4^{th} Grade Texas Mathematics: Unpacked Content

What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?

Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

At A Glance:

New to 4th Grade:

- Rather than using place value to read, write, compare, and order whole numbers, students are expected to *interpret* the value of each place value position as 10 times the position to the right or as one-tenth of the value of the place to its left.
- Students should interpret place value and compare whole numbers to the billions place.
- Students use expanded notation and numerals to represent the value of digits in whole numbers through 1 billion and decimals to the hundredths place.
- Students should round whole numbers to the hundred thousands place.
- Students should represent decimals to the hundreds place using concrete and visual models and money.
- Determine measurements to the hundredths place using a number line.
- When investigating fractions other than unit fractions, in which the numerator is 1, students should be able to join (compose) or separate (decompose) the fractions of the same whole in more than one way. (Example: 2/3=1/3+1/3)
- Students should be able to compare two fractions without using models or pictures.
- Students should be able to add and subtract fractions with common denominators, first using objects and pictorial models, and building toward using a number line and properties of operations.
- Students should add and subtract fractions using benchmark fractions (0, ¼, ½, & 3/4) and evaluate the reasonableness.
- Add and subtract whole numbers and decimals to the hundreds place using the standard algorithms-not just concrete and pictorial models.
- Students should represent the product of 2 two-digit numbers using arrays, area models, or equations including perfect squares through 15 by 15.
- Using strategies such as *commutative, associative, and distributive* properties, mental math and partial products along with standard algorithms, students will multiply up to a 4-digit number by a 1-digit number and 2-digit numbers by 2-digit numbers.
- Students will divide using 4-digit dividends using strategies and algorithms.
- Students will solve multiplication and division problems and interpret remainders without the use of pictorial representation.
- Solve multi-step problems involving the 4 operations of whole numbers using strip diagrams and equations with variables.
- Use and/or create an input/output table to generate a pattern and a rule.
- Students use models to discover the formulas for perimeter and area of rectangles and squares and solve problems related to perimeter and area of squares and rectangles.
- Identify points, lines, line segments, rays, angles, and perpendicular and parallel lines without pictorial or concrete models.
- Identify and draw one or more lines of symmetry in 2 dimensional figures. (no longer using reflections to determine symmetry)
- Apply knowledge of right angles to identify acute, right, and obtuse triangles.
- Classify 2 dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of specified sides.

- Students understand that there are 360 degrees in a circle, and angles are measured to the nearest whole number as “slices” out of the circle where the center of the circle is the vertex of the angle.
- Students measure and draw angles to the nearest whole number using a protractor.
- Determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.
- Represent data with whole numbers and fractions on a frequency table, dot plot, or stem and leaf plot.
- Solve one and two step problems using data and whole number, decimal, and fraction form in a frequency table, dot plot, stem and leaf plot.
- Distinguish between fixed and variable expenses.
- Calculate profit in a given situation.
- Compare the advantages and disadvantages of various savings options.
- Describe how to allocate a weekly allowance among spending, saving, including for college, and sharing.
- Describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending money.

Moved from 4th Grade:

- Recall and apply multiplication facts through 12 by 12
- Use patterns and relationships to develop strategies to remember basic multiplication and division facts.
- Demonstrate translations, reflections, and rotations using concrete models.
- Use translations, reflections, and rotations to verify that two shapes are congruent.

Instructional Implications for 2013-14:

Identify Gaps such as:

- Since place value is moving to the billions, students will need to be taught through the billions in the 13-14 school year
- Students will need to be taught angle measurements in 13-14 school year since this concept is moving from 6th grade to 4th
- Fractions is a major concept in 4th grade and goes beyond concrete and pictorial models; students will need to compare without pictures; sums & differences using benchmark fractions; decomposing fractions
- Multiply & Divide a 4 digit by one digit number
- Represent multi-step problems with strip diagrams and equations
- Stem & Leaf plots have moved down from 5th grade

Professional Learning Implications for 2013-14:

- Teachers will need to identify the gaps that will need to be addressed in the 2013-14 school year
- Embed the process standards into instruction and application
- Identify academic vocabulary
- PD and resources regarding Personal Financial Literacy
- Initial learning of the teachers' grade level TEKS (teachers unpacking the TEKS at their grade level)
- Vertical study of the strands to know how the TEKS align and progress from 3rd through 5th grade

Grade 4 Primary Focal Areas:

The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

The primary focal areas in Grade 4 are use of operations, fractions, and decimals and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and represent points on a number line that correspond to a given fraction or terminating decimal. In algebraic reasoning, students will represent and solve multi-step problems involving the four operations with whole numbers with expressions and equations and generate and analyze patterns. In geometry and measurement, students will classify two-dimensional figures, measure angles, and convert units of measure. In data analysis, students will represent and interpret data.

Mathematical process standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

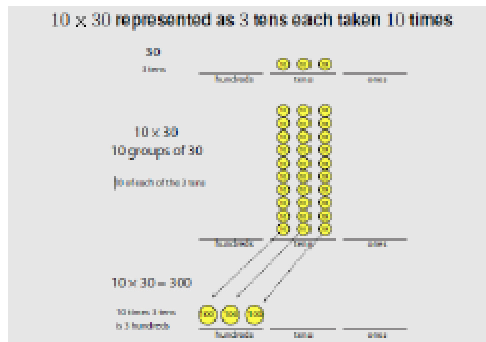
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

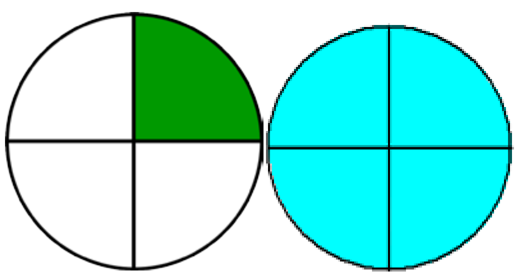
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

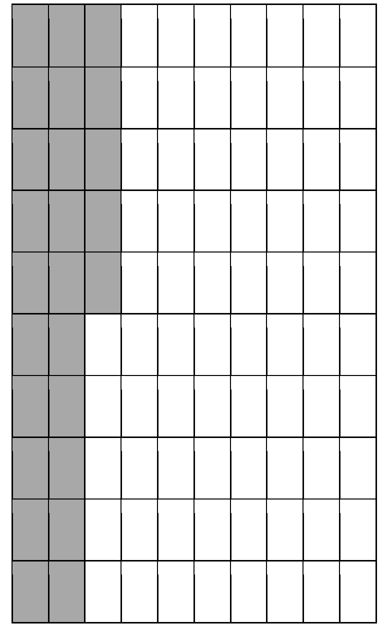
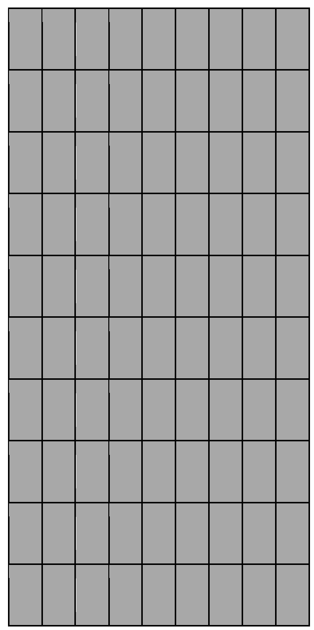
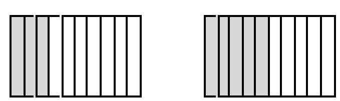
(E) create and use representations to organize, record, and communicate mathematical ideas;

