Advances in PID Control

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Thanks for Stimulating Collaboration!

- Interactive learning modules
- PID control
- Feedforward control
- Tuning and adaptation
- Event based control

Tore and I are very grateful for the excellent interactions.

Reseach Goals

Primary goals:

- New edition of Advanced PID Control
- Develop basis for a new generation of auto-tuners
- Understand trade-offs between performance (load disturbance attenuation, measurement noise injection) and robustness
- Understand when FOTD models are sufficient and when better modeling is required - Can design be based on FOTD models

Secondary goals:

- Understand current tuning rules: Lambda, SIMC, AMIGO
- Design rules for noise filtering
- Suitable optimization methods

Some Recent Papers

- Hast, Åström, Bernhardsson, Boyd PID Design by Convex Concave Optimization. ECC 2013
- Garpinger, Åström, Hägglund Performance and robustness trade-offs in PID Control. Journal of Process Control, 24:5(2014) 568-577.
- Romero Segovia, Hägglund Åström Measurement noise filtering for PID controllers JPC 24(2014) 299-313
- Romero Segovia, Hägglund Åström Measurement noise filtering for common PID tuning rules CEP 32(2014) 43-63
- Berner, Åström , Hägglund Towards a New Generation of Relay Autotuners IFAC World Gongress 2014
- Garpinger, Hägglund Modeling for Optimal PID Design. IFAC World Congress 2014
- Boyd, Hast, Åström. MIMO PID Tuning via Iterated LMI Restriction. Submitted Automatica 2014

Outline

Design Criteria

- A trade-off between conflicting requirements
  - Load disturbance attenuation
  - Robustness to process uncertainty
  - Measurement noise
  - Setpoint response
- Set-point response can be treated separately (2 DOF setpoint weighting)

Performance:

$$IE = \int_0^\infty e(t)dt = 1/k_i, \qquad IAE = \int_0^\infty |e(t)|dt$$

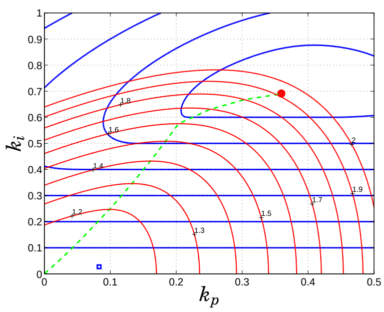
Robustness:

$$M_s = \max_\omega \left|\frac{1}{1+P(i\omega)C(i\omega)}\right|, \qquad M_t = \max_\omega \left|\frac{P(i\omega)C(i\omega)}{1+P(i\omega)C(i\omega)}\right|$$

Measurement noise: Noise gain, SDU, filtering

Level Curves for Performance and Robustness

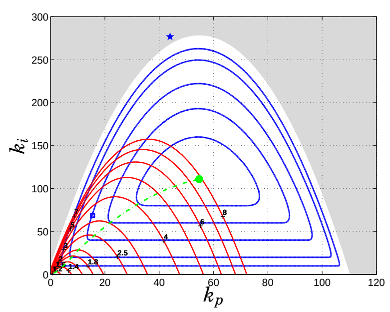
- Performance (IAE= $1/k_i$ blue) and robustness (M_s, M_t red)
- IE level curves are horizontal lines



Approximately: k_i gives performance and k_p sets robustness

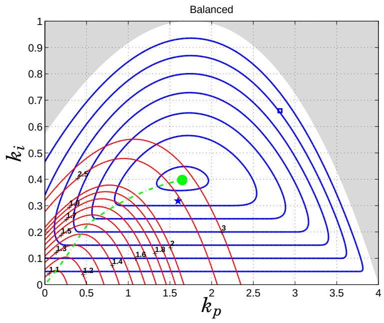
PI Control – Lag-Dominated Dynamics

- $P(s) = 1/((s+1)(0.1s+1)(0.01s+1)(0.001s+1))$, $\tau = 0.067$
- Unconstrained optimal controller poor robustness $M_s = 8$!
- ZN step □ and ZN frequency ⋆



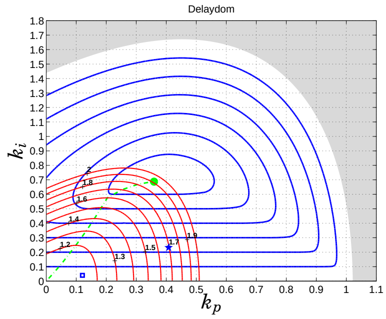
PI Control – Balanced Dynamics

- $P(s) = 1/(s+1)^4$, $\tau = 0.33$
- Unconstrained optimal controller has robustness $M_s = 2.8$
- ZN step □ and ZN frequency ⋆

