Texture Analysis

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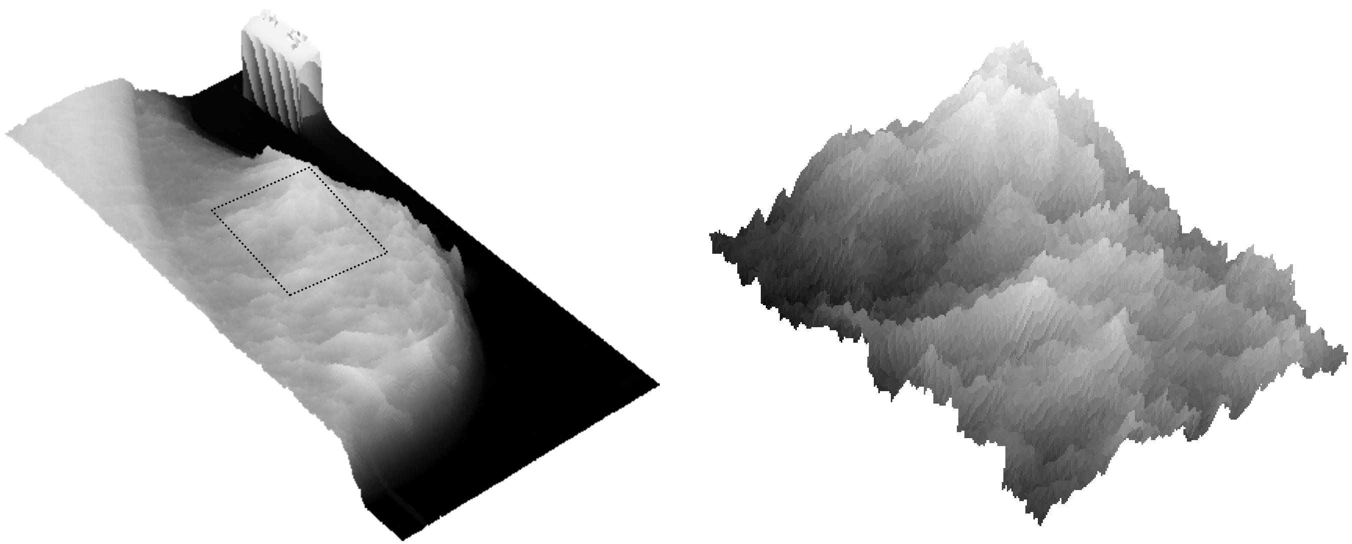
What is Texture?

- Texture is a feature used to partition images into regions of interest and to classify those regions.
- Texture provides information in the spatial arrangement of colours or intensities in an image.
- Texture is characterized by the spatial distribution of intensity levels in a neighborhood.

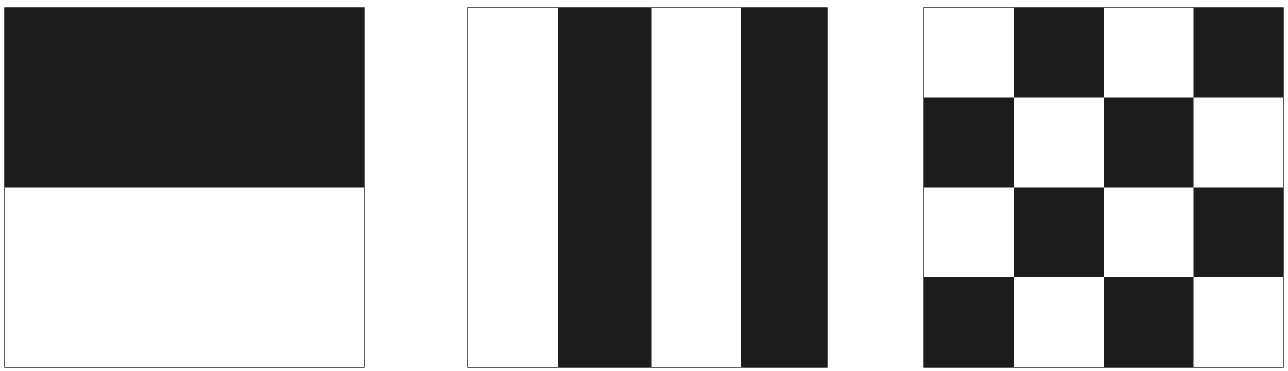
What is Texture?

- Texture is a repeating pattern of local variations in image intensity:
  - Texture cannot be defined for a point.



What is Texture?

- For example, an image has a 50% black and 50% white distribution of pixels.



- Three different images with the same intensity distribution, but with different textures.

Texture

- Texture consists of texture primitives or texture elements, sometimes called texels.
  - Texture can be described as fine, coarse, grained, smooth, etc.
  - Such features are found in the *tone* and *structure* of a texture.
  - Tone is based on pixel intensity properties in the texel, whilst structure represents the spatial relationship between texels.

Texture

– If texels are small and tonal differences between texels are large a fine texture results.
– If texels are large and consist of several pixels, a coarse texture results.

Texture Analysis

- There are two primary issues in texture analysis:
  - ❶ texture classification
  - ❷ texture segmentation
- Texture segmentation is concerned with automatically determining the boundaries between various texture regions in an image.

- Reed, T.R. and J.M.H. Dubuf, *CVGIP: Image Understanding*, **57**: pp. 359-372. 1993.

Texture Classification

- Texture classification is concerned with identifying a given textured region from a given set of texture classes.
  - Each of these regions has unique texture characteristics.
  - Statistical methods are extensively used.

    e.g. GLCM, contrast, entropy, homogeneity

Defining Texture

- There are three approaches to defining exactly what texture is:
  ① Structural: texture is a set of primitive texels in some regular or repeated relationship.
  ② Statistical: texture is a quantitative measure of the arrangement of intensities in a region. This set of measurements is called a *feature vector*.
  ③ Modelling: texture modelling techniques involve constructing models to specify textures.

Defining Texture

- Statistical methods are particularly useful when the texture primitives are small, resulting in microtextures.
- When the size of the texture primitive is large, first determine the shape and properties of the basic primitive and the rules which govern the placement of these primitives, forming macrotextures.

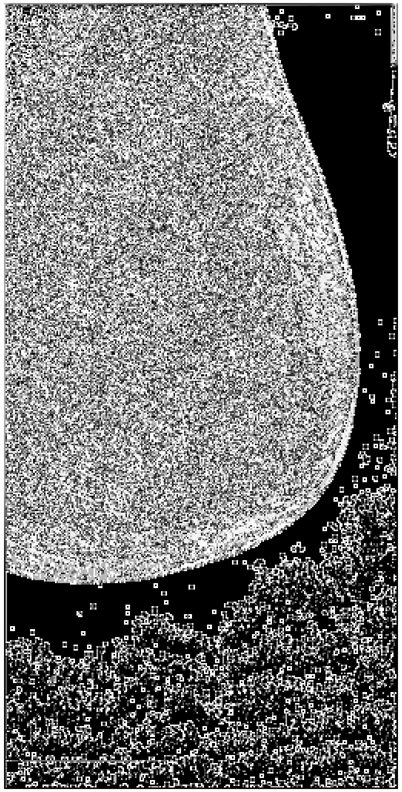
Simple Analysis of Texture

Range

- One of the simplest of the texture operators is the range or difference between maximum and minimum intensity values in a neighborhood.
  - The range operator converts the original image to one in which brightness represents texture.