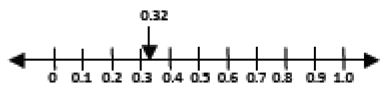
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

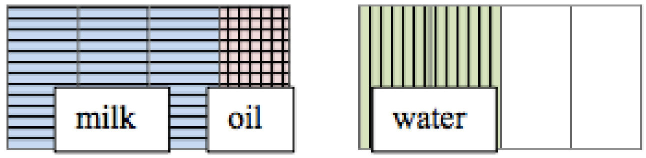
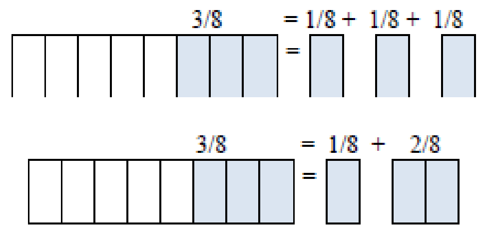
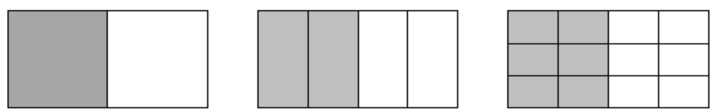
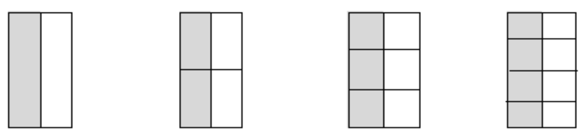
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

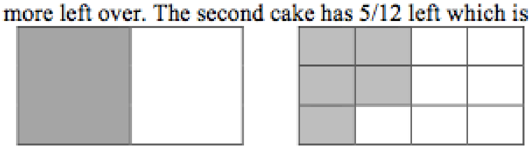
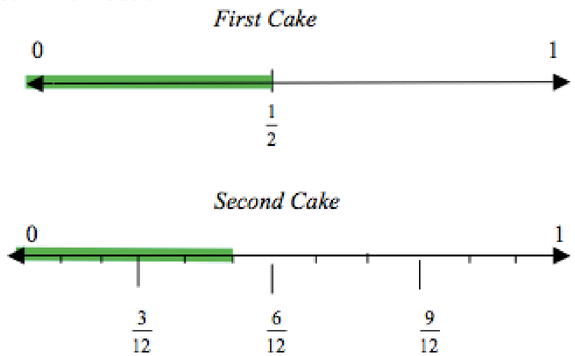
Number and Operations: TEKS 4.2	**The student applies mathematical process standards to represent and explain fractional units. The student is expected to:**
4.2 (A) Interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left	What do these standards mean a child will know and be able to do? This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with. In this base ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.  *Each of the 3 tens becomes a hundred and moves to the left. In the product, the 3 in the tens place of 30 is shifted one place to the left to represent 3 hundreds. In 300 divided by 10 the 3 is shifted one place to the right in the quotient to represent 3 tens.*

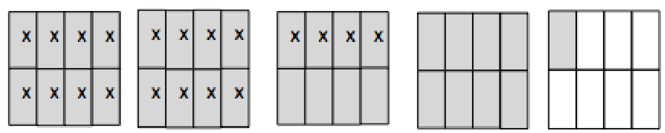
4.2 (B) Represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals.	This standard refers to various ways to write numbers. Students should have flexibility with the differeent number forms. Traditional expanded form is 285=200+80+5. Written form or number name is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.
4.2 (C)Compare and order whole numbers to one billion and represent comparisons using the symbols <,>, =	Students should be able to compare two multi-digit whole numbers using appropriate symbols. Examples: 7,456,345,201 < 7,457,201,000
4.2 (D) Round whole numbers to a given place value to the hundred thousands place	Example: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel? Some typical estimation strategies for this problem: Student 1 I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. Student 2 I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500. Student 3 I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.
4.2 (E) Represent decimals, including tenths and hundredths, using concrete and visual models and money.	Matthew has $1.25. Create two visual models that show the same amount. Examples: 

	 
4.2 (F) Compare and order decimals using concrete and visual models to the hundredths.	When the wholes are the same, the decimals or fractions can be compared. Example: Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths. 

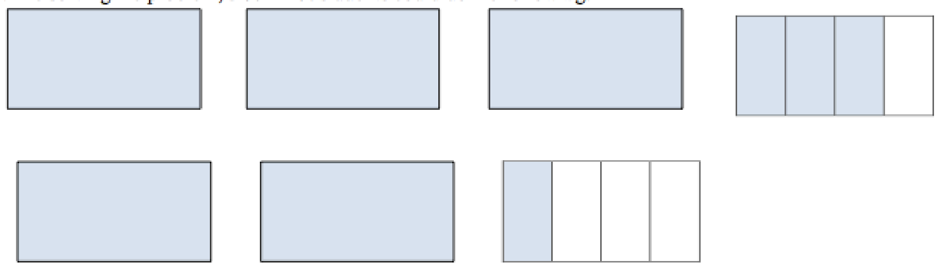
<table>
<tr><td>4.2 (G) Relate decimals to fractions that name tenths and hundredths.</td><td>Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.
<table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th><th>•</th><th>Tenths</th><th>Hundredths</th></tr><tr><td></td><td></td><td></td><td>•</td><td>3</td><td>2</td></tr></table></td></tr>
<tr><td>4.2 (H) Determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.</td><td>Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value. 
</td></tr>
<tr><td>Number and Operations: TEKS 4.3</td><td>The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:</td></tr>
<tr><td>4.3 (A) Represent a fraction a/b as a sum of fractions 1/b, where a and b are whole numbers and b is greater than zero, including when a is greater than b.</td><td>A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.
Example: 2/3 = 1/3 + 1/3
Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.
Example of word problems:
Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?
Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 +1/6 + 1/6 or 5/6 of the whole pizza.
Example with mixed numbers:
A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?</td></tr>
</table>

	 **3/4 + 1/4 + 2/4 = 6/4 = 1 2/4**
4.3 (B) Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations.	Students should justify their breaking apart (decomposing) of fractions using visual fraction models. Example: 
4.3 (C)Determine if two given fractions are equivalent using a variety of methods	This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts. Example:  **1/2 = 2/4 = 6/12** Students should begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions. 1/2 x 2/2 = 2/4.  $\frac{1}{2}$ $\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$ $\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$ $\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$

4.3 (D) Compare two fractions with different numerators and different denominators and represent the comparison using the symbols <, >, =	This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and 1/8 of two medium pizza's is very different from ½ of one medium and 1/8 of one large). Example: There are two cakes on the counter that are the same size. The first cake has half of it left. The second cake has 5/12 left. Which cake has more left? Student 1 Area model: The first cake has more left over. The second cake has 5/12 left which is smaller than ½.  Student 2 Number Line model:  Student 3 verbal explanation: I know that 6/12 equals ½. Therefore, the second cake which has 5/12 left is less than ½.