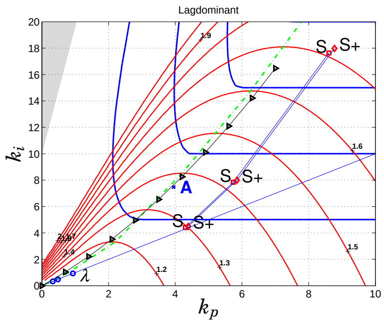


PI Control – Delay-Dominated Dynamics

- $P_1(s) = e^{-s}/(1+0.05s)^2$, $\tau = 0.92$
- Unconstrained optimal controller has robustness $M_s = 2!$
- IE and IAE minimization equivalent for small M_s, M_t

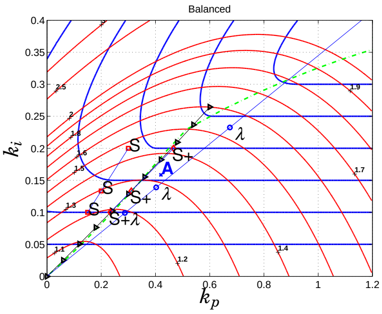


Tuning – Lag-Dominated Dynamics



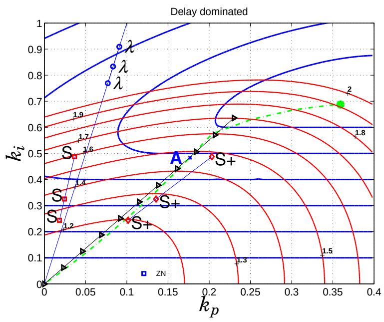
- Lambda tuning has very low gains
- S and S+ give similar tuning
- Lambda tuning gives constant integral time $T_i = k_p/k_i$

Tuning – Balanced Dynamics



- Tuning methods S+, A and λ gives similar results
- All controllers have constant integral time $T_i = k_p/k_i$

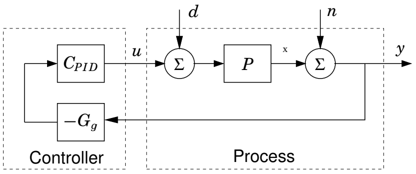
Tuning Delay-Dominated Dynamics



- Lambda tuning too high integral gain
- Obvious why Skogestad modified his method
- All controllers have constant integral time $T_i = k_p/k_i$

Outline

Measurement Noise



Controller transfer function

$$G_f = \frac{1}{1 + sT_f + s^2T_f^2/2} \qquad C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s, \qquad C = C_{PID}G_f$$

Transfer function from noise to control signal

$$-G_{un}(s) = \frac{C}{1+PC} = SC$$

For controllers with integral action we have $G_{un}(0) = 1/K$ where $P(0) = K$.

Approximation of G_{un}

We have $S \approx \dfrac{s}{s + Kk_i}$ for s small, and $S \approx 1$ large s

$$-G_{un}(s) = \frac{C}{1+PC} = SC \approx \frac{k_i + k_p s + k_d s^2}{(s + Kk_i)(1 + sT_f + (sT_f)^2/2)}$$

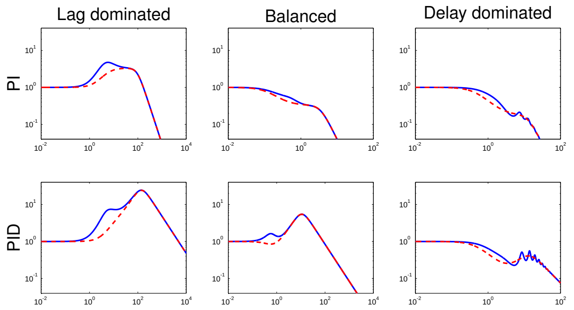
For low frequencies (small s) the numerator of G_{un} is dominated by the integral gain k_i and we have

$$C_{PID}(s) \approx \frac{k_i}{s}, \quad G_f(s) \approx 1, \quad S(s) \approx \frac{s}{s + Kk_i}.$$

Hence

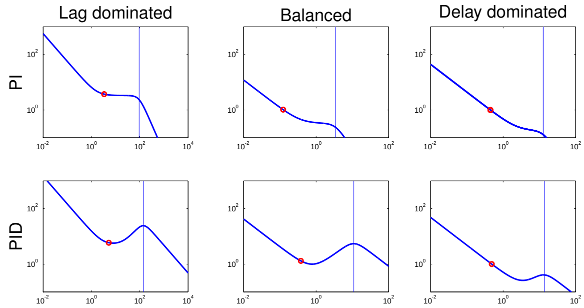
$$G_{un}(s) \approx G_{un}^{PID} = -\frac{k_i + k_p s + k_d s^2}{(s + Kk_i)(1 + sT_f + (sT_f)^2/2)}$$