Variance

- Another estimator of texture is the variance in neighborhood regions.
  - This is the sum of the squares of the differences between the intensity of the central pixel and its neighbours.



Quantitative Texture Measures

- Numeric quantities or statistics that describe a texture can be calculated from the intensities (or colours) themselves

Grey Level Co-occurrence

- The statistical measures described so far are easy to calculate, but do not provide any information about the repeating nature of texture.
- A gray level co-occurrence matrix (GLCM) contains information about the positions of pixels having similar gray level values.

GLCM

- A co-occurrence matrix is a two-dimensional array, **P**, in which both the rows and the columns represent a set of possible image values.
  - A GLCM $\mathbf{P_d}[i,j]$ is defined by first specifying a displacement vector **d**=(dx,dy) and counting all pairs of pixels separated by **d** having gray levels i and j.
  - The GLCM is defined by:

$$P_d[i,j] = n_{ij}$$

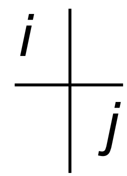
GLCM

- where n_{ij} is the number of occurrences of the pixel values (i,j) lying at distance **d** in the image.
- The co-occurrence matrix $\mathbf{P_d}$ has dimension $n \times n$, where n is the number of gray levels in the image.

GLCM

- For example, if **d**=(1,1)

2	1	2	0	1
0	2	1	1	2
0	1	2	2	0
1	2	2	0	1
2	0	1	0	1



$$P_d = \begin{array}{|ccc|cc} 0 & 2 & 2 & 0 & \\ 2 & 1 & 2 & 1 & i \\ 2 & 3 & 2 & 2 & \\ \hline 0 & 1 & 2 & & \\ & j & & & \end{array}$$

there are 16 pairs of pixels in the image which satisfy this spatial separation. Since there are only three gray levels, P[i,j] is a 3×3 matrix.

GLCM

Algorithm:

- Count all pairs of pixels in which the first pixel has a value *i,* and its matching pair displaced from the first pixel by **d** has a value of *j*.
- This count is entered in the i^{th} row and j^{th} column of the matrix $P_d[i,j]$
- Note that $P_d[i,j]$ is not symmetric, since the number of pairs of pixels having gray levels [i,j] does not necessarily equal the number of pixel pairs having gray levels [j,i].

Normalised GLCM

- The elements of $P_d[i,j]$ can be normalised by dividing each entry by the total number of pixel pairs.

  Normalised GLCM; ***N*[i,j]**, defined by:

$$N[i,j] = \frac{P[i,j]}{\sum_i \sum_j P[i,j]}$$

  which normalises the co-occurrence values to lie between 0 and 1, and allows them to be thought of as probabilities.

Numeric Features of GLCM

- Gray level co-occurrence matrices capture properties of a texture but they are not directly useful for further analysis, such as the comparison of two textures.
- Numeric features are computed from the co-occurrence matrix that can be used to represent the texture more compactly.

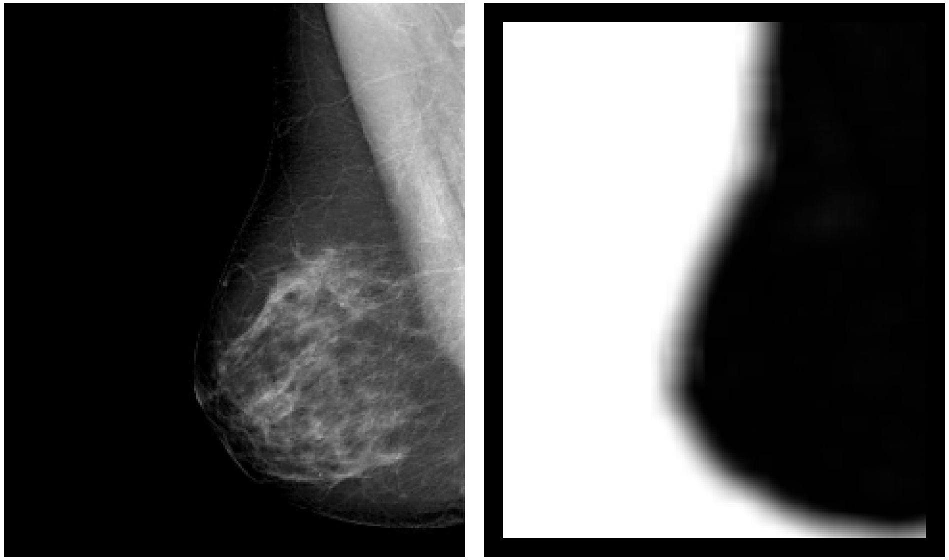
Maximum Probability

- This is simply the largest entry in the matrix, and corresponds to the strongest response.
  - This could be the maximum in any of the matrices or the maximum overall.

$$C_m = \max_{i,j} P_d[i, j]$$

Maximum Probability

- Maximum probability with w=21, and **d**=(2,2)



Moments

- The order *k* element difference moment can be defined as:

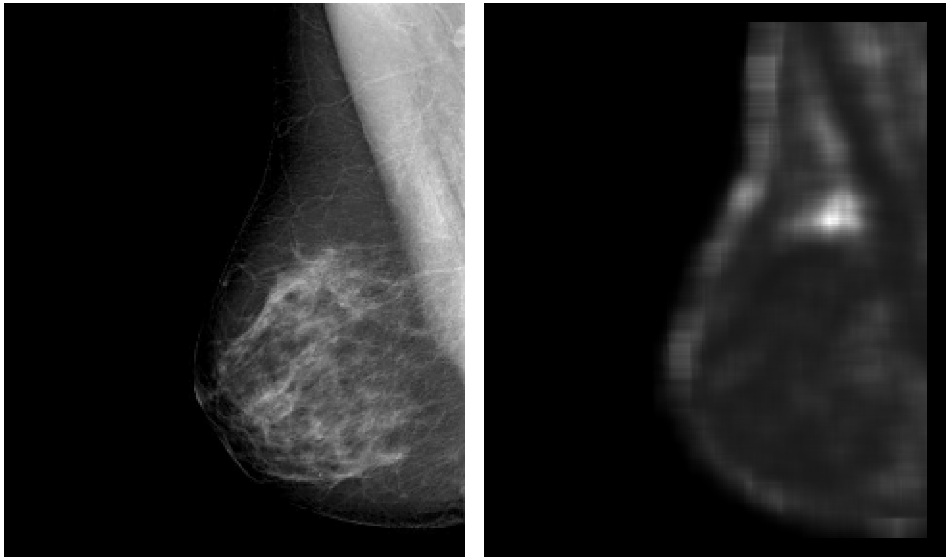
$$Mom_k = \sum_i \sum_j (i-j)^k P_d[i,j]$$