4.3 (E) Represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations	A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. Example: Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not. The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot. Example: Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend? Possible solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas.  Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator.

Example:
While solving the problem, 3 ¾ + 2 ¼ students could do the following:



Student 1
3 + 2 = 5 and ¾ + ¼ = 1 so 5+ 1 = 6

Student 2
3 ¾ + 2 = 5 ¾ so 5 ¾ + ¼ = 6

Student 3
3 ¾ = 15/4 and 2 ¼ = 9/4 so 15/4 + 9/4 = 24/4 = 6

Fourth Grade students should be able to decompose and compose fractions with the same denominator. They add fractions with the same denominator.
Example:

$$\frac{7}{5}+\frac{4}{5} = \overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{7}+\overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{4} = \frac{\overbrace{1+1+\cdots+1}^{7+4}}{5} = \frac{7+4}{5}.$$

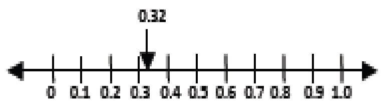
Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract 5/6 from 17/6, they decompose.
Example:

$$\frac{12}{6}+\frac{5}{6}, \quad \text{so} \quad \frac{17}{6}-\frac{5}{6}=\frac{17-5}{6}=\frac{12}{6}=2$$

Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalen fraction with the same denominator as the fraction.
Example:

$$7\frac{1}{5}=7+\frac{1}{5}=\frac{35}{5}+\frac{1}{5}=\frac{36}{5}.$$

Students use this method to add mixed numbers with like denominators. Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition.

4.3 (F) Evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, ¼, ½, ¾, and 1 referring to the same whole	Students use benchmark fractions to estimate and examine the reasonableness of their answers. Students will recognize that comparisons are valid only when the two fractions refer to the same whole. For example, when students are asked if – – –, they should use benchmark fractions to realize that – is greater than – so therefore, the equation cannot be true.
4.3 (G) Represent fractions and decimals to the tenths and hundredths as distances from zero on a number line.	Students represent values such as 0.32 or — on a number line. — is more than — (or —) and less than — (or —). It is closer to — so it would be placed on the number line near that value. 
Number and Operations: TEK 4.4	**The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:**
4.4 (A) Add and subtract whole numbers and decimals to the hundredths place using the standard algorithm	This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previous learned strategies are still appropriate for students to use. In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable. As with addition and subtraction, students should use methods they understand and can explain. Computation algorithm is a set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. A computation strategy is purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and maybe aimed at converting one problem into another.

Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). It can help to put 0s in places so that all numbers show the same number of places to the right of the decimal point. Although whole numbers are not usually written with a decimal point, but that a decimal point with 0s on its right can be inserted (e.g., 16 can also be written as 16.0 or 16.00). The process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers. For example, students can write digits representing new units below on the addition or subtraction line, and they can decompose units wherever needed before subtracting.

Examples:

- 3.6 + 1.7

A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½.

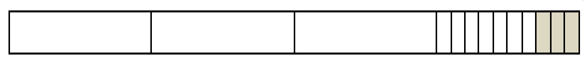
- 5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

Example: 4 - 0.3

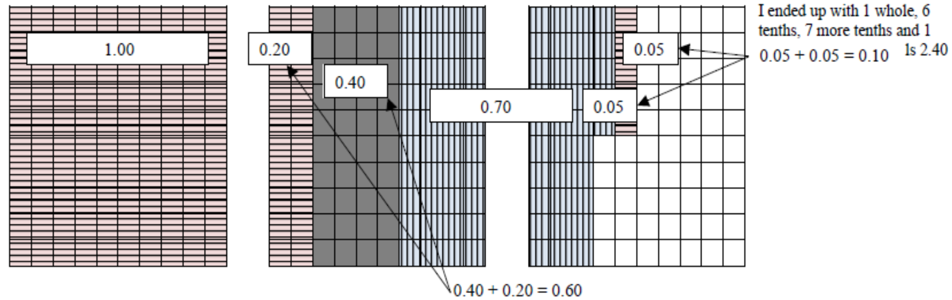
3 tenths subtracted from 4 wholes. The wholes must be divided into tenths. (solution is 3 and 7/10 or 3.7)

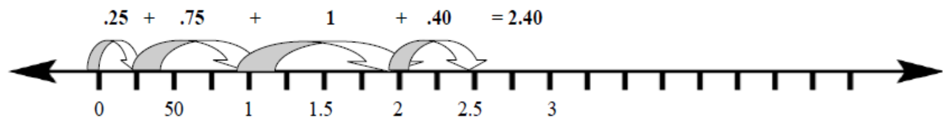


Additional examples on next page.

Example:

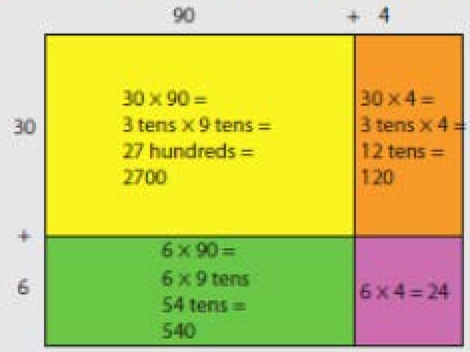
A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

<table>
<tr>
<td></td>
<td>
Student 1
1.25 + 0.40 + 0.75
First, I broke the numbers apart:
I broke 1.25 into 1.00 + 0.20 + 0.05
I left 0.40 like it was.
I broke 0.75 into 0.70 + 0.05
I combined my two 0.05s to get 0.10.
I combined 0.40 and 0.20 to get 0.60.
I added the 1 whole from 1.25.


Student 2
I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole.
I then added the 2 wholes and the 0.40 to get 2.40.


</td>
</tr>
<tr>
<td>4.4(B) Determine products of a number and 10 or a 100 using properties of operations and place value understandings</td>
<td>Multiplying by a 100 will shift every digit of the multiplicand two places to the left in the product (the product is 100 times larger). Multiplying by a 10 will shift every digit of the multiplicand one place to the left in the product (the product is 10 times larger).

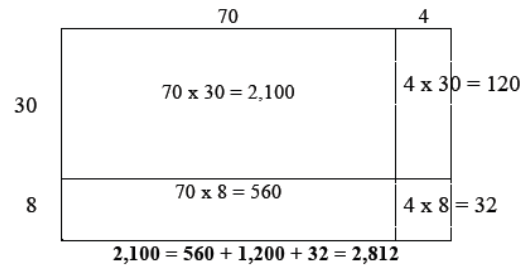
Example:
Multiplying by 104 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left.</td>
</tr>
</table>

4.4 (C) Represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15.	Students use visual representations to represent products of 2 two-digit numbers. Students should show fluency in multiplication facts (from 3rd grade). **Computation of 36×94 connected with an area model**  *The products of like base-ten units are shown as parts of a rectangular region.*

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

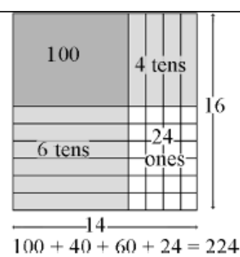
Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

Example:
What would an array area model of 74 x 38 look like?