Bode Plots of Noise Transfer Function G_{un}



- Validity of approximation (error in mid frequency range M_s peak)
- Differences PI/PID lag dominated/delay dominated

Bode Plots Controller Transfer Function C



- Gain crossover frequency
- Frequency $\omega_f = \sqrt{2}/T_f$

Stochastic Modeling

Measurement noise stationary with spectral density $\Phi(\omega)$

$$\sigma_u^2 = \int_{-\infty}^{\infty} |G_{un}(i\omega)|^2 \Phi(\omega) d\omega$$

$$\sigma_{y_f}^2 = \int_{-\infty}^{\infty} |G_f(i\omega)|^2 \Phi(\omega) d\omega$$

$$G_{un}(s) \approx -\frac{k_i + k_p s + k_d s^2}{(s + Kk_i)(1 + sT_f + (sT_f)^2/2)}$$

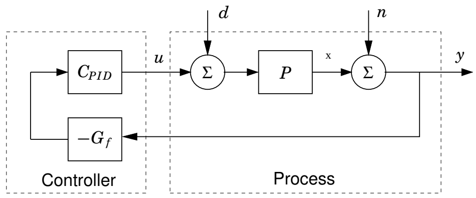
White noise

$$\sigma_u^2 \approx \pi \left(\frac{k_i}{K} + \frac{k_p^2 - 2k_i k_d}{T_f} + 2\frac{k_d^2}{T_f^3} \right) \Phi_0, \qquad \sigma_{y_f}^2 = \frac{\pi}{T_f} \Phi_0$$

Noise gain

$$k_{nw} = \frac{\sigma_u}{\sigma_{y_f}} \approx \sqrt{\frac{k_i T_f}{K} + k_p^2 - 2k_i k_d + 2\frac{k_d^2}{T_f^2}}.$$

Finding a Suitable Filter Time Constant



$$G_f = \frac{1}{1 + sT_f + s^2T_f^2/2} \qquad C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s, \quad C = C_{PID} G_f$$

- Develop sound design procedure for PI and PID control of a given process
- Apply procedure to a representative test batch
- Analyse results to find insights and understanding
- Explore and try to find simple design rules

Finding a Suitable Filter Time Constant

An iterative design procedure

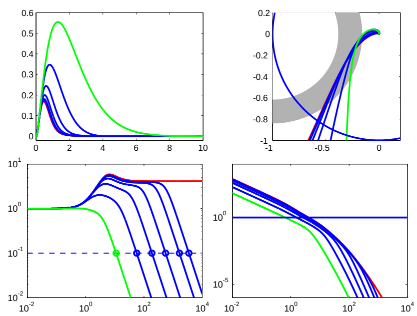
1. Design controler for nominal process P_0 e.g. by minimizing IAE subject to robustness constraints, $G_f = 1$.
2. Compute ω_{gc} for PG_f
3. Choose $T_f = \alpha/\omega_{gc}$, $\alpha = 0.01, 0.02, 0.05, 0.01, 0.15, 0.2$
4. Repeat from 2 with until convergence
5. Make trade-off plots (load disturbance attenuation-noise injection)

Can be applied to any design procedure, particularly simple for design methods based on the FOTD model.

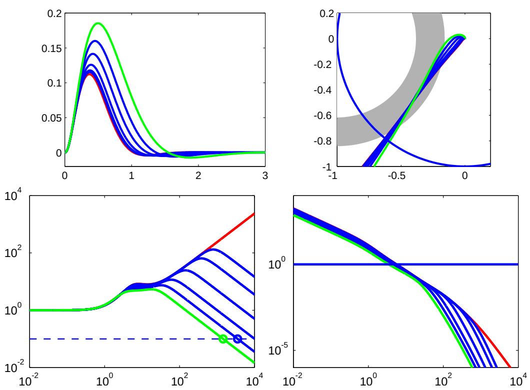
PI Control Lag-dominant Dynamics

$$P_1(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)}$$

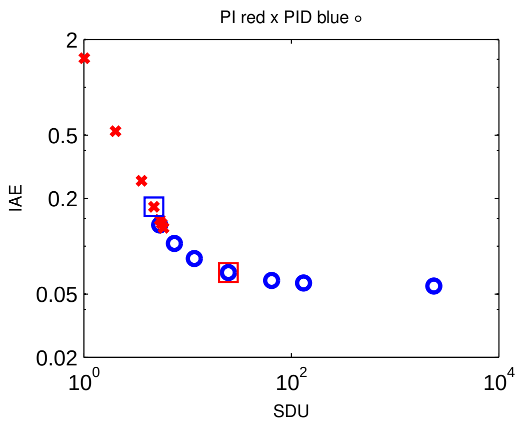
FOTD parameters: $K = 1$, $T = 1.04$, $L = 0.08$, and $\tau = 0.07$