- This descriptor has small values in cases where the largest elements in *P* are along the principal diagonal. The opposite effect can be achieved using the inverse moment.

$$Mom_k = \sum_i \sum_j \frac{P_d[i,j]}{(i-j)^k}, \; i \neq j$$

Moments

- Moments with w=21, and $\mathbf{d}$=(2,2)



Contrast

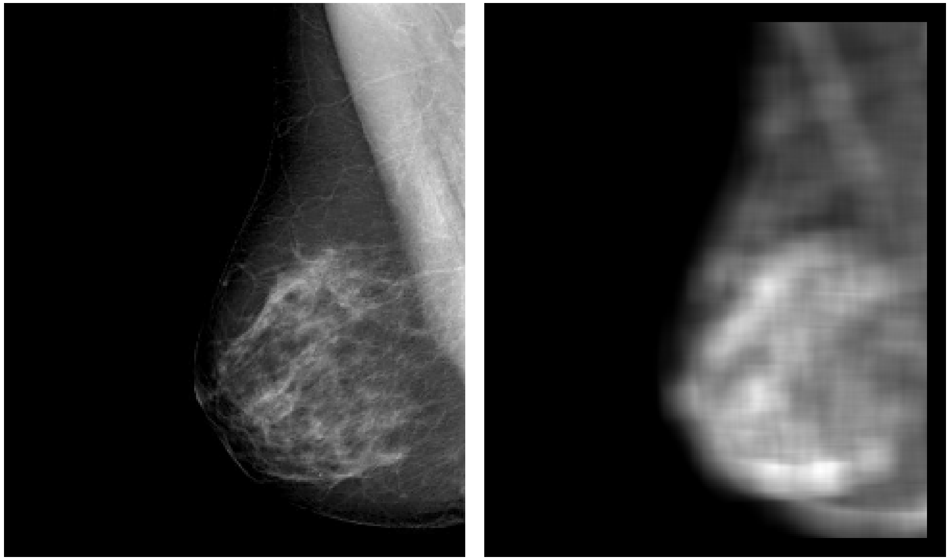
- Contrast is a measure of the local variations present in an image.

$$C(k,n) = \sum_i \sum_j (i - j)^k P_d[i,j]^n$$

  - If there is a large amount of variation in an image the P[i,j]'s will be concentrated away from the main diagonal and contrast will be high.
  - (typically k=2, n=1)

Contrast

- Contrast with w=21, and **d**=(2,2)



Homogeneity

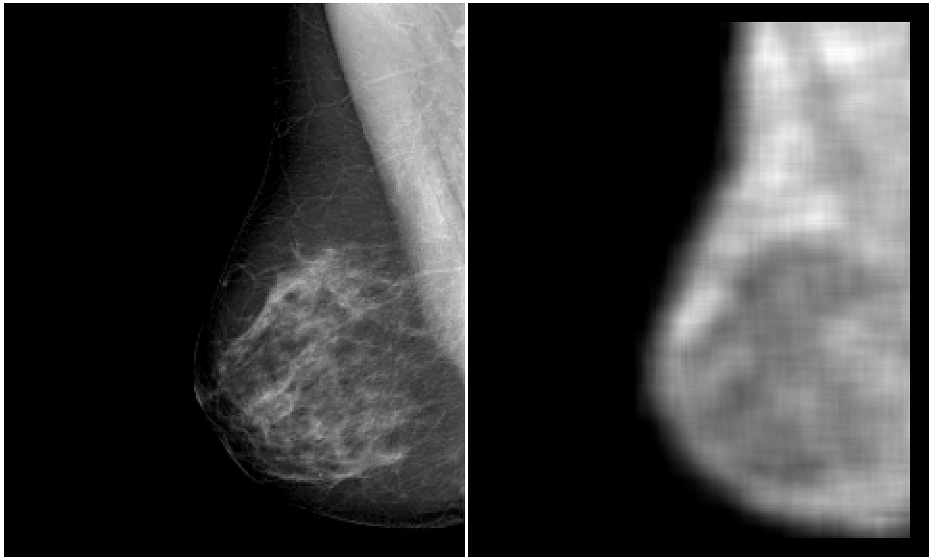
- A homogeneous image will result in a co-occurrence matrix with a combination of high and low P[i,j]'s.

$$C_h = \sum_i \sum_j \frac{P_d[i,j]}{1 + |i - j|}$$

  - Where the range of graylevels is small the P[i,j] will tend to be clustered around the main diagonal.
  - A heterogeneous image will result in an even spread of P[i,j]'s.

Homogeneity

- Homogeneity with w=21, and $\mathbf{d}$=(2,2)



Entropy

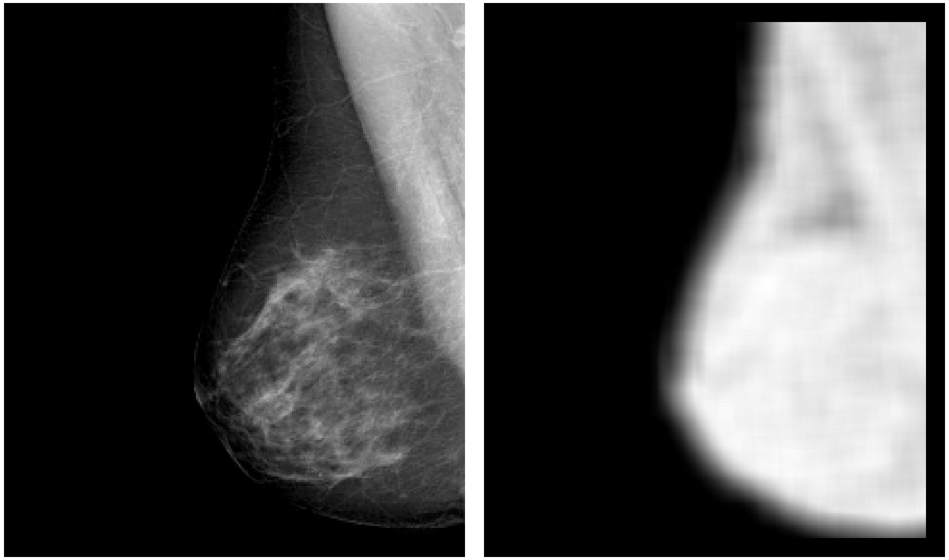
- Entropy is a measure of information content. It measures the randomness of intensity distribution.

$$C_e = -\sum_i \sum_j P_d[i,j] \ln P_d[i,j]$$

  - Such a matrix corresponds to an image in which there are no preferred graylevel pairs for the distance vector **d**.
  - Entropy is highest when all entries in P[i,j] are of similar magnitude, and small when the entries in P[i,j] are unequal.