Example:
To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.

The area model below shows the partial products.
14 x 16 = 224



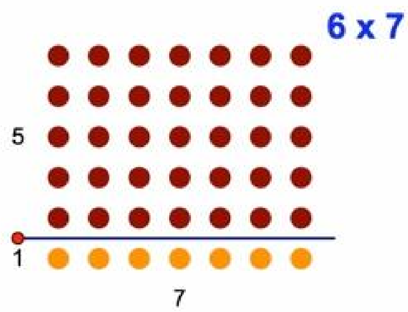
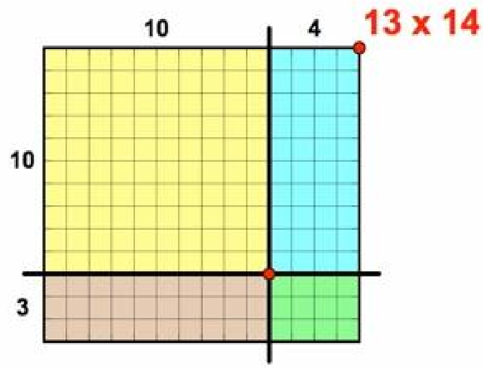
Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

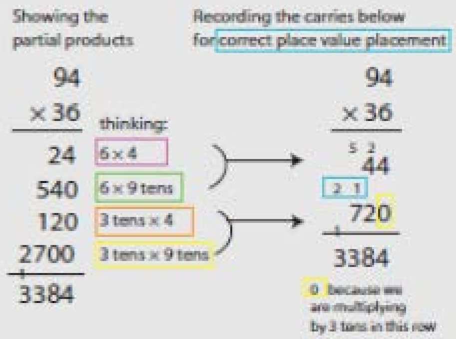
They use different strategies to record this type of thinking.

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

```
  25
 x24
 400 (20 x 20)
 100 (20 x 5)
  80 (4 x 20)
  20 (4 x 5)
 600
```

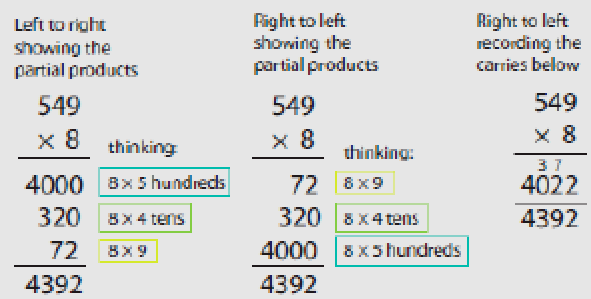
	 
4.4 (D) Use Strategies and algorithms, including the standard algorithm, to multiply up to a 4 digit number by a 1 digit number and to multiply a 2 digit number by a 2 digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties.	Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc, when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5^{th} grade. Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units. Example: $36 \times 94 = (30 + 6) \times (90 + 4)$ $= (30 + 6) \times 90 + (30 + 6) \times 4$ $= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4.$ Example: To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.

Computation of 36×94: Ways to record general methods



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from $30 \times 4 = 120$ is placed correctly in the hundreds place and the digit 2 from $30 \times 90 = 2700$ is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by 30 (not 3).

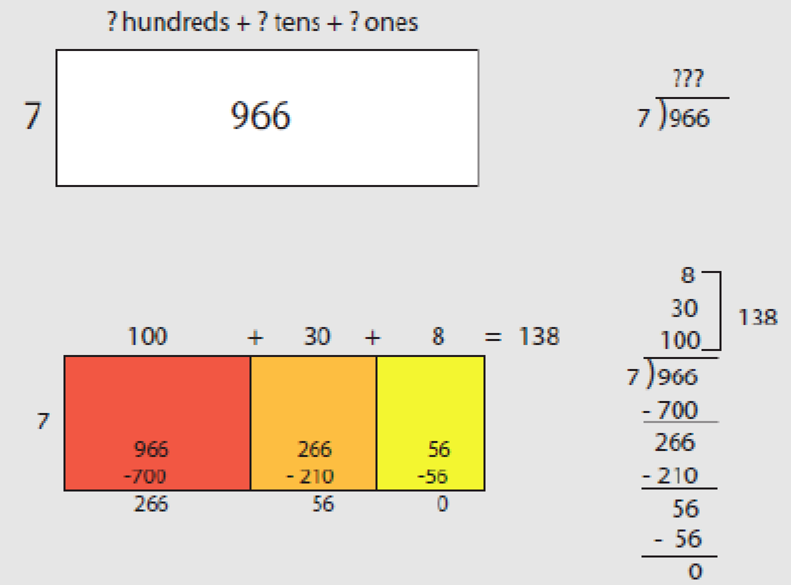
Computation of 8×549: Ways to record general methods



The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from $8 \times 9 = 72$ is written diagonally to the left of the 2 rather than above the 4 in 549.

4.4(E) Represent the quotient of up to a 4 digit whole number divided by a 1 digit whole number using arrays, area models, or equations.	General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understands as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 6 is related to 420 6 and 4200 6. Students can draw on their work with multiplication and they can also reason that 4200 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group. Another component of understanding general methods for multi-digit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller unites. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups).

Division as finding side length



966 ÷ 7 is viewed as finding the unknown side length of a rectangular region with area 966 square units and a side of length 7 units. The amount of hundreds is found, then tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, and 7 by 8. It can be connected with the decomposition of 966 as $7 \times 100 + 7 \times 30 + 7 \times 8$. *By the distributive property, this is* $7 \times (100 + 30 + 8)$, *so the unknown side length is 138. In the recording on the right, amounts of hundreds, tens, and ones are represented by numbers rather than by digits, e.g., 700 instead of 7.*

Example:
Using and Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

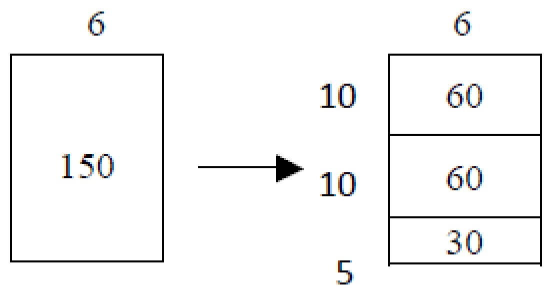
1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used

120 of the 150 so they have 30 left.