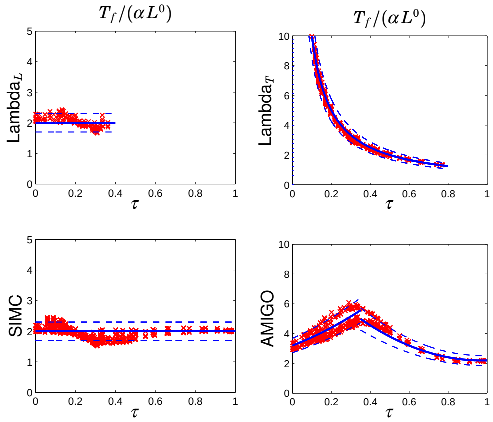
PID Control Lag-dominant Dynamics



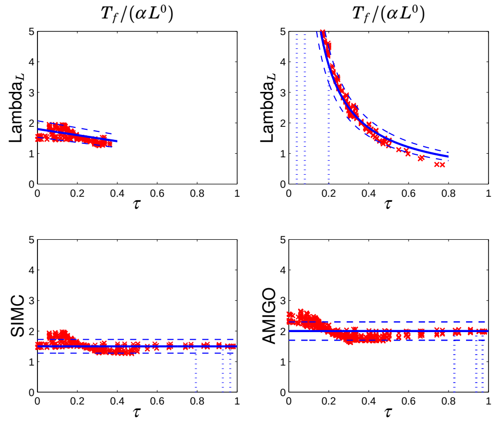
Trade-offs



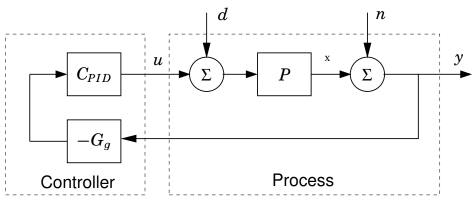
PI Control - FOTD Correlations



PID Control - FOTD Correlations



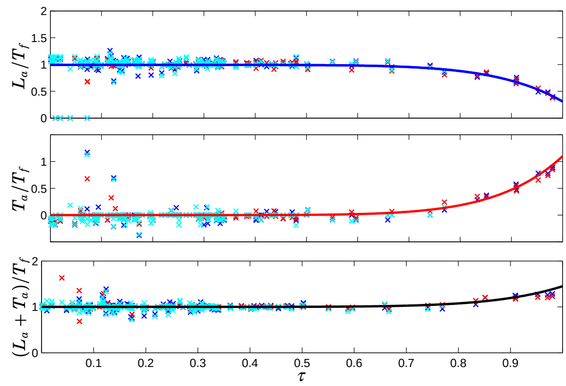
Effect of T_f on FOTD Parameters



- With filtering the effective process dynamics changes from P to PG_f
- How to determine the FOTD parameters?
- The step response method

$$L = L_0 + (1 - 0.65\,\tau^8)T_f, \qquad T = T_0 + 1.1\,\tau^8 T_f.$$

Effect of T on FOTD Parameters



Outline

1. Introduction
2. Performance and Robustness
3. Performance and Measurement Noise
4. Optimization
5. Next Generation Auto-tuners
6. Summary

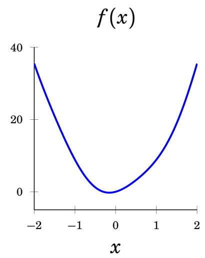
Convex Optimization

The basic convex optimization:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le 0, \quad i = 1, \dots, m \\ & h_j(x) = 0, \quad j = 1, \dots, n, \end{aligned}$$

$f_i(x)$, convex functions, $h(x)$ affine functions of x.

- If a local minimum exists it is a global minimum
- Efficient and fast numerical algorithms
- Good software tools CVX



Convex-Concave Procedure

Replace concave part by linearization around current solution point x_k

$$f(x) - g(x) \approx f(x) - g(x_k) - \nabla g(x_k)^T(x - x_k)$$

The approximated problem

$$\begin{aligned} \text{minimize} \quad & f_0(x) - g_0(x_k) - \nabla g_0(x_k)^T(x - x_k) \\ \text{subject to} \quad & f_i(x) - g_i(x_k) - \nabla g_i(x_k)^T(x - x_k) \le 0, \quad i = 1, \dots, m \end{aligned}$$

is convex and solved to generate a new solution point x_{k+1}. Iterate until convergence.

- Composition always possible if Hessian of $f(x) - g(x)$ is bounded
- Converges to a local minimum or saddle-point.
- Sacrifices global optimality but gains convexity and hence speed.
- Feasible starting point is needed.