Entropy

- Entropy with w=21, and **d**=(2,2)



Correlation

- Correlation is a measure of image linearity

$$C_c = \frac{\sum_i \sum_j \left[ijP_d[i,j] \right] - \mu_i \mu_j}{\sigma_i \sigma_j}$$

$$\mu_i = \sum iP_d[i,j], \;\; \sigma_i^2 = \sum i^2 P_d[i,j] - \mu_i^2$$

- Correlation will be high if an image contains a considerable amount of linear structure.

GLCM - References

- Carlson, G.E. and W.J. Ebel. "Co-occurrence matrix modification for small region texture measurement and comparison". in *IGARSS'88- Remote Sensing: Moving Towards the 21st Century*, pp.519-520, IEEE, Edinburgh, Scotland. 1988.
- Argenti, F., L. Alparone, and G. Benelli, "Fast algorithms for texture analysis using co-occurrence matrices"*. IEE Proceedings, Part F: Radar and SIgnal Processing*, **137**(6): pp. 443-448. 1990.
- Gotlieb, C.C. and H.E. Kreyszig, "textur descriptors based on co-occurrence matrices"*. Computer Vision, Graphics and Image Processing*, **51**(1): pp. 70-86. 1990.

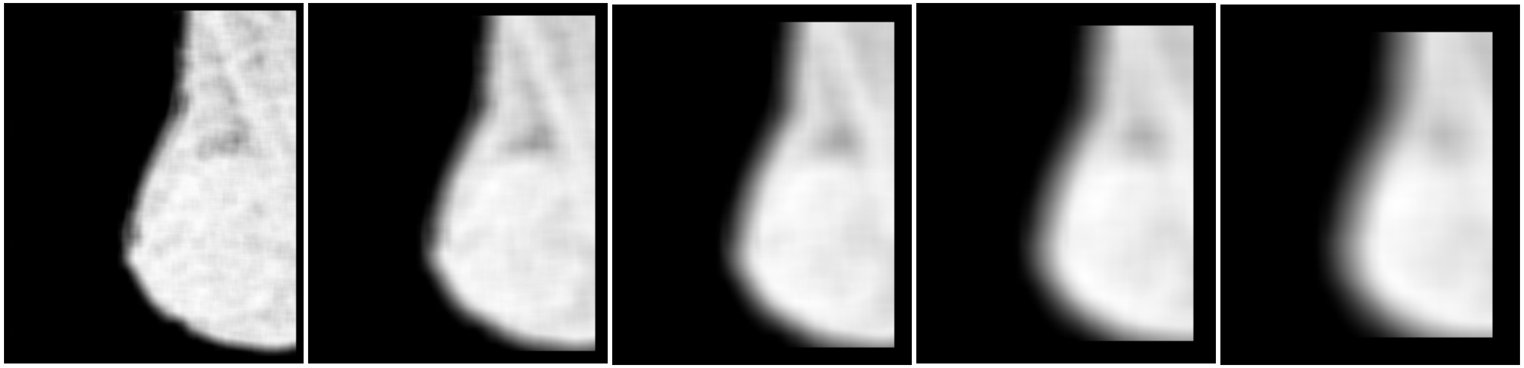
Problems with GLCM

- One problem with deriving texture measures from co-occurrence matrices is how to choose the displacement vector **d**.
  - The choice of the displacement vector is an important parameter in the definition of the GLCM.
  - Occasionally the GLCM is computed from several values of **d** and the one which maximises a statistical measure computed from P[i,j] is used.
  - Zucker and Terzopoulos used a χ^2 measure to select the values of **d** that have the most structure; i.e. to maximise the value:

$$\chi^2(d) = \sum_i \sum_j \frac{P_d^2[i,j]}{P_d[i]P_d[j]} - 1$$

Windowing

- Algorithms for texture analysis are applied to an image in a series of windows of size *w*, each centered on a pixel (i,j).
  - The value of the resulting statistical measure are assigned to the position (i,j) in the new pixel.



Haralick Texture Operator

- Haralick et al. suggested a set of 14 textural features which can be extracted from the co-occurrence matrix, and which contain information about image textural characteristics such as homogeneity, linearity, and contrast.

- Haralick, R.M., K. Shanmugam, and I. Dinstein, "Textural features for image classification". *IEEE Transactions on Systems, Man and Cybernetics*: pp. 610-621. 1973.

Graylevel Difference Statistics

- Grey-level differences are based on absoute differences between pairs of grey-levels.
- The grey-level differences are contained in a 256-element vector, and are computed by taking the absolute differences of all possible pairs of grey levels distance **d** apart at angle θ, and counting the number of times the difference is 0,1,…,255

Graylevel Difference Statistics

- Let **d**=(dx,dy) be the displacement vector between two image pixels, and g(d) the gray-level difference at distance **d**.

$$g(d) = \left| f(i,j) - f(i+dx, j+dy) \right|$$

- $p_g(g,d)$ is the histogram of the gray-level differences at the specific distance, **d**. One distinct histogram exists for each distance **d**.

Graylevel Difference Statistics

- The difference statistics are then normalized by dividing each element of the vector by the number of possible pixel pairs.
- Several texture measures can be extracted from the histogram of graylevel differences:

Graylevel Difference Statistics

- Mean:

$$\mu_d = \sum_{k=1}^{N} g_k p_g(g_k, d)$$

  - Small mean values μ_d indicate coarse texture having a grain size equal to or larger than the magnitude of the displacement vector.

- Entropy:

$$H_d = -\sum_{k=1}^{N} p_g(g_k, d) \ln p_g(g_k, d)$$

  - This is a measure of the homogeneity of the histogram. It is maximised for uniform histograms.

Graylevel Difference Statistics

- Variance:

$$\sigma_d^2 = \sum_{k=1}^{N} (g_k - \mu_d)^2 p_g(g_k, d)$$

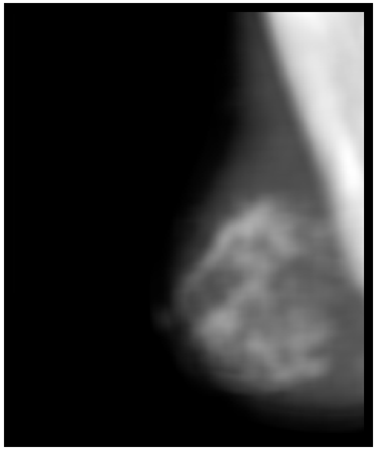
  - The variance is a measure of the dispersion of the gray-level differences at a certain distance, **d**.