3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways. Here is one example:

- 150
-60 (6x10)
90
-60 (6x10)
30
-30 (6x5)
0

Example 150 6



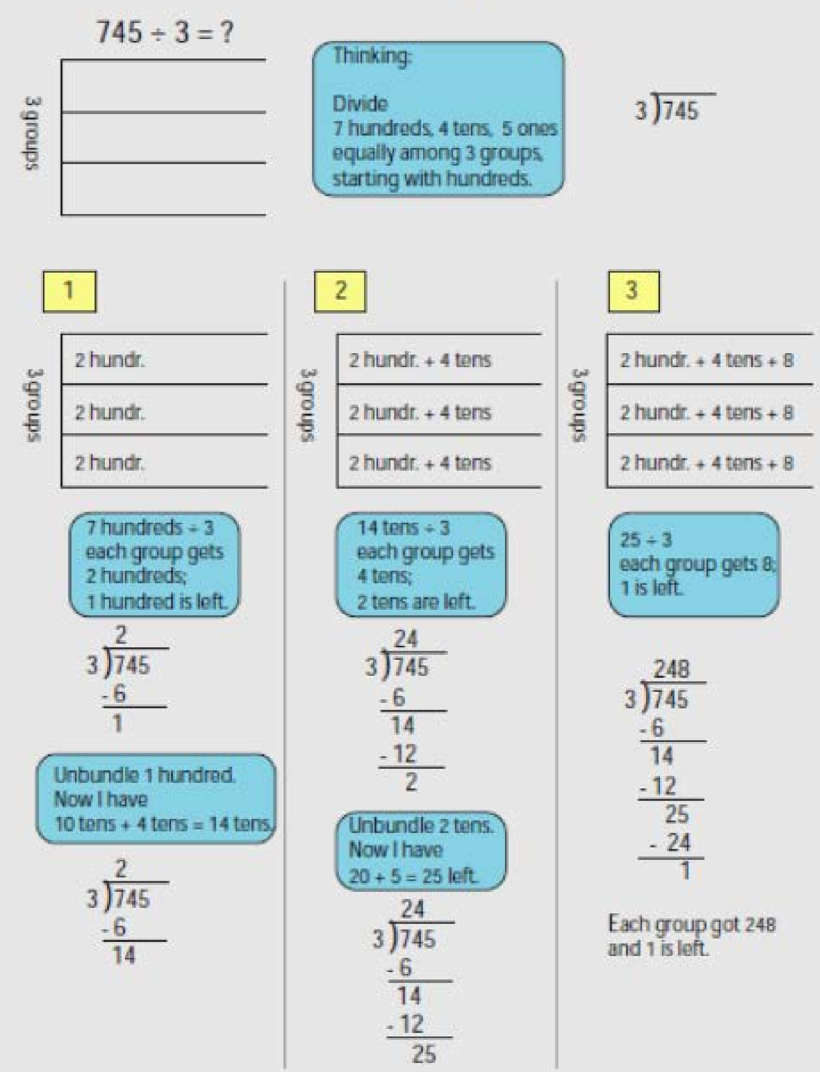
4.4(F) Use strategies and algorithms, including the standard algorithm, to divide up to 4 digit dividend and a 1 digit divisor

This standard calls for students to explore division through various strategies.

Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

Student 1	Student 2			Student 3
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	592 -400 192 -160 32 -32 0	50 20 4	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

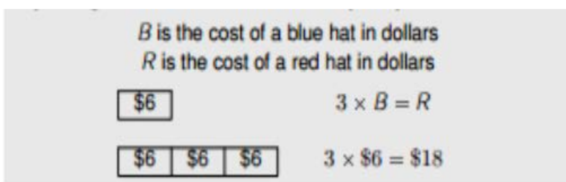
Division as finding group size



$745 \div 3$ can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In Step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated, and the 1 is the number of hundreds not allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.

4.4(G) Round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers	This standard refers to place understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundred chart as tools to support their work with rounding. Example: Round 368 to the nearest hundred. This will either be 300 or 400, since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400  300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500. When I added 300, 200 and 30, I know my answer will be about 530. The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer. is 100. Then I need 2 more hundreds. So we still need 250 bottles. 60 = 240, so we need about 240 more bottles.
4.4(H) Solve with fluency 1 and 2 step problems involving multiplication and division including interpreting remainders	This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders: • Remain as a left over • Partitioned into fractions or decimals • Discarded leaving only the whole number answer • Increase the whole number answer up one • Round to the nearest whole number for an approximate result

	Example: Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as: Problem A: 7 Problem B: 7 r 2 Problem C: 8 Problem D: 7 or 8 Problem E: $7\frac{2}{6}$ possible solutions: Problem **A: 7.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.* Problem **B: 7 r 2.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.* Problem **C: 8.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.* Problem **D: 7 or 8.** Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.* Problem **E:** $7\frac{2}{6}$. Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $7\frac{2}{6}$ Example: There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).* Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over.

Algebraic Reasoning **TEK 4.5**	**The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:**
4.5(A) Represent multi-step problems involving the 4 operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity	 In a multiplicative comparison, the underlying question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one*

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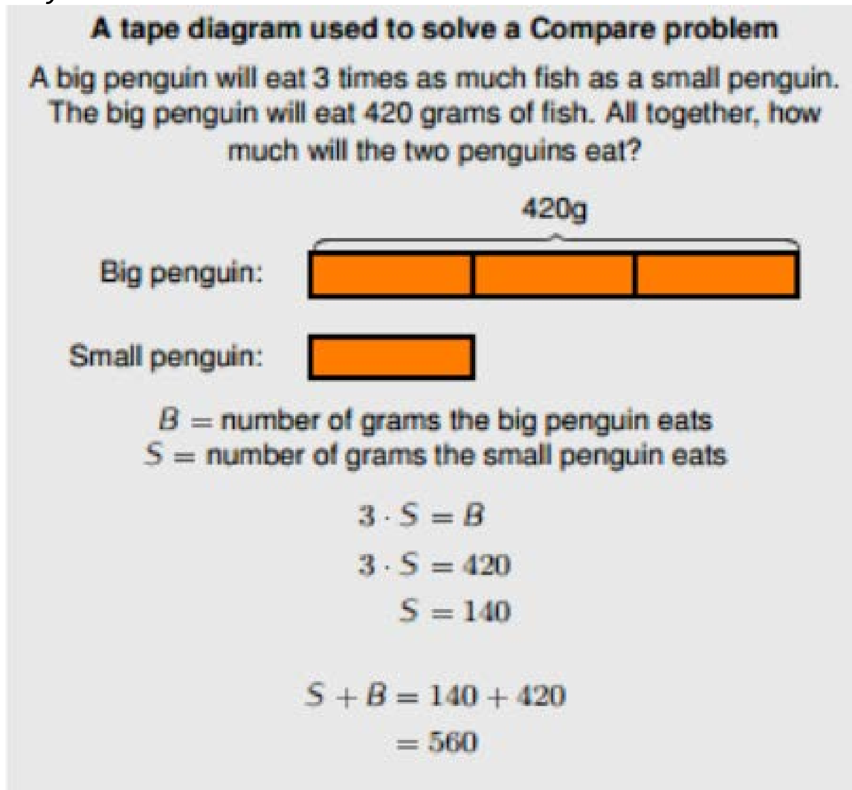
quantity in order to result in the other.