Convex-Concave Optimization for PID Control

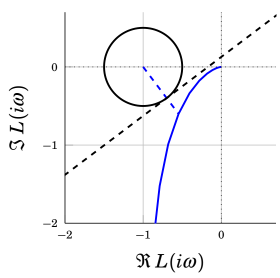
Loop transfer function

$$G_l = PG_f\left(k_p + \frac{k_i}{s} + k_d s\right)$$

is linear in the parameters k_p, k_i, k_d. The robustness constraint that $G_l(i\omega)$ is outside the circle

$$r - |G_l - c| \le 0$$

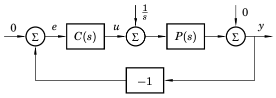
does not give a convex problem. Convex-concave optimization can be applied since G_l is linear in the parameters. For each frequency the constraint to be outside the circle is replaced by being outside a half plane (the dashed line)



Heat Rod

$P(s) = e^{-\sqrt{s}}$

$C(s) = k_p + \frac{k_i}{s} + k_d s$



Optimization problem

maximize k_i

subject to $|S(i\omega)| \le 1.4$

$|T(i\omega)| \le 1.4$

Convex approximation

max. k_i

s.t. $1/1.4 - \Re\left(\frac{(L_k+1)^*}{|L_k+1|}(L+1)\right) \le 0$

$r_T - \Re\left(\frac{(L_k-c_T)^*}{|L_k-c_T|}(L-c_T)\right) \le 0$

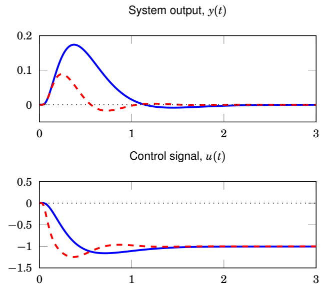
Nyquist Plot and Load Step Response

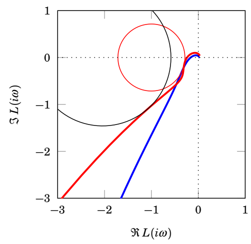
$C_{\mathrm{PI}}(s) = 2.94 + \frac{11.54}{s}$

IE = 0.086, IAE = 0.10

$C_{\mathrm{PID}}(s) = 7.40 + \frac{48.25}{s} + 0.46s$

IE = 0.021, IAE = 0.031



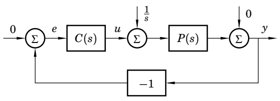


IE or IAE

Intuitively it may seem like optimization of IE or IAE will give the same result provided the system is well damped,

$P(s) = \frac{1}{(s+1)^3}$

$C(s) = k_p + \frac{k_i}{s} + k_d s$



Optimization problem

maximize k_i

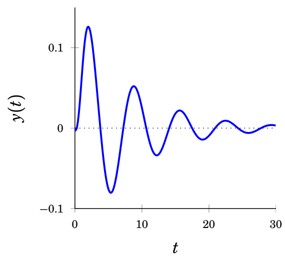
subject to $|S(i\omega)| \le 1.4$

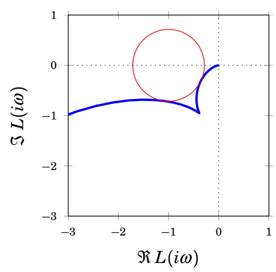
Convex approximation

max. k_i

s.t. $1/1.4 - \Re\left(\frac{(L_k+1)^*}{|L_k+1|}(L+1)\right) \le 0$

Nyquist Plot and Step Responses





The oscillatory behavior related to cusp in Nyquist curve

Adding a Curvature Constraint

$P(s) = \frac{1}{(s+1)^3}$

$C(s) = k_p + \frac{k_i}{s} + k_d s$

Optimization problem

maximize k_i

subject to $|S(i\omega)| \le 1.4$

$\kappa \le 1/1.4$

Convex approximation

max. k_i

s.t. $1/1.4 - \Re\left(\frac{(L_k+1)^*}{|L_k+1|}(L+1)\right) \le 0$

$x^T Q x + A_k x + b_k \le 0$

- A grid of 1000 frequencies between 10^{-2} and 10^2 rad/s.
- Solved using CVX in MATLAB.
- Converges within twelve iterations (4 s).