- Contrast:

$$C_d = \sum_{k=1}^{N} g_k^2 p_g(g_k, d)$$

Graylevel Difference Statistics

Mean



Standard Deviation

Entropy

Contrast

Runlength Statistics

- The lengths of texture primitives in different directions can serve as a texture description.
  - A *run length* is a set of constant intensity pixels located in a line.
- Runlength statistics are calculated by counting the number of runs of a given length (from 1 to *n*) for each grey level.

- Galloway, M.M., "Texture classification using gray level run lengths". *Computer Graphics and Image Processing*, **4**(2): pp. 172-179. 1975.

Runlength Statistics

- In a course texture it is expected that long runs will occur relatively often, whereas a fine texture will contain a higher proportion of short runs.
- Statistical measures:
  - Let $B(a,r)$ be the number of primitives of all directions having length r, and grey-level a, m and n the image dimensions, and L the number of intensity values.
  - Let K be the number of runs:

$$K = \sum_{a=1}^{L} \sum_{r=1}^{Nr} B(a,r)$$

Runlength Statistics

① Long-run emphasis:

$$S_{lr} = \frac{1}{K}\sum_{a=1}^{L}\sum_{r=1}^{Nr} B(a,r)r^2$$

– This is a measure that emphasizes the long-runs of a gray-level image. Long-run emphasis will be large when there are lots of long runs of the same intensity.

② Short-run emphasis: $S_{sr} = \frac{1}{K}\sum_{a=1}^{L}\sum_{r=1}^{Nr} \frac{B(a,r)}{r^2}$

– This is a measure that emphasizes the short-runs of a gray-level image. short-run emphasis will be large when there are lots of short runs of the same intensity.

Runlength Statistics

③ Grey-level distribution:

$$S_d = \frac{1}{K}\sum_{a=1}^{L}\left[\sum_{r=1}^{Nr} B(a,r)r^2\right]^2$$

- The sum in [ ] gives the total number of runs for a certain gray-level value grey-level *a*. The distribution will be large when runs are not evenly distributed over the different intensities.

Runlength Statistics

④ Run-length distribution:

$$S_{rd} = \frac{1}{K}\sum_{r=1}^{Nr}\left[\sum_{a=1}^{L} B(a,r)r^2\right]^2$$

– The sum in [ ] gives the total number of occurrences of a certain run length *l* for any gray level. s for a certain gray-level value grey-level *r*.

⑤ Run percentage:

$$S_{rp} = \frac{K}{mn}$$

Edges and Texture

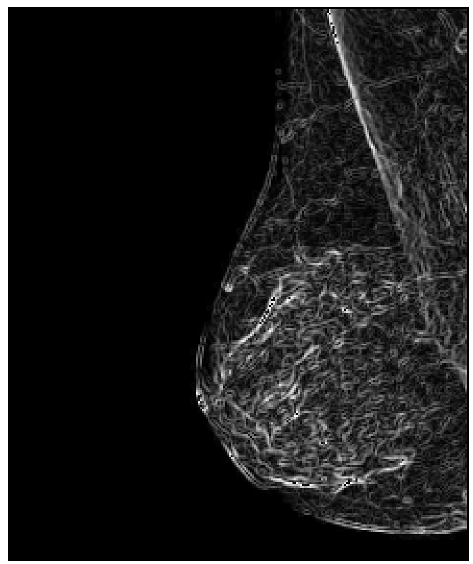
- It should be possible to locate the edges that result from the intensity transitions along the boundary of the texture.
  - Since a texture will have large numbers of texels, there should be a property of the edge pixels that can be used to characterise the texture.
    - a set of common directions
    - a measure of the locadensity of the edge pixels
- Compute the co-occurrence matrix of an edge-enhanced image.

- Davis, L.S. and A. Mitiche, "Edge detection in textures". *Computer Graphics and Image Processing*, **12**: pp. 25-39. 1980.

Edges and Texture

Original

Sobel-Enhanced



Contrast

Energy and Texture

- One approach to generating texture features is to use local kernels to detect various types of texture.
- Laws† developed a texture-energy approach that measures the amount of variation within a fixed-size window.

- † Laws, K.I. "Rapid texture identification". in *SPIE Image Processing for Missile Guidance*, pp.376-380. 1980.

Laws

- A set of convolution kernels are used to compute texture energy.
- The kernels are computed from the following vectors:

$$
\begin{aligned}
L5 &= \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix} \\
E5 &= \begin{bmatrix} -1 & -2 & 0 & 2 & 1 \end{bmatrix} \\
S5 &= \begin{bmatrix} -1 & 0 & 2 & 0 & -1 \end{bmatrix} \\
R5 &= \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \end{bmatrix} \\
W5 &= \begin{bmatrix} -1 & 2 & 0 & -2 & -1 \end{bmatrix}
\end{aligned}
$$

