3} \times \frac{3}{4} + 51\frac{1}{4} \times \frac{4}{5} + 61\frac{1}{5} \times \frac{5}{6}$.

5. A staircase has 10 steps. How many ways are there for Tommy to walk from the first step to the 10^{th} step if either 2 or 3 steps are taken each time?

6. A car travels from Town A to Town B at a constant speed. If it increases its speed by 20%, it can arrive one hour ahead of schedule. If it increases its speed by 25% after travelling the first 120 km at the usual speed, it can arrive 36 minutes ahead of schedule. Find the distance between Town A and Town B.

WEEK 8

Name: ____________ Date: ____________

Class: ____________ Marks: ____________/24

Solve these questions. Show your working clearly. Each question carries 4 marks.

1. A small hotel has 12 rooms that can accommodate a total of 80 guests.
A big room can accommodate 8 persons and a mid-sized room can accommodate 7 persons. 5 persons can stay in a small room. How many big, mid-sized and small rooms are there in the hotel?

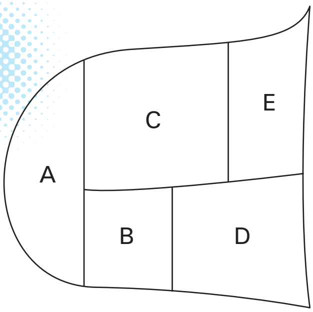
2. The numbers between 1 to 200 that are not multiples of 3 or 5 are arranged from the smallest to the largest. Find the 95^{th} number.

3. David misread the ones digit of a number A and the product of A and B became 407. Sophia misread the tens digit of A and the product of A and B became 451. Find the value of A $\times$ B.

4. Evaluate $A = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + ... + \frac{1}{49 \times 50}$.

$\left(\text{Hint: } \frac{1}{n \times (n + 1)} = \frac{1}{n} - \frac{1}{n + 1}\right)$

5.



In how many ways can the figure shown above be coloured using four different colours so that the adjacent regions have different colours?

6. A journalist needs to travel to another country. He can choose to drive for 15 hours and then travel the remaining journey by train for another 20 hours. Otherwise, he can drive for 8 hours and then travel the remaining journey by train for another 34 hours. How many hours must he drive, at least, if he chooses to drive for the whole journey and given that the time spent is a whole number?