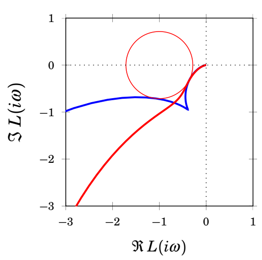
Nyquist Plot and Load Step Responses

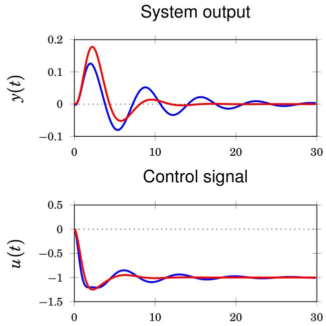
$C(s) = 3.31 + \frac{6.62}{s} + 6.26s$

IE = 0.15, IAE = 0.74

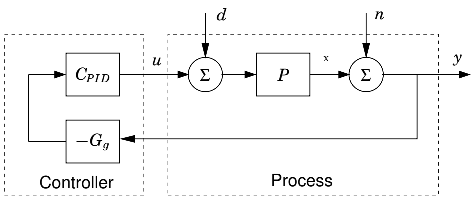
$C(s) = 3.61 + \frac{3.20}{s} + 3.34s$

IE = 0.31, IAE = 0.57





Multivariable PID Controllers



Controller transfer function

$G_f = \frac{1}{1 + sT_f + s^2T_f^2/2} \qquad C_{PID}(s) = K_p + K_i\frac{1}{s} + K_d s, \qquad C = C_{PID}G_f$

Optimization: Minimize $\|(P(0)K_{\mathrm{I}})^{-1}\|$ subject to

$\|S\|_\infty \le S_{\max}, \quad \|T\|_\infty \le T_{\max}, \quad \|Q = CS\|_\infty \le Q_{\max}$

The Wood-Berry Distillation Column

Process model

$$P(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.2+1} \end{bmatrix}$$

Optimization

$S_{\max} = 1.4, \quad T_{\max} = 1.4, \quad Q_{\max} = 3/\sigma_{\min}(P(0)) = 0.738.$

- Derivative action time constant: $\tau = 0.3$
- Sampled with $N = 300$ logarithmically spaced frequency samples in the interval $[10^{-3}, 10^3]$
- Initialization: $K_{\mathrm{P}} = 0, \quad K_{\mathrm{I}} = \epsilon P(0)^\dagger, \quad K_{\mathrm{D}} = 0, \quad \epsilon = 0.01.$

Wood and his Column

General and Diagonal PID Controllers

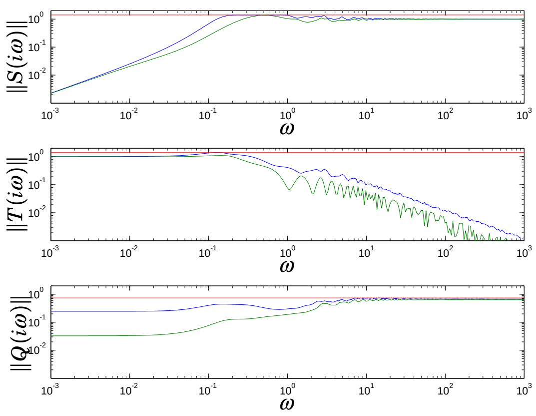
Optimal PID controller (converged in 7 iterations)
$\|(P(0)K_\mathrm{I})^{-1}\| = 2.25$.

$$K_\mathrm{P} = \begin{bmatrix} 0.1750 & -0.0470 \\ -0.0751 & -0.0709 \end{bmatrix}, \quad K_\mathrm{I} = \begin{bmatrix} 0.0913 & -0.0345 \\ 0.0402 & -0.0328 \end{bmatrix},$$
$$K_\mathrm{D} = \begin{bmatrix} 0.1601 & -0.0051 \\ 0.0201 & -0.1768 \end{bmatrix},$$

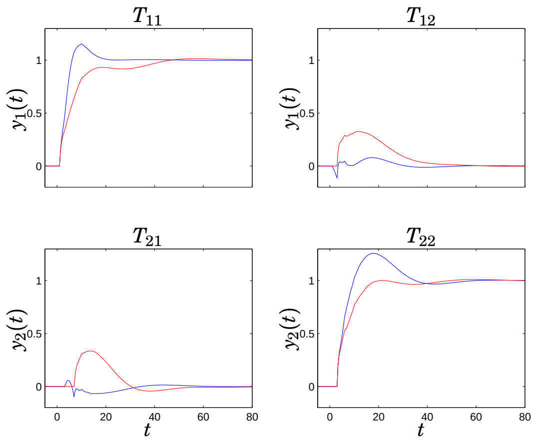
Diagonal PID controller (converged in 8 iterations)
$\|(P(0)K_\mathrm{I})^{-1}\| = 13.36$,

$$K_\mathrm{P} = \begin{bmatrix} 0.1535 & 0 \\ 0 & -0.0692 \end{bmatrix}, \quad K_\mathrm{I} = \begin{bmatrix} 0.0210 & 0 \\ 0 & -0.0136 \end{bmatrix},$$
$$K_\mathrm{D} = \begin{bmatrix} 0.1714 & 0 \\ 0 & -0.1725 \end{bmatrix},$$

General PID Controller



Step Responses



Extensions

- Exchanging objectives and constraints
- Frequency dependen bounds
- Other closed loop transfer functions
- Low frequency disturbance attenuations
  $S(s)P(s) \approx s(P(0)K_\mathrm{I})^{-1}P(0)$
- High frequency roll-off
- Unstable plants
- Robustness to plant variations
- More general controllers