Laws

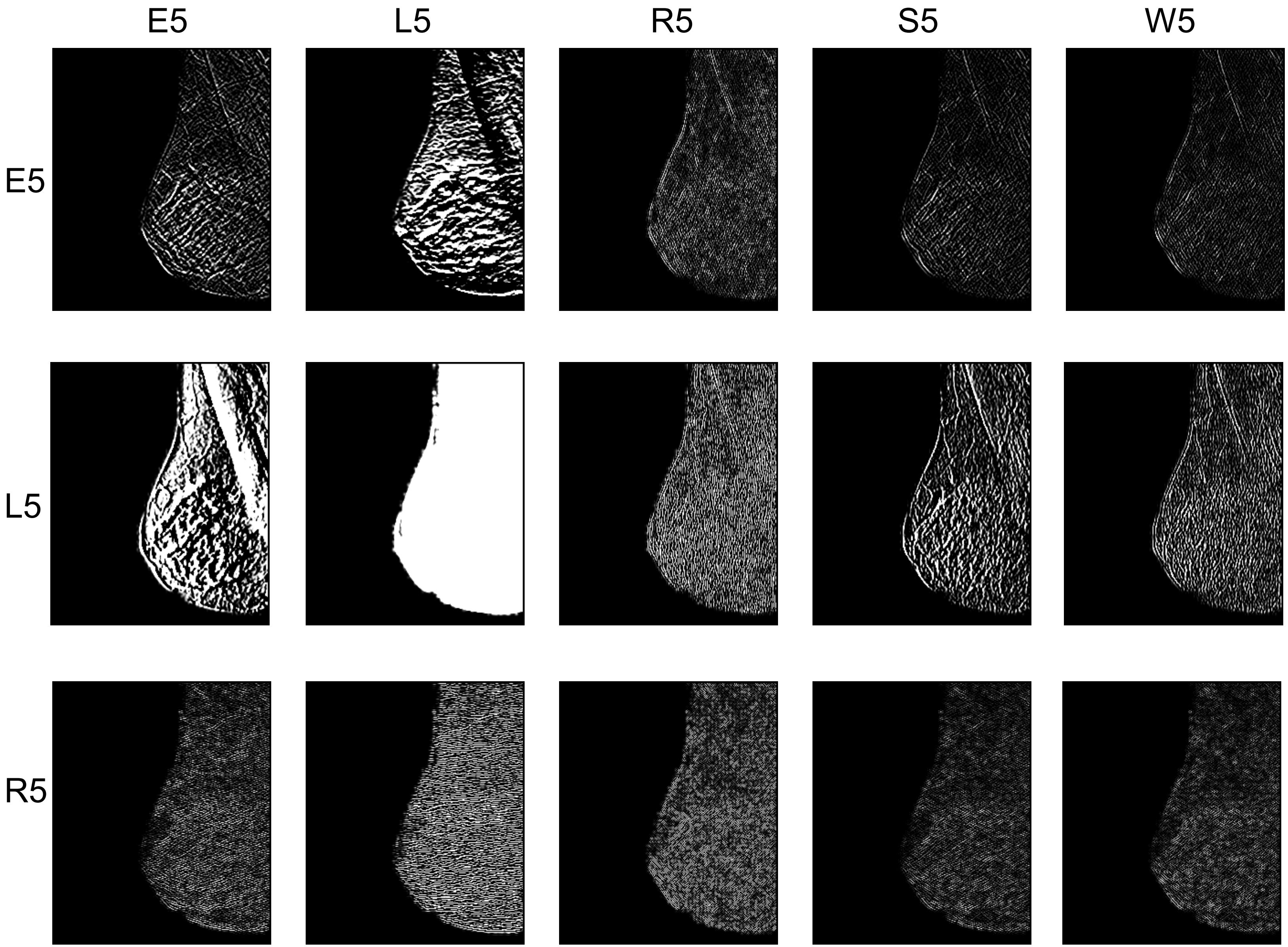
- The **L5** (level) vector gives a centre-weighted local average. The **E5** (edge) vector detects edges, the **S5** (spot) vector detects spots, the **R5** (ripple) vector detects ripples, and the **W5** (wave) vector detects waves.
- The two-dimensional convolution kernels are obtained by computing the outer product of a pair of vectors.

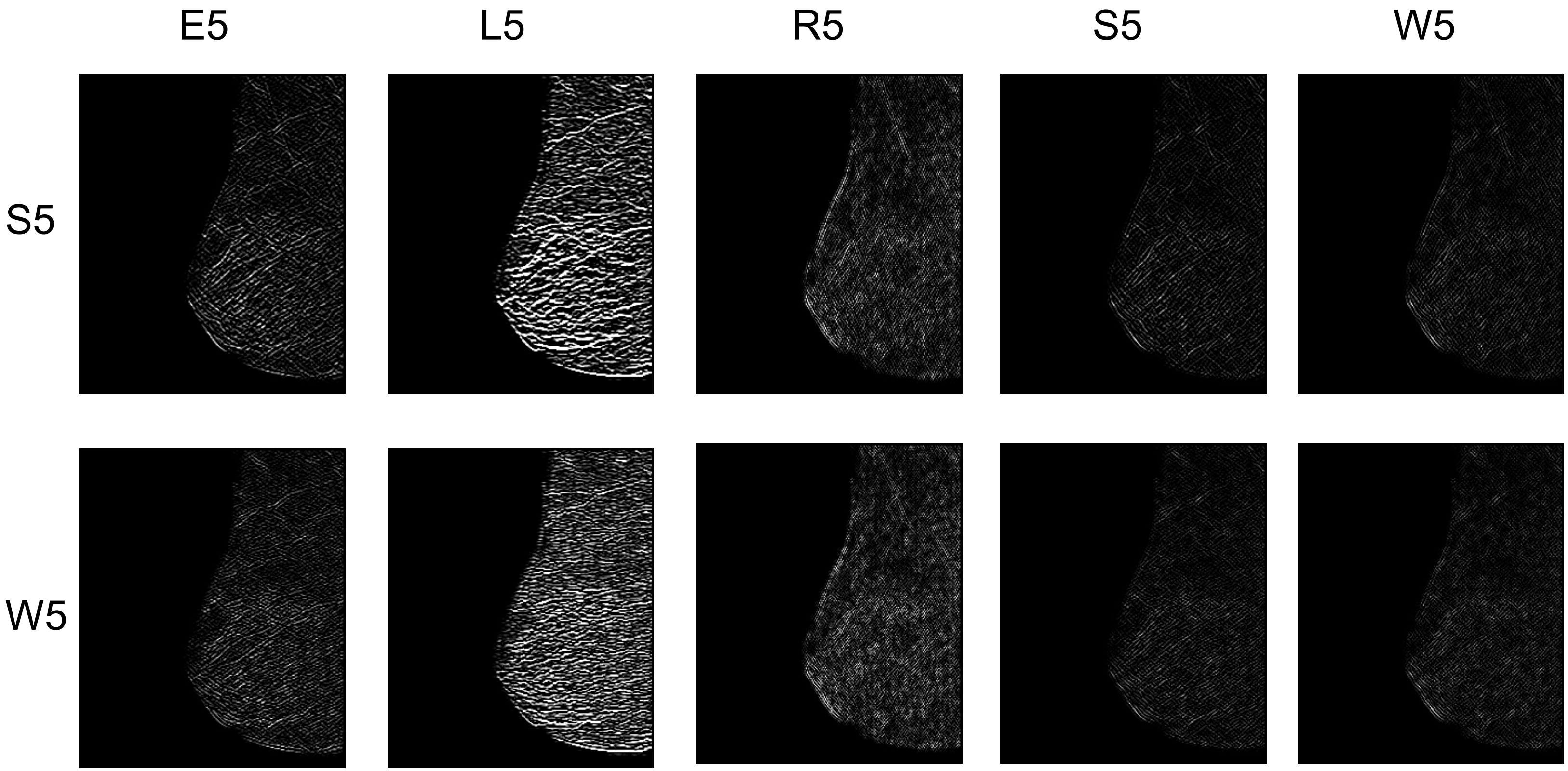
Laws

e.g. **E5L5** is computed as the product of **E5** and **L5** as follows:

$$\begin{bmatrix} -1 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -4 & -6 & -4 & -1 \\ -2 & -8 & -12 & -8 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 8 & 12 & 8 & 2 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

- This results in 25 5×5 kernels, 24 of the kernels are zero-sum, the L5L5 is not.