Examples:
Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p).
Group Size Unknown: A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? (18 ÷ p = 3 or 3 x p = 18).
Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18).

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<tr>
<td>4.5(B) Represent problems using an input-output table in</td>
<td>This standard begins with a small focus on reasoning about a number or shape pattern, connecting a rule for a given pattern with its sequence of numbers or shapes. Patterns that consist of repeated sequences of shapes or</td>
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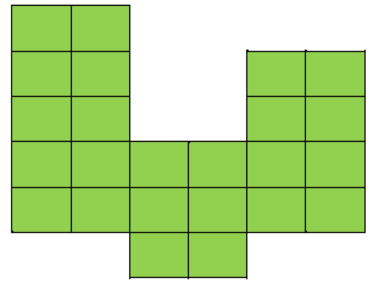
numerical expressions to generate a number pattern that follows a given rule to generate a table to represent the relationship of the values in the resulting sequence in their position in the sequence	growing sequences of designs can be appropriate for the grade. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and they could reason about how the dots are organized in the design to determine the total number of dots in the 100th design. In examining numerical sequences, fourth graders can explore rules of repeatedly adding the same whole number or repeatedly multiplying by the same whole number. Properties of repeating patterns of shapes can be explored with division. For example, to determine the 100th shape in a patter that consists of repetitions of the sequence "square, circle, triangle," the fact that when we divide 100 by 3 the whole number quotient is 33 with remainder 1 tells us that after 33 full repeats, the 99th shape will be a triangle (the last shape in the repeating pattern), so the 100th shape is the first shape in the pattern, which is a square. Notice that the Standards do not require students to infer or guess the underlying rule for a pattern, but rather ask them to generate a pattern from a given rule and identify features of the given pattern. This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns. Example: There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days? Day | Operation | Beans 0 | 3 x 0 + 4 | 4 1 | 3 x 1 + 4 | 7 2 | 3 x 2 + 4 | 10 3 | 3 x 3 + 4 | 13 4 | 3 x 4 + 4 | 16 5 | 3 x 5 + 4 | 19 **Example:** Use the rule "add 3" to write a sequence of numbers. Starting with a 0, students write 0, 3, 6, 9, 12, . . . Use the rule "add 6" to write a sequence of numbers. Starting with 0, students write 0, 6, 12, 18, 24, . . .
4.5(C) Use models to determine the formulas for a perimeter of a rectangle (l+w+l+w or 2L+2w), including the special form for perimeter of a square (4s) and a area of a rectangle (l x w)	Students develop an understanding of the concept of perimeter through various experiences, such as walking around the perimeter of a room, using rubber bands to represent the perimeter of a plan figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. Students should also strategically use tools, such as geoboards, tiles, and graph paper to find all of the possible rectangles that have a given perimeter (eg., find the rectangles with a perimeter of 14cm). They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Following this experience, students can reason about connections

	between their representations, side lengths, and the perimeter of the rectangles. • The formula is a generalization of the understanding, that, given a unit of length, a rectangle whose sides have length *w* units and *l* units, can be partitioned into *w* rows of unit squares with *l* squares in each row. The product *l* x *w* gives the number of unit squares in the partition, thus the area measurement is *l* x *w* square units. These square units are derived from the length unit. • For example, $P = 2l + 2w$ has two multiplications and one addition, but $P = 2(l + w)$, which has one addition and one multiplication, involves fewer calculations. The latter formula is also useful when generating all possible rectangles with a given perimeter. The length and width vary across all possible pairs whose sum is half of the perimeter (e.g., for a perimeter of 20, the length and width are all of the pairs of numbers with sum 10).
4.5(D) Solve problems related to perimeter and area of rectangles where dimensions are whole numbers.	Students learn to apply these understandings and formulas to the solution of real-world and mathematical problems. Example: A rectangular garden has an area of 80 square feet. It is 5 feet wide. How long is the garden? Here, specifying the area and the width creates an unknown factor problem. Similarly, students could solve perimeter problems that give the perimeter and the length of one side and ask the length of the adjacent side. Students should be challenged to solve multistep problems. Example: A plan for a house includes rectangular room with an area of 60 square meters and a perimeter of 32 meters. What are the length and the width of the room? In fourth grade and beyond, the mental visual images for perimeter and area from third grade can support students in problem solving with these concepts. When engaging in the mathematical practice of reasoning abstractly and quantitatively in work with area and perimeter, students think of the situation and perhaps make a drawing. Then they recreate the "formula" with specific numbers and one unknown number as a situation equation for this particular numerical situation. "Apply the formula" does NOT mean write down a memorize formula and put in known values because in fourth grade students do not evaluate expressions (they begin this type of work in Grade 6). In fourth grade, working with perimeter and area of rectangles is still grounded in specific visualizations and numbers. These numbers can now be any of the numbers used in fourth grade (for addition and subtraction for perimeter and for multiplication and division for area). By repeatedly reasoning about constructing situation equations for perimeter and area involving specific numbers and an unknown number, students will build a foundation for applying area, perimeter, and other formulas by substituting specific values of the variables in later grades.

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?

1-foot square
of carpet



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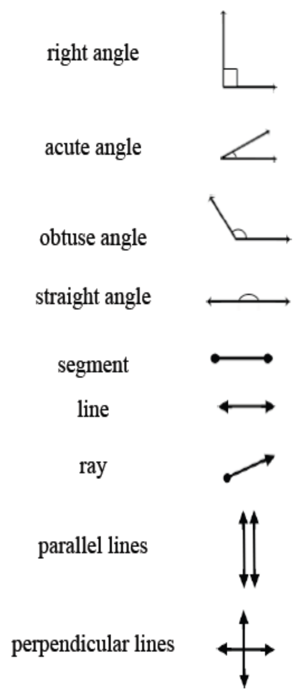
<tr><th>Geometry and Measurement
TEK 4.6</th><th>The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:</th></tr>
<tr><td>4.6(A) Identify points, lines, line segments, rays, angles, and perpendicular and parallel lines</td><td>

This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students may not easily identify lines and rays because they are more abstract.



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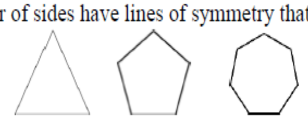
<tr><td>4.6(B) Identify and draw one or more lines of symmetry, if they exist, for a 2 dimensional figure</td><td>

Students need experiences with figures which are symmetrical and on-symmetrical. Figures include both regular and on-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

This standard only includes line symmetry not rotational symmetry.

Example:
For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.
Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.