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Reflections

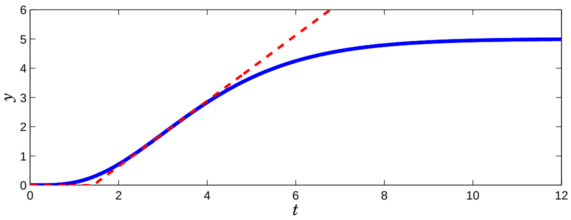
Putting it all together

- Relay feedback
- Good excitation - the secret to good modeling
  - The classic, integrators, filters, asymmetry, adaptive hysteresis
- Short experimentation time
  - Avoid waiting for steady state!
  - How short can it be?
- Design of identification experiment
  - Input signal and excitation essential!!
  - The beauty of the relay-autotuner
  - How to design the second phase? Chirp + pulse??
- How to assess a model?
  - Behavior in closed loop the primary goal!
  - Fitting error, cross-validation, AIC, Vinnicombe
- Computational issues?
  - Matlab, Python, FMI
- Implementation: Coding, box, DCS, web, cloud

Key Issues

- A long term plan - include what we have learned
- Auto-tuners for building simulation
- Auto-tuners for controllers
  - Boxes, PLCs, DCS systems
  - Simple version: PI control
  - Complex version: Selection of PI or PID and better modeling
- Criteria
  - Short experiments
  - Good robust tuning rules with design parameter
- Implementation issues
  - Stand alone box: Matlab, Python, FMI
  - SoftwaremWeb, cloud

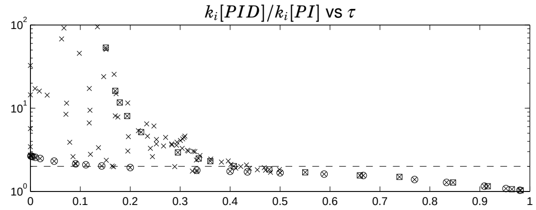
Modeling Issues



- True time delay is a fundamental limitation
- FOTD lumps true time delay and high order dynamics
- Does not matter for $\tau > 0.4$
- Better models than FOTD are required for PID control and $\tau < 0.4$

Modeling for PI & PID Control

AMIGO Tuning - complete testbatch



circles: $P(s) = \dfrac{K}{1+sT}e^{-sL}$, squares: $P(s) = \dfrac{K}{(1+sT)^2}e^{-sL}$

- FOTD OK for $\tau > 0.4$ better model required for smaller τ!
- Derivative action small improvement for $\tau > 0.8$

Models

Two parameter models

$$P(s) = \frac{b}{s+a}, \quad P(s) = K\,e^{-sL}$$

Three parameter models

$$P(s) = \frac{b}{s^2+a_1s+a_2}, \quad P(s) = \frac{b}{s+a}e^{-sL}, \quad P(s) = \frac{K}{1+sT}e^{-sL}$$

$$P(s) = \frac{K}{(1+sT)^2}e^{-sL}$$

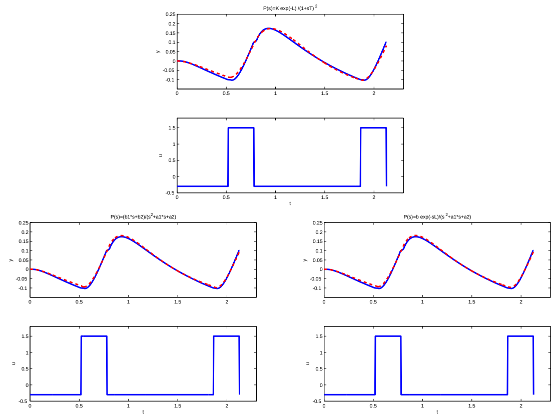
Four parameter models

$$P(s) = \frac{b_1s+b_2}{s^2+a_1s+a_2}, \quad P(s) = \frac{b}{s^2+a_1s+a_2}e^{-sL}$$