E5
L5
R5
S5
W5
E5
L5
R5


E5
L5
R5
S5
W5
S5
W5

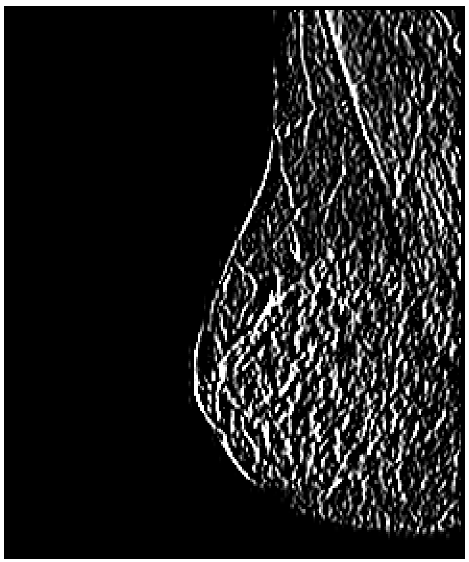
Laws

- Bias from the "directionality" of textures can be removed by combining symmetric pairs of features, making them rotationally invariant.

  e.g. S5L5(H) + L5S5(V) = L5S5R

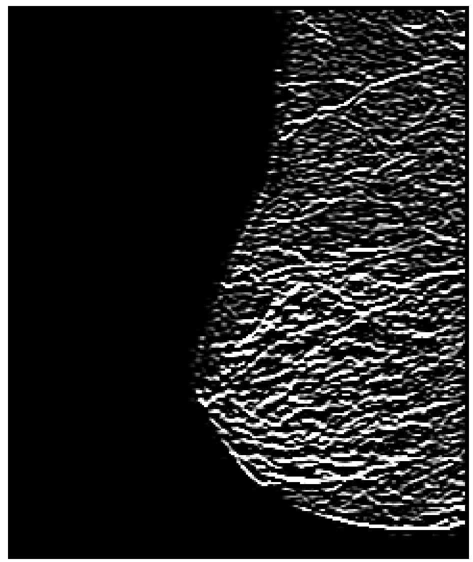
Laws

L5S5



+

S5L5



=

Laws

- After the colvolution with the specified kernel, the texture energy measure (TEM) is computed by summing the absolute values in a local neghborhood:

$$L_e = \sum_{i=1}^{m} \sum_{j=1}^{n} \left| C(i,j) \right|$$

- If *n* kernels are applied, the result is an *n*-dimensional feature vector at each pixel of the image being analysed.

Laws

ALGORITHM:

- (1) Apply convolution kernels
  (2) Calculate the texture energy measure (TEM) at each pixel. This is achieved by summing the absolute values in a local neighborhood.
  (3) Normalise features - use L5L5 to normalise the TEM images

Fractal Dimension

- Fractal geometry can be used to discriminate between textures.
- The fractal dimension **D** of a set of pixels I is specified by the relationship:

$$1 = Nr^D$$

where the image I has been broken up into *N* overlapping copies of a basic shape, each one scaled by a factor, *r*.

- Russ, J.C., "Surface characterisation: Fractal dimensions, Hurst coefficients, and frequency transforms". *Journal of Computer Assisted Microscopy*, **2**: pp. 249-257. 1990.

Fractal Dimension

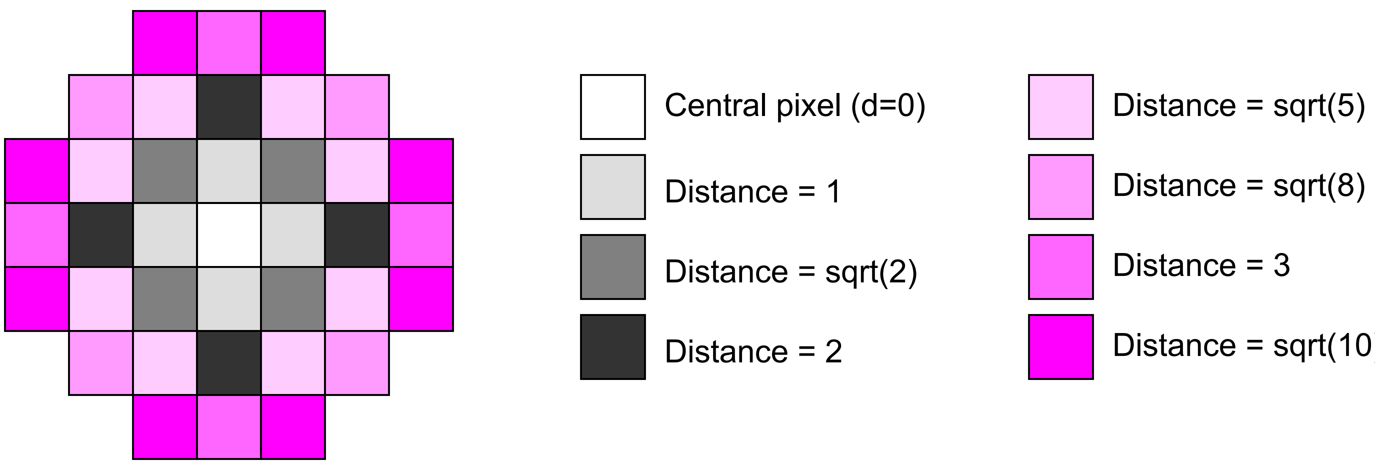
- **D** can be estimated by the Hurst coefficient:

$$D = \frac{\log N}{\log(\frac{1}{r})}$$

- There is a Log-Log relationship between *N* and *r*. If log(N) were plotted against log(r) the result should be a straight line whose slope is approximately **D**.

Hurst

- The Hurst-coefficient is an approximation that makes use of this relationship.
  - Consider a 7×7 pixel region which is marked according to the distance of each pixel from the central pixel.
  - There are eight groups of pixels, corresponding to the eight difference distances that are possible



Hurst

- Within each group, the largest difference in intensity is found, this is the same as subtracting the minimum value from the maximum value.
- The central pixel is ignored, and a straight line is fitted to the Log of the maximum difference (y-coord), and the Log og the distance from the central pixel (x-coord).
- The slope of this line is the Hurst coefficient, and replaces the pixel at the centre of the region.

Hurst

85	70	86	92	60	102	202
91	81	98	113	86	119	189
96	86	102	107	74	107	194
101	91	113	107	83	118	198
99	68	107	107	76	107	194
107	94	93	115	83	115	198
94	98	98	107	81	115	194