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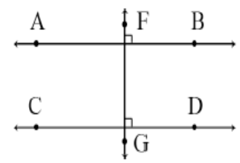
4.6(C) Apply knowledge of right angles to identify acute, right and obtuse triangles	Given a set of different triangles, students measure the angles (using a protractor or geometry exploration software) and classify triangles according to their properties. Students organize their data in a table and analyze patterns to match triangles with the most appropriate name. Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangles has no congruent sides.
4.6(D) Classify 2 dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of specified sides	Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular or by angle measurement. **Parallel or Perpendicular Lines:** Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles ( ). Example: Draw two different types of quadrilaterals that have two pairs of parallel sides? Is it possible to have an acute right triangle? Justify your reasoning using pictures and words. Example: How many acute, obtuse and right angles are in this shape?  Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

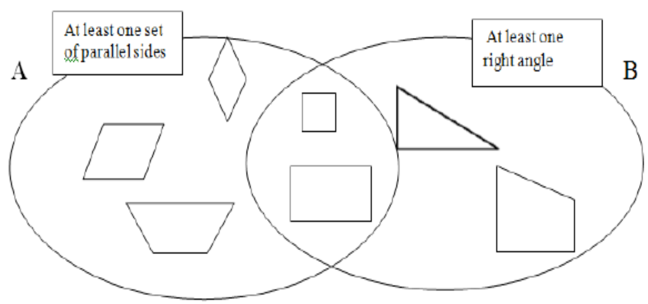
Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

A **kite** is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (*adjacent to)* each other.

Parallel and perpendicular lines are shown below:

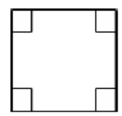


This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.
Example:
Which figure in the Venn diagram below is in the wrong place, explain how do you know?



Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

	Example: For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example. • A parallelogram with exactly one right angle. • An isosceles right triangle. • A rectangle that is *not* a parallelogram. *(impossible)* • Every square is a quadrilateral. • Every trapezoid is a parallelogram. Example: Identify which of these shapes have perpendicular or parallel sides and justify your selection.  A possible justification that students might give is: The square has perpendicular lines because the sides meet at a corner, forming right angles. 
Geometry and Measurement **TEK 4.7**	**The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:**
4.7(A) Illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers.	Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a rotation about a point makes a complete circle to recognize and sketch angles that measure approximately and . They extend this understanding and recognize and sketch angles that measure approximately and . They use appropriate terminology (acute, right and obtuse) to describe angles and rays (perpendicular). This standard brings up a connection between angles and circular measurement (360 degrees). Angle measure is a "turning point" in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An angle is the union of two rays, *a* and *b*, with the same initial point *P*. The rays can be made to coincide by rotating one to the other about *P*, this rotation determines the size of the angle between *a* and *b.* The rays are sometimes called the *sides* of the angles.

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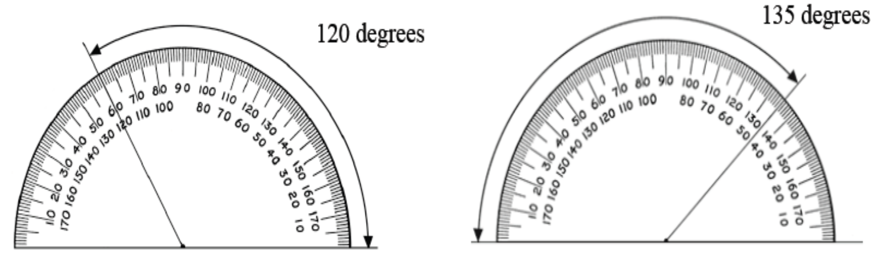
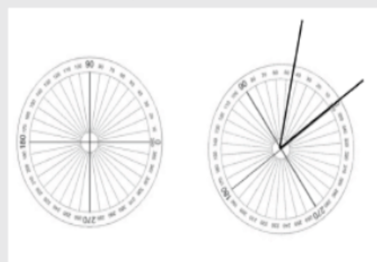
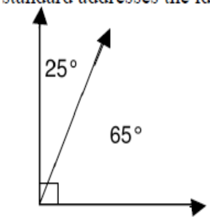
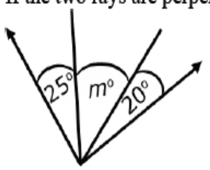
<td>4.7(B) Illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that “cuts” n/360 out of any circle whose center is at the angles vertex has a measure of n degrees. Angle measures are limited to whole numbers.</td>
<td>Another way of saying this is that each ray determines a direction and the angle size measures the change from one direction to the other. Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360 degrees.
Two angles are called complementary if their measurements have the sum of   . Two angles are called supplementary if their measurements have the sum of   . Two angles with the same vertex that overlap only at a boundary (ie share a side) are called adjacent angles. These terms may come up in classroom discussion. This concept is developed thoroughly in middle school.

An angle
<table>
<tr><th>name</th><th>measurement</th></tr>
<tr><td>right angle</td><td>90°</td></tr>
<tr><td>straight angle</td><td>180°</td></tr>
<tr><td>acute angle</td><td>between 0 and 90°</td></tr>
<tr><td>obtuse angle</td><td>between 90° and 180°</td></tr>
<tr><td>reflex angle</td><td>between 180° and 360°</td></tr>
</table>
Like length, area and volume, angle measure is additive. The sum of measurements of adjacent angles is the measurement of the angle formed by their union. This leads to other important properties. If a right angle is decomposed into two adjacent angles, the sum is   , thus they are complementary. Two adjacent angles that compose a “straight angle” of   must be supplementary.</td>
</tr>
</table>

4.7(C) Determine the approximate measures of angles in degrees to the nearest whole number using a protractor. 4.7(D) Draw an angle with a given measure	Students should measure angles and sketch angles  As with all measureable attributes, students must first recognize the attribute of angle measure, and distinguish it from other attributes. As with other concepts students need varied examples and explicit discussions to avoid learning limited ideas about measuring angles (e.g., misconceptions that a right angle is an angle that points to the right, or two right angles represented with different orientations are not equal in measure). If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all slanted lines with 45º measures and horizontal and vertical lines with measures of 90º. Others believe angles can be "read off" a protractor in "standard" position, that is, a base is horizontal, even if neither ray of the angle is horizontal. Measuring and then sketching many angles with no horizontal or vertical ray perhaps initially using circular 360º protractors can help students avoid such limited conceptions. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23) **A 360° protractor and its use**  The figure on the right shows a protractor being used to measure a 45° angle. The protractor is placed so that one side of the angle lies on the line corresponding to 0° on the protractor and the other side of the angle is located by a clockwise rotation from that line.

4.7(E) Determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.	This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.  Example: A lawn water sprinkler rotates 65 degress and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees? Example: If the two rays are perpendicular, what is the value of m?  Example: Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4? Students can develop more accurate and useful angle and angle measure concepts if presented with angles in a variety of situations. They learn to find the common features of superficially different situations such as turns in navigation, slopes, bends, corners, and openings. With guidance, they learn to represent an angle in any of these contexts as two rays, even when both rays are not explicitly represented in the context; for example, the horizontal or vertical in situations that involve slope (e.g., roads or ramps), or the angle determined by looking up from the horizon to a tree or mountain-top. Eventually they abstract the common attributes of the situations as angles (which are represented with rays and a vertex,) and angle measurements.
Data Analysis TEKS 4.8	**The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:**
4.8(A) Identify relative sizes of units within the customary and metric systems	Students develop benchmarks and mental images about meter and a kilometer and they also understand that "kilo" means a thousand, so 3000m is equivalent to 3km. Ex: … about the height of a tall chair, the length of 10 football fields, the distance a person might walk in about 12 min.