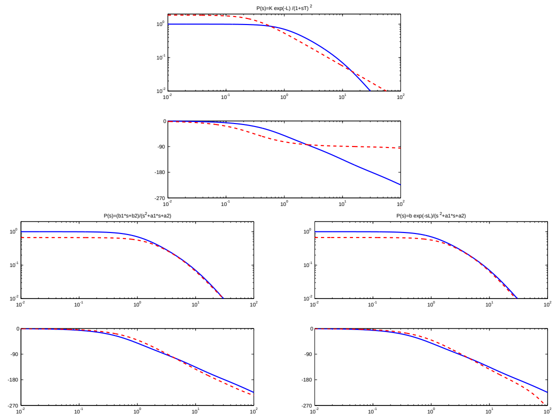
Five parameter model

$$P(s) = \frac{b_1s+b_2}{s^2+a_1s+a_2}e^{-sL}$$

Three and Four Parameter Models



Three and Four Parameter Models



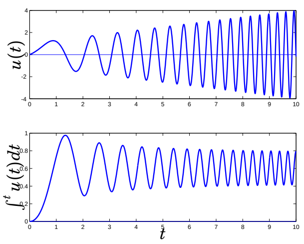
Excitation

- The key to successful system identification
- Symmetric relay dominant at one frequency
- The asymmetric relay has the dominant frequency at the period T_p, a low frequency component and some high frequencies
- Highly desirable to have excitation at other frequencies
- Modifications of the relay
  - Integrator
  - Filters
  - Change hysteresis: Ulf Holmberg
  - Mix of integrator and relay: Waller
  - Flat spectrum: Kristian
  - Asymmetric relay
- Chirp signals

The Chirp Signal

$$u(t) = (a + b\,t)\sin(c + d\,t)t$$

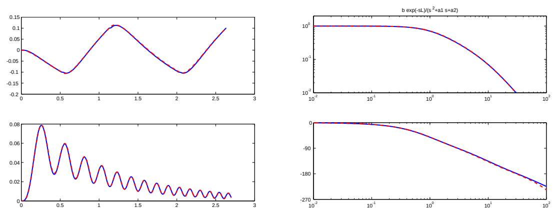
Frequency varies between a and $c + d\,t_{max}$ amplitude between $a + b\,t_{max}$



Notice both high and low frequency excitation

Asymmetric Relay and Chirp

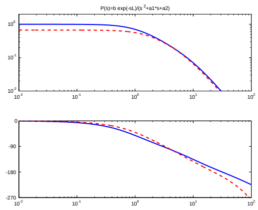
- Asymmetrical relay experiment combined chirp signal experiment
- Double experiment time. Constant amplitude, $L = 0.01, w = 15*(1+0.5*t), t_{max} = 2.7$, $0.15 \le \omega\,L \le 0.35$



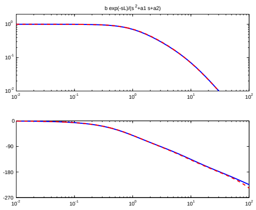
Parameters: $a_1 = 10.366 \pm 0.033$, $a_2 = 9.574 \pm 0.028$, $b = 9.566 \pm 0.027$, $L = 0.0109 \pm 0.0002$

Effect of Proper Excitation

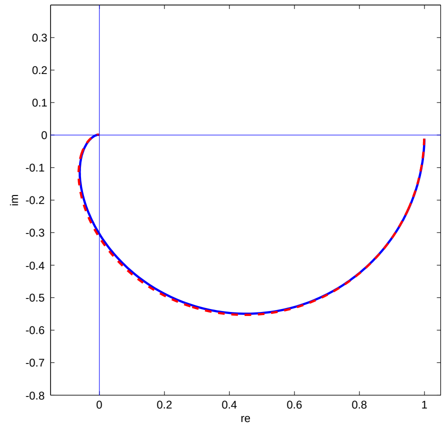
Only relay



Relay and chirp



Nyquist Plots

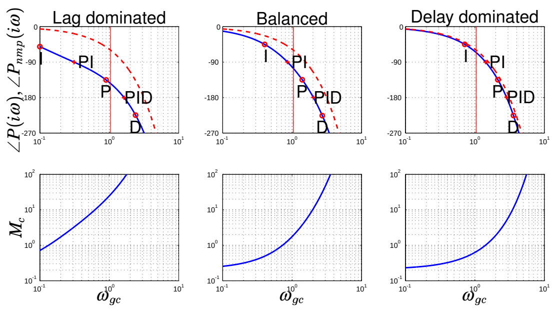


Outline

Summary

- Trade-off plots give a lot of insight
  - Effect of parameters k_i, k_p, τ
  - Assessment of tuning rules (close to green line)
- Rational ways of designing filters
  - Simple rules related to FOTD or T_i, T_d
  - The equation for noise gain
- Feedforward (not covered in the talk)
- Computations
  - PID Design Tool
  - Interactive Learning Modules
  - Convex optimization
- Automatic tuning
  - Better excitation: asymmetric relay and chirp
- How to package the results
  - Simple tuners
  - Elaborate tuners with extensive computations - cloud?
- Assessment plots

Assessment Plots



$$\omega_{gc} \approx P(0)k_i, \qquad M_c = \frac{1}{|P(i\omega_{gc})|} e^{\gamma(-\arg P(i\omega_{gc}) - \pi + \varphi_m)}, \quad \gamma \approx 1$$

KJÅ European Journal of Control 6:1 (2000) 2-20