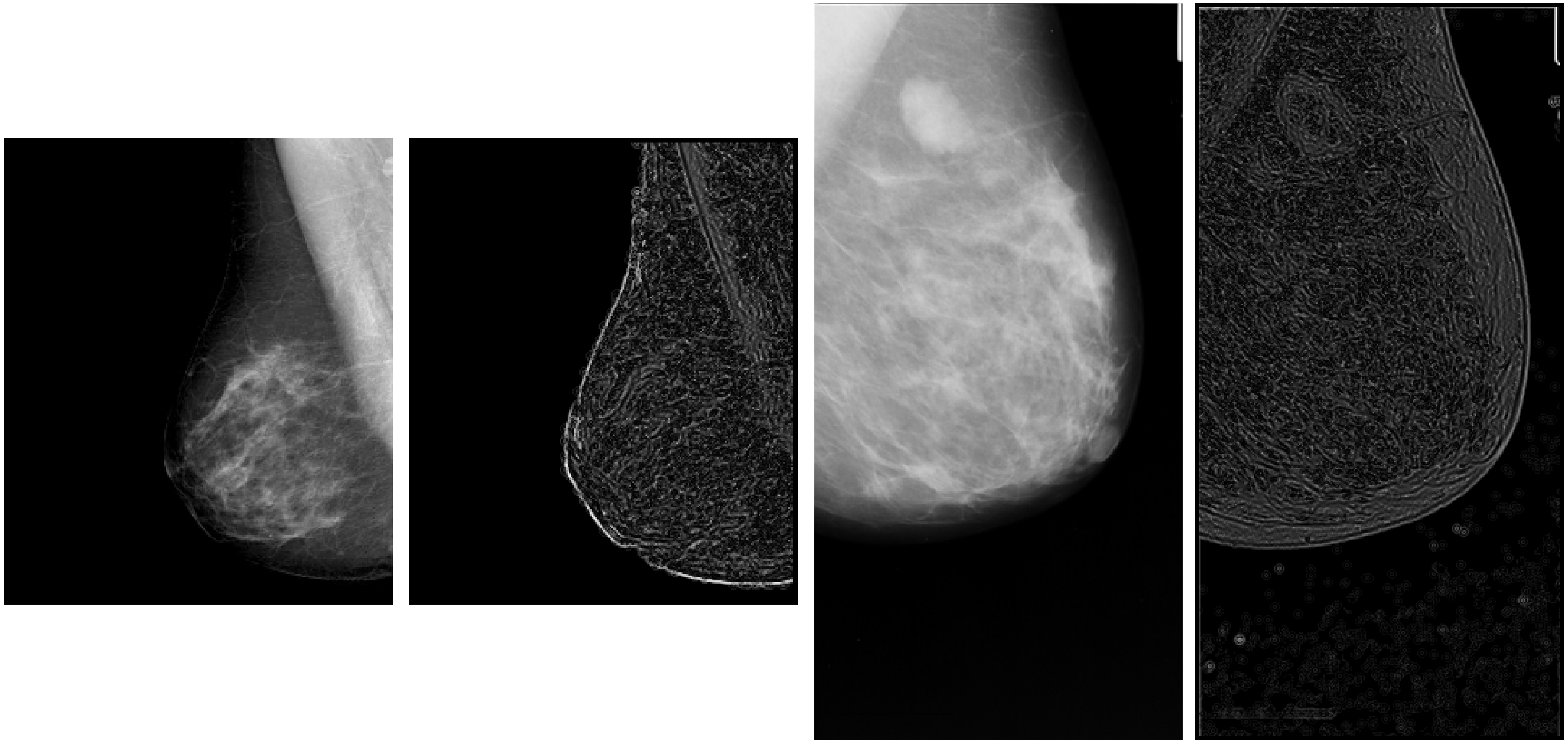
Distance	d=1	d=$\sqrt{2}$	$d = 2$	$d = \sqrt{5}$	$d = \sqrt{8}$	$d = 3$	$d = \sqrt{10}$
Difference	30	39	44	50	51	130	138

Log(dis)	0.0	0.347	0.693	0.805	1.04	1.099	1.151
Log(dif)	3.401	3.664	3.784	3.912	3.932	4.868	4.927

$$y = 1.145x + 3.229$$

- The slope of the line, *m*=1.145 is the Hurst coefficient

Hurst



Surfaces and Texture

- There are some algorithms that are based on a view of the gray-level image as a three-dimensional surface, where intensity is the third dimension.
  - Vector dispersion
    - Matalas, I., S. Roberts, and H. Hatzakis. "A set of multiresolution texture features suitable for unsupervised image segmentation". in *SIgnal Processing VIII: Theories and Applications*. 1996.
  - Surface curvature
    - Peet, F.G. and T.S. Sahota, "Surface curvature as a measure of image texture". IEEE Transactions on Pattern Analysis and Machine Intelligence, **7**(6): pp. 734-738. 1985.

Model-based Methods

- Model-based methods for texture analysis is an approach used to characterize texture which determines an analytical model of the textured image being analyzed.
  - Such models have a set of parameters.
  - The values of these parameters determine the properties of the texture, which may be synthesized by applying the model.

    e.g. Markov random fields

Model-based Methods

- Markov Random Fields (MRF) have been extensively studied as a model for texture.
- In the discrete Gauss-Markov random field model, the gray level at any pixel is modeled as a linear combination of gray levels of its neighbors plus an additive noise, as defined by:

$$f(i,j) = \sum_{k,l} f(i-k, j-l)h(k,l) + n(i,j)$$

Model-based Methods

- The summation is carried out over a specified set of pixels which are neighbors to the pixel (i,j)
- The parameters of this model are the weights h(k,l).
- These parameters are computed from the given texture using least-squares method.
- These estimated parameters are then compared with those of the known texture class to determine the class of the particular texture being analysed.

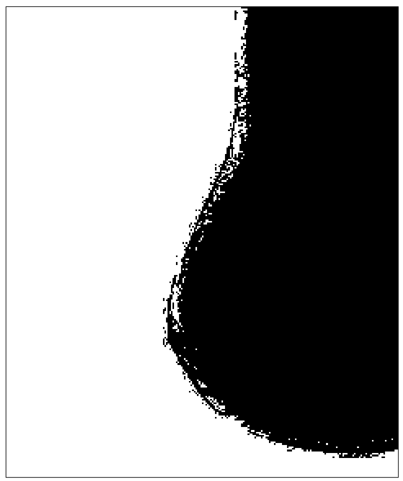
Texture Segmentation

- Any texture measure that provides a value, or a vector of values at each pixel, describing the texture in a neighborhood of that pixel can be used to segment an image into regions of similar textures.
- There are two major categories:
  - ① Region-based: attempt to group or cluster pixels with similar texture properties
  - ② Boundary-based: attempt to find "texture-edges" between pixels from different texture distributions

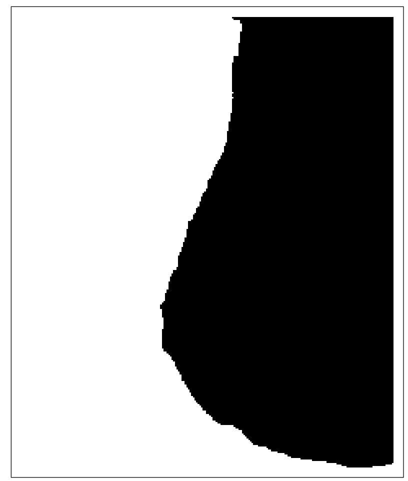
Texture Segmentation

GLC: Entropy (Wsize =13)

Original T=4



Entropy T=12



Breast Contour Approximation