<table>
<tr>
<td>4.8(B) Convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table.</td>
<td>Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.
Example:
Customary length conversion table
<table>
<tr><th>Yards</th><th>Feet</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>n</td><td>n x 3</td></tr>
</table>
Foundational understandings to help with measure concepts:
Understand that larger units can be subdivided into equivalent units (partition).
Understand that the same unit can be repeated to determine the measure (iteration).
Understand the relationship between the size of a unit and the number of units needed (compensatory principal).</td>
</tr>
</table>

4.8(C) Solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication or division as appropriate

Students will use the four operations to solve word problems involving distance, volume, mass and money.

Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?
possible solution: Charlie plus 10 friends = 11 total people
11 people x 8 ounces (glass of milk) = 88 total ounces
1 quart = 2 pints = 4 cups = 32 ounces

Therefore 1 quart = 2 pints = 4 cups = 32 ounces
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces

If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional Examples with various operations:
Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?
Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)

Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

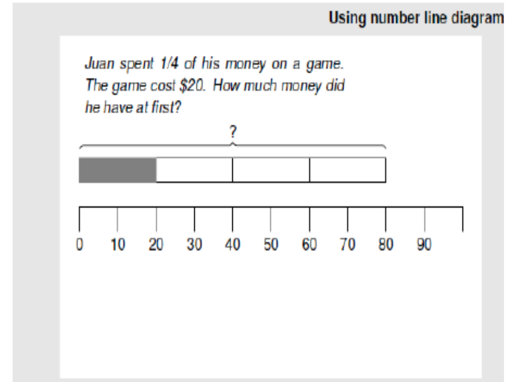
Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought

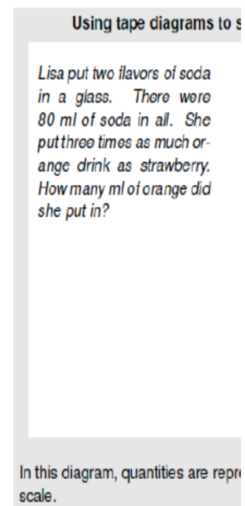
2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

Example:



Students also combine competencies from different domain
arithmetic operations, addition, subtraction, multiplication,
Example: “How many liters of juice does the class need to l
Students may use tape or number line diagrams for solving
Example:



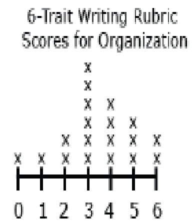
<table>
<tr>
<th>Data Analysis
TEK 4.9</th>
<th>The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:</th>
</tr>
<tr>
<td>4.9(A) Represent data on a frequency table, dot plot, or stem and leaf plot marked with whole numbers and fractions.</td>
<td>

Students need to be able to identify different types of graphs. Students need to be able to collect data to put into charts and graphs. Students need to be able to create each of the following types of graphs:

- Frequency Table
- Dot plots/Line graphs
- Stem & Lear Plot

Dot plots are simple plots on a number line where each dot (X) represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

Example 1:
Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?
Solution:



A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram.

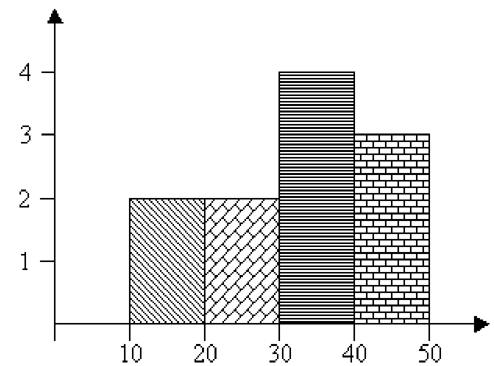
</td>
</tr>
</table>

Stem-and-leaf plots are a method for showing the frequency with which certain classes of values occur. You could make a frequency distribution table or a histogram for the values, or you can use a stem-and-leaf plot and let the numbers themselves to show pretty much the same information.

For instance, suppose you have the following list of values: 12, 13, 21, 27, 33, 34, 35, 37, 40, 40, 41. You could make a frequency distribution table showing how many tens, twenties, thirties, and forties you have:

Frequency Class	Frequency
10 - 19	2
20 - 29	2
30 - 39	4
40 - 49	3

You could make a histogram, which is a bar-graph showing the number of occurrences, with the classes being numbers in the tens, twenties, thirties, and forties:



On the other hand, you could make a stem-and-leaf plot for the same data:

<table>
<tr><td></td><td>
<table>
<tr><th>stem</th><th>leaf</th></tr>
<tr><td>1</td><td>2 3</td></tr>
<tr><td>2</td><td>1 7</td></tr>
<tr><td>3</td><td>3 4 5 7</td></tr>
<tr><td>4</td><td>0 0 1</td></tr>
</table>
The “stem” is the left-handed column which contains the tens digit. The “leaves’ are the lists in the right-handed column, showing all the ones digits for each of the tens, twenties, thirties, and forties. As you can see the original values can still be determined; you can tell where 40, 40, and 41 are in the stem-and-leaf plot.</td></tr>
<tr><td>4.9(B) Solve one and two step problems using data and whole number, decimal, and fraction form in a frequency table, dot plot, stem & leaf plot</td><td>Students need to be able to read graphs and analyze the data. Students need to be able to make predictions from analyzing graphs.</td></tr>
<tr><td>Personal Financial Literacy TEK 4.10</td><td>The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:</td></tr>
<tr><td>4.10(A) Distinguish between fixed and variable expenses</td><td>Fixed costs are those that do not fluctuate with changes in production such as rent, insurance, salary.

Variable costs are those that respond directly to the actively level of a company such as hourly wages, utilities, mailing and shipping, office supplies.

It is important to understand why variable costs can fluctuate but fixed costs may not.

Example Lessons: http://smartertexas.org/?page_id=914</td></tr>
<tr><td>4.10(B) Calculate profit in a given situation</td><td>Profit is equal to the money made minus the costs incurred. If a student wants to calculate profit, then they must subtract the cost from money made.

Example:
If I sell something for $75 and my cost was $40, I make $35 profit.
http://smartertexas.org/?page_id=914</td></tr>
</table>

4.10(C) Compare the advantages and disadvantages of various savings options	Students will compare criteria used to compare savings account options. Sample Lessons: http://www.treasurydirect.gov/indiv/tools/tools_moneymath.pdf http://smartertexas.org/?page_id=914
4.10(D) Describe how to allocate a weekly allowance among spending, saving, including for college, and sharing	Allowances teach students how to earn, save, invest, budget and make wise purchasing decisions. These are all skills needed to live a sound financial life as an adult. Students need to see what happens if they save their money over time. Students need to also practice spending and understand poor purchases (due to costs, etc). Sample Lessons: http://smartertexas.org/?page_id=914
4.10(E) Describe the basic purpose of financial institutions. Including keeping money safe, borrowing money, and lending	People put their money in a bank for safekeeping. Bank pay interest on the money people put in the bank for extended periods of time. Banks lend the money to borrowers and investors. Banks charge interest on the money they lend. Sample Lessons: http://smartertexas.org/?page_